Constraints on Anaphoric Determiners

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Constraints on anaphoric functions

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Abstract

Some constraints on functions from sets and relations to sets are studied. Such constraints are satisfied by anaphoric functions, that is functions denoted by anaphoric determiners. These constraints are generalisations of anaphor conditions known from the study of simpler cases of nominal anaphors. In addition a generalisation of the notion of conservativity as applied to anaphoric functions is proposed. Two classes of anaphoric determiners found in NLs are discussed as examples.

1 Introduction

Progress in the study of the logical properties of NLs (Natural languages) is closely related to the study of various constraints that must be satisfied by functions interpreting functional expressions in NLs. We know that such expressions do not denote arbitrarily and thus that functions interpreting them obey various specific constraints of a logical nature. The most prominent results obtained in this context are results obtained in generalised quantifiers theory and they concern functions corresponding to various types of quantifiers. The constraint on quantifiers which has been extensively studied from theoretical and empirical points of view is the constraint of conservativity concerning the denotations of various determiners found in NLs.

The purpose of this paper is to exhibit some specific properties of anaphoric functions, that is functions denoted by anaphoric determiners. Some anaphoric functions have been studied in the context of (nominal) anaphors (Keenan 1988, 2007, Keenan and Westerståhl 1997). In the simplest case anaphors are NP-like expressions which occur as grammatical objects of sentences referentially dependent on their subjects, as *Every poet admires himself.* Here *himself* is the anaphoric NP and can be correctly interpreted as the function $SELF$, which maps a binary relation like $ADMIRE$ to the set $\{x : \langle x, x\rangle \in ADMIRE\}$. Then $EVERY(POET)$ maps that set to $TRUE$ iff $POET$ is a subset of it, as usual. Crucially the above mentioned authors show that the $SELF$ function is not a natural extension of a generalised quantifier that maps unary relations (properties) to truth values - that is, the possible denotations for subjects in one place predicates. Complex anaphors such as *everybody except himself* or *five students including herself* also denote anaphoric functions that lie outside the class of generalised quantifiers.

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The primary focus of the present study is that of "anaphoric determiners" and their denotations. Anaphoric determiners combine with common nouns (like non-anaphoric determiners) to build complex NPs which denote anaphoric functions, ones like the simple cases above, that lie outside the class of the standard conservative functions that build generalised quantifier denoting expressions. An example from Polish illustrating such an anaphoric determiner is in (1a), which contrasts with the non-anaphoric one in (1b):

(1a) Piotr nienawidzi *swego* sąsiada
(Piotr hates his+own neighbour)
(1b) Piotr nienawidzi jego sąsiada
(Piotr hates his neighbour), ('his'= not the one of Piotr)

We show that, in addition to properly "anaphoric properties", the determiner *swego* (his+own) satisfies a natural generalisation of the conservativity property characteristic of non-anaphoric determiners.

In the next section a detailed description of anaphoric determiners and of their denotations, *anaphoric functions*, is provided. Then, in the next section two important classes of anaphoric determiners are discussed. Finally, a generalisation of conservativity is proposed and discussed in the context of these examples.

2 Anaphoric functions

In this section we provide some formal notions allowing us to clarify the properties and functions which we are going to discuss. We will be interested in functions which are not quantifiers though they can be seen as related to quantifiers. They are not generalised quantifiers since they do not map properties (unary relations) to truth-values, though like generalised quantifiers they do map n+2-ary relations to n+1-ary ones (Keenan and Westerståhl 1997). More generally the type of functions we study will be noted $\langle 1^n, k : k\text{-}1\rangle$, for $n \geq 0$ and $k \geq 2$. Thus these are functions which have n sets and a k-ary relation as arguments and a k$-$1-ary relation as results.

To be more precise, we will be interested in the particular case when $k = 2$ and thus we will study basically functions of type $\langle 1, 2 : 1\rangle$ and functions of type $\langle 1^2, 2 : 1\rangle$. Such functions, when satisfying some specific constraints to be specified, will be called *anaphoric functions*.

From the empirical point of view anaphoric functions are denoted by anaphors and by anaphoric (unary or n-ary) determiners. Anaphoric determiners are expressions which take common nouns as arguments and give NP-like expressions, or anaphors, as result. These NPs are not typical NPs because in particular either they do not occur in subject positions at all or if they do, they are not interpreted as anaphors bound to non-subject arguments of the verb. This is related to their anaphoric character. Semantically they are referentially dependent on the extensions of the subject NPs. More precisely they denote functions from relations to sets which satisfy the anaphor condition (Keenan 2007) and thus they are nominal (accusative) anaphors. In fact nominal anaphors even satisfy an additional condition: the functions that they denote are not extensions of quantifiers. In that sense they are "proper" anaphors.

So in this paper we do not discuss anaphors as such but determiners forming anaphors when applied to common nouns. The functions that they denote also satisfy specific conditions. The reason is precisely that they form anaphors which, as indicated, satisfy specific anaphoric conditions. In order to formulate anaphoric conditions for anaphoric determiners let me define the anaphor condition (AC) which characterises anaphors in more detail. We are interested in the interpretation of simple sentences of the form NP_1 TVP NP_2. In such sentences NP_1 is interpreted by a type $\langle 1\rangle$ quantifier, which is a set of sets, and $TVPs$ is interpreted by a binary relation. Concerning NP_2 there are two possibilities; it can be interpreted either by a function which is an accusative extension of a type $\langle 1\rangle$ quantifier or by a function (from binary relations to sets) which is not an accusative extension of a type $\langle 1\rangle$ quantifier. An accusative extension Q_{acc} of a type $\langle 1\rangle$ quantifier Q is defined as follows (Keenan 1988):

Definition 1: For R a binary relation over the universe E and $a \in E$, write aR for $\{b : \langle a, b\rangle \in R\}$. Then for each type $\langle 1\rangle$ quantifier Q, $Q_{acc}(R) = \{a : Q(aR) = 1\}$.

Thus the accusative extension of a quantifier is a function from binary relations to sets induced by the quantifier in the way indicated in definition 1. Accusative extensions of quantifiers permit one to compute directly denotations of verb phrases formed from transitive verb phrases and a noun phrase in the position of the direct object.

Functions which are accusative case extensions are specific in the sense that they satisfy the following accusative extension condition:

AEC: A function F from binary relations to sets satisfies the AEC iff for R and S binary relations, and $a, b \in E$, if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$.

The following two simple conditions can be used to decide whether functions from binary relations to sets to satisfy or not the AEC condition:

Fact 1: A function F from binary to unary relations does not satisfy the AEC if there exists a set B and $a, b \in B$ such that $a \in F(E \times B) \wedge b \notin F(E \times B)$
Fact 2: If a function F from binary relations to sets satisfies AEC then for any $A \subseteq E$ one has $F(E \times A) = \emptyset$ or $F(E \times A) = E$.

Not every function from binary relations to sets satisfies AEC. The best known example is the function $SELF$ which can be used to interpret the reflexive pronoun *himself/herself*. This function satisfies the anaphor condition AC, which is weaker than AEC:

AC: A function F from binary relations to sets satisfies the *anaphor condition* iff for R and S binary relations, and $a \in E$, if $aR = aS$ then $a \in F(R)$ iff $a \in F(S)$.

Obviously, functions which are accusative extensions of some quantifiers also satisfy AC. This means that anaphoric functions should not satisfy AEC:

Definition 2: A function from binary relations to sets (i.e. a function of type $\langle 2:1\rangle$) is *anaphoric* iff it satisfies AC and fails AEC.

One can check (Keenan 2007) that $SELF$ is an anaphoric function of type $\langle 2:1\rangle$ as is the function $NOBODY$-$EXCEPT$-$SELF$.

The AC applies to functions of type $\langle 2:1\rangle$. We need a similar condition for functions denoted by unary and binary anaphoric determiners, which are functions of type $\langle 1, 2:1\rangle$ and functions of type $\langle 1^2, 2:1\rangle$ respectively. A class of type $\langle 1, 2:1\rangle$ functions is given by the accusative extension of type $\langle 1, 1\rangle$ quantifiers. Thus if D is a type $\langle 1, 1\rangle$ quantifier then the function $F(X, R) = D(X)_{acc}(R)$ is a type $\langle 1, 2:1\rangle$ function.

Similarly, a class of type $\langle 1^2, 2:1\rangle$ functions is given by the accusative extension of type $\langle 1^2, 1\rangle$ quantifiers. Such quantifiers are functions which take two sets as arguments and give a type $\langle 1\rangle$ quantifier as result (Keenan and Moss 1985). For instance the function denoted by the binary determiner *More...then ...* is a type $\langle 1^2, 1\rangle$ quantifier. Now, if D is a type $\langle 1^2, 1\rangle$ quantifier then the function $G(X_1, X_2, R) = D(X_1.X_2)_{acc}(R)$ is a type $\langle 1^2, 2:1\rangle$ function. In the next section we will briefly discuss anaphoric type $\langle 1^2, 2:1\rangle$ functions denoted by Polish binary anaphoric determiners obtained by the combination of "ordinary" binary determiners with the pronoun *swoj*.

For functions of type $\langle 1, 2:1\rangle$ we have the following condition:

ACD1: A function F of type $\langle 1, 2:1\rangle$ satisfies *anaphor condition for unary determiners* (ACD1) iff for any $a \in E$, $X \subseteq E$ and R, S binary relations, if $a((E \times X) \cap R) = a((E \times X) \cap S)$ then $a \in F(X, R)$ iff $a \in F(X, S)$.

The following property gives a justification of condition ACD1:

Fact 3: If the function F of type $\langle 1, 2:1\rangle$ satisfies ACD1 then the function G^A of type $\langle 2:1\rangle$ defined as $G^A(R) = F(A, R)$ satisfies AC.

What fact 3 informally says is that functions satisfying ACD1 are those from which we get functions satisfying AC when fixing their set argument. For instance as long as anaphoric determiner such as *swego* (cf. (1a) above) satisfy ACD1, then the complex anaphoric NPs such as *swego sşiada* 'his own neighbour' are guaranteed to satisfy the AC condition just as lexical reflexive like *himself* satisfy that condition.

We have a similar condition for type $\langle 1^2, 2:1\rangle$ functions:

ACD2: A function F of type $\langle 1^2, 2:1\rangle$ satisfies *anaphor condition for binary determiners* (ACD2) iff for any $a \in E$, $X, Y \subseteq E$ and R, S binary relations, if $a((E \times X) \cap R) = a((E \times X) \cap S)$ and $a((E \times Y) \cap R) = a((E \times Y) \cap S)$ then $a \in F(X, Y, R)$ iff $a \in F(X, Y, S)$.

We have a similar "justifying" property for ACD2 condition given above:

Fact 4: If the function F of type $\langle 1^2, 2:1\rangle$ satisfies ACD2 then the function $G^{A,B}$ of type $\langle 2:1\rangle$ defined as $G^{A,B}(R) = F(A,B,R)$ satisfies AC.

We can use the same method of fixing "nominal" arguments to define anaphoric functions of type $\langle 1, 2:1\rangle$ and of type $\langle 1^2, 2:1\rangle$. Thus we have:

Definition 3: A function F of type $\langle 1, 2:1\rangle$ is *anaphoric* iff it satisfies the condition ACD1 and the function G^A of type $\langle 2:1\rangle$ is anaphoric (in the sense of D2) for $A \neq \emptyset$ (where $G^A(R) = F(A,R)$) .
Definition 4: A function F of type $\langle 1^2, 2:1\rangle$ is *anaphoric* iff it satisfies the condition ACD2 and the function $G^{A,B}$ of type $\langle 2:1\rangle$ is anaphoric for any $A \neq \emptyset$ and $B \neq \emptyset$ (where $G^{A,B}(R) = F(A,B,R)$).

In the above definitions anaphoricity of type $\langle 1, 2:1\rangle$ and type $\langle 1^2, 2:1\rangle$ functions is reduced to anaphoricity of type $\langle 2:1\rangle$ functions defined in D2. The condition for A and B to be non-empty is necessary because otherwise $F(\emptyset, R)$ and $F(\emptyset, \emptyset, R)$ would satisfy AEC and thus there would be no anaphoric functions of type $\langle 1, 2:1\rangle$ and of type $\langle 1^2, 2:1\rangle$. In the next section various examples of anaphoric determiners denoting anaphoric functions are given.

3 Anaphoric determiners

To illustrate some of the properties discussed above we present in this section two classes of anaphoric determiners and at the end of the section a "natural" type $\langle 1, 2:1\rangle$ function which is not anaphoric.

An example belonging to the first class of anaphoric determiners have already been briefly mentioned. This is the expression *every... but himself/herself* as it occurs in (2a). A similar example with *no...except himself* is given is (2b)

(2a) Leo shaved every student but himself.
(2b) Lea admires no linguist except herself.

Let us show that the function *NO...BUT-SELF* interpreting the anaphoric determiner in (2b) is anaphoric. In order to show that a function from binary to unary relations does not satisfy the AEC, the condition given in fact 1 can be used. We want to show first that the function *NO*(A)-*BUT-SELF*, as specified in (3), does not satisfy the AEC for any $A \neq \emptyset$:

(3) *NO*(A)-*BUT-SELF*(R) $= \{x : A \cap xR = \{x\}\}$

If $A = E$ we get the type $\langle 2:1\rangle$ function *NOBODY-BUT-SELF* which is anaphoric. Suppose now that $A \neq \emptyset$ and $A \neq E$. Choose as B a supersubset of A which differs from A just by one element: that is $A \subset B$ and for some $a \in E$, $A' \cap B = \{a\}$ (where A' is the complement of A. Obviously for some $b \in A$ one has $a, b \in B$ (and $a \notin A$). It follows from this and from (3) that (4) is true:

(4) $a \in NO(A)\text{-}BUT\text{-}SELF(E \times B) \wedge b \notin NO(A)\text{-}BUT\text{-}SELF(E \times B)$

Thus $NO(A)\text{-}BUT\text{-}SELF$ does not satisfy the AEC for non trivial A.

It remains to show that this function satisfies the AC condition. But this is obvious given the description in (3). Indeed, suppose that $aR = aS$ for some binary relations S and R and that $a \in \{x : A \cap xR = \{x\}\}$. This means that $A \cap aR = \{a\}$. Since $aR = aS$, we have also $A \cap bR = \{b\}$ and thus $b \in \{x : A \cap xR = \{x\}\}$.

The above examples of anaphoric determiners can be informally characterised as those which are obtained from the combination of the reflexive *himself/herself* with the exclusive determiners. We can add here many other examples in which the exclusion clause is a Boolean compound in which one of its Boolean components is *himself/herself*. Thus *every...except himself and Bill*, *no...but Lea or himself*, etc. are other examples of anaphoric determiners. Some properties of functions denoted by such determiners are discussed in section 5.

Along with exclusive determiners natural languages display a kind of parallel determiner called *inclusion* determiners (Zuber 1998). These are complex determiners like *Most...including Leo*, *five...including Leo*, *three other... in addition to Leo and Lea*, *Leo and some other/five other/most other...*. We can also use these determiners to form anaphoric determiners, as in the following examples:

(5) Leo talked to five other linguists in addition to himself.
(6) Lea hates most vegetarians including herself
(7) Martin washed no student, not even himself.

Inclusive anaphoric determiners, like the one in (6), along with their semantics, are discussed in the next section. Complex anaphoric determiners formed from parts of non-anaphoric determiners (basically exclusion and inclusion determiners) and the pronoun *himself/herself* will be called *self*-type anaphoric determiners in what follows.

We are not going to specify the exact way in which *self* is combined with determiners to form anaphoric determiners. Obviously such a combination cannot be arbitrary. For instance we would not consider that *Some... and himself* or *No... and not even himself* are anaphoric determiners when they occur in the following examples:

(8) Leo washed some logicians and himself.
(9) Leo admires no logician and not even himself.

Semantically speaking in these examples we have Boolean meets of functions from binary to unary relations (such functions form a Boolean algebra) and thus not functions taking unary and binary relations as arguments. One observes in this context that when we have anaphoric determiners then not only are the noun phrases they form referentially dependent on the subject NPs but also that they indicate in addition that both NPs, the subject NP and the object NP share a common property. Consider the following examples from this point of view:

(10) Leo washed some logicians in addition to/besides himself.
(11) Leo washed no logician, not even himself.
(12) Five logicians admire ten other vegetarians in addition to themselves

Sentences in (10) and in (11), in opposition to those in (8) and (9) entail that Leo is a logician. Similarly (12) entails that there are five logicians who are vegetarians. Thus in all cases the (denotations) of the anaphoric NPs and their antecedent NPs share a (non-trivial) property.

Anaphoric determiners belonging to the first class of determiners, the *self*-type anaphoric determiners, are in some sense language independent since similar examples can presumably be constructed in any language having a reflexive pronoun. For instance in Polish, using the reflexive pronoun *siebie* one can construct virtually all *self*-type anaphoric determiners discussed above.

We are going now to present briefly a second type of examples of anaphoric determiners. They will be called *Slavic anaphoric determiners*.

Observe that *self*-type anaphoric determiners are not morphologically simple determiners: they are specific compositions of the nominal anaphor *self* with parts "ordinary" determiners. Some languages, apparently in opposition to English, have, however, morphologically simple anaphoric determiners. This is in particular the case of some Slavic and Scandinavian languages, of Latin, etc. Such determiners in addition involve a possession relation, in opposition to *self*-type determiners. In what follows we illustrate this class of determiners by the lexical anaphoric determiner $SVOJ$ found in Polish. For a more detailed description of see Zuber (2010a).

In Polish, the third person possessive pronoun *swoj* takes common nouns as arguments and forms (nominal) anaphors. So it behaves in some respects as a determiner. Its meaning is roughly, though not exactly, *his/her/their own*. It will be glosed by HOWN. Since the expressions formed with *swoj* form nominal anaphors they cannot be used in NPs occuring in subject positions and consequently they are not used in Polish in the nominative case.

In addition the Polish determiner (pronoun) *swój* can combine with virtually any "ordinary" determiner to form a complex anaphoric determiner which, when applied to a common noun, gives a nominal anaphora. Such complex anaphoric determiners also involve in their semantics a possession relation. Thus one has *większość swoich...*/most HOWN, *10 swoich...*/10 HOWN, *żaden ze swoich...*/no HOWN, *conajmniej 5 swoich...*/at least 5 HOWN, *wszystkie swoje... oprócz Kazia*/every HOWN, except K., *niektóre swoje..., włącznie z Kaziem*/ some HOWN, including K., etc. These lexically complex determiners have a Boolean structure, and consequently one can form their Boolean compounds. In particular bare *swój* has the negative form *nieswój*/NOT-HOWN. The negative forms in their turn can combine with "ordinary" determiners to give complex anaphoric determiners. Both bare, negative and positive, forms can occur in such complex anaphiric determiners. Thus we have *Niektóre nieswoje...*/some NOT HOWN, *5 swoich i 6 nieswoich...*/5 HOWN and 6 NOT-HOWN, etc.

As indicated in the introduction NLs also have binary or even n-ary determiners. These are expressions which take two or more CNs as arguments and which form NPs. There are two types of such determiners (cf. Keenan and Moss 1985, Beghelli 1994): reducible and non-reducible ones. The Polish determiner *swój* can also combine with binary (or even n-ary) determiners to give binary (or n-ary) anaphoric determiners of both types. For instance the determiner *swój* can combine with binary irreducible determiners corresponding to the English *more ...than...*. Thus, *swój* combined with the binary determiner *wiecej...niż* (*more...then*) gives the binary anaphoric determiner *wiecej swoich... niż swoich...* (*more HOWN... than HOWN...*). In (13) and (14) we have Polish examples of such binary quantifiers:

(13) Piotr spalił więcej swoich obrazów niż (swoich) listów.
Piotr burnt more HOWN paintings than (HOWN) letters
'Piotr burnt more of his own paintings than (his own) letters'

(14) Leon sprzedał proporcjonalnie więcej swoich obrazów niż książek.
Leon sold proportionally more OWN paintings than books
'Leon sold proportionally more of his (own) paintings than books'

Let us briefly present the semantics of the unary anaphoric determiner *swój* (HOWN) and of complex determiners containing *swój*. To account for the possessive nature of such determiners we need in addition the binary relation POS, which expresses the possessor relation (which needs not to be just ownership or authorship relation). Such a relation, contextually determined, is needed for the semantics of "ordinary" possessives as well (Peters and Westerståhl 2006). We will suppose that POS is anti-reflexive: for evey x, $\langle x, x\rangle \notin POS$. In other words no object is in a possessor relation with itself.

Our goal now is to represent the possessive aspects of the semantics of simple and complex anaphoric determiners whose empirical properties we have seen. Concerning bare determiners, it is necessary to analyse separately the meaning of the singular and plural forms. The semantics of *swój* in singular is simple if one supposes that the singular presupposes the unicity of the possessed object. Consider (15):

(15) Piotr admires HOWN neighbour

On its most natural reading (15) entails (presupposes) that Piotr has just one neighbour. Formally we express such unicity of objects having a specific property using the description operator *iota* ι. More specifically, the description noted $\iota x(x \in A)$ designates the unique object x which has the property A, if such an object exists. With the help of this notation the semantics of bare singular *swój* can be represented as follows:

(16) $SVOJ(A, R) = \{x : |xPOS \cap A| = 1 \wedge \langle x, \iota y(y \in xPOS \cap A)\rangle \in R\}$

It is not difficult to show that $SVOJ$ is anaphoric. First, it obviously satisfies the anaphor condition ACD1. Furthermore, using Fact 2 we can show that $SVOJ(A, R)$ is not an extension of a type $\langle 1\rangle$ quantifier for any $A \neq \emptyset$.

Let us see now the semantics of complex Slavic anaphoric determiners. Suppose that Let D is a type $\langle 1, 1\rangle$ quantifier, the denotation of some unary determiner *Det*. D_S is the denotation of the anaphoric determiner *Det swój* obtained by combining *Det* with *swój* according to the rules underlying the above examples. Such a determiner applies to a CN and gives an anaphor. Thus D_S can be considered as a function taking two arguments, a set and a relation, and giving as result a set (the denotation of the whole VP). Then:

(17) $D_S(A, R) = \{x : xPOS \cap A \neq \emptyset \wedge D(xPOS \cap A)(xR) = 1\}$

The clause $xPOS \cap A \neq \emptyset$ expresses the existential presupposition that possessives induce. The remaining part shows how the anaphoricity is expressed by the accusative case extension of the type $\langle 1 \rangle$ quantifier formed with D applied to A which is modified with the help of POS. Thus *Kazio hates most of his mistresses* is true if K is a member of the set $\{x : xPOS \cap M \neq \emptyset \wedge MOST(xPOS \cap M)(xH) = 1\}$.

Thus (17) indicates that one can associate with any type $\langle 1, 1 \rangle$ quantifier (and a possessor relation $POSS$) a Slavic type anaphoric function.

Given the above analysis one can show that the function $F_A(R) = D_S(A, R)$ is not an extension of a type $\langle 1 \rangle$ quantifier (for any $A \neq \emptyset$) and that it satisfies anaphor condition AC. Thus NP-like expressions formed from complex anaphoric determiners (that is NPs formed from anaphoric determiners applied to CNs) are proper anaphors.

All anaphoric determiners we have considered "contain", in some sense, an "ordinary" determiner or part of it. In the next section we show how one can associate a class of anaphoric determiners to a specific class of non-anaphoric determiners.

To conclude this section let me give an example of an expression which looks as if it could denote a type $\langle 1, 2 : 1 \rangle$ anaphoric function but the function it denotes does not satisfy the ACD1 condition. Consider the determiner *the greatest number of* as it occurs in (18). The type $\langle 1, 2 : 1 \rangle$ function $NSUP$ interpreting this determiner is given in (19). Since (18) is equivalent to (20) one could think that the function in (19) is anaphoric:

(18) Leo knows the greatest number of languages.
(19) $NSUP(X, R) = \{x : \forall y (y \neq x \rightarrow |xR \cap X| > |yR \cap X|\}$
(20) Leo knows more languages than anybody else.

It is easy to check that for no $A \neq \emptyset$ the function $F_A(R) = NSUP(A, R)$ satisfies the AC condition. For instance suppose that Leo studies precisely those languages that he knows. It does not follow from this that *Leo studies the greatest number of languages* is equivalent to (18). Thus $NSUP$ is not an anaphoric function. However, this function satisfies the constraint of *argument invariance* (Keenan and Westerståhl, 1997) proper to comparatives and superlatives.

4 Generalising conservativity

Conservativity is a property of some classes of quantifiers. A non-trivial notion of conservativity applies to functions from sets or/and relations to truth-values which take at least two arguments. Many quantifiers are precisely such functions. In particular quantifiers denoted by unary and n-ary determiners can be said to be conservative. We have seen that anaphoric functions (that we consider here) are systematically related to quantifiers (of type $\langle 1,1 \rangle$ or of type $\langle \langle 1,1 \rangle 1 \rangle$). So it is quite natural to ask whether and in what sense anaphoric functions are conservative.

Let us recall first the notion of conservativity for type $\langle 1,1 \rangle$ quantifiers. A well-known by now definition is as follows:

Definition 5: $F \in CONS$ iff for any property X, Y one has $F(X)(Y) = F(X)(X \cap Y)$

Conservativity of type $\langle 1,1 \rangle$ quantifiers can additionally be formulated in two different ways:

Fact 5 (cf. Keenan and Faltz 1986) : F is conservative or $F \in CONS$ iff for any property X, Y and Z if $X \cap Y = X \cap Z$ then $F(X)(Y) = F(X)(Z)$
Fact 6 (Zuber 2005): $F \in CONS$ iff for any property X, Y one has $F(X)(Y) = F(X)(X' \cup Y)$

It is also possible to define conservativity for the whole class of type $\langle 1,1,2 \rangle$ quantifiers. In this case we have the following definition (cf. Westerståhl 2004):

Definition 6: A type $\langle 1,1,2 \rangle$ quantifier F is *conservative* iff for any sets A, B and any binary relation R one has $F(A, B, R) = F(A, B, (A \times B) \cap R)$

As in the case of "simple" type $\langle 1,1 \rangle$ quantifiers it is possible to give an equivalent defining condition for conservativity of type $\langle 1,1,2 \rangle$ quantifiers to hold. Thus we have:

Proposition 1: A type $\langle 1,1,2 \rangle$ quantifier is conservative iff $F(A, B, R_1) = F(A, B, R_2)$ whenever $(A \times B) \cap R_1 = (A \times B) \cap R_2$

Clearly none of the above definitions of conservativity applies directly to an anphoric function. However, proposition 1 and fact 5 give us a hint as to what form the definition of conservativity of type $\langle 1,2:1 \rangle$ functions should take. Here is the definition:

Definition 7: Let F be a type $\langle 1,2:1 \rangle$ function. Then F is *conservative* iff for all $X \subseteq E$ and R_1, R_2 binary relations, if $(E \times X) \cap R_1 = (E \times X) \cap R_2$ then $F(X, R_1) = F(X, R_2)$.

By analogy with fact 1 and definition 1 conservativity of type $\langle 1,2:1 \rangle$ functions can be defined equivalently as the following proposition shows:

Proposition 2: A function F of type $\langle 1,2:1 \rangle$ is conservative iff $F(X, R) = F(X, (E \times X) \cap R)$

The following property gives additional plausibility to the above definitions of generalised conservativity:

Proposition 3: Let D be a type $\langle 1,1\rangle$ quantifier and F a type $\langle 1,2:1\rangle$ function defined as: $F(X,R)=D(X)_{acc}(R)$. Then F is conservative iff D is conservative.

Proof:
Suppose *a contrario* that F is conservative and D is not. Thus for some $X,Y \in E$, $D(X)(Y) \neq D(X)(X \cap Y)$. Let $R = E \times Y$. Then:

$F(X,R) = D(X)_{acc}(R) = \{a : D(X)(aR) = 1\} = \{a : D(X)(a(E \times Y)) = 1\}$
$F(X,(E\times X)\cap R) = D(X)_{acc}(E\times(X\cap Y)) = \{a : D(X)(a(E\times(X\cap Y))) = 1\}$
Since $D(X)(a(E\times Y)) = D(X)(Y)$ and $D(X)(a(E\times(X\cap Y))) = D(X)(X\cap Y)$, this means that $F(X,R) \neq F(X,(E\times X)\cap R)$, which is impossible given that F is conservative.
Suppose now that D is conservative. Then:
$F(X,R) = D(X)_{acc}(R) = \{a : D(X)(aR) = 1\}=$
$=\{a : D(X)(X \cap aR) = 1\}$, since D is conservative
$=\{a : D(X)(a((E\times X)\cap R)) = 1\}$, since $X = a(E\times X)$ and $a(R\cap S) = aR\cap aS)$
$=D(X)_{acc}((E\times X)\cap R) = F(X,(E\times X)\cap R)$

Thus the generalised conservativity of functions induced by type $\langle 1,1\rangle$ quantifiers, when they are used in the accusative extension of a type $\langle 1\rangle$ quantifier, is strictly related to the "classical" conservativity of the inducing quantifier.

Let us recall now some properties of denotations of binary determiners, that is quantifiers of type $\langle\langle 1,1\rangle 1\rangle$. We have the following definition of conservativity (Keenan and Moss 1985, Zuber 2005):

Definition 8: A type $\langle\langle 1,1\rangle 1\rangle$ quantifier is *conservative* iff for any $X_1, X_2, Y_1, Y_2 \subseteq E$, if $X_1 \cap Y_1 = X_1 \cap Y_2$ and $X_2 \cap Y_1 = X_2 \cap Y_2$ then $F(X_1,X_2)(Y_1) = F(X_1,X_2)(Y_2)$.

The following proposition shows the equivalent way to define conservativity for type $\langle\langle 1,1\rangle 1\rangle$ quantifiers:

Proposition 4: A type $\langle\langle 1,1\rangle 1\rangle$ quantifier is conservative iff for any $X_1, X_2, Y \subseteq E$ one has $F(X_1,X_2)(Y) = F(X_1,X_2)(Y \cap (X_1 \cup X_2))$.

Definition 8 and proposition 4 can be used as basis for generalising conservativity to type $\langle 1^2, 2:1\rangle$ functions:

Definition 9: A type $\langle 1^2, 2:1\rangle$ function F is conservative iff for any $X_1, X_2 \subseteq E$ and any binary relations R_1 and R_2, if $(E\times X_1)\cap R_1 = (E\times X_1)\cap R_2$ and $(E\times X_2)\cap R_1 = (E\times X_2)\cap R_2$ then $F(X_1,X_2,R_1) = F(X_1,X_2,R_2)$.

The corresponding equivalent property is indicated in the following proposition:

Proposition 5: A type $\langle 1^2, 2:1\rangle$ function F is conservative iff for any $X_1, X_2 \subseteq E$ and binary relation R one has $F(X_1, X_2, R) = F(X_1, X_2, (E\times(X_1\cup X_2))\cap R)$.

It is easy to check that various examples of type $\langle 1, 2:1\rangle$ anaphoric functions and type $\langle 1^2, 2:1\rangle$ anaphoric functions discussed above are conservative. In the next section we discuss in more detail two sub-classes of *self*-type conservative anaphoric functions which are denotable (in English). Now I would like to present briefly a class of *self*-type anaphoric conservative functions denoted by anaphoric inclusive determiners. These determiners are expressions of the form *Det, including himself/herself* or of the form *Det, including NP and himself/herself*, where *Det* is ordinary unary determiner denoting a type $\langle 1, 1\rangle$ quantifier which is monotone increasing on the second argument. Thus the following expressions are examples of anaphoric inclusion determiners: *Every...including herself, Most...including some Albanians and himself, Five...including herself and ten Japanese*, etc. Obviously the semantics of such expressions depends on the semantics of the determiner *Det*: if *Det* denotes D then the anaphoric determiner of the form *Det, including himself/herself* denotes the function in (21) and the anaphoric determiners of the form *Det, including NP and himself/herself*, denote the function in (22), where Q is the denotation of the *NP*:

(21) $F(X, R) = \{x : x \in (X \cap xR) \wedge D(X)(xR) = 1\}$
(22) $G(X, R) = \{x : x \in X \wedge D(X)(xR) = 1 \wedge Q(xR) = 1\}$

It is easy to check that if D is conservative, functions in (21) and (22) are conservative

Concerning "Slavic" unary anaphoric determiners denoting the function D_S given in (17) we have the following general result:

Proposition 6: If a type $\langle 1, 1\rangle$ quantifier D is conservative then type $\langle 1, 2:1\rangle$ function D_S is conservative (where $D_S(A, R) = \{x : xPOS\cap A \neq \emptyset \wedge D(xPOS\cap A)(xR) = 1\}$).

When the determiner D is not conservative the "Slavic" anaphoric determiner to which it gives rise may denote a non-conservative type $\langle 1, 2:1\rangle$ function. For simplicity consider the English example in (23):

(23) Leo likes only his own students.

This sentence is probably ambiguous: depending on whether the focus is on "only" or on "own", it can mean that either Leo likes his own students and nothing else or that Leo likes his own students and not other students. Thus the following two type $\langle 1, 2:1\rangle$ functions are involved in the semantics of (23):

(24a) $F(A, R) = \{x : xR \subseteq xPOS \cap A\}$
(24b) $G(A, R) = \{x : xR \cap A \subseteq xPOS \cap A\}$

One can check that the function in (24b) is conservative and that the one in

(24a) is not.

Consider now a type $\langle 1^2, 2:1\rangle$ function $F = \textit{MORE-HOWN-THAN}$, which is involved in the semantics of examples like the one in (13); it is defined in (25):

(25) $\textit{MORE-HOWN-THAN}(X_1, X_2, R) = \{x : |xPOS \cap xR \cap X_1| > |xPOS \cap xR \cap X_2|\}$, where POS is the possessor relation.

Again, it is easy to check that this function is conservative.

5 Other constraints

Generalised conservativity introduced in the previous section in definitions 7 and 9 concerns type $\langle 1, 2:1\rangle$ and type $\langle 1^2, 2:1\rangle$ functions in general and not only anaphoric functions. As we have seen some non anaphoric functions (in the sense defined here) also satisfy the generalised conservativity. Moreover generalised consevativity is independent of anaphor conditions ACD1 and ACD2 proper for anaphoric functions. What is interesting is the fact that anaphoric functions satisfy in addition other constraints, some of which are stronger than generalised conservativity.

It is well-known that various natural language quantifiers can satisfy stronger constraints than conservativity (Keenan and Westerståhl, 1997). In particular they can be intersective or co-intersective. The question thus arises whether one can generalise the notion of intersectivity or co-intersectivity to anaphoric functions as well. In what follows I show briefly how it can be done.

Recall (Keenan and Westerståhl, 1997) that a type $\langle 1, 1\rangle$ quantifier D is intersective (resp. co-intersective) iff $D(X_1, Y_1) = D(X_2, Y_2)$ whenever $X_1 \cap Y_1 = X_2 \cap Y_2$ (resp. $X_1 \cap Y_1' = X_2 \cap Y_2'$). This leads to the following definition of intersective or co-intersective anaphoric functions (Zuber 2010b):

Definition 10: A type $\langle 1, 2:1\rangle$ function is *intersective* (resp. *co-intersective*) iff $F(X_1, R_1) = F(X_2, R_2)$ whenever $(E \times X_1) \cap R_1 = (E \times X_2) \cap R_2$ (resp. $(E \times X_1) \cap R_1' = (E \times X_2) \cap R_2'$).

The following proposition, similar to Proposition 3, can be considered as justifying the above definition:

Proposition 6: Let D be a type $\langle 1, 1\rangle$ quantifier and F a type $\langle 1, 2:1\rangle$ function defined as: $F(X, R) = D(X)_{acc}(R)$. Then F is intersective (resp. co-intersective) iff D is intersective (resp. co-intersective).

In order to give various examples of denotable intersective and co-intersective functions let us first recall some examples of "ordinary" determiners denoting type $\langle 1, 1\rangle$ intersective or co-intersective quantifiers. Anaphoric determiners will be based on such "ordinary" determiners in the way already suggested above.

Intersective and co-intersective type $\langle 1, 1\rangle$ quantifiers form atomic Boolean algebras. Atoms of these algebras are uniquely determined by sets. More precisely atoms of the intersective algebra are functions At_A such that $At_A(X)(Y) = 1$ iff $X \cap Y = A$ and atoms of the co-intersective algebra are functions At_B such that $At_B(X)(Y) = 1$ iff $X \cap Y' = B$, $(A, B, X, Y \subseteq E)$.

In NLs there are many expressions denoting (various) atoms of intersective and co-intersective algebras. Thus, roughly speaking, exception determiners with *No* (and *No* itself) denote atoms of the intersective algebra and exception determiners with *Every* (and *Every* itself) denote atoms of the co-intersective algebra. For instance the determiner *No...except Leo* denotes the atomic intersective quantifier determined by the singleton $\{L\}$ whose only element is Leo and the determiner *No...except Albanians* (as it occurs in *No student except Albanians (albanian students) danced*) denotes the atom of intersective functions determined by the set $ALBANIAN$. Similarly, the determiner *Every* denotes the atom (of co-intersective quantifiers) which is determined by the empty set.

We can give now various examples of intersective and co-intersective anaphoric functions and anaphoric determiners which denote them. Let At_A be the (intersective or co-intersective) atom determined by the set A. The type $\langle 1.2 : 1\rangle$ function F_{At_A} given in (26) is an anaphoric function based on the atomic quantifier At_A. Furthermore, if At_A is intersective then F_{At_A} is intersective and if At_A is co-intersective then F_{At_A} is co-intersective:

(26) $F_{At_A}(X, R) = \{x : x \notin A \wedge At_{A\cup\{x\}}(X)(xR) = 1\}$

The fact that functions in (26) are anaphoric is easy to establish. Let us see some functions which are instances of (26) for illustration. Consider the type $\langle 1, 1\rangle$ quantifier NO. It is the atomic intersective quantifier determined by the empty set. Thus $A = \emptyset$, $At_\emptyset = NO$ and consequently, given the values of NO, the anaphoric function F_{NO} based on NO is given in (27):

(27) $F_{NO}(X, R) = \{x : X \cap xR = \{x\}\}$

The function in (27) is the same as the one in (3) above and thus is the denotation of the anaphoric determiner *No...except himself/herself*.

If $At_A = NO\text{-}BUT\text{-}\{L\}$ then the anaphoric function based on $NO\text{-}BUT\text{-}\{L\}$ is given in (28). This function is the denotation of the anaphoric determiner *No... except Leo and himself* (if *Leo* refers to L):

(28) $F_{NO-BUT-\{L\}}(X, R) = \{x : X \cap xR = \{x, L\}\}$

In the similar way by considering atoms of co-intersective quantifiers we obtain from (26) anaphoric functions denoted by the anaphoric determiners like *Every...except himself, Every...except herself and Leo*, etc. Thus we can say that (26) gives us a class of anaphoric intersective or co-intersective functions which are denotable (say, in English).

It is possible to further restrict the conservativity of type $\langle 1, 2 : 1\rangle$ anaphoric functions and define anaphoric cardinal or co-cardinal functions with the help of the cardinal and co-cardinal type $\langle 1, 2 : 1\rangle$ functions (see Zuber 2010b). We will not attempt here to make such generalisations considering that they should be preceded by some empirical justifications.

Examples of *self*-type anaphoric determiners discussed above suggest that functions they denote satisfy a constraint stronger than conservativity. Observe that anaphoric functions given in (21), (22) and (26) all have the property given in (29):

(29) $F(A, R) \subseteq A$.

This is also true of denotations of anaphoric determiners formed with *self* and other connectives than *except* or *including*. It is easy to see that the determiner like *Five..., in addition to Lea and himself* denotes a function which satisfies the condition given in (29).

Interestingly, the anaphoric condition ACD1, (generalised) conservativity and the condition given in (29) entail a specific version of conservativity, *anaphoric conservativity* (or a-conservativity) , specific to *self*-type anaphoric determiners. It is defined as follows:

Definition 11: A type $\langle 1, 2 : 1\rangle$ function F is *a-conservative* iff $F(X, R) = F(X, (X \times X) \cap R)$.

The following proposition makes clearer what a-conservativity is:

Proposition 7: A type $\langle 1, 2 : 1\rangle$ function F is *a-conservative* iff for any $X \subseteq E$ and any binary relations R_1 and R_2 if $(X \times X) \cap R_1 = (X \times X) \cap R_2$ then $F(X, R_1) = F(X, R_2)$.

Obviously any a-conservative function is conservative. For anaphoric and conservative functions satisfying condition in (29) we have the following proposition:

Proposition 8: A type $\langle 1, 2 : 1\rangle$ anaphoric and conservative function F such that $F(X, R) \subseteq X$ is a-conservative.

Proof: Suppose *a contrario* that for some $X \subseteq E$, $F(X, R) \neq F(X, (X\times X)\cap R)$ and thus that (by conservativity) $F(X, (E\times X)\cap R) \neq F(X, (X\times X)\cap R)$. This means that for some $a \in X$, $a \in F(X, (E\times X)\cap R)$ and $a \notin F(X, (X\times X)\cap R)$ (or $a \notin F(X, (E \times X) \cap R)$ and $a \in F(X, (X \times X) \cap R)$). This is, however, impossible given that F is anaphoric and the fact that in this case $a((E \times X) \cap R) = a((X \times X) \cap R))$.

We can thus suppose that *self* type anaphoric determiners denote a-conservative functions. This is not the case with with Slavic type anaphoric determiners. However, if we suppose that the possessor relation POS is anty-reflexive, functions denoted by Slavic anaphoric determiners satisfy the condition given in (30):

(30) $G(A, R) \subseteq \{x : xPOS \cap A\}'$

There is thus an important formal difference between *self*-type anaphoric determiners and Slavic ones.

6 Conclusion

In this article we have studied some simple properties of functions needed for the semantic description of (complex) anaphors in simple syntactic constructions. To be more precise, we have considered anaphoric functions applied to binary relations and giving sets as output. The case of anaphoric functions applied to relations of higher arity was not taken into consideration since *a priori* one cannot exclude some additional technical problems for such cases (cf. Ben Shalom 2003)

The existence of anaphors in NLs shows that the expressive power of English (and other languages) would be less that it is if the only noun phrases we need were ones interpretable as subjects of main clause intransitive verbs. The reason is that anaphors like *himself, herself* must be interpreted by functions from relations to sets which lie outside the class of generalised quantifiers as classically defined (Keenan 1987, 1988, 2007). This does not mean, however, that the semantics of NLs necessitates the whole class of functions from relations to sets since functions denoted by anaphors satisfy specific constraints belonging to the group of invariance conditions (Keenan and Westerståhl 1997). In this paper we extended such constraints to functions denoted by unary and binary anaphoric determiners. These constraints are not trivial and can be considered as giving rise to specific inference patterns (cf. Keenan 2007 for the case of nominal anaphors).

We also proposed a second type of restrictions on anaphoric functions: they are conservative, in a generalised sense, and their conservativity is related to the conservativity of quantifiers which are "parts" of them. The notion of generalised conservativity proposed here applies to all type $\langle 1, 2 : 1 \rangle$ functions, not just anaphoric ones. For instance it is easy to show that the non-anaphoric function $NSUP$ defined in (19) is conservative.

Conservativity is a very natural property. In simple cases it has both empirical and theoretical justifications. It is well-known that generalised conservativity, and related notions, are needed anyway in order to account for some semantic peculiarities of NLs (cf. Kuroda 2008, Zuber 2004). In that respect, our use of non-English examples should not be considered as accidental if we want to talk non-metaphorically about differences between specific natural languages.

References

[1] Beghelli, F. (1994) Structured Quantifiers, *in* Kanazawa, M. and Piñon, Ch. (eds.) *Dynamics, Polarity, and Quantification*, CSLI Publications, 119-145

[2] Ben Shalom, D. (2003) A Characterisation of the Linguistic Quantifier SELF, *Research on Language and Computation* 1: 99-104

[3] Keenan, E. L. (1987) Semantic Case Theory, *in* Groenendijk, J. and Stokhof, M. (eds.) *Proc. of the Sixth Amsterdam Colloquium*

[4] Keenan, E. L. (1988) On Semantics and the Binding Theory, *in* Hawkins, J. (ed.) *Explaining Language Universals*, Blackwell, pp. 105-144

[5] Keenan, E. L. (2007) On the denotations of anaphors. *Research on Language and Computation* 5-1: 5-17

[6] Keenan, E. L. and Faltz, L. M. (1985) *Boolean Semantics for Natural Language*, D. Reidel Publishing Company, Dordrecht.

[7] Keenan, E. L. and Moss, L. (1985) Generalized quantifiers and the expressive power of natural language, *in* J. van Benthem and A. ter Meulen (eds.) *Generalized Quantifiers*, Foris, Dordrecht, pp.73-124

[8] Keenan, E. L. and Westerståhl, D. (1997) Generalized Quantifiers in Linguistics and Logic, *in* van Benthem, J. and ter Meulen, A. (eds.) *Handbook of logic and language*, Elsevier, Amsterdam, pp. 837-893

[9] Kuroda, S-Y. (2008) Head Internal Relative Clauses, Quantifier Float, the Definitness Effect and the Mathematics of Determiners, *San Diego Linguistics Papers* 3, pp. 126-183

[10] Peters, S. and Wasterstahl, D. (2006) *Quantifiers in Language and Logic*, Oxford University Press

[11] Westerståhl, D. (1994) Iterated Quantifiers, *in* Kanazawa, M. and Pinon, Ch. (eds.) *Dynamics, Polarity and Quantification*, CSLI, Stanford University, pp. 173-209

[12] Zuber, R. (1998) On the Semantics of Exclusion and Inclusion Phrases, *in* Lawson, A. (ed.) *SALT8*, Cornell University Press, pp. 267-283

[13] Zuber, R. (2004) A class of non-conservative determiners in Polish, *Linguisticae Investigationes*, XXVII : 1, pp. 147-165

[14] Zuber, R. (2005) More Algebras for Determiners, *in* P. Blache and E. Stabler (eds.) *Logical Aspects of Computational Linguistics 5*, LNAI, vol. 3492, Springer-Verlag, pp. 363-378

[15] Zuber, R. (2010a) Semantics of Slavic anaphoric possessive determiners, Proceedings of SALT 19, forthcoming

[16] Zuber, R. (2010b) Generalising Conservativity, in Dawar, A. and de Queiroz, R. (eds.) *WoLLIC 2010*, LNAI 6188, Springer Verlag, pp. 247-258