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# GRIBOV PROGRAM OF UNDERSTANDING CONFINEMENT

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## 1 Introduction

For V.N. Gribov, the problem of confinement as the problem of understanding the dynamics of vacuum fluctuations, and of the structure of hadrons as physical states of the theory, was always inseparable from the problem of understanding the physics of high energy hadron scattering (the Pomeron picture). Hand-written notes of one of his talks on confinement started with the sentence

*Confinement — older than quarks themselves.*

and ended with

... [the picture] *can be checked* [in high energy scattering] *on nuclei.*

My aim is to introduce you to the Gribov scenario, and theory, of the quark confinement. But first, a short excursion into the history of high energy physics, and Gribov's place in it.<sup>a</sup>

## 2 High energy hadron scattering phenomena

In the late fifties, when Gribov, then a young researcher at the Ioffe Physico-Technical Institute, became interested in the physics of strong hadron interactions, there was no consistent picture of high-energy scattering processes, not to mention *a theory*. Apart from the Pomernanchuk theorem — an asymptotic equality of particle and antiparticle cross sections<sup>2</sup> — not much was theoretically understood about processes at high energies.

<sup>a</sup>Based on the preface to the book<sup>1</sup>, written in cooperation with Leonid Frankfurt.

## 2.1 Asymptotic behaviour $s^{\alpha(t)}$

Gribov's 1960 paper "*Asymptotic behaviour of the scattering amplitude at high energies*" in which he proved an inconsistency of the so-called black-disk model of diffractive hadron-hadron scattering, may be considered a first building block of the modern theory of high-energy particle interactions.<sup>3</sup>

Gribov's use of the so-called double-dispersion representation for the scattering amplitude, suggested by S. Mandelstam back in 1958,<sup>4</sup> demonstrated the combined power of the general principles of relativistic quantum theory — unitarity (conservation of probability), analyticity (causality) and the relativistic nature (crossing symmetry) — as applied to high energy interactions.

By studying the analytic properties in the cross channels, he showed that the imaginary part of the scattering amplitude in the form

$$A_1(s, t) = s f(t) \tag{1}$$

that constituted the black-disk model of diffraction in the physical region of  $s$ -channel scattering, contradicts the unitarity relation for partial waves in the crossing  $t$ -channel. To solve the puzzle, Gribov suggested the behaviour of the amplitude (for large  $s$  and finite  $t$ ) in the general form

$$A_1(s, t) = s^{q(t)} B_t(\ln s), \tag{2}$$

(equation (16) in <sup>3</sup>), where  $B_t$  is a slow (non-exponential) function of  $\ln s$  (decreasing fast with  $t$ ) and  $q(0) = 1$  ensures the approximate constancy of the total cross section with energy,  $\sigma^{\text{tot}}(s) \simeq \text{const}$ .

In this first paper Gribov analyzed the constant exponent,  $q(t) = 1$ , and proved that the cross section in this case has to decrease at high energies,  $B_t(\ln s) < 1/\ln s$ , to be consistent with the  $t$ -channel unitarity. He remarked on the possibility  $q(t) \neq \text{const}$  as "*extremely unlikely*" since, considering the  $t$ -dependence of the scattering amplitude, this would correspond to a strange picture of the radius of a hadron infinitely increasing with energy. He decided to "*postpone the treatment of such rapidly changing functions until a more detailed investigation is carried out*".

He published the results of such an investigation the next year in the letter to ZhETF "*Possible asymptotic behaviour of elastic scattering*". In his letter Gribov discussed the asymptotic behaviour "*which in spite of having a few unusual features is theoretically feasible and does not contradict the experimental data*".<sup>5</sup> Gribov was already aware of the finding by T. Regge<sup>6</sup> that in non-relativistic quantum mechanics

$$A(s, t) \propto t^{\ell(s)}, \tag{3}$$

in the unphysical region  $|t| \gg s$  (corresponding to large imaginary scattering angles  $\cos \Theta \rightarrow \infty$ ), where  $\ell(s)$  is the position of the pole of the partial wave  $f_\ell$  in the complex plane of the orbital momentum  $\ell$ .

T. Regge found that the poles of the amplitude in the complex  $\ell$ -plane were intimately related with bound states/resonances. It is this aspect of the Regge behaviour that initially attracted the most attention:

*“S. Mandelstam has suggested and emphasized repeatedly since 1960 that the Regge behaviour would permit a simple description of dynamical states (private discussions). Similar remarks have been made by R. Blankenbecker and M.L. Goldberger and by K. Wilson”* (quoted from<sup>7</sup>).

Gribov apparently learned about the Regge results from a paper by G. Chew and S. Frautschi of 1960<sup>8</sup> which contained a *footnote* describing the main Regge findings. In their paper, Chew and Frautschi had advocated the standard black-disk diffraction model (1), and referred to Regge only in the context of the “*connection between asymptotic behaviour in  $t$  and the maximum  $\ell$  that interacts strongly*”.

The structure of the Regge amplitude (3) motivated Gribov to return to the consideration of the case of the  $t$ -dependent exponent in his general high-energy ansatz (2) that was dictated by  $t$ -channel unitarity.

By then M. Froissart had already proved his famous theorem that limits the asymptotic behaviour of the total cross sections,<sup>9</sup>

$$\sigma^{\text{tot}} \propto s^{-1} |A_1(s, 0)| < C \ln^2 s. \quad (4)$$

Thus, having accepted  $\ell(0) = 1$  for the rightmost pole in the  $\ell$ -plane as the condition “*that the strongest possible interaction is realized*”, Gribov formulated “*the main properties of such an asymptotic scattering behaviour*”:

- the total interaction cross section is constant at high energies,
- the elastic cross section tends to zero as  $1/\ln s$ ,
- the scattering amplitude is essentially imaginary,
- the significant region of momentum transfer in elastic scattering shrinks with energy increasing,  $\sqrt{-t} \propto (\ln s)^{-1/2}$ .

He also analysed the  $s$ -channel partial waves to show that for small impact parameters  $\rho < R$  their amplitudes fall as  $1/\ln s$ , while the interaction radius  $R$  increases with energy as  $\rho \propto \sqrt{\ln s}$ . He concluded:

*“this behaviour means that the particles become grey with respect to high-energy interaction, but increase in size, so that the total cross section remains constant”.*

These were the key features of what has become known, quite ironically, as the “Regge theory” of strong interactions at high energies. On the opposite side of the Iron Curtain, the basic properties of the Regge pole picture of forward/backward scattering were formulated half a year later by G. Chew and S. Frautschi in<sup>10</sup>. In particular, they suggested “*the possibility that the recently discovered  $\rho$  meson is associated with a Regge pole whose internal quantum numbers are those of an  $I = 1$  two-pion configuration*”, and conjectured the universal high-energy behaviour of backward  $\pi^+\pi^0$ ,  $K^+K^0$  and  $pn$  scattering due to  $\rho$ -reggeon exchange. G. Chew and S. Frautschi also stressed that the hypothetical Regge pole with  $\alpha(0) = 1$  responsible for forward scattering possesses quantum numbers of the *vacuum*.

Dominance of the vacuum pole automatically satisfies the Pomeranchuk theorem. The name “Pomeron” for this vacuum pole was coined by Murray Gell-Mann, who referred to Geoffrey Chew as an inventor.

*Shrinkage of the diffractive peak* was predicted, and was experimentally verified at particle accelerator experiments in Russia (IHEP, Serpukhov), Switzerland (CERN) and the US (FNAL, Chicago), as were the general relations between the cross sections of different processes, that followed from the Gribov factorization theorem.<sup>11</sup>

## 2.2 Complex angular momenta in relativistic theory

In non-relativistic quantum mechanics the interaction Hamiltonian allows for scattering partial waves to be considered as analytic functions of complex angular momentum  $\ell$  (provided the interaction potential is analytic in  $r$ ).

Gribov’s paper “*Partial waves with complex orbital angular momenta and the asymptotic behaviour of the scattering amplitude*” showed that the partial waves with complex angular momenta can be introduced in a relativistic theory as well, on the basis of the Mandelstam double dispersion representation.

Here it is the unitarity in the crossing channel that *replaces* the Hamiltonian and leads to analyticity of the partial waves in  $\ell$ . The corresponding construction is known as the “Gribov-Froissart projection”.<sup>12</sup>

Few months later Gribov demonstrated that the simplest (two-particle)  $t$ -channel unitarity condition indeed generates the moving *pole* singularities in the complex  $\ell$ -plane. This was the *proof* of the Regge hypothesis in relativistic theory.<sup>13</sup>

The “Regge trajectories”  $\alpha(t)$  combine hadrons into families:  $s_n = \alpha(m_n^2)$ ,

where  $s_h$  and  $m_h$  are the spin and the mass of a hadron (hadronic resonance) with given quantum numbers (baryon number, isotopic spin, strangeness, etc.).<sup>10</sup> Moreover, at negative values of  $t$ , that is in the physical region of the  $s$ -channel, the very same function  $\alpha(t)$  determines the scattering amplitude, according to (2). It looks *as if* high-energy scattering was due to  $t$ -channel exchange of a “particle” with spin  $\alpha(t)$  that varies with momentum transfer  $t$  — the “reggeon”.

Thus, the high-energy behaviour of the scattering process  $a + b \rightarrow c + d$  is linked with the spectrum of excitations (resonances) of low-energy scattering in the dual channel,  $a + \bar{c} \rightarrow \bar{b} + d$ . This intriguing relation triggered many new ideas (bootstrap, the concept of duality). Backed by the mysterious *linearity* of Regge trajectories relating spins and squared masses of observed hadrons, the duality ideas, via the famous Veneziano amplitude, gave rise to the concept of hadronic strings and to development of string theories in general.

### 2.3 Interacting Pomerons

A number of theoretical efforts was devoted to understanding the approximately constant behaviour of the total cross sections at high energies.

To construct a full theory that would include the Pomeron trajectory with the maximal “intercept” that respects the Froissart bound,  $\alpha_P(0) = 1$ , and would be consistent with unitarity and analyticity proved to be very difficult. This is because multi-Pomeron exchanges become essential, which generate branch points in the complex plane of angular momentum  $\ell$ . These branch point singularities were first discovered by Mandelstam in his seminal paper of 1963<sup>14</sup>.

Moreover, the study of particle production processes with large rapidity gaps led V.N. Gribov, I.Ya. Pomeranchuk and K.A. Ter-Martirosyan to the concept of interacting reggeons.

By the end of the 60-s V. Gribov had developed the general theory known as Gribov Reggeon Calculus. He formulated the rules for constructing the field theory of interacting Pomerons — the Gribov Reggeon Field Theory (RFT) — and developed the corresponding diagram technique. Gribov RFT reduces the problem of high energy scattering to a non-relativistic quantum field theory of interacting particles in 2+1 dimensions.

### 2.4 Gribov partons and Feynman partons

One of Gribov’s most important contributions to high energy hadron physics was the understanding of the space-time evolution of high energy hadron-

hadron and lepton-hadron processes, in particular the nature of the reggeon exchange from the  $s$ -channel point of view.

In his lecture at the LNPI Winter School in 1973,<sup>15</sup> Gribov outlined the general phenomena and typical features that were characteristic for high energy processes in any quantum field theory. This lecture gives a perfect insight into Gribov's extraordinary way of approaching complicated physical problems of general nature. The power of Gribov's approach lies in applying the universal picture of fluctuating hadrons to both *soft* and *hard* interactions.

To understand the structure of hard (deep inelastic) photon-hadron interactions Feynman suggested the idea of partons — point-like constituents of hadronic matter.

Gribov's partons are constituents of hadron matter, components of long-living fluctuations of the hadron projectile, which are responsible for soft hadron-hadron interactions: total cross sections, diffraction, multiparticle production, etc.

Feynman defined partons in the infinite momentum frame to suppress vacuum fluctuations whose presence would have undermined the notion of the parton wave function of a hadron.<sup>16</sup>

Gribov's earlier work "Interaction of  $\gamma$ -quanta and electrons with nuclei at high energies"<sup>17</sup> had been a precursor to the famous Feynman paper. Gribov described the photon interaction in the rest frame of the target nucleus. An incident real photon (or a virtual photon in the deep inelastic scattering case) fluctuates into hadrons before the target, at the longitudinal distance  $L$  increasing with energy. (For the  $e^-p$  deep inelastic scattering B.L. Ioffe has shown that the assumption of Bjorken scaling implies  $L \sim 1/2xm_N$ , with  $x$  the Bjorken variable and  $m_N$  the nucleon mass.<sup>18</sup>) Therefore, at sufficiently large energy, when the fluctuation distance exceeds the size of the target, the photon no longer behaves as a point-like weakly interacting particle. Its interaction resembles that of a hadron and becomes "black", corresponding to complete absorption on a large nucleus. This paper can be viewed as the generalization of the VDM (vector dominance model) to high energy processes.

Being formally equivalent to Feynman's treatment, Gribov's approach is better suited for the analysis of deep inelastic phenomena at very small Bjorken  $x$ , where the interaction becomes actually strong, and the perturbative QCD treatment is bound to fail.

Gribov diffusion in the impact parameter space giving rise to energy increase of the interaction radius and to the reggeon exchange amplitude, co-existing fluctuations as a source of branch cuts, duality between hadrons and partons, a common basis for hard and soft elastic, diffractive and inelastic process — these are some of the key features of high energy phenomena in

quantum field theories, which are still too hard a nut for QCD to crack.

### 2.5 Gribov reggeon field theory

Two best known applications of the Gribov RFT are

- general quantitative relation between the shadowing phenomenon in hadron-hadron scattering, the cross section of diffractive processes and inelastic multi-particle production, known as “Abramovsky-Gribov-Kanchelli (AGK) cutting rules”,<sup>19</sup> and
- the essential revision by Gribov of the Glauber theory of nuclear shadowing in hadron-nucleus interactions.<sup>20</sup>

In 1968 V.N. Gribov and A.A. Migdal demonstrated that the scaling behaviour of the Green functions emerged in field theory in the strong coupling regime.<sup>21</sup> Their technique helped to build the quantitative theory of second order phase transitions and to analyse critical indices characterizing the long-range fluctuations near the critical point.

The problem of high energy behaviour of soft interactions remained unsolved, although some viable options were suggested. In particular, in “*Properties of Pomeron Poles, diffraction scattering and asymptotic equality of total cross sections*”<sup>22</sup> Gribov has shown that a possible consistent solution of the RFT in the so-called *weak coupling* regime calls for the formal asymptotic equality of *all* total cross sections of strongly interacting particles.

Gribov’s last work in this subject was devoted to the intermediate energy range and dealt with interacting hadron fluctuations.<sup>23</sup>

The study of the *strong coupling* regime of interacting reggeons (pioneered by A.B. Kaidalov and K.A.Ter-Martirosyan) led to the introduction of the *bare* Pomeron with  $\alpha_P(0) > 1$ . The RFT based on *t*-channel unitarity should enforce the *s*-channel unitarity as well. The combination of increasing interaction radius and the amplitudes in the impact parameter space which did not fall as  $1/\ln s$  (as in the one-Pomeron picture) led to logarithmically increasing asymptotic cross sections, resembling the Froissart regime (and respecting the Froissart bound (4)). The popularity of the notion of the bare Pomeron with  $\alpha_P(0) > 1$  is based on experiment (increasing total hadron cross sections). Psychologically, it is also supported by the perturbative QCD finding that the (small) scattering cross section of two small-transverse-size objects should increase with energy in a power-like fashion in the restricted energy range (the so-called hard BFKL regime<sup>24</sup>).



## 2.6 Reggeization and the Pomeron singularity in gauge theories

In the mid-sixties V.N. Gribov initiated the study of double logarithmic asymptotics of various processes in quantum electrodynamics (QED), making use of the powerful technique he had developed for the analysis of the asymptotic behaviour of Feynman diagrams in the limit  $s \rightarrow \infty$  and  $t = \text{const.}$ <sup>25</sup>

In particular, in 1975 V.N. Gribov, L.N. Lipatov and G.V. Frolov studied the high energy behaviour of QED processes from the point of view of “Regge theory”. High energy scattering amplitudes with exchange of an *electron* in the  $t$ -channel acquire, in higher orders in QED coupling, a characteristic behaviour  $A \propto s^{j(t)}$  with  $j(m_e^2) = 1/2$ , that is, electron becomes a part of the Regge trajectory: *reggeizes*. (So do quarks and gluons in QCD; V.S. Fadin, L.L. Frankfurt, L.N.Lipatov, V.E. Sherman, 1976.) For the *vacuum channel*, however, Gribov, Lipatov and Frolov found<sup>26</sup> that the rightmost singularity in the complex  $j$ -plane is not a moving pole (as it is, e.g., for electron) but, instead, a fixed branch point singularity positioned to the right from unity,  $j = 1 + \alpha^2 > 1$ . This was a precursor of a similar result found later by Fadin, Lipatov and Kuraev<sup>24</sup> in non-Abelian theories, and QCD in particular. The problem of apparent anti-Froissart behaviour of the perturbative “hard Pomeron” in QCD still awaits resolution.

With the advent of quantum chromodynamics as a microscopic theory of hadrons and their interactions, the focus of theoretical studies has temporarily shifted away from Gribov-Regge problematics to “hard” small-distance phenomena.

## 3 Gribov as QCD apprentice

V.N. Gribov became interested in non-Abelian field theories relatively late, in 1976. His very first study, as a QCD apprentice, produced amazing results.

In February 1977 in the proceedings of the 12<sup>th</sup> PNPI Winter School he published two lectures which were to change forever the non-Abelian landscape.

In the first lecture “*Instability of non-Abelian gauge fields and impossibility of the choice of the Coulomb gauge*”<sup>27</sup> he showed that the three-dimensional transversality condition

$$(\partial \cdot \mathbf{B}) \equiv \frac{\partial B_i}{\partial x_i} = 0, \quad i = 1, 2, 3 \quad (5)$$

which is usually imposed on the fields to describe massless vector particles (the Coulomb gauge), *does not* solve the problem of gauge fixing. Due to essential non-linearity of gauge transformation, a “transverse” field (5) may

actually happen to be a pure gauge field which should not be counted as a physical degree of freedom.

Gribov explicitly constructed such “transversal gauge fields” for the  $SU(2)$  gauge group and showed that the *uncertainty* in gauge fixing arises when the effective magnitude of the field becomes large,

$$L \cdot B \sim \frac{1}{g_s},$$

or, in other words, when the effective interaction strength (QCD coupling) becomes of the order of unity, that is, in the non-perturbative region.

He also gave an elegant physically transparent explanation of the *anti-screening* phenomenon within the Hamiltonian approach, which had been observed by I. Khriplovich back in 1969<sup>28</sup> in the ghost-free Coulomb gauge and then (re)discovered and coined as the *asymptotic freedom* by D. Gross & F. Wilczek and H.D. Politzer in 1973.

In the Hamiltonian language, there are (or, better to say, seem to be)  $N^2 - 1$  massless  $\mathbf{B}$  fields (transverse gluons), and, as in QED, an additional Coulomb field mediating interaction between colour charges. Unlike QED, the non-Abelian Coulomb quantum has a colour charge of its own. Therefore, traversing the space between two external charge, it may virtually decay into two transverse fields,

$$\mathbf{0} \rightarrow \perp + \perp \rightarrow \mathbf{0},$$

or into a  $q\bar{q}$  pair

$$\mathbf{0} \rightarrow \mathbf{q} + \bar{\mathbf{q}} \rightarrow \mathbf{0},$$

in the same manner as a QED Coulomb quantum fluctuates in the vacuum into an  $e^+e^-$  pair. Both these effects lead to *screening* of the colour charge of the external sources, in a perfect accord with one’s physical intuition. The fact that there are “physical” fields in the intermediate state — quarks and/or transverse gluons — fixes the sign of the virtual correction to correspond to *screening*, via the unitarity relation.

Technically speaking, these virtual decay processes contribute to the QCD  $\beta$ -function as

$$\left\{ \frac{d\alpha_s^{-1}(R)}{d\ln R} \right\}^{\text{phys}} \propto \frac{1}{3}N + \frac{2}{3}n_f, \quad (6a)$$

that is, make the effective (running) coupling *decrease* at *large* distances  $R$  between the external charges.

Where then the *anti-screening* comes from? It originates from another, specifically non-Abelian, effect namely, interaction of the Coulomb quantum with the field of “zero-fluctuations” of transverse gluons in the vacuum,

$$\left[ \mathbf{0} + \perp \rightarrow \mathbf{0} \right]^n.$$

In a course of such multiple rescattering, the Coulomb quantum preserves its identity as an *instantaneous* interaction mediator, and therefore is not affected by the unitarity constraints. Statistical average over the transverse vacuum fields in the second order of perturbation theory ( $n = 2$ ; the  $n = 1$  contribution vanishes upon averaging) results in an additional contribution to the Coulomb interaction energy which, translated into the running coupling language, gives

$$\left\{ \frac{d\alpha_s^{-1}(R)}{d\ln R} \right\}^{\text{stat}} \propto -4N \quad (6b)$$

Taken together, the two contributions (6) combine into the standard QCD  $\beta$ -function. An anti-intuitive minus sign in (6b) has its own simple explanation. It is of the same origin as the minus sign in the shift of the energy of the ground state of a quantum-mechanical system under the second order of perturbation:

$$\delta E \equiv E - E_0 = \sum_n \frac{|\langle 0 | \delta V | n \rangle|^2}{E_0 - E_n} < 0.$$

The rôle of perturbation  $\delta V$  is played here by transverse vacuum fields.

Propagation of a Coulomb field  $C^a$  in the “external” transverse gluon field  $\mathbf{B}_\perp^a$  is described by the operator which resembles the Faddeev–Popov ghost propagator,

$$G^{-1}[\mathbf{B}_\perp] = (\mathbf{D} \cdot \partial) \equiv \partial_i^2 + ig_s [B_{i\perp}, \partial_i], \quad (7)$$

with  $\mathbf{D}$  the covariant derivative,

$$D_i[\mathbf{B}_\perp] \psi = \partial_i \psi + ig_s [B_{i\perp}, \psi].$$

For small vacuum fields,  $g_s \mathbf{B}_\perp / \partial \ll 1$ , expanding perturbatively the Coulomb propagator  $G$  to the second order in  $g_s$  produces the perturbative anti-screening effect as stated in (6b).

If we take, however, the gluon field in the QCD vacuum as large as

$$g_s \mathbf{B}_\perp / \partial \sim g_s \mathbf{B}_\perp \cdot L \sim 1,$$

a qualitatively new phenomenon takes place namely, the Coulomb (ghost) propagator may become singular:

$$G^{-1}[\mathbf{B}_\perp] C_0 = C_0 (\partial_i^2 C_0 + ig_s [B_{i\perp}, \partial_i C_0]) = 0. \quad (8)$$

Such a “zero-mode” solution  $C_0$  in the external field is a sign of an infrared instability of the theory. On the other hand, the “surface”  $G^{-1}[\mathbf{B}_\perp] = 0$  in the functional  $\mathbf{B}_\perp$ -space is that very border beyond which the solution of the gauge-fixing equation (5) becomes copious.

The fact that the Coulomb propagator develops singularity does not necessarily mean that the Faddeev–Popov ghost “rises from the dead” by pretending to propagate as a particle. It rather tells us that we have failed to formulate the quantum theory of interacting vector fields, to properly fix physical degrees of freedom.

The existence of “Gribov copies” means that the standard Faddeev–Popov prescription for quantizing non-Abelian gauge theories is, strictly speaking, incomplete and should be modified.

Gribov addressed the problem of possible modification of the quantization procedure in the second lecture “Quantization of non-Abelian gauge theories”<sup>29</sup>. The paper under the same title based on these two lectures is now a universally accepted (though disturbing) truth and during 25 odd years since its appearance in 1978<sup>30</sup> was being cited more than 20 times per year, in average. It goes without saying, that this paper was initially rejected by a NPB referee, who wrote (as far as I can remember) that *the author raises the problem of confinement which problem had been already solved and isn’t worth talking about*.

To properly formulate non-Abelian field dynamics, Gribov suggested to limit the integration over the fields in the functional integral to the so-called *fundamental domain*, where the Faddeev–Popov determinant is strictly positive (the region in the functional space of transverse fields  $\mathbf{B}_\perp$  *before* the first zero mode (8) emerges).

In the second lecture, Gribov produced qualitative arguments in favour of the characteristic modification of the gluon propagator, due to the new restriction imposed on the functional integral. Effective suppression of large gluon field results, semi-quantitatively, in an infrared singular polarization operator

$$D^{-1}(k) \simeq k^2 + \frac{\langle G^2 \rangle}{k^2}, \quad (9a)$$

which coincides with the perturbative gluon Green function at large momenta (small distances),  $D(k) \propto k^{-2}$  but makes it *vanish* of at  $k = 0$ , instead of having a pole corresponding to massless gluons:

$$D(k) \propto \frac{k^2}{k^4 + \langle G^2 \rangle}. \quad (9b)$$

The new non-perturbative parameter  $\langle G^2 \rangle$  in (9) has dimension (and the meaning) of the familiar vacuum gluon condensate.

Literally speaking, the ansatz (9b) cannot be correct since such a Green function would violate causality.<sup>b</sup> In reality, the gluon (as well as the quark) propagator should have a more sophisticated analytic structure with singularities on unphysical sheets, which would correspond, in the standard field-theoretical language, to *unstable* particles.

In spite of many attempts, the problem of *Gribov copies* (“Gribov horizon”, “Gribov uncertainties”) remains open today and plagues the dynamics of any essentially non-linear gauge invariant systems. A promising recent attempt to implement the Gribov fundamental domain restriction in pure gluodynamics, and its prehistory, can be found in<sup>31</sup>.

Gribov did not pursue this goal himself not because of severe difficulties in describing the fundamental domain in the functional space: he always had his ways around technical obstacles. He convinced himself (though not yet the physics community at large) that the solution to the confinement problem lies not in the understanding of the interaction of “large gluon fields” but instead in the understanding of how the QCD dynamics can be arranged as to prevent the non-Abelian fields from growing real big.

#### 4 Gribov light quark confinement

By 1990 V. Gribov formulated the “light-quark confinement scenario” which was essentially based on the existence of two very light (practically) massless quarks in the theory.

##### 4.1 Super-critical binding of fermions

As a result of the search for a possible solution of the confinement puzzle, which he pursued with unmatched intensity and depth for more than 10 years after 1977, Gribov formulated for himself the key ingredients of the problem and, correspondingly, the lines to approach it:

- The question of interest is not of “a” confinement, but that of “the” confinement in the real world, namely, in the world with two very light quarks ( $u$  and  $d$ ) whose Compton wave lengths are much larger than the characteristic confinement scale ( $m_q \sim 5 - 10 \text{ MeV} \ll 1 \text{ GeV}$ ).

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<sup>b</sup>It is unfortunate, therefore, that the form (9b) which Gribov suggested and discussed for illustrative purposes only, is often referred to in the literature as “the Gribov propagator”.

- No mechanism for binding massless *bosons* (gluons) seems to exist in QFT, while the Pauli exclusion principle may provide means for binding together massless *fermions* (light quarks).
- The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently infrared-unstable dynamics: the ultraviolet and infrared regimes of the theory may be closely linked. Example: the pion field as a Goldstone boson emerging due to spontaneous chiral symmetry breaking (short distances) and as a quark bound state (large distances).
- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects. Feynman's famous  $i\epsilon$  prescription was designed for (and is applicable only to) the theories with stable perturbative vacua. To understand and describe a physical process in a confining theory, it is necessary to take into consideration the response of the vacuum, which leads to essential modifications of the quark and gluon Green functions.

There was a deep reason for this turn, which Gribov formulated in the following words:

“I found I don't know how to bind massless *bosons*”

(read: how to dynamically construct *glueballs*). As for fermions, there is a corresponding mechanism provided by the Fermi–Dirac statistics and the concept of the “Dirac sea”. Spin-1/2 particles, even massless which are difficult to localize, can be held together simply by the fact that, if pulled apart, they would correspond to the free-fermion states that are *occupied* as belonging to the Dirac sea. In a pure perturbative (non-interacting) picture, the empty fermion states have *positive energies*, while the *negative-energy* states are all filled. With account of interaction the situation may change, *provided* two *positive-energy* fermions (quarks) were tempted to form a bound state with a *negative* total energy. In such a case, the true vacuum of the theory would contain *positive kinetic energy* quarks hidden inside the *negative energy* pairs, thus preventing positive-energy quarks from flying free.

A similar physical phenomenon is known in QED under the name of super-critical binding in ultra-heavy nuclei. Dirac energy levels of an electron in an external static field created by the large electric charge  $Z > 137$  become complex. This means instability: classically, the electron “falls on the centre”. In QFT the instability develops when the energy  $\epsilon$  of an empty atomic electron level falls, with increase of  $Z$ , below  $-m_e c^2$ . An  $e^+e^-$  pair pops up from the

vacuum, with the vacuum electron occupying the level: the super-critically charged ion decays into an “atom” (the ion with the smaller charge,  $Z - 1$ ) and a real positron

$$A_Z \implies A_{Z-1} + e^+, \quad \text{for } Z > Z_{\text{crit}}.$$

In the QCD context, the increase of the running quark-gluon coupling at large distances replaces large  $Z$  of the QED problem.

Gribov generalized the problem of super-critical binding in the field of an infinitely heavy source to the case of two massless fermions interacting via Coulomb-like exchange. He showed that in this case the super-critical phenomenon develops much earlier.

In the Lund University preprint<sup>32</sup> Gribov has shown that a *pair* of light fermions interacting in a Coulomb-like manner develops super-critical behaviour if the coupling hits a definite critical value

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = 1 - \sqrt{\frac{2}{3}}. \quad (10)$$

In QCD, with account of the colour Casimir operator, the criticality condition translates into

$$\frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1} \left[ 1 - \sqrt{\frac{2}{3}} \right] \simeq 0.137. \quad (11)$$

This number, apart from being easy to remember, has another interesting property: it is numerically *small*, which opens an intriguing, and perfectly heretic, possibility of understanding, and describing (at least semi-quantitatively) the full wealth of hadron spectra and interactions in the perturbative language of quarks and gluons.

#### 4.2 Gribov equation

Gribov constructed the equation for the quark Green function as an approximation to the general corresponding Schwinger-Dyson equation.

This approximation took into account the most singular (logarithmically enhanced) infrared and ultraviolet effects due to quark-gluon interactions and resulted in a non-linear differential equation which possesses a rich non-perturbative structure.

An amazing simplicity of the Gribov construction makes one wonder, why such an equation had not been discovered 15–20 years earlier when a lot of effort was applied in a search for non-perturbative phenomena of

the super-conductivity type in QED (Nambu–Jona-Lasinio; Baker–Johnson; Fomin–Miransky et al.; ...).

Take the first order self-energy diagram (which is easier for you to imagine than for me to draw): a fermion (quark/electron) with momentum  $q$  virtually decays into a quark (electron) with momentum  $q'$  and a massless vector boson (gluon/photon) with momentum  $q - q'$ :

$$\Sigma(q) = [C_F] \frac{\alpha}{\pi} \int \frac{d^4 q'}{4\pi^2 i} [\gamma_\mu G(q') \gamma_\mu] D(q - q'), \quad D_0(k) = \frac{1}{k^2 + i\epsilon}, \quad (12)$$

with  $G$  and  $D$  the fermion and boson propagators, respectively. This Feynman diagram diverges linearly at  $q' \rightarrow \infty$ . To kill the ultraviolet divergences (both linear and logarithmic), it suffices to *differentiate* it twice over the external momentum  $q$ .

The first Gribov's observation was that  $1/k^2$  happens to be the Green function of the four-dimensional Laplace operator,

$$\partial^2 \frac{1}{(q - q')^2 + i\epsilon} = -4\pi^2 i \delta(q - q'), \quad \partial_\mu \equiv \frac{\partial}{\partial q_\mu},$$

where  $\partial_\mu$  now denotes the momentum differentiation. Therefore, the operation  $\partial^2$  applied to (12) takes away the internal momentum integration and leads to an expression which is *local* in the momentum space,

$$\partial^2 \Sigma_1(q) = [C_F] \frac{\alpha}{\pi} \gamma_\mu G_0(q) \gamma_\mu, \quad k = q - q' = 0. \quad (13)$$

This is the “Born approximation”. With account of higher order radiative corrections, the first thing that happens is that the bare fermion propagator  $G_0$  dresses up,  $G_0(q) \rightarrow G(q)$ , and so do the Born vertices  $\gamma_\mu \rightarrow \Gamma_\mu(q, q, 0)$ . The second crucial observation was that the exact vertex function  $\Gamma_\mu(q, q - k, k)$  describing the emission of a *zero momentum* vector boson,  $k_\mu \equiv 0$ , is not an independent entity but is related with the fermion propagator by the Ward identity,

$$\Gamma_\mu(q, q, 0) = -\partial_\mu G^{-1}(q), \quad (14)$$

which statement is *literally* true in Abelian theory (QED), and, after some reflection, *can be made* true in the non-Abelian case (QCD) as well.<sup>c</sup>

Thus, we have arrived to the Gribov equation for the quark Green function<sup>32,33</sup>

$$\partial^2 G^{-1}(q) = g \partial_\mu G^{-1}(q) G(q) \partial_\mu G^{-1}(q) + \dots, \quad g \equiv C_F \frac{\alpha_s}{\pi}, \quad (15)$$

<sup>c</sup>I successfully fought the temptation to call it an “illiterally true” statement.



where ... stand for less singular  $\mathcal{O}(g^2)$  integral terms.

Yet another set of higher order corrections makes the coupling run,  $g \rightarrow g(q^2)$ . In the  $|q^2| \rightarrow \infty$  limit the QCD coupling vanishes due to the asymptotic freedom, and (15) turns into the free equation,  $\partial^2 G^{-1} = 0$ , whose solution has the form

$$G^{-1}(q) = Z^{-1} \left[ (m - \hat{q}) + \frac{\nu_1^3}{q^2} + \frac{\nu_2^4 \hat{q}}{q^4} \right]. \quad (16)$$

This general perturbative solution has two new arbitrary parameters  $\nu_1$  and  $\nu_2$  in addition to the familiar two (mass  $m$  and the wave function renormalization constant  $Z$ ), since the master equation is now the *second* order differential equation, unlike the standard renormalization group (RG) approach.

Since the new terms are singular at small  $q$ , in QED, for example, we simply drop them as unwanted, thus returning to the RG structure. Such a prescription, however, exploits the knowledge that nothing dramatic happens in the infrared domain, so that the real electron in the physical spectrum of the theory, whose propagation we seek to describe, is inherently that very same object that we put into the Lagrangian as a fundamental bare field.

In an infrared unstable theory (QCD) we better wait and see.

Indeed, Gribov found that if the coupling in the infrared region exceeded the critical value (11), a bifurcation occurred in (15) and a new phase emerged corresponding to *spontaneous breaking of the chiral symmetry*.

### 4.3 Quarks, pions and confinement

As far as *confinement* is concerned, the approximation (15) turned out to be insufficient. A numerical study of the Gribov equation carried out by Carlo Ewerz showed<sup>34</sup> that the corresponding quark Green function does not possess an analytic structure that would correspond to a confined object.

Given the dynamical chiral symmetry breaking, however, the Goldstone phenomenon takes place bringing pions to life. In his last paper<sup>35</sup> Gribov argued that the effects that Goldstone pions induce, in turn, on the propagation of quarks is likely to lead to confinement of light quarks and, as a result, to confinement of any colour states.

The approximate equation for the Green function of a massless quark, which accommodates a feed-back from Goldstone pions reads<sup>35</sup>

$$\begin{aligned} \partial^2 G^{-1}(q) = & g(q) \partial_\mu G^{-1}(q) G(q) \partial_\mu G^{-1}(q) \\ & - \frac{3}{16\pi^2 f_\pi^2} \{i\gamma_5, G^{-1}(q)\} G(q) \{i\gamma_5, G^{-1}(q)\}. \end{aligned} \quad (17)$$

The modified Gribov equation (17) still awaits a detailed study aiming at the analytic structure of its solutions.

Another important open problem is to construct and to analyse a similar equation in the gluon sector, from which a consistent picture of the coupling  $g(q)$  rising above the critical value in the infrared momentum region should emerge.

It is important to notice that since pions have emerged *dynamically* in the theory, their coupling to quarks is not arbitrary but is tightly linked with the quark propagator itself (search for an anti-commutator of  $\gamma_5$  with  $G^{-1}$  in (17)). Moreover, the pion-axial current transition constant  $f_\pi$  is not arbitrary either, but has to satisfy a definite relation which, once again, is driven by the behaviour of the exact quark Green function:

$$f_\pi^2 = \frac{1}{8} \int \frac{d^4 q}{(2\pi)^4 i} \text{Tr} \left[ \{i\gamma_5, G^{-1}\} G \{i\gamma_5, G^{-1}\} G (\partial_\mu G^{-1} G)^2 \right] + \frac{1}{64\pi^2 f_\pi^2} \int \frac{d^4 q}{(2\pi)^4 i} \text{Tr} \left[ (\{i\gamma_5, G^{-1}\} G)^4 \right]. \quad (18)$$

The second of the two papers<sup>33,35</sup> concluding Gribov's study of the light-quark super-critical confinement theory remained unfinished. It ends abruptly in the middle of the discussion of the most intriguing question, namely, what is the meaning, and practical realization, of unitarity in a confining theory.

Gribov works on gauge theories and, in particular, all his papers, talks and lectures devoted to anomalies and the QCD confinement (including some lectures that were translated from Russian for the first time) were collected in the book<sup>1</sup> due to be published in Moscow by the end of 2001.

From these papers, an interested reader will be able to derive the Gribov equation and to study the properties of its (perturbative and non-perturbative) solutions, as well as to formulate the open problems awaiting analysis and resolution (markedly, how to construct an equation for the running coupling, similar to that for the quark Green function). Gribov lectures will also give an opportunity to grasp the physical picture of the super-critical QCD binding which includes the notion of an "inversely populated" Dirac sea of light quarks, and to think about phenomenological aspects, and verification, of the Gribov confinement scenario.

Speaking of Gribov heritage, another must-to-have for each university/educational body in theoretical physics is Gribov lectures on Quantum Electrodynamics which can be considered, and used as, an introductory course into relativistic field theory in general.<sup>36</sup>

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