PROJECT SELECTION CONSIDERING DELAYED ACCEPTANCE OF INVESTMENT PROJECTS

A THESIS

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PROJECT SELECTION CONSIDERING DELAYED ACCEPTANCE OF INVESTMENT PROJECTS

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SUMMARY

When a company is planning to enter a new product market, it is possible to follow one of three basic strategies:

A) Be the first to enter the market.

B) Follow the leader.

C) Wait more time and be "one more" in that product market.

The firm faces different risk-reward alternatives as a function of the time to launch a product. This product market behavior can be extended to many other types of investment decisions that the company faces year by year.

This research includes the construction of a project selection process, with the consideration of the timing effect as the main objective, which combines an important number of real world characteristics:a stochastic sequential decision model with new projects every year, those projects which were not selected can be carried forward to the next years, correlated cash flows among projects, and budget and project contingency constraints.

The following pattern is assumed for the projects: In proportion to the implementation time of the project, the expected value of the elements in the cash flow stream decrease, and also their variability.

The selection process is based in the expected present worth as a measure of reward and its variability as a measure of risk. Different levels of the main parameters, risk-aversion factor, delaying project acceptance and annual budget are tested in three project sets.The problem is solved analytically and simulated.

The most important result of this research effort is the realization of a model which combines capital budgeting theory, new-product development theory, and mathematical and computational tools into a practical and realistic sequential procedure for project selection. Such a model would be useful to any decision maker who faces the problem of allocating limited financial resources of the firm in a periodic sequential decision making environment.

CHAPTER I

INTRODUCTION

Background

When a company is planning to enter a new product market, it is possible to follow one of three basic strategies.

A) Be the first to enter the market.

B) Follow the leader (be the second to enter the market) or

C) Wait more time and be "one more" in that product market.

The first strategy is the most risky, but also is the most likely to result in higher payoff. It has been found that sometimes the second firm to enter the market will achieve only half the sales of the first firm, and the third firm half the sales of the second firm (13). It is clear that timing is important, and rewards from entering a market at an opportune moment could be substantial. On the other hand, there are more risks associated with entering a market early: substantial resource and development (R&D) cost may not be recovered, and there is more uncertainty regarding product acceptance. A firm that waits can avoid exessive R&D costs and can avoid products that do not sell well. Therefore the firm faces different risk-reward alternatives as a function of the time to launch a product.

This product-market behavior can be extended to many other types of investment decisions that the company faces year by year: cost-reduction measures, plant and warehouse location, installation of environmental controls, etc. Each of these proposals has associated with it a risk-reward relation depending upon the time at which it is implemented.

Different models have been proposed in the literature to solve this problem. Some authors assume certainty conditions and use deterministic models, others use stochastic models under uncertainty conditions and make one decision at one point in time for the planning horizon. Others propose a sequential decision procedure with new projects considered every year. No one, however, has developed a project selection process which combines an important number of real world characteristics: a stochastic sequential decision model with new projects considered every year, those projects which were not selected can be carried forward to the next years, correlated cash flows among projects, and budget and project contingency constrains.

Purpose

With the consideration of the timing effect as the main objective of this work, the purpose of this research is:

1) To develop a project selection technique which considers sequential decision points, variability of the cash flows, and this variability dependent on timing.

2) To obtain computational experience with this project selection technique,testing the effects of different project evaluation criteria.

Method of Approach

The approach of the research will be to postulate a fixed planning period of five years with annual investment decisions, generate cash flow streams for investment projects, and apply different project evaluation criteria to select the projects. The following pattern will be assumed for investment projects: in proportion to the implementation time of the project, the expected value of the elements in the cash flow stream will decrease, and also their variability. Typical patterns will be based on literature concerning marketing of new products(22). Projects which are not selected one year might be available for selection the following one or two years. The generation of the streams will be done using uncertainty conditions for different cases of correlated cash flows: complete independence, perfect correlation, partially correlated, and cross-correlated flows. Also, there will be considered contingency and budget constrains.

In order to structure the project selection technique it will be necessary to obtain the expected present worth as a measure of reward and associate the variance of the present worth as a measure of risk. Then there will be tested different levels of risk-aversion in order to represent aggresive and conservative project selection strategies. The resulting sets of selected projects will then represent different points on an "efficiency frontier". Also, the model will give additional information to support the decision process; ie,the amount of cash every year, the amount of cash at the horizon, the total cost of each selected decision alternative, etc.

In order to solve the problem, a sequential analysis through the planning period will be done. This process will include the selection of the projects, the computation of the expected present worth and its variability for a set of projects, and a simulation to determine project outcomes for that particular year. A comparison of the results with and without the option of delaying project acceptance will also be made

It is expected that the results gained from this research will yield a more realistic and practical decision making technique dealing with the variabilities of the cash flows dependent upon the timing of project acceptance.

CHAPTER II

LITERATURE SURVEY

In the literature can be found many different approaches to the problem of allocating limited cash resources to the proposed alternatives a company faces each period of time. These approaches range from models considering certainty conditions (deterministic models) to models considering a probabilistic future, and models with different kinds of interrelationships between projects. Depending upon the size of the firm, the amount of money involved in project selection and the accuaracy required of the models, each firm attempts to select a technique or model appropiate for its needs.

Deterministic Models

Among the models assuming certainty conditions, the most comprehensive treatment of the problem has been by Weingartner(35). He uses a mathematical programming approach that deals with the set of investment alternatives, borrowing and lending activities, and complex interrelationships among projects. The form of his Basic Horizon model is:

$$\text{Maximize:} \sum_j \hat{a}_j x_j + v_T - w_T \qquad (2\text{-}1)$$

$$\text{Subject :} \sum_j a_{1j} x_j + v_1 - w_1 \leq D_1 \qquad (2\text{-}2)$$

$$\sum_j a_{tj}x_j-(1+r)v_{t-1}+v_t+(1+r)w_{t-1}$$

$$-w_t \leq D_t \qquad t=2,3,4...T \qquad (2-3)$$

$$0 \leq x_j \leq 1 \qquad j=1,2,3,..n \qquad (2-4)$$

$$v_t, w_t \geq 0 \qquad t=1,2,3...T \qquad (2-5)$$

where, a_{tj}=cash outflow for project j at time t.

$\hat{a}_{tj}$=time T value of post-horizon cash flows.

D_t =cash available at time t from other sources.

v_t =lending from t to t+1 at rate r.

w_t =borrowing from t to t+1 at rate r.

This linear programming model maximizes the net value of assets at the horizon. These consist of the funds available for lending at that time and the discounted streams of net revenues past the horizon. The model assumes all interest is payable at the end of the year, and new loans can be immediately made to cover any cash shortages. To the four restrictions above it is possible to add others expressing relationships of complementarity and competitiveness between projects.

Bernhard(2) made a comprehensive review of the mathematical programming models, surveying, extending, criticizing, and building a generalized deterministic model. He considers various cases and some relationships of other models proposed in the literature, such as those by Baumol and Quandt, Weingartner, and Lorie and Savage, etc. However, the principal shortcoming of these approaches is the assumption of complete information, because in most investment situations the future is not known with certainty.

Non-Deterministic Models

In a more realistic world, the decisions are based usually on predictions about the future.The problem then focuses on the variations in the outcomes of the alternatives. If it is possible to know or assume some probability distribution about the outcomes, the decision will be under risk, on the other hand, if it is not possible to associate any probability distribution to the project outcomes the decision will be under uncertainty (30).

The Concept of Risk

Usually the variability of the future outcomes is used as a concept of risk. Some authors, as Markowitz (19) and Tobin (31), measure this risk by the variance or the standard deviation of the return. Markowitz discusses the risk-vs-return problem within the context of securities investments. The problem is one of determining the optimal set of securities (a portfolio) from a large number of prospective investment opportunities. Optimality is based upon two criteria: expected return (E) , and variance of return (σ^2). Given the probabilistic estimates of the future performance of securities,an efficient set of portfolios is determined. Then from that set a portfolio is selected which best reflects the decision maker's preferences. Markowitz selects the variance of return (σ^2) as a measure of risk. However, he says that the standard deviation(σ) or the coefficient of dispersion (σ/E) could also be used as measures of risk, and any of the three measures will result in the set of efficient portfolios. Mao (16) compares this concept with an alternative one, the semivariance, which he defines as:

$$S_h = E\left[(R-h)^-\right]^2 \tag{2-6}$$

where: R=is a random variable with known probability distribution.

h=is a critical value which R should exceed.

E=is an expectation operator.

and,

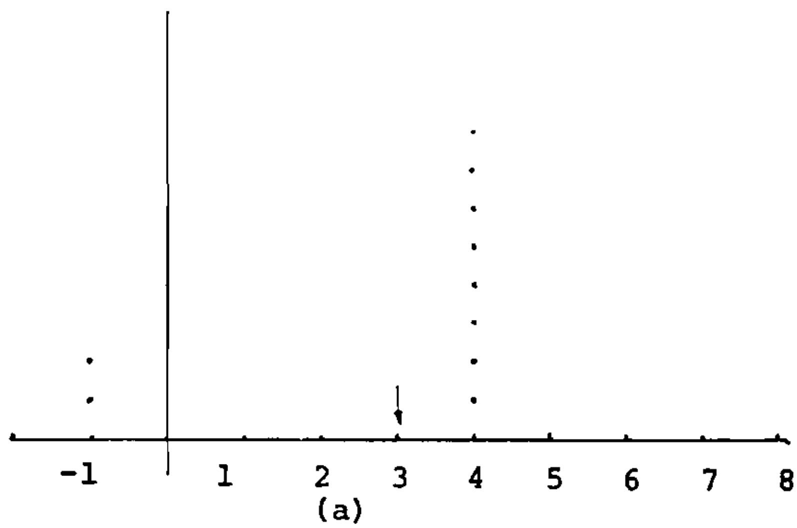
$$(R-h)^- = (R-h) \qquad \text{if } (R-h) \leq 0$$

$$(R-h)^- = 0 \qquad \text{if } (R-h) > 0$$

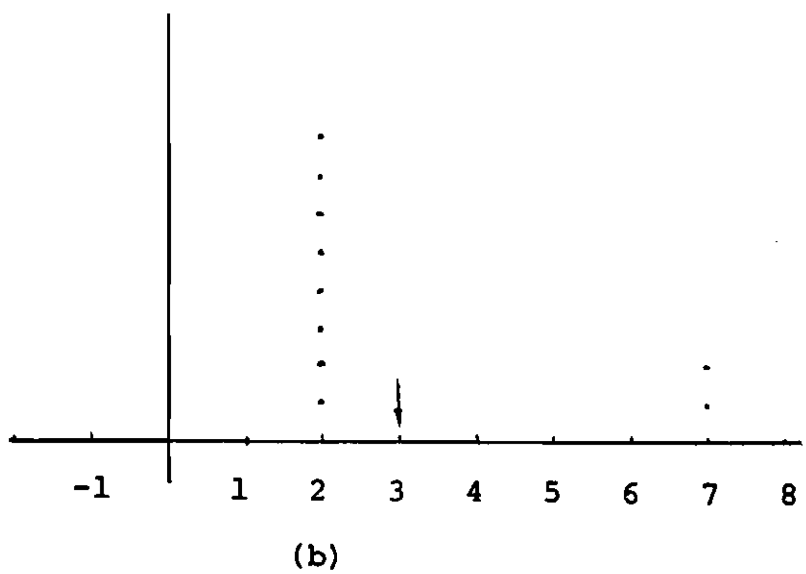
Alternativley, it can be expressed as:

$$S_h = E\left[\min(R-h), 0\right]^2 \tag{2-7}$$

The effect is to measure the downside (unfavorable) variability. Both the variance and semivariance criteria will pick the same solution for investments involving only symmetric distributions. However, the two criteria may indicate different solutions if returns from investments are asymmetric. Mao illustrated this with a skewed distribution, figure 2-1(a), and its reflection about the mean figure 2-1(b), where each point represents one possible investment outcome. It easy to see that both distributions (a) and (b) have the



E=3 V=4 S_h=3.2 h=3



E=3 V=4 S_h=0.8 h=3

Figure 2-1. Difference between Variance and Semivariance.

same means and variances; therefore the variance criterion will evaluate the two proposals as equally attractive. However, an investor interested in reducing losses will prefer (b) to (a). The semivariance criterion will also pick (b) because the distribution (a) has an S_h of 3.2, and distribution (b) has an S_h of only 0.8. The variance seems to be too conservative because of the fact that any extreme (below or above of the expected return) is undesirable. However, it is a more popular measure of risk than the semivariance, because of its familiarity and ease of computation.

Dealing With Risk

There are numerous approaches for compensating for risk in the project selection process. Among the simplest ones are:

I) The payback period:number of years required to recover the initial cash investment.

II) The risk-adjusted discount rate: the riskless rate and a premium for risk,ie,

$$i_a = i + i_r$$

where i_r denotes the incremental return required to compensate for risk. And,

III) The variation of project life as a measure for adjusting risk,ie, a very risky ten year project may be reduced to an eight or seven year project to compensate for risk.

The main disadvantage of the payback period is that this criterion gives equal importance to all cash flows ocurring before the project recovers its initial investment and no importance to flows ocurring after that time. It has the virtue of promoting the liquidity of the firm, but at the same time, some good projects with high returns in the future may be seriously underrated. On the other hand, Van Horne (32) shows that the disadvantage of the risk-adjusted discount rate is the difficulty of determining the appropriate one for each particular alternative. Also,he discusses(33) the drawbacks of using project life as a mean for adjusting for risk.

Robicheck and Myers (26) recommend the concept of certainty-equivalent,defined as a certain amount equivalent to the outcome of a risky situation, or, in other words, a certain amount such that an investor is indifferent between this amount for certain and a chance on the outcome of the risky situation. With this method, distibutions of possible cash flow outcomes are specified period by period and a certainty equivalent is substituted for each of the distributions. Van Horne(32) explains that the difficulties of this approach are: a) The specification of the appropriate certainty-equivalents period by period for an investment opportunity and b) Being consistent in these specifications from project to project.

Baumol(1) introduces a modification to the variance criterion, named Expected Gain Confidence Limit Criterion (EGCL). This model involves the calculation of a critical point on which every alternative decision should be based. The basic equation in his approach is:

$$CP=EV-\phi\sigma \tag{2-8}$$

where; EV=expected value of return.

σ=standard deviation of expected return.

ϕ=degree of risk aversion (a number of standard deviations on the low side of EV, below which values can not be tolerated.

The value of ϕ is selected by the investor or portfolio manager based on risk preferences - ϕ and CP vary inversely. For example, assuming returns are normally distibuted, if the investors are willing to accept a 0.25 chance that the portfolio return is below CP, they should set ϕ=2. If less chance of a low return is desired, this may be achieved by setting ϕ=3.

A more elaborate approach which considers the probability distributions of the project outcomes over time is the method of Hillier (9). Period by period the project outcome is treated as a random variable with known mean and standard deviation. Then the mean and variance of the "figure of merit" (net present value, equivalent uniform annual cost, or internal rate of return) are determined analytically. Thus, Markowitz' method for single-period investments is extended to multiple periods. Furthermore, Hillier incorporates the concepts of perfect independence and perfect and partial correlation among cash flows. Later in 1971(11) Hillier reexamined the problem from the view point of expected utility of present worth. His solution procedure consists of an approximate linear programming approach and an exact Branch-and-Bound algorithm. The utility functions considered are: I) A basic model,(figure 2-2),where the expression for Utility of present worth is given by a hyperbola

$$U(p)= \frac{(a_1+b_1p)+(a_2+b_2p)-Q}{2} \tag{2-9}$$

where;

$$Q= \sqrt{\left[(a_1+b_1p)+(a_2+b_2p)\right]^2-4p\left[a_1+b_1b_2p+a_2\right]}$$

$$a_1=d(1-b_1) \qquad a_2=d(b_2-1)$$

II) And a high risk aversion model for U(p),(figure 2-3), which differs from the above only in the behavior of the utility function as p grows very large in the negative direction. The algebraic form of the function is:

$$U(p)= a_1+b_1p-a_1e^{-\left[(1-b_1)/a_1\right]p} \tag{2-10}$$

Using Hillier's results, many other authors have extended his ideas and studied various general cases of investment situations, as Kahak and Owen(12), Canada and Wadsworth(5), Mantell(15), Young and Contreras(37), etc. An important drawback of Hillier's and related methods is the difficulty of implementing the analytical procedures necessary to derive the mean and variance of the present worth of the selected projects. The complexity of some real world problems

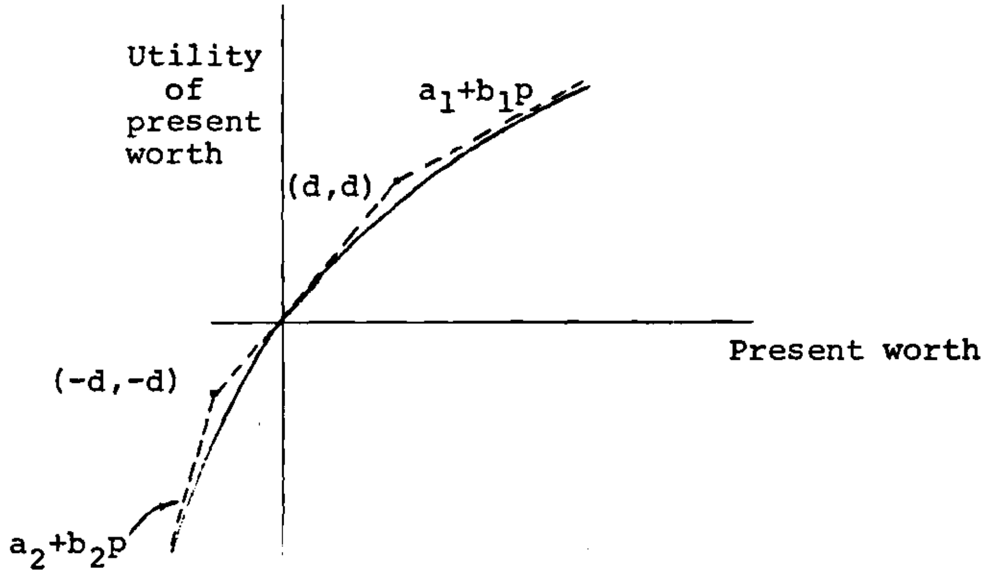


Figure 2-2. The Basic Model for Utility of present worth, U(p).

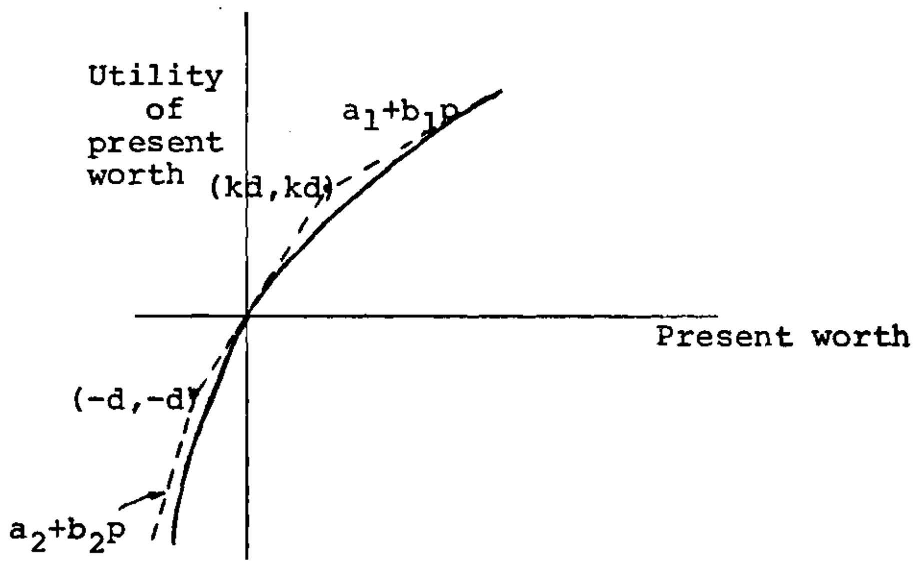


Figure 2-3.The High Risk-Aversion Model for Utility of present worth,U(p).

precludes the use of these methods.

On the other hand, Hertz(8) in 1964 uses Monte Carlo simulation to deal with the riskiness of an investment. As in the case of Hillier's models, the objective of the computer simulation is to generate a probabilistic distribution for the present worth. This enables the decision maker to compare expected returns and their variabilities for two or more alternatives. Even though Hertz makes a distinction between "risk of investment" (probability that the project will result in a loss) and "variability of return on investment" (dispersion of the probability distribution for the present worth), most other authors only use the variance of return as a measure of project risk. A feature of the Hertz approach is that computer simulation always results in a distribution for the present worth of the selected projects. The stochastic models discussed by Hillier do not always generate directly a probability distribution, but instead use the means and variances of the cash flows to obtain the mean and variance of the present worth of the selected projects.

Lately, in the fall of 1977 Bey and Porter(3) wrote a paper which deals with the evaluation of capital budgeting portfolio models by using simulated data. In their work they point out that while decision rules as payback, internal rate of return, and net present value may deal effectively with some of the problems which the decision maker faces,ie, large number of available alternatives, interrelationships among projects, constraints on capital resources,etc.,"they have the common shortcoming of considering projects only on an individual basis and, therefore, fail to consider the statistical interrelationships among the set of proposals". They also cited some other authors who have suggested a portfolio approach to capital budgeting, Lintner(14), Naslund(20), Salazar and Sen(27), and Quirin(25).

In their paper Bey and Porter make an empirical study of the performance of several of the major portfolio approaches to capital budgeting. The portfolio models studied were:

1) A modification to the mean-variance model (EV-I) as adapted to capital budgeting by Weingartner(36).

2) Porter's (23) extension of the Lintner (14) single-period case (EV-II).

3) A mean-semivariance model(ES_h).

4) A chance-constrained model (CCP).

Their study assumed one decision at one point in time and uses as a standard of comparision the second degree stochastic dominance model, because of its conceptual superiority (24). Then they simulated several decision environments and found that the results of the decision models are highly dependent upon whether the project cash flows are positively or negatively correlated. For the positively correlated cash flows the mean-semivariance model (ES_h) clearly outperformed all the others. The next best performance was accomplished by the chance-constrained model (CCP), follow by the EV-II and EV-I. Even though a direct comparison of the NPV model with the others is not easy, because this model selects only one set for the efficiency frontier, consisting of all those with NPV greater than or equal to zero, the study clasifies its performance as quite poor. On other hand, for negatively correlated cash flows the ranking of the models depends of how the comparison is made. However, in general the only change in the ranking of performance is in the EV-II and EV-I models which interchange their places. Bey and Porter suggest at the end of the study that: a) The set of projects selected will depend on which portfolio model was used and b) There is no benefit in attempting to match decision environments and capital budgeting models.

Uncertainty Resolution

It is possible to find in the literature two major approaches which deal with the concept of uncertainty resolution in an explicit manner: the payback period method, and the certainty-equivalent method. Uncertainty resolution describes the situation in which information needed to formulate or assume probability distributions of possible events is unknown.

Even though uncertainty resolution has been discussed by several authors, as Robicheck and Myers(26), Percival and Westerfield(21), Bierman and Hansman(4), it has not been found very useful in the allocation of the firm's resources among competing alternatives. For example, a major difficulty in the certainty-equivalent approach is the development of an appropriate utility function to identify the time preferences of consumption. In particular, an individual's time preference for future consumption depends on what investment opportunities this individual would have in the future. However, in most real investment situations, the ocurrence, timing, and characteristics of future investment opportunities are difficult to predict with certainty. On other hand, in the use of the payback method as a basis for measuring uncertainty resolution, it is difficult to find a meaningful index representing the rate of the resolution of uncertainty through time, when the cash flows of a proposal are expressed in terms of a probability tree. It is possible to compute the expected payback period and variability about the expectation for a proposal. However, the interpretation of the statistic in terms of uncertainty resolution over time is rather vague.

Product Development

Up to now the literature search has dealt with the problem of allocating limited money resources to different project proposals. Although the work done in this research may apply to all types of investment proposals, as cost-reduction measures, plant and warehouse location, and installation of environmental controls, the timing in the launching of a new product is of particular importance. Therefore, part of this literature search also treats this concept. Unfortunately, the literature in this field is not as rich as the literature of capital budgeting.

In 1972 Seavoy(28) said that "new-product marketing is an art, a science, a gamble", and classifies the risks in five areas: risk in the product, risk in production, risk in the market,risk in distribution, and risk in commercialization. He really points out the importance of timing, saying: "if you're late or early (in the market), the market will pass you by like a speeding jet".

FitzRoy(7) proposes three basic product strategies:

1) Be the first in the market (or market leadership).
This is a high risk strategy, but the company has the possibility of high income. In order to be a successful company of this type, the firm has to be inventive, high risk oriented, development oriented, and also should have the resources required to absorb possibles losses.

2) Follow the leader (second in the market).
In this strategy, the firm chooses to be the second one in the market. Here the firm takes advantage of the mistakes made by the leader, and then it may launch a better product. This kind of behavior is a lower risk strategy, but the potential revenues are lower too.

3) Me-too.
In this strategy the company goes into an established market. This choice has, generally, the lowest risk. But in order to

generate some profits, the firm requires superior product positioning and because most of the time those markets have severe price competition, the company must have production and distribution strengths.

There are some other aspects the company must examine before choosing a strategy, as: the market opportunities (advantage of the firm relative to the competition), the maximum utilization of resources, and corporate stability (overall level of risk).

In 1966 Pessemier (22), combining the product life cycle concept (figure 2-4) and the timing concept, shows the effects on investments,sales and profits of two different companies when they enter the market with similar products but at different times (fig.2-5). This figure shows how the success of a product entering the market will depend on the degree to which its entry leads or follows similar products. Company A, the first to go into the market, spends and risks more money than company B, but assuming good planning and management control, company A will get more profits, as shown in the figure.

Kotler (13) said that the first firm to enter the market will enjoy, if its product is perfected, a substantial advantage over the second one. It is estimated that the second firm to enter the market will achieve only half the sales of the first firm,and a third firm entering the market would achieve only half the sales of the second firm. He also points out that, when the firm which enters the market first

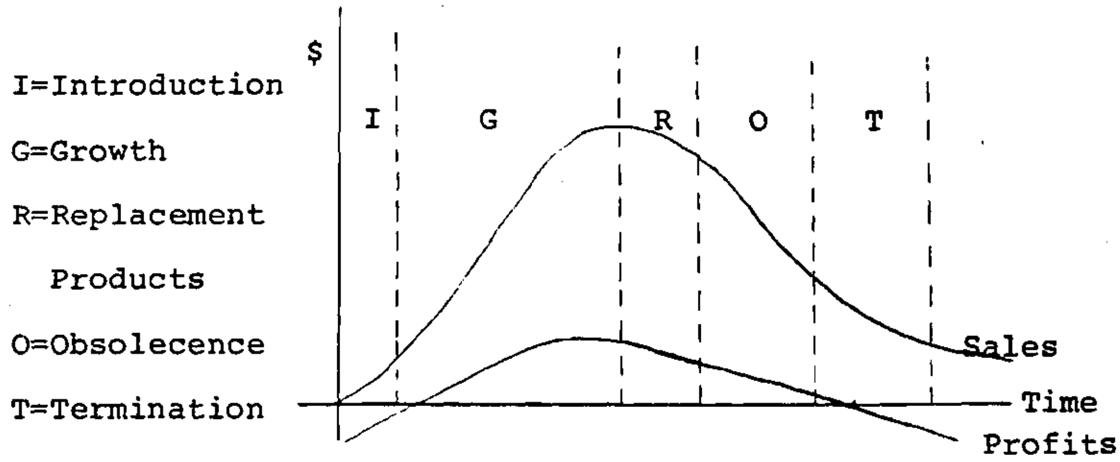


Figure 2-4. Typical Product Life Cycle

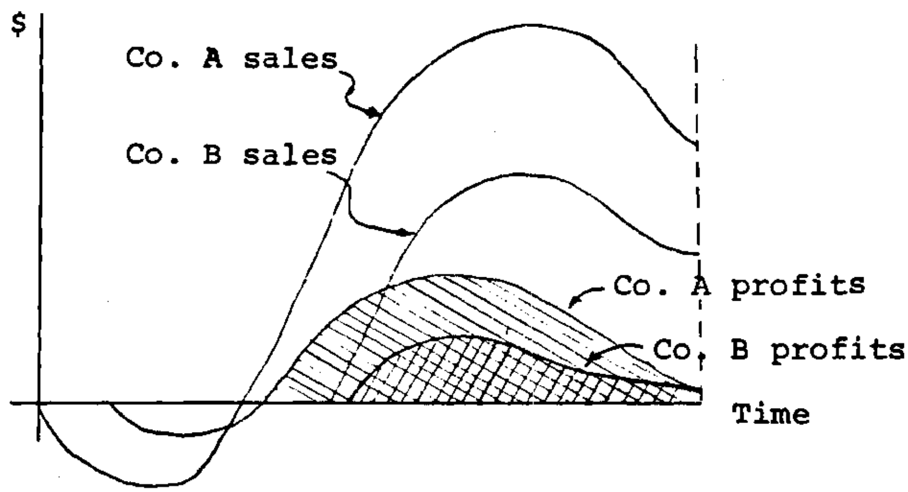


Figure 2-5. Illustration of the Effect of Timing of Entry of two Similar New Products Offered by Competing Companies.

has a poor version of the product, it may spoil its share of the market. The quality and/or suitability of the product is a function of the passage of time and the money spent on R&D.

Is clear then, that timing is important and the rewards from entering a market at an opportune moment can be substantial. However, it should be said that sometimes products are placed on the market prematurely and fail, losing the market leadership, and, sometimes even worse, going out of the market losing a great deal of money. This occurred in the Bowmar case: they had the initial advantage in the hand-held electronic computer market, and they lost it because of factors related to this "premature concept"(28).

The above covers the literature survey of the two principal areas upon which this work is based: capital budgeting and product development. In the next section the model used in this research will be established.

CHAPTER III

DESCRIPTION OF THE MODEL

Overview

In the evaluation of investment projects, whether new products or any other kind of investment porposal, the projected cash flow streams represent the major determinants of project worth in the evaluation process. Although in the past many decision makers assumed certainty conditions for analytic purposes, today many planners recognize that probabilistic formulations of project outcomes add considerable quantitative information for project evaluation and selection. However, this type of formulation introduces some additional problems not found in the deterministic case.

Before presenting the detailed methods related to this formulation, is necessary to describe the general model, including the sequential decision process, the linear programming model, and the assumptions made in the model, in order to give a clearer idea of the main purpose pursued throughout this research.

The general model, which is described in a flow chart in figures 3-1 and 3-2, begins with three cash flow estimates (the optimistic, most likely, and pessimistic ones), for each year for each project as principal data. This is done in the context of a fixed planning period with annual

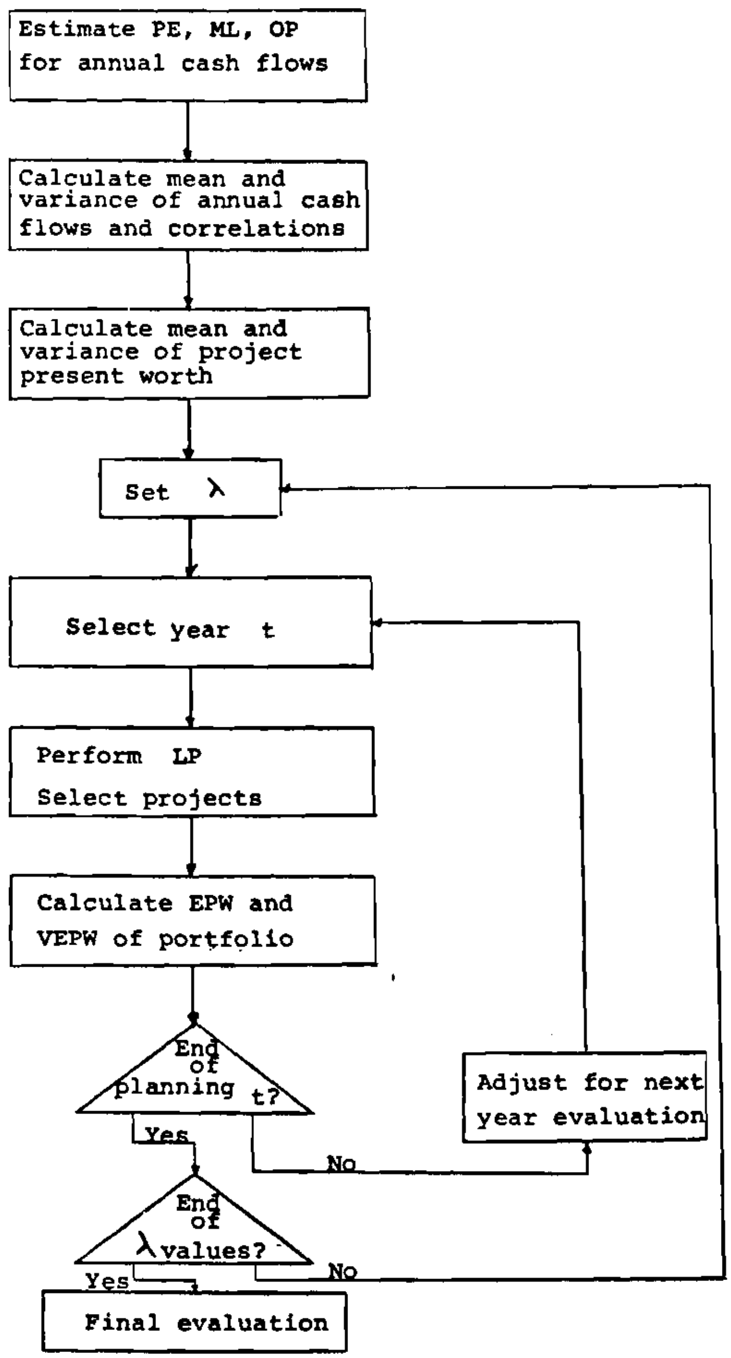


Figure 3-1. Analytical Solution.

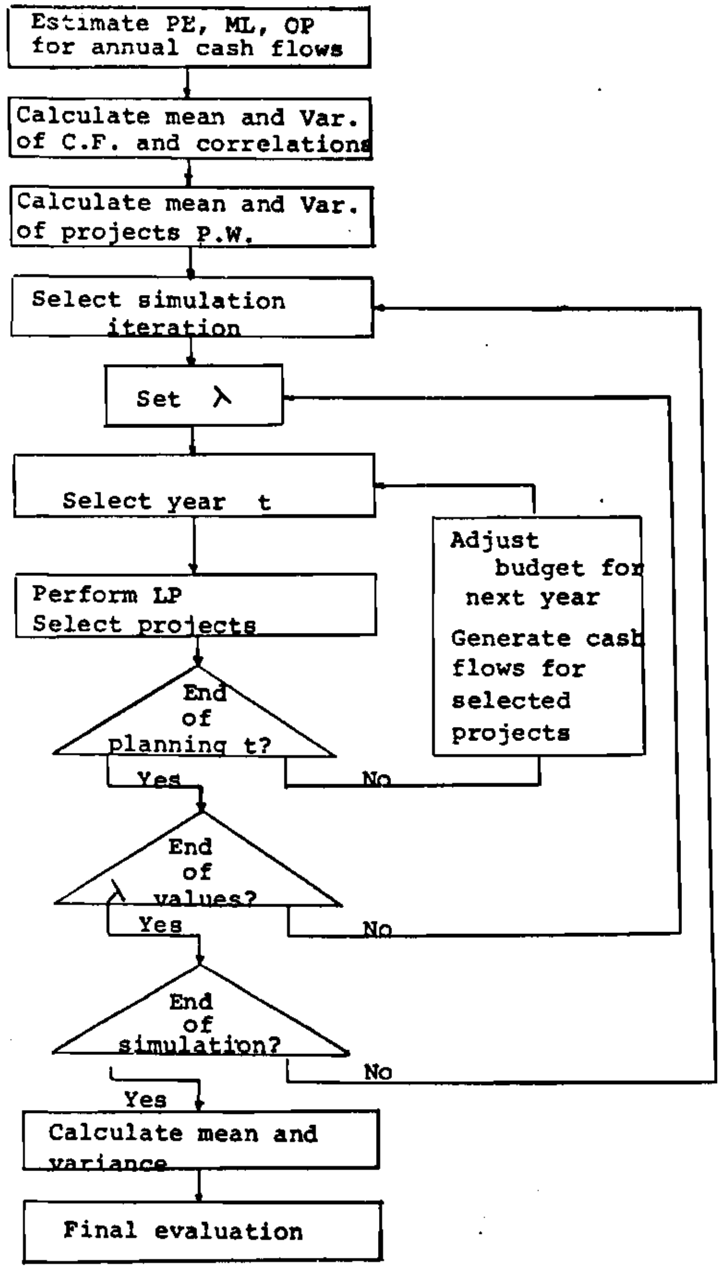


Figure 3-2. Simulation

investment decisions. Each proposal may be considered for selection during each of three years, the year in which the project is proposed for the first time, and the next two. In general, it is assumed that as the time of implementation is delayed, the expected value of the elements in the cash flow steams will decrease, and also their variability. This pattern is based on articles by Kotler(13) and Pessemier(22) about the marketing of new products.

It is convenient to assume a Beta distribution for each annual cash flow for each project. The mean and variance of the cash flows are readily calculated using well-known formulas. Then the expected present worth and the variance of the expected present worth is obtained for each project. However, in the calculation of the variance of the expected present worth, the model includes the different cases of correlated flows, which are explained in detail in the next section. The discount factor is assumed constant through time.

With all of this information a linear programming (LP) model is used as follows:

$$\text{Maximize: } \sum_{i=1}^{N} EPW_i x_i - \lambda \sum_{i=1}^{N} VEPW_i x_i \qquad (3\text{-}1)$$

$$\text{st: } \sum_{i=1}^{N} c_{ti} \;\; K_t \qquad \text{(Budget constraint)} \qquad (3\text{-}2)$$

Contingency Constraints (3-3)

$$0 \leq x_i \leq 1 \qquad (3\text{-}4)$$

All Vars$\geq$0 (3-5)

where: EPW_i=Expected present worth of project i.

$VEPW_i$=Variance of the expected present worth of project i.

λ=Risk aversion factor.

K_t=Budget in year t

From the LP model a set of projects is obtained for the first year and a value of the variance of present worth for the set of projects (portfolio) including cross-correlation effects. As was mentioned earlier, the non-selected projects are considered then with the projects of the next year and the sequential process is done through all the planning period (a project may be selected only once). Because of the fact that the LP model is not an integer programming algorithm, the decision process assumes an arbitrary x value, ie; $x\geq0.7$ for the acceptance of fractional projects. Deviations from the original budget are carried forward to the next year, assuming lending or borrowing at some interest rate i, as necessary. Project returns are assumed to be invested elsewhere in the company.

The model gives additional useful information for the decision maker, as: the amount of cash at every year, the amount of cash at the horizon, the total cost of the alternatives, etc. The solution procedure can be performed in two ways: analytically and simulated. Analytically means that the model will work with the values give by the parameters of the Beta distribution, ignore cross-correlation effects, and assume no budget deviations. Simulated implies using a Beta random number subroutine to simulate the cash flow values, including cross-correlation effects, and borrowing and lending to adjust for budget deviations.

One of the main advantages of the algorithm is that the decision maker can "play" with the sequential process. He can change the budget for every year, the value of the risk aversion factor (λ), and the decision rules for project acceptance (ie.; the model permits the selection of projects only in the first year, or the second, or in any of the first three years after the project is identified). With this the decision maker ends with a series of different alternatives, and each set of projects selected (portfolio) can be represented as a point on an "efficiency frontier". Therefore, depending upon the specific considerations of each firm (budget,aggressiveness,etc.) the selection of the investment alternatives can be made.

It is possible that some of the concepts just exposed here may not be very clear. The next chapter explains in detail how the model can be used. The rest of this chapter is dedicated to describing the theory upon which the sequential model is based.

Probabilistic Consideration of the Cash Flow

Assuming probabilistic conditions, the net present value of any project is a random variable. Considering a stream of random net cash flow increments A_{tj}, generated by a project j (j=1,2,3,...n) at times t (t=0,1,2...n) using i_k as a discount rate, the net present value of the cash flow stream will be:

$$NPV_j = \sum_{t=0}^{n} \left[\frac{A_{tj}}{\prod_{k=0}^{t} (1+i_k)} \right] \tag{3-6}$$

where NPV_j is the discounted net present value of project j.

A very common assumption in capital budgeting problems is the assumption of the discount rate i_k as constant over the planning period, and also known with certainty, reducing equation (3-6) to the form:

$$NPV_j = \sum_{t=0}^{n} \frac{A_{tj}}{(1+i)^t} \tag{3-7}$$

This is the formula used throughout the analysis.

Since a random process governs the values taken by A_{tj}, this can be represented by discrete or continous density functions such as those illustrated in figure 3-3. In figure 3-3 (a) the mass function $f(A_{tj})$ describes the relative frequency of each discrete value of outcomes, while in 3-3 (b) the expression

$$p(A_{tj})_{x,y} = \int_{A=x}^{y} G(A_{tj})\,dA \tag{3-8}$$

gives aproximately the relative frequency over a small range

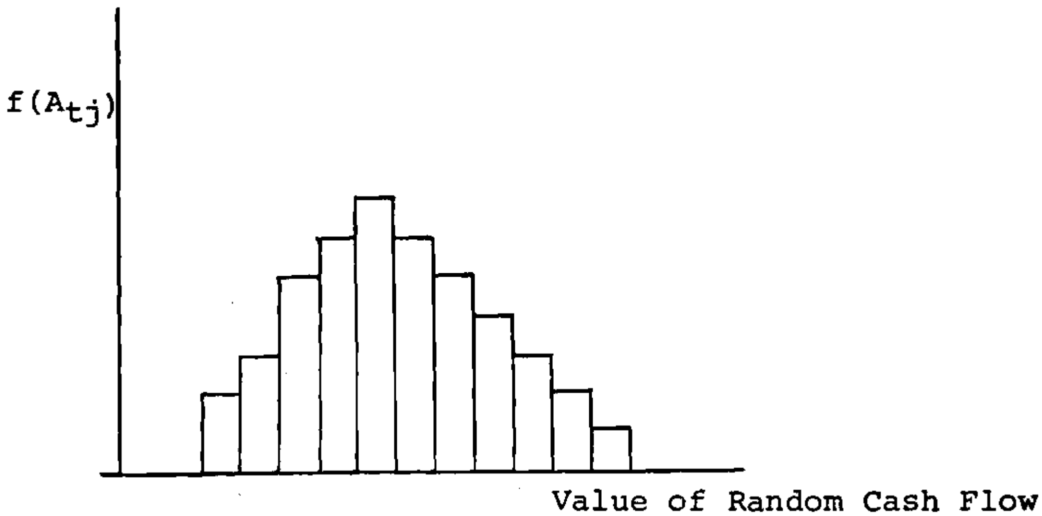


Figure 3-3(a) Probability Function for a Discrete Random Cash Flow.

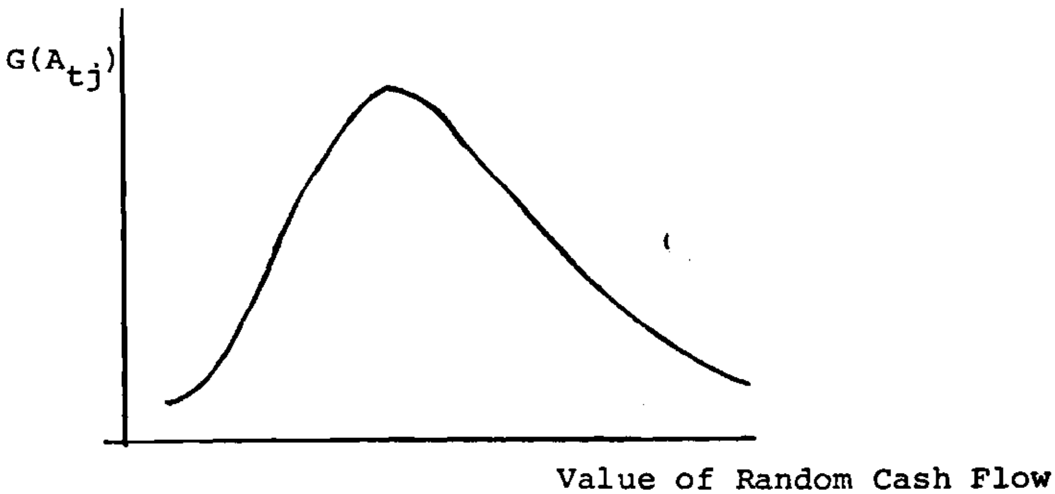


Figure 3-3(b) Probability Density Function for a Continuous Cash Flow.

of outcomes for a continuously distributed A_{tj}, where $G(A_{tj})$ is the probability density function of the random cash flow. It is convenient to represent each random cash flow using the mean and variance of a distribution, such as the Beta distribution. This approach, proposed by Wagle (34) and summarized by Hillier (10),has the advantage that it is patterned after the PERT technique, which has achieved considerable success in evaluation of research and development program schedules. Another advantage is that it is very easy to estimate the Beta distribution parameters. This technique needs three estimates by the analyst: an optimistic one, which represents a cash flow if the project goes as well as reasonably possible, a pessimistic one, assuming the project goes as poorly as reasonable possible, and a most like estimate. These three values are assumed to correspond to the upper bound, lower bound and the mode of the Beta distribution, respectively. This Beta distribution resembles a Normal distribution with two principal exceptions:

1) The Beta distribution is truncated at the tails, while the Normal distribution continues indefinitely.

2) The Beta distribution may be skewed right or left, instead of being symmetric as the Normal.

The second condition may be present because the most likely estimate may take any value between the other two estimates, depending upon the analyst's judgement, and not necessarily midway between the extreme bounds. Under these assumptions the mean and the variance of each cash flow element in any period t for a project j can be found by (10):

$$E(A_{tj}) = (1/6)(PE_{tj}+4ML_{tj}+OP_{tj}) \quad (3\text{-}9)$$

and

$$V(A_{tj}) = \left[(1/6)(OP_{tj}-PE_{tj})\right]^2 \quad (3\text{-}10)$$

where: $E(A_{tj})$ = mean of the cash flow for period t and project j.

$V(A_{tj})$ = variance of the cash flow for period t and project j.

PE_{tj} = pessimistic estimate of cash flow in period t and project j.

ML_{tj} = most likely estimate of cash flow in period t and project j.

OP_{tj} = optimistic estimate of cash flow in period t and project j.

Present Value Of Each Proposal

The general definition of the present value of a project is: the sum of the discounted cash flows throughout the project life. In the non-deterministic case the effect of randomness can be expressed through the mean and variance of the distribution of A_{tj}. The summation of these discounted random outcomes is also a random variable described by the formula (3-7), where NPV_j is the random net present value for project j, A_{tj} is the random cash flow in period t for project j, and i is the discount rate. So for the discrete case as well as the continuous one, the random net present value for the project will have a mean net present value $E(NPV_j)$ and a variance of net present value $V(NPV_j)$. This is very important, because it permits one to relate the unknown NPV_j to the random cash flow elements of the project. The mean net present value of the project is simply the sum of the discounted cash flow elements:

$$E(NPV_j) = \sum_{t=0}^{n} \frac{E(A_{tj})}{(1+i)^t} \tag{3-11}$$

On the other hand, the value of the variance will depend on the relationships among the cash flows of the project. Several kinds of this relationship may exist,ie: complete independence, complete dependence, partial dependence, and combinations of these.

Case Of Complete Independence

When the variability of a project outcome is due to random elements without any causative or consequential relationship with any other outcome in the cash flow stream, the cash flow for that project is said to be independent.

For this case the variance of the project net present value is obtained form the formula for the variance of the weighted sum of independent random variables (29).

$$Var(ax+by) = a^2 Var(x) + b^2 Var(y) \tag{3-12}$$

or

$$V(NPV_j) = \sum_{t=0}^{n} \frac{\sigma^2 A_{tj}}{(1+i)^{2t}} \tag{3-13}$$

where $\sigma^2_{A_{tj}}$ = variance of the t^{th} cash flow element, project j.

Case Of Complete Dependence

Complete dependence, or perfect correlation, exists when the random cash flows have a "one to one" relationship among events in succeding periods, ie.: marketing expenses varying directly with sales.

The mean net present value $E(NPV_j)$ is calculated exacly the same way for the independent case, because the present value does not depend on the dependence-independence assumptions. To calculate the variance of the net present value it is necessary to use the relation:

$$Var(ax+by) = a^2 Var(x) + b^2 Var(y) + 2ab Cov(x,y) \quad (3\text{-}14)$$

Considering that;

$$Cov(x,y) = \rho_{xy}\sigma_x\sigma_y \quad (3\text{-}15)$$

the variance can be found as follows:

$$V(NPV_j) = V(A_{0j}) + \frac{V(A_{1j})}{(1+i)^2} + \frac{V(A_{2j})}{(1+i)^4} + \ldots + \frac{V(A_{nj})}{(1+i)^{2n}} + \frac{2Cov(A_{0j},A_{1j})}{(1+i)} + \frac{2Cov(A_{0j},A_{2j})}{(1+i)^2} + \ldots + \frac{2Cov(A_{n-1},A_{nj})}{(1+i)^{2n-1}} \quad (3\text{-}16)$$

substituting $\sigma^2_{tj} = V(A_{tj})$

$$V(NPV_j) = \sum_{t=0}^{n} \frac{\sigma_{tj}}{(1+i)^{2t}} + 2 \sum_{x=0}^{n-1} \sum_{y=0}^{n} \frac{\rho_{x_j y_j} \sigma_{x_j} \sigma_{y_j}}{(1+i)^{x+y}} \tag{3-17}$$

where $\rho_{xy}=1$ because of perfect correlation. Then the calculation is reduced to:

$$V(NPV_j) = \left[\sum \frac{\sigma_{tj}^2}{(1+i)^t} \right]^2 \tag{3-18}$$

Case Of Partial Dependence

There are cases when the outcomes of a project are neither independent nor perfectly correlated. This is the case of partial correlation. The mean net present value does not represent any problem, and it is calculated by the same formula used before. For the calculation of the variance the formula used is equation (3-17). However, in this case ρ_{xy} is not one any more, so the problem is to find a good way to estimate ρ_{xy}. Using two common restrictive assumtions, this calculations became fairly simple.

Assumption 1: The random variables are Markov-dependent through time. In other words, whatever influences the cash flow in period t, derives only from the preceding period t-1, so the partial correlation between lag time periods of two or more is zero.

Assumption 2: The correlation coefficient for the cash flow in time t and the cash flow in t-1 is the same as for the cash flow in time t+a and the cash flow in time t+a-1.

Then, using some early work by Mood and Cramer (18,6), and assuming that A_{0j} and A_{1j} are partially correlated, with a given value for A_{0j}, then the estimate of the expected value of A_{1j} given A_{0j} is:

$$E(A_{1j}|A_{0j}=x)=E(A_{0j})+\rho_{A_{0j}A_{1j}}\left(\frac{\sigma_{A_{1j}}}{\sigma_{A_{0j}}}\right)(x-E(A_{0j})) \quad (3\text{-}19)$$

then

$$\frac{E(A_{1j}|A_{0j}=x)-E(A_{1j})}{\sigma_{A_{1j}}}=\rho_{A_{0j},A_{1j}}\left(\frac{x-E(A_{0j})}{\sigma_{A_{0j}}}\right) \quad (3\text{-}20)$$

By obtaining estimates of A_{1j} conditional on A_{0j}, an estimate of the correlation coefficient can be made. The deviation of A_{1j} from its unconditional mean is related to the deviation of the given value x of A_{0j} from the unconditional mean for A_{0j}, by the correlation coefficient $\rho_{A_{0j},A_{1j}}$. Mood points out that if A_{0j} and A_{1j} are bivariate normal, the procedure gives the best unbiased estimate of ρ. Cramer says that otherwise it gives the best linear estimate according to the principle of least squares (18,6).

To use the method, it is necessary to select given values of A_{0j} and then estimate the expected values of A_{1j} given the A_{0j}'s. It is helpful to select the given values of A_{0j} as being 3σ above and below $E(A_{0j})$, and then use the formula for estimating the mean of a Beta distribution. It is possible to average all the resulting values of ρ's and then construct the correlation matrix.

$$
\begin{array}{c|cccccc|}
t= & 0 & 1 & 2 & 3 & \cdots & n \\
0 & 1 & \rho & \rho^2 & \rho^3 & & \rho^n \\
1 & \rho & 1 & \rho & \rho^2 & & \\
2 & \rho^2 & \rho & 1 & \rho & \cdots & \cdots \\
\vdots & & & & & \ddots & \vdots \\
n & \rho^n & & \cdots & \cdots & & 1
\end{array}
$$

With this correlation matrix, and equation (3-17), the variance of the project net present value for partially correlated cash flows is obtained.

Case Of Independence And Partial Or Perfect Correlation.

Sometimes it is possible to have the initial investment of a project j independent of the rest of the cash flows stream, but at the same time, the remaining cash flow stream may be partially or perfectly correlated itself. In this case, like in the others, the mean net present value is found exactly the same way, by adding the discounted cash flow elements. However,the calculation of the variance of project net present value has some minor changes.

I) Initial investment independent, and the rest of the cash flow stream partially correlated.

The only difference in the calculation of the variance in this case, with respect to the case in which all the cash flows are partially correlated, is in the values of the first column and the first row of the correlation matrix. Here, both the first column and the first row take values of zero with the exception of the first element, which is 1. The rest of the calculations are exacly the same.

II) Initial investment independent and the rest of the cash flow stream perfectly correlated.

A simple combination of the perfect independence situation and the perfectly correlated case is used to obtain the variance of the net present value in this case. The resulting expression is:

$$V(NPV_j)=V(A_{0j})+\left(\sum_{t=1}^{n} \frac{\sigma_{A_{tj}}}{(1+i)^t}\right)^2 \qquad (3\text{-}21)$$

<u>Correlation Between Projects j And k.</u>

Sometimes the projects can be affected in their cash flows by changes in economic or political conditions. When this happens it is said that the net present value of projects j and k are cross-correlated. For the projects which are affected one can pairwise combine the statistical parameters into one set,(one for the mean and one for the variance) for each pair.

The calculation of the mean net present value for such pair is fairly simple; just add the two net present values of the projects.

$$E(NPV_{j,k})=E(NPV_j)+E(NPV_k) \qquad (3\text{-}22)$$

In the computation of the variance, the case of independence or dependence between projects must be considered. The mathematical calculations for these two cases is not simple. However, assuming two conditions the computation can be straightfoward.

Assumption 1: The economic or political conditions will push the cash flows up or down simultaneously, rather than in different time periods. This permits one to assume the correlation coefficient ρ_{ikt} to be the same between projects j and k.

Assumption 2: This correlation coefficient will be the same through time ($\rho_{jkt} = \rho_{jkt+1}$ for t=1,2,3,...n). Thus we need only to define ρ_{jk}.

With these two assumptions, and by methods analogous to equation 3-17, the variance is expressed as;

$$V(NPV_{jk}) = \sum_{t=0}^{n} \frac{\sigma_{tj}^{2}}{(1+i)^{2t}} + \sum_{t=0}^{n} \frac{\sigma_{tk}^{2}}{(1+i)^{2t}} + 2\rho_{jk} \sum_{t=0}^{n} \frac{\sigma_{tj}\sigma_{tk}}{(1+i)^{2t}} \qquad (3\text{-}23)$$

From this formula, the assumption of independence or dependence between the projects will just change the last term of expression 3-23. For cross-correlated projects ρ_{jk} will take values between 0 and 1 depending upon the degree of correlation. For projects not cross-correlated ρ_{jk} will be equal to zero and the equation 3-23 reduces to:

$$V(NPV_{jk}) = \sum_{t=0}^{n} \frac{\sigma_{tj}^2}{(1+i)^{2t}} + \sum_{t=0}^{n} \frac{\sigma_{tk}^2}{(1+i)^{2t}} \quad (3\text{-}24)$$

Case Of Time-Wise And Project-Wise Correlation

In this case the correlations occur not only within the cash flow streams of two different projects(auto-correlation), but also between the cash flow elements of the projects (cross-correlation).

The mean net present value is just the sum of the mean cash flow elements of both projects, equation 3-22. By the combination of the formulas used in the preceding cases, recalling that the auto-correlations are Markovian and the cross-correlations are zero lagged, the formula for the computation of the variance of net present value is:

$$V(NPV_{jk}) = \sum_{t=0}^{n_j} \frac{V(A_{tj})}{(1+i)^{2t}} + \sum_{t=0}^{n_k} \frac{V(A_{tj})}{(1+i)^{2t}} +$$

$$2\rho_{jk} \sum_{t=0}^{\min(n_j,n_k)} \frac{\sigma_{tj}\sigma_{tk}}{(1+i)^{2t}} +$$

$$2\sum_{\substack{x=0 \\ x<y}}^{n_j-1} \sum_{y=1}^{n_j} \frac{\rho_{xy,j}\sigma_{xj}\sigma_{yj}}{(1+i)^{x+y}} +$$

$$2\sum_{\substack{x=0 \\ x<y}}^{n_k-1} \sum_{y=1}^{n_k} \frac{\rho_{xy,k}\sigma_{xk}\sigma_{yk}}{(1+i)^{x+y}} \quad (3\text{-}25)$$

This completes the exposition of the correlated model used as a part of the overall decision model developed in this work. In the next chapter a detailed description of the solution procedure will be presented.

CHAPTER IV

SOLUTION PROCEDURE AND COMPUTATIONAL EXPERIENCE

Project Generation And Input Data

For testing the solution procedure three sets of projects were generated. Each one assumed fifteen new investment alternatives every year, with a project life of ten years, and a planning period of five years. Based on marketing literature, the expected return of a project and its variability were assumed to be decreasing functions of the delay in acceptance of the project. With this in mind, three cash flow estimates for each proposal were made. A complete list of the input data needed for the algorithm, as well as the parameters used, follows:

A) Pessimistic, most likely and optimistic estimates of annual project cash flows.

B) Number of projects:
Fifteen new investments available every year.

C) Time horizon:
The tenth year.

D) Autocorrelation coefficient x:
$0 \leq x \leq 1$ distributed roughly according to a uniform distribution.

E) Initial investment coefficient y:

[illegible]

Parameter used: I) Initial investment independent of the rest of cash flows: y=0

II) Initial investment with the same correlation as the rest of cash flows: y=1

F) Decision rules for project selection:

Parameter used: FR=0 Project can be selected in any of the first three years after becoming available.

FR=1 Project can be selected only in the first year.

FR=2 Project can be selected only in the second year.

G) Analytical or simulated solution:

Parameter used: ANA=0 means analytical solution.

ANA=1 means simulated solution.

H) Risk-aversion factor:

Parameter used: Lambda value in the objective function of the LP model.

I) Delta for lambda values:

A delta of 0.25 was used, which means that the values range from 1 to 0, ie.: 1.0,0.75,0.5, 0.25,0.0.

J) Annual investment budget:

Parameter used: $2000,$4000,$6000 for the first and second project set.

\$500,\$1000,\$1500 for the third project set.

K) Discount rate:

10%

L) Cross-correlation index w;

$0 \leq w \leq 1$ distributed roughly according to a uniform destribution.

M) Contingency constraints:

In a matrix form, 5 or 6 constraints per year.

Program Language

Two programs were used to solve the problem, both coded in Fortran IV for use on the CDC Cyber 74 at the Georgia Institute of Technology. (Appendix B).

Characteristics Of The Projects

Table 4-1 shows a sample of projects used to test the solution procedure. Through these two projects it is shown how the value of the cash flows decrease as the acceptance of the projects is delayed one or two years. Equations (3-9) and (3-10) were used to obtain the mean and variance of the cash flows in each year as follows:

Project 1 t=1

$$E(A_{tj})=(1/6)(PE_{tj}+4ML_{tj}+OP_{tj})$$

$$E(A_{tj})=(1/6)(-710+4(-700)-600)=-698.33$$

and

$$V(A_{tj})=((1/6)(OP_{tj}-PE_{tj}))^2$$

$$V(A_{tj})=[((1/6)((-680)-(710))]^2=25$$

Table 4-1. Example of Project Cash Flows, Means and Variances.

PROJECT 1
Acceptance in:

	FIRST YEAR			SECOND YEAR			THIRD YEAR			F.Y.		S.Y.		T.Y.	
t	PE	ML	OP	PE	ML	OP	PE	ML	OP	E	V	E	V	E	V
0	-710	-700	-680	0	0	0	0	0	0	-698	25	0	0	0	0
1	580	600	615	-700	-690	-685	0	0	0	599	34	-690	6	0	0
2	580	600	615	285	300	310	-653	-650	-640	599	34	299	17	-648	4
3	480	500	515	285	300	310	145	150	155	499	34	299	17	150	2
4	480	500	515	235	250	260	170	175	180	499	34	249	17	175	2
5	980	1000	1015	235	250	260	140	150	155	999	34	249	17	149	6
6	880	900	915	480	500	510	140	150	155	899	34	498	25	149	6
7	870	900	915	380	400	410	235	250	255	899	56	398	25	248	11
8	870	900	915	480	500	510	185	200	205	897	56	498	25	198	11
9	870	900	915	480	500	510	235	250	255	897	56	498	25	248	11
10	870	900	915	480	500	510	235	250	255	897	56	498	25	248	11

PROJECT 2
Acceptance in:

	FIRST YEAR			SECOND YEAR			THIRD YEAR			F.Y.		S.Y.		T.Y.	
t	PE	ML	OP	PE	ML	OP	PE	ML	OP	E	V	E	V	E	V
0	-220	-200	-190	0	0	0	0	0	0	-201	25	0	0	0	0
1	490	500	520	-205	-200	-195	0	0	0	501	25	-200	2	0	0
2	490	500	520	297	300	305	-203	-200	-196	501	25	300	1	-199	1
3	490	500	520	297	300	305	148	150	152	501	25	300	1	150	0
4	485	500	520	296	300	306	148	150	152	500	34	300	2	150	0
5	485	500	520	296	300	305	148	150	153	500	34	300	2	150	0
6	485	500	525	295	300	305	147	150	153	501	44	300	2	150	1
7	485	500	525	295	300	305	146	150	153	501	44	300	2	149	1
8	480	500	525	293	300	305	145	150	153	500	56	299	4	149	1
9	480	500	525	293	300	305	144	150	153	500	56	299	4	141	2
10	480	500	525	290	300	305	143	150	153	500	56	299	6	149	2

PE=Pessimistic value
ML=Most likely value
OP= Optimistic value
E=Mean
V=Variance

F.R.=First year
S.Y.=Second year
T.Y.=Third year

where A_{tj}= Stream of random net cash flows generated by a project j at the end of present and future time periods t.

PE_{tj}=Pessimistic estimate of cash flow in period t and project j.

ML_{tj}=Most likely estimate of cash flow in period t and project j.

OP_{tj}=Optimistic estimate of cash flow in period t and project j.

Various patterns of project cash flows were made (figure 4-1): good at the begining, uniform, variable, good at the end, etc., in order to test the procedure under realistic circumstances.

In order to make clearer the characteristics of each project set, and to help understand some of the results obtained in the computational experience, a variability ratio is defined as:

$$VR=\frac{1/n \sum_{j=1}^{n} \text{(Variance of total expected present worth for project j)}}{1/n \sum_{j=1}^{n} \text{(Total expected present worth for project j)}}$$

Thus, for each project set:

Set 1; VR=4973/1131=4.38

Set 2; VR=7154/997=7.17

Set 3; $VR=2.75*10^{10}/1164=2.36*10^{7}$

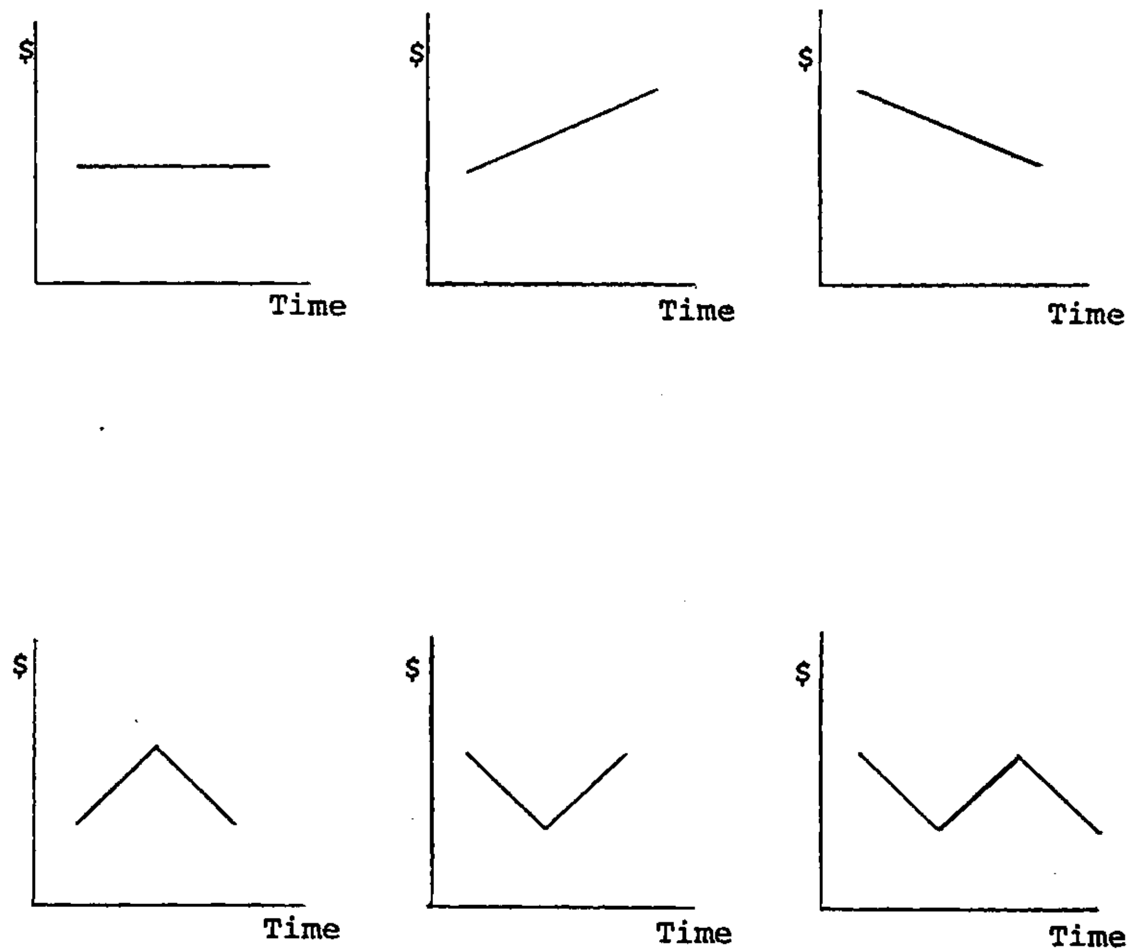


Figure 4-1 Different Types of Project Cash Flow Patterns Used.

These numbers point out clearly the high degree of variability of project set 3 compared with project sets one and two.

Flexibility Of The Algorithm And Program Runs

Before going into the details of the analytical and simulation procedures, it is important to show the flexibility of the algorithm for changing key values. This enables the analyst to obtain a wide span of decision environments. This flexibility is presented in figure 4-2. After the decision maker has obtained the three basic estimates of the cash flows, he can easily change the following items:

A) Decision rules for project selection.

B) Risk-aversion factor (lambda value).

C) Annual budget.

D) Solve analytically or simulate.

Table 4-2 shows how the analysis was structured, presented in the format of a fractional design of experiments, in order to perform the program runs and obtain meaningful comparative results. Thus, cell 1 represents the program values obtained when I) The projects may be selected in their first, second or third year, II) The lambda value in the objective function of the LP model is 1, III) The annual budget is $2000 and IV) The first set of projects is used. The total number of cells obtained is given by:

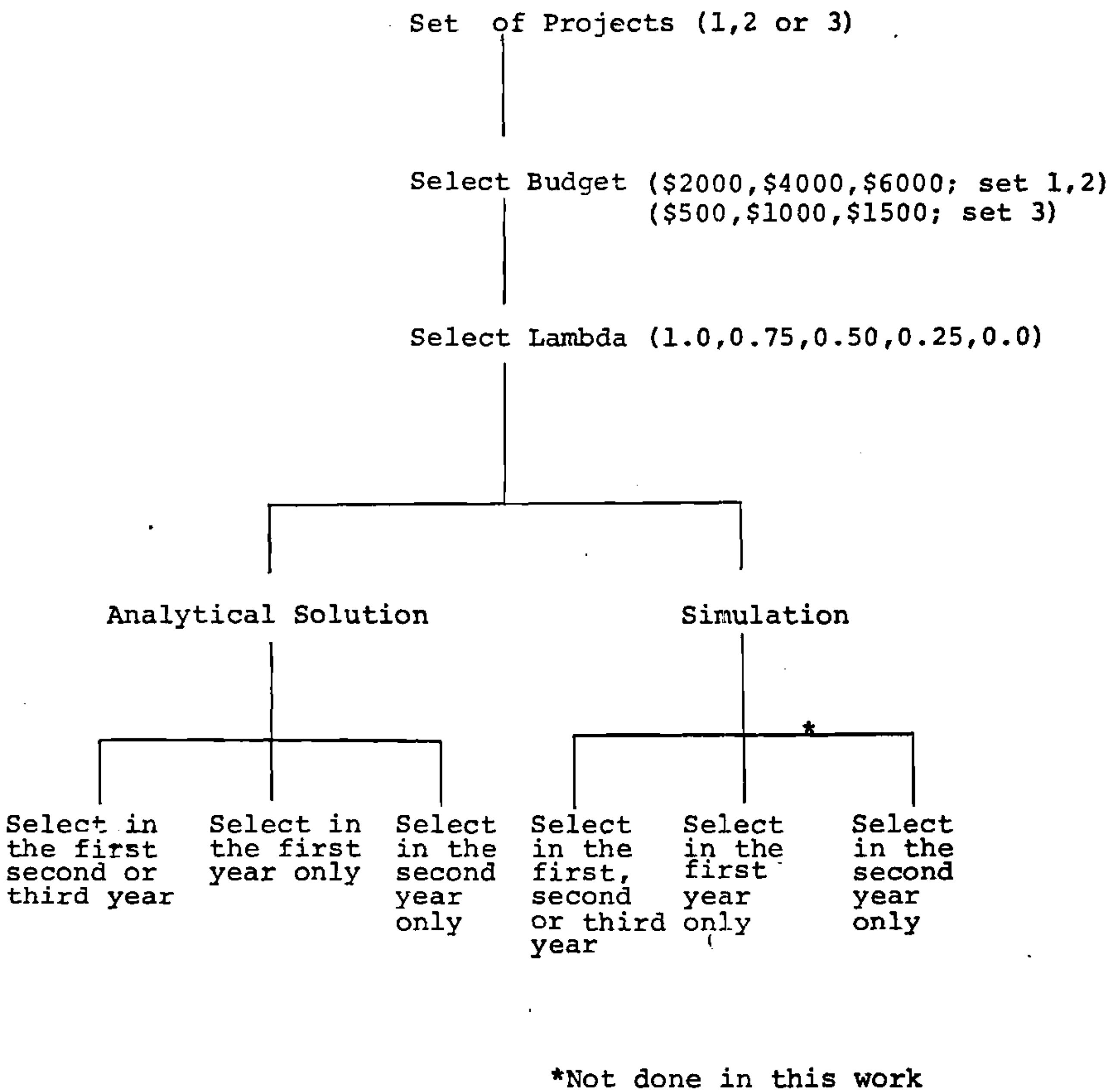


Figure 4-2. Flexibility of the Algorithm.

Table 4-2. Structure of The Analysis.

Lambda	Decision Rule 1 Projects can be selected in the first, second or third year.							Decision Rule 2 "only first year"	Decision Rule 3 "only second year"
1	Budget 1			Budget 2			Budget 3		
	Set 1	Set 2	Set 3	Set 1	Set 2	Set 3			
	cell 1	cell 2	cell 3	cell 4	cell 5	cell 6			
.75									
. . . . . .									
0									

$$\begin{bmatrix}\text{Decision rules} \\ \text{for project} \\ \text{selection (A)} \\ \text{thus;}\end{bmatrix} * \begin{bmatrix}\text{Lambda} \\ \text{values} \\ \text{(B)}\end{bmatrix} * \begin{bmatrix}\text{Initial} \\ \text{budget} \\ \text{(C)}\end{bmatrix} * \begin{bmatrix}\text{Set of} \\ \text{projects}\end{bmatrix} = \text{Number of cells} \quad (4\text{-}1)$$

(D)

(3)*(10)*(3)*(3)= 270 cells.

Each cell represents a five-year planning period. Consequently five LP problems are solved per cell. Therefore, the total number of LP's solved is:

270 cells * 5 LP/cell= 1350 LP's.

For simulated solutions this number is given by equation 4-1 times the 5 LP/cell,times the number of simulations.

Therefore, for set 1:

From 4-1 A=3, B=10, C=3, D=1, simulations =20

thus,

(A)*(B)*(C)*(D)*5*20= 9000 LP's solved

for set 2:

From 4-1 A=3, B=10, C=3, D=1, simulations =20

thus,

(A)*(B)*(C)*(D)*5*20= 9000 LP's solved

for set 3:

From 4-1 A=2, B=3, C=1, D=1, simulations =50

thus,

(A)*(B)*(C)*(D)*5*50= 1500 LP's solved.

Computational Experience

Changing the values of the parameters mentioned above according to figure 4-2 and table 4-2, computational experience was obtained with the three sets of generated projects.

Analytical Results

The results obtained from the analytical solution are summarized in tables A-1 through A-9 (appendix). Observing these tables and the behavior of the total expected present worth (TEPW) , its standard deviation(SD), the total cost of each alternative (TC) and the amount of cash at the horizon(CH), as a function of each of the parameters, some major conclusions can be drawn.

I) Effect of Changing the Decision Rules for Project Selection.

For the three sets of projects, the largest amounts of total expected present worth and cash at the horizon were obtained when the program is allowed to select projects in "the first, second or third year", followed by "only the first year", and "only the second year" decision rules, in that order. This result would be expected from an optimal selection procedure.Also, it was found that the total investment cost of each project portfolio is not very sensitive to changes in the decision rules. Thus, the cost of each strategy is almost the same for the same values of all other parameters. Furthermore, in some cases these values were lower for the "first, second or third year" than for the other two decision rules. On the other hand, the values obtained for the standard deviation behave as expected: the largest values are for the portfolios with the largest amounts of money. Generally, the results show that the strategy of always being the first in the market, or being aggresive and accepting only projects in the first year, may not give the highest expected returns. These results are shown in table 4-3 and tables A-1 through A-3.

II) Effect of Changing the Value of Lambda.

A singular result, obtained only because of the specific structure of project sets one and two, was the conclusion that being totally indifferent to risk would always be the best strategy. Comparing the 3σ limits of each possible choice, for these two sets, the selection of the 0 lambda value is in all cases the best strategy. In the first set of projects, table A-1 shows that with a budget of \$6000, the total expected present worth for $\lambda=0.25$ is \$180,300, with a standard deviation of 257. The corresponfing values for $\lambda=0$ are \$200,700 total expected present worth with standard deviation of 1259. Then, according to statistical principles, the firm might receive with $\lambda=0$:

Amount	Probability	Limits
\$200,700 $\pm$ 1259	63.3%	σ
\$200,700 $\pm$ 2591	95.0%	2σ
\$200,700 $\pm$ 3885	99.8%	3σ

Thus, the worst thing that could happen for the firm is to

Table 4-3. Selected Results for Project Set 1, Budget of $6000.

	Select in 1st, 2nd, or 3rd year	Select in 1st year only	Select in 2nd year only
TEPW			
λ=0.75	156,600	152,600	91,200
λ=0.25	180,300	164,900	107,500
CH			
λ=0.75	492,000	460,700	301,700
λ=0.25	532,700	493,300	342,000
TC			
λ=0.75	29,600	30,700	30,000
λ=0.25	30,100	30,200	28,900
SD			
λ=0.75	149	220	151
λ=0.25	256	305	246

TEPW= Total expected present worth
CH = Cash at the horizon
TC = Total cost
SD = Standard deviation

receive \$196,775 (200,660-3σ), which is better than the best value for lambda 0.25, which is 181,100 (180,300+3σ). This always happens in project sets 1 and 2. Therefore, in such cases the projects selected with $\lambda=0$ are always better than the projects for all other values of lambda.

However, this is not true for project set three. The projects in this set have a significantly greater variability in their cash flows than the first two sets. Therefore, the selection of the strategy will depend on the degree of risk the decision maker allows in his selection process. Here, for example, with an annual budget of \$1000 and the "first, second or third year" decision rule (table A-7), the decision maker will have the following alternatives:

Lambda values	Total expected present worth	Limits σ	2σ	3σ
1.00	65,464	± 5,275	±10,550	±15,825
.75	68,205	± 5,832	±11,727	±17,590
.50	75,367	±10,681	±21,363	±32,045
.25	75,367	±10,681	±21,363	±32,045
.00	90,027	$\pm 2 * 10^6$	$\pm 5 * 10^6$	$\pm 8 * 10^6$

One thing can definitely be concluded: the value of $\lambda=0$ is not likely to be chosen by any decision maker because of its high degree of variability, or risk. Also, it can be observed that for the lambda values of 0.5 and 0.25, there is no difference in the table values. This kind of behavior was found also for the other two decision rules, "only the first year" and "only the second year", of this third project set. Furthermore, for these two last ones the values were also the same for $\lambda=0.75$, which means that this project set is not highly sensitive to intermediate values of lambda.

Another criterion that may help the decision maker is the amount of cash at the horizon and the total cost of each project portfolio. In most cases both of them increase as the lambda value decreases from one to zero.

III) Effect of Changing the Annual Budget.

Here, the three project sets behave in the same way as the annual budget increases, from $2000 to $6000 for the first and second sets, and from $500 to $1500 for the third. The total expected present worth and the amount of cash at the horizon increase, while keeping the same values of lambda and the same decision rules for project selection. This is a logical result, because as the budget increases, more projects can be selected. Consequently, the increments in the values of the total expected present worth and cash at the horizon occur.

However, an important observation is that, even though the standard deviations change in the same directions as the expected present worth and cash at the horizon, the increment in this value (the standard deviation) is by far smoother than the other values, as shown in the following example:

Project set	Increase in Budget	Lambda λ	Increase in CH	Increase in TEPW	Increase in SD
1	4,000	0	317,900	106,000	78
2	4,000	0	291,600	85,600	600
3	1,000	0.5	106,600	36,800	560

On other hand, the sensitivity of project sets one and two, measured by changes in the project portfolio, to changes in the lambda value was found to be higer as the initial budget increased. In some cases where the budget was $2000 the projects selected were the same for lambda values of 1.0,0.75, and 0.5.

This behavior can be explained by the thightness of the budget at small amounts: it does not easily permit changes in the projects selected. However, as the budget is increased, the number of projects eligible for selection also increases, making the lambda value important in the selection process. However, this did not happen with the third project set; this set was always insensitive, as mentioned earlier, to intermediate values of lambda, despite the budget amount.

IV) Finally, observing the tables, in can be seen that some values do not follow the general behavior of the others, **ie.**: in table A-5 the lambda value of 0.75 gives lower expected values than the lambda value of 1.0: 47,500 versus 49,900 for a budget of 2000, etc., these cases are due to the approximation made by the linear programming model used throughout work in the selection process. All project variables with value grater than of equal to 0.7 were rounded to 1, and values less than 0.7 were rounded to zero.

Simulation Results

I) Simulation of Project Sets 1 and 2

A simulation was performed for two of the decision rules for project selection, "first, second or third year" and "first year only", for project sets one and two (see figure 4-2). The process was simulated 20 times each for most of the possible selection alternatives (20A,27,30A);some alternatives were excluded because of insensitivity to parameters.

The complete results obtained from this simulation are given in tables A- 9 to A-13, and selected results are shown in table 4-4. Comparing the values obtained in the anlytical solution with those obtained in the simulation, some differences can be observed. This raises some questions, as: are the differences significant?,why do they exist?, which method, analytical or simulation, is better?. Before trying to answer these questions some statistical principles are reviewed.

In the problem formulation both the total expected present worth and the cash at the horizon are random variables which are sums of Beta distributed variables. The Central Limit Theorem states that if a random variable M may be represented as the sum of n independent random variables, then for a sufficiently large n , M is approximately Normally

Table 4-4. Comparison of Selected Results for Project Set 1, Budget of \$6000, Select in the 1st, 2nd, or 3rd Year Decision Rule.

	Analytical Results	Simulation Results
TEPW		
λ= 1.00	156,600	156,645
λ= 0.75	164,900	166,800
λ= 0.50	171,500	172,200
λ= 0.25	180,300	180,300
λ= 0.00	200,600	200,900
CH		
λ= 1.00	470,700	471,000
λ= 0.75	492,000	497,000
λ= 0.50	509,600	512,100
λ= 0.25	532,700	532,900
λ= 0.00	585,300	586,500

TEPW= Total expected present worth
CH = Cash at the horizon

distributed(37A). For correlated random variables, M can also be considered Normally distributed(10). Both the total expected present worth and the cash at the horizon thus behave as Normally distributed. Assuming that the simulation provides a sample of size 20, it is possible to perform a Test of Hypothesis for each case, the TEPW and CH,

$$H_o: \; u = u_o$$

$$H_1: \; u \neq u_o$$

with a t distribution(due to the size of the sample). The results of these tests are in table 4-5.

Now, after the statistical principles have been reviewed, the comparison between the analytical results and the simulations can be made.

Analytical Solution Vs. Simulation For Sets 1 And 2

The results obtained from the Hypothesis Tests show that the differences between the analytical and simulation procedures are significant in most cases at levels of $\alpha=0.01$ or $\alpha=0.05$ (see table 4-5). There are two major reasons which explain this type of behavior:

1) During the simulation the variance of the total expected present worth is calculated including the cross-correlation between projects. This is not done in the analytical procedure.

2) During the simulation the amount of money available for subsequent annual budgets may change according to the random values obtained from the project cash flows.

Table 4-5. Hypothesis Tests for the Simulation of Sets 1 and 2.

Set 1 Projects 1st, 2nd, or 3rd year.

λ	TEPW	CH	TEPW	CH	TEPW	CH
1.00	-	-	0.00	12.95**	0.00	3.52**
.75	-	-	4.35**	3.49**	6.90**	9.66**
.50	45.8**	12.7**	4.35**	3.43**	4.67**	6.35**
.25	4.3**	3.9**	4.04**	3.87**	0.00	6.05**
.00	4.4**	3.9**	4.25**	0.17	4.35**	3.66**

Set 1 Projects "only the first year"

1.00	-	-	0.08	2.03*	0.00	1.94*
.75	-	-	4.37**	5.77**	4.40**	1.95*
.50	4.36**	3.52**	4.36**	3.47**	0.00	3.25**
.25	4.35**	4.34**	1.45*	1.63	4.36**	0.72
.00	6.34**	11.01**	4.47**	3.67**	4.40**	1.15

Set 2 Projects 1st, 2nd, or 3rd year.

1.00	-	-	5.42**	4.90**	3.27**	16.29**
.75	-	-	3.97**	3.07**	3.04**	3.84**
.50	0.00	3.72**	0.00	0.30	7.76**	1.40*
.25	0.00	3.27**	0.27	1.20	4.16**	3.56**
.00	4.20**	4.24**	0.97	2.04*	18.19**	18.10**

Set 2 Projects "only the first year"

1.00	-	-	0.00	1.80*	0.00	1.94*
.75	-	-	0.00	1.06	0.00	2.70**
.50	4.36**	9.88**	4.34**	2.85*	0.00	6.16**
.25	6.77**	6.34**	0.00	0.87	0.00	0.32
.00	4.36**	6.70**	4.39**	17.62**	4.36	3.72**

$H_0: u=u_o$ $\qquad t=\frac{x-u}{s/\sqrt{n}}$ $\qquad$ Critical values:

$H_1: u \neq u_0$

5% t=1.753 (*)
1% t=2.600 (**)

This could change the projects selected and the cash at the horizon.

A very interesting, and important, result obtained with these two project sets is the fact that the project portfolios selected by the analytical procedure are nearly the same as those chosen by the simulation. As an example of this behavior tables B-1 and B-2 in the appendix show in vector form the projects selected by the analytical procedure for two decision environments:

I) Project set two, budget \$6000, λ=0.75

II) Project set two, budget \$6000, λ=0.25

Tables B-3 and B-4 show the results from the simulation for the same decision environments. It can be observed that even though the Test of Hypothesis generally reveals significant differences between the two solutions, the projects selected by the two solution procedures were the same, except for one or two projects. This type of behavior is found in all cases for these two project sets. Therefore, it is possible to say that in this case both the analytical solution procedure and the simulation give basically the same result with respect to project selection. Furthermore, figure 4-3 shows the patterns followed by the simulation for project set 1, budget of \$6000, and decision rule "first, second or third year". This gives a very good idea of the changes in the values of total expected present worth and its variance during the simulation process. As can be seen in the figure,

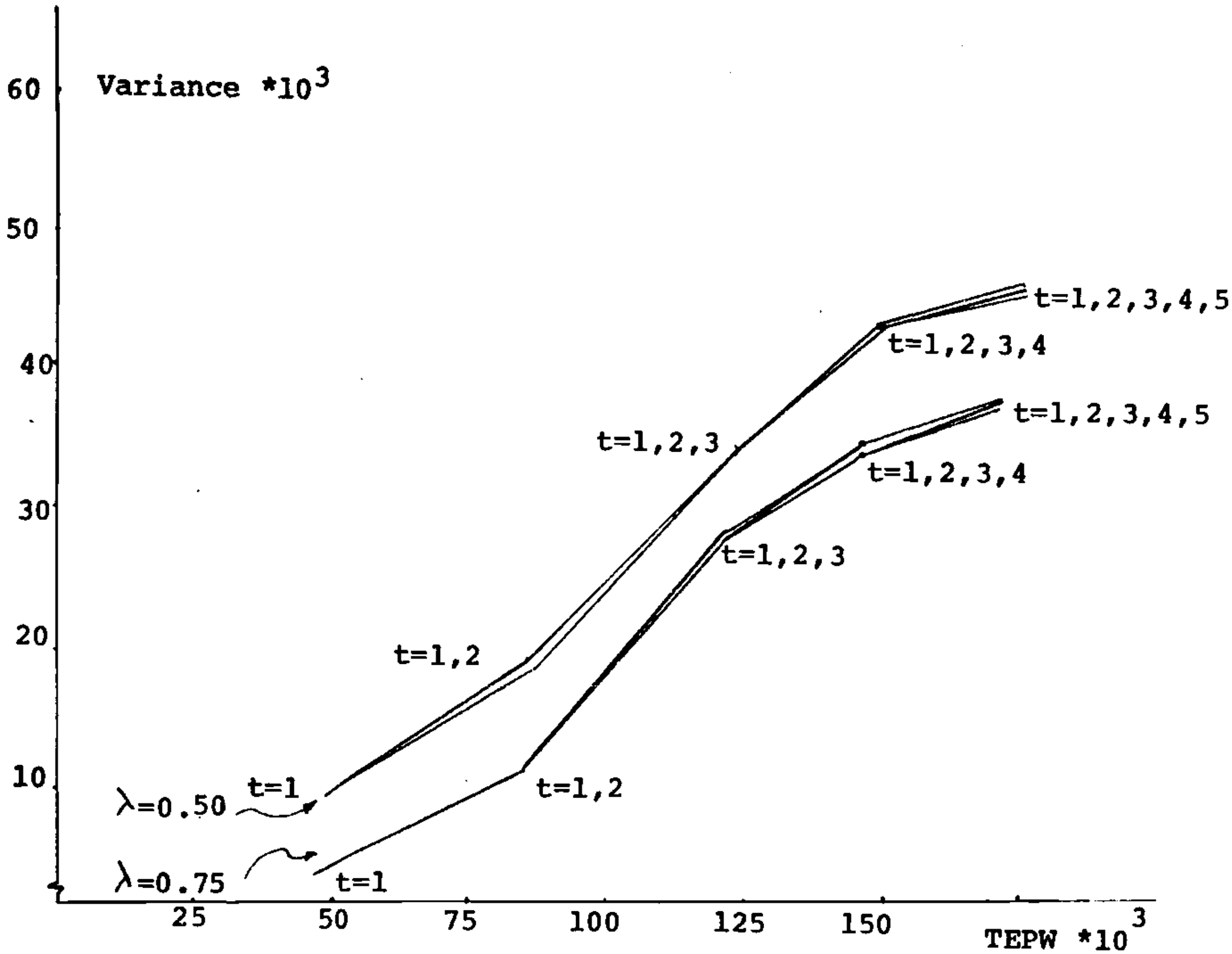


Figure 4-3. "Patterns Followed by the Simulation".
Set 1, Budget $6000 and "1st, 2nd, or 3rd year", Rule.

the patterns followed from t=1 to t=5 are quite smooth, indicating that the mean values obtained from the simulations are reliable for decision making.

Simulation of Set 3

Due to the magnitude of the values of the variance found in the analytical solutions for project set three, fifty simulations were performed for each decision environment tested, instead of twenty. The decision environments were:

Selection rules	Lambda	Budget	Simulations
"first,second or third year"	1.00	1500	50
"first,second or third year"	0.75	1500	50
"first,second or third year"	0.50	1500	50
"first,second or third year"	0.25	1500	50
"first,second of third year"	0.00	1500	50
"only the first year"	1.00	1500	50
"only the first year"	0.75	1500	50
"only the first year"	0.50	1500	50
"only the first year"	0.25	1500	50
"only the first year"	0.00	1500	50

These alternatives were chosen because of the fact that they combine two factors relating to decision environments, five lambda values, and a tighter budget that forces more comptition among the projects. The results are presented in table A-14. The differences between the analytical solution and the simulation are quite evident (see table 4-6). This is because of the two reasons mentioned earlier, and because of the high variability of the project cash flows. Furthermore tables B-5 and B-6 in the appendix show the differences in portfolios chosen by the two procedures. There are similarities in portfolios, but there are enough differences to prevent the decision maker from simply using the analytical procedure.

Efficiency Frontiers

The values of expected present worth and variance for different lambda values can be plotted to obtain a graphical representation of the efficiency frontier. Figure 4-4 shows the efficiency frontiers as time progresses for one of the situations. Each point represents a specific portfolio of projects selected by the LP model as a function of the lambda value. The leftmost curve represents the values of TEPW and SD after making decision at t=1. The next curve represents the values cumulative for t=1 and t=2. As time progresses the cumulative curves shift to the right and up.

Figure 4-5 shows the final efficiency frontiers (t=1,2,3,4,5) for the three decision rules for set 1 and a budget of $6000. It can clearly be seen that "select in the first, second or third year" dominates "select in first year". It would also dominate "select in second year" where it not

Table 4-6. Comparison of Selected Results for Project Set 3, Budget of $1500, Select in the 1st, 2nd, or 3rd Year, Decision Rule

	Analytical Results	Simulation Results
TEPW		
= 1.00	82,600	76,600
= 0.75	86,210	79,200
= 0.50	92,100	82,300
= 0.25	93,600	84,300
= 0.00	109,200	93.800
CH		
= 1.00	230,600	218,900
= 0.75	240,300	273,200
= 0.50	255,800	273,900
= 0.25	259,700	274,200
= 0.00	298,900	276,700

TEPW= Total expected present worth
CH = Cash at the horizon

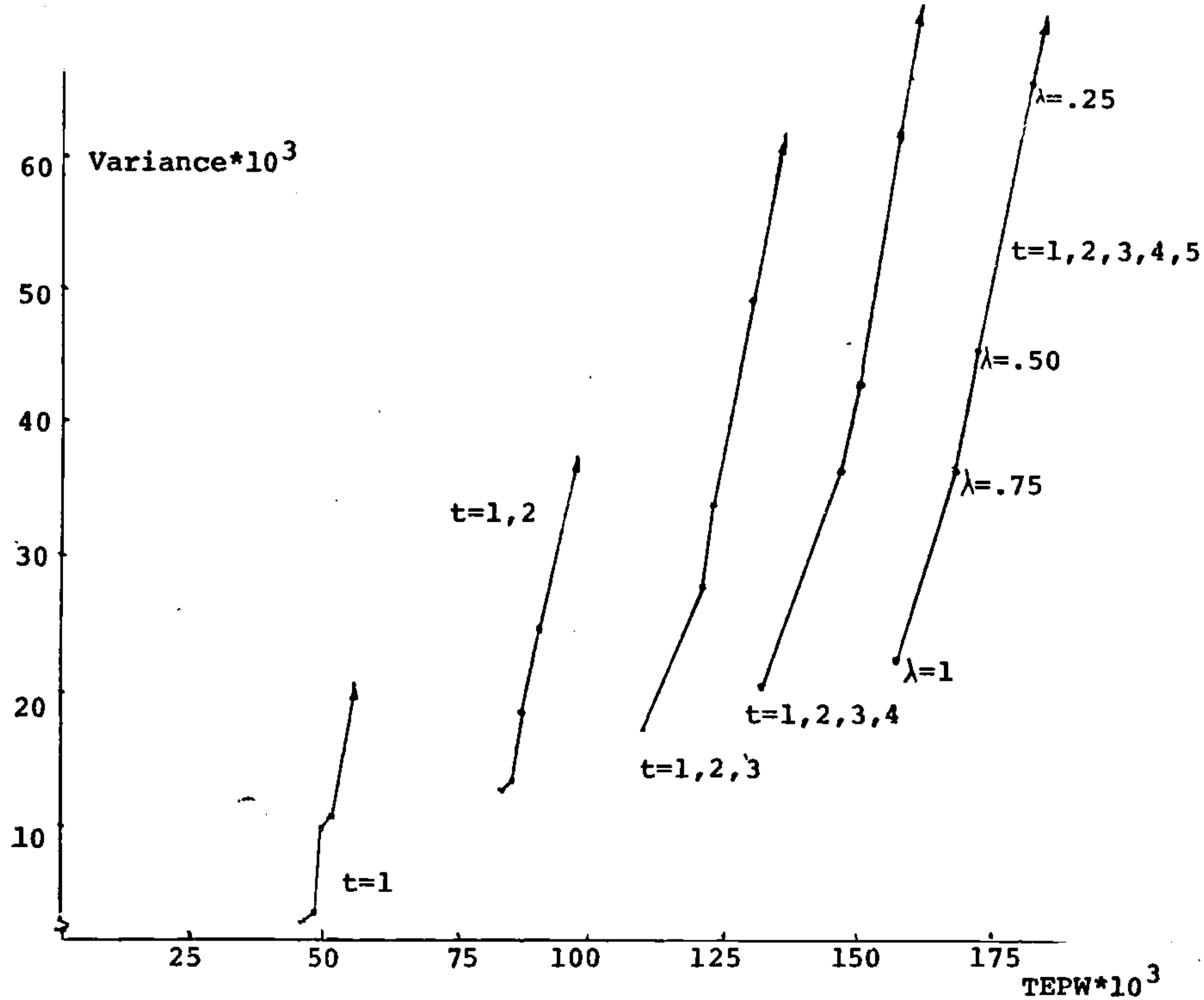


Figure 4-4. Time Progression of Efficiency Frontiers, Set 1, Budget of $6000, Select in 1st, 2nd, or 3rd Year.

for the one point at λ=1.0. The overall frontier is given by the frontier for "select in first, second or third year" plus the dashed line in figure 4-5.

Computational Statistics

The program uses a core memory of 74,000, although this could be reduced by reprogramming. Also, for the analytical procedure the "average run" uses 26 sec. of CPU time (CDC Cyber 74); therefore, for each project set the total computation time is :

26 sec * 45 runs = 1170 sec.

On the other hand, for the simulation, the "average run" uses 130 sec. of CPU time. Thus, for project sets 1 and 2 the total computation time for each is:

130 sec * 26 runs = 3380 sec.

and for project set 3:

130 sec * 10 runs = 1300 sec.

Summary

Three project sets were generated to test the model of chapter 3. This test included an analytical and simulation procedure.

During the analytical solution the main parameters of the model were changed in order to provide the decision maker with a wide span of decision environments. The key values values changed were: I) The decision rules for project selection, II) The risk aversion factor, and III)

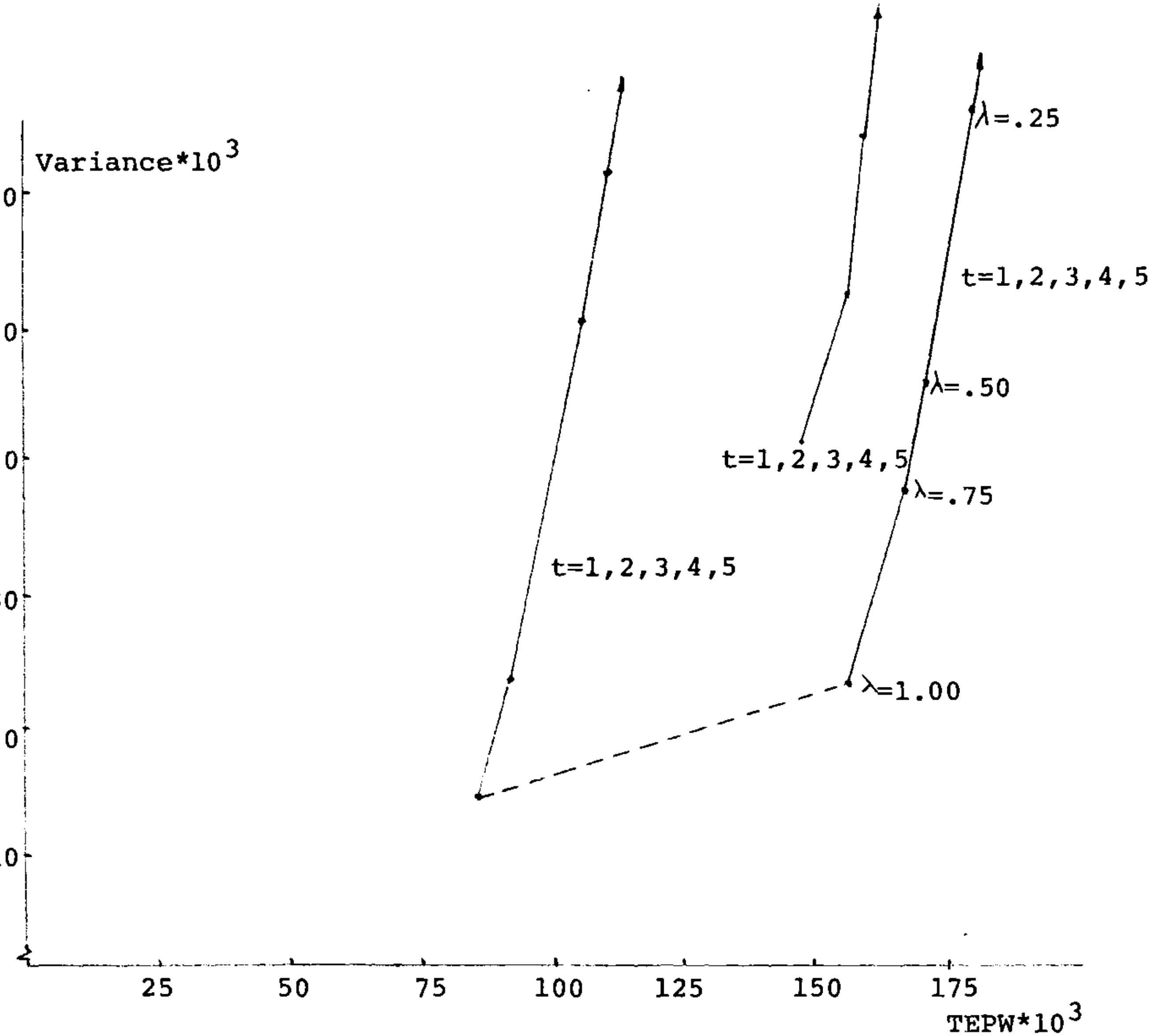


Figure 4-5. Comparison of Final Efficiency Frontiers for Three Decision Rules, Set 1, Budget of $6000.

The annual budget. From these changes some major conclusions were drawn, and then the more interesting decision environments of each project set were simulated.

For project sets one and two the analytical and simulation procedures gave the same results with respect to project selection. On other hand, for project set three, there are enough differences to prevent the decision maker from using only the analytical procedure.

CHAPTER V

CONCLUSIONS AND RECOMENDATIONS

Conclusions

The objectives of this research were:

I) Construct a decision making procedure for selecting investment projects where the returns and variabilities of return depend on the timing of project acceptance.

II) Develop a solution algorithm for this procedure,and

III) Gain some computational experience with the algorithm.

In chapter three the model was described. Specific characteristic considered were flexibility of the model, inclusion of correlated cash flows, and inclusion of a risk-aversion parameter. The model uses linear programming to solve periodic selection problems subject to one budget constraint and several contingency constraints. The solution procedure was further developed and tested in chapter four with three sets of projects. Each one assumed fifteen new investment alternatives every year, with a project life of ten years, and a planning period of five years. The results show that the model can give a very good set of different decision alternatives from which the decision maker can select the one which fullfills his goals.

The most important result of this research effort is the realization of a model which combines capital budgeting theory, new-product development theory, and mathematical and computational tools into a practical and realistic sequential procedure for project selection. Such a model would be useful to any decision maker who faces the problem of allocating limited financial resources of the firm in a periodic sequential decision making environment.

For the first two project sets tested in this work, the ones with small variability ratio (4.38 and 7.17 respectively), the analytical procedure and the simulation give basically the same results. This was not the case for the third project set. Here the large value of the variability ratio ($2.36*10^7$) produces enough differences between the portfolios selected by the analytical procedure and the simulation to prevent the decision maker from simply using the analytical procedure.

The best decisions were achieved with the decision rule: select in the first, second of third year. Thus, an aggressive marketing policy, characterized by market leadership in every new product, may lead to suboptimal results. For extremely risk-averse companies, however, other decision rules may be attractive. The efficiency frontiers for "select in the first, second or third year" do not dominate completely those for the other decision rules, and to obtain the best overall frontier, one must usually include portfolios selected by two decision rules.

Recommendations

After making basic assumptions about the model and working with these assumptions, specific recommendations can be made based on difficulties and successes with developing a solution procedure and testing it on problems. These recommendations are:

1) An effort should be made to obtain the most realistic estimates of the annual project cash flows, because these are the basic data upon the model is based.

2) The same effort should be given to obtaining autocorrelation and cross-correlation indexes, this will help obtain more realistic solution alternatives.

3) Although the interest rate was considered to be the same for discounting the cash flows and for borrowing and lending small amounts of budget money from one year to another, the model can easily accomodate the use of different rates.

APPENDIX A

COMPLETE RESULTS FOR EACH SET (Tables)

Table A-1. Set 1, "first, second or third year" Decision Rule.

	Budget 2000			Budget 4000			Budget 6000		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	76,400	99	219,400	123,200	125	363,000	156,600	149	470,724
0.75	76,400	99	219,400	122,900	127	361,700	164,900	191	492,000
0.50	76,400	99	219,400	133,300	189	388,900	171,500	213	509,600
0.25	80,190	109	230,500	137,900	232	400,700	180,300	256	532,700
0.00	94,600	1,217	267,400	156,600	1,261	449,600	200,600	1,295	585,300

	Budget 2000		Budget 4000		Budget 6000	
λ	TC	B	TC	B	TC	B
1.00	9,800	261	20,100	-108	29,700	284
0.75	9,800	261	19,900	282	29,600	387
0.50	9,800	261	19,900	104	29,900	140
0.25	10,500	-545	19,900	85	30,100	-125
0.00	10,200	-270	20,000	-119	30,000	20

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH=Cash at the Horizon.
TC=Total Cost.
B= Budget Money at the End of Planning Period.

Table A-2. Set 1,"only first year" Decision Rule.

	Budget 2000			Budget 4000			Budget 6000		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	69,900	90	202,500	115,100	152	342,300	148,200	202	499,300
0.75	69,900	90	202,500	117,600	180	347,700	152,600	220	460,700
0.50	76,900	137	221,800	120,500	177	356,400	155,700	225	468,600
0.25	76,900	138	221,500	121,600	201	358,500	164,900	305	493,300
0.00	91,300	1,200	258,500	150,000	1,200	432,600	190,200	1,270	558,500

	Budget 2000		Budget 4000		Budget 6000	
λ	TC	B	TC	B	TC	B
1.00	9,800	226	20,400	- 244	30,700	220
0.75	9,800	226	19,600	357	30,700	- 35
0.50	10,300	- 368	20,200	- 323	30,600	281
0.25	10,000	- 214	19,800	207	30,200	-266
0.00	10,000	- 57	20,000	- 138	30,100	- 61

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-3. Set 1,"only second year only" Decision Rule.

	Budget 2000			Budget 4000			Budget 6000		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	49,400	71	149,700	73,000	96	233,000	82,500	106	276,000
0.75	49,400	71	150,000	76,400	125	242,000	91,200	151	301,700
0.50	49,400	71	150,000	79,300	148	248,500	103,600	223	331,600
0.25	56,000	154	166,700	87,900	207	271,100	107,500	246	342,000
0.00	61,400	294	181,400	95,600	343	290,900	114,700	358	360,300

	Budget 2000		Budget 4000		Budget 6000	
λ	TC	B	TC	B	TC	B
1.00	9,900	69	20,200	-155	30,200	1744
0.75	10,000	- 55	20,200	-310	30,000	-131
0.50	10,000	- 55	19,200	313	28,800	1310
0.25	9,800	174	19,800	200	28,900	1146
0.00	10,300	-324	19,700	284	28,800	1300

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-4. Set 2, "first, second or third year" Decision Rule.

	Budget 2000			Budget 4000			Budget 6000		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	63,700	99	187,400	102,900	139	310,800	124,500	153	388,200
0.75	65,900	106	193,100	102,300	149	308,300	130,600	183	404,100
0.50	68,000	124	198,700	105,900	159	318,000	139,500	228	427,100
0.25	68,000	124	198,700	113,700	236	337,800	149,600	279	452,400
0.00	83,400	936	237,500	135,500	1,048	395,100	179,000	1,500	529,100

	Budget 2000		Budget 4000		Budget 6000	
λ	TC	B	TC	B	TC	B
1.00	10,300	- 347	20,200	- 314	30,100	- 261
0.75	10,300	- 335	19,800	139	30,200	- 202
0.50	10,300	- 373	20,000	- 74	30,500	- 315
0.25	10,300	- 373	19,900	128	29,600	440
0.00	9,700	291	20,300	- 263	30,200	24

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-5. Set 2,"only first year" Decision Rule.

	Budget 2000			Budget 4000			Budget 6000		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	49,900	105	151,600	73,500	151	230,300	73,500	151	230,300
0.75	47,500	104	144,200	78,200	188	246,300	87,600	199	276,200
0.50	52,200	139	157,300	86,000	208	266,100	100,200	244	314,200
0.25	50,200	144	151,300	95,300	285	290,400	123,200	362	383,400
0.00	77,900	942	233,600	130,800	1,454	382,700	159,500	1,470	479,200

	Budget 2000		Budget 4000		Budget 6000	
λ	TC	B	TC	B	TC	B
1.00	10,200	279	19,000	2,174	19,000	15,605
0.75	9,500	490	20,400	- 45	23,600	9,916
0.50	10,100	-148	20,000	203	25,900	6,703
0.25	9,600	353	20,000	-1	30,200	716
0.00	9,800	168	20,100	- 180	30,000	- 214

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-6. Set 2, "only second year" Decision Rule.

	Budget 2000			Budget 4000			Budget 6000		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	39,400	74	123,500	57,900	105	191,500	65,700	115	232,200
0.75	39,700	75	125,000	59,300	115	195,400	69,200	133	242,400
0.50	41,500	89	129,400	65,400	151	211,200	87,000	223	292,100
0.25	46,700	144	142,900	74,000	211	233,300	92,400	251	306,400
0.00	52,300	304	157,200	81,400	338	252,500	99,300	361	323,100

	Budget 2000		Budget 4000		Budget 6000	
λ	TC	B	TC	B	TC	B
1.00	9,900	142	19,000	1223	30,000	1982
0.75	10,000	131	19,100	1041	30,600	1217
0.50	9,900	- 3	19,000	1076	30,800	-823
0.25	10,000	- 58	18,800	1359	30,800	-1047
0.00	9,900	42	18,800	1232	30,400	-389

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-7. Set 3, "first, second or third year" Decision Rule.

	Budget 500			Budget 1000			Budget 1500		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	46,100	4,929	124,300	65,400	5,275	180,400	82,600	5,616	230,600
0.75	50,700	5,490	137,500	68,200	5,830	187,800	86,210	6,230	240,300
0.50	55,300	10,273	149,200	75,300	10,681	206,400	92,100	10,830	255,800
0.25	55,300	10,273	149,200	75,300	10,681	206,400	93,600	13,062	259,700
0.00	67,600	$3*10^{6}$	181,100	90,000	$3*10^{6}$	243,900	109,200	$3*10^{6}$	298,900

	Budget 500		Budget 1000		Budget 1500	
λ	TC	B	TC	B	TC	B
1.00	2,033	501	4,959	72	7,529	-85
0.75	2,837	-359	5,059	-60	7,778	-292
0.50	2,727	-205	5,072	-81	7,902	-472
0.25	2,727	-205	5,072	-81	7,895	-383
0.00	2,735	-238	4,807	237	7,139	341

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-8. Set3, "only first year" Decision Rule.

	Budget 500			Budget 1000			Budget 1500		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	27,100	2,411	75,300	50,300	4,965	141,100	66,700	5,217	189,500
0.75	32,000	3,477	89,200	54,700	5,567	153,400	70,600	5,840	200,200
0.50	32,000	3,477	89,200	54,700	5,567	153,400	70,600	5,840	200,200
0.25	32,000	3,477	89,200	54,700	5,567	153,400	70,600	5,840	200,200
0.00	35,800	$4*10^5$	98,200	65,000	$3*10^6$	179,500	87,300	$3*10^6$	243,600

	Budget 500		Budget 1000		Budget 1500	
λ	TC	B	TC	B	TC	B
1.00	2,249	309	4,894	133	7,531	- 66
0.75	2,921	-440	5,236	-322	7,841	-430
0.50	2,921	-440	5,236	-322	7,841	-430
0.25	2,921	-440	5,236	-322	7,841	-430
0.00	2,465	112	4,892	88	7,942	-482

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-9. Set 3, "only second year" Decision Rule.

	Budget 500			Budget 1000			Budget 1500		
λ	TEPW	SD	CH	TEPW	SD	CH	TEPW	SD	CH
1.00	26,600	2,051	74,800	38,900	2,992	111,700	48,500	3,208	142,000
0.75	26,200	2,174	73,200	39,900	3,068	114,900	48,500	3,208	142,000
0.50	26,200	2,174	73,200	39,900	3,068	114,900	48,500	3,208	142,000
0.25	26,200	2,174	73,200	39,900	3,068	114,900	47,900	3,238	140,700
0.00	30,700	$2*10^5$	84,500	47,000	$2*10^5$	133,000	59,600	$3*10^5$	171,200

	Budget 500		Budget 1000		Budget 1500	
λ	TC	B	TC	B	TC	B
1.00	2,595	-158	5,047	- 11	7,471	19
0.75	2,459	121	5,234	-242	7,471	19
0.50	2,459	121	5,234	-242	7,471	19
0.25	2,459	121	5,234	-242	7,571	-95
0.00	2,179	336	5,090	- 97	7,709	-149

TEPW= Total Expected Present Worth.
SD= Standard Deviation.
CH= Cash at the Horizon.
TC= Total Cost.
B= Budget Money at the End of Planning Period.

Table A-10. Simulation Results, Set 1
"first, second or third year"
Decision Rule.

	Budget 2000				Budget 4000				Budget 6000			
λ	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD
1.00	-	-	-	-	123,204	0	363,200	5,600	156,600	0	471,000	360
0.75	-	-	-	-	123,190	288	362,780	1,300	166,890	1,260	497,855	2,682
0.50	77,018	58	221,288	657	132,900	450	387,920	1,324	172,200	698	512,148	1,800
0.25	78,790	1,430	226,383	4,662	137,337	558	399,599	1,333	180,300	0	532,963	144
0.00	93,600	990	264,700	3,000	156,600	30	449,700	1,000	200,900	300	586,500	1,430

MTEPW= Mean of the Total Expected Present Worth.
SDD= Standard Deviation of the MTEPW.
MCH= Mean of the Cash at the Horizon.
SD= Standard Deviation of Cash at the Horizon.

Table A-11. Simulation Results, Set 1, "Only first year" Decision Rule.

	Budget 2000				Budget 4000				Budget 6000			
λ	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD
1.00	-	-	-	-	115,130	104	342,100	408	148,200	0	449,200	424
0.75	-	-	-	-	117,924	311	349,100	1,038	152,540	62	460,900	384
0.50	76,300	675	219,900	2,330	119,800	800	354,300	2,740	155,800	0	468,700	400
0.25	75,500	1,480	217,600	4,080	122,250	1,670	360,400	5,140	164,800	120	493,200	271
0.00	90,680	445	256,640	755	150,000	29	423,900	202	190,200	133	558,300	546

MTEPW= Mean of the Total Expected Present Worth.
SDD= Standard Deviation of the MTEPW.
MCH= Mean of the Cash at the Horizon.
SD= Standard Deviation of the Cash at the Horizon.

Table A-12. Simulation Results, Set 2, "first,second or third year" - Decision Rule.

	Budget 2000				Budget 4000				Budget 6000			
λ	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD
1.00	-	-	-	-	102,000	980	307,200	3,320	124,550	90	388,300	35
0.75	-	-	-	-	102,500	224	308,900	700	130,250	614	402,900	1,395
0.50	68,000	0	198,700	50	105,900	0	318,000	59	139,400	26	427,000	217
0.25	68,000	0	198,700	50	112,900	795	338,500	2,660	150,120	470	454,400	2,456
0.00	83,950	589	240,000	2,690	135,800	1,520	396,650	3,390	174,550	1,100	519,200	2,450

MTEPW= Mean of the Total Expected Present Worth.
SDD= Standard Deviation of the MTEPW.
MCH= Mean of the Cash at the Horizon.
SD= Standard Deviation of the Cash at the Horizon.

Table A-13. Simulation Results, Set 2
"only first year"
Decision Rule.

	Budget 2000				Budget 4000				Budget 6000			
λ	TEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD	MTEPW	SDD	MCH	SD
1.00	-	-	-	-	73,480	0	230,300	96	73,480	0	230,307	100
0.75	-	-	-	-	78,300	0	246,300	160	87,650	0	276,400	208
0.50	52,170	70	157,100	128	86,150	102	266,550	660	100,250	0	314,550	240
0.25	53,370	2,900	160,300	6,270	95,290	0	290,400	307	123,250	0	383,451	309
0.00	78,007	8	224,280	403	130,550	223	383,590	221	159,300	322	480,820	1,900

MTEPW= Mean of the Total Expected Present Worth.
SDD= Standard Deviation of the MTEPW.
MCH= Mean of the Cash at the Horizon.
SD= Standard Deviation of the Cash at the Horizon.

Table A-14(a). Simulation Results for Set 3, "first, second or third year" Decision Rule, Budget 1500.

λ	MTEPW	SDD	MCH	SD
1.00	76,663	6,780	218,960	4,750
0.75	79,215	5,730	273,170	7,670
0.50	82,325	9,788	273,974	12,363
0.25	84,321	11,585	274,214	13,122
0.00	93,767	487,724	276,753	19,673

MTEPW= Mean of the Total Expected Present Worth.
SDD= Standard Deviation for the MTEPW.
MCH= Mean of the Cash at the Horizon.
SD= Standard Deviation of MCH.

Table A-14(b). Simulation Results for Set 3, "first year only" Decision Rule, Budget 1500.

λ	MTEPW	SDD	MCH	SD
1.00	62,273	5,910	183,770	788
0.75	60,469	5,538	184,246	7,201
0.50	60,902	5,890	184,318	15,183
0.25	61,753	5,958	184,328	15,250
0.00	74,722	590,129	218,541	21,918

MTEPW= Mean of the Total Expected Present Worth.
SDD= Standard Deviation for the MTEPW.
MCH= Mean of the Cash at the Horizon.
SD= Standard Deviation of MCH.

APPENDIX B

PROGRAM USED TO SOLVE THE PROBLEM

Table B-1. Projects Selected, Analytical Solution
Set 2, Budget 6000, Lambda 0.25

.............STATISTICS OF THE SIMULATION........

MEAN OF THE CASH IN THE HORIZONT 452426.14

OF THE CASH IN THE HORIZONT = 0.00

89

PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMULATION

YEAR 1

100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00
0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	100.00
0.00	0.00	0.00	0.00	0.00					

YEAR 2

0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	100.00
0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
100.00	0.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00	0.00
0.00	100.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	100.00
100.00	0.00	0.00	100.00	0.00					

YEAR 3

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00
0.00	100.00	0.00	100.00	100.00					

YEAR 4

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00	100.00	0.00
0.00	100.00	0.00	100.00	0.00					

YEAR 5

0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	0.00	100.00	0.00	0.00
0.00	0.00	0.00	100.00	100.00					

MEAN OF THE P.W. 149604.36 MEAN OF THE VARIANCE OF P.W. 77967.79

Table B-2. Projects Selected, Analytical Solution, Set 2, Budget 6000, Lambda 0.75.

..............STATISTICS OF THE SIMULATION........

MEAN OF THE CASH IN THE HORIZONT 484121.63

90

OF THE CASH IN THE HORIZONT 0.00

PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMULATION

YEAR 1

100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00
0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00
0.00	0.00	100.00	0.00	0.00					

YEAR 2

0.00	0.00	0.00	100.00	0.00	100.00	0.00	0.00	0.00	0.00
0.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
0.00	0.00	0.00	100.00	100.00	0.00	100.00	0.00	0.00	0.00
0.00	100.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00	100.00
100.00	0.00	0.00	100.00	0.00					

YEAR 3

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	100.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00
0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
100.00	100.00	100.00	100.00	0.00	0.00	0.00	100.00	0.00	0.00
0.00	100.00	0.00	100.00	100.00					

YEAR 4

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	100.00
0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	100.00	100.00	0.00	0.00	0.00	100.00	0.00	100.00	0.00
100.00	100.00	100.00	0.00	0.00					

YEAR 5

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	0.00	100.00	0.00	0.00
0.00	100.00	0.00	100.00	100.00					

MEAN OF THE P.W. 130661.57 MEAN OF THE VARIANCE OF P.W. 33733.71

Table B-3. Projects Selected, Simulation, Set 2, Budget 6000, Lambda 0.0

............STATISTICS OF THE SIMULATION........

91

MEAN OF THE CASH IN THE HORIZCNT 454381.62

OF THE CASH IN THE HORIZCNT 2456.82

PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMULATION

YEAR 1

100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00
0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	[illegible]
100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	100.00
0.00	0.00	0.00	0.00	0.00					

YEAR 2

0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	100.00
0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
100.00	0.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00	0.00
0.00	100.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	100.00
100.00	0.00	0.00	100.00	0.00					

YEAR 3

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	70.00
70.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00	100.00	30.00
0.00	100.00	0.00	100.00	100.00					

YEAR 4

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00	100.00	0.00
0.00	100.00	0.00	100.00	0.00					

YEAR 5

0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	45.00	0.00	0.00	30.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	0.00	100.00	65.00	0.00
0.00	0.00	35.00	45.00	100.00					

MEAN OF THE P.W. 150119.28 MEAN OF THE VARIANCE OF P.W. 80762.92

VAR P.W. 467.12

Table B-4. Project Selected, Simulation, Set 2, Budget 6000, Lambda 0.75

............STATISTICS OF THE SIMULATION........

92

MEAN OF THE CASH IN THE HORIZONT . 482923.84

F THE CASH IN THE HORIZONT 1395.68

PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMULATION

YEAR 1

100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00	100.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	100.00	0.00	0.00	100.00
0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00
0.00	0.00	100.00	0.00	0.00					

YEAR 2

0.00	0.00	0.00	100.00	0.00	100.00	0.00	0.00	0.00	0.00
0.00	100.00	100.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
0.00	0.00	0.00	100.00	70.00	0.00	100.00	0.00	0.00	0.00
0.00	100.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00	100.00
100.00	0.00	0.00	100.00	0.00					

YEAR 3

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	30.00
0.00	0.00	100.00	0.00	30.00	0.00	0.00	100.00	0.00	0.00
0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
100.00	100.00	100.00	100.00	0.00	0.00	0.00	100.00	0.00	0.00
0.00	100.00	0.00	100.00	100.00					

YEAR 4

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	0.00	0.00	75.00	0.00	0.00	0.00	0.00	100.00
0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	100.00	100.00	0.00	0.00	0.00	100.00	0.00	100.00	0.00
100.00	100.00	75.00	0.00	0.00					

YEAR 5

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	95.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100.00	0.00	100.00	100.00	100.00	100.00	0.00	100.00	0.00	0.00
0.00	100.00	0.00	95.00	100.00					

MEAN OF THE P.W. + 138241.71 MEAN OF THE VARIANCE OF P.W. 23839.72

VAR P.W. 614.76

Table B-5. Projects Selected, Analytical Solution, Set 3, Budget 1500, Lambda 0.25.

STATISTICS OF THE SIMULATION [illegible]

93

MEAN OF THE CASH IN THE HORIZONT 268272.28

VARIANCE OF THE CASH IN THE HORIZONT 0.00

PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMULATION

YEAR 1

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
100.00	0.00	0.00	100.00	0.00	0.00	100.00	0.00	0.00	0.00
0.00	100.00	0.00	0.00	0.00					

YEAR 2

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	100.00	[illegible]	0.00	100.00

YEAR 3

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
100.00	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	100.00	0.00	0.00
[illegible]	[illegible]	[illegible]	100.00	[illegible]					

YEAR 4

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.00	100.00	0.00	[illegible]	0.00	0.00	100.00	100.00	0.00	[illegible]

YEAR 5

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0.00	0.00	0.00	0.00	100.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

MEAN OF THE P.W. 70691.75 MEAN OF THE VARIANCE OF P.W. 34188732.17

VAR P.W. 0.00 VAR V.P.W. 0.00

Table B-6. Projects Selected, Simulation, Set 3, Budget 1500, Lambda 0.75

..............STATISTICS OF THE SIMULATION.........

MEAN OF THE CASH IN THE HORIZON1 ....184246.86

94

VARIANCE OF THE CASH IN THE HORIZON1 ....7201.54

PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMULATION

YEAR 1

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	[illegible]
0.00	0.00	[illegible]	[illegible]	0.00	0.00	0.00	0.00	0.00	[illegible]
100.00	0.00	0.00	100.00	0.00	0.00	100.00	0.00	0.00	[illegible]
0.00	100.00	0.00	0.00	0.00					

YEAR 2

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	0.00	0.00	0.00
0.00	0.00	[illegible]	[illegible]	0.00	0.00	0.00	0.00	0.00	0.00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	38.00	82.00	15.00	0.00	74.00
34.00	72.00	24.00	42.00	28.00					

YEAR 3

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	[illegible]	[illegible]
46.00	10.00	20.00	0.00	12.00	0.00	6.00	56.00	0.00	[illegible]
6.00	48.00	0.00	64.00	2.00					

YEAR 4

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	0.00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	[illegible]
[illegible]	25.00	[illegible]	[illegible]	[illegible]	0.00	62.00	60.00	20.00	[illegible]
0.00	42.00	36.00	0.00	0.00					

YEAR 5

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24.00	0.00	0.00	0.00	74.00					

ACAGKTI. 78/10/19. GEORGIA TECH CYBER 74.

17.54.12.JOB.T2180.
17.54.12.USER.IE306B1..
17.54.15.NEWS(DEP,FL=60)
17.54.15. NEWS COMPLETE.
17.54.16.ATTACH.TAPE9.

```
      PROGRAM THESIS 1(INPUT,OUTPUT,TAPE3,TAPE4,TAPE6=OUTPUT,TAPE5=INPUT)
     1)
      REAL INT,ML
      INTEGER M,N
      DIMENSION PE(380,20),ML(380,20),OP(380,20),R(380),IND(380),PHI(
     1380),E(380,20),V(380,20),EPW(380),RO(20,20),VPW(380),BDGT(10,10)
     2,NE(10),EE(380,20)
C*****THIS PROGRAM COMPUTES THE EXPECTED PRESENT WORTH OF ALL PROJETS
C*****AND THEIR VARIABILITY(WITH AUTOCORRELATIONS WITHIN EACH PROJECT
      READ(5,*)M
      READ(5,*)N
      READ(5,*)MM1
      READ(5,*)MMN
      READ(5,*)((BDGT(I,J),J=1,MM1),I=1,MMN)
      READ(5,*)(NE(I),I=1,MM1)
      MM=M+1
      M22=MM
      WRITE(3,1040)MM1
      WRITE(3,1041)N
      WRITE(3,1042)M22
      WRITE(3,1043)(NE(I),I=1,MM1)
      WRYTE(3,1044)(BDGT(I,J),J=1,MM1)
 1040 FORMAT(I3)
 1041 FORMAT(I5)
 1042 FORMAT(I3)
 1043 FORMAT(5I5)
 1044 FORMAT(5F10.2)
      DO 533 I=1,N
      READ(5,*)(PE(I,J),J=1,MM)
      READ(5,*)(ML(I,J),J=1,MM)
      READ(5,*)(OP(I,J),J=1,MM)
  533 CONTINUE
      READ(5,*)INT
      READ(5,*)(IND(I),I=1,N)
      READ(5,*)(R(I),I=1,N)
      DO 544 I=1,N
      PHI(I)=R(I)
  544 CONTINUE
      WRITE(6,1000)M
      WRITE(6,1001)N
      WRITE(6,1008)INT
      DO 1 I=1,N
      WRITE(6,998)I
  998 FORMAT(10X,*PROJECT NO.*,I5,//)
      WRITE(6,1022)
      DO 2 J=1,MM
      JW=J-1
      WRITE(6,1023)JW,PE(I,J),ML(I,J),OP(I,J)
    2 CONTINUE
 1022 FORMAT(6X,*Y*,4X,*PESS.VALUE*,4X,*M.L. VALUE*,4X,*OPP. VALUE*)
 1023 FORMAT(5X,I2,4X,F10.2,4X,F10.2,4X,F10.2)
      WRITE(6,1009)R(I)
      WRITE(6,1010)PHI(I)
      WRITE(6,1011)IND(I)
      IF(R(I) .EQ. 0)WRITE(6,330)
      IF(R(I) .EQ. 1)WRITE(6,331)
      IF(R(I) .LT. 1)GO TO 644
      GO TO 665
  644 IF(R(I) .GT. 0)WRITE(6,332)
  665 IF(IND(I) .EQ. 0)WRITE(6,333)
      IF(IND(I) .EQ. 1)WRITE(6,334)
    1 CONTINUE
  330 FORMAT(5X,*THE PROJECT IS INDEPENDENT*)
  331 FORMAT(5X,*THE PROJECT IS PERFECT CORRELATED*)
  332 FORMAT(5X,*THE PROJECT IS PARTIAL CORRELATED*)
  333 FORMAT(5X,*AND THE INITIAL CASH FLOW IS INDEPENDENT OF THE REST
     1OF CASH FLOWS*,///)
  334 FORMAT(5X,*AND THE INITIAL CASH FLOW HAS THE SAME RELATION OF THE
     1REST OF CASH FLOWS*,///)
 1000 FORMAT(10X,*TIME HORIZONT*,I5,/)
 1001 FORMAT(10X,*NUMBER OF PROJECTS*,I5,/)
 1008 FORMAT(10X,9HINTEREST=,2X,F10.2,/)
 1009 FORMAT(5X,*AUTOCORRELATION INDEX*,2X,F10.2)
 1010 FORMAT(5X,*AUTOCORRELATION VALUE*,2X,F10.2)
 1011 FORMAT(5X,*INDEPENDENCE INDEX INITIAL C.F.*,I5)
      MM=M+2
      DO 10 I=1,N
      WRITE(6,1090)I
      WRITE(6,1091)
      REPW=0.
      DO 11 J=2,MM
      L=J-1
      LL=J-2
      E(I,L)=1./6.*(PE(I,L)+4.*ML(I,L)+OP(I,L))
      EE(I,L)=1./6.*(PE(I,L)+4.*ML(I,L)+OP(I,L))
      V(I,L)=(1./6.*(OP(I,L)-PE(I,L)))**2
      WRITE(6,801)LL,E(I,L),V(I,L)
      EPW(I)=REPW+((E(I,L))/(INT**LL))
      REPW=EPW(I)
   11 CONTINUE
      EPW(I)=REPW
   10 CONTINUE
      M22=MM-1
      WRITE(3,1049)((E(KW,KWZ),KWZ=1,M22),KW=1,N)
      WRITE(4,1049)((EE(KW,KWZ),KWZ=1,M22),KW=1,N)
      WRITE(4,1049)((V(KW,KWZ),KWZ=1,M22),KW=1,N)
 1049 FORMAT(2X,11F10.2)
      TT=0
      DO 690 I=1,N
```

```
      WRITE(4,1049)((EE(KW,KWZ),KWZ=1,M22),KW=1,N)
      WRITE(4,1049)((V(KW,KWZ),KWZ=1,M22),KW=1,N)
 1049 FORMAT(2X,11F10.2)
      TT=0
      DO 690 I=1,N
      WRITE(6,802)I,EPW(I)
      TT=TT+EPW(I)
  690 CONTINUE
      TTX=TT/N
      WRITE(6,7654)TTX
 7654 FORMAT(2X,*AVERAGE EPW*,F20.3,//)
      WRITE(3,1050)(EPW(KL),KL=1,N)
      WRITE(4,1050)(EPW(KL),KL=1,N)
 1050 FORMAT(2X,11F10.2)
      RR=0
      DO 12 I=1,N
      RVPW=0.
      DO 13 J=2,MM
      L=J-1
      IF(PE(I,L) .EQ. 0) GO TO 777
      GO TO 888
  777 IF(ML(I,L) .EQ. 0) GO TO 778
      GO TO 888
  778 IF(OP(I,L) .EQ. 0) GO TO 13
  888 LL=J-2
      IF(R(I) .NE. 0.) GO TO 100
      VPW(I)=RVPW+((V(I,L))/(INT**(2*LL)))
      RVPW=VPW(I)
      GO TO 13
  100 IF(R(I) .NE. 1.) GO TO 101
      IF(IND(I) .NE. 0) GO TO 102
      VPW(I)=RVPW+V(I,L)**2
      IND(I)=1
      VPWW=VPW(I)
      GO TO 13
  102 VPW(I)=RVPW+((V(I,L)**0.5)/(INT**LL))
      RVPW=VPW(I)
      GO TO 13
  101 RR=ABS(R(I))
      IF(RR .GE. 1) WRITE(6,800)
      DO 14 KK=2,MM
      K=KK-1
      DO 15 KKL=2,MM
      KL=KKL-1
      KLL=KL-K
      IEXP=IABS(KLL)
      RO(K,KL)=PHI(I)**IEXP
   15 CONTINUE
   14 CONTINUE
      VPW(I)=RVPW+((V(I,L))/(INT**(2*LL)))
      RVPW=VPW(I)
   13 CONTINUE
      VPW(I)=RVPW -
      IF(R(I) .EQ. 0) GO TO 104
      IF(R(I) .NE. 1.) GO TO 104
      VPW(I)=VPW(I)**2.+VPWW
      GO TO 106
  104 SUM=0.0
      IF(IND(I) .NE. 0) GO TO 103
      DO 16 KK=2,MM
      RO(KK,1)=0
   16 CONTINUE
      DO 17 KKL=2,MM
      RO(1,KKL)=0
   17 CONTINUE
      IND(I)=0
  103 MM11=M+1
      DO 18 KK=2,MM11
      K=KK-1
      DO 19 KL=2,MM
      KLT=KL-K
      IF(KLT .LE. 0) GO TO 19
      IF(KLT .EQ. 0) GO TO 19
      KR=K+KL-2
      SUM=SUM+((RO(K,KL)*V(I,K)**0.5*V(I,KL)**0.5)/(INT**(KR)))
   19 CONTINUE
   18 CONTINUE
      VPW(I)=VPW(I)+2.*SUM
  106 WRITE(6,803)I,VPW(I)
   12 CONTINUE
      DO 2211 I=1,N
      RR=RR+VPW(I)
 2211 CONTINUE
      RRX=RR/N
      WRITE(6,9876)RRX
 9876 FORMAT(2X,*AVERAGE VAR EPW*,F20.3)
      WRITE(3,1051)(VPW(IT),IT=1,N)
      WRITE(4,1051)(VPW(IT),IT=1,N)
      WRITE(4,1051)((PE(I,J),J=1,M22),I=1,N)
      WRITE(4,1051)((ML(I,J),J=1,M22),I=1,N)
      WRITE(4,1051)((OP(I,J),J=1,M22),I=1,N)
 1051 FORMAT(2X,11F10.2)
  800 FORMAT(10X,21HCHECK DATA CORR INDEX)
  801 FORMAT(4X,I2,3X,F14.2,5X,F20.2)
 1091 FORMAT(5X,*T*,3X,*EXP. CASH FLOW*,5X,*EXP. VARIANCE C.FLOW*)
 1090 FORMAT(7,5X,*PROJECT NO.*,I5)
  802 FORMAT(5X,8HPROJECT=,2X,I4,2X,4HEPW=,2X,F20.2)
  803 FORMAT(5X,8HPROJECT=,I4,2X,4HVPW=,F20.2)
      ENDFILE 4
      ENDFILE 3
      REWIND 4
      REWIND 3
      STOP
```

96

97

```
      PROGRAM TESIS(INPUT,OUTPUT,TAPE4,TAPE6=OUTPUT,TAPE5=INPUT)
      REAL INT,ML,LEM,LAM,MT
      INTEGER MA,NA
      DIMENSION XLEM(10),NE(50),B(60),XSTAROP(50,5),EPW(375),VPW(375)
     2,CCR(45,45,5),TEPP(20),TVPP(20),ML(375,12),PE(375,12),OP(375,12),
     3CASH(15),EYCASH(15),A(60,105),BDGT(6),BDOTT(6),V(375,12),EE(375,
     412),INFIX(8),TOL(4),KOUT(7),ERR(P),JM(60),X(60),P(60),Y(60),
     5KB(101),E(60,60),PSELEC(45,5),PPSEL(45,5),XXTCH(20)
C  THIS PROGRAM ********IF FR=1 MEANS THE PROJETS ONLY ACCEPTED IN THE FIRST
C  YEAR*******IF FR=2 MEANS THE PROJECTS ONLY ACCEPTED IN THE SECOND YEAR
C  AND IF FR=0 MEANS THE PROJECTS MAY BE ACCEPTED IN THE FIRST,SECOND AND
C THIRD YEAR**************************************************
C  WHEN ANA=0 MEANS THE PROGRAM WILL SIMULATE,AND WHEN ANA  EQ 1 THEN
C THE PROGRAM WILL TAKE THE MEAN OF EACH YEAR(ANALITICAL SOLUTION).
      FR=0
      ANA=0
C************************************************************************
      IF(FR .EQ. 0)WRITE(6,5400)
      IF(FR .EQ. 1)WRITE(6,5401)
      IF(FR .EQ. 2)WRITE(6,5402)
      IF(ANA .EQ. 1)WRITE(6,5403)
      IF(ANA .EQ. 0)WRITE(6,5404)
 5400 FORMAT(2X,*THIS RUN ALLOWS THE PROGRAM TO SELECT PROJECTS IN
     1THE FIRST,SECOND, OR THIRD YEAR OF THEIR LIFE*,///)
 5401 FORMAT(2X,*THIS RUN ALLOWS THE PROGRAM TO SELECT PROJECTS IN
     1 ONLY THE FIRST YEAR OF THEIR LIFE*,///)
 5402 FORMAT(2X,*THIS RUN ALLOWS THE PROGRAM TO SELECT PROJECTS IN
     1 ONLY THE SECOND YEAR OF THEIR LIFE*,///)
 5403 FORMAT(2X,*THIS RUN SOLVES THE PROBLEM ANALITICALLY*,///)
 5404 FORMAT(2X,*THIS RUN SOLVES AND SIMULATE *,///)
      READ(5,*)Z
      WRITE(6,888)Z
  888 FORMAT(2X,*NUMBER OF SIMULATIONS*,F10.2,//)
      READ(5,*)LSTR
      READ(5,*)M
      READ(5,*)N
      MM=M+1
      READ(5,*)MM1
      READ(4,887)((EE(I,J),J=1,MM),I=1,N)
      READ(4,4433)((V(I,J),J=1,MM),I=1,N)
      READ(5,*)(XLEM(L),L=1,LSTR)
      READ(5,*)(NE(J),J=1,MM1)
      READ(5,*)(BDOTT(J),J=1,MM1)
      READ(4,887)(EPW(I),I=1,N)
      READ(4,4433)(VPW(I),I=1,N)
 4433 FORMAT(2X,11F10.2)
      READ(5,*)INT
      READ(4,887)((PE(I,J),J=1,MM),I=1,N)
      READ(4,887)((ML(I,J),J=1,MM),I=1,N)
      READ(4,887)((OP(I,J),J=1,MM),I=1,N)
  887 FORMAT(2X,11F10.2)
      DO 181 JJ=1,MM1
      DO 18 LI=1,44
      IPT=LI+1
      READ(5,*)(CCR(LI,JT,JJ),JT=IPT,45)
   18 CONTINUE
  181 CONTINUE
      L=0
C234567***CHOOSING A VALUE OF LAMMDA
      MM=M+2
  131 L=L+1
      DO 6973 KWA=1,MM1
      DO 6974 KWW=1,45
      PSELEC(KWW,KWA)=0
      PPSEL(KWW,KWA)=0
 6974 CONTINUE
 6973 CONTINUE
      RA=0.69315
      RB=0.15773
      SS=1
  251 J=1
      TTCOST=0
      DO 6971 KWA=1,MM1
      DO 6972 KWW=1,50
      XSTAROP(KWW,KWA)=0
 6972 CONTINUE
 6971 CONTINUE
      DO 3 LI2=1,MM1
      BDGT(LI2)=$DOTT(LI2)
    3 CONTINUE
      WRITE(6,9284)(BDGT(IU),IU=1,MM1)
 9284 FORMAT(2X,*INITIAL BUDGET EVERY YEAR*,5F10.2)
      DO 20 I=1,MM
      EYCASH(I)=0.
   20 CONTINUE
      XLAM=XLEM(L)
      WRITE(6,804)XLAM
      XNEXBUD=0.
C******SETTTING TIME    AND SIMULATION********
C*****PARAMETERS FOR THE I.P.
  109 NA=NE(J)
      DO 909 LF=1,8
      INFIX(LF)=0
  909 CONTINUE
      DO 910 LF=I,60
      DO 911 LL=1,101
      A(LF,LL)=0
  911 CONTINUE
```

```
 7204 FORMAT([illegible]
      DO 20 I=1,MM
      EYCASH(I)=0.
   20 CONTINUE
      XLAM=XLEM(L)
      WRITE(6,804)XLAM
      XNEXBUD=0.
C*******SETTTING TIME    AND SIMULATION********
C*****PARAMETERS FOR THE I.P.
  109 NA=NE(J)
      DO 909 LF=1,8
      INFIX(LF)=0
  909 CONTINUE
      DO 910 LF=1,60
      DO 911 LL=1,101
      A(LF,LL)=0
  911 CONTINUE
  910 CONTINUE
      DO 912 LF=1,57
      B(LF)=0
  912 CONTINUE
      DO 914 LF=1,7
      KOUT(LF)=0
  914 CONTINUE
      DO 913 LF=1,4
      ERR(LF)=0
  913 CONTINUE
      DO 915 LF=1,57
      JH(LF)=0
      P(LF)=0
      X(LF)=0
      Y(LF)=0
  915 CONTINUE
      DO 916 LF=1,101
      KB(LF)=0
  916 CONTINUE
      DO 917 LF=1,57
      DO 918 LL=1,57
      E(LF,LL)=0
  918 CONTINUE
  917 CONTINUE
      DO 1 LJ=1,101
      IF(J .EQ. 1)LJA=LJ
      IF(J .EQ. 2)LJA=LJ+60
      IF(J .EQ. 3)LJA=LJ+135
      IF(J .EQ. 4)LJA=LJ+225
      IF(J .EQ. 5)LJA=LJ+330
      IF(LJ .GT. 45) GO TO 2
      A(1,LJ)=-(EPW(LJA)-(XLEM(L)*VPW(LJA)))
      GO TO 1
    2 A(1,LJ)=0
    1 CONTINUE
      DO 83 LJ=1,101
      IF(J .EQ. 1)LJA=LJ
      IF(J .EQ. 2)LJA=LJ+60
      IF(J .EQ. 3)LJA=LJ+135
      IF(J .EQ. 4)LJA=LJ+225
      IF(J .EQ. 5)LJA=LJ+330
      IF(LJ .GT. 45)GO TO 4
      IF(ANA .EQ. 1)GO TO 7989
      IF(J .NE. 1)GO TO 6783
 7989 A(2,LJ)=-EE(LJA,J)
      GO TO 83
 6783 MT=ML(LJA,J)
      AT=PE(LJA,J)
      BT=OP(LJA,J)
      CALL RVALUE(MT,AT,BT,RA,RB,BETA)
      A(2,LJ)=-BETA
      GO TO 83
    4 IF(LJ .EQ. 46)GO TO 5
      A(2,LJ)=0
      GO TO 83
    5 A(2,LJ)=1
   83 CONTINUE
      DO 50 LL=3,12
      DO 51 LL1=1,101
      A(LL,LL1)=0
   51 CONTINUE
   50 CONTINUE
      IF(J .EQ. 1)GO TO 901
      IF(J .EQ. 2)GO TO 902
      IF(J .EQ. 3)GO TO 903
      IF(J .EQ. 4)GO TO 904
      IF(J .EQ. 5)GO TO 905
  901 A(3,1)=-1
      A(3,8)=1
      A(3,47)=1
      A(4,13)=-1
      A(4,15)=1
      A(4,48)=1
      A(5,18)=-1
      A(5,30)=1
      A(5,49)=1
      A(6,33)=-1
      A(6,35)=1
      A(6,50)=1
      A(7,43)=-1
      A(7,45)=1
      A(7,51)=1
      GO TO 906
  902 A(3,3)=-1
      A(3,15)=1
      A(3,47)=1
```

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```
 902 A([illegible])=-1
     A([illegible]5)=1
     A([illegible]7)=1
     A([illegible]8)=-1
     A(4[illegible]0)=1
     A(4[illegible]8)=1
     A(5,[illegible]8)=-1
     A(5,[illegible])=1
     A(5,[illegible])=1
     A(6,[illegible])=-1
     A(6,[illegible])=1
     A(6,5[illegible])=1
     GO TO 906
 903 A(3,3)=-1
     A(3,5)=1
     A(3,47)=1
     A(4,13)=-1
     A(4,15)=1
     A(4,48)=1
     A(5,25)=-1
     A(5,29)=1
     A(5,49)=1
     A(6,35)=-1
     A(6,39)=1
     A(6,50)=1
     GO TO 906
 904 A(3,10)=-1
     A(3,14)=1
     A(3,47)=1
     A(4,20)=-1
     A(4,25)=1
     A(4,48)=1
     A(5,31)=-1
     A(5,35)=1
     A(5,49)=1
     A(6,40)=-1
     A(6,45)=1
     A(6,50)=1
     GO TO 906
 905 A(3,5)=-1
     A(3,10)=1
     A(3,47)=1
     A(4,16)=-1
     A(4,20)=1
     A(4,48)=1
     A(5,25)=-1
     A(5,30)=1
     A(5,49)=1
     A(6,39)=-1
     A(6,43)=1
     A(6,50)=1
 906 CONTINUE
     DO 7 LI=13,57
     DO 8 LJ=1,45
     LJ2=LI-12
     IF(LJ .EQ. LJ2)GO TO 9
     A(LI,LJ)=0
     GO TO 8
   9 A(LI,LJ)=1
   8 CONTINUE
   7 CONTINUE
     DO 10 LI=13,57
     DO 11 LJ=46,56
     A(LI,LJ)=0
  11 CONTINUE
  10 CONTINUE
     DO 12 LI=13,57
     DO 14 LJ=57,101
     LJ22=LI-12
     LJ33=LJ-56
     IF(LJ33 .EQ. LJ22)GO TO 15
     A(LI,LJ)=0
     GO TO 14
  15 A(LI,LJ)=1
  14 CONTINUE
  12 CONTINUE
     B(1)=0
     B(2)=BDGT(J)
     DO 16 LI=3,12
     B(LI)=0
  16 CONTINUE
 908 CONTINUE
     DO 17 LI=13,57
     B(LI)=1
  17 CONTINUE
     INFIX(1)=4
     INFIX(2)=101
     INFIX(3)=60
     INFIX(4)=57
     INFIX(5)=2
     INFIX(6)=1
     INFIX(7)=100
     INFIX(8)=0
     TOL(1)=0.00001
     TOL(2)=0.00001
     TOL(3)=-0.001
     TOL(4)=0.0000000001
     PRN=0
     IF(J .EQ. 1)GO TO 1071
     CALL ANYPRO(NA,J,XSTAROP,A,B)
1071 IF(FR .EQ. 1)GO TO 107
     IF(FR .EQ. 2)GO TO 1091
```

```
      CALL ANYPRO(NA,J,XSTAROP,A,B)
 1071 IF(FR .EQ. 1)GO TO 107
      IF(FR .EQ. 2)GO TO 1091
      GO TO 106
  107 DO 1088 KT=1,30
      A(1,KT)=0
 1088 CONTINUE
      DO 1099 KT=13,42
      B(KT)=0
 1099 CONTINUE
      GO TO 106
 1091 DO 1118 KT=1,15
      A(1,KT)=0
 1118 CONTINUE
      DO 1119 KT=31,45
      A(1,KT)=0
 1119 CONTINUE
      DO 1120 KT=13,27
      B(KT)=0
 1120 CONTINUE
      DO 1121 KT=43,57
      B(KT)=0
 1121 CONTINUE
  106 CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E)
      IF(KOUT(1) .EQ. 4)WRITE(6,1000)
      IF(KOUT(1) .EQ. 6)WRITE(6,1001)
 1000 FORMAT(2X,*INFEASIBLE SOLUTION*)
 1001 FORMAT(2X,*ITERATION LIMIT EXX*)
      IF(KOUT(1) .EQ. 4)STOP
      IF(ANA .EQ. 0)GO TO 5777
      WRITE(6,8748)
 8748 FORMAT(1H1)
      WRITE(6,7000)J
 5777 CONTINUE
      DO 21 MML1=1,45
      LV1=KB(MML1)
      IF(X(LV1) .GT. 1)WRITE(6,1002)
 1002 FORMAT(2X,*SOLUTION GR THAN 1*)
      IF(X(LV1) .GE. .5)GO TO 22
      GO TO 21
   22 XSTAROP(MML1,J)=1
      PSELEC(MML1,J)=XSTAROP(MML1,J)+PSELEC(MML1,J)
 7000 FORMAT(///,10X,*.....YEAR OF ANALYSIS=*,I5,*.....*,//)
      IF(ANA .EQ. 0)GO TO 21
      WRITE(6,77)MML1,X(LV1)
   77 FORMAT(2X,*PROJECT SELECTED*,I4,2X,*VALUE OF THE PROJECT*,F10.5)
   21 CONTINUE
C*****COMPUTATION OF THE EXPECTED PRESENT WORTH VALUE OF THE SER
C*****OF PROJECTS AND THEIR VARIABILITY (WITH CROSS-CORRELATIONS).
      CALL EXPECT(MM,XSTAROP,NA,J,V,CCR,EPW,VPW,INT,TEPW,TVPW)
      TEPP(SS)=TEPW+TEPP(SS)
      TVPP(SS)=TVPW+TVPP(SS)
      TEPPP=TEPW+TEPPP
      TVPPP=TVPW+TVPPP
C*****SIMULATION OF THE CASH FLOWS ******************
C******                            !!*********!!!****!!
      MMMM=M+1
  555 FORMAT(2X,5F10.2)
      IF(ANA .EQ. 0)GO TO 6763
      CALL ANAL(J,MM,NA,XSTAROP,EE,CASH,TCOST)
      GO TO 6764
 6763 CALL SIMUL(J,MM,NA,XSTAROP,RA,RB,ML,PE,OP,CASH,TCOST)
 6764 CALL NYB(XSTAROP,NA,J,BDGT,TCOST,INT,XNEXBUD)
      TTCOST=TTCOST+TCOST
      WRITE(6,815)XNEXBUD
  815 FORMAT(10X,*ADDITION OR SUBSTACTION TO THE NEXT YEAR BUDGET=*,
     1F20.2)
      JT=J+1
C********SUMMATION OF CASH IN EVERY YEAR FOR ALL PROJECTS
      DO 29 JCA=JT,MM
      JC=JCA-1
      EYCASH(JC)=EYCASH(JC)+CASH(JC)
   29 CONTINUE
      IF(J .EQ. MM1) GO TO 108
      J=J+1
      GO TO 109
  108 WRITE(6,8631)
 8631 FORMAT(1H1)
      WRITE(6,8632)TTCOST
 8632 FORMAT(///,5X,*TOTAL COST OF THE PROJECTS=*,F10.2)
      WRITE(6,812)(EYCASH(JC),JC=1,MMMM)
      TCHHT=0.
C*****CASH IN THE HORIZONT*******
      DO 30 MM5=1,MMMM
      MM55=MMMM-MM5
      TCHHT=TCHHT+(EYCASH(MM5)*(INT**MM55))
   30 CONTINUE
      XXTCH(SS)=TCHHT
      XTCH=XTCH+TCHHT
      WRITE(6,813)TCHHT
      IF(SS .EQ. Z) GO TO 250
      SS=SS+1
      GO TO 251
  250 WRITE(6,7090)
 7090 FORMAT(1H1,///,5X,*..............STATISTICS OF THE SIMULATION.............
     1..*,////)
      PXTCH=XTCH/SS
      WRITE(6,6688)PXTCH
 6688 FORMAT(5X,*MEAN OF THE CASH IN THE HORIZONT*,F20.2)
      XXSUM=0
      MSS=SS
      DO 6785 KX=1,MSS
      HABS=ABS(PXTCH-(XTCH/MSS))
```

```
  250 WRITE(6,7090)
 7090 FORMAT(1H1,///,5X,*..............STATISTICS OF THE SIMULATION..............
     1..*,/////)
      PXTCH=XTCH/SS
      WRITE(6,6688)PXTCH
 6688 FORMAT(5X,*MEAN OF THE CASH IN THE HORIZONT*,F20.2)
      XXSUM=0
      MSS=SS
      DO 6785 KX=1,MSS
      VABS=ABS(PXTCH-XXTCH(MSS))
      XXSUM=XXSUM+(VABS**2)
 6785 CONTINUE
      SSS=SS-1
      IF(ANA .EQ. 1)SSS=2
      XX22=XXSUM/SSS
      RXX22=XX22**.5
      WRITE(6,6786)RXX22
 6786 FORMAT(//,5X,*VARIANCE OF THE CASH IN THE HORIZONT*,F20.2)
      WRITE(6,7091)
 7091 FORMAT(///,5X,*PERC. OF THE TIMES EACH PROJECT IS SELECTED IN SIMU
     1LATION*,//)
      DO 7001 KY=1,MM1
      DO 7002 KYA=1,45
      PPSEL(KYA,KY)=(PSELEC(KYA,KY)/SS)*100
 7002 CONTINUE
      WRITE(6,7015)KY
 7015 FORMAT(///,5X,*YEAR*,I4,/)
      WRITE(6,7003)(PPSEL(KYA,KY),KYA=1,45)
 7003 FORMAT(2X,10F10.2)
 7001 CONTINUE
      TTTPW=TEPPP/SS
      TTVPW=TVPPP/SS
      WRITE(6,7020)TTTPW,TTVPW
 7020 FORMAT(//,2X,*MEAN OF THE P.W.*,F20.2,5X,*MEAN OF THE VARIANCE OF THE
     1 P.W.*,F20.2,//)
      DO 7010 KY=1,MSS
      VVABS=ABS(TTTPW-TEPP(MSS))
      VVABS1=ABS(TTVPW-TVPP(MSS))
      YXSUM1=YXSUM1+(VVABS**2)
      YXSUM2=YXSUM2+(VVABS1**2)
 7010 CONTINUE
      YY1=YXSUM1/SSS
      YY2=YXSUM2/SSS
      RYY1=YY1**.5
      RYY2=YY2**.5
      WRITE(6,7011)RYY1,RYY2
 7011 FORMAT(2X,*VAR P.W.*,F20.2,2X,*VAR V.P.W.*,F20.2)
      IF(L .EQ. LSTR) GO TO 130
      GO TO 131
  130 CONTINUE
      STOP
  804 FORMAT(5X,*LAM=*,F10.3,/)
  805 FORMAT(5X,*MA=*,I4,2X,*NA=*,I4,2X,*NS=*,I4,2X,*NTO=*,I4,2X,*IO=*,I
     14,2X,*IPCT=*,I4,2X,*NSC=*,I4,2X,*IBD=*,I4,2X,*IAUG=*,I4,/)
  806 FORMAT(5X,*BS=*,2X,F20.3,/)
  807 FORMAT(5X,*C=*,2X,F20.3,/)
  808 FORMAT(5X,*A=*,2X,8F10.2)
  809 FORMAT(5X,*XSTAROP=*,2X,F20.3,/)
  810 FORMAT(5X,*J=*,I4,2X,*TEPP=*,2X,F20.3,2X,*TVPP=*,2X,F20.3,/)
  811 FORMAT(5X,*EYCASH=*,2X,F20.3,/)
  812 FORMAT(5X,*EYCASHSUM=*,2X,F20.3,/)
  813 FORMAT(5X,*TCHHT=*,2X,F20.3,/)
  814 FORMAT(5X,*J=*,I4,///)
      STOP
      END
      SUBROUTINE EXPECT(MM,XSTAROP,NA,J,V,CCR,EPW,VPW,INT,TEPW,TVPW)
      REAL INT
      DIMENSION XSTAROP(50,5),CCR(45,45,5),EPW(375),VPW(375),V(375,12)
      TEPW=0.
      TVPW=0.
      CCF=0.
      DO 31 I=1,NA
      IF(XSTAROP(I,J) . EQ. 1.) GO TO 110
      GO TO 31
  110 IF(J .EQ. 1)IWO=I
      IF(J .EQ. 2)IWO=I+60
      IF(J .EQ. 3)IWO=I+135
      IF(J .EQ. 4)IWO=I+225
      IF(J .EQ. 5)IWO=I+330
      TEPW=TEPW+EPW(IWO)
      TVPW=TVPW+VPW(IWO)
   31 CONTINUE
      TVVPW=TVPW
      WRITE(6,543)TEPW,TVPW
  543 FORMAT(//,10X,*TEPW=*,F10.2,2X,*TVPW=*,F20.2,//)
      NNA=NA-1
      DO 32 I=1,NNA
      IF(XSTAROP(I,J) .EQ. 1)GO TO 111
      GO TO 32
  111 IW=I+1
      DO 33 JA=IW,NA
      IF(XSTAROP(IW,J) .EQ. 1)GO TO 112
      GO TO 33
  112 IF(J .EQ. 1)NTJ=I
      IF(J .EQ. 1)NTJJ=JA
      IF(J .EQ. 2)NTJ=I+60
      IF(J .EQ. 2)NTJJ=JA+60
      IF(J .EQ. 3)NTJ=I+135
      IF(J .EQ. 3)NTJJ=JA+135
      IF(J .EQ. 4)NTJ=I+225
      IF(J .EQ. 4)NTJJ=JA+225
      IF(J .EQ. 5)NTJ=I+330
```

```
      IF(J .EQ. 1)NTJJ=JA
      IF(J .EQ. 2)NTJ=I+60
      IF(J .EQ. 2)NTJJ=JA+60
      IF(J .EQ. 3)NTJ=I+135
      IF(J .EQ. 3)NTJJ=JA+135
      IF(J .EQ. 4)NTJ=I+225
      IF(J .EQ. 4)NTJJ=JA+225
      IF(J .EQ. 5)NTJ=I+330
      IF(J .EQ. 5)NTJJ=JA+330
      IF(CCR(I,IW,J) .EQ. 0)GO TO 33
      DO 34 K=2,MM
      K1=K-1
      K2=K-2
      RV1=V(NTJ,K1)**.5
      RV2=V(NTJJ,K1)**.5
      CCF=CCF+((RV1*RV2)/(INT**(2*K2)))
   34 CONTINUE
      CCF=2*CCR(NTJ,NTJJ,J)*CCF
      TVVPW=TVVPW+CCF
      CCF=0
   33 CONTINUE
   32 CONTINUE
      WRITE(6,544)TVVPW
  544 FORMAT(10X,*TVPW WITH CCR=*,F20.2)
      RETURN
      END
      SUBROUTINE SIMUL(J,MM,NA,XSTAROP,RA,RB,ML,PE,OP,CASH,TCOST)
      REAL ML,M
      DIMENSION XSTAROP(50,5),ML(375,12),PE(375,12),OP(375,12),CASH(
     115),ESTX(375,12)
      DO 6 I=1,MM
      CASH(I)=0.
    6 CONTINUE
      TCOST=0.
      DO 700 II=1,NA
      COSTPRY=0.
      IF(XSTAROP(II,J) .EQ. 1) GO TO 701
      GO TO 700
  701 IF(J .EQ. 1)NR=0
      IF(J .EQ. 2)NR=60
      IF(J .EQ. 3)NR=135
      IF(J .EQ. 4)NR=225
      IF(J .EQ. 5)NR=330
      I=II+NR
      DO 702 J11=2,MM
      J1=J11-1
      M=ML(I,J1)
      A=PE(I,J1)
      B=OP(I,J1)
      IF(B .EQ. 0) GO TO 444
      GO TO 450
  444 IF(A .EQ. 0) GO TO 445
      GO TO 450
  445 IF(M .EQ. 0) GO TO 702
  450 CONTINUE
      CALL RVALUE(M,A,B,RA,RB,BETA)
  333 FORMAT(5X,*BETA=*,F10.2)
      ESTX(I,J1)=BETA
      IF(J1 .EQ. J) GO TO 703
      CASH(J1)=CASH(J1)+ESTX(I,J1)
      GO TO 702
  703 CASH(J1)=0.
  702 CONTINUE
      COSTPRY=COSTPRY+ESTX(I,J)
      TCOST=TCOST+ESTX(I,J)
  700 CONTINUE
      WRITE(6,707)J,TCOST
      RETURN
  704 FORMAT(5X,*J1=*,I4,2X,*CASH(J1)=*,F20.3,/)
  705 FORMAT(5X,*II=*,I4,2X,*COSTPRY=*,F20.3,/)
  706 FORMAT(5X,*J1=*,I4,2X,*CASH(1)=*,F20.3,/)
  707 FORMAT(5X,*J=*,I4,2X,*TCOST=*,F20.3,/)
      RETURN
      END
      SUBROUTINE RVALUE(M,A,B,RA,RB,BETA)
      REAL M
      XMU=(4*M+A+B)/6.0
      XVAR=((B-A)*(B-A))/36.0
      BMEAN=(XMU-A)/(B-A)
      BVAR=XVAR/((B-A)*(B-A))
      XK1=BMEAN*(BMEAN*(1.0-BMEAN)/BVAR-1.0)
      XK2=XK1*((1.0-BMEAN)/BMEAN)
      CALL GAMMARN(XK1,RA,RB,GAM1)
      CALL GAMMARN(XK2,RA,RB,GAM2)
      BETA=(GAM1/(GAM1+GAM2))*(B-A)+A
      RETURN
      END
      SUBROUTINE GAMMARN(TK,RA,RB,GAM)
      GAMMA=1.0
      K1=TK
      TK1=K1
      CALL RANDU(RA,RB,R1)
      IF(R1-(TK-TK1)) 10,10,20
   10 K1=K1+1
   20 DO 303 IW=1,K1
      CALL RANDU(RA,RB,R1)
  303 GAMMA=GAMMA*R1
      GAM= -ALOG(GAMMA)
      RETURN
      END
      SUBROUTINE RANDU(RA,RB,YFL)
      TEMP=RA+RB
      IF(TEMP .LE. [illegible]
```

```
          K1=TK
          TK1=K1
          CALL RANDU(RA,RB,R1)
          IF(R1-(TK-TK1)) 10,10,20
       10 K1=K1+1
       20 DO 303 IW=1,K1
          CALL RANDU(RA,RB,R1)
      303 GAMMA=GAMMA*R1
          GAM= -ALOG(GAMMA)
          RETURN
          END
          SUBROUTINE RANDU(RA,RB,YFL)
          TEMP=RA+RB
          IF(TEMP .LE. 1.) GO TO 555
          TEMP=TEMP-1.0
      555 YFL=TEMP
          RA=RB
          RB=TEMP
          RETURN
          END
          SUBROUTINE NYB(XSTAROP,NA,J,BDGT,TCOST,INT,XNEXBUD)
          REAL INT
          DIMENSION XSTAROP(50,5),BDGT(6)
          XYZ=ABS(TCOST)
          XXT=BDGT(J)-XYZ
          IF(XXT .LE. 0) WRITE(6,747)
      747 FORMAT(10X,*NEED MONEY*)
          XXTT=XXT*INT
          J12=J+1
          BDGT(J12)=BDGT(J12)+XXTT
          XNEXBUD=XXTT
          RETURN
          END
          SUBROUTINE ANYPRO(NA,J,XSTAROP,A,B)
          DIMENSION XSTAROP(50,5),A(60,105),B(60)
          DO 25 MTA=2,J
          MT1=MTA-1
          DO 26 MT2=16,NA
          IF(XSTAROP(MT2,MT1) .EQ. 1) GO TO 107
          GO TO 26
      107 MTT3=MT2-15
          A(1,MTT3)=0
          MTT2=MTT3+12
          B(MT22)=0
       26 CONTINUE
       25 CONTINUE
          RETURN
          END
          SUBROUTINE VER ( A, B, JH, X, E, KB, Y, N, NE, M, MF, INVC,
         1                    NUMVR, NUMPV, INFS, LA, TPIV, TECOL, M2 )
C
C***************************************************************************
C
C               VER TAKES THE BASIS SET OF COLUMNS (AS INDICATED BY
C               KB), ALONG WITH WHATEVER ARTIFICIAL COLUMNS ARE
C               NECESSARY, AND FORMS AN INVERSE (E). OTHER OUTPUT
C               INCLUDES: VALUE OF BASIC VARIABLES (X), CHANGES TO
C               BASIS SET OF COLUMNS (KB), AND THE ITERATION STATISTICS
C               (NUMVR, NUMPV, INFS, INVC). IT MAY HAPPEN THAT
C               SOME OF THE REAL COLUMNS (AS INDICATED BY KB)
C               CANNOT BE PIVOTED INTO THE BASIS, IN WHICH CASE THEY
C               ARE REPLACED BY AN ARTIFICIAL COLUMN.
C
C               THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT RETURNED
C               TO SIMPLX. SUBROUTINES CALLED BY THIS SUBROUTINE ARE
C               JMY (UPDATE OF ENTERING COLUMN) AND PIV (PERFORMS
C               ACTUAL PIVOT).
C
C               INPUT: A,B,JH,KB,N,NE,M,MF,NUMVR,NUMPV,X,Y
C
C               OUTPUT: X,E,KB,INVC,NUMVR,NUMPV,INFS
C
C***************************************************************************
C
          DIMENSION JH(1), X(1), E(1), KB(1), Y(1), A(1), B(1)

C                        INITIATE
          IF (LA)  1121, 1121, 1122
     1121 INVC = 0
     1122 NUMVR    =  NUMVR  +1
           DO 1101  I = 1, M2
     1101  E(I)=0.0
          MM=1
           DO 1113  I = 1, M
          E(MM) =1.0
          X(I) = B(I)
     1113 MM = MM + M + 1
            DO 1110  I = MF, M
          IF (JH(I)) 1111, 1110, 1111
     1111 JH(I) = 12345
     1110 CONTINUE
          INFS = 1
C                        FORM INVERSE
            DO 1102  J = 1, M
          IF ( KB(J) )  600 , 1102 , 600
      600  CALL        JMY ( J, A, E, M, Y, ME )
C                                CHOOSE PIVOT
     1114 TY = 0.0
            DO 1104  I = MF, M
          IF (JH(I)  - 12345 )  1104, 1105, 1104
     1105 IF (ABS(Y(I))-TY) 1104,1104,1106
     1106 IR    = I
          TY=ABS(Y(I))
```

```
      DO 1102  J = 1, N
      IF ( KB(J) )  400 , 1102 , 400
 400  CALL       JMY ( J, A, E, M, Y, ME )
C                         CHOOSE PIVOT
1114 TY = 0.0
        DO 1104  I = MF, M
      IF (JH(I)  - 12345 )  1104, 1105, 1104
1105 IF (ABS(Y(I))-TY) 1104,1104,1106
1106 IR    = I
      TY=ABS(Y(I))
1104     CONTINUE
C                         TEST PIVOT
      IF(TY-TPIV) 1107,1108,1108
C         BAD  PIVOT,  ROW IR,  COLUMN J
1107 KB(J) = 0
      GO TO 1102
C                         PIVOT
1108 JH(IR) = J
      KB(J)  = IR
 900  CALL       PIV  ( IR, Y, M, E, X, NUMPV, TECOL )
1102      CONTINUE
C                     RESET ARTIFICIALS
        DO 1109  I = 1, M
      IF ( JH(I) - 12345  )  1109, 1112, 1109
1112 JH(I) = 0
1109   CONTINUE
      RETURN
      END
      SUBROUTINE NEW (M,N, JH, KB, A, B, MF, ME )
C
C**********************************************************************
C
C           NEW SCANS 'A' OF THE INITIALLY FORMULATED TABLEAU OF A
C           NEW PROBLEM TO SEE IF THERE ARE ANY COLUMNS THAT CAN BE
C           USED INSTEAD OF ARTIFICIAL COLUMNS IN THE INITIAL BASIS.
C           TO BE ELIGIBLE, A COLUMN MUST HAVE ONLY ONE NON-ZERO
C           ELEMENT, AND IT MUST HAVE THE SAME SIGN AS THE CORRE-
C           SPONDING RIGHT HAND SIDE ELEMENT. (THIS MEANS THAT ANY
C           NEGATIVE BASIC VARIABLE MUST BE ARTIFICIAL.)
C
C           THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT RETURNED
C           TO SIMPLX.
C
C           INPUT: M,N,A,B,MF,ME
C
C           OUTPUT: JH, KB
C
C**********************************************************************
C

      DIMENSION JH(1), KB(1), A(1), B(1)
1400   DO 1401 I = 1, M
1401 JH(I) = 0
C                     INSTALL SINGLETONS
      KT  =  0
       DO 1402  J = 1, N
      KB(J) = 0
      KTA = KT + MF
      KTB = KT + M
C                                   TALLY ENTRIES IN CONSTRAINTS
      KQ  = 0
      DO  1403 L = KTA,KTB
      IF (A(L)) 1404, 1403, 1404
1404 KQ = KQ+1
      LQ = L
1403   CONTINUE
C                                   CHECK WHETHER J IS CANDIDATE
       IF(KQ -1) 1402,1405,1402
1405 IQ = LQ- KT
         IF ( JH(IQ) ) 1402, 1406, 1402
1406 IF (A(LQ)*B(IQ)) 1402, 1407, 1407
C                                   J IS CANDIDATE. INSTALL
1407 JH(IQ) = J
      KB(J)  = IQ
1402 KT = KT + ME
      RETURN
      END
      SUBROUTINE MIN ( JT, N, M, A, P, KB, ME, TCOST, IR, TPIV, Y, JIN )
C
C**********************************************************************
C
C           MIN SELECTS THE COLUMN TO ENTER THE BASIS. IT SELECTS
C           THE COLUMN WITH THE MOST NEGATIVE REDUCED COST (AS
C           COMPUTED IN SUBROUTINE DEL). THE COLUMN NUMBER SELECTED
C           (JT) IS RETURNED TO THE CALLING SUBROUTINE.
C
C           THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT IS RETURNED
C           TO SIMPLX. SUBROUTINE DEL IS CALLED TO COMPUTE THE
C           REDUCED COST OF EACH COLUMN.
C
C           INPUT: N,M,A,P,K,B,ME,TCOST,IR,TPIV,Y
C
C           OUTPUT: JT, JIN
C
C**********************************************************************
C
      DIMENSION P(1), KB(1), Y(1), A(1)
 700 JT = 0
      DA = TCOST
      IQ=0
      PIV = -TPIV
      JIN = 0 -TPIV
      PIVO [illegible]
      [illegible]
```

```
C
      IF(KQ -1) 1402,1405,1402
 1405 IQ = LQ- KT
         IF ( JH(IQ) ) 1402, 1406, 1402
 1406 IF (A(LQ)*B(IQ)) 1402, 1407, 1407
C                                   J IS CANDIDATE. INSTALL
 1407 JH(IQ) = J
      KB(J)  = IQ
 1402 KT = KT + ME
      RETURN
      END
      SUBROUTINE MIN ( JT, N, M, A, P, KB, ME, TCOST, IR, TPIV, Y, JIN )
C
C***********************************************************************
C
C          MIN SELECTS THE COLUMN TO ENTER THE BASIS. IT SELECTS
C          THE COLUMN WITH THE MOST NEGATIVE REDUCED COST (AS
C          COMPUTED IN SUBROUTINE DEL). THE COLUMN NUMBER SELECTED
C          (JT) IS RETURNED TO THE CALLING SUBROUTINE.
C
C          THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT IS RETURNED
C          TO SIMPLX. SUBROUTINE DEL IS CALLED TO COMPUTE THE
C          REDUCED COST OF EACH COLUMN.
C
C          INPUT: N,M,A,P,K,B,ME,TCOST,IR,TPIV,Y
C
C          OUTPUT: JT, JIN
C
C***********************************************************************
C
      DIMENSION P(1), KB(1), Y(1), A(1)
  700 JT = 0
      DA = TCOST
      IS=0
      PIV = -TPIV
      JIN = 0
      PIVO = -TPIV
      AA = -1.0E+20
C
  701 DO  702  JM = 1, N
C                                   SKIP COLUMNS IN BASIS
  703 IF ( KB(JM) )  702, 300, 702
  300  CALL      DEL (  JM, DT, N, A, P, ME, IR, DP, Y )
      IF ( IR - 1 )  705,705,2705
C                  DUALS   RATIO TEST
 2705 IF ( ABS(DT) + TCOST )  2706,2708,2708
C                           ZERO RATIO - USE MOST NEGATIVE PIVOT ELEMENT
 2706 IF ( DP - PIV )  2707,702,702
 2707 PIV = DP
      JT = JM
      IS = 1
      GO TO 702
C                           NONZERO RATIO
 2708 IF ( IS )  702,2709,702
C                           IF DUAL INFEASIBLE, SET JIN AND EXIT
 2709 IF ( DT )  2710,2711,2711
 2710 JT = JM
      JIN = 1
      GO TO 2702
C                           SKIP_POSITIVE (I NEAR ZERO) PIVOT ELEMENTS
 2711 IF ( DP + TPIV ) 2712,702,702
 2712 RATIO = DT/DP
C                           SAVE MINIMUM RATIO
      IF ( RATIO - AA )  702,2713,2715
C                           IF RATIO TIE, USE MOST NEGATIVE PIVOT ELEMENT
 2713 IF ( DP - PIVO ) 2714,702,702
 2714 PIVO = DP
 2715 AA = RATIO
      JT = JM
      GO TO 702
C                  PRIMALS     DJ TEST
  705 IF   ( DT - DA )  708, 702, 702
  708 DA = DT
      JT = JM
  702    CONTINUE
 2702 CONTINUE
      RETURN
      END
      SUBROUTINE JMY (JT, A, E, M, Y, ME )
C
C***********************************************************************
C
C          JMY UPDATES COLUMN JT OF THE ORIGINAL CONSTRAINT-SET
C          BY PREMULTIPLYING IT BY THE CURRENT INVERSE. THE
C          UPDATED COLUMN IS RETURNED AS Y. THE MULTIPLICATION
C          IS DONE IN COLUMN RATHER THAN ROW ORDER.
C
C          THE SUBROUTINE IS USED BY SIMPLX AND VER, AND RETURNS
C          OUTPUT TO CALLING SUBROUTINE.
C
C          INPUT: JT,A,E,M,ME
C
C          OUTPUT: Y
C
C***********************************************************************
C
      DIMENSION E(1), Y(1), A(1)
C
  600 DO 610  I= 1,M
  610 Y(I) =0.0
      LP  =  JT*ME - ME
      LL = 0
      DO 605    I= 1,M
```

```
C
C          INPUT: JT,A,E,M,ME
C
C          OUTPUT: Y
C
C***********************************************************************
C
      DIMENSION E(1), Y(1), A(1)
C
  600 DO 610  I= 1,M
  610 Y(I) =0.0
      LP  =  JT*ME - ME
      LL = 0
      DO 605   I= 1,M
      LP = LP + 1
      IF  (A(LP))   601, 602, 601
  601 DO 606  J  =  1,M
      LL = LL + 1
  606 Y(J) = Y(J) + A(LP) * E(LL)
      GO TO 605
  602 LL = LL + M
  605 CONTINUE
  699 RETURN
      END
      SUBROUTINE PIV  ( IR, Y, M, E, X, NUMPV, TECOL )
C
C***********************************************************************
C
C          PIV, USING AN UPDATED COLUMN, Y, PIVOTS THE COLUMN
C          INTO THE BASIS, PIVOTING ON ROW:IR (ESTABLISHED IN
C          SUBROUTINE ROW). AFTER PIVOTING, THE BASIC VARIABLES
C          ARE UPDATED.
C
C          THE SUBROUTINE IS USED BY SIMPLX AND VER, AND RETURNS
C          OUTPUT TO CALLING SUBROUTINE.
C
C          INPUT: IR,Y,M,E,X,NUMPV,TECOL
C
C          OUTPUT: E,X,NUMPV
C
C***********************************************************************
C
      DIMENSION Y(1), E(1), X(I)
C
  900 NUMPV  =  NUMPV  +  1
C
      T2 = -Y(IR)
      Y(IR) = -1.0
      LL = 0
C                                   TRANSFORM INVERSE
  903 DO 904  JP= 1, M
      L = LL + IR
      IF(ABS(E(L))-TECOL) 914,914,905
  914 LL = LL + M
      GO TO 904
  905 T3 = E(L) / T2
      E(L) =0.0
      DO 906  I  = 1, M
      LL= LL +1
  906 E(LL) = E(LL) +T3* Y(I)
  904 CONTINUE
C                                   TRANSFORM X
       T3 = X(IR) / T2
      X(IR) = 0.0
      DO 908  I  = 1, M
  908 X(I) = X(I) +T3* Y(I)
C                              RESTORE Y(IR)
      Y(IR) = -T2
C
  999 RETURN
      END
      SUBROUTINE DEL (  JM, DT, M, A, P, ME, IR, DP, Y )
C
C***********************************************************************
C
C          DEL COMPUTES THE REDUCED COST FOR THE COLUMN JM.
C
C          THIS SUBROUTINE IS USED BY MIN AND OUTPUT IS RETURNED
C          TO MIN.
C
C          INPUT: JM,M,A,P,ME,IR,Y
C
C          OUTPUT: DT
C
C          THE ARGUMENT VARIABLES USED IN THIS SUBROUTINE ARE AS
C          FOLLOWS:
C          DT= REDUCED COST
C          P=  SIMPLEX MULTIPLIERS (PI)
C
C***********************************************************************
C
      DIMENSION P(1), Y(1), A(1)
  300 DT = 0.0
      DP=0.0
      KDEL = (JM - 1) * ME
C
  301 DO  303  IDEL = 1, M
      KDEL=KDEL+1
      IF ( A(KDEL))304, 303, 304
  304  IF ( P(IDEL) )  302,2303, 302
  302 DT = DT + P(IDEL) * A(KDEL)
C          DO SECOND PRICING VECTOR (IF NONZERO COEFFS)
[illegible]
```

```
      [illegible]
  300 DT = 0.0
      DP=0.0
      KDEL = (JM - 1) * ME
C
  301 DO  303  IDEL = 1, M
      KDEL=KDEL+1
      IF ( A(KDEL))304, 303, 304
  304  IF ( P(IDEL) )  302,2303, 302
  302 DT = DT + P(IDEL) * A(KDEL)
C           DO SECOND PRICING VECTOR (IF NONZERO COEFFS)
 2303 IF (IR-1)  303,303,2304
 2304 IF (Y(IDEL))    2305,303,2305
 2305 DP=DP+Y(IDEL)*A(KDEL)
  303 CONTINUE
C
  399 RETURN
      END
      SUBROUTINE ERR ( M, A, B, TERR, JH, X, P, Y, ME, LA )
C
C*************************************************************************
C
C          ERR CALCULATES THE SOLUTION ERROR ACCUMULATED AND
C          STORES IT SO THAT IT MAY BE PRINTED IN THE OUTPUT,
C          IF DESIRED.
C
C          THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT RETURNED
C          TO SIMPLX.
C
C          INPUT: M,A,B,TERR,JH,X,P,Y,ME,LA
C
C          OUTPUT: TERR
C
C          THE ARGUMENT TERR IS EQUIVALENCED TO ERR IN THE
C          SIMPLX SUBROUTINE.
C
C*************************************************************************
C
      DIMENSION JH(1), X(1), P(1), Y(1), TERR(8), A(1), B(1)
      DO 401 I = 1, M
  401 Y(I) =-B(I)
      DO 402 I = 1, M
      JA = JH(I)
      IF (JA) 403, 402, 403
  403 IA=ME*(JA-1)
      DO 405 IT = 1, M
      IA = IA + 1
      IF(A(IA) )  415, 405, 415
  415 Y(IT) =Y(IT) +X(I) * A(IA)
  405 CONTINUE
  402 CONTINUE
C                                    FIND SUM AND MAXIMUM OF ERRORS
      DO 481  I = 1, M
      YI  = Y(I)
      IF ( JH(I) )  472, 471, 472
  471  YI = YI + X(I)
  472 TERR(LA+1)=TERR(LA+1)+ABS(YI)
      IF(ABS(TERR(LA+2))-ABS(YI)) 482,481,481
  482 TERR(LA+2) = YI
  481  CONTINUE
C                                    STORE P TIMES BASIS AT DT
         IR = 0
      DO 411  I = 1, M
      JM = JH(I)
      IF ( JM     )  300 , 411 , 300
  300  CALL      DEL (  JM, DT, M, A, P, ME, IR, DP, Y )
  410 TERR(LA+3)=TERR(LA+3)+ABS(DT)
      IF(ABS(TERR(LA+4))-ABS(DT)) 413,411,411
  413  TERR(LA+4)  =  DT
  411 CONTINUE
      RETURN
      END
      SUBROUTINE BET ( M, MC, MF, JH, X, Y, P, E, IR, PMIX )
C
C*************************************************************************
C
C          BET OBTAINS THE VECTOR OF SIMPLEX MULTIPLIERS TO BE
C          USED TO CALCULATE REDUCED COSTS. IN A NORMAL, PHASE 2
C          ITERATION, THIS IS JUST THE FIRST ROW OF THE CURRENT
C          INVERSE. IN PHASE 1, HOWEVER, SPECIAL PROCEDURES ARE
C          USED TO AVOID EXPLICITLY STATING THE INFEASIBILITY FORM.
C
C          THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT RETURNED
C          TO SIMPLX.
C
C          INPUT: M,MC,MF,JH,X,E,IR,PMIX
C
C          OUTPUT: Y, P
C
C*************************************************************************
C
      DIMENSION JH(1), X(1), P(1), E(1), Y(1)
  500 MMM = MC
      MMI = IR
      PS = 1.0
      IF ( JH(IR) )   502,2502,502
 2502 PS = -1.0
      IF ( X(IR) )  3502,502,502
 3502 PS = 1.0
C                              PRIMAL PRICES
  502 DO 503 J = 1, M
      P(J) = E(MMM)
      IF  (  IR - 1 )    503,503,2503
```

```
      P(J) = E(MMM)
      IF  (  IR - 1 )   503,503,2503
 2503 Y(J) = PS$E(MMI)
      MMI = MMI + M
  503 MMM =MMM + M
      IF ( IR - 1 )  599,501,599
C                        COMPOSITE PRICES
  501 DO 504  J = 1, M
  504 P(J) = P(J)$ PMIX
      DO 505  I = MF, M
      MMM =I
      IF  ( X(I) )  506, 507, 507
  506 DO 508  J = 1, M
      P(J) = P(J) + E(MMM)
  508 MMM = MMM + M
      GO TO 505
  507 IF (JH(I))  505, 509, 505
  509 DO 510  J = 1, M
      P(J) = P(J) - E(MMM)
  510 MMM = MMM +M
  505   CONTINUE
C
  599 RETURN
      END
      SUBROUTINE XCK ( M, MF, JH, X, TZERO, JIN )
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C           XCK EXAMINES THE CURRENT BASIC SOLUTION (X) FOR TWO
C           TESTS. FIRST, IT LOOKS FOR ANY VARIABLES THAT CAN BE
C           SET TO ZERO BY COMPARING TO THE ESTABLISHED ZERO
C           TOLERANCE (TZERO). SECOND, IT DETERMINES IF ANY ARTI-
C           FICIAL VARIABLES ARE NON-ZERO. IN THE SECOND TEST,
C           THE ROW INDEX OF THE NON-ZERO ARTIFICIAL VARIABLE (JIN)
C           IS RETURNED TO THE CALLING SUBROUTINE AS AN INDICATION
C           OF THE PHASE THE ALGORITHM IS IN.
C
C           THIS SUBROUTINE IS USED BY SIMPLX AND OUTPUT RETURNED
C           TO SIMPLX.
C
C           INPUT: M,MF,JH,X,TZERO
C
C           OUTPUT: JIN
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
      DIMENSION JH(1), X(1)
C
C             RESET X AND CHECK FOR INFEASIBILITIES
 1212   JIN  =  0
      XI = TZERO
      DO 1201 I = MF, M
      IF(ABS(X(I))-TZERO) 1202,1203,1203
 1202 X(I) = 0.0
      GO TO 1201
 1203 IF ( X(I) )  1204, 1201, 1205
 1205 IF ( JH(I) )  1201, 1206, 1201
 1206 IF ( XI - ABS(X(I)) )  1207,1207,1201
 1207 XI = ABS(X(I))
      JIN = I
 1201   CONTINUE
      RETURN
      END
      SUBROUTINE  ROW  ( IR, M, MF, JH, X, Y, TPIV )
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C           ROW PERFORMS THE OPERATION FOR THE EXIT CRITERION OF
C           THE PRIMAL SIMPLX ONLY. THE ROW CHOSEN IS DETERMINED
C           IN THE FOLLOWING ORDER:
C             1) X(IR)=0, ARTIFICIAL
C             2) X(IR)=0, REAL Y(IR)>0
C             3) X(IR)=NON-ZERO, X(IR)/Y(IR)= MIN(I)FOR
C                              (X(I)/Y(I):(X(I)/Y(I))>0)
C
C           THE SUBROUTINE IS USED BY SIMPLX AND OUTPUT IS RETURNED
C           TO SIMPLX.
C
C           INPUT: M,MF,JH,X,Y,TPIV
C
C           OUTPUT: IR
C
C           THE ARGUMENT IR IS THE INDEX OF THE ROW SELECTED TO
C           EXIT THE BASIS.
C
C$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
      DIMENSION JH(1), X(1), Y(1)
 1000 IR = 0
      AA = 0.0
      IA  =  0
      DO 1050  I = MF,  M
      IF ( X(I) )  1050, 1041, 1050
 1041 YI=ABS(Y(I))
      IF  (  YI  -  TPIV   )   1050, 1050, 1042
 1042 IF  ( JH(I) )  1043, 1044, 1043
 1043 IF  (IA)   1050, 1048, 1050
 1048  IF  ( Y(I) )  1050, 1050, 1045
 1044 IF   (IA)   1045, 1046, 1045
 1045 IF   (  YI    -  AA   )   1050, 1050, 1047
 1046    IA = 1
 1047 AA  =  YI
      IT  =  I
```

```
 1047 AA   =  TA
      IR   =  I
 1050 CONTINUE
      IF (IR)1099,1001,1099
 1001 AA = 1.0E+20
C                FIND MIN. PIVOT AMONG POSITIVE EQUATIONS
      DO 1010 IT = MF , M
      IF  ( Y(IT)  -  TPIV )  1010, 1010, 1002
 1002 IF  ( X(IT) ) 1010, 1010, 1003
 1003 XY = X(IT) / Y(IT)
      IF ( XY - AA )  1004, 1005, 1010
 1005 IF ( JH(IT))  1010, 1004, 1010
 1004 AA = XY
      IR = IT
 1010 CONTINUE
C  FIND PIVOT AMONG NEGATIVE EQUATIONS, IN WHICH X/Y IS LESS THAN THE
C MINIMUM X/Y IN THE POSITIVE EQUATIONS, THAT HAS THE LARGEST ABSF(Y)  M
 1016 BB = - TPIV
      DO 1030 I  = MF , M
      IF (X(I))  1012, 1030, 1030
 1012  IF  ( Y(I)     -  BB  )   1022, 1030, 1030
 1022 IF  (  Y(I) * AA  -  X(I)   )   1024
C*************************************************************************
, 1024, 1030
 1024 BB  =  Y(I)
      IR  =  I
 1030 CONTINUE
 1099 RETURN
      END
      SUBROUTINE SIMPLX (INFIX,A,B,TOL,PRM,KOUT,ERS,JH,X,P,Y,KB,E,KPRNT)
C
C*************************************************************************
C
C           SIMPLX IS THE PRINCIPAL CONTROLLING SUBROUTINE FOR
C           THE SOLUTION OF THE SIMPLEX ALGORITHM. ONLY THE PRIMAL
C           SOLUTION CAN BE OBTAINED BY USING SIMPLX IN ITS PRESENT
C           FORM. MODIFICATION CAN BE MADE TO FIND THE DUAL
C           SOLUTION.
C
C           THE SUBROUTINE RECEIVES AND OUTPUTS DATA AS INSTRUCTED
C           BY AN EXTERNAL INPUT/OUTPUT PROGRAM. SUBROUTINES USED
C           IN SOLVING THE SIMPLEX ALGORITHM ARE AS FOLLOWS:
C           1) NEW
C           2) VER
C           3) XCK
C           4) GET
C           5) MIN
C           6) JMY
C           7) ROW
C           8) PIV
C           9) ERR
C
C           INPUT: INFIX,A,B,TOL,PRM
C
C           INTERMEDIATE: P,Y,E
C
C           OUTPUT: KOUT,ERR,JH,X,KB
C
C           ARGUMENTS WHICH MUST BE INITIALIZED ARE AS FOLLOWS:
C
C            1) INFIX= AN INTEGER VECTOR CONTAINING 8 INPUT QUANTITIES
C               REPRESENTING THE FOLLOWING VALUES:
C              (1) INFLAG= INPUT CONDITION: 0 OR 4 MEANS NEW PROBLEM.
C              (2) N= THE NUMBER OF COLUMNS IN THE 'A' MATRIX.
C              (3) ME= THE LENGTH OF ONE COLUMN IN THE 'A' MATRIX.
C                      (THE FIRST DIMENSION OF THE 'A' MATRIX)
C              (4) M= THE ROW NUMBER OF THE FINAL CONSTRAINTS IN
C                     THE 'A' MATRIX (M<OR=ME).
C              (5) MF= THE ROW NUMBER OF THE FIRST CONSTRAINT IN
C                      THE 'A' MATRIX (MF<OR=M).
C              (6) MC= THE ROW NUMBER OF THE OBJECTIVE FORM (COSTS)
C                      IN THE 'A' MATRIX (MF>MC>0).
C              (7) NCUT= THE MAXIMUM NUMBER OF ITERATIONS THAT
C                        WILL BE ALLOWED TO SOLVE THE PROBLEM.
C              (8) NVER= THE REINVERSION FREQUENCY (NVER=0 MEANS
C                        DO NOT REINVERT).
C            2) A= REAL VALUED COEFFICIENT MATRIX, STORED IN
C                  COLUMN ORDER.
C            3) B= REAL VALUED VALUES OF RIGHT HAND SIDE.
C            4) TOL= A VECTOR CONTAINING THE 4 ALLOWABLE REAL
C                    VALUED TOLLERANCES AS FOLLOWS:
C              (1) TPIV= PIVOT TOLERANCE.
C              (2) TZERO= TOLERANCE FOR SETTING 'X' TO ZERO.
C              (3) TCOST= REDUCED COST IS CONSIDERED TO BE NEGATIVE
C                         ONLY IF IT IS BELOW THIS QUANTITY.
C              (4) TECOL= QUANTITIES IN THE PIVOT ROW OF THE INVERSE
C                         ARE ASSUMED ZERO IF MAGNITUDE BELOW THIS
C                         QUANTITY (USED ONLY IN VERSION 2 OF THE
C                         SUBROUTINE).
C            5) PRM= REAL VALUED MIXED PRICING COEFFICIENT.
C
C           INTERMEDIATE ARGUMENTS USED ARE AS FOLLOWS:
C
C            1) P= REAL VALUED CONSTRAINT PRICES (PI).
C            2) Y= REAL VALUED TEMPORARY WORKING AND STORAGE VECTOR.
C            3) E= REAL VALUED VECTOR CONTAINING THE CURRENT
C                  INVERSE IN COLUMN ORDER.
C
C           ARGUMENTS RETURNED AS OUTPUT TO INITIATING PROGRAM
C           ARE AS FOLLOWS:
C
C            1) KOUT= AN INTEGER VECTOR CONTAINING 7 OUTPUT QUANTITIES
C                     REPRESENTING THE FOLLOWING VALUES:
```

```
C
C         ARGUMENTS RETURNED AS OUTPUT TO INITIATING PROGRAM
C         ARE AS FOLLOWS:
C
C           1) KOUT= AN INTEGER VECTOR CONTAINING 7 OUTPUT QUANTITIES
C                    REPRESENTING THE FOLLOWING VALUES:
C             (1) K= OUTPUT CONDITION:
C                    3= FEASIBLE AND OPTIMAL.
C                    4= NO FEASIBLE SOLUTION.
C                    5= NO PIVOT PERFORMED, INFINITE SOLUTION.
C                    6= ITERATION LIMIT EXCEEDED.
C                    7= ILLEGAL INPUT QUANTITY.
C             (2) ITER= NUMBER OF ITERATIONS TAKEN.
C             (3) INVC= NUMBER OF ITERATIONS SINCE LAST INVERSION.
C             (4) NUMVR= NUMBER OF INVERSIONS PERFORMED.
C                      (INCLUDING INITIAL AND FINAL INVERSIONS.)
C             (5) NUMPV= NUMBER OF PIVOTS PERFORMED.
C             (6) INFS= INFEASIBILITY FLAG:
C                      1= INFEASIBLE
C                      0= FEASIBLE.
C             (7) JT= FINAL PIVOT COLUMN SELECTED.
C           2) ERR= A REAL VALUED VECTOR CONTAINING 8 OUTPUT QUANTITIES
C                   OF  THE CALCULATED ERRORS ACCUMULATED IN THE
C                   OPERATIONS REPRESENTING THE FOLLOWING VALUES:
C             (1) SUM OF THE FEASIBILITY ERRORS.
C             (2) MAXIMUM FEASIBILITY ERROR.
C             (3) SUM OF THE REDUCED COSTS IN THE BASIS.
C             (4) MAXIMUM REDUCED COST (IN ABSOLUTE VALUE) IN THE BASIS.
C              IF A FINAL INVERSION IS PERFORMED, THEN THE PARAMETERS
C              (1) THRU (4) WILL BE ERRORS BEFORE THE INVERSION AND
C              PARAMETERS (5) THRU (8) WILL BE THE CORRESPONDING
C              ERRORS AFTER THE INVERSION.
C           3) JH= AN 'M' ELEMENT INTEGER VECTOR CONTAINING THE
C                  REAL INDEX OF THE BASIC VARIABLES. EXAMPLE:JH(3)=27
C                  MEANS THAT THE 3RD BASIC VARIABLE IS THE 27TH
C                  VARIABLE IN THE CONSTRAINT MATRIX. IF JH(I)=0,
C                  THEN THE 'I'TH BASIC VARIABLE IS ARTIFICIAL.
C           4) X= REAL VALUED SOLUTION VECTOR.
C           5) KB= AN 'N' ELEMENT INTEGER VECTOR CONTAINING THE
C                  BASIS INDEX OF THE REAL VARIABLES. EXAMPLE:KB(27)=3
C                  MEANS THAT THE 27TH VARIABLE IN THE CONSTRAINT
C                  MATRIX IS THE THIRD BASIC VARIABLE. IF KB(J)=0,
C                  THEN THE 'J'TH VARIABLE IS NON-BASIC.
C
C*********************************************************************
C

      INTEGER XXAUX,OUTPUT,IIII
       DIMENSION INFIX(8), KOUT(7), ERS(8), ZZ(4), IOFIX(16), TERR(8), T
     1OL(4)
       DIMENSION A(1), B(1), JH(1), X(1), P(1), Y(1), KB(1), E(1)
       EQUIVALENCE (INFLAG,IOFIX(1)),(N,IOFIX(2)),
     1            (ME,IOFIX(3) ),  (M,IOFIX(4)), (MF,IOFIX(5)),
     2  (MC, IOFIX(6) ),  ( MCUT, IOFIX(7) ) , ( NVER, IOFIX(8) ),
     3 ( K, IOFIX(9) ), (ITER, IOFIX(10) ), (INVC , IOFIX(11) ) ,
     4 (NUMVR, IOFIX(12) ), ( NUMPV, IOFIX(13) ) ,
     5  (INFS, IOFIX(14)    ) , ( JT,  IOFIX(15) ) ,( LA  , IOFIX(16) ),
     6(TPIV,ZZ(1)),(TZERO,ZZ(2)),(TCOST,ZZ(3)),(TECOL,ZZ(4))
      OUTPUT=6
      DO 1340  I= 1, 8
       TERR(I) = 0.0
      IOFIX(I+8)  =  0
 1340  IOFIX(I)  =  INFIX(I)
       DO 1308 I=1,4
       ZZ(I)=TOL(I)

 1308  CONTINUE
      PMIX = PRM
      TCOST=-ABS(TCOST)
      IPRNT = 1
      M2 = M**2
      INFS = 1
      LA   =   0
C                       CHECK FOR ILLEGAL INPUT
       NIL=ME-M
       WRITE(6,26) NIL
   26  FORMAT(I8)
      IF (N)  1304, 1304, 1371
 1371  IF (M - MF ) 1304, 1372, 1372
 1372  IF (MF - MC)  1304, 1304, 1373
 1373  IF ( MC ) 1304 , 1304, 1374
 1374  IF (ME - M ) 1304, 1375, 1375
 1304 K  =  7
      GO TO 1392
 1375  XXAUX=INFLAG
   41   FORMAT(15H OK TO 1375 SI )
      IF( MOD(XXAUX, 4 ) - 1 ) 1400, 1320, 100
 1400  CALL      NEW (M,N, JH, KB, A, B, MF, ME )
 1320 IF(KPRNT.GT.2)WRITE(6,5000)
   42   FORMAT(20H OK TO 1320 SIMPLX    )
 5000 FORMAT ('0INVERT')
      IPRNT=1
       CALL      VER ( A, B, JH, X, E, KB, Y, N, ME, M, MF, INVC,
     1                   NUMVR, NUMPV, INFS, LA, TPIV, TECOL, M2 )
C                              PERFORM ONE ITERATION
  100 TA = TZERO
      IF ( INFLAG - 8 )  101,2101,2101
 2101 TA = TCOST
  101  CALL      XCK ( M, MF, JH, X, TA, IR )
      IF ( INFLAG - 8 )  102,2102,2102
 2102 IF ( IR )  2103,2103,500
 2103  CALL      GET ( M, MC, MF, JH, X, Y, P, E, IR, PMIX )
      GO TO 203
```

```
 2101 TA = TCOST
  101 CALL      XCK ( M, MF, JH, X, TA, IR )
      IF ( INFLAG - 8 )  102,2102,2102
 2102 IF ( IR )  2103,2103,500
 2103 CALL      GET ( M, MC, MF, JH, X, Y, P, E, IR, PMIX )
      GO TO 203
  102 JIN = (IR+M)/(M+1)
      IR = JIN
C
C              CHECK CHANGE OF PHASE.. GO BACK TO INVERT IF GONE INFEAS.
      IF (INFS - JIN ) 1320,  500,  200
C                     BECOME FEASIBLE
  200 INFS = 0
      IR = 0
  201 PMIX  =  0.0
  500 CALL      GET ( M, MC, MF, JH, X, Y, P, E, IR, PMIX )
      CALL      MIN ( JT, N, M, A, P, KB, ME, TCOST, IR, TPIV, Y, JIN )
      JM  =  JT
      J = JM
      IF ( IR - 1 )  202,202,2202
  202   IF  (JM)    203, 203, 222
C              ALL COSTS NON-NEGATIVE... K = 3 OR 4
  203 K = 3 + INFS
      DO TO 257
 2202 IF ( INFS - JIN )   1320,2204,2203
 2203 INFS = 0
 2204 IF ( JM ) 207,207,222
C                                   NORMAL CYCLE
  222 CALL      JMY ( J, A, E, M, Y, ME )
      IF ( IR - 1 )  223,223,2223
 2223 IF ( INFS )  210,210,2224
 2224 YI = TPIV
      IR = 0
      DO 2226 I = MF,M
      IF ( Y(I) - YI )   2226,2225,2225
 2225 IR = I
 2226 CONTINUE
      DO TO 206
  223 CALL      ROW  ( IR, M, MF, JM, X, Y, TPIV )
C                                   TEST PIVOT
  206   IF( IR )   207, 207, 210
C                                   NO PIVOT
  207 K = 5
  257 IF (PMIX)   201, 400, 201
C                            ITERATION LIMIT FOR CUT OFF
  210 IF (ITER -NCUT )    208,  160,  160
C                                   PIVOT FOUND
  208 CALL      PIV  ( IR, Y, M, E, X, NUMPV, TECOL )
  221 JOLD  = JH(IR)
      IF (JOLD) 213, 213, 214
  214 KB(JOLD) = 0
  213 KB(JM)    = IR
      JH(IR)    = JM
      LA  =  0
      ITER   = ITER   +1
      INVC   = INVC   +1
      IF(KPRNT.LE.2)GO TO 2214
      IF ( IPRNT ) 2212,2213,2212
 2212 WRITE (OUTPUT,5501)
 5501 FORMAT (' ITER',2X,'PHASE',5X,'IN',4X,'OUT',4X,'ROW',7X,'OBJ')
      IPRNT = 0
 2213 IPHS = 2 - INFS
      OBJ = -X(MC)
      WRITE (OUTPUT,5502) ITER,IPHS,JM,JOLD,IR,OBJ
 5502 FORMAT (1X,I4,4(2X,I5),2X,E14.7)
C                            INVERSION FREQUENCY
 2214 IF (INVC - NVER  )  100, 1320, 100
C                                         CUT OFF ... TOO MANY ITERATIONS
  160 K = 6
  400 CALL      ERR ( M, A, B, TERR, JH, X, P, Y, ME, LA )
      IF (LA)    193, 191,  193
  191 LA  =  4
      XXAUX = INFLAG
      IF ( XXAUX - 8 )  192,190,190
  190 XXAUX = XXAUX - 8
  192 IF ( XXAUX  -  4  )  1320,  193,  193
  193 IF  (K-5)     1392,  194,  1392
  194 CALL      JMY ( J, A, E, M, Y, ME )
C                   SET EXIT VALUES
 1392  DO  1309  I=  1, 8
 1309 ERS(I)            =   TERR(I)
       DO 1329  I = 1, 7
 1329 KOUT(I)   =   IOFIX(I+8)
C     WRITE (OUTPUT,10 06)(P(IXX),IXX=1,M)
C1006 FORMAT(' P',5X,11E10.2/(7X,11E10.2))
C     DO 802 IXX=1,M
C802  WRITE (OUTPUT,10 08)(E(IXX+(JXX-1)*M),JXX=1,M)
C1008 FORMAT(' E',5X,11E10.2/(7X,11E10.2))
C     WRITE (OUTPUT,10 09)(X(IXX),IXX=1,M)
C1009 FORMAT(' X',5X,11E10.2/(7X,11E10.2))
C     WRITE (OUTPUT,10 10)(JH(IXX),IXX=1,M)
C1010 FORMAT(' JHBMP',22I5/(6X,22I5))
      RETURN
      END
      SUBROUTINE ANAL(J,MM,MA,XSTAROP,EE,CASH,TCOST)
      REAL ML,M
      DIMENSION XSTAROP(50,5),CASH(15),ESTX(375,12),EE(375,12)
      DO 6 I=1,MM
      CASH(I)=0.
    6 CONTINUE
      WRITE(6,712)
  712 FORMAT(1H1)
      TCOST=0.
```

```
C                            SET EXIT VALUES
     1392  DO  1309  I=  1, 8
     1309 ERS(I)           =    TERR(I)
           DO 1329  I = 1, 7
     1329 KOUT(I)   =   IOFIX(I+8)
C         WRITE (OUTPUT,10 06)(P(IXX),IXX=1,M)
C1006 FORMAT(' P',5X,11E10.2/(7X,11E10.2))
C         DO 802 IXX=1,M
C802  WRITE (OUTPUT,10 08)(E(IXX+(JXX-1)*M),JXX=1,M)
C1008 FORMAT(' E',5X,11E10.2/(7X,11E10.2))
C         WRITE (OUTPUT,10 09)(X(IXX),IXX=1,M)
C1009 FORMAT(' X',5X,11E10.2/(7X,11E10.2))
C         WRITE (OUTPUT,10 10)(JH(IXX),IXX=1,M)
C1010 FORMAT(' JHSMP',22I5/(4X,22I5))
          RETURN
          END
          SUBROUTINE ANAL(J,MM,NA,XSTAROP,EE,CASH,TCOST)
          REAL ML,M
          DIMENSION XSTAROP(50,5),CASH(15),ESTX(375,12),EE(375,12)
          DO 6 I=1,MM
          CASH(I)=0.
        6 CONTINUE
          WRITE(6,712)
      712 FORMAT(1H1)
          TCOST=0.
          DO 700 II=1,NA
          COSTPRY=0.
          IF(XSTAROP(II,J) .EQ. 1) GO TO 701
          GO TO 700
      701 IF(J .EQ. 1)NR=0
          IF(J .EQ. 2)NR=60
          IF(J .EQ. 3)NR=135
          IF(J .EQ. 4)NR=225
          IF(J .EQ. 5)NR=330
          I=II+NR
          DO 702 J11=2,MM
          J1=J11-1
          IF(EE(I,J1) .EQ. 0)GO TO 702
          ESTX(I,J1)=EE(I,J1)
          IF(J1 .EQ. J) GO TO 703
          CASH(J1)=CASH(J1)+ESTX(I,J1)
          GO TO 702
      703 CASH(J1)=0.
      702 CONTINUE
          WRITE(6,710)
          MWT=MM-1
          WRITE(6,709)II,(ESTX(I,J1A),J1A=1,MWT)
      710 FORMAT(///,20X,*...........CASH FLOWS..........*,/)
      709 FORMAT(5X,*PROJECT*,I4,2X,11F10.2)
          COSTPRY=COSTPRY+ESTX(I,J)
          TCOST=TCOST+ESTX(I,J)
      700 CONTINUE
          WRITE(6,707)J,TCOST
          RETURN
      704 FORMAT(5X,*J1=*,I4,2X,*CASH(J1)=*,F20.3,/)
      705 FORMAT(5X,*II=*,I4,2X,*COSTPRY=*,F20.3,/)
      706 FORMAT(5X,*J1=*,I4,2X,*CASH(1)=*,F20.3,/)
      707 FORMAT(//,5X,*YEAR*,I4,3X,*TOTAL COST OF THE YEAR*,F20.2,/)
          RETURN
          END
SCAN 1429 EOR    1429
0 T
```

APPENDIX C

EFFICIENCY FRONTIERS

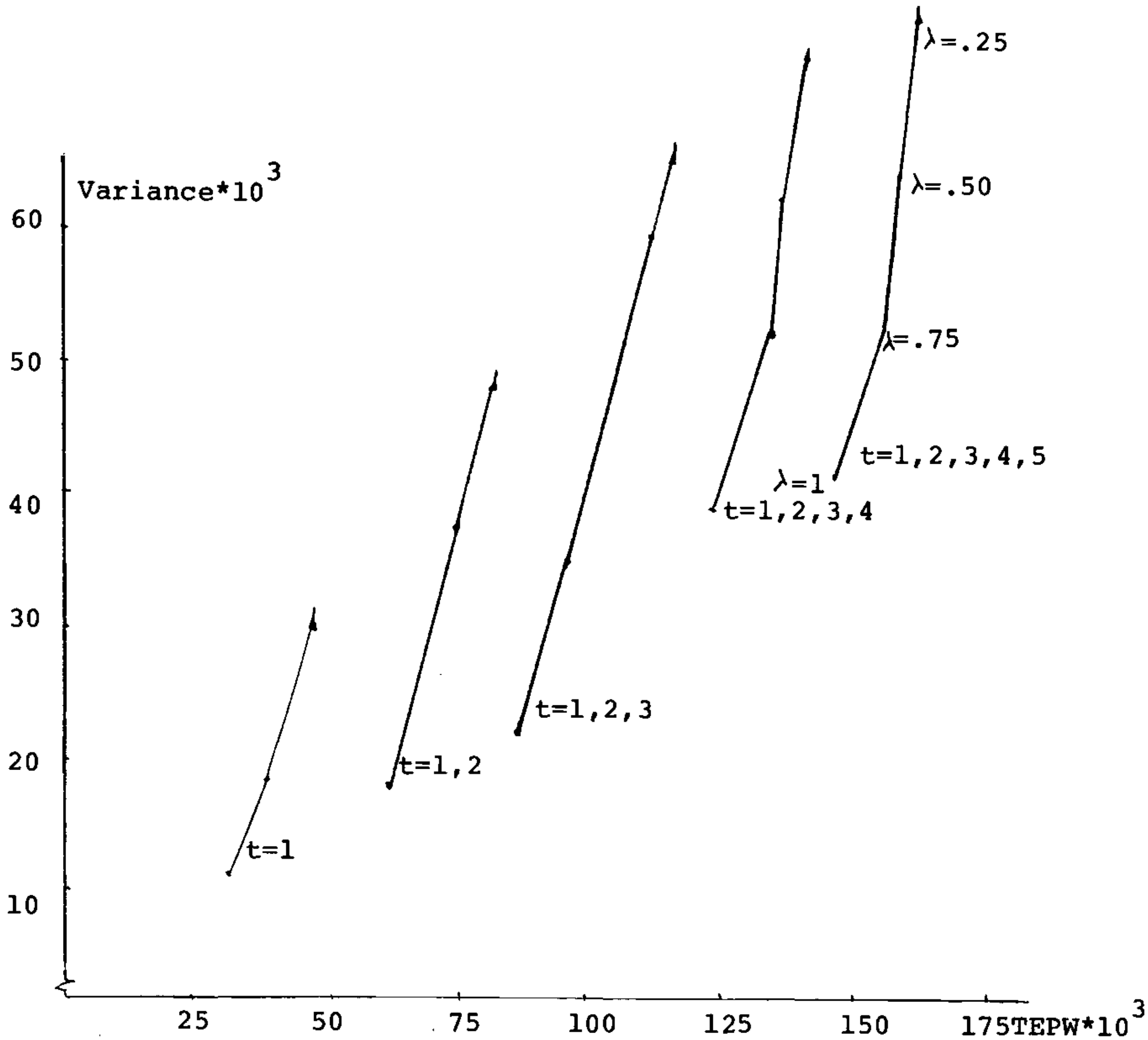

Figure C-1. Efficiency Frontiers for Set 1, Budget $6000, "first year only" Decision Rule.

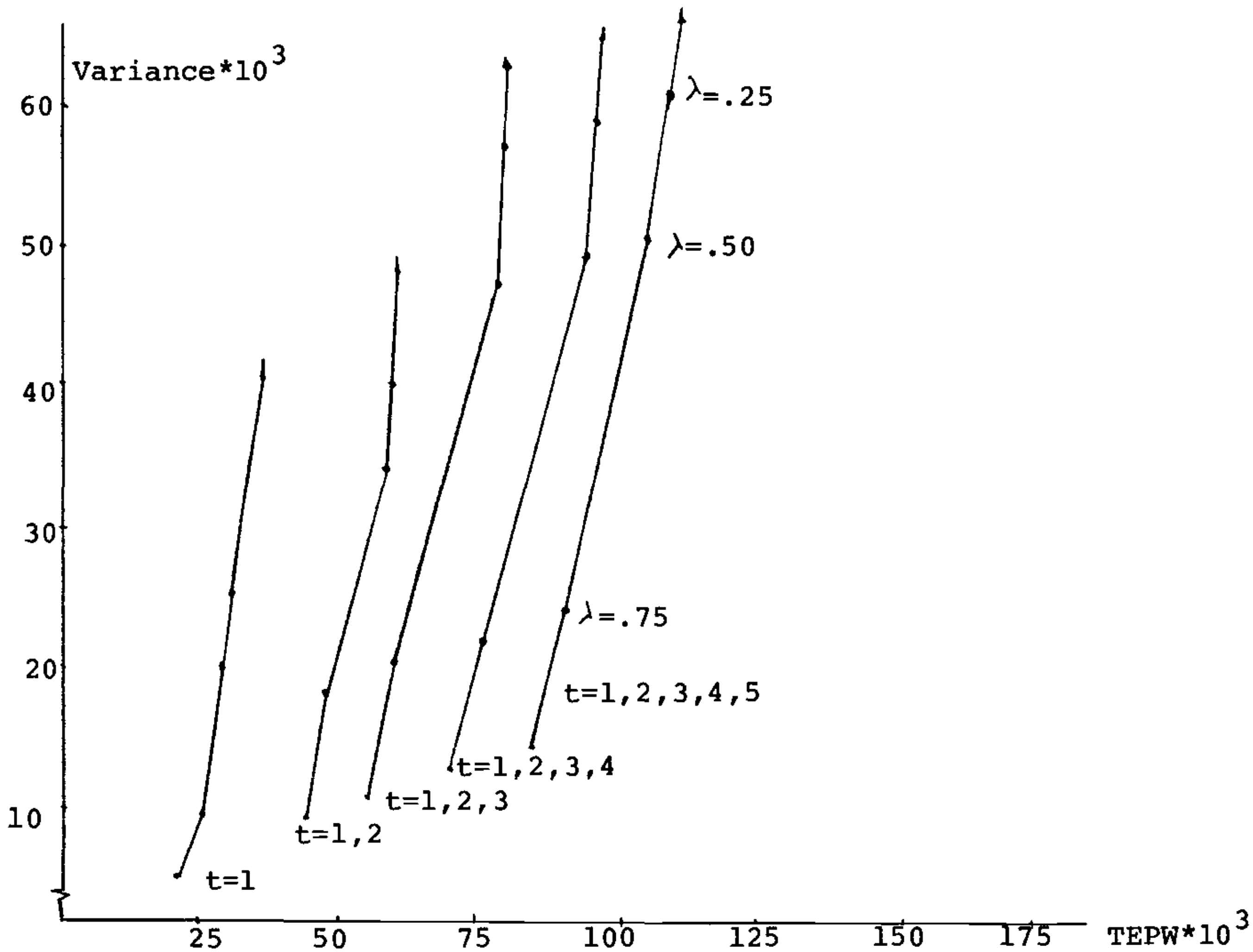

Figure C-2. Efficiency Frontiers for Set 1, Budget $6000, "second year only" Decision Rule.

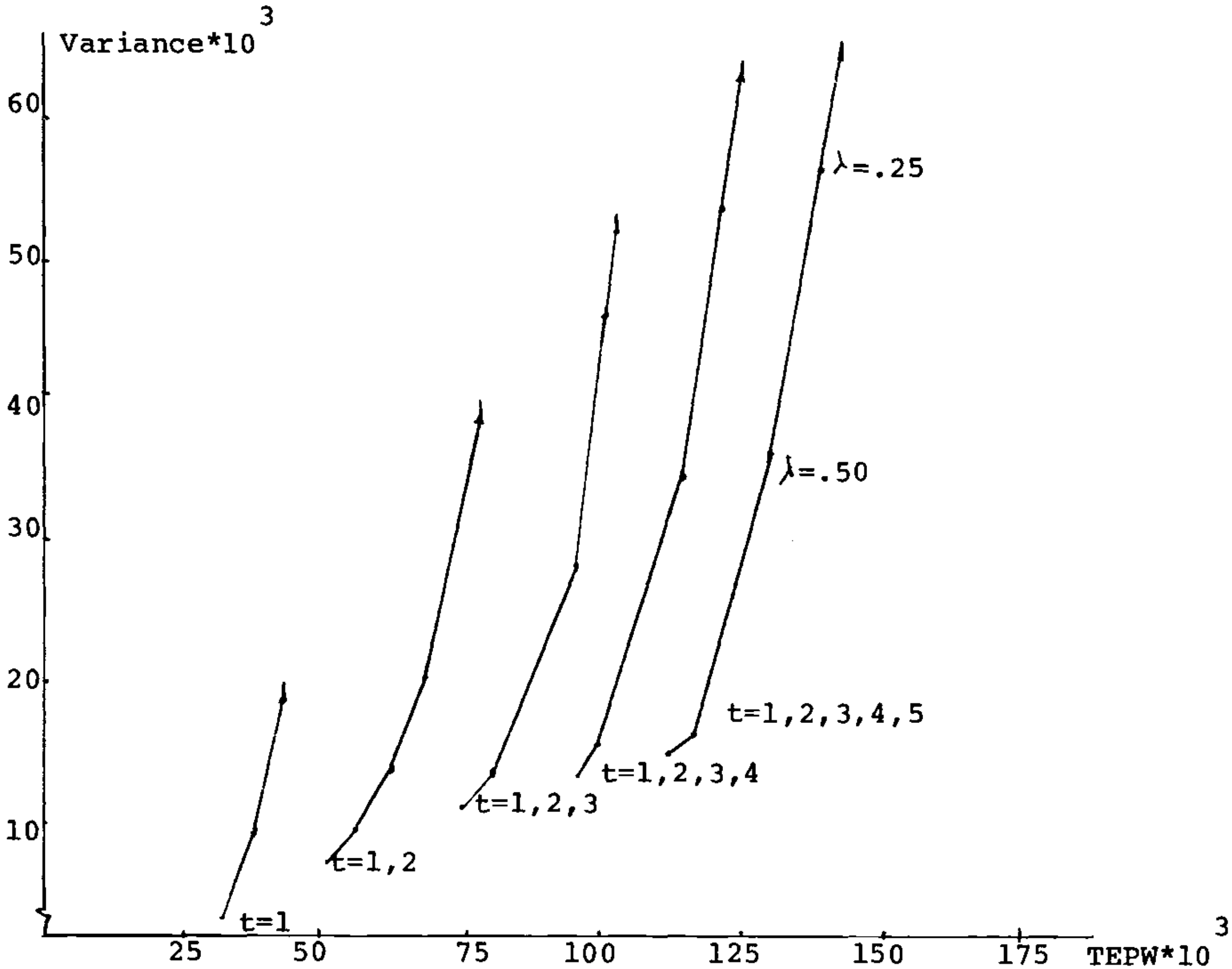

Figure C-3. Efficiency Frontiers for Set 1, Budget $4000, "first, second or third year" Decision Rule.

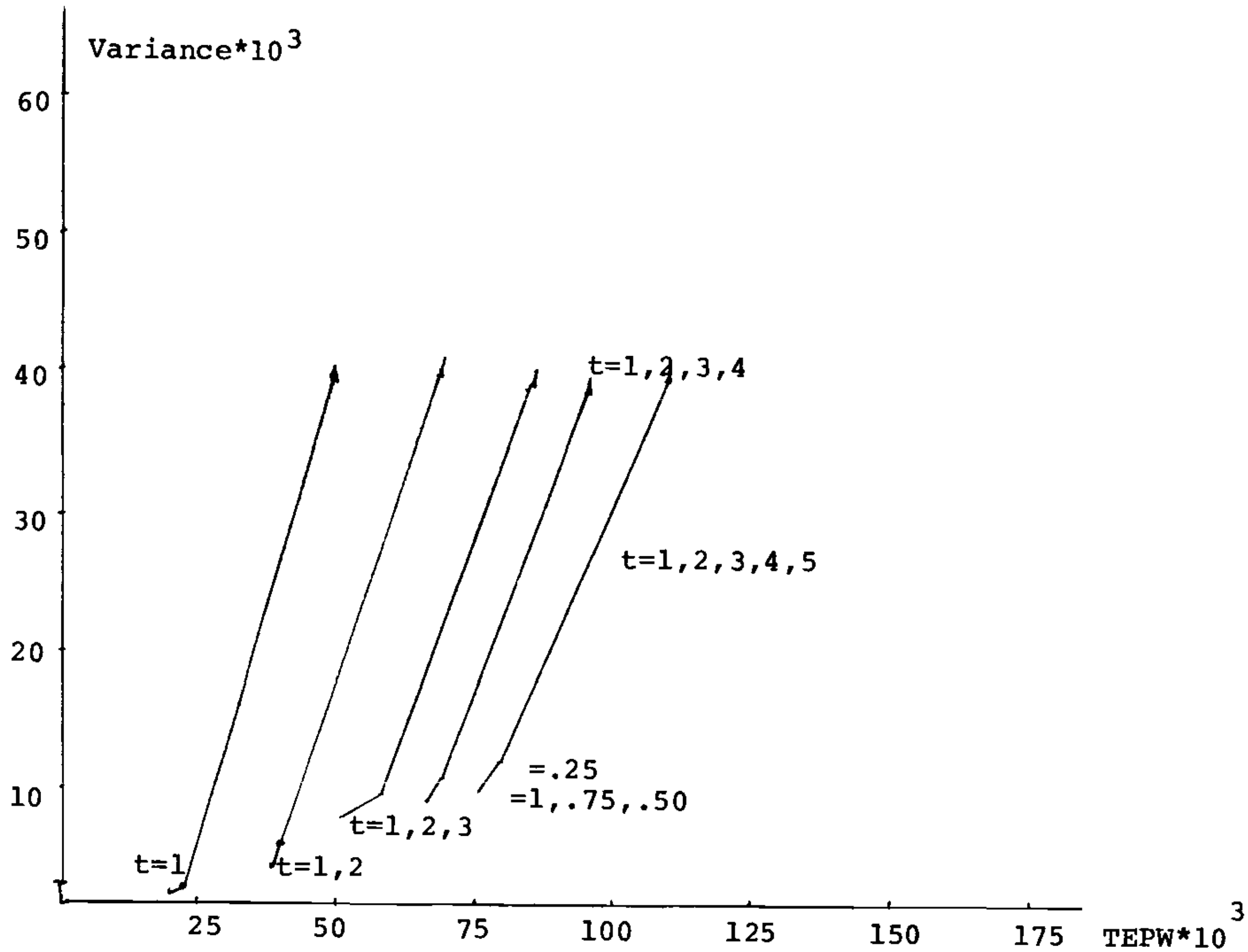

Figure C-4. Efficiency Frontiers for Set 1,Budget $2000,
"first, second or third year" Decision Rule.

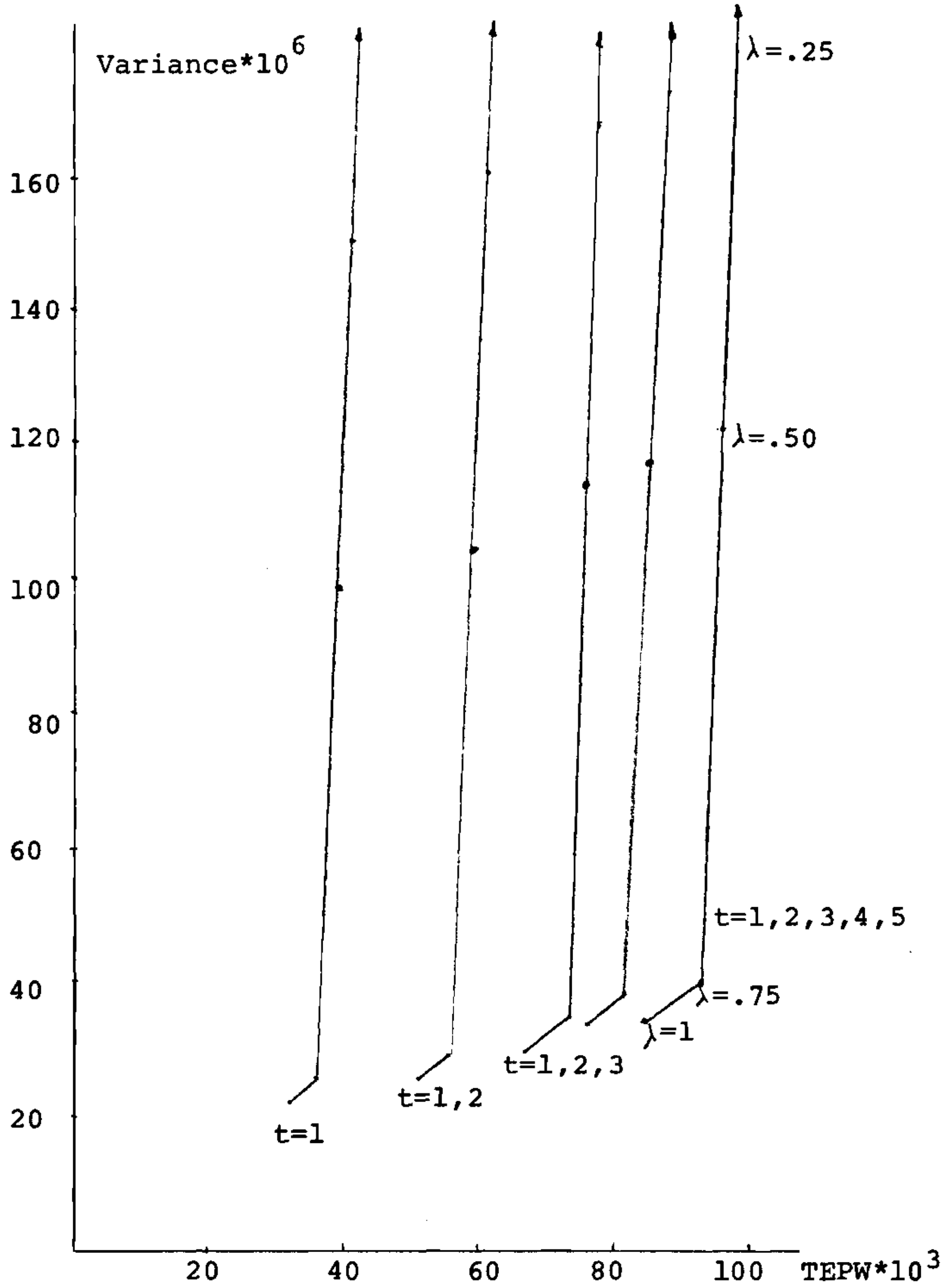

Figure C-5. Efficiency Frontiers for Set 3,Budget $1500,"first,second or third" Decision Rule.

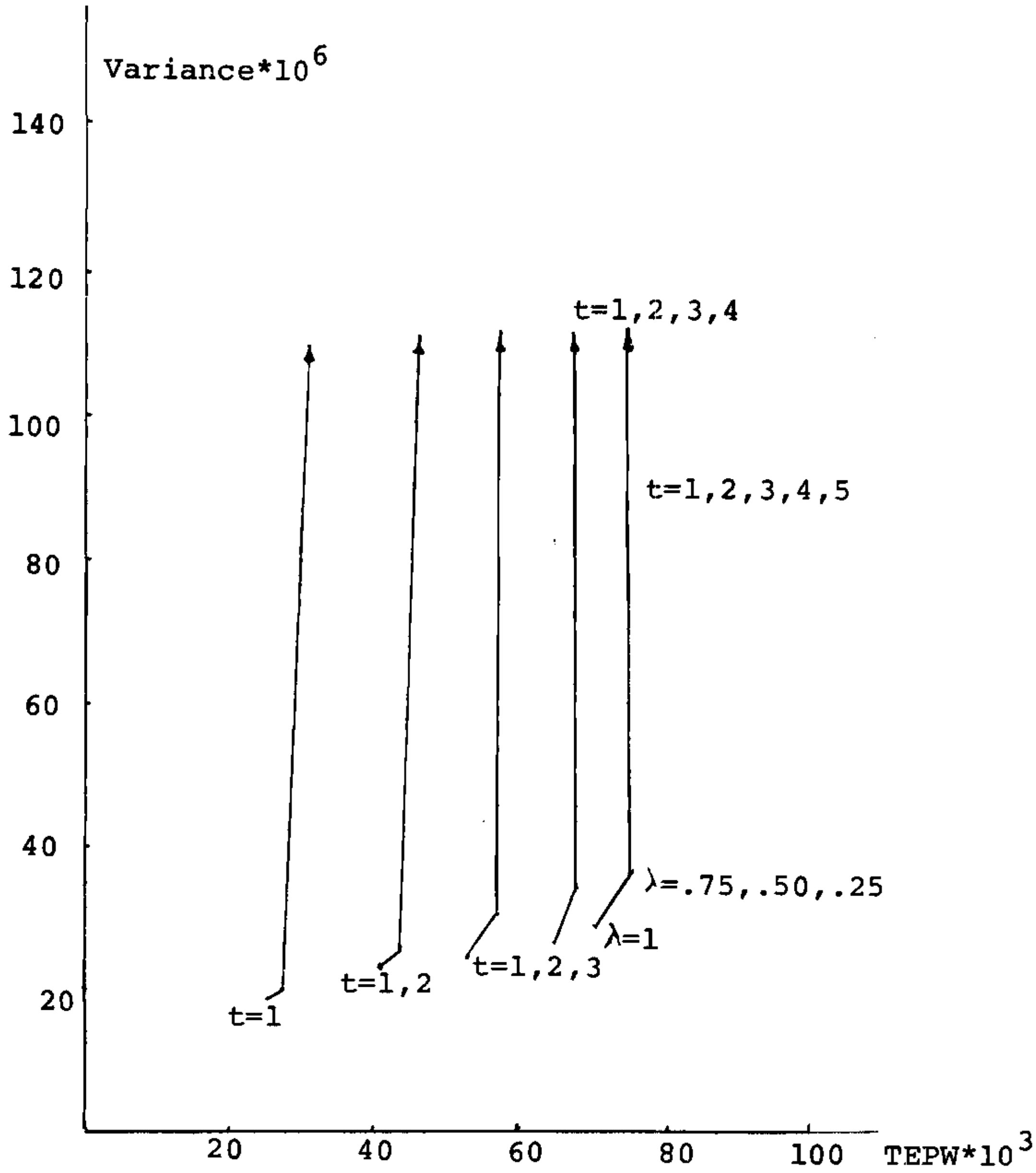

Figure C-6. Efficiency Frontiers for Set 3, Budget $1500, "only first year" Decision Rule.

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