Correlations between two particles in jets

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Abstract: We study the correlation between two soft particles in QCD jets. We extend the Fong–Webber analysis to the region away from the hump in the single inclusive energy spectrum and show that the correlation function should flatten off and then decrease for large values of $\ell = \ln(1/x)$.

Introduction

Perturbative QCD (pQCD) successfully predicted inclusive particle spectra in jets (for review see [1] and references therein). It sufficed to make one step beyond the leading double log approximation (DLA) and to analyse parton cascades with the next-to-leading accuracy by taking into account angular ordering in soft gluon multiplication, quark–gluon transitions, exact DGLAP parton splitting functions and running coupling effects. The corresponding MLLA (modified leading log approximation) "*evolution Hamiltonian*" has the accuracy $\gamma_0 + \gamma_0^2$, where γ_0 is the characteristic parameter of the pQCD expansion (the DLA multiplicity anomalous dimension) given by

$$\gamma_0^2 = \frac{4N_c\,\alpha_s(k_\perp^2)}{2\pi} = \frac{4N_c}{\beta\,Y}\,; \quad \beta = \frac{11N_c}{3} - \frac{4}{3}T_R\,,\;\; T_R = \frac{1}{2}n_f.$$

Here $Y = \ln(Q/2\Lambda)$, with Q the hardness of the jet production process (e^+e^- c.m.s. annihilation energy) and Λ the QCD scale. For the LEP-I energy, $Q = 91.2$ GeV (Z^0 peak) we have $Y \simeq 5.2$ (using $\Lambda \simeq 0.25$ GeV).

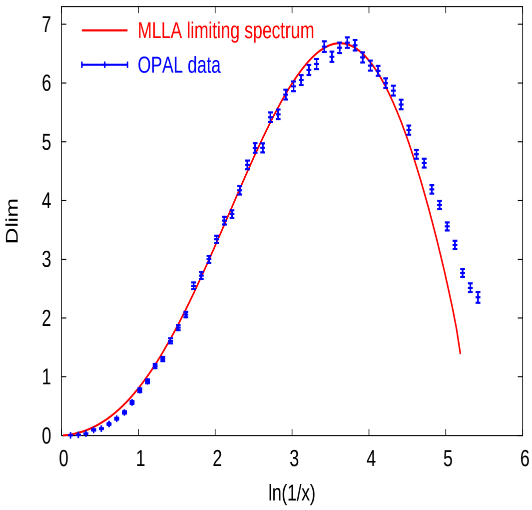


Fig.1: Inclusive spectrum for $Q = 91.2$ GeV.

In spite of expansion parameter being numerically large,

$$\gamma_0 \simeq 0.5\,, \tag{1}$$

the *shape* of the inclusive energy distribution of soft hadrons turned out to be well described by the analytic QCD curve — the so-called limiting spectrum

$$D^{\lim}(\ell, Y) = \frac{1}{\sigma}\frac{d\sigma}{d\ell}\,; \quad \ell = \ln\frac{1}{x}\,,$$

that one derives pushing the minimal transverse momentum in partonic cascades Q_0 down to Λ.

Overall normalization in Fig. 1 (number of hadrons per gluon) is a non-perturbative parameter that one determines phenomenologically. This parameter should cancel in the ratio

$$C_{G,Q}(\ell_1, \ell_2, Y) = \frac{D^{(2)}_{G,Q}(\ell_1, \ell_2, Y)}{D_{G,Q}(\ell_1, Y)\; D_{G,Q}(\ell_2, Y)}\,, \tag{2}$$

so that one could expect the two-particle correlation function (2) to provide a more stringent test of parton dynamics. However, for a long time, particle correlations are known to be poorly treated by pQCD.

Here we revisit two-particle correlations and report the results of an improved pQCD analysis of the problem [2].

Fong–Webber approximation

The first (and only) pQCD analysis of two-particle correlations in jets beyond the DLA was performed by Fong and Webber in 1990. In [3] the next-to-leading $\mathcal{O}(\gamma_0)$ correction, $C = 1 + \sqrt{\alpha_s} + \cdots$, to the normalized two-particle correlator was calculated. For a system of two quark jets produced in e^+e^- annihilation,

$$R(\ell_1,\ell_2,Y) \;=\; \frac{1}{2} + \frac{1}{2} C_Q(\ell_1,\ell_2,Y)\,,$$

the Fong–Webber result reads ($n_f = 3$)

$$R \;=\; 1.375 - 1.125\left(\frac{\ell_1-\ell_2}{Y}\right)^2 - \left[1.262 - 0.877\frac{(\ell_1+\ell_2)}{Y}\right]\frac{1}{\sqrt{Y}}. \qquad (3)$$

The first two terms in (3) are of the DLA origin while the third one constitutes an $\mathcal{O}(\gamma_0)$ MLLA correction. This expression was derived in the region $|\ell_1 - \ell_2|/Y \ll 1$, that is when the energies of the registered particles are relatively close to each other (and to the maximum of the inclusive distribution, see Fig.1). In this approximation the correlation function is quadratic in the difference $\ell_1 - \ell_2$ and increases linearly with the sum, $\ell_1 + \ell_2$.

Particle correlations from MLLA evolution equations

To analyse two-particle correlations we write down the evolution equations for $D^{(2)}_{G,Q}(\ell_1,\ell_2,Y)$ that follow from the general MLLA evolution equations for jet generating functionals [4]. Unlike the case of the inclusive spectrum, equations for $D^{(2)}$ are inhomogeneous due to the presence of the product of one-particle distributions $D(\ell_1)D(\ell_2)$. In the small-x limit, by approximating the energy fraction integrals of the parton splitting functions as follows,

$$\int_0^\ell d\ell'\, P(x')\, F(\ell-\ell') \;\simeq\; \int_0^\ell d\ell'\,\left[\,c_1 - c_2\delta(\ell')\,\right]\, F(\ell-\ell')\,,$$

it is straightforward to reduce the original integral equations to a system of two linear (inhomogeneous) second order differential equations in variables ℓ_1 and y_2, where $y \equiv \ln(k_\perp/\Lambda)$ and we keep the ratio of particle energies fixed, $\ell_1 - \ell_2 =$const.

We then substitute the product $C\cdot D(\ell_1)D(\ell_2)$ for $D^{(2)}$ and solve the equations iteratively, using the fact that the normalized correlator (2) is a slowly changing function as compared with the distributions themselves. The range of applicability of the solution so obtained is not restricted to the vicinity of the hump as in [3]. Actually, the new solution can be trusted for arbitrary values of ℓ_i, as long as $\ell_1, \ell_2 > 2$ (to respect the adopted soft approximation, $x \ll 1$).

For example, the answer for the two-particle correlation inside a gluon jet reads

$$C_G(\ell_1,\ell_2,Y) - 1 = \frac{1 - b\left(\psi_{1,\ell} + \psi_{2,\ell} - [\beta\gamma_0^2]\right) - \delta_1 - [a\chi_\ell + \delta_2]}{1 + \Delta + \delta_1 + [a(\chi_\ell + \beta\gamma_0^2) + \delta_2]}. \qquad (4)$$

The function Δ is given by

$$\Delta \;=\; \gamma_0^{-2}\left(\psi_{1,\ell}\psi_{2,y}+\psi_{1,y}\psi_{2,\ell}\right), \tag{5}$$

where $\psi_i = \ln D(\ell_i, Y)$, and the subscripts ℓ and y mark its respective derivatives. Since logarithmic derivatives of the inclusive spectrum are, typically, of the order of $\psi_\ell \sim \psi_y = \mathcal{O}(\gamma_0)$, we have $\Delta = \mathcal{O}(1)$. This term is already present in the DLA, see [5], and is responsible for the fall-off of the correlation with increase of $\eta = |\ell_1 - \ell_2|$.

The term $b\left(\psi_{1,\ell}+\psi_{2,\ell}\right)$, where $b = \frac{1}{4N_c}\left[\frac{11}{3}N_c - \frac{4}{3}T_R\left(1-\frac{2C_F}{N_c}\right)^2\right]$ in the numerator of (4) is the next-to-leading (MLLA) correction $\mathcal{O}(\gamma_0)$. The term δ_1 is given by

$$\delta_1 = \gamma_0^{-2}\left[\,\chi_\ell(\psi_{1,y}+\psi_{2,y}) + \chi_y(\psi_{1,\ell}+\psi_{2,\ell})\,\right], \qquad \chi \equiv \ln[C_G]. \tag{6}$$

It is $\mathcal{O}(\gamma_0)$ and constitutes a MLLA correction as well, since $\chi_\ell \sim \chi_y = \mathcal{O}\left(\gamma_0^2\right)$.

The correction term δ_2 is given by

$$\delta_2 = \gamma_0^{-2}\left(\chi_\ell\chi_y + \chi_{\ell y}\right), \qquad \chi_{\ell y} \sim \chi_\ell\chi_y = \mathcal{O}\left(\gamma_0^4\right). \tag{7}$$

It is $\mathcal{O}\left(\gamma_0^2\right)$ and constitutes a NMLLA correction, as well as the term $a\chi_\ell$. Thus, all terms in the square brackets in (4) are of the order of γ_0^2. Being formally negligible within the MLLA accuracy, we nevertheless keep them in the answer. By so doing we follow the logic that was used in the derivation of the single inclusive MLLA spectra namely, solving *exactly* the *approximate* (MLLA) evolution equations.

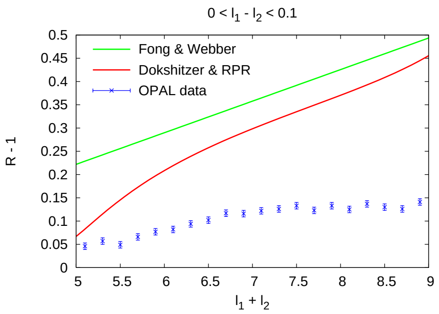
The correlator C_G itself enters on the r.h.s. of (4) via (6) and (7). Substituting

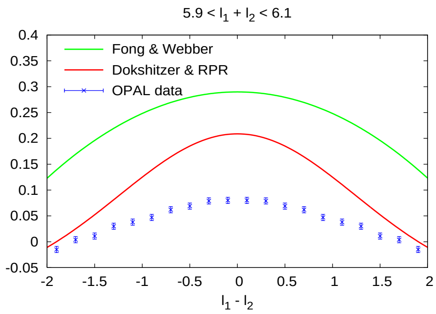
$$\chi = \ln[C_G] \simeq \ln\left\{1 + \frac{1 - b\left(\psi_{1,\ell}+\psi_{2,\ell} - [\beta\gamma_0^2]\right)]}{1+\Delta+[a\beta\gamma_0^2]}\right\}$$

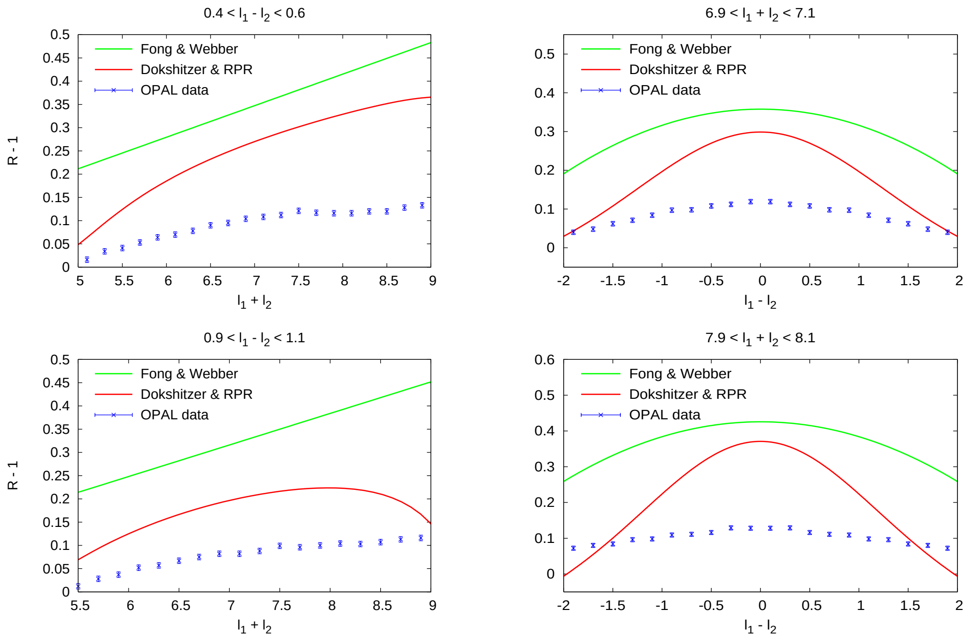
into the expressions for δ_1 and δ_2 provides then an *iterative solution*. The sum of the correction terms $\delta_1+\delta_2+a\chi_\ell$ is very small, $\mathcal{O}\left(10^{-2}\right)$, near the hump ($\ell_1 \simeq \ell_2$) and increases away from it.

The correlators for the quark jet and for e^+e^- annihilation (two quark jets) can be obtained from that of the gluon jet.

Below the results are presented for four different bands of the (ℓ_1, ℓ_2) plane, in comparison with the Fong–Webber approximation and the OPAL data [6]. In the left column we show $R-1$ as a function of the sum $\ell_1+\ell_2$ for three values of the difference, and on the right column, vice versa, as a function of the difference for fixed sum.







Conclusions

The correlation should be strongest when the two particles have the same energy, $\ell_1 = \ell_2$, in agreement with the Fong–Webber analysis.

At the same time, the correlation function that we derived directly from the evolution equations without expanding the answer in the formal perturbation parameter γ_0, no longer increases linearly but flattens off (and then tends to decrease) with $\ell_1 + \ell_2$. Such a behaviour is in accord with general theoretical expectations. Indeed, the smallest energy gluon with $\ell \to Y$ is pushed at large angles, $\Theta \sim 1$, and, in virtue of the QCD coherence, should be radiated independently of the rest of the parton ensemble.

Though our curves are somewhat closer to the OPAL measurements than the Fong–Webber results, the discrepancy remains substantial. Whether this discrepancy is due to higher order perturbative corrections or rather due to non-trivial hadronization effects that have not been seen in the inclusive one-particle spectra, remains to be studied. Forthcoming experimental data on two-particle correlations in g/q jets produced in pp collisions (CDF) should elucidate this problem.

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