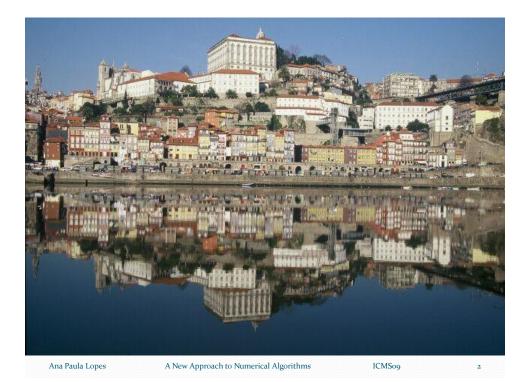


Ana Paula Lopes

A New Approach to Numerical Algorithms



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Introduction

• In this paper we developed a new Lanczos algorithm on the Grassmann manifold.

• This work comes in the wake of the article by A. Edelman, T. A. Arias and S. T. Smith, *"The geometry of algorithms with orthogonality constraints"*

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Introdu	ction		
	mann and Stiefel manifold ality constraints	s are based on	
	zos method and the conjug re closely related	ate gradients	
	e main problems of the Lar f orthogonality	iczos method is	1

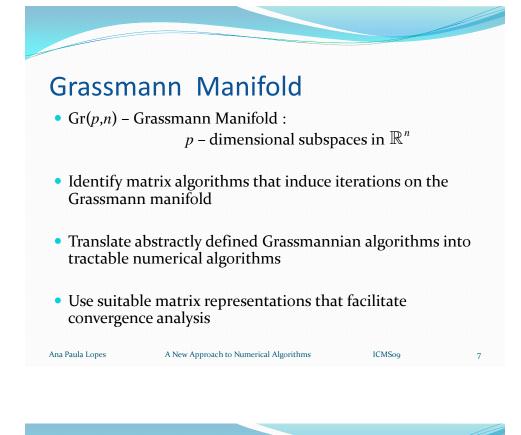
Numerical linear algebra problems

- The problem of computing eigenvalues, eigenvectors and invariant subspaces is always present in areas as diverse as Engineering, Physics, Computer Sciences and Mathematics.
- Lately, it has been verified that the iterations of eigenvalues and eigenvectors problems are best analyzed in some special spaces.
- A bridge between the geometry of the abstract spaces and the well known algorithms of numerical linear algebra.

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Optimization Problem

- The optimization problem of the estimative of the invariant subspaces is made explicit with a geometric approach.
- However a geometrical treatment on the Grassmann manifold appropriate for numerical linear algebra is not present in standard references.



Iterations on the Grassmann Manifold

• One possible way to represent numerically an element \mathcal{Y} of Gr(p,n) consists of specifying an $n \times p$ full column rank matrix Y whose columns span the space \mathcal{Y} , and we can write

$$\mathcal{Y}=span(Y)$$

- *Y* is called the column space of *Y*
- The set of all the matrices that have the same column space is a fiber over *Y*

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Iterations on the Grassmann Manifold

- We have an iteration on the Grassmann Manifold if a fiber is mapped into a fiber
- The concept of the fiber bundle structure allows us to describe the relationship between subspaces and matrices representations

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Lanczos on the Grassmann manifold

- The Lanczos algorithm is a method for computing some eigenvalues of a large symmetric matrix *A* and their eigenvectors
- The idea consists in building a sequence of nested subspaces span *x*, *Ax*, *A*²*x*,... and solving the eigenproblem reduced to these subspaces



Algorithm Grassmann-Lanczos (GL)

Let *A* be an symmetric matrix $n \times n$ Consider *y* an *p*-dimensional subspace of \mathbb{R}^n i.e.,

 $\mathcal{Y} \in \operatorname{Gr}(p, n)$

The algorithm produce a sequence of subspaces

 $\begin{array}{ccc} \operatorname{Gr}(p,n) \to \operatorname{Gr}(p,n) \\ \mathcal{Y} & \mapsto & \mathcal{Y}_{\!\scriptscriptstyle +} \end{array}$

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Algorithm Grassmann-Lanczos (GL)

- 1. Pick an orthonormal matrix $Y, n \times p$, being $\mathcal{Y} = \operatorname{span}(Y)$
- 2. Create an orthonormal basis Q for the Krylov subspace

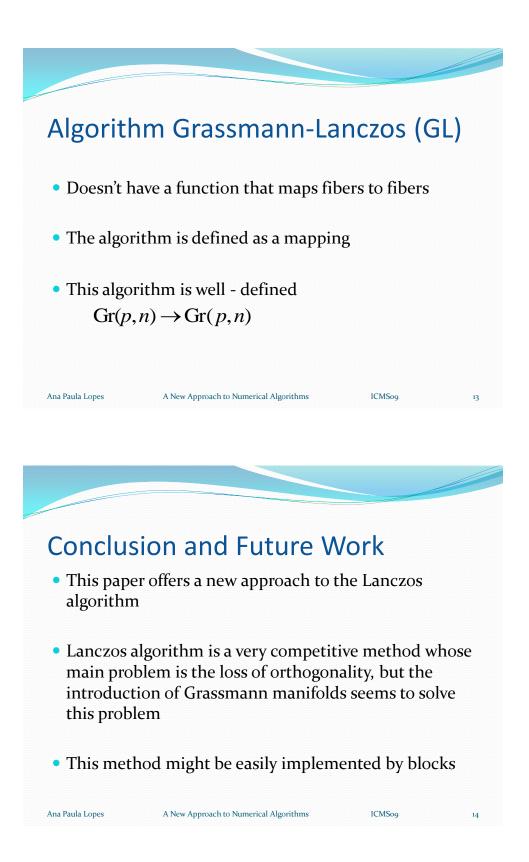
 $\mathcal{K}_m(Y) = span \ Y, AY, ..., A^{m-1}Y$

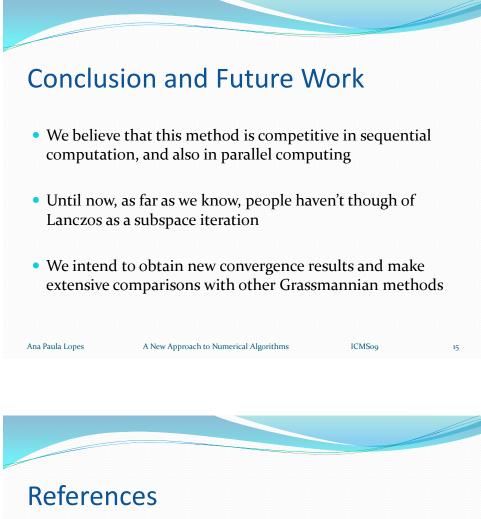
- 3. Calculate the matrix Rayleigh quotient $M = Q^T A Q$ that represents the projection of A into $\mathcal{K}_{w}(Y)$
- 4. Calculate *X*, an orthonormal basis for the *p*-dimensional dominated eigenspace of *M*
- 5. Let y_1 be the span of QX

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	www.paulalopes.co	<u>)m</u>	
aploj	<u>pes@iscap.ipp.pt</u> - Ana I	Paula Lopes	
	Thank you Teşekkür ederim Obrigado		

Ana Paula Lopes

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