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'DAY NUMBER': A PROMOTER ROUTINE OF FLEXIBILITY AND CONCEPTUAL UNDERSTANDING

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This paper is part of the Project "Adaptive thinking and flexible computation: Critical issues". In this paper we discuss different perspectives of flexibility and adaptive thinking in literature. We also discuss the idea of proceptual thinking and how this idea is important in our perspective of adaptive thinking. The paper analyses a situation developed with a first grade classroom and its teacher named the day number. It is a daily activity at the beginning of the school day. It consists on to look for the date number and to think about different ways of writing it using the four arithmetic operations. The analyzed activity was developed on March 19, so the challenge was to write 19 in several ways. The data show the pupils' enthusiasm and their efforts to find different ways of writing the number. Some used large numbers and division, which they were just starting to learn. The pupils presented symbolic expressions of 19, decomposing and recomposing it in a flexible manner.

THEORETICAL FRAMEWORK

The relevance of adaptive expertise and flexibility became a topic under a strong debate in mathematics education since the reform movement in 1980's. There are several perspectives of flexibility. Star and Newton (2009) define it as knowing multiple solutions as well as the capacity to choose the most appropriated for a given problem. Other authors see the flexibility as the use of efficient strategies (Heirdsfield & Cooper, 2004). Threlfall (2009) reject these perspectives advocating that the strategy is not selected but emerges from a process that is not fully conscious or rational involving a connection between what the pupil notices about the specific features of the numbers presented in the proposed problem and what he knows about numbers and their relationships (*zeroing-in*). Baroody and Rosu (2006) refer that flexibility in calculation is related to the discovering of patterns and relations, as children develop number sense, thus building a network of relationships. Rechtsteiner-Merz and Rathgeb-Schnierer (2015) clarify that flexible mental calculation involves both flexibility (the ability to switch between different tools of solution) and adaptivity (the ability to select the most appropriate strategy). For them, "adaptivity in flexible mental calculation is related to recognition of problem characteristics, number patterns and numerical relationships" (p. 2).

In our project "Adaptive thinking and flexible computation: Critical issues", we adopt an integrative approach to develop flexibility in mental calculation (Brocardo, 2014), focusing on the relationship between conceptual knowledge and procedural one. In this approach, we integrate the views of Tall (2013) of operational symbolism, of Sfard (1991) of concept formation, and of Threlfall (2009) of zeroing-in behind calculation-strategy-flexibility.

We assume, as Sfard (1991), that processes and mathematical objects are two sides of the same coin. According to Tall (2013), it is essential to consider the cognitive combination of process and concept, proposing the *procept* construct as an amalgam of three components: (1) a *process*; (2) a mathematical *object* produced by the process; and (3) a *symbol* representative of either process or object (the same notation represents the duality of process and concept). Furthermore, the author refers to three types of abstraction: (1) *operational abstraction* focused on actions that become operations (actions on objects becoming thematised objects of thought); (2) *structural abstraction* focused on the structure of objects (properties of objects becoming thematised objects of thought); and (3) *formal abstraction* focused on definitions formulated linguistically (deducing from definitions to prove other properties to construct formal objects of thought). The operational abstraction and structural abstraction resonate to operational and structural conceptions formulated by Sfard (1991) and "build from perceptual ideas that become conceptualized as mathematical concepts" (Tall, 2013, p. 13).

The *proceptual thinking,* as the combination of conceptual and procedural thinking, includes the flexible way as the symbolism can be manipulated as process or object, that is to say, as procedural action or mental object that at a higher level may be manipulated, decomposed or recomposed. Moreover, for Tall (2013), "the symbols themselves may be seen not only as operations to be performed but also compressed into mental number concepts that can be manipulated in the mind" (p. 4). In the *compression* of knowledge, immediate conceptual links replace lengthy operations.

So the adaptive thinking involves the development of a flexible and relational understanding enabling the pupils to compress mathematical ideas into more flexible forms, making them simpler (Tall, 2013). In this perspective, what matters is the ability to produce new known facts from old ones, acting as an autonomous knowledge generator and not the ability to efficiently produce answers from a memory network as advocated by Baroody and Rosu (2006).

METHODOLOGY

This study follows a qualitative approach within an interpretive paradigm. It aims describing and interpreting an educational phenomenon (Erickson, 1986). It is part of the project "Adaptive thinking and flexible computation: Critical issues" being developed by the Schools of Education of Lisbon, Setúbal and Portalegre. The data for this paper was collected through participant observation of a first grade classroom and its teacher, which was videotaped and transcribed.

The 'Day number' is a routine in this class, which is developed every day, at the beginning of the day. It is conducted orally by the teacher interacting with all pupils. The challenge is to present different symbolic representations of the number of the day, in this case the 19, as the date was March 19. On the left side of the blackboard is affixed the hundred-square which often works as a calculation aid. Pupils are challenged to represent 19 in different ways using the four arithmetic operations. The teacher records on the blackboard and asks for explanations. When a pupil knows a different representation he/she puts his/her hand up and waits for his/her turn. This activity usually takes about 20 minutes. Pupils are engaged in presenting different symbolic expressions representing the number. The pupils' names are fictitious for ethical reasons.

RESULTS

The teacher starts the dialogue with the pupils doing the respective records on the blackboard.

Teacher: 19 is next to what number, Maria?

Maria: 20.

Teacher: Before or after of 20, António?

António: 10+10+1.

Teacher: What gives 10+10?

Pupils: 20.

Teacher: Renato, is 19 double or almost double?

Renato: Almost double.

Teacher: What is the double following 19?

Renato: 20.

These pupils had already worked with doubles and almost double of a number, teacher's questions serve to clarify the thinking and to enlarge the understanding of the situation. Pupils continue to express their solutions:

Lúcia: 3x5+4.

Dario: 38:2

Teacher: Why?

Dario: Because 19+19 is 38.

Teacher: How did you get 38?

Dario: Because 18+18 is 36.

Teacher: How did you get 19+19?

Dario: It´s +2.

(...)

Teacher: Look at the table [hundred square]. How do you do quickly 19+19, Daniel?

Dario: 19+10=29, coming down, 29+10=39, 39-1=38.

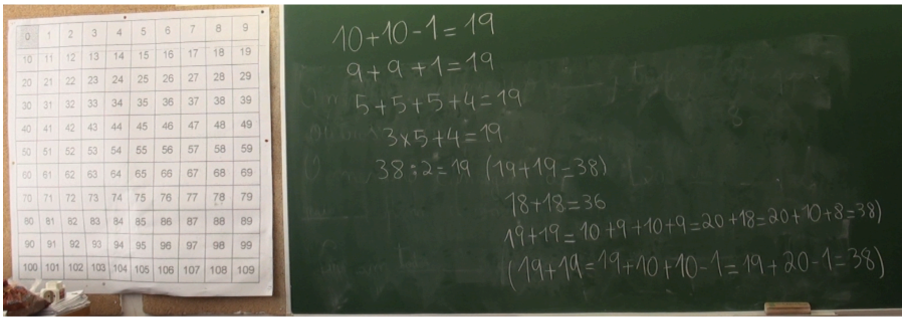

Figure 1: The hundred square and the records of expressions representing 19

The teacher continues the dialog writing on the board what the pupils say as for instance: 3x6+1=19, 10+5+4=19, 100:100+18=19, 2x9+1=19. A pupil said 1000:1000+18 =19 and another one, 2000:2000+18. They use large numbers for express a generalization: 1 is the result of dividing any number by itself. It should be noted that these pupils were beginning the study of multiplication and the division have only appeared informally. Despite this, they could apply appropriately the two operations and start to make generalizations. A pupil starts to do: 4+4+ , but hesitates, the teacher asks:

Teacher: Is 19 even or odd?

Pupils: Odd.

Teacher: What does happen when I do every four counts? Do I get an even or odd number? Let's count.

Pupils: 4, 8, 12, 16, 20

One pupil: So, 19 is odd, that is 4+4+4+3.

Again the teacher appeals to something pupils already knew and they established relationships.

FINAL REMARKS

The pupils think about numbers as mathematical objects without connection with life contexts. Here the context of date is just a motif for the work of generating multiple symbolic expressions of the number of the day. These expressions represent the number as an object but represent also the process of manipulating several numbers using the four arithmetic operations in order to get the given number (Sfard, 1991). The pupils use the operations in a related way. They represent the number using the multiplication and the division, which they were just starting to learn, from the additive structure. The procept 19 includes a collection of other representations that are obtained through different processes although representing the same object. Each symbolic expression operates both as a process (the involved operations) and a concept (the result of the operations) (Tall, 2013). The pupils seem to grasp more easily the idea of double, using it as a thinkable object in a flexible way to derive new relationships. Although they are pupils of 1st grade, they already show great flexibility in decomposition and recomposition of 19 (Threlfall, 2009).

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