Monetary and Fiscal Policies Interactions in a Monetary Union With Country-size Asymmetry

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Abstract

Within a country-size asymmetric monetary union, idiosyncratic shocks and national fiscal stabilization policies cause asymmetric cross-border effects. These effects are a source of strategic interactions between non-coordinated fiscal and monetary policies: on the one hand, due to larger externalities imposed on the union, large countries face less incentives to develop free-riding fiscal policies; on the other hand, a larger strategic position vis-à-vis the central bank incentives the use of fiscal policy to, deliberately, influence monetary policy. Additionally, the existence of non-distortionary government financing may also shape policy interactions. As a result, optimal policy regimes may diverge not only across the union members, but also between the latter and the monetary union.

In a two-country micro-founded New-Keynesian model for a monetary union, we consider two fiscal policy scenarios: (i) lump-sum taxes are raised to fully finance the government budget and (ii) lump-sum taxes do not ensure balanced budgets in each period; therefore, fiscal and monetary policies are expected to impinge on debt sustainability. For several degrees of country-size asymmetry, we compute optimal discretionary and dynamic non-cooperative policy games and compare their stabilization performance using a union-wide welfare measure. We also assess whether these outcomes could be improved, for the monetary union, through institutional policy arrangements.

We find that, in the presence of government indebtedness, monetary policy optimally deviates from macroeconomic to debt stabilization. We also find that policy cooperation is always welfare increasing for the monetary union as a whole; however, indebted large countries may strongly oppose to this arrangement in favour of fiscal leadership. In this case, delegation of monetary policy to a conservative central bank proves to be fruitful to improve the union's welfare.

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1 Introduction

In the European Monetary Union (EMU), monetary policy is decided at a centralized level but fiscal policy is determined at the national level by each member country's government. Small and large countries coexist and are expected to experience different stabilization policy trade-offs and to have different interests concerning institutional policy arrangements. In effect, within a country-size asymmetric monetary union, idiosyncratic shocks and national fiscal stabilization policies cause asymmetric cross-border effects. These spill-over effects are a source of strategic interactions between non-coordinated fiscal and monetary policies: on the one hand, due to larger externalities imposed on the union, large countries face less incentives to develop free-riding fiscal policies; on the other hand, a larger strategic position vis-à-vis the central bank incentives the use of fiscal policy to, deliberately, influence monetary policy. In turn, small countries face the opposite incentives. Additionally, the available mix of fiscal policy instruments, namely if whether or not it includes non-distortionary sources of government financing, is also critical in shaping monetary and fiscal policy interactions. As a result, optimal stabilization policy regimes may diverge not only across the union members, but also between the latter and the monetary union as a whole. Institutional policy arrangements that would improve the union's welfare may lack support from the large countries.

In the context of these challenging research issues raised by country-size asymmetry in a monetary union, our main objective is to characterize optimizing stabilization policies in a monetary union when policymakers may act strategically and to assess whether institutional policy arrangements, such as policy cooperation or monetary policy delegation to a weight-conservative central bank, can improve the stabilization outcomes. In particular, we intend to examine how country-size asymmetry and the inexistence of non-distortionary sources of government financing shape these outcomes and how they determine possible recommendations on monetary and fiscal policy arrangements to be applied, for instance, in the EMU context.

To address the above mentioned issues we begin by setting a baseline framework: a two-country micro-founded macroeconomic model for a closed monetary union with monopolistic competition and sticky prices, in line with the ones firstly developed by Benigno (2004) for monetary policy analysis, and extended by Beetsma and Jensen (2004, 2005) to include fiscal policy. We consider two scenarios for the framework of fiscal policy. In the first scenario lump-sum taxes are raised to fully finance the government budget and, thus, monetary policy does not interfere with the government sources of financing. In the second scenario, lump-sum taxes do not ensure balanced budgets in each period; therefore, fiscal and monetary policies are expected to impinge on debt sustainability. Following the recent work of Leith and Wren-Lewis (2007a, 2007b), we allow the model to include two fiscal policy instruments yielding both demand and supply-side effects, respectively, home-biased government spending and distortionary taxes.

We derive a welfare criterion to allow the derivation of optimal stabilization policies and the ranking of the alternative policy outcomes under different strategic set-ups. This relies on a quadratic approximation to the union-weighted average of the representative households' welfare where linear terms are removed through the use of a subsidy fully financed by lump-sum taxes, as in Rotemberg and Woodford (1998), for instance.[1]

The characterization of optimal stabilization policies under non-cooperative and dynamic settings requires the model to be solved numerically using appropriate algorithms that reflect the various timing structures of the policy games: Nash, monetary leadership and fiscal leadership. We follow the methodology developed in the recent work of Kirsanova and co-authors (Blake and Kirsanova, 2006, for a closed-economy setup and Kirsanova et al., 2005, for an open-economy setup) to find the leadership discretionary equilibrium with dynamic rational expectations macroeconomic models.[2]

This paper is organized as follows. In Section 2 we develop the policy setup for policy analysis, which includes the description of the economic structure, the policy environment and the policy games, and calibration. In Section 3 we perform policy analysis related with dynamic responses and welfare evaluation of the different policy regimes. Finally, in Section 4 we present concluding remarks and suggest extensions for future work.

2 Setup for Policy Analysis

To capture strategic interactions between monetary and fiscal policies, we closely follow Beetsma and Jensen (2004, 2005). The model is extended to capture country-size asymmetry, to allow for a more generic case of cross-country consumption elasticity and to include different fiscal policy scenarios.

A monetary union is modelled as a closed area with two countries, H (home) and F (foreign), populated by a continuum of agents $\in [0,1]$. The population on the segment $[0,n)$ belongs to country H, while agents on $[n,1]$ live in country F. The countries are assumed to have identical economic structures: each country is characterized by two private sectors - households and firms -, one fiscal authority, and is subject to a common monetary policy. Nevertheless, countries may face idiosyncratic shocks.

[1] Benigno and Woodford (2004, 2005, 2006), for a closed-economy, and Ferrero (2007), for a monetary union, present an alternative way to remove the linear terms of the social loss function, in the presence of a distorted steady-state. They focus on timeless optimal commitment policies and they need to compute second-order approximations to the structural equations of the model to get a purely quadratic loss. Schmitt-Grohé and Uribe (2004a,b), Correia et al (2003) and Lambertini (2006) illustrate the so-called Ramsey approach, which configures an alternative to the joint design of optimal policies. Neither of these approaches is compatible with the study of the policy problem under discretion.

[2] Adam and Billi (2006), for a closed-economy setup, present an alternative computational method that delivers second-order accurate welfare expressions for economies with a distorted steady-state within the linear-quadratic approach.

2.1 Households

Throughout this section we will address the optimization problem of the representative Home (H)-household, bearing in mind that the representative Foreign (F)-household behaves similarly. The representative H-household seeks to maximize the following lifetime utility (U_0^j).

$$U_0^j = E_t \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^j, \overline{C}_t^H\right) + V\left(G_t^H\right) - v\left(L_t^j\right)\right] \tag{1}$$

where

$$u\left(C_t^j, \overline{C}_t^H\right) = \tfrac{\sigma}{\sigma-1}\left(C_t^j\right)^{\frac{\sigma-1}{\sigma}}\left(\overline{C}_t^H\right)^{\frac{1}{\sigma}}$$

$$V\left(G_t^H\right) = \delta\tfrac{\psi}{\psi-1}\left(G_t^H\right)^{\frac{\psi-1}{\psi}}$$

$$v\left(L_t^j\right) = \tfrac{d}{1+\eta}\left(L_t^j\right)^{1+\eta}$$

The index j refers to a specific household, while the index H refers to the country H where j lives.

Each household delivers utility from consuming across a basket of home- and foreign-produced goods $\left(C_t^j\right)$, from own-country *per capita* government consumption on domestically produced goods $\left(G_t^H\right)$, while she receives disutility from labour effort (L_t^j, measured in hours). $\overline{C}^H$ is a bounded exogenous disturbance[3] and C^j is a real consumption Dixit-Stiglitz index defined (as in Benigno and Benigno, 2006, or Lombardo and Sutherland, 2004) by

$$C^j \equiv \left[n^{\frac{1}{\rho}}\left(C_H^j\right)^{\frac{\rho-1}{\rho}} + (1-n)^{\frac{1}{\rho}}\left(C_F^j\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{2}$$

In turn, C_H^j and C_F^j are also Dixit-Stiglitz indexes of consumption across a continuum of differentiated goods produced, respectively, in country H and F:

$$C_{H,t}^j \equiv \left[\left(\tfrac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n c_t^j(h)^{\frac{\theta-1}{\theta}}dh\right]^{\frac{\theta}{\theta-1}} ;\ C_{F,t}^j \equiv \left[\left(\tfrac{1}{1-n}\right)^{\frac{1}{\theta}}\int_n^1 c_t^j(f)^{\frac{\theta-1}{\theta}}df\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

The elasticity of substitution between goods produced in each country (θ) may differ from the elasticity of substitution between Home and Foreign consumption baskets (ρ).

Maximization of (1) is, as usual, subject to a budget constraint. The flow budget constraint for the representative Home household is

$$P_t C_t^j + E_t\left(Q_{t,t+1}D_{t+1}^j\right) = W_t(j)\,L_t^j + \int_0^n \Pi_t^j(z)\,dz - P_t T_t^H + D_t^j \tag{4}$$

[3] We introduce a country specific demand shock by letting the marginal utility of consumption be stochastic.

where P is the consumption-based price index defined below, $W(j)$ is the nominal wage rate of labour of type j, $\Pi^j(z)$ is the share of profits of domestic firm z going to household j in country H and T^H is a *per capita* lump sum tax levied by the domestic government on its citizens. Household j has access to a complete set of state-contingent securities that span all possible states of nature and are traded across the union. D^j_{t+1} denotes the nominal payoff of a portfolio of state-contingent securities, purchased by the representative home household at date t, while $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs, common across countries.

Assuming no trade barriers and given the structure of preferences, purchasing power parity holds, and the underlying consumption-based price index (P_t) is defined as

$$P_t \equiv \left[nP_{H,t}^{1-\rho} + (1-n)\,P_{F,t}^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{5}$$

while the country-specific price indexes P_H and P_F are given by

$$P_{H,t} \equiv \left[\frac{1}{n}\int_0^n p_t(h)^{1-\theta}\,dh\right]^{\frac{1}{1-\theta}} \;;\; P_{F,t} \equiv \left[\frac{1}{1-n}\int_n^1 p_t(f)^{1-\theta}\,df\right]^{\frac{1}{1-\theta}} \tag{6}$$

where $p(h)$ and $p(f)$ are the prices of typical goods h and f produced in country H and F, respectively.

The problem of the representative household can be split into an intertemporal and an intratemporal problem. In regards to the household's intratemporal problem, it requires choosing the allocation of a given level of expenditure across the differentiated goods to maximize the consumption index, C^j. Plugging into the appropriate output aggregators the resulting individual demands and the optimal government spending allocation across domestically produced goods, we obtain the national aggregate demands, Y^H and Y^F,

$$Y_t^H = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} C_t^W + G_t^H \tag{7H}$$

$$Y_t^F = \left(\frac{P_{F,t}}{P_t}\right)^{-\rho} C_t^W + G_t^F \tag{7F}$$

where the union-wide consumption, C^W, is defined as $C^W \equiv \int_0^1 C^j dj$, and

$$\left(\frac{P_H}{P}\right)^{\rho-1} = n + (1-n)\,T^{1-\rho} \;;\; \left(\frac{P_F}{P}\right)^{\rho-1} = nT^{\rho-1} + (1-n) \tag{8}$$

The variable T stands for the terms-of-trade, defined as the relative price of the F-bundle of goods in terms of the H-bundle of goods ($T \equiv P_F/P_H$). According to (8), changes in the terms-of-trade imply a larger response in a country's aggregate demand the smaller the size of the country, *i.e.*, the larger the degree of openness.

As for the household's intertemporal problem, the household chooses the set of processes $\left\{C_t^j, L_t^j; D_{t+1}^j\right\}_{t=0}^{\infty}$, taking as given all the other processes and the initial wealth, as to maximize the intertemporal utility function (1) subject to (4). Solution for this problem yields the familiar Euler equation

$$u_c\left(C_t^j, \overline{C}_t^H\right) = \beta\left(1+i_t\right) E_t\left\{\left(\frac{P_t}{P_{t+1}}\right) u_c\left(C_{t+1}^j, \overline{C}_{t+1}^H\right)\right\}, \tag{9}$$

where $1+i_t = \frac{1}{E_t Q_{t,t+1}}$ is the gross risk-free nominal interest rate. Moreover, assuming that the initial state-contingent distribution of nominal bonds is such that the life-time budget constraints of all households are identical, the risk-sharing condition implies that

$$u_c\left(C_t^H, \overline{C}_t^H\right) = u_c\left(C_t^F, \overline{C}_t^F\right) \tag{10}$$

Finally, the labour supply decision determines that the real wage for labour type j is given by

$$\frac{W_t\left(j\right)}{P_t} = \mu_{w,t}^H * \frac{v_L\left(L_t^j\right)}{u_c\left(C_t^j, \overline{C}_t^H\right)} \tag{11}$$

where $\mu_{w,t}^H \geqslant 1$ is an exogenous Home-specific wage markup that is used as a device to introduce the possibility of "pure cost-push shocks" that affects the equilibrium price behaviour but does not change the efficient output, as in Benigno and Woodford (2003, 2005).

2.2 Firms

There are a continuum of firms in country H and in country F. The production function for the differentiated consumption good y, indexed by $h \in [0, n)$ in country H and by $f \in [n, 1]$ in country F, is described, for $y(h)$, by

$$y_t\left(h\right) = a_t^H L_t\left(h\right) \tag{12}$$

where a_t^H is an exogenous H-specific technology shock, common to all H-firms, and $L_t\left(h\right)$ is the firm-specific labour input offered by a continuum of H-households, indexed in the unit interval. In a symmetric equilibrium, the work effort chosen by the household $\left(L_t^h\right)$ equals the aggregate labour input $\left(L_t\left(h\right)\right)$.

To introduce price stickiness, we assume that firms set prices according to the process defined in Calvo (1983). Each period, a randomly selected fraction of firms at H, $1-\alpha^H$, have the opportunity to change their prices, independently of the time that has elapsed since the last price-resetting, while the remaining firms keep the prices of the previous period. If it has the chance to reset prices in period t, an optimizing h-firm will set $p_t^o\left(h\right)$ in order to maximize the expected future profits, subject to the demand for its product and the production technology. The first order condition for this optimizing wage-taker firm can be expressed as

$$\left(\frac{p_t^o(h)}{P_{H,t}}\right)^{1+\theta\eta} = \frac{\frac{\theta}{\theta-1} E_t \sum_{s=t}^{\infty} \left(\alpha^H \beta\right)^{s-t} \mu_{w,s}^H \left(1-\zeta^H\right) v_y\left(Y_s^H; a_s^H\right) \left(\frac{P_{H,s}}{P_{H,t}}\right)^{\theta(1+\eta)} Y_s^H}{E_t \sum_{s=t}^{\infty} \left(\alpha^H \beta\right)^{s-t} \left(1-\tau_s^H\right) u_c\left(C_s^H, \overline{C}_s^H\right) \left(\frac{P_{H,s}}{P_{H,t}}\right)^{\theta-1} \left(\frac{P_{H,s}}{P_s}\right) Y_s^H} \tag{13}$$

where $p_t^o(h)$ still applies at s, τ_s^H is a proportional tax rate on sales with the non-zero steady-state level $\overline{\tau}^H$ and ζ^H is an employment subsidy fully financed by lump sum taxes that, removing average monopolistic and tax rate distortions, ensures the efficiency of the steady-state output-level.[4] The price index P_H evolves according to the law of motion

$$P_{H,t}^{1-\theta} = \alpha^H P_{H,t-1}^{1-\theta} + \left(1-\alpha^H\right) p_t^o(h)^{1-\theta} \tag{14}$$

2.3 Policy Environment

To close the model presentation, description of the policy environment is in order. In this section, we describe the instruments and constraints for the monetary and fiscal policies and present a set of meaningful objective functions facing the policy authorities. These policy functions have a twofold purpose: (i) to enable the derivation of optimal discretionary policy rules across several regimes of monetary and fiscal policies interactions and (ii) to assess the welfare impacts of the different policy regimes.

2.3.1 Policy instruments and constraints

In our model, the nominal interest rate, i_t, is the single instrument through which the common monetary policy operates.

As for fiscal policy, we assume two alternative policy scenarios. In a first set-up, lump-sum taxes $\left(T^H\right)$ are raised in sufficient amount to fully finance, in each period, an employment subsidy $\left(\zeta^H\right)$ and the instruments used for stabilization purposes – the home-biased government spending $\left(G^H\right)$ and the sales tax rate $\left(\tau^H\right)$.[5] Here, fiscal policy is balanced-budget and Ricardian equivalence holds. In a second scenario, lump-sum taxes only adjust to fully accommodate the employment subsidy and the government inter-temporal solvency condition appears as an additional binding constraint to the set of possible equilibrium paths of the endogenous variables. In this case, the sources of strategic interactions between monetary and the fiscal authorities are large, because both policies impinge on debt sustainability. Stabilization fiscal policy instruments

[4] Following Leith and Wren-Lewis (2007a, 2007b), we use this employment subsidy as a device to eliminate linear terms in the social welfare function without loosing the possibility of using the sales tax rates as fiscal policy instruments.

[5] For simplicity, we admit that government debt is zero in this scenario.

are the same as in the first scenario, G^H and τ^H and, thus, fiscal policy encompasses demand and supply-side effects.[6] The budget constraints for the fiscal authorities can be written as:

$$B_t^H = (1+i_{t-1})\, B_{t-1}^H + P_{H,t}G_t^H - \tau_t^H P_{H,t}Y_t^H \tag{15H}$$
$$B_t^F = (1+i_{t-1})\, B_{t-1}^F + P_{F,t}G_t^F - \tau_t^F P_{F,t}Y_t^F \tag{15F}$$

where B_t^H and B_t^F represent the *per capita* nominal government debt of country H and F, respectively.

With asset markets clearing only at the monetary union level, the sole public sector inter-temporal budget constraint is the union-wide consolidated debt. However, in the context of a monetary union with an institutional arrangement like the EMU, there are arguments to impose the verification of this inter-temporal budget constraint at the national levels. Accordingly,

$$b_t^i = (1+i_t)\left(b_{t-1}^i \frac{P_{t-1}}{P_t} + \frac{P_{i,t}}{P_t}G_t^i - \tau_t^i \frac{P_{i,t}}{P_t}Y_t^i\right), \quad i = H, F \tag{16}$$

where the variable $b_t^i \equiv \frac{(1+i_t)B_t^i}{P_t}$ denotes the real value of debt at maturity in *per capita* terms.

2.3.2 Equilibrium Conditions

To solve for the optimal policy, authorities have to take into account both the private sector behaviour as well as the budget constraints, described above. These conditions can be log-linearized and written in gap form as

$$E_t c_{t+1}^w = c_t^w + \sigma\left(i_t - E_t \pi_{t+1}^w\right) \tag{17}$$

$$y_t^H = s_c \rho (1-n)\, q_t + (1-s_c)\, g_t^H + s_c c_t^w \tag{18H}$$

$$y_t^F = -s_c \rho n q_t + (1-s_c)\, g_t^F + s_c c_t^w \tag{18F}$$

$$\pi_t^H = \beta E_t \pi_{t+1}^H + k^H\,(1+s_c\rho\eta)\,(1\text{-}n)\,q_t + k^H \frac{1+s_c\sigma\eta}{\sigma} c_t^w + k^H\,(1\text{-}s_c)\,\eta g_t^H + k^H \frac{\overline{\tau}^H}{\left(1\text{-}\overline{\tau}^H\right)}\tau_t^H \tag{19H}$$

$$\pi_t^F = \beta E_t \pi_{t+1}^F \text{-} k^F\,(1+s_c\rho\eta)\, n q_t + k^F \frac{1+s_c\sigma\eta}{\sigma} c_t^w + k^F\,(1\text{-}s_c)\,\eta g_t^F + k^F \frac{\overline{\tau}^F}{\left(1\text{-}\overline{\tau}^F\right)}\tau_t^F \tag{19F}$$

[6] While it is consensual to treat the interest rate as the monetary policy instrument, it is recognized that fiscal policy has many dimensions and that the several fiscal policy instruments have different effects. Beetsma and Jensen (2004, 2005) and Gali and Monacelli (2007) assume that the fiscal policy instrument is public spending financed by lump sum taxes, Ferrero (2007) presents a model where fiscal policy is conducted through distortionary taxation and public debt and Leith and Wren-Lewis (2007b) consider three potential fiscal instruments - government spending, labour income taxes and revenue taxes.

$$q_t = q_{t-1} + \pi_t^F - \pi_t^H - \left(\widetilde{T}_t - \widetilde{T}_{t-1}\right) \tag{20}$$

$$\widehat{b}_t^H = i_t + \frac{1}{\beta}\left\{\widehat{b}_{t-1}^H \text{-} \pi_t + (1\text{-}n)(1\text{-}\beta) q_t + \frac{\overline{Y}}{\overline{b}^H}\left[(1\text{-}s_c) g_t^H \text{-} \overline{\tau}^H y_t^H \text{-} \overline{\tau}^H \tau_t^H\right]\right\} + \widehat{\varepsilon}_{b^H,t} \tag{21H}$$

$$\widehat{b}_t^F = i_t + \frac{1}{\beta}\left\{\widehat{b}_{t-1}^F \text{-} \pi_t \text{-} n(1\text{-}\beta) q_t + \frac{\overline{Y}}{\overline{b}^F}\left[(1\text{-}s_c) g_t^F \text{-} \overline{\tau}^F y_t^F \text{-} \overline{\tau}^F \tau_t^F\right]\right\} + \widehat{\varepsilon}_{b^F,t} \tag{21F}$$

where

$$k^H \equiv \frac{\left(1-\alpha^H\right)\left(1-\alpha^H\beta\right)}{\alpha^H(1+\theta\eta)};\ k^F \equiv \frac{\left(1-\alpha^F\right)\left(1-\alpha^F\beta\right)}{\alpha^F(1+\theta\eta)},$$

$\widehat{\varepsilon}_{b^H,t}$ and $\widehat{\varepsilon}_{b^F,t}$ are composite shocks defined as

$$\begin{aligned}
\widehat{\varepsilon}_{b^H,t} &= \widetilde{i}_t + \frac{1}{\beta}\left\{(1\text{-}n)(1\text{-}\beta)\widetilde{T}_t + \frac{\overline{Y}}{\overline{b}}\left[(1\text{-}s_c)\widetilde{G}_t^H \text{-} \overline{\tau}^H \widetilde{Y}_t^H + \left(1\text{-}\overline{\tau}^H\right)\widehat{\mu}_{w,t}^H\right]\right\} \\
\widehat{\varepsilon}_{b^F,t} &= \widetilde{i}_t + \frac{1}{\beta}\left\{-n(1\text{-}\beta)\widetilde{T}_t + \frac{\overline{Y}}{\overline{b}}\left[(1\text{-}s_c)\widetilde{G}_t^F \text{-} \overline{\tau}^F \widetilde{Y}_t^F + \left(1\text{-}\overline{\tau}^F\right)\widehat{\mu}_{w,t}^F\right]\right\}
\end{aligned}$$

and where lower case variables refer to variables in gaps. For a generic variable, X_t, its gap is defined as $x_t = \widehat{X}_t - \widetilde{X}_t$, where $\widehat{X}_t$ and $\widetilde{X}_t$ denote, respectively, their effective and efficient flexible-price values, in log-deviations from the zero-inflation efficient steady-state (see, section 2.3.3, below). A "union-wide" variable, X^w, is defined as $X^w \equiv nX^H + (1-n)X^F$.

Equation (17) refers to the IS equation, written in terms of the union consumption[7] and nominal interest-rate gaps. Equations (18H) and (18F) are country-specific aggregate demand equations, with s_c being the steady-state consumption share of output and q_t being the terms-of-trade gap ($\equiv \widehat{T}_t - \widetilde{T}_t$). These three equations constitute the aggregate demand-side block of the model and were derived from log-linearization of equations (7H), (7F), (8), (9) and (10).

The aggregate supply-side block of the model was obtained from the log-linear approximation of equations (13) and (14), as well as from their Foreign counterparts, around the efficient steady-state equilibrium. Equations (19H) and (19F) are open-economy Phillips curves, describing the pure New-Keynesian aggregate supply (AS) in each country. Positive gaps on the terms-of-trade, consumption and public spending have inflationary consequences at H: an increase in the demand for H-produced goods leads to more work effort, and, thus, raises marginal costs. Moreover, the positive gaps on the terms-of-trade and on the consumption exert an additional inflationary pressure as they reduce the marginal utility of nominal income for households. The efficient tax rate $\widetilde{\tau}_t^i$,

[7] Assuming that the initial state-contingent securities distribution is such that the lifetime budget constraints of all households are identical, the risk-sharing condition implies that $c_t^w = c_t^H = c_t^F$.

used to compute the tax rate gap ($\tau_t^i = \widehat{\tau}_t^i - \widetilde{\tau}_t^i$) in country i, is defined as the tax rate required to fully offset the impact of an idiosyncratic "cost-push" (wage markup) shock.[8] Equation (20) is the terms-of-trade gap's identity, reflecting the inflation differential and the one-period change in the efficient level of the terms-of-trade ($\widetilde{T}_t - \widetilde{T}_{t-1}$).

The final equations, (21H) and (21F), are the government budget constraints relevant for the equilibrium allocation only in the second fiscal policy scenario.[9]

In sum, in the first balanced-budget policy scenario, given the path for policy instruments and the initial value of $\widehat{T}_{t-1}$, the system including equations (17)-(20) provides solutions for the endogenous variables c_t^w, y_t^H, y_t^F, π_t^H, π_t^F and q_t. In the second policy scenario, where policymakers are constrained to ensure debt sustainability, equations (21H) and (21F) add to the previous system to describe the economic structure of the economy.

2.3.3 Policy Objectives - The Social Planner's Problem

The optimal allocation for the monetary union as a whole, in any given period t, can be described as the solution to the following social planner's problem, where the single policy authority is willing to maximize the discounted sum of the utility flows of the households belonging to the whole union (W):

$$\max_{\substack{C_{H,t}^H,\, C_{H,t}^F,\, C_{F,t}^H,\\ C_{F,t}^F,\, G_t^H,\, G_t^F}} W = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [n w_t^H + (1-n)\, w_t^F] \right\}, \tag{22}$$

$$\text{with } w_t^H = u\left(C_t^H, \overline{C}_t^H\right) + V\left(G_t^H\right) - \frac{1}{n}\int_0^n v\left(L_t^j\right) dj$$

$$\text{and } w_t^F = u\left(C_t^F, \overline{C}_t^F\right) + V\left(G_t^F\right) - \frac{1}{1-n}\int_n^1 v\left(L_t^j\right) dj$$

s.t.

(production functions) $\quad Y_t^H = a_t^H L_t^H$

$Y_t^F = a_t^F L_t^F$

(resource constraints) $\quad nY_t^H = nC_{H,t}^H + (1-n)\, C_{H,t}^F + nG_t^H$

$(1-n)\, Y_t^F = nC_{F,t}^H + (1-n)\, C_{F,t}^F + (1-n)\, G_t^F$

(consumption indexes) $\quad C_t^H \equiv \left[n^{\frac{1}{\rho}} \left(C_{H,t}^H\right)^{\frac{\rho-1}{\rho}} + (1-n)^{\frac{1}{\rho}} \left(C_{F,t}^H\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$

$C_t^F \equiv \left[n^{\frac{1}{\rho}} \left(C_{H,t}^F\right)^{\frac{\rho-1}{\rho}} + (1-n)^{\frac{1}{\rho}} \left(C_{F,t}^F\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$

[8] The steady-state tax rates are given by $\overline{\tau}^i = (1-\beta)\frac{\overline{b}^i}{\overline{Y}} + (1-s_c)$ and the efficient tax rates by $\widetilde{\tau}_t^i = -\frac{1-\overline{\tau}^i}{\overline{\tau}^i}\widehat{\mu}_{w,t}^i$, for $i = H, F$.

[9] The derivations of all these equations are available upon request.

The social planner will choose to produce equal quantities of the different goods in each country. Moreover, the aggregation over all agents (households, governments and central bank) cancels out the budget constraints and, thus, the social planner's solution is not constrained by them.

Maximization program in (22) yields the following optimallity conditions

$$
\begin{aligned}
u_c\left(C_t^H,\overline{C}_t^H\right) n^{\frac{1}{\rho}} \left(\frac{C_{H,t}^H}{C_t^H}\right)^{-\frac{1}{\rho}} &= v_y\left(Y_t^H; a_t^H\right) && (23)\\
u_c\left(C_t^H,\overline{C}_t^H\right) (1-n)^{\frac{1}{\rho}} \left(\frac{C_{F,t}^H}{C_t^H}\right)^{-\frac{1}{\rho}} &= v_y\left(Y_t^F; a_t^F\right) && (24)\\
u_c\left(C_t^F,\overline{C}_t^F\right) n^{\frac{1}{\rho}} \left(\frac{C_{H,t}^F}{C_t^F}\right)^{-\frac{1}{\rho}} &= v_y\left(Y_t^H; a_t^H\right) && (25)\\
u_c\left(C_t^F,\overline{C}_t^F\right) (1-n)^{\frac{1}{\rho}} \left(\frac{C_{F,t}^F}{C_t^F}\right)^{-\frac{1}{\rho}} &= v_y\left(Y_t^F; a_t^F\right) && (26)\\
V_G\left(G_t^H\right) &= v_y\left(Y_t^H, a_t^H\right) && (27)\\
V_G\left(G_t^F\right) &= v_y\left(Y_t^F, a_t^F\right) && (28)
\end{aligned}
$$

Efficient equilibrium In a symmetric efficient steady-state equilibrium, it follows that $\overline{Y}^H = \overline{Y}^F = \overline{Y}$; $\overline{C^H} = \overline{C^F} = \overline{C}$; $\overline{C_H^H} = \overline{C_H^F} = n\overline{C}$; $\overline{C_F^H} = \overline{C_F^F} = (1-n)\,\overline{C}$ and $\overline{G^H} = \overline{G^F} = \overline{G}$.

The complete solution for the efficient equilibrium is given by the following expressions (29-32)

$$
\widetilde{C}_t^w = \frac{1}{1+\eta\left[s_c\sigma + (1-s_c)\,\psi\right]}\left\{\left[1+(1-s_c)\,\psi\eta\right]\widehat{\overline{C}}_t^w + (1+\eta)\,\sigma\widehat{a}_t^w\right\} \quad (29)
$$

$$
\widetilde{C}_{H,t}^H - \widetilde{C}_{F,t}^H = \widetilde{C}_{H,t}^F - \widetilde{C}_{F,t}^F = -\frac{\rho\,(1+\eta)}{1+\eta\left[s_c\rho + (1-s_c)\,\psi\right]}\left(\widehat{a}_t^F - \widehat{a}_t^H\right) \quad (30)
$$

$$
\widetilde{G}_t^w = \frac{\psi}{1+\eta\left[s_c\sigma + (1-s_c)\,\psi\right]}\left[-\eta s_c\widehat{\overline{C}}_t^w + (1+\eta)\,\widehat{a}_t^w\right] \quad (31)
$$

$$
\widetilde{G}_t^F - \widetilde{G}_t^H = \frac{(1+\eta)\,\psi}{1+\eta\left[s_c\rho + (1-s_c)\,\psi\right]}\left(\widehat{a}_t^F - \widehat{a}_t^H\right) \quad (32)
$$

To fully define the gap variables described in section above, we need to determine the efficient interest rate and terms-of-trade levels. The former follows directly from the Euler equation, while the latter results from the combination of equation (30) with the optimal intratemporal household's allocations

$$
\widetilde{i}_t = \frac{1}{\sigma}E_t\left[\left(\widetilde{C}_{t+1}^w - \widetilde{C}_t^w\right) - \left(\widehat{\overline{C}}_{t+1}^w - \widehat{\overline{C}}_t^w\right)\right] \quad (33)
$$

$$\widetilde{T}_t = -\frac{1+\eta}{1+\eta\left[s_c\rho+(1-s_c)\psi\right]}\left(\widehat{a}_t^F-\widehat{a}_t^H\right). \tag{34}$$

In the first fiscal policy scenario (lump-sum taxes warrant balanced budgets) this efficient allocation corresponds to the decentralized flexible-price equilibrium when monopolistic and tax distortions are removed through an employment subsidy and the implemented government spending rules agree with those derived under the social planner's optimization. However, in the second fiscal policy scenario, that union-wide optimal allocation may not be supported as a flexible-price equilibrium, since fiscal policy instruments may have to deviate from those rules to ensure fiscal solvency. Anyway, the policy problem will be formulated with variables in gaps defined in terms of the efficient outcomes and the two steady-state equilibriums coincide.

Steady-state equilibrium In order to avoid the traditional inflationary bias problem arising from an inefficiently low steady-state output level, we will assume the existence of an employment subsidy that removes average monopolistic and tax rate distortions. To compute this employment subsidy, observe that the profit-maximizing H-firms, in a flexible-price setup, choose the same price $p_t(h) = P_{H,t}$ such that

$$u_c\left(C_t^H,\overline{C}_t^H\right) = \frac{\theta}{(\theta-1)\left(1-\tau_t^H\right)}\mu_{w,t}^H\left(1-\zeta^H\right)\left[n+(1-n)T_t^{1-\rho}\right]^{\frac{1}{1-\rho}}v_y\left(Y_t^H,a_t^H\right)$$

and, the Foreign counterpart of this price-setting behaviour is given by

$$u_c\left(C_t^F,\overline{C}_t^F\right) = \frac{\theta}{(\theta-1)\left(1-\tau_t^F\right)}\mu_{w,t}^F\left(1-\zeta^F\right)\left[nT_t^{\rho-1}+(1-n)\right]^{\frac{1}{1-\rho}}v_y\left(Y_t^F;a_t^F\right)$$

To get symmetry in the steady-state levels of the output, consumption, government spending and prices in both countries, we need to impose that $\frac{\theta}{(\theta-1)\left(1-\overline{\tau}^H\right)}\overline{\mu}_w\left(1-\zeta^H\right) = \frac{\theta}{(\theta-1)\left(1-\overline{\tau}^F\right)}\overline{\mu}_w\left(1-\zeta^F\right) = \overline{\mu}$ where, as we have already remarked, the employment subsidy ζ^i is fully financed by lump sum taxes.[10]

In steady-state, we verify that

$$u_c\left(\overline{C},\overline{C}\right) = \overline{\mu}v_y\left(\overline{Y},\overline{a}\right)$$

and, if the employment subsidy ζ^i is set to match $\overline{\mu}=1$, the efficient steady-state output-level holds. Hence, the employment subsidy in country $i=H,F$ is assumed to take the value

$$\zeta^i = 1-\frac{(\theta-1)\left(1-\overline{\tau}^i\right)}{\theta\overline{\mu}_w} \tag{35}$$

The steady-state nominal (and real) interest rate is $\overline{i} = 1/\beta - 1$.

[10] Following Leith and Wren-Lewis (2007a, 2007b), we use this employment subsidy as a device to eliminate linear terms in the social welfare function without losing the possibility of using the sales tax rates as fiscal policy instruments. This employment subsidy is financed using lump-sum taxes.

2.3.4 Policy Objectives - The Social Loss Function

Benevolent authorities seek to maximize the social (whole union) loss function, W, given, now, the set of equations describing the effective economic structure dynamics: (17)-(20), in the first policy scenario; and (17)-(21F), in the second policy scenario. Moreover, full cooperation between monetary and fiscal authorities characterizes the policy regime. This environment enables the derivation of union-wide optimal stabilization policies, but serves also as a benchmark to assess alternative policy regimes.

Following Rotemberg and Woodford (1998, 1999), Woodford (2003), Benigno (2004), Amato and Laubach (2003), Steinsson (2003) and Beetsma and Jensen (2004, 2005), we compute a quadratic (second-order Taylor series) approximation of W around a deterministic steady-state. Ignoring the terms independent of policy as well as terms of, the approximation yields:[11]

$$W \simeq -\Omega E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\}, \tag{36}$$

where

$$\begin{aligned} L_t = {} & \Lambda_c \left(c_t^w\right)^2 + \Lambda_g \left[n \left(g_t^H\right)^2 + (1\text{-}n) \left(g_t^F\right)^2 \right] + \Lambda_{gc} \left(c_t^w\right) \left[n \left(g_t^H\right) + (1\text{-}n) \left(g_t^F\right) \right] \\ & + \Lambda_T q_t^2 - \Lambda_{gT} \left(g_t^F\text{-}g_t^H\right) q_t + n \Lambda_\pi^H \left(\pi_t^H\right)^2 + (1\text{-}n) \Lambda_\pi^F \left(\pi_t^F\right)^2 \end{aligned} \tag{37}$$

and

$$\begin{aligned} \Lambda_c & \equiv s_c \left(\frac{1}{\sigma} + s_c \eta \right), \quad \Lambda_g \equiv (1\text{-}s_c) \left(\frac{1}{\psi} + (1\text{-}s_c)\eta \right), \quad \Lambda_{gc} \equiv 2 s_c (1\text{-}s_c) \eta, \\ \Lambda_T & \equiv n (1\text{-}n) s_c \rho (1 + s_c \rho \eta), \quad \Lambda_{gT} \equiv 2n (1\text{-}n) s_c (1\text{-}s_c) \rho \eta, \\ \Lambda_\pi^H & \equiv \frac{\theta (1+\theta\eta) \alpha^H}{(1\text{-}\alpha^H \beta)(1\text{-}\alpha^H)}, \ \Lambda_\pi^F \equiv \frac{\theta (1+\theta\eta) \alpha^F}{(1\text{-}\alpha^F \beta)(1\text{-}\alpha^F)} \end{aligned}$$

Fluctuations in the consumption and the public spending gaps imply welfare losses (in line with households', respective, risk aversion, $1/\sigma$ and $1/\psi$), as well as fluctuations in work effort (η). Inflation at H is more costly the higher the degree of nominal rigidity $\left(\alpha^H\right)$, the higher the elasticity of substitution between H-produced goods (θ) and the higher the elasticity of disutility with respect to work effort (η). The welfare cost of inflation vanishes $\left(\Lambda_\pi^H\right)$ when prices are fully flexible $\left(\alpha^H = 0\right)$.

At the monetary union level, misallocation of goods also applies for deviations of the terms-of-trade from the respective efficient level. The costs of this distortion (Λ_T) increase with the elasticity of substitution between Home and Foreign produced goods (ρ), with the steady-state consumption share on output (s_c), with η and decrease with country-size asymmetry. Following an

[11] The derivation of the social loss function is available upon request.

asymmetric technology shock, efficiency requires prices to change as to shift the adjustment burden "equally" across the two countries (Benigno and López-Salido, 2006). This creates a trade-off for the monetary authority between the stabilization of relative prices to the correspondent efficient levels and the stabilization of inflation in both countries and, thus, provides a rational for the stabilization role of fiscal policy.

The cross-term between the consumption gap and the weighted average government spending gap occurs because positive co-movements between these two variables cause undesirable fluctuations in the work effort for the monetary union as a whole, in addition to the effort fluctuations caused by each of these variables *per se*. There is also a negative cross-term between the terms of trade gap and the relative spending gap that is increasing (in absolute value) with η and ρ, while decreasing with country-size asymmetry. This negative co-movement arises because a positive terms-of-trade gap rises H-competitiveness which, combined with a negative relative public spending gap (higher public spending at H than at F), shifts demand towards H-produced goods. As a consequence, work effort shifts from F- towards H- households (cf. Beetsma and Jensen 2004 and 2005, for these arguments).

2.3.5 Other policy objectives

We also consider that policymakers may have divergent policy objectives. This is a valid assumption since it is reasonable to conjecture that national (fiscal) authorities are mainly concerned with their own citizens and so, their objective functions should only comprise the utility of the respective constituencies. Pragmatically, we approximate the national welfare criteria through welfare losses obtained from splitting the union-wide loss function. We let for future work the proper derivation of the national welfare functions.[12]

We will also consider the case of the delegation of monetary policy to a weight-conservative central bank by distorting the weights on the inflation and the output terms of the social loss function. Delegating monetary policy to a weight-conservative central bank is usually seen as a potential solution to reduce the time-inconsistency problems of policy stabilization, which can be aggravated by specific incentives of the fiscal authorities.

The table below summarizes the policy environments we will analyze.

[12] The derivation of the appropriate utility-based loss functions for independent and non-cooperative fiscal authorities requires extra computations to avoid linear terms. Benigno and Benigno (2006) obtain loss functions, for cooperative and non-cooperative monetary policy regimes, that are formally identical to ours but different regarding the targets and the relative weights.

Benevolent Cooperative Policymakers
$L_t^{H,F}=L_t$
$L_t^{M}=L_t$
Benevolent non-Cooperative Policymakers
$L_t^H=\Lambda_c\left(c_t^w\right)^2+\Lambda_g\left(g_t^H\right)^2+\Lambda_{gc}c_t^w g_t^H+\Lambda_T q_t^2+\frac{1}{n}\Lambda_{gT}g_t^H q_t+\Lambda_\pi^H\left(\pi_t^H\right)^2$
$L_t^F=\Lambda_c\left(c_t^w\right)^2+\Lambda_g\left(g_t^F\right)^2+\Lambda_{gc}c_t^w g_t^F+\Lambda_T q_t^2-\frac{1}{1-n}\Lambda_{gT}g_t^F q_t+\Lambda_\pi^F\left(\pi_t^F\right)^2$
$L_t^{M}=L_t$
Conservative Central Bank
$L_t^H;L_t^F$
$L_t^M=(1-\rho^c)\left\{\Lambda_c\left(c_t^w\right)^2+\Lambda_g\left[n\left(g_t^H\right)^2+(1-n)\left(g_t^F\right)^2\right]+\Lambda_{gc}c_t^w\left[n\left(g_t^H\right)+(1-n)\left(g_t^F\right)\right]+\right.$
$\left.+\Lambda_T q_t^2-\Lambda_{gT}\left(g_t^F-g_t^H\right)q_t\right\}+\rho^c\left\{n\Lambda_\pi^H\left(\pi_t^H\right)^2+(1-n)\Lambda_\pi^F\left(\pi_t^F\right)^2\right\}$

2.4 Policy Games

We assume that fiscal and monetary authorities set their policy instruments in order to minimize the respective loss functions, given the dynamic structure of the economies, and that they can engage themselves in various policy games. We will consider, as a benchmark case for policy analysis, that policymakers are benevolent and cooperate under discretion. To assess the importance of the time-consistency, we also compute the optimal policy solution under commitment. These two optimizing problems will be solved by using the algorithms in Soderlind (1999).

We also consider discretionary non-cooperative policy games and, depending on the time of events, we can obtain Nash or leadership equilibria. In these different setups, the timing of the events is as following: 1) the private sector forms expectations; 2) the shocks are realized; 3a) the central bank sets the interest rate; 3b) the fiscal authorities choose the right amount of fiscal policy instruments. If 3a) and 3b) occur simultaneously we get a Nash equilibrium; if 3a) occurs before the central bank chooses its policy and the latter is aware of the fiscal policy reaction, we get a monetary leadership equilibrium; if the order of the occurrences is reversed, we have fiscal leadership equilibria. We will also assume that the fiscal authorities act at the same time, playing Nash. To solve for these dynamic policy games we use the methodology developed by Blake and Kirsanova (2006) for a closed-economy setup and by Kirsanova et al. (2005) for an open-economy model.

To illustrate the methodology involved, we next present the case of a full non-cooperative discretionary game with monetary leadership.[13]

We have five strategic agents in the game. There are three explicit players, the monetary and the two fiscal authorities, and two implicit players, the private

[13] Also as an example, we present, in the appendix, a numerical algorithm for the solution of this regime.

sector of both countries. In this type of game, the monetary authority moves first and sets the interest rate. Then, the two fiscal authorities decide the amount of their policy instruments. Finally, the last players are the private sectors of both countries.

To solve for this type of game, inversion of the order of playing is required: we start by solving the optimization for the last player ending up with the optimization for the leader (first player). The private sector's optimization problem is already solved out - the system of the structural equations of the model - and can be represented by the system:

$$\left[\begin{array}{c} Y_{t+1} \\ X_{t+1} \end{array}\right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \left[\begin{array}{c} Y_t \\ X_t \end{array}\right] + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \left[\begin{array}{c} U_t^H \\ U_t^F \end{array}\right] + \left[\begin{array}{c} D_1 \\ D_2 \end{array}\right] U_t^M + \left[\begin{array}{c} \varepsilon_{t+1} \\ \mathbf{O} \end{array}\right] \tag{38}$$

where Y_t are predetermined state variables and X_t are the effective instruments of private sectors, the non-predetermined or jump variables (consumption and the two inflation rates, in our model). The policy instruments are represented by U_t^H, U_t^F and U_t^M. U_t^H and U_t^F stand for the instruments of the followers which are, respectively, the Home and the Foreign fiscal authorities, while U_t^M represents the instrument of the leader, which is the monetary authority. ε_{t+1} is a vector of innovations to Y_t with covariance matrix Σ. This system describes the evolution of the economy as observed by policymakers.

In the discretionary case, the three policymakers reoptimize every period by taking the process by which private agents form their expectations as given - and where the expectations are consistent with actual policies (Söderlind 1999). The two Nash fiscal authorities minimize their loss functions treating the monetary policy instrument as parametric but incorporating the reaction functions of the private sectors. Assuming that the fiscal authority of the H country has the following objective function:

$$\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left(G_t^{H\prime} Q^H G_t^H \right) = \frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \left(Z_t^{\prime} \mathcal{Q}^H Z_t + Z_t^{\prime} \mathcal{P}^H U_t + U_t^{\prime} \mathcal{P}^{H\prime} Z_t + U_t^{\prime} \mathcal{R}^H U_t \right) \tag{39}$$

where G_t^H is the target variables for the H fiscal authority while Q^H is the corresponding matrix of weights. The target variables can be rewritten in terms of the predetermined and non-predetermined state variables collected on vector Z_t, in terms of the policy instruments (U_t) and in terms of combinations of these two variables. Being a follower, the H fiscal authority observes monetary authority's actions and reacts to them. In a linear-quadratic setup, the optimal solution belongs to the class of linear feedback rules of the form:

$$U_t^H = -F^H Y_t - L^H U_t^M \tag{40}$$

where F^H denotes feedback coefficients on the predetermined state variables and L^H is the leadership parameter. The other fiscal authority solves a similar problem and get:

$$U_t^F = -F^F Y_t - L^F U_t^M \tag{41}$$

Being in a Nash game, the two fiscal authorities do not respond to each other's actions.

The monetary leadership authority takes into account these fiscal policy reaction functions as well as the private sector's optimal conditions, when solves its optimization problem. Thus, the leader can manipulate the follower by changing its policy instrument. The monetary leadership reaction function takes the form of:

$$U_t^M = -F^M Y_t \tag{42}$$

2.5 Calibration

Our baseline calibration was chosen taking as reference Beetsma and Jensen (2004, 2005), Benigno and Benigno (2006), Benigno (2004), Benigno and López-Salido (2006) and Ferrero (2007).

As it is common in the literature, we assume that each period corresponds to a one quarter of a year. The one period discount factor of the private sector and policy makers β is set to 0.99, which implies a four percent annual basis steady-state interest rate.

The parameter θ, the elasticity of substitution between goods produced in the same country, is set such that the price mark-up is equal to 10%. We thus set θ equal to 11, which is a high value than the one found in the literature where distortions come only from monopolistic competition in the goods market[14]. The elasticity of substitution between Home and the Foreign produced goods ρ is set to 4.5, as in Benigno and Benigno (2006). These authors remark that, when this intratemporal elasticity is higher than the intertemporal elasticity of substitution in consumption (σ), the home and the foreign goods are substitutes in the utility. We follow Beetsma and Jensen (2004, 2005) and set the coefficient of the intertemporal elasticity of substitution in consumption σ at 0.4, which implies a coefficient of risk aversion for private consumption equal to 2.5. This is also the value we adopt for the coefficient of risk aversion for public spending $(1/\psi = 2.5)$. The steady-state value of consumption over output $\left(s_c = \overline{C}/\overline{Y}\right)$ is set at 0.75 in our baseline calibration.

Following Benigno and Benigno (2006) and Ferrero (2007), the inverse of the Frisch elasticity of labour supply to real wage, η, is assumed to be 0.47.[15] Our benchmark calibration intends to reflect a perfectly symmetric setup from which we can diverge and assess how asymmetries affect the results. Hence, we begin by assuming that the two economies in the monetary union have an equal size $(n = 0.5)$, have identical degrees of nominal rigidities $\left(\alpha^H = \alpha^F\right)$. We select a

[14] See Ferrero (2007) on this.

[15] Beetsma and Jensen (2004, 2005) emphasize the dilemma of choosing reasonable values for this parameter and for the mark-up and getting realistic magnitudes on the inflation response to changes in real variables. They set $\eta = 0.3$ and $\eta = 10$ on their papers of 2005 and 2004, respectively.

value for α equal to 0.75, in order to get an average length of price contracts equal to one year. To match the numerical constraint of the Maastricht Treaty, the yearly steady-state debt-output ratio is calibrated to 60%, in the second policy scenario where budgetary constraints are binding.[16] However, we also explore the implications of alternative assumptions regarding the relative dimensions of the two countries and the initial steady-state debt stock.[17]

Finally, we assume that the consumption and the technology shocks follow an uncorrelated AR(1) process with common persistence of 0.85, while the wage mark-up shocks are i.i.d., and the standard deviation of the innovations are equal to 0.01.

In what follows, we will broadly assume that policymakers engage in optimizing discretionary fiscal and monetary policy games. We attempt to draw welfare implications arising from different policy regimes under the two fiscal policy scenarios - with and without debt constraints. In particular, special attention will be given to the analysis across several degrees of country size asymmetry (from $n^H = 0.5$, for a symmetric monetary union, to $n^H = 0.9$) and across meaningful debt levels (yearly debt-to-output ratios from 50% to 100%).

2.6 Discretionary policy outcomes under cooperation

Strategic interactions among fiscal and monetary authorities in a monetary union are absent when they agree to maximize the union-wide social welfare. However, if policymakers cannot commit relative to the private sector, there can be meaningful strategic interactions between the former and the latter leading to substantial discrepancy between discretionary and commitment cooperative policy outcomes. Within our fiscal policy scenarios, time-inconsistency problems only reveal to be significant and critical to understand the discretionary outcomes, when stabilization policies face debt constraints.

In effect, in the balanced-budget scenario, the solutions under discretion and commitment coincide. Moreover, within this scenario, only asymmetric technology shocks impose welfare costs and require fiscal policy instruments deviating from their efficient levels.[18] Hence, monetary policy does not face stabilization trade-offs. Furthermore, given that the marginal costs and the inflation rates of the smaller (and more open economies) are affected, to a larger extent, by changes in the relative prices, small countries have to perform more active fiscal policies[19] than the larger ones and, even so, face worse stabilization performance.

[16] We calibrate debt to be zero, in the balanced-budget policy scenario.

[17] Leith and Wren-Lewis (2007a) have shown that the optimal discretionary stabilization policy plan depends crucially on the level of the debt-output ratio. The relative efficiency of the monetary and fiscal policy instruments to accomplish the short-run and the long-run stabilization assignments depends on the size of the debt stock: the tax rate reveals to increase its short-run stabilization performance with the raise of the debt-output ratio at the same time as it becomes less effective on the satisfaction of the government budget constraint.

[18] From Tables 1 and 2 it is clear that the feedback coefficients on symmetric shocks and on mark-up shocks of the fiscal and monetary policy rules are zero and that only fiscal policy feeds back on asymmetric technology shocks.

[19] We will consider that a policy is more active if it increases, in absolute value, the deviation

Figure 1 details the responses of key endogenous variables to a 1% negative technology shock hitting the large country (H) which, in the balanced-budget fiscal policy scenario, is equivalent to a positive technology shock hitting the small one (F). It is apparent that this shock, with a direct positive effect on the terms-of-trade gap and inefficiently shifting demand from F to H, justifies a large increase in the government spending gap and in the tax rate gap of the small country to lessen the asymmetric effect of the shock. Notwithstanding, we can also observe that this is not enough to avoid the higher (relative to the large country) variability of its inflation and output gap, under optimal discretionary or committed cooperative policies.

A shock, such as a negative symmetric technology shock that could be fully stabilized, under a balanced-budget policy scenario, leads to policy trade-offs and welfare stabilization costs, when policy instruments have to be adjusted to ensure fiscal solvency. The structure of discounting embedded in the welfare criterion, determining that smaller but more permanent gaps on welfare-related variables deliver lower welfare costs than larger, although transitory, gaps, determines that it would be optimal to let debt accommodate the shocks and to adjust fiscal policy instruments just to sustain the new (higher) debt stock levels. As a result, there would be long-lasting (negative) gaps on consumption, government spending and output. However, in the first period, once forward-looking expectations have been formed, policymakers may face the incentive to adopt policies that reduce those permanent effects because their consequences for short-run macroeconomic volatility could be slighter. Actually, as Leith and Wren-Lewis (2007a) have demonstrated, this conduces to a first-period policy, under commitment, that guarantees smaller permanent effects on the debt-to-output ratios, relative to a policy that could not benefit of the existence of predetermined expectations. The temptation to adopt the same policy they implemented at the first-period, if policymakers could re-optimise thereafter - that characterizes the time-inconsistency problem of the optimal policy under commitment -, will only disappear when permanent disequilibria are fully eliminated and the debt-to-output ratios return to their pre-shock levels. Hence, government debt is a source of considerable time-consistency problems which reveal decisive to explain discretionary policy outcomes and their large discrepancy relative to the ones obtained under commitment. Effectively, we observe that, in accordance with the findings of Leith and Wren-Lewis (2007a, 2007b), under the optimal discretionary stabilization policy plan, all variables return to their efficient pre-shock levels at expenses of higher short-run volatility, whereas an optimal commitment policy plan would give rise to a more permanent disequilibrium in the debt-output ratios and in some welfare-related variables but lower short-run variability (see Figure 2). Here, the aggressive policy response to shocks, in order to control future expectations and improve stabilization of current variables, manifests in small but inertial deviations of the policy instruments from their efficient values and permanent variations of government debts.

of the policy instruments from their efficient values.

Additionally, since the level of government indebtedness affects the relative effectiveness of the fiscal and monetary policy instruments on debt stabilization, the elimination of the long-term debt consequences is achieved diversely in small and large public debt scenarios, under discretion. In fact, the larger the steady-state debt-output ratios, the larger the impact of monetary policy in debt-service costs and,thus, the incentive to shift monetary policy conducting towards debt stabilization increases with the level of the latter; conversely, fiscal policy instruments – particularly, the tax rate gaps – lose efficacy to control debt and become relatively more apt to offset the inflationary consequences. For the considered (large) steady-state levels of public debt, the optimal discretionary policy entails a first-period cut in the interest rate gap in response to a symmetric shock that raises simultaneously debt and inflation, in a discretionary full cooperative policy regime.[20] This response is complemented, initially, with a decrease of the government spending gaps while, depending on the magnitude of the large debt-output ratios, the tax rate gaps may increase, to help debt stabilization, or may decrease to offset inflationary consequences.[21] The resulting decline of the debt induces policymakers to move policy instruments in opposite direction in the subsequent period (see the adjustments to a negative technology symmetric shock, b = 60% vs. b = 80%, in Figure 3).

Thus, the presence of government debt and the need to ensure fiscal solvency lead to time-consistency problems that materialize in a bias towards debt stabilization and a worse short-run macroeconomic stabilization performance, when policy is conducted in a period-by-period optimizing way. In contrast with the balanced-budget policy scenario, the welfare consequences of symmetric and idiosyncratic mark-up shocks can no longer be fully eliminated, as it is apparent from inspection of the policy feedback coefficients on these shocks (see Tables 1-4). Likewise, the time-consistent policy response to a negative technology shock at H, requiring, in the first period, an increase of the tax rate gap at H and a fall in the interest rate gap and in the tax rate gap at F, magnifies the effects on the inflation rates and on the consumption gap. This is evident from comparison of Figures 4 and 1 which illustrate the case of a negative technology shock hitting a large country (H) under cooperation in the two fiscal policy scenarios. The difference for a country-size symmetric monetary union is on the relatively larger short-run fluctuation experienced by the small country.

As expected, these adjustments influence the computations of the social loss under the two policy scenarios. In effect, by examination of Tables 5-6, it is easy to check that, under full cooperation, the welfare costs of the shocks are large when policy stabilization can not benefit from the existence of non-distortionary government sources of financing. These costs diminish with the degree of country-size asymmetry[22] and the representative household of the large

[20] For sufficiently small levels of public debt, the interest rate gap could augment in response to a shock that raises debt and boosts inflation.

[21] Under our model calibration, a negative symmetric technology shock requires a decrease on the tax rate gaps for steady-state debt-output ratios larger or equal to 65%.

[22] In practice, large country-size asymmetry implies a more symmetric structure of shocks at the union level.

country is clearly better-off, relative to the one living in the small country. This asymmetric distribution of the stabilization burden between the large and the small country amplifies with the level of government indebtedness: the monetary union and its large country profit with the raise of the debt-to-output ratios, while the small country loses.

2.7 Discretionary policy outcomes under non-cooperative regimes

The non-cooperative set up introduces the possibility of strategic interactions between the policymakers. Differences in the policy objectives, in the order of playing - Nash, monetary leadership or fiscal leadership - and in the relative size of each country may shape such strategic interactions.

Balanced-budget policies In face of an asymmetric technology shock, the tax rate and the government spending responses alleviate the impact on domestic inflation rates but accentuate the effect of this type of shock on the terms-of-trade gap. The latter produces a negative externality which, if not fully internalized by national authorities, implies a more active use of fiscal policy instruments.

With equal-size countries, this free-riding behaviour between fiscal authorities does not aggravate a potential free-riding problem between them and the central bank because the effects of their (symmetric) actions on union-wide variables cancel out. When it leads, the central bank anticipates this outcome and, thus, the monetary leadership and the Nash solutions coincide. On the other hand, under fiscal leadership, each fiscal authority ignores that the other government will set a symmetric policy, but it perceives that the central bank, internalizing the negative fiscal policy externalities, will react to an excessive policy response. As a consequence, both governments moderate their fiscal policy responses, reducing the free-riding problem (Cf. the fiscal policy feedback coefficients on a^H in Table 1). Therefore, among the non-cooperative regimes, the fiscal leadership delivers the lowest welfare stabilization costs.

Conversely, in a monetary union with country-size asymmetry, small countries suffer to a greater extent the impact of country-specific shocks and cause smaller cross-border effects; thus, their incentives may differ from those experienced by fiscal authorities of the large countries. The smaller a country is, the smaller are its externalities and the larger are the incentives of its government to free-ride; the large country, imposing larger spill over effects, will be more cautious in using its fiscal policy. This asymmetric conduct impinges on union-wide variables and forces a reaction of monetary policy to idiosyncratic technology shocks. From inspection of Table 2, one can see the extra fiscal policy activism of the small country (F) under Nash compared with the cooperative solution, as well as the different fiscal policy conducting of the large country. Being relatively more active, the fiscal policy of the small country determines the effect on aggregate fiscal policy instruments. Hence, since government spending gap and the tax rate gap increase at F in response to a negative technology shock at H, the interest rate gap increases to alleviate inflationary consequences at the union-wide level (see Figure 1, for a comparison of the dynamics under cooperation and Nash). Non-cooperation clearly benefits the small country, since it can achieve a better stabilization of its inflation rate making use of a costless policy instrument - the tax rate gap -, at the expenses of the large country.

With country-size asymmetry, and relative to Nash, fiscal leadership further exacerbates the "indiscipline" of the country that has more incentives to free-ride - the small country - while moderating the large country's fiscal policy reaction; the large country perceives that its policy largely impacts on aggregate variables to which the central bank reacts. Under monetary leadership, the central bank, perceiving the opposite incentives of the fiscal authorities, counteracts the aggregate effects of the small country's fiscal policy. This moderates fiscal policy at F but it leads to a more active fiscal policy at H. The welfare ranking of the two policy regimes depends on the degree of country-size asymmetry and on the balance of the different incentives. For strong country-size asymmetry ($n^H \geq 0.85$) there are welfare gains from having a monetary leadership that extend to all countries.

Table 5 shows that policy cooperation dominates non-cooperation for the monetary union, independently of its degree of country-size asymmetry, but cooperation reveals to be worse for the small country. In general, the latter prefers the fiscal leadership regime, except if it is too small. The preferences of the large country are in accordance with those for the monetary union as a whole. It benefits from being in a full cooperative regime and, among the non-cooperative regimes, it profits when it leads relative to the central bank, as long as the degree of country-size asymmetry is not too high.

Binding government budget constraints In this scenario, the need to ensure fiscal solvency amplifies the sources of strategic interactions between monetary and fiscal policies.

In an equal-sized monetary union, the incentives each fiscal authority face are similar: 1) they use more (less) actively the fiscal policy instruments that cause negative (positive) cross-border effects; 2) they free-ride on monetary policy to accommodate debt and, thus, react less to debt-disequilibria. Relative to the cooperative policy, and in face of a negative technology shock at H, this materializes in a smaller variation of the tax rate gaps and in a larger response of the government spending and of the interest rate gaps. Comparative to Nash equilibrium, where policymakers act simultaneously, fiscal leadership accentuates the free-riding of fiscal policy relative to monetary policy whereas monetary leadership controls it better. For instance, in face of a negative country-specific technology shock, aggregate fiscal policy and monetary policy turn out to be looser under fiscal leadership relative to the outcome under monetary leadership.[23]

[23] This can be checked by computing, for the various policy regimes, the aggregate government spending and tax rate responses to an idiosyncratic negative technologic shock at H,

In fact, monetary policy becomes more debt-accommodative across all non-cooperative regimes. Apparently, non-cooperation between domestically-oriented fiscal authorities cannot mitigate the time-consistency problem of both monetary and fiscal policies. By non-internalizing the cross-border fiscal policy effects and aggravating the time-consistency problem of the monetary policy, the non-cooperative regimes inflict larger welfare stabilization costs than cooperation, in spite of their positive effect on the time-consistency problem of fiscal policy. These welfare costs are attenuated under monetary leadership while magnified under fiscal leadership. Consequently, monetary leadership displays the lowest welfare costs among the non-cooperative policy games whilst fiscal leadership delivers the worse stabilization outcome (see Table 6, for $n^H = 0.5$).

Considering now country-size asymmetry, the incentives that each government faces are the result of the type but also of the size of the externalities it causes. As in the balanced-budget policy scenario, small countries, causing small externalities, have incentives to engage in more active fiscal policies than under cooperation. However, as the additional activism of the fiscal policy response moves towards debt-stabilization, it has negative consequences for the macroeconomic stabilization of the small countries. Conversely, the large countries are more likely to implement relatively less active fiscal policies under non-cooperation; a moderated fiscal policy response to shocks, *i.e*, less activeness on the control of the domestic budgetary consequences, leads to a better macroeconomic stabilization performance. Hence, in practice, this reasoning pairs with the argument that large countries, expecting domestic debt accommodation from the monetary policy, have less incentives to use fiscal policy instruments towards debt control and engage in fiscal policies that aim at achieving a better domestic macroeconomic stabilization. Likewise, a small country, relying to a less extent on the monetary policy accommodation of debt, becomes more cautious towards the use of fiscal policy in order to control for its domestic budgetary consequences and achieves a worse stabilization performance. To mitigate these asymmetric welfare consequences, the union-wide benevolent central bank accommodates relatively more the budgetary consequences of the small country than it would do in a cooperative policy regime and takes converse attitude relative to the large country (cf. feedback coefficients on Table 4). In this policy context and in comparison with the Nash equilibrium, the fiscal leadership scenario aggravates the free-riding problem between the large country's fiscal authority and the central bank, whilst the monetary leadership moderates it.[24]

In fact, we find that for the small country and the monetary union as a whole, enhancing policy cooperation is welfare-improving, unless there is a markedly high degree of country-size asymmetry (see Table 6). Policy cooperation can be

using the feedback coefficients on Table 3.

[24] From inspection of Table 4, we verify that, relative to Nash and in response to a negative technology shock at H, the government spending gap falls less (more) and the tax rate gap decreases by more (less) at H in fiscal leadership (monetary leadership). Hence, in fiscal leadership (monetary leadership) the fiscal policy of the large country is globally more loose (tight).

counterproductive in a monetary union where monetary policy accommodates proportionally the budgetary consequences of an excessively large country, in a large-debt monetary union. Moreover, if fiscal policies focus on national interests and do not cooperate, there are obvious social welfare stabilization gains from having a benevolent central bank that moves first. However, the large country – enhancing welfare under non-cooperation - prefers fiscal leadership, the policy regime where it can exploit its larger strategic position vis-à-vis the central bank. Hence, in a large-debt monetary union and for a sufficiently high degree of country-size asymmetry ($n^H \geq 0.6$), the policy regimes that deliver a better stabilization performance for the union may hardly emerge, since indebted large countries may strongly oppose to them.

Summing-up The outcomes of the discretionary policy games depend, crucially, on the type of incentives that the existence or not of non-distortionary sources of government financing creates. For the country that inflicts larger externalities – the large country – the welfare rankings match those of the union-wide, in the balanced-budget scenario. Conversely, when monetary policy moves towards debt-accommodation, it is the small country that has coincident welfare rankings with the union.

Under balanced-budget policies, monetary policy stabilization trade-offs arise only in presence of domestically oriented small country's governments that, benefiting from free-rider behaviours, perform more active fiscal policies. The policy regime that performs a better stabilization performance for the monetary union as a whole – policy cooperation – also dominates for the large countries. When debt constraints apply, monetary policy accommodates budgetary consequences in a large-debt monetary union and the free-riding incentives of the large countries dominate; so it may be hard to implement the socially desirable policy regime. In this case, the fiscal leadership regime could be more likely to emerge.

Additionally, under non-cooperation, the union's welfare decreases when the steady-state debt-to-output ratios increase symmetrically across countries (see Table 7). Moreover, this welfare reduction impacts exclusively in the small country, while the large country achieves a better stabilization outcome in higher-debt scenarios. Hence, focusing exclusively on welfare consequences of the stabilization policies, large countries are better-off in a large indebted monetary union, under a fiscal leadership regime; if this outcome prevails, it will lead to the worst stabilization performance for the union as a whole.[25]

2.8 The case for a conservative central bank

In the debt-constrained framework, the large country may oppose to the cooperative solution, which, among the discretionary policy regimes, broadly delivers the best union-wide welfare outcome. Additionally, fiscal leadership is the most preferred regime for the large country. Since this non-cooperative regime is more

[25] The welfare losses for the yearly debt-to-output ratios from 50% to 100% are available upon request.

likely to emerge, we assess if the existence can improve on this outcome. Relying in the existent literature, the welfare gains from this institutional arrangement are uncontroversial in the context of monetary policy models (see, among others, Rogoff, 1985, and Clarida *et al*, 1999). However, in the context of models that integrate monetary and fiscal policies, the presence of a conservative central bank is not unambiguously positive (see, for instance, Dixit and Lambertini, 2001, 2003a, 2003b, Adam and Billi, 2006, and Blake and Kirsanova, 2006).

In the balanced-budget scenario, where cooperative solution under commitment coincides with that under discretion, a weight-conservative central bank may be seen as a device to attenuate distortions arising only from the lack of policy cooperation. The eventual welfare gains of implementing this institutional policy arrangement in this policy scenario are just marginal. In fact, these gains proved to be null under monetary leadership. Under fiscal leadership, monetary conservatism only turns to be welfare improving, if the degree of country-size asymmetry in the monetary union is not too high[26] ($n^H < 0.7$ by inspection of Table 5).

In the presence of binding government budget constraints, delegating monetary policy to a conservative central bank gains an additional rationale: it can also mitigates distortions generated by the lack of fiscal and monetary commitment, which are important in this policy scenario. Intuitively, an inflation-averse central bank is more effective in controlling inflation expectations and, thus, it may improve the short-run trade-off between inflation and output. However, central bank conservatism can have a perverse effect as it may strengthen the incentives to reduce the permanent effects on debt and real welfare-relevant variables, amplifying the time-consistency problems of monetary and fiscal policies; moreover, it may exacerbate the strategic interactions between fiscal and monetary authorities due to the conflict of objectives. In fact, our experiments suggest that the desirability of monetary conservatism is ambiguous and that it depends on the timing of policy moves (fiscal leadership vs. monetary leadership), on the country-size asymmetry, and on the magnitude of the steady-state debt-to-output ratios, for instance.

With country-size symmetry, the comparison of the welfare losses under the benevolent and the conservative central bank (Table 6) shows that the latter delivers a worse stabilization outcome, except under fiscal leadership. In this case, fiscal authorities, internalizing that monetary policy could be less debt accommodative and over-reactive to inflationary consequences, moderate their free-riding behaviours.[27] Looking at the discretionary policy feedback coefficients on shocks, a weight-conservative scenario accentuates the budgetary accommodation stance of the monetary policy, particularly under monetary leadership (cf. Table 8). Even so, inflation variability diminishes, with the assistance of

[26] The conservative central bank moderates the large country's fiscal policy reaction to shocks, but it exacerbates the small country's fiscal policy.

[27] We have also computed the welfare losses when fiscal authorities cooperate against a conservative central bank and when all policymakers share the same inflation-averse policy objective. The losses under all these policy scenarios are higher than under the correspondent benevolent scenarios. These results are available upon request

the tax rates, but the welfare gains only emerge, when a conservative central bank contributes to reduce meaningful distortions generated by home-biased fiscal policy objectives, under fiscal leadership.

In general, these results also apply to the case of country-size asymmetry: delegating monetary policy to a conservative central bank improves the welfare of the union only under fiscal leadership. However, in this case, the incentives each fiscal authority faces do not parallel and, therefore, the welfare implications do not spread proportionally across countries. For instance, with our calibration, a conservative central bank may produce welfare gains for the union as a whole and for its small countries at expenses of a worse stabilization performance for the larger ones (cf. Table 6).[28] Apparently, the presence of a conservative central bank reduces the strategic power of the larger country.

3 Concluding Remarks

This work explored the interactions between monetary and fiscal stabilization policies in a micro-founded macroeconomic dynamic model for a monetary union with country-size asymmetry, under two fiscal policy scenarios – with and without debt constraints. We assessed how country-size asymmetry and the need to ensure fiscal policy solvency shape the strategic interactions between monetary and fiscal policies and determine welfare stabilization evaluation of the different policy regimes.

We concluded that it may be misleading to use the simplifying assumption of balanced budget fiscal policies in the analysis of the monetary and fiscal policy interactions under discretion. Debt raises substantially the problems of time-inconsistency and, by introducing additional sources of strategic interactions, it crucially affects the structure of incentives for the small and the large countries.

We found that small countries perform more active fiscal policies than large countries. Moreover, while in the balanced-budget scenario, macroeconomic stabilization is the only common policy concern of fiscal and monetary authorities, in the presence of government indebtedness, the latter optimally specializes on debt stabilization. We also found that policy cooperation is welfare increasing for the monetary union as a whole. When no debt constraints apply, incentives to free-ride prevail for the small countries, but cooperation dominates for the larger ones; thus, the best outcome for the union is more likely to emerge. However, in the second scenario, indebted large countries may strongly oppose to this arrangement in favour of fiscal leadership: given their large strategic power in face of a monetary policy that accommodates debt stabilization. In this case, delegation of monetary policy to a more conservative central bank could be a fruitful device to improve the welfare of the union.

[28] The gains of a conservative central bank under fiscal leadership are highly dependent on the type of shocks that prevail on the economies and on the degree of their persistence. For instance, for degrees of persistence of the technology shocks below 0.7, the conservative central bank impinges higher welfare costs on the union and on their large countries in spite of the better stabilization performance of the small countries.

In future work we intend to derive the benevolent non-cooperative country-specific loss functions and, additionally, include micro-founded political economy motivations to mimic the current behaviour of fiscal policy authorities. Other extension stems from the need to represent more realistically a monetary union composed by many small countries and few large ones. A two-country model, as we have used as a good starting point for representing a monetary union with country-size asymmetry, can be improved by describing part of the union as a continuum of small open economies, as Gali and Monacelli (2007) do for the whole monetary union. In the EMU, the majority of the country-members are small comparing to the union as a whole, and so, taken in isolation, their policy decisions have very little impact.

Appendix: Monetary leadership and Nash between the fiscal authorities

This appendix summarizes the iterative dynamic programming algorithm for the discretionary monetary leadership case when fiscal authorities play a Nash between them. This is an extension of the algorithms developed by Oudiz and Sachs (1985) and Backus and Driffill (1986) and popularized by Söderlind (1999). It closely follows the one developed by Kirsanova et al. (2005).

There are five strategic agents in the game: three explicit players - the monetary and the two fiscal authorities - and two implicit players - the private sector of both countries - that always act in last. In this type of game, the monetary authority moves first and sets the interest rate. Then the two fiscal authorities decide the levels of their fiscal policy instruments. Finally, the private sector in both countries reacts being the ultimate follower.

To solve this type of game, one inverts the order of playing and begins by solving the optimization of the last player, ending up with the optimization of the leader (the first player). The private sector's optimization problem is already solved out - the system of equations in section 2 - and can be represented by the following system, written in a state space form:

$$\widetilde{A}_0 \begin{bmatrix} I_{n_1} & \mathbf{O}_{n_1 x n_2} \\ \mathbf{O}_{n_2 x n_1} & H_{n_2 x n_2} \end{bmatrix} \begin{bmatrix} Y_{t+1} \\ E_t X_{t+1} \end{bmatrix} = \widetilde{A} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \widetilde{B} \begin{bmatrix} U_t^H \\ U_t^F \end{bmatrix} + \widetilde{D} U_t^M + \widetilde{C} \widetilde{\varepsilon}_{t+1} \tag{43}$$

where Y_t is an n_1-vector of predetermined state variables, Y_0 is given, and X_t are the effective instruments of the private sector, an n_2-vector of non-predetermined or forward-looking variables ($n = n_1 + n_2$). The policy instruments are represented by U_t^H, U_t^F and U_t^M. U_t^H and U_t^F stand for the instruments of the followers which are, respectively, the Home and the Foreign fiscal authorities, while U_t^M represents the instrument of the leader, which is the monetary authority. ε_{t+1} is an n_ε-vector of exogenous zero-mean *iid* shocks with an identity covariance matrix. Premultiplying (43) by $\widetilde{A}_0^{-1}$ we get

$$\begin{bmatrix} Y_{t+1} \\ HE_tX_{t+1} \end{bmatrix} = A \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + B \begin{bmatrix} U_t^H \\ U_t^F \end{bmatrix} + DU_t^M + C\varepsilon_{t+1} \tag{44}$$

where $A = \widetilde{A}_0^{-1}\widetilde{A}$, $B = \widetilde{A}_0^{-1}\widetilde{B}$, $D = \widetilde{A}_0^{-1}\widetilde{D}$ and $C = \widetilde{A}_0^{-1}\widetilde{C}$. The covariance matrix of the shocks to Y_{t+1} is CC' and matrices A, B, C, and D are partioned conformably with Y_t and X_t as

$$\begin{aligned} A &\equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix};\ B \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \\ D &\equiv \begin{bmatrix} D_1 \\ D_2 \end{bmatrix};\ C \equiv \begin{bmatrix} C_1 \\ \mathbf{O} \end{bmatrix} \end{aligned}$$

A common special case is when $H \equiv I$, but in general this matrix need not to be invertible. This system describes the evolution of the economy as observed by policymakers.

The followers' optimization problem

In the discretionary case, the three policymakers reoptimize every period by taking the process by which private agents form their expectations as given - and where the expectations are consistent with actual policies (Söderlind 1999). The two Nash fiscal authorities minimize their loss functions treating the monetary policy instrument as parametric but incorporating the reaction functions of the private sectors. Assuming that the fiscal authority of the Home country has the following objective function:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(G_t^{H\prime}Q^HG_t^H\right) = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(Z_t'\mathcal{Q}^HZ_t + Z_t'\mathcal{P}^HU_t + U_t'\mathcal{P}^{H\prime}Z_t + U_t'\mathcal{R}^HU_t\right) \tag{45}$$

where $G_t^{H\prime}$ is the target variables for the Home fiscal authority while Q^H is the corresponding matrix of weights. The target variables can be rewritten in terms of the predetermined and non-predetermined state variables collected on vector Z_t, in terms of the policy instruments (U_t) and in terms of combinations of these two variables.

The fiscal authority in H optimizes every period, taking into account that she will be able to reoptimize next period. The model is linear-quadratic, thus the solution in $t+1$ gives a period return which is quadratic in the state variables, $W_{t+1}^H \equiv Y_{t+1}^{\prime}S_H^{t+1}Y_{t+1} + w_{t+1}^H$, where S_H^{t+1} is a positive semidefinite matrix and w_{t+1}^H is a scalar independent of Y_{t+1}. Moreover, the forward looking variables must be linear functions of the state variables, $X_{t+1} = -N_{t+1}Y_{t+1}$. Hence, the value function of the fiscal authority of H in t will then satisfy the Bellman equation:

$$W_t^H = \min_{U_t^H}\frac{1}{2}\left[\left(Z_t'\mathcal{Q}^HZ_t + Z_t'\mathcal{P}^HU_t + U_t'\mathcal{P}^{H\prime}Z_t + U_t'\mathcal{R}^HU_t\right) + \beta E_t\left(W_{t+1}^H\right)\right] \tag{46}$$

s.t. $E_t X_{t+1} = -N_{t+1} E_t Y_{t+1}$, $W^H_{t+1} \equiv Y'_{t+1} S^{t+1}_H Y_{t+1} + w^H_{t+1}$, eq. (44) and Y_t given.

Rewriting the system by using $E_t X_{t+1} = -N_{t+1} E_t Y_{t+1}$ Using the expression above to substitute into the upper block of (44), we get

$$E_t X_{t+1} = -N_{t+1}\left[A_{11} Y_t + A_{12} X_t + B_{11} U^H_t + B_{12} U^F_t + D_1 U^M_t\right]$$

while the lower block of (44) is

$$H E_t X_{t+1} = A_{21} Y_t + A_{22} X_t + B_{21} U^H_t + B_{22} U^F_t + D_2 U^M_t$$

Multiplying the former equation by H, setting the result equal to the latter equation and solving for X_t we obtain

$$\begin{aligned} X_t \quad = \quad & \text{-}\underbrace{(A_{22}+HN_{t+1}A_{12})^{-1}(A_{21}+HN_{t+1}A_{11})}_{J_t} Y_t \text{-} \underbrace{(A_{22}+HN_{t+1}A_{12})^{-1}(B_{21}+HN_{t+1}B_{11})}_{K^H_t} U^H_t \\ & \text{-}\underbrace{(A_{22}+HN_{t+1}A_{12})^{-1}(B_{22}+HN_{t+1}B_{12})}_{K^F_t} U^F_t \text{-} \underbrace{(A_{22}+HN_{t+1}A_{12})^{-1}(D_2+HN_{t+1}D_1)}_{K^M_t} U^M_t \end{aligned}$$

$$X_t = -J_t Y_t - K^H_t U^H_t - K^F_t U^F_t - K^M_t U^M_t \tag{47}$$

where J_t is n_2xn_1, K^H_t is n_2xk_H, K^F_t is n_2xk_F and K^M_t is n_2xk_M (k_H and k_F stand respectively for the number of fiscal policy instruments of H and F, while k_M stands for the number of monetary policy instruments)[29].

The evolution of Y_t Use (47) in the first n_1 equations in the system(44) to get the reduced form evolution of the predetermined variables

$$\begin{aligned} Y_{t+1} \quad = \quad & \underbrace{[A_{11} - A_{12} J_t]}_{O_{Y_t}} Y_t + \underbrace{\left[B_{11} - A_{12} K^H_t\right]}_{O_{H_t}} U^H_t \\ & + \underbrace{\left[B_{12} - A_{12} K^F_t\right]}_{O_{F_t}} U^F_t + \underbrace{\left[D_1 - B_{12} L^F_t\right]}_{O_{M_t}} U^M_t + C_1 \varepsilon_{t+1} \end{aligned}$$

$$Y_{t+1} = O_{Y_t} Y_t + O_{H_t} U^H_t + O_{F_t} U^F_t + O_{M_t} U^M_t + C_1 \varepsilon_{t+1} \tag{48}$$

Being a follower, the Home fiscal authority observes monetary authority's actions and reacts to them. In a linear-quadratic setup, the optimal solution belongs to the class of linear feedback rules of the form:

[29] It is assumed that $A_{22} + HN_{t+1} A_{12}$ is invertible.

$$U_t^H = -F_t^H Y_t - L_t^H U_t^M \tag{49}$$

where F_t^H denotes feedback coefficients on the predetermined state variables and L_t^H is the leadership parameter. The other fiscal authority solves a similar problem and get:

$$U_t^F = -F_t^F Y_t - L_t^F U_t^M \tag{50}$$

Being in a Nash game, the two fiscal authorities do not respond to each other's actions.

The monetary leadership authority takes into account these fiscal policy reaction functions as well as the private sector's optimal conditions, when solves its optimization problem. Thus, the leader can manipulate the follower by changing its policy instrument. The monetary leadership reaction function takes the form of:

$$U_t^M = -F_t^M Y_t \tag{51}$$

Reformulated optimization problem Therefore we can substitute eqs. (47) and (48) into (46) to obtain an equivalent minimization problem[30]:

$$\begin{aligned}
2\widetilde{W}_t^H \quad \equiv \quad & \min_{U_t^H} \Big\{ Y_t' \left[Q_H^S + \beta O_{Y_t}' S_H^{t+1} O_{Y_t}\right] Y_t + U_t^{H'} \left[\mathcal{U}_H^{S,H'} + \beta O_{H_t}' S_H^{t+1} O_{Y_t}\right] Y_t \\
& + Y_t' \left[\mathcal{U}_H^{S,H} + \beta O_{Y_t}' S_H^{t+1} O_{H_t}\right] U_t^H + U_t^{F'} \left[\mathcal{U}_F^{S,H'} + \beta O_{F_t}' S_H^{t+1} O_{Y_t}\right] Y_t \\
& + Y_t' \left[\mathcal{U}_F^{S,H} + \beta O_{Y_t}' S_H^{t+1} O_{F_t}\right] U_t^F + U_t^{M'} \left[\mathcal{U}_M^{S,H'} + \beta O_{M_t}' S_H^{t+1} O_{Y_t}\right] Y_t \\
& + Y_t' \left[\mathcal{U}_M^{S,H} + \beta O_{Y_t}' S_H^{t+1} O_{M_t}\right] U_t^M + U_t^{H'} \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\
& + U_t^{F'} \left[\mathcal{R}_F^{S,H} + \beta O_{F_t}' S_H^{t+1} O_{F_t}\right] U_t^F + U_t^{M'} \left[\mathcal{R}_M^{S,H} + \beta O_{M_t}' S_H^{t+1} O_{M_t}\right] U_t^M \\
& + U_t^{H'} \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{F_t}\right] U_t^F + U_t^{F'} \left[\mathcal{P}_{HF}^{S,H'} + \beta O_{F_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\
& + U_t^{H'} \left[\mathcal{P}_{HM}^{S,H} + \beta O_{H_t}' S_H^{t+1} O_{M_t}\right] U_t^M + U_t^{M'} \left[\mathcal{P}_{HM}^{S,H'} + \beta O_{M_t}' S_H^{t+1} O_{H_t}\right] U_t^H \\
& + U_t^{F'} \left[\mathcal{P}_{FM}^{S,H} + \beta O_{F_t}' S_H^{t+1} O_{M_t}\right] U_t^M + U_t^{M'} \left[\mathcal{P}_{FM}^{S,H'} + \beta O_{M_t}' S_H^{t+1} O_{F_t}\right] U_t^F \Big\}
\end{aligned} \tag{52}$$

where

[30] We have make use of the fact that w_{t+1}^H is independent of Y_{t+1} and $E_t \varepsilon_{t+1} = 0$.

$$
\begin{aligned}
Q_H^S &= \mathcal{Q}_{11}^H - J_t^{'}\mathcal{Q}_{21}^H - \mathcal{Q}_{12}^H J_t + J_t^{'}\mathcal{Q}_{22}^H J_t \\
\mathcal{U}_H^{S,H} &= J_t^{'}\mathcal{Q}_{22}^H K_t^H - \mathcal{Q}_{12}^H K_t^H + \mathcal{P}_{12}^H - J_t^{'}\mathcal{P}_{22}^H \\
\mathcal{U}_F^{S,H} &= J_t^{'}\mathcal{Q}_{22}^H K_t^F - \mathcal{Q}_{12}^H K_t^F + \mathcal{P}_{13}^H - J_t^{'}\mathcal{P}_{23}^H \\
\mathcal{U}_M^{S,H} &= J_t^{'}\mathcal{Q}_{22}^H K_t^M - \mathcal{Q}_{12}^H K_t^M + \mathcal{P}_{11}^H - J_t^{'}\mathcal{P}_{21}^H \\
\mathcal{R}_H^{S,H} &= K_t^{H\prime}\mathcal{Q}_{22}^H K_t^H - K_t^{H\prime}\mathcal{P}_{22}^H - \mathcal{P}_{22}^{H\prime} K_t^H + \mathcal{R}_{22}^H \\
\mathcal{R}_F^{S,H} &= K_t^{F\prime}\mathcal{Q}_{22}^H K_t^F - K_t^{F\prime}\mathcal{P}_{23}^H - \mathcal{P}_{23}^{H\prime} K_t^F + \mathcal{R}_{33}^H \\
\mathcal{R}_M^{S,H} &= K_t^{M\prime}\mathcal{Q}_{22}^H K_t^M - K_t^{M\prime}\mathcal{P}_{21}^H - \mathcal{P}_{21}^{H\prime} K_t^M + \mathcal{R}_{11}^H \\
\mathcal{P}_{HF}^{S,H} &= K_t^{H\prime}\mathcal{Q}_{22}^H K_t^F - K_t^{H\prime}\mathcal{P}_{23}^H - \mathcal{P}_{22}^{H\prime} K_t^F + \mathcal{R}_{23}^H \\
\mathcal{P}_{HM}^{S,H} &= K_t^{H\prime}\mathcal{Q}_{22}^H K_t^M - K_t^{H\prime}\mathcal{P}_{21}^H - \mathcal{P}_{22}^{H\prime} K_t^M + \mathcal{R}_{21}^H \\
\mathcal{P}_{FM}^{S,H} &= K_t^{F\prime}\mathcal{Q}_{22}^H K_t^M - K_t^{F\prime}\mathcal{P}_{21}^H - \mathcal{P}_{23}^{H\prime} K_t^M + \mathcal{R}_{31}^H
\end{aligned}
$$

Hence, the problem faced by the Home fiscal authority has been transformed to a standard linear-quadratic regulator problem without forward looking variables but with time varying parameters. The first-order condition is

$$
\begin{aligned}
0 &= \left[\mathcal{U}_H^{S,H\prime} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{Y_t}\right] Y_t + \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{H_t}\right] U_t^H \\
&\quad + \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{F_t}\right] U_t^F + \left[\mathcal{P}_{HM}^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{M_t}\right] U_t^M
\end{aligned}
$$

Since $U_t^H = -F_t^H Y_t - L_t^H U_t^M$ and $U_t^F = -F_t^F Y_t - L_t^F U_t^M$, the first-order condition can be solved for the feedback coefficients of the reaction function of the Home fiscal authority:

$$
F_t^H \equiv \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{H_t}\right]^{-1} \left\{ \begin{array}{l} \left[\mathcal{U}_H^{S,H\prime} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{Y_t}\right] \\ - \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{F_t}\right] F_t^F \end{array} \right. \tag{53}
$$

$$
L_t^H \equiv \left[\mathcal{R}_H^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{H_t}\right]^{-1} \left\{ \begin{array}{l} \left[\mathcal{P}_{HM}^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{M_t}\right] \\ - \left[\mathcal{P}_{HF}^{S,H} + \beta O_{H_t}^{\prime} S_H^{t+1} O_{F_t}\right] L_t^F \end{array} \right. \tag{54}
$$

Finding the recursive equation for S_H^t Substituting the decision rules (49) , (50) and (51) into (52) we obtain the recursive equations for

$$
S_H^t \equiv T_{0,t}^H + \beta T_t^{H\prime} S_H^{t+1} T_t^H \tag{55}
$$

$$
\begin{aligned}
T_{0,t}^{H} \;=\; & Q_{H}^{S}-\mathcal{U}_{H}^{S,H}\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)-\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)'\mathcal{U}_{H}^{S,H'}-\mathcal{U}_{F}^{S,H}\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)\\
&-\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)'\mathcal{U}_{F}^{S,H'}-\mathcal{U}_{M}^{S,H}F_{t}^{M}-F_{t}^{M'}\mathcal{U}_{M}^{S,H'}+\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)'\mathcal{R}_{H}^{S,H}\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)\\
&+\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)'\mathcal{R}_{F}^{S,H}\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)+F_{t}^{M\prime}\mathcal{R}_{M}^{S,H}F_{t}^{M}\\
&+\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)'\mathcal{P}_{HF}^{S,H}\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)+\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)'\mathcal{P}_{HF}^{S,H'}\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)\\
&+\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)'\mathcal{P}_{HM}^{S,H}F_{t}^{M}+F_{t}^{M'\prime}\mathcal{P}_{HM}^{S,H'}\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)\\
&+\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)'\mathcal{P}_{FM}^{S,H}F_{t}^{M}+F_{t}^{M'\prime}\mathcal{P}_{FM}^{S,H'}\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)
\end{aligned}
$$

and

$$
T_{t}^{H}=O_{Y_t}-O_{H_t}\left(F_{t}^{H}-L_{t}^{H}F_{t}^{M}\right)-O_{F_t}\left(F_{t}^{F}-L_{t}^{F}F_{t}^{M}\right)-O_{M_t}F_{t}^{M}
$$

Similar formulae can be derived for country F.

The leader's optimization problem

This part of the problem is the standard optimization problem when the system under control evolves as

$$
\begin{bmatrix} Y_{t+1} \\ HE_tX_{t+1} \end{bmatrix}=\begin{bmatrix} A11\text{-}B11F_t^H\text{-}B12F_t^F & A12 \\ A21\text{-}B21F_t^H\text{-}B22F_t^F & A22 \end{bmatrix}\begin{bmatrix} Y_t \\ X_t \end{bmatrix}+\begin{bmatrix} D11\text{-}B11L_t^H\text{-}B12L_t^F \\ D21\text{-}B21L_t^H\text{-}B22L_t^F \end{bmatrix}U_t^M+C\varepsilon_{t+1} \tag{56}
$$

The monetary authority loss function is

$$
\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(G_t^{M'}\mathrm{Q}^M G_t^M\right)
$$

But, since the leadership integrates the followers' reaction functions - $U_t^H=-F_t^HY_t-L_t^HU_t^M$ and $U_t^F=-F_t^FY_t-L_t^FU_t^M$ - into its optimization problem, the leadership's loss function as to be rewritten in terms of the relevant variables for the leadership authority. Since

$$
\begin{bmatrix} Y_t \\ X_t \\ U_t^M \\ U_t^H \\ U_t^F \end{bmatrix}=\underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -F_t^H & 0 & -L_t^H \\ -F_t^F & 0 & -L_t^F \end{bmatrix}}_{\mathcal{C}}\begin{bmatrix} Y_t \\ X_t \\ U_t^M \end{bmatrix}
$$

we can set $G_t^{M'}\mathrm{Q}^M G_t^M=\begin{bmatrix} Y_t' & X_t' & U_t^{M\prime} \end{bmatrix}\widetilde{\mathcal{K}}^M\begin{bmatrix} Y_t \\ X_t \\ U_t^M \end{bmatrix}$ where $\widetilde{\mathcal{K}}^M=\mathcal{C}'\underbrace{C^{M\prime}\mathrm{Q}^M C^M}_{\mathcal{K}^M}\mathcal{C}$ and $\widetilde{\mathcal{K}}^M$ have to partioned conformably with $\begin{pmatrix} Y_t' & X_t' & U_t^{M\prime} \end{pmatrix}'$.

The iterative procedure

We start with initial approximation for the monetary policy rule, $F^M_{(0)}$, symmetric positive definite matrices (usually, identity matrices), $S^{(0)}_H$ and $S^{(0)}_F$, some (e.g. a matrix of zeros) $N_{(0)}$ and solve the follower's problem, using Eq. $(53-55)$ for country H and equivalent equations for country F. We get $F^H_{(0)}$ and $L^H_{(0)}$, as well as $F^F_{(0)}$ and $L^F_{(0)}$ and updated matrices $S^{(1)}_H$ and $S^{(1)}_F$. We then take into account the policy reaction functions of fiscal authorities and compute new matrices in Eq. (56), updated target variable $\left(G^M_t = C^M \mathcal{C} \begin{pmatrix} Y'_t & X'_t & U^{M\prime}_t \end{pmatrix}'\right)$ and solve the problem for the monetary authority. This will give us the monetary policy reaction function, $F^M_{(1)}$, and updated matrices $N_{(1)}$ and $S^{(1)}_M$. Then, we again solve the problem for the fiscal authorities to update $S^{(2)}_H$ and $S^{(2)}_F$ and $F^H_{(1)}$, $L^H_{(1)}$, $F^F_{(1)}$ and $L^F_{(1)}$ and so on. The fixed point is found when the policy rules and the matrices converge towards constants for a given level of tolerance.

Blake and Kirsanova (2007) have examined the existence of multiple discretionary equilibria in dynamic linear quadratic rational expectations models. They have concluded that linear quadratic discretionary problems can only have isolated stable equilibria. Even when the number of stable eigenvalues exceed the number of pre-determined variables in the model, there is no indeterminacy because the time-consistency property of the discretionary equilibria rules out that possibility. However, there could be multiple but isolated discretionary equilibria. To check this hypothesis (only when the number of explosive eigenvalues is smaller than the number of non-predetermined variables) it is necessary to initialize the algorithm with different matrices and see if the solutions obtained are or not distinct.

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Tables and Figures

Baseline: σ=ψ=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; α^H=α^F=0.75; $\rho_a = 0.85$; n = 0.5								
		a_t^H	a_t^F	μ_t^H	μ_t^F	a_{t-1}^H	a_{t-1}^F	q_{t-1}
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1622	-0.1622	0.0000	0.0000	-0.1532	0.1532	-0.2745
Coop	τ_t^H	1.4001	-1.4001	0.0000	0.0000	-1.3224	1.3224	-2.3688
	g_t^F	-0.1622	0.1622	0.0000	0.0000	0.1532	-0.1532	0.2745
	τ_t^F	-1.4001	1.4001	0.0000	0.0000	1.3224	-1.3224	2.3688
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1655	-0.1655	0.0000	0.0000	-0.1600	0.1600	-0.2866
Nash	τ_t^H	1.7869	-1.7869	0.0000	0.0000	-1.7855	1.7855	-3.1985
	g_t^F	-0.1655	0.1655	0.0000	0.0000	0.1600	-0.1600	0.2866
	τ_t^F	-1.7869	1.7869	0.0000	0.0000	1.7855	-1.7855	3.1985
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1607	-0.1607	0.0000	0.0000	-0.1481	0.1481	-0.2653
FL	τ_t^H	1.7178	-1.7178	0.0000	0.0000	-1.7120	1.7120	-3.0668
	g_t^F	-0.1607	0.1607	0.0000	0.0000	0.1481	-0.1481	0.2653
	τ_t^F	-1.7178	1.7178	0.0000	0.0000	1.7120	-1.7120	3.0668
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.1655	-0.1655	0.0000	0.0000	-0.1600	0.1600	-0.2866
ML	τ_t^H	1.7869	-1.7869	0.0000	0.0000	-1.7855	1.7855	-3.1985
	g_t^F	-0.1655	0.1655	0.0000	0.0000	0.1600	-0.1600	0.2866
	τ_t^F	-1.7869	1.7869	0.0000	0.0000	1.7855	-1.7855	3.1985

Table 1: Policy reaction functions - balanced-budget, nH = 0.5

Baseline: σ=ψ=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; α^H=α^F=0.75; $\rho_a = 0.85$; n = 0.8								
		a_t^H	a_t^F	μ_t^H	μ_t^F	a_{t-1}^H	a_{t-1}^F	q_{t-1}
	i_t	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	g_t^H	0.0649	-0.0649	0.0000	0.0000	-0.0613	0.0613	-0.1098
Coop	τ_t^H	0.5601	-0.5601	0.0000	0.0000	-0.5289	0.5289	-0.9475
	g_t^F	-0.2596	0.2596	0.0000	0.0000	0.2452	-0.2452	0.4392
	τ_t^F	-2.2402	2.2402	0.0000	0.0000	2.1158	-2.1158	3.7900
	i_t	-0.0209	0.0209	0.0000	0.0000	0.0205	-0.0205	0.0367
	g_t^H	0.0615	-0.0615	0.0000	0.0000	-0.0489	0.0489	-0.0876
Nash	τ_t^H	0.3149	-0.3149	0.0000	0.0000	0.3419	-0.3419	0.6124
	g_t^F	-0.2719	0.2719	0.0000	0.0000	0.2746	-0.2746	0.4919
	τ_t^F	-3.4803	3.4803	0.0000	0.0000	4.0882	-4.0882	7.3232
	i_t	-0.0234	0.0234	0.0000	0.0000	0.0234	-0.0234	0.0419
	g_t^H	0.0592	-0.0592	0.0000	0.0000	-0.0422	0.0422	-0.0755
FL	τ_t^H	0.2580	-0.2580	0.0000	0.0000	0.4488	-0.4488	0.8040
	g_t^F	-0.2704	0.2704	0.0000	0.0000	0.2696	-0.2696	0.4830
	τ_t^F	-3.4857	3.4857	0.0000	0.0000	4.1320	-4.1320	7.4017
	i_t	-0.0021	0.0021	0.0000	0.0000	0.0074	-0.0074	0.0133
	g_t^H	0.0667	-0.0667	0.0000	0.0000	-0.0648	0.0648	-0.1161
ML	τ_t^H	0.6477	-0.6477	0.0000	0.0000	-0.6695	0.6695	-1.1993
	g_t^F	-0.2667	0.2667	0.0000	0.0000	0.2592	-0.2592	0.4643
	τ_t^F	-3.1542	3.1542	0.0000	0.0000	3.1011	-3.1011	5.5550

Table 2: Policy reaction functions - balanced-budget, nH = 0.8

Baseline: σ=ψ=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; α^H=α^F=0.75; λ^H=λ^F=1; $\rho_a = 0.85$; n = 0.5; B/4Y=60%											
		a_t^H	a_t^F	μ_t^H	μ_t^F	$\overline{c}_t^w$	a_{t-1}^H	a_{t-1}^F	q_{t-1}	b_{t-1}^H	b_{t-1}^F
	i_t	0.6826	0.6826	-0.1734	-0.1734	0.2921	-0.0000	0.0000	-0.0000	-0.5733	-0.5733
	g_t^H	0.3391	-0.1862	-0.0497	0.0109	0.0327	-0.2053	0.2053	-0.3678	-0.1644	0.0359
Coop	τ_t^H	-2.4114	2.0082	1.0910	-0.9886	-0.0863	0.6206	-0.6206	1.1117	3.6068	-3.2681
	g_t^F	-0.1862	0.3391	0.0109	-0.0497	0.0327	0.2053	-0.2053	0.3678	0.0359	-0.1644
	τ_t^F	2.0082	-2.4114	-0.9886	1.0910	-0.0863	-0.6206	0.6206	-1.1117	-3.2681	3.6068
	i_t	0.7520	0.7520	-0.1911	-0.1911	0.3218	-0.0000	0.0000	-0.0000	-0.6316	-0.6316
	g_t^H	0.3594	-0.2256	-0.0471	0.0131	0.0286	-0.2269	0.2269	-0.4065	-0.1557	0.0433
Nash	τ_t^H	-0.0943	1.1465	0.5464	-0.8137	0.2251	-1.8627	1.8627	-3.3366	1.8061	-2.6899
	g_t^F	-0.2256	0.3594	0.0131	-0.0471	0.0286	0.2269	-0.2269	0.4065	0.0433	-0.1557
	τ_t^F	1.1465	-0.0943	-0.8137	0.5464	0.2251	1.8627	-1.8627	3.3366	-2.6899	1.8061
	i_t	0.7524	0.7524	-0.1912	-0.1912	0.3220	0.0000	-0.0000	0.0000	-0.6320	-0.6320
	g_t^H	0.3396	-0.2056	-0.0437	0.0097	0.0287	-0.1830	0.1830	-0.3278	-0.1445	0.0320
FL	τ_t^H	0.3780	0.6837	0.4446	-0.7143	0.2272	-2.6778	2.6778	-4.7968	1.4696	-2.3613
	g_t^F	-0.2056	0.3396	0.0097	-0.0437	0.0287	0.1830	-0.1830	0.3278	0.0320	-0.1445
	τ_t^F	0.6837	0.3780	-0.7143	0.4446	0.2272	2.6778	-2.6778	4.7968	-2.3613	1.4696
	i_t	0.6932	0.6932	-0.1761	-0.1761	0.2966	-0.0000	0.0000	-0.0000	-0.5823	-0.5823
	g_t^H	0.3627	-0.2208	-0.0479	0.0119	0.0304	-0.2350	0.2350	-0.4210	-0.1584	0.0392
ML	τ_t^H	-0.3669	1.0894	0.6066	-0.7901	0.1546	-1.5500	1.5500	-2.7766	2.0052	-2.6120
	g_t^F	-0.2208	0.3627	0.0119	-0.0479	0.0304	0.2350	-0.2350	0.4210	0.0392	-0.1584
	τ_t^F	1.0894	-0.3669	-0.7901	0.6066	0.1546	1.5500	-1.5500	2.7766	-2.6120	2.0052

Table 3: Policy reaction functions - debt, nH = 0.5

Baseline: σ=ψ=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; $\alpha^H=\alpha^F$=0.75; $\lambda^H=\lambda^F$=1; $\rho_a=0.85$; n = 0.8; B/4Y=60%											
		a_t^H	a_t^F	μ_t^H	μ_t^F	$\overline{c}_t^w$	a_{t-1}^H	a_{t-1}^F	q_{t-1}	b_{t-1}^H	b_{t-1}^F
	i_t	1.0921	0.2730	-0.2775	-0.0694	0.2921	-0.0000	0.0000	-0.0000	-0.9173	-0.2293
	g_t^H	0.2274	-0.0745	-0.0432	0.0043	0.0327	-0.0821	0.0821	-0.1471	-0.1428	0.0144
Coop	τ_t^H	-1.2065	0.8033	0.4979	-0.3954	-0.0863	0.2482	-0.2482	0.4447	1.6459	-1.3072
	g_t^F	-0.2978	0.4508	0.0174	-0.0562	0.0327	0.3285	-0.3285	0.5885	0.0574	-0.1859
	τ_t^F	3.2131	-3.6163	-1.5817	1.6842	-0.0863	-0.9929	0.9929	-1.7786	-5.2289	5.5676
	i_t	0.9293	0.5793	-0.2338	-0.1495	0.3228	-3.1787	3.1787	-5.6940	-0.7730	-0.4942
	g_t^H	0.1456	-0.0272	-0.0242	-0.0059	0.0253	-0.3065	0.3065	-0.5490	-0.0800	-0.0194
Nash	τ_t^H	1.4286	-0.5334	-0.0721	-0.1553	0.1915	-3.5439	3.5439	-6.3482	-0.2384	-0.5135
	g_t^F	-0.4296	0.5744	0.0210	-0.0578	0.0310	0.4076	-0.4076	0.7302	0.0694	-0.1910
	τ_t^F	6.8594	-5.5344	-1.6558	1.3191	0.2835	-16.9330	16.9330	-30.3325	-5.4738	4.3608
	i_t	0.9368	0.5749	-0.2294	-0.1547	0.3234	-2.9852	2.9852	-5.3476	-0.7584	-0.5113
	g_t^H	0.1364	-0.0178	-0.0226	-0.0075	0.0254	-0.2692	0.2692	-0.4823	-0.0747	-0.0249
FL	τ_t^H	1.7470	-0.8120	-0.1151	-0.1225	0.2001	-4.0777	4.0777	-7.3045	-0.3805	-0.4048
	g_t^F	-0.4037	0.5484	0.0172	-0.0539	0.0310	0.3051	-0.3051	0.5466	0.0568	-0.1783
	τ_t^F	6.1141	-4.7821	-1.5078	1.1694	0.2850	-13.0557	13.0557	-23.3871	-4.9846	3.8657
	i_t	0.6527	0.7404	-0.1957	-0.1582	0.2981	-3.0052	3.0052	-5.3833	-0.6471	-0.5231
	g_t^H	0.1457	-0.0265	-0.0239	-0.0064	0.0255	-0.3578	0.3578	-0.6408	-0.0789	-0.0212
ML	τ_t^H	1.2850	-0.4787	-0.0305	-0.1744	0.1725	-3.7175	3.7175	-6.6592	-0.1008	-0.5765
	g_t^F	-0.4309	0.5880	0.0230	-0.0629	0.0336	0.3051	-0.3051	0.5465	0.0761	-0.2080
	τ_t^F	6.3608	-5.4950	-1.6520	1.4320	0.1853	-14.6393	14.6393	-26.2237	-5.4611	4.7339

Table 4: Policy reaction functions - debt, nH = 0.8

Baseline:σ=ψ=0.4; ρ=4.5;θ=11; η=0.47;β=0.99; $\alpha^{H}=\alpha^{F}$=0.75; $\lambda^{H}=\lambda^{F}$=1;$\rho_a = \rho_c = 0.85$								
Union-wide loss	$n^H = 0.5$	$n^H = 0.55$	$n^H = 0.6$	$n^H = 0.65$	$n^H = 0.7$	$n^H = 0.75$	$n^H = 0.8$	$n^H = 0.9$
LCoop*100	3.8078	3.7697	3.6555	3.4651	3.1985	2.8558	2.4370	1.3708
LML*100	3.9479	3.9129	3.8075	3.6306	3.3800	3.0526	2.6438	1.5534
LFL*100	3.8942	3.8617	3.7635	3.5973	3.3595	3.0450	2.6470	1.5639
LN*100	3.9479	3.9137	3.8106	3.6371	3.3904	3.0665	2.6599	1.5658
conservative central bank is done for $\rho = 0.75; 1-\rho = 0.25$								
LMLconservative	3.9479	3.9129	3.8075	3.6306	3.3800	3.0526	2.6438	1.5534
LFLconservative	3.8523	3.8233	3.7346	3.5822	3.3597	3.0587	2.6694	1.5827
Loss of a representative household country H and F								
LHCoop*100	3.8078	3.6989	3.5296	3.2999	3.0097	2.6592	2.2482	1.2449
LHML*100	3.9479	3.9334	3.8441	3.6790	3.4358	3.1114	2.7011	1.5931
LHFL*100	3.8942	3.8829	3.8013	3.6473	3.4173	3.1060	2.7063	1.6046
LHN*100	3.9479	3.9352	3.8490	3.6878	3.4487	3.1278	2.7194	1.6065
LFCoop*100	3.8078	3.8562	3.8442	3.7719	3.6390	3.4458	3.1921	2.5036
LFML*100	3.9479	3.8878	3.7526	3.5407	3.2498	2.8762	2.4148	1.1966
LFFL*100	3.8942	3.8358	3.7067	3.5043	3.2247	2.8622	2.4098	1.1973
LFN*100	3.9479	3.8874	3.7530	3.5429	3.2542	2.8824	2.4216	1.1989
conservative central bank								
LHMLconservative	3.9479	3.9334	3.8441	3.6790	3.4358	3.1114	2.7011	1.5931
LHFLconservative	3.8523	3.8238	3.7391	3.5950	3.3839	3.0943	2.7130	1.6214
LFMLconservative	3.9479	3.8878	3.7526	3.5407	3.2498	2.8762	2.4148	1.1966
LFFLconservative	3.8523	3.8226	3.7279	3.5584	3.3033	2.9519	2.4948	1.2346

Table 5: Losses: union-wide, H and F - balanced-budget

Baseline: $\sigma=\psi$=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; $\alpha^H=\alpha^F$=0.75; $\lambda^H=\lambda^F$=1; $\rho_a=\rho_c=0.85$; B/4Y =60%								
Union-wide loss	$n^H = 0.5$	$n^H = 0.55$	$n^H = 0.6$	$n^H = 0.65$	$n^H = 0.7$	$n^H = 0.75$	$n^H = 0.8$	$n^H = 0.9$
LCoop*100	4.8950	4.8533	4.7280	4.5192	4.2269	3.8511	3.3917	2.2225
LML*100	5.0697	5.3056	5.2641	4.9940	4.5837	4.0676	3.4625	2.0293
LFL*100	5.3826	6.2510	5.8882	5.4233	4.8733	4.2497	3.5623	2.0405
LN*100	5.1264	5.9954	5.6737	5.2541	4.7494	4.1681	3.5174	2.0416
conservative central bank is done for $\rho=0.75; 1-\rho=0.25$								
LMLconservative	5.2812	5.4786	5.4105	5.1252	4.6957	4.1573	3.5302	2.0665
LFLconservative	5.2958	5.4233	5.3344	5.0430	4.6150	4.0824	3.4638	2.0228
Loss of a representative household country H and F								
LHCoop*100	4.8950	4.6041	4.2850	3.9378	3.5624	3.1589	2.7272	1.7795
LHML*100	5.0697	4.2078	3.8818	3.5961	3.2889	2.9435	2.5531	1.6287
LHFL*100	5.3826	3.8105	3.6292	3.4088	3.1435	2.8309	2.4701	1.6067
LHN*100	5.1264	3.7865	3.6356	3.4368	3.1862	2.8819	2.5232	1.6433
LFCoop*100	4.8950	5.1578	5.3925	5.5990	5.7774	5.9276	6.0497	6.2094
LFML*100	5.0697	6.6474	7.3375	7.5900	7.6049	7.4398	7.1002	5.6352
LFFL*100	5.3826	9.2339	9.2766	9.1647	8.9096	8.5062	7.9314	5.9452
LFN*100	5.1264	8.6952	8.7308	8.6292	8.3971	8.0270	7.4944	5.6260
conservative central bank								
LHMLconservative	5.2812	4.4176	4.0945	3.7840	3.4398	3.0539	2.6257	1.6530
LHFLconservative	5.2958	4.3213	3.9224	3.5913	3.2549	2.8940	2.5010	1.6053
LFMLconservative	5.2812	6.7754	7.3844	7.6158	7.6261	7.4675	7.1485	5.7886
LFFLconservative	5.2958	6.7701	7.4525	7.7389	7.7884	7.6477	7.3151	5.7806

Table 6: Losses: union-wide, H and F - debt = 0.6

Baseline: $\sigma=\psi$=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; $\alpha^H=\alpha^F$=0.75; $\lambda^H=\lambda^F$=1; $\rho_a=\rho_c=0.85$; B/4Y =80%								
Union-wide loss	n^H = 0.5	n^H = 0.55	n^H =0.6	n^H = 0.65	n^H = 0.7	n^H = 0.75	n^H = 0.8	n^H = 0.9
LCoop*100	4.8679	4.8259	4.7001	4.4903	4.1967	3.8191	3.3577	2.1831
LML*100	5.3503	6.2595	5.8606	5.3816	4.8305	4.2134	3.5360	2.0277
LFL*100	5.5226	6.4957	6.0542	5.5346	4.9448	4.2925	3.5855	2.0477
LN*100	5.3746	6.3835	5.9553	5.4529	4.8824	4.2496	3.5608	2.0478
conservative central bank is done for $\rho = 0.75; 1-\rho = 0.25$								
LMLconservative	5.4704	6.3186	5.9989	5.5383	4.9771	4.3356	3.6276	2.0585
LFLconservative	5.3986	5.7262	5.5260	5.1578	4.6777	4.1092	3.4665	2.0030
Loss of a representative household country H and F								
LHCoop*100	4.8679	4.5583	4.2243	3.8659	3.4830	3.0757	2.6440	1.7073
LHML*100	5.3503	3.6857	3.5485	3.3607	3.1195	2.8236	2.4721	1.6046
LHFL*100	5.5226	3.5372	3.4255	3.2599	3.0394	2.7638	2.4332	1.6103
LHN*100	5.3746	3.5620	3.4564	3.2947	3.0761	2.8002	2.4671	1.6313
LFCoop*100	4.8679	5.1530	5.4137	5.6500	5.8618	6.0492	6.2122	6.4649
LFML*100	5.3503	9.4052	9.3288	9.1349	8.8228	8.3830	7.7913	5.8363
LFFL*100	5.5226	10.1115	9.9973	9.7591	9.3908	8.8787	8.1951	5.9842
LFN*100	5.3746	9.8320	9.7036	9.4609	9.0970	8.5977	7.9352	5.7964
conservative central bank								
LHMLconservative	5.4704	4.2457	3.9919	3.7001	3.3658	2.9877	2.5662	1.6022
LHFLconservative	5.3986	4.0279	3.7292	3.4495	3.1499	2.8181	2.4483	1.5821
LFMLconservative	5.4704	8.8522	9.0093	8.9521	8.7366	8.3795	7.8736	6.1652
LFFLconservative	5.3986	7.8020	8.2211	8.3304	8.2426	7.9826	7.5392	5.7914

Table 7: Losses: union-wide, H and F - debt =0.8

Baseline: $\sigma=\psi$=0.4; ρ=4.5; θ=11; η=0.47; β=0.99; $\alpha^H=\alpha^F$=0.75; $\lambda^H=\lambda^F$=1; ρ_a= 0.85; n = 0.5; B/4Y=60%											
		a_t^H	a_t^F	μ_t^H	μ_t^F	$\overline{c}_t^w$	a_{t-1}^H	a_{t-1}^F	q_{t-1}	b_{t-1}^H	b_{t-1}^F
Conservative central bank											
	i_t	0.8456	0.8456	-0.2148	-0.2148	0.3618	-0.0000	0.0000	-0.0000	-0.7102	-0.7102
	g_t^H	0.3288	-0.1970	-0.0417	0.0082	0.0282	-0.1840	0.1840	-0.3297	-0.1377	0.0270
FL	τ_t^H	0.4267	1.0540	0.4307	-0.8069	0.3168	-2.4089	2.4089	-4.3150	1.4239	-2.6676
	g_t^F	-0.1970	0.3288	0.0082	-0.0417	0.0282	0.1840	-0.1840	0.3297	0.0270	-0.1377
	τ_t^F	1.0540	0.4267	-0.8069	0.4307	0.3168	2.4089	-2.4089	4.3150	-2.6676	1.4239
	i_t	0.8895	0.8895	-0.2260	-0.2260	0.3807	0.0000	-0.0000	0.0000	-0.7472	-0.7472
	g_t^H	0.3523	-0.2355	-0.0453	0.0156	0.0250	-0.2119	0.2119	-0.3796	-0.1497	0.0516
ML	τ_t^H	0.5041	1.3272	0.4082	-0.8735	0.3918	-2.3922	2.3922	-4.2853	1.3495	-2.8878
	g_t^F	-0.2355	0.3523	0.0156	-0.0453	0.0250	0.2119	-0.2119	0.3796	0.0516	-0.1497
	τ_t^F	1.3272	0.5041	-0.8735	0.4082	0.3918	2.3922	-2.3922	4.2853	-2.8878	1.3495

Table 8: Policy reaction functions - debt, nH = 0.5, conservative central bank

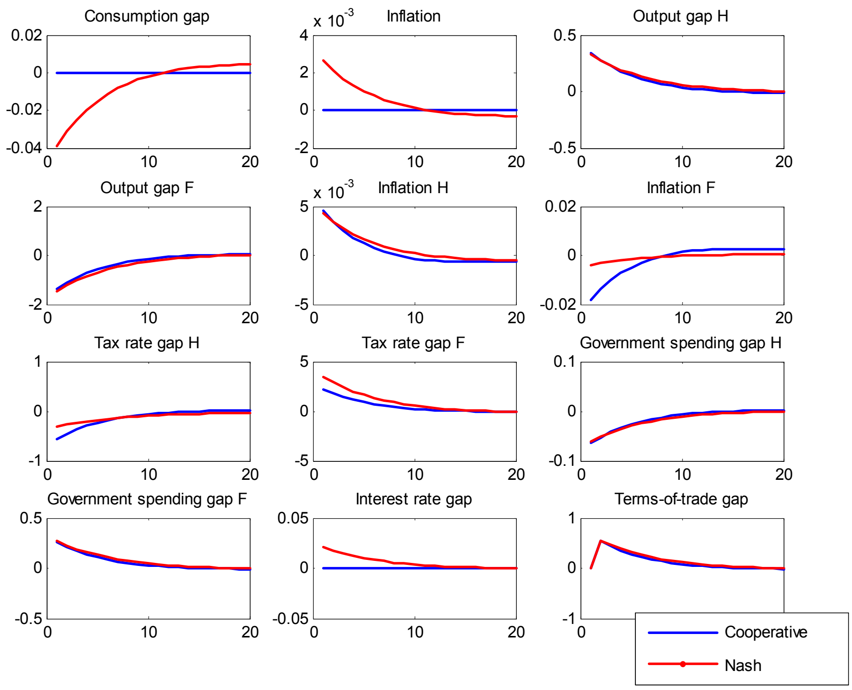

Figure 1: Responses to a 1% negative technology shock at H, nH = 0.8 - Cooperation vs Nash - Balanced-Budget

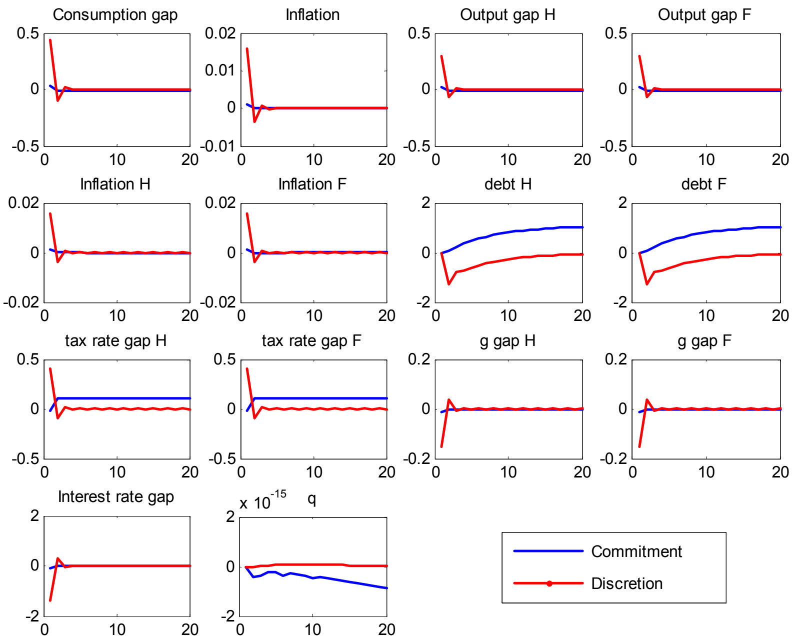

Figure 2: Responses to a 1% negative symmetric technology shock, nH = 0.5, Commitment vs Discretion - Debt

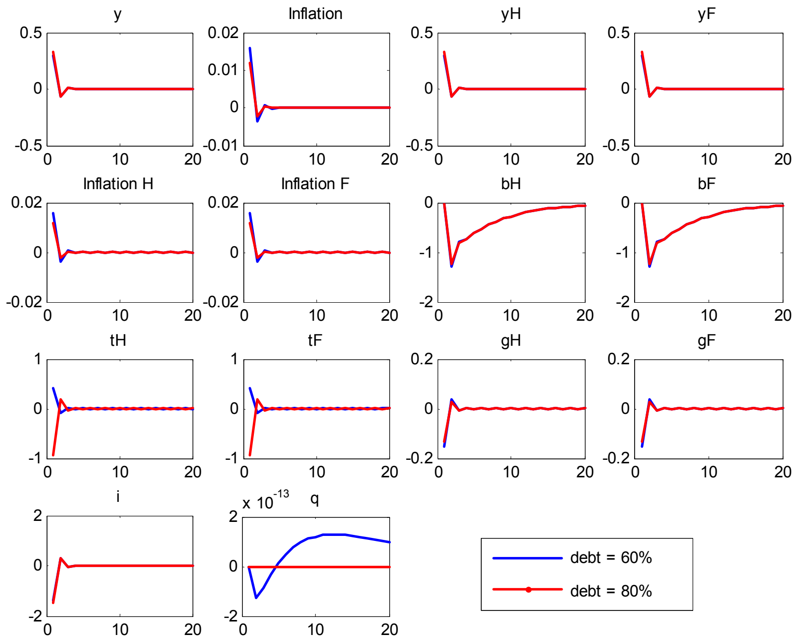

Figure 3: Responses to a 1% negative symmetric technology shock, nH = 0.5, Cooperative discretion

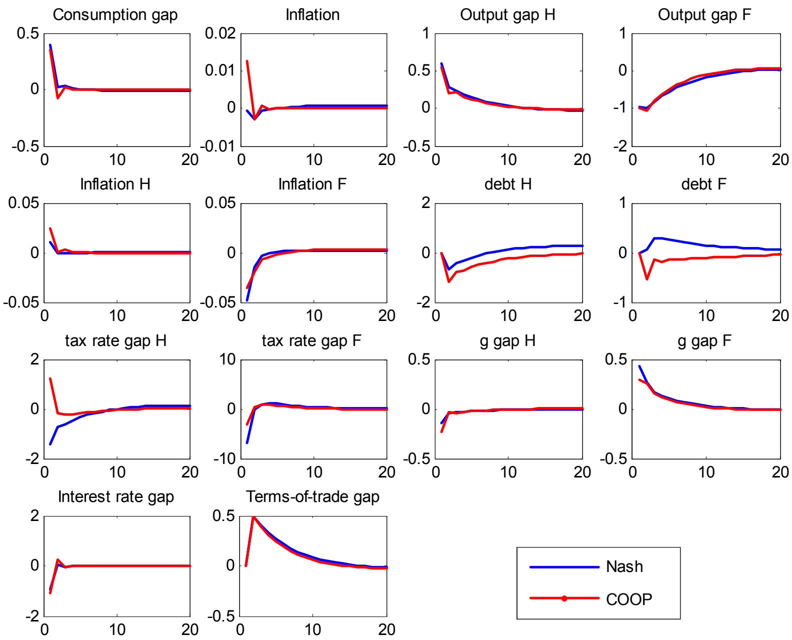

Figure 4: Responses to a 1% negative technology shock at H, nH = 0.8, Cooperative vs Nash - Debt