# Localization method for device-to-device through user movement 

The Dang Huynh, Chung Shue Chen, Siu-Wai Ho

## To cite this version:

The Dang Huynh, Chung Shue Chen, Siu-Wai Ho. Localization method for device-to-device through user movement. IEEE International Conference on Communications (ICC), 2015, London, United Kingdom. 2015, <10.1109/ICCW.2015.7247279>. <hal-01216870>

HAL Id: hal-01216870<br>https://hal.inria.fr/hal-01216870

Submitted on 17 Oct 2015

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Localization Method for Device-to-Device through User Movement 

The Dang Huynh*, Chung Shue Chen*, and Siu-Wai Ho ${ }^{\dagger}$<br>*Alcatel-Lucent Bell Labs, Centre de Villarceaux, 91620 Nozay, France<br>${ }^{\dagger}$ Institute for Telecommunications Research, University of South Australia, Australia<br>Email: \{The_Dang.Huynh, cs.chen\}@alcatel-lucent.com, siuwai.ho@unisa.edu.au


#### Abstract

Indoor positioning system is a key component for developing various location based services such as indoor navigation in large complex buildings (e.g., commercial center and hospital). Meanwhile, it is challenging to design a cost effective solution which is able to provide high accuracy. A new method, namely Two-Step Movement (2SM), was proposed in [1] to demonstrate how to build a positioning system which requires only one Reference Point (RP) by exploiting user movement. The method can offer good precision and minimize the number of RPs required so as to reduce system implementation cost. Built on 2SM, here we first improve the positioning performance through multisampling technique to combat measurement noise. Secondly, we propose the Generalized Two-Step Movement (G2SM) method for device-to-device (D2D) systems in which both the mobile terminal (MT) and RP can be mobile device. The mobile user's position can be derived analytically and given in simple closedform expression. Its effectiveness in the presence of noise is shown in simulation results.


Index Terms-Positioning system, localization algorithm, user movement, multi-sampling, mobile device, smart applications.

## I. Introduction

Positioning systems play an important role in providing a wide range of today's services, including indoor navigation in a hospital building, location-based advertisement in a commercial center, and outdoor vehicle navigation system. It helps to locate objects or people carrying the objects and to provide geographic information for facilitating many activities. For instance, vehicle navigation systems are indispensable for drivers in big cities to find the best route to a destination or be guided to bypass traffic jam intelligently. Locationbased services are also deployed in shopping malls so that customers can get navigation while walking in a complex environment and receive promotion information from shops nearby. Positioning systems have become an essential part of our digital society. The market of location-based services has been rapidly growing over the last decade.

Global positioning system (GPS) is the most popular outdoor positioning system. GPS chips have been installed in a vast majority of today's smart devices and the positioning has high precision. However, it does not function well in indoor environment because GPS signal would be severely degraded due to the lack of line-of-sight or even be totally blocked due to walls and buildings. Thus, many indoor positioning systems have been proposed so far, see for example [2]-[5] and references therein, which basically can be classified into four main categories:

- Triangulation \& Trilateration make use of distance measurement between RPs and MT to localize the MT [6]. The number of RPs required vary according to the technology used [2], e.g., one RP with the help of an antenna array for providing the angle of arrival (AoA) can be sufficient to find the position of a MT. However, it is clear that the implementation of an antenna array is often cumbersome and expensive.
- Fingerprinting estimates the device position by comparing the current received signal strength with pre-measured location data that have been stored at Wi-Fi hotspot or access point (AP) database, see for example [7].
- Scene analysis infers the device position based on a set of images or scenes recorded by one or multiple cameras, see [3], [8].
- Proximity is usually used to detect if a MT is nearby or in the coverage area of a RP. An example of popular commercial products that provide proximity information is iBeacon [9].
Recently, the authors proposed a new method called TwoStep Movement (2SM) [1] which is an improvement of Triangulation method for localizing the MT by requiring only one RP. The basic idea is to exploit the useful information of user gesture (e.g., the distance and angle of movement). Combined with the measured distance between the RP and MT, the position of the mobile device can be determined. Simulation result shows that the average positioning error is within $10 \%$ of the distance between the RP and MT.

To improve the previous result, this paper demonstrates the use of multi-sampling technique so that many measurements will be conducted during the movement of the MT instead of using only one at the end of each movement (see [1]) to combat measurement errors and improve the positioning performance. Secondly, we will generalize the 2SM localization method for device-to-device (D2D) systems in which the RP and MT are both mobile terminals and thus call it Generalized Two-Step Movement (G2SM). In this context, many interesting applications can be found in reality. For instance, a device can thus locate itself through a peer mobile device or a mobile RP. In consequence, the positioning can be carried on from device to device for a large system and in a multi-hop manner.

The paper is organized as follows. Section [I presents 2SM method and the effectiveness of proposed multi-sampling
approach. Section IIII presents the generalized 2SM (G2SM) method and derives the algorithms. Analytical and simulation results are provided. Finally, Section IV concludes the paper.


Fig. 1: Two-Step Movement (2SM).

## II. Two-Step Movement using Multi-Sampling

## A. System Design

To begin with, we recall our system design and the 2SM positioning method which requires only one RP (for minimal system implementation cost). We determine user position by exploiting its movement, e.g., in walking or by waving his/her hand-held device.

The methodology is depicted in Fig. 1 Consider a movement of MT and its trajectory is from position $B$ to $C$ and then $D$. The RP is at position $A$. It is assumed that the MT is capable of measuring the distance between itself and the RP (e.g., by received signal strength or some standard techniques) and is capable of measuring the distance and the angle (direction) of its movement (e.g., via embedded sensor) such that the angle $\alpha$ which is the angle measured from the positive $x$-axis counterclockwise (see Fig. 11) and the distances $A B, A C$ and $B C$ are determined. By using the geometry of $\triangle A B C$, in general we will find two possible solutions of the MT's location, say $B\left(x_{B}, y_{B}\right)$ and $B^{\prime}\left(x_{B}^{\prime}, y_{B}^{\prime}\right)$, cf. [1, Theorem 1]. Similarly, by using $\triangle A B D$, we will also find two possible locations of the MT, i.e., $B\left(x_{B}, y_{B}\right)$ and $B^{\prime \prime}\left(x_{B}^{\prime \prime}, y_{B}^{\prime \prime}\right)$. By combining the above results, one can determine the location of the MT, i.e., $B\left(x_{B}, y_{B}\right)$. Clearly, in practice with environmental noise or system imperfection, we will see error in the estimation.

## B. Motivation of Multi-Sampling

The precision of the estimation is depending on the accuracy in the measurement phase (e.g., $\alpha, A B, A C, B C$ ). In reality, the limitation of hardware technology and presence of noise
may severely degrade the quality of the inputs to our algorithm and then leads to poor position estimation. To reduce the impact of noise, naturally one can think of making many measurements and then combine them to produce better result. The intuition is that in the simplest case where noise follows a zeromean distribution, we can expect the output of our algorithm also to have roughly a zero-mean error distribution. Another idea is that one could probably use a set of measurements to infer or pick out a better result.

Recall that in 2SM [1], all measurements are done only once at the end of each movement step (e.g., at $C$ ). Here, we propose a multi-sampling 2 SM such that many measurements are carried out along the path so that the MT continuously keeps track of the movement and the distance to the RP. In other words, the step $B C$ is considered as a series of small steps and we will use all the data obtained from these steps for positioning (see Fig. 2). In the first movement from $B$ to $C$ (see Fig. 2a), measurements are taken at the intermediate points $C 1, \ldots, C_{n}$ and each of them allows to compute two possible positions of $B$, denoted by $B 1$ and $B 2$. Obviously, $n$ intermediate measurements give us two sets of $B 1$ and $B 2$, denoted by $S_{B 1}$ and $S_{B 2}$ with $\left|S_{B 1}\right|=\left|S_{B 2}\right|=n$. One can simply take the middle points of $S_{B 1}$ and $S_{B 2}$ respectively or formulate an optimization problem to find the best estimates of $\overline{B 1}$ and $\overline{B 2}$. Similarly, the above process is applied to the second step movement (see Fig. 2b) such that we can find two estimates, denoted by $\overline{B 3}$ and $\overline{B 4}$, respectively. In general, we will have four sets of points as shown in Fig. 3. To resolve the ambiguity, we can then for example take the mid-point of the pair which have the minimum Euclidean distance (i.e., the mid-point of $\overline{B 1}$ and $\overline{B 3}$ in Fig. 3) as the MT's position or formulate an optimization problem to minimize the error.

## C. Numerical Studies

Simulation is performed to investigate the performance of the above multi-sampling 2SM in comparison to that in [1], say single-sampling 2 SM . The simulation set-up is as follows:

- RP is placed at the center of a room, i.e., $A(0,0)$.
- The initial position of MT is called $B$ and its distance to the RP is set to three values: 1,5 , and 10 meters. Its position on a corresponding circle is randomly generated.
- Measurement error of distance is denoted by $e_{d}$ and bounded by $[-1 \%, 1 \%],[-2 \%, 2 \%]$, and $[-5 \%, 5 \%]$ with respect to its true value. Meanwhile, measurement error of angle is denoted by $e_{a}$ and bounded by $\left[-1^{\circ}, 1^{\circ}\right]$, $\left[-2^{\circ}, 2^{\circ}\right]$, and $\left[-5^{\circ}, 5^{\circ}\right]$.
Note that there are several ways to choose the sampling intervals. For example, one may simply use uniform sampling, i.e., $B C$ is divided into $n$ intervals of equal distance such that $B C_{1}=C_{i} C_{i+1}$, where $1 \leq i \leq n-1$. However, we observe that it is better to do sampling when the movement distance is relatively large, i.e., closer to $C$ in the first movement (and $D$ in the second movement). Indeed, this coincides with the result obtained in [1] that the larger the $B C_{i}$ in the first move, the more accurately the second move can help to determine the true MT position.

(a) The first movement with multi-sample

(b) The second movement with multi-sample

Fig. 2: Two-Step Movement (2SM) with multi-sampling near the end points.

|  | $e_{d}=1 \%, e_{a}=1^{\circ}$ |  | $e_{d}=2 \%, e_{a}=2^{\circ}$ |  | $e_{d}=5 \%, e_{a}=5^{\circ}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Single-sampling | Multi-sampling | Single-sampling | Multi-sampling | Single-sampling | Multi-sampling |
| $A B=1(B C=C D=0.1 A B)$ | 0.1412 | 0.1154 | 0.2640 | 0.2164 | 0.5691 | 0.4090 |
| $A B=1(B C=C D=0.2 A B)$ | 0.0808 | 0.0666 | 0.1508 | 0.1370 | 0.3417 | 0.2638 |
| $A B=1(B C=C D=0.5 A B)$ | 0.0484 | 0.0416 | 0.0804 | 0.0792 | 0.1868 | 0.1541 |
| $A B=5(B C=C D=0.1 A B)$ | 0.7194 | 0.5858 | 1.3012 | 1.0269 | 2.9226 | 2.1480 |
| $A B=5(B C=C D=0.2 A B)$ | 0.3957 | 0.3623 | 0.7480 | 0.6062 | 1.6587 | 1.2861 |
| $A B=5(B C=C D=0.5 A B)$ | 0.2193 | 0.2108 | 0.4412 | 0.3393 | 0.9134 | 0.8067 |
| $A B=10(B C=C D=0.1 A B)$ | 1.4165 | 1.1650 | 2.6257 | 1.9102 | 5.8929 | 4.0634 |
| $A B=10(B C=C D=0.2 A B)$ | 0.8006 | 0.5777 | 1.1580 | 1.1238 | 3.3873 | 2.5696 |
| $A B=10(B C=C D=0.5 A B)$ | 0.4987 | 0.4325 | 0.8798 | 0.7959 | 1.8750 | 1.5831 |

TABLE I: Average estimation error (in meter) due to the single-sampling and multi-sampling 2SM methods under various $A B$, $B C, C D$, and noise levels.


Fig. 3: The possible solutions in the presence of noise.

Table I compares the performance of the single-sampling and multi-sampling 2 SM . The result is obtained by $10^{5}$ runs. In multi-sampling 2 SM , we perform 1000 sampling near the end point of each movement (see Fig. 2). Table $\mathbb{1}$ shows that the multi-sampling has effectively reduced the position estimation error by $15 \%-30 \%$. Besides, it is also interesting to see that when the positioning error resulted in the single-sampling is larger, the improvement thanks to multi-sampling is even more significant.

## III. Generalization of the Two-Step Movement (G2SM) to Device-to-Device Context

In this section, we show how to determine the position of the MT when the RP is also moving. It is a generalization of the 2 SM method, thus called Generalized 2SM (G2SM).

## A. System Design and Basic Idea

We consider the following system and generalization:

- The RP is also mobile (movable).
- The MT is capable of measuring the distance between itself and the RP.
- The MT is capable of measuring the distance and the angle (direction) of the movement it has done.
Figure 4a depicts the following technical details:
- The RP is initially located at $A\left(x_{A}, y_{A}\right)$ which is known.
- The MT is initially at position $B$, which is however unknown, denoted by coordinates $\left(x_{B}, y_{B}\right)$.
- $C$ and $D$ are the positions of the MT and RP, respectively, during their movement. Assume that RP can localize itself, so that $D\left(x_{D}, y_{D}\right)$ is known. However, $C\left(x_{C}, y_{C}\right)$ is not given.
- MT is capable of measuring the distance between itself and the RP, i.e., distances $A B$ and $D C$ are deterministic. For example, this can be done by measuring the received signal strength or using other standard techniques.
- MT is capable of measuring the distance and angle of its movement so that $B C$ and the angle $\alpha \in(0,2 \pi]$ (with respect to the $x$-axis) are deterministic.

Theorem 1 If $A\left(x_{A}, y_{A}\right), D\left(x_{D}, y_{D}\right), A B, D C, B C$, and $\alpha$ are known, Algorithm $\square$ gives two possible solutions of $B$,


Fig. 4: Generalization of the Two-Step Movement (G2SM).
denoted by $\left(x_{B}, y_{B}\right)$, satisfying

$$
\begin{align*}
& x_{B}\left(B C \cos \alpha-x_{D}+x_{A}\right)+y_{B}\left(B C \sin \alpha-y_{D}+y_{A}\right) \\
& =x_{D} B C \cos \alpha+y_{D} B C \sin \alpha \\
& -\frac{A B^{2}+B C^{2}-D C^{2}-\left(x_{A}^{2}+y_{A}^{2}\right)+\left(x_{D}^{2}+y_{D}^{2}\right)}{2} \tag{1}
\end{align*}
$$

```
Algorithm 1 Generalized One-Step Movement (G1SM)
Require: \(A\left(x_{A}, y_{A}\right), D\left(x_{D}, y_{D}\right), A B, D C, B C, \alpha\);
    function \(\operatorname{GenOneStep}\left(A\left(x_{A}, y_{A}\right), \quad D\left(x_{D}, y_{D}\right), A B\right.\),
    \(D C, B C, \alpha)\)
            \(\triangleright\) Precompute \(a, b, c\) such that \(a x_{B}+b y_{B}=c\);
            \(a=B C \cos \alpha-x_{D}+x_{A} ;\)
            \(b=B C \sin \alpha-y_{D}+y_{A}\);
            \(c \quad=\quad x_{D} B C \cos \alpha+y_{D} B C \sin \alpha \quad-\)
    \(\left(A B^{2}+B C^{2}-D C^{2}-\left(x_{A}^{2}+y_{A}^{2}\right)+\left(x_{D}^{2}+y_{D}^{2}\right)\right) / 2 ;\)
        \(\triangleright\) Compute \(x_{B}, y_{B}\);
        if \(b==0\) then
            \(x_{B}=c / a ;\)
            \(y_{B 1}=y_{A}+\sqrt{A B^{2}-\left(x_{B}-x_{A}\right)^{2}} ;\)
            \(y_{B 2}=y_{A}-\sqrt{A B^{2}-\left(x_{B}-x_{A}\right)^{2}}\);
            return \(\left\{B 1\left(x_{B}, y_{B 1}\right), B 2\left(x_{B}, y_{B 2}\right)\right\}\);
        else
            \(d=-a / b ;\)
            \(e=c / b ;\)
            \(\Delta=\left(x_{A}-d\left(e-y_{A}\right)\right)^{2}-\left(1+d^{2}\right)\left(x_{A}^{2}+(e-\right.\)
    \(\left.\left.y_{A}\right)^{2}-A B^{2}\right) ;\)
            \(x_{B 1}=\left(x_{A}-d\left(e-y_{A}\right)+\sqrt{\Delta}\right) /\left(1+d^{2}\right) ;\)
            \(y_{B 1}=d x_{B 1}+e ;\)
            \(x_{B 2}=\left(x_{A}-d\left(e-y_{A}\right)-\sqrt{\Delta}\right) /\left(1+d^{2}\right) ;\)
            \(y_{B 2}=d x_{B 2}+e\);
            return \(\left\{B 1\left(x_{B 1}, y_{B 1}\right), B 2\left(x_{B 2}, y_{B 2}\right)\right\}\);
        end if
    end function
```

Proof: The equations of the two circles centered at $A\left(x_{A}, y_{A}\right)$ and $D\left(x_{D}, y_{D}\right)$ after the first movement (see

Fig. (4a) can be written as

$$
\begin{align*}
& A B^{2}=\left(x_{B}-x_{A}\right)^{2}+\left(y_{B}-y_{A}\right)^{2} \\
& D C^{2}=\left(x_{C}-x_{D}\right)^{2}+\left(y_{C}-y_{D}\right)^{2} \tag{2}
\end{align*}
$$

where

$$
\begin{align*}
& x_{C}=x_{B}+B C \cos \alpha \\
& y_{C}=y_{B}+B C \sin \alpha \tag{3}
\end{align*}
$$

By substituting (3) to (2), we have

$$
\begin{align*}
& A B^{2}=\left(x_{B}-x_{A}\right)^{2}+\left(y_{B}-y_{A}\right)^{2} \\
& D C^{2}=\left(x_{B}+B C \cos \alpha-x_{D}\right)^{2}+\left(y_{B}+B C \sin \alpha-y_{D}\right)^{2} \tag{4}
\end{align*}
$$

which gives the equality

$$
\begin{aligned}
D C^{2}-A B^{2} & =2 x_{B}\left(B C \cos \alpha-x_{D}+x_{A}\right)-x_{A}^{2} \\
& +2 y_{B}\left(B C \sin \alpha-y_{D}+y_{A}\right)-y_{A}^{2} \\
& +\left(B C \cos \alpha-x_{D}\right)^{2}+\left(B C \sin \alpha-y_{D}\right)^{2}
\end{aligned}
$$

that can be re-written as

$$
\begin{aligned}
& 2 x_{B}\left(B C \cos \alpha-x_{D}+x_{A}\right)+2 y_{B}\left(B C \sin \alpha-y_{D}+y_{A}\right) \\
& =2 x_{D} B C \cos \alpha+2 y_{D} B C \sin \alpha+D C^{2}-A B^{2}-B C^{2} \\
& +\left(x_{A}^{2}+y_{A}^{2}\right)-\left(x_{D}^{2}+y_{D}^{2}\right) .
\end{aligned}
$$

Hence,

$$
\begin{aligned}
& x_{B}\left(B C \cos \alpha-x_{D}+x_{A}\right)+y_{B}\left(B C \sin \alpha-y_{D}+y_{A}\right) \\
& =x_{D} B C \cos \alpha+y_{D} B C \sin \alpha \\
& -\frac{A B^{2}+B C^{2}-D C^{2}-\left(x_{A}^{2}+y_{A}^{2}\right)+\left(x_{D}^{2}+y_{D}^{2}\right)}{2}
\end{aligned}
$$

Let $a=B C \cos \alpha-x_{D}+x_{A}, b=B C \sin \alpha-$ $y_{D}+y_{A}$, and $c=x_{D} B C \cos \alpha+y_{D} B C \sin \alpha-$ $\left(A B^{2}+B C^{2}-D C^{2}-\left(x_{A}^{2}+y_{A}^{2}\right)+\left(x_{D}^{2}+y_{D}^{2}\right)\right) / 2$. So, Eqn. (1) can be re-written in the form $a x_{B}+b y_{B}=c$ and be solved as follows:

- If $b=0$ (or $B C \sin \alpha=y_{D}-y_{A}$ ), it is clear that $x_{B}=$ $c / a$ and $y_{B}=y_{A} \pm \sqrt{A B^{2}-\left(x_{B}-x_{A}\right)^{2}}$.
- If $b \neq 0$, let $d=-a / b$ and $e=c / b$, we see that now $y_{B}$ is expressible as a function of $x_{B}$ such that $y_{B}=d x_{B}+e$. Substituting $y_{B}$ to the first equation in (2), we have

```
Algorithm 2 Generalized Two-Step Movement (G2SM)
Require: \(A\left(x_{A}, y_{A}\right)\);
    function \(\operatorname{GenTwoStep}\left(A\left(x_{A}, y_{A}\right)\right)\)
        RP makes the first movement from \(A\) to \(D\); obtaining \(D\left(x_{D}, y_{D}\right)\);
        In parallel, MT makes the first movement from \(B\) to \(C\); measuring \(A B, D C, B C, \alpha\);
        \(\left\{B 1\left(x_{B 1}, y_{B 1}\right), B 2\left(x_{B 2}, y_{B 2}\right)\right\}=\operatorname{GENONESTEP}\left(A\left(x_{A}, y_{A}\right), D\left(x_{D}, y_{D}\right), A B, D C, B C, \alpha\right) ; \triangleright\) Compute two locations
    \(B 1\) and \(B 2\);
        RP makes the second movement from \(D\) to \(E\); obtaining \(E\left(x_{E}, y_{E}\right)\);
        In parallel, MT makes the second movement from \(C\) to \(F\); measuring \(C F, E F, \beta\); make sure that \(\beta \neq \alpha\) and \(\beta \neq \alpha \pm \pi\);
        \(X=B C \cos \alpha+C F \cos \beta ; \quad \triangleright\) Change in \(x\)-coordinate after the second move;
        \(Y=B C \sin \alpha+C F \sin \beta ; \quad \triangleright\) Change in \(y\)-coordinate after the second move;
        \(B F=\sqrt{X^{2}+Y^{2}}\);
        \(\cos \gamma=X / B F ;\)
        \(\sin \gamma=Y / B F\);
        Compute \(\gamma \in[0 ; 2 \pi)\) from \(\cos \gamma\) and \(\sin \gamma\);
        \(\left\{B 3\left(x_{B 3}, y_{B 3}\right), B 4\left(x_{B 4}, y_{B 4}\right)\right\}=\operatorname{GenOneStep}\left(A\left(x_{A}, y_{A}\right), E\left(x_{E}, y_{E}\right), A B, E F, B F, \gamma\right)\); Compute two locations
    \(B 3\) and \(B 4\);
        \(B\left(x_{B}, y_{B}\right)=\left\{B 1\left(x_{B 1}, y_{B 1}\right), B 2\left(x_{B 2}, y_{B 2}\right)\right\} \cap\left\{B 3\left(x_{B 3}, y_{B 3}\right), B 4\left(x_{B 4}, y_{B 4}\right)\right\} ; \quad \triangleright\) Determine the true MT location
    \(B\left(x_{B}, y_{B}\right)\) from the set of \(B 1, B 2, B 3\) and \(B 4\);
        return \(B\left(x_{B}, y_{B}\right)\);
    end function
```

$$
\left(x_{B}-x_{A}\right)^{2}+\left(d x_{B}+e-y_{A}\right)^{2}=A B^{2}
$$

which is a quadratic equation
$\left(1+d^{2}\right) x_{B}^{2}-2\left(x_{A}-d\left(e-y_{A}\right)\right) x_{B}+x_{A}^{2}+\left(e-y_{A}\right)^{2}-A B^{2}=0$
that can be solved easily.
Algorithm 11 shows step-by-step how to compute the possible solutions of $B$. It outputs two points $B 1\left(x_{B 1}, y_{B 1}\right)$ and $B 2\left(x_{B 2}, y_{B 2}\right)$.

Remark 1 It is clear that one of the two points, $B 1\left(x_{B 1}, y_{B 1}\right)$ and $B 2\left(x_{B 2}, y_{B 2}\right)$, must be the position of the MT (or both of them are the position of MT if B1 and B2 are identical).

Remark 2 In the special case when the RP is fixed (i.e., $A D=0$ such that $x_{A}=x_{D}$ and $y_{A}=y_{D}$ ), the above generalized Algorithm $\square$ (GlSM) will become the 1SM Algorithm of [1] and Eqn. (1) can be simplified as:

$$
\begin{aligned}
x_{B} \cos \alpha+y_{B} \sin \alpha & =x_{A} \cos \alpha+y_{A} \sin \alpha \\
& -\frac{\left(A B^{2}+B C^{2}-A C^{2}\right)}{2 B C} .
\end{aligned}
$$

## B. Generalized Two-Step Movement (G2SM)

After the first movement, we have two possible locations of the MT given by G1SM using Algorithm [1 but we cannot determine which one is the true location. To resolve this ambiguity, it is natural to think of using a second movement. The basic idea is simple: a Generalized Two-Step Movement (G2SM) is a combination of two consecutive G1SM's where each move would give two possible positions (in which one of these two positions must be the true position). It is clear that by comparing the results from two G1SM's, we can determine
the location of the MT if the results from the two G1SM's are not redundant.

Figure 4b depicts how G2SM works. While the RP moves from $D$ to $E$, the MT carries out the second movement from $C$ to $F$ in the direction of angle $\beta$, which is measured from the positive $x$-axis counter-clockwise. The distance $C F$ and $\beta$ are known by the MT, whereas the distance $E F$ from the MT to the new position of RP is measured from the received signal strength using standard techniques. The underlying idea is that we can now consider the second movement similarly as that of G1SM case in which the starting point of MT is $B$ and the ending point is $F$ with respect to the two positions of $\mathrm{RP}, A$ and $E$. We can compute the distance $B F$ and the angle $\gamma$ analytically (see Algorithm 2) line 7-12) and then use the method of Algorithm 1 to determine $B$. Algorithm 2 shows the above step-by-step. By comparing the results from the two G1SM's, we determine the location of the MT.

Remark 3 The G2SM requires the MT to change the moving direction such that $\beta \neq \alpha$ and $\beta \neq \alpha \pm \pi$ (or the $R P$ is changing its direction), otherwise the ambiguity cannot be eliminated since the system of equations obtained from the second movement would be equivalent to that of the first one. However, this statement can be relaxed in reality since it is difficult to move in straight line and with the presence of noise.

In practice with estimation error or system imperfection, say the existence of noise, we may not obtain a common solution from the two G1SM's computations, i.e., the two possible solutions obtained from the first movement are different from the solutions obtained from the second movement (see Fig 3). To solve this problem, we can choose the pair of points in $\{\overline{B 1}$, $\overline{B 2}, \overline{B 3}, \overline{B 4}\}$ that have the minimum Euclidean distance, i.e.,

|  | $A D=D E=[0.1,0.2] \times A B$ |  |  | $A D=D E=[0.2,0.5] \times A B$ |  |  | $A D=D E=[0.5,1] \times A B$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} e_{d}=1 \% \\ e_{a}=1^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} e_{d}=2 \% \\ e_{a}=2^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} e_{d}=5 \% \\ e_{a}=5^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} e_{d}=1 \% \\ e_{a}=1^{\circ} \\ \hline \end{gathered}$ | $\begin{aligned} & e_{d}=2 \% \\ & e_{a}=2^{\circ} \\ & \hline \end{aligned}$ | $\begin{gathered} e_{d}=5 \% \\ e_{a}=5^{\circ} \\ \hline \end{gathered}$ | $\begin{gathered} e_{d}=1 \% \\ e_{a}=1^{\circ} \\ \hline \end{gathered}$ | $\begin{aligned} & e_{d}=2 \% \\ & e_{a}=2^{\circ} \\ & \hline \end{aligned}$ | $\begin{gathered} e_{d}=5 \% \\ e_{a}=5^{\circ} \\ \hline \end{gathered}$ |
| $A B=1(B C=C F=0.1 A B)$ | 0.1139 | 0.2011 | 0.4121 | 0.0677 | 0.1189 | 0.2736 | 0.0461 | 0.0820 | 0.1808 |
| $A B=1(B C=C F=0.2 A B)$ | 0.0855 | 0.1518 | 0.3310 | 0.0597 | 0.1126 | 0.2432 | 0.0473 | 0.0832 | 0.1803 |
| $A B=1(B C=C F=0.5 A B)$ | 0.0509 | 0.0918 | 0.2119 | 0.0462 | 0.0877 | 0.1802 | 0.0413 | 0.0765 | 0.1678 |
| $A B=5(B C=C F=0.1 A B)$ | 0.5433 | 0.9654 | 2.1378 | 0.3346 | 0.6336 | 1.3726 | 0.2431 | 0.4196 | 0.9274 |
| $A B=5(B C=C D=0.2 A B)$ | 0.4162 | 0.7548 | 1.6035 | 0.3058 | 0.5562 | 1.1936 | 0.2361 | 0.4100 | 0.9044 |
| $A B=5(B C=C F=0.5 A B)$ | 0.2567 | 0.4452 | 1.0373 | 0.2349 | 0.4349 | 0.9156 | 0.2148 | 0.3852 | 0.8364 |
| $A B=10(B C=C F=0.1 A B)$ | 1.1319 | 1.9817 | 4.1819 | 0.7184 | 1.1993 | 2.7077 | 0.4064 | 0.7965 | 1.8415 |
| $A B=10(B C=C F=0.2 A B)$ | 0.8090 | 1.5358 | 3.2446 | 0.6189 | 1.2082 | 2.4429 | 0.4292 | 0.8366 | 1.8521 |
| $A B=10(B C=C F=0.5 A B)$ | 0.5059 | 0.9164 | 2.0490 | 0.4579 | 0.8819 | 1.9196 | 0.4539 | 0.7590 | 1.6926 |

TABLE II: G2SM with multi-sampling: the average positioning error (in meter) under various $A B, A D, B C$, and noise levels.
solving $\min \left\{d\left(P_{i}, P_{j}\right) \mid P_{i} \neq P_{j}\right\}$, where $P_{i}, P_{j} \in\{\overline{B 1}, \overline{B 2}$, $\overline{B 3}, \overline{B 4}\}$ and $d\left(P_{i}, P_{j}\right)$ is used to denote the Euclidean distance of points $P_{i}$ and $P_{j}$, and then takes their mean position. One may also consider a possible optimization problem to improve the result in combining the original data sets instead of the above.

## C. Simulation Result

The performance of the G2SM method is investigated by simulation. Parameters used are the same as those used when studying 2SM (see Section II-C). Since the RP is also mobile (movable), we have to set the values of $A D$ and $D E$. Here, we consider they are proportional to $A B$ and in three movement ranges: $[0.1,0.2] \times A B$ (i.e., small move), $[0.2,0.5] \times A B$ (i.e., medium move), and $[0.5,1] \times A B$ (i.e., large move). In the simulation, we consider that the RP moves in a distance which is at most $A B$ (assuming that $A B$ is the signal coverage range of the RP). Secondly, we consider $A D=D E$, for simplicity. However, the movement direction of the RP does not need to be fixed and we thus generate it to be uniformly distributed in $(0,2 \pi]$. Note that the proposed algorithms are not limited to above numerical settings.

Table 【 $\llbracket$ shows the simulation result of G2SM localization with multi-sampling. For each setup, we conduct $10^{5}$ runs of simulation to obtain the average performance. Same as 2 SM with multi-sampling, we perform 1000 sampling near the end point of each movement at G2SM. As expected, it can be seen that the estimation error in determining the position of the MT increases as the noise power increases. However, there is a correlation between the movement distance of the two devices (the MT and RP) and the resulting error. If the RP moves a bit (see $A D=D E=[0.1,0.2] \times A B$ ) but the MT moves a lot (see the case $B C=C F=0.5 A B$ ), there is substantial error decrease. However, when RP moves a lot (see $A D=D E=[0.5,1] \times A B$ ), the error appears independent of how much the MT moves. Another interesting observation is that a substantial movement of either the MT or the RP is sufficient for achieving good performance. Overall, the average error is within about $15 \%$ of $A B$. In the best case, the average error is less than $5 \%$ of $A B$.

## IV. Conclusion

In this paper, we first combine the 2 SM method with multisampling technique to improve the positioning performance.

Simulation result shows an error decrease of $15 \%-30 \%$. Secondly, we propose the generalized localization method G2SM by utilizing device movement in which both the MT and RP are allowed to move. The position of MT can be determined analytically and in simple closed-form expression. Simulations are conducted to study its performance under various setup and noise levels. Results show that an average error within about $15 \%$ of the distance between the MT and RP can be realized. Since G2SM would allow a MT to locate itself through a peer mobile device, it has potential applications in future large D2D or multi-hop systems.

## AcKNOWLEDGMENT

The work presented in this paper was supported by ANR project IDEFIX under grant number ANR-13-INFR-0006 and has been carried out at LINCS (www.lincs.fr). The authors would like to thank Fabien Mathieu and Philippe Jacquet for their valuable comments.

## REFERENCES

[1] T. D. Huynh, C. S. Chen, and S.-W. Ho, "Exploiting user movement for position detection," in IEEE Consumer Communications and Networking Conference, Las Vegas, Jan. 2015, pp. 1-6.
[2] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," IEEE Trans. Sys. Man Cyber - Part C, vol. 37, no. 6, pp. 1067-1080, Nov. 2007.
[3] Y. Gu, A. Lo, and I. Niemegeers, "A survey of indoor positioning systems for wireless personal networks," IEEE Commun. Surveys Tuts., vol. 11, no. 1, pp. 13-32, Jan. 2009.
[4] K. Al Nuaimi and H. Kamel, "A survey of indoor positioning systems and algorithms," in International Conference on Innovations in Information Technology, April 2011, pp. 185-190.
[5] R. Harle, "A survey of indoor inertial positioning systems for pedestrians," IEEE Commun. Surveys Tuts., vol. 15, no. 3, 2013.
[6] B. Kim, W. Bong, and Y. Kim, "Indoor localization for Wi-Fi devices by cross-monitoring AP and weighted triangulation," in IEEE Consumer Communications and Networking Conference, Jan 2011, pp. 933-936.
[7] P. Mirowski, D. Milioris, P. Whiting, and T. K. Ho, "Probabilistic radiofrequency fingerprinting and localization on the run," Bell Labs Technical Journal, vol. 18, no. 4, pp. 111-133, Feb. 2014.
[8] A. Papapostolou and H. Chaouchi, "Scene analysis indoor positioning enhancements," Annals of Telecommunications, vol. 66, no. 9-10, pp. 519533, 2011.
[9] A. Cavallini, iBeacons Bible, http://meetingofideas.files.wordpress.com/ 2013/12/ibeacons-bible-1-0.pdf.

