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► To cite this version:

Marie-José Durand-Richard. Peacock's "History of Arithmetic", an Attempt to reconcile empiricism to universality. 2010. <halshs-00530818>

HAL Id: halshs-00530818

<https://halshs.archives-ouvertes.fr/halshs-00530818>

Submitted on 5 Nov 2010

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PEACOCK'S « HISTORY OF ARITHMETIC », AN ATTEMPT TO RECONCILE EMPIRICISM TO UNIVERSALITY*

MARIE-JOSÉ DURAND-RICHARD**

When the Whig Anglican algebraist Rev. George Peacock (1791-1858) conceived of his new abstract view of Symbolical Algebra in the 1830s, he had already written an impressive little known « History of Arithmetic » for the *Encyclopaedia Metropolitana*, eventually published in 1845, back in the 1820s. This paper studies why this « History of Arithmetic » was conceived and how it reinforced Peacock's general view of algebra as a symbolizing process. As a fellow, tutor and lecturer at Trinity College since 1814, Peacock was already involved in the renewal of mathematics curriculum and mathematical research in Cambridge, as well as in the cultivation and the diffusion of science. As a reformer, Peacock along with his colleagues in Cambridge, faced the Industrial Revolution, its varied pressures on the country's academic institutions, and its concern with transformation processes. As soon as the 1820s, Peacock sought out a universal genesis from arithmetic to algebra, founded on the mathematical language of operations, and he launched his « History of Arithmetic » as a large inquiry into the vocabulary that all known tribes and nations used for elementary computations. In this way, he supported a moderate empiricist approach to science, deeply rooted in Locke's philosophy of human understanding. With a comparative and philological approach to numeral languages in hand, Peacock presented first arithmetic and then algebra as the progressive developments of abstract calculating languages, symbolising algorithmical processes. This view accounted for the special place he gave to Indian and Arabic arithmetics in his exposition of contemporaneous knowledge on numbers.

Key words : arithmetic, algebra, symbolical, history.

* A first version of this paper was read at the International Congress of History of Science, held in Beijing in 2005, both with Dhruv Raina (Jawaharlal Nehru University, New Delhi) and Agathe Keller (SPHERE-REHSEIS) first version papers.

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PRESENTATION : WHAT WAS AT STAKE ?

In 1826, the Whig Anglican algebraist Rev. George Peacock (1791-1858) wrote an impressive « History of Arithmetic » for the *Encyclopaedia Metropolitana*¹, which was only published in 1845. As historians of mathematics rather identified Peacock as one of the British algebraists who impulsed a new abstract way to conceive algebra in the first half of the XIXth century, they generally ignore this encyclopaedic paper of 154 double-column pages. Since at least four decades, numerous historians of mathematics explored what was really brought at this turning point in algebra. For instance, Lubos Novy detailed each contribution of these British algebraists setting up the main realms of modern algebra², and Walter Cannon situated the first generation of them as the core of what he named « the network of Cambridge »³. More recently, historians of science explored moreover the contextual conditions of birth of this trend of thought.

The different stages of the contribution of this network for the renewal of algebra are now famous. Its first generation, formed by Charles Babbage (1791-1871), with John F.W. Herschel (1791-1871), Peacock and some today less known students, founded *The Analytical Society* in 1812, in order to enforce the introduction of the Leibnizian notation of the infinitesimal calculus in Cambridge⁴. They worked in publishing papers on this topics, and new textbooks for Cambridge examinations⁵, in

¹ G. Peacock, (1826) 1845, « Arithmetic », *Encyclopaedia Metropolitana*, vol. I : Pure Sciences. London, Smedley, & Rose. I, 369-523.

² L., Novy, 1968, « L'Ecole Algébrique Anglaise », *Revue de Synthèse*, III° S., n°49-52, janv. déc. 1968, 211-222 ; and 1973, *Origins of Modern Algebra*, Leyden, Noordhoff International Publishing, translated by Jaroslave Tauer, pp. 187-199.

³ W. F. K. Cannon, 1964, « Scientists and Broadchurchmen : An Early Intellectual Network », *Journal of British Studies*, IV, n° 1, 65-88.

⁴ When moderator for the Senate House examination in 1817, 1819 and 1821, Peacock forced its adoption by writing his questions with the Leibnizian notation. And Babbage's sentence on the dot-age and the d-ism is very often quoted. S. Bachelard, 1967, *La représentation géométrique des quantités imaginaires au début du XIXème siècle*, Paris, Conférences du Palais de la Découverte. Becher, H. W., 1980, 1980, « Woodhouse, Babbage, Peacock and modern algebra », *Historia Mathematica*, 7, 389-400. P. Enros, 1981, « Cambridge University and the adoption of analytics in early 19th century England ». *Social history of mathematics*. H. Mehrtens, H. Bos, I. Schneider, eds., Boston, Birkhäuser, 135-47; and 1983, « The Analytical Society (1812-1813) : Precursor of the renewal of Cambridge Mathematics », *Historia Mathematica*, 10, 24-47. M. V. Wilkes, 1990, « Herschel, Peacock, Babbage and the Development of the Cambridge Curriculum », *Notes Rec. Royal Soc. Lond.*, 44, 205-19.

⁵ In 1816, Babbage, Herschel and Peacock translated Lacroix's *Elementary Treatise of Integral and Differential Calculus*, with abundant notes by Peacock, and each of them prepared a special *Collection of Examples* in 1820. This need for new textbooks was abundantly referred to by these three students in their correspondance. Cf. Royal Society Library. *Herschel Manuscripts*, Hs.2.69,

order to «reimport in England ... a century of foreign improvement»⁶, and to found a view of algebra which could make it independent from geometry. This view was specially voiced by Peacock, who presented a purely symbolical view of Algebra, firstly for students in 1830 in *A Treatise of Algebra*, and then for scientists in 1833, in his *Report on the recent progress and actual state of certain branches of analysis*, pronounced at Cambridge for the third meeting of the newly founded *British Association for the Advancement of Science*. With the second generation of this network came on the stage : Augustus de Morgan (1806-1871), Duncan F. Gregory (1813-44), George Boole (1815-64), Arthur Cayley (1821-95) and James S. Sylvester (1814-97), and a nebula of less well known mathematicians around them⁷. With these followers, Peacock's symbolical approach was at first expanded as a «calculus of operations», and then diversified in producing new methods and objets, from a calculus on differential operators⁸ and logic⁹, to matrices and octonions¹⁰. That production of new objects, beyond quantitative entities, is held as one of the main contributions which impuled a radical change on the object of Algebra : previously considered as an investigation for a general theory of the resolution of equations, Algebra could then begin to stand as the study of abstract structures¹¹.

In any case, historians often considered the birth of so modern an approach to algebra in Great-Britain as surprising. For the XVIIIth century and the very beginning of the XIXth century, Continental mathematicians were more aknowledged than the British ones. And even if the examination for the B.A. degree was concentrated on mathematics

Herschel to Babbage : 24.12.1816, Hs.13.246, Peacock to Herschel : 13.11.1816, Hs.13.249, Peacock to Herschel : 17.03.1817, Hs.13.250, Peacock to Herschel : 30.05.1817.

⁶ Babbage, C., 1813, « Preface », *Memoirs of the Analytical Society*, Cambridge, p. iv.

⁷ Koppelman, Elaine H., 1969, *Calculus of Operations : French Influence on the British Mathematics in the first half on the nineteenth century*, Ph. D. Diss., John Hopkins University, 1969. In his 1968 paper (op. cit.), L. Novy included William R. Hamilton (1805-65) among them, but that is a retrospective error, as Hmailton was an Irish man, and above all, as he situated himself as a « theoretical » scholar, excluded from the « philological school ». Hamilton, W. R., 1837a, « Theory of conjugate functions, or algebraic couples; with a preliminary and elementary essay on algebra as the science of pure time », *T.R.I.A.*, 17, 203-422. *Math. Papers*, 3, 4-96.

⁸ D.F. Gregory, 1839, « Demonstrations of theorems in the differential calculus and calculus of finite differences », *Cambridge Mathematical Journal* (1839), 1, 212-24, *Mathematical Writings*, Cambridge, 1865, 108-23.

⁹ G. Boole, 1847, *The Mathematical Analysis of Logic, being an Essay towards a Calculus of Deductive Reasoning*, Cambridge; 1854, *An Investigation of the Laws of Thought, on which are founded the mathematical Theories of Logic and Probabilities*, London.

¹⁰ A. Cayley, « A Memoir on the Theory of Matrices », *Philosophical Transactions* (1858), 148, 17-37, in *Collected Mathematical Papers* (ed. A. Cayley and A. Forsyth), 14 vols, Cambridge, 2, 475-96.

¹¹ N. Bourbaki, *Eléments d'Histoire des Mathématiques*, Paris, 1969, 74; A., Mac Farlane, 1916, "George Peacock (1791-1858)", *Lectures on ten British Mathematicians*, New York, p. 7-18.

in Cambridge, its Geometrical and Newtonian approach seemed to be outrun, when compared to the algebraical developments of Laplace's *Mécanique Céleste* and *Théorie des Probabilités*. General histories of mathematics regularly referred this faithfulness on Newtonian notation to the quarrel of priority between Newton and Leibniz for the invention of the Calculus. But this reason is too meagre a cause to explain a century of specific development of mathematics in Great-Britain¹². More precise studies of the Cambridge university context showed that this faithfulness was linked to an attachment to more permanent¹³ forms of knowledge. If Cambridge educated an elite of gentlemen for the future governing class, it will remain a branch of the Church of England until 1871. In the Anglican universities of Cambridge and Oxford, the obligation of faithful oaths – both in Colleges and University, as for undergraduates to obtain degrees as much as for professors to get a chair¹⁴ – structured a traditional conservative way of thinking knowledge as legitimated by cultural values rooted in the past¹⁵.

Of course, facing the Industrial Revolution, what was previously conceived as the educational system of the governing class in order to warrant stability in the whole nation became dangerous manifestations of inertia. Debates in *The Edinburgh Review* showed how the hasty upheavals induced by the industrial world profoundly threatened the ancient equilibrium provided by Anglican universities.

Therefore, the astonishment facing the new symbolical view of algebra sustained first by young Cambridge students stems from a retrospective view of history of mathematics, being only concerned with what announces our present knowledge. As Leo Corry recently urged it, we have to pay attention to the fact that « the image of knowledge »¹⁶ – here that of algebra – was not immediately the one which was developed in the XXth century as investigating abstract structures. The attention on images of knowledge and their different territories – both chronological, geographical, social and conceptual – shed a new light on some recurring issues on the history of British algebraists, such as: why was there such a long time between the early identification of the properties of a field by De Morgan in 1842, and of a group by

¹² Garland, M. McMackin, 1980, *Cambridge before Darwin, The Ideal of a Liberal Education, 1800-1860*, Cambridge, Cambridge University Press.

¹³ As William Whewell will name them some decades after. Whewell, W., (1845), 1850, *On a liberal education in general, and with particular reference to the leading studies in the University of Cambridge*, Cambridge.

¹⁴ These oaths induced such an interiorisation of these values that any attempt to reform the system could be accused as a perjury.

¹⁵ Paley, William, 1794, *A View on the Evidences of Christianity*, London, Faulder, 6^{ème} éd., and 1802, *Natural Theology*, London. Cambridge, 1832, *A Collection of Cambridge Senate House Papers in Homer, Virgil, Locke, and Paley's Philosophy and Evidences, as given at the examination of B. A. degrees*, Cambridge, Hall & Hankins.

¹⁶ Corry, L., *Modern Algebra and the Rise of Mathematical Structures*, Basel-Boston-Berlin, 2004, pp. 2-4.

Cayley in 1854, and the late development of Abstract Algebra in the 1930s, more than seventy years after? Focusing on the historical background of Peacock's symbolical view of abstraction can help us to answer such questions.

The first part of this paper endeavours to show what was at stake at Cambridge university when this renewed approach to algebra was conceived. Therefore, it will first establish how Peacock's mathematical thought was profoundly involved in his reforming commitment in that institutional and scientific context. Peacock was always close to the Whig policy which echoed utilitarian criticisms on Anglican universities. If he remained a moderate reformer, he nevertheless expressed a constant admiration for the educational institutions born in France with the Political Revolution, and was willing to transform Cambridge from a « seminary of sound learning and religious education » to a « national University ». With such a mind, Peacock was debating on what may be for Cambridge a professional formation in mathematics at a higher level¹⁷. His whole life was entirely devoted to this reform, until he died as one of the members of the Executive Commission who undertook the first important reformation of Cambridge university in the 1850s¹⁸. His general purpose was to ground law as objective and rational without religious implications, and so, to express it in such a way that it can involve all possible renewals in human practices.

Meanwhile, Peacock also developed an analogous scheme of thought for mathematics. His own philosophy of algebra was sensitive both to inventive mathematical practices and to the deductive form of mathematical reasoning. He tried to establish a mediate path between conflicting trends about mathematics, viewed either as a foundational or as a progressive matter. The « silent perseverance » of his moderate reforming commitment¹⁹ can be perceived nowadays as a « case of creative indecision »²⁰. But Peacock remained a Reverend, who took orders in 1817, and became deacon of Ely cathedral in 1839, keeping the Lowndean Professorship of Geometry and Astronomy he obtained only in 1836. And Joan Richards precisely analysed how the

¹⁷ G. Peacock, 1841, *Observations on the Statutes of the University of Cambridge*, Cambridge, p. 92, 100, 166. *Trinity Colleg Library*, Peacock to Whewell, 10th of march 1842, Add. Mss. a. 210¹⁰⁸.

¹⁸ Herschel, J. F. W. 1858, "Obituary Notice on G. Peacock", *Proceedings of the Royal Society*, 9; 536-541. Tillyard, A.I., 1913, *A History of University Reform from 1800 to the Present Time, with suggestions towards a complete scheme for the University of Cambridge*, Camb. Un. Press, Cambridge.

¹⁹ « It is by silent perseverance only that we can hope to reduce the many-headed monster of prejudice, and make the University answer her character as the loving mother of good learning and science ». Royal Society Library. *Herschel's Correspondance*, Hs.13.249, Peacock to Herschel : 17.03.1817.

²⁰ M. Fisch, 'The Making of Peacock's Treatise of Algebra : a Case of Creative Indecision', *Archive for History of Exact Sciences* (1999), 54, 137-179 ; and « 'The emergency which has arrived' : the problematic history of nineteenth-century British algebra - a programmatic outline », *BJHS*, 1994, 27, 247-76.

religious view of absolute truth was impressed on this network of algebraists²¹. In this paper, we shall focus on the way by which Peacock managed to harmonize this view with his political will to involve new practices – in the university as well as in mathematics. It will be shown that his conceptual view of the genesis of algebra, as built from arithmetical practices, was deeply fostered by Locke's philosophy of language, as still taught at this period in Cambridge university²². Locke accepted the major gap between « nature » and the knowledge of nature which can be expressed by the language, and his philosophy afforded an essential role to the symbolical function of representation of language. As we shall see, in the same way as Locke presented the different stages of how the operations of mind worked, Peacock conceived the genesis of operations of algebra, from arithmetical practices to abstract operative laws. In that prospect, Peacock's *History of Arithmetic* belonged completely to his enterprise, and it will be assessed with the same importance during the 1820s and during the 1840s, when *A Treatise of Algebra* was reedited, as both of them had once more to be defended. Examining the language of arithmetic in very numerous countries and periods, Peacock was anxious to value arithmetic amongst mathematicians : he wanted to show the universality of this operative way of thinking, and presents it as the first step of abstraction in the construction of algebra as the « science of general reasoning by symbolical language »²³. Peacock appeared to his contemporaries as the more philosophical mathematician among all of them²⁴, and it will be shown how he precisely mobilised all the resources of rhetorical argumentation to convince his readers of its symbolical function as an essential feature of mathematical language.

PEACOCK AS A WHIG ANGLICAN ALGEBRAIST

As Wilkes insisted on²⁵, the initiatives of this network²⁶ were not the first attempt to renew Cambridge's curriculum. What was really changing now was a

²¹ Richards, J.L., 1980, « The Art and the Science of British Algebra : a Study in the perception of Mathematical Truth », *Historia Mathematica*, 7, 343-65 ; and 2002, « 'In a rational world all radicals would be exterminated', Mathematics, Logic and Secular Thinking in Augustus De Morgan's England », *Science in Context*, 15 (1), 137-164.

²² Durand, M.J., 1990, « Genèse de l'Algèbre Symbolique en Angleterre : une Influence Possible de John Locke », *Revue d'Histoire des Sciences*, 43, n°2-3, 129-80.

²³ G. Peacock, *A Treatise of Algebra*, Cambridge, 1830, 1.

²⁴ Herschel, J. F. W., 1858, « Obituary Notice on G. Peacock », *Proceedings of the Royal Society*, 9; 536-541.

²⁵ Wilkes, M.V., 1990, « Herschel, Peacock, Babbage and the Development of the Cambridge Curriculum », *Notes Rec. Royal Soc. Lond.*, 44, 205-19.

²⁶ As Ivor Grattan-Guinness indicated to me in 1998 in Wuhan (China), « network » (Cannon's denomination) is a better term than « school » (Novy's denomination) as a name for this group of

coordinate political determination, as its members attempted to reconcile learned men and practical men, maintaining social cohesion between academics and industrial, whose science could be the cement. From its beginning, it was essentially organized by a common vision of the necessity of profound unifying reforms, in order to adapt the old institutions to the effects of the Industrial Revolution, and to avoid such a turmoil as the French Revolution. Many of its members could also be found among « the Gentlemen of Science », who managed the *British Association for the Advancement of Science* during its first twenty years of existence²⁷. Its most committed reformers were close to Whigs and Radicals²⁸. Peacock and his friends were very much concerned by the gap between the traditional education in the Anglican Universities of Cambridge and Oxford, and the new conceptions of knowledge, fostered by utilitarianism and empiricism, firmly praised by the criticisms²⁹ of the new Whig journal *the Edinburgh Review* born in 1802. They directly worked to understand the epistemological consequences of the Industrial Revolution, and to adapt the old institutions, offering their reflection to the governing class.

1. Peacock and the renewal of scientific institutions

Although Peacock appeared as a moderate man, he constantly manifested a very firm determination in this reforming enterprise. Matriculated as a sizar at Trinity College in 1809, this modest Anglican vicar's son was second wrangler after Herschel for the B. A. degree and the Smith Prize in 1813. The letters he wrote to his elder brother in London Stock Exchange during that whole period soon personally criticised the Radical thinkers such as W. Cobbett, J. Horne Tooke, J. Cartwright and Francis Burdett, although he felt very close to them when he entered Trinity³⁰. He quickly became more temperate facing also reformers in Cambridge³¹. Nevertheless, Peacock assessed that he

algebraists. It did not include a unique Master and his followers, but rather several authors working on algebra as marked by the symbolical function of language, but with different directions of research.

²⁷ J. Morrell and A. Thackray, *Gentlemen of Science, Early Years of the British Association for the Advancement of Science*, Oxford, 1981.

²⁸ Smith, Charles, & Wise, M. Norton, 1989, *Energy and Empire : A Biographical Study of Lord Kelvin*, Cambridge, Cambridge University Press, ch. 6.

²⁹ J. Playfair, « Review of Laplace's *Traité de Mécanique Céleste* », *Ed. Review* (1808), **11**, 249-84 ; (Anonymous), Review : « A Reply to the Calumnies of the Edinburgh Review against Oxford ; containing an Account of Studies pursued in that University. Oxford », *Edinburgh Review* (1810), **16**, 158-87

³⁰ Trin. Coll. Library, *Peacock Manuscripts*, Peacock to his brother William, 21.08.1810, P 3⁷ ; 30.09.1810, P 3⁸.

³¹ Trin. Coll. Library, *Peacock Manuscripts*, Peacock to his brother William, 02.03.1811, P 3¹⁰.

will « never cease to exert [himself] to the utmost of the cause of reform »³². What he effectively did it, both inside and outside Cambridge, maintaining a constant moderate, but straightforward reforming ambition.

Just as he introduced Leibnizian notation in Cambridge examinations, Peacock was directly involved in the creation of the Cambridge Philosophical Society (1819), the Royal Astronomical Society (1820), and the British Association for the Advancement of Science (1831), where he will often be an officer and referee. These creations were planned to bring a new equilibrium between the learned societies recently founded in the new industrial towns, and the old institutions of knowledge, where severe religious restrictions of admission to the degrees kept out a lot of students. On that ground too, Peacock was constantly extolling a rational faith³³, and worked to loosen the relationship between the religious and the educational roles of Cambridge. Always sustaining students against the compulsory attendance to the daily Chapel service³⁴, he was also one of the four organisers of the public petition submitted to the Parliament in 1834 with sixty-four signatures, asking for the abolition of religious tests in Cambridge examinations³⁵.

The various Syndicates which were formed in Cambridge in order to develop the study of the different branches of natural philosophy found Peacock among their members. He contributed in that way to the creation of the Cambridge Observatory (1816-1823), the Pit Press, the Fitwilliam Museum, and the extension of the Cambridge buildings (1829-1842). Peacock will offer the students « the acquisition of accurate knowledge not confined to Classical or Mathematics, [but for] other sciences, whether natural, political, or moral »³⁶. Arguing for a professional education in Cambridge, Peacock was in a constant opposition with William Whewell (1794-1866), the Master of Trinity from 1841, who conceived Liberal education as general rather than specialized.

³² Royal Society Library. *Herschel Correspondance*, Hs.13.249, Peacock to Herschel : 17.03.1817.

³³ For instance, travelling in Italy, he considered Catholic manifestations of faith, such as processions or adoration of Turin's shroud, as fanatic attitudes. Trinity College Library, *Peacock Manuscripts*, Peacock to his father, 27.07.1816, P 3⁴³ ; Peacock to his sister, 28.07.1839, P 3⁸⁹.

³⁴ Anonymous, « The late Dean of Ely », *Cambridge Independant Press*, from *The Times*, 13.11.1858, p. 6...

³⁵ But this action failed in House of Commons. Anonymous, *Cambridge Chronicle*, 28.03.1834. Anonymous, *Cambridge Independant Press*, 20.11.1859, p. 7. M.-J. Durand(-Richard), 1985, « George Peacock (1791-1858) : La Synthèse Algébrique comme loi symbolique dans l'Angleterre des Réformes (1830) », *Thèse pour le doctorat de l'E.H.E.S.S.*, pp. 236-239.

³⁶ G. Peacock, *Observations upon the Report made by a Syndicate appointed to confer with the Architexts who were desired to furnish the University with designs for a new Library*, Cambridge, p. 13.

With all these commitments, Peacock was in his time a scientist of national importance, who spoke all along his life as an actor of the social transformations to which he was confronted. His letters are those of an advised man, very well informed on economical questions³⁷. His correspondance gives evidence of his relationships with the progressive governing class, from the time when he was in charge with its sons as a tutor in Trinity College (1814-1836), and directly with numerous high political or governing personalities, such as Sir G. Grey or M. Spring Rice, later Chancellor of the Exchequer. Lord Melbourne supported him for his nominations both as Lowndean professor – against Whewell – and as deacon of Ely. In his numerous responsabilites, he rather served the national institutions or the University³⁸ rather than the religious Colleges. With his *Observations on the Statutes of the University of Cambridge* (1841), Peacock paved the way for the direct intervention of the Crown in the university affairs, and to its political role in the Cambridge Reform. In this book, the statutes for permanent functions of the University were carefully distinguished from those for possible local evolutions, and the necessity of their secularisation was insisted on. At the climax of his career, in 1850, he was one of the five members of the Cambridge University Commission chosen by Lord Russell to inquire on the best way to manage the reform, in a way to reinforce the University power on the Colleges one. It is worth to point out that the Commissioners Report foresaw to involve engineering studies in mathematical ones. Despite a strong resistance of the Colleges – Whewell spoke of a violation of the right of property – , the Cambridge University Bill received the Royal agreement in 1855, and an Executive Commission of eight members had to make it effective. Peacock was one of them³⁹. But the complete separation between religious and professional education would be imposed only in 1871.

³⁷ In Paris during the political Revolution of 1830, he appreciated the new Monarchy, writing : *I yesterday saw the Chamber of Deputies proceed to the Palais Royal to offer the crown to the Duc d'Orléans : it is however quite clear that he unites the votes of all the well informed classes of men in France and that his elevation will meet with universal acquiescence at least if not with universal approbation... The French justly dread a civil war and it is surprizing to observe the readings with which they are ... to sacrifice their personal opinions and wishes to the cause of public union and peace.* Meanwhile, Peacock informed his brother of the transactions in Paris Stock Exchange. Trinity College Library, *Peacock Manuscripts*, Peacock to his brother William, 01.08.1830, P 3⁷⁰ ; 05.08.1830, P 3⁷¹, s.d. received on the 10.08.1830, P 3⁷². In 1842, Peacock also published in 1846, *Upon the Probable Influence of the Corn Laws upon the Trade of Corn*, and, travelling in Madeira for his health in 1850, « A Review on the State of Agriculture » in that island. As deacon of Ely, Peacock also exerted his administration according to three essenital axes of the Whig program : health, justice and education.

³⁸ Peacock was secretary of Cambridge University Chancellor, the duke of Gloucester (1831), and then, secretary (1837-39) and president (1844) of the British Association.

³⁹ Durand(-Richard), 1985, op. cit., pp. 192-276, pp. 315-344. A similar strand of reforms took place in Oxford a the same period.

So, this Whig Anglican algebraist partook the general reforming desire for a Liberal education, but if he was much committed to the professionalisation of University, he was searching the conditions by which the Reform could effectively maintain a common language and permanent law on the local contingent evolutions. Even if the Board of Mathematics in 1848 separated the Mathematical Tripos between two parts, and introduced different Tripos, the curriculum and the Senate House Examination for Mathematical Honours were reinforced in mathematics, and the foundational knowledge in Cambridge remained a general one⁴⁰.

2. Algebra as a symbolizing process

At the beginning of the XIXth century, strong debates took place both in Cambridge, in Oxford, and between the two, in order to determine what would be the permanent foundations of the new inductive sciences, such as physics or political economy. In Oxford, the Noetics raised the same fundamental questions about logic⁴¹ as the Analytics did in Cambridge about mathematics⁴². Strongly impressed by the empiricist criticisms of the *Edinburgh Review*, the reformers want to grant the place of practices and experiments in the constitution of knowledge. In mathematics, the Analytics and their followers will insist to accept negative and impossible quantities⁴³, as well as differential operators⁴⁴, and to carry out operations on them. Moreover, they were looking at better foundations than just an analogy with operations on arithmetical quantities – as it was the case since the developments of algebra, as John Playfair (1748-1819) recently did for instance⁴⁵. The Analytics attempted to make explicit the conditions by which algebra could be expressed really as a science – a science of

⁴⁰ Durand-Richard, M.-J., 1996, « L'Ecole Algébrique Anglaise : les conditions conceptuelles et institutionnelles d'un calcul symbolique comme fondement de la connaissance », in (éds) Goldstein, C., Gray, J., Ritter, J., *L'Europe Mathématique - Mythes, histoires, identité. Mathematical Europe - Myth, History, Identity*, Paris, Eds M.S.H, 445-77. Warwick, A., 2003, *Masters of Theory, Cambridge and the Rise of Mathematical Physics*, Chicago and London, The University of Chicago Press.

⁴¹ P. Corsi, *The heritage of Dugald Stewart : Oxford Philosophy and the method of political economy*, Firenze, 1988.

⁴² M.-J. Durand-Richard, « L'Ecole Algébrique Anglaise : les conditions conceptuelles et institutionnelles d'un calcul symbolique comme fondement de la connaissance », in *L'Europe mathématique - Mythes, histoires, identités*(ed.) C. Goldstein, J. Gray and J. Ritter, Paris, 445-498.

⁴³ H. Pycior, *Symbols, Impossible Numbers, and Geometric Entanglements*, Cambridge, 1997.

⁴⁴ M. Panteki, « The Mathematical Background of George Boole's *Mathematical Analysis of Logic* (1847) », in *Anthology on Boole* (ed.) James Gasser, Waterloo, 2000, 167-212.

⁴⁵ Playfair, J., 1778, « On the Arithmetic of Impossible Quantities », *Philosophical Transactions of the Royal Society*, 68, 318-343.

necessary truths –rather than a judicious notation which simply authorizes a mechanical treatment of operations. To secure this view, they will assert the preeminent role of operations, whatsoever could be the symbols on which they were applied on. So, operations could be defined, no more by their results, but only by their laws of combination.

In that context, Peacock's main goal was to enforce Algebra over Geometry as the fundamental knowledge in the curriculum. For this reason, he first needed to establish that Algebra was not only a tool, born from the writing of letters in place of numbers with successive extensions of arithmetical practices, but a Science, as such characterised by its deductive rigour and its universality. What he had to make explicit – and even natural – was the logical part of algebra. But what was essential for this Whig reformer was to preserve the link between algebra and its inventive practices. His project was clearly to banish what appeared as a parrot-fashion transmission of a long established knowledge, which the Senate House Examination compelled to restate mechanically for the obtention of the Bachelor of Arts degree, at least for the Wranglers candidates.

The path was difficult to face this twofold requirement, and Peacock elaborated a very specific one. Firstly, he introduced a radical separation between the meaning of the algebraic symbols, and the logic of operations. This radical breaking off constitutes the cornerstone of this Symbolical approach⁴⁶, for him and his followers. Operations are no more rooted on their possible results, but on their properties. But what was complicated was to preserve too the submission of experience to the unity of mind's work.

Peacock presented algebra in a constructive epistemological way, as the third step of a historical reconstruction of a genetical process. The first one was Arithmetic, the « science of measure and quantity », a very practical one, but on which he did not insist in this presentation undoubtedly because of his 1826 paper. The second one was Arithmetical Algebra, where the symbols were « general in their form, but not in their value ». It did not correspond to the actual state of Algebra, whose logic often failed because of analogical practices, when the algebraist was confusing the necessary truth – the logic of operations – and the contingent one – the truth of the results. Peacock precisely focused to separate them. His Arithmetical Algebra was a logical reconstruction⁴⁷, where arithmetical quantities such as $(a - b)$ and $\sqrt{a - b}$ exist only if

⁴⁶ Peacock already referred explicitly to syntax of language at the end of his « History of Arithmetic » [481 § 280].

⁴⁷ This logical reconstruction of Arithmetical Algebra stood as an attention paid to criticism of the Radical William Frend (1757-1841) – the future father-in-law of De Morgan –, who asked to exclude negative and impossible quantities from algebra, because they did not correspond to any experience, and introduced logical contradictions because the meaning of operations was therefore changing. W. Frend, *The Principles of Algebra*, Cambridge, 1796.

($a \geq b$). Peacock insisted on the algorithmical character of arithmetical operations⁴⁸. The third step was Symbolical Algebra – the sole universal –, whose symbols were « general in their form, and in their value ». This « language of symbolical reasoning » made explicit some combining properties of operations, which Peacock gave as the « laws of combination » on these arbitrary symbols. The « business of algebra » was precisely conceived to discover these general forms of the results.

So, in Peacock's work, the relationship between Arithmetical Algebra and Symbolical Algebra was particularly ambiguous, essentially because the first one had a twofold epistemological statute. It worked first as the « science of suggestion » for Symbolical Algebra, in the sense that its results, expressed by general symbols, make « signs » – in the first meaning of this word – for the mathematician, helping him to guess the general symbolical laws behind them⁴⁹. But it works too, afterwards, as one of the possible contingent « interpretations » which gave meanings to the symbols of Symbolical Algebra. Consequently, Arithmetical Algebra was logically subordinate to Symbolical Algebra.

3. A symbolizing process rooted in Locke's philosophy of language

So, the relationship between Arithmetical Algebra and Symbolical Algebra is somewhat troublesome for the modern reader. Peacock pointed out that the laws of combination were obtained neither by some extension of those in arithmetic – which induced a change in words meaning – nor by some analogy with them, because analogy was not part of the deductive reasoning. He did not prohibited the use of analogy, but he made it the perceptive part of a more essential principle, the famous principle of equivalent forms⁵⁰. This principle of equivalent forms authorized Peacock not to deduce all the resulting forms in Symbolical Algebra, but only to take them from Arithmetical Algebra, as long as their forms were absolutely general. It is the main reason why he did not give an axiomatic deductive presentation of Symbolical Algebra⁵¹. This double principle assessed :

(A) : Whatever form is algebraically equivalent to another when expressed in general symbols, must continue to be equivalent, whatever those symbols denote."

⁴⁸ For instance, in his new edition of the 1830's *Treatise*, where he separated in two volumes Arithmetical and Symbolical algebras, Peacock insisted particularly on « ineliminable quotients », giving decimal results with the same process of division as entire ones. Clearly, Peacock needed to reassess his position in front of Whewell's one in his *History of Inductive Sciences* (1837) and *Philosophy of Inductive Sciences* (1840).. G. Peacock, *Treatise of Algebra*, 2nd ed. Cambridge, 1842-45, II, 26.

⁴⁹ F. Duchesneau, *L'empirisme de Locke*, La Haye, 1973, 200-202.

⁵⁰ Peacock, 1830, op. cit., p. 108.

⁵¹ Durand(-Richard), 1985, op. cit. Fisch, 1994, op. cit., 264.

(B) : *Converse Proposition* : Whatever equivalent form is discoverable in arithmetical algebra considered as the science of suggestion, when the symbols are general in their form, though specific in their value, will continue to be an equivalent form when the symbols are general in their nature as well as their form⁵².

If a retrospective view is not resorted to, this twofold principle cannot be read as any anticipation of a modern inclusion of Arithmetical Algebra in Symbolical Algebra, which a modern reader could be tempted to look at. For a deeper understanding of Peacock's conception of this relationship, and of Symbolical Algebra, it is essential to underline that he insisted on these two points : (1) its universality was assured by the arbitrariness of its general symbols, (2) the meaning of the symbols was a contingent fact : it could exist, as in Arithmetical Algebra, or it could exist not, as for (-1) or $\sqrt{(-1)}$. Meaning was only a possible interpretation, and it was considered as too much linked with experience to take part to the universality of algebra. Also, Peacock conceived algebra as a « purely demonstrative science », which was not at all concerned with the adequacy to the physical⁵³.

The arbitrariness of the symbols, the combining character of the operations, and above all, this kind of relationship between demonstration and truth, were essential features of Locke's *Essay on Human Understanding*. As included in Cambridge curriculum and examinations, Locke's philosophy belonged to the common background of those algebraists in Cambridge. So, if the term « operation » is heard in relationship with the operations of mind, it can be shown that Peacock's vocabulary and methodology were consistent with those of Locke's *Essay*, where algebra was praised as the moral sciences for giving the sole universal truth, because both of them defined freely their words, in consequence of what their « real essence » was confounded with their « nominal one »⁵⁴. For Locke, general ideas and words just concern the mind, and not at all Nature, or the substance of things, which is unknowable ; mathematical propositions are the only ones which could be considered as universal truths, because they were abstract and general signs, and only with this condition, Already, Robert Woodhouse (1773-1827) at the very turn of the century, and Babbage in 1813, sustained Locke's formal conception of demonstration as in accordance with the search of a theory of invention⁵⁵.

⁵² Peacock, 1833, 194.

⁵³ Peacock, 1833, 187.

⁵⁴ M.-J. Durand(-Richard), 1991, « Genèse de l'Algèbre Symbolique en Angleterre : une Influence Possible de John Locke », *Revue d'Histoire des Sciences* (1990), 43, 129-80. J. Locke, *Essay on Human Understanding*, 2nd ed., London, 1694, IV.5.2.

⁵⁵ Durand-Richard, M.-J., 2001, « Révolution industrielle : logique et signification de l'opérateur », *Mélanges en l'honneur d'Ernest Coumet*, Paris, n° spécial de la *Revue de Synthèse*, « Histoire des jeux, jeux de l'histoire », T. 122, 4^e S. n° 2-3-4, avril-décembre 2001, Centre International de Synthèse, Albin Michel, pp. 319-346.

So, Peacock is not exactly an anxious or undecided theoretician⁵⁶. He tried to keep together the inventive and the deductive processes of reasoning. As Locke, and as a member of the Anglican Church, he can assess that operations in algebra were not just mechanical rules to be applied mechanically as a machine. He could support them by the operations of mind, and so, he could be sure that algebraical operations still made sense as they were linked to God's creation. Here stands what authorized Peacock to be satisfied with what is now perceived as a formal view of algebra because its close relationship to arithmetical practices in Peacock's view is lost. The combining processes of the faculties of mind was sufficient for Peacock to endorse his theological and teleological view of truth⁵⁷.

PEACOCK'S SYMBOLICAL VIEW OF ARITHMETIC'S HISTORY

If Peacock did not present the details of the first step of his historical reconstruction of algebra in his 1830s works, the main reason was that they were previously developed in his extensive paper written for the *Encyclopædia Metropolitana*, and its analysis will help to reinforce his philosophical view of algebra as a symbolising process of operations. Both its writing and its publication took place respectively in 1826 when Peacock developed his specific way of conceiving Algebra, and in 1845 when he had to reinforce it facing the rising influence of Whewell's view of the curriculum in Cambridge.

Peacock's unifying view of mathematics could meet the design of the *Encyclopædia Metropolitana*. It was launched by Samuel Coleridge (1772-1834) in 1817. The publication of the 21 volumes and 8 of plates began in the 1820s, but it is difficult to precise the exact date of each paper in it⁵⁸. As a tory propagandist, Coleridge deeply opposed the mechanistic trend of empiricist philosophy, and the danger of plebification and desintegration of knowledge. When the *British Association* was formed in 1831, he advocated the formation of a « clerisy », a body of theologians, scholars and men of science, in charge to protect its unity⁵⁹. Distinguishing between understanding and reason, the last one being the sole to found wisdom by organizing men's thoughts, Coleridge gave a classification of sciences, built on mathematics as a formal and pure

⁵⁶ M. Fisch, 1999, « The Making of Peacock's Treatise of Algebra : a Case of Creative Indecision », *Archive for History of Exact Sciences* (1999), 54, 137-179.

⁵⁷ J. L. Richards, J.L., « The Art and the Science of British Algebra : a Study in the perception of Mathematical Truth », *Historia Mathematica* (1980), 7, 343-65.

⁵⁸ I. Grattan-Guinness, « Mathematics and mathematical physics from Cambridge, 1815-40 : a survey of the achievements and of the French influences », in *Wranglers and Physicists : Studies on Cambridge physics in the nineteenth century* (ed. P. M. Harman), Manchester, 1985, 84-111.

⁵⁹ R. Yeo, 1993, *Defining science, William Whewell. Natural knowledge and public debate in early Victorian Britain*, Cambridge, 1993, 44.

science. With this classification, he preferred a thematic order rather than the alphabetical order of the French *Encyclopédie*, which he condemned for its disorganising form⁶⁰. Finding here Babbage as one of Coleridge's counsellors, and Peacock as author, enlightened both the scope of this clerisy, and their own commitment in a general investigations for a new unifying view of science.

Peacock's « History of Arithmetic » gathered together his own early researches on languages and arithmetical notations, given at the very first meetings of the *Cambridge Philosophical Society*⁶¹, and gave a special witness of the continuity of his commitment in supporting a symbolical view of algebra. This impressive paper was not just intended to give full informations about arithmetic through ages and human groups. It constituted the first milestone of Peacock's undertaking to conceive Algebra as a pure Symbolical language and to exhibit it as a constructive process founded on the Lockean natural operations of mind. As Peacock gave a genetical presentation of Symbolical Algebra in 1830, he first called up history as a fundamental argument supporting this philosophical thesis on the development of mathematics⁶². His main goal was to confer a full acceptance of arithmetical and algebraic practices in the University, and to change its epistemological status. He wanted it to drop the status of a counting tool just for human affairs, even if he profoundly respected it. He investigated what he thought as the universal aspects of human experience so induced, in order to found mathematics as a universal language for other sciences. In this way, Algebra could appear temporarily as the best outcome of the symbolizing process in mathematics⁶³. We can find there the

⁶⁰ S. Coleridge, « General Introduction, or Preliminary Treatise on Method », in *Encyclopædia Metropolitana*. (ed. Smedley and Rose) London, I, 1-43.

⁶¹ Peacock speaking of 'the present year 1826' in his *History of Arithmetic* confirms this point [note p. 413 § 66], as well as the Appendix to his paper. Yet, some bibliographical references indicate that Peacock also completed it after 1826. The *Minutes of the Cambridge Philosophical Society* for 1826 and 1827 give the titles – but alas only the titles – of Peacock's unpublished communications : « on Greek arithmetical notation » (27.01.1826, 13.03.1821), « On the Origin of Arabic Numerals », and the date of their Introduction in Europe » (10.04.1826, 24.04.1826), « On the numerals of the South American Languages » (11.12.1826), « On the Discoveries recently made on the subject of the Hieroglyphics » (12.03.1827, 21.05.1827), « Account of the Representations occurring in Egyptian Monuments, of the Duties of that Country, and of the funeral Rituals » (07.02.1828). There is no trace of them on Peacock's papers in Cambridge Trinity College Library. Peacock pursued this trend of thought when he published his Durand(-Richard), 1985, op. cit., 244-245.

⁶² The inequality in the three parts of the paper pointed out to the foundational turn of the project : 114 pages beared on the historical notice of the different methods of numeration, 22 pages on the operations on abstract numbers, and 19 pages on the operations on concrete numbers.

⁶³ Some years later, in 1836, De Morgan would present his « Calculus of Functions » in the same *Encyclopædia Metropolitana*, as the next step in this symbolisation process of mind operations. A. De Morgan, « Calculus of functions », *Encyclopædia Metropolitana*, London, [1836] 1845, II, 303-92.

main reason why Peacock does not say anything on the theory of numbers : he is working on the experiential foundations of mathematics, not on the development of theoretical parts of Arithmetic⁶⁴.

I would like to show, in this paper, how Peacock used every kind of rhetorical arguments in order to convince the reader of the validity of his view. He wrote as an ethnologist, a philologist, an historian and a philosopher of mathematics, following a reconstructed approach to the development of Algebra. He undertook to persuade his contemporaries that Algebra was the universal language of mathematics, obtained from arithmetical practices and language.

1. Peacock as an ethnologist and a philologist

The paper first started with what Peacock called a « metaphysical question ». It « forms a natural introduction to an historical notice of the different methods of numeration, which have been adopted by different nations at different periods of the world » [369 § 2].

The question bore on how a child acquires for instance the idea of the number « four », as distinct from « four horses », or « four cows ». Peacock immediately linked this process to language and to the faculty of abstraction. All along the paper, abstraction will be characterized by unicity, simplicity, and universality :

Abstraction is the creature of language, and without the aid of language, he (the child) will never separate the idea of any number from the qualities of the objects with which it is associated...

We are thus lead to the distinction of numbers into abstract and concrete, though the abstraction exist merely in the word by which it is represented in different arithmetical systems [369 § 2].

Thanks to numeral words, the child can keep this idea in mind, and he can pronounce it without associating it to a peculiar thing. So, because the words precede signs in the development of arithmetic, Peacock was going to examine, through words, the traces of arithmetical experiences. He was investigating their universal part, hidden by their contingent diversity.

Peacock founded his remarks on the more recent works about how to understand the origins of language. He covered a much larger scope than British Indologists, adding many recent reviews on foreign numeral languages – such as those in W. von Humboldt, or in diplomats and Jesuit missionaries writings – to Playfair and

⁶⁴ There was a specific paper on the theory of numbers in the *Encyclopædia Metropolitana*, written by Peter Barlow (1776-1862). Peacock precised that he did not develop operating practices when they are founded on specific algebraic knowledge, and he attributes the confusion of some of its methods to the ignorance of this underlying algebraic knowledge [437 § 128-129].

Colebrooke investigations. It will be praised by his old friend Herschel as the « most learned history on this subject »⁶⁵. And this structural methodological approach was explicitly fostered by the new investigations on the origin of languages [371-372 § 8-10]. Following the tracks of comparative grammar⁶⁶, he insisted on this specific methodology in order to assess that affinities between different languages must be observed by means of grammatical identities rather than from resemblance between words :

The more philosophical of modern Philologists, indeed, have ceased to regard affinity of the roots as a decisive proof of the affinity of languages; it may arise from the mere mixture of languages, and from the intercourse of the people by whom they are spoken, but it by no means demonstrates them to be of common origin, unless accompanied also by a corresponding affinity of grammatical structure [372 § 10].

Pursuing this idea for numerical languages, Peacock wanted explicitly to prove that, for the arithmetical language, « amongst all nations, practical methods of numeration have preceded the formation of numerical languages ». Moreover, he considered that numeral words depended on operations for counting, which preceded them. So – but this is a discussed assumption to day – he assessed that numeral words came directly from these counting methods, which correspond for arithmetic to the grammatical structure just referred to in language.

Arithmetical language is then investigated in order to show that it is firstly founded on the perception of numeration methods. Methodologically, Peacock gave a very large series of examples and counter-examples, with a lot of comparison tables, in order to prove his thesis by furnishing a knowledge as probable as possible of the historical and epistemological development of numeral words. His approach was very close to his way of conceiving language in general, and mathematical language in particular : practice comes first ; people directly play with numeration methods ; then they perceive how these methods are organized, and afterwards, express them in verbal language, before special symbols were found to write them down. So, language is rooted in practice, and the word itself comes from a mental process of abstraction.

⁶⁵ J. F. W. Herschel, « Obiturary Notice on G. Peacock », *Proceedings of the Royal Society* (1858), **9**, 536-541.

⁶⁶ Comparative grammar was then one of the favourite subjects of *The Apostles* in Cambridge, and Peacock was close to them, and he also had epistolical exchanges with W.D. Conybeare on that subject. *Trinity College Library*, Cambridge, correspondence of Peacock, P.1⁸⁸.

2. Peacock as a philosopher of mathematics

According to this queries regarding how to investigate arithmetical words, so as to reinforce his demonstration, Peacock chose first to examine languages of « the most primitive and barbarous people », because he considers that they may be more unaltered than others. Consequently, they can furnish better informations on structural affinities of languages, in spite of some unexplained cases. In fact, Peacock was going to assess that all languages kept the marks of the original decimal scale, and that its universal presence arose from its natural origin : the organisation of the human body, with its symmetry and the ten fingers, was considered as the first « **natural abacus** » [370 § 4]. That first instrument for counting could be praised as universal, because of its « natural » origin, which makes it existing before human thinking itself. Establishing this natural character of the decimal scale was a very strong argument for Peacock in assessing that arithmetical language was legitimately founded :

It will be found, upon an examination of the numerical words of different languages, that they have been formed upon regular principles, subordinate to those methods of numeration which have been suggested by nature herself, and which we may suppose to have been more or less practised amongst all primitive people ; for in what manner can we account for the very general adoption of the decimal system of notation, and what other origin can we assign to it than the very natural practice of numbering by the fingers on the two hands [370 § 4].

Referring to Aristotle, and preventing both the mathematician and the philosopher against the mystical aspect of very ancient loose analogies – particularly Pythagoreans and Platinists ones –, Peacock specified that « natural abacus » as the material demonstration of a general law of nature :

The universality of the decimal scale proves, according to Aristote, that its adoption was not accidental, but had its foundation in some general law of nature. This is a most philosophical principle of reasoning, which leads in the present instance of the correct conclusion, notwithstanding the Pythagorean and Platonic dreams about the perfection and properties of the number ten, which are thrown out as conjectures to account otherwise for its general adoption [383 § 22].

The traces of that starting point are first explored through the words which, in different languages, before the ciphers, denominate numbers. Those words indicate « the regular principles by which numeral systems are formed upon ». From § 8 to § 33, Peacock pursued the inquiry in order to prove the preceding assessment through a multitude of people : Tibet, China, the Indian Archipelago – essentially Malasian and Javanese –, Celtic languages – including Basque –, and numerous tribes of South and North America, from Polar American to African tribes. He did not forget singular

counting specificities, like those expressing 19 or 29 from the superior ten, which was a rather widespread method in an-hands counting.

Concerning the natural aspect, Peacock showed that if other natural scales of numeration exist, they are yet related to 10 : either 5 and 10 are sub-scales of 20, or 5 is a sub-scale of 10. And the scale 20 often comes from people who counted both on their fingers and on their toes. All these scales can be reduced to 10, and have been superseded by the scale 10, either from inside with the own natural progress of knowledge, or « from other nations through commercial intercourse, colonization, or conquest » [371 § 8]. Here stood precisely for Peacock the distinction between « tribes and nations » : the counting process is natural in the first case, it is more consciously organised in the second one.

In such a way, Peacock considered that scales such as 12 or 2 were established later, as more philosophical ones, issued from a more advanced stage of arithmetical knowledge [371 § 7] : they do not reveal any kind of original practice. More specifically, he devoted several paragraphs to the attention paid by Leibniz to the scale 2, and for the correspondance that he underlined with the hexagrams, which he named « the Cova, or the lineations of Fohi, the founder of the Empire ». But Peacock did not agree with Leibniz metaphysical interpretation⁶⁷ of the 0 and 1, that he called « metaphysical dreaming » [392 § 34], concluding that this scale was already a symbolical arithmetic, but was not suitable for the ordinary wants of every day life. For that reason, it was adopted by a sole genius man, and not by a nation. There, a special attention was immediately given to Chinese numeral words, because in that case, as Peacock wrote : « Chinese expressions for numerals are in all cases symbolical ». They are simply specific keys of the ideographic language [376 §13].

However, Peacock selected both what he named « the local value » (nowadays « positional value ») and the invention of zero as the specific characters of any effecient system of numeration. He therefore considered Indian numeral system as the first complete invention of decimal system, and followed the philologists in ascribing to the sanskrit language the origin of the classical language of Europe⁶⁸ :

The intimate analogy in the grammatical structure, and in many of the roots of the classical language of Europe with the Sanskrit, combined with the evidence furnished by historical and other monuments, point out the East as the origin of those tribes, whose progress to the West was attended by civilisation and empire, and amongst whom the powers of the human mind have received their highest degree of

⁶⁷ As unity was considered the symbol of the Deity, this formation of all nbs from zero and unity was considered in that age of metaphysical dreaming, as an apt image of the creation of the world by God from chaos [392 § 34]

⁶⁸ But Peacock did not decide on the question of the unique or multiple origin of all languages. What is more interesting for him are the conditions of development of arithmetical language. Only in that case, he can decide of the unique origin, because of the natural abacus.

development' [372 § 12].

Finally, Peacock concluded on § 33 the demonstration he began on § 8 : that the natural scales are founded on the decimal one, and that : '**The natural scales of numeration alone have ever met with general adoption**' [371 § 8, emphasis mine].

3. Peacock as a Whig actor in society

At this point of the demonstration, the arguments on the unification of scales of numeration met a political argument that Peacock already employed for the unification of languages⁶⁹, insisting on the political role of the nations to sustain both of them, claiming that : '**The natural scales only are national**' [391 § 33, emphasis mine].

Already when speaking about Chinese and Indian numeral systems, Peacock underlined how « nations » played an essential part in their development. He used the existence of an organised society to sustain an argument on utility, which was a very widespread argument for the utilitarian trend to which he was linked. For him, the existence of a useful, powerful system of numeration was closely linked to that of civilization and of a strong state, for example about the developing needs they sustained for writing large numbers [377 § 16]. About the Aztecks, Mexicans, Muyscas and Peruvians for instance, Peacock clearly linked the perfection of their numeral systems and the existence of organised governments, even if they expressed numbers with the vicenary scale – the base 20 – which, he assessed, was derived from the base 10 :

The Mexicans, Muyscas and Peruvians constitute the only three nations of Ancient America, who possessed government regularly organised, and who had made considerable progress in many of the arts of civilized life, in architecture, sculpture, and painting. They were the only people, in short, in that vast continent, who could be considered as possessing literary or historical monuments. On this account alone their numeral systems would merit very particular attention ; but still more so from their perfect development. The first presents the most complete example that we possess of the vicenary scale, with the quinary and denary subordinate to it. The second, of the same scale, with the denary alone subordinate to it ; whilst the third, or Peruvian, is strictly denary, and is equally remarkable for its extent and regularity of construction [389 § 29].

There can also be stressed the special care with which Peacock spoke of the Peruvian Quipus, which constitute for him a very perfect and material representation of numbers, in a decimal scale, « incomparably superior to those of any other American nation » [390 § 290]. Quipus indeed authorize the recording of numbers and the practice

⁶⁹ This argument also met Peacock's political position in Cambridge, when he urged the unifying role of the Crown in Cambridge, for supporting University against the Colleges.

of arithmetical operations with a particular rapidity and accuracy. They form a system of knots on different coloured strings, and constitute an excellent administrative tool reserved to their guardians⁷⁰. Here was recognisable the insistence of the reforming network, particularly sensible in Babbage, Peacock and De Morgan positions – as members of the Decimal Commission, but also in their public undertakings – on the necessary commitment of national organisations in the progress of knowledge⁷¹.

After his investigation of numeral words, Peacock turned to the invention of numeral symbols, which he named « symbolical arithmetic ». That one is no more linked with the most primitive people, but with historical periods, and organized societies. Once more, his presentation was directly linked with that of operations, not only the four elementary operations of arithmetic, but also the extraction of square and other roots of numbers. He presented all of them with numerous examples.

Peacock began this presentation with a very long paragraph on the arithmetical notation of the Greeks, because « they cultivated the sciences for the greatest success » [394 § 38]. He noted Delambre's disappointment not to see them developing the decimal notation : announcing his own way of thinking operations, he praised the too strong attachment of the Greeks to their alphabetical notation [405 § 41]. Nevertheless, in order to stand out from Delambre, and to show that Greek inventivity was not absent, Peacock appealed to Archimedes' *Arenary*, which gave the means to overtake the limitations of the initial system. On this point, he considered Stifel and Stevinus contributions in the XVIth century as an extension of this work of Archimedes, and insisted on the fact that progress is not there if the utility of notation is not socially perceived :

There are many of the artifices of notation employed in this work, which if pursued and properly generalised, would have given, increased symmetry as well as extent to their symbolical Arithmetic.

The only reason which can easily be assigned why this extension of their notation had not been generally adopted for all the symbols, when once applied to those of the nine digits, appears to have been, that as they merely proposed by it, in the first instance, to make their notation coextensive with the terms of their numeral language, they paused when that objet was effected ; and, however simple its extension to all the other symbols may have been, it was not likely to be adopted when the utility of it was not felt ; the advantage indeed of a simple and expressive notation addressed to the eye, as distinct from language, were in no respect understood by the ancient geometers ; and it is only

⁷⁰ What Peacock forgot to indicate, is that these Quipus are much more remarkable because they were found in societies without scripture. Investigations on Quipus still question researchers today.

⁷¹ Babbage's *Reflections on the Decline of Science in England, and some of its Causes* (1830) and Peacock's *Observations on the statutes of the University of Cambridge* (1841) can be specially praised on that topic.

in modern times that the power of symbolical language have been completely appreciated [397 § 38]

On the same way, among the Greeks, Peacock payed a special attention to Ptolemy, because of his preference for the sexagesimal notation. According to Peacock, Ptolemy's choice was clearly linked with astronomy, because it allowed the divisions of the cercle to be « nearly equal to the days in the year » [401 § 39]. Peacock explained how it was essentially used to avoid fractions, because of the numerous factors of 60. Therefore, this system too was considered as a refined state of development of the numeral systems, which was largely used before the introduction of the « Hindu notation »⁷². Contrary to Theon of Alexandria's assessment, Peacock dared to consider that the invention of sexagesimal arithmetics preceded that of Ptolemy, concluding :

Whoever, however, was the author, it must be considered as the greatest improvement in the science of calculation which preceded the introduction of the Hindoo notation ; it enables astronomers at once to get rid of fractions, the treatment of which in their ordinary arithmetic, was so extremely embarrassing : and enables them to extend their approximations, particularly in the construction of tables, to say required degree of accuracy [401 § 39].

Following then a chronological order, and the supremacy of nations, Peacock attributed the importance of roman ciphers to the domination of Romans. Still underlining the above parallel between the low state of development of different nations and the lack of notation, Peacock gave a large place to what he named « Palpable Arithmetic », which practice authorized operations when fitted notations were missing. He presented there all kinds of abacus : Roman abacus, Chinese Swan Pan, *Logistica Tabula* (tablet strewed by sand), and the Greek Abacus [407-410 § 51-57]. He insisted on the general use of such counters in Europe until the end of the XVth century in Italy, and until the XVIth and XVIIth century in France and England, mentioning notably Saunderson's calculating board for the blinds, or Napier's multiplicative rods. Peacock alluded there to Leibniz's arithmetical machine and to Babbage's Difference Engine as extensions of this trend of counting :

The existence of systems of symbolical Arithmetic implies some considerable progress in the arts of life ; and we, consequently, cannot expect that such systems should be numerous, particularly when we consider how few are the nations with whom civilization had been of native growth [407 § 51]

⁷² « Hindu » is the modern writing of the term « Hindoo » used by Peacock. It is written here between quotation marks, because nowadays, it could have a religious connotation, and seems to exclude non-Hindu Indians. Besides, using the term « Indian » could seem to exclude geographical places as Pakistan, Bengladesh or Sri Lanka.

Following his view on utility, Peacock pointed out treatises such as *Arithmetic, or the Ground of Arts* (1540) of Robert Recorde, and *Arithmetica Practica* (1662) of Gaspari Schotti, because they indicated the operating rules of this «Calcular Arithmetic», which was so much in use among the merchants that it was named *Arithmetica Mercatoria*. Merchants are specially praised there for their organising action. Peacock tackled the argument of utility not only with an historical point of view, but with a technical one. It can be seen there that he was not really interested by enquiring on the origins on Arithmetic, but only on how history testified of the natural and universal character of Arithmetic, in order to legitimate the importance of Arithmetic as a natural foundation of Algebra, a language with special notation. According to Peacock, the importance paid to the «Hindu» decimal notation came essentially from the fact that it had superseded the preceding ones, and that it authorized plainly the transition from palpable arithmetic to written arithmetic.

As Dhruv Raina will analyze in his following paper, what is essential to Peacock is, firstly, to assert that the «Hindu» Arithmetic is at least as ancient as Diophantus' one [413 § 66], and secondly, to show that this notation was adopted by the Arabs from «Mohammed ben Musa, the Khuwarezmitte» [413 § 67], and transmitted by them, as early as the Xth century, in all the countries where the Arabic language was known, and to Europ. And he devoted special historical attention to the conditions of that transmission.

4. Peacock as a historian

From there, Peacock worked essentially as a genuine historian of mathematics, worrying to write history with other glasses than those of his own time, and even expressing the appropriate criteria for such a methodological way of thinking. He gave evidence of the birth of the history of mathematics as a new discipline, independent of traditional reviews of diplomats and missionaries, whose investigations he analysed yet⁷³.

Firstly, Peacock fostered his presentation by a large range of examples, very carefully chosen in order to precise the whole scale of numeration and operation methods. In that way, he opened the way for the reader to get his own view, giving a complete access to his own sources, discussing manuscripts⁷⁴ as well as original books. For instance, when he tried to determine as exactly as possible the dates when the Arabic

⁷³ Peacock will be also more generally concerned by history. When he becomes deacon of Ely, from 1839 to his death, he managed the restoration of the cathedral, and studied documents on its history, newly found during the works, et dated from 1374. *Cambridge University Library*. Peacock's correspondance with R. Willis. Add. Ms. 5026 ff 24-30.

⁷⁴ Peacock refers to manuscripts of the British Library as well as those of the Bodleian Library in Oxford.

ciphers replaced the Roman ones, Peacock refused to observe just the datations, and preferred to investigate the calendar computations through *Computatio Ecclesiastico*, which was a printed book. Studying John Wallis (1616-1703), who is still well known for his interest in ancient and Arabic texts, Peacock put him on his guard against the possible confusion between the moment when symbols are found and the moment when they are really used, because of the delay between the real practice and the writing of the copist, which occurred before the development of printing. Peacock discussed manuscript dates, places where the manuscripts were found, and did not hesitate to dispute the scholars views, such as Delambre's one about the Greeks. For instance, on the role of Gerbert d'Aurillac in the introduction of the Arabic ciphers from Spain to France, Peacock was not so convinced as Colebrooke by Wallis arguments. He discussed also the real period when Leonardo Pisano (*Liber Abaci* in 1220) was living and working. Moreover, as a Whig reformer, Peacock underlined very carefully the inertia of traditions in this process of adopting the decimals, in educated colleges as well as on the market place.

What was more important for Peacock was the extension of arithmetical practices to various types of trades. He studied it in Simon Stevinus in *La Disme* (1590), which marked the introduction of the decimal system in merchant practices [440 § 136]. His insistence on Stevinus's work is very remarkable, because this work is alas not so much praised generally in classical histories of mathematics nowadays⁷⁵, and there, this reveals once more Peacock's perspective. *La Disme* introduced for him an important progress in universality and abstraction, in a time when arithmetical practices were often linked with concrete numbers, and where the subdivisions of systems of measures were established relatively to the human body, and therefore to hand-made labour. On the contrary, Peacock asserted that if Spain was important for the adoption of this decimal system, it was confined to the translation of the astronomical texts from Arabic in Latin, before that the « contests which distracted this country » concluded with the « final expulsion of the Moors » [414 § 70]. Spain was then relayed by Italy, which played a more important role in realising the advices of Stevinus.

Here, Peacock insisted on the role in return of these new practices on the development of arithmetic, and more precisely on the role of commercial and banking practices in improving arithmetical operations, explaining them with great details. Bills of exchange and book-keeping received a special attention. While their practice tended to make their users considered as usurers, Peacock worked to establish the contrary and to restore their social dignity. The reforming commitment of Peacock appeared clearly here. As a Whig whose eldest brother worked in Stock Exchange⁷⁶, Peacock was

⁷⁵ J.T. Merz praised Peacock's review of Stevinus as the best one at the end on the XIXth century. J. T. Merz, *An History of Scientific European Thought in the nineteenth century*, 4 vols, Gloucester (1898) 1976, I, 645.

⁷⁶ Peacock's correspondance with his brother William, between 1807 to 1838. Trinity College Library, Cambridge, Peacock's papers, P 3^x. Also in Durand(-Richard), 1985, op. cit., 543-555.

particularly concerned with the new ways by which mathematics can govern economy, in Quattrocento Italy as well as in Great-Britain at his time :

The Tuscans generally and the Florentines in particular, whose city was the cradle of the literature and arts of the XIIIth and XIVth centuries, were celebrated for their knowledge of Arithmetic : the method of book-keeping, which is called especially Italian, was invented by them ; and the operations of Arithmetic, which were so necessary too the proper conduct of their extensive commerce, appear to have been cultivated and improved by them with particular care ; to them we are indebted for our present processes for the multiplication and division of whole numbers, and also for the formal introduction into books of Arithmetic, under distinct heads, of questions in the single and double rule of three, loss and gain, fellowship, exchange, simple interest, discount, compound interest, and so on ; in short, we find in those books, every evidence of the early maturity of this science, and of its diligent cultivation ; and all these considerations combine to show that the Italians were in familiar possession of Algorithm long before the other nations of Europe [415 § 70]

The word « algorithm » was here specially analysed by Peacock, who observed the transformation of its meaning, once more by way of mathematical practice. He even laughed at Stifelius who ignored the Arabic origin of this word, when he insisted on the fact that it was born in the same time and at the same place that the word « Algebra » :

The term algorithm, which originally meant the notation by nine figures and zero, subsequently received a much more extensive signification, and was applied to denote any species of notation, whatever for the purpose of expressing the assigned relations of numbers or quantities to each other [438 § 132].

And Peacock devoted a long place to the improvement of the approximation processes that the decimal system brought, thanks to the decimal fractions, and to the dot notation. Peacock's conclusion on this point was really essential to his design about notation :

In general, however, it may be remarked, that the invention of a distinct, expressive, and comprehensive notation, is the last step which is taken in the improvement of analytical and other sciences ; and it is only when the complexity of the relations which are sought to be expressed in a problem is so great as to surpass the powers of language, that we find such expedients estimated [438 § 130].

My conclusive point would bear on the long detailed review that Peacock presented about the new French measures system, in other words the adoption of the metrical system of weights and measures during the French Revolution, from 'purely philosophical principles', which were in accordance with his Whig's position on the

progressive unifying power of nations⁷⁷. It reflected Peacock's profound admiration of the new French institutions born with the Revolution :

If ever an opportunity itself for the establishment of a system of weights and measures upon perfectly philosophical principles, it undoubtedly occurred in the early part of the French revolution, when the entire subversion of all the old establishments, and the hatred of all associations connected with them, had created a passion for universal change [446 § 171].

This review began with the measure of the pendulum vibrating seconds by Richer in 1671, the discussions of Cassini, and De la Condamine during the XVIIIth century, and finally, the decisions of the revolutionary commission in 1794 and 1798. If Peacock did not hide the difficulties of this adoption, and the disadvantages that it supports for the centesimal division of the quadrant, he insisted on the universal aspect of the new unity, quoting in French this part of the report of the 10 Prairial 1798 to the two councils of the legislative body :

Cette unité, tirée du plus grand et du plus invariable des corps que l'homme puisse mesurer, a l'avantage de ne pas différer considérablement de la demi-toise et des plusieurs autres mesures usitées dans les différens pays : elle ne choque point l'opinion commune. Elle offre un aspect qui n'est pas sans intérêt. Il y a quelque plaisir pour un père de famille à pouvoir se dire : "Le champ qui fait subsister mes enfans est une telle portion du globe. Je suis dans cette proportion conpropriétaire du monde" [448 § 174]

Moreover, this adoption enabled the union of operations on concrete numbers and operations on abstract numbers, contributing to the unification of the numerical realm. Once more, and as Babbage will later do for his engines, Peacock praised the political intervention to help the unification of knowledge.

CONCLUSION

This *History of Arithmetic* can really stand as a very well documented reference about what was known and discussed about the historical construction of arithmetic as a human undertaking. Nevertheless, in spite of his ethnological, philological, philosophical and historical enlarged positions, it is clear that his own starting conceptions about the construction of knowledge through language lead him to some limitations. For example, Peacock did not try to recompose the own processes of Indian and Arabic algorithms, notably for the rule of approximation of surd numbers, or for the

⁷⁷ From § 150 to 170, Peacock firstly shows the enormous diversity of ancient systems of weights and measures

rule of alligation and of position. In several places, he substitutes for them their algebraical outcome [436 § 125 & 463 § 238], what appears nowadays as a crucial aspect of a recurring position. His representation of algebra helps us to understand his comparative and philological approach to different numeral languages, and the respective places afforded particularly to Arabic and Indian arithmetic in the exposition of contemporaneous knowledge on numbers. As an Anglican Whig mathematician Peacock conceived these processes both as natural, and supported by the constitution of nations, writing a really « social study » of arithmetic, impressed with utilitarianist values on the organising role of State, even on structuring knowledge. Consequently, if Peacock's view maintained some recurring interpretation of arithmetical processes, it introduced a really constructivist one, notwithstanding the faculties of mind.

Peacock's main goal in this paper was to bring dignity to the arithmetical practices – which was truly new in academic circles – so as to connect together the scholarly knowledge and the tradesman's knowledge, by showing their universal sources, founding them on the common experience of all people facing the world everywhere. Peacock's representation of mathematics committed an empiricist conception of knowledge, even a mathematical one : the development of algebra is traced from arithmetical practices founded on the decimal system of numeration, and this one is conceived as a natural, and therefore as an universal one. Such a knowledge was founded on experience and organized by successive steps of symbolization, until Symbolical Algebra, and that view was deeply rooted in Locke's philosophy. But Locke's empiricism was still a very moderate empiricism, where the operative faculties of mind still stand as innate. In such a way, this *History of Arithmetic* stands as an integral part of Peacock's whole prospect of a very particular view of Algebra, which attempted to hold together contingent mathematical practices and universal necessary truth of deductive mathematical reasoning. Peacock can be praised for this powerful attempt : with his Symbolical Algebra, he tried to solve philosophically the difficult issue of the nature of algebra, with just the mathematical means of the first half of the XIXth century.

Acknowledgments

We would like to thank Karine Chemla and REHSEIS for their constant moral and financial support all along the préparation of our papers.

Josipa Petrunic worked a lot on the English way to write correctly this paper.

Special notice

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