

ESSAYS IN INVENTORY DECISIONS UNDER UNCERTAINTY

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ESSAYS IN INVENTORY DECISIONS UNDER UNCERTAINTY

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LIST OF SYMBOLS AND ABBREVIATIONS

Chapter 2:

p	Selling price
c	Actual cost to purchase in the period 0
\bar{c}	Expected cost to purchase in the period 0
h	Holding cost for one period
y	Order-up-to level for period 0
x	Sum of average period demand to forward buy
w	Total order-up-to-level for GOGA ($w=x+y$)
i	Inventory position (on hand plus on order)
Φ^{-1}	Inverse CDF (cumulative distribution function) of demand

Chapter 3:

i	Supplier index, $i=1, \dots, n$.
x	Random variable for demand
$f(x)$	probability density function of demand
$F(x)$	CDF of demand
$\phi(x)$	probability density function of demand for normal distribution
$\Phi(x)$	CDF of demand for normal distribution
q_0	The centralized make to stock quantity
q_i	The make to stock quantity at supplier i
m	The minimum quantity from all suppliers and the retailer's capacity decision.
c_i	The cost to produce one unit at supplier i
w_i	The wholesale selling price of one unit from supplier i to the retailer
s_i	The salvage value for one unsold unit at supplier i
k	The cost per unit to buy capacity at the retailer
α	Salvage value for unused capacity at the retailer
$\gamma(q)_0$	The loss function in the centralized control case, where
$\gamma(q)_0 = \int_q^{\infty} (x - q) f(x) dx$	
$\gamma(.)_i$	The loss function for supplier i
μ	Mean of demand
σ	Standard deviation of demand
δ_i	The salvage manipulator from retailer to supplier i (may be negative)
$\Delta = \sum_{i=1}^n \delta_i$	The total salvage manipulation the retailer must pay (or receive if negative) to the n suppliers

Chapter 4:

c_S	Cost to make one unit at the supplier (raw material and production value add)
c_R	Additional cost to retailer per one unit above supplier costs (value added, landed cost including shipping)
s_S	Salvage value of a unit at the supplier
s_R	Salvage value of a unit at the retailer
β	Goodwill (shortage) cost at the retailer per unit of unmet customer demand
p	Retailer selling price
w	Wholesale price (supplier selling price to the retailer per unit)
q_S	Production quantity at the supplier (decision variable)
q_R	Retailer component purchase and production quantity (decision variable)
L_S	Production lead time at the supplier
L_R	Production lead time at the retailer
L_T	Transportation lead time between the supplier and the retailer
x	Random demand prior to information update
μ	Demand mean prior to information update
σ	Standard deviation of demand prior to information update
x_e	Random pre-order demand signal during supplier lead time, i.e. “New Market Information” as described in Donohue (2000). x_e is ordered such that $x_e^1 < x_e^2$ implies $G(x x_e^1) \geq G(x x_e^2)$.

μ_e	Pre-order mean (advanced demand information mean)
σ_e	Standard deviation of the advanced demand information
$f(.)$	pdf/pmf of demand as viewed at the first decision point (supplier tier)
$g(x x_e)$	pdf/pmf of demand as viewed at the second decision point, given x_e
$F(.)$	CDF of demand as viewed at the first decision point
$G(.)$	CDF of demand as viewed at the second decision point, given x_e
$\gamma(.)$	The retailer loss function (i.e. expected shortage) if the demand is more than the production quantity where $\gamma(q_R) = \int_{q_R}^{\infty} (x - q_R) g(x x_e) dx$

SUMMARY

Uncertainty is a norm in business decisions. In this research, we focus on the inventory decisions for companies with uncertain customer demands. First, we investigate forward buying strategies for single stage inventory decisions. The situation is common in commodity industry where prices often fluctuate significantly from one purchasing opportunity to the next and demands are random. We propose a combined heuristic to determine the optimal number of future periods a firm should purchase at each ordering opportunity in order to maximize total expected profit when there is uncertainty in future demand and future buying price.

Second, we study the complexities added by having bundling of products in an Assemble-To-Order (ATO) environment. The assembler/retailer must decide how much assembly capacity to acquire before the selling season. Similarly, each supplier must decide how much to make prior to the selling season. All players are interdependent and the quantity of bundles for sale, by definition, cannot exceed the lowest quantity at any of the players. We outline a salvage manipulator mechanism that coordinates the decentralized supply chain.

Third, we extend our salvage manipulator mechanism to a two stage supply chain with a long cumulative lead time. With significant lead times, the assumption that the suppliers all see the same demand distribution as the retailer cannot be used. We find that optimal profits are achieved through our subsidy mechanism.

CHAPTER 1

INTRODUCTION

Uncertainty is common in all business. For example, customer demands are very uncertain from time to time. It is uncertain when a customer commit to a purchase and how much the customer may order. Uncertainty increases the complexity of business decisions such as inventory quantity determination. Companies that can better deal with uncertainty generally generate better profit. In this research, we focus on the inventory decisions for company with uncertain customer demands.

First, we investigate forward buying strategies for single stage inventory decisions. The situation is common in commodity industry where prices often fluctuate significantly from one purchasing opportunity to the next and demands are random. These fluctuations allow firms to benefit from forward buying (buying for future demand in addition to current demand) when prices are low. We propose a combined heuristic to determine the optimal number of future periods a firm should purchase at each ordering opportunity in order to maximize total expected profit when there is uncertainty in future demand and future buying price. We compare our heuristic with existing methods via simulation using real demand data from BlueLinx, a two-stage distributor of building products. The preliminary results show that our combined heuristic performs better than any existing methods considering forward buying or safety stock separately. We also compare our heuristic to the optimal inventory management policy by full enumeration for a smaller data set. The proposed heuristic results also show to be close to optimal. This study is the first to decide both the optimal number of future periods to buy for

uncertain purchase price and the appropriate purchasing quantity with safety stock for uncertain demand simultaneously. The experience suggests that the proposed combined heuristic is simple and can be very beneficial for any company where forward buying is possible.

We next investigate the complexities added by having bundling of products in an Assemble-To-Order (ATO) environment. Building on Pasternack's (1985) seminal work, Gerchak and Wang (2004) extend the channel coordination idea to an ATO environment. We build upon their work by allowing risk at both the supplier and the retailer echelon. The retailer must decide how much assembly capacity to acquire before the selling season. Similarly, the n suppliers must decide how much to make prior to the selling season. All players are interdependent and the quantity of bundles for sale, by definition, cannot exceed the lowest quantity at any of the players. We outline a salvage manipulator mechanism that coordinates the decentralized supply chain.

We then extend our salvage manipulator mechanism to a two stage decision making environment with demand information updating. After the first stage production decision, some demand uncertainty is resolved through pre-sales. The second stage purchase decision is made with more information, but upper bounded by the supplier's production output.

With these three studies, we show that inventory decisions under various kinds of uncertainty can be improved to increase, if not maximize, expected total supply chain profit. Note that equation numbers begin with number one in each chapter and appendix.

CHAPTER 2

BLUELINX CAN BENEFIT FROM INNOVATIVE INVENTORY MANAGEMENT METHODS FOR COMMODITY FORWARD BUYS

2.1 Introduction

This paper describes a heuristic developed to improve the purchasing decisions of BlueLinx Corporation, a two-stage distributor of building product materials with annual revenues around eight billion U.S. dollars. Purchasing and selling commodities at BlueLinx is a complex process due to both fluctuating purchase prices and highly seasonal and uncertain customer demand. They purchase bulk commodities from suppliers such as lumber mills and sell smaller truckloads to customers as requested. The customers do not procure commodities directly from the mill because they 1) do not purchase enough volume at one time to satisfy the minimum mill quantity requirement, or 2) they do not want to give up the flexibility of shipment size and destination that is absorbed by the two-stage distributor.

BlueLinx can charge a positive margin by absorbing lead-times, breaking bulk, and providing fast deliveries. However, a highly variable portion of their profit or loss is derived solely from the difference between the price they purchase the commodities at versus the price they sell them at. Due to the competitive nature of their business, the price BlueLinx can charge for its product is determined by market forces and may be considered exogenous to BlueLinx's decision making. Thus, strategic purchasing that minimizes the cost of acquiring the product provides BlueLinx with the largest opportunity for improving profits.

In this paper, we provide insights into BlueLinx's problem by modeling a two-stage distributor that has a purchasing opportunity at a known, current cost with forecasts for future demands and a known distribution for future costs. The distributor's decision is whether to buy enough products to satisfy demand only in the period 0 (first or current period where inventory on hand can be increased by a purchase) or to also buy to meet demand in future periods (forward buy periods beyond the vendor delivery lead-time). We propose a heuristic for this problem that is a combination of two existing methods for determining the optimal number of future periods to buy and the order-up-to levels under an uncertain cost and demand environment. The goal is to maximize the total expected profit. We use actual sales data (simulated through a bootstrapping technique) from BlueLinx for the years 2001-2005 to demonstrate the effectiveness of the proposed heuristic. The results show that our combined heuristic performs better than any existing methods considering forward buying or safety stock separately. We also compare our heuristic to the optimal inventory management policy by full enumeration for a smaller data set. It shows that the proposed heuristics is close to optimal. This study is the first to decide both the optimal number of future periods to buy for uncertain purchase price and the appropriate purchasing quantity with safety stock for uncertain demand simultaneously. The study suggests that the proposed combined heuristic is simple and can be very beneficial for any company where forward buying is possible.

We begin by describing BlueLinx's purchasing environment.

2.1.1 BlueLinx Purchasing Environment

The following conditions describe the purchasing environment of BlueLinx and are based on discussions about procurement practices with the current and former directors of supply chain procurement at BlueLinx Corporation.

Condition 1: BlueLinx is a price taker. BlueLinx exists in a highly fragmented market where the largest player comprises only 10% of the total market and where there are many small players with no influence at all. Moreover, the company has little price flexibility. Selling prices cannot be raised to cover prior high priced purchases; rather, selling prices are a constant marginal addition to current market prices for the products. In fact, all players in the industry are price takers.

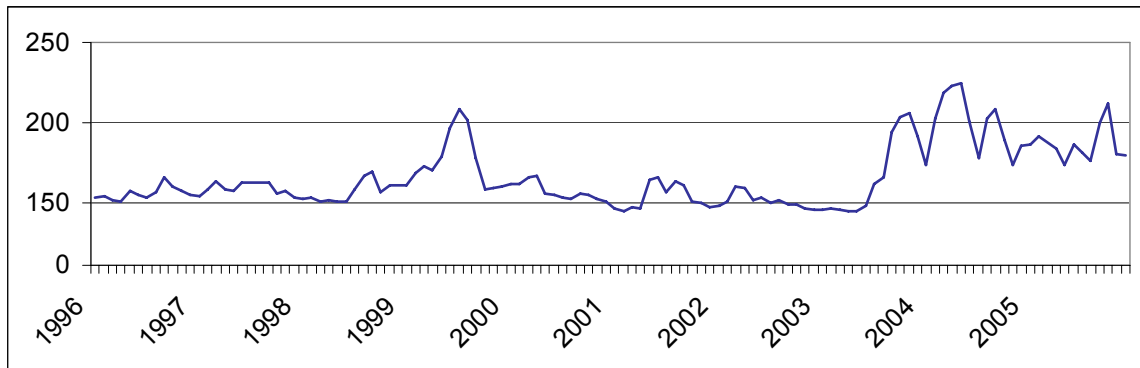
Condition 2: Demand is stochastic with a known distribution. The demand distribution is non-stationary given the highly seasonal nature of building product demand.

Condition 3: The demand forecast is unbiased. Tracking signals demonstrate that the forecasting method is unbiased for the commodity products at BlueLinx.

Condition 4: The purchase price exhibits randomness as shown in Figure 2.1 for the price of plywood (Economagic, 2006). It is the nature of commodity goods to fluctuate in price daily, and even hour to hour depending on conditions of supply and demand. Figure 1 below shows 10 years of monthly data. Each monthly figure is reported as the average of the daily price close. Note the spike near the end of 2003 has many theories from analysts; 1) U. S. Military placed large orders in August for the First Armored Division in Iraq to build barracks, 2) hurricane Isabel caused significant

demand for plywood to reinforce windows, 3) strong single-family home builder demand with insufficient stock.

Figure 2.1: Historical Plywood Prices 1996-2005



Condition 5: Since lead-times are significant, an order must be placed before demand is realized. Products such as rebar are often purchased internationally, requiring significant lead times. U.S. demand for rebar, for example, exceeds domestic production. Thus, local spot purchases are not available if the original order quantity falls below realized demand. Therefore, the current period (period 0) in all models is really the first period that can have inventory increased by a purchase. For plywood at BlueLinX, the period 0 is 3 months from today initially. This period rolls forward during the horizon.

Condition 6: There are no viable substitute products. Customers (builders and industrial manufacturer) have specifications calling for certain materials and so they will not use different grades or variants. If the company is out of a particular commodity, it cannot fill demand with a substitute product; for example, a builder requiring Oriented Strand Board (OSB) would not substitute plywood for his application.

Condition 7: Demand is independent between periods and unmet demand is lost. Demand in one period does not affect other periods since any unmet demand is filled by

another player in the market. The customer cannot wait for BlueLinx if it is out of stock. He will find the materials he needs for his current demands from whoever has them in stock. Currently, BlueLinx estimates expected price trends but they buy just enough to cover the demand point forecasted.

2.1.2 Forecasts

We used Holt-Winters with additive seasonality to forecast future demand. Given the highly seasonal nature of building products, this method fits the data well and is currently used by BlueLinx. We applied the seasonal indices to the wood and metal products that were computed based on the prior four years of sales. The smoothing parameters were kept constant during the simulation, as BlueLinx adjusts them infrequently. Holt-Winters method with seasonality is used by BlueLinx to produce the monthly (period) point forecasts. The standard deviation of demand is calculated each month for plywood based on the prior rolling year of sales orders. Because the vendor lead-time is three months, it is necessary to use increasing prediction intervals since the demand distribution today is narrower than the wider distribution (more uncertainty) three months in the future.

We were able to fit autoregressive functions to historical prices for plywood that achieved normally distributed errors and low Mean Absolute Deviations. ARIMA (3,0,0) worked best for the wood price data. ARIMA is Autoregressive Integrated Moving Average time series forecasting (Makridakis et al., 1998). The first parameter (3) specifies that three prior data points are used to correlate to the forecasted data point. The second parameter specifies the degree of first differencing involved and the third

parameter specifies the order of the moving average portion. The plywood data used is from January 1996 through May 2001. Our simulation starts with June 2001 so that we can be certain the price forecast parameters are based only on the historical data that would have been available at the time. For wood, the last known price $-.1 * (\text{two periods ago}) + .1 * (\text{three periods ago})$ produced the lowest error. The price forecast each period is a combination of the prior three price data points. Manikas (2007) finds that, for plywood historical prices during 1996 through 2001, autocorrelation functions can accurately forecast future prices.

2.2 Literature Review

The model developed in this paper combines two existing procurement methods. Assuming demand is deterministic, one method is the optimal forward buying algorithm from Golabi (1985). The other method incorporates uncertain demand and buys safety stock that may be used for future periods as outlined in Gavirneni (2004). We discuss these two methods and give an example of each in the following subsections.

2.2.1 Forward Buying with Deterministic Future Demand

Golabi (1985) proposes a method whereby material for future periods is bought as long as the marginal cost is less than the marginal savings. For example, given that the current ordering cost in a period is \$180 per unit, in the next period the ordering cost is expected to be \$200. If demand in the next period is expected to be one, and holding cost for that one unit is less than \$20, it would be beneficial to purchase in the first period and hold the stock to fill demand in the second period. This differential is translated into a

series of non-increasing price thresholds. If the realized price is less than or equal to the threshold, then it is optimal to buy for that many periods forward and incur the holding costs. Golabi's model assumes concave holding costs for a single item with deterministic demands. Ordering prices in each period are random with a known distribution.

Magirou (1982) uses a very similar method to Golabi with the addition of allowing a fixed storage capacity and selling beyond the forecast demand in the commodities market for oil.

Golabi's model finds the price points in a current period so that a decision maker could determine the optimal number of periods to forward buy in order to minimize total expected cost. Golabi's equation accounts for the probability that the next period price will be less than the current price plus the benefit of locking in the prior price minus the holding costs of one period. Equation (1) below is the corrected equation (9) from Golabi's paper that specifies the next price point such that forward buying n periods is optimal. A_n is the threshold price per unit such that buying n periods ahead is optimal. If the current purchase price is less than or equal to A_n , it is optimal to buy for the current period plus n periods ahead. Let x be the purchase price, $F(x)$ be the known cumulative price distribution for each period and h be the cost to hold one unit of stock for one period. A_0 is the highest possible purchase price since Golabi assumes all demand must be met for the current period (period 0). Each additional threshold price is computed according to

$$A_{n+1} = \int_0^{A_n} x dF(x) + \int_{A_n}^{\infty} A_n dF(x) - h \quad (1)$$

Given the probability the price falls in the future, the first integral in (1) is the opportunity cost of not being able to take advantage of a lower purchase cost in the next period, should it materialize. The second integral is the benefit of locking in at the A_n purchase cost. The final term is the holding cost for buying inventory in period n for use in period $n+1$.

We illustrate this heuristic with a stationary, uniform price distributions for ease of understanding. However, the simulation and real situation at the company are with non-stationary price distributions. Assume that the price at any buying opportunity can be \$25, \$50, \$75 or \$100 – each with equal probability. Assume the holding cost for one period is \$5. Since we must buy to cover demand in the current period (0), A_0 equals the highest possible price. Thus $A_0 = 100$, the highest possible purchase price of our distribution.

A_1 is the expected price lower than or equal to A_0 plus the benefit of locking in at the price A_0 minus the one period holding cost h .

The expected price lower than or equal to A_0 is $\$25 * 25\% + \$50 * 25\% + \$75 * 25\% + \$100 * 25\% = \$62.50$

The expected benefit of locking in the price of A_0 is \$0 (since the price cannot go higher than \$100). $A_1 = \$62.50 + \$0 - \$5 = \57.50 . If the current purchase price is \$57.50 or lower, buy for the current period demand plus the demand for next period.

Likewise, the expected price lower than or equal to A_1 is $\$25 * 50\% + \$50 * 25\% = \$18.75$. $\$57.50 * 50\% = \28.75 as the benefit of locking at $A_1 = \$57.50$. Therefore, $A_2 = \$18.75 + \$28.75 - \$5 = \42.50

Similarly, $\$25 * 25\%$ is $\$6.25$, and $A_2 * 75\%$ is $\$31.88$. Therefore, $A_3 = \$6.25 + \$31.88 - \$5 = \33.13 .

$A_4 = \$6.25 + \$24.84 - \$5 = \26.09 , and $A_5 = \$19.57$ which is below the possible price range for the distribution so we can be certain that we will never buy for more than four periods in advance.

This method answers the question of how many periods in advance to buy for to satisfy all predicted demand and to minimize total expected costs. Given our price distribution, Table 2.1 summarizes the number of periods to forward buy for each possible realization of the purchasing cost. A period is any unit of time for the fixed review period procurement. BlueLinx uses a month as its period for plywood.

Table 2.1: Price Breaks to Forward Buy

		A₀	A₁	A₂	A₃	A₄	Calculated Value
		Current Period	Buy 1 period ahead	Buy 2 periods ahead	Buy 3 periods ahead	Buy 4 periods ahead	
		\$100.00	\$57.50	\$42.50	\$33.13	\$26.09	
\$	100	Yes					
\$	75	Yes					
\$	50	Yes	Yes				
\$	25	Yes	Yes	Yes	Yes	Yes	
Purchase Price							

Based on Table 2.1, if the current purchase price is \$75 or \$100; only buy for the current period's demand. If the current price is \$50; buy for the current period's demand plus the demand for one additional period. If the current price is at \$25; it is optimal to buy for the current period's demand plus the demand for the next four periods.

As an example, assume the forecasts for the next 6 months are as shown in Table 2.2, where 0 is the current month, 1 denotes one period in the future, etc.:

Table 2.2: Example Monthly Demand Forecasts

Period	0	1	2	3	4	5
Demand	100	110	105	90	120	115

Assume the current price to purchase the item is \$50. Given the price thresholds previously calculated (shown in Table 2.1), this price falls between A_1 and A_2 .

Therefore, we should buy one period forward since we have passed the A_1 threshold but not the A_2 two-period threshold. We always include the current period 0 in the order up to level equation. In this example, given that the items can be purchased for \$50 each, Golabi's method would recommend we order enough units to get our inventory up to $100 + 110 = 210$ units.

This method assumes deterministic demand and instantaneous replenishment. However, BlueLinx's demand data appears to be normally distributed around the forecast mean and thus Golabi's method does not take into account the uncertainty of demand. Additionally, if prices fluctuate less than the one-period holding costs, forward buying will never occur under this method. We also provide for non-zero lead times by moving the current period 0 out by three months to account for transit time, for example, from China to the USA. Therefore, period 0, 1, 2, 3 in the above example are 3, 4, 5, 6 months into the future for plywood at BlueLinx.

2.2.2 Buying with Uncertain Future Demand

The second method from Gavirneni (2004) accounts for demand uncertainty. We use the forecast error to estimate the distribution of demand in each period instead of

using just the point forecast of demand. We choose the normal distribution because the shape of the forecast error closely follows a normal bell curve in the real demand data from BlueLinux. However, for ease of explanation we have used a more simple uniform distribution in the examples in the prior section.

Gavirneni proposes a myopic heuristic based on a newsvendor ratio to calculate the quantity to procure each period. Gavirneni shows the closed form of the order up equation where the purchasing cost is constant, however the equation cannot easily be integrated when purchasing costs fluctuate so the myopic heuristic is a practical approximation. The newsvendor equation with the profit margin as cost of underage and the holding costs as the cost of overage for myopic buying provides the critical ratio. This ratio is applied to the demand distribution to offer the optimal amount to procure given the demand distribution. The notation for this method is:

p	Selling price
c	Actual cost to purchase in the period 0
\bar{c}	Expected cost to purchase in the period 0
h	Holding cost for one period
y	Order-up-to level for period 0
x	Sum of average period demand to forward buy
w	Total order-up-to-level for GOGA ($w=x+y$)
i	Inventory position (on hand plus on order)
Φ^{-1}	Inverse CDF (cumulative distribution function) of demand

The standard newsvendor equation where the overage is simply the one period holding costs has the following order-up-to level as shown in (2).

$$y = \Phi^{-1}\left(\frac{p - c}{p + h - c}\right) \quad (2)$$

As an example, assume the same monthly forecasts from Table 2.2. The current purchase cost (c) is \$50 and the selling price (p) for this item is \$101, and the holding cost is \$5 per period. Using the forecast error as a surrogate for the variability of demand, a demand distribution can be generated about the point forecast. For period 0, the point forecast is 100. For the sake of clarity in this example, we will assume that the demand distribution is uniform centered on the point forecast of 100, $\sim U(50, 150)$.

The critical ratio is calculated as:

$$\left(\frac{p - c}{p + h - c}\right) = \left(\frac{101 - 50}{101 + 5 - 50}\right) = \left(\frac{51}{56}\right) = 91\% \quad (3)$$

The order up to quantity y is given by the equation:

$$y = \Phi^{-1}(.91) \quad (4)$$

Therefore, $y = (150 - 50) * .91 + 50 = 141$ units. The standard newsvendor would suggest we buy enough units to have 141 units in the period 0 instead of just the 100 units point forecast. The extra 41 units are safety stock and may be used to fill future demand as well, but they have been calculated solely based on the demand distribution in the period 0.

Gavirneni modifies the standard newsvendor to use the expected cost in the denominator and the realized purchasing cost in the numerator as shown in (5).

$$y = \Phi^{-1}\left(\frac{p - c}{p + h - \bar{c}}\right) \quad (5)$$

Using the same example as we did above for the standard newsvendor, but with the expected cost (\bar{c}) to buy is \$62.50, the average of prices from our known distribution of \$25, \$50, \$75, \$100. The critical ratio is calculated as:

$$\left(\frac{p - c}{p + h - \bar{c}} \right) = \left(\frac{101 - 50}{101 + 5 - 62.50} \right) = \left(\frac{51}{43.5} \right) = 117.2\% \quad (6)$$

Because the holding cost is 10% of the purchase cost, yet the current realized procurement cost is 20% below expected, the critical ratio is very large. Planning to hold safety stock to cover 100% of uncertainty in a period is the maximum safety, therefore if (6) mathematically yields a number greater than 100%, we force the ratio to be 100%. The order up to quantity y is given by the (7) below:

$$y = \Phi^{-1}(1.00) \quad (7)$$

Therefore, $y = (150 - 50) * 1.00 + 50 = 150$ units rounded up. Gavirneni's method would recommend we buy enough units to have 150 on hand in the current period. The extra 50 units above the point forecast are safety stock worth holding given our profits and holding costs for one period. Gavirneni's method applies this modified newsvendor ratio to each period 0's demand distribution iteratively, but his method does explicitly buy for future period demand distributions. Gavirneni and Morton (1999) look at stochastic demand with a one-time price increase for speculative buying. However, period to period prices may decrease or increase when looking to forward buy. Therefore, we cannot directly apply their dynamic programming solution to BlueLinx's problem.

Gavirneni's method takes into account demand and price uncertainty, but it also involves buying safety stock rather than taking into account the predicted demand in future periods. BlueLinx's demand is highly seasonal, therefore the shape and size of each period's demand distribution is important for forward buys.

2.3 Problem Statement and Proposed Heuristic

2.3.1 Problem

A company that procures commodities knows the current prices to purchase at. At each ordering opportunity a company needs to decide how much to order to cover both current demand and possible future demand. The tradeoff to such a decision is that the company would be buying more and thus incurring holding costs (warehouse space, capital tied up, etc.) to offset the possibility of paying higher purchase prices closer to when demand will be realized.

We examine a two stage distributor's ordering strategy given forecasted customer demand, forecasted purchase prices and linear holding costs. Unfilled demand is assumed lost to competitors.

2.3.2 Proposed Heuristic

Our proposed method, which combines the work of Golabi (1985) and Gavirneni (2004), is named GOGA out of respect for their prior contributions. We use the price breaks per Golabi in (1) to determine how many periods to forward buy. For each period, the newsvendor from Gavirneni is used to account for uncertainty in the demand. For forward buys, only the mean forecasted demand is bought, just as Golabi does for all

periods including period 0. In essence, we are using Gavirneni's method on the demand distribution for period 0, then we use Golabi's method for forward buys for additional periods using the point forecast of demand.

The Holt-Winters model with additive seasonality is used to produce point forecasts for future sales of the commodity. A forecasting method with seasonality needs to be used given the clear seasonality of the consumption of building products. Plywood is used extensively in construction, which is clearly a seasonal activity. (For a thorough discussion on the Holt-Winters forecasting method see Chatfield (1978).) Given our work with the two stage distributor BlueLinx, we feel that additive seasonality is appropriate for the forecast models. The Holt-Winters forecast is used to generate the mean future forecast. The lead time from purchase to receipt is explicitly taken into account by shifting the current period to the forecast 3 months from now to account for transit time to the USA from China. To account for the increasing uncertainty in time, the demand distribution prediction intervals increase. In the case of BlueLinx, the lead time is three months, so the point forecast of demand three months out is used as the current period demand, with the distribution expanded by applying Yar and Chatfield's (1990) prediction interval equations for three steps ahead. Those equations are beyond the scope of our paper, but similar accounting for the non-stationary distribution of demand for long lead-time items is suggested for companies implementing our proposed heuristic.

Although commodities are not perishable, we balance the lost profit versus cost of capital to derive the critical ratio when forward buying. The cost of underage (C_u) is the profit lost by not selling the product. The cost of overage (C_o) is the one period holding

cost (h). The critical fractile over the demand distribution D is given by (3). Since the critical ratio is to buy safety stock and since we have buying opportunities each period, the ratio is only used for the current period and not for a future period during a forward buy. The modified newsvendor ratio is shown in (8) where p is the expected selling price, h is the holding cost for one period (holding % times c), c is the current procurement cost, and \bar{c} is the expected price. We modify the expected price to be the expected cost next period rather than the forecasted cost this period as Gavirneni (2004) does. This minor modification results in significantly better results. We believe the improvement comes from using a estimate of next period's cost, thus including the most recent realized cost and where the expected cost this period is based on the prior three months of data. The optimal order up to inventory y in each current period is given in equation (9).

$$F(y) = \frac{C_u}{C_u + C_o} = \frac{p - c}{p + h - \bar{c}} \quad (8)$$

$$y = F^{-1}\left(\frac{C_u}{C_u + C_o}\right) \quad (9)$$

For the normally distributed demand with mean of μ and standard deviation σ , $y = \mu + z * \sigma$, where z is the inverse standard distribution of CDF computed in (8). For plywood at BlueLinx, the demand appeared normally distributed.

As a simple numerical example, assume a company buys a commodity at (c) \$95 per unit and sells it for (p) \$100 per unit, plus they have a per period holding cost (h) of 10% annually / 12 months; thus, the holding cost is .83% of the buying cost or \$0.79 per month. For one period ahead, underage (C_u) is \$5 and overage (C_o) is \$0.79; therefore,

the critical ratio is calculated at 86.3%. If the demand is normally distributed, using a standard normal inverse table, this translates to a factor (z) of 1.09. Therefore, the optimal order up to inventory y calculated from (9) becomes:

$$y = \mu + 1.09 * \sigma \quad (10)$$

In our application, the mean (μ) comes from the Holt-Winters forecast of demand, and the factor 1.09 comes from the t-value of the Student's one-tailed t-distribution as a function of the prior forecast error and the degrees of freedom assuming that we do not have σ available.

In contrast to our heuristic, the Golabi (1985) method would buy only the point forecast during every period. Likewise, the Gavirneni (1985) method would buy the optimal y each period but it would not forward buy unless under the speculative assumption.

We will use a numerical example to demonstrate how our method works. Holding (h) is \$5, selling price (p) is \$101, the current purchasing cost (c) is \$50, and the expected purchasing cost (\bar{c}) is \$62.50. Using equation (8) we get a ratio over 100%, which we truncate this to 100%. The demand distribution for period zero is distributed uniformly (50,150). The forecasts for the next six months are as shown previously in Table 2.2. (Note that if the holding cost (h) is more than \$12.5, the ratio from equation (8) will be less than 100 %.)

Using Gavirneni's formula from equation (8) we apply a ratio of 100% to the demand distribution giving $y=150$ units. Next, we use Golabi's price thresholds to find that we should also forward buy one period (since the purchasing cost \$50 is less than A_1 \$57.50 and greater than A_2 42.50 from Table 2.1). Therefore, we also buy next period's

point forecast of 110 units ($x=110$). The total purchase under our method will be 260 units this period ($150 + 110 \text{ units} = 260 \text{ units for } w$). Note that in all methods, the calculated amount is an order up to quantity. Therefore, if the inventory position (on hand plus open purchase orders) is 260 or more units, zero additional units will be purchased. If the inventory position is 190 units, 70 more will be purchased.

In summary, the proposed heuristic and the cost and revenue are performed according to the steps below each period throughout the planning horizon:

1. Compute y and x . y is the order up level for period 0 (current period) based on the modified Gavirneni's method shown in equation (8). x is the sum of point forecasts corresponding to additional periods beyond period 0 specified by Golabi's method for forward buys. Recall, the total order up quantity $w=x+y$
2. Order $w-i$ if i is less than w , where i is the inventory position in period 0. Otherwise, order nothing. The cost $c^*(w-i)$ is added to the total cost if i is less than w . The ordered units arrive with deterministic lead-time, so inventory position increases to w in period 0.
3. Let D be the demand realized. Sell minimum of D and i . Total Revenue has $c * (1 + \varepsilon) * \min(D, i)$ added to it, where ε is the profit margin. We assume that each period we maintain a constant percent margin beyond the current purchase cost that is valid for commodities. Inventory position i is decremented by $\min(D, i)$.
4. Each remaining unsold unit incurs a holding cost h if D is less than i . Total cost has $h * (i-D)$ added to it.

2.4 Simulation Method

We tested the effectiveness of our proposed procurement method using simulation. We ran 100 replications using four years of daily demand supplied by BlueLinx. The daily sales orders were aggregated into monthly demand to correspond to BlueLinx's fixed review period for purchasing plywood. For each replication, we ran nine parameter combinations of holding cost and profit margin. We choose 100 replications because it gave us a relative error of 3.5%. One data set was the actual invoice data from BlueLinx and the other 99 were created through bootstrapping (Davidson and Hinkley; Demirel and Willemain, 2002).

Our objective was to maximize expected profits given that purchase prices fluctuate from period to period. Selling prices are some percent profit ε of the selling period purchase price regardless of at what price the inventory was purchased at. Holding costs of 20% are typical for this industry due to the inclusion of obsolescence, shrinkage and other miscellaneous costs. A holding cost based solely on the cost of capital would be around 8%. We also included 14% as a number between these two extremes. Plywood gross margins in this industry range from 14% to 22%. We used margins of 14%, 18% and 22% to test the wood data. At the request of BlueLinx, their specific holding costs and profit margin are not specified, but do lie within the parameter ranges used. The commodity building product business is highly competitive and the company feels part of its competitive edge is the lack of public knowledge regarding its commodity sales. The parameter values are summarized in Table 2.3.

Table 2.3: Parameter Values for Simulation

Holding Cost % h	8%	14%	20%
Plywood Profit Margin ε	14%	18%	22%

Actual invoiced daily demands over four years for plywood were used. To create a new test case, the number of selling days in a month was calculated; subsequently, the same number of random draws with replacement was performed to make a new possible sales month for the item. Since every day had sales except weekends and holidays, we did not need to determine time between orders; we only needed to determine the quantity of each selling day per month. Note that ordering costs were omitted, as they are insignificant compared to the material purchase costs.

For comparison, five methods were used for each test data set: 1) no forward buy or safety stock purchased, 2) safety stock using the normal newsvendor ratio, 3) Golabi's (1985) method for forward buys, 4) Gavirneni's (2004) newsvendor method for safety stock, and 5) our proposed method, called GOGA, which has forward buys and the newsvendor model for safety stock.

2.5 Results

For the 100 simulation runs of the wood product, the profits for each of the parameter settings for the five methods are shown below in Table 2.4. Note that the four year profit has been adjusted so that the lowest profit method shows \$1000 and all other methods are scaled accordingly. This profit scaling was done at the request of BlueLinx

to ensure that neither volume nor profit of commodity products are communicated to competitors from this article.

Table 2.4: Comparison of Buying Methods for Wood

	$\epsilon = 14\%$		$\epsilon = 18\%$		$\epsilon = 22\%$		
h = 8%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,264	\$	1,294	\$	1,319	Newsvendor SS
	\$	1,287	\$	1,232	\$	1,198	Golabi
	\$	1,454	\$	1,432	\$	1,411	Gavirneni
	\$	1,564	\$	1,517	\$	1,508	GOGA
h = 14%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,207	\$	1,244	\$	1,269	Newsvendor SS
	\$	1,099	\$	1,080	\$	1,068	Golabi
	\$	1,318	\$	1,316	\$	1,327	Gavirneni
	\$	1,373	\$	1,367	\$	1,369	GOGA
h = 20%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,165	\$	1,204	\$	1,231	Newsvendor SS
	\$	1,000	\$	1,000	\$	1,000	Golabi
	\$	1,232	\$	1,253	\$	1,269	Gavirneni
	\$	1,232	\$	1,253	\$	1,269	GOGA

For the highest holding cost (20%), it was not optimal to ever forward buy.

Therefore, the GOGA and Gavirneni methods are equally effective. For lower holding costs, our GOGA method outperforms the other methods as shown in Table 2.5.

Table 2.5: Profit % improvement over no FB/no SS

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	29.2%	23.9%	43.2%	53.0%
h = 14%	24.0%	8.2%	32.0%	37.0%
h = 20%	20.0%	0.0%	25.1%	25.1%
$\epsilon = 14\%$	21.2%	12.9%	33.5%	39.0%
$\epsilon = 18\%$	24.7%	10.4%	33.4%	37.9%
$\epsilon = 22\%$	27.3%	8.9%	33.6%	38.2%

The results above demonstrate that, by taking into account demand uncertainty, the normal newsvendor equation and Gavirneni's modified newsvendor equation give additional profit beyond the no forward buying-no safety stock base-case. Forward

buying under Golabi's method is most beneficial under lower holding costs which allow more speculative stock to be purchased. However, at a holding cost of 20%, forward buying offers no benefit and Golabi's method's performs the same as the base-case method with no forward buying and no safety stock. Our GOGA method combines the modified newsvendor of Gavirneni to handle demand uncertainty and Golabi's view of price uncertainty to forward buy. When the distribution of future prices is known, this information can be exploited for forward buys along with the demand uncertainty, thereby making the GOGA method superior for commodity forward buys in comparison to existing methods. In Gavirneni's (1985) paper, he found results that were very close to optimal for three different distributions. His method does well with this real data as well, but not at the same level. A difference with the test data he used is that it had five price levels ranging from \$5 to \$25 which is a large swing in commodity prices, not present in the historical plywood data during the past 10 years. Another difference from his results that does not hold with our price data is the highly predictable spot price movements found in Random Walk, Mean Reverting, or Momentum distributions.

As a final test, we compute the optimal expected profit attainable for small datasets. We used uniformly distributed prices between \$10 and \$100, and uniformly distributed demand levels. Only a small number of periods were used since the number of combinations to compute grows quickly. Table 2.6 shows how close to optimal the GOGA heuristic achieved. In all cases tested, the expected profits from GOGA are at most off the optimal profits by less than 6%.

Table 2.6: GOGA Profit Compared to Optimal FB/SS

Periods	Demand Levels	Price Levels	% E[Optimal]
2	2	2	94.0%
2	3	3	96.0%
2	4	4	95.0%
2	5	5	96.0%
3	2	2	100.0%
3	3	3	100.0%
2	2	2	94.0%
3	2	2	100.0%
4	2	2	100.0%
5	2	2	100.0%

2.6 Conclusions and Discussion

Given the fluctuating prices of commodities, forward buys make sense. Even though wood and other commodities are non-perishable, the newsvendor formula can be applied to the demand distribution to balance profit and holding costs. Golabi's method should be used to determine the number of periods to forward buy given supplier lead times, current purchase price and expected future purchase prices. However, the quantity to procure should not be the point forecast but rather it should be based upon the distribution of demand. The distribution of demand should reflect more uncertainty through increasing prediction intervals. By utilizing Gavirneni's application of the newsvendor equation to the increasingly spread out demand distribution as part of our GOGA heuristic, we achieve better results overall than either Golabi or Gavirneni's methods achieve in isolation.

Forward buys as outlined in Golabi's method clearly increases profits over not doing so as shown in the Table 2.4 and Table 2.5. The modified newsvendor equation of Gavirneni provides advantages over buying no safety stock as shown in these same two

tables. Our GOGA method combines forward buys and safety stock based upon a modified newsvendor equation for maximum profit improvement. For plywood purchasing the GOGA method has the best expected profit overall. In Table 2.6, we showed that the GOGA heuristic achieves near optimal or optimal for smaller datasets. The real world demand and price data are much larger and less predictable, so the GOGA results when applied to the BlueLinx data cannot be said to be either close or far from optimal with any certainty.

The Director of Global Sourcing at BlueLinx has written that he is convinced by the simulation results that his company will be able to significantly improve the bottom line with this new method. He has formally request BlueLinx's IT resources to review options for developing programs utilizing these formulas into their ERP (Enterprise Resource Planning) system. The company believes that this more formal approach will improve their profitability on commodity buys but has requested that only the method be shared publicly, not their expected dollar profit improvement or volume of commodity sales as those are competitive secrets. An article in the Atlanta Journal-Constitution (AJC Online, 2002) on November 2, 2006 stated, "The slump in the housing industry is causing real pain, and not just among home builders...In a conference call with analysts, CEO Stephen E. Macadam said BlueLinx eliminated about 8 percent of its work force, including 175 salaried employees and 100 hourly workers." The timeline for the implementation of the GOGA method is now uncertain.

This new method will be programmed into the existing routines in the homegrown ERP system. A table needs to be added to store the historical commodity price data to allow future price forecasting via an autoregressive function. The role of adding and

ownership of the data in this new table will be assigned as additional duties to an employee in the purchasing department. Demand forecasting and order up to formulas currently exist in the ERP routines, but do require modifications to perform the new GOGA method. The change to the calculations is behind the scenes and will result in new quantities in the recommended purchase quantity field. Because product specialists already order the system recommended amount on a PO, no new training is required for existing procurement staff.

It is important to note that although this method was demonstrated on plywood for one particular company, it is applicable to any industry where purchases prices fluctuate from purchase period to purchase period. To model the long lead time vendor, we shift the Golabi period 0 out three months and use wider prediction intervals to reflect the increased demand uncertainty. Some real examples from companies in different industries are; Boeing's Commercial Aircraft Group requires titanium to build the frame for the cockpit. Potash Corporation buys natural gas to process the potash into dry, granular fertilizer. A plastic film converter faces highly variable prices of mill rolls of polystyrene from Dow because this plastic is petroleum based. RR Donnelly is faced with volatile paper prices from mills to print and bind books. Pepsi-Cola General bottlers must decide how much sugar to buy and when. As these few examples illustrate, for many products, and in many industries, fluctuating purchases prices are a normal part of business decision making. The GOGA method can be applied to all these situations for profit improvement.

2.7 Extension for Volume Price Discounts

We now demonstrate the calculations under volume price discounts. We again assume that the list price at any buying opportunity can be \$25, \$50, \$75 or \$100 – each with equal probability. Assume the holding cost for one period is \$5. We have the following threshold price breaks shown in Table 2.7.

Table 2.7: Quantity Discount Thresholds

Threshold	% Discount
250	10%
300	15%
350	20%

As an example, assume the forecasts for the next 6 months are as shown in Table 2.8 where 0 is the current month, 1 denotes one period in the future, etc.:

Table 2.8: Example Monthly Demand Forecasts

Period	0	1	2	3	4	5
Demand	150	110	105	90	175	190

To account for volume discounts and surcharges, we modify (1) as below, where Δd is the change in discount rate from the last threshold A_n .

$$A_{n+1} = \int_0^{\frac{A_n}{1-\Delta d}} x dF(x) + \int_{\frac{A_n}{1-\Delta d}}^{\infty} \frac{A_n}{1-\Delta d} dF(x) - h \quad (11)$$

Using (11) we can create new price thresholds as shown in Table 2.9, where *Cum. Dem* is the cumulative volume of demand, d is the percent discount or surcharge, and Δd is the change in percent discount from the last threshold.

Table 2.9: Forward Buying with Discounts

	A0	A1	A2	A3	A4	A5
	Current	Buy 1	Buy 2	Buy 3	Buy 4	Buy 5
	Period	Period	Period	Period	Period	Period
	Ahead	Ahead	Ahead	Ahead	Ahead	Ahead
	\$ 100.00	\$ 64.44	\$ 51.64	\$ 39.98	\$ 31.23	\$ 24.67
\$ 25.00	YES	YES	YES	YES	YES	
\$ 50.00	YES	YES	YES			
\$ 75.00	YES					
\$ 100.00	YES					
Cum. Demand	150	260	365	455	630	820
d	0%	10%	20%	20%	20%	20%
Δd	0%	10%	10%	0%	0%	0%

Now it is beneficial, with a current price of \$50, to buy 2 periods ahead compared with 1 period in the base case shown in Table 2. Notice that at a quantity of 300 we have a 15% discount, but also a 20% discount at 20%. Since the additional demand of 105 for period 2 takes the total to 365, we apply just the 20% discount for d , for a net change of 10% (20% - 10%) on the $A2$ row.

As expected, the price thresholds are higher starting in period 1 since we are eligible for a 10% discount off the entire purchase. Notice that even though there are no future price discounts for additional quantities starting in period 3, the price thresholds $A3$, $A4$ and $A5$ are all higher than in the base case. This is due to the lowered realized price threshold in period 0 as we cross price thresholds when buying ahead for period 1 and period 2.

As an extreme example of surcharges, the next example in Table 2.10 shows a 30% surcharge for purchases of 200 or more at one time.

Table 2.10: Forward Buying with Surcharge

	Calculated Value	\$ 25.00	\$ 50.00	\$ 75.00	\$ 100.00	Cum Dem.	d	Δd
A0	\$ 100.00	Yes	Yes	Yes	Yes	150	0%	0%
A1	\$ 43.08	Yes				260	10%	10%
A2	\$ 35.29	Yes				365	20%	10%
A3	\$ 27.72	Yes				455	20%	0%

Notice that the threshold to buy for period 1 has lowered significantly from the base case from \$57.50 to \$43.08. Again, all additional forward buys are lowered as well. In the base case it was possible to buy four periods ahead, while this surcharge example makes only three periods feasible (since the fourth period \$22.04 threshold is not possible given the lowest possible price of \$25 per unit). Now for a current price to buy of \$50, it does not make sense to buy beyond the current period. The current price would have to be \$43.08 or lower to recommend buying ahead one period.

Table 2.11: Parameter Levels

Holding Cost % h	8%	14%	20%		
Plywood profit margin ε	14%	18%	22%		
Price Discount	-1%	0%	1%	2%	5%

We used the actual invoiced daily demand for plywood over four years. To create a new test case, the number of selling days in a month were calculated; subsequently, the same number of random draws with replacement were performed to make a new possible sales month for the item. Since every day had sales except weekends and holidays, we did not need to determine time between orders; we only needed to determine the quantity of each selling day per month. Note that ordering costs were omitted, as they are insignificant compared to the material purchase costs.

Five methods were used for each test data set: 1) no forward buy or safety stock purchased, 2) safety stock using the normal newsvendor ratio, 3) Golabi's (1985) method for forward buys, 4) Gavirneni's (2004) newsvendor method for safety stock, and 5) our proposed method, called GOGA, which has forward buys and the newsvendor model for safety stock. Each run included no volume discount, 1% volume surcharge, 1% volume discount, 2% volume discount, and 5% volume discount.

2.7.1 Results

For the 100 simulation runs of the wood product, the profits for each of the parameter settings for the five methods are shown below in Table 2.12 for a zero percent discount (effectively, a no volume discount scenario). Note that the four year profit has been adjusted so that the lowest profit method shows \$1000 and all other methods are scaled accordingly.

Table 2.12: Comparison of Buying Methods for Wood

	$\epsilon = 14\%$		$\epsilon = 18\%$		$\epsilon = 22\%$		
h = 8%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,264	\$	1,294	\$	1,319	Newsvendor SS
	\$	1,287	\$	1,232	\$	1,198	Golabi
	\$	1,454	\$	1,432	\$	1,411	Gavirneni
	\$	1,564	\$	1,517	\$	1,508	GOGA
h = 18%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,207	\$	1,244	\$	1,269	Newsvendor SS
	\$	1,099	\$	1,080	\$	1,068	Golabi
	\$	1,318	\$	1,316	\$	1,327	Gavirneni
	\$	1,373	\$	1,367	\$	1,369	GOGA
h = 20%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,165	\$	1,204	\$	1,231	Newsvendor SS
	\$	1,000	\$	1,000	\$	1,000	Golabi
	\$	1,232	\$	1,253	\$	1,269	Gavirneni
	\$	1,232	\$	1,253	\$	1,269	GOGA

For the highest holding cost (20%), it was not optimal to ever forward buy. Therefore, the GOGA and Gavirneni methods are equally effective. For lower holding costs, the profit from our GOGA method was increased by using the forward buys from Golabi as one can see in Table 2.13 below.

Table 2.13: Profit % improvement over no FB/no SS

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	29.2%	23.9%	43.2%	53.0%
h = 18%	24.0%	8.2%	32.0%	37.0%
h = 20%	20.0%	0.0%	25.1%	25.1%
$\epsilon = 14\%$	21.2%	12.9%	33.5%	39.0%
$\epsilon = 18\%$	24.7%	10.4%	33.4%	37.9%
$\epsilon = 22\%$	27.3%	8.9%	33.6%	38.2%

The results above demonstrate that, by taking into account demand uncertainty, the normal newsvendor equation and Gavirneni's modified newsvendor equation give additional profit beyond no safety stock. Forward buying under Golabi is most beneficial when holding costs are lowest because these low holding costs allow more speculative stock to be purchased. However, at the holding cost of 20%, no forward buying was done and thus Golabi's methods are equal to the method with no forward buying and no safety stock. Our GOGA method combines the modified newsvendor of Gavirneni to handle demand uncertainty with Golabi's view of price uncertainty to forward buy. When the distribution of future prices is known, this information can be exploited for forward buys along with the demand uncertainty, thereby making the GOGA method superior for commodity forward buys in comparison to existing methods. This base case is developed and the results are discussed in detail in Manikas, Chang and Ferguson (2006).

We now examine price discounts for volume purchases. The threshold quantity was set to twice the average monthly invoiced sales over the four year data set from BlueLinx. This amount allows forward buying and safety stock buying to be even more advantageous over buying the mean forecast every period. Discounts are typically small for commodity purchases and they can be an immediate price break, year end rebate, or other more favorable terms. Since immediate price breaks and favorable terms are easily translated into a single percent discount off the current purchase price, we focus on those price breaks here. We first examine the case where, at a threshold, the price drops by 1%. If this threshold is met, all units purchased are at the discounted price. There are no additional tiers of further discounting. Table 2.14 below shows the improvement over no forward buying and no safety stock from the various methods with a 1% discount. The results of holding and margin combinations are listed in the appendix B.

Table 2.14: Profit % improvement over no FB/no SS with 1% Discount

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	30.3%	26.3%	41.9%	53.8%
h = 18%	24.9%	9.0%	31.2%	36.7%
h = 20%	20.9%	0.0%	24.7%	24.7%
$\epsilon = 14\%$	22.3%	14.2%	32.1%	38.8%
$\epsilon = 18\%$	25.7%	11.4%	32.7%	38.0%
$\epsilon = 22\%$	28.1%	9.7%	33.1%	38.4%

We also consider the effects of a 2% discount and a 5% discount to meet the threshold purchase quantity in one period as shown in tables 2.15 and 2.16 respectively.

Table 2.15: Profit % improvement over no FB/no SS with 2% Discount

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	31.3%	32.1%	43.9%	57.6%
h = 18%	25.8%	9.7%	32.2%	38.2%
h = 20%	21.7%	0.0%	22.7%	22.7%
$\epsilon = 14\%$	23.4%	16.8%	32.3%	40.0%
$\epsilon = 18\%$	26.6%	13.5%	33.0%	39.0%
$\epsilon = 22\%$	28.8%	11.5%	33.5%	39.5%

Table 2.16: Profit % improvement over no FB/no SS with 5% Discount

	News vendor	Golabi	Gavirneni	GOGA
h = 8%	34.3%	40.8%	50.0%	69.0%
h = 18%	28.6%	11.8%	38.3%	45.8%
h = 20%	24.3%	0.0%	26.8%	26.8%
$\epsilon = 14\%$	26.8%	21.4%	39.2%	49.6%
$\epsilon = 18\%$	29.3%	17.0%	38.3%	46.5%
$\epsilon = 22\%$	31.2%	14.2%	37.7%	45.6%

There is a possibility that exceeding a certain threshold will actually cause prices to increase. Given the typical high capacity nature of mills it may allocate a certain volume of stock to the buyer, and any excess will have to be met by withholding stock from another customer. It may also be the case that the excess quantity has to be bought from an alternative supply source, which is equivalent to a volume surcharge. In table 2.17, we show the results for a 1% surcharge for meeting the threshold.

Table 2.17: Profit % improvement over no FB/no SS with 1% Surcharge

	News vendor	Golabi	Gavirneni	GOGA
h = 8%	28.2%	21.5%	35.0%	51.5%
h = 18%	23.0%	7.5%	29.0%	33.0%
h = 20%	19.2%	0.0%	25.0%	25.0%
$\epsilon = 14\%$	20.1%	11.6%	27.8%	36.7%
$\epsilon = 18\%$	23.8%	9.4%	29.7%	36.3%
$\epsilon = 22\%$	26.5%	8.0%	31.5%	36.5%

At quantities less than half the average monthly demand, the methods all take advantage of discounts and the results trend toward the base case with no discount as shown in Table 2.13. Similarly, at thresholds greater than four times the average monthly demand, none of the methods trigger discounts and the results again trend toward the base case with no discount.

The GOGA method performs as well as or better than the other methods tested here for a wide range of profit margins and holding costs, even in the presence of volume price discounts or surcharges. In all situations, the new GOGA heuristic is likely to increase profits of plywood buying and selling.

2.7.2 Conclusions and Discussion

Given the fluctuating prices of commodities, forward buys and safety stock make sense in certain situations. Golabi's method works well to determine the number of periods to forward buy given current and expected future prices. However, the quantity to procure in the current period should not be the point forecast but rather it should include safety stock based upon the holding costs and price differential in the current period versus expected price. By utilizing both Gavirneni's application of the newsvendor equation to the normal distribution and Golabi's price thresholds for forward buys, our GOGA heuristic achieves better results overall than either of those methods alone achieves.

Forward buys as outlined in Golabi's method clearly increases profits over not doing so, as shown in the Tables 2.8-2.12. However, as holding costs increase and profit margins decrease, this benefit is reduced or nullified. The modified newsvendor equation of Gavirneni provides advantages over buying no safety stock. Tables 2.8-2.12 show that over the range of price discounts, holding costs, and profit margins tested, using Gavirneni's modified newsvendor formula is advantageous to total profit. Our GOGA method combines aspects of these two methods for maximum profit improvement.

2.8 Equation for the GOGA Heuristic

The maximum number of periods to forward buy j , is obtained from solving the equation below. Note that unlike the cost distribution in Golabi (1985), we are using the data note (Manikas, 2007) to predict the point cost C_j .

$$\text{Max } j : \left(\sum_{i=0}^j (C_j - C_0) - j * H \right) > 0 \quad j \geq 0 \quad (12)$$

Once the number of periods to forward buy is calculated, the order up to level y can be determined using the following equation:

$$y = \Phi^{-1} \left(\frac{P - C_0}{P - \bar{C} + H} \right) + \sum_{i=0}^j D_i \quad (13)$$

CHAPTER 3

PRACTICAL COORDINATION MECHANISMS FOR ASSEMBLE-TO-ORDER SUPPLY CHAINS

3.1 Introduction and Literature

An Assemble-To-Order (ATO) system is where an assembler packages a number of components from different suppliers into final products and sells them to end-customers. Such systems are very common in the business world and can be found in the fashion industry (e.g. perfume gift sets), in high technology manufacturing (e.g. computers and printers), and in the service industry (e.g. all-inclusive travel packages). Due to the presence of many stakeholders each with their localized objectives, these systems are very difficult to manage. Fugate et al. (2006) note that often in business each supply chain member attempts to myopically optimize without consideration of the full system. Maximum efficiency can be achieved if a central decision maker decides all quantities, but many participants are unwilling to give up control of their organizations. As a result, many of these systems are run in a decentralized manner and that results in a significant loss in overall efficiency. To overcome this inefficiency it is necessary to identify mechanisms that give the participants control over their local entity while at the same time enable them to make decisions that achieve centralized efficiency. Such mechanisms, when they can be clearly identified, are called supply chain coordination contracts (Cachon (2003)).

The inefficiency caused by decentralized control was observed by Pasternack (1985) and he proposed a coordination mechanism that provides partial credit for unsold

goods at the retailer. Since then a number of authors have identified other contracts that can achieve coordination in supply chains. Examples of these coordinating contracts are buy-back contracts (He et al. (2006)), revenue sharing contracts (Dana and Spier (2001), Cachon and Lariviere (2005)), and mid-term returns (Taylor (2001)). However, revenue sharing as a coordination mechanism has a downside because it allows a retailer to “cheat” (Wang et al. (2004)) or to give less sales effort (Cachon and Lariviere (2005)). All these contracts were developed with a two-stage serial system (similar to the one in Bollapragada et. al. (2004)) as the basic supply chain setting.

The literature on coordination contracts for ATO systems is rather sparse. Gerchak and Wang (2004) noticed that a revenue sharing contract does not coordinate a three member (2 suppliers, 1 retailer) ATO supply chain. They propose a subsidy mechanism by which the retailer helps the two suppliers with the excess inventory at their locations. In their system, the retailer does not face any uncertainty and is clearly the profit leader of the supply chain. As will become apparent later, that is not true in the setting we consider. Bernstein and DeCroix (2006) analyze a three player ATO supply chain in a multi-period setting, establish the effectiveness of a mechanism in which subsidies flow from the retailer to the suppliers, and transfer payments flow from the suppliers to the retailer. In our setting the subsidies can flow in either direction (i.e. from the retailer to a supplier or from a supplier to the retailer) or concurrently in both directions. Gurnani and Gerchak (2007) propose a penalty mechanism for each supplier proportional to the amount deficient to the needs of the assembler firm. We utilize a reward rather than penalty structure that allows each supplier to gain additional expected profit. In addition, our model allows the assembler to be rewarded; i.e. we do not *a priori*

assume that the power lies solely with either the retailer (assembler) or the supplier echelon.

Our paper extends the existing literature on coordination contracts for ATO supply chains in many aspects. We consider a generic n -supplier, 1-retailer supply chain while most of the existing ATO literature has focused on a 2-supplier, 1-retailer setting. We model the fact that the retailer also faces uncertainty and has to make a capacity acquisition decision at the same time when the suppliers are making their production decisions. With the retailer capacity acquired, assembly requires an insignificant lead time and can be initiated upon demand realization (as done in Wang and Gerchak (2003)). There is no presumed profit leader in our model and any of the suppliers or the retailer could be the profit leader. For this type of supply chain, in order to achieve coordination, we propose a *salvage manipulation* mechanism by which the lower risk participants support the higher risk participants by promising additional salvage value for their leftover inventories or capacity. We provide a simple computational mechanism to obtain the exact magnitudes of the salvage manipulation flows across the supply chain. We show that these supplemental salvage values can flow in either direction and in some cases in both directions. We extend our salvage manipulation options to include transfer pricing to prevent firms deviating from the optimal solution for the supply chain (as done in Cachon and Zipkin (1999)).

3.2 Model and Notation

We investigate a supply chain with a single retailer that assembles the finished good (kit) upon realized customer demand which is only known in distribution when the production decisions (at the suppliers) and the capacity acquisition decision (at the

retailer) are made at the beginning of the period. Components (without loss of generality one each) from n suppliers comprise the finished good. The product and the components in the supply chain have a single selling period and must be salvaged at a reduced price at the end of the season. While at first the single period assumption appears to be very restrictive, it is valid for many products that have a well-defined selling season (e.g. travel packages, holiday perfume sets, and any seasonal products) and whose production has significant lead-time enabling only a single production decision. We assume (as was done in Barnes-Schuster et al. (2002)) that a single product is sold to consumers at a fixed market price that is exogenously specified for a single selling season. Fixed prices are common for catalog goods where the price has to be published and is not easily altered during the selling season (Emmons and Gilbert (1998)). The retailer has to make a capacity acquisition decision at the beginning of the season (concurrent with when the suppliers are making their production decisions) and due to the demand uncertainty, the acquired capacity may not be enough or some of it may go unused.

3.2.1 Sequence of Events

The sequence of events for the supply chain members is as follows:

Before Selling Season:

1. The forecasted demand distribution is viewed by all members.
2. The retailer decides how much assembly capacity to buy and acquires it.

Concurrently with the retailer's decision, each supplier determines his or her production quantity, makes the components and stocks them.

During Selling Season:

3. Actual end customer demand x is realized at the retailer.

4. Retailer uses components from each supplier to assemble finished goods, sell them, collect revenue, and pays suppliers for the components used.
5. Salvage values, if any, are recovered at the suppliers and at the retailer.

3.2.2 Assumptions

We assume (as was done in much of existing literature (e.g. Rao et al. (2005)) that the production costs, selling prices, and salvage values for all the components and the final product are exogenously given and known to all participants in the supply chain. We also follow their assumption that the unmet demand is lost. The customer demand distribution parameters are known at the beginning of the season by the retailer and all suppliers. We further assume that, for each of the suppliers, the wholesale selling price to the retailer must be greater than the production cost to ensure selling is profitable. Additionally, the salvage value at each supplier must be less than his or her respective cost to produce the component or there is no penalty for over-production. We use the features of capacity decision, delayed final assembly, and no finished goods held at the retailer similar to that done in Anderson, Morrice and Lundeen (2006). The capacity acquisition cost and its salvage value at the retailer are known to all the participants. In addition, the salvage value must be less than the cost to acquire that capacity in order to induce a risk of over-acquisition by the retailer. The component suppliers own and manage their component inventory, and the retailer assembles components into finished goods as demand arrives. This is similar to the Vendor Managed Consignment Inventory (VMCI) strategy proposed by Fang et al. (2007). It is assumed, without loss of generality, that the final product requires one component from each of the n suppliers.

3.2.3 Notation

The following notation is used to model our problems.

i	Index, for the suppliers ($i=1, \dots, n$), and for the retailer ($i=0$)
x	Random variable for final product demand from end customers
μ	Mean of demand x
$f(x)$	probability density or mass function of demand x
$F(x)$	CDF of demand x
q_c	The centralized quantity
q_i	The make to stock quantity at supplier i ($i=1, \dots, n$), and the assembly capacity at the retailer ($i=0$)
m	The minimum quantity from all players, i.e. minimum of q_i , $i=0, \dots, n$
c_i	The cost to produce one unit at supplier i ($i=1, \dots, n$) and the cost to acquire one unit of assembly capacity at the retailer ($i=0$)
p	The selling price of the final product by the retailer
w_i	The selling price of one component from supplier i to the retailer
s_i	The salvage value for one unsold unit at supplier i ($i=1, \dots, n$) and the salvage value of one unit of unused capacity at the retailer ($i=0$)
$\gamma(q_c)$	The loss function (i.e. expected shortage) in the centralized control case if the demand x is more than the production quantity q_c , where $\gamma(q_c) = \int_{q_c}^{\infty} (x - q_c) f(x) dx$
$\gamma(q_i)$	The loss function for player i ($i=0, \dots, n$) in the decentralized control case
δ_i	The salvage manipulation from the retailer to supplier i ($i=1, \dots, n$)

Based on our discussion above, all the costs must be non-negative and satisfy the following conditions in order to make sense in a business context. Salvage values may be positive or negative (in the case of disposal costs).

$$p > w_i > c_i > 0 \quad (1)$$

The retailer must stand to make a profit above his capacity acquisition cost and the components' costs to be in the business.

$$p > c_0 + \sum_{i=1}^n w_i \quad (2)$$

3.3 Centralized and Decentralized Supply Chain Operation

In this section, we determine the optimal inventory control policies when this supply chain is operated under centralized control and then under decentralized control. First, we deal with the centralized decision making environment.

3.3.1 Centralized Control Case

Solving the centralized case enables us to determine the maximum expected supply chain profit that we use as a baseline for evaluating the performance of the decentralized supply chain. When a single entity owns or controls (as in vertical integration) all suppliers and the retailer/assembler, double marginalization (Spengler (1950)) can be eliminated. The company decides on a single quantity to stock at each supplier and the retailer also will acquire this amount of assembly capacity.

Proposition 3.1: The total supply chain profit is maximized when all players select the same quantity.

Proof: By contradiction. Assume that $\{q_0, q_1, \dots, q_n\}$ with $q_i \neq q_j$, for some pair i, j be an optimal solution. Let m be the minimum of $\{q_0, q_1, \dots, q_n\}$. Consider an alternate solution in which every player in the supply chain acquired only m units. The total revenue associated with the new solution would be no different from the revenue associated with the original solution since only m units of final product can be produced. However, the total cost associated with the new solution is lower than that associated with the original

solution where there are excess units for some players. Thus either (i) the solution in which every player orders m units is an alternate optimal solution; or (ii) the original solution was not optimal implying a contradiction. \square

The total expected centralized profit of the centralized supply chain, $E[\Pi_C]$, can be computed as follows:

$$E[\Pi_C] = -q_c \sum_{i=0}^n c_i + \int_{-\infty}^{q_c} \left(\sum_{i=0}^n s_i (q_c - x) + px \right) f(x) dx + pq_c \int_{q_c}^{\infty} f(x) dx \quad (3)$$

The terms in (3) above represent the component production costs, the capacity acquisition cost at the retailer, the capacity and component salvage value, the retailer revenue when demand is less than q_c , and the retailer revenue when demand is greater than or equal to q_c . We can rewrite (3) as:

$$E[\Pi_C] = -q_c \sum_{i=0}^n c_i + \sum_{i=0}^n s_i \left(\int_{-\infty}^{\infty} (q_c - x) f(x) dx + \int_{q_c}^{\infty} (x - q_c) f(x) dx \right) + p \left(\int_{-\infty}^{\infty} x f(x) dx - \int_{q_c}^{\infty} (x - q_c) f(x) dx \right) \quad (4)$$

Recall that μ is the average demand and $\gamma(q_c) = \int_{q_c}^{\infty} (x - q_c) f(x) dx$, and therefore, the total

expected supply chain profit becomes

$$E[\Pi_C] = \left(p - \sum_{i=0}^n s_i \right) \mu - q_c \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) - \gamma \left(p - \sum_{i=0}^n s_i \right) \quad (5)$$

We take its first derivative and set it to zero to give the critical fractile shown below, where q_c^* is the optimal order quantity for the centralized system and $F(q_c^*)$ is its corresponding CDF of the demand.

$$\frac{\partial E[\Pi_C]}{\partial q_c} = - \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) + (1 - F(q_c)) \left(p - \sum_{i=0}^n s_i \right) = 0 \quad (6)$$

Solving (6) gives the critical fractile:

$$F(q_c^*) = \frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \quad (7)$$

The ratio in (7) is always less than 100% given the conditions in (1). Figure 3.1 below shows an example of the centralized control supply chain.

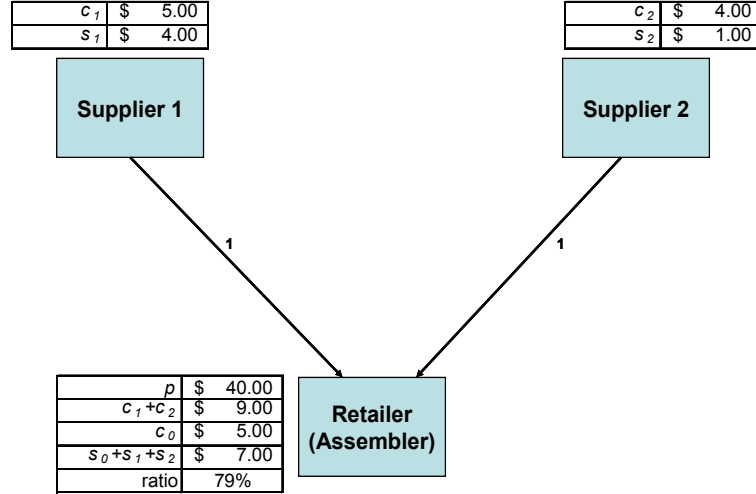


Figure 3.1: The Centralized Solution for an ATO with 1 Retailer and 2 Suppliers ($n=2$).

For this supply chain, if the demand is uniformly distributed between 0 and 99, the total expected profit will be \$1011.20 given the values for p , c_i , and s_i as shown in Figure 3.1.

In the expected profit function in (5), we use the subscript C to denote centralized control. Subsequently, we will use D to denote the decentralized case, and M to denote the salvage manipulation coordinated case. For a standardized uniform demand distribution (without loss of generality) over the range $[0, 1]$, we know the probability

density function is $f(x) = \frac{1}{1-0} = 1$ and $\mu = 1/2$. Using this probability density function

in (5) with the loss function $\gamma(q_c) = \frac{1}{2}(1 - q_c)^2$ gives the expression below.

$$E[\Pi_C] = \frac{1}{2} \left(p - \sum_{i=0}^n s_i \right) - q_c \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) - \left(p - \sum_{i=0}^n s_i \right) \frac{1}{2} (1 - q_c)^2 \quad (8)$$

For brevity, the equivalent expressions for exponential and normal demands have been omitted. We provide the profit function for the uniform distribution here because that is the distribution we use in most of our examples. However, it is worth noting that all our results are established without any assumption on the demand distribution.

3.3.2 Decentralized Case

We now look at the case where all the suppliers and the retailer act independently in an attempt to maximize their local profits without implicit or explicit agreements amongst them. The retailer's objective is to maximize his profit (denoted by the following equation) by selecting the capacity to acquire (q_0) with known values for all other variables except demand (assuming $q_i \geq q_0, i = 1, \dots, n$).

$$E[\Pi_0] = -q_0 c_0 + \left(p - \sum_{i=1}^n w_i \right) \int_{-\infty}^{q_0} x f(x) dx + \left(p - \sum_{i=1}^n w_i \right) q_0 \int_{q_0}^{\infty} f(x) dx + s_0 \int_{-\infty}^{q_0} (q_0 - x) f(x) dx \quad (9)$$

Taking the first derivative of the above profit function with respect to q_0 and setting it equal to zero allows us to solve for the critical fractile and hence the optimal capacity q_0^* to maximize his expected profit.

$$\frac{\partial E[\Pi_0]}{\partial q_0} = -c_0 + \left(p - \sum_{i=1}^n w_i \right) (1 - F(q_0)) + s_0 F(q_0) = 0 \quad (10)$$

Solving (10) gives the critical fractile:

$$F(q_0^*) = \frac{p - \sum_{i=1}^n w_i - c_0}{p - \sum_{i=1}^n w_i - s_0} \quad (11)$$

Proposition 3.2: The centralized critical fractile is greater than the decentralized

retailer's local critical fractile if $\sum_{i=1}^n s_i (c_0 - p) + \sum_{i=1}^n w_i \sum_{i=0}^n s_i < \sum_{i=1}^n c_i (s_0 - p) + \sum_{i=1}^n w_i \sum_{i=0}^n c_i$.

Proof: It is easy to prove the condition by setting (7)>(11) given the assumptions in (1) hold. □

For the decentralized supply chain, the suppliers need to decide on their production quantity prior to the selling season for the retailer. The retailer will only pay for the units it needs once the demand is realized. The supplier may get a salvage value ($s_i < c_i$) for each unsold unit at the end of the selling season. Assuming that the retailer has sufficient capacity to use any quantity that each supplier produces, the supplier i 's expected profit is shown below (assuming $q_j \geq q_i, i \neq j$):

$$E[\Pi_i] = -q_i c_i + w_i \int_{-\infty}^{q_i} x f(x) dx + s_i \int_{-\infty}^{q_i} (q_i - x) f(x) dx + w_i q_i \int_{q_i}^{\infty} f(x) dx \quad (12)$$

Again, taking the first derivative of (12) above with respect to q_i and setting it equal to zero allows us to solve for the critical fractile and hence the optimal production quantity q_i^* for each supplier in the decentralized situation.

$$\frac{\partial E[\Pi_i]}{\partial q_i} = -c_i + s_i F(q_i) + w_i (1 - F(q_i)) = 0 \quad (13)$$

Solving (13) gives the critical fractile:

$$F(q_i^*) = \frac{w_i - c_i}{w_i - s_i} \quad (14)$$

Similarly, the ratio in (14) is always less than 100% given the conditions in (1). However, if c_i is close enough to s_i , the ratio will be close to 100%. The ratio in (14) may be more or less than the ratio in (7). That is, depending on the values of the parameters, each supplier may have a higher or lower ratio or production quantity under decentralized decision making than under centralized control. The following proposition shows the

condition that the decentralized supplier's critical fractile is greater than the centralized supplier's critical fractile.

Proposition 3.3: Supplier i 's local critical fractile is higher than the centralized critical

fractile if $(w_i - c_i) \left(p - \sum_{i=1}^n s_i + s_0 \right) > (w_i - s_i) \left(p - \sum_{i=1}^n c_i + c_0 \right)$.

Proof: Again the proof is straightforward by setting (14)>(7) given the assumptions in (1) hold. \square

Provided that the retailer makes its quantity decision q_0^* based on (11) and each supplier makes its quantity decision q_i^* based on (14), the final quantity of finished product the retailer can make will be the minimum of those, denoted as m . It is worth mentioning here that we also analyzed the case in which the participants did not know the costs present at the other participants. The performance of the decentralized supply chain is much worse in that setting and as a result, the benefits of our proposed mechanism will be much more pronounced. By making an assumption of common knowledge of cost parameters, we are only estimating a lower bound on the performance of the salvage manipulation mechanism. This lower bound of the total expected profit for the decentralized supply chain can be expressed as follows:

$$E[\Pi_D] = -\sum_{i=0}^n mc_i + p \int_{-\infty}^m xf(x)dx + pm \int_m^{\infty} f(x)dx + \sum_{i=0}^n s_i \left(\int_{-\infty}^m (m-x)f(x)dx + \int_m^{\infty} (x-m)f(x)dx \right) \quad (15)$$

The total expected decentralized supply chain profit can be rewritten as below, where

$$\gamma(m) = \int_m^{\infty} (x-m)f(x)dx.$$

$$E[\Pi_D] = \left(p - \sum_{i=0}^n s_i \right) \mu - m \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) - \gamma \left(p - \sum_{i=0}^n s_i \right) \quad (16)$$

We will now find the expected supply chain profit for the demand distribution that is uniform $[0, 1]$, $\mu = \frac{1}{2}$, $\gamma(m) = \frac{1}{2}(1-m)^2$. The profit equation becomes:

$$E[\Pi_D] = \frac{1}{2} \left(p - \sum_{i=0}^n s_i \right) - m \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) - \frac{1}{2} \left(p - \sum_{i=0}^n s_i \right) (1-m)^2 \quad (17)$$

Figure 3.2 shows the same supply chain as in Figure 3.1, but under decentralized decision-making. Even though Supplier 1 has a ratio of 88%, no more components can be sold than the quantity corresponding to 40% on the CDF of demand since Supplier 1's components must be mated with those of Supplier 2.

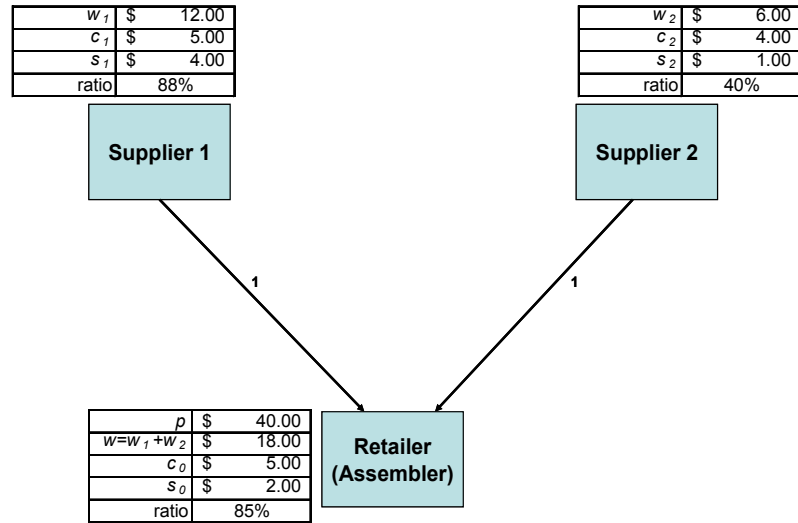


Figure 3.2: The Representative Supply Chain Under Decentralized Control.

Given uniform demand $[0,99]$, using (16) gives $E[\Pi_D]=\$769.40$. This expected profit is under the assumption that each player in the supply chain is rational and will maximize his/her expected profit with full knowledge of the other player's information (as done in Cachon and Lariviere (2001)).

However, if the cost information regarding other players was not known or not utilized during decentralized quantity decision-making, the $E[\Pi_D]=\$586.40$ from (9) and (12) where $q_i \neq m$ for every i . The loss of total supply chain profit is the value of

information. Using all known information, the profit is better than ignoring information, however it is still less than the expected profit under centralized control.

Proposition 3.4: $E[\Pi_D] \leq E[\Pi_C]$.

Proof: We can prove this in two steps:

(i). There is at least one critical ratio in (11) or (14) less than or equal to the critical ratio in (7). This can then be translated to $m \leq q_c$.

(ii). The profit functions in (5) and (17) are concave.

We can prove (i) by contradiction.

Let us assume that all ratios in (11) and (14) are greater than that in (7). Let $a_0, a_1, \dots, a_n, b_0, b_1, \dots, b_n$ and x and y be positive numbers. If $a_i/b_i > x/y$ for $i=0, \dots, n$, it is relatively straightforward to prove that $(a_0+a_1+\dots+a_n)/(b_0+b_1+\dots+b_n) > x/y$ by simple algebraic manipulations. However, if we perform similar operation for all numerators and denominators in (11) and (14) for all $i=0, \dots, n$, it will be exactly equivalent to the ratio in (7). That is a direct violation of $(a_0+a_1+\dots+a_n)/(b_0+b_1+\dots+b_n) > x/y$. Hence, we can not have all ratios in (11) and (14) are greater than that in (7). That is, there is at least one $q_i \leq q_c$, and therefore $m \leq q_c$.

For (ii), we can take the second derivative in (6) and prove that it is less than 0.

Therefore, both the profit functions in (5) and (17) are concave.

Combining (i) and (ii), we can conclude that $E[\Pi_D] \leq E[\Pi_C]$.

□

3.4 Coordinated Decentralized Supply Chain

It is not realistic to assume that all players will be owned and controlled by one entity. It is most commonly the case that they remain decentralized decision makers each

with their local profit function. However, the decentralized expected profit has been shown to be no more than the expected profit under centralized control. It would be desirable to have a mechanism that enables all the decision makers to optimize their local profit functions and yet make decisions that lead to globalized efficiency. This is the concept of supply chain coordination and we achieve that here with a method we call salvage manipulation.

The inefficiency in the system is mainly due to the disparity of different profitabilities associated with each of the participants. If all of them had the same profitability, the localized critical fractiles would be equal to the globalized critical fractile and the decentralized supply chain would have the same total profit as the centralized supply chain. One way to overcome this disparity would be for the more profitable participants to support the less profitable ones and do so in a way that enables every one in the supply chain to order exactly the same quantity as in the centralized solution. We achieve this by proposing a salvage manipulation scheme in which the more profitable participants promise to help with salvage of leftover (if any) inventories or capacity at the less profitable participants. Let us denote by δ_i the additional (to the salvage value that was already available to him/her) salvage value that the retailer promises to supplier i for the leftover inventory at his/her location. Notice that δ_i can be either positive or negative. If it is positive, that means the retailer is more profitable than supplier i and if it is negative, it means that supplier i is more profitable than the retailer. For the coordinated case, the expected profit for supplier i is:

$$E[\Pi_i] = -q_i c_i + w_i \int_{-\infty}^{q_i} x f(x) dx + (s_i + \delta_i) \int_{-\infty}^{q_i} (q_i - x) f(x) dx + w_i q_i \int_{q_i}^{\infty} f(x) dx \quad (18)$$

Taking the first derivative with respect to q_i and setting it equal to zero gives:

$$\frac{\partial E[\Pi_i]}{\partial q_i} = -c_i + (s_i + \delta_i)F(q_i) + w_i(1 - F(q_i)) = 0 \quad (19)$$

Solving (19) for the supplier i 's fractile and setting it equal to the centralized fractile in (7), we can find the salvage manipulation (δ_i) for supplier i from the following equality.

$$\frac{w_i - c_i}{w_i - s_i - \delta_i} = \frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \quad (20)$$

The expected profit for the retailer is:

$$E[\Pi_0] = -mc_0 + \left(p - \sum_{i=1}^n w_i \right) \int_{-\infty}^m xf(x)dx + \left(s_0 - \sum_{i=1}^n \delta_i \right) \int_{-\infty}^m (m-x)f(x)dx + \left(p - \sum_{i=1}^n w_i \right) m \int_m^{\infty} f(x)dx \quad (21)$$

Taking the first derivative of (21) with respect to m and setting it to zero gives:

$$\frac{\partial E[\Pi_0]}{\partial m} = -c_0 + \left(p - \sum_{i=1}^n w_i \right) (1 - F(m)) + \sum_{i=1}^n \delta_i (1 - F(m)) + s_0 F(m) = 0 \quad (22)$$

Solving the critical ratio in (22) and setting it equal to the critical ratio from the centralized control case as the right hand side of the formulation, we get the following equality.

$$\frac{p - \sum_{i=1}^n w_i - c_0}{p - \sum_{i=1}^n w_i + \sum_{i=1}^n \delta_i - s_0} = \frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \quad (23)$$

The retailer has an additional overage cost of $\sum_{i=1}^n \delta_i$. If the sum of salvage manipulations

is negative, the retailer is being offered the salvage manipulation against his capacity cost c_0 by one or more of the suppliers.

Solving n equation (20) allows us to calculate the salvage manipulation that the retailer promises to each supplier i :

$$\delta_i = \frac{-c_i p - s_i \sum_{i=0}^n c_i + p s_i + c_i \sum_{i=0}^n s_i + w_i \sum_{i=0}^n c_i - w_i \sum_{i=0}^n s_i}{\sum_{i=0}^n c_i - p} \quad (24)$$

According to conditions in (1) and (2), the denominator of the above equation will be non-zero. Therefore, we are assured that the salvage manipulation will never be undefined. In addition, this salvage manipulation calculation is independent of the demand distribution.

Proposition 3.5: If the δ_i 's are chosen such that the retailer and all the suppliers have the same critical ratio, then that common ratio is equal to the centralized ratio.

Proof: Let δ_i 's be defined such that for some θ ,

$$\frac{w_i - c_i}{w_i - s_i - \delta_i} = \theta \text{ for } i=1, \dots, n, \text{ and} \quad (25)$$

$$\frac{p - \sum_{i=1}^n w_i - c_0}{p - \sum_{i=1}^n w_i + \sum_{i=1}^n \delta_i - s_0} = \theta \quad (26)$$

$$\text{From (25) above, we get } w_i - c_i = \theta w_i - \theta s_i - \theta \delta_i \quad (27)$$

Summing up over i , we get the expression for the n suppliers:

$$\sum_{i=1}^n w_i - \sum_{i=1}^n c_i = \theta \sum_{i=1}^n w_i - \theta \sum_{i=1}^n s_i - \theta \sum_{i=1}^n \delta_i \quad (28)$$

From (26) above, we get

$$p - \sum_{i=1}^n w_i - c_0 = \theta p - \theta \sum_{i=1}^n w_i + \theta \sum_{i=1}^n \delta_i - \theta s_0 \quad (29)$$

Adding (28) and (29) on both sides we get

$$p - \sum_{i=0}^n c_i = \theta p - \theta \sum_{i=0}^n s_i \quad (30)$$

Leading us to :

$$\frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} = \theta \text{ which means that } \theta \text{ must be the centralized ratio.}$$

□

The total expected supply chain profit is:

$$E[\Pi_M] = \left(p - \sum_{i=0}^n s_i \right) \mu - m \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) - \gamma \left(p - \sum_{i=0}^n s_i \right) \quad (31)$$

Similar to the decentralized control case in section 3.2, the expected total supply chain profit for normal, uniform and exponential can be determined. The profit under coordination in (31) is identical to that of (5) in the decentralized case with $m = q_c$.

Proposition 3.6: $E[\Pi_M] = E[\Pi_C]$. The proposed salvage manipulation mechanism coordinates the supply chain and improves the overall expected profit.

Proof: As proved in Proposition 3.5, the common ratio after the salvage manipulation is equal to the centralized ratio in (7). That implies that $m = q_c$ and the expected profit in (31) is equal to that in (5). □

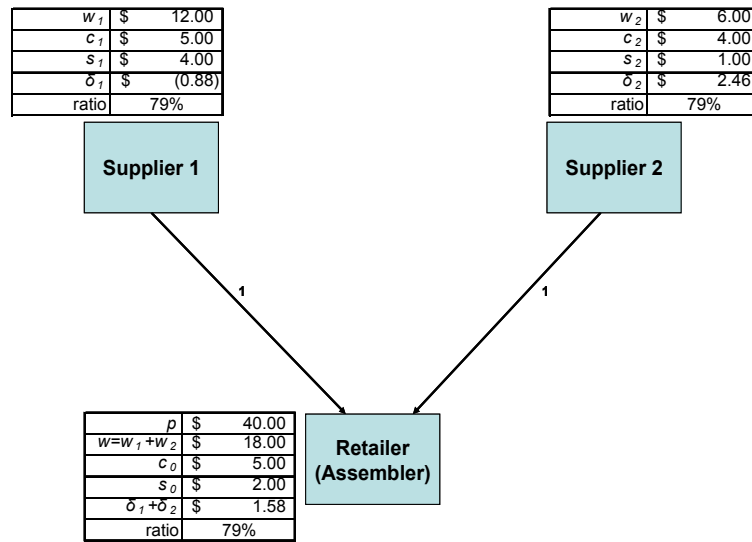


Figure 3.3: Results of Salvage Manipulation for the Previous Supply Chain.

Figure 3.3 illustrates a salvage manipulation result for the previous example. The manipulations are obtained from (24). Given uniform demand $[0,99]$, $E[\Pi_M] = \$1011.20$ from (31) using salvage manipulations and it is identical to the profit under centralized control. Recall that in Figure 3.2, the retailer had a ratio of 85%, supplier 1 had a ratio of 88% and supplier 2 had a ratio of 40%. To coordinate this supply chain for maximum efficiency, each player needs to select the same ratio as under the centralized control case. Since supplier 1 has a ratio higher than the centralized ratio, he offers salvage manipulation of \$0.88 to the retailer for any unsold capacity. Supplier 2 originally had a ratio of 40%, and therefore the retailer offers supplier 2 a salvage manipulation of \$2.46 to offset the cost of his leftover units. Notice that the retailer in this example is receiving salvage manipulation as well as giving salvage manipulation. The retailer's net salvage manipulation is a promise to pay out \$1.58 to the suppliers for every unit of leftover inventory.

We can examine the salvage manipulation equation for each retailer-supplier relationship to determine what happens to the amount of salvage manipulation flow to

supplier i when the parameters change. The effects that parameters have on the salvage manipulation from the retailer to a supplier are shown in Table 3.1. For example, δ_i will increase when retailer final price p increases or supplier component production cost c_i increases; δ_i will decrease when supplier selling price w_i increases or supplier salvage value s_i increases.

Table 3.1: Parameter Effects on a Salvage Manipulation

Parameter		δ_i	Comments
Retail price (p)	↑	↑	
Capacity cost (c_0)	↑	↓	
Capacity salvage (s_0)	↑	↑	
Component manufacturing cost (c_i)	↑	↑	$i=1, \dots, n$
Sum of components costs ($\sum c_i$)	↑	↓	$i=1, \dots, n$, not including c_i
Wholesale price (w_i)	↑	↓	$i=1, \dots, n$
Component salvage (s_i)	↑	↓	$i=1, \dots, n$
Sum of component salvage ($\sum s_i$)	↑	↑	$i=1, \dots, n$, not including s_i

Recall that a negative salvage manipulation δ_i signifies that a promise is made by supplier i against unused capacity at the retailer, while a positive value denotes that a promise is made by the retailer toward unsold inventory at supplier i .

3.4.1 Participation Options

We have proved that the salvage manipulation can enhance and coordinate the overall expected profit for the entire decentralized supply chain. However, there are four options for firms in the supply chain to participate in the salvage manipulation mechanism to ensure that all players will do at least as well as they would under the decentralized case. These options are described and discussed below.

1) “Minimum”Option - No Salvage Manipulation:

This option is where the players are not willing to participate in coordination. There are no practical implementation issues with this option since it is the default decentralized control result. We now determine conditions under which this may happen.

Define y as the difference between the centralized quantity and a decentralized quantity:

$$y = F^{-1} \left(\frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \right) - m = q_c^* - m \quad (32)$$

The following proposition describes the condition when a participant will gain benefit by using the salvage manipulation.

Proposition 3.7: **As compared to non-coordinated decentralized case,** the retailer's expected profit increases via the salvage manipulation if:

$$-yc_0 + \left(p - \sum_{i=1}^n w_i \right) \int_m^{q_c} xf(x)dx - \left(p - \sum_{i=1}^n w_i \right) m \int_m^{q_c} f(x)dx + (s_0 - \sum_{i=1}^n \delta_i) \int_m^{q_c} (q_c - x)f(x)dx > 0 \quad (33)$$

and supplier i 's expected profit increases via salvage manipulation if:

$$-yc_i + w_i \int_m^{q_c} xf(x)dx - w_i m \int_m^{q_c} f(x)dx + (s_i + \delta_i) \int_m^{q_c} (q_c - x)f(x)dx > 0 \quad (34)$$

Proof: The left-hand-side (LHS) of (21) will increase as long as (33) holds. Similarly, the LHS of (18) will increase as long as (34) holds. \square

When one or more of these inequalities are not satisfied, there could be a subset of supply chain participants who would be unwilling to sacrifice their expected profits in order to increase total expected supply chain profit. In such a case, the rational choice is not to use salvage manipulation and everyone realizes the same expected profit as in the decentralized setting. When one player's critical ratio is at an extreme value, this scenario can arise. Table 3.2 shows the situation where the retailer, a department store, has very low risk (high ratio) compared to the risks of the suppliers. The perfume manufacturer and body lotion supplier both own the items and assume all risk for unsold/unused items. It takes a bottle of perfume and one body lotion to make a bundle

for the customer. Assume that $p=400$, $c_0=5$, $w_1=150$, $c_1=40$, $w_2=70$, $c_2=20$ and all other parameters are zero. In this situation, the department store may not sacrifice its profit by offering the full salvage manipulation.

Table 3.2: Minimum Participation as the Only Option for the Retailer.

	Centralized		Minimum	Full
	Ratio	Profit	Profit	Profit
Dept. Store	84%	\$ 13,860.00	\$ 7,824.20	\$ 7,240.30
Perfume			\$ 3,976.00	\$ 4,551.04
Body lotion			\$ 1,760.80	\$ 2,068.66
		\$ 13,860.00	\$ 13,561.00	\$ 13,860.00

2) **“Full” Option - Everyone is better off:**

With this option, each player uses the full salvage manipulation to achieve the optimal total expected supply chain profits. For n suppliers, n salvage manipulation agreements are constructed. If salvage manipulation increases the expected profit of all players while coordinating the supply chain this option makes perfect sense. There are a wide range of parameters where this occurs. Table 3.3 below uses the scenario depicted in Figure 3.3 to show the centralized control option versus the Full and Minimum options. In this case the full salvage manipulation benefits all participants.

Table 3.3: Comparison of Centralized versus Minimum and Full Option Profits.

	Centralized		Minimum	Full
	Ratio	Profit	Profit	Profit
Dept. Store	79%	\$ 1,011.20	\$ 516.00	\$ 661.17
Perfume			\$ 214.40	\$ 272.25
Body lotion			\$ 39.00	\$ 77.78
		\$ 1,011.20	\$ 769.40	\$ 1,011.20

3) **“Partial” Option - Use Partial Salvage Manipulation:**

With this option, the players may choose to use partial manipulation to achieve coordination effect. When one or more of the inequalities in proposition 3.7 are not satisfied, it may be possible to identify a salvage manipulation mechanism that achieves

partial coordination without diminishing the profit of any firm. Unfortunately, there is no shortcut to compute the magnitudes of salvage manipulation. Rather, an appropriate method to find the optimal salvage manipulation is via line search starting at $m+1$ and going up to q_c in reasonable increments.

Figure 3.4 below shows that the department store can increase his profit via the Partial option. The department store is willing to participate in partial salvage manipulation to make the perfume supplier's ratio between 51% and 57% because the department store would be no worse off than the Minimum option in this range. The department store maximizes his expected profit by promising salvage manipulation to where the perfume supplier's ratio is 55% (marked by the vertical dashed line in Figure 3.4 below). This situation is probably more suitable when there is a “power retailer” (Raju and Zhang (2005)).

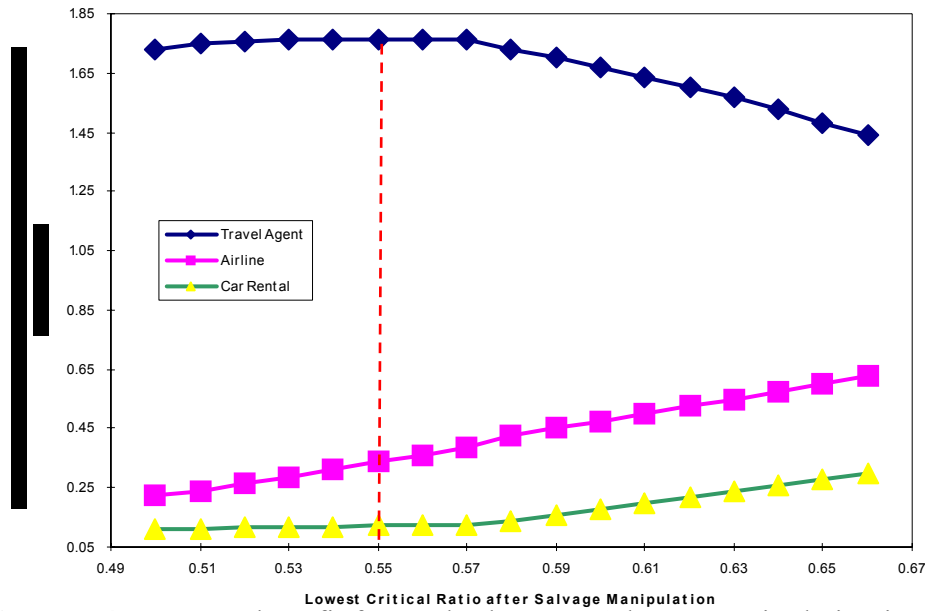


Figure 3.4: Expected Profit for Each Player as Salvage Manipulation is Applied.

It must be stressed that although the Partial option is always to the benefit of any player that had a decentralized ratio above the centralized ratio, the simplicity of using the Full

option can outweigh the minor profit improvement for the single player. Furthermore, only under the Full option is the total supply chain profit equivalent to the centralized supply chain profit. Figure 3.5 below shows the critical ratios for each player as salvage manipulation is applied to achieve the expected profit curves shown in Figure 3.4.

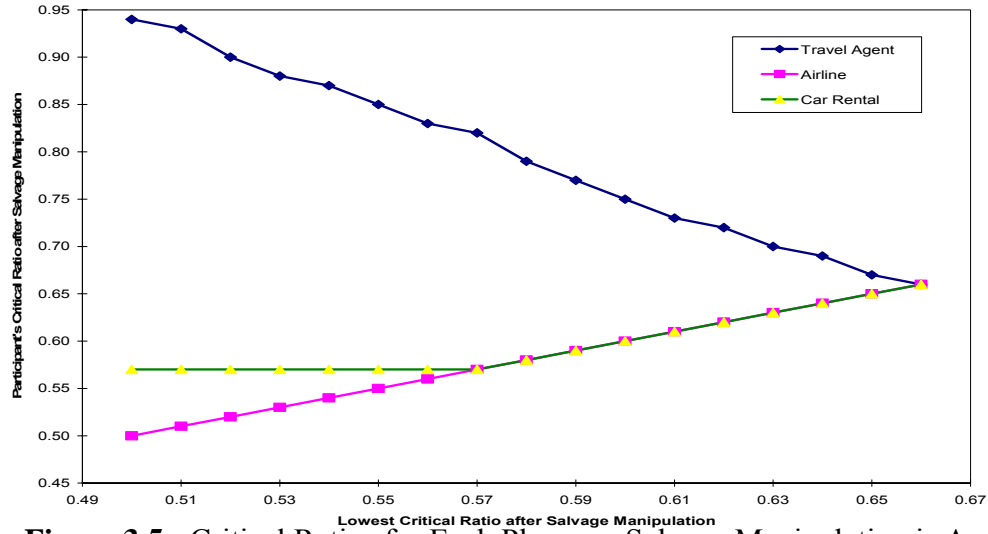


Figure 3.5: Critical Ratios for Each Player as Salvage Manipulation is Applied.

The low risk participant can choose any point along the x-axis as long as his/her profits are not hurt. In this example, it so happens that he/she will choose the lowest critical ratio of 0.55 as shown in Figure 3.4. In fact, we can show that the low-risk player never maximizes his/her profits by choosing the salvage manipulation that sets the minimum critical ratio to the centralized critical ratio. The following proposition describes this situation.

Proposition 3.8: The expected profit of the highest ratio player is maximized at a quantity less than the centralized control quantity when salvage manipulation is promised to lower ratio players.

Proof: Either (i) the quantity corresponding to the centralized critical fractile corresponds to the optimal profit for the highest ratio player providing salvage manipulation; or (ii) a

quantity corresponding to a fractile smaller than the centralized control fractile maximizes expected profit for the highest ratio player. Since the expected profit function is concave, the slope must be zero at its maximum point. We examine the slope at the quantity corresponding to the centralized control ratio where (i) the retailer has the highest ratio, (ii) one or more suppliers has the highest ratio.

For (i), a power retailer provides salvage manipulation to the supplier(s). By definition

$\sum_{i=1}^n \delta_i > 0$ if the retailer has a localized critical fractile higher than the centralized control

critical fractile. Substituting optimal centralized critical ratio from (7) into the coordinated retailer's first order equation in (23) gives (35) below:

$$Slope = -c_0 + \left(p - \sum_{i=1}^n w_i \right) \left(\frac{p - \sum_{i=0}^n c_i}{\sum_{i=0}^n s_i - p} \right) + \sum_{i=1}^n \delta_i \left(\frac{p - \sum_{i=0}^n c_i}{\sum_{i=0}^n s_i - p} \right) + s_0 \left(\frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \right) \quad (35)$$

The first term ($-c_0$) dominates the final term since $c_0 > s_0$ and the ratio in the final bracket is at most 1. The middle terms are composed of a positive term multiplied by a negative term. Therefore, the slope of the retailer's expected profit function is negative at the quantity corresponding to the centralized control ratio. There must exist a ratio smaller than the centralized ratio where the retailer's expected function is maximized.

(ii) Let there be a single supplier (without loss of generality) with the highest ratio denoted as supplier i . Substituting (7) into the coordinated supplier's first order equation in (19) gives (36) below:

$$slope = -c_i + s_i \left(\frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \right) + \delta_i \left(\frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \right) + w_i \left(\frac{p - \sum_{i=0}^n c_i}{\sum_{i=0}^n s_i - p} \right) \quad (36)$$

The first term is clearly negative. Furthermore, since $c_i > s_i$ and the critical ratio in brackets is at most 1, we know that the first term dominates the second term. By definition, δ_i is negative when the supplier has the highest ratio. w_i is positive, but it is multiplied by a negative term in the final bracket. We know that the slope equation shown in (36) is always negative. Therefore, the supplier with the highest local ratio will always have a maximum expected profit at a quantity corresponding to a ratio less than the centralized control ratio. \square

We simulated 10,000 problems where the parameters $p, c_0, c_1, c_2, w_1, w_2, s_0, s_1$, and s_2 were randomly generated ensuring that the retailer's ratio was higher than the centralized ratio (power retailer). Specifically, salvage value was $\sim U(0, 9)$ (Uniform from 0 to 9), component cost was $\sim U(1+s_i, 51+s_i)$, component price was $\sim U(1+c_i, 51+s_i)$. Retailer capacity salvage was $\sim U(0, 4)$ and capacity cost was $\sim U(1+s_0, 11+s_0)$. Selling price was $\sim U(1+c_0+w_1+w_2, 101+c_0+w_1+w_2)$. Demand was distributed uniformly from 0 to 99. For the retailer, using Full salvage manipulation achieved 93.34% of the optimal profit under Minimum or Partial options for each test case. If the Minimum option is not the best option, the Full option achieved 94.46% of the profits as under the Partial option. Note that the Minimum option was the best option in 19.89% of the cases.

4) “Transfer Payment” option – Use post coordination side payments:

With this option, transfer payments will be made in order to take advantage of coordination via the full salvage manipulation. If transfer payment agreements can be made such that all players are no worse off than using either the Minimum or Partial option, the Full option can be used to achieve this coordination. An increase or decrease in realized profit would be split amongst the players, effectively sharing both risk and

reward. This option can be illustrated by modifying our department store problem parameters. Let $p=250$, $c_0=5$, $s_0=0$, $w_1=100$, $c_1=50$, $s_1=0$, $w_2=70$, $c_2=30$, $s_2=0$. Therefore, the salvage manipulations needed to fully coordinated the supply chain are $\delta_1=24.24$, $\delta_2=9.39$. There are infinite combinations of percentage splits among players for the Transfer Payment option. An equitable split would be to agree on splitting the realized profit according the percentage of expected profit for each player under the decentralized setting. This option requires two contracts per supplier/retailer relationship and therefore is more complicated in practice. In addition to salvage manipulation, players may provide a side payment according to prearranged profit margin values (e.g. split in relation to the share of expected profit under the Minimum option).

Table 3.4 below shows the expected profits under the Minimum, Full, Partial, and Transfer Payment options. In this example, we chose to have the perfume and body lotion suppliers make up the expected profit difference (coordinated minus partial option) for the department store in proportion to their respective percentage profit improvements from Full coordination. A transfer payment agreement will work provided that no player is worse off after transfer payments than if they did not participate.

Table 3.4: Comparison of the Participation Options.

	Centralized		Minimum	Full	Partial (55%)	Transfer Payment	
	Ratio	Profit	Profit	Profit	Profit	Amount	Profit
Dept. Store	66%	\$ 5,362.50	\$ 2,730.00	\$ 2,437.50	\$ 2,764.63	\$ 327.13	\$ 2,764.63
Perfume			\$ 1,225.00	\$ 1,625.00	\$ 1,312.27	\$(215.91)	\$ 1,409.09
Body lotion			\$ 1,107.50	\$ 1,300.00	\$ 1,120.50	\$(111.22)	\$ 1,188.78
		\$ 5,362.50	\$ 5,062.50	\$ 5,362.50	\$ 5,197.40	\$ 0.00	\$ 5,362.50

3.5 Conclusion and Discussion

Centralized control of an ATO supply chain results in the highest overall expected profit for the n -supplier and one assembler supply chain. However, it is typically not

practical or not feasible to have a single company that owns or controls the entire supply chain. Therefore, a decentralized supply chain is the business setting in which the retailer and suppliers have to operate. With each player acting independently, we have shown that the expected profits of the total supply chain may be reduced due to asymmetries in the critical ratios amongst the players even with perfect information availability.

We have introduced a new coordinating mechanism called salvage manipulation that allows the supply chain to obtain the same expected profit as under centralized control. Salvage manipulation is a promise from the low risk participants to offset costs of overage at the high risk members, but do not result in cash flow if demand is not less than the decision quantity. It is possible that a full application of salvage manipulation makes one or more players worse off than under decentralized control. If this is the case, players may choose partial salvage manipulation, decentralized quantities (Minimum option), or a contract involving transfer payments. These options offer the supply chain participants to consider when and how to take advantage of the salvage manipulation.

Our contribution comes from allowing risk at any player in the supply chain rather than assuming a power retailer or dominant supplier base. In addition, our proposed coordination mechanism allows simultaneous bi-directional subsidies. This allows our model to be more robust than prior models that require either no-risk suppliers or a no-risk assembler.

There are many ways to extend our proposed mechanism. First, we can further investigate under what kind of conditions the various salvage manipulation options are desirable and what benefits they offer. Second, we can study the interaction and competition of more than one retailer or alternate suppliers using this salvage

manipulation mechanism. Third, we always can try to extend our proposed mechanism to multiple supplier levels in the supply chain.

CHAPTER 4

COORDINATION OF A TWO-TIER SUPPLY CHAIN WITH DEMAND INFORMATION UPDATING

4.1 Introduction and Motivation

We investigate a two tier serial supply chain with two players that make production decisions in sequence. The supplier decides his production quantity, then produces and stocks the goods. During the production lead time, some demand uncertainty is resolved at the retailer via advanced sales information (e.g. pre-orders). The second player, the retailer, then selects how many units to buy from the supplier and may incur additional cost to convert the components to finished goods. Fugate et al. (2006) note that often in business each supply chain member attempts to myopically optimize without consideration of the full system. Maximum efficiency can be achieved if a central decision maker decides all quantities, but in practice, this is not desirable or possible. As a result, many decentralized systems are run in a manner that results in a significant loss in overall efficiency. To overcome this inefficiency it is necessary to identify mechanisms such that participants make decisions that achieve centralized efficiency. Such mechanisms, when they can be clearly identified, are called supply chain coordination contracts (Cachon (2003)). We contribute to the research by (i) applying a single salvage manipulation mechanism, previously utilized in single period settings only, to this two period extension, (ii) specifying the parameter effects on the salvage manipulation to the retailer, and (iii) investigating the parameter effects of demand uncertainty on the disparity between uncoordinated and coordinated decision making.

The existing literature is rich in two tier supply chain coordination models where the supplier and retailer view the same demand distribution (stationary demand distributions). Pasternack (1985) models a two-stage supply chain where the manufacturer offers retailers a partial credit for all unsold goods. he proposed a coordination mechanism that provides partial credit for unsold goods at the retailer. Since then a number of authors have identified other contracts that can achieve coordination in supply chains. Buy-back contracts (He et al. (2006)), revenue sharing contracts (Dana and Spier (2001), Cachon and Lariviere (2005)), and mid-term returns (Taylor (2001)) are examples of coordinating contracts. A downside to the revenue sharing mechanism is that it allows a retailer to “cheat” (Wang et al. (2004)) or to exert less sales effort (Cachon and Lariviere (2005)). These contracts were all developed with a two-stage serial system (similar to the one in Bollapragada et. al. (2004)) as the supply chain setting.

Coordination has been extended to two period problems with demand information updating. Gurnani and Tang (1999) allow information updating between the first and second production periods where the production cost in the second period is random. Donohue (2000) models two production modes with a contract based on wholesale prices and return price for excess stock at the retailer. Chen et al. (2006) modify Donohue’s work to be a single production quantity in period one and a retailer procurement decision in period two. They suggest a contract where the retailer provides additional salvage value to the supplier for excess components and the supplier provides a partial credit to the retailer for excess stock at the end of the selling season. We thoroughly investigate this scenario and provide insights into the incentives for both

parties to participate. We use a single subsidy mechanism, salvage manipulation as introduced in Chapter 3, to coordinate the supply chain. We use numerical examples to illustrate the benefits of coordination under a variety of cost parameters, presales ranges, and demand distribution variance resolutions.

The problem of coordinating multiple independent decision makers in a supply chain is widespread in practice. Commonly, the lead time between stages (tiers) of the supply chain is such that additional demand information is learned between decision points. This additional market information can be used to update the demand forecast prior to the next decision.

4.2 Model and Notation

We focus on a two-stage supply chain for a single selling period situation. The situation is further complicated when the demand distributions viewed by each player are not identical. The supplier makes his production quantity decision with the most demand uncertainty. The retailer resolves some demand uncertainty through pre-sales prior to making her purchase quantity decision bounded by the supplier's stock level. See Figure 4.1 for a representative two-tier serial supply chain.

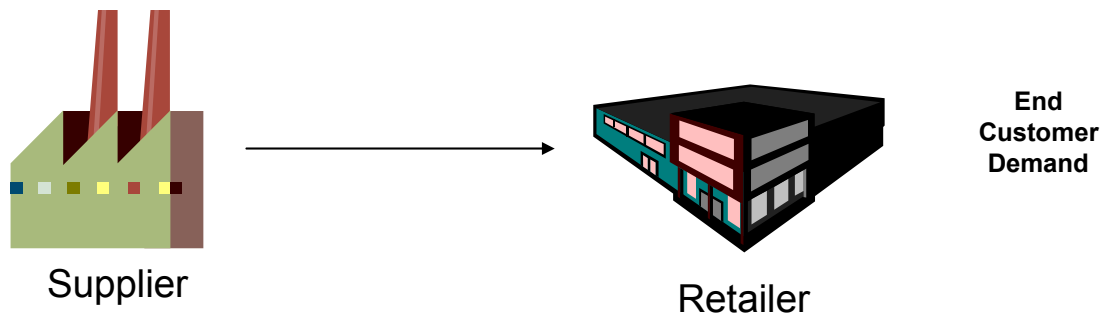


Figure 4.1: Representative Two-Tier Serial Supply Chain

At both stages of the supply chain value may be added in the form of material and labor. The retailer's purchase quantity is constrained by the stock at the supplier. The supplier has no constraint on the amount of raw material it can procure or production capacity it may utilize. There are no wealth restrictions in the supply chain, meaning that the supplier can produce any quantity it desires and the retailer can procure any quantity available (Eeckhoudt (1995)). The lead time between the supplier and retailer is deterministic. For long lead times, such as a supplier in China shipping a component to a company in North America via ocean freight, the lead time variance is very small compared to the average lead time. Therefore, a deterministic lead time based on the average lead time does not severely limit the practicality of our model. We assume (as was done in Barnes-Schuster et al. (2002)) that a single product is sold to consumers at a fixed market price that is exogenously specified for a single selling season at the retailer. Fixed prices are common in practice (e.g. catalog goods) where the price has to be published and is not easily altered during the selling season (Emmons and Gilbert (1998)).

This is an important problem to investigate because we seek to maximize the expected total supply chain profit without artificially assuming that a central control structure is implemented. It is neither practical nor desirable to have a single controlling entity in a global supply chain environment. Additionally, demand signals prior to the selling season do occur in practice through presales or other market indicators. A rational retailer will adjust her procurement quantity accordingly.

4.2.1 Notation

c_S	Cost to make one unit at the supplier (raw material and production value add)
c_R	Additional cost to retailer per one unit above supplier costs (value added, landed cost including shipping)
s_S	salvage value of a unit at the supplier
s_R	Salvage value of a unit at the retailer
β	Goodwill (shortage) cost at the retailer per unit of unmet customer demand
p	Retailer selling price
w	Wholesale price (supplier selling price to the retailer per unit)
q_S	Production quantity at the supplier (decision variable)
q_R	Retailer component purchase and production quantity (decision variable)
L_S	Production lead time at the supplier
L_R	Production lead time at the retailer
L_T	Transportation lead time between the supplier and the retailer
x	Random demand prior to information update
μ	Demand mean prior to information update
σ	Standard deviation of demand prior to information update
x_e	Random pre-order demand signal during supplier lead time, i.e. “New Market Information” as described in Donohue (2000). x_e is ordered such that $x_e^1 < x_e^2$ implies $G(x x_e^1) \geq G(x x_e^2)$.
μ_e	Pre-order mean (advanced demand information mean)

σ_e	Standard deviation of the advanced demand information
$f(.)$	pdf/pmf of demand as viewed at the first decision point (supplier tier)
$g(x x_e)$	pdf/pmf of demand as viewed at the second decision point, given x_e
$F(.)$	CDF of demand as viewed at the first decision point
$G(.)$	CDF of demand as viewed at the second decision point, given x_e
$\gamma(.)$	The retailer loss function (i.e. expected shortage) if the demand is more than the production quantity where $\gamma(q_R) = \int_{q_R}^{\infty} (x - q_R) g(x x_e) dx$

4.2.2 Sequence of Events

At time $t=0$:

- 1) Supplier makes quantity production decision q_S to maximize his expected profit
- 2) Supplier begins production
- 3) Retailer may begin collecting advanced demand information

At time $t=L_S$:

- 4) Supplier stocks product at his location
- 5) Retailer uses pre-sales information (x_e) to determine new demand distribution
- 6) Retailer makes her quantity procurement decision q_R in order to maximize her expected profit, upper bounded by the supplier's on hand inventory
- 7) Supplier recovers salvage value (if any) for unsold goods

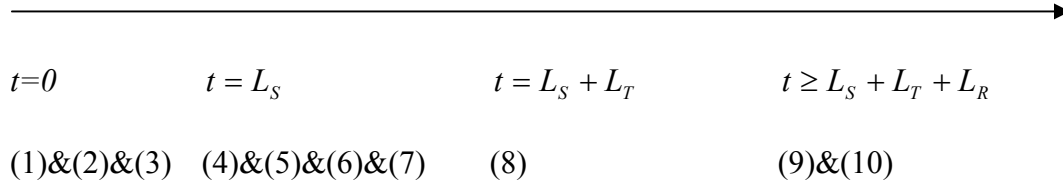
At time $t=L_S + L_T$:

- 8) Retailer receives components from supplier and then produces the finished goods (c_R represents all costs incurred by retailer beyond wholesale purchase price)

At time $t \geq L_S + L_T + L_R$:

9) End customer demand is realized

10) Retailer has goodwill penalty for unmet demand (if any). Retailer recovers salvage value (if any) for unsold products



4.2.3 Assumptions

Once the selling season begins at the retailer, no additional products can be produced or shipping in time from the supplier to meet the demand. Any unmet demand is lost, while excess stock is discarded, possibly for a salvage value.

We assume that the retail price is exogenous. All products are sold for a single price through one retailer. The retailer sells this one product only and the product's lifecycle extends through the entire horizon. Therefore, we do not have to explicitly account for product decline or new product introduction with possible cannibalization effects (Roberts and McEvily (2005)). In the centralized model, the retailer controls all stages of the supply chain and thus the supply chain avoids double marginalization (Lee et. al (1997), Spengler (1950)). The decision makers in the supply chain are rational and do not suffer from misperceptions of information flowing through the supply chain (Sternan (1987)).

Figure 4.2 below shows representative views of end customer demand in time. When the supplier makes his production quantity decision, the distribution is widest indicating the most uncertainty. During the production lead time, market information such as pre-sales resolves some demand uncertainty. When the retailer makes her procurement decision at $t = L_S$, the mean has been updated and standard deviation reduced based upon pre-orders received at the retailer. The third distribution is the narrowest demand distribution seen at the start of the selling season at the retailer, and therefore occurs after the decision points in our model. The retailer has already procured and readied the goods for selling prior to this point in time.

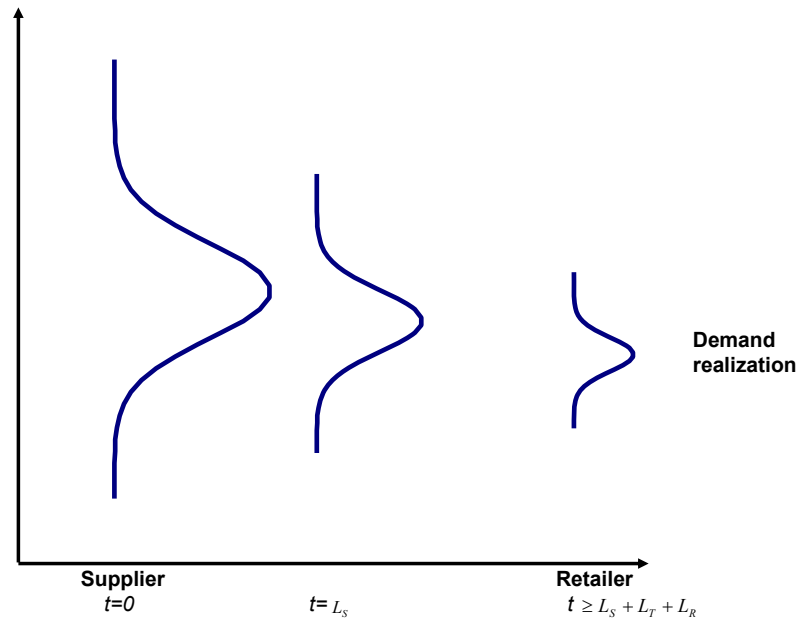


Figure 4.2: Demand Distribution Resolution in Time

Note that the retailer may place her order at time $t=0$, however this would mean she forgoes the opportunity to utilize advance demand information she may receive during the supplier production lead time. The supplier cannot ship goods before $t = L_S$ regardless. If $L_S = L_T = 0$ then the problem compresses to the conventional single

period newsvendor with both decisions viewing the same customer demand distribution. This simpler problem has been solved in Pasternack (1985) and subsequent research.

4.3 Supply Chain Operation Scenarios

We now analyze the centralized and decentralized control scenarios to illustrate how myopic decision making introduces inefficiencies.

4.3.1 Analysis of the Centralized Control Case

An example of the demand uncertainty resolution is shown in Figure 4.3 below where one of three smaller distributions will be viewed by the retailer based on market information during the supplier production lead time. Retailer distributions may overlap and be many in number (not limited to the three as illustrated). At the supplier's decision point, a wider distribution exists reflecting greater uncertainty about customer demand at the retailer.

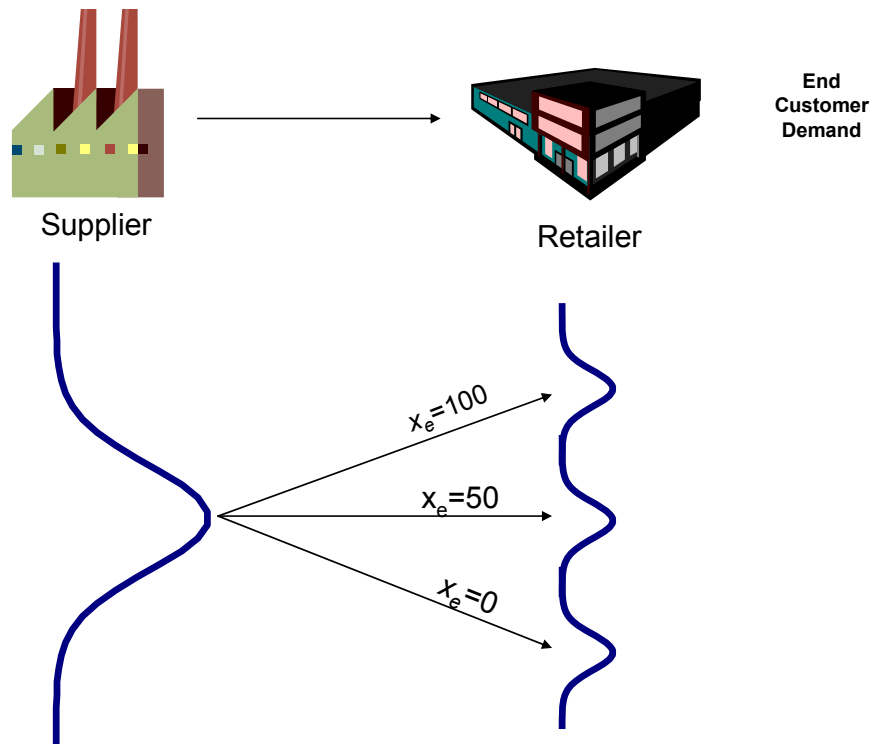


Figure 4.3: Centralized Control Supply Chain

For centralized control, there is no intermediate selling price (wholesale). The price is simply the selling price to the customer from the retailer. In addition, the total costs are composed of the raw material costs and value added costs to the point where the product is in the value chain. The product may be salvaged at the supplier if it is not sent to the retailer for further processing and sold to a customer.

The superscript C denotes centralized quantity decisions. To solve this problem we investigate the decisions backwards starting with the optimal second stage quantity decision at the retailer, followed by the optimal procurement and production quantity at the supplier.

Second Stage Problem

Given the sunken production cost at the supplier with q_S^C on hand, the retailer in the second stage wants to maximize the expected profit. The retailer quantity decision to maximize profit depends on (1) below:

$$\Pi_R^C(q_R^C | x_e) = E[p \min(x, q_R^C) + s_R(q_R^C - x)^+ + s_S(q_S^C - q_R^C) - \beta(x - q_R^C)^+ - c_R q_R^C] \quad (1)$$

Equation (1) contains the revenue plus the salvage value for goods not sold in the second period and the salvage for supplier goods not sent to the retailer minus the goodwill/shortage cost for unmet demand minus the retailer production costs. The maximal expected profit can be determined for the second period decision at the retailer from (11). Let Z^C denote the optimal order quantity for the retailer in period two given the advance demand information x_e . We will solve for Z^C below for two possible cases.

Case 1: $Z^C < q_S^C$

We first assume that the second period order quantity is less than the first period production. Instead of subscript R for retailer, we will use I for this first case where the second period retailer purchase quantity is less than the first period supplier production quantity. Subsequently, we will use subscript 2 to denote the case where the second period purchase quantity is equal to the first period production quantity (when the optimal buy quantity is equal or greater than the supplier stock on hand).

$$E[\Pi_1^C] = p \int_0^{Z^C} xg(x | x_e)dx + pZ^C \int_{Z^C}^{\infty} g(x | x_e)dx + s_R \int_0^{Z^C} (Z^C - x)g(x | x_e)dx + (q_S^C - Z^C)s_S - \beta \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx - c_R Z^C \quad (2)$$

$$E[\Pi_1^C] = -p \int_0^{\infty} xg(x | x_e)dx - p \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx + s_R \int_0^{\infty} (Z^C - x)g(x | x_e)dx + s_R \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx + (q_S^C - Z^C)s_S - \beta \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx - c_R Z^C \quad (3)$$

$$E[\Pi_1^C] = -(c_R - s_R + s_S)Z^C + q_S^C s_S + (p - s_R)\mu_e - \gamma(p + \beta - s_R) \quad (4)$$

Taking the first derivative of the expected profit in (4) with respect to quantity Z^C and setting it to zero gives:

$$\frac{\partial E[\Pi_1^C]}{\partial Z^C} = -(c_R - s_S) + (1 - G(Z^C))(p + \beta - s_R) = 0 \quad (5)$$

$$G(Z^C | x_e) = \frac{p + \beta - c_R - s_S}{p + \beta - s_R} \quad (6)$$

Lemma 4.1: Given $Z^C < q_S^C$, the expected profit function (4) is concave in the retailer quantity Z^C

Given the first derivative in (5), we can find that the second derivative of the expected profit function as shown in (7) below. Given the assumptions on the parameters, the equation will always be negative. Therefore, the expected profit function curve is concave.

$$\frac{\partial^2 E[\Pi_1^C]}{\partial Z^2} = -(p + \beta) + s_R < 0 \quad (7)$$

□

If the retailer had bought an additional unit from the supplier, the opportunity to receive the salvage value at the supplier for that unit is forgone. Therefore, the profit margin of an additional unit sold by the retailer is reduced by the lost salvage value at the supplier (s_S). Likewise, if the retailer is over stock an additional unit, this indicates that she bought the unit from the supplier, so the opportunity cost of not collecting salvage value at the supplier must be added to the overage cost. Equation (6) is the same as the conventional newsvendor formulation with the following cost of underage and cost of overage:

Cost of underage is $u = p + \beta - c_R - s_S$

Cost of overage $o = c_R - s_R + s_S$

$$Z^C = G^{-1}\left(\frac{u}{u + o}\right) = G^{-1}\left(\frac{p + \beta - c_R - s_S}{p + \beta - s_R}\right) \quad (8)$$

Case 2: $Z^C \geq q_S^C$

When the second period optimal order quantity is equal to or greater than the first period production quantity, the salvage term in the retailer profit function and critical ratio drops

out as shown below. The retailer is limited to procure exactly q_s^C components from the supplier.

$$E[\Pi_2^C] = p \int_0^{Z^C} xg(x | x_e)dx + pZ^C \int_{Z^C}^{\infty} g(x | x_e)dx +$$

$$s_R \int_0^{Z^C} (Z^C - x)g(x | x_e)dx - \beta \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx - c_R Z^C \quad (9)$$

$$E[\Pi_2^C] = p \int_0^{\infty} xg(x | x_e)dx - p \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx + s_R \int_0^{\infty} (Z^C - x)g(x | x_e)dx +$$

$$s_R \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx - \beta \int_{Z^C}^{\infty} (x - Z^C)g(x | x_e)dx - c_R Z^C \quad (10)$$

$$E[\Pi_2^C] = -(c_R - s_R)Z^C + (p - s_R)\mu_e - \gamma(p + \beta - s_R) \quad (11)$$

Taking the first derivative of the expected profit in (11) with respect to quantity Z^C and setting it to zero gives:

$$\frac{\partial E[\Pi_2^C]}{\partial Z^C} = -c_R + (1 - G(Z^C))(p + \beta - s_R) = 0 \quad (12)$$

$$G(Z^C | x_e) = \frac{p + \beta - c_R}{p + \beta - s_R} \quad (13)$$

For the retailer, the costs of underage and overage from (13) are shown below. Since the retailer is buying all the stock from the supplier in this case, there is no supplier salvage term:

Cost of underage is $u = p + \beta - c_R$

Cost of overage $o = c_R - s_R$

Lemma 4.2: Given $Z^C \geq q_S^C$, the expected profit function (11) is concave in the retailer quantity Z^C

We now find the second derivative of the expected profit function from the first derivative shown in (12). Given the assumptions on the parameters, the equation will always be negative. Therefore, the expected profit function curve is concave.

$$\frac{\partial^2 E[\Pi_2^C]}{\partial Z^2} = -(p + \beta) + s_R < 0 \quad (14)$$

□

Therefore, (13) gives the solution to the optimal retailer quantity as shown below:

$$Z^C = G^{-1}\left(\frac{u}{u + o}\right) = G^{-1}\left(\frac{p + \beta - c_R}{p + \beta - s_R}\right) \quad (15)$$

First Stage Problem (Supplier Production Quantity Decision):

The production cost for q_S^C units is $c_S q_S^C$.

Because we have accounted for the salvage at the supplier already in the profit realized in the second stage, there is only one profit equation for the first stage quantity decision.

Borrowing the notation from Chen et al. (2006), let

$$k(q_S^C) = \begin{cases} \max \left\{ x_e \mid Z^C \leq q_S^C : G(Z^C \mid x_e) = \frac{p + \beta - c_R - s_S}{p + \beta - s_R} \right\} & \text{if the set is non-empty,} \\ 0 & \text{otherwise.} \end{cases}$$

Figure 4.4 below illustrates what k represents. Given the possible final distributions (G) seen by the retailer given advanced demand information (x_e), k corresponds to the point where the Z quantity from the critical ratio is equal to the supplier on hand (q_S).

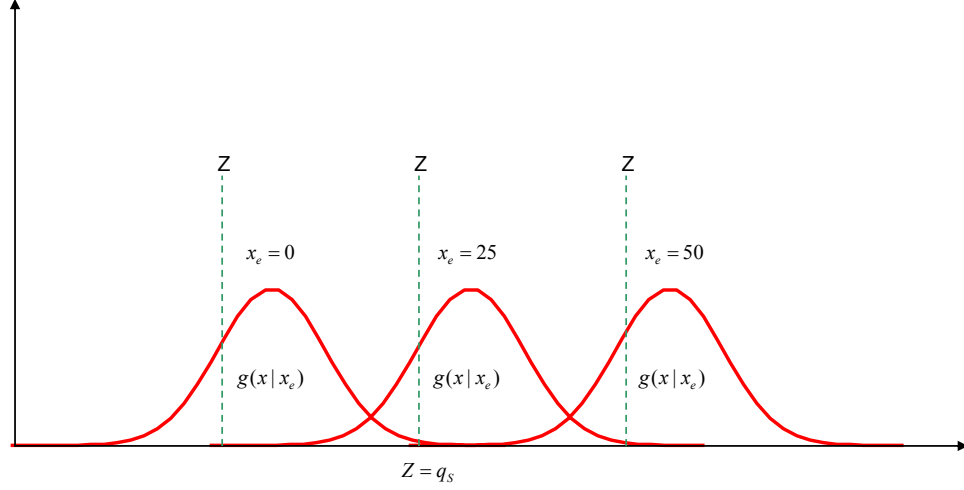


Figure 4.4: Illustration of k on possible retailer distributions

This single profit equation combines the two cases from stage two. The centralized control supply chain profit, where the first stage production quantity is q_s^C , can be written as:

$$\Pi_s^C = \int_0^{\infty} (\Pi^C(q_s^C | x_e) g(x | x_e)) dx - c_s q_s^C \quad (16)$$

From (4) and (11) we can rewrite (16) as

$$\Pi_s^C = \int_0^{k(q_s^C)} \Pi_1^C(q_s^C) g(x | x_e) dx_e + \int_{k(q_s^C)}^{\infty} \Pi_2^C(q_s^C) g(x | x_e) dx - c_s q_s^C \quad (17)$$

A closed form solution to equation (17) cannot be found easily even for normal distributions for end customer demand and presales information, but the optimal stage one quantity can be computed via simulation.

Proposition 4.1: There exists an optimal profit for stage one, and therefore an optimal production quantity in stage one.

Proof: This follows from equation (17)'s concavity. The first derivative of the terms in (17) with respect to q_S is $s_S - c_R + (1 - G(q_S))(p + \beta - s_R) - c_S$. The second derivative with respect to q_S is $-(p + \beta) + s_R < 0$. \square

During the supplier's production lead time (L_S) to make the q_S^* units, pre-orders will be taken at the retailer that resolve some demand uncertainty. The retailer's decision quantity will be bound above by q_S^* . Salvage may occur at the supplier for production units that the retailer realizes are no longer ideal to purchase given updated demand information. In this situation, the units will be salvaged at the supplier and not incur value added costs at the retailer. Likewise, the retailer may desire more units that produced by the supplier. However, the retailer is bounded above by the units in stock at the supplier.

4.3.1.1 Numerical Example

We now demonstrate the effect of applying equations (8) and (15) for a uniform demand distribution. For exposition, we allow the advanced demand signal x_e to take two values; 0 (low) or 1 (high). If x_e is 0 then the end customer demand distribution is $\sim U(0,99)$, while if x_e is 1, the end customer demand distribution is $\sim U(100,199)$.

Let $p = 60, \beta = 0, c_R = 10, s_R = 4, w = 20, c_S = 6, s_S = 2$. Therefore, the critical ratio that the retailer will select from (8), is:

$$\frac{p + \beta - c_R - s_S}{p + \beta - s_R} = \frac{60 - 10 - 2}{60 - 4} = \frac{48}{56} = 85.7\% \quad (18)$$

The supplier, with his knowledge that x_e of 0 and x_e of 1 each have a 50% probability, knows that the retailer will want to buy either 85 or 185 units. The net benefit for another

unit for the supplier to produce is positive at 171 units, but turns negative at 172 units.

This is equivalent to the combined net benefit for the two stages, where the first two terms in (19) are the retailer having bought a unit acquired from the supplier and the third term represents the units left over for the units beyond 85:

$$\Pr(sell)(p + \beta - c_R - c_S) - \Pr(notsell)(p + \beta - s_R) - \Pr(nobuy)(c_S - s_S) \quad (19)$$

Since $x_e = 1$ has a 50% probability, the retailer will want to buy 185 units with 50% probability (for the first bracket), and with 50% probability not buy 185 units (buy 85 only), leaving the supplier to salvage components (in excess of 85) in the second bracket below.

$$50\% * [29\% * c_U^R - 71\% * c_O^R] - 50\%[c_O^S] \quad (20)$$

$$50\% * [29\% * 44 - 71\% * 8] - 50\%[4] = 50\% * [4.24] - 50\%[4] = 0.12$$

Similarly, for 186 units:

$$50\% * [28\% * c_U^R - 72\% * c_O^R] - 50\%[c_O^S] \quad (21)$$

$$50\% * [28\% * 44 - 72\% * 8] - 50\%[4] = 50\% * [3.68] - 50\%[4] = (0.16)$$

In this example, there are two possible advanced demand signals. For $x_e = 0$, the retailer learns that customer demand is $\sim U(0,99)$. Therefore, applying his critical ratio, she buys 85 of the 171 units available at supplier. The supplier will salvage the remaining 86 units. The centralized profit is \$1,349.20. If $x_e = 1$, the retailer updates the customer demand distribution to $\sim U(100,99)$. She wants to buy 185 units from the supplier but can procure only 171. The centralized profit is \$6,092.64 with this higher advanced demand signal. Under decentralized control, for the two demand signals, profit would be \$1,140.40 and \$5,940.40 respectively.

4.3.2 Decentralized Control Case

Under decentralized decision making, the supplier makes his production decision based on the lower of (i) his myopically calculated production quantity, (ii) his estimate of the retailer's future purchase quantity.

The superscript D denotes decentralized quantity decisions. Similar to our solution method under centralized control, we will solve the second stage problem followed by the first stage problem.

Second Stage Problem (Retailer's quantity decision):

Retailer receives pre-sale orders during the supplier's production lead time (L_s). Retailer then modifies the end customer demand distribution according to information gained from customer demand pre-ordered. At the end of the supplier production lead time, the retailer updates the customer demand distribution and then makes her quantity decision per (27) below:

$$E[\Pi_R^D] = p \int_0^{q_R^D} xg(x | x_e)dx + pq_R^D \int_{q_R^D}^{\infty} g(x | x_e)dx + s_R \int_0^{q_R^D} (q_R^D - x)g(x | x_e)dx - \beta \int_{q_R^D}^{\infty} (x - q_R^D)g(x | x_e)dx - (c_R + w)q_R^D \quad (22)$$

$$\frac{\partial E[\Pi_R^D]}{\partial q_R^D} = -c_R - w + (p + \beta)(1 - G(q_R^D)) + s_R G(q_R^D) \quad (23)$$

Lemma 4.3: In the decentralized case, the expected profit function (22) is concave in the retailer quantity q_R^D

The second derivative of (23) is shown below. Given the assumptions on the parameters, the equation will always be negative. Therefore, the expected profit function curve is concave.

$$\frac{\partial^2 E[\Pi_R^C]}{\partial q_R^2} = -(p + \beta - s_R)g(q_R^D) < 0 \quad (24)$$

□

Setting (23) equal to zero, we get

$$G(q_R^D) = \frac{p + \beta - c_R - w}{p + \beta - s_R} \quad (25)$$

$$q_R^* = G^{-1}\left(\frac{p + \beta - c_R - w}{p + \beta - s_R}\right) \quad (26)$$

$$q_R^D = \min(q_S^D, q_R^*) \quad (27)$$

Total profit for the retailer is shown below:

$$E[\Pi_R^D] = -(c_R + w - s_R)q_R^D + (p - s_R)\mu_e - \gamma(p + \beta - s_R) \quad (28)$$

First Stage Problem (Supplier Production Quantity):

Supplier uses the expected distribution of retailer's problem as his distribution for the production quantity decision. The supplier's profits follows the same two cases; case 1 where the retailer buy less than the supplier has on hand (shown in (29)), and case 2 where the retailer buys all of the supplier stock (shown in (30)) where $q_R^D = q_S^D$.

$$\Pi_1^D = wq_R^D + s_s(q_S^D - q_R^D) - c_s q_S^D \quad (29)$$

$$\Pi_2^D = wq_R^D - c_s q_S^D \quad (30)$$

$$\Pi_S^D = \int_0^{k(q_S^D)} \Pi_1^D(q_S^D)g(x | x_e)dx + \int_{k(q_S^D)}^{\infty} \Pi_2^D(q_S^D)g(x | x_e)dx \quad (31)$$

The first derivative of (29) with respect to q_s^D for case 1 is

$$\frac{\partial[\Pi_s^D]}{\partial q_s^D} = s_s - c_s \quad (32)$$

The first derivative of (30) with respect to q_s^D for case 2 is

$$\frac{\partial[\Pi_s^D]}{\partial q_s^D} = w + s_s - c_s \quad (33)$$

Substituting (32) and (33) into (31) gives:

$$\frac{\partial \Pi_s^D}{\partial q_s^2} = \int_0^{k(q_s^D)} (s_s - c_s) g(x | x_e) dx + \int_{k(q_s^D)}^{\infty} (w + s_s - c_s) g(x | x_e) dx - c_s q_s^D + s_s (q_s^D - Z^D) \quad (34)$$

Setting (34) equal to zero and solving it gives the optimal supplier production quantity (q_s^D).

The total supply chain profit in the decentralized case is simply the sum of (28) and (34).

This expected profit can be compared to that in (17).

Proposition 4.2: $q_R^C \geq q_R^D$

Proof: This follows from a comparison of the retailer centralized ratio in (8) and (13) for the two cases with the decentralized retailer ratio in (26). Since the ratio is larger under centralized control, the retailer purchase quantity is thus larger. Therefore, the expected profit under decentralized control can be no greater than that under centralized control.

□

4.4 Coordinated Decentralized Supply Chain

The superscript M denotes salvage manipulation influenced quantity decisions. Again we solve the problem backwards starting with the second stage first.

Retailer's Problem:

The retailer may receive pre-orders during the supplier's production lead time and modify the customer demand distribution accordingly.

Second Stage Problem (Retailer's buy Quantity Decision):

The retailer receives pre-sale orders during the supplier's production lead time (L_s). The retailer can use this advance demand information to update the end customer distribution, then makes her quantity decision per (41) below. Salvage manipulation (δ) is a promise between the supplier and the retailer. A positive value signifies that the retailer is promising to ease the financial burden of excess stock at the supplier, while a negative value indicates that the supplier must promise additional salvage value for excess inventory at the retailer.

$$E[\Pi_R^M] = p \int_0^{q_R^M} xg(x|x_e)dx + pq_R^M \int_{q_R^M}^{\infty} g(x|x_e)dx + (s_R - \delta) \int_0^{q_R^M} (q_R^M - x)g(x|x_e)dx - \beta \int_{q_R^M}^{\infty} (x - q_R^M)g(x|x_e)dx - (w + c_R)q_R^M \quad (35)$$

$$\frac{\partial E[\Pi_R^M]}{\partial q_R^M} = -(w + c_R) + (p + \beta)(1 - G(q_R^M)) + (s_R - \delta)G(q_R^M) \quad (36)$$

Similar to (28), the retailer's profit is:

$$E[\Pi_R^M] = -(c_R + w - s_R - \delta)q_R^M + (p - s_R - \delta)\mu_e - \gamma(p + \beta - s_R - \delta) \quad (37)$$

Therefore, for the two cases as used in (29) and (30), we have two possible profits as shown in (38) and (39):

$$\Pi_1^M = wq_R^M + s_s(q_s^M - q_R^M) - c_s q_s^M + \delta \int_0^{q_R^M} (q_R^M - x)g(x|x_e)dx \quad (38)$$

$$\Pi_2^M = wq_R^M - c_S q_S^M + \delta \int_0^{q_R^M} (q_R^M - x)g(x | x_e)dx \quad (39)$$

Lemma 4.4: In the coordinated case, the expected profit function (37) is concave in the retailer quantity q_R^D

The second derivative of (37) is shown below. Given the assumptions on the parameters, the equation will always be negative. Therefore, the expected profit function is concave.

$$\frac{\partial^2 E[\Pi_R^M]}{\partial q_R^2} = -(p + \beta - s_R + \delta)g(q_R^M) < 0 \quad (40)$$

□

Setting (36) equal to zero gives

$$G(q_R^M) = \frac{p + \beta - w - c_R - s_S}{p + \beta - s_R + \delta} \quad (41)$$

$$q_R^* = G^{-1}\left(\frac{p + \beta - w - c_R - s_S}{p + \beta - s_R + \delta}\right) \quad (42)$$

$$q_R^M = \max(q_R^M, q_R^*) \quad (43)$$

We need to set δ so that equation (42) is equal to the CSC equation (8).

$$\frac{p + \beta - c_R - s_S}{p + \beta - s_R} = \frac{p + \beta - w - c_R}{p + \beta - s_R + \delta} \quad (44)$$

Therefore, the salvage manipulation term (retailer promise to supplier) is:

$$\delta = \frac{(s_S - w)(\beta + p - s_R)}{\beta - c_R + p - s_S} \quad (45)$$

Proposition 4.3: From the retailer's point of view, the salvage manipulation always is a promise from the supplier to the retailer ($\delta \leq 0$).

Proof: This follows from our assumptions regarding the parameters in (45). The denominator is positive and the numerator contains a negative term multiplied by a positive term. The retailer's critical ratio (8) and (15) under centralized control is always greater than the ratio under decentralized control in (26). Therefore, to achieve optimal profits, the retailer must be promised additional salvage for excess units for her to procure the quantity optimal for the entire supply chain. The supplier takes on this extra risk to boost the total expected supply chain profit to optimal. By adjusting wholesale price, the supplier may adjust the proportion of total profits he receives compared to the retailer under coordination.

□

First Stage Problem (for the Supplier Production Quantity):

The supplier uses the distribution of the retailer's problem as his distribution for the production quantity decision.

$$E[\Pi_S^M] = wZ^M + (s_S + \delta)(q_S^M - Z^M) - c_S q_S^M \quad (46)$$

Using the two profit cases from (38) and (39) we can assemble the entire profit equation as shown below.

$$E[\Pi_S^M] = \int_0^{k(q_S^M)} (\Pi_1^M(q_S^M)) g(x | x_e) dx + \int_{k(q_S^M)}^{\infty} (\Pi_2^M(q_S^M)) g(x | x_e) dx - c_S q_S^M + wZ^M + (s_S + \delta)(q_S^M - Z^M) \quad (47)$$

q_S^M is solved by setting the first derivative of (47) to zero. However, since the k term depends on the supplier ratio and demand distribution after information updating, (47) needs to be solved via iteration. We give an example of this below.

As an illustration of the supplier's control of profits via the wholesale price, we now show a numerical example. $p = 100, \beta = 10, c_R = 8, s_R = 5, c_S = 5, s_S = 2$. Note that

in every case below, with proper selection of salvage manipulation according to (45) by the supplier, the total supply chain profit is optimal. However, by adjusting his wholesale price (w) and the corresponding salvage manipulation, the supplier is able to split the total supply chain profits between himself and the retailer in any manner he desires. This profit split is the incentive mechanism for the supplier to participate in a salvage manipulation agreement with the retailer. The retailer's incentive comes solely from the salvage manipulator mechanism. The Retailer % Profit is $\frac{(37)}{(37) + (47)}$ and the Supplier

% Profit is $\frac{(47)}{(37) + (47)}$.

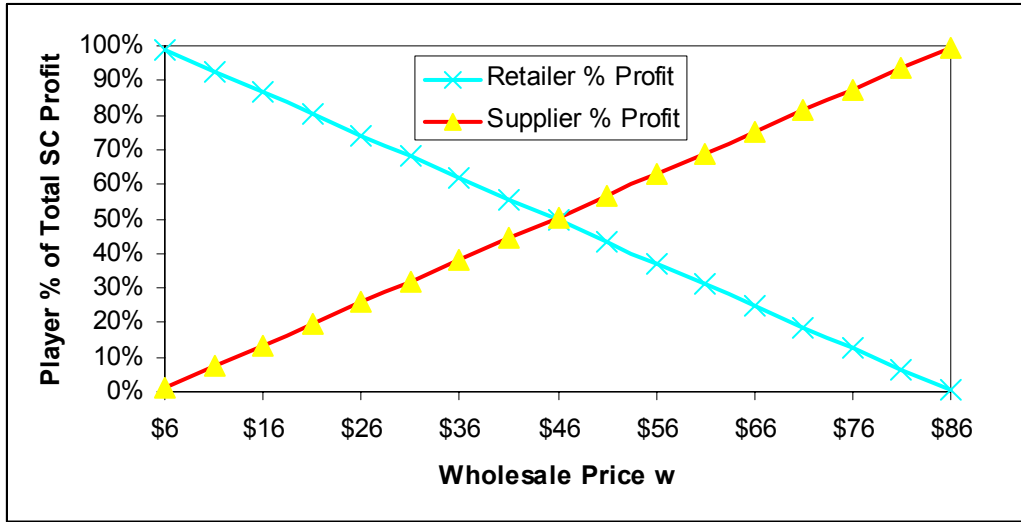


Figure 4.5: Wholesale price affecting profit percentage

Proposition 4.4: $q_R^D = q_R^C$

Proof: The δ term is chosen to make the critical ratio for the retailer under decentralized decision making equal to the critical ratio under centralized decision making, therefore $q_R^D = q_R^C$. Given the retailer wants the same quantity, the supplier can produce a quantity

such that the total profits achieved under coordination are equivalent to those achieved under centralized control. \square

We can examine what effect changes in parameters have on the subsidy offered by the supplier to the retailer in order to coordinate the supply chain. Table 4.1 shows how the negativity of the salvage manipulation changes when parameters change in (45). Recall that the more negative δ is, the higher the subsidy that the supplier must promise to the retailer to achieve coordination. The supplier provides a promise of $-\delta$ per unit left over at the retailer at the end of the selling season.

Table 4.1: Subsidy effect when parameters change

Parameter		δ negativity
Retail price (p)	\uparrow	\downarrow
Goodwill (β)	\uparrow	\uparrow
Wholesale price (w)	\uparrow	\uparrow
Retailer production cost (c_R)	\uparrow	\uparrow
Supplier production cost (c_S)	\uparrow	no effect
Retailer salvage value (s_R)	\uparrow	\downarrow
Supplier salvage value (s_S)	\uparrow	\downarrow

In the above table 4.1, for example, the higher the retailer selling price p , the less negative δ is, thus the supplier promises less salvage manipulation to the retailer. Similarly, the higher the retailer goodwill/shortage cost, the more negative δ is, thus the supplier must promise more salvage manipulation to the retailer to coordinate the supply chain.

4.5 Experimental Setup, Computation Results and Managerial Insights

In this section, we demonstrate the disparity between coordinated profit and non-coordinated profit to demonstrate the importance of using a coordination mechanism such as our proposed salvage manipulation mechanism. First, let us assume the following simple example:

$x_e \sim N(5, 1.3\bar{3})$, effectively giving a range of zero through 10.

$$x \sim N(150, x_e * 10)$$

Also let $p = 85, \beta = 14, c_R = 12, s_R = 5, w = 20, c_S = 9, s_S = 5$

In the centralized setting, the ideal quantity for the supplier to produce (q_S^C) is 182 units.

The retailer will buy the 71% point on whatever the stage two distribution is after observing presales (x_e). Assume that the advanced demand information is $x_e = 7$.

By solving (8), (15) and (17), we find the centralized total profit. With this advanced demand information, the retailer would like to buy 189 units, but is limited to buy all 182 units from the supplier in the centralized and coordinated scenarios.

Centralized control profit = \$9,996.00

By solving (28) and (34), we find the decentralized total profit.

Decentralized control profit = \$8,028.00 total
= \$6,386.00 retailer
= \$1,642.00 supplier

In the decentralized scenario, the retailer buys 158 units from the supplier given the same advance demand information. This is because her local ratio is smaller than under centralized decision making. Our salvage manipulation coordination mechanism resolves this discrepancy. By solving (37) and (47), we find the coordinated total profit.

Coordinated via Salvage Manipulation Profit = \$9,996.00 total
= \$7,994.00 retailer
= \$2,002.00 supplier

In this example, both players are better off through salvage manipulation even though it flows from supplier to retailer only. However, as shown in Figure 4.5, the supplier may manipulate his expected profit via his wholesale selling price to the retailer.

To thoroughly test the disparity between coordinated and non-coordinated profits based on variance reduction through advanced demand information and also based on the myopic retailer ratio let us use the following parameter ranges for simulation.

Let: $\eta \sim U(100,1000)$, where $\sigma_2 = \frac{\sigma_1}{\eta\%}$ to decrease variance in stage two

$$c_S \sim U(1,50)$$

$$s_S \sim U(0, c_S - 1)$$

$$w \sim U(c_S + 1, c_S + 51)$$

$$c_R \sim U(1,50)$$

$$s_R \sim U(0, c_R - 1)$$

$$p \sim U(w + c_R + 1, w + c_R + 101)$$

$$\beta \sim U(0, p - 1)$$

The results for these 10,000 random scenarios was an average decentralized supply chain profit loss of \$389.72. The centralized control case and coordination via salvage manipulation case both had an average supply chain profit of \$4,525.02. Taking the absolute percentage improvement in profit between coordination and decentralized decision making (where 0% is equivalence), coordinated achieved a 409% average improvement in dollars. This number is skewed to such a large value by some coordinated total profits that are small, while the decentralized case had a large negative

profit. Regardless of the magnitude of inefficiency, decentralized control is clearly not a profitable strategy since salvage manipulation can provide such a significant upside.

Somewhat counter intuitively, regression shows that as the standard deviation reduction for the second decision point (retailer) is increased, the difference between coordinated and non-coordinated profits appears random (p-value of .39). The localized retailer critical ratio may be correlated to the disparity between coordinated and non-coordinated profits. With a p-value of .07, the disparity decreases as the ratio increases. This could be due in part to the reduced quantities available for coordinating when at a higher ratio (e.g., a decentralized retailer ratio of 95%, can only be raised up to 5% more via coordination).

With advanced demand information, the second stage has better information to make procurement and production decisions. However, the supply chain profit depends on the interdependence of the two players because the stage one production quantity limits the stage two purchase quantity, and the stage one inventory on hand will necessarily be excess if the stage two retailer chooses to purchase less than the full amount on hand at the supplier.

We have shown that the centralized retailer quantity is always equal to or greater than the quantity the retailer will select under decentralized control without incentive. The supplier may also want to select less than the optimal quantity under myopic decision making and therefore requires an incentive to produce more. Because the retailer buys and owns the stock from the supplier, she assumes the risk of unsold goods she has bought. Therefore, the supplier will need to provide a subsidy to the retailer if he wants the retailer to procure the optimal quantity of goods from him. The supplier transforms

the optimal retailer procurement quantity to optimal via the salvage manipulation mechanism – a promise to the retailer to provide additional salvage value for unsold goods at the end of the selling season. However, the supplier cannot expect a reduction in his risk from a third party promising salvage manipulation to him. Therefore, for him to produce and stock the amount optimal for the entire supply chain, he will manipulate the wholesale price such that he receives an appropriate percentage of the total coordinated supply chain profits. This result is similar to that found in Pasternack (1985) for a single stage problem.

4.6 Conclusion and Future Research

We have developed a single subsidy parameter (salvage manipulation) to coordinate this two stage newsvendor problem. The subsidy that the supplier promises to the retailer for excess stock at the end of the selling season can be calculated by (45). We have also shown the effect that the cost parameters for both players have on the subsidy value.

Through numerical example and simulation, we have shown the magnitude of profit loss through decentralized decision making. Coordinating the supply chain is essential to profit maximization. Although a single owner (centralized decision maker) will coordinate this supply chain, however, in practice this is not practical. Our salvage manipulation mechanism allows retention of decentralized decision making, but with coordinated supply chain profits.

This research can be extended in several important ways; rather than a single supplier, multiple component suppliers to an assemble-to-order retailer would enhance the problem (as done in Chapter 3). The supplier could have his own supplier. Having

multiple tiers of suppliers could be accommodated with a series of salvage manipulation terms, rather than one as done here. For a single retailer and n suppliers, n salvage manipulation terms would be required. Other extensions may include having competing suppliers, multiple production modes, or multiple retailers with competition.

APPENDIX A

DERIVATIONS FOR CHAPTER 3 OPTIMAL QUANTITY

With centralized supply chain control, given selling price p at the retailer, every supplier will produce the same quantity $q_i = q_0, \forall i$. Total profit is given below:

$$\Pi(x) = \begin{cases} -q_0 \sum_{i=0}^n c_i + px + \sum_{i=0}^n s_i (q_0 - x), & x \leq q_0 \\ -q_0 \sum_{i=0}^n c_i + pq_0, & x > q_0 \end{cases} \quad (1)$$

and the expected profit is

$$E[\Pi] = -q_0 \sum_{i=0}^n c_i + \int_{-\infty}^{q_0} \left(\sum_{i=0}^n s_i (q_0 - x) + px \right) f(x) dx + pq_0 \int_{q_0}^{\infty} f(x) dx \quad (2)$$

This is equivalent to

$$E[\Pi] = -q_0 \sum_{i=0}^n c_i + \sum_{i=0}^n s_i \left(\int_{-\infty}^{\infty} (q_0 - x) f(x) dx + \int_{q_0}^{\infty} (x - q_0) f(x) dx \right) + p \left(\int_{-\infty}^{\infty} x f(x) dx - \int_{q_0}^{\infty} (x - q_0) f(x) dx \right) \quad (3)$$

If the retailer acquires capacity greater than the quantity of items produced at the suppliers ($q_R > m$), his expected profit equation below, will change

$$E[\Pi_R] - E[\Pi_r] = -(q_R - m)c_0 + s_0 \int_m^{q_R} (q_R - x) f(x) dx \quad (4)$$

If c_0 and s_0 are zero the equation above is zero, therefore, there is no penalty for capacity acquisition. However, if $c_0 > s_0 > 0$, then the above equation is negative. Therefore, to maximize his expect profit, the retailer will acquire m units of capacity.

We can now write the total expected profit equation where all suppliers choose the same quantity to make, and the retailer purchases an equivalent amount of capacity.

Recall that μ is the average demand and $\gamma(q_0) = \int_{q_0}^{\infty} (x - q_0) f(x) dx$, therefore, the total expected supply chain profit, becomes

$$E[\Pi] = \left(p - \sum_{i=0}^n s_i \right) \mu - q_0 \left(\sum_{i=0}^n c_i - \sum_{i=0}^n s_i \right) + \gamma_0 \left(\sum_{i=0}^n s_i - p \right) \quad (5)$$

To obtain the optimal expected profit for the supply chain, we take its first derivative and set it to zero and it gives the critical fractile shown below, where q_0^* is the optimal order quantity for the centralized system and $F(q_0^*)$ is its corresponding CDF of the demand.

$$\frac{\partial E[\Pi]}{\partial q} = -\sum_{i=0}^n c_i + \sum_{i=0}^n s_i F(q) + p(1 - F(q)) = 0 \quad (6)$$

$$p - \sum_{i=0}^n c_i = \left(p - \sum_{i=0}^n s_i \right) F(q) \quad (7)$$

Giving the critical fractile here

$$F(q_0^*) = \frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \quad (8)$$

Each supplier has the same critical fractile from above; therefore, each supplier stocks the same amount. The critical fractile above can be rewritten in general terms, shown here, taking into account the cost of underage/expedite (C_u) and the cost of overage (C_o).

$$F(q_0^*) = \frac{C_u}{C_u + C_o} \quad (9)$$

Where the cost of underage/expedite $C_u = p - \sum_{i=0}^n c_i$ and the cost of

overage $C_o = \sum_{i=0}^n (c_i - s_i)$. Note, that the situation where the retailer has unlimited

assembly capacity (no risk) can be modeled by setting $c_0 = 0, s_0 = 0$.

Under the decentralized control scenario, the retailer chooses the capacity to buy prior to the selling season. This reflects his ability to deliver end-bundles of product to the customers. Additionally, the manufacturer chooses the quantity of units to make and stock prior to the selling season. The retailer may sell up to the minimum of his capacity q_R and the minimum q_i on hand at the manufacturers. All demand beyond the minimum of these quantities is assumed lost. The manufacturers receive salvage value for unsold units at the end of the selling period. The retailer's expected profit is shown below where q_R is the quantity of capacity bought prior to the selling season. For this ratio, we will first assume that the manufacturers make at least q_R units so that there is no constraint on q_R .

$$\Pi_R(x) = \begin{cases} -q_R c_0 + \left(p - \sum_{i=1}^n w_i \right) x + s_0 (q_R - x), & x \leq q_R \\ -q_R c_0 + \left(p - \sum_{i=1}^n w_i \right) q_R, & x > q_R \end{cases} \quad (10)$$

and the expected retailer's profit is

$$E[\Pi_R] = -q_R c_0 + \left(p - \sum_{i=1}^n w_i \right) \int_{-\infty}^{q_R} x f(x) dx + \left(p - \sum_{i=1}^n w_i \right) q_R \int_{q_R}^{\infty} f(x) dx + s_0 \int_{-\infty}^{q_R} (q_R - x) f(x) dx \quad (11)$$

Taking the partial derivative of the above profit equation with respect to q_R and setting it to zero allows us to solve for the critical fractile for retailer in the decentralized case.

$$\frac{\partial E[\Pi_R]}{\partial q_R} = -c_0 + \left(p - \sum_{i=1}^n w_i \right) (1 - F(q_R)) + s_0 F(q_R) = 0 \quad (12)$$

$$\left(p - \sum_{i=1}^n w_i + s_0 \right) F(q_R) = p - \sum_{i=1}^n w_i - c_0 \quad (13)$$

Giving the critical fractile below

$$F(q_R^*) = \frac{p - \sum_{i=1}^n w_i - c_0}{p - \sum_{i=1}^n w_i - s_0} \quad (14)$$

The manufacturer has to decide on the quantity of units to make and stock prior to the selling season. The retailer will only pay for the units he needs once demand is realized. The manufacturer may get a salvage value ($s_i < c_i$) for each unsold unit at the end of the selling season. Manufacturer i 's expected profit is shown below assuming that the retailer has sufficient capacity to use any quantity he provides.

$$\Pi_i(x) = \begin{cases} -q_i c_i + w_i x + s_i (q_i - x), & x \leq q_i \\ -q_i c_i + w_i q_i, & x > q_i \end{cases} \quad (15)$$

and the expected profit for manufacturer i is

$$E[\Pi_i] = -q_i c_i + w_i \int_{-\infty}^{q_i} x f(x) dx + s_i \int_{-\infty}^{q_i} (q_i - x) f(x) dx + w_i q_i \int_{q_i}^{\infty} f(x) dx \quad (16)$$

Taking the partial derivative of the above profit equation with respect to q_i and setting it to zero allows us to solve for the critical fractile for decentralized case.

$$\frac{\partial E[\Pi_i]}{\partial q_i} = -c_i + s_i F(q_i) + w_i (1 - F(q_i)) = 0 \quad (17)$$

$$(w_i - s_i) F(q_i) = w_i - c_i$$

(18)

Giving the critical fractile here

$$F(q_i^*) = \frac{w_i - c_i}{w_i - s_i} \quad (19)$$

We can relate the critical fractile (26) to the cost of overage (C_o) and the cost of underage/expedite (C_u) since q_i , c_i , and p are not affected by demand being above or below x .

$$\begin{aligned} C_u &= w_i - c_i \\ C_o &= c_i - s_i \end{aligned} \quad (20)$$

The critical fractile for each supplier i is shown below. Since each component costs (c_i) and salvage cost (s_i) can be different per supplier, the fractile may be different for each supplier.

$$F(q_i^*) = \frac{C_u}{C_u + C_o} = \frac{w_i - c_i}{w_i - s_i} \quad (21)$$

For the coordinated case, the manufacturer's expected profit is shown below.

$$\Pi_i(x) = \begin{cases} -q_i c_i + w_i x + (s_i + \delta_i)(q_i - x), & x \leq q_i \\ -q_i c_i + w_i q_i, & x > q_i \end{cases} \quad (22)$$

The expected profit for manufacturer i is

$$E[\Pi_i] = -q_i c_i + w_i \int_{-\infty}^{q_i} x f(x) dx + (s_i + \delta_i) \int_{-\infty}^{q_i} (q_i - x) f(x) dx + w_i q_i \int_{q_i}^{\infty} f(x) dx \quad (23)$$

Taking the first derivative with respect to q_i and setting it to zero gives

$$\frac{\partial E[\Pi_i]}{\partial q_i} = -c_i + (s_i + \delta_i)F(q_i) + w_i(1 - F(q_i)) = 0 \quad (24)$$

$$w_i - c_i = (w_i - s_i - \delta_i)F(q_i) \quad (25)$$

We can solve the following equality to find the salvage manipulator (δ_i) for each manufacturer.

$$\frac{w_i - c_i}{w_i - s_i - \delta_i} = \frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \quad (26)$$

Therefore, the salvage manipulator that supplier i promises to the retailer is:

$$\delta_i = \frac{-c_i p - s_i \sum_{i=0}^n c_i + p s_i + c_i \sum_{i=0}^n s_i + w_i \sum_{i=0}^n c_i - w_i \sum_{i=0}^n s_i}{\sum_{i=0}^n c_i - p} \quad (27)$$

The retailer's expected profit is shown below.

$$\Pi_R(x) = \begin{cases} -qc_0 + \left(p - \sum_{i=1}^n w_i\right)x + s_0(q - x), & x \leq q \\ -qc_0 + \left(p - \sum_{i=1}^n w_i\right)q - \sum_{i=1}^n \delta_i(x - q), & x > q \end{cases} \quad (28)$$

The expected profit for the retailer is

$$\begin{aligned} E[\Pi_R] = & -qc_0 + \left(p - \sum_{i=1}^n w_i\right) \int_{-\infty}^q x f(x) dx + s_0 \int_{-\infty}^q (q - x) f(x) dx + \\ & \left(p - \sum_{i=1}^n w_i\right) q \int_q^{\infty} f(x) dx - \sum_{i=1}^n \delta_i \int_q^{\infty} (x - q) f(x) dx \end{aligned} \quad (29)$$

Taking the first derivative with respect to q and setting it to zero gives

$$\frac{\partial E[\Pi_R]}{\partial q} = -c_0 + \left(p - \sum_{i=1}^n w_i \right) (1 - F(q)) + \sum_{i=1}^n \delta_i (1 - F(q)) + s_0 F(q) = 0 \quad (30)$$

$$-c_0 + p - \sum_{i=1}^n w_i + \sum_{i=1}^n \delta_i = \left(p - \sum_{i=1}^n w_i + \sum_{i=1}^n \delta_i - s_0 \right) F(q) \quad (31)$$

Using the critical ratio from the centralized control case as the right hand side of the formulation, we get the equality shown below.

$$\frac{p - \sum_{i=1}^n w_i - c_0}{p - \sum_{i=1}^n w_i + \sum_{i=1}^n \delta_i - s_0} = \frac{p - \sum_{i=0}^n c_i}{p - \sum_{i=0}^n s_i} \quad (32)$$

We can solve the above equation to find the salvage manipulators against the retailer capacity. However, we already know each δ_i , so we can just sum those to find the net payment to manufacturers (may be negative).

APPENDIX B

HOLDING AND PROFIT COMBINATION RESULTS FOR

VOLUME DISCOUNT FORWARD BUYS

Table B1: 1% Discount Results

	$\epsilon = 14\%$	$\epsilon = 18\%$	$\epsilon = 22\%$	
h = 8%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,276	\$ 1,304	\$ 1,328	Newsvendor SS
	\$ 1,317	\$ 1,255	\$ 1,216	Golabi
	\$ 1,436	\$ 1,420	\$ 1,401	Gavirneni
	\$ 1,572	\$ 1,525	\$ 1,517	GOGA
h = 18%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,218	\$ 1,253	\$ 1,277	Newsvendor SS
	\$ 1,108	\$ 1,087	\$ 1,074	Golabi
	\$ 1,301	\$ 1,310	\$ 1,326	Gavirneni
	\$ 1,366	\$ 1,365	\$ 1,371	GOGA
h = 20%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,175	\$ 1,213	\$ 1,238	Newsvendor SS
	\$ 1,000	\$ 1,000	\$ 1,000	Golabi
	\$ 1,227	\$ 1,250	\$ 1,265	Gavirneni
	\$ 1,227	\$ 1,250	\$ 1,265	GOGA

Table B2: 2% Discount Results

	ε = 14%		ε = 18%		ε = 22%		
h = 8%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,288	\$	1,314	\$	1,336	Newsvendor SS
	\$	1,386	\$	1,311	\$	1,265	Golabi
	\$	1,464	\$	1,439	\$	1,415	Gavirneni
	\$	1,620	\$	1,561	\$	1,546	GOGA
h = 18%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,229	\$	1,262	\$	1,284	Newsvendor SS
	\$	1,117	\$	1,094	\$	1,079	Golabi
	\$	1,311	\$	1,320	\$	1,335	Gavirneni
	\$	1,384	\$	1,379	\$	1,384	GOGA
h = 20%	\$	1,000	\$	1,000	\$	1,000	No forward buys/SS
	\$	1,185	\$	1,222	\$	1,245	Newsvendor SS
	\$	1,000	\$	1,000	\$	1,000	Golabi
	\$	1,195	\$	1,231	\$	1,255	Gavirneni
	\$	1,195	\$	1,231	\$	1,255	GOGA

Table B3: 5% Discount Results

	$\epsilon = 14\%$	$\epsilon = 18\%$	$\epsilon = 22\%$	
h = 8%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,325	\$ 1,343	\$ 1,362	Newsvendor SS
	\$ 1,497	\$ 1,395	\$ 1,331	Golabi
	\$ 1,547	\$ 1,497	\$ 1,456	Gavirneni
	\$ 1,763	\$ 1,672	\$ 1,635	GOGA
h = 18%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,262	\$ 1,289	\$ 1,307	Newsvendor SS
	\$ 1,144	\$ 1,115	\$ 1,096	Golabi
	\$ 1,383	\$ 1,380	\$ 1,386	Gavirneni
	\$ 1,479	\$ 1,451	\$ 1,444	GOGA
h = 20%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,216	\$ 1,247	\$ 1,267	Newsvendor SS
	\$ 1,000	\$ 1,000	\$ 1,000	Golabi
	\$ 1,245	\$ 1,272	\$ 1,288	Gavirneni
	\$ 1,245	\$ 1,272	\$ 1,288	GOGA

Table B4: 1% Surcharge Results

	$\epsilon = 14\%$	$\epsilon = 18\%$	$\epsilon = 22\%$	
h = 8%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,252	\$ 1,284	\$ 1,310	Newsvendor SS
	\$ 1,258	\$ 1,209	\$ 1,179	Golabi
	\$ 1,332	\$ 1,347	\$ 1,371	Gavirneni
	\$ 1,545	\$ 1,508	\$ 1,491	GOGA
h = 18%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,195	\$ 1,234	\$ 1,261	Newsvendor SS
	\$ 1,090	\$ 1,073	\$ 1,062	Golabi
	\$ 1,272	\$ 1,291	\$ 1,307	Gavirneni
	\$ 1,325	\$ 1,329	\$ 1,337	GOGA
h = 20%	\$ 1,000	\$ 1,000	\$ 1,000	No forward buys/SS
	\$ 1,155	\$ 1,196	\$ 1,224	Newsvendor SS
	\$ 1,000	\$ 1,000	\$ 1,000	Golabi
	\$ 1,231	\$ 1,252	\$ 1,267	Gavirneni
	\$ 1,231	\$ 1,252	\$ 1,267	GOGA

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