TELMA Cross Experiment Guidelines

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Rapporto Interno 01/07, Istituto Tecnologie Didattiche - C.N.R.
Genova, May 2007

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Preface

This document contains the guidelines developed by members of TELMA[1] as a means for planning, conducting, and analysing a cross experiment aimed at contributing to the construction of a shared research perspective among TELMA teams[2]. This is the product of the PhD students and young researchers that brought forward the whole activity. The actual experimental phase was proceeded by a reflective phase in which an agreement was achieved on what research questions to address during the experiment. On this basis the first version of the guidelines document was built, containing all the research questions to be addressed, but also the experimental plans for each team. This included the employed *didactical functionalities*[4] (Cerulli, Pedemonte, Robotti 2006) of the considered ICT tools, indications of the experimental settings, and the methods of data collection and analysis. During the whole experimental phase, the document was constantly updated, and shared among the involved persons which were periodically required to compare the different activities and reflections brought forward by all the teams. The last version of the guidelines, reported here, contains also some reflections on the experiment, however deeper analysis of the results of the project, in terms of developing a methodology for integrating researches and theoretical frameworks can be found in Artigue et al. (2007) and Cerulli et. al. (2007)[3].

M. Cerulli, B. Pedemonte, E. Robotti

[1] TELMA (Technology Enhanced Learning in Mathematics, http://telma.noe-kaleidoscope.org/) is a European Research Team (ERT) established as one of the activities of Kaleidoscope, a Network of Excellence (IST–507838) supported by the European Community (www.noe-kaleidoscope.org).

[2] (DIDIREM) University Paris 7 Denis Diderot - DIDIREM, France; (ETL-NKUA) National Kapodistrian University of Athens - Educational Technology Lab - Athens, Greece; (CNR-ITD) Consiglio Nazionale delle Ricerche - Instituto Tecnologie Didattiche - Genova, Italy; (LIG) MeTAH-LIG-University of Grenoble, France; (UNILON) University of London - Institute of Education - London, UK; (UNISI) University of Siena - Department of Mathematics, Italy.

[3] See also TELMA papers on the Kaleidoscope archive:
http://www.telearn.org/browse/publications/collection/?&collection=28

1 Introduction (Cerulli, M., Pedemonte B., Robotti, E.)

This work is the result of an on-line collaboration among students and young researchers of TELMA teams. The main aim of this collaboration is the integration of TELMA teams by means of a cross experiment. The object of this experiment is a deeper understanding the potential of the ICT tools employed by TELMA teams in a variety of educational situations. Moreover it will address aspects related to *representations*, *learning contexts*, *theoretical framework*, which have been selected as the main themes of interest of TELMA project.
For the experiment we chose to use tools (see Table 1) that are developed by some TELMA teams, and that can be employed to address a common mathematical knowledge domain: algebra.

Tool	Developer's team	Experimenting TEAM
Aplusix	MeTAH-Grenoble	ITD, Università di Siena
E-Slate	ETL-NKUA	UNILON
ARI-LAB 2	ITD	MeTAH-Grenoble, DIDIREM- Paris VII, ETL-NKUA

Table 1 The tools employed by TELMA teams in the cross experiment.

TELMA teams agreed on the duration of the experiment (more or less one month) and on the average school level (8/9 level).
This document provides a set of guidelines for the design of the experiment. The guidelines are built on the basis of a three sets of questions, selected collaboratively by the involved teams, that refers to the three main themes of TELMA. Each set of questions is a selection of a bigger set of scientifically relevant questions that were indicated in a document produced by an expert TELMA researcher of the theme (see appendix 1). The final questions were selected with an on-line collaborative process according to the following main criterions:

- relevance for the largest number of TELMA teams
- interests of TELMA teams
- feasibility (considered also time and pupils' age constrains)

The selected questions are presented in this document in bold italic, and are interlaced with comments aiming at clarifying the meanings of the questions and at organizing them. Each team will have to answer the questions according to the indications provided in this guideline. We begin presenting a key question concerning the research goals of the experiments.

1.1 Research goals

The question we consider in this section can be interpreted both as referring to research aims or educational aims. For this reason, we present it twice starting focusing on the research aims.

What are the precise aims of your experiment and the questions you want to focus on?

Below we list the aims of this research:

- studying how differences of theoretical frameworks, contexts and educational approaches may influence the effectivenes and potentialities of a tool. In particular for each tool we are interested in the comparison between the practical experiment and the scenarios indicated by the developers of the tool. The analysis can be based on a comparison between the theoretical framework used by the designer team and that uesd by experimenting team; the

learning context considered by the team designer and that put in practice by the experimenting team; the caractheristics of representation assigned to the tool by the team designer and those exploited by the experimenting team
- deepening the theoretical construct of *didactical functionnalities*[4] on the base of the previous analysis.
- ……*[if you think, you can add others research aims that can be shared by the teams]*

1.2 Tools

In order to study each tool and its educational potentialities we need to know its key elements and the key ideas underlying its design; this information will be required to the teams that developed the tools. Such information will also be useful to help experimenting teams in designing their activity.
Each team will also have to answer the 9 selected questions concerning the experimented tool. The questions have been classified according to an a-priori versus a-posteriori criterion:

- 4 a-priori (with respect to the experiment) questions aims at collecting information concerning the design of the experiment.
- 2 a-posteriori (with respect to the experiment) questions aims at individuating, collecting and gathering the results of the experiment.
- 3 a-priori/posteriori (with respect to the experiment) are questions for which we aspect a-priori answers that can be integrated/compared also with a-posteriori (or in itinere) answers.

Answers to the a-priori questions have to be provided before the beginning of the experiments, and will constitute themselves an integrated part of the guidelines. Also the first answers to the a-priori/posteriori will need to be provided before the beginning of the experiment. Finally a-posteriori answers will be provided after the experiment. The scheme of the questions to be addressed for each tool is represented in Table 2, followed by a section for each tool with indications of the questions each team has to answer before and after the experiment.

[4] Given an ICT tool, and an educational goal, it is possible to identify its *didactical functionalities* (Cerulli, Pedemonte, Robotti, 2006):
"With didactical functionalities we mean those properties (or characteristics) of a given ICT, and/or its (or their) modalities of employment, which may favor or enhance teaching/learning processes according to a specific educational goal.
The three key elements of the definition of the *didactical functionalities* of an ICT tool are:
1. a set of *features/characteristics of the tool;*
2. a specific *educational goal;*
3. a set of *modalities of employing* the tool in a teaching/learning process referred to the chosen educational goal."

<table>
<tr><th colspan="5">TOOL</th></tr>
<tr><td colspan="2">Developer's team</td><td colspan="3">○ Brief description of the instrument, explaining its key ideas
○ Indicate the theoretical framework employed to design and implement the software</td></tr>
<tr><td colspan="2"></td><td>Questions about Theoretical framework</td><td>Questions about Representation</td><td>Questions about Learning Context</td></tr>
<tr><td rowspan="3">Experimenting team</td><td>A priori</td><td>• What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?
This question is relatively "open" and the answers should be very instructive for us about how each team works with his own culture.
• What are the precise aims of your experiment and the questions you want to focus on?
This question refers to the didactical aims of the experiment. For this reason, in order to answer this question, it is necessasry to have an idea about the design of the experiment</td><td></td><td>• What is the type of research that you follow (e.g. classroom based, case srudies) and how is this related to the kind of your research focus;
• Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?</td></tr>
<tr><td>A posteriori</td><td>• In your opinion, in which ways do your theoretical choices have influenced:
– the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employement including the configuration of the software)?
– the conception of the experiment?
– the choices of the data and their analysis?
the results you obtain and the conclusions you draw from these?</td><td>• Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?</td><td></td></tr>
<tr><td>A priori and posteriori</td><td></td><td>• What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?
• What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?</td><td>• How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?</td></tr>
</table>

Table 2 The questions to be addressed for each tool by developer and experimenting teams.

2 Aplusix

2.1 Developer's team: LIG (Bouhineau, D., Nicaud, J.-F.)

- *Brief description of the instrument, explaining its key ideas*
- *Indicate the theoretical framework employed to design and implement the software*

The main goal of the developer's team has been to design and develop an interactive learning environment for algebra which would allow students to freely build and transform algebraic expressions and would also provide feedback to help them to learn. The APLUSIX system has been developed to incorporate these ideas. These are the key ideas used to design and implement the software.

APLUSIX also includes a Computer Algebra System aspect in that commands corresponding to the types of problems encountered by students in school algebra (e.g. expand or factor an expression, solve an equation) are available. The expectation is that the commands may help solve the exercises but they do not solve them directly. Parameterisation of the system allows these commands to be hidden if the teacher does not wish to permit the student this sort of support. The software is intended to be usable either in a classroom situation or by a student working independently. These are less important ideas about the design and implementation of the software.

Students build their own expressions; it doesn't mean that the implementation of the software relies on the constructivist theoretical framework. Powerful feedbacks are given to the students; it doesn't mean that the implementation of the software relies on the behaviourist theoretical framework.

The APLUSIX software has been designed and implemented taking into account various theoretical frameworks, and no one in particular. It has been designed and implemented, such that it could be used in most of the theoretical frameworks ((socio)constructivism, (neo)behaviourism, instrumental approach, theory of didactic situations, …).

2.2 Experimenting team 1: ITD *(Cerulli, M., Pedemonte B., Robotti, E.)*

2.2.1 A priori (with respect to the experiment) questions aiming at collecting information concerning the design of the experiment.

- General:

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

Research developed by ITD team is aimed at studying how new technologies, if inserted in suited contexts, can contribute to the construction of innovative environments that can enhance learning processes and can also change traditional approach to school teaching.
We consider Aplusix tool in this perspective.

A software tool can have an important role as an artifact mediating teaching and learning processes, but it is only one of the components of the whole environment. Not less important are the pedagogic activities in which the use of the tool is integrated, the way in which these activities evolves, the social interactions that take place, and the way in which the work is organised and embedded in the general structure of the school and of the educational institution.

The attention to the learning environment as a whole has brought us to progressively refer to theories that highlight the importance of studying the relations among individuals, mediating tools, and the social groups. ITD team have made reference, in particular, to Activity Theory that gives us a framework, namely terms and notions associated to those terms, that ITD team considered useful for analysing the learning environment where the ICT tools ITD team have developed are integrated. Referring to Activity Theory, ITD team interpret a learning environment as constituted by the enactment of a teaching and learning activity oriented to an educational object, involving students, teachers, and tools. A learning environment is not something that is assigned or planned in advance, but it is negotiated and built by participants in the enactment of a teaching and learning activity and evolves during its development.

Activity Theory has given us a reference for making explicit and for analysing the main components that shape technology mediated learning environments, and has suggested a way to examine how such components interrelate. More specifically, ITD team use the Cole and Engeström model of the complex relationships between elements in an activity (Cole and Engeström, 1991). This model allows us to develop a methodology for performing the analysis of the learning environments where ARI-LAB is integrated (see activity theory in annexes).
ITD team has defined a methodology for analysing learning environments mediated by technology through the identification of three main directions of analysis on the basis of the Cole and Engeström's model of activity. The three directions identified are the following:

- How the educational technology used can mediate new ways for the learner of accessing, representing, and interact with the concepts, procedures, and rules that are involved in the acquisition of a given mathematics knowledge which constitutes a learning object for a teaching and learning activity.
- How the educational technology used can contribute to the design and the enactment of didactical practices aimed at an evolution in the use of the rules related to the knowledge to be learnt and to the construction of appropriate ways and meanings for using them.

- How the educational technology used can contribute to mediate the assumption of new and old roles by participants in the didactical practice.

- <u>Analysis of Aplusix tool</u>

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

The main kind of feedback that we are going to exploit concerns the relationships between the statements/expressions inserted by the user in the system and the following statements/expressions produced by the user starting from them. In this case the software provides a 3 values feedback concerning the correctness of the statemente/expression the user is writing. The 3 values are: correct (the connecting lines are black); uncorrect (the connecting lines are red with a cross); unknown (the connecting lines are blue with a cross).
This kind of feedback may enable the student to accomplish a task and validate his/her solution of a problem with the aid of the computer, without the intervention of the teacher. Moreover, because this feedback is given constantly, at any moment of the interaction, we hypothesise that the user may be constantly stimulated to reflect on each single step. Moreover we believe that ad hoc designed activities with Aplusix may help the pupil to foster/develop his/her own control systems. For these reason we believe that this tool is suitable for supporting pupils with difficulties, however with some limits that we will discuss further on.
Before we discuss the limits we would like to observe that two kinds of "objects" can be inserted in the software: algebraic expressions and statemnents (e.g. those that include the symbols "=", "<", ">"). In the first case, the provided feedback basically informs constantly the user about the equivalence or not between the original expression and the expression that is being produced by the user. The second case instead, is a peculiar one, because the control on the "correctness", and the related feedback, is based on the truth values associated to the statements, in the sense that if the produced statement has the same truth values of the original statement, the step is indicated as correct, and vice versa is indicated as uncorrect if the truth values are not the same. As the truth values can be only "true" and "false", any "true" statement can be followed by any other "true" statement which may don't even be related to the original one; in the same way any "false" statement can be followed by any other "false" statement which may don't even be related to the original one. More over, the software does not provide any indication on the truth values of a single statement, this implies that in order to assess the "true"-ness of a statement one has to firstly introduce a statement that is surely true, and then introduce the considered statement and see if they hold the same truth values.

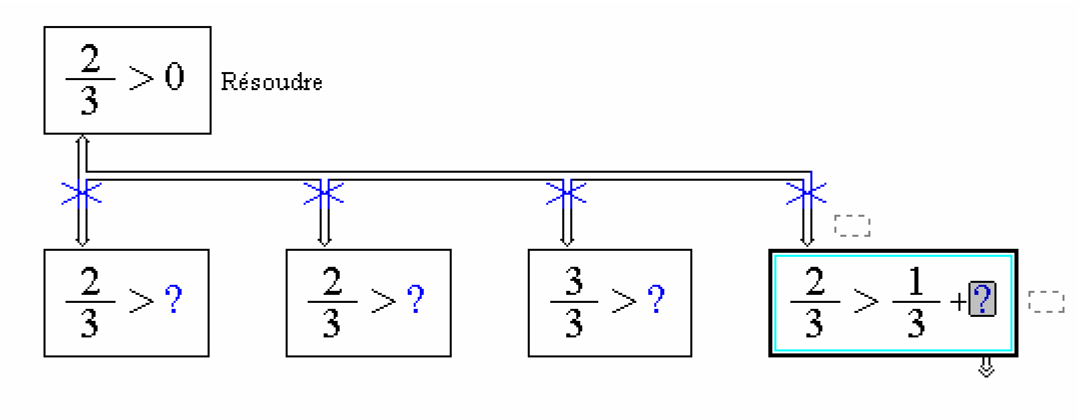


Fig. 1 Feedback provided concerning the truth values of statements

This issue has strong implications in the design of tasks to be proposed to pupils. For instance if one wants pupils to produce a fraction that is smaller then 2/3, in order to exploit the feedback provided by Aplusix, one has to previously introduce a true statement and propose the statement with the unknown fraction as a follow up, as show in Fig. 1.

The example shown in Fig. 1 introduces another important issue that we see as a limit of the tool, that is the fact that it doesn't allow to freely insert any kind of expression/statement, and that doesn't allow manipulations without previously indicating the typology of task, to be chosen among a very short list of prebuilt tasks: Calculer, Developpér, Factoriser, Résoudre. In particular we observe that each typology of task admits only some specific kinds of expressions or statements, which means that if an expression or statement is does not correspond to certain not well specified characteristics, it simply cannot be inserted. However the software provides textual feedback with hints concerning what kind of expressions/statements can be associated to each kind of task. We believe that it would be very useful to admit the insertion and manipulation of expressions and statements without the constrains of the built-in typologies of tasks. In fact this constrain is a strong one and affects any attempt to design new activities forcing researchers/teachers to overcome this specific obstacles with the help of their own creativity. In particular in the example of Fig. 1 the intended task to be proposed to pupils is "fill in the table substituting the question marks with fractions", in this case when the pupil tries to insert a fraction the feedback provided by Aplusix will help him/her validating the correctness of his/her solution; however, the only way to insert a statements like "2/3>0" (and then producing such a table with empty slots) was to firstly chose the task "resoudre" (which means "solve") which has nothing to do with the task actually proposed to pupils, thus the teacher will have to tell pupils to ignore the writing "resoudre" and will have to provide a written or spoken introduction to the task.

Among the functionalities that we plan to use for the experiment, we also plan to exploit:

- the possibility for the user to insert comments at each step of the manipulations
- the possibility to insert "open" statements/tasks like for instance the one shown in Fig. 1, where "empty" values are represented by question marks
- the possibility to record the actions performed by the user and to see them a posteriori, this may be exploited either for research issues, either to provide pupils a tool to analyse their own work or other pupils' work.

Finally we would like to observe that the strong limits on the kinds of expressions/statements that can be considered influenced strongly the choices of our experiment. In fact in the software it is not possible to insert polynomials with degree higher then 4 and it is not possible to insert fractions with letter at the denominators. This put, in our case limits on:

the idea of introducing algebra in terms of generalizing arithmetic.

The idea of generalizing the properties of fractions and powers

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

The key distance that we consider is based on the fact that in paper and pencil one has total freedom to write any kind of task, and to perform any kind of manipulation on algebraic objects, whilst in Aplusix there are strong constraints, especially on the tasks, the expressions and the statements that can be written/chose. On the other hand, Aplusix provided the feedback that we described above, that cannot be provided by the paper and pencil environment. As we previously suggested, this could be exploited as a mean to develop pupils systems of control, but according to our theoretical frameworks, this cannot be taken as granted; in particular we believe that specific activities should be designed in order to help the pupil becoming independent from the tool; in fact in the paper and pencil environment the control on correctness is under the responsibility of the pupil, whilst in Aplusix is under the responsibility of the tool. For this reason, we plan to exploit both, Aplusix and the paper and pencil environment.

Design of the Teaching Experiment

Describe briefly the key ideas of your experiment and then answer to the following questions

The key idea is that of exploiting the feedback provided by Aplusix as a means for supporting pupils in remedial activities concerning numerical fractions. In particular we hypothesize that the control systems embedded in Aplusix, under specific conditions, may be acquired by pupils reinforcing their self control systems concerning operations with fraction. We present a draft of our teaching experiment in an annex.

What are the precise aims of your experiment and the questions you want to focus on?

Specific educational goals

Reinforcing pupils operational skills with fractions

Reinforcing relationships concepts such as equivalence of fractions and ordering of fractions

Reinforcing pupils self control systems concerning the handling of fractions. The objective is that pupils become able to enact strategies to recognize errors, correct errors and avoid errors.

Specific research goals

Can, and how, Aplusix (with it's *control feature feedback*) be exploited in order to:

reinforce pupils operational skills with fractions

reinforce relationships concepts such as equivalence of fractions and ordering of fractions

help pupils in controlling and reflect on their actions and to develop strategies for overcoming mistakes.

reinforce pupils self control systems concerning the handling of fractions.

Is it possible to exploit the *trace feature* as follows:

letting the pupils use it as a means to analyse their work, commenting their productions step by step, finding and commenting mistakes.

as a starting point for setting up class discussions.

Individuate and define classes of educational activities that exploit Aplusix and that result to be effective in order to reach the above mentioned educational goals. As a consequence specific *didactical functionalities* for Aplusix will be defined.

What is the type of research that you follow (e.g. classroom based, case studies) and how is this related to the kind of your research focus;

The research is classroom based. In particular the remedial activities will involve all the class as a means to recall the contents of the previous school year. However hopefully each pupils will have chances to work on a computer on his/her own, because the aim is that each single pupil reinforces his/her own abilities and control systems. We also plan group activities, such as class discussions, where concepts will be shared.

Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?

For this experiment we are not concerned with the generation of new meanings, rather we focus on reinforcement and revision of meanings already studied by pupils in the previous school year. Moreover we focus on the construction and reinforcement of control systems. With this respect the characteristics of the activities and tasks that we plan to exploit are:

Tasks are open, in the sense that pupils are let free to explore and/or try different solutions. This is possible because of Aplusix's control feature, that takes the responsibility of controlling the correctness of solutions, letting pupils free to focus on the production and analysis of the solutions themselves.

The tasks are designed in order to favour the construction of relationships between concepts and aspects related to fractions. For instance in Fig. 1 the ordering of fractions is put in relationship with sum of fractions.

The tasks are designed according to non standard approaches to concepts related to fractions. For instance pupils may be asked to decompose a fraction into a sum of 2 or more fractions like in the example in Fig. 1.
How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?
- analysis of solutions produced by pupils with Aplusix and the related traces recorder with the *trace feature*.
- Field notes
- analysis of solutions produce by pupils with paper and pencil with particular focus on any possible reference to, or trace of, the use of Aplusix.
- analysis of class discussions with particular focus on any possible reference to, or trace of, the use of Aplusix.
- Eventual clinical interviews of pupils and/or teachers I focusing on aspects that we individuate as relevant during the experiment.

2.2.2 A posteriori (with respect to the experiment) questions aiming at individuating, collecting and gathering the results of the experiment.

The experiment was experimented in two classes A and B (age 11-12) of two Schools. The experiment were experimented in the school computers laboratory of each school. Such laboratories are equipped with PC.
The experiment has been experimented within the ordinary school time (no extra time was required).

The classes A involved in the teaching experiment was of 14 students grouped into 7 groups. The classes B involved in the teaching experiment was of 19 students grouped into 9 groups.

Students of the two classes participated to a session of two hours per week. The experiment covered a total of 10 hours. The experimentation began at the end of October and ended at the end of November.
Each students group had at his/her disposal a PC where Aplusix was installed. Two persons of ITD staff was always present during each session. Teacher was present too.

Is there any difference in the answers that you gave during the a priori analysis at the following questions?

The answers given a-posteriori try to take into account what has been observed during the experiment; in a sense the answers given a-priori are to be interpreted as “plans” and “hypothesis”, whilst the answers given a-posteriori are to be interpreted as observations of fact and verifications/confutation of hypothesis. Here we avoided to repeat things that are unchanged from a-priori answers (unless they weren’t particularly important), indicating references to the documents containing a-priori answers.
Our a-priori answers contained some hypothesis, here we are going to discuss two of them for which we obtained some clear results from the experiment.

Hypothesis 1 (Aplusix tasks constrain might affect its impact/effectiveness/easiness?): in our a-priori answers we observed that Aplusix has strong constrains on the kinds of “tasks” that it accepts, which are only for, non of which corresponding to the tasks we were planning to propose. This was overcome by telling the pupils to communicate to Aplusix that they were accomplishing one of the tasks he recognizes, but to accomplish instead the tasks we were proposing them orally. We were afraid that this could confuse pupils, but actually it turned out not to be a problem, as pupils simply

ignored Aplusix's request to specify a standard "task": it was taken as a start up procedure, not affecting the rest of the activity.
We had a similar problem with the fact that in order to have a correctness feedback for statements (within certain tree graphs) one has to previously insert a correct feedback[5]; in this case the ready-made statement was given as part of the starting conditions of the task, and again this issue didn't influence relevantly on pupils' accomplishment of the tasks.

Hypothesis 2 (constant correctness/equivalence feedback implies stimulus to reflect?): in our a-priori answers, considering the equivalence (or correctness) feedback of Aplusix, we hypothesised :

> *"because this feedback is given constantly, at any moment of the interaction, we hypothesize that the user may be constantly stimulated to reflect on each single step. Moreover we believe that ad hoc designed activities with Aplusix may help the pupil to foster/develop his/her own control systems. For these reason we believe that this tool is suitable for supporting pupils with difficulties […]".*

With this respect the experiment as a whole (including software, educational goals and modalities of employing Aplusix), suggests that actually Aplusix can be of help in fostering pupils' control systems, and can be suitable for supporting pupils with difficulties. However, the experiment also showed that this is possible only under certain conditions. In fact the first part of our hypothesis resulted to be false, in the sense that *the considered feedback* ***did not*** *stimulate constantly pupils to reflect on each single step*; on the contrary, it resulted to be an incentive for pupils to "random alike" or "trial and error" strategies, simply because it is easy to try out many solutions, and sooner or later one will guess the right one. Pupils' reflections on what they were doing was instead obtained by means of specific tasks requiring them to formulate and discuss the strategies they employed for solving the proposed problems; in this way, even those who solved the problem "by chance" or by "trial and error" had to develop and validate a strategy; without such a request, many students wouldn't have reflected on their solutions at all.

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

The main kind of feedback that we was used concerns the relationships between the statements/expressions inserted by the user in the system and the following statements/expressions produced by the user starting from them. In this case the software provides a 3 values feedback concerning the correctness of the statement/expression the user is writing. The 3 values are: correct (the connecting lines are black); incorrect (the connecting lines are red with a cross); unknown (the connecting lines are blue with a cross).
This kind of feedback helped students to accomplish the proposed tasks and to validate the solutions developed in paper and pencil, without the intervention of the teacher.
This kind of feedback may enable the student to accomplish a task and validate his/her solution of a problem with the aid of the computer, without the intervention of the teacher.
In particular in our experiment this feedback has been interpreted also in terms of a feedback concerning the equivalence of expressions. If we interpret to consecutive expressions in Aplusix, instead of considering them as one consequence of the other, we considered them as two expressions to be compared; thus the feedback provided by Aplusix can be interpreted also as follows: equivalent expressions[6] (the connecting lines are black); non-equivalent expressions (the connecting lines are red with a cross); unknown (the connecting lines are blue with a cross).

[5] (see TASK 4: http://vds.univ-lemans.fr/groups/TELMA/forum/main/349273376748)
[6] It works either with expressions or statements including the symbols <, > and =, as we explained in our a-priori answer to this questions (see TASK 4: http://vds.univ-lemans.fr/groups/TELMA/forum/main/349273376748)

We observed that such equivalence feedback was effective in order let pupils test, try out or explore solutions for the proposed problems. However such solutions may fail to be abstracted into strategies, and with this respect it turned out to be very important the request for pupils to verbalize their strategies inserting comments in Aplusix (we admitted also paper and pencil). However, it remained the issues of validating the developed strategies, and such validation could be not offered by the tool, thus needed the teacher's intervention, this leads us to the last 2 kinds of effective activities:

1. The teacher[7] proposes pupils to apply the developed strategies for solving new problems; some times the new problems are ad hoc designed for the specific strategies considered.
2. The teacher orchestrates a class discussion where the different strategies are discussed and thus validated, or not validated, by the class.

Pupils working in pairs (or small groups) fostered the elaboration of strategies that had to be shared by the members of the pair.
The possibility of the teacher to interact with pairs while they were involved in the activities resulted to be crucial for stimulating the discussions among members of pairs, for stimulating verbalization of strategies, and for discussing the validation of developed strategies. Aplusix, in fact, is not able to validate the whole strategy of resolution but rather the result of the resolution.
The possibility for the teacher to orchestrate class discussions (which we could instantiate only in class B) concerning the strategies resulted to be effective for validating and institutionalizing the found strategies.

In some cases the solutions of problems were validated by pupils in terms of confrontations with other groups, or in terms of confrontation with paper and pencil computations.

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

Distance in terms of freedom
In paper and pencil pupils are more free to write whatever they want, and to use computations procedures they are confident with. On Aplusix there are constrains on what can be written and how it can be written (for instance one cannot decide the "shape" and dimensions of an expression); however, on the other hand with Aplusix it is possible to easily delete a value or expression and substitute it with a new one, which makes easier to try out several solutions in a little amount of time

Distance in terms of validation
In paper and pencil there is no built in validation system, pupils can validate their solutions only by executing new computation, or by reflecting on their solutions and discussing them, or by asking the teacher to validate them.
Aplusix, instead, with respect to the proposed tasks, gave validations of the numerical solutions found by pupils (but did not provide validations of the employed strategies, for which teacher's interventions could not be avoided).

Distance in terms tasks
Aplusix, when a session is opened, admits only 4 kinds of tasks, none of which corresponds to the tasks we proposed, this obliged us to chose one of the given tasks in order to enter the system, and

[7] In the experiment these interventions were done either by the teachers or by the researchers acting like teachers.

ask the pupils to ignore the task indicated by Aplusix and accomplish the task provided by as orally or in paper and pencil. However such distance didn't seem to affect the experiment.

Distance in terms of representations of objects
It is possible to represent "unfilled" expressions both in paper and pencil and in Aplusix, but on the computer it is possible at any time to fill the empty boxes and if needed to delete them or substitute the inserted number.
In Aplusix it is possible to build dynamically the structure of an expression (even made only of empty boxes), while in paper and pencil one has to "design" the structure of the expression before writing it, as, once written, it is static and cannot be changed. This also implies that in Aplusix it is possible to change at any moment the structure of an expression (or statement), which is not possible in paper and pencil.
The structure of an expression in Aplusix is built by inserting one operations after another (in the expression), which results in an automatic change of the sizes of boxes, of fraction lines, brackets, etc.; such a procedure is very hard in paper and pencil because automatic resize is not available, and one needs to plan the complete structure of the expression (with sizes) before writing it.
Such procedure also obliges to clarify ambiguities related to precedence issues and to commutative properties, as such ambiguities can lead to build an expression which is different from the one the user wants to write.

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

The evaluation data which we considered are the following:

- **Evaluation tables** (to evaluate each group of student in each session). Tables concerns: facility of use, impact and efficacy. Evaluation tables are specific for each session. In total we have 5 evaluation tables for each class, one for each session.
- **Video recorder** (to evaluate a particular group for each session)
- **A recorder** (to evaluate two students groups for each session)

In the two classes evaluation tables had been completed for each students group and for each session. In this way, at the end of the experiment, it has been possible to evaluate the learning evolution of each students group comparing the results of the tables. Moreover, has been possible to make an analysis to evaluate the general learning evolution of the two classes.

Evaluation tables have been constructed on the basis of the a-priori analysis of the tasks of each session with the aim to provide answers to the questions of guidelines. The first column of the table describes the contents corresponding to the mathematical educational goal of the task. The second column highlights the characteristics/features of the software that were involved in the pupils' accomplishment of the task. The tables were filled (by researchers) during the sessions and used as means for evaluating: if students were able to use the characteristics of the tool; if their competences evolved during the experiment; if and how learning of the topics considered had been supported by the tool.

These characteristics have been evaluated considering three parameters: easy of use, impact and effectiveness.

With the parameter "easy of use" we evaluate the easiness/difficulty which user could have in the interaction with a particular characteristic of the software. For example, we evaluate the difficulty to use the insertion of expressions, or the use of tree graphs.

With the parameter "impact" we evaluate how pupils react to a given characteristic of the software at the moment of its use. For example, we evaluated the impact of the red equal of Aplusix (which corresponds to an error made by the user) considering for instance: if it is clear or if it is sufficient to help student to change strategy.

With the parameter "effectiveness" we evaluate how much the characteristic of the software is useful to achieve the educational goals described in the first column. For example, to evaluate the effectiveness of the feedback feature we evaluate how much it appears to be good in terms of helping pupils overcoming and understanding errors.

For each parameter 4 values have been assigned:
1 = a fail marks
2 = a pass mark
3 = a good mark
4 = an excellent mark

Below is an example of evaluation table referred to the fourth session: ordering, multiplication and division of fractions.

	Characteristics and properties	Easiness	Impact	Effectiveness
Insertion of expressions	Virtual keyboard n =	1 2 3 4	1 2 3 4	1 2 3 4
	Standard keyboard n =	1 2 3 4	1 2 3 4	1 2 3 4
Manipulation	Equivalence Feedback	1 2 3 4	1 2 3 4	1 2 3 4
	Multiplication and division of fractions	1 2 3 4	1 2 3 4	1 2 3 4
Using tree graphs	Ordering of fractions	1 2 3 4	1 2 3 4	1 2 3 4
	Multiplication and division of fractions	1 2 3 4	1 2 3 4	1 2 3 4
Using other means of representation	Paper and pencil n =			
Comments				

The analysis of the collected data has been conducted in terms of:

- comparison with the a-priori analysis of each task
- comparison between the experiences conducted in the two different classes; in fact the age were the same, but the teacher, and modalities of conducting sessions, were different

- discussions with the teachers.
- Discussions among researchers.

Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?

In the two analysed classes (class A and class B) the modes of representation not provided by the tool but used by the students are:

- Paper and pencil representations
- Calculator representations

Paper and pencil

The groups of pupils of Class A used paper and pencil representations regularly, whilst the groups of pupils of class B used it now and then.
Paper and pencil representations appeared to have 4 specific objectives:

- Support for solving the proposed problems:
  - in class A we observed 3 groups over 7 solving the problems in paper and pencil, and then transcribing them in Aplusix mainly because they were required to work with Aplusix, as if (we might hypothesize) they were doing it because of the class' didactical contract.
  - In class B, 2 groups over 9, in each session, used to execute computations in paper and pencil, and to exploit Aplusix for validating the correctness of their computations.

- Support for computations in order to avoid mental computations[8]:
  - In class A 7 groups over 7 always used paper and pencil to execute computations manually.
  - In class B, 3 groups over 9 sometimes used paper and pencil for the same reason (in particular when computations resulted to be difficult). In this class Aplusix was then used to check the correctness of the computations executed in paper and pencil.

- Support for writing extended comments:
  - In class A, 3 groups[9] over 7 used paper and pencil for writing the comments that were too big to be inserted in the Aplusix's dedicated comment cells (they could contain a limited number of characters).

- Support for taking notes:
  - In class B, all the groups sometimes used paper and pencil for taking notes derived from class discussions or from interactions (and institutionalizations) guided by the teachers and the researchers.

Calculator

In class A one group used a hand calculator as a support for executing computations.

[8] Notice that Aplusix doesn't execute computations, unless a specific command is not used, which was not the case of our experiment, as in class A pupils simply didn't know that such command existed, and in class B it was inhibited by didactical contract.

[9] The same groups as above

In class B, 4 groups over 9, used the Windows built in calculator as a support for executing computations, and one group uses a hand calculator for the same purposes. After 2 sessions some pupils realize that Aplusix has a computation command, but the teacher and researchers inhibit the use of such command and of calculators. Nevertheless they go back to use hand calculators and windows calculators.

In your opinion, in which ways do your theoretical choices have influenced:

a. the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employment including the configuration of the software)?

b. the conception of the experiment?

c. the choices of the data and their analysis?

d. the results you obtain and the conclusions you draw from these?

We are going to answer the first two questions in a narrative form, describing how our experiment was originated.

Preliminary constrains

Because the joint experiment was planned by the TELMA group as a whole, it had to take into account the following constrains:

- The joint experiment constrain concerning the tool to be used
- TELMA constrains indicating the age range and the focus on fractions and early algebra the researchers' will: we had previously been informed by the Siena team about an experiment that they developed and in which Aplusix resulted to be an effective tool for recovery activities

The educational goal

First of all we started from a generic educational goal concerning the recovery of aspects concerning fractions and powers with 7^{th} grade pupils. This first choice was taken in agreement with the teachers that were going to instantiate the experiment, and was due basically to the following issues:

- the experiment was to be developed at the beginning of a school year, thus a period which is suitable for recovery of what was done in the previous year
- the teachers' will;

Given this general aim, in agreement with the teacher, we decide not to focus on the mere execution of computational techniques, but on understanding of such techniques, and of the properties of powers and fractions. As it was supposed to be a recovery activity, this assumed that somehow/somewhere previous standard activities, based on acquisition of computational techniques, had produced some lack of understanding, or however, "something to recover". For this reason we decided to avoid standard computational activities and to look for other kinds of activities to be addressed to Aplusix. Notice that Aplusix has been on purpose designed to support standard activities, thus we had to look for alternative ways to employ it.

The analysis of the tool Aplusix

On the basis of the above discussion, we began to explore Aplusix, and try out activities, looking for the possibilities to set up open ended tasks; the reasons for this choice are to be individuated in:

- the particular educational goal, as we wanted to focus on comprehension rather then in execution of computational techniques and of properties of fractions and powers.
- A socio-constructivist hypothesis concerning pupils active involvement in knowledge construction and negotiation of meanings through social interaction

- Open ended activities would have helped the production of different solutions to the problems, giving a chance to instantiate discussion among groups and class discussions. This being of course related to the above socio-constructivist hypothesis.

The analysis of the software was conducted by the researchers by exploring its possibilities trying out what it was possible to do with it.
First of all it was noticed that Aplusix doesn't allow to accomplish any task without previously indicating what kind of task one wants to accomplish, choosing among 4 kinds of tasks (compute, solve, factorise, develop). Such pre-defined kinds tasks correspond to standard computational tasks, thus they correspond exactly to the tasks that we wanted to avoid in the experiment; moreover, the software doesn't allowing defining new tasks. For this reason it was decided that in the experiment we would have presented the tasks orally or in written forms, outside Aplusix, and asking the pupils to ignore the specification of task required by Aplusix. In this way it was possible to present to the pupils any kind of task and to skip the constrains provided by the tool.

One of the first things that we tried it was to explore the possibilities of manipulating numerical and literal expressions in order to work toward generalization of rules for computing with fractions and with powers. This implied that we needed to be able to write and manipulate expressions where letters could be situated in any place and in any kind of expression with powers and fractions. For instance, in order to express and cope with the definition of power, we wanted to be able to write expressions such as "a^b" and "a^6", which weren't allowed by Aplusix; the same happened also with fractional expressions with letters as denominator. This put very strong limits to the possibilities of manipulating expressions with a general aim of abstracting computational rules. For this reason we abandoned the idea of interpreting generalization in terms of expressing algebraic statements with letters. Moreover, we abandoned the idea of working with powers because only power up to the fourth grade were allowed in Aplusix, and this we believed was a too much strong constrain because it would leave very narrow space to explore the rules for computing with powers, as only the exponents 1,2,3 and 4 were admitted. Once we restricted to fraction, we still had strong constrains on the possibilities of using letters with fractions, thus we decided to abandon the idea of working with literal expressions for generalising numerical expressions and for generalising properties of fractions and of the operation with fractions.

What came next is probably the result of the discussion on the limits and features of Aplusix and of a bunch of other reasons. One central feature of Aplusix is a feedback on the correctness of the steps produced by users while accomplishing one of the pre-defined kinds tasks. We realised that such feedback could actually be interpreted as a feedback concerning the equivalence relationship between the expressions written on different lines: instead of interpreting two expressions as one the consequence of the other via a computational step, it was possible to interpret them as two expressions to be compared, and the feedback provided by Aplusix would state if the two expressions were equivalent or not. Notice that this is merely a matter of giving to a feature of a tool a meaning rather then another one. Among the reasons for which we realised that this shift was possible, and we decided to adopt it, we can consider:

- the educational goal, as we had decided not to focus on the execution and correctness of computation techniques, thus for us it was not important to interpret the mentioned feedback as a feedback concerning correctness/uncorrectness of computational steps;
- one of the researchers (Cerulli 2004) has a strong experience in experiments on the teaching of algebra were the concept of equivalence is central and which are based on the idea of comparing expressions; it may be the case that he was naturally looking toward that direction.

- Questioning/exploring/producing equivalence relationships seemed to be a class of tasks that were likely to be proposed in an open ended form, which was the main typology of activity that we were looking for (see above discussion).

At this point we concentrated on the possibilities offered by Aplusix to express/explore/produce equivalence statements by means of open ended activities, and we individuated some more key features (see also Fig. 2):

- the software allows to leave "unfilled" some boxes, thus it permits to construct expressions with placeholders (represented by a square with a question mark inside), thus allowing to propose to pupils tasks where they are asked to fill the place holders.
- The software allows the construction of trees of expressions (which could be "unfilled" too) showing the equivalence relationships among them
- When the root of a tree is a statement, instead of an expression, if the leafs of the tree are statements too (these too can be left "unfilled"), then the "equivalence" feedback can be interpreted in terms of equivalence of truth values of the statements; this allows to enlarge the class of tasks to tasks including the production/comparison/exploration of ordering statements between fractions.

Each of these features was seen by the researches as a chance for setting up open ended activities. We also individuated another particular feature, which we believed to be important in terms of shifting pupils attention from providing correct solutions, to elaborating/formulating strategies for solving the proposed problems; it is the case of the possibility to insert comments to each line.

Typologies of activities and tasks

Following the tool analysis, we restricted our educational goal to the recovery of key aspects concerning fractions, such as the equivalence of fractions, the ordering of fractions, and the four operations with fraction. We then designed a set of activities which were partially negotiated with the experimenting teachers, in what follows we describe the rational behind the design of the activities and the setting of the experiment.

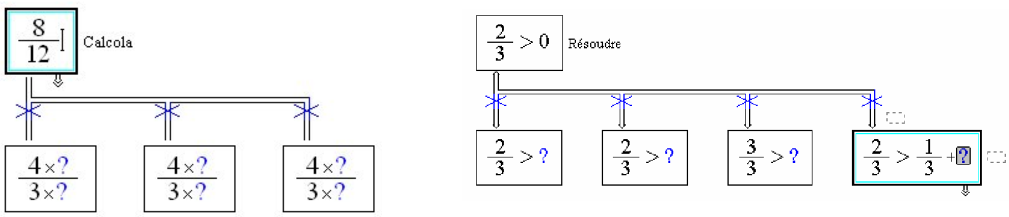


Fig. 2 Examples of threes with empty placeholders; the pupils were required to fill them. On the left the tasks concerns equivalence of fractions and multiplications of fractions. The example on the right concerns the ordering of fractions. On the left the feedback can be interpreted in terms of equivalence of fractions; on the right the feedback can be interpreted in terms of equivalence of statements.

First of all we adopted mainly open ended/explorative tasks of the kind "fill the boxes" and "write the strategy you followed", chosen according to hour constructivist hypothesis, as we previously stated. However, we may observe that each task required pupils also to write the strategy followed to fill the boxes; this was required because of the following reasons:

- a theoretical assumption that what is learnt by pupils through open ended activities, may not be necessarily coherent with the teachers educational goal.
- Our focus was on strategies which we assumed to be a means for abstracting the properties of fractions, the educational goal didn't aim at the production of correct solutions, but to the production of strategies to be abstracted: the solutions to the "fill the boxes" tasks could not

necessarily be abstracted into strategies, considered also the fact that the nature of the tool favoured also "random" strategies.

- A theoretical assumption that learning happens also through semiotic processes and through social processes of knowledge sharing; we believed that for such processes to occur, focusing on strategies, it was a necessary condition that the strategies themselves were formulated into written or oral text.

To accomplish these tasks pupils were subdivided into groups of at least 2 because of our socio-constructivist assumption that knowledge building happens also through a process of social interaction.

We also adopted some closed ended activity, but this was due mainly to:

- to try out some standard ways of using Aplusix
- to try out if the software could reinforce computation skills directly
- because of cultural scholastic bias (in italian we would say "per dovere")

Also in the case of closed ended tasks pupils were asked to accomplish a "write your strategy" task, for the same reasons explained above.

It is important to observe that on the one hand Aplusix doesn't provide any validation feedback concerning strategies employed/developed by pupils, while on the other hand, by theoretical hypothesis, we assumed that pupils' strategies could be or could not be coherent with the teachers educational goal. In other words we believed that there was a need for the teacher to put into discussion such strategies and guide their evolution by means of social interaction and communicative strategies (Mariotti....). For these reasons we planned that at certain points of the experimental sessions the teachers would intervene to collect pupils answers, strategies and observations and would set up a class discussion; for the same reasons, we allowed, and encouraged, the teachers' (and researches acting as teachers) continuous interactions with the group of pupils, in particular proposing them tasks of the kind "try out your strategy". This was a way to validate or put into discussion the strategies developed by each single group. Of course these choices are due also to the peculiar educational objective.

Finally, we planned in advance 5 sessions (Introduction to Aplusix, Equivalent fractions and expressions with fractions, Ordering of fractions, Product ad division among fractios, Addition and subtraction among fractions), together with a set tasks to be used the sessions. However, at the end session we analysed roughly the session with the teacher (based on field observations), and planned the main lines for the following session. Then the following session was designed using, and adjusting, the tasks that we had previously defined, and eventually including new, ad hoc designed tasks. Thus, the experiment was refined in itinere, and behind each specific session there were a discussion and a set of decisions based on field observations. A detailed analysis of how these decisions were taken could be possible after we have analysed in detail the data of the experiment.

Setting and interaction

The experiment was developed in two classes in Genoa; 40 students altogether, grade 7. In each class the experiment lasted 5 weeks: a session of 2 hours for each week. Students worked in their school’s computer laboratory with the teachers. Two researchers were always present during the experiment either to observe, to solve technocal problems, and to act as teachers too (helping the official teachers).

Pupils were subdivided into groups of at least 2 because of our socio-constructivist assumption that knowledge building happens also through a process of social interaction.

The tasks were given orally or in the form of written text on paper:

- tasks such as the "try out your strategy", or other tasks aiming at questioning/highlight/validate strategies were given orally; this was due to the fact that it was not possible to foresee in advance when and how it would be necessary to propose such tasks. They are basically fruits of the interactions teachers/pupils during the development of the activities.
- Tasks of the kind "fill the boxes" were planned in advance, and thus were given in the form of written text on paper; the reason why they weren't given in the form of some text, or whatever else, within the software Aplusix, it was that Aplusix simply doesn't allow tasks that are different from its 4 pre-defined tasks that we previously introduced.
- Class discussions were of course set up orally in some cases by the teacher, in other by the researchers; the second case happened when the teacher didn't feel confident enough with conducting class discussions.

During the activities the teachers interacted with pairs (groups) because in this way they could be stimulated: discussions within pairs, verbalization of strategies, discussion on the validation of developed strategies. Moreover, the teachers orchestrated some class discussions as a means for validating and institutionalize the strategies developed by pupils. Such behaviours of the teachers was motivated by the assumption that pupils learning in the participation to practical activities may not necessarily lead to the teachers' educational goals, thus there is a need of the intervention of the teacher who can guide the evolution of pupils learning through special communication strategies. However, we would like to observe that, behind the generic indications given by our theoretical assumptions, it may be the case that during the experiment some actions were due to improvisation or to contextual events.

For what concerns the "write your strategy" tasks, pupils were left free to write them on paper, or to write them in the form of comments in Aplusix, however the second options sometimes resulted tricky (and with some limitations on the numbers of characters), thus pupils often opted for paper and pencil.

Pupils were left free to executes any computation they wanted on paper and pencil (and in fact they did it massively) because we meant to employ Aplusix mainly as a means for stimulating pupils' production of a variety of solutions and for validating their solution, and as a support; but we didn't mean to employ Aplusix as the only tool to be used.
Moreover, we firstly inhibited pupils' use of Aplusix computational facilities, or other similar tools (ex the computer's calculator, or hand calculators), but during the experiment we decided to let the pupils free to use whatever they want. The reason why we firstly wanted to inhibit calculation facilities is that we wanted pupils to do computations on their owns; in fact we thought that this would have been a meaningful and significant practice for our educational goals and did not want to de-charge pupils with the responsibility of executing computation. However as soon as the experiment began, we realized that the tasks of "filling the boxes" and "write your strategy" were already heavy, significant, and meaningful enough for the pupils, and Aplusix was working only as a support, and didn't do the work instead of the pupils. The focus on the tasks was not on executing computations, thus it was actually not important if the computations were executed by means of computation facilities; computations were only one of the tools pupils had as means for solving the open ended tasks of filling the boxes and formulating their strategies. It is curious to observe that, even if we planned the kinds of tasks before the beginning of the experiment, we needed to see pupils in action before realizing that computation really were not the central element of the tasks, and thus it was fair enough to let pupils free to use computation tools. Probably, while designing the

experiment, we were under the influence of cultural scholastic constrains, and could not foresee how effective it was the switch of focus from computation to production and comparison of expressions/fractions.

Collected data and analysis of the experiment

We still have to conduct a deep analysis of the data. For the moment we have only done a surface analysis based on our field observations during the experiment; such an analysis was a means for adjusting the experiment in itinere.

For the experiment we planned to collect audio and video recordings and pupils protocols (written on paper, and Aplusix files). This was because, following Vigotskian hypothesis, we intended to observe the teaching/learning process, included class discussions, and discussion among groups of pupils (also when interacting with the teacher); we were not interested in a quantitative analysis of the results, thus we didn't set up a pre test and a post test. Our analysis will be focused on the process and will aim at finding out how the didactical functionalities of Aplusix that we defined affected the teaching/learning process. We recall that our **research aim** was *to develop and testify new didactical functionalities of Aplusix in order to use it as a means for supporting pupils in revisiting and consolidating some mathematical concepts already learned*.

We also developed, for each session, a table of issues to be observed, an "evaluation scheme" for Aplusix. The scheme was different for each session because it depended on the educational objective of the session, and on the involved features of the tool. The schema focused on effectiveness with respect to the educational goal, on easy of use, and on impact.

Tool's features with respect to the tasks		**Easy of use**	**Impact**	**Effectiveness**
Inserting expressions	Virtuale keyboard n =	1 2 3 4	1 2 3 4	1 2 3 4
	Reale keyboard n =	1 2 3 4	1 2 3 4	1 2 3 4
Manipulation	Equivalence Feedback	1 2 3 4	1 2 3 4	1 2 3 4
	Expressions with fractions	1 2 3 4	1 2 3 4	1 2 3 4
Threes with empty placeholders	Equivalent Fractions	1 2 3 4	1 2 3 4	1 2 3 4
	Ordering of fractions	1 2 3 4	1 2 3 4	1 2 3 4
Use of other representation systems	Carta e penna n =			
Comments				

Table 3. A researcher, during each experimental session, filled a form like this for each pair (or group) of pupils. Easy of use, effectiveness, and impacts are classified from bad/low/negative (1) to good/high/positive (4). This form belongs to the session concerning "Ordering of Fractions". The upper part of the table (above the black line) is used to evaluate the tools' features with respect to the addressed tasks. In particular we find evaluation concerning the input of expressions, the manipulation of expressions, and the threes with empty placeholders. In the bottom parte of the table, the "use of other representation systems" item is due to one of the questions of the joint experiment guidelines.

We decided to collect this kind of data because we needed to do a first evaluation of the features of the tool that we individuated, and employed, with respect to the didactical functionalities that we defined. This methodology of evaluation of the tool is derived from a similar methodology that was used by Robotti and Pedemonte in a previous project (ITALES - Innovative Teaching And Learning Environment for School, IST-2000-26356).

Didactial functionalities

All the above discussions provide motivations for the choices (at least for a set of them!) that underlie the didactical functionalities that we defined. Below we are going to resume how the main didactical functionality that we used can be characterised.

Characteristics of the tool

- The software provided a feedback which could be interpreted in terms of equivalence or not (or unknown) of expressions;
- the software allows to leave "unfilled" some boxes, thus it permits to construct expressions with placeholders (represented by a square with a question mark inside), thus allowing to propose to pupils tasks where they are asked to fill the place holders.
- The software allows the construction of trees of expressions (which could be "unfilled" too) showing the equivalence relationships among them
- When the root of a tree is a statement, instead of an expression, if the leafs of the tree are statements too (these too can be left "unfilled"), then the "equivalence" feedback can be interpreted in terms of equivalence of truth values of the statements.

Educational goal

Recovery and consolidation of key concepts related to fractions such as equivalence, operations with fractions, and ordering of fractions.

Modalities of employment of the tool

This includes some details on the setting, the typologies of activities, and a specific general educational strategy[10]:

- **Setting:** pupils work at least in pairs; the teacher interacts with the pairs (or groups) during the activities in order to question/validate their strategies and to stimulate/support their production; the teacher orchestrates class discussions for the same reasons and to institutionalize/socialize findings[11].
- **Typologies of activities**: open ended activities, such as “fill the boxes”[12], that exploit the validation/equivalence feedback of Aplusix; verbalization activities such as “write you strategy”; “try out your strategy” or other tasks provided ad hoc by the teacher to question/validate pairs’ strategies; class discussions highlighting and discussing the emerged strategies.
- **General educational strategy**: to enable pupils to explore open ended problems and to try out solutions to be verbalized, validated and institutionalized.

Obtained results and drawn conclusions

This will be answered after a deeper analysis of the data.

[10] A detailed discussion of the reasons why we chose all these elements are to be found in the previous parts of the document.

[11] This resulted to be less easy to be implemented and to be effective if compared to the interaction teacher/pairs.

[12] See Fig. 2 for examples.

2.3 Experimenting team 2: UNISI *(Maffei, L., Maracci, M.)*

2.3.1 A priori (with respect to the experiment) questions aiming at collecting information concerning the design of the experiment.

- General:

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

We adopt the Vygotskian theory in order to study how the control offered by Aplusix can influence the behavior of the students towards errors and impasse. According to this theory, we formulate the hypothesis that the feedbacks provided in the microworld determines a change in the attitude towards errors and impasse.

- Analysis of Aplusix tool

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

In the *training mode*, two fundamental feedbacks are provided: the correctness of the calculation and the correct end of the exercise. In the *test mode*, no feedback is provided. Aplusix records the students' actions, and the command Replay system allows both the student and the teacher to observe the student work off-line, step by step. As a consequence, in the *training mode* the solutions are validated by the tool step by step; in the *test mode*, the tool allows the student to make a self-correction (in this revision, the system indicates the errors like in the training mode and pupils are asked to correct them).

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

The objects are represented in the same way as in paper and pencil, but to manipulate them is required to respect one constraint: the equivalence between expressions. Infact, the feedback provided by the software is based on the control of the equivalence between two consequent step. In addition, the use of the independent line is useful as a means of control and as support to perform the transformations required.

Design of the Teaching Experiment
Describe briefly the key ideas of your experiment and then answer to the following questions

We intend to use Aplusix as a diagnostic tool in a twofold sense. Infact, the microworld allows teachers (researchers), to observe students' difficulties, thanks to the command Replay system, but a self-diagnosis is also possible. Regarding to this second aspect, specific hypotheses at a metacognitive level have been formulated in terms of self confidence, self control and consciousness of the students about their own difficulties. The research project is conducted with the collaboration of a group of teachers, among them the experimenting teachers, in charge of realising in the classroom the sequences of activities designed by the project. The interaction between teachers and researchers will be constant and active in any phase of the project. Before being presented in the classroom, educational activities will be discussed by teachers and researchers; similarly, the material produced by pupils will be analysed by the whole research group.

What are the precise aims of your experiment and the questions you want to focus on?
Two are the aims of our experimentation: by one hand, the didactical problem aims to help students to become conscious of their difficulties in order to find, thanks to the well-structured environment, strategies to overcome these; by the other hand, the research problem investigates the new relationship, between the machine and the student, which should play a crucial role in reaching a self-consciousness of the encountered difficulties.
What is the type of research that you follow (e.g. classroom based, case srudies) and how is this related to the kind of your research focus;
Our study involves two classes of the 9th grade: one experimental class and one control class. The research starts from detecting all the difficulties encountered by the whole class, and then, tries to study how each pupil (with his specific difficulties) may take an advantage from the interaction with the microworld.
Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?
We think that Aplusix can be used not at the comprehension level, but a metacognitive level, as we said before. It can be an useful tool if used in order to generate cosciousness of what a pupil have learnt and what he haven't yet learnt.
How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?
A researcher is present in the classroom in order to control and collect the experimental data;
After each session students are requested to report on what they think they have learnt and comment on the use of Aplusix;
The researcher observes the student work off-line, step by step;
The analysis of the collected data is made with the collaboration of the whole research group (teachers and researchers).

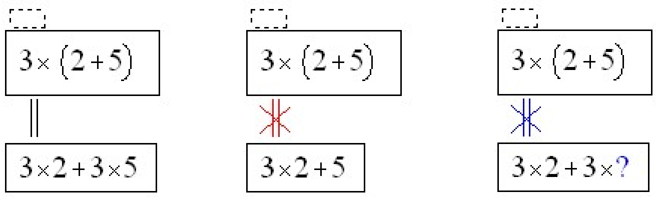
2.3.2 A posteriori (with respect to the experiment) questions aiming at individuating, collecting and gathering the results of the experiment.

Is there any difference in the answers that you gave during the a priori analysis at the following questions?
Generally speaking the answers given to these a-posteriori questions are more detailed but consistent with the anserws given a-priori. The most relevant difference concerns the analysis of the distance between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil.(question 3 in this document). In the present document, partly as a consequence of a first analysis of the collected data, more differences, between the objects manipulation within the two environments, are stressed.

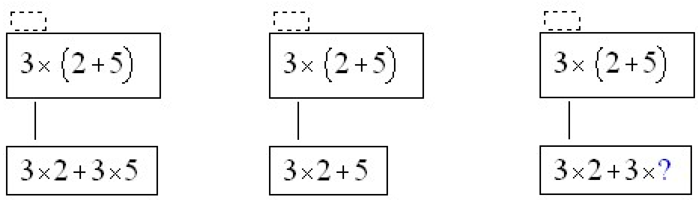
What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?
Within the paper and pencil environment:
As for the initial and final test, no immediate feedbacks were provided to pupils, and their solutions were validated by their teachers.
On the contrary as for the pre-test, at the end of the test itself (one hour) pupils were given a text with the “results” (depending on the task, the result of a numerical expression or the largest number out a set of given numbers...) of the single tasks and were asked to review and possibly modify their own solutions. The ultimate validation rested on the teachers.

Within the tool environment:
The tool was set up to function in two different ways depending on the kind of tasks given: “with control” mode and “without control” mode.

In the former mode, the tool provided immediate feedbacks on the equivalence of two successive expressions or relations. In detail, the tool provides the following 3 feedbacks: if two consecutive expressions or relations are equivalent the lines connecting them are black; if they are not equivalent, the connecting lines are red and crossed; and if one of the expressions or equations is not well formed (in a mathematical sense) the connecting lines are blue and crossed. Moreover the tool can be set up in such a way that users can not ignore these feedbacks, that is users can be allowed to write down a third expression only if the two previous ones (in the same tree branch) are equivalent.



Whereas in the latter mode, no immediate feedbacks were provided concerning the equivalence of two consecutive expressions or relations. In this case, two consecutive expressions or relations are connected with a single black line.



When the tool was set up as just specified, pupils were asked to review and possibly modify their solutions using the “self correction” environment , which allows to see one's solution step by step and modify it. Within this environment the tool functions in “with control” mode.
In both modes of functioning the tool points out when an expression is not mathematically well formed (e.g. a question mark appear if an argument of an operator is missed).

The validation of pupils’ solutions rested on the tool both in the ‘with control’ mode and ‘without control’ mode.

Finally, when working within the tool one can open a work space independent from the task work space named the detached step. The environment – the detached step – may be set up both with the control feedbacks and without them.

What is the “distance” between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?
The tasks proposed to pupils during the experimentation involved manipulation of numerical expressions and ordering of rational numbers (integers, fractions and decimals). The signifiers provided by the tool are completely consistent with those used in paper and pencil.

Some differences of different kind emerge when considering the means of manipulating the signifiers.
As for the structure of the worksheet:

the tool providing a not linear edition, allows pupils to substitute a specific sub-expression with another sub-expression; on the contrary in paper and pencil pupils have to re-write all the expression even if they want to change only some terms.
paper and pencil differs from the tool environment for the possible spatial organization of expressions and numbers (for instance in paper and pencil numbers can be put in column to perform the usual multiplication algorithm)
As for direct manipulation of expressions
the tool points out when an expression is not mathematically well formed (e.g. a question mark appear if an argument of an operator is missed),
the tool does not allow to select part of an expression which is not a well-formed expression itself (i.e. a sub-expression),
when deleting an argument of a sum (selecting and pressing backspace key), the sum operator itself is deleted.

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?
The main means to capture the role of the tool is given by the analysis of the tool log files. The tool is endowed with a user-friendly log file viewer (Replay System) which allows to follow pupils' solutions step by step (including the different possible trials and errors that pupils made and erase from their final solution). Unfortunately, such viewer does not work the kind of tasks labelled "problem" but only with the "exercise" kind of tasks.
Moreover one can investigate whether some traces of possible strategies developed in the interaction with the tool can be found in pupils' paper and pencil productions (pre-final and final tests).
Interesting elements are expected to arise also from the reports (questions-driven) which pupils are asked to write at the end of each lab session.
Finally some pupils have been video-taped with the aims of collecting more elements to investigate how they interacted with the tool, unfortunately such recordings reveal almost useless because of interferences between the camera and pupils' monitors.

Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?
Let us remind that the experiment was organized as follows:

Initial test (1 hour): students worked individually in paper and pencil environment
Lab sessions (three 2 hours sessions):
(a) researchers introduced the activities;
(b) students worked in pairs with Aplusix;
(c) students wrote individual report based on a set of questions.
Pre Final Test (2 hours): half of the students worked individually with Aplusix and half in paper and pencil. A new report is required.
Delayed Final test (1 hour): students work in paper and pencil environment.

Pupils were required to solve the tasks in paper and pencil in the test sessions (as for the so called "pre-test", half pupils were asked to use paper and pencil and half to use the tool environment) and to use paper and pencil to report on the lab activities.
During the lab activities pupils were asked to solve the tasks within the tool environment, only during the first one they were allowed to use paper and pencil too.

For the successive sessions the use of paper and pencil was forbidden: we wanted pupils to use only the tool because it provides researchers with the possibility of reviewing (and so analyzing) pupils' solutions step by step.
During the first lab session, just a small number of pupils used paper and pencil environment too.
Paper and pencil have been exclusively used to perform calculations.
The use of calculators was not allowed; seemingly when pupils did not feel confident with their calculation skills, they resorted to the paper and pencil environment to perform calculations (for instance implementing the usual arithmetic algorithms): the obtained results were imported within the tool workspace.

In your opinion, in which ways do your theoretical choices have influenced:

a. the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employment including the configuration of the software)?
b. the conception of the experiment?
c. the choices of the data and their analysis?
d. the results you obtain and the conclusions you draw from these?

Before facing the task of answering these a-posteriori questions, we want to say first of all that in our opinion, research problems, theoretical frameworks, educational goals, ICT tools analysis and so on constitute a system whose elements influence each other. In this sense, it may be difficult to isolate the influence of one of these elements over the other ones – in our case, for instance, the influence of the theoretical framework over the educational goal.

In your opinion, in which ways do your theoretical choices have influenced:

- the analysis of the software and the identification of didactical functionalities (software features, educational aims, modalities of employment including the configuration of the software)?

According to the Vygotskian approach (semiotic) mediation plays a crucial role in the learning process and more in general in the development of the higher psychological functions. In particular, we share the view that “the use of auxiliary signs breaks up the fusion of the sensory field and the motor system and thus makes of new kinds of behaviour possible” (Vygotsky, 1978) which we think can be extended also to the use of tools, and ICT tools in particular (we won't address here the question of the relationship between signs and tools).
As far as we know this point of the Vygotskian theory has not been fully developed in mathematics education, as a consequence the analysis of the data of the experimentation may make the need of enriching our theoretical framework emerge.

Anyway this general assumption has led us in the processes of identifying specific didactical functionalities of the tool Aplusix – sets of features of the tool, educational goals and modalities of employment of those sets of features – and of designing the whole experimentation.

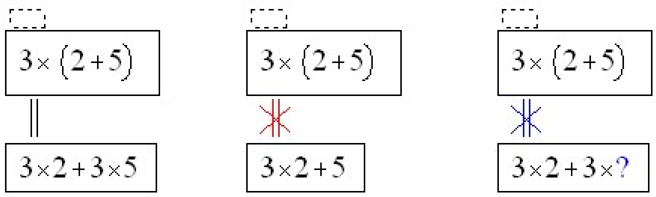
When referring to didactical functionalities, we refer to a system whose elements (software features, educational aims, modalities of employment) are strictly interrelated rather than juxtaposed. Constituting a system, the elements influence each other. The influence of the chosen theoretical frameworks over each of these elements may reflect upon the other ones, and conversely our choices (either the choice of the educational goals and of the software features to exploit) are only partly directly affected by our theoretical framework and partly they affect each other.

To isolate the influence of the chosen theoretical framework over each of these elements appears a really hard task.

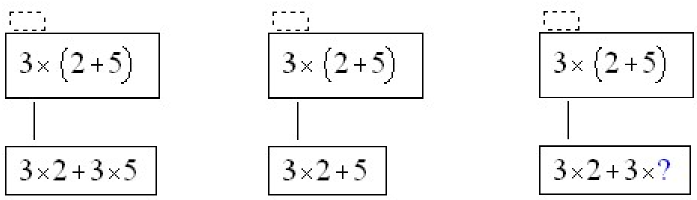
Aplusix is an interactive learning environment which allows students to build and transform numerical and algebraic expressions and relations. Within this environment pupils can be presented with some exercises involving manipulation of numerical and algebraic expressions (such as “expand or factorize an expression”, “solve an equation”, usually included in the Italian national curricula of the secondary upper school).

In the analysis of the tool, we investigate the characteristics of the mediation offered by the artefact focusing on those constrains capable of breaking up pupils' automatisms when accomplishing tasks. In fact, according to our theoretical hypothesis, the rupture of automatisms (in routinized tasks) and the emergence of unexpected obstacles may demand students to develop new and deeper reflections on their actions and on the tool feedbacks in order to (a) explain the occurred ruptures, and (b) overcome the arisen obstacles. As a consequence, pupils may be led to reflect on their actions and behaviour and possibly to construct new kinds of behaviour.
On the contrary, if the interaction between pupils and tool does not result in ruptures and obstacles, pupils are confirmed about the efficiency of their actions and behaviour, and may gain more and more confidence in their skills. Pupils' successes in solving tasks may result in a reinforcement of their skills.

One main feature emerged from our analysis is the presence of feedbacks concerning the equivalence of two consecutive (algebraic or numerical) expressions or relations. More in detail, the tool provides the following 3 feedbacks: if two consecutive expressions or relations are equivalent the lines connecting them are black; if they are not equivalent, the connecting lines are red and crossed; and if one of the expressions or equations is not well formed (in a mathematical sense) the connecting lines are blue and crossed. Moreover the tool can be set up in such a way that users can not ignore these feedbacks, that is users can be allowed to write down a third expression only if the two previous ones (in the same tree branch) are equivalent.



This particular feature seems to fit well with our general theoretical assumption. More precisely, coherently with the Vygotskian theory, we make the hypothesis that the use of the particular means of control provided by the tool may (a) on the one hand improve the specific performances, (b) and on the other influence the development of general abilities concerning consciousness and control of one’s activity. And as a consequence it may determine changes in the attitude towards errors and impasse.



Within Aplusix environment, the teacher can also propose tasks in which the control feedbacks are not available (in this case two consecutive expressions or relations

are connected with a single black line).
We exploit this feature of the tool, coupled with the possibility for the users to observed a-posteriori and, in case, modify the final product of their own work within an environment provided with the above described control feedbacks (self-observation and self-correction environments). Even in this case the pupils who successfully perform the tasks are confirmed in the efficiency of their computational skills, whereas pupils who commit errors are in a sense obliged to reflect upon their actions to detect and overcome their errors, or at least by-pass them.

Finally, when working within the tool one can open a work space independent from the task work space named the detached step. The environment – the detached step – may be set up both with the control feedbacks and without them.

Premised that we limit ourselves to the arithmetical domain, the above sketched analysis of the tool makes at least a couple of possible educational goals emerge, which indeed we address in our experimentation: (a) to reinforce pupils' operational and ordering skills in arithmetic, and (b) to enable pupils to develop abilities for controlling and reflecting on their own work, to develop strategies for detecting and overcoming errors, and for anticipating possible difficulties. The tool would help pupils to become conscious of their failures and of the need to activate means for overcoming them.
As for the identification of the former goal, our theoretical perspective has a minor influence. We speak of "reinforcing" - rather than acquiring - operational and ordering skills because our intention is to introduce pupils with Aplusix at the entrance of the secondary upper school as an occasion for pupils themselves to re-gain confidence with kinds of tasks they are expected to be familiar with and with computational and ordering skills they are expected to have built on (for the teacher it is an occasion to investigate pupils' skills in solving those tasks). Pupils' skills may result reinforced as a consequence of repeatedly successful tasks solving sessions.
The latter goal is more directly linked to our general hypothesis according to which the rupture of automatisms and the contextual emergence of obstacles (which a pupil may experience when makes errors within Aplusix) obliges the individual to reconsider his/her actions in order to detect the reasons for such rupture and overcome the arisen obstacles. According to the Vygotskijan approach this process could influence the development of higher psychological functions concerning monitoring, evaluation and anticipation of one's problem solving activity. Incidentally, let us note that the rupture of automatisms and the emergence of obstacles turn a routinized exercise into a "true" problem.

Let us now come to the modalities of employment of the sets of the software features which we follow in order to hopefully reach the stated educational goals.
How we intend to employ the described set of features has been already sketched and laid down above. In order to reach our educational goals, and coherently with the theoretical perspective adopted, we need to create occasions in which (a) pupils' automatisms are broken up when causing errors, (b) the presence of errors can not be ignored, (c) pupils have to detect their errors, and (d) to overcome them, (e) observation and reflection upon one's work are fostered.
Two different modalities of employment are identified, which contribute to create such occasions:

1. The environment within which pupils have to solve the given tasks is endowed with the control feedbacks described above. In addition, the tool is set up in such a way that users are not allowed to write down a third expression if the two previous ones (in the same tree branch) are not equivalent. (Briefly **"with control"-mode**)
2. Pupils solve the posed tasks in the environment without the control feedbacks (Briefly **"without control"-mode**), but are required to observe and, in case, correct their own work

a-posteriori. For the subsequent self-correction activity the tool is set up to function as described in item 1.

Let us remark that even if one cannot ignore that he/she has made errors (b), and is able to detect them (c), he/she may not be obliged to overcome his/her errors if he/she is able to by-pass them.

We conclude the discussion concerning the modalities of employment of the features of the tool, remarking that in both cases the pupils are thought to work in pairs with the tool without the teacher's support.

The choice of students working without teacher's support is motivated by the wish to make students take responsibility of detecting and overcoming their errors.

The importance of students taking responsibility of their own errors and of overcoming them is also confirmed by studies in metacognition

> "Even if the teacher recognises the student's error and intervenes, it is up to the student to modify his behaviour: but if the student is to significantly change his behaviour he first has to be convinced that the change has to be made, that the existing behaviour lead to failure." (Zan, 2002a)

The choice of students working in pairs is not directly motivated by - though it does not conflict at all with – our theoretical framework nor by the educational goals. On the one hand, it is due to contingent reasons: the "small" number of computers in the laboratories together with the impossibility of dividing the classes for separate lab sessions

On the other hand we think that the analysis of the interaction between students (some pairs are audio and video recorded) can provide us with a more deep insight in their thinking processes.

To sum up:

Educational Goals	(a) to reinforce pupils' computational and ordering skills	
	(b) to enable pupils to develop strategies for monitoring, detecting, overcoming and anticipating errors	
Features of the tool	"with control" - mode	"without control"- mode
	detached step with control	detached step without control
		self-observation and self-correction "with control" - mode
Modalities of employment	Pupils working in pairs without the teacher's support	
	Tool set up: (a) control feedbacks active, (b) error message cannot be disregarded.	Tool set up: (a) control feedbacks not available, (b) self-observation and self-correction environment available.

- the conception of the experiment

The experiment has been conducted in three 9th grade classes (first year of secondary scientific school). The math domain has been chosen according to the teachers' needs to allow students to go over operational skills - whose pupils are expected to be familiar with - before starting the new curriculum. More specifically, the math domain has concerned arithmetic calculation and ordering of integer and rational numbers (context constraint).

The activity in class starts with an initial test in paper and pencil (one - hour); the same test will be repeated at the end of the teaching experiment, with the aim to evaluate the improvement in pupils' performances. Then different phases of intervention (four two - hours sessions) are planned, centred on the use of Aplusix.

Description of the school activity:
Initial test (1 hour): students work individually in paper and pencil environment
Lab sessions (three 2 hours sessions):

(a) introduction of the activities;
(b) students work in pairs with Aplusix;
(c) report based on a set of questions.

Pre Final Test (2 hours): half of the students work individually with Aplusix and half in paper and pencil. A new report is required.
Final test (1 hour): students work in paper and pencil environment.

We plan an initial and a final test with the aim to investigate whether and how pupils' activity within the tool environment reflects on their activity within the "usual" paper and pencil environment.
The initial test is not only a means to evaluate (through comparison with the final test) the effect of the experimentation on the paper and pencil environment. It also aims to identify possible initial difficulties and errors of the pupils involved in the experimentation, as a basis for constructing tasks for and designing the following lab sessions. For this reason too, paper and pencil is highly suitable because ensures that the diagnosed difficulties and errors are not due to the use of unfamiliar (even if user friendly) environments.

During the tests and the Lab sessions, pupils are presented with different kinds of task involving computation and ordering of integer, decimal and rational numbers.
In the Lab sessions pupils are required to work without their teacher's support because we want them to autonomously confront with their failures and to take responsibility of detecting and overcoming their errors.
As previously argued, only if pupils attain consciousness of their failures they can seriously engage themselves in changing those behaviours that bring them to fail. Following this general assumption, when planning the teaching experiment we concern the organization of a context where pupils are led to detect and modify, in an autonomous way, those behaviours causing their own failures.
In our opinion the rupture of automatisms (in routinized tasks) and the emergence of unexpected obstacles (when solving tasks within the tool environment) may contribute to make students aware of their failures, and at the same time stimulate them to engage themselves in detecting *what* is wrong and in correcting it. According to the Vygotskian perspective, this process of detecting and overcoming errors may determine the development of particular ways to accomplish specific tasks which may result in solution strategies in the microworld but also in the paper and pencil enviromment. The importance of students taking responsibility of their own errors and of overcoming them is also confirmed by studies in metacognition, as we have just underlined in the previous answer.

The study has had a twofold goal. On the one hand, a didactical goal consisting of the retrieval of specific skills in arithmetic calculation; on the other hand, a research goal concerning the study of the role played by the specific microworld in reaching the didactical goal: if, and in affirmative case, how interacting with Aplusix may help to overcome the encountered difficulties. In particular, attention focused on investigating the functioning of the tool in the meta-cognitive processes related to become aware of one's own difficulties and to manage one's own resources to improve calculation performances.

In the general frame of a meta-cognitive approach we are interested to investigate the specific role played by ICT tool – and in particular by Aplusix features – in the evolution of self awareness and control on ones own resources.

In fact, metacognition tells us that pupils have to achieve awareness and control; we report again Zan's words:

> *"Even if the teacher recognises the student's error and intervenes, it is up to the student to modify his behaviour: but if the student is to significantly change his behaviour he first has to be convinced that the change has to be made, that the existing behaviour lead to failure."* (Zan, 2002a)

According to *this* assumption, the teacher's / researcher's intervention have to concern the organization of a context where pupils are led to modify, in an autonomous way, those behaviours that bring them to fail. Zan identifies two essential processes that a teacher /researcher has to foster and to strengthen: the attainment of consciousness and the possibility to activate personal control processes (Zan, 2002b). Consistently with this hypothesis, we assume that the interaction with the machine contribute to make students aware that something is wrong, but at the same time stimulate them to engage themselves in detecting *what* is wrong and in correcting it. This process of detecting and overcoming errors may determine the development of particular ways to accomplish specific tasks which may result in solution strategies in the microworld but also in the paper and pencil enviromment.

Let us note that the theoretical framework suggests more than what we could take into consideration because of the constraints of the experimentation. For instance, the need of a short term experiment does not fit well with the theoretical assumptions concerning long term processes and the importance of alternating pupils' autonomous work with pupils-teacher interaction.
We detail now a bit more on the kinds of the task proposed during the tests and the lab sessions, and on the reason why we choose such kinds of task. We report an example for each of the kind or tasks proposed in the lab sessions.

1. Transform the given expression in another equivalent expression and then compute the result.
   $16-9+16\times(-9)$
2. Compute the results of the following expression
   $$\frac{(-2)^2 \times 2^3}{2^4}$$
3. How many integers are there between the given numbers? (Write "0", "1", "2" and so on instead of "none", "one", "two", and so on)
   $$\frac{2}{4};\ \frac{9}{4}.$$
   If there are 2 or more than 2 integers, write the smallest and the largest. If there is just one integer write it, otherwise write "n".
4. Write the smallest and the greatest numbers out of the following. Write the smallest first and separate the number with the symbol <.
   $$\frac{5}{3};\ 2;\ \frac{5}{4};\ \frac{4}{3}.$$
5. Compute the following expression in at least two different ways (use the tree-structure)
   $$\left(-2+\frac{3}{8}\right)\times\frac{3}{8}-\frac{3}{4}$$
6. Say whether the following equalities are true or not and justify your answer (write "t" if true and "f" if false)
   $$0,75\left(\frac{1}{4}(-5)\right)=\left(0,75\times\frac{1}{4}\right)(0,75\times(-5))$$

So far we did not mention the tool features for editing tasks. The moment has come to spend some words about editing. Briefly, within the tool environment two “macro-kinds” of task can be inserted (by the administrator): “exercises” and “problems” – as they are named within the tool.
The former ones are rigidly pre-defined kinds of task: compute an expression, solve an equation and so on. The latter ones are somewhat more flexible, anyway the administrator when editing the task has to specify the expected correct answer, which as to be a well formed algebraic mathematical expression or relation.
Tasks 1, 2 and 5in the box above are “exercises” in the tool environment, whereas tasks 3, 4 and 6 are “problems”.
The someway not natural instructions on how formulating the answers to the tasks 3, 4 and 6 are due to the above mentioned constraints on the task editing.

To conclude, we remark that all the presented tasks are mainly closed ended tasks with just a single correct answer. This choice is only indirectly motivated by our theoretical framework and our educational goals. In fact, according to our theoretical framework and in order to achieve the stated educational goals pupils need to interact with the tool without their teacher’s mediation. This implies the need that the tasks can be as much as possible consistently posed within the tool and that their solutions can be validated by tool alone. Such constraints lead us to choose the kinds of task reported.

- the choices of the data and their analysis?

During the experimentation we collected the following data:

- students’ written productions from the initial and final tests and from the pre-test (paper and pencil environment);
- log files of Aplusix from the Lab sessions, including the pre-test
- written reports at the end of each Lab sessions and at the end of the pre-test
- video and audio records of some pairs work from the Lab sessions;

	initial test	**lab sessions**	**pre- final test**	**final test**
Written solutions	X		X	X
log files of Aplusix		X	X	
video and audio records		X	X	
Written report		X	X	

As mentioned in the description of the teaching experiment, the collection of pupils’ solutions of the initial and final tests should allow to investigate whether and how pupils' activity within the tool environment reflects on their activity within the “usual” paper and pencil environment. The comparison between pupils' performances in the two tests – in terms of number and typologies of failures – should provide some hints about the short-time effect of pupils' work with Aplusix.
Moreover, according to our theoretical framework, the use of the tool can influence pupils’ performances because the control provided by the tool itself can be internalized. Traces of such

process of internalization will be looked for in pupils' productions in the paper and pencil environment.

Aplusix is provided with a user friendly log files viewer which shows one's work step by step as it appeared on the user's screen. This feature allows the researcher (as well as the teacher) to analytically observe the whole pupils' work. Pupils' errors and impasse are easily detachable as well as their strategies to overcome or by-pass such errors or impasse. The detailed analyses of pupils' strategies to overcome or even by-pass difficulties and errors and of the possible evolution of such strategies are precisely among our main focuses.
Unfortunately this log files viewer is still in development and it does not work well with the log files of the tasks "problem" of Aplusix (within Aplusix two different kinds of task are available: exercises and problems). As a consequence the data concerning pupils' work with "problems" are less detailed than those concerning pupils' work with "exercises"; even the analysis of those data is less accurate.
According to the Vygotskijan approach, consciousness plays a relevant role in the development of higher psychological functions. Because we are interested in the development of abilities of controlling, monitoring, anticipating difficulties, detecting and overcoming errors, we are also interested in investigating the development of pupils' level of consciousness of their possible difficulties, of their available strategies, of their own processes and abilities.
In order to investigate this aspect we decided to ask pupils to write reports at the end of each lab session. More in detail, the reports are based on a set of two or three questions from which pupils are asked to elaborate and detail their view and perception on the kind of activity, on the functioning of the tool, on the difference – if any – between the tool environment and paper and pencil, and between Aplusix feedbacks and the teacher's feedbacks.
Some of the questions are also devoted to investigate the user-friendless of the tool.

Audio and video records aim at gaining some information about the interaction between pupils, from which indications may emerge related to the development of both pupils' strategies and their consciousness. Whereas the analysis of the log files does not provide any hints of how pupils' cooperation develops.

Last but not least, our interest in the development of higher psychological functions and the focus on pupils' consciousness of their possible difficulties, of their available strategies, of their own abilities, leads us to carry on a qualitative study rather than a quantitative statistical one. In fact we are convinced that at the early stage of such a research detailed qualitative analysis of pupils' behaviours may be more inspiring than statistical comparisons of data.

- the results you obtain and the conclusions you draw from these?

We thing that the question how our theoretical choices have influenced the results we obtain can be addressed under different perspectives. Trivially, results are determined by the whole planning and design of the teaching experiment and thus they are influenced by our theoretical choices in a really indirect way.
In a less trivial sense, our theoretical choices influence what we recognize as a result of our experimentation. Even less trivially, our choices exercise their influence on what we consider as evidence supporting those results.
In this report we face the posed question trying to outline what can be considered a result and what supports such result, according to our theoretical perspective.

We have to prove our results against the stated educational aims: (a) to reinforce pupils' computational and ordering skills, and (b) to enable pupils to develop strategies for monitoring, detecting, overcoming and anticipating errors.
Let us remark that a result may be, in a sense, "positive", that is it may confirm our a priori hypotheses and highlight the coherency of the planned teaching experiment with the posed educational goals; or "negative", that is it may question our hypotheses and the efficiency of the conception of the teaching experiment with respect to the posed educational goals.

As for the educational goal (a), we stated above that a reinforcement of pupils' computational and ordering skills can result from the interaction between pupils and tool whenever such interaction does not originate ruptures and obstacles to pupils' actions.
A first result would emerge from the comparison of pupils' performances (in terms of successes and failures) in the initial and the final tests as well as from the analysis of the evolution of pupils' performances along the lab sessions. Anyway because of the different kinds of tasks and activities proposed to pupils in the different lab sessions, the need emerges of defining criteria for comparing pupils' successes and failures in those sessions.
Besides the possible improvement of pupils' performances, we consider as results also those elements supporting the realization of the hypothesized process. More precisely, as argued when describing the data collected, according to our theoretical framework, the use of the tool can influence pupils' performances because the control provided by the tool itself can be internalized. Traces of such process of internalization will be looked for in pupils' production in the paper and pencil environment.

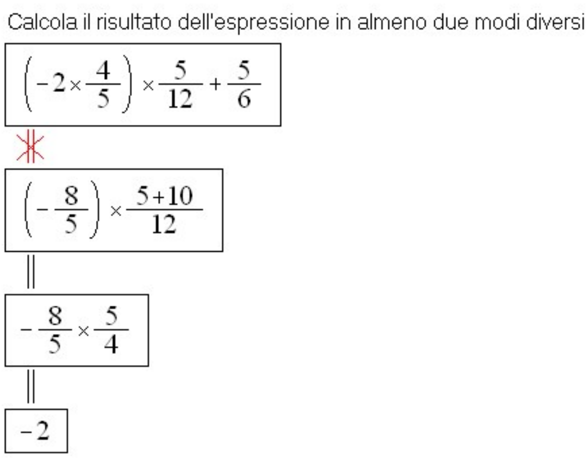
As for the educational aim (b), a first result consists in observing whether and to what extent pupils take the responsibility of detecting and overcoming their own errors.

More results can arise from the investigation of the construction, the emergence of strategies for:

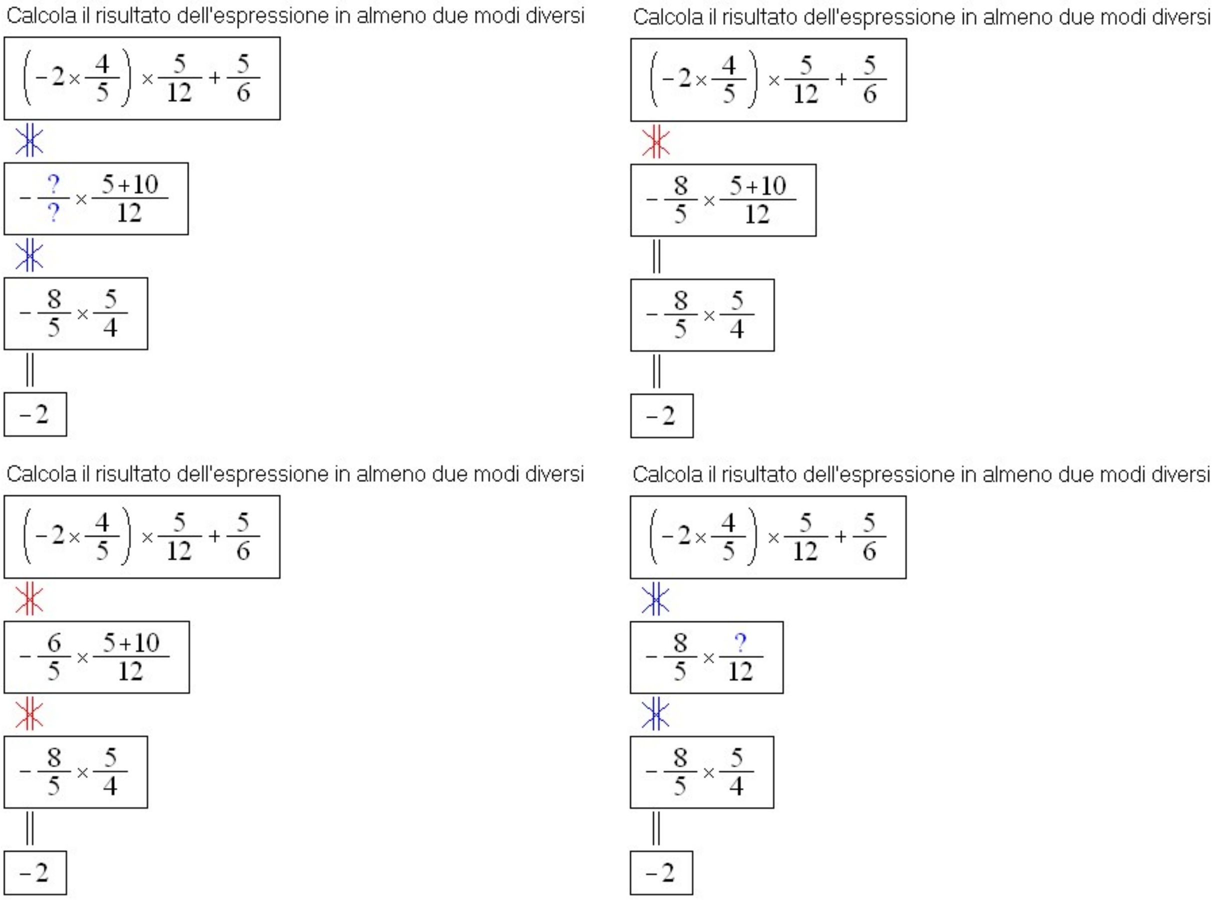
- detecting and overcoming errors,
- anticipating difficulties,
- bypassing errors.
-

The construction and interiorization of strategies are long term processes. Within the limits of this experimentation we can only hopefully find some traces of the first steps in the development of such strategies.

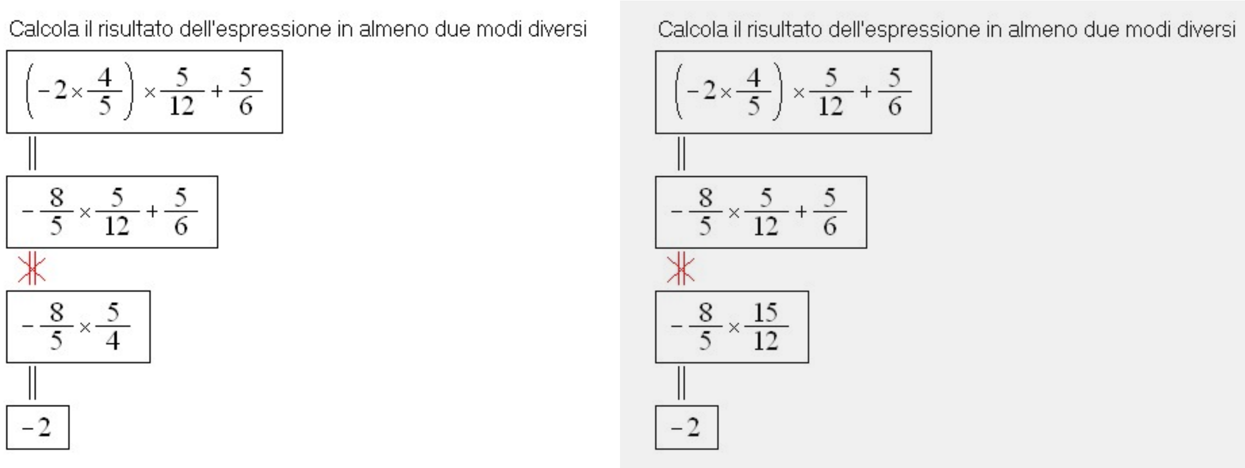
Here is an example of what we consider as the first development of a strategy for detecting errors.
The task of computing an expression in at least two different way was given in the without control mode, then pupils were asked to observe their work and in case correct it in the self-correction environment (with control mode). This first figure shows pupils' production as displayed in the self-correction environment.
As one can see the non equivalence of the first two expressions is marked.



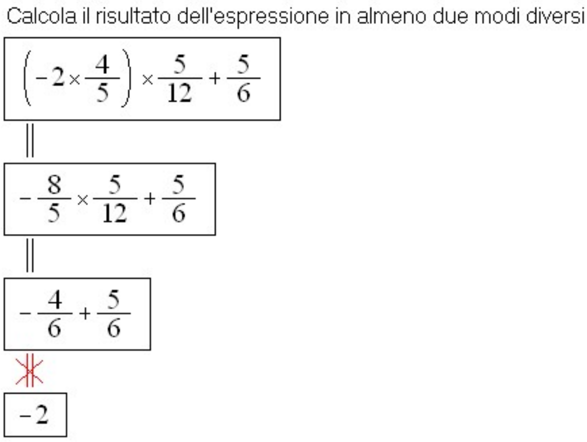
At first, the two pupils seem worried about having committed errors in calculation, as a matter of fact they control whether they computed correctly -2 times 4/5 and 5/12+5/6.

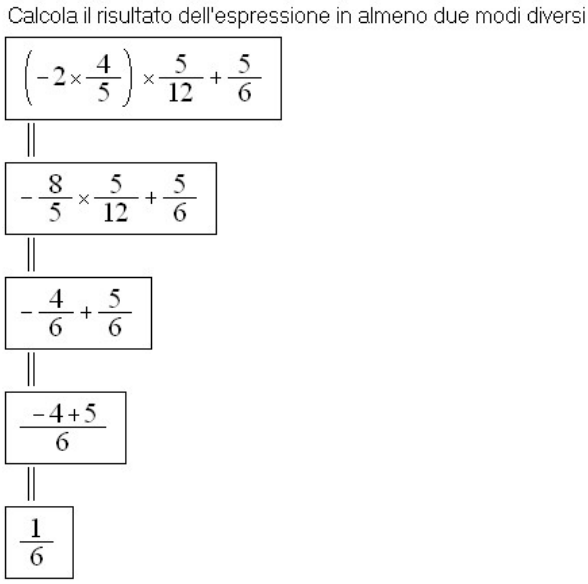


After many minutes they write -8/5 as result of -2 times 4/5 and give up to compute 5/12+5/6: the resulting expression is equivalent to the given one. Anyway they do not detect their error yet (neither they overcome it) and in fact they make the same error when writing the third expression.



It will take some more minutes and unsuccessful tries before pupils detect and overcome their error and conclude positively the task.





Moreover more problems can be posed related to the construction of strategies:

- whether the arisen strategies are only occasionally or systematically mobilized by pupils,
- to what extent such strategies are dependent on the tool environment and whether they can be mobilized in different environments (e.g. paper and pencil).

Unfortunately because of the limits of the experimentation carried on, we think that results related to these topics hardly emerge.

One more interesting point concerns the development of pupils' consciousness of their own possible difficulties and of their strategies to anticipate, bypass and overcome them. Here again a short term experimentation may not suffice to fully explore this dimension. Any way , some hints or traces may emerge, be found in the reports pupils edit.
We report below two extracts from the reports of two pupils (of different schools) who comment on the control provided by the tool and whose view is perfectly consistent with our a priori hypotheses.

In your opinion, is Aplusix control useful?

Nic: I think it is very useful because the automatic control allows us to detect our errors and then it obliges us to reason in order to understand where the errors are.

Do you find more easy to understand and correct your errors with Aplusix self-correction or with your teacher's correction? Which one is more useful in your opinion?

Bru: [during the self-correction] we are obliged to correct the committed errors and so they are better assimilated and we can learn the rule we forgot. It is just in this way that we could better learn the error and better remember it

Finally, in answering the first question we remarked that Vygotskij's hint concerning the effect of the use of signs (and tools) on the development of higher psychological functions has not so far been further developed.

As a consequence one more result – which we are not able to anticipate yet – should be to identify to what extent the present development of the Vygotskijan approach can support the analysis of the construction of abilities of monitoring, control and self-correction.

3 ARI-LAB2

3.1 Developer's team: ITD *(Pedemonte, B., Robotti, E.)*

- *Brief description of the instrument, explaining its key ideas*

ARI-LAB-2 is a multi-environment open system that supports different and complementary pedagogical opportunities.
It is a stand-alone system running on local networks. It can be used in a classroom or in a school laboratory to enable arithmetic problem solving activities in an interactive and collaborative way.
In ARI-LAB-2, the teacher plans and structures educational activities for his or her students (s/he edits texts of problems, s/he builds examples of solution…), and the student solves arithmetic problems using manipulation tools, representation features and communication features provided by the different environments which make up the system.

ARI-LAB-2 comprises the following tools:

Solution Sheet environment. It allows the solution to be described and presented.

Microworlds environment. Microworlds embody an abstract domain of knowledge described in a model, and offer a variety of ways to achieve a goal. Within microworlds, the user can create and manipulate computational objects to develop the solution strategy for a problem. While interacting with these computational objects, the user receives various kinds of feedback that may foster the emergence of goals for problem solution and the construction of meanings for the strategies developed. The available microworlds are: Euro, Abacus, Calendar, Number Building, Number Line, Graphs, Simplified Spreadsheet, Operations, Fractions and Arithmetic Manipulator.

Communications environment. It enables the exchange of messages and solutions among users.

Problem solving, and, in particular, Arithmetic problem solving, is a field in which primary and lower secondary school pupils tend to have considerable difficulties. We know that mathematics knowledge acquisition cannot be approached purely from a symbolic perspective. Moreover, we know that making reference to students' own experiences in the real world is necessary as well as attaching importance to concrete approaches to ideas and concepts.
ARI-LAB-2 addresses this need by offering teachers tools to build constructive learning activities and giving students the possibility to interact with rich constructive environments to carry out these activities. In particular for each proposed task the teacher can define the set of microworlds of ARI-LAB-2 that can be used according to her educational needs/goals.
For example, by using ARI-LAB-2 microworlds, the student can create and manipulate concrete representations to which he or she can assign a mathematical meaning in order to develop the solution strategy

- ***Indicate the theoretical framework employed to design and implement the software***

The aim of our design system is to support the construction of class teaching and learning activities in arithmetic problem solving (at primary and lower secondary school level) making reference to a **social constructive** perspective. In this perspective, learning is the result of the student's exploration and active construction mediated by the **tools**, and by the social interaction developed in the **activity** the student is engaged in.

In particular we refer to the Activity Theory framework which offers us an appropriate tool to instantiate the main relationships that characterized a learning environment (we adopt the term *learning environment*, to consider the teaching and learning situation as a whole).
In the Activity Theory the nature of any artefact can be understood only within the context of human activity, by identifying the ways people use this artefact, the needs it serves, and history of its development. Human usually uses an ICT tool because he wants to reach a goal. But the use of a particular tool can change the structure of activity and can result in new goals to be satisfied. Activity theory allows to analyse this change.
For what concerning the design of ICT tools, "*Activity theory can make an important impact on the development of design support tools. The design of a new ICT tool involves the design of a new activity. However, even the perfect design of an ideal activity does not guarantee the success of a system. The transformation of an activity from an initial target state can be difficult and even painful. Activity theory can be used to develop a representational framework that will help designers to capture current practice and build predictive models of activity dynamics. Such conceptual tools would enable designers to achieve appropriate design solutions, especially during the early phases of design*." (Kaptelinin, V. 1997)

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3.2 Experimenting team 1: LIG *(Trgalova, J., Chaachoua, H.)*

3.2.1 A priori (with respect to the experiment) questions aiming at collecting information concerning the design of the experiment.

- General:

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

Artefact/instrument
Our goal is to study the effects of using a computer-based tool (ARI-LAB2) on the learning of the concept of fraction. Verillon and Rabardel (1995) stress that a tool, an "artefact", is not immediately an instrument. A person who wants to use an artefact builds up her/his relation with it:s/he develops uses of the artefact (instrumentalisation) and builds instrument utilisation schemes to control these uses (instrumentation). This process is called "instrumental genesis". Within this theoretical framework, we are interested in studying instrumental genesis in pupils working with the fraction microworld of ARI-LAB2 software.

Anthropological theory (concept of praxeology) (Chevallard 1992)
According to Lagrange (1999), tasks, techniques and their relationship with the instrumental genesis are a key point in the use of technology to teach and learn mathematics. The author points out that "the organization of the tasks and associated techniques must comply with the constraints of that [instrumental] genesis and direct it in a productive way: schemes cannot develop arbitrarily and not all combinations of schemes are able to produce mathematical meaning". Therefore, our purpose is first, to investigate the types of tasks that can be given and that are meaningful in the computer-based environment and, second, to search for tasks and techniques that allow developing an appropriate instrumental genesis for functions.

- Analysis of ARI-LAB2 tool

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

- Design of the Teaching Experiment

- *Describe briefly the key ideas of your experiment and then answer to the following questions*

What are the precise aims of your experiment and the questions you want to focus on?

What is the type of research that you follow (e.g. classroom based, case srudies) and how is this related to the kind of your research focus;

Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

3.2.2 A posteriori (with respect to the experiment) questions aiming at individuating, collecting and gathering the results of the experiment.

Recall that for our experiment, we chose Ari-Lab2 software. The mathematical concept we decided to deal with was the notion of fraction. The experiment was carried out in one elementary school class comprising two levels, Grades 4 and 5. Grade 4 pupils have just been introduced to the notion of fraction (sharing a unit in equal parts), while Grade 5 pupils have learnt the meaning of fractions last year in the traditional paper and pencil environment. Given the pupils school level, we only used the "Fraction" microworld of Ari-Lab2 software.

Is there any difference in the answers that you gave during the a priori analysis at the following questions?

Taking into account the limited feedback of the tool in the first kind of activities (see the previous question), at the end of each activity, we asked the pupils to reflect on their actions. We expected that the pupils would search for the reasons of their possible mistakes and difficulties. Unfortunately, it was not the case. The pupils just corrected their errors committed in the anticipation phase on paper according to the answer provided by the tool and moved ahead to the next question. The computer environment did not allow them to interpret and explain the feedback of the tool. Moreover, when the feedback was provided in the form of the correct answer, the situation lost all of its interest.

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

The feedback provided in the "Fraction" microworld consists in the fact that a number (e.g. 7/3) can be built in various ways (e.g. 7/3, 2 + 1/3, 14/6, ...) and that the tool displays labels keeping trace of the way it has been built. This kind of feedback seemed particularly interesting for our experiment because first, it allows to introduce the idea that different expressions can represent a same number, and second, it allows working techniques related to the arithmetic operations with fractions. Both these aspects are conform with the French curriculum for the Grades 4 and 5. The pupils answers given in the paper and pencil environment were validated afterwards by the tool only.

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

It turned out that the "distance" between the fraction implemented in the Ari-Lab2 "Fraction" microworld and the fraction taught and learned in the French primary school is rather big. This distance is due to the following three issues:

1. Meaning of fraction: in the tool, a fraction appears as a number placed on the number line, while in the primary school, it is introduced in the context of sharing a unit. During the experiment, the teacher needed sometimes to switch from Ari-Lab2 to a context familiar to pupils (e.g. sharing pizzas) in order to help them overcome their difficulties.
2. Construction technique: in the tool, the fraction construction is based on the geometric projection method which is not available at the primary school level. Our pupils were used to construct fractions by means of sharing units on the number line. Therefore, in the experiment, we decided not to explain the construction technique of the tool to the pupils, but rather use it as a black box. As a consequence, the construction technique could not contribute to make sense of fraction construction.

3. Execution of the technique: the “division” button is associated to the fraction construction. Thus the underlying meaning of the fraction a/b is that of a quotient. In the French primary school, the way fractions are introduced leads to seeing the fraction a/b as a × 1/b. During the experiment, we noticed that the pupils had developed a double language: e.g., in the Ari-Lab2 environment, they were referring to the fraction ¾ as “three divided by four”, while in paper and pencil environment, they were saying “three fourths”.

To sum up:

	Ari-Lab2 “Fraction” microworld	***French primary school***
Fraction	Number on a number line	New number allowing to express results of measurements (sharing a unit context)
Construction technique	Thales property (projection method)	Sharing units on the number line
Meaning of the fraction a/b	Quotient a/b	$a \times 1/b$

Table. Distance between the fraction implemented in the tool and the fraction taught in French primary school.

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

As we mentioned previously, the role of Ari-Lab2 was to validate the pupils answers provided in the paper and pencil environment. In our experiment, we distinguished two kinds of activities according to the role of the tool:

1. The constructed number is the answer to the question (e.g. locate a fraction on the number line, compare two fractions). In this case, the role of the tool is very limited because the feedback is provided in the form of the answer to the question, therefore it does not stimulate the solving process.
2. The constructed number is not the answer to the question (e.g. add up two fractions). In this case, the tool provides a feedback just telling whether the answer is correct or not without revealing the correct answer, so that the solving process can start again.

Clearly, the situations of the second kind are much more appropriate to learning fractions.

Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?

In our experimental activities, we combined the uses of paper and pencil and Ari-Lab2 environments. Although the representations of fractions we used in both environments (representation of a fraction on a number line, fraction expression) are very similar, the two environments turned out to be complementary. The pupils were asked to anticipate their answers in the paper and pencil environment and these were validated with Ari-Lab2. The anticipation phase which is essential to the learning, is infeasible with Ari-Lab2 in most of our activities. For example, in our first activity, we asked the pupils to locate given fractions on the number line. In the Ari-Lab2 “Fraction” microworld, it is not possible to place a fraction freely, one must construct it, therefore the fraction is placed correctly. The role of the paper and pencil environment was to allow the pupils to engage in the solving of the exercises freely, with their own knowledge and without the software constraints. The role of the computer environment was to validate the pupils answers.

In your opinion, in which ways do your theoretical choices have influenced:

a. the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employment including the configuration of the software)?

b. the conception of the experiment?

c. the choices of the data and their analysis?

d. the results you obtain and the conclusions you draw from these?

Introduction

In this contribution, we try to clarify how the choice of the theoretical frameworks have influenced the analysis of the software and the identification of didactical functionalities (software features, educational aims, modalities of employment including the configuration of the software) on the one hand, and the design of the exepriment on the other hand. The answer to the next two questions (influence of the theoretical frameworks to the choice of the data and their analysis, and to the interpretation of the results) requires a further analysis which is in progress.

For our experiment, we have chosen Ari-Lab2 software developed by the CNR-ITD research team. Ari-Lab2 consists of several microworlds designed to support activites in arithmetic problem solving and in the transition to algebra. The mathematical concept we decided to deal with was the notion of fraction. The experiment was carried out in one elementary school class comprising two levels, Grades 4 and 5. Grade 4 pupils have just been introduced to the notion of fraction (sharing a unit in equal parts), while Grade 5 pupils have learnt the meaning of fractions last year in the traditional paper and pencil environment. Among the Ari-Lab2 microworlds, two are suitable for working with fractions: Fraction microworld and Symbolic manipulator microworld. The latter provides tools for proving rules given a set of axioms. Given the pupils school level, we considered this microworld not adapted.

In the following section, we will focus on the Fraction microworld that has been used in our experiment.

The software analysis and identification of didactical functionalities

The Fraction microworld provides a graphical representation of fractions on the number line. The fraction construction technique implemented in the tool is based on the projection principle (Thales theorem). Commands allowing to perform basic arithmetic operations on fractions (addition, subtraction and multiplication) are available (Figure 1).

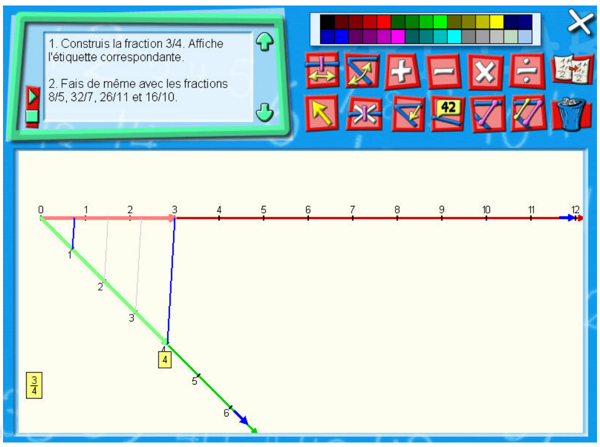

Figure 1. Ari-Lab 2, Fraction microworld interface : Creating a fraction ¾.

Analysis of the Fraction microworld of the Ari-Lab 2 software

The software analysis has been guided by two main theoretical frameworks: the theory of didactic situations (TDS) and the anthropological theory. According to the TDS framework, learning occurs in the pupil while interacting with a "milieu". Thus the "milieu", and especially the feedback it provides, plays a key role in the learning process. The feedback should be rich enough so that the pupil, or a group of pupils, can work autonomously and construct knowledge by adapting themselves to the "milieu". The teacher's role is minimised in order to avoid the effects of the didactic contract. Therefore, in the Fraction microworld analysis, we have investigated its potential in terms of the feedback it is able to provide. Within the anthropological theory, we have explored the types of tasks related to the concept of fraction that can be given and that are meaningful in the microworld. Identifying the relevant tasks led us to define the educational goals for our experiment.

In terms of feedback provided in the Fraction microworld, the fact that a number (e.g. 7/3) can be built in various ways and that the tool displays labels keeping trace of the way it has been built (Figure 2a), seemed particularly interesting. First, it allows to introduce the idea that different expressions can represent a same number, and second, it allows working techniques related to the arithmetic operations with fractions (Figure 2b).

Moreover, the fact that a fraction the user builds with Ari-Lab 2 is placed on the number line can be considered as a feedback validating the user's anticipated answer. Such a feedback is useful in the tasks of placing a fraction on the number line, comparing and ordering fractions, situating a fraction between two consecutive integers, and inserting a fraction between two given fractions. The educational goals for the experiment could have been defined:

(1) recognize various expressions that represent a same number;
(2) situate a fraction between two consecutive integers;
(3) compare and order fractions;
(4) add and subtract simple fractions, multiply a simple fraction by an integer.

These goals are conform with the French curriculum for the Grades 4 and 5 (See Appendix 1). Therefore, the teacher could integrate Ari-Lab 2 to the teaching of fractions without changing the mathematics organization of her teaching project.

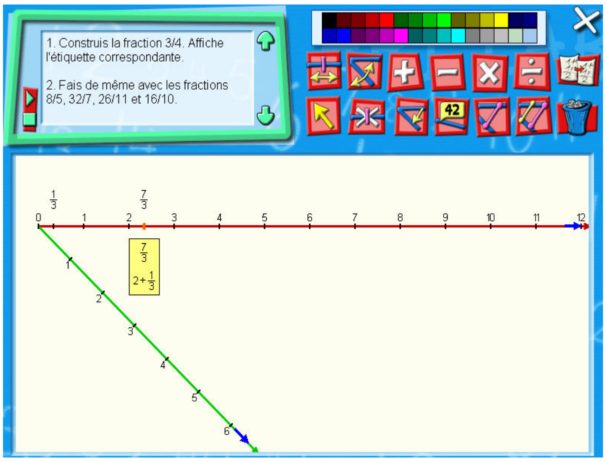

Figure 2a. 7/3 and 2+1/3 represent a same number.

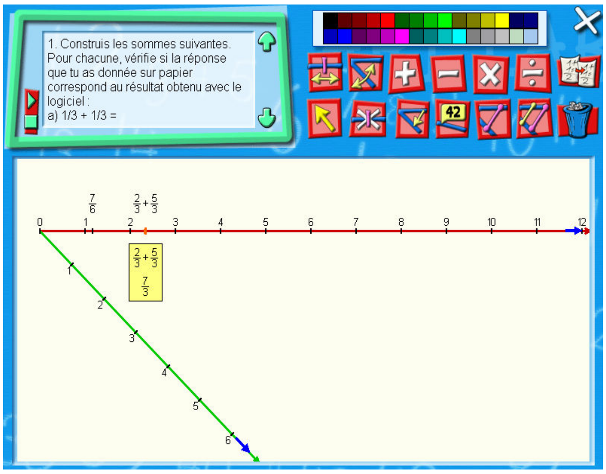

Figure 2b. The sum 2/3 + 5/3 is equal to 7/3.

Didactical functionalities

The analysis of the tool in terms of feedback and the types of tasks that are meaningful in this environment allowed us, on the one hand, to identify the features and characteristics of the tool that are to be used in the experimental activities, and, on the other hand, to define the educational goals of the experiment.

As regards the modalities of employment of the tool, the pupils are thought to work in pairs with the tool without the teacher's intervention. The choice of pupils working in pairs is motivated by the socio-constructivist hypothesis underlying the TDS according to which the social interactions between pupils can contribute to the learning. The teacher is purposly taken distant while the pupils are solving activities. The reason for that is twofold: (1) as was mentioned above, we wish to avoid the pupils respond according to the didactic contract, and (2) in order to learn, the pupils have to encounter a cognitive conflict while solving a problem. This conflict raises from a contradiction between an anticipation and a denial, coming either from the "milieu", or from peers.

In the table below, we summarize the three dimensions of the didactical functionalities of the tool:

Educational goals	(1) recognize various expressions that represent a same number (2) give different expressions of a given number (3) situate a fraction between two consecutive integers (4) compare and order fractions (5) add and subtract simple fractions, multiply a simple fraction by an integer
Features/characteristics of the tool	Creating a fraction on the number line Displaying labels keeping trace of the way the number has been built
Modalities of employing the tool	Pupils working in pairs without teacher's intervention Class discussion of the pupils answers

Table 1. Didactical functionalities of the Ari-Lab 2 Fraction microworld.

The design of the experiment

The experiment has been carried out in one primary school class comprising two levels: Grade 4 and Grade 5. The teacher was just working on fractions at both levels: the Grade 4 pupils were introduced to the notion of fraction by means of problems where a unit needs to be subdivided (e.g. length measurement). Grade 5 pupils have already been introduced to the notion of fraction last year and at the time of the experiment; they were learning how to locate a fraction on the number line.

The experiment was planned to consist of 3 phases:

(1) A pre-test in a paper/pencil environment, aiming at collecting information about the knowledge of fractions in the pupils prior to the experiment. We tried to make the pupils explicit their representations of the notion of fraction, their ways to operate with fractions (comparing two fractions, adding two fractions, multiplying a fraction by an integer), and their ability to locate a fraction on a numerical line.

(2) The experimental teaching sequence involving the software consisted of two phases: familiarization with the software, mainly the representation of fractions embedded in the fraction microworld of the software, and a series of activities aiming at the learning of fractions (comparing fractions, locating a fraction on a number line, various written forms of a given fraction, operations with fractions).

(3) A post-test in a paper/pencil environment aiming at measuring the effects of using the software on the learning of fractions.

During the familiarization phase, one of the researchers was manipulating a computer with the Ari-Lab 2 Fraction microworld. His screen has been projected so that the pupils were able to see his actions and do the same. In what follows, we describe the design of the experimental teaching sequence and provide a rationale for each question.

The first series of activities (see below) aimed at reaching the first 3 goals dealing with locating a fraction on the number line, and recognizing or giving different expressions representing a same number. The structure of the tasks is similar: the pupils have first to anticipate the answer, then verify it with the tool and correct errors, and finally explain their procedure. Such a structure is motivated by an attempt to provoke a cognitive conflict coming from confronting the anticipated answer, possibly incorrect, with the answer provided by the tool. The feedback provided by the milieu in this case consists in validating the pupils' answers. We hope that when the pupils encounter such a conflict, they will not satisfy themselves with identifying and correcting them, but that they will also try to overcome them.

I) Fractions, numerical expressions involving fractions

1) *Building fractions*

a) Using a blue pen, indicate where the following fractions are situated : ¾, 8/5, 32/7, 26/11, 16/10.

b) Build the same fractions with Ari-Lab2. Compare your answers with those provided by the computer. Correct your errors, if any, in red.

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2) *Different expressions representing a same number*

a) Among the following expressions, encircle with a blue pen those that represent a same number.

i)	¾	9/12	15/16	6/10	45/60
ii)	2+1/3	3/5	7/3	21/9	1+5/3

b)Verify your answer with the computer. Correct your errors, if any, in red.
c) Explain how you decide whether two expressions represent a same number or not.

---

3) *Different expressions representing a given number*
a) Give three different expressions of the number 7/3.
b) Verify your answer with the computer. Correct your errors, if any, in red.

The next series of activities (see below) aim at comparing fractions, either to the closest integers, or to each other. The structure of the tasks is similar to the previous ones. The feedback provided by the tool allows validating the pupils' answers.

II) Comparing fractions
4) *Situating fractions between two consecutive integers*
a) Situate each of the following fractions between two consecutive integers:
… < 18/10 < …
… < ¾ < …
… < 17/3 < …
… < 45/100 < …
… < 356/100 < …
… < 25/2 < …
b) Verify your answer with the computer. Correct your errors, if any.
c) Explain how you decide between what two integers a given fraction is comprised.

---

5) Comparing fractions
a) Among the 5 children,
- who has the most of chocolate ?
- who has the least of chocolate ?
- do some have the same quantity as some other?
Tom : 8/3 kg Théo : 7/2 kg Lou : 14/4 kg Léo: 7/5 kg Lola : 8/5 kg
b) Order the quantities of chocolate the children have from the smallest to the greatest one.
c) Verify your answer with the computer. Correct your errors, if any.
d) Your Grade 4 classmate is asking you how you do to compare two fractions. How would you explain it to him?

The last series of questions aim at learning techniques related to the basic arithmetic operations with fractions: adding and subtracting two fractions, and multiplying a fraction by an integer. The structure of the task is again the same: anticipation of the answer, validation with the computer, correction of errors and explanation of the procedures. The feedback provided by the tool in these activities is more likely to support the pupils' procedures evolution. Let us illustrate this in an example. The pupil has to calculate ½ + 1/3. Suppose that his procedure consists in adding the numerators and the denominators of the two fractions thus obtaining 2/5 as a result (frequent strategy in primary school pupils). In order to verify his answer with the tool, he will construct 2/5 on the other hand, and ½ + 1/3 on the other hand. Unlike to the previous activities, the feedback provided by the tool will simply indicate that the answer is not correct, without providing the right answer. The pupil will realize that his initial strategy is not correct and will need to search for another one that allows finding the fraction which is equal to the sum ½ + 1/3.

III) Operations with fractions

6) *Calculation*

a) Calculate :

1/3 + 1/3 = ...	½ + 3/2 =
2/3 + 4/3 =	1/3 + ½ =
4/10 + 2/5 =	3/10 + 7/10 =
3 × ½ =	5 × 1/3 =

b) Verify your answer with the computer. Correct your errors, if any.

c) Your Grade 4 classmate is asking you how you do to add two fractions and to multiply a fraction by an integer. How would you explain it to him?

To add two fractions:

To multiply a fraction by an integer:

Each series of activities is followed by a discussion among pupils orchestrated by the teacher pertaining to the pupils' responses and strategies aiming at a collective validation of the correct ones.

To conclude, the design of our experimental activities was guided by the chosen theoretical frameworks. On the one hand, we attempted to create a milieu integrating the tool in the way for the pupils to be able to engage in the proposed tasks without a teacher's intervention. This required to guarantee a feedback coming either from the milieu itself or from the pupils interactions with peers. Moreover, the feedback should be rich enough to support the pupils initial knowledge evolution. On the other hand, the tasks we have chosen were conform both to the French curriculum and to the mathematics organization of the teacher's teaching project.

3.3 Experimenting team 2: DIDIREM *(Cazes, C., Georget, J.-P., Haspekian, M., Souchard, L., Vandebrouck, F.)*

3.3.1 A priori (with respect to the experiment) questions aiming at collecting information concerning the design of the experiment.

- General:

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

For this a priori phase, we use 2 frames :

- *ergonomic approach : Tricot (ergonomic psychology), Scapin, Bastien (computer science)*
- *instrumental approach (cf. deliverable)*

Ergonomic approach

This frame is not developped in the deliverable on theoretical frames but it is used in the E-learning domain. Actually, we only know french references, please tell us if you know references in english language.

For more information, you can watch on:
http://www.ergoweb.ca/criteres.html
http://perso.wanadoo.fr/andre.tricot/Tricot-et-al_EIAHStrasbourg.pdf.

This frame use 3 key concepts which are linked:

usefulness
usability
acceptability

Usefulness means that we must watch if the tool is conform to the original aim of the authors. In a certain way, it's a part of a final evaluation of a tool. A priori, it is an important thing to think about it but in our case we don't know exactly what they are. So, following this frame, we must define our own aims and design precisely our experiment to work on this point. We can't do it actually.

Usability: it is the possibilities of manipulating the tool. In this, we must watch on provided feedbacks, [cf. doc bastien]. It's a large part of an a priori analysis and during the process of development of a tool.

Acceptability: possibilities to access and to use the tool. Often, it is more an a part of an a posteriori work, because we don't always know how the things will happen and why the users use the tool and if they really want to use it.

The frame is a general one which permit to integrate different frameworks and results in mathematic education, psychology, ergonomy, computer science.

In the point of view of ergonomy, the 3 concepts cannot efficiently be studied independently. For example, the usability analysis can permit some hypothesis about the acceptability aspect but it is not sufficient. In fact, we can see that some tools that have a "bad" usability (in our point of view) and which are well accepted in the classes As a concrete example with AriLab, one can't undo actions nor redo them as for many other softwares. If we only make a usability analysis, it can be interpreted as a bad point. But, in terms of acceptability or usefulness, one can see here that it is expensive in time and in energy to not think enough before doing things with the tool: the students must be more reflexive on their way of solving the problems. Here, an usefulness and acceptability analysis can occur better after the experiment.

Instrumental approach

Our questions are "how the students will use the tools (mainly euro and operations tools in this short experiment) and even will they use them?". We want to study if there is a instrumental genesis and which instrument (in this frame) are developed.

For recalling the concepts, we cite (Artigue, 2002): "The instrument is differentiated from the object, material or symbolic, on which it is based and for which is used the term "artefact". Thus an instrument is a mixed entity, part artefact, part cognitive schemes which make it an instrument. For a given individual, the artefact becomes an instrument through a process, called instrumental genesis, involving the construction of personal schemes or, more generally, the appropriation of social pre-existing schemes. Instrumental genesis works in two directions. Firstly, it is directed towards the artefact, loading it progressively with potentialities, and eventually transforming it for specific uses; this is called the instrumentalisation of the artefact. Secondly, instrumental genesis is directed towards the subject, leading to the development or appropriation of schemes of instrumented action which progressively take shape as techniques that permit an effective response to given tasks. The latter direction is properly called instrumentation."

- Analysis of AriLab tool

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

We began to study the operations microworld. When we make an error, the tool don't write the error (the digit we want to print on the screen). In other words, we strike a key but nothing is written except the word "Error" (not at the place of the cursor). The second point is that after the three first errors, the tool give us an explanation of what we have to do and how. Then, it doesn't give more explanations for the next errors. For example, for the result of an addition after the 3 first errors, we try each of the digit (0, 1, 2, etc.) for each column of the sum without the need to understand how the algorithm works. Here, the validation is made automatically by the tool itself.

In the euro microworld, the student can select a set of coins and then call a wizard to obtain the value of the set. There is no visual feedback but only a sound feedback.

Our analysis is not finished because we don't want (and because we had little time to do it) to analyse all the tool but we want to concentrate on the microworlds that could be used by the teachers/students in our experiment. It's only a first approach of our work.
What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?
In the numbers microworld, we think that we find a bug :
560,400 is read as 560 thousand 400 by the wizard

560,4 is read as 560 comma 4
560,40 is read as 560 virgule 40

But we find this feature interesting because this feedback can not easily occur in a paper-and-pencil environment. We often see students who write a number and when the teacher or another student pronounce the number he realize that he make a mistake (for example, he forgot a zero or add an additional one).

Design of the Teaching Experiment
Describe briefly the key ideas of your experiment and then answer to the following questions
What are the precise aims of your experiment and the questions you want to focus on?
What is the type of research that you follow (e.g. classroom based, case srudies) and how is this related to the kind of your research focus;
Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?
How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

3.3.2 A posteriori (with respect to the experiment) questions aiming at individuating, collecting and gathering the results of the experiment.

As we said in the a priori document, the purpose of the experiment is to make pupils meet several techniques of subtractions, to give sense to these techniques and to go towards the expert technique by looking for the most effective. AriLab permits to build a situation where the choice of two microworlds is offered: euros and abacus. The first technique in the euro microworld is to decompose the first term of the subtraction in order to obtain the second and its complement as the result of the subtraction. The second technique in the abacus microworld is closer to the expert technique of subtraction where the decomposition are based on the decimal system.

We have prepared 3 sessions: 2 for discovering each of microworlds and a final problem session (see our work for task#6). Our task 1 in session 1 is in a euro context “you have the following banknotes (50+50+20+5) and you want to spend 90 euros. How do you do?”. Our task 2 in session 2 is in the abacus context “Use abacuses to make 267-78 and explain every stage by drawing below”. In session 3, there are initially two problems. The first one is explicitly in a euro context “I have 541 euro and I spend 175. How much does it remain me?”. Nevertheless, we think that it is more easily resolved with the abacus microworld. The second problem is a problem with trains but it doesn’t occur because of time. The name of pupils are Marius, Gaspard, Gregoire, Charline and Lea.

Is there any difference in the answers that you gave during the a priori analysis at the following questions?

There is a difference between our answers at the questions as we explain above and finally validation have to be made collectively within the students and the teacher.

Without any other experiments, we always think what we have planned is adequat in terms of distance. In our experiment the time for the instrumental genesis is a "limiting factor" for achieving an adequate use of the tools by the pupils and to permit to them to go beyond their actual capabilities of computing with the whole numbers.

Some ergonomic features and probably some bugs have also play a role in the experiment but we didn't go further on this aspect.

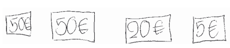
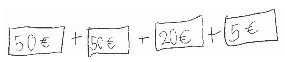
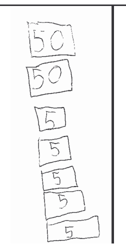
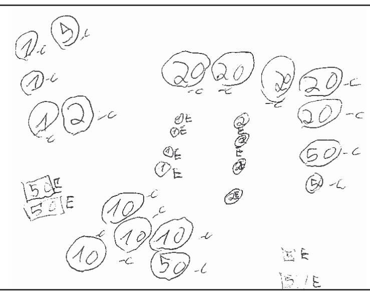
Can you please tell me if it's more clear now (if not give some precisions if you can as we are more able to explain precisely) ?

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

As we saw in the a priori questions, Arilab provides a feedback in the two microworlds selected in our experiment. In the microworld euro, the feedback concerns the exchange: "there are few" or "there are too many". In the microworld abacus, the feedback is only a validation "yes" or "no" and it doesn't help student to overcome his difficulty. For instance, during the problem session, Marius is stopped because the abacus microworld doesn't want him to exchange 1 unit with 1 ten. It is impossible but Marius doesn't know why.

The feedback seems useful only when simple tasks are asked to pupils. Even for such tasks, some pupils don't enter their answer because they prefer show it to their teacher before. For others pupils, the feedback works as we anticipated.

Example of the efficient of the feedback in the case of a simple task in session 1: from 50 euros, complete to have 125 euros.

				
Charline	Léa	Marius	Grégoire	Gaspard

All results are different; pupils know that their own answer is correct.

For more complex tasks, the difficulty for pupils is to conceive their process of solution and the feedback can not help them is this previous phase. Most of time, pupils need the help of the teacher. The validation come from the teacher or from the peers.

Example of validation by peers in the case of the problem "I have 541 euros and I spend 175. How much does it remain me? Explain how you do." Two pupils Gregoire and Gaspard found different results and they compare each other. So Gaspard decide to compute again with the abacus microworld. Here is his last screen but Gaspard doesn't know if his answer is correct.

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

Each of pupils tries to solve the problem firstly in the euro micro word and then goes to work with the abacus micro word (excepted Gregoire as we have seen previously). When they work in the euro microworld, we observed that none of them uses the first awaited technique. Pupils working with euro use an hybrid technique by trying to take off 175 from 541 step by step. It seems easier for them to work in the abacus microworld. The distance is probably closer with the paper and pencil tasks.

For example, we observed a specific difficulty in the Charline's work. She doesn't give any sense to 541 in the microworld. Whereas, in the abacus microworld, she can express 541 and even exchange 1 hundred with 10 tens. Even in the first session, when the task is to compute 125-90 with the first technique, she has difficulty to translate from the euro microworld to the mathematic signification. Her answer is the right one:

Il reste 35 €	Il me résté 20 5 10
Léa	Charline

For Charline, abacus microworld seems to be well adapted to help her to give sense to numbers. At the end of the session 3, the exchanges are well understood by pupils but the use of AriLab is not integrated in their mind for solving tasks. Pupils know how to do the exchanges in the two microworlds but they can't call these procedures by themselves without any help.

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

We adress this point with the help of the video tapes and the notes of the observers during the experiment. Firstly, we notice some points that we find interesting to present in the frame of our short experiment.

We have tried to explain how our experiment was not so sucessful as we anticipated in the a priori analysis. Mainly, it was because of instrumental genesis reason (impossible to etablish in a so short time and we were aware about this point before the experimentation) ***and*** because of the "lack" of knowledge of the students which we were not aware of (we discover this point during the experimentation).

Consequently and in a certain way, the role of the tools is minor in front of the type of the task asked to the pupils as we said at the end of our document for TASK7.

We encountered some obstacles which have not permitted the students to use the tools efficiently.

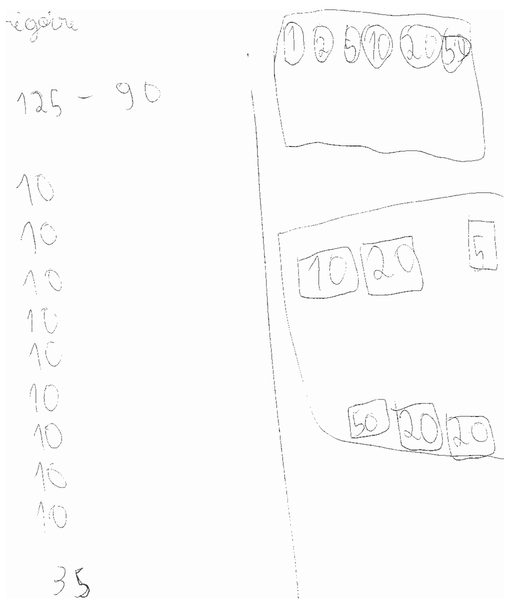
Firstly and to try to be more clear about both cases given in our document, there is the "incident" of Gregoire which is (finally !) a good mental calculator and mainly uses the tool only to present the results (which are wrong) and not to construct them. For him, the tool is only a way to give a representation of the result in a pleasant manner.

Otherwise, the others pupils use the tool to try to solve the problem. Gaspard is the only one to succed using the abacus microworld of the tool .

The other aspect that we adress in our work is the case of Marius which is trying to use both microworlds but does not success and then tries a paper-and-pencil technique which also does not success. The tool does not give appropriate feedbacks to the difficult task asked to the pupils, it gaves only feedback to simple tasks so it cannot help pupils to find a process to solve the problem.

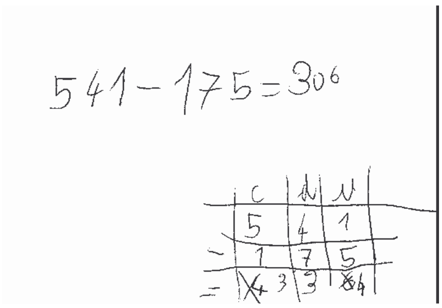
Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?

One pupil Gregoire seems to be a good mental calculator. During the first and the last session, he prefers to solve the tasks in his mind or in paper and pencil environment and use AriLab as a board to present his results. In the first session (task “compute 125-90”), he uses a paper-and-penciel technique and present his result as in the euro microworld.



In the problem session, he also uses a paper and pencil technique, makes an error: his result is 541-175=365 and he uses the euro microworld to present this wrong result.

Another pupil, Marius, during the problem session, has trying to solve the task with the two microworlds successively. Finally, he uses a paper-and-pencil technique which is wrong.



In your opinion, in which ways do your theoretical choices have influenced:

a. the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employment including the configuration of the software)?

b. the conception of the experiment?

c. the choices of the data and their analysis?

d. the results you obtain and the conclusions you draw from these?

We want to understand/describe/analyze the impact of the theoretical frames in experimention's design and how it goes off. At the same time, the perspective of an ITC's natural integration in a “normal “ class is a part of a french global context of research that we want to cope with.

The DIDIREM team has chosen for this experimentation the tool Ari-Lab produced by the team ITD-CNR. We first present analysis of the tool and the different didactic functionalities and selected some of these. Then we explain the conception of the experiment. We also try to clarify and illustrate the exact role theoretical frames have played in both parts.

Analysis of the tool and identification of the didactical functionalities

Introduction

The DIDIREM team has chosen for this experimentation the tool Ari-Lab produced by the team ITD-CNR. The information provided by the designers of Ari-Lab present this tool as a set of inter-connected microworlds. The first inspection of Ari-Lab confirm this affirmation. As in any microworld, some abstract concepts are reified into the microworld and embodied action on these abstract concepts is accessible through the direct manipulation of their representations. The interface looks attractive and its design seems especially well adapted to the elementary school.

A specific emphasis has been put in this tool on the development of interaction capabilities both for collaboration between students and interaction between teachers and students. These characteristics of the tool will suggest two ways for the didactic exploitation of the tool in the group: one giving the priority to the interaction between microworlds in problem solving, the other to the interaction between students for the collective elaboration of solutions of a complex problem and a reflexive work on these solutions. Our usual theoretical frames don't easily permit to analyse in depth the collaborative phases. Moreover, due of the short time allocated to the experimentation, the team had finally decided to focus on the interaction between microworlds and didn't analyse the communication features of the software. We more deeply explored the ten microworlds (euros, abacus, number building, number line, calendar, graphs, simplified spreadsheet, operations, fractions and arithmetic manipulator) searching for those a priori best adapted to this experimentation.

Selection of the microworlds

The selection was based on an analysis supported more or less consciously by the following frames:

- The instrumental and the ergonomic approach
- The theory of the didactical situations (TDS)
- The anthropological theory
- The epistemological and didactical knowledge.

This analysis of the different microworlds has shown many interesting features and interesting modalities of employment for most of them. We develop here some key elements about this analysis separating by the lens of theoretical frames. Obviously there are linked and weren't used in a chronological manner and separatly.

The instrumental and the ergonomic approach

The inspection of Ari-Lab, from an ergonomic perspective, mainly concerned its “usability”. Issues related to “acceptability”[13] could also have been taken in charge by this ergonomic analysis and by the concepts of the anthropological approach. Being aware of the fact that the duration of an instrumental genesis is necessarily longer than this micro-experimentation, we tried to have moderate and realistic ambitions in that respect from the beginning. This also made us sensitive to the fact that, in such a short experiment, we could not really study the potential for mathematics learning of a tool as complex as Ari-Lab, but only some very limited facets of this potential. For instance the interface seems quite simple and intuitive in the “euros” microworld. We can easily drag and drop the coins and the banknotes in a frame and can change them and the representation of the coins and the banknotes are realistic. The “abacus” microworld offers more features but its *utilisability* is less intuitive. For example, it is necessary to click on the rubber each time we want to (des)activate: that is not the case in the other microworlds where the rubber self-desactivates when used. It's also the case of the calendar microworld use a lot of messages (in english) and many actions to achieve some elementary tasks.

[13] We refer here to the distinction established in (Tricot, 2003) between three dimensions for ergonomic analysis : « utilisability » evaluates the tool according to its accessibility and facility of use, « utility » evaluates if the tool really does what it is supposed to do, « acceptability » evaluates its acceptability by its prospective users (persons or institutions).

The theory of the didactical situations (TDS)

This frame is an important part of the french didactic culture that we share in the DIDIREM team. Generaly, we pay a particular attention to the possibilities of action offered to the pupils, to the nature of the feedback possibly received, and to the interpretations in terms of "milieu".

We distinguish feedbacks consisting in just a validation of pupils' answers and feedbacks more elaborated, and feedbacks more likely to support pupils' strategies evolution, and mathematics knowledge development. Some microworlds were eliminated because the system of feedback they proposed was too much limited as compared with what is generally expected from a "milieu" offering a-didactic potential for learning. For instance, an informative feedback is provided when the rest si greater than the divisor in the operations microworld. When changing money in the euros microworld, the software not only answers in terms of "yes or no" but indicate if there is "too few" or "too much" money to proceed to the change.

The anthropological theory

The team has used the anthropological theory to take in charge the institutionnal analysis mainly in the first two dimensions of the notion of didactic functionality. In our opinion, the importance given to issues of didactic legitimacy and institutionnal distance attests the role played by this theory in out approach of the experimentation.

Some microworlds were eliminated for reasons of distance from an institutional point of view (Chevallard). We found quite interesting the fractions microworld based on the representation of rational numbers on the real line. The underlying mathematics refer to the Thalès theorem (as it is called in France) which is taught later in the academic year. We estimated that it was not realistic to ask a teacher to substantially change its mathematics organization of the academic year, just to be involved in a micro-experimentation.

The euros and abacus microworlds both permit to perfom subtraction between whole numbers but due to different features, the techniques (Chevallard) used are different. In the euros microworld, we can make decomposition of the numbers in action (by changing the money) and isolate one term of the subtraction to find the result. The type of decompositions depends of the numbers involved. On the other side, the abacus microworld mainly permit decomposition based on the decimal system (by decomposition of group of tens balls). Working on the diversity of the techniques (personal and expert ones) used in arithmetic situations is an important educational goal in the french curricum at the elementary level. So these microworlds seem to be complementary for the subtraction's approach.

The epistemological and didactical knowledge

Beside these different theoretical frames illustrated above, a background of epistemological and didactic knowledge as regard the numerical conceptual field had also a certain influence. for example, the abacus world is epistemologically coherent: the decompositions and recompositions made in it are are used in one of the expert techniques of resolution of sustractions. It is supposed to be a facilitator for the students to make links between the microworlds and the techniques they already know for solving subtraction problem.

Conclusion of the analysis and the selection of the microworlds

The analysis and the selection of microworlds has been a progressive process. Due to the attractive design of the software, we decided to work towards an experiment at the elementary level. We wanted to involve at most two microworlds to favour interactions between them and to provide a sufficiently rich "milieu". Regarding to the previous elements illustrated above, two microworlds were finally selected: “euros” and “abacus”. The language's obstacle is partly avoid by the realistic representation of the coins and the banknotes in the euros world and the small number of messages provided by the both can be easily explain to the students. They have feedbacks of different type and they are complementary in terms of educational goals and techniques involved. Their modalities of employment seems also rather similar.

Conception of the experiment

As we have though of the conception from the beginning of our work, we have paid a particular attention to it when we found a class for the experiment. After presenting our work on the tool, we present now our management of the conception of the experiment.

The chosen subject is the notion of subtraction at the grade 2. For these pupils, what is at stake at this time is the extension of the field of numbers towards numbers greater than 100 and the preparation of the algorithm for subtraction through the development of personal techniques.

The theoretical frameworks are implied in many aspects of this experiment and we have tried to isolate their main roles.

the anthropological approach

- for the identification of the different institutional techniques of subtraction and the associated type of tasks. The theory permits to describe the most economic and efficient techniques with regard to the personal techniques or more contextualized (here in the context of the euros). According to this frame, the multiplicity of techniques proposed to the pupils favors the conceptualization of the subtraction.

the TDS

- for the importance of the processes of devolution of tasks to the pupils, the formulation of the solutions, the decontextualisation and the institutionalization phases. The importance of the milieu and the feedbacks that we have discussed above.
- the instrumental and ergonomic approaches for the consideration of the instrument in the construction of the knowledge, the long processes of the instrumental genesis, the instrumental distances, the instrumented techniques and paper and pencil techniques.

Toward several techniques of subtraction

The purpose of the experiment is to make pupils meet several *techniques* of subtraction, to give sense to these techniques and to go towards the expert technique by looking for the most effective. The new mathematics syllabus for elementary school asks teachers not to limit to the canonical decompositions using the decimal numeration and to encourage diversity in the use of decompositions. The goal is to prepare the automatization of calculation without penalizing the flexibility which is necessary to mental calculation or to what it is called “calcul réfléchi”.

AriLab permit to build a situation where the choice of two microworlds is offered to pupils: euros and abacus. Pupils can solve the same subtractive problem within the two microworlds. They can reflect about which one is the best adapted to the problem knowing that each of them favorize the development of a specific technique.

The first technique in the euros microworld is to decompose the first term of the subtraction in order to obtain the second and his complement as the result of the subtraction. This technique is relied to the breaking of banknotes.

The second technique in the abaccus microworld is closer from an expert technique of subtraction taught in France. With those, the decompositions are based on the decimal system without allowing

however the representation of the carry over phenomena. The *instrumental distance* from the expert technique is thus fewer with the second technique than with the first one.
To oblige pupils to use the second technique, it is necessary to exceed the domain of the numeric competence of the pupils in situations of subtractions. The pupils have already met the situations of subtractions on small numbers by counting for example whith the help of their fingers (technique of the "countdown" or the complement for example). These techniques are not anymore adapted if we propose higher numbers. It's a typical use of the concept of didactical variable in TDS.

The problem 1 (see appendices) is explicitly a problem in an euros context. Nevertheless, we think that it is more easily resolved with the abacus microworld. The problem 2 is a problem in a totally different context (problem with trains) playing with the same didactic variables. The succession of both problems would allow pupils to realize that the expert technique is the most effective (in term of time and automatization of the resolution) with the numbers provided.
The work in both microworlds allows to supply a richer "milieu", especially as regards to the feedbacks. Indeed, to resolve the same subtraction with the same numerical values in both microworlds is going to lead pupils to make comparisons. The comparisons constitute a new feedback by itself.

Scenario: general structure
Thus, also regarding to the local constraints and the teacher of the class, we have prepared 3 sessions about 30 to 45 min. each: 2 for discovering of each of microworlds and 1 final session for the aimed problem. Each of them are led by a researcher of our team and only 5 students (supposed to not have too much difficulties) chosen by the teacher have participated to the experiment.

The first 2 sessions are structured in the same way based on theoretical considerations:
- presence of a phase of familiarization of 2 microworlds which is not exclusively centred on the features but embarks mathematical knowledge. The instrumental approach leads us to mix both the mathematical knowledge and some elements of the instrumentalisation process. During these phases of familiarization, feedbacks are put well in evidence to insure the pupils identify them.
- devolution of one or two simple situations more or less similar to the one of the phase of familiarization and individual work of the pupils. The pupils confront with feedbacks to resolve the situations.
- formulation and explanation by the pupils of their solutions, this collective elaboration of the solutions allows to homogenize the knowledge (institutionalisation) and the personal geneses.
  Being aware of the fact that the duration of an instrumental genesis is necessarily longer than this micro-experimentation, we tried to accelerate the process of instrumental genesis at every stage of the experiment.
- paper documents with screenshots (see appendices) were distributed to illustrate the instrumented techniques
- sometimes, the situations proposed to the pupils required mathematical adaptations (notably more stages) with regard to the mathematically simple situations given in the phases of familiarization.
- a vocabulary specific and adapted to the pupils was invented to indicate the various components of the microworlds: "distributor", " brown table", "counter", etc.
- the collaboration between students has been favorized to permit to find solutions in a reasonable amount of time and motivate discussions on the different strategies used by pupils.

The sessions: session 1
Familiarisation with the euros microworld

a) The teacher distributes the document 1 (see appendices)
b) He shows how to exchange a 100€, exactly as on the document distributed to the pupils.
c) He explains the feedback when one makes a mistake.
d) He announces the task and distributes the sheet to be completed. Devolution of the situation, research phase. Individual task: find 125-90
e) Every pupil reports on its solution. It is necessary to understand that there are several solutions. The teacher points out that there are several solutions according to the pupils but that the final result is always the same.

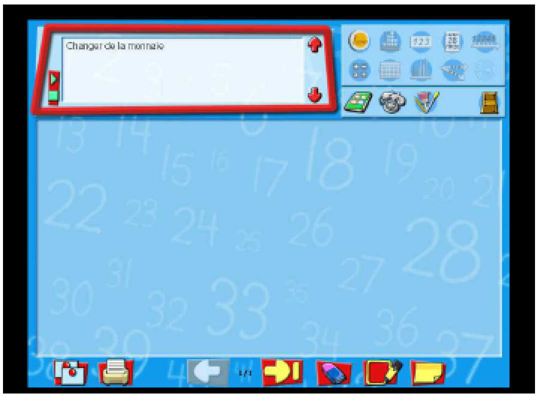
The sessions: session 2
Familiarisation with the Abaccus microworld

a) The teacher distributes the document 2.
b) He shows how to make 125 on the abacus.
c) He shows to remove 20 for example, or 25 but explain the problem he wants to remove 90 as when we had 125 euro. Reference to session 1.
d) He explains how to exchange hundred from 125 to 10 dozens.
e) He stops explaining how to remove 90 - 125
f) Reference to the session 1
g) Individual task : 267– 78
h) The teacher annonce the result and make links with the first session. The question of the best microworld for a problem is posed.

The sessions: session 3
The third session is dedicated to the resolution of both problems (euro and train). The problems are devolved to the pupils and the pupils are going to work alone and to choose the microworld which they prefer. The pupils can consult documents distributed in the previous 2 sessions. The subtraction to be made is in both cases 541 - 175.
A commun moment is led at the end to go towards an institutionalization of the expert technique.

Partial conclusion
We didn't recall here what we have already written in the previous report about the collected data and our first results as it is still a work in progress.

3.3.3 Appendix 1 : Notice of use and activity sheet for session 1



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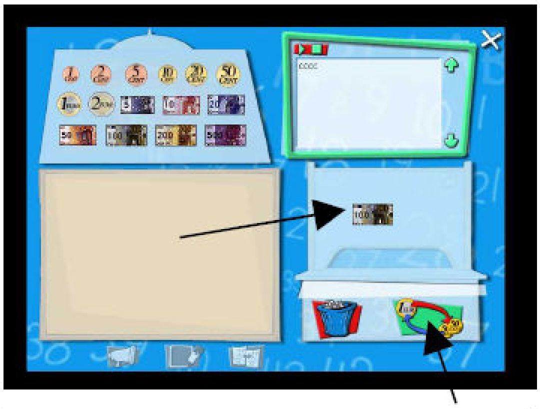

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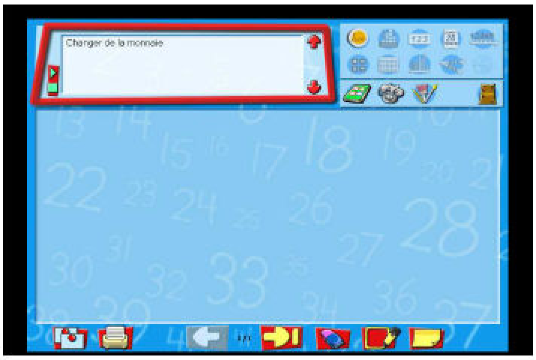
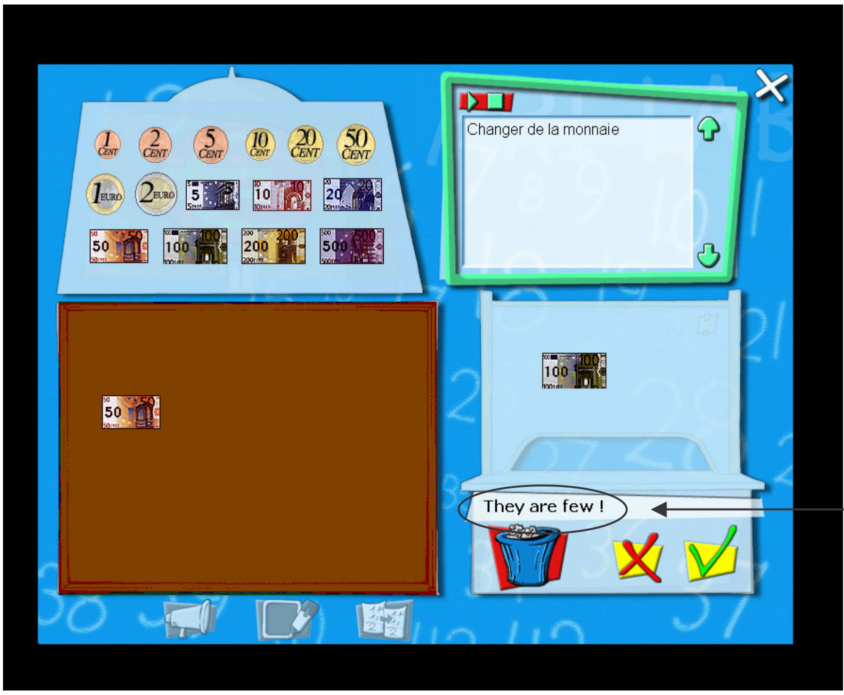


8



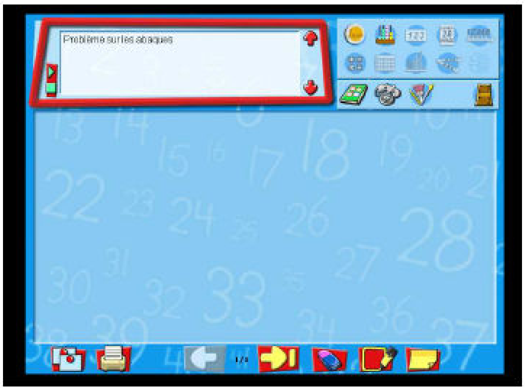
Pas assez de billets sur la table marron

Trop de billets sur la table marron

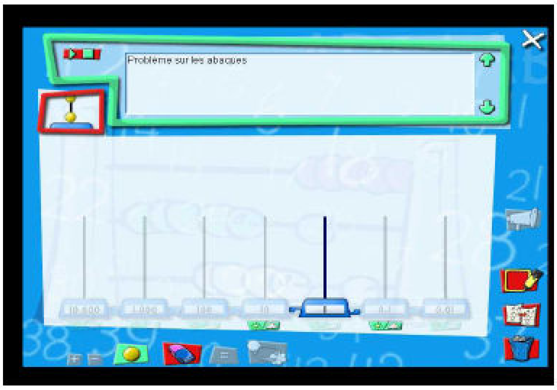
Complete on the brown table so that the exchange is possible:

You have the following banknotes and you want to spend 90 euro. How do you do?

3.3.4 Appendix 2: Notice of use and activity sheet for session 2



1



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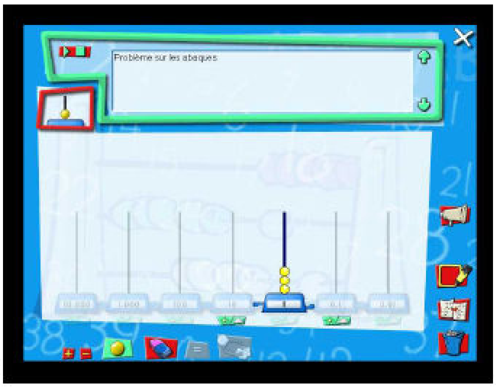

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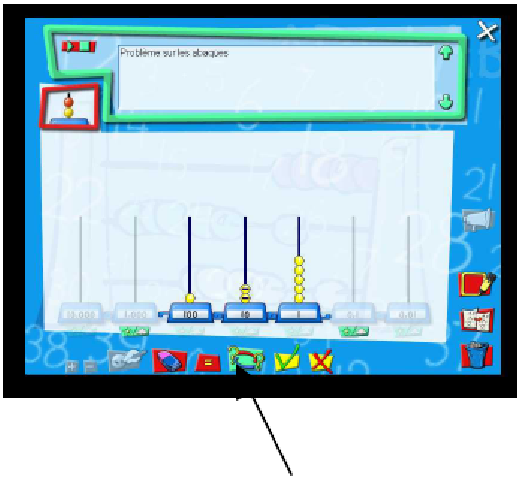
7

We want to make 125 minus 90.
How to do?


Problème sur les abaques

8


Problème sur les abaques

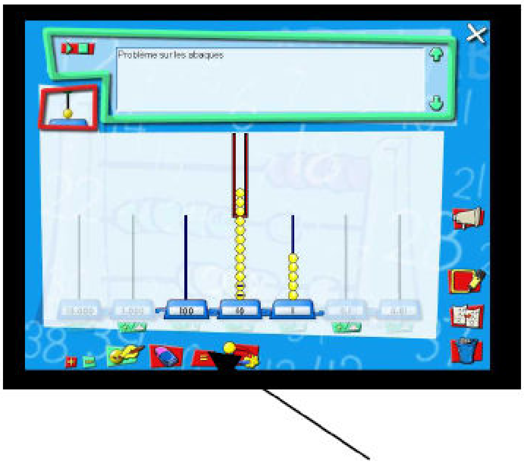
9


Problème sur les abaques

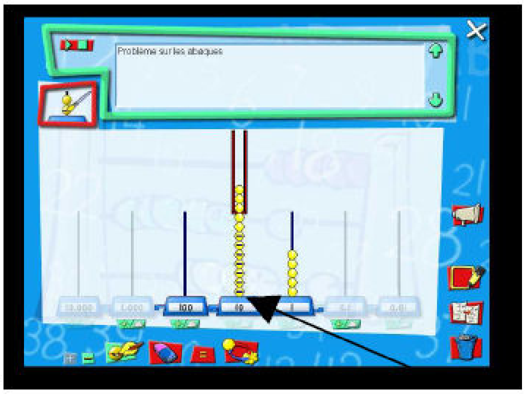
10


Problème sur les abaques

11

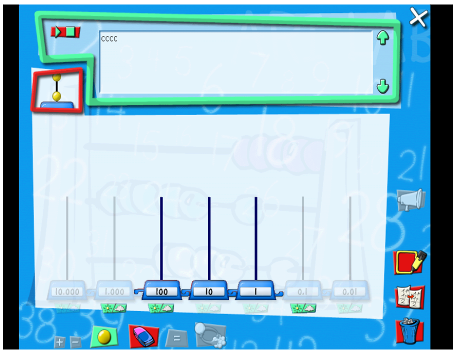

Problème sur les abaques

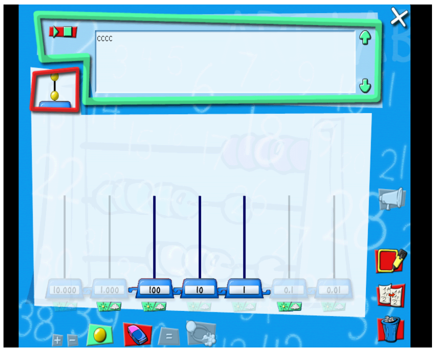
12

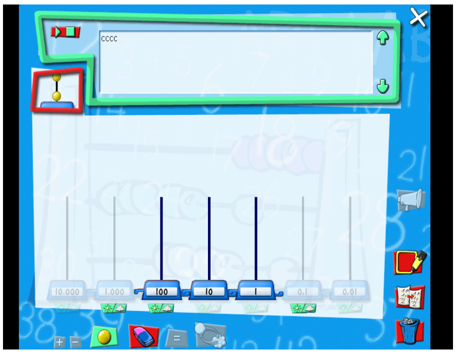

Problème sur les abaques

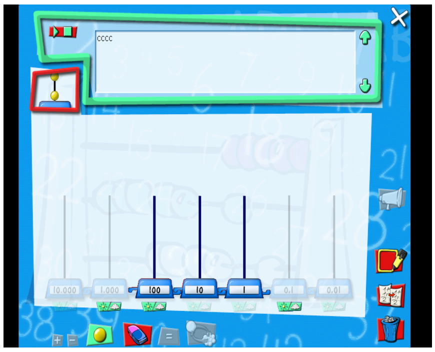
13

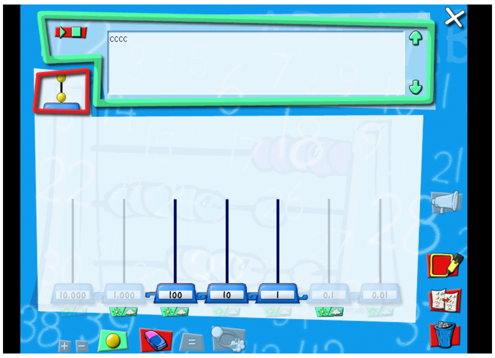
Use abacuses to make 267-78 and explain every stage by drawings below:

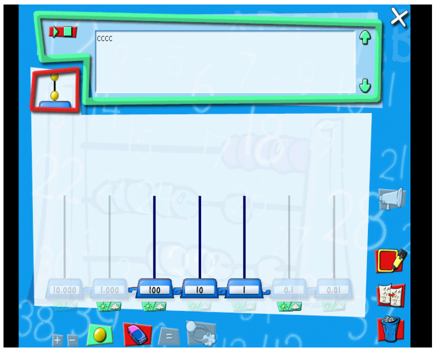
1 

2 

3 

4 

5 

6 

3.3.5 Appendix 3 : problems 1 and 2 for the session 3

Problem 1:

I have 541 euro and I spend 175. How much does it remain me?
Explain how you do.

Problem 2:

There are 541 passengers in a train and 175 go down. How many passengers does it stay in the train? Explain how you do.

3.4 Experimenting team 3: ETL-NKUA *(Psycharis, G., Latsi, M., Gavrilis, K., Keisoglou, S.)*

3.4.1 A priori (with respect to the experiment) questions aiming at collecting information concerning the design of the experiment.

- General:

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

The main ideas behind the theoretical frameworks used by ETL refer to constructionism (Harel & Papert, 1991), to sociocultural approaches (Crook, 1994), to situateed abstraction (Noss et al 1997), to semiotic mediation, and instrumentalisation (Vygotksy 1997 , Mariotti 2002, Verillon & Rabardel 1995, Artigue 2002)

This choice is motivated by our estimation that both theoretical frameworks seem to be bringing into the foreground some basic for us issues in the process of learning and teaching: a) the role of the social setting where the learning activity is integrated b) an active role for the students during knowledge construction c) acknowledgement and investigation of the role of tools and representations in the process of learning and teaching.

The potential of these frameworks is related with our estimation that they seem to provide a different point of view –that of the tools and the representations- in the learning process taking into account the role of the social setting where this process takes place.

The limitations entailed in the selection of a (/or more) theoretical frameworks is that it focuses on some aspects and leaves outside some other aspects of the subject under investigation. Those will be better illustrated at the end of the experiment.

- Analysis of AriLab tool

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

Using the fractions' microworld students get only phenomenological and arithmetic cues based on Thales' theorem. The feedback provided by the microworld of the arilab "Fractions" is symbolic and arithmetic, through the use of number half –line and the multiplication or partition half line. So it's students' responsibility to act, to choose the right microworld - from those available, so as to solve the problem at hand-, to experiment and to validate the results of their experimentation in the microworld. Finally, they have to choose the representation that they think as right and appropriate to be incorporated in their solution sheet. At a second level students' solution can be validated by their co-students or by their teacher.

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

The 'fraction' microworld gives students the chance to explore rational numbers interacting with a graphical representation model based on Thales' theorem. Students can build fractions on the number half-line and make operations with fractions working with lengths selected on the number half-line. The symbolic – fractional notation, which is given automatically, is interwoven with a kinesthetic approach of the graphical representation.

Although the graphical representation could be produced easily in paper and pencil didactical situations, the microworld gives students the chance to experiment easily with it, to have a more clear, accurate and attractive graphical representation which is easily modified and manipulated. Moreover, one of its great advantages is that it connects the graphical and kinesthetic with the automatic mathematical notation. As a result the 'distance' between the objects and the means of manipulation provided by the microworld and those used in paper-and-pencil based work within the target domain could be considered important.

- Design of the Teaching Experiment
- *Describe briefly the key ideas of your experiment and then answer to the following questions*

What are the precise aims of your experiment and the questions you want to focus on?

The aim is to use Arilab so as to teach fractions and to investigate this mathematical notion. Interest will be focused on the meanings generated and structured by the interplay between learners' actions, available tools / representations and activities designed.

- What kind of meanings will be constructed by students in relation to fractions?
- How the software will be appropriated by students and which features of it will be catalytic in the construction of meanings?
- How the representations of fractions supported by the software might be involved in the generation of student meanings

What is the type of research that you follow (e.g. classroom based, case studies) and how is this related to the kind of your research focus;

Our research is informed by the socio-cultural theoretical perspective, which emphasizes the importance of culturally situated and socially shared activity, of discourse and of mediational means for learning. Within this framework we take the approach that the potential use of the technological tools is tightly related to the ways these will be shaped by practitioners in their respective roles in the school system. The type of research that we follow is mainly classroom based. The classroom activities are perceived as innovative for the actors involved since they consist of small group project work based on the use of exploratory software. In accordance with the socio-cultural perspective we are looking at meaning making as a process of interaction between people participating in communities and cultures. The focus of our research far from the typical classroom practice includes also the potential transformation by the use of ICT. This methodological tenet translates in:

(a) inducing the transformation that we wish to observe through pedagogical intervention;
(b) focusing our analysis on conceptually significant learning instances and patterns that illuminate what the activity could be shaped to become.

Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?

T he designed activities would take the scenario form which constitutes the basis of the method and strategy analysis according to which our team proposes to apply the educational activities in the classroom but also of the structure of collaboration among different groups (the classroom as a whole, small groups of pupils in the same classroom or in different ones). According to our experience the characteristics of the activities that they support the generation of meanings stem from the utilization of the different representations and the feedback that they can provide so as to provoke children:

(a) to work in open-ended exploratory tasks without single answers
(b) to collaborate with other groups for a common goal.

(c) to communicate with other pupil's work.

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

Gaining entry to observe in the classroom would be based on a relationship of mutual trust and interest that we have cultivated with specific schools in the course of a long term collaboration in research projects. For gaining detailed access to pupil's work the team has developed a data collection scheme recording classroom activities as they happen in real time. In the classroom we will set a video-camera and one microphone on one group of students who would be our focus (focus group), occasionally moving the camera to capture other significant instances as they occurre in other groups' work. Concurrently with the video-recordings, we should take notes describing the overall classroom atmosphere and focusing on potentially significant details that may capture our attention in teacher practice, student groupwork and student communication.

The analysis will include the transcription of video-recorded observational data as well as the sorting and archiving of the corpus of pupil's work. For the analysis we adopt a generative stance (Goetz and LeCompte, 1984) allowing for the data to shape the structure of the results and the clarification of the research issues. In our analysis, we will identify critical episodes, i.e. moments in time which have particular and characteristic bearing on the pupil's interaction with the available tools accompanied with the constructed mathematical meanings. We will use these as the main means of presenting and discussing the data, taking into account that even though they do not represent some quantifiable entity, each one would be chosen to represent clearly the kind of activity that was going on in that classroom.

3.4.2 A posteriori (with respect to the experiment) questions aiming at individuating, collecting and gathering the results of the experiment.

In the context of ETL's participation in the common TELMA project of cross-experimentation our team has chosen AriLab2. The approach of ETL was to design tasks aiming to facilitate pupil's interactions between the intuitive, the formal and the procedural aspects of mathematical concepts within the conceptual field of fractions in processes of engaging in meaningful activities. Our task design was thus centered on the utilization of the different representations and the feedback that they can provide so as to provoke multiple decisions within open-ended exploratory tasks concerning the equivalence and the ordering of fractions as well as operations with fractions. The idea was that instead of seeing the numbers on the real line as static measures corresponding to specific points to include them in a context of motion in the space. This was achieved by interpreting the feedback concerning the position of fractions on the line as feedback concerning the representation of a distance in an everyday context. The above choices led to a set of activities based on integrating the kinesthetic aspects of the representation of division with pupils moving ('walking') on the real line to reach authentic places (e.g. their homes, their schools, a playground, a supermarket etc.).

Is there any difference in the answers that you gave during the a priori analysis at the following questions?

We will try to answer this question having in mind a similar one: "If we were to design a new experiment in the future aiming at the same mathematical educational goal and employing AriLab2, which would be the necessary conditions for the experiment to be successful?" The conceptualisation of AriLab2 by our team was closely related -and thus limited- both to the task design and the theoretical origins underlying our research approach. After the experiment it is clear that we have formed a more elaborated view about the added value as well as the limitation of the educational exploitation of the representations and functionalities provided by AriLab2.

As far as the methodology of the research due to time and other constrains we adopted a case study approach. However, interaction with mathematical representations is not by itself sufficient for effective learning. Students need to make sense of their experience of manipulating representations in the context of social interaction. In retrospect we think that the choice of conducting case studies deprived our experiment of the more fruitful social interactions that could have taken place in the context of classroom related to the interpretation of the phenomena observed on the screen not only within the teams but also among the teams. It could be put forward that under teachers' guidance in the context of a classroom pupils' experiences would be better consolidated and unified and that the inevitable gap between mathematical meanings related to fractions and computer phenomenology could be further exploited.

As far as the general design of the experiment we would keep the same main directions, namely the tool's characteristics, typologies of activities and educational strategies. However, gaining the experience of the micro-experiment we could elaborate in a different way some of the parameters related to the above parts of the research. More specifically, the multiple ways by which the software supported the geometrical representation of fractions as parts of the number line could further challenge the assumptions about what and how can be 'normally' taught in the primary school. Although we used experimentation and exploration to acquaint students with the ordering of fractions in the number-line –a representation neglected in greek primary school curriculum, though considered as basic- we feel that we have bypassed the exploitation of other kinds of representations available in AriLab2. For instance, we could have focused more on the construction of fractions in relation to the geometrical representation based on Thale's theorem of dividing certain parts of the number-line in equal segments. Although Thale's theorem is typically a theorem taught in an abstract way in secondary education, we think that the way it is related to fractions and 'concretised' by AriLab2 makes it accessible by younger students even though in a more intuitive and informal way.

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

Before answering the question we shall note that the role of feedback is central to the ETL's approach in exploiting computational environments with integrated multiple representations and functionalities for the teaching and learning of mathematics. The choice of AriLab2 was partly based on the experience of ETL team in the design and implementation of a series of microworlds that combine symbolic expression of mathematical relationships with dynamic manipulation of graphics. Given this experience, the NKUA team is used not to focus on mere execution of computational techniques but on the connections made by the students between mathematical situations they are dealing with and the ways in which they use the available representations to construct meanings for them.

In the Fraction microworld the provided feedback combines arithmetic as well as geometric aspects of the notion of fraction represented by using two half number lines: one horizontal and one slanted (called multiplication or partition number line). More specifically, the construction of a fraction is realized as a quotient of a division: the divider and divisor are selected from the horizontal and the slanted lines respectively. The arithmetic notation of each fraction is automatically given near its representing point on the line. In AriLab2 there's a thus a simultaneous representation of fraction as quotient, measuring number (point on the number line) and operator. Another geometrical feature of the fractions in the Ari-Lab2 concerns its vector-like representation of the respective segment on the horizontal line.

Other features of Ari-Lab2 like the labels' button for representing the number corresponding to a defined fraction and the ability to change the size of the unit on both lines, are important for the visual imagery and manipulative aspects of the tool. Key elements in the available representation are:

- the representation of fraction as a point on the number line.
- the possibility to modify dynamically the unit of measure of the two half-lines.
- the association of a post-it to every point constructed on the two half lines. In every post it the system automatically inserts the symbolic expressions related to the construction of the point performed by the user interacting with the microworld.

The feedback provided by Ari-Lab does not validate student's actions. Neither does it signal mistakes, nor does it give definite answers. The feedback given to students comes as a result of their interaction with the representations and the mathematical content integrated in the 'Fraction Microworld'. In our experiment students' actions and the solutions suggested in relation to the task that they carried out were validated at many levels:

- self-validation after testing their conjectures and hypotheses with the graphical and arithmetical representations provided by the software
- validation of the answers reached by pen and pencil using the microworld and vice versa
- peer-validation after discussion and argumentation
- teacher-researcher validation (although an effort was made to restrict this kind of validation as much as possible)

The data analysis revealed that most of the pupil's decisions concerning the role of feedback came as a result of peer-validation and discussion among the members of each team. This finding seems to be mainly related to the open-ended nature of the tasks which were designed to leave space for multiple choices. In this context AriLab2 was exploited as a 'tool to think with' and as a field of experimentation, testing and exploration.

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

AriLab offers new kinds of access to the static representations of fractions in a number-line as well as new kinds of dynamically connected representations. In the 'Fraction microworld' students can build fractions on the number half-line and make operations with fractions working with lengths selected on the number half-line. Although the graphical representation of fractions can be produced easily in paper and pencil didactical situations, the microworld gives students a more clear, accurate and attractive graphical representation of fractions providing also the chance to connect it with with the correspondent mathematical notation as well as to modify and and manipulate it easily. As a result the 'distance' between the objects and the means of manipulation provided by the microworld and those used in paper-and-pencil based work within the target domain could be considered important.

Our team deeply explored the different objects and the associated representations, possible actions on these objects and some evaluation of the 'distance' between these objects and representations and those unfamiliar to pupils due to curricular constraints. For example, the interpretation of a fraction within part-whole relations is the first and probably the most dominant facet of the concept presented to students at the primary level. Due to the efficiency in arithmetic operations the mathematics curriculum provides a mechanistic use of fractions in calculations which leaves obscure all the other aspects of the concept especially those concerning fraction as division as well as the ordering of fractions on the real line. As a result students learn to manipulate fractions in calculation tasks at a typical level, without understanding the connections between their different representations.

Our analysis revealed indications that this kind of 'distance' between the "curriculum mathematics" and those embedded in computational environments like AriLab can be bridged with pupils' intuitions in a context of experimentation and exploration. For instance, a group of pupils in our experiment constructed meanings related to the notion of infinity of rational numbers and numbers' continuity on the number line which are considered abstract and they are normally taught at the secondary level. Using the tool a group of pupils realized for the first time (although not so clear) that several numbers exist between 6/4 and 7/4.

537.	R	Good. Can it be here? [She points to the segment between 6/4 and 7/4.
538.	S1	Yes.
539.	R	Where is it? [The kids point at the Arilab2 4 different places after 6/4.]
540.	S2	There can be more…
541.	R	How many?
543.	S2	Too many!
544.	R	1000 maybe?
549.	S1	…More than that.

Excerpt 1. Pupils' using AriLab2 to construct meaning for the infinity of numbers.

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

Espousing an instrumental approach on the way we capture/analyse the tool we believe that a tool does not exist in itself. In contrast, it evolves as an instrument in the perspective of its utilisation by individual or social groups in the context of specific activities and for specific purposes. It follows that the role of AriLab2 is interwoven with the sociomathematical norms of greek primary schools, the activities that children carried out and the utilisation schemes they developed.

Due to the time needed for a class to get used to Arilab2 research team preferred a case study instead of a classroom based research. The micro-experiment took place at the computer laboratory of the school. We choose 4 students, two boys and two girls divided in two working groups (G1 and G2) consisted of one boy and one girl each. Each pair of students was assigned to one computer. They were asked to collaborate in the solving of the given problems, to express their ideas aloud, to assist each other in understanding what they were doing, and to take as much time as they needed. The researchers intervened as less as possible, asking questions to promote discussion while not giving definite answers. In the classroom we set a video-camera on one group of students occasionally moving it to capture other significant instances as they occurred in the other groups' work which was captured by the use of a tape-recording. Data collection included verbatim transcription of video recordings of our focus group (G2) and transcripts of protocols of the other group (G1). Concurrently with the video-recordings, the researchers took notes describing the overall atmosphere and focusing on potentially significant details that captured their attention in student groupwork and communication.

For the analysis we adopted a generative stance (Goetz and LeCompte, 1984) allowing for the data to shape the structure of the results and the clarification of the research issues. The identified critical episodes can be defined as moments in time which have particular and characteristic bearing on the pupil's interaction with the available tools accompanied with the constructed mathematical meanings. In future, we will use these episodes as the main means of presenting and discussing the data, taking into account that even though they do not represent some quantifiable entity, each one would be chosen to represent clearly the kind of activity that was going on in that classroom.

Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?

Several times during the experiment students recurred to paper and pencil in order to solve the problems. Analyzing the collected data we identified 9 episodes showing that students of both groups (G1, G2) had used and other modes of representations, except those provided by the tool. The following table presents the number of 'paper-and-pencil' episodes for each group and each activity respectively.

	1st activity	2nd activity	3rd activity	4th activity	5th activity	6th activity	Total
G1	3	0	0	0	0	1	**4**
G2	1	0	1	0	1	2	**5**
Total	4	0	1	0	1	3	**9**

Table 1: Number of 'paper-and-pencil' episodes per activity and group.

The 9 episodes have been divided in 3 different categories concerning their function as alternative problem solving strategies performed by students:

Category 1: Working with integers (4 episodes).
Category 2: Doing arithmetic operations (like addition, subtraction and reduction of fractions) (3 episodes).
Category 3: Making drawings (2 episodes).

The episodes included in category 1 concerned mainly pupil's attempts to convert fractions to integers, so that they could be more easily manipulated. Since our tasks had to do with kilometers and students tried to convert kilometer into segments (meters) by multiplying with 1000 without thinking of the kilometer as a unit that could be divided into fractions (Figure 1).

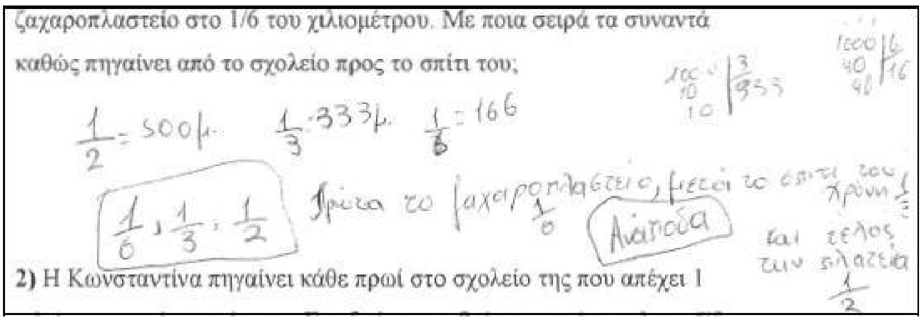


Figure 1: Students convert fractions to kilometers in order to solve the first problem.

Additionally, due to certain representations and funcionalities of AriLab2 both groups seemed to prefer the use of pen and paper in order to have the result of an operation (Figure 2) in the form of a new fraction (e.g. 1/3+1/2 = 5/6) (Category 2) since AriLab2 provides only a label including only the first part of a calculation (e.g. 1/3+1/2).

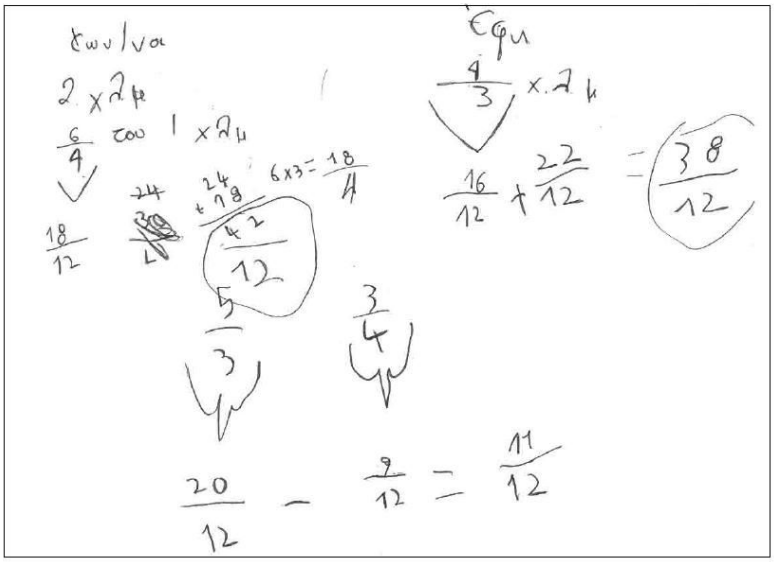


Figure 2. Students making calculations with paper and pencil.

The episodes of the category 3 indicate that the drawings made by students were used as means of conceptualization and visualization of the problems. The following example (Figure 3) is presented as an indicative example of the way drawings were used by the groups of students in their efforts to conceptualize certain aspects of the tasks. In this case pupils decided to make their own number line on the paper with more information concerning the problem situation. We observed that this kind of episodes appeared only at the beginning of the micro-experiment (specifically, during the activity 1 in the group G2), so it may be reasonable to assume that at that time students were trying to bridge the representation of fractions in AriLab2 with their conceptualisation of the given tasks. At later phases of their exploration these pupils seemed to focus only on the number line on the screen.

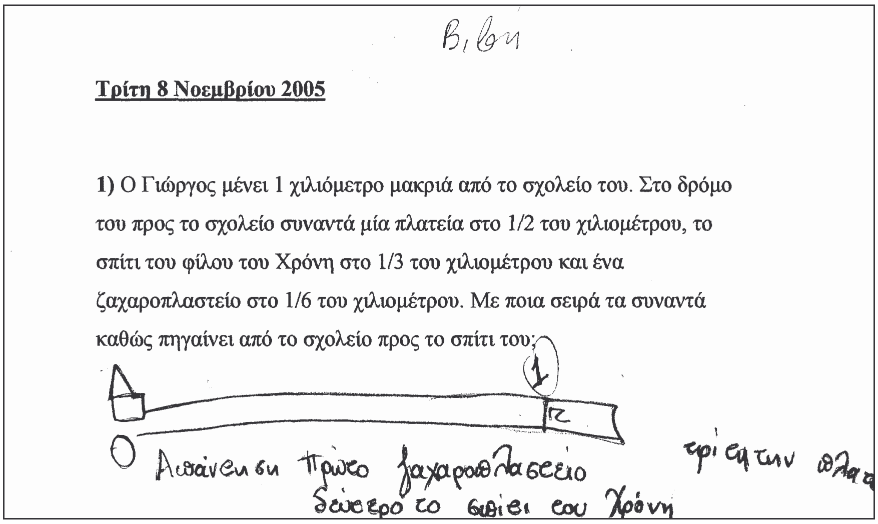


Figure 3: The 'number line' as an iconic representation including topological information (design of school, home and pupil's comments).

In your opinion, in which ways do your theoretical choices have influenced:

a. the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employment including the configuration of the software)?

b. the conception of the experiment?

c. the choices of the data and their analysis?

d. the results you obtain and the conclusions you draw from these?

4 E-slate microworlds

4.1 Developer's team: ETL-NKUA *(Psycharis, G., Latsi, M., Gavrilis, K., Keisoglou, S.)*

- *Brief description of the instrument, explaining its key ideas*
- *Indicate the theoretical framework employed to design and implement the software*

ICT systems used by NKUA team are E-slate microworlds. E-slate is a construction kit for educational software, addressing both technical and non-technical users. It is made up of components, which are pieces of software designed to be as generic as possible, a custom made desktop environment providing services for construction, layout and use, and two distinct ways of connecting components, that is, either using prefabricated connections or defining your own through a programming language. The'Sliders' microworld is based on a script to define functional relationships between the values of slider buttons.

The design and use of such technological environment have been highly influenced by the constructionism and socioconstructivism paradigms. In particular, there are three central points to e-slate design rationale:

- User empowerment by building educational software through authoring in component environments,
- Socially- grounded approach to the building of user communities which is seen as part and parcel of the strategy fro the development and use of component environments,
- Long-term sustainability, as crucial for the component movement to progress, but also as constituting its real strength in relation to standalone technologies for computational media.

A special attention is also placed on how the use and the affordances of artefacts interact with students' mathematizations in problem situations in real classroom settings. The NKUA team studies the ways students use representations to express mathematical meaning in E-slate microworlds. In order to understand student meaning generation the team has found the constructs of 'situated abstractions' (Noss and Hoyles, 1996) and classroom norms (Cobb & Yackel, 1996) useful tools.

4.2 *Experimenting team 1: UNILON* *(Morgan, C.)*

4.2.1 A priori (with respect to the experiment) questions aiming at collecting information concerning the design of the experiment.

- General:

What theoretical frame(s) do you use and what motivated your choice? How do you see their potential and eventually limitations for this project?

The focus of our research is shaped by a socio-cultural and social semiotic theoretical framework. This assumes that the meanings available to participants in a particular activity or setting are structured by the semiotic resources available and by the contexts of situation and of culture.

In studying teachers' and students' use of the E-Slate tool in the classroom we are interested to see how the new semiotic resources available to the classroom participants are taken up and coordinated with other, more familiar, means of representation. Specifically, we have been looking at the ways in which the 'slider' representation of fraction is employed through physical manipulation and through oral and written means of communication.

- Analysis of Aplusix tool

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

Design of the Teaching Experiment ***(note: this has been answered a-posteriori)***
Describe briefly the key ideas of your experiment and then answer to the following questions

- *The tasks given to students included:*
- *deciding which of two fractions was the larger*
- *finding a fraction between two given fractions*
- *making an estimate of the decimal representation of a fraction*
- *deciding whether a fraction is closer to ¼ or to ½*
- *explaining methods for achieving the above tasks and justifying answers*

Use of the microworld was introduced by each teacher at the beginning of each lesson using an interactive whiteboard to demonstrate/ remind the students how to use the microworld and to discuss with the class the kinds of reasoning that might be relevant to the tasks. Students were then set tasks on paper-based worksheets. The structure of these worksheets differed between the two teachers. Teacher A set a number of repetitively structured tasks (e.g. 15 examples of the type "Find a fraction between x and y"), while teacher B set a smaller number of more substantial but varied tasks. The pedagogy in classroom A was more strongly framed: the teacher controlled the pacing of the lessons, introducing one type of task at a time and reviewing the answers with the whole class. Students in the classroom B were allowed more independence to move from task to task at their own pace and to evaluate their own outcomes; a brief review at the end of each lesson addressed general outcomes and strategies rather than specific answers.

In both cases, tasks were presented to the students in traditional forms, using familiar notations but with the instruction to "use the sliders to find …"

What are the precise aims of your experiment and the questions you want to focus on? ***(note: this has been answered a-posteriori)***
The learning objectives decided by the teachers involved in the experiment were related to developing students' understanding of the relative sizes of fractions (initially focusing on fractions less than 1).

What is the type of research that you follow (e.g. classroom based, case srudies) and how is this related to the kind of your research focus; ***(note: this has been answered a-posteriori)***
We used classroom-based research, studying two classes in different schools. In school A, the students were in Year 7 (11-12 years) in a 'mixed ability' class; in school B they were in Year 8 (12-13 years). In school A, a single lesson with a Year 8 group of above average attaining students was also observed using the microworld but complete data was not collected for this class.

The two teachers involved in the experiment are both students on the Masters course in Mathematics Education at IoE. They were fully aware of the aims of the research and took responsibility for the planning of the lessons and the collection of data. They will also be invited to respond to a presentation of the initial analysis of data. The learning objectives were decided in discussion with these teachers and they worked together to adapt the microworld to their needs and to decide the general type of activity that they would offer to their students.

However, the two teachers have rather different teaching styles, resulting in differences in the specific form of pedagogy in the two classrooms (as described above). Their two schools also integrate technology in rather different ways: in school A all students have their own laptops which are carried to all lessons and used as needed; in the school B, special arrangements had to be made to take the class to a computer room where they worked in pairs.

We consider these two classrooms to be case studies and they are expected to yield different results. The pedagogy as well as the student activity will be subject to analysis, focusing on teacher use of the tool and the means of representation that it offers.

Which characteristics of the activities and tasks do you think they support the generation of meanings in a constructionist or experimental or even playful way?
How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

4.2.2 A posteriori (with respect to the experiment) questions aiming at individuating, collecting and gathering the results of the experiment.

Is there any difference in the answers that you gave during the a priori analysis at the following questions?

What forms of feedback are provided? How are solutions validated and by whom (e.g. by the tool itself, by a teacher, by peer- or self-validation?

What is the "distance" between the objects and the means of manipulating provided by the tool and those used in paper-and-pencil based work within the target domain?

How do you capture/analyse the role of the tools in pupils problem solving processes or solutions?

The data collected includes:
Students' written responses to a pre-task.
Lesson plans for each of the lessons in which the tool is to be used, including descriptions of activities both using the microworld and using other resources.
Copies of paper-based resources used in each lesson.
Students' written work produced during each of the lessons.
Audio/video recordings of some teacher-student and student-student interaction.
Teachers' notes, reflecting on the experimental lessons and their outcomes (not yet collected).

Analysis will identify the forms of representation of fractions and relationships between fractions employed by teachers and by students. The strategies used by students to solve set tasks will be categorized. A characterization of the representations and strategies present in each classroom will be constructed and we will seek relationships between them.
Do users also use other modes of representation not provided by the tool itself (e.g. paper-and-pencil representations, calculator)? What are these and what does their function appear to be? How do these modes of representation relate to those provided by the tool?

In your opinion, in which ways do your theoretical choices have influenced:

a. the analysis of the software and the identification of its didactic functionalities (software features, educational aims, modalities of employment including the configuration of the software)?

b. the conception of the experiment?

c. the choices of the data and their analysis?

d. the results you obtain and the conclusions you draw from these?

Our social semiotic perspective leads us to focus on the sign systems and meaning potential available within the learning context. Introducing a new semiotic tool into a classroom introduces new meaning potentials afforded by the tool and participants' interactions with it. This introduction cannot be considered in isolation but can only be understood in relation to other resources within the immediate situation and the wider cultural context.
In conducting the cross experimentation, this perspective leads us to reject a conventional 'scientific' approach to experimentation that would attempt to define and control all the variables in the situation. This is partly because we believe such an approach, in its pure form, to be fundamentally impossible in research in the social sciences (though approximations to it can yield useful results). More positively, we consider that studying a situation while taking into account and analysing both the immediate context and the discursive resources that participants bring with them from the context of culture yields insight into the processes and complexities of meaning making.
The experiment was thus conceived in collaboration with two teacher-researchers at two levels. At the first level, each of the teachers worked with their own class to investigate the use made by students of the tool and the representations it afforded. At the second level, the data collected in both teachers' classrooms provide opportunities to consider how the different contexts of situation and of culture may affect the types of meanings constructed by both students and teachers while using the same tool with broadly similar didactical functionalities.

Analysis of the software and the identification of didactical functionalities

The E-Slate software provides a variety of types of components that can be linked and manipulated by users. In the context of this experimentation we have made use of a small microworld, built within E-Slate, provided by the developers. This microworld includes 'sliders' whose behaviour is governed by a set of Logo procedures. These procedures link the values displayed on dependent sliders to the value of the control slider.[14] There are also Cartesian graph components which display plots of the values displayed on each of the dependent sliders against the value of the control slider. While this microworld could in principle be used to explore a wide range of types of functional relationship, for the purposes of the cross-experimentation it was decided to focus only on multiplicative relationships between the control slider value and the values of the dependent sliders, using these specifically to address the area of fractions.

Features of the software

Our analysis of the features of the software focuses on the semiotic resources it provides and the meaning potential offered by these resources. A significant characteristic of the microworld (and even more so of E-Slate itself) is its multi-semiotic nature. In other words, it makes use of several different semiotic systems, specifically:

Linguistic and symbolic (algebraic and numerical) systems in the context of the Logo procedures;

The slider components (these could themselves be considered multi-semiotic as the sliders can be labelled with numbers);

Cartesian graphs.

Additionally, the E-Slate environment makes use of:

Linguistic resources in menus;

Linguistic, numerical and iconic (in the form of check boxes) systems in control panels for settings of the components;

Icons for selecting components;

Other graphic features such as colour, font size, component size.

It is the first three systems listed above that provide the most obvious potential for mathematical meaning and it is primarily these that our analysis of didactic functionalities takes into account. In accordance with Halliday's social semiotic approach, applied to a multi-semiotic text[15] (O'Halloran, 2005), we wish to consider the ideational, interpersonal and textual aspects of the microworld as a whole. We achieve this by analysing each of the three main systems first and then considering the relationships between them. The questions we use to interrogate the text are adapted from those posed in more general terms in Morgan (2006):

What is a fraction as it is represented in this text?

What other mathematical objects are present?

What relationships are there between fractions and other mathematical objects?

What processes are represented in the text and who or what are the actors in these processes?

What forms of causality are represented?

What role is there for human agency?

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[14] The term *control slider* emerged at the development stage of the experimentation as the team struggled to make sense of the tool. An alternative term, *base slider*, was also used at early stages of the process but was not used by the time the classroom work started. This was not a deliberate decision but has to be considered significant for the way the relationships between sliders and their behaviour were represented in the classroom. The term *base* is used in other mathematical contexts (for example, exponential functions, vector geometry) to indicate the roles that various parts of a structure play in relation to one another and to the structure as a whole; its use thus focuses on the mathematical relationship between the sliders. The term *control*, on the other hand, does not come from mathematical discourse but might be construed as a link with discourses of other types of computer applications, for example, gaming. It focuses on the activity of the user and the physical, visual and technical aspects of the microworld rather than on the mathematical relationships.

[15] We consider the microworld as a 'text' in the sense of a socially coherent unit of communication (Hodge & Kress, 1988)

What roles or identities are available for the 'reader' (or 'user')? What relationship does s/he have to the author of the text and to the mathematics represented in it?

The analysis of the text cannot be undertaken completely abstracted from the context of its use, as the meanings readers/users make from a text are always mediated by the resources they bring with them. When analysing a priori we take into account the resources commonly available within discourses of school mathematics (as experienced by the target group of students in the lower secondary school in England) but remain aware that in practice other discursive resources, including those of everyday practices (e.g. computer gaming) may come into play.

Exemplification of analysis of software features

Educational goals

The choice of fractions as the subject domain and the initial adaptation of the microworld to address this domain was undertaken, with the help of the developing team, in order to fit with the general mathematical theme of the TELMA cross-experimentation. The identification of more specific educational goals was undertaken in collaboration with the teacher-researchers during joint exploration of the software to determine its functioning and its potential for classroom use. We were constrained by the expectations of the National Curriculum (DfEE, 1999) and the National Framework for Teaching Mathematics (DfES, 2001) as these are implemented in the two schools involved (though there are differences between schools in the ways they interpret the strictures of these documents). The novel representation of fractions offered by the sliders, especially the possibilities for dynamic linking of multiple fractions, led us to focus on students' developing understandings of comparative sizes of fractions. In terms of curricular learning objectives, the teacher-researchers adapted the software and designed lesson plans and supplementary resources to enable students to:

Compare the sizes of fractions (less than 1);

Arrange fractions in order of size;

Understand the effects of changing the numerator and/or denominator of a fraction;

Predict a fraction that will lie between two given fractions.

The multi-semiotic nature of the software with its alternative representations of fraction raised the question of how students would make connections between the different representations. We were particularly interested in connections with more conventional symbolic representations of fraction in numerator/denominator form. The open and constructive nature of the software would allow students to explore and address problems using trial and error validated by visual inspection of the outcomes of experimentation but we hypothesised that the links between the symbolic representation of fraction required to change the parameters of the Logo procedure and the visual representation provided by the sliders would lead to new kinds of understanding of the symbolic form and a learning trajectory from trial and error to symbolic reasoning. By expanding the semiotic resources available to students to make sense of fractions, the potential meanings they may make are also transformed.

Modalities of employment

As we said at the beginning, our theoretical framework leads us to study 'naturalistic' situations, taking account of contextual issues at the point of analysis. For this reason, although the teacher-researchers involved were informed of the aims of the study and had full access to TELMA documents, including those related to theoretical frameworks, their planning was not explicitly informed by these (except to the extent that the stated aim of the experimentation was to investigate students' representations of fraction and the ways they made use of these while working with the software). Of course, this is not to say that the ways they made use of the software, incorporating it into their lessons, was not theoretically informed. Both teacher-researchers are students on the MA course in Mathematics Education at the Institute of Education and have been introduced to theories of learning during their course. In particular, immediately prior to joining the project, both had been studying a course module on New Technologies in Mathematics Education, during which they were introduced to constructionism.

Any evidence of this influencing their planning?

In addition, the ways in which the teacher-researchers planned and taught was strongly influenced by the implicit theories of learning embodied in the discourse of the National Curriculum and official guidance for teachers and by the more local discourses of teaching and learning current in their various schools, departments, professional associations etc. as well as by everyday non-specialised discourses. As practitioner researchers, the teachers were engaged in research but simultaneously engaged in their professional role as teachers. Where they used resources from discourses of research, including those of theoretical frameworks, these were thus recontextualised (Bernstein, 1996), acquiring new types of meaning as they were put to new purposes within the practice of teaching. This may be illustrated by the fact that, although the two teacher-researchers discussed the software, the aims of the research, the learning objectives for the students and the sorts of task students might do and were provided with similar guidelines for the conduct of the experiment, the implemented modes of employment had significant differences. Some of these differences were clearly due to organisational, resourcing and cultural differences between the two schools. Others may be traceable to differences in theoretical orientation on the part of the two teachers.

Examples to illustrate different modes of employment

	Teacher 1	Teacher 2
Student-computer configuration	All students worked with individual laptops which they carried with them throughout the school day, using them as required in any curriculum area and for recreational purposes.	Students worked in pairs in a computer laboratory, not their usual classroom. Some pairs had two computers, some shared a single computer.
Whole class introduction and review – both teachers used an interactive whiteboard to introduce the software to the whole class and to support whole class discussion at various points during the lessons.	Prepared a PowerPoint presentation as an introduction to each lesson, including written questions and links to the software.	Used the software only, posing questions orally
Configuration of the software – during early stages of the project, the teachers had used versions of the microworld with the numerical scale visible or obscured. There was some discussion of the possible differences for student meaning construction in these two modes.	The numerical scale was visible on the sliders.	The sliders were coloured black to hide the numerical scale. (In practice, many students in this class discovered how to change the colour of the slider, revealing the numbers.)
	Separate microworlds were provided with the required number of sliders for each task set.	Students were given instructions for bringing up additional sliders as required.
Tasks were set by both teachers in the form of printed worksheets.	Sets of repetitive tasks with variation in the numbers, requiring simple numerical or	A smaller number of unique tasks, including some requiring written reflection or

	one word answers.	justification of answers.

The conception of the experiment

The research aim of the experiment was to investigate the representations of fraction employed and they ways in which they were used in the classroom during lessons in which the E-Slate fractions microworld was available. As was said above, this was conceived at two levels: for the teacher-researchers, the focus of interest was on the representations used by their students; for the university researcher, the focus was on representations used by both students and the teachers themselves. The ways in which the teachers adapted the software and integrated it into their lessons was itself an object of analysis.

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