SAR calculation using FDTD simulations

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Abstract: *The main intend of this work, is to determinate the Specific Absorption Rate (SAR) on human head tissues exposed to radiation caused by sources of 900 and 1800MHz, since those are the typical frequencies for mobile communications systems nowadays. In order to determinate the SAR, has been used the FDTD (Finite Difference Time Domain), which is a numeric method in time domain, obtained from the Maxwell equations in differential mode. In order to do this, a computational model from the human head in two dimensions made with cells of the smallest possible size was implemented, respecting the limits from computational processing. It was possible to verify the very good efficiency of the FDTD method in the resolution of those types of problems.*

Key words: FDTD, SAR.

1. INTRODUCTION

Related with the exponential growth of mobile communications in the last decade, we also find the growing interest of scientific community in the study of the interaction of electromagnetic waves with the human body. The most typical example is the use of a mobile phone with its respective antenna near the human head. In this case, we find the need to characterize and quantify the distribution of the electromagnetic fields and the energy levels absorbed by the human tissues. The measurement of this type of units on the inside of the human body is extremely complicated and almost impossible, and due to this fact, it is imperative to use numerical methods implemented in software, so that we can have an idea of the values involved in those situations.

For non-ionizing radiation, like the case of radiofrequency electromagnetic radiation, (the radiofrequency scope of the electromagnetic spectrum is in the interval between 3KHz and 300GHz), the biological effects known are reasonably well quantified and are mainly related with the warming of the biological tissues, and because of this those effects are known as "thermal effects". When the warming level of the biological tissues exceeds the natural capacity for thermal regulation of the human organism, damages can occur in those same tissues. Besides the thermal effects resulting of the exposing to radiofrequency electromagnetic radiation, there is also the possibility of occurring non thermal effects, although about those effects there isn't still many discussion and conclusions. Actually, the only confirmed mechanism as a potential generator of health prejudicial effects resulting from the exposing to radiofrequency radiation is the warming of the biological tissues. Therefore, it's based in these mechanisms that the security limits are imposed to the radiofrequency bands.

However, it's still in discussion if this is the appropriated approach to this question, since there is the possibility of the appearance of non thermal effects in a long term perspective that can also be prejudicial to health. In order to characterize the absorbed radiation by the human body, it is necessary to use an appropriated measurement parameter. The parameter that is used is the Specific Absorption Rate (SAR) that represents the levels which is absorbed by a mass unit of tissue. The units for the SAR are Watt by kilogram of tissue exposed [W/kg]. That's why, for the radiofrequency radiation, the security limits are established by the SAR parameter.

With the intent to have an estimated value for SAR levels in the tissues of human head, it will be used a numerical method, software implemented to resolve the Maxwell equations. When we use numerical methods in order to resolve Maxwell equations, we find ourselves in the computational electromagnetic area, that is in our days a very useful tool in the study of electromagnetic waves propagation, not only in telecommunications areas, but also in medicine areas (medical systems of radiotherapy and analysis) and in biology.

The recent growth those methods have accomplished is a result from the growth of the informatics industries, which have permitted the simulation of several complex problems without having the necessity to use "super computers".

The simulation of electromagnetic phenomena trough this process, allows us to very reliable simulations. Like that, the computational electromagnetic has been revealed in the last few years, as a very powerful tool in the study of electromagnetic phenomena.

Within the different methods existing in this area, the FDTD method has been clearly distinguished as one of the most powerful, mostly by its relatively easy implementation and also by the results produced.

The FDTD method has had an exponential growing on the last decades and we can find today a large number of published papers and works that use it. The method has been used in several different areas of science, including the interaction of electromagnetic fields with biological tissues. We can refer as an example studies on the SAR levels observed in human tissues has a result of GSM radiation [1], studies on the detection of malign tumours [2] and breast cancer [3], studies on the effects of the exposure of biological cells to electromagnetic impulses [4] and the simulation for healing cancer cells trough hyperthermia [5].

2. THE FDTD

The FDTD method is used in the resolution of electromagnetic problems, most of them very complex and where the analytics solutions are not viable, and therefore it's necessary to use a numeric method of calculus. Originally developed by Kane Yee [6] in 1966, this method consists in transforming the Maxwell equations in its differential form into explicit equations in the time domain and in a well defined point of the computational domain. In order to do this, the time and space derivative are replaced by finite differences obtained by the basic definition of derivative of a function in a certain point. Using this principle, we obtain a set of explicit equations for each component of the electromagnetic field that allows to calculate the electric field in an instant and the magnetic field in the immediately instant after, alternating this process until the stationary state is obtained.
In conclusion, this method allows that we can have a time evolution from any of the field components in any point of the computational domain defined.

2.1 The Maxwell equations

The Maxwell equations in their differential form are given by:

$$\nabla \times \vec{E} = -\mu \cdot \frac{\partial \vec{H}}{\partial t} \tag{1}$$

$$\nabla \times \vec{H} = -\varepsilon \cdot \frac{\partial \vec{E}}{\partial t} + \sigma \cdot \vec{E} \tag{2}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \tag{4}$$

Where μ represents the magnetic permeability in [H/m], ε represents the electrical permittivity in [F/m] and σ represents the electrical conductivity in [S/m]. Using a cartesian coordinate system, equations (1) and (2) will result in the set of the following six equations:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}\right) \tag{5}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \tag{6}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{7}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right) \tag{8}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \tag{9}$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \tag{10}$$

2.2 Kane Yee algorithm

Equations (5) to (10) are the base of the FDTD algorithm, where the different derivates are replaced by finite differences. Yee has defined a time and space function calculated in a generic point of the pre defined matrix as:

$$F^n(i,j,k) = F(i\Delta x, j\Delta y, k\Delta z, n\Delta t) \tag{11}$$

where i, j and k are integers; Δx, Δy e Δz are the cells dimensions according the directions from the ordinate axis; Δt is the time increment, assumed uniform in all the observation time and n an integer. From this explanation, we verify that before we can use this method, we must first proceed with the discrimination of the computational space in cells. Yee has defined the Yee cell, where we can see the different components of the electromagnetic field, has represented in Picture 1.
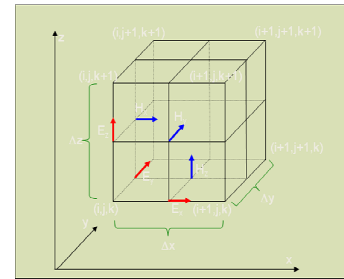
Picture 1 –Yee cell

If we apply the definition of a derivative in a point to the equations (5) to (10), we will obtain the base equations for the FDTD. For instance, we can use equation (5) to demonstrate that the magnetic field along the x axis is given by [7]:

$$H_x^{n+\frac{1}{2}}(i,j,k) = \left(\frac{1 - \frac{\rho'_{i,j,k} \cdot \Delta t}{2 \cdot \mu_{i,j,k}}}{1 + \frac{\rho'_{i,j,k} \cdot \Delta t}{2 \cdot \mu_{i,j,k}}} \right) H_x^{n-\frac{1}{2}}(i,j,k) + \qquad (12)$$

$$\left(\frac{\frac{\Delta t}{\mu_{i,j,k}}}{1 + \frac{\rho'_{i,j,k} \cdot \Delta t}{2 \cdot \mu_{i,j,k}}} \right) \left(\frac{E_y^n(i,j,k+1/2) - E_y^n(i,j,k-1/2)}{\Delta z} - \frac{E_z^n(i,j+1/2,k) - E_z^n(i,j-1/2,k)}{\Delta y} \right)$$

Using equation (8) and the same method, we obtain the electric field component along the *z* axis:

$$E_z^{n+1}(i,j,k) = \left(\frac{1 - \frac{\sigma_{i,j,k} \cdot \Delta t}{2 \cdot \varepsilon_{i,j,k}}}{1 + \frac{\sigma_{i,j,k} \cdot \Delta t}{2 \cdot \varepsilon_{i,j,k}}} \right) E_z^n(i,j,k) + \qquad (13)$$

$$\left(\frac{\frac{\Delta t}{\varepsilon_{i,j,k}}}{1 + \frac{\sigma_{i,j,k} \cdot \Delta t}{2 \cdot \varepsilon_{i,j,k}}} \right) \left(\frac{H_y^{n+1/2}(i+1/2,j,k) - H_y^{n+1/2}(i-1/2,j,k)}{\Delta x} - \frac{H_x^{n+1/2}(i,j+1/2,k) - E_z^{n+1/2}(i,j-1/2,k)}{\Delta y} \right)$$

The cells size and the value for time increment are related trough the stability condition known as Courant limit, which guarantees the algorithm stability and it's defined by [7]:

$$\Delta t \leq \frac{1}{c_0 \cdot \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} \qquad (14)$$

Trough the analysis of equations (12) and (13), we can conclude that the calculation of any one of the different fields components will only depend on its own value in the previous time instant and on the values of the other field component in adjacent points and in the immediate time instant before, and therefore those values are available in memory.

2.3 SAR calculation

We will now proceed to the SAR calculation on the tissues of a human head. In order to do so, it has been created a model from the human head based in medical images obtained from a magnetic resonance. Those images are showed in top left corner of Figure2. Considering that different zones from a human head are composed by homogeny materials, we have obtained the model used in our FDTD simulation, represented in the top right corner of Figure 2. Due to this assumption it was possible to assume the discrimination from a sagital cut of a human head into Yee cells, as represented in the lower right corner of Figure 2.
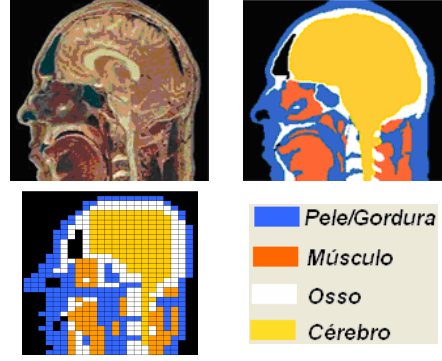

Figure 2- Sagital cut for a human head and the respective composing tissues for the different areas.

In order to obtain a more exhaustive study, it has been studied three different cuttings of a human head: frontal, superior and sagital. Figure 3 gives a representation of those different cuts included in the simulation area used for the simulations.
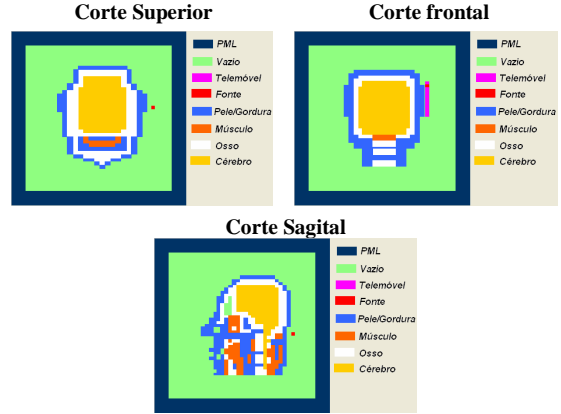

Figure 3 – Composition of the simulation area for the three different human head cuts used in the simulations.

The value of SAR that quantifies the electromagnetic energy that is absorbed by a mass unit of tissue, was obtained by equation 15, where σ, ρ and E_i are electric conductivity, volumic mass of the tissues and the absolute value of the electrical field in a given cell of the FDTD mesh.

$$SAR = \frac{\sigma}{2\rho} \cdot \left| E_i^n \right|^2 \qquad (15)$$

The simulation was performed within the 900 and 1800 MHz, since those are the typical frequencies of mobile communications. The usual values for the different parameters needed to calculate SAR are presented in Tables 1 and 2.

Table 2 - Electromagnetic proprieties for human head tissues at 900MHz [8].

Tissue	(ε_r)	σ [S/m]	ρ [kg/l]
Skin/Fat	23.000	0.630	1.000
Muscle	59.000	1.260	1.000
Bone	5.400	0.045	1.200
Brain	51.00	1.460	1.050

Table 2 - Electromagnetic proprieties for human head tissues at 1800MHz [8].

Tissue	(ε_r)	σ [S/m]	ρ [kg/l]
Skin/Fat	38.870	1.190	1.000
Muscle	59.370	2.040	1.000
Bone	11.500	0.230	1.200
Brain	43.54	1.150	1.050

3. SAR Simulation results

Figures 4 and 5 show the results for SAR levels in a human head at two different time moments, obtained due to radiation sources with 900 and 1800 MHz.
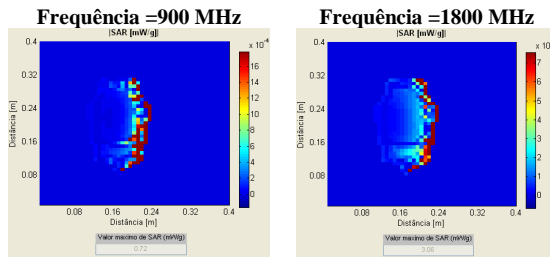
Superior Cut (Time Step = 150):

Figure 4 – SAR levels in a human head (superior cut) for 900 MHz and 1800 MHz (Time step=150)

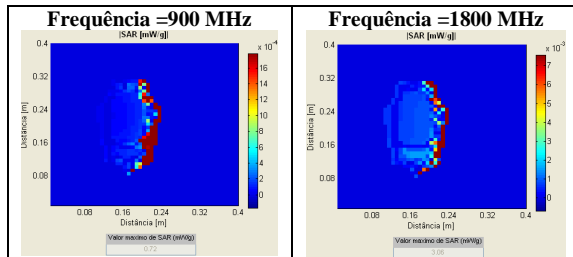**Superior Cut (Time Step = 320):**

Figura 5 – SAR levels in a human head (superior cut) for 900 MHz and 1800 MHz (Time step=150).

Figures 6 and 7 show the same SAR levels simulated in the same circumstances, but this time it was used the frontal cut of the human head.
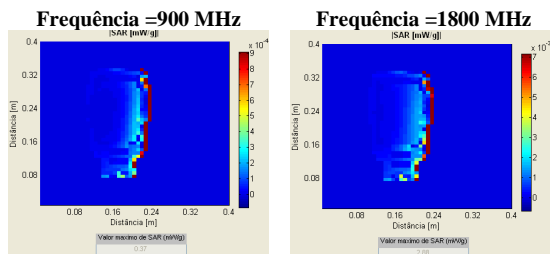
Frontal Cut (Time Step = 150):

Figure 6 – SAR levels in a human head (frontal cut) for 900 MHz and 1800 MHz (Time step=150).
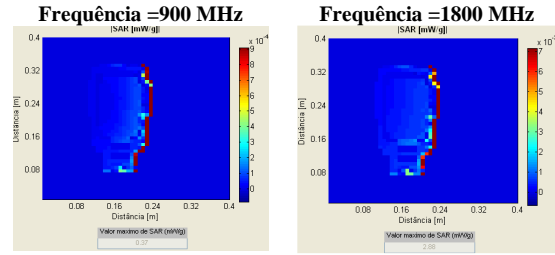
Frontal Cut (Time Step = 320):

Figure 7 – SAR levels in a human head (frontal cut) for 900 MHz and 1800 MHz (Time step=150).

The simulated results on the SAR levels for the sagital cut, present very similar results to the two previous simulations.

4. Conclusions

Trough the analysis of pictures 4 to 7 we can verify the efficiency of the FDTD method in the determination of SAR levels in a human head. It was shown that those levels are superior in the case of 1800MHz when compared with the same levels for 900MHz. This was an identical result for all of the human head cuttings used.
We can also conclude that the SAR levels tend to grow as we increase the exposing time to the radiation. Another conclusion is the fact that the higher SAR levels are found in the peripherical tissues of the human head side which is exposed to the radiation source.

6. References

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