When the fiction of learning is kept: A case of networking two theoretical views

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WHEN THE FICTION OF LEARNING IS KEPT: A CASE OF NETWORKING TWO THEORETICAL VIEWS

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A case of networking two didactic phenomena - the Topaze effect and the Funnel communications pattern - through analyses of the same episode from two theoretical perspectives is presented. It shows how this networked analysis deepens insight into the kind of learning of the same episode through complementary views, how it strengthens the theoretical understanding the two phenomena as parts of the theories and how it uncovers blind spots of the theories themselves.

INTRODUCTION

Networking of theories is a new approach of connecting different theories in the same empirical study (Bikner-Ahsbahs et al., 2010). Recent research has shown that the networking of theories may capitalize on the theories' strengths (Gellert, Barbé, & Espinoza, 2012). It usually deepens insight into the theories involved and their concepts (Kidron, 2008), identifies blind spots and boundaries (Font, Trigueros, Badillo, & Rubio, 2012), leads to new methodological considerations (Bikner-Ahsbahs et al., 2010), and possibly to locally integrated theoretical parts (Arzarello, Bikner-Ahsbahs, & Sabena, 2009; Gellert et al., 2012). Networking also may yield enriched research outcomes and deepened insights (Artigue 2009; Drijvers, Dodino, Font, & Trouch, 2013). Different methodologies have been developed for supporting networking activities such as the comparison of research praxeologies (Artigue, Bosch, & Gascon, 2011), cross-experiments (Artigue, 2009) and cross-analyses of the same data. In this paper, we report about a case of networking two theories based on cross-analysis (Bikner-Ahsbahs et al., 2010). On the methodological level, we will show how the cross-analysis of one episode to which researchers associate two different phenomena, the Topaze effect and the Funnel effect, leads to mutually inform the two theories by a deeper a understanding of both phenomena, and reveals strengths and weaknesses of the two theories. The empirical part of the paper attempts to answer the questions: Do the two phenomena have a common ground that is deeply rooted in the practice of teaching and learning? How can this ground be described?

METHODOLOGICAL CONSIDERATIONS

There are two theories involved in this case study: the Theory of Didactic Situations (Brousseau, 1997) and the social constructivist view focussing on social interactions developed by Bauersfeld (1978) and further developed by others (e.g. Voigt, 1983). The method of cross-analyses is used to analyze the same episode respectively from

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the two theoretical perspectives, followed by an exchange of the results mutually enriching understanding the episode. In order to deepen theoretical insight, a process of networking the two analyses and their results in terms of the two theories is conducted using the networking strategies comparing, contrasting and coordinating the theoretical understanding of the episode (Bikner-Ahsbahs & Prediger, 2010).

The episode

Two grade 10 Italian students G and C and their teacher (T) are discussing on what happens to the exponential function for very big x. Before, the students explored the behavior of exponential functions and its slope by building a secant for small differences of x-values with the computer (Fig. 1): the computer screen shows a secant built by two points very close to each other leading to a quasi-tangent line. The transcript presents the discourse and some gestures.

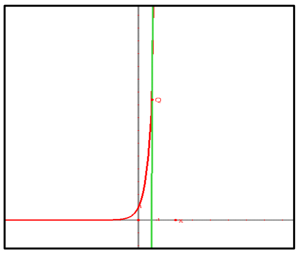
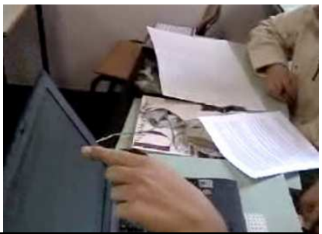


Figure 1. The graph on the computer screen.

1 G: but always for a very big this straight line (00:02), when they meet each other, there it is again…that is it approximates the, the function very well, because…

2 T: what straight line, sorry?

3 G: this here (pointing at the screen), for x very, very (00:14) big



00:02 G is pointing at the line in the screen

00:14 G's hand goes upwards

00:19 G: two two forefingers touching each other

00:28 T: right hand vertically raised

4 T: will they meet each other ? [challenging connotation]

5 G: that is [cioè], yes, yes they meet each other (00:19)

6 T: but after their meeting, what happens? (continuing to keep the hands in the same configuration as in 00:19)

7 G: eh…eh, eh no…, it makes so (00:24: G crossed the left hand over the right one. T keeps the previous gesture)

8 T: ah, ok, this then continues, this, the vertical straight line (00:28), has a well fixed x, hasn't it? The exponential function later goes on increasing the x, doesn't it ? Do you agree? Or not?

9 G: yes […]

10 T (addressing C): He [G] was saying that this vertical straight line (pointing at the line in the screen) approximates very well the exponential function

11 G: that is, but for x that are very…very big

12 T: and for how big x? 100 billions? (00:51) x = 100 billions?

13 G: because at a certain point…, that is, if the function (00:57) increases more and more, more and more (00:59), then it also becomes almost a vertical straight line (1:03)

00:57 G raises his left hand

00:59 G moves his hand upwards

01:03 final position of G’s hand after moving upwards.

01:13 T raises his right hand

14 T: eh, this is what it seems to you by looking at; but imagine that if you have x = 100 billions, there is this barrier…is it overcome sooner or later, or not? [connotation: suggesting the answer yes]

15 G: yes

16 T: and so when it is overcome), this x 100 billions (01:13), how many x do you still have at disposal, after 100 billions? (01:14 like 01:13)

17 G: infinite

18 T: infinite… and how much can you go ahead after 100 billion (repeating the gesture as in 01:14)?

19 G: infinite points

20 T: then the exponential function goes ahead on its own, doesn't it?

TWO VIEWS ON THE SAME EPISODE

Looking at this episode, two of the co-authors familiar with the Theory of Didactic Situations (TDS) immediately identified a Topaze effect, one of the paradoxes of the didactic contract. The third co-author familiar with Bauersfeld’s interactionism saw a Funnel pattern, while Sabena and Arzarello who have proposed the data presented an interpretation in terms of a semiotic game (see Arzarello et. al, 2009). This created an evident need for mutual clarification. In what follows, we focus on the first two interpretations resulting in a networking process.

Topaze Effect

For making clear the reasons for an interpretation in terms of Topaze effect, we associated with the usual discursive description of Topaze effect (Brousseau, 1997) four criteria characterizing a Topaze effect:

- The teacher has a precise expectation in terms of students’ answers.

- There is a substantial distance between the students' initial productions and utterances and these expectations.
- One can observe a succession of questions or dialogue piloted by the teacher for obtaining the expected answer drastically reducing the mathematical meaning of it.
- When the expected answer is produced, the teacher tries to maintain the fiction that the answer is really significant and that the didactical contract has not been broken.

Up to what point does the teacher-students interaction in this episode fulfil these criteria? Regarding the first criteria, complementary data collected show that the teacher has precise expectations: students should express that when x increases, the slope of the exponential function also increases and, with some help, that its evolution is also exponential. However, the Topaze effect if any is not linked to this expectation but to G's first utterance. It is understood by the teacher as expressing that an exponential curve can be approximated by a vertical line and have a vertical asymptote. Regarding this point, the teacher has a precise expectation: he wants the students to reject this claim. Concerning the second criteria, G's utterance is certainly distant from the teacher's expectation, but its exact meaning is not clear. G speaks about *secant* and and *almost vertical line* whereas the teacher immediately interprets G's idea as an approximation of the vertical straight line and thus turns the discussion towards the rejection of an asymptotic behavior of the exponential. Regarding the third criteria, we indeed observe a succession of questions piloted by the teacher who develops an argumentation to which students are more asked to adhere than to contribute. At the end of the episode, G's contribution consists merely of words: "yes", "infinite", "infinite points" directly induced by the teacher's questions, which looks like a Topaze effect. The teacher closes the episode. He goes beyond what has been already said, expressing the fact that the line and the curve must separate, and once again looks for the students' agreement. In this episode, we thus find some characteristics of a Topaze effect, however the teacher does neither really hide his expectations nor his arguments to the students. Moreover, in the last exchanges (lines 15-19), the drastic reduction of G's contribution seems to result more from the fact that he has given up and does not want to break the didactic relation than because he cannot contribute anymore to the mathematical exchange. For all these reasons, the final conclusion was that, even if some criteria of a Topaze effect are fulfilled here, interpreting this episode just as a Topaze effect, one would miss essential characteristics of it.

This analysis conducted the co-authors towards deepening the inquiry about the Topaze effect. This inquiry showed that, in the research literature, discussions about the Topaze effect hardly exist; detailed examples are scarce and only partially fulfil the criteria listed above. They more often show teachers who reduce the students' mathematical work to solve simple and isolated tasks than teachers who face the paradoxical characteristics of the didactic contract by the absence of or unexpected answer (see for instance Novotna & Hospesova, 2007). Such an extension of the idea of Topaze effect is questionable. It could make us forget the didactic joint action between students and teachers inherent in classroom functioning (Sensevy, 2012), which imposes some "didactic reticence" to the teacher, but also requires from her to regularly relax this didactic reticence for making the interaction of the students with the *milieu* cognitively productive.

The Funnel pattern

In 1978, Bauersfeld identified a Funnel pattern being built in the process of communication between teacher and students in mathematics classrooms. The Funnel pattern starts with an open question followed by four steps of *actions narrowing by answer expectations* (Bauersfeld, 1978, 162, own translation):

- The student does not recognize the mathematical operation or is not able to draw an adequate conclusion. The teacher asks an additional question but gets a false answer or does not get any answer.
- The teacher continues his effort to get at least part of the expected answer. Understanding is not anymore approached basically.
- Missing the expected answer the teacher tends to narrow his efforts aiming to just saying what is expected, no matter who says that. Self-determined behavior of the student decreases and at the same time the situation becomes more and more emotionalized.
- The process is finished as soon as the answer occurs no matter whether the student or the teacher has produced it.

Up to what point does the episode show a Funnel pattern? The analysis method consists of turn-by-turn analyses in which both the teacher and the student are driven by zugzwangs to react and reach the aim (Voigt 1983). G begins to answer the question how the exponential function grows for very big x (1) but the teacher interrupts him. The teacher takes the terms *straight line* and *approximates* as key words indicating the mistake that a vertical straight line approximates the graph for big x (10). But G does not say vertical (1), he later talks about almost vertical (13) and disagrees (11, 13) expanding what he observes at the computer screen as a result of a DGS-construction for big x. In line 10, the teacher phrases G's utterance as a claim to be falsified. G's resistance dries down when the teacher evaluates the construction on the screen as misleading (14). In line 10 the teacher begins to socially construct a proof by contradiction. This is not done by narrowing actions to produce the expected mathematical answer but by stage-managing the argumentation process demanding agreement for clear facts. G's reduced answers (15, 17, 19) only partly fulfil the third criteria of a Funnel pattern, since the teacher does not narrow his expectations towards producing the proof. He produces the proof himself to convince the students and seems to approach this goal by approved routines, such as depowering the student to act by depriving him from his argumentation base and demanding confirmation to undeniable facts. This way, another interaction pattern (e.g. Voigt 1983) that reduces the students' contributions is constituted accepting that G maintains his view on a backstage-level.

Networking the two approaches

Connecting the two different analyses revealed interesting similarities but also differences. The analyses led to the conclusion that the episode finally shows neither a Topaze effect nor a Funnel pattern; and brought to the fore that a *fiction* of having learnt mathematics is maintained: the students produced answers but not necessarily with insight into mathematics. The differences in the way these negative conclusions have been obtained also clarify the complementary nature of the two theories. Thus the networking process progressed in at least three directions: (1) by having a more mature and dense understanding of each theory, and a clearer awareness of its limitations, (2) by enlarging the units of analysis taking into account new views on the data; (3) by comparing phenomena that turned out to be close and complementary. As an example for the first direction, networking led us to deconstruct, then reconstruct the Topaze concept, and to replace dichotomic considerations by an idea of a degree of proximity with this theoretical object. In the same manner, networking showed that looking only at social interaction as it is methodically done in the interactionist approach may be foreshortened because there can be different levels of acting. Routine actions are actions at the surface level whereas additional views of insight supported for example by individual interest can be underneath but not shown within the social interactions. Limitations were experienced because data constructed from another perspective withstood our analyses demanding to deepen the argumentation and its theoretical foundation. G's resistance, and what it reveals about argumentation in this classroom culture, are only partially captured by TDS constructs. Conversely, TDS's epistemic perspective attests the limitation of the milieu for supporting the expected proof by contradiction. This limitation cannot be identified in a pure interactionist view. Regarding the second way of progressing, TDS cannot totally explain why the situation does not degenerate into a complete Topaze effect. The debate between the researchers assisted the TDS researchers to tackle the data in a way, more sensitive to the classroom culture and to the "emotive dimension" that might explain why finally G gave up. Regarding the last direction some fundamental differences in the principles between the two theories were identified. The analyses neither address the same focuses, nor do they have the same granularity. Bauersfeld's theory is an interactionist approach, reasoning primarily in terms of routines, rooted in patterns of interaction which partially escape consciousness; epistemological concerns are thus not easily captured. Through contrasting, the epistemic strength of TDS became apparent but also up to what point TDS was less sensitive to non-epistemic characteristics of the social interaction. Conversely, the idea of the didactic contract that produces mutual obligations for social interaction and the insight that an insufficient milieu forces the teacher to act deepened the understanding of the episode from the interactionist view: here, the teacher moves the didactic contract and tries to make the students enter a new game - a proof by contradiction that ends with the final alignment of the two actors. Stage-managing a proof by contradiction that itself is part of an interaction pattern "socially correcting a mistake" reduces the student's contributions to one word sentences which gave the impression of a Funnel pattern. At a more general level, the importance of maintaining the didactic contract in the TDS offers an insight into interaction patterns: teacher and students trying to keep their roles may at the level of social interactions orient themselves towards the supposed expectation of the teacher, but the student may at the backstage level have kept his view alive. Referring to the Funnel pattern, Bauersfeld proposed that the teacher should change the didactical model. This way, he goes into the direction of changing the milieu. However, the interactionist view does not offer a frame of constructing such a milieu as opposed to the TDS-perspective, which might recommend zooming in another screen to evidence that the tangent line never can be vertical.

CONCLUSION

The networking process reached a joint improved understanding of the episode by the strategy of coordinating the analyses of the two theoretical frames. Cross-analyses showed up to what point the two theories could complement and consolidate each other. It made both research groups progress in their analyses in a way that demanded deepening the understanding of the respective theories by, for instance, re-questioning their theoretical constructs. The two analyses in terms of Topaze effect and Funnel pattern also led to become aware of the resonances between the two phenomena, their respective strengths but also limits. The characteristics of the episode contributed in a large part in making this methodology productive, as it addresses a problem that may often occur in teaching practice. The networking process produced a distinction between the surface level of social interaction and the depth level of epistemic insight. It grasped the same situation with different sensibilities and finally converged in a coherent picture being condensed in a common ground: *the fiction that learning took place*. In our opinion, such a fiction can exist to some degree each time the milieu and forms of joint action (Sensevy, 2012) are not sufficient to produce the new knowledge and to lead the teacher to give substantial responsibility to the students.

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