A MULTIFACTOR APPROACH TO THE SOCIAL DISCOUNT RATE[1]

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Abstract

This work focuses on the appraisal of public and environmental projects and, more specifically, on the calculation of the social discount rate (SDR) for this kind of very long-term investment projects. As a rule, we can state that the instantaneous discount rate must be equal to the hazard rate of the public good or to the mortality rate of the population that the project is intended to. The hazard can be due to technical failures of the system, but, in this paper, we are going to consider different independent variables that can cause the hazard. That is, we are going to consider a multivariate hazard rate. In our empirical application, the Spanish forest surface will be the system and the forest fire will be the fail that can be caused by several factors. The aim of this work is to integrate the different variables that produce the fail in the calculation of the SDR from a multivariate hazard rate approach.

Key-words: social discount rate, multivariate hazard rate, forest fires, public and environmental projects.

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1. INTRODUCTION.

A hazard function describes mathematically the effect that increases in waiting time have on the risk that something will happen to prevent an event from occurring (Gross and Clark, 1975). In the framework of temporal discounting, the fail represents the probability of an event occurring at *t* (or during an interval starting at *t*) that will prevent the receipt of a reward, divided by the probability of the event not occurring until *t*, that is the conditioned probability of fail. In this paper, we are going to study the discounting function including the risk, considered as the hazard rate of a group (in general the reliability of a system). Consequently, we propose a discounting function for investment appraisal based on the system's reliability. So, the discounting function at *t* will be (Cruz-Rambaud and Muñoz-Torrecillas, 2005):

$$R(t) = 1 - F(t) = e^{-\int_0^t h(x)dx}, \tag{1}$$

where $R(t)$ is the system's reliability, $F(t)$ the distribution function, at instant 0, of the random useful life of the system, and $h(x)$ the instantaneous hazard rate at instant *x*, $0 \le x \le t$. In effect, let *T* be the random variable that represents the life of an investment in a public good, for example, a sewage treatment plant. Let us suppose that the distribution function of the random variable, $F(t) = P(T \le t)$ (with density function $f(t)$), represents the probability that the public good will stop working before *t* years after starting its useful life (time 0).

The *reliability of the system* at year *t*, $R(t)$, is the probability that the life of the system will be greater than *t*:

$$P(T > t) = 1 - F(t) = R(t),\ t > 0. \tag{2}$$

Therefore,

$$h(t) = \frac{f(t)}{R(t)} \tag{3}$$

represents the proportion of units that fail in the interval $(t, t+dt)$ with respect to the units that continue working at year *t*. This is the well-known concept of *hazard rate*, and it can be shown that:

$$h(t) = -\frac{R'(t)}{R(t)}, \tag{4}$$

from which we can obtain the expression of the system reliability, seen before in equation (1):

$$R(t) = \exp\left[-\int_0^t h(x)dx\right].$$

In general, if we make the hazard rate of the random variable *T*, defined in an interval of the form $[0,+\infty)$, equal to the instantaneous rate of a discounting function from *d* to $d+t$ ($t \geq 0$), say $A(d,t)$, we can deduce that (Cruz-Rambaud, 1995):

$$A(d,t) = \frac{1-F(d+t)}{1-F(d)} \ . \tag{5}$$

In the specific case in which *d* = 0, as *F*(0) = 0, it must be (see Maravall, 1970):

$$A(0,t) =: A(t) = 1 - F(t) = R(t), \ t \geq 0 \,. \tag{6}$$

Our findings can be also derived from Gollier (2002a, 2002b). See Cruz-Rambaud and Muñoz-Torrecillas (2005) for a complete demonstration. Observe that the discounting function can be composed by several components. In effect, assume that, in the expression of a discounting function $A(d,t)$, *n* causes are implicit (among them, the hazard). This makes that, at first, *A* is a function of *n* functions denoted by $A_1, A_2, \ldots, A_n$, that is:

$$F = \Psi(A_1, A_2, \ldots, A_n).$$

However, this general treatment of the hazard can be very difficult, whereby we are going to adopt the following simplifying assumption: any discounting function can be decomposed into the product of the discounting function derived from the hazard rate and the discounting function due to the remainder causes. Therefore,

$$A(d,t) = \hat{A}(d,t) \cdot \tilde{A}(d,t),$$

where:

- $\hat{A}(d,t)$ represents the discounting function derived from the hazard rate of the system, and
- $\tilde{A}(d,t)$ denotes the discounting function due to the rest of causes involved in the valuation process.

Nevertheless, in this paper, we only consider the problem of determine the discounting function $\hat{A}(d,t)$, leaving the study of the influence of the rest of causes to a future research. Thus, the next step in our research will be to consider the hazard rate of all components in a system, and to study the way in which all of them can be joined in only one hazard rate representing the entire system. To do this, we are going to start with the concept of multivariate hazard rate.

Following Navarro (2008), if (T_1, T_2) is a random vector (usually representing the lifetime of two units or two components in a system) with absolutely continuous reliability function defined by $R(t_1,t_2) = \Pr(T_1 > t_1, T_2 > t_2)$ and density function $f(t_1,t_2)$, the univariate failure rate (or hazard rate) can be extended to the bivariate set-up by using different ways.

The first one uses the *bivariate failure rate function* defined in Basu (1971)[1] by:

$$h(t_1,t_2) = \frac{f(t_1,t_2)}{R(t_1,t_2)}, \tag{7}$$

for all (t_1,t_2), such that $R(t_1,t_2) > 0$.

We can also extend it to the case of a multivariate failure rate function in the following way:

[1] Puri and Rubin (1974) defined a multivariate hazard rate in a similar way.

$$h(t_1,t_2,\dots,t_n)=\frac{f(t_1,t_2,\dots,t_n)}{R(t_1,t_2,\dots,t_n)}, \tag{8}$$

for all $(t_1,t_2,\dots,t_n)$, such that $R(t_1,t_2,\dots,t_n)>0$.

The second option is to use the *hazard (failure) gradient* defined by Johnson and Kotz (1975), Barlow and Proschan (1976) and Marshall (1975) as:

$$h(t_1,t_2)=(h_1(t_1,t_2),h_2(t_1,t_2))=-\mathrm{grad}\ln R(t_1,t_2),$$

where

$$h_i(t_1,t_2)=-\frac{\partial}{\partial t_i}\ln R(t_1,t_2), \tag{9}$$

for $i=1,2$ and for all (t_1,t_2), such that $R(t_1,t_2)>0$.

From a multivariate approach, we will have:

$$h(t_1,t_2,\dots,t_n)=(h_1(t_1,t_2,\dots,t_n),h_2(t_1,t_2,\dots,t_n),\dots,h_n(t_1,t_2,\dots,t_n))$$

$$=-\mathrm{grad}\ln R(t_1,t_2,\dots,t_n),$$

where

$$h_i(t_1,t_2,\dots,t_n)=-\frac{\partial}{\partial t_i}\ln R(t_1,t_2,\dots,t_n), \tag{10}$$

for $i=1,2,\dots,n$ and for all $(t_1,t_2,\dots,t_n)$, such that $R(t_1,t_2,\dots,t_n)>0$.

Johnson and Kotz (1975) introduced the concept of a vector multivariate hazard rate and Marshall (1975) showed that this vector function determines the multivariate failure distribution uniquely and is the natural generalization of the corresponding univariate concept. In this way, some recent characterizations using the hazard gradient were given by Navarro and Ruiz (2004), Kotz *et al*. (2007) and Navarro *et al*. (2007).

Johnson and Kotz (1975) state that: “For a concept such as “hazard rate”, it is unreasonable to expect a single value to represent this aspect of a multivariate distribution. The basic idea underlying the univariate definition is that of rate of decrease in “survivors” with increase in value (t) of T (as in a life table where the hazard rate is in fact the force of mortality). When there are two or more variates this rate depends on which variate is changed (or, more generally, the proportions in which different variates are changed) and we need a different "rate" for each variate." So they defined the (joint) multivariate hazard rate of m absolutely continuous random variables $T_1, T_2, \ldots, T_m$ as a vector, called the *vector* (or *gradient*) *multivariate increasing/ decreasing hazard rate* (*IHR*/*DHR*).

However, our aim in this paper is to aggregate all the components of a multivariate hazard rate in a unique hazard rate describing the whole system. Thus, the organization of the paper is the following. After justifying the mathematical expression of the discounting function associated to the hazard rate of a system or population, in Section 2 the formula of the discounting function associated to the weighted mean of n hazard rates due to several causes is deduced. Section 3 describes the fitting process of data coming from empirical data to a theoretical probability distribution, more specifically, a Weibull distribution. In Section 4 we apply the results obtained in Section 3 to the data provided by the *Ministerio de Medio Ambiente y Medio Rural y Marino* which includes the environmental Spanish agency, obtaining the corresponding empirical discounting function. Finally, Section 5 summarizes and concludes.

2. MULTIVARIATE HAZARD RATE AND DISCOUNTING.

In the Introduction we have described the attempts of several authors to define a multivariate hazard rate, but we are looking for a single value of the hazard rate representing the entire system, not a vector. To do this, we are going to consider the approach of Barlow and Proschan (1996) for structures of nonidentical components. More specifically, we are going to focus on the particular case when the system's hazard rate is the weighted average of the hazard rates of the n system's components, that is:

$$h = \alpha_1 \cdot h_1 + \alpha_2 \cdot h_2 + \cdots + \alpha_n \cdot h_n = \sum_{k=1}^{n} \alpha_k \cdot h_k \,. \tag{11}$$

Taking into account that the general structure of the system's fail has the following form (Barlow and Proschan, 1996):

$$h(t) = \sum_{k=1}^{n} \frac{R_k \cdot \partial R / \partial R_k}{R} \cdot h_k(t), \tag{12}$$

we can deduce that:

$$R_k \frac{\partial R / \partial R_k}{R} = \alpha_k ;\ k = 1,2,\ldots,n . \tag{13}$$

We will find the expression of *h* by induction over *k*. This way, for $k = 1$,

$$R_1 \frac{\partial R / \partial R_1}{R} = \alpha_1 ,$$

or equivalently,

$$\frac{\partial R / \partial R_1}{R} = \frac{\alpha_1}{R_1} .$$

By integrating both members with respect to R_1, we will have:

$$\ln R + \Phi_{n-1}(R_2, R_3, \ldots, R_n) = \alpha_1 \cdot \ln R_1 .$$

To determine $\Phi_{n-1}(R_2, \ldots, R_n)$, we can differentiate the previous equality with respect to R_2, resulting in:

$$\frac{\partial R / \partial R_2}{R} + \frac{\partial \Phi_{n-1}(R_2, R_3, \ldots, R_n)}{\partial R_2} = 0 .$$

As $\frac{\partial R / \partial R_2}{R} = \frac{\alpha_2}{R_2}$, if we substitute in the previous formula, we will have:

$$\frac{\partial \Phi_{n-1}(R_2, R_3, \ldots, R_n)}{\partial R_2} = -\frac{\alpha_2}{R_2} ,$$

from where:

$$\Phi_{n-1}(R_2, R_3, \ldots, R_n) = -\alpha_2 \cdot \ln R_2 + \Phi_{n-2}(R_3, \ldots, R_n) ,$$

that is,

$$\ln R + \Phi_{n-2}(R_3, \ldots, R_n) = \alpha_1 \cdot \ln R_1 + \alpha_2 \cdot \ln R_2 .$$

Le us suppose now that ($k < n$):

$$\ln R + \Phi_{n-(k-1)}(R_k, \ldots, R_n) = \sum_{i=1}^{k-1} \alpha_i \cdot \ln R_i \ .$$

By a similar process, we can demonstrate that:

$$\ln R + \Phi_{n-k}(R_{k+1}, \ldots, R_n) = \sum_{i=1}^{k} \alpha_i \cdot \ln R_i \ ,$$

therefore we can finally deduce that:

$$\ln R + \Phi = \sum_{k=1}^{n} \alpha_k \cdot \ln R_k \ ,$$

where Φ is a constant. Obviously, for $t = 0$, $R_k = 0$, for every *k*, whereby $\Phi = 0$ and

$$R = R_1^{\alpha_1} \cdot R_2^{\alpha_2} \cdots R_n^{\alpha_n} = \prod_{k=1}^{n} R_k^{\alpha_k} \ . \qquad (14)$$

Observe that, in this particular case, last expression can be directly derived from equation (1) by simple integration of equation (11).

3. FITTING DATA TO A THEORETICAL DISTRIBUTION.

In this section, we will use a Weibull distribution whose cumulative distribution function takes the following form:

$$F(t) = 1 - e^{-a(t-\mu)^b} \ , \qquad (15)$$

where $t \geq \mu$, $a > 0$ and $b > 0$. In order to apply the ordinary least squares (OLS) method, we are going to transform this equation into a linear relation, taking Napierian logarithms twice:

$$1 - F(t) = e^{-a(t-\mu)^b} \ ,$$

$$\ln[1 - F(t)] = -a(t-\mu)^b \ ,$$

$$\ln \frac{1}{1 - F(t)} = a(t-\mu)^b \ ,$$

$$\ln \ln \frac{1}{1-F(t)} = \ln a + b \ln(t-\mu) . \tag{16}$$

More specifically, if we have the experimental data of forest fires, in percentage terms:

$$(t_1, p_1), (t_2, p_2), \ldots, (t_n, p_n) ,$$

we can build the experimental values of the distribution function:

$$(t_1, F^*(t_1)), (t_2, F^*(t_2)), \ldots, (t_n, F^*(t_n)) ,$$

where:

- $F^*(t_1) = p_1$,
- $F^*(t_2) = p_1 + p_2$,

  $\vdots$

- $F^*(t_n) = p_1 + p_2 + \cdots + p_n$.

To apply this methodology, it is necessary to be sure that we have calculated all percentages $p_1, p_2, \ldots, p_n$ over the same initial number of hectares, that is, that p_k is the percentage of burnt hectares in the interval $]t_{k-1}, t_k]$, not over the number of hectares that were unburned at t_{k-1}, but over the hectares initially available in 0. We could have also the percentages over the hectares that remain without burning at the end of the previous year. In this case, these percentages will directly represent the hazard rates that, in the case of the Weibull, are represented by the following expression:

$$h(t) = ab(t-\mu)^{b-1} , \tag{17}$$

from where, taking Napierian logarithms,

$$\ln h(t) = \ln(ab) + (b-1) \ln(t-\mu) .$$

Moreover, it is not a problem that afforestation has occurred in the forest extensions under consideration. Following with the solution of the initial problem, we will make the regression with a linear function:

$$\ln \ln \frac{1}{1-F(t)} = \ln a + b \ln(t-\mu) . \qquad (18)$$

For this purpose, we will make the following changes of variables:

- $\ln \ln \frac{1}{1-F(t)} = y$,
- $\ln(t-\mu) = x$, and
- $\ln a = A$,

resulting in:

$$y = A + bx ,$$

where we have to find *A* and *b*. This way, once we have calculated the values of *x* and *y*:

X	y
$\ln(t_1-\mu)$	$\ln \ln \frac{1}{1-F^*(t_1)}$
$\ln(t_2-\mu)$	$\ln \ln \frac{1}{1-F^*(t_2)}$
$\vdots$	$\vdots$
$\ln(t_n-\mu)$	$\ln \ln \frac{1}{1-F^*(t_n)}$

we will be able to obtain a straight line from the regression:

$$y = A^* + b^* x ,$$

from where we can obtain a^*:

$$\ln a^* = A^* \Rightarrow a^* = e^{A^*} .$$

When there are several causes, it is necessary to fit each one prior to applying the formula.

4. EMPIRICAL APPLICATION.

In order to apply our model, we have considered the Spanish forest surface as the system and the forest fire as the fail of the system. To do this, taking into account that the fail can be produced by different causes, we have used a multivariate hazard rate. The *Ministerio de Medio Ambiente y Medio Rural y Marino* is the Spanish Environmental Department which publishes the forest fires data every year, including the burnt surface due to different causes. Five causes have been identified:

- lightning (bolt of lightning),
- negligence and other accidental causes,
- deliberate (arson),
- unknown, and
- reproduction.

The information provided by the *Ministerio de Medio Ambiente y Medio Rural y Marino* have been divided into sure and estimated causes. The identification of each of these causes refers to the determination of the agent who originates the fire. Depending on the type of reason, this agent can be either a person, in the case of deliberate fires or arson, or an object, like a machine or a tool, etc. The percentage of identified agents with regard to the whole of events is variable and depends principally on the type of reason. This way, for example, for the disasters originated by lightning, the fact of managing to determine with certainty the reason, implies the identification of the cause (the own bolt of lightning).

In general, there is a bigger percentage of agents identified when the reasons are associated to negligence or accidents than to deliberate reasons. Thus, the deliberate fires supposed in 2008 45.26% of the fires, burning 60.39% of the forest surface, followed by the fires caused by negligence or accidents with 35.41% of the total number of fires. Regarding deliberate fires, among the most frequent motivations, we find traditional agricultural burnings, pasture renewal and slash burning. These traditional practices are generally carried out inadequately, without administrative authorization and in periods of high risk of fire.

The following table reflects surfaces affected by forest fires in Spain in 2008 classified by reasons:

CAUSE	**NUMBER OF FIRES** Type of cause			**TOTAL FORESTAL SURFACE (Hectares)**
	Sure	**Supposed**	**Total**	
Lightning (bolt of lightning)	347	24	371	362,71
Negligence and other accidental causes	2.607	1521	4.128	16.091,59
Deliberate (arson)	1.871	3404	5.275	30.389,17
Unknown	1.789	0	1.789	3.318,44
Reproduction	61	32	93	159,43
Total	**6.675**	**4.981**	**11.656**	**50.321,34**

Table 1. Surfaces affected by forest fires in Spain in 2008 classified by reasons. Source: *Ministerio de Medio Ambiente y Medio Rural y Marino*

We could weight these causes based on, for example, the government budget to prevent each of them. But, currently, we do not have the data of the Spanish government budget to prevent forest fires, detailed by cause. So, we will have to make some assumptions in order to choose the weighting coefficients (α_k) for the corresponding hazard rates:

1. Let us weight the causes due to natural reasons, that is lightning and reproduction, with $\alpha_k = 1$. So, $\alpha_1 = 1$ and $\alpha_5 = 1$.

2. Assume that increasing the advertising campaigns to prevent fires caused by negligence and other accidental causes, can result in a 5% decrease in the fires originated by this cause. Therefore, $\alpha_2 = 1 - 0.05 = 0.95$.

3. Let us assume that increasing the advertising campaigns to prevent deliberated fires and also the efforts in pursuing and punishing this kind of actions, can result in a 10% decrease in the fires originated by this cause. Therefore, $\alpha_3 = 1 - 0.1 = 0.9$.

4. Finally, suppose that increasing the budget for research on forest fire causes, we could decrease a 3% the unknown causes. So, $\alpha_4 = 1 - 0.03 = 0.97$.

To calculate the hazard rate, we have used the data of forest fires in Spain in the period from 1998 to 2008. More specifically, we have taken the data[2] of burnt forest surface (measured in hectares) by cause over the total forest surface.

The total forest surface is given by the Third National Forest Inventory (*Inventario Forestal Nacional*: IFN). The IFN is a research work repeated every ten years. It was established as an indispensable tool to adequately know the structure and the forests dynamics in order to take the necessary actions to handle and conserve them. The Third National Forest Inventory corresponds to the period 1997-2007.

Next, we are going to explain the methodology used to obtain the discount function from the hazard rate. We are going to study the fitting of the forest fires data for Spain from 1998 to 2008, using a Weibull distribution, as explained in Section 3.

In the following table we show in detail the calculations for cause 1. First of all, we will make the regression from data of burnt forest surface (measured in hectares), over the total forest surface.

Year t_i	% Burnt surface	p_i	$F^*(t_i)$	$x_i^* = \ln t_i$	$y_i^* = \ln\ln\dfrac{1}{1-F^*(t_i)}$
1998	0.009474	0.000095	0.000095	7.599902	–9.264307
1999	0.005958	0.000060	0.000154	7.600402	–8.776363
2000	0.015559	0.000156	0.000310	7.600902	–8.079055
2001	0.034541	0.000345	0.000655	7.601402	–7.330055
2002	0.012326	0.000123	0.000779	7.601902	–7.157641
2003	0.111520	0.001115	0.001894	7.602401	–6.268230
2004	0.011823	0.000118	0.002012	7.602900	–6.207611
2005	0.048811	0.000488	0.002500	7.603399	–5.990163
2006	0.018307	0.000183	0.002683	7.603898	–5.919405
2007	0.009919	0.000099	0.002782	7.604396	–5.883055
2008	0.001396	0.000014	0.002796	7.604894	–5.878044

Table 2. Regression data.

[2] Source: Website of the *Ministerio de Medio Ambiente y Medio Rural y Marino*: http://www.mma.es/portal/secciones/biodiversidad/defensa_incendios/estadisticas_incendios/index.htm

Plotting the concrete couples:

$$(x_1^*, y_1^*), (x_2^*, y_2^*), \ldots, (x_n^*, y_n^*),$$

we can observe the existence of a vertical asymptote for $x = \ln 1997$, which theoretically can be justified as follows:

$$\lim_{t \to 1997^-} = \ln \ln \frac{1}{1 - F(t)} = -\infty .$$

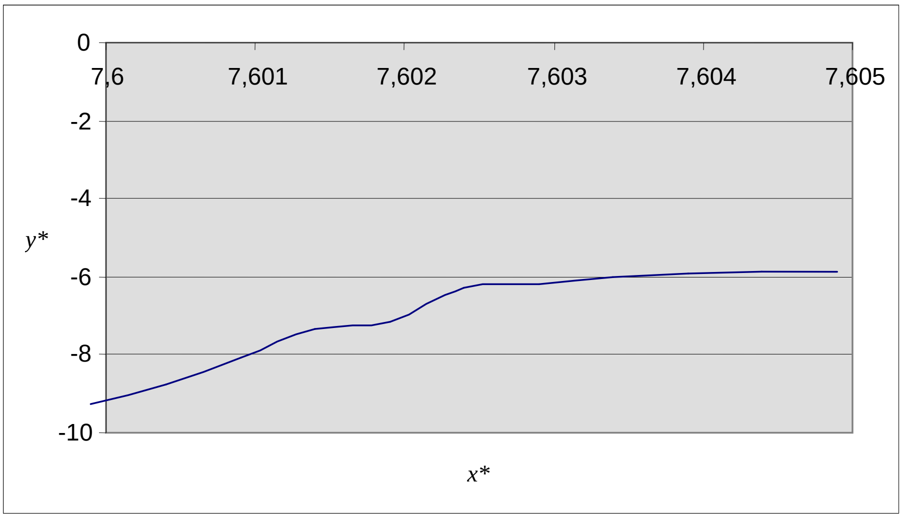

Figure 1. Representation of the regression data.

what lead us to take $\mu = 1997$ and make a new regression over $\ln(t - 1997)$ values:

$\ln(t - 1997)$	$\ln \ln \dfrac{1}{1 - F^*(t_i)}$
0	–9.264307
0.693147	–8.776363
1.098612	–8.079055
1.386294	–7.330055
1.609438	–7.157641
1.791759	–6.268230
1.945910	–6.207611
2.079442	–5.990163
2.197225	–5.919405
2.302585	–5.883055

2.397895	–5.878044

Table 3. Data of the regression over the log values.

obtaining:

$$y = 1.6337\,x - 9.577,$$

with a coefficient of determination R^2 = 96.01%:

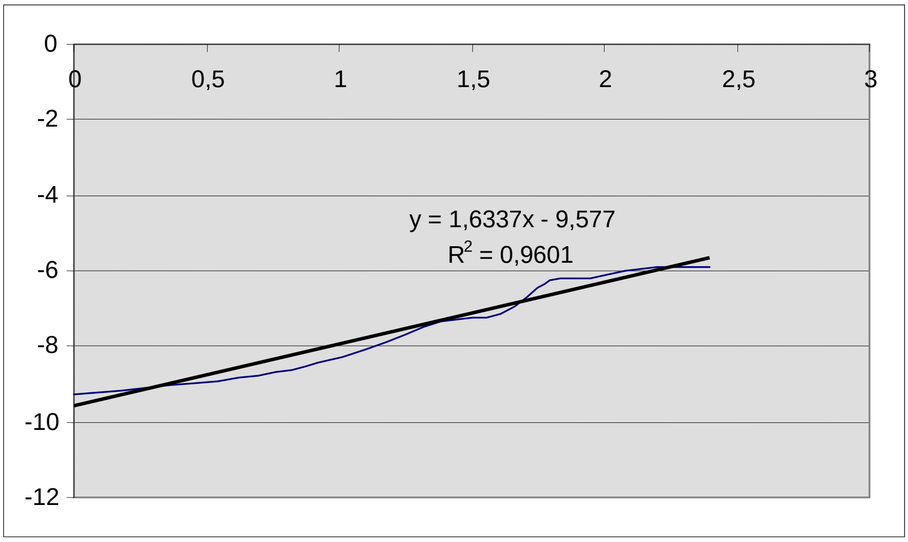

Figure 2. Graphic of the new regression.

Thus,

$$a_1^* = e^{-9.577} = 0.0000693045$$

and

$$b_1^* = 1.6337\,.$$

Therefore,

$$\hat{A}_1(t) = \hat{R}_1(t) = e^{-0.0000693045(t-1997)^{1.6337}}\,.$$

With the rest of the causes, we will proceed similarly. We summarize the results in the following table:

	Cause 1	Cause 2	Cause 3	Cause 4	Cause 5
α	1	0.95	0.9	0.97	1
Reg.	$y = 1.6337x - 9.577$	$y = 0.941x - 6.6082$	$y = 1.0294x - 5.9046$	$y = 1.0901x - 7.7352$	$y = 0,.8578x - 9.2696$
a^*	6.93045E-05	0.001349259	0.00272769	0.000437165	9.42462E-05
b^*	1.6337	0.941	1.0294	1.0901	0.8578

Table 4. Results of the regression over each cause.

Finally, using formula (14), we can obtain the mathematical expression of the discounting function including the effects of the five fire causes:

$$A(t,1997) = \left[\hat{R}_1(t)\right]^1 \cdot \left[\hat{R}_2(t)\right]^{0.95} \cdot \left[\hat{R}_3(t)\right]^{0.9} \cdot \left[\hat{R}_4(t)\right]^{0.97} \cdot \left[\hat{R}_5(t)\right]^1 =$$

$$= \mathrm{EXP}\Big[-0.00006930(t-1997)^{1.6337} - \left(0.00134926(t-1997)^{0.941}\right)^{0.95} - \left(0.00272769(t-1997)^{1.0294}\right)^{0.9}$$

$$- \left(0.00043717(t-1997)^{1.0901}\right)^{0.97} - 0.00009425(t-1997)^{0.8578}\Big] .$$

Starting from this formula, we can calculate the discounting function and the corresponding annual discount rate for a certain period of time, for example 20 years (see Table 5).

Year	Discounting function	Discount rate (%)
1998	0.99568501	0.43336850
1999	0.99127729	0.44465081
2000	0.98681231	0.45246544
2001	0.98230459	0.45889161
2002	0.97776280	0.46450882
2003	0.97319290	0.46957842
2004	0.96859936	0.47424521
2005	0.96398573	0.47859938
2006	0.95935491	0.48270167
2007	0.95470934	0.48659529

2008	0.95005112	0.49031227
2009	0.94538209	0.49387716
2010	0.94070389	0.49730928
2011	0.93601795	0.50062411
2012	0.93132562	0.50383434
2013	0.92662807	0.50695050
2014	0.92192642	0.50998144
2015	0.91722167	0.51293470
2016	0.91251476	0.51581674
2017	0.90780658	0.51863318
2018	0.90309793	0.52138893

Table 5. Discounting functions and discount rates associated to different delays.

5. Conclusion.

Observe that the hazard rate of the random variable T is slightly increasing, which can be interpreted as a slight annual increase in the number of burnt hectares over the remaining ones. This leads to an increasing instantaneous rate of discount associated to the discounting function (which coincides with the hazard rate of the random variable T). Therefore, the more distant the cash-flows are, the higher the average discount rate is. We can interpret this result as an increasing risk of forest fires over time, which implies that the present value of the profits performed by the public good decreases as they are situated in a more distant future, resulting in an increasing average annual discount rate.

There is a wide range of economic literature on the constant versus variable discount rate. Our work is positioned in the school of thought favorable to a variable discount rate over the investment horizon, as observed in the results from the empirical application. Many authors, as Harvey (1986), state that the discount rate for long-term projects must be decreasing. We agree with them, although in our case this should occur after an initial period. The policies of prevention and public awareness will reverse the evolution of the hazard rate, leading to a decreasing hazard rate in a very long-term delay. That is to say, for a certain investment, the discount rate could be increasing in the early period mainly due to an inadequate conservation policy or to a lack of adaptation in the first years of the investment, etc.

Our hazard rate approach to calculate the discount rate responds to the consideration of the risk associated to the waiting time until obtaining the reward (or cash flows' maturities). This risk can be considered as the hazard of the system in which we will invest, the mortality of the population that will enjoy the investment and, even, the decrease in the income's marginal utility. Nevertheless, we have focused on the risk as the hazard of the investment project. The justification of our approach is that the instantaneous discount rate to discount the future cash flows must be equal to the instantaneous hazard rate of the system (the investment). As it can be understood, it is very difficult to support a constant hazard rate hypothesis that will imply to assume a constant risk associated to the obtaining of the investment's future cash flows over time.

6. References.

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