

IS STOCK MARKET VOLATILITY PERSISTENT? A FRACTIONALLY INTEGRATED APPROACH

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Abstract

This paper seeks to study the persistence in the G7's stock market volatility, which is carried out using the GARCH, IGARCH and FIGARCH models. The data set consists of the daily returns of the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 indexes over the period 1999-2009. The results evidences long memory in volatility, which is more pronounced in Germany, Italy and France. On the other hand, Japan appears as the country where this phenomenon is less obvious; nevertheless, the persistence prevails but with minor intensity.

Keywords: long memory, volatility, persistence, IGARCH Model, FIGARCH Model

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1. INTRODUCTION

A common finding in most of the empirical studies using financial data concerns the apparent persistence of shocks, or long memory, for the estimates of volatility. The presence of this property implies that the market does not immediately respond to an amount of information flowing into the financial market, but reacts to it gradually over time. Therefore, past price changes can be used as significant information for predicting future price changes. One implication of this is that shocks to the volatility process tend to have long-lasting effects. In addition, it also provides negative evidence as well as a new perspective to the Efficient Market Hypothesis (EMH) (Fama, 1970; Sharpe, 1970).

The origin of interest in long memory does not, however, lie in the economics/finance arena but instead appears to come out from a very different world – hydrology – and was first provided by Hurst (1951) while studying the flow of the river Nile. Hurst (1951, 1956) analyzed 900 geophysical time series and was partly motivated through the desire to understand the persistence of the stream flow and, thus, the design of reservoirs. Interestingly, after his seminal work, several authors have found the same pattern in many other domains of science, such as, biology, geophysics, climatology and other natural sciences as well (*e.g.* Mandelbrot and Wallis, 1968 and McLeod and Hipel, 1978). In economics, this phenomenon was first observed by Mandelbrot and van Ness (1968) while modelling asset price dynamics. Since then, the Hurst exponent H , has been calculated for many financial time series, such as, stock prices, stock indexes and currency exchange rates (Peters, 1994; Vandewalle and M. Ausloos, 1997 and Grau-Carles, 2000). In most cases, a Hurst exponent $1/2 < H < 1$ has been found, indicating long memory correlations in the observed data (Grau-Carles, 2000). Particularly, while analyzing stock market returns an interesting picture seems to arise (Matteo *et al.*, 2003; Grau-Carles, 2000): large and more developed markets (*e.g.*,

NYSE and LSE), usually tend to have H equal to, or slightly less than $1/2$, whereas less developed markets show a tendency to evidence $1/2 < H < 1$. In other words, large markets seem to be efficient in the sense that $H \approx 1/2$, whereas less developed markets tend to exhibit long-range correlations. A possible explanation for this is that smaller markets are more prone to experience correlated fluctuations and, therefore, more susceptible to be influenced by aggressive investors, which may in part explain a Hurst exponent greater than $1/2$.

Given the above, several reasons have been advanced for the apparent widespread finding of persistence in financial time series. For instance, Porteba and Summers (1987), have argued that for multiperiod assets like stocks, shocks have to persist for a long period for a time-varying risk premium to be able to explain the large fluctuations observed in stock market. In fact, if volatility changes are only transitory, no significant adjustments to the risk premium will be made by the market; hence, no significant changes in the discount factor or in the price of a stock, as determined by the net present value of the future expected cash flow, will occur. In addition, Schwert and Seguin (1990) have found out a common source of time-varying volatility across the disaggregated stock portfolios suggesting that portfolios might be co-persistent in the terms of Bollerslev and Engle (1993). On the other hand, Engle and Gonzalez-Rivera (1991) have noticed that persistence in variance seems to be related to the size of the business, with smaller businesses having a lower persistence than the larger corporations studied by Engle and Mustafa (1992).

Additionally, Chambers (1998) has not only observed long memory in the volatility of individual stocks and of aggregate indexes, but also that the degree of persistence was invariant with respect to the frequency of the data. In fact, the null hypothesis of a unit root in variance is not rejected by several authors using different sets of stock market data (French *et al.*, 1987; Chou, 1988 and Pagan and Schwert, 1990). A distinct explanation based on the interaction in the market of agents with different time horizons was also provided by Muller *et al.* (1997). According to the authors, long memory arises from the reaction of short-term dealers to the dynamics of a proxy for the expected volatility trend (coarse volatility), which causes persistence in the mean higher frequency volatility process (fine volatility). On the other hand, long-

term dealers base their decisions on the fundamentals of the market ignoring short-term movements. The empirical literature on long memory is vast and relies mainly on the estimation of the ARCH-type models (Vilasuso, 2002; Bentes *et al.* 2008; Mendes and Kolev, 2008; Oh *et al.*, 2008; Kang *et al.*, 2009; Kasman *et al.*, 2009; *inter alia*). In this context, the FIGARCH process, introduced by Baillie *et al.* (1996), seems to be of particular relevance since it constitutes a more flexible form than the traditional GARCH or IGARCH processes and accommodates both of them as special cases.

In order to determine whether the G7's stock market volatility exhibits long memory this paper focuses on the daily returns of the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 indexes. The analysis is carried out based on the GARCH, IGARCH and FIGARCH models. The results points out to the existence of long memory, which is more pronounced in Germany, Italy and France.

The remainder of the paper is organized as follows. Section 2 clarifies the meaning of long memory. Section 3 describes the methodology. Section 4 presents the statistical characteristics of the sample data. Section 5 discusses the empirical results and, finally, Section 6 draws the conclusions.

2. LONG MEMORY: SOME DEFINITIONS

Before proceeding any further a clarification about its meaning is necessary. Generally speaking, it is considered that long memory is related to a high degree of persistence of the observed data; hence, these two terms are used as synonymous. There are, however, several ways of defining it. Basically, it can be expressed either in the time domain or in the frequency domain. In the time domain, long memory manifests itself as hyperbolically decaying autocorrelation functions. This means that observations far from each other are still strongly correlated and decays at a slow rate. In other words, a stationary discrete time series process is defined to exhibit long memory if the autocorrelation function ρ_j at lag j satisfies

$$\lim_{j \rightarrow \infty} \frac{\rho_j}{c_\rho j^{-\alpha}} = 1, \quad (1)$$

for some constants $0 < c_\rho < \infty$ and $0 < \alpha < 1$. In contrast, we say that a weakly stationary process has short memory when its autocorrelation function is geometrically bounded (Brockwell and Davis, 1987)

$$|\rho_j| \leq c_\rho r^{|j|}, \quad (2)$$

for $c_\rho > 0$, $0 < r < 1$.

A more generalized definition of expression (1) was presented by McLeod and Hipel (1978)

$$\lim_{n \rightarrow \infty} \sum_{j=-n}^n |\rho_j| = \infty, \quad (3)$$

where n denotes the number of observations.

In the frequency domain, the same information comes in a form of a spectrum showing all the information within the interval $-\pi$ to π . In this context, a stationary time series is said to exhibit long memory if the spectral density f behaves as

$$\lim_{\lambda \rightarrow 0} \frac{f(\lambda)}{c_f |\lambda|^{-\beta}} = 1, \quad (4)$$

for some constants $0 < c_f < \infty$ and $0 < \beta < 1$. A connection between expressions (1) and (4) and the Hurst exponent, H , were also found (Beran, 1994): if $1/2 < H < 1$, then $\alpha = 2 - 2H$ and $\beta = 2H - 1$, which characterizes a classical process of long memory. On the contrary, negative memory or antipersistence occurs when $-1 < \beta < 0$ holds.

3. METHODOLOGY

In order to shed some light into the long memory process of stock market volatility the ARCH, GARCH, IGARCH and FIGARCH framework are theoretically described. Some of its main characteristics and its advantages/shortcomings are also discussed.

3.1 ARCH MODEL

One of the most popular models when dealing with this property is the ARCH(q) model derived by Engle (1982). Consider the time series y_t and the associated prediction error $\varepsilon_t \equiv y_t - E_{t-1}y_t$ where E_{t-1} is the expectations operator conditioned on time $t-1$ information. By definition, ε_t is serially uncorrelated with mean zero but the conditional variance of the process σ_t^2 is changing over time. In the classic ARCH(q) process proposed by Engle (1982) σ_t^2 is postulated to be a linear function of the lagged squared innovations implying Markovian dependence dating back only q periods; that is, ε_{t-i}^2 for $i = 1, 2, \dots, q$.

3.2 GARCH MODEL

A Generalized Autoregressive Conditional Heteroskedasticity (GARCH) was defined by Bollerslev (1986) so that $\varepsilon_t = z_t \sigma_t$, z_t is i.i.d., with zero mean and unit variance

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, \quad (5)$$

where $\omega > 0$, $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator L ($L^i x_t = x_{t-i}$) of order q and p , respectively. For stability and covariance stationarity of the ε_t process, all the roots of $[1 - \alpha(L) - \beta(L)]$ and $[1 - \beta(L)]$ are constrained to lie outside the unit circle.

Expression (5) may also be rewritten in the form of an ARMA(m, p) process in ε_t^2 ,

$$[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t, \quad (6)$$

where $m \equiv \max\{p, q\}$, and $v_t \equiv \varepsilon_t^2 - \sigma_t^2$ is mean zero serially uncorrelated. One characteristic of the GARCH model is that the effect of the past squared innovations on the current conditional variance decays exponentially with the lag length. In applied work, it has been frequently demonstrated that the GARCH(1,1) process is able to represent the majority of financial time series. In fact, a data set that requires a model of order greater than GARCH(1,2) or GARCH(2,1) is very rare (Bera and Higgins, 1993).

3.3 IGARCH MODEL

A common empirical regularity that has been found in many studies using financial data concerns the apparent persistence implied by the estimates for the conditional variance functions. In the GARCH model that is manifested by the presence of an approximate unit root in the autoregressive polynomial; *i.e.*, $\alpha_1 + \dots + \alpha_q + \beta_1 + \dots + \beta_p = 1$. Engle and Bollerslev (1986) was the first to refer to this class of models as integrated on variance or IGARCH (Integrated GARCH). Its specification is given succinctly by

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [(1-\beta(L))]v_t. \quad (7)$$

The authors pointed out the similarity between IGARCH processes and processes that are integrated in the mean. For a process that is integrated in the mean (one that must be differenced to induce stationarity) a shock in the current period affects the level of the series into the indefinite future. As in the martingale model for conditional means, current information remains important for forecasts of the conditional variance for all time horizons.

However, the idea of an infinite unconditional variance distribution in characterizing financial data is not new to the IGARCH class of models. Mandelbrot (1963) and Fama (1965) both have suggested the stable Paretian class of distributions with characteristic exponent less than two as providing a good description of the distributional properties of speculative prices.

3.4 FIGARCH MODEL

Despite its insight, the IGARCH process is not entirely satisfactory in modeling long memory in stock market volatility since it assumes infinite memory (Vilasuso, 2002). This, allied to the fact that the extreme degree of persistence found in many empirical studies might be contrary to the observed pricing behaviour led to the introduction of the Fractional IGARCH (FIGARCH(p,d,q)) model (Baillie *et al.* 1996).

Mathematically, the FIGARCH process can be written as follows

$$\Phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]v_t, \quad (8)$$

where $0 \leq d \leq 1$ is the fractional differential parameter. The FIGARCH model provides greater flexibility for modeling the conditional variance, because it nests the covariance stationary GARCH when $d = 0$, and the IGARCH when $d = 1$, as special cases. For the FIGARCH the persistence of shocks to the conditional variance or the degree of long memory is measured by the fractional differencing parameter d . Thus its attraction lies on the fact that for $0 < d < 1$, the model is sufficiently flexible to allow for an intermediate range of persistence. In particular, the FIGARCH model implies a slow hyperbolic rate of decay for the lagged squared innovations in the conditional variance function, although the cumulative impulse response weights associated with the influence of a volatility shock on the optimal forecasts of the future conditional variance eventually tend to zero.

A common approach for estimating ARCH models assumes a conditional normality process. Under this assumption the parameters of the FIGARCH model can be estimated using nonlinear optimization procedures to maximize the logarithm of the Gaussian likelihood function. Considering the random variable $z_t \sim N(0,1)$, the log likelihood of Gaussian or normal distribution (L_{Norm}) can be expressed as

$$L_{Norm} = -\frac{1}{2} \sum_{t=1}^T [\ln(2\pi) + \ln(\sigma_t^2) + z_t^2], \quad (9)$$

where T is the number of observations. It is worthy to note that the estimation procedure of the FIGARCH model requires a minimum number of observations. This

minimum number is related to the truncation order of the fractional differencing operator $(1-L)^d$.

4. DATA

This study focuses on the daily closing prices of S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500, spanning over a period from 4th January 1999 to 21st January 2009. All indexes were collected from Datastream, resulting in 2623 observations for each series. To perform our analysis the daily sample prices were converted into a daily nominal percentage return series (not adjusted for dividends), given by

$$r_t = 100 \ln(P_t/P_{t-1}), \quad (10)$$

for $t=1, \dots, T$, where r_t denotes the return at time t , P_t the current price and P_{t-1} the previous day's price. Expression (10) can be rewritten as

$$r_t = 100 [\ln P_t - \ln P_{t-1}]. \quad (11)$$

In accordance with Morana and Beltrati (2004) the motivations underlying the use of daily observations were two-fold: (i) from a statistical point of view, computing daily returns yields to a sample which is large enough to make statistical meaningful analysis; and, (ii) from a practical point of view the daily frequency is of utmost importance to the financial industry and to investors. For instance, risk management needs accurate forecast of daily and weekly volatility to implement value-at-risk models. Furthermore, in the case of quantitative asset allocation models, investors are interested in risk measurement at the daily or even lower frequencies. This is worthy to note as there is a general tendency in empirical studies to single out the advantages of high frequency data neglecting, somehow, the potentialities of lower frequencies. The closing prices dynamics and returns are depicted in Figs. 1 and 2, respectively.

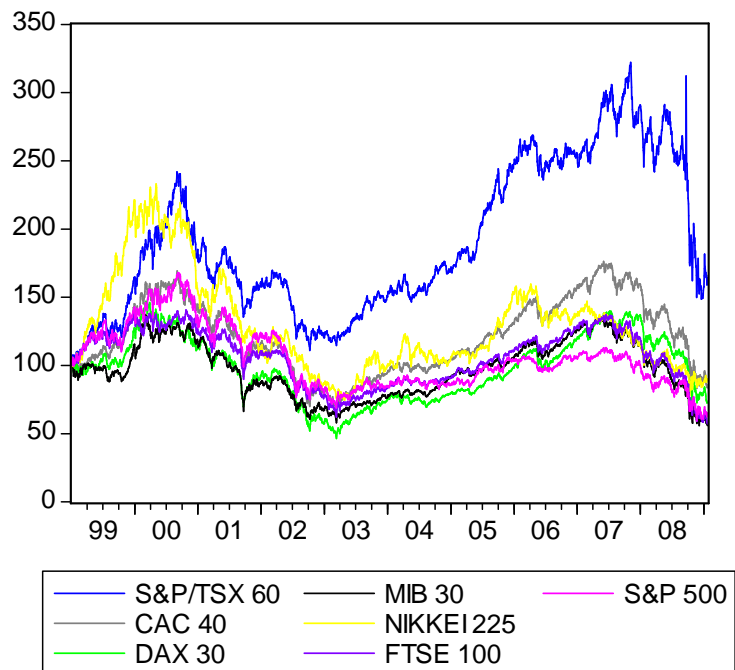


Fig. 1. Daily closing prices of the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 indexes in the period ranging from 4th January 1999 to 21st January 2009.

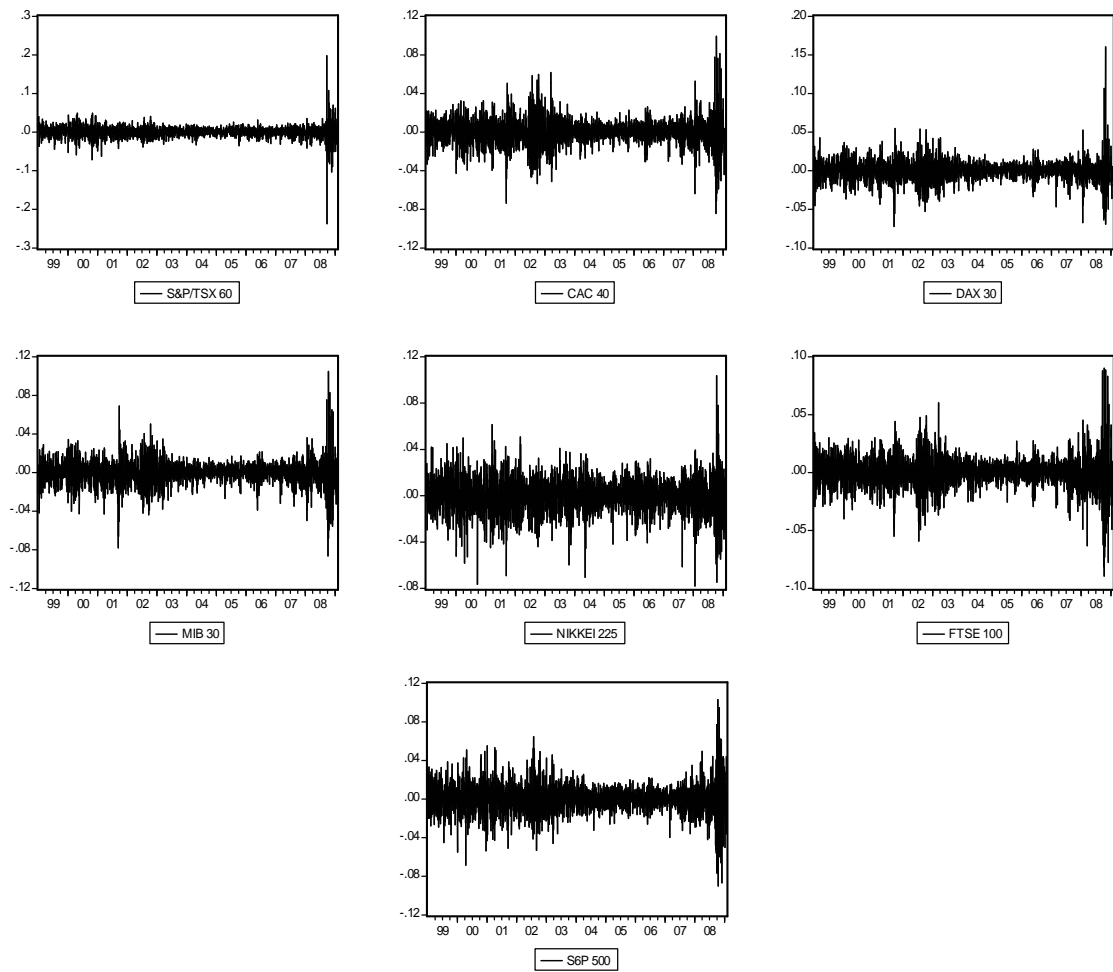


Fig. 2. Daily returns of the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 indexes in the period ranging from 4th January 1999 to 21st January 2009.

Table 1 summarizes the descriptive statistics for returns. The results show that the sample mean is positive, but close to zero. To evaluate the significance of this outcome a statistical test with the null of zero mean was conducted. The p -values (S&P/TSX 60 – 0.5167; CAC 40 – 0.7961; DAX 30 – 0.6396, MIB 30 – 0.3645, NIKKEI 225 – 0.8939, FTSE 100 – 0.4160 and S&P 500 – 0.5788) show that the null is not rejected at the 1% significance level. A slightly different pattern seems to arise while analyzing the standard-deviation, where the null of zero standard-deviation is rejected at the 1% significance level. This clearly reveals different volatilities.

In addition, our results also show that the NIKKEI 225 is the most volatile index. This is not surprising as the Japanese stock market was subjected to a severe

instability over the period under consideration, showing a non-increasing long-run trend in the raw prices and quite sharp oscillations over time. This was, thus, transmitted to returns and translates into abnormally large oscillations or high volatility, as observed. Next, the S&P 500 shows the second highest volatility followed by the S&P/TSX 60. The German DAX 30 and the Italian MIB 30 exhibit the lowest volatility as measured by the standard-deviation.

Consistent with a plethora of studies on the stylized facts of stock market volatility, returns are non-normally distributed with fat tails, as indicated by the skewness, kurtosis and Jarque-Bera test. In fact, all series display negative asymmetry, except Germany, which is positively skewed. Similarly, they are all leptokurtic with a kurtosis greater than 3. Likewise, the Jarque-Bera test also indicates significant departures from normality.

Additionally, the null hypothesis of a white-noise process for the sample returns was also assessed. According to the Ljung-Box statistics the DAX 30 and MIB 30 returns are not serially correlated, which seems to confirm that the log-prices follows a martingale. For the S&P/TSX 60, CAC 40, MIB 30, FTSE 100 and S&P 500, however, there is significant evidence of serial dependence, which can be removed by fitting an AR(5), AR(6), AR(5), AR(6) and AR(2) model, respectively. In order to test the presence of conditional heteroskedasticity and the evidence of ARCH effects the Lagrange Multiplier ARCH test was performed. The results show that all indexes' returns exhibit ARCH effects since the null of no ARCH was rejected for the time series under consideration.

Table 1

Descriptive Statistics for the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 returns

| Series | S&P/TSX 60 | CAC 40 | DAX 30 | MIB 30 | Nikkei 225 | FTSE 100 | S&P 500 |
|------------------------|---------------|------------|------------|------------|------------|------------|------------|
| Mean | 0,000190 | -6,87E-05 | -0,000121 | -0,000222 | -3,92E-05 | -0,000213 | -0,000163 |
| Median | 0,000708 | 0,000271 | 0,000445 | 7,69E-05 | 0,000165 | 8,22E-05 | 0,000172 |
| Maximum | 0,197644 | 0,099199 | 0,160461 | 0,104822 | 0,103666 | 0,090037 | 0,103077 |
| Minimum | -0,236878 | -0,084287 | -0,072114 | -0,086364 | -0,077852 | -0,089287 | -0,090098 |
| Standard- Deviation | 0,014979 | 0,013610 | 0,013181 | 0,012528 | 0,015043 | 0,013428 | 0,015007 |
| Skewness | -0,939692 | -0,056967 | 0,505864 | -0,129414 | -0,207952 | -0,175215 | -0,011419 |
| Kurtosis | 42,74492 | 8,285171 | 15,25676 | 10,29054 | 5,761197 | 9,418414 | 7,678499 |
| Jarque-Bera | 172963.6** | 3053,102** | 16524.24** | 5814.168** | 851.168** | 4514.084** | 2391.359** |
| Q(5) | 60.368 | 27.330 | 8.5671 | 45.577 | 3.1907 | 44.975 | 25.624 |
| Q(10) | 72.499 | 45.699 | 14.154 | 70.575 | 10.951 | 61.951 | 39.827 |
| Q _s (5) | 0.2527 | 0.1244 | - | 0.2937 | - | 0.0803 | 3.7048 |
| Q _s (10) | 6.3675 | 6.8479 | - | 17.206 | - | 5.3865 | 16.405 |
| LM-ARCH | 101.235** | 63.571** | 27.193** | 71.169** | 34.732** | 90.523** | 72.221** |

Notes: The Jarque-Bera corresponds to the test statistic for the null hypothesis of normality in sample return distribution. The Ljung-Box statistics, $Q(n)$ and $Q_s(n)$, check for the serial correlation of the return series and the squared returns up to the n th order, respectively. The LM-ARCH denotes the ARCH test with lag 10.

** indicates the rejection of the null hypothesis at the 1% significance level.

* indicates the rejection of the null hypothesis at the 5% significance level.

Prior to testing for the long memory property in volatility, all the sample returns were subjected to 2 unit root tests, ADF (Augmented Dickey-Fuller) and KPSS (Kwiatkowski, Phillips, Schmidt and Shin), in order to determine whether stationarity holds. These tests differ in the null hypothesis. Thus, for the ADF test the null is that a time series contains a unit root, $I(1)$ process, whereas the KPSS has the null of stationarity, *i.e.*, $I(0)$. Table 2 illustrates the empirical results of the unit root tests.

Table 2

Unit Root test for the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 returns

| Returns | ADF | KPSS |
|-----------------------|-------------|-------------|
| S&P/TSX 60 | -24,72529** | 0,301588 |
| CAC 40 | -51,57925** | 0,305330 |
| DAX 30 | -50,42855** | 0,186591 |
| MIB 30 | -23,18508** | 0,323470 |
| NIKKEI 225 | -51,41697** | 0,259831 |
| FTSE 100 | -23,67382** | 0,425302 |
| S&P 500 | -55,56673** | 0,206709 |

Notes: MacKinnon critical values: -3.43 (1%) and -2.86 (5%) for constant and -3.96 (1%) and -3.41 (5%) for constant and linear trend. Kwiatkowski-Philips-Schmidt-Shin critical values: 0.739 (1%) and 0.463 (5%) for constant and 0.216 (1%) and 0.1446 (5%) for constant and linear trend.

** indicates the rejection of the null hypothesis at the 1% significance level.

* indicates the rejection of the null hypothesis at the 5% significance level.

For the ADF test, large negative values for all cases support the rejection of the null of a unit root at the 1% significance level, whereas the statistics of the KPSS show that the return series are insignificant for the rejection of the null hypothesis of stationarity, implying that they are stationary processes. Hence, all series are suitable for subsequent analysis in this study.

5. EMPIRICAL RESULTS

In this Section we estimate the GARCH (1,1), IGARCH (1,1) and FIGARCH (1, d ,1) models and compare their performance. The estimation results, listed in Tables 3, 5 and 6, have been produced using the Maximum Likelihood Estimation (MLE) method with the Generalized Error Distribution (GED).

Table 3

Maximum likelihood estimates of the GARCH(1,1) with GED

| Series | S&P/TSX 60 | CAC 40 | DAX 30 | MIB 30 | NIKKEI 225 | FTSE 100 | S&P 500 |
|------------------|---------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\hat{\omega}$ | 7,29E-07* (3,03E-07) | 1,42E-06** (4,19E-07) | 1,59E-06** (4,14E-07) | 1,39E-06** (4,07E-07) | 3,62E-06** (1,16E-06) | 1,40E-06** (4,54E-07) | 9,33E-07** (3,15E-07) |
| $\hat{\alpha}_1$ | 0,066785** (0,0009092) | 0,095208** (0,011338) | 0,107301** (0,013528) | 0,105619** (0,013138) | 0,077715** (0,012027) | 0,099643** (0,012411) | 0,053777** (0,008021) |
| $\hat{\beta}_1$ | 0,929492** (0,009142) | 0,898933** (0,011595) | 0,887383** (0,013372) | 0,887719** (0,013470) | 0,908478** (0,013956) | 0,894714** (0,012384) | 0,942955** (0,008389) |
| <i>GED</i> | 1,381377** (0,048237) | 1,538070** (0,050186) | 1,365483** (0,040428) | 1,379187** (0,051252) | 1,508999** (0,049865) | 1,580459** (0,055901) | 1,514688** (0,052089) |
| <i>Log-L</i> | 7359,269 | 8085,075 | 8198,237 | 8369,754 | 7485,422 | 8145,663 | 7742,724 |
| <i>SIC</i> | -6,415955 | -6,148160 | -6,238412 | -6,366378 | -5,694693 | -6,194481 | -5,889447 |
| <i>AIC</i> | -6,441077 | -6,172840 | -6,249609 | -6,388807 | -5,705890 | -6,219161 | -5,905133 |

Notes: ** indicates the rejection of the null hypothesis at the 1% significance level.

* indicates the rejection of the null hypothesis at the 5% significance level.

The conclusions are similar for all the returns. Specifically, the GARCH (1,1) estimates reveals the presence of volatility clustering in the conditional variance, since the estimated parameters are all significant at 1%. Also, like most financial applications using high frequency returns, the sum of the estimated parameters of the lagged variance and the lagged squared residuals in the GARCH(1,1) process is close to one ($\alpha + \beta \simeq 1$). This might suggest that volatility is highly persistent, *i.e.*, shocks tend to have a permanent influence on the conditional variance, a fact that favours the IGARCH(1,1) specification. Nevertheless, to assess the significance of this outcome a Wald test to the sum of the parameters was performed. Accordingly, the null and alternative hypothesis were specified as follows

$$\begin{cases} H_0 : \alpha_1 + \beta_1 = 1 \\ H_a : \alpha_1 + \beta_1 < 1 \end{cases}$$

The results of the Wald test are reported in Table 4.

Table 4

Results of the Wald test to the sum of the lagged variance with the lagged squared residuals

| <i>Wald Test</i> | S&P/TSX | | DAX 30 | MIB 30 | NIKKEI | | S&P 500 |
|------------------|--------------------|---------------|---------------|---------------|---------------|-----------------|--------------------|
| | 60 | CAC 40 | | | 225 | FTSE 100 | |
| χ^2_a | 5,960998* | 1,278926 | 0,743907 | 1,092194 | 7,662850** | 0,998128 | 6,200133* |

Notes: The χ^2 test was estimated with 1 lag and 1 degree of freedom.

** indicates the rejection of the null hypothesis at the 1% significance level.

* indicates the rejection of the null hypothesis at the 5% significance level.

The rejection of the null at the 5% significance level, for the S&P/TSX 60 and S&P 500, and at the 1% for the NIKKEI 225 suggests that these returns are not highly persistent. Thus, the IGARCH (1,1) model was estimated for the remainder series (Table 5).

Table 5

Maximum likelihood estimates of the IGARCH(1,1) with GED

| Series | S&P/TSX 60 | CAC 40 | DAX 30 | MIB 30 | NIKKEI 225 | FTSE 100 | S&P 500 |
|------------------|---------------------------|---------------|---------------|---------------|-----------------------|-----------------|--------------------|
| $\hat{\omega}$ | - | 0,010844** | 0,013774** | 0,011297** | - | 0,010770** | - |
| | - | (0,0035368) | (0,0046686) | (0,0037685) | - | (0,0035301) | - |
| $\hat{\alpha}_1$ | - | 0,096630** | 0,112039** | 0,109404** | - | 0,100781** | - |
| | - | (0,012471) | (0,012275) | (0,015959) | - | (0,014815) | - |
| $\hat{\beta}_1$ | - | 0,903370 | 0,887961 | 0,890596 | - | 0,899219 | - |
| | - | - | - | - | - | - | - |
| <i>GED</i> | - | 1,517003** | 1,352310** | 1,355676** | - | 1,562021** | - |
| | - | (0,074558) | (0,081166) | (0,061987) | - | (0,072183) | - |
| <i>Log-L</i> | - | 8105,026 | 8197,047 | 8385,193 | - | 8165,877 | - |
| <i>SIC</i> | - | -6,152302 | -6,240506 | -6,369009 | - | -6,198717 | - |
| <i>AIC</i> | - | -6,174696 | -6,249464 | -6,389164 | - | -6,221111 | - |

Notes: ** indicates the rejection of the null hypothesis at the 1% significance level.

* indicates the rejection of the null hypothesis at the 5% significance level.

The results given on Table 5 show that though ω , α_1 and GED are statistically significant at 1%, the same does not occur with β_1 , which is not significant at any of the conventional levels. It also emerges while comparing Tables 3 and 5 that there is little discernible difference for these returns between the GARCH and IGARCH models. This is not surprising as the IGARCH(1,1) specification nests the GARCH (1,1) (*e.g.*, Vilasuso, 2002, Kang *et al.*, 2009).

The next step was then to adjust the FIGARCH model (1, d ,1) with the restrictions $d \neq 0$, $d \neq 1$. Table 6 summarizes the results.

Table 6
Maximum likelihood estimates of the FIGARCH(1, d ,1) with GED

| Series | S&P/TSX 60 | CAC 40 | DAX 30 | MIB 30 | NIKKEI 225 | FTSE 100 | S&P 500 |
|------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\hat{\omega}$ | 2,579237* (1,2177) | 3,496927* (1,5484) | 5,571936 (2,4065) | 4,267883* (2,1695) | 3,208807** (0,98780) | 3,153874* (1,3604) | 3,317692** (1,1903) |
| $\hat{\alpha}_1$ | 0,145704** (0,047175) | 0,079608 (0,046211) | 0,060435 (0,041596) | 0,046432 (0,049529) | 0,121331 (0,087791) | 0,061955 (0,053911) | 0,150769** (0,056288) |
| $\hat{\beta}_1$ | 0,689087** (0,049783) | 0,634176** (0,067334) | 0,650559** (0,057723) | 0,590615** (0,069073) | 0,480313** (0,11719) | 0,536630** (0,073075) | 0,614842** (0,090704) |
| \hat{d} | 0,571926** (0,060333) | 0,588910** (0,057982) | 0,623683** (0,059569) | 0,592794** (0,058795) | 0,399785** (0,057646) | 0,535808** (0,048787) | 0,483263** (0,067574) |
| GED | 1,388687** (0,067704) | 1,532205** (0,074964) | 1,357048** (0,084732) | 1,379116** (0,062769) | 1,506296** (0,071506) | 1,586665** (0,073075) | 1,506983** (0,076568) |
| Log-L | 7382,398 | 8112,191 | 8206,130 | 8394,823 | 7489,988 | 8174,546 | 7754,758 |
| SIC | -6,418763 | -6,151763 | -6,241430 | -6,370350 | -5,708610 | -6,199325 | -5,891130 |
| AIC | -6,446348 | -6,178635 | -6,254866 | -6,394983 | -5,708610 | -6,226198 | -5,909045 |

Notes: ** indicates the rejection of the null hypothesis at the 1% significance level.

* indicates the rejection of the null hypothesis at the 5% significance level.

The first diagnostic concerns the mixed significance of the estimated parameters: thus, while β_1 , GED and d estimates are all significant at 1%, distinct patterns seem to arise for ω and α_1 estimates, which ranges from no significance at all to 1% or 5% significance.

A second diagnostic refers to the estimated fractional differencing parameter d , which spans from 0.399785 for the NIKKEI 225 to 0.623683 in the case of the DAX 30; thus, rejecting the null hypothesis of GARCH ($d=0$) and IGARCH ($d=1$) models. Hence, the estimated FIGARCH parameters are consistent with a long-memory process, which more realistically describes the return dynamic properties.

Notwithstanding, it is worthy to note that, NIKKEI 225, the most volatile index according to the standard-deviation, exhibits the lowest persistence. Similarly, DAX 30, one of the less volatile ones, displays the highest memory, suggesting somehow that there is an inverse relation between these two measurements. This might be explained by the fact that smaller markets characterized by less liquidity, like the DAX30 or MIB 30, are less efficient in the sense of the EMH, thus exhibiting higher persistence. This is consistent with the findings of Matteo *et al.* (2003) and Grau-Carles (2000).

Subsequently, following Mittnik and Paoella (2003), the maximum log-likelihood value, the bias-corrected Akaike Information Criterion (AIC) and the Schwartz Information Criterion (SIC) were used to discriminate between models. These criteria are also recommended by Sin and White (1996) to take a final decision. Overall, the results described in Tables 3, 5 and 6, strongly indicate that the FIGARCH (1, d ,1) is the best model to capture the dependence in the variance.

6. CONCLUSIONS

In this paper we have examined the persistence in volatility of the G7's stock market indexes. The daily returns of the S&P/TSX 60, CAC 40, DAX 30, MIB 30, NIKKEI 225, FTSE 100 and S&P 500 were modeled using a GARCH(1,1), IGARCH (1,1) and FIGARCH (1, d ,1) framework. As suggested by Baillie *et al.* (1996), FIGARCH is better suited to capture persistence in volatility than the GARCH or IGARCH models since it is a more flexible form, which nests both processes as special cases. The results suggest that the financial industry and investors should consider persistence in the volatility of all indexes.

Another interesting feature which seems to arise in this study is that, the NIKKEI 225, the most volatile index according to the standard-deviation, exhibits the lowest persistence. Analogously, the DAX 30, one of the less volatile ones, displays the highest persistence, suggesting an inverse relation between these two measurements. This might be explained by the fact that smaller markets are less liquid, less efficient, and more prone to experience correlated fluctuations and, therefore, more susceptible to be influenced by aggressive investors.

To conclude we shall mention that though the estimated fractional differencing parameter evidences distinct persistence across the G7's countries, which is more pronounced in Germany, Italy and France, there is no relevant difference among them, suggesting that the returns tend to some kind of homogeneity, which can be viewed as a result of globalization.

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