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UNLOCKING THE STANDARD MODEL

I . 1 GENERATION OF QUARKS . SYMMETRIES

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Abstract: A very specific two-Higgs-doublet extension of the Glashow-Salam-Weinberg model for one generation of quarks is advocated for, in which the two doublets are parity transformed of each other and both isomorphic to the Higgs doublet of the Standard Model. The chiral group $U(2)_L \times U(2)_R$ gets broken down to $U(1) \times U(1)_{em}$. In there, the first diagonal $U(1)$ is directly connected to parity through the $U(1)_L \times U(1)_R$ algebra. Both chiral and weak symmetry breaking can be accounted for, together with their relevant degrees of freedom. The two Higgs doublets are demonstrated to be in one-to-one correspondence with bilinear quark operators.

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1   Introduction

It is well known that the genuine Glashow-Salam-Weinberg (GSW) model [1] cannot account for both chiral and weak symmetry breaking. For composite Higgses, the failure of the most simple scheme of dynamical symmetry breaking bred "technicolor" models [2] [3] [4], in which at least one higher scale and extra heavy fermions are needed. They unfortunately face themselves many problems. Guided by a long quest [5] for all complex doublets isomorphic to the Higgs doublet of the Standard Model, I propose here a very simple and natural two-Higgs-doublet extension in which, in particular, parity is restored to the primary role that it is expected to play and where the issues evoked above are solved. Last, an isomorphism is demonstrated which connects the two proposed Higgs doublets and bilinear quark-antiquark operators. This works therefore achieves a synthesis of two-Higgs-doublet models and dynamical symmetry breaking. The arguments that I present are essentially based on elementary considerations concerning symmetries. For the sake of simplicity, only the case of one generation of quarks is dealt with here.

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2 The Glashow-Salam-Weinberg model and its single Higgs doublet

A $SU(2)_L$ transformation we shall write

$$\mathcal{U}_L = e^{-i\alpha_i T_L^i}, \quad i = 1, 2, 3. \tag{1}$$

Eq. (1) shows that we adopt here the convention of physicists and consider the hermitian T^i's as the generators of $SU(2)_L$. So defined, they satisfy the commutation relations

$$[T_L^i, T_L^j] = i\,\epsilon_{ijk}T_L^k, \tag{2}$$

and they write

$$\vec{T}_L = \frac{1}{2}\vec{\tau}, \tag{3}$$

where the $\vec{\tau}$ are the Pauli matrices

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

The T_L^i's are accordingly

$$T_L^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T_L^+ = T_L^1 + iT_L^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T_L^- = T_L^1 - iT_L^2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{5}$$

With respect to $SU(2)_L$, left-handed flavor fermions are cast into doublets $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$, while right-handed fermions are singlets. These doublets belong to the fundamental representation of $SU(2)_L$ and the generators T_L^i act on them accordingly by

$$T_L^i.\begin{pmatrix} u_L \\ d_L \end{pmatrix} = T_L^i\begin{pmatrix} u_L \\ d_L \end{pmatrix}. \tag{6}$$

The Higgs doublet H of the GSW model is a complex doublet

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi^1 + i\chi^2 \\ \chi^0 - ik^3 \end{pmatrix}, \quad <\chi^0> = v \Leftrightarrow <H> = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{7}$$

also in the fundamental representation of $SU(2)_L$. It is built with four real scalar fields $\chi_0 = \frac{v}{\sqrt{2}} + \chi, \chi^1, \chi^2, k^3 \equiv -i\chi^3$. We have set $\chi^3 = ik^3$ in (7) to emphasize that it is complex. The action of T_L^i on H writes

$$T_L^i.H = T_L^i H. \tag{8}$$

H is used to give a mass to the three gauge bosons and also, through Yukawa couplings, to the d-type quarks. To give a mass to u-type quarks, a second (non-independent) complex doublet $\tilde{H}$

$$\tilde{H} = i\tau^2 H^* = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi^0 + ik^3 \\ -(\chi^1 - i\chi^2) \end{pmatrix}, \quad <\tilde{H}> = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}, \tag{9}$$

is used which is also in the fundamental representation of $SU(2)_L$. It has the same law of transformation (8) [1].

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[1] If one considers $\tilde{H} = \beta_j T^j H^*$ and request that, by a transformation $e^{-i\alpha_i T_L^i}$, $\delta\tilde{H} = -i\alpha_i T_L^i.\tilde{H} = -i\alpha_i T_L^i \tilde{H}$, one gets the condition $\beta_1 = 0 = \beta_3$. β_2 is undetermined and can be taken to be $\beta_2 = 2i$.

We define the transformed $T_L^i \cdot \chi^\alpha, i = 1, 2, 3, \alpha = 0, 1, 2, 3$ of the components χ^α by

$$T_L^i . H = \frac{1}{\sqrt{2}} \begin{pmatrix} T_L^i \cdot \chi^1 + i T_L^i \cdot \chi^2 \\ T_L^i \cdot \chi^0 - T_L^i \cdot \chi^3 \end{pmatrix} \tag{10}$$

and the same for $\tilde{H}$. The law of transformation (8), when applied to both H and $\tilde{H}$, is equivalent to

$$\begin{aligned}
T_L^1 \cdot \chi^0 &= +\tfrac{i}{2}\,\chi^2, & T_L^2 \cdot \chi^0 &= +\tfrac{i}{2}\,\chi^1, & T_L^3 \cdot \chi^0 &= +\tfrac{1}{2}\,\chi^3, \\
T_L^1 \cdot \chi^1 &= -\tfrac{1}{2}\,\chi^3, & T_L^2 \cdot \chi^1 &= -\tfrac{i}{2}\,\chi^0, & T_L^3 \cdot \chi^1 &= +\tfrac{i}{2}\,h^2, \\
T_L^1 \cdot \chi^2 &= -\tfrac{i}{2}\,\chi^0, & T_L^2 \cdot \chi^2 &= +\tfrac{1}{2}\,\chi^3, & T_L^3 \cdot \chi^2 &= -\tfrac{i}{2}\,\chi^1, \\
T_L^1 \cdot \chi^3 &= -\tfrac{1}{2}\,\chi^1, & T_L^2 \cdot \chi^3 &= +\tfrac{1}{2}\,\chi^2, & T_L^3 \cdot \chi^3 &= +\tfrac{1}{2}\,\chi^0.
\end{aligned} \tag{11}$$

Let us make the following substitutions

$$\chi^0 \to -h^3, \quad \chi^1 \to h^1, \quad \chi^2 \to -h^2, \quad \chi^3 \to h^0 \tag{12}$$

such that, in terms of the h^i's, H and $\tilde{H}$ write

$$H = \begin{pmatrix} h^1 - ih^2 \\ -(h^0 + h^3) \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} h^0 - h^3 \\ -(h^1 + ih^2) \end{pmatrix}. \tag{13}$$

Then, the laws of transformation (11) rewrite simply

$$\boxed{\begin{aligned}
T_L^i . h^j &= -\tfrac{1}{2}\left(i\,\epsilon_{ijk}h^k + \delta_{ij}\,h^0\right) \\
T_L^i . h^0 &= -\tfrac{1}{2}\,h^i
\end{aligned}} \tag{14}$$

which is our main formula.

Acting a second time on the χ^α according to the rules (11) yields

$$T_L^i.(T_L^j \cdot \chi^\alpha) - T_L^j.(T_L^i \cdot \chi^\alpha) = -i\,\epsilon_{ijk}T_L^k \cdot \chi^\alpha = -[T_L^i, T_L^j].\chi^\alpha, \quad \alpha = 0, 1, 2, 3. \tag{15}$$

After (10), it is natural to define $(T_L^i T_L^j).\chi^\alpha$ by

$$(T_L^i T_L^j).H = \frac{1}{\sqrt{2}} \begin{pmatrix} (T_L^i T_L^j).\chi^1 + i(T_L^i T_L^j).\chi^2 \\ (T_L^i T_L^j).\chi^0 - (T_L^i T_L^j).\chi^3 \end{pmatrix}, \tag{16}$$

in which $T_L^i T_L^j$ stands for the product of the two corresponding matrices. Since they satisfy (2) and by using (11), one gets straightforwardly [2]

$$[T_L^i, T_L^j].\chi^\alpha = +i\,\epsilon_{ijk}T_L^k.\chi^\alpha, \quad \alpha = 0, 1, 2, 3. \tag{18}$$

3   Its two-Higgs-doublet avatar

In analogy with (13), we shall introduce two complex $SU(2)_L$ Higgs doublets H and K

$$K = \begin{pmatrix} \mathfrak{p}^1 - i\mathfrak{p}^2 \\ -(\mathfrak{s}^0 + \mathfrak{p}^3) \end{pmatrix} \tag{19}$$

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[2]Eqs. (15) and (18) require

$$(T_L^i T_L^j).\chi^\alpha = -T_L^i.(T_L^j.\chi^\alpha). \tag{17}$$

It is easy to trace the origin of the "$-$" sign in (17) back to the definition (1) and (3) of the generators of $SU(2)$ as hermitian matrices. If one instead defines a group transformation without the "i" in the exponential, which yields anti-hermitian generators, one gets identity of $(T_L^i T_L^j).\chi^\alpha$ and $T_L^i.(T_L^j.\chi^\alpha)$. The "$i$" in the r.h.s. of the commutation relation (2) is then also canceled.

and

$$H = \begin{pmatrix} \mathfrak{s}^1 - i\mathfrak{s}^2 \\ -(\mathfrak{p}^0 + \mathfrak{s}^3) \end{pmatrix} \tag{20}$$

in which we defined $\mathfrak{p}^\pm = \mathfrak{p}^1 \pm i\mathfrak{p}^2$ and $\mathfrak{s}^\pm = \mathfrak{s}^1 \pm i\mathfrak{s}^2$. We shall also indifferently speak of K as of the quadruplet

$$K = (\mathfrak{s}^0, \mathfrak{p}^3, \mathfrak{p}^+, \mathfrak{p}^-), \tag{21}$$

and of H as of the quadruplet

$$H = (\mathfrak{p}^0, \mathfrak{s}^3, \mathfrak{s}^+, \mathfrak{s}^-). \tag{22}$$

K and H have the same laws of transformations (14) by $SU(2)_L$, in which, now h stands generically for $\mathfrak{s}$ or $\mathfrak{p}$, Each component of the type $\mathfrak{s}$ is hereafter considered as a scalar and each $\mathfrak{p}$ as a pseudoscalar, such that H and K are parity transformed of each other.

K and H are also stable by $SU(2)_R$, the generators T_R^i of which being defined like in (5), and transform according to

$$\boxed{\begin{aligned} T_R^i . h^j &= -\tfrac{1}{2}\left(i\,\epsilon_{ijk}h^k - \delta_{ij}\,h^0\right) \\ T_R^i . h^0 &= +\tfrac{1}{2}\,h^i \end{aligned}} \tag{23}$$

Since h^0 and $\vec{h}$ have opposite parity, (14) and (23) mix scalars and pseudoscalars.

Last, let us define the action of the generators I_L and I_R of $U(1)_L$ and $U(1)_R$ on K and H by (this is true component by component)

$$\boxed{I_L . K = -H, \quad I_L . H = -K, \quad I_R . K = H, \quad I_R . H = K} \tag{24}$$

they simply swap parity with the appropriate signs.

From eqs. (14), (23) and (24), one deduces the laws of transformations of K and H by the diagonal $U(2)$

$$\begin{aligned} T^i . h^j &= -i\,\epsilon_{ijk}h^k, \\ T^i . h^0 &= 0, \\ I . h^{0,i} &= 0. \end{aligned} \tag{25}$$

K and H thus decompose into a singlet h^0 + a triplet $\vec{h}$ of the diagonal $SU(2)$. They are stable by $SU(2)_L$ and $SU(2)_R$ but not by $U(1)_L$ nor by $U(1)_R$. They stay unchanged when acted upon by the diagonal $U(1)$. The union $K \cup H$ is stable by the whole chiral group $U(2)_L \times U(2)_R$.

It is instructive to represent the $\vec{T}_L, \vec{T}_R$ generators, and also the $\vec{T}$ of the diagonal $SU(2)_V$ symmetry in the basis the elements of which are the four entries (h^0, h^3, h^+, h^-) of K and H (see appendix A for explicit formulæ). The $SU(2)_{L,R}$ generators satisfy the commutation relations

$$[T_{L,R}^+, T_{L,R}^-] = -2T_{L,R}^3, \quad [T_{L,R}^3, T_{L,R}^+] = -T_{L,R}^+, \quad [T_{L,R}^3, T_{L,R}^-] = T_{L,R}^-, \tag{26}$$

which become the customary $SU(2)$ commutation relations when the roles of T^+ and T^-, or of h^+ and h^-, are swapped. Left and right generators of course commute. They also satisfy the anticommutation relations

$$\{T_{L,R}^i, T_{L,R}^j\} = \frac{1}{2}\delta_{ij}I_{L,R}, \tag{27}$$

which makes them like 4×4 Pauli matrices. The three generators of the diagonal $SU(2)$ have the same commutation relation but no peculiar anticommutation relation.

K and H are not eigenvectors of $\vec{T}_L$, $\vec{T}_R$ or $\vec{T}$. Calculating the eigenvectors of the $SU(2)_L$ generators $\vec{T}_L$ and $\vec{T}_R$ given by (A) and (A) leads to 4-vectors gathering the two doublets $\begin{pmatrix} h^- \\ -(h^0 + h^3) \end{pmatrix}$ and $\begin{pmatrix} h^0 - h^3 \\ -h^+ \end{pmatrix}$ isomorphic respectively to H and $\tilde{H}$ (13) of the GSW model.

In this 4-basis, the third generator $T^3 = T_L^3 + T_R^3$ of the diagonal $SU(2)$ symmetry coincides with the electric charge [3] Q, with eigenvalues 0 (twice), $+1$ and -1. Then, the Gell-Mann-Nishijima relation $Y = Q - T_L^3$ identifies the weak hypercharge Y with T_R^3.

4 Symmetries and their breaking

In association with a suitable potential, the gauge symmetry gets spontaneously broken by the presence of non-vanishing vacuum expectation value(s) (VEV(s)). These VEV's should be electrically neutral. Secondly, even if, since we deal with a parity violating theory, pseudoscalar fields can also be expected to "condense" in the vacuum, such VEV's can reasonably be expected only at higher order such that, classically, it is legitimate to restrict to scalar VEV's. This leaves the two $\mathfrak{s}^0$ in K and $\mathfrak{s}^3$ in H. We shall write accordingly

$$\mathfrak{s}^0 = \frac{v}{\sqrt{2}} + s, \quad \mathfrak{s}^3 = \frac{\sigma}{\sqrt{2}} + \xi, \tag{28}$$

such that [4]

$$< K >= -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad < H >= -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \end{pmatrix}. \tag{29}$$

Let us first investigate the breaking of the chiral group $U(2)_L \times U(2)_R$. Two $U(1)$ groups are left unbroken by the vacuum. The first is the diagonal $U(1)$, which is the multiplication of fermions by a phase. We have seen that its generator, the unit matrix, is the sum of I_L and I_R which swap parities of the scalar fields. The second is the electromagnetic $U(1)_{em}$. Indeed, only T^3, that is, the electric charge Q, gives 0 when acting on $\mathfrak{s}^0$ and $\mathfrak{s}^3$. $U(2)_L \times U(2)_R$ gets accordingly spontaneously broken down to $U(1) \times U(1)_{em}$, such that six Goldstone bosons are expected. Three degrees of freedom should become the three longitudinal $\vec{W}_\parallel$. The three others are expected to be physical particles and to acquire mass by a "soft" breaking of $U(2)_L \times U(2)_R$. This role is held by the $SU(2)_L$ invariant Yukawa couplings (we shall study them more at length in a subsequent work [6] and only give a few remarks at the end of this section).

The weak group $SU(2)_L$ has been considered *de facto* as a subgroup of the chiral group $U(2)_L \times U(2)_R$. This can always be done [5] and creates connections among Goldstone bosons. One among the four Goldstones (two charged and two neutral) which arise from the breaking of $U(2)_L \times U(2)_R$ down to its diagonal $U(2)$ subgroup is identical to the neutral Goldstone of the breaking of $SU(2)_L$ (which generates three Goldstones) and gets accordingly eaten by the massive W^3. The spectrum that results is therefore composed of the three massive $\vec{W}$'s, of the three remaining physical (pseudo)Goldstone bosons of the breaking $U(2)_L \times U(2)_R \to U(2)$ (which are also those of the breaking of $SU(2)_L \times SU(2)_R$ down to $SU(2)$), and of two Higgs bosons.

Since, for one generation of fermions, there is no distinction between the diagonal $SU(2)$ and the $SU(2)$ of flavor, the pseudoscalar triplet in K cannot but be "pion-like" and the pseudoscalar singlet in H be "η-like" (it is similar to η for one generation but can be the pseudoscalar singlet or another combination for more generations).

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[3]That the electric charge is the 3rd generator of an $SU(2)$ group is a prerequisite for charge quantization.

[4]The sign of the VEV's is not relevant, neither for the masses of the gauge bosons, which depend on their squares, nor for the fermions since, for example, the sign of the Yukawa couplings can always be adapted.

[5]and it can even always be done when going to N generations and to the chiral group $U(2N)_L \times U(2N)_R$. The embedding only needs to be suitably chosen so as to match the weak Lagrangian and dictates the way chiral and weak symmetries get entangled [5].

This degree of freedom, which disappears to the benefit of the massive neutral W^3, is presumably the one that plays a dual role with respect to the chiral and weak symmetry breakings.

The goal being to build a spontaneously broken $SU(2)_L$ theory of weak interactions, let us make a few more remarks concerning the breaking of $SU(2)_L$. All generators T_L^i acting non-trivially on $\mathfrak{s}^0$ (see for example appendix A), $SU(2)_L$ gets fully broken by $< K > \neq 0$, which yields three Goldstone bosons inside K. In the genuine GSW model, they would become the three longitudinal $\vec{W}_\parallel$'s. The situation is now changed as K gets instead connected to chiral breaking. This is where H enters the game since $SU(2)_L$ gets also fully broken by $< \mathfrak{s}^3 > \neq 0$, which generates the largest part of the $\vec{W}$ mass. Note that the same argumentation can be applied to $SU(2)_R$ since K and H are also stable multiplets of $SU(2)_R$.

These considerations fix the three Goldstones $(\mathfrak{p}, \mathfrak{s}^+, \mathfrak{s}^-)$ in H, two charged scalars and one neutral pseudoscalar, as the ones doomed to become the three longitudinal $W_\parallel$'s. This establishes H as the Higgs multiplet the closest to that of the GSW model and K as the additional "chiral" multiplet. The three $\vec{\mathfrak{p}}$ in there are pion-like, and s is a second Higgs boson.

Would parity be unbroken, the two Higgs multiplets H and K would be equivalent. This would entail in particular that $\mathfrak{s}^0$ and $\mathfrak{p}^0$ have identical VEV's. This is not the case. The two VEV's v and σ do not belong to fields that are parity transformed of each other but to the neutral scalars $\mathfrak{s}^0$ and $\mathfrak{s}^3$. They are independent parameters, controlling respectively the chiral and weak symmetry breaking. As far as the diagonal $SU(2)$ is concerned, it gets broken down to its electromagnetic $U(1)_{em}$ subgroup.

Soft chiral breaking is expected to provide low masses for the three $\vec{\mathfrak{p}}$. This process is achieved through $SU(2)_L$ invariant Yukawa couplings of fermions to both Higgs multiplets K and H. At high energy (m_W) these are standard renormalizable couplings between scalar fields and two fermions. At low energy, they can be rewritten (bosonised) by using the Partially Conserved Axial Current (PCAC) hypothesis [7][8][9]. So doing, they yield [6] in particular terms which are quadratic in the three $\vec{\mathfrak{p}}$ Goldstones and which match pion-like mass terms in agreement with the Gell-Mann-Oakes-Renner relation [10][9].

The situation is therefore different from when one sticks to a unique Higgs doublet in that we have enough degrees of freedom to accommodate for both chiral and weak physics of one generation of quarks.

This pattern of symmetry breaking also constrains the quartic scalar potential $V(H, K)$ that one introduces as a spontaneous-symmetry-breaking tool. Since the three $\vec{\mathfrak{p}}$ in K should match the three Goldstones of the chiral breaking $SU(2)_L \times SU(2)_R \rightarrow SU(2)$, no mass difference between the neutral and charged components should be generated in V. Other constraints arise from the requirement that scalar-pseudoscalar transitions should also be avoided at this (classical) level. Together, they largely simplify the expression of $V(H, K)$, which also receives, at low energy, additional contributions from the bosonised Yukawa couplings.

5 The isomorphism between the two Higgs doublets and bilinear quark operators

Like for $SU(2)_L$, we define a $SU(2)_R$ transformation by

$$\mathcal{U}_R = e^{-i\beta_j T_L^j}, \quad j = 1, 2, 3, \tag{30}$$

in which the generators T_R^j are given by the same hermitian matrices as in (5). The $U(2)_L \times U(2)_R$ algebra is completed by the two generators I_L and I_R, each one being represented by the unit 2×2 matrix.

Any quark bilinear car be represented as $\bar{\psi}\mathbb{M}\psi$ or $\bar{\psi}\mathbb{M}\gamma_5\psi$, where $\mathbb{M}$ is also a 2×2 matrix, or, equivalently, as "even" and "odd" composite operators $\bar{\psi}\frac{1+\gamma_5}{2}\mathbb{M}\psi$ and $\bar{\psi}\frac{1-\gamma_5}{2}\mathbb{M}\psi$.

The laws of transformations of (even and odd) fermion bilinears by an element $\mathcal{U} = \mathcal{U}_L \times \mathcal{U}_R$ of the chiral group are defined as follows:

$$(\mathcal{U}_L \times \mathcal{U}_R) \cdot \bar{\psi} \frac{1 + \gamma_5}{2} \mathbb{M} \psi = \bar{\psi} \, \mathcal{U}_L^{-1} \, \mathbb{M} \, \mathcal{U}_R \, \frac{1 + \gamma_5}{2} \psi,$$

$$(\mathcal{U}_L \times \mathcal{U}_R) \cdot \bar{\psi} \frac{1 - \gamma_5}{2} \mathbb{M} \psi = \bar{\psi} \, \mathcal{U}_R^{-1} \, \mathbb{M} \, \mathcal{U}_L \, \frac{1 - \gamma_5}{2} \psi, \tag{31}$$

which gives, by expanding $\mathcal{U}_L = 1 - i\beta_j T_L^j + \dots$ and $\mathcal{U}_R = 1 - i\kappa_j T_R^j + \dots$

$$T_L^j \cdot \bar{\psi} \frac{1 + \gamma_5}{2} \mathbb{M} \psi = -\bar{\psi} \, T^j \mathbb{M} \, \frac{1 + \gamma_5}{2} \psi,$$

$$T_L^j \cdot \bar{\psi} \frac{1 - \gamma_5}{2} \mathbb{M} \psi = +\bar{\psi} \, \mathbb{M} T^j \, \frac{1 - \gamma_5}{2} \psi,$$

$$T_R^j \cdot \bar{\psi} \frac{1 + \gamma_5}{2} \mathbb{M} \psi = +\bar{\psi} \, \mathbb{M} T^j \, \frac{1 + \gamma_5}{2} \psi,$$

$$T_R^j \cdot \bar{\psi} \frac{1 - \gamma_5}{2} \mathbb{M} \psi = -\bar{\psi} \, T^j \mathbb{M} \, \frac{1 - \gamma_5}{2} \psi. \tag{32}$$

The T^j's are seen to simply act by left- or right-multiplication on the matrix $\mathbb{M}$. Eqs. (32) give, for scalar and pseudoscalar bilinears

$$T_L^j \cdot \bar{\psi} \mathbb{M} \psi = -\frac{1}{2} \left(\bar{\psi} \, [T^j, \mathbb{M}] \, \psi + \bar{\psi} \, \{T^j, \mathbb{M}\} \, \gamma_5 \psi \right),$$

$$T_L^j \cdot \bar{\psi} \mathbb{M} \gamma_5 \psi = -\frac{1}{2} \left(\bar{\psi} \, [T^j, \mathbb{M}] \, \gamma_5 \psi + \bar{\psi} \, \{T^j, \mathbb{M}\} \, \psi \right),$$

$$T_R^j \cdot \bar{\psi} \mathbb{M} \psi = -\frac{1}{2} \left(\bar{\psi} \, [T^j, \mathbb{M}] \, \psi - \bar{\psi} \, \{T^j, \mathbb{M}\} \, \gamma_5 \psi \right),$$

$$T_R^j \cdot \bar{\psi} \mathbb{M} \gamma_5 \psi = -\frac{1}{2} \left(\bar{\psi} \, [T^j, \mathbb{M}] \, \gamma_5 \psi - \bar{\psi} \, \{T^j, \mathbb{M}\} \, \psi \right), \tag{33}$$

in which $\{,\}$ stands for the anticommutator of two matrices. Then, by using commutation and anticommutation relations of the Pauli matrices, one notices that the set of 2×2 matrices $(I, \vec{T})$ is stable by these two operations. One then gets from (33)

$$T_L^i \cdot \bar{\psi} I \psi = -\frac{1}{2} \bar{\psi} \gamma_5 2 T^i \psi,$$

$$T_L^i \cdot \bar{\psi} \gamma_5 2 T^j \psi = -\frac{1}{2} \left(i \, \epsilon_{ijk} \, \bar{\psi} \gamma_5 2 T^k \psi + \delta_{ij} \, \bar{\psi} I \psi \right), \tag{34}$$

and

$$T_L^i \cdot \bar{\psi} \gamma_5 I \psi = -\frac{1}{2} \bar{\psi} 2 T^i \psi,$$

$$T_L^i \cdot \bar{\psi} 2 T^j \psi = -\frac{1}{2} \left(i \, \epsilon_{ijk} \, \bar{\psi} 2 T^k \psi + \delta_{ij} \, \bar{\psi} \gamma_5 I \psi \right), \tag{35}$$

which show that the two quadruplets, which are parity-transformed of each other

$$\Phi = (\phi^0, \vec{\phi}) = \bar{\psi} \left(I, 2\gamma_5 \vec{T} \right) \psi \tag{36}$$

and

$$\Xi = (\xi^0, \vec{\xi}) = \bar{\psi}\left(\gamma_5 I, 2\vec{T}\right)\psi \tag{37}$$

are stable by $SU(2)_L$ and have the following laws of transformation (we write them for Φ, the ones for Ξ are identical)

$$\boxed{\begin{aligned} T_L^i \cdot \phi^j &= -\tfrac{1}{2}\left(i\,\epsilon_{ijk}\phi^k + \delta_{ij}\,\phi^0\right) \\ T_L^i \cdot \phi^0 &= -\tfrac{1}{2}\,\phi^i \end{aligned}} \tag{38}$$

The same considerations can be applied to $SU(2)_R$, which leads to

$$\boxed{\begin{aligned} T_R^i \cdot \phi^j &= -\tfrac{1}{2}\left(i\,\epsilon_{ijk}\phi^k - \delta_{ij}\,\phi^0\right) \\ T_R^i \cdot \phi^0 &= +\tfrac{1}{2}\,\phi^i \end{aligned}} \tag{39}$$

Acting with the generators I_L and I_R of $U(1)_L$ and $U(1)_R$ on any scalar $S = \bar{\psi}\mathbb{M}\psi$ of pseudoscalar $P = \bar{\psi}\mathbb{M}\gamma_5\psi$ according to (33) yields [6]

$$\boxed{I_L \cdot S = -P, \quad I_L \cdot P = -S, \quad I_R \cdot S = P, \quad I_R \cdot P = S} \tag{41}$$

Eqs. (38), (39) and (41, being identical to (14), (23) and (24), establish the isomorphism between K and H and the composite multiplets Ξ and Φ. It is completed by going from Φ and Ξ to multiplets $\mathfrak{K}$ and $\mathfrak{H}$ which have dimension $[mass]$ like K and H:

$$\begin{aligned}
\mathfrak{K} &= \frac{1}{\sqrt{2}}\frac{v}{\mu^3}\begin{pmatrix} \phi^1 - i\phi^2 \\ -(\phi^0 + \phi^3) \end{pmatrix} = \frac{v\sqrt{2}}{\mu^3}\begin{pmatrix} \bar{d}\gamma_5 u \\ -\tfrac{1}{2}(\bar{u}u + \bar{d}d) - \tfrac{1}{2}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) \end{pmatrix} \equiv \begin{pmatrix} \mathfrak{k}^1 - i\mathfrak{k}^2 \\ -(\mathfrak{k}^0 + \mathfrak{k}^3) \end{pmatrix}, \\
&\qquad <\bar{u}u + \bar{d}d> = \mu^3,
\end{aligned}$$

$$\begin{aligned}
\mathfrak{H} &= \frac{1}{\sqrt{2}}\frac{\sigma}{\nu^3}\begin{pmatrix} \xi^1 - i\xi^2 \\ -(\xi^0 + \xi^3) \end{pmatrix} = \frac{\sigma\sqrt{2}}{\nu^3}\begin{pmatrix} \bar{d}u \\ -\tfrac{1}{2}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) - \tfrac{1}{2}(\bar{u}u - \bar{d}d) \end{pmatrix} \equiv \begin{pmatrix} \mathfrak{h}^1 - i\mathfrak{h}^2 \\ -(\mathfrak{h}^0 + \mathfrak{h}^3) \end{pmatrix}, \\
&\qquad <\bar{u}u - \bar{d}d> = \nu^3.
\end{aligned} \tag{42}$$

In particular, as far as the Higgs bosons are concerned, the isomorphism writes

$$\mathfrak{s}^0 \leftrightarrow \frac{v}{\sqrt{2}\mu^3}\left(\bar{u}u + \bar{d}d\right), \quad \mathfrak{s}^3 \leftrightarrow \frac{\sigma}{\sqrt{2}\nu^3}\left(\bar{u}u - \bar{d}d\right). \tag{43}$$

The components of $\mathfrak{K}$ and $\mathfrak{H}$ have of course the same laws of transformations (38) and (39) as the ones of Φ and Ξ. In this framework, the VEV's of the scalar bilinear fermion operators $\bar{u}u$ and $\bar{d}d$ act as catalysts for both chiral and weak symmetry breaking.

6   Conclusion and prospects

We advocate for a minimal extension of the Glashow-Salam-Weinberg model for one generation of fermions which simply endows it with two Higgs multiplets instead of one, the two of them being parity transformed

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[6]At the group level, S and P transform according to

$$\begin{aligned}
e^{-i\alpha I_L} \cdot (S, P) &= \cos\alpha\,(S, P) + i\sin\alpha\,(P, S), \\
e^{-i\alpha I_R} \cdot (S, P) &= \cos\alpha\,(S, P) - i\sin\alpha\,(P, S).
\end{aligned} \tag{40}$$

As expected since the phases of ψ and $\bar{\psi}$ cancel, by a transformation $e^{-i\alpha I}$ of the diagonal $U(1)$, any S and P is left invariant.

of each other [7]. This procedure unlocks the Standard Model in the sense that both chiral and weak symmetry breaking can now be accounted for and that enough degrees of freedom become available to describe the two corresponding scales of physics: a first Higgs multiplet carries the three would-be longitudinal $\vec{W}_\parallel$ + a scalar Higgs boson which is very much "standard-like", and a second multiplet carries the three (pseudo)Goldstones of the broken $SU(2)_L \times SU(2)_R$ chiral symmetry into the diagonal $SU(2)$ + an additional Higgs boson s. The six Goldstones can be traced back to the breaking of $U(2)_L \times U(2)_R$ down to $U(1) \times U(1)_{em}$. The first $U(1)$, which is the diagonal subgroup of $U(1)_L \times U(1)_R$, is tightly related to parity. The eight components of the two Higgs multiplets are in one-to-one correspondence with the eight scalar and pseudoscalar bilinear quark operators that can be built with one generation of quarks.

A second work [6] will be devoted to the mass spectrum of fermions, gauge, Higgs and (pseudo)Goldstone bosons, to their couplings, and to the peculiar properties of the second Higgs boson s. The case of N generations will also be evoked; there exist in this case $2N^2$ Higgs multiplets isomorphic to the one of the GSW model, which should *a priori* all be incorporated, like we did above for H and K; fermion mixing can then be taken care of in a natural way in the embedding of the weak $SU(2)_L$ group into the chiral group $U(2N)_L \times U(2N)_R$ [5].

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Appendix

A Expression of $SU(2)_L$ and $SU(2)_R$ generators in the basis of the four components of K or H

They are 4×4 matrices which act on 4-vectors with basis

$$
h^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \; h^3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \; h^+ = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \; h^- = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.
$$

$*$ The three $SU(2)_L$ generators write

$$
T_L^3 = \begin{pmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \quad T_L^+ = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad T_L^- = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \end{pmatrix};
$$

$*$ the three $SU(2)_R$ generators write

$$
T_R^3 = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}, \quad T_R^+ = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad T_R^- = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \end{pmatrix};
$$

$*$ the three $SU(2)_V$ generators $\vec{T} = \vec{T}_L + \vec{T}_R$ are accordingly

$$
T^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = Q, \quad T^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad T^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},
$$