# Tools for testing the Solvency Capital Requirement for life insurance

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**Abstract.** Longevity risk is one of the major risks that an insurance company or a pension fund has to deal with and it is expected that its importance will grow in the near future. In agreement with these considerations, in Solvency II regulation the Standard formula furnished for calculating the Solvency Capital Requirement explicitly considers this kind of risk. According to the new European rules in our paper we suggest a multiperiod approach to evaluate the SCR for longevity risk. We propose a backtesting framework for measuring the consistency of SCR calculations for life insurance policies.

Key words: Longevity risk, Solvency Capital Requirements, Life annuity, Beck test.

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## 1. INTRODUCTION

The determination of capital requirements represents the first Pillar of Solvency II. In this framework the Solvency Capital Requirement (SCR) is defined as the amount of capital that an insurer needs in order to remain viable in the market and maintain its default probability below a certain level. The main purpose of the new solvency regulation is to obtain a more realistic modelling and assessment of the different risks insurance companies are exposed to. According to this regulation the SCR calculation could rely on a standard formula, full internal models or partial internal models coupled with some parts of the standard model.

The basic principal is that the SCR will be determined as the 99.5% Value at risk (VaR) of the Available Capital over one-year time horizon.

Insurance companies are encouraged to enforce (stochastic) internal models since they should provide a more accurate assessment of the insurance risks. Unfortunately such models are rather expensive and sophisticated, therefore small and medium-size companies could prefer to rely on the standard model, even if also larger companies could prefer to implement a few modules for their (partial) internal models. For these reasons the European Commission has furnished a standard model that insurance companies are allowed to use for approximating the capital requirements. This Standard model is based on a modular approach: the overall risk is split into several risks (modules) for each of them risk sub-modules are considered. Modules' and sub-modules' SCRs are computed separately and then aggregated according to a pre-specified correlation matrices.

The European Commission for calibrating this standard model has recently published the Technical Specification of 5$^{TH}$ Quantitative Impact Study (QIS5) which maybe represents the last oppoutunity for insurance company for evaluating the capital amount necessary to satisfy the new solvency regulations.

Longevity risk, i.e. the risk that the trend of longevity improvements significantly change in the future, is one of the main risks insurers or pension funds providers have to front. Whereas in most industrialized countries the fall in benefits from public pay as you go pension schemes, and in general the uncertainty in the public pension systems, it is expected that the relevance of the longevity phenomenon will increase in the next future. In agreement with these considerations, longevity risk is explicitly considered in Solvency II standard formula representing a sub-module of the life underwriting risk module.

In this paper we refer exactly to the sub-module of longevity risk and we suggest a multiperiod forward approach, that is, we estimate at issue time the solvency adequacy along the overall portfolio contract duration. We propose a backtesting framework for measuring the consistency of SCR calculations for life insurance policies. In particular to evaluate the performances of the SCR calculation methodologies we quantify the convergence of SCR forecasts through the time. Finally graphical analysis and numerical evidences are provided.

A wide literature has been recently interested in these issues. Some authors analyzed capital requirement for certain portfolios but considering approaches different from the 1-year 99.5% VaR of Solvency II, for example Hary et al (2008) and Olivieri and Pitacco (2008). Others considered the impact and the significance of longevity risk on annuity or pension fund portfolios but they did not relate to capital requirement under a given solvency regime. More recently (2010) Börger analyzed the adequacy of the longevity shock specified in QIS4 standard formula comparing the resulting capital requirement to the VaR based on a stochastic mortality model. He found structural shortcomings and proposed a modified longevity shock for Solvency II standard model.

The paper is organised as follows: in section 2 we investigate the longevity phenomenon, in section 3 we discuss the SCR calculation in a multiperiod forward approach subject to the QIS5 guidelines, in section 4 we propose a backtesting framework for measuring the consistency of SCR calculations for life insurance policies.

## 2. LONGEVITY RISK

It is very challenging to capture the tendency of the future mortality pattern, in particular at retirement ages when the rectangularization phenomenon and the random marked fluctuations are combined.

The risk connected to the mortality trend comes out in different ways.

As concerns the former, one individual may live longer than the average lifetime in the reference population. It corresponds to possible deviations around expected mortality rates. It is related to the individual position and it becomes negligible in respect of the large portfolios because of the pooling effect.

As concerns the latter, the average lifetime of a population may differ from what it is expected. It refers to the deviations from expected values, rather than around them. It reveals its systematic nature. This component matches up with *longevity risk* and it considers the aggregate mortality phenomenon. Its effect may be significant if referred to portfolios of long duration life contracts such as pension annuities, characterized by a multiplicity of payments. Therefore the correct assessment of the longevity risk firstly involves a stochastic representation of mortality for measuring the possible impact on the future payments and on annual outflows. Risk management tools for dealing with longevity risk include reinsurance arrangements and alternative risk transfers as securitization and in particular mortality-linked securities.

From the insurer point of view, the adoption of internal models addressing the longevity risk is needed to operate an appropriate capital allocation policies.

## 3. SOLVENCY CAPITAL REQUIREMENT FOR LONGEVITY RISK: THE STANDARD FORMULA.

The SCR calculation according to Solvency II standard formula is based on a modular approach which allows to obtain the SCR summing the Capital Requirement for operational risk ($SCR_{op}$) and Adjustment for the risk absorbing effect of technical provisions and deffered taxes ($SCR_{Adj}$) to the BSCR (Basic SCR).

The BSCR is the Solvency Capital Requirements before any adjustment and it is computed combining, on the basis of a pre-specified correlation matrix *Corr*, capital requirement for six main risk categories (modules): Market risk, Health underwriting risk, Default risk, Life underwriting risk, Non- life underwriting risk, Intangible assets risk, so it follows:

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} SCR_i SCR_j} + SCR_{\text{int}\,angibles} \qquad (1)$$

where:

$Corr_{i,j}$ = item set out in row i and column j of the correlation matrix

$SCR_i, SCR_j$ = capital requirements for the individual SCR risks according to the rows and coloumns of the correlation matrix.

Each modules listed above consists of several sub-modules whose corresponding SCRs are calculated aggregating the sub-modules' SCRs according to a given correlation matrix.

Given the purpose of this paper, we focus on the longevity risk representing a specific sub-module of *Life underwriting risk* module. It covers the risk of losses or adverse changes in value of insurance liabilities resulting from changes in level, in the trend or in the volatility of mortality rates, where a decrease in death rate lead to an increase in value of the insurer's liabilities. According to Solvency II standard formula capital charge for longevity risk ($SCR_{long}$) should be calculated as the change in Net Assets Value (NAV) due to a longevity shock under a specific survival scenario at time t=0. Hence we have:

$$SCR_{long} = \left(\Delta NAV \middle| longevity\ shock\right) \qquad (2)$$

The longevity shock is represented by a 20% permanent reduction of the mortality rates for each age and contract linked to longevity risk. As specified in Solvency II regulations the parameters and the assumptions used for SCR calculation are calibrated "to correspond to the VaR of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one year period."

It is worth stressing that CEIOPS, 2010 defined the NAV as the difference between the market value of assets and liabilities. As well known, the market value of liabilities is difficult to determine, therefore it stated that it can be approximated by the so called Technical Provisions which consists of the Best Estimate Liabilities (BEL) and Risk Margin (RM).

The Risk Margin can be interpreted as loading for facing all residual risk in respect of those met by the SCR. It is calculated via a cost of capital (*CoC*) approach and in our case considering only the longevity risk it results:

$$RM_t = \sum_{h \geq 0} \frac{CoC \cdot SCR_{long,t+h}}{(1+r_f)^{-h}}$$

where $r_f$ is the risk free interest rate.

In order to solve the evident situation of circularity, CEIOPS 2010 specifies that for SCR calculation liabilities should not include Risk Margin. Therefore we have:

$$NAV_t = A_t - BEL_t \qquad (3)$$

where $A_t$ represents the market value of Assets at time t.

The Solvency II capital requirement are defined according to a balance sheet framenwork looking at the insurer obligations over one single year. In this paper we are interested in evaluating capital requirements at the beginning of each year with respect to the duration of the contract. In this way, given a specific scenario, the insurer may estimate today, t = 0, what will be the amount of capital necessary to meet its future obligations year by year till the contract will be in force.

## 4. SCR BACK TESTING APPROACH

To investigate the predictive power of the $SCR_{long}$ model, we propose a *contracting horizon back testing* approach in a risk management perspective.

A key element of backtesting that differentiates it from other forms of historical testing is that back testing calculates how a strategy would have performed if it had *actually* been applied in the past. This requires the backtest to replicate the conditions of the time in question in order to get an accurate result.

In the context under consideration, the back testing framework is designed to measure from time to time if the insurer has allocate more capital to support his in-force business, with adverse effects on free reserves and profitability.

As shown in Dowd et al 2010, a *good* model should produce forecasts that perform well out-of-the sample, as well as provide good fits to the historical data and plausible forecasts ex ante.

Generally, this kind of model performance is specified in the VaR validation analysis and recently for verify the goodness of mortality models.
In evaluating the capital amount necessary to satisfy the new solvency regulations, the metric of interest is the $SCR_{long}$ which is a complex multifactor value based on a given financial and demographic scenario.
The steps of the test are the following:

- selection of *lookback* window;
- selection *lookforward* window;
- comparison of forecasts with realized outcomes.

About the first one, the historical horizon is chosen, taking into account that if the window length is *n* and the evaluation time is *t*, we use observations from years $t-n$ to $t-1$.
As regards the second one, it is represented by the prediction interval. In the third one, the method of evaluating the projections against the realized results is detected.
We measure the accuracy of the projections by contracting the horizon backtest over the time. In other words we fix a start date *t*, i.e. the date the forecasts are made (he "stepping-off" year), and an end date N (the forecast date). We estimate the $SCR_{long}$ on the basis of the information from time $t-n$ up to $t-1$. As t moves towards N, we re-estimate the $SCR_{long}$ formula by using the same numbers of historical observations on the basis of the best estimate we chose for representing the mortality dynamics. For consistent forecasts we would expect that consecutive forecasts will converge to the realized $SCR_{long}$ value as the stepping off date approach the forecast year.

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