

In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

1 1 97 -
James W. ...

7/25/68

A SIMULATION MODEL FOR THE INTERACTION BETWEEN
THE AGE STRUCTURE OF THE POPULATION AND
ECONOMIC GROWTH

A THESIS

Presented to

The Faculty of the Graduate Division

by

Walter Ludwig Zimmermann

In Partial Fulfillment
of the Requirements for the Degree
Master of Science
in the School of Industrial Engineering

Georgia Institute of Technology

December, 1968

A SIMULATION MODEL FOR THE INTERACTION BETWEEN
THE AGE STRUCTURE OF THE POPULATION AND
ECONOMIC GROWTH

Approved:

[Handwritten signature]
Chairman

[Handwritten signature]

[Handwritten signature]

Date approved by Chairman: JAN. 9, 1969

ACKNOWLEDGMENTS

The author would like to express grateful acknowledgment to Dr. Joseph Krol, the thesis advisor, for his permanent guidance, encouragement, and experienced criticism of the work.

Thanks are also due to Dr. Joseph Talavage and Dr. Kong Chu, members of the thesis advisory committee, for their interest and their constructive discussion of the project.

Mrs. Betty Sims deserves thanks for her careful work as typist.

Finally, the author would like to express his gratitude to the Georgia Tech World Student Fund, whose scholarship permitted him to spend a fruitful year of graduate study at the Georgia Institute of Technology.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS.	ii
LIST OF TABLES	v
LIST OF ILLUSTRATIONS.	vi
Chapter	
I. INTRODUCTION.	1
Preliminary Remarks	
Literature Survey	
Definition of the Problem	
Scope and Limitations	
II. SELECTION OF A METHOD OF SOLUTION	9
Possible Methods of Solution	
The Technique of Simulation	
III. THE STRUCTURE OF THE MODEL.	15
Preliminary Remarks	
Formulation of a Mathematical Model	
Flows Represented in the Model	
The Effects of the Population on the Economy	
Summary of the Assumptions Made in the Model	
IV. THE MATHEMATICAL MODEL.	35
The Simulation Language DYNAMO	
Sources of Statistical Data	
Model Validation	
The Population, Education, and Labor Force Sector	
The Public Sector	
The Private Sector	
The Business Sector	
V. EXPERIMENTAL RESULTS.	69
Experimental Design	
Effects of the Test Inputs on Population, Labor Force, School Enrollment and Expenditures for Education	

Chapter	Page
V. EXPERIMENTAL RESULTS (Continued)	
Effects of the Test Inputs on Economic Growth The Behavior of the Model Variables over Time	
VI. CONCLUSIONS AND RECOMMENDATIONS	89
APPENDIX	92
The Computer Program A Sample of a PLOT-Output Some Results of the Model Validation	
BIBLIOGRAPHY	106

LIST OF TABLES

Table	Page
1. Test Inputs and Input Functions	70
2. Total Population (PTL) in Millions as a Function of the Test Inputs for the Birthrate.	71
3. Labor Force (PWRK) in Millions as a Function of the Birthrate and the Participation in the Labor Force.	72
4. Total School Enrollment (PSTUD) in Millions as a Function of the Test Inputs for the Birthrate and the Participation in College Education.	73
5. Expenditures for Education (EDUC) in Billions of Dollars as a Function of the Test Inputs for the Birthrate and the Participation in College Education.	75
6. Gross National Product (GNP), Business Output (PPRD), Business Capital Stock (BKST), Personal Disposable Income (CPI) as a Function of Selected Test Inputs.	76
7. Growth Factors of the Gross National Product (GNP), the Private Income (PINC), and the Per Capita Income (CPI), as a Function of Selected Test Inputs.	77
8. Some Results of the Model Validation.	104

LIST OF ILLUSTRATIONS

Figure		Page
1.	The Basic Economic Relations.	17
2.	Flow of Money	19
3.	Flow of Goods	20
4.	Effects of the Population on the Output of Goods and Services.	23
5.	Effects of the Population on the Demand for Goods and Services.	27
6.	Time Notation in DYNAMO	36
7.	The Population Model.	39
8.	Education and Labor Force	44
9.	The Public Sector	53
10.	The Private Sector.	59
11.	The Business Sector	62
12.	Time Behavior of Some Variables at Rising Birthrate . . .	82
13.	Time Behavior of Some Variable Ratios at Rising Birthrate.	83
14.	Time Behavior of Some Variables at Falling Birthrate. . .	85
15.	Time Behavior of Some Variable Ratios at Falling Birthrate	86

SUMMARY

The study presents a simulation model of a closed national economy with special emphasis on the interaction between the population structure and the performance of the economy.

The model is built on the following assumptions: (a) The size of the labor force depends on the age-structure of the population and a participation rate; (b) the quality of the labor force can be adjusted on the basis of the average number of years of formal education; (c) public spending is a linear function of the size of the population; (d) private spending depends on private disposable income and the average propensity to consume; (e) business investments depend on the expectations for future production and, finally, (f) the stabilization policy of the government guarantees sufficient demand at all times. The output of the economy is computed from the adjusted size of the labor force and the business capital stock by means of a Cobb-Douglas production function.

The model behavior is simulated for a period of 40 years, starting with the initial values of the U. S. economy in 1960. The period from 1960 to 1968 is used to validate the model. The computer program is written in DYNAMO.

The effects on the model behavior are examined in a series of experiments consisting of changes in four exogenous variables, i.e., the birthrate, the participation in the labor force, the participation

in college education, and the average propensity to consume. The test inputs reflect the extremes of the possible behavior of the test variables or a "middle of the way" behavior. The reaction of about ten model variables to the test inputs is discussed in detail with particular reference to the size of the population and the labor force, school enrollment, gross national product, personal disposable and per capita income, savings, and the growth of average personal wealth.

The results illustrate significantly strong effect of the time lag between birth and the entry into the labor force on per capita income in the case of a changing birthrate and the importance of the average propensity to consume in a closed economy.

A number of proposals for further experiments with the model and for possible extensions is made at the end of the study.

CHAPTER I

INTRODUCTION

Preliminary Remarks

Using the definition of economics by Samuelson (21), we may say that the purpose of an economy is to use scarce productive resources to produce various commodities over time and to distribute them for consumption, now and in the future, among the various people and groups in the society.

This economic process is only partly under the control of distinct decision makers, such as the government or the management of private enterprise. A basic influence is exerted by the characteristics of the population, i.e., its age structure, education and spending behavior. The age structure has a dominating influence on the demand for education and the size of the labor force. The technological level of the production is closely related to the training received by the labor force. If the economy is assumed to be closed, then the spending and saving attitudes determine the amount of funds available for investments and thus influence the future growth of the economy.

The author was attracted to this problem area by the situation in his native Germany. In this country, two world wars have strongly distorted the normal age composition of the population through the elimination of considerable parts of two generations. As a result, the

supply of young labor into the labor force, of students into education, and of old persons into retirement has been subject to strong changes over a period of about 30 years. In addition, a shorter workweek and longer compulsory education has been introduced in the past ten years. The coincidence of these events created problems of adjustments that could be solved only by the import of foreign labor.

The experience of Germany is not unique. Most economic problems of the underdeveloped countries can be reduced to the fact that their populations are rapidly growing and that their labor force lacks adequate training. In highly industrialized economies, the effects of the population are less dominating but still far from being negligible. Two problems particular to advanced nations that are strongly affected by the age structure of the population are the social security and retirement plans.

Literature Survey

The interactions between the population and the performance of the economy have occupied economists and political thinkers at least since the first industrial revolution. Adam Smith (23), Ricardo (18), Malthus (11) and Marx (12) have contributed most to the revolution of thought in this field. Due to the social situation in the early stages of industrialization, their concern centered mainly around the problems of the distribution of wealth, the future of poverty, and full employment. Consequently, it dealt largely with the social and philosophical issues involved.

The new situation created by the advent of general affluence in the industrial nations is treated by Galbraith (7). Based on an analysis of the modern industrial system, he proposes new social aims, such as more education and leisure instead of a further increase of consumption that he considers unnecessary at the present rate.

A common characteristic of all these works is that they do not quantify the economic relations they describe. Their purpose is to discover and predict trends in the economic development and to propose directions. This can be an important function. Should for example Galbraith's proposals become generally accepted, the effects on the economy would be considerable.

It is only in recent years that attempts to build large quantitative models of economic systems have been undertaken. This is not an accident. Only the great progress in the areas of economic statistics and computers made quantification possible. The most ambitious effort to describe a total economy at a macroeconomic level of aggregation is represented by "The Brookings SSRC Quarterly Econometric Model of the United States" (5). This model consists of more than 250 definitional and behavioral equations and is intended to make short term predictions. It is still too early to comment on the success of this particular experiment. Another example is the simulation model of the Indian economy by Holland and Gillespie (9). They subdivide the economy into six main sectors. Several economic policies are tested on their effect on gross national product, the balance of payments, and other endogenous variables.

Orcutt, Greenberger, Korbel, and Rivlin (14) present a stochastic microanalytic model of national economy. Individuals and combinations of individuals (spending units), such as families, serve as the basic components of the model. Runs are made for the period from 1950 to 1960 with operating characteristics based on the available statistical data. Comparison of the results with observed aggregate data shows that the microanalytic method does not yet provide reliable predictions. This situation may improve once detailed statistics about the behavior of the microcomponents are available.

The work that comes closest to the intentions of this study has been done by Denison (2) in 1959. He analyzes the growth rates of the gross national product over a period of 50 years, ending in 1959. He divides the total growth rate between four sources: the inputs of capital, land, and labor, and changes in the rate of output per unit of input. Education is considered by adjusting the quality of labor according to their average number of years of formal education. The basis of his allocation is the marginal productivity theory, saying that the contribution of a certain input to production is proportional to the share of the total income created by this input. Thus, if capital income like interests or dividends account for 20 per cent of the national income, then the use of capital contributes equally 20 per cent to the production of goods and services. This theory assumes free competition and equilibrium in the economy.

To obtain his results, Denison makes three further assumptions: he assumes that 60 per cent of the differences in income are due to

differences in education. He assumes further that the effects of the economics of scale increase the growth due to all other causes by 10 per cent, and, finally, the advance of knowledge is assumed to contribute 0.75 percentage points to the growth rate each year. The final allocation of the growth rate among its different sources is projected into the period from 1960 to 1980. Combined with the forecasts for the inputs of land, labor, and capital, this yields predictions for the gross national product in this time period. For the observable years up to 1968, these predictions turn out to be sensibly below reality. The reason may be that the projections were made on the basis of data with great economic disturbances in the form of depressions and wars.

Definition of the Problem

The previous discussion shows that age structure, education, and spending attitudes of the population have a considerable influence on the performance of the economy.

Starting from an initial situation, only the birthrates and the deathrates determine the future age composition of the population. The rates of participation in education and in the active labor force then influence the future quality and size of the labor force. Assuming constant deathrates, and neglecting the effects of spending habits for the moment, the effects of the population on the economy then depend on the behavior of only three variables: the birthrate, the rate of participation in education, and the rate of participation in the labor force.

In a free society, none of the three variables is subject to direct control, but is the result of the sum of a very great number of individual decisions. Only the lower levels of education have compulsory attendance; participation in higher education is voluntary.

Except by simple extrapolations, it is almost impossible to give a long term prediction of the birthrate. The number of factors it depends on is large and, for example, in the case of the change in moral attitudes, hard to quantify. The same holds true to a lesser extent for the participation in education and in the labor force. However, the upper and lower limits of the future behavior of each of the three variables can be estimated. Their actual course will then be somewhere between these limits.

The spending habits of the private sector of the economy can be described by the average propensity to consume (APC). This coefficient indicates how much of the income of a period is spent in the aggregate. A constant APC implies that the level of consumption depends only on the level of current income. Although such an assumption neglects a number of other factors, it describes the long term behavior of aggregate spending quite well. The APC in the U. S. has been fairly constant at 0.93 in the last ten years, but values deviating in either direction by up to 4 per cent seem still possible.

The objective of this study is to compare the growth of a closed national economy when the birthrate, the participation in education, the participation in the labor force, and the average propensity to consume follow different patterns. In more general terms, this

objective means to formulate a dynamic model of an economic system and to compare the effects of four test inputs on the behavior of the endogenous variables of the system.

In order to make valid comparisons, the assumptions about the effects of the population changes have to be justified by statistical evidence or by experience. A test of the whole system with past data that allows us to compare the theoretical results with the real life results is necessary to verify the validity of the assumptions.

Because of the better availability of statistical data, this study is based on the U. S. economy. However, it could easily be adapted to other economies by adjusting the initial values and parameters of the system.

Scope and Limitations

The actual system of a national economy is extremely complex. Ideally, the behavior of each microcomponent--families, individuals, firms, institutions--would have to be considered separately. In this study only aggregate values of the whole economy are used. Single industries or persons may differ from the average economic behavior that is examined here.

By considering both production and demand, this study contains the two major components that determine the level of activity in a free economy. It thus avoids the over-simplified assumption of total economic balance.

However, an assumption about the control of the aggregate output has to be made. Economic control in the predominant present view is one

of the tasks of the federal government. The main tools, public investments and taxation, are applied by Congress individually in each situation. In the study, a stabilization policy fixed by a constant formula has to be applied. This formula may not always reflect the actions that would have been taken in a real world situation. However, it has the great advantage to provide a constant basis for the study of the effects of the various test inputs.

To sum up, since the knowledge about the nature and intensity of the interactions at the aggregate level of an economy is incomplete, a considerable number of assumptions have to be made. These assumptions are summarized in a list at the end of Chapter III.

CHAPTER II

SELECTION OF A METHOD OF SOLUTION

Possible Methods of Solution

The objective of this study is to test the quantitative effects of changes in the behavior and composition of the population on economic growth. Since there are several possible methods of attack on the problem, a selection has to be made.

Two main approaches have been found to be applicable. The first one has previously been used by Denison (2) and might be called the "Growth Function Method." After the contributions of the individual sources to the total economic growth have been quantified by an analysis of past data, a single relation can be set up that describes the gross national product as a function of the inputs of land, labor, and capital with an allowance for education, technical progress and the economies of scale. The response of gross national product to a time series of test inputs can then be calculated.

This method has some major shortcomings. One is that population affects only the size and quality of the labor force but not the consumer demand. But a more important limitation is the fact that by projecting the inputs separately, all interactions that exist between them are neglected. For example, a larger amount of capital is available for investments if the population decides to save more.

The second approach is to describe the interactions between population and the economic growth in a dynamic mathematical model and to simulate the model behavior on a computer. Such a model takes the form of a number of simultaneous equations, some of which govern the model behavior over time by means of time lags or time differentials.

A large number of economic models is already in existence. Naylor, Balintfy, Burdick, and Chu (13) distinguish three main groups: macrodynamic, econometric, and simulation models. In order to provide better perspective for the method later employed in this study a short outline of their discussion is given below.

Macrodynamic models characteristically describe entirely closed systems. To express feedback within the system, they normally take the form of a small number of simultaneous differential equations. They can be solved explicitly with the tools of mathematics. However, the complexity of the models is very limited due to the requirements of mathematical analysis. Because of their high degree of aggregation, these models have mainly pedagogical value and are unsuitable for decision making. A list of some of the more important models of this kind includes Kalecki (10), Samuelson (20), Domar (3), Hicks (8), and Phillips (15).

Simulation models are characterized by a large number of equations and by the fact that they may contain exogenous variables. Because of their size, they depend on electronic computers for a study of their behavior. Simulation in this context may be defined as proposed by Balintfy, Naylor, Burdick, and Chu (13) as:

Simulation is a numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical models that describe the behavior of a business or economic system (or some component thereof) over extended periods of real time.

To study the consequences of alternative policies, analytical solutions of a model are unnecessary. A simulation model will provide the time path of the endogenous variables. It can be made as realistic as the available data permit.

Econometric models differ, according to Cohen (1), from simulation models primarily in the treatment of lagged endogenous variables: "In an economic model, lagged variables are, in effect, treated as exogenous variables. They are viewed to be predetermined by outside forces rather than by earlier applications of the mechanism specified in the model."

An econometric model is used to compute the value of the endogenous variables of one future period. For the next period, all lagged variables will be readjusted to the observed reality.

A simulation model, in contrast, updates the value of its lagged variables within the system by recursion and runs over extended periods of time. It is thus not self-correcting and errors in the initial values are carried on over the whole run.

The preceding discussion shows that for the pseudo-experiments with the economy that are to be carried out in this study, the simulation approach offers considerable advantages over Denison's method. It especially meets three requirements: different policies can easily be tested, interactions and feedback between variables can be included,

and few practical limits to the complexity of the model are imposed. Because of these attributes, the simulation method is used in this study.

The Technique of Simulation

Since simulation is a relatively new technique for problem solving, there are a number of different philosophies about the features of a simulation model for a economic system. Two main issues are concerned with questions whether a model should be entirely closed and how much disaggregation is necessary to obtain meaningful results.

Forrester (6) argues that since economic and industrial systems are closed-loop, information-feedback systems, the models of such a system have to be entirely closed. He states that an independent variable is used as if the response of the system under study had no coupling with the independent variable. In this case, the purpose of the exogenous variable is not to create a more realistic model behavior but to set arbitrarily the external conditions under which the system is to be observed. Consequently, Forrester permits exogenous variables only as test inputs.

Forrester's philosophy does not consider that in economic, industrial, and military models the behavior of some variables may be purposefully predetermined by the policy makers as part of a strategy. The purpose of the simulation is then to determine the best strategy among the alternatives by comparing their effects on the behavior of the system.

In other cases the practical formulation of an entirely closed system is very difficult, if not impossible, because not enough statistical data are available to set up a dependable functional relationship between some variables. In this case, it seems preferable to introduce exogenous variables whose behavior is predicted independently from the model. For example, it can be supposed that the birthrate is mainly a function of income, education, moral attitudes, and progress in birth control, but such a relationship presently cannot be quantified. In such a case it is certainly safer to project the birthrate on the basis of its past behavior.

In the issue of disaggregation, two opposite positions are possible. The microeconomic side argues that each of the microcomponents of an economy (i.e., groups of individuals, single industries, institutions) has a different economic behavior (spending, saving, contribution to production). If each of these groups is considered separately, the predictions for the aggregate behavior can be expected to be more accurate. However, aggregating by summing up the behavior of the microcomponents also adds up all the errors that are included in the prediction not only of the behavior of the microcomponents but also of their importance relative to the total economy. This approach can only be successful if for all the microcomponents sufficient statistical data are available. This is rarely the case at present.

Sasser and Naylor (19) consider three conditions important for successful modelbuilding: experience, trial-and-error methods, and a considerable amount of luck. This viewpoint conforms with the wide-

spread opinion that modelbuilding is as much an art as a science. They permit predetermined input variables that may be controlable (policy variables) or uncontrollable and in both cases are independent from the model variables. The degree of disaggregation should be a trade-off between model realism and model simplicity. This philosophy thus favors a pragmatic approach to modelbuilding and therefore tolerates different model structures within a wide range.

In the same paper the authors present a rationale for the use of simulation in economics. The following nine steps are recommended:

1. Formulation of the problem.
2. Collection and processing of real world data.
3. Formulation of a mathematical model.
4. Estimation of the parameters of the operation characteristics of the model.
5. Evaluation of the model and parameter estimates.
6. Formulation of a computer program.
7. Validation of the model.
8. Experimental design.
9. Analysis of simulated data.

Each of these nine steps is necessary although their order may be flexible. The rationale is followed in this study. Comments on each step will be made along with their execution in the next chapter.

CHAPTER III

THE STRUCTURE OF THE MODEL

Preliminary Remarks

Data collection, formulation of a model, estimation of parameters, and the validation of the model is a closely interrelated process. The available data may have to be adjusted to the needs of the model, but equally the model has to be set up in such a way that it depends only on the most reliable data. This approach increases the probability that the hypotheses underlying the behavioral equations are as tenable as possible. It is also necessary to ensure that only meaningful variables are considered, and that the model is sufficiently disaggregated to include all important variables. All these aims can only be satisfied by an extensive use of trial-and-error methods. The result is finally a model that meets the requirements of the study. This final model is presented in the remainder of this chapter.

Formulation of a Mathematical Model

The model is divided into four sectors:

1. The Population, Education, and Labor Force Sector.
2. The Private Sector.
3. The Public Sector.
4. The Business Sector.

The first sector describes the changes in the age structure of the population, enrollment in all schools, employment and spending in education, and the size and quality of the labor force. To assure mutually exclusive definitions, the Population Sector could be regarded as a subdivision of the Public Sector.

The Private Sector deals with income and spending by private persons, excluding private business. This sector corresponds to what is often referred to as "Household Sector."

The Public Sector, sometimes also called Government Sector, describes income, spending, and employment on all levels of government, i.e., the federal, state, and local level. Government spending includes the expenditures for the public part of the educational system.

The Business Sector describes production and demand in private business and business investments.

Before going into the details of each sector, the structure of the model is presented by showing its main flows and its logical connections in diagrammatic form. Since the computer program is written in DYNAMO, the diagrams mostly use the symbols of Industrial Dynamics. Details of this symbolic representation can be found in (6).

Flows Represented in the Model

The model considers three flows: money, goods, and people.

Figure 1 shows in a simple diagram how these three flows connect the basic sectors of the economy.

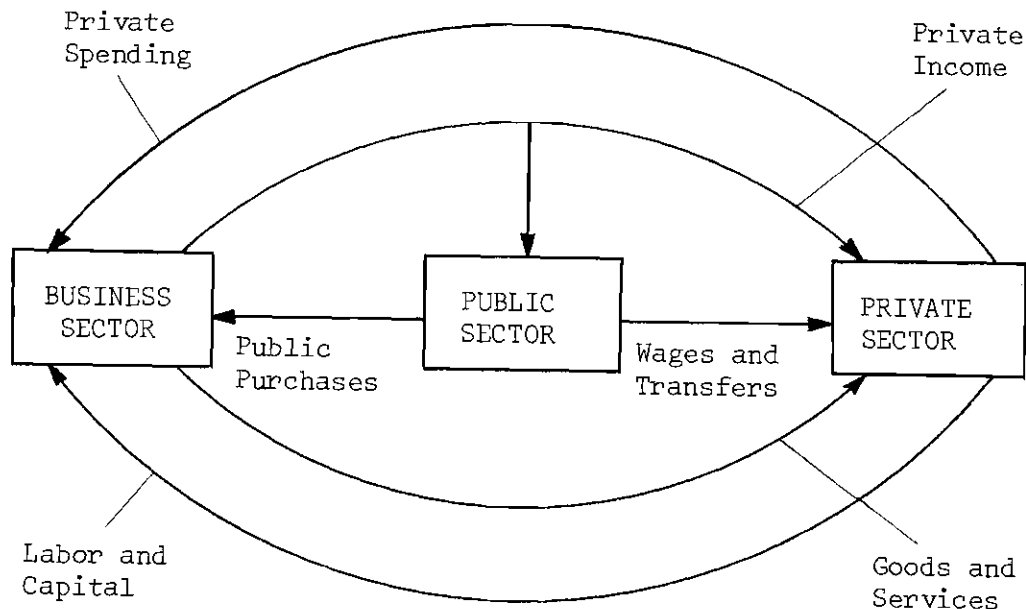


Figure 1. The Basic Economic Relations

The private sector provides the business sector with labor and capital and receives a money income in the form of wages, interest, and dividends. This income is respent for goods and services. Taxes and social security contributions are branched off the private income flow and represent the income of the government. They are respent for public purchases, the public wage bill and transfer payments. Since this is a dynamic process, time enters as a variable. Usually, the assumption is made, that the spending of the private sector in one period flows back as income in the same period, and that this income is partly respent in the next period. This basic model is incomplete,

because it does not consider savings or the possibility of an unbalanced government budget.

In the model used in this study, savings and loan accounts are added to the business, public, and private sectors. Net business investments and government deficits are financed by the savings that take place in the economy. If the total savings are equal to the total net investments, including public deficits, then the economy is in equilibrium in the sense of the classical theory of economics. More savings than investments mean deflation, less savings than investments cause inflation.

Flow of Money

Figure 2 shows the flow of money between the sectors of the model. Since money is accumulated only in the savings and loan accounts, inflow has to be equal to outflow in the private, business, and public sectors. This condition permits to specify one flow in each of the three sectors by a definitional equation. All other money flows have to be specified by behavioral equations. The assumed interactions between the population and the economy are expressed in the relationships. Outputs of specific interest in this study are the flows of private income and spending and the expenditures for education that are a part of government spending.

Flow of Goods

Figure 3 shows the flow of goods and services in the model. Goods and services are produced in the business sector with the inputs of labor and capital. Goods can be stored between the time periods in

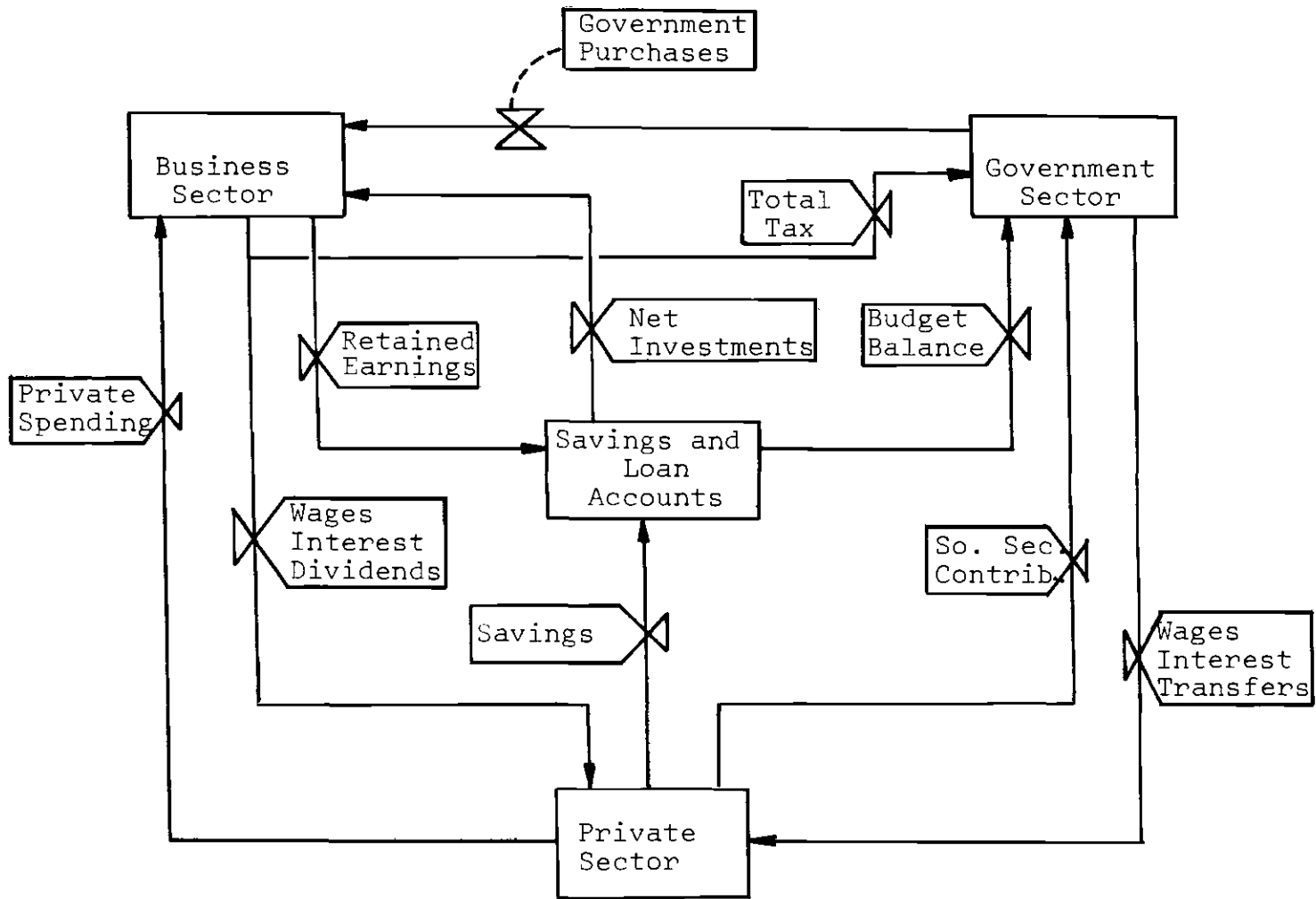


Figure 2. Flow of Money.

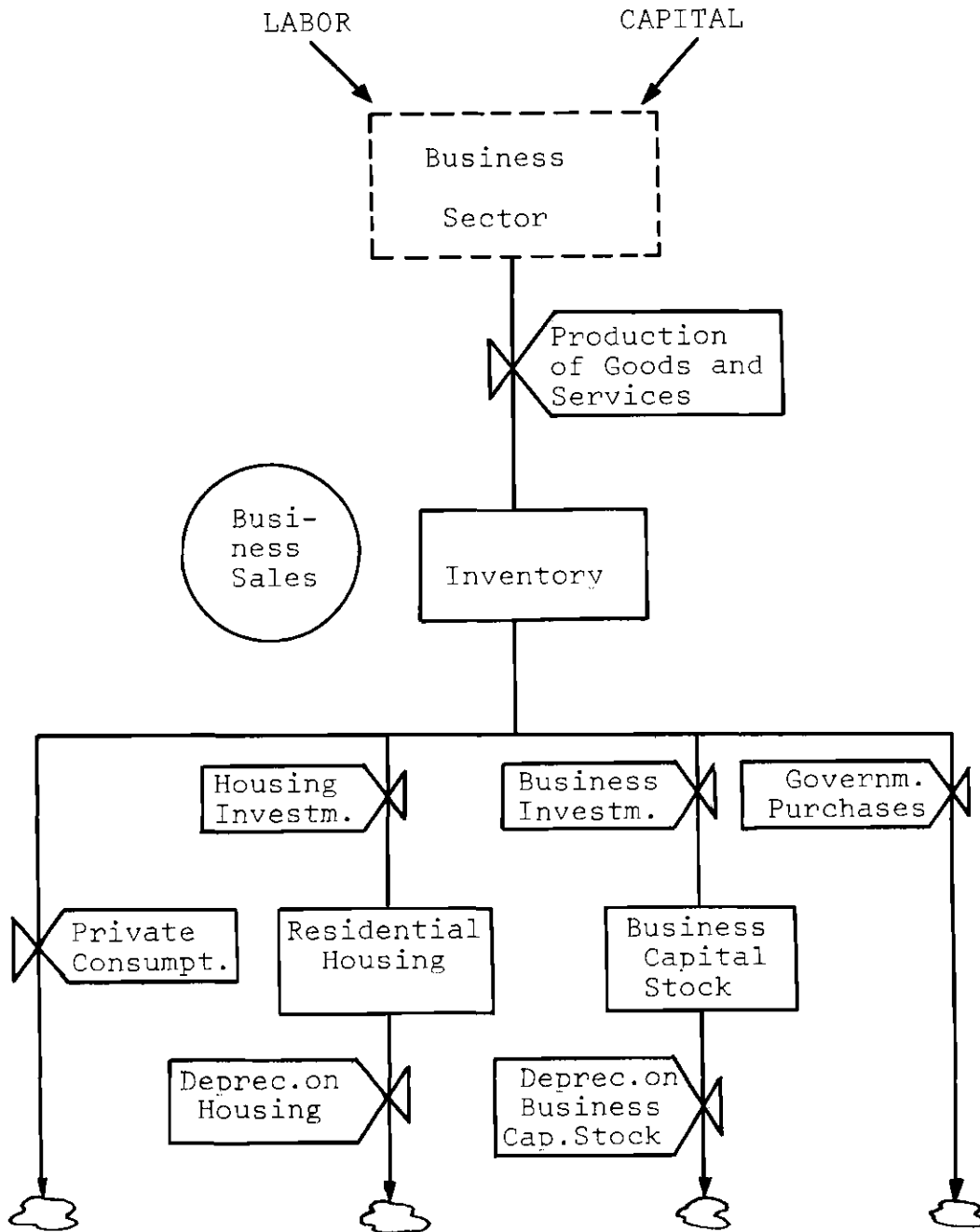


Figure 3. Flow of Goods.

a stock. The production is consumed partly by the government and the private sector, and part is invested into the stock of business capital and into private housing. Depreciation is accounted for both business capital stock and private housing.

Flow of People

The third flow considered in this study is the flow of people. Two distinct flows are set up as shown in Figures 7 and 8, respectively. In a first chain, the birth of people and their movement through the age groups is modeled. This chain updates the age composition of the population that is determined by its initial values and the birth and death occurring as time proceeds. A second chain models the movement of people through all levels of education into the labor force and on into retirement. The school and labor force model is closely connected to the model of the age composition of the population. For example, the rate of students entering elementary school is governed by the number of people that are six years old in the same time period.

The Effects of the Population on the Economy

This section presents the basic logic behind the model used in this study. It also lists and defines the assumptions that have been made. Numerical equations are introduced later in the description of the sectors of the model.

Changes in the composition and the decisions of the population affect the economy in two ways: there is an effect on the possible output of goods and services and a second effect on the demand for goods and services. The two effects are not independent, since

production and demand are closely interrelated.

Effects on the Output of Goods and Services

Figure 4 shows schematically how the output of the economy depends on the population: given an initial age composition of a population, the birthrates and the deathrates determine the age composition of the population over time. The number of students in the various levels of education then depends on the size of the corresponding age groups and, if school attendance is not compulsory, on the rates of participation.

Students graduating and not going on to higher levels of education are assumed to enter the labor force. A part of these graduates, such as fulltime housewives, does not join the labor force and has to be deducted. A very detailed model also would have to consider that people re-enter the labor force at higher ages. However, the number of these entries is small enough to be neglected in this study.

The size of the labor force is closely related to the number of people in working age and thus depends directly on the age structure of the population. Statistics show that for the period after 1950 the American labor force comprised constantly about 60 per cent of the population older than 15 years. Because of its high degree of constancy, this relation is used in the model to determine the size of the labor force. As a result, the number of retirements is computed from the size of labor force and the number of new entries. A straightforward but less practical model would determine the size of the labor force from the number of entries and retirements and the initial value.

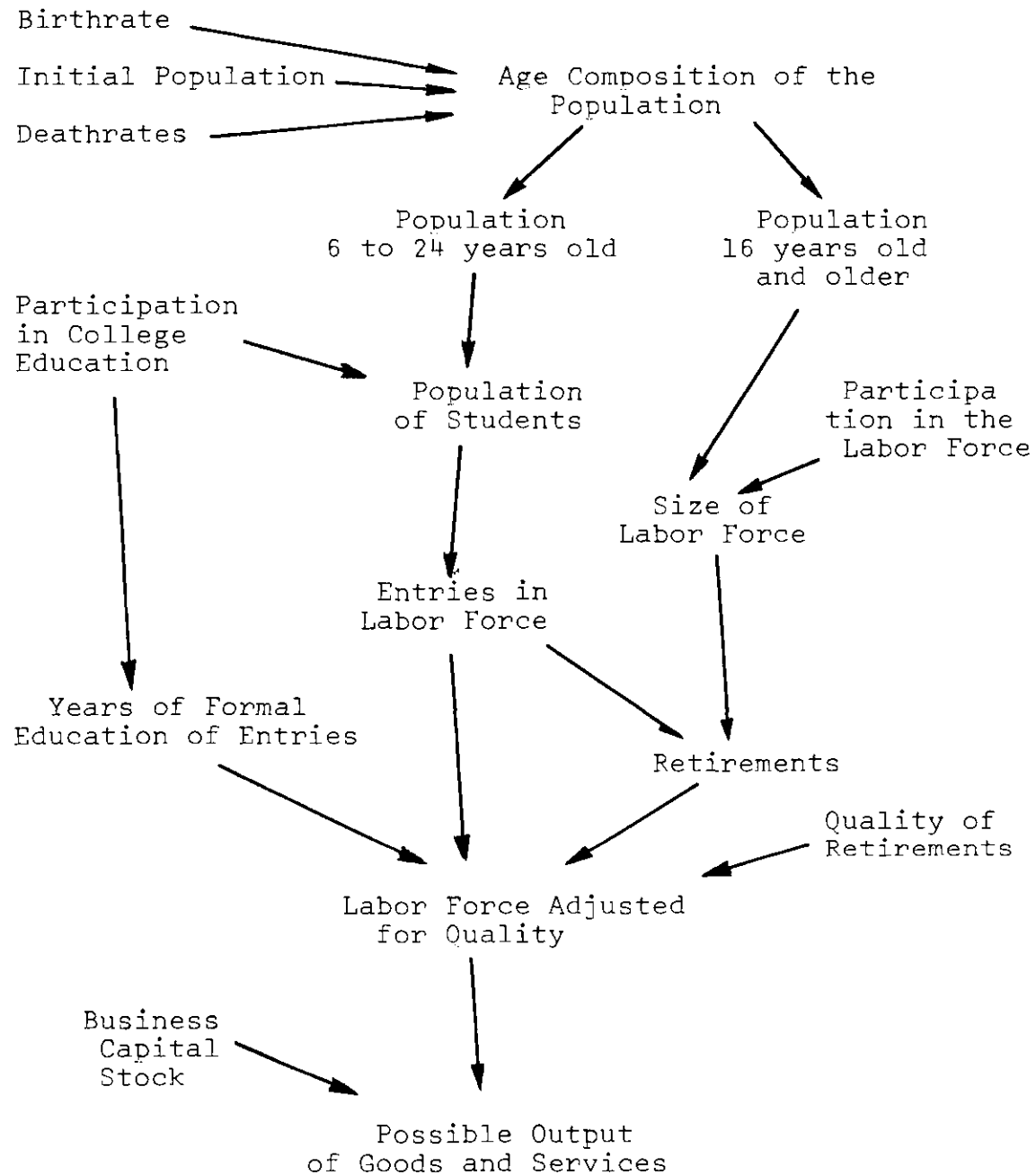


Figure 4. Effects of the Population on the Output of Goods and Services.

The participation rates for smaller groups vary widely with age and sex and are not constant over time. Typically, 98 per cent of the 40 year old men and 50 per cent of the women are at work. While the rates for men in their fifties show a decreasing trend, the rate for women are still growing. A detailed model of the labor force with independent variables consisting of age, sex, and time did not appear to be feasible since a prediction of the trends in the labor force seems impossible.

Studies of past economic growth attribute an important part of the increase in output to improvements in the quality of the labor force. Quality in this context is seen as the ability of the labor force to produce more with the same number of men and the same equipment due to improved training. The model, therefore, adjusts the quality of the labor force on the basis of the average years of school attendance.

It is assumed that an increase in the average number of years in school by 10 per cent increases the quality of the labor force by 9 per cent. This assumption is based on the statistical observation that individuals with a 10 per cent longer than average education have a 9 per cent higher than average income. Since income reflects an individual's contribution to production, it can be concluded that education has the assumed effect on production through the quality of the labor force.

The years of formal education of the entries to the labor force is derived from their school background. Statistical data are available

for the average amount of formal education of the retirements. With this information, an adjustment factor is computed that indicates how much better the labor force is at any time in the simulation as compared to the initial quality of the labor force. The adjusted labor force, together with the capital stock of the economy determine the possible production.

The birthrate, the labor force participation rate, and the rate of participation in higher education are exogenous variables in the model since there is not enough statistical evidence to connect them with the endogenous variables of the model.

The future behavior of the above three variables can be predicted by extrapolation of past data or by estimation of all factors that influence the variable. These predictions may differ considerably.

In general, the participation in higher education is expected to grow. A prediction for the trend of the birthrate is much more difficult to make. It is certainly affected by the number of marriages, which can be related to the age structure of the population. But the important influence of changes in moral attitudes and improvements in birth control methods are impossible to quantify at the present time. In the case of the labor force participation rate, predictions of a decrease that have been made in the past did not materialize if the whole labor force is considered. Earlier retirement of men has been offset by a higher participation of women in the labor force. Also, the predictions for the future shortening of the workweek differ considerably.

No attempt is made in this study to obtain precise predictions for the birthrate, the rate of participation in higher education or the labor force participation rate. The behavior of the three variables is instead given by different test inputs that reflect their probable behavior or the extremes of the possible behavior.

Population and the Demand for Goods and Services

Setting up an aggregate demand function for a whole economy is a difficult task. The reason is that in the sectors of the model (government, business, private) the demand is closely related to the economic activity of the other sectors. Figure 5 shows the interactions that are incorporated in this model.

The aggregate demand for goods and services is composed of the demands of government, business, and the private sector.

The government's purchases of goods and services are the part of the public expenditures that is not directly spent for wages, transfer payments, or interest. The expenditures concerned are for education, services, and public investments. No difference is made in the model between federal, state, and local spending.

Each of the expenditures has to be connected with other variables of the model by an assumed relationship. A rather obvious assumption links the expenditures for education to the number of students and the cost per student. The student population can be easily derived from the population model with the rate of participation in education as test input. Reliable time series are available for the

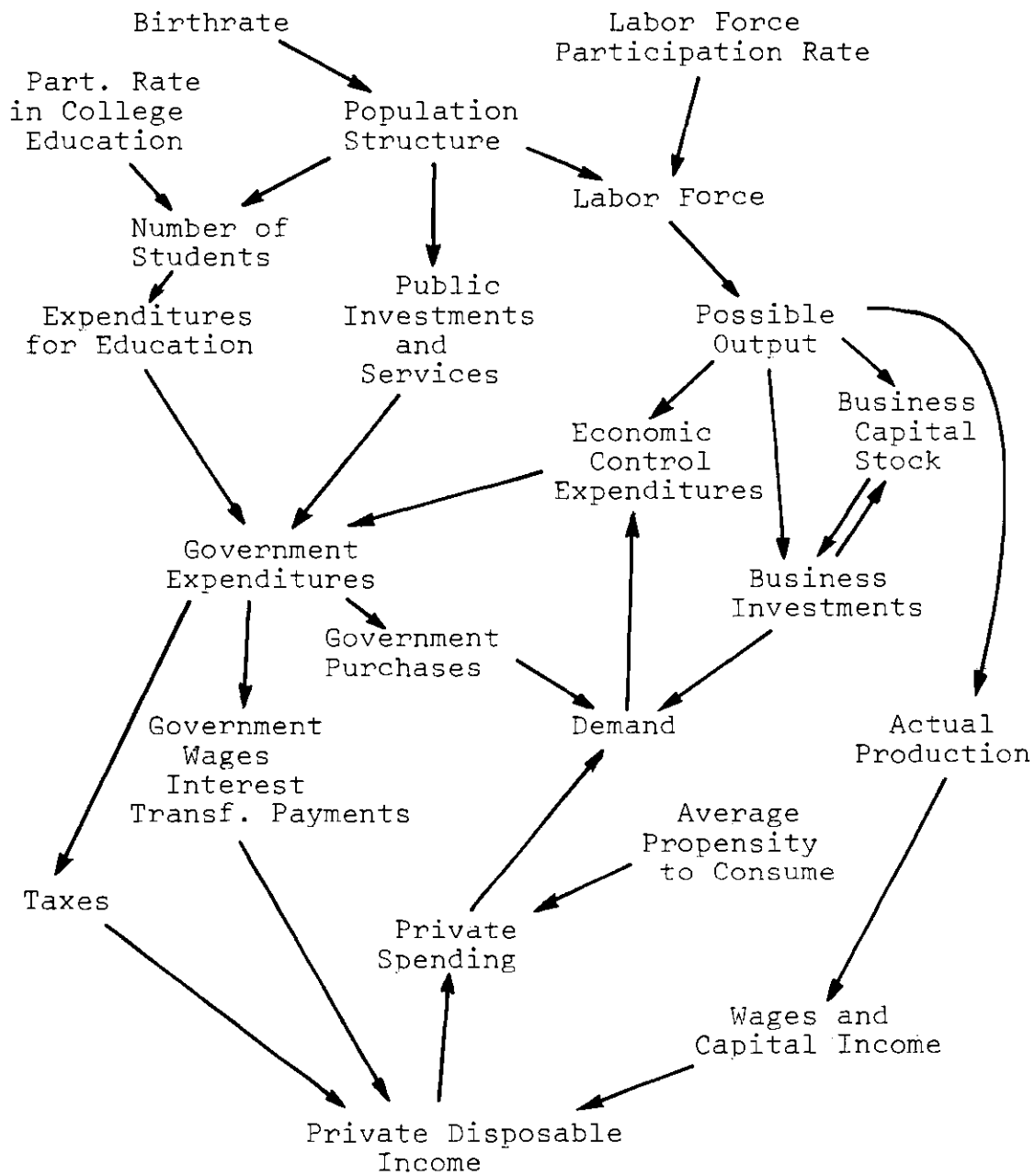


Figure 5. Effects of the Population on the Demand for Goods and Services.

average cost per student in elementary and higher education. These series are extrapolated into the future.

Public services include all government activities other than investments or education. The major items are the cost of administration, police protection, national security, and jurisdiction. The assumption is made that these expenditures have a linear relationship with the size of the population. The function used in the model is based on a graphical fit to the actual expenditures for public services between 1950 and 1960.

Public investments typically are for roads, parks, buildings and equipments. They show a direct linear relation to the size of the total population. In addition, the model assumes that the control of the aggregate output that is exerted by the government is achieved partly by direct public investments. Public investments thus are composed of a basic part that is a function of the total population and an additional part that depends on the economic policy of the government. It is assumed that the government automatically closes the gap between the possible production and the demand from all sources by public investments. The other form of economic control by fiscal policies is through taxation. Tax cuts are used in the model combined with public investments in order to stimulate private demand. Economic stabilization by monetary means is not considered in this model, since it serves mainly to control minor short term fluctuations.

The assumed control mechanism deals only with insufficient demand, not with inflation. If public investments are at their lower

limit and the aggregate demand is still greater than the supply, then the price level will rise. The real amount of goods purchased then is deflated with the new price index, since the model strictly works with constant prices. The price index simply is the ratio between desired and possible sales.

Real world economic stabilization involves many more factors. In particular, stabilizing effects are delayed by a recognition, decision, and execution lag. The actions taken also are not based on the simple result of a rigid formula, since the legislative branch wants to keep government spending under control. The simple model of economic control is preferred to the totally unrealistic assumption of a balanced budget policy and an economy with planned demand.

Business demand originates in the depreciation of the existing capital stock and in the need for additional net investments. The depreciation allowances are a fixed percentage of the capital stock. New investments depend on the expectations for future production. In the model, the control policy of the government assures that the actual production is always equal to the possible production. If an increase in output of 5 per cent is expected, then the size of the business capital stock has to be equally increased by 5 per cent, if the ratio between output and capital input is to be maintained.

Since the economy is assumed to be closed, the amount of investments is limited by the available productive capacity. Only the part of the total production that is not used by the private or public sector can be reinvested into business.

Two further factors influence business investments in real life. In the classical theory of economics, the interest rate was believed to be the regulating force between savings and investments. Modern economics deny this dominating influence of the cost of money on investments, partly because financing from retained earnings has become a widespread practice.

The second factor, technological progress, can force single industries into big investments. Examples are the appearance of semiconductors in electronics or of the jet plane in air transportation. In the aggregate of the whole economy, these jumps are equaled by slowdowns in other industries. In the model, technological progress is considered in the function for the future possible production. Since business investments depend on the future production, it is indirectly affected by technological progress.

Private spending in any period depends only on the income in the previous period and the average propensity to consume. Although this is a very simple form of an aggregate consumption function, it describes the actual dependency of consumption well in an economic situation without extreme fluctuations.

The consumption function creates the dominating feedback loop in the model. Income depends on production, production follows demand, and demand again depends on the income of the previous period. The growth of the system is limited only by the inputs of labor and capital. Depending on the initial conditions this system is not always self-stabilizing. For this reason, government stabilization policy has to assure that the demand always follows the possible output.

The Aggregate Production Function

The central relationship of the model is the aggregate production function. It relates the inputs of labor and capital to the output of goods and services in an entire economy.

The usefulness of such a function has been widely discussed by economists. The main argument against it is that the behavior of a great number of different sectors of an economy cannot be aggregated in a single function. However, since separate production functions exist only for a small part of these sectors, the aggregate production function is at the moment the only tool to express the total output of the economy as a function of the inputs.

Solow (22) set up a production function of the Cobb-Douglas type for the U. S. economy between 1900 and 1955. A Cobb-Douglas function takes the following form:

$$Q = c \cdot L^a \cdot C^b$$

where Q = Total output.

L = Input of labor.

C = Input of capital.

a, b, c = Constants.

Derivation of the function gives:

$$\frac{\Delta Q}{Q} = a \frac{\Delta L}{L} + b \frac{\Delta C}{C}$$

Thus, if a 1 per cent increase of the labor input leads to a 0.7 per cent increase in output, and if a 1 per cent increase in capital input leads to a 0.3 per cent growth of the output, then the production function is

$$O = c.L^{0.7}.C^{0.3}$$

A trend component, z , may be added to represent the annual change due to other sources such as technical progress or "the human factor." Expressed in terms of annual change this yields

$$\frac{\Delta O}{O} = a \frac{\Delta L}{L} + b \frac{\Delta C}{C} + z$$

Solow estimated the value of a as 0.65, of b as 0.35, and of z as 0.015.

In a different approach, Denison (2) adjusts the labor force for quality and assumes that the economies of scale increase the growth due to other factors by 10 per cent. He determines the contributions of the factors to the output with the marginal productivity theory and arrives at a share of 73 per cent for labor, 19.3 per cent for capital, and 6.3 per cent for land. Technical progress and the advance of knowledge other than through quality improvements of the labor force are assumed to contribute 0.75 percentage points to growth each year.

For the model, values of $a = 0.71$, $b = 0.39$, and $z = 0.02$ have been determined by test runs over the period from 1960 to 1968. The

coefficients are thus slightly above Solow's values.

Summary of the Assumptions Made in the Model

To give a summary of the limitations of the model, all major assumptions are listed below:

The economy is closed, i.e., import and export of goods and services as well as of capital are always in balance.

The output of goods and services by private business is computed from the inputs of labor and capital by means of a Cobb-Douglas production function, using a constant trend component to consider autonomous growth and technological progress.

The quality of the labor force is adjusted on the basis of the average number of years of formal education of its members. The size of the labor force is based on the size of the population older than 15 years.

Business investments depend on the expectations for future production, computed by extrapolating the production of the past four years, and on the availability of capital.

A government policy of economic stabilization guarantees sufficient demand at all times.

Public spending for investments and services is a function of the size of the labor force.

Spending of the private (household) sector depends only on the disposable income and the average propensity to consume.

A further simplification of the model is due to the fact that all money flows and levels are expressed strictly in constant prices.

In other words, no prediction about inflation is made.

Assumptions of minor importance to the structure of the model are explained in their respective context.

CHAPTER IV

THE MATHEMATICAL MODEL

The Simulation Language DYNAMO

For the computer simulation of the model behavior, DYNAMO has been chosen as the language. The advantages of DYNAMO that justify its use rather than that of a general purpose language such as FORTRAN or ALGOL are the simplicity of programming, the detailed error detection by the compiler, and the fact that plots of the output can easily be obtained.

The standard version of DYNAMO has been developed by Alexander Pugh III at M.I.T. for an IBM 709 computer. This version is described in the "DYNAMO User's Manual" (17). The experiments for this study were run on a Burroughs B5500 computer with a somewhat modified DYNAMO compiler. One of its advantages is that the standard equation forms that are required for the IBM computer do not have to be used. However, the program still distinguishes between level, rate, and auxiliary equations.

This equation classification is closely connected with the time notation used in DYNAMO which is shown in Figure 6. In a DYNAMO program, each variable name is followed by a time subscript that indicates, to which time or time period in the simulation the variable refers.

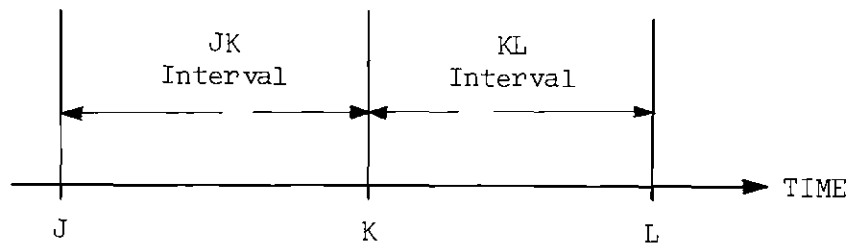


Figure 6. Time Notation in DYNAMO

The time for which the calculations are currently made is called time K. The previous time for which calculations were made is called time J, and the next instant for which calculations will be made is called time L. The intervals between these times are called JK and KL. The length of an interval is DT .

Level equations are equivalent to state variables of a system. In DYNAMO the value of a quantity at time J is updated to its new value at time K, depending on other quantities at time K or in the JK interval. Rates are discrete analogs of derivatives of the type $\Delta X/DT$. They are computed at time K for the interval KL from levels or auxiliaries at time K.

Auxiliary equations tie together the flow of information in the model and perform algebraic operations with rates at time K. They could be substituted into the rate equations but they normally have some physical meaning and thus facilitate the understanding of the model. Due to some characteristics of the Burroughs compiler, rates are sometimes expressed by auxiliary equations in this program.

A special feature of DYNAMO are BOXCAR functions that permit the storage of historical data over more than one period DT.

The equations are identified in the program by a character for the section of the program and a number for the individual equation. A single character to the left of the equation indicates the equation form: R = Rate, L = Level, A = Auxiliary, N = Initial Value, C = Constant, and B = Boxcar.

Sources of Statistical Data

In the formulation of the model, initial values, constants and parameters must be derived from statistical data. The following five sources contain all pertinent information about population, employment, school enrollment, spending, income, production and other items of interest:

1. Statistical Abstract of the United States, 1968.
U. S. Bureau of the Census, Washington, D. C. (1968),
U. S. Government Printing Office.
2. Census of the Population, 1960, U. S. Bureau of the Census,
Washington, D. C. (1962), U. S. Government Printing Office.
3. Manpower Report of the President, U. S. Government Printing Office,
Washington, D. C. (1966).
4. Projection of Educational Statistics to 1976-1977, U. S. Education
Office, Series 10000, No. 36-67. U. S. Government Printing
Office, Washington, D. C. (1968).

5. Economic Report of the President, 1967, U. S. Congress, Joint Economic Committee. U. S. Government Printing Office, Washington, D. C. (1967).

Model Validation

To test the validity of the model, experiments were made with the initial data of the U. S. economy in 1960 and compared with the behavior of the real economy in the observable period from 1960 to 1968. For this purpose, the test inputs were replaced by their actual values. Fifty-four variables were printed. Systematic trial-and-error runs permitted to adjust the equations and constants of the model in such a way that the simulated values of 16 key variables deviated less than 8 per cent from their actual values. In addition, running the model over a period of 60 years permitted to examine whether any variable would behave in an unlikely or impossible way.

A table comparing the simulated and the actual values of a number of variables in the period between 1960 and 1967 can be found in the Appendix.

The Population, Education, and Labor Force Sector

Population

The population sector of the model is shown in Figure 7. Since the desired results of this study depend on changes in the population structure, this sector is very detailed and has more than 100 equations. The following terminology is used in this part of the model:

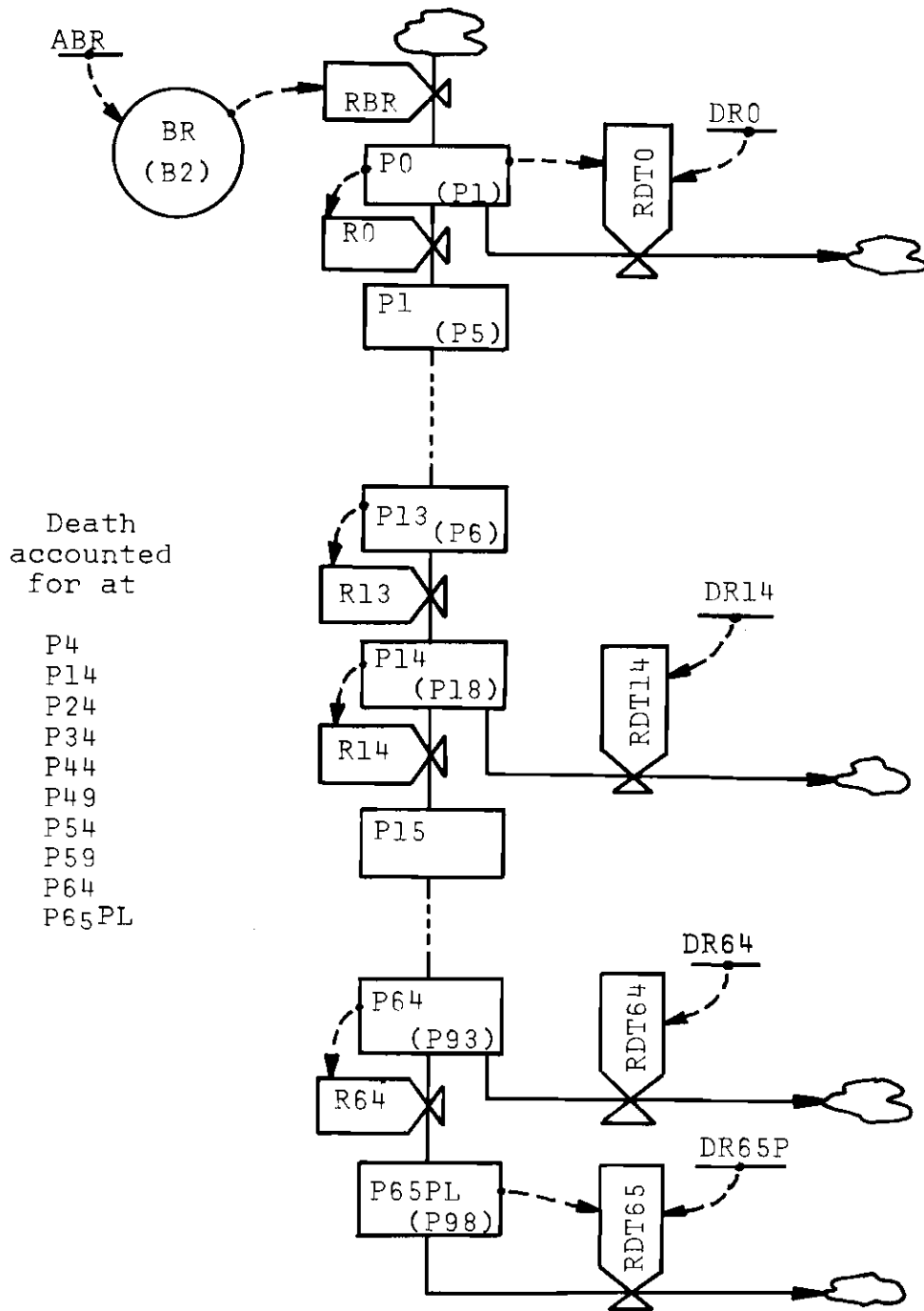


Figure 7. Population Model.

ABR	Asymptotical level of the birthrate (dimensionless)
BR	Birthrate (dimensionless)
BXi	($i=1,2,3,4$); Auxiliary boxcar function
DRi	($i=0,4,14,24,34,44,49,54,59,64$); Deathrate at age i (dimensionless)
Pi	($i=1,\dots,64$); Population i years old (people)
P65PL	Population 65 years old and older (people)
P16PL	Population 16 years old and older (people)
PKID	Population 0 to 5 years old (people)
PPUP	Population 6 to 12 years old (people)
PTL	Total population (people)
Ri	($i=0,1,\dots,64$); Rate of people shifting from age i to age $i+1$ (people/year)
RBR	Number of births (people/year)
RDTi	($i=0,1,4,14,24,34,44,49,54,59,64$); Number of deaths at age i (people/year)

The age distribution of the population, regardless of sex, is modeled by subdividing the population under 65 years old into 65 age groups, one group for each year of age. The population older than 65 years of age (P65PL) is represented in a single group.

A chain of level and rate equations shifts the people from the time of their birth at age zero through all age groups until they leave the final group and die. In the model, death can occur at the ages 0,4,14,24,34,44,49,54,59,64, and 65 plus. The equations that perform the shifts and subtract the losses caused by death are similar at all ages. They are shown below for age three and four:

$$L \quad P3.K = P3.J + (DT)(R2.JK - R3.JK) \quad (P9)$$

$$R \quad R3.KL = P3.K \quad (P10)$$

$$L \quad P4.K = P4.J + (DT)(R3.JK - R4.JK - RDT4.JK) \quad (P11)$$

$$R \quad RDT4.KL = (DR4)(P1.K + P2.K + P3.K + P4.K) \quad (P12)$$

$$C \quad DR4 = 0.0010$$

To cut down on the number of equations, the shift of people between age groups is sometimes performed by boxcar functions, e.g. between the ages 4 and 13:

$$B \quad BX1 = \text{BOXLIN}(10, 1) \quad (P14)$$

$$A \quad BX1 * 1.K = P4.K \quad (P15)$$

$$A \quad P13.K = BX1 * 10.K \quad (P16)$$

BX1 is a boxcar function with ten cars that shifts its cars once in every period DT. Thus, if the size of the four-year-old population is fed into the first car, then after nine annual shifts this age group will be 13 years old and can be found in the tenth car in the train. The boxtrain is then cut off to permit the deduction of the deaths occurring in the age groups from 4 to 13 years old.

The total population and some auxiliary subtotals are found by adding up the size of the individual age groups.

A	$PKID.K = PO.K + \dots + P5.K$	(P99)
A	$PPUP.K = P6.K + \dots + P14.K$	(P100)
A	$P16PL.K = P16.K + \dots + P65PL.K$	(P101)
A	$P2564.K = P25.K + \dots + P64.K$	(P102)
A	$PTL.K = PO.K + \dots + P65PL.K$	(P103)

The initial values of the age groups are taken from the 1960 Census of the population and represent the state of 1960. The same source also provides the values of the deathrates for the 11 subgroups of the population for which death is computed separately. Since the deathrates show only small variations over time, they are represented by constants in the model.

The birthrate is a test input. Its behavior is described by a third-order delay that changes the initial value of the birthrate smoothly towards an asymptotical level with a delay constant of ten years.

R	$BR.K = DELAY3(ABR, 10)$	(P2-1)
N	$BR = 0.0236$	(P2-2)
C	$ABR = 0.0236; 0.030; 0.015$	(P2-3)

Starting with $BR = 0.0236$, which is the value of the birthrate in 1960, three alternative test inputs are examined. First, a constant

birthrate $BR=0.0236$; second, an increase to $BR=0.03$; and third, a decrease to $BR=0.015$.

These three test inputs seem feasible. Actually, the birthrate in the U. S. dropped to a low of $BR=0.017$ in 1968. An international comparison places Hungary lowest with $BR=0.014$ and Brazil highest with $BR=0.04$. For the validation of the model the actual birthrates in the U. S. between 1960 and 1968 were fed into the program with a table function.

Education

The participation of students in education has four economic effects. First, the quality of the student as a potential member of the labor force is increased. Second, his entry into the labor force is delayed. Third, expenditures for schools and teachers are created. And fourth, the teaching and auxiliary staff required by educational institutions is taken away from other production or service areas.

Figure 8 shows how the model evaluates these effects.

The following terminology is used in this part of the model:

CL_i	$(i=1,2,\dots,6)$; Enrollment in the i th year of college (students)
D_1	Dropout rate in high schools (decimal fraction)
DP_i	$(i=1,2,3)$; Droupouts after i th year of high school (students/year)
ECL	New entries to college education (students/year)
$EDUC$	Total expenditures for education (dollars/year)
GBS	Graduates from four-year institutions (students/year)
GHS	Graduates from high schools (students/year)

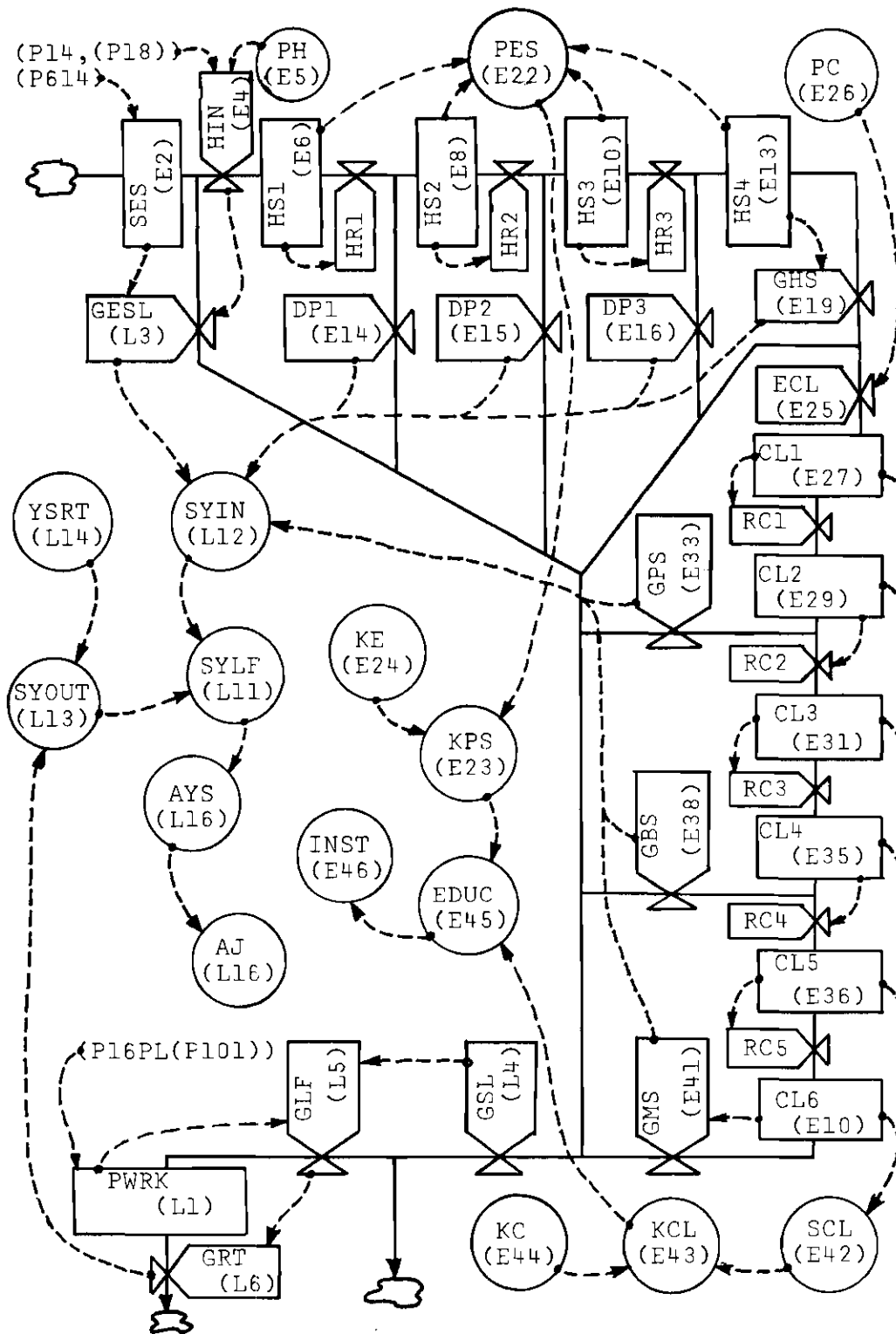


Figure 8. Population and Labor Force.

GES	Graduates from elementary schools (students/year)
GMS	Graduates from six-year institutions (students/year)
GPS	Graduates from two-year institutions (students/year)
HS _i	($i=1,2,3,4$); Enrollment in i th year of high school (students)
HR _i	($i=1,2,3,4$); Shift from i th to ($i+1$)th year in high school (students/year)
HIN	New entries to high schools (students/year)
HDP	High school dropouts (students/year)
INST	Instructional staff, all schools (persons)
KC	Cost per student in college (dollars/student)
KCL	Cost of college education (dollars/year)
KE	Cost per student in lower education (dollars/year)
KPS	Cost of elementary and high school education (dollars/year)
PBS	Graduation rate in four-year institutions (dimensionless)
PC	Participation rate in college (dimensionless)
PCA	Asymptotical level of PC (dimensionless)
PES	Enrollment in elementary and high schools (students)
PH	Participation rate in high schools (dimensionless)
PP	Graduation rate in two-year institutions (dimensionless)
PSTUD	Total enrollment in all educational institutions (students)
RC _i	($i=1,\dots,5$); Shift from i th to ($i+1$)th year of college (students/year)
RS	Ratio of students to total population (dimensionless)
SES	Enrollment in elementary schools (students)
TR	Teacher-student ratio (dimensionless)
T	Time (years)

The number of students in elementary schools (SES) is computed as a constant multiplier of the corresponding age groups:

$$A \quad \text{SES.K}=(1.02)(\text{P6.K}+\dots+\text{P13.K}) \quad (\text{E2})$$

A series of equations analogous to those in the population sector updates the size of all high school and college classes, depending on the numbers of new entries, dropouts, and graduates. The new entries (HIN) into high school are expressed as the 14-year-old population times a participation rate (PH). High school participation, which was 94 per cent in 1960 is assumed to grow to 98 per cent following a third-order delay with delay coefficient ten. The enrollments in each class are updated by a level and a rate equation. The equations for the first class are shown below:

$$R \quad \text{HIN.KL}=(\text{P14.K})(\text{PH.K}) \quad (\text{E4})$$

$$L \quad \text{PH.K}=\text{PH.J}+(\text{DT})(1/10)(0.98-\text{PH.J}) \quad (\text{E5-1})$$

$$N \quad \text{PH}=0.94 \quad (\text{E5-2})$$

$$L \quad \text{HS1.K}=\text{HS1.J}+(\text{DT})(\text{HIN.JK}-\text{HR1.JK}) \quad (\text{E6})$$

$$R \quad \text{HR1.KL}=\text{HS1.K} \quad (\text{E7})$$

Dropout rates from high schools (D_{Pi}) decline slowly with time. Initially, they are about 2.4 per cent in the first year, 7.2 per cent in the second year, and 6.0 per cent in the third year. These figures are statistical averages for the period between 1950 and 1960.

$$A \quad D1.K=0.24-(0.0005)(T.K) \quad (E14)$$

$$A \quad DP1.K=(D1.K)(HS1.K) \quad (E15)$$

$$A \quad DP2.K=(D1.K)(HS2.K)(3.0) \quad (E16)$$

$$A \quad DP3.K=(D1.K)(HS3.K)(2.5) \quad (E17)$$

The number of dropouts (HDP) and the total enrollment in high schools (SHS) is obtained by addition:

$$A \quad HDP.K=DP1.K+DP2.K+DP3.K$$

$$A \quad SHS.K=HS1.K+HS2.K+HS3.K+HS4.K$$

College education is modeled in the same way as high school education. The entries into college (ECL) are computed by a third-order delay equation, dependent on the size of the 17-year-old group (P17), the initial participation rate (PC) and the asymptotical level of the participation rate (PCA).

$$R \quad PC.K=DELAY3(PCA,20) \quad (E26)$$

$$N \quad PC=0.45 \quad (E26-2)$$

$$C \quad PCA=0.55; 0.75 \quad (E26-3)$$

$$A \quad ECL.K=(PC.K)(P17.K) \quad (E25)$$

$$L \quad CL1.K=CL1.J+(DT)(ECL.JK-RC1.JK) \quad (E27)$$

$$R \quad RC1.KL=CL1.K) \quad (E29)$$

Participation in college education is a test input. Two test values of PCA, i.e., the asymptotical level of the participation rate, are examined: PCA=0.55, which reflects approximately the increase in college attendance that can actually be expected; and PCA=0.75, which results in an unlikely, but still feasible high rate of attendance.

Six years of college education are included in the model. Graduates leave after two years (GPS), four years (GBS), and six years (GMS). Doctoral programs are not considered because the number of students is too small to have an impact in relation to other programs. Addition of the enrollment in the six classes gives the total number of college students (SCL).

$$R \quad GPS.KL=(CL2.K)(PP.K) \quad (E34)$$

$$A \quad PP.K=0.45-(0.002)(T.K) \quad (E35)$$

$$R \quad GBS.KL=(CL4.KL)(PBS.K) \quad (E38)$$

$$A \quad PBS.K=0.79-(0.004)(T.K) \quad (E39)$$

$$R \quad GMS.KL=CL6.K \quad (E41)$$

$$A \quad SCL.K=CL1.K+\dots+CL6.K \quad (E42)$$

The number of teachers and staff members (INST) in all educational institutions is computed from the student-teacher ratio (TR). This ratio is considered separately for lower and higher education and is adjusted for time-dependent changes.

$$A \quad \text{INST.K}=(\text{PES.K}+(2)(\text{SCL.K}))(\text{TR.K}) \quad (\text{E46})$$

$$A \quad \text{TR.K}=0.046+(0.003)(\text{T.K}) \quad (\text{E46-1})$$

Labor Force

The model determines the size and the quality of the labor force with the aid of assumptions that have been explained earlier. The following terminology is used in this part of the model:

APR	Asymptotical level of PR (dimensionless)
AJ	Quality adjustment coefficient (dimensionless)
AYS	Average time spent in schools by the members of the labor force (years of school/person)
BXL8	Auxiliary boxcar function
GESL	Entries to the labor force from elementary schools (persons/year)
GLF	Adjusted total entries to the labor force (persons/year)
GSL	Entries to the labor force from all schools (persons/year)
PR	Participation rate in the labor force (dimensionless)
PWRK	Size of the labor force (persons)
SYIN	Total time spent in schools by the entries to the labor force (years)
SYLF	Total time spent in schools by the labor force (years)
SYOUT	Total time spent in schools by the retirees (years)
YSGL	Average time spent in schools by the new entries (years/person)
YSRT	Average time spent in schools by the retirees (years/person)

The size of the labor force (PWRK) is expressed as the product of the population 16 years old and older (P16PL) times the participation rate (PR). Over the last ten years this rate has been constant at 60 per cent with deviations of less than one percentage point.

The participation in the labor force is a test input of the model. In addition to the case of a constant participation factor, the effects of a factor that falls slowly from 60 per cent to an asymptotical rate (APR) of 55 per cent over a period of ten years are examined. A third-order delay is used to describe this falling rate.

$$A \quad PWRK.K=(P16PL.K)(PR.K) \quad (L1)$$

$$R \quad PR.KL=DELAY3(APR,10) \quad (L2-1)$$

$$N \quad PR=0.6 \quad (L2-2)$$

$$C \quad APR=0.60; 0.55 \quad (L2-3)$$

The potential number of entries into the labor force (GSL) results from adding up the number of graduates from all levels of education. Since not all graduates join the labor force, an adjustment of the number of entries by a factor 0.94 is made. People entering the labor force at times other than immediately after graduation from school are not considered in the model.

The number of retirements (GRT) is determined from the number of new entries and the size of the labor force. For this operation the storage of the size of the labor force in the previous year (BXL8*2) is required.

A	$GESL.K = GES.K - (PH.K)(P13.K)$	(L3)
A	$GSL.K = GESL.K + GHS.K - ECL.K + GPS.K + GBS.K + GMS.K + HDP.K$	(L4)
A	$GLF.K = (GSL.K)(0.94)$	(L5)
A	$GRT.K = GLF.K - PWRK.K + BXL8*2.K$	(L6)
B	$BXL8 = BOXLIN(2,1)$	(L7)
A	$BXL8* = 71.5/70.8$	(L8)
A	$BXL8*1.K = PWRK.K$	(L9)

The quality of the labor force is adjusted on the basis of the average years of formal education of its members. The total number of school years of all entries (SYIN) is computed by multiplying the number of graduates at each school level by the corresponding number of years in school, adding the products and adjusting the sum for those graduates that do not join the labor force.

The total number of years in school of the retirees (SYOUT) is the product of the number of retirements (GRT) and their average number of years in school (YSRT) which is based on a retirement age of 65 years and is approximated by a first-order delay function. The total number of years spent in school by the members of the labor force (SYLF) is updated each year by adding the school years of the new entries and subtracting those of the retirees.

$$A \quad SYIN.K=(0.94)((GESL.K)(8)+(GHS.K)(12)+(GPS.K)(14) \quad (L10) \\ + (GBS.K)(16)+(GMS.K)(18)+(HDP.K)(9)-(ECL.K)(12))$$

$$A \quad SYOUT.K=(GRT.K)(YSRT.K) \quad (L11)$$

$$L \quad YSRT.K=YSRT.J+(DT)(1/50)(12.8-YSRT.J) \quad (L14)$$

$$N \quad YSRT=8.5 \quad (L14-1)$$

$$L \quad SYLF.K=SYLF.J+(DT)(SYIN.K-SYOUT.K) \quad (L12)$$

$$N \quad SYLF=(PWRK)(11.2) \quad (L12-1)$$

The average number of years of school attendance of the members of the labor force (AYS) is obtained by dividing the total years of school attendance represented in the labor force (SYLF) by the size of the labor force (PWRK). The quality adjustment factor (AJ) is computed on the assumption that a 10 per cent increase in formal education over the initial value results in a 9 per cent increase in quality. The initial value of the average school attendance is 11.2 years.

$$A \quad AYS.K=SYLF.K/PWRK.K \quad (L16)$$

$$A \quad AJ.K=((AYS.K)(0.9)/11.2)+0.1 \quad (L17)$$

The Public Sector

The public sector describes aggregate income spending and employment on all levels of public administration. Figure 9 shows the diagram of the sector. The following terminology is used in this part of the model:

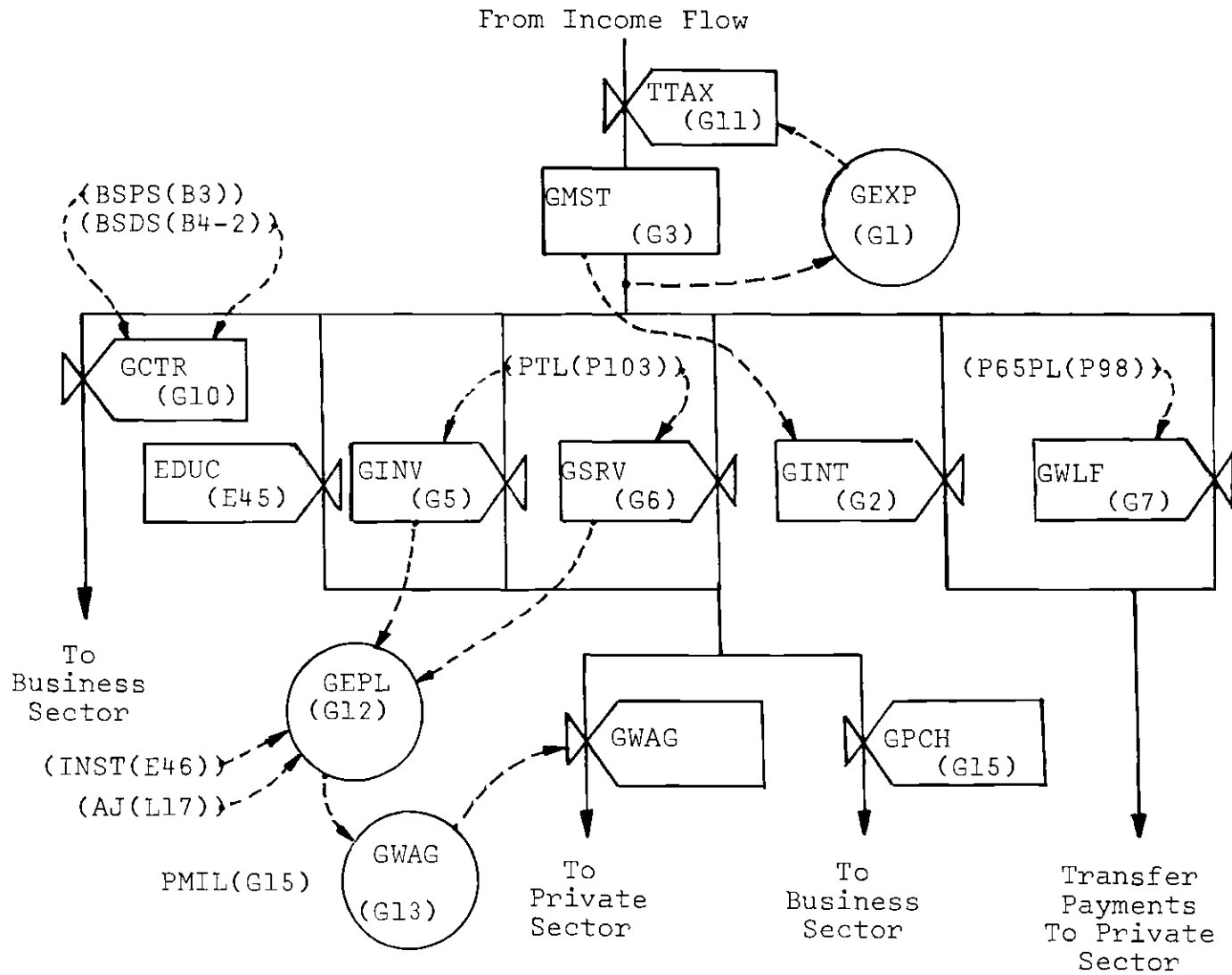


Figure 9. The Public Sector.

GCTR	Expenditures to control aggregate demand (dollars/year)
GEPL	Government employment (people)
GEXP	Public expenditures (dollars/year)
GINT	Interest on public debt (dollars/year)
GINV	Public investments (dollars/year)
GMST	Public debt (dollars)
GPCH	Public purchases from business sector (dollars/year)
GSRV	Expenditures for public services (dollars/year)
GWAG	Public wage bill (dollars/year)
GWLF	Public expenditures for welfare and social security (dollars/year)
PINS	Private payments to social security (dollars/year)
PMIL	Military personnel (people)
TTAX	Total taxes (dollars/year)
WLFP	Payments for welfare and social security (dollars/year)

Total spending is subdivided into public services, investments, education, welfare, interest payments, and expenditures for the stabilization of total demand. The behavior of each of these expense items follows a particular assumption.

The annual interest payments (GINT) are a constant percentage of the public debt (GMST). The public debt is updated by a level equation.

$$A \quad GINT.K = (-0.038)(GMST.K) \quad (G2)$$

$$L \quad GMST.K = GMST.J + (DT)(TTAX.K - GEXP.K) \quad (G3)$$

$$N \quad GMST = -320 \quad (G3-1)$$

Expenditures for public services (GSRV) which cover administration, justice, police, defense, and similar items are separated from public investments (GINV) which cover expenditures for roads, buildings, research, and other long term programs. However, both (GSRV) and (GINV) are assumed to be linear functions of the size of the population (PTL). The growth of public investments is further increased by multiplying (GINV) by one-half of the growth index (GROW) of the gross national product. This assumption seems intuitively reasonable.

$$A \quad GINV.K = -21 + (0.268)(PTL.K)(GROW.K+1)^2 \quad (G5)$$

$$A \quad GSRV.K = 40 + (0.60)(PTL.K) \quad (G6)$$

Expenditures for education (as explained in the education sector, Equation E45) depend on the number of students and the cost per student and are an increasing function of time. It should be noted that both private and public education are included in this term.

The public expenditures for welfare and social security (GWLF) are the difference between the private sector's receipts (WLFP) and its payments to social security (PINS). Both money flows are assumed to be linear functions of the size of the population 65 years old or older. It seems evident that the number of aged people has a great effect on social security expenditures. The size of welfare payments depends strongly on social legislation. Since legal decisions cannot be forecast, this influence is not considered in the model. However, various policies could easily be used as test inputs.

$$A \quad \text{GWLF.K} = \text{WLFP.JK} - \text{PINS.JK} \quad (\text{G7})$$

$$R \quad \text{WLFP.KL} = -90 + (7.8)(\text{P65PL.K}) \quad (\text{G8})$$

$$R \quad \text{PINS.KL} = -60 + (4.1)(\text{P65PL.K}) \quad (\text{G9})$$

Stabilization and control of the aggregate demand in the economy is one of the responsibilities of government. It is realized in the model in two ways: (a) by additional spending, and (b) by lowering of taxes. If the desired demand (BSDS) from all sources (private, business, government) is smaller than the possible supply (BSPS) then additional public investments (GCTR) are assumed to take place. They raise the actual demand to the level of possible supply. No such investments are planned when the desired demand exceeds the supply.

$$A \quad \text{GCTR.K} = \text{MAX}(\text{BSPS.K} - \text{BSDS.K}, 0) \quad (\text{G10})$$

Summation of the six individual items of public spending, i.e., (GSRV), (GINV), (GWLF), (GINT), GCTR), and (EDUC)(0.5) gives the total public expenditure (GEXP). It is worth mentioning that only one-half of the expenditures for education is included in the public expenditures. The second half originates in private educational institutions. Taxation (TTAX) is such that all public expenditures (GEXP) are covered except those for economic control (GCTR), which have to be financed by deficit spending in order to be effective. In the case of insufficient demand, the taxation is further lowered by using the double value of (GCTR). This policy increases private disposable income and stimulates future private demand. Trial runs show that this assumption is

necessary to avoid a permanent growth of public spending for economic control at the expense of the private sector.

$$A \quad GEXP.K = (EDUC.K)^2 + GSRV.K + GINV.K + GWLF.K + GINT.K + GCTR.K \quad (G1)$$

$$A \quad TTAX.K = GEXP.K - (GCTR.K)(2) \quad (G11)$$

The assumed form of economic control by the government assures that no output is lost due to insufficient demand. This assumption means that all labor and capital resources are permanently in full use. Real world economic control has not reached such a state of perfection as exemplified by the unemployment figures. However, considerable progress has been made in applying fiscal controls effectively and it is possible that these assumptions may become quite realistic in the future.

The total public expenditures (GEXP), after subtraction of the pure transfer payments (GWLE) for welfare, social security and interest (GINT) correspond to a production of goods and services. A part of it is created within the public sector by the employees. This part amounts to the total wage bill (GWAG). The rest, that is (GPCH), is bought from the business sector and thus becomes a part of its sales.

Public employment (GEPL) is assumed to be a function of the expenditures for public services (GSRV) and investments (GINV) plus the number of staff members in public education (INST). This term is adjusted for the change in quality of the labor force. The public wage bill (GWAG) equals the average annual salary of \$4,700 times the civilian public employment (GEPL), with a constant number of 2.5

million military personnel added (PMIL). The average salary is assumed to grow at a rate equal to one-half of the growth of the gross national product $(GROW+1)/2$.

$$A \quad GEPL.K=(INST.K+(0.066)(GSRV.K+GINV.K))/AJ.K \quad (G12)$$

$$A \quad GWAG.K=(GEPL.K+PMIL)(4.7)(GROW.K+1)/2 \quad (G13)$$

$$A \quad GPCH.K=GINV.K+GSRV.K+EDUC.K/2-GWAG.K \quad (G14)$$

$$C \quad PMIL=2.5 \quad (G14-1)$$

The Private Sector

The private sector, as shown in Figure 10, describes private income, spending, and savings. The following terminology is used in this part of the model:

APC	Average Propensity to Consume (dimensionless)
HST	Stock of residential housing (dollars)
HDPR	Depreciation on the housing stock (dollars/year)
PCONS	Private consumption (dollars/year)
PINC	Private disposable income (dollars/year)
PINV	Investment in residential housing (dollars/year)
PMST	Accumulated private savings (dollars)
PPCH	Total private spending (dollars/year)
PPCR	Deflated private spending (dollars/year)
PSAV	Private savings (dollars/year)

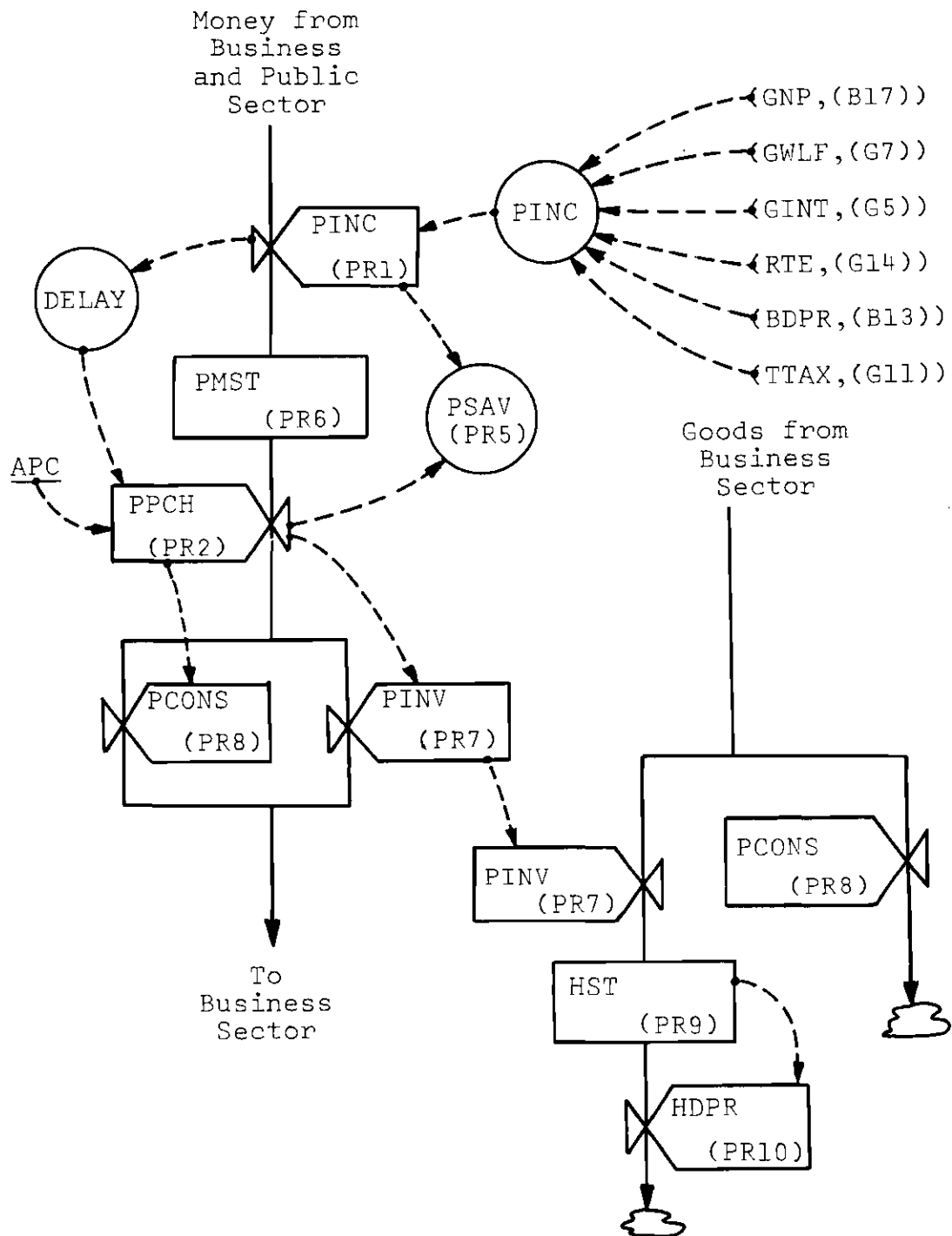


Figure 10. The Private Sector.

Private income, on the basis of the economic cycle shown in Figure 1 equals the total output of the economy less all the leakages of the cycle. The total output or the Gross National Product (GNP) is the total production of the business sector plus the output created in the public sector, expressed by the public wage bill. The leakages include the depreciation and retained earnings of the business sector and the total tax income of the public sector. However, part of the taxes flows back into the private sector as transfer payments for welfare and social security.

$$A \quad \text{PINC.K} = \text{GNP.K} + \text{GWL.F.K} + \text{GINT.K} - \text{RTE.K} - \text{BDPR.K} - \text{TTAX.K} \quad (\text{PR1})$$

Total private spending (PPCH) is assumed to be last year's income (PINC) multiplied by the average propensity to consume (APC). When the supply in the economy is insufficient because of excessive demand, inflation will raise the price level. Deflated spending (PPCR) is computed by dividing (PPCH) by the inflation coefficient (IC) which is the ratio of demand (BSDS) over supply (BSPS). The deflated value of private spending permits comparisons at constant prices.

$$L \quad \text{PPCH.K} = \text{PPCH.J} + (\text{DT})((\text{APC})(\text{PINC.K}) - \text{PINC.J}) \quad (\text{PR2-1})$$

$$N \quad \text{PPCH} = 316 \quad (\text{PR2-2})$$

$$C \quad \text{APC} = 0.93; 0.96; 0.89 \quad (\text{PR3})$$

$$A \quad \text{PPCR.K} = \text{PPCH.K} / \text{IC.K} \quad (\text{PR4})$$

$$A \quad \text{IC.K} = \text{MAX}(\text{BSDS.K} / \text{BSPS.K}, 1) \quad (\text{B22})$$

Personal saving (PSAV) is defined as the difference between income (PINC) and spending (PPCH). Savings (PSAV) are accumulated by a level equation to measure the total financial assets of the private sector (PMST).

$$A \quad PSAV.K = PINC.K - PPCH.K \quad (PR5)$$

$$L \quad PMST.K = PMST.J + (DT)(PSAV.K) \quad (PR6-1)$$

$$N \quad PMST = 210 \quad (PR6-2)$$

Private spending (PCONS) is divided into investments for residential housing, which is assumed to be 15 per cent of total spending, and spending for other purposes. This division permits to obtain an estimate of the per capita value of residential housing by adding investments (PINV) and subtracting depreciation (HDPR) from the previous value.

$$A \quad PINV.K = (0.15)(PPCH.K) \quad (PR7)$$

$$A \quad PCONS.K = (0.85)(PPCH.K) \quad (PR8)$$

$$L \quad HST.K = HST.J + (DT)(PINV.K - HDPR.K) \quad (PR9-1)$$

$$N \quad HST = 750 \quad (PR9-2)$$

$$A \quad HDPR.K = (0.035)(HST.K) \quad (PR10)$$

The Business Sector

The model of the business sector is shown in Figure 11.

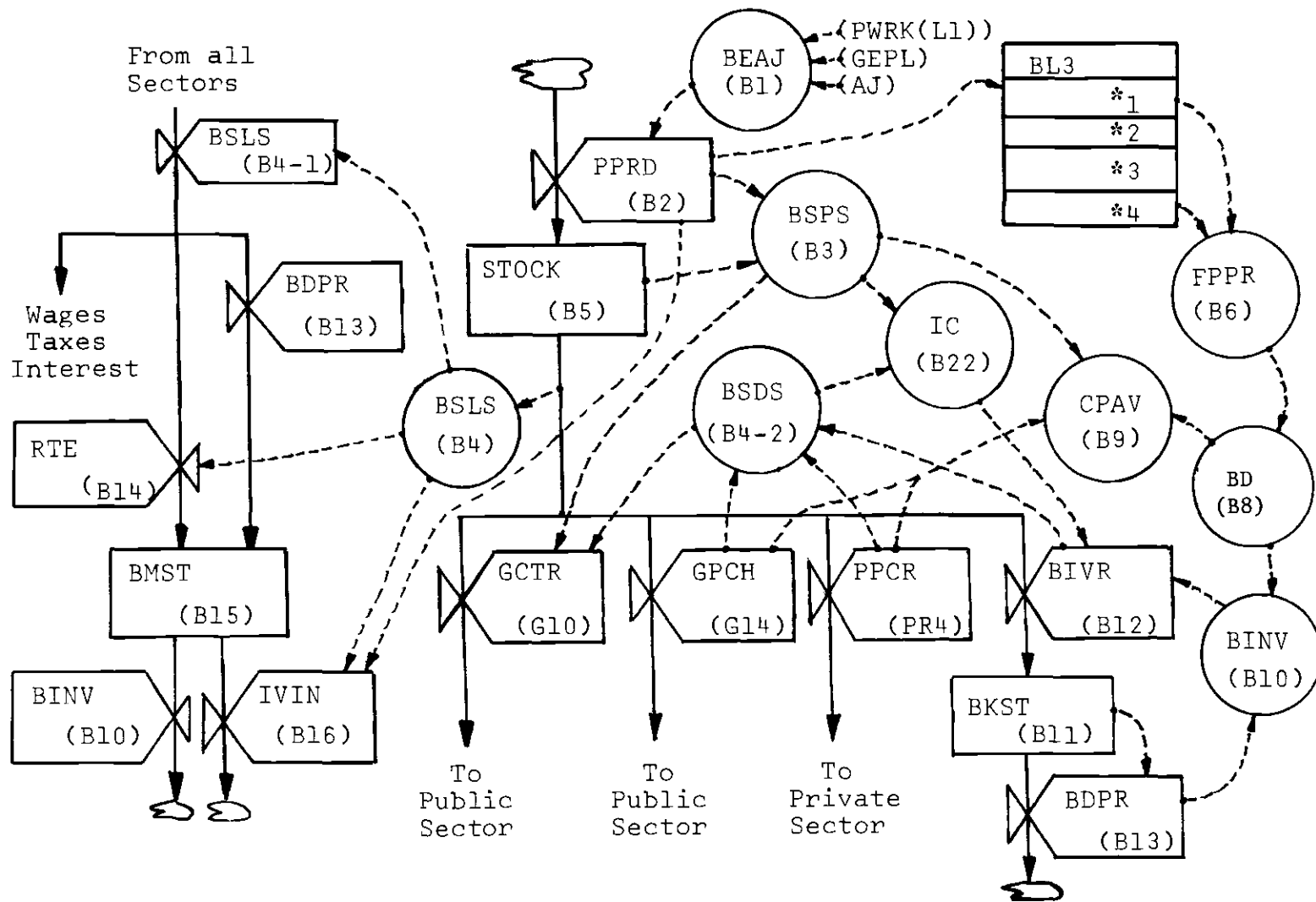


Figure 11. The Business Sector.

It describes production and distribution of goods and services and the flow of money through the sector. The following terminology is used in this part of the model:

BD	Business investment due to depreciation and expected demand (dollars/year)
BEAJ	Business employment adjusted for quality (people)
BDPR	Depreciation on the business capital stock (dollars/year)
BINV	Business investments (dollars/year)
BIVR	Delated business investment (dollars/year)
BKST	Business capital stock (dollars)
BL3	Auxiliary boxcar function
BMST	Business income account (dollars)
BPRD	Actual business production (dollars/year)
BSDS	Desired demand for goods and services (dollars/year)
BSLS	Sales of goods and services (dollars/year)
BSPS	Possible supply of goods and services (dollars/year)
BXLI	Auxiliary boxcar function
CI	Ratio of demand over supply (dimensionless)
CPAV	Free capacity without economic control (dollars/year)
GNP	Gross national product (dollars/year)
BR	Growth rate of the gross national product (dimensionless)
GROW	Growth index of the gross national product (dimensionless)
IC	Inflation coefficient (dimensionless)
IVIN	Increase of inventory (dollars/year)
PPRD	Possible production of goods and services (dollars/year)
PPIN	Auxiliary term in the function for PPRD (dollars/year)

The possible output of goods and services is expressed by a Cobb-Douglas production function (PPRD and PPIN) which depends on the inputs of labor (BEA) and capital (BKST). The labor input (BEAJ) is the available labor force less the civilian and military government employees, adjusted for quality. The exponents for the labor and capital terms in the production function and the trend component are based on Solow's values,^{*} but empirically adjusted by trian runs.

$$A \quad BEAJ.K=(PWRK.K-GEPL.K-PMIL)(AJ.K) \quad (B1)$$

$$L \quad PPRD.K=PPRD.J+(DT)(PPIN.K-(0.980)(PPRD.J)) \quad (B2-1)$$

$$N \quad PPRD=440 \quad (B2-2)$$

$$A \quad PPIN.K=(2.4)EXP(0.71)LOGN(BEAJ.K)+(0.39)LOGN(BKST.K)) \quad (B2-3)$$

The model assumes that 4 per cent of the production is stored as inventory. The possible sales (BSPS) are then equal to 96 per cent of the production (PPRD) plus the existing inventory (STOCK). The stabilization policy of the government assures that the actual sales of goods and services (BSLS) are always equal to the supply (BSPS). Consequently, the actual production (BPRD) is always equal to the possible production (PPRD). The total demand (BSDS) is the sum of the demand of the private sector (PPCH), the public sector (GPCH), and the business sector (BINV). The inventory (STOCK) is updated by a level equation.

^{*}See Chapter III, page 32.

A	$BSPS.K = (0.96)(PPRD.K) + STOCK.K$	(B3)
A	$BSLS.K = BSPS.K$	(B4-1)
A	$BSDS.K = PPCH.K + GPCH.K + BINV.K$	(B4-2)
R	$BPRD.KL = PPRD.K$	(B4-3)
L	$STOCK.K = STOCK.J + (DT)(PPRD.K - BSLS.K)$	(B5-1)
N	$STOCK = 20$	(B5-2)

Gross business investment (BINV) is a function of depreciation (BDPR), expectation for future production (FPPR), and the availability of free capacity (CPAV). The auxiliary term (BD) expresses the effect of depreciation (BDPR) and future possible production (FPPR) on business investments (BINV). First, investments replace the depreciated part of the business capital stock (BDPR). In addition, industry is assumed to increase the business capital stock in line with the expected increase in production in such a way that the growth of the business capital stock is equal to one and one-half times the expected growth of the business production. The model then computes whether any free capacity (CPAV) is still available in the economy at this rate of investment (BD). If this is the case, a decision is made to use one-half of the free capacity for further investments. This assumption seems reasonable since at the same time government lowers taxes to stimulate demand.

A	$BD.K = BDPR.K + (BKST.K)(FPPR.K - PPRD.K)(1.5)/PPRD.K$	(B8)
---	---	------

$$A \quad CPAV.K = BSPS.K - GPCH.K - BD.K \quad (B9)$$

$$A \quad BINV.K = BD.K + CPAV.K / 2 \quad (B10)$$

Future possible production (FPPR) is computed by extrapolation of the past production and expressed as the present possible production (PPRD) plus the average increase in the past three years. The data about the past are stored in a boxcar function (BL3).

$$A \quad FPPR.K = PPRD.K + (BL3*1.K - BL3*4.K) / 3 \quad (B6)$$

$$B \quad BL3 - \text{BOXLIN}(4, 1) \quad (B7-1)$$

$$A \quad BL3*1.K = PPRD.K \quad (B7-2)$$

$$C \quad BL3* = 420 / 390 / 380 / 365 \quad (B7-3)$$

The business capital stock (BKST) is updated by adding new investments (BIVR) and subtracting depreciation (BDPR). In the case of inflation, business investments (BINV) are deflated by means of the ratio of demand over supply (IC). Depreciation (BDPR) is described as a fixed percentage of the capital stock.

$$L \quad BKST.K = BKST.J + (DT)(BIVR.K - BDPR.K) \quad (B11)$$

$$N \quad BKST = 400 \quad (B11-2)$$

$$A \quad BIVR.K = BINV.K / IC.K \quad (B12)$$

$$A \quad BDPR.K = (0.050)(BKST.K) \quad (B13)$$

The left part of Figure 11 shows the flow of money through the business sector. Business income is equal to the total sales. The major part of this income is respent for wages, taxes, interests, and dividends. A part of the income is retained to pay for depreciation (BDPR) and for retained earnings (RTE), which are assumed to be a constant percentage of the total sales (BSLS). This income is spent for investments (BINV) and inventory increase (IVIN). The balance of income and spending of the business sector is accumulated in the business income account (BMST).

$$A \quad RTE.K=(0.03)(BSLS.K) \quad (B14)$$

$$L \quad BMST.K=BMST.J+(DT)(RTE.K+BDPR.K-BINV.K-IVIN.K) \quad (B15-1)$$

$$N \quad BMST=-180 \quad (B15-2)$$

$$A \quad IVIN.K=PPRD.K-BSLS.K \quad (B16)$$

Some supplementary equations not appearing in the diagram shown in Figure 11 are listed within the business sector.

$$A \quad GNP.K=PPRD.K+GWAG.K \quad (B17)$$

$$A \quad GR.K=BXL1*1.K/BXL1*2.K \quad (B18)$$

$$B \quad BXL1=BOXLIN(2,1) \quad (B19-1)$$

$$A \quad BXL1*1.K=GNP.K \quad (B19-2)$$

$$A \quad BXL1*=487/475 \quad (B19-3)$$

$$L \quad \text{GROW.K} = \text{GROW.J} + (\text{DT})((\text{GNP.K}/489) - \text{GROW.J}) \quad (\text{B20-1})$$

$$N \quad \text{GROW} = 1 \quad (\text{B20-2})$$

$$A \quad \text{CI.K} = \text{BSDS.K} / \text{BSPS.K} \quad (\text{B21})$$

$$A \quad \text{IC.K} = \text{MAX}(\text{CI.K}, 1) \quad (\text{B22})$$

The Gross National Product (GNP) is the total output of goods and services in the economy. It consists of the production of the business sector (PPRD) and the wages of the public sector (GWAG). The growth rate (GR) of the (GNP) is defined as the ratio between the present and the previous (GNP). This operation requires a boxcar function in DYNAMO.

A level equation (GROW) accumulates all growth rates as a measure of the total growth relative to the initial output. The demand-supply ratio (CI) is an indicator of the state of balance of the economy.

CHAPTER V

EXPERIMENTAL RESULTS

Experimental Design

Table 1 combines the test inputs and the test input functions in tabular form.

A total of 36 runs resulted from the combination of three test inputs each for the birthrate and two test inputs each for the participation in the labor force and the participation in college education.

All 36 runs have been tested on the computer. The tables which follow present a summary and comparison of the most interesting. The results obtained show how a number of endogenous variables is affected by the four test input variables. These tables are the basis for the discussion of the results.

Only a small sample of the 234 variables of the model is represented in the tables of the results. The variables selected include the absolute growth of the population and the labor force, the growth of enrollment and spending in education, and a number of variables that characterize the absolute and per capita growth of the economy. In addition, growth indices for the gross national product and the private disposable income are presented. All results must be regarded as conditional statements that are true only if the assumptions behind the model are realized.

In the tables, numerical values for expenditures are in billions of dollars; populations are shown in millions. Money values are in constant (1958) dollars. Time $T=0$ represents 1960.

Table 1. Test Inputs and Input Functions

1)	<u>Birthrate</u>	
	Input function:	R BR.KL=DELAY3(ABR,10)
		N BR=0.0236
	Test inputs:	C ABR=0.0236; 0.03; 0.015.
2)	<u>Participation in the Labor Force</u>	
	Input function:	R PR.KL=DELAY3(APR,10)
		N APR=0.6; 0.55.
3)	<u>Participation in College Education</u>	
	Input function:	R PC.KL=DELAY3(PCA,20)
		N PC=0.45
	Test inputs:	C PCA=0.55;0.75.
4)	<u>Private Spending (Average Propensity to Consume)</u>	
	Input function:	A PPCH.K=(APC)(PINC.J)
	Test inputs:	C APC=0.93;0.96;0.89.

Effects of the Test Inputs on Population, Labor Force,
School Enrollment and Expenditures for Education

Effects on the Total Population

The size of the total population is strongly affected by the birthrate. Differences caused by changes in the birthrate grow exponentially with time and are shown in Table 2.

At a constant birthrate $BR=0.0236$, the population changes from 179.9 million in 1960 to 240.2 million after 20 years and to 329.9 million after 40 years. In comparison, in the case of the falling

Table 2. Total Population (PTL) in Millions as a Function of the Test Inputs for the Birthrate

ABR	0.015	0.0236	0.030
Time			
0	179.6	179.6	179.6
5	191.8	191.8	191.8
10	204.3	206.6	208.3
15	213.9	222.7	229.4
20	221.8	240.2	254.8
25	228.5	259.4	284.7
30	235.2	280.6	319.2
35	242.2	304.0	358.6
40	249.8	329.9	403.7

NOTE: Time T=0 represents 1960.

birthrate with ABR=0.015 the total population grows only to 221.8 million after 20 years and to 249.8 million after 40 years. If the birthrate grows according to ABR=0.030, the population grows to 254.8 million after 20 years and to 403.7 million after 40 years. The absolute differences caused changes in the birthrate are considerable in the long run. After 20 years, these differences amount to about 7 per cent of the population at constant birthrate. After 40 years this percentage changes to about 24 per cent.

Effects on the Labor Force

The effects of the birthrate and the labor force participation rate are summarized in Table 3.

Table 3. Labor Force (PWRK) in Millions as a Function of the Birthrate and the Participation in the Labor Force

ABR	0.015		0.0236		0.030	
APR	0.6	0.55	0.6	0.55	0.6	0.55
Time						
0	75.9	75.9	75.9	75.9	75.9	75.9
5	80.2	79.6	80.2	79.6	80.2	79.6
10	85.7	82.5	86.2	83.0	86.5	83.3
15	92.3	85.9	93.3	86.7	94.0	87.4
20	99.4	91.6	100.7	92.8	101.8	93.8
25	105.9	97.2	108.8	99.8	111.2	102.0
30	110.8	101.6	117.8	108.0	123.4	113.2
35	114.9	105.3	127.8	117.1	138.4	126.9
40	118.7	108.8	138.8	127.3	155.9	142.9

NOTE: Time T=0 represents 1960.

It can be observed that changes in the birthrate affect the labor force after a delay of about 18 years. Thus, after 20 years the size of the labor force for the low, constant, and high birthrate and APR=0.6 is 99.4, 100.7, and 101.8 millions, respectively. The differences between these figures are about 1 per cent.

After 40 years, the corresponding values for the size of the labor force are 118.7, 138.8, and 155.9 millions. In this case, the differences have increased to about 14 per cent of the size of the labor force at constant birthrate.

Changing the labor force participation rate from APR=0.6 to the lower level APR=0.55 means that in the long run only 55 per cent of the

population older than 15 years are in the labor force, compared to 60 per cent for $APR=0.6$. The effect is a decrease in the size of the labor force by the corresponding amount of 8.4 per cent in the long run. About half of this decrease becomes effective within 12 years.

Effects on School Enrollment

School enrollment, as shown in Table 4, is affected by the test inputs for the birthrate and for the participation in college education.

Table 4. Total School Enrollment (PSTUD) in Millions as a Function of the Test Inputs for the Birthrate and the Participation in College Education

ABR	0.015		0.0236		0.030		
	PCA	0.55	0.75	0.55	0.75	0.55	0.75
Time							
0		41.1	41.1	41.1	41.1	41.1	41.1
5		47.3	47.3	47.3	47.3	47.3	47.3
10		52.7	52.9	52.7	52.9	52.7	52.9
15		55.8	56.5	57.4	58.0	58.5	59.2
20		54.7	56.1	62.0	63.4	67.6	68.9
25		51.2	56.3	67.3	69.4	80.0	82.1
30		48.2	50.4	73.1	75.8	93.6	97.6
35		47.3	49.5	79.4	82.7	107.3	111.4
40		47.9	50.3	86.2	90.1	121.5	126.6

NOTE: Time $T=0$ represents 1960.

Changes in the birthrate have a very strong effect that appears after a delay of seven years, caused by the time lag between birth and the entry into school. For the low participation in college education $PCA=0.55$ and the decreasing birthrate $ABR=0.015$, enrollment in school

increase by only 16 per cent from 41.1 to 47.9 million in 40 years. School enrollment more than doubles from 41.1 to 86.2 million in the same period if the birthrate stays constant at $ABR=0.0236$. It almost triples from 41.1 to 121.5 in the case of the growing birthrate $ABR=0.030$.

An increase in the participation in college education from $PCA=0.55$ to $PCA=0.75$ has only a small effect on the total school enrollment. $PCA=0.55$ means that the percentage of the 18-year-old population entering college rises gradually from 45 to 55 per cent; it rises to 75 per cent for $PCA=0.75$.

The increased college attendance raises total school enrollment by 2.2 per cent after 20 years and by 4.5 per cent after 40 years.

Effects on Expenditures for Education

Table 5 shows that the effects of the birthrate and the participation in college education on expenditures for education are similar to the results obtained for school enrollment shown in Table 4.

Table 5. Expenditures for Education (EDUC) in Billions of Dollars as a Function of the Test Inputs for the Birthrate and the Participation in College Education

ABR	0.015		0.0236		0.030	
PCA	0.55	0.75	0.55	0.75	0.55	0.75
Time						
0	24.2	24.2	24.2	24.2	24.2	24.2
5	36.6	36.6	36.6	36.6	36.6	36.6
10	51.2	51.6	51.2	51.6	51.2	51.6
15	65.1	67.1	66.5	68.5	67.7	69.9
20	76.0	80.6	83.9	88.5	89.9	94.5
25	83.3	90.9	103.7	111.4	119.7	127.6
30	87.5	96.8	126.5	137.9	158.3	171.3
35	93.5	103.1	152.5	167.7	202.8	222.0
40	103.1	114.9	181.9	201.2	253.1	278.8

NOTE: Time T=0 represents 1960.

Effects of the Test Inputs on Economic Growth

The following section shows in condensed form how some of the endogenous variables of the model change over time under the influence of the test inputs. The results are presented in three tables. Table 6 shows the absolute changes of gross national product (GNP), possible business production (PPRD), business capital stock (BKST), personal disposable income (PINC), and per capita disposable income (CPI) as a function of nine different combinations of test inputs.

Table 6. Gross National Product (GNP), Business Output (PPRD), Business Capital Stock (BKST), Personal Disposable Income (PINC), and Per Capita Disposable Income (CPI) as a Function of Selected Test Inputs (In Billions of Dollars, CPI in Thousands)

APC			0.93					0.96	0.89	
ABR		0.015	0.0236			0.030		0.236		
PCA		0.55	0.75	0.55		0.55		0.55		
APR		0.6	0.55	0.6	0.6	0.55	0.6	0.55	0.6	
	Time									
GNP	0	489	489	489	489	489	489	489	489	
	20	1307	1217	1314	1315	1224	1323	1230	1255	1396
	40	2285	2105	2600	2565	2343	2829	2562	2373	2809
PPRD	0	440	440	440	440	440	440	440	440	440
	20	1168	1091	1160	1162	1084	1158	1080	1108	1234
	40	2008	1862	2136	2109	1945	2172	1995	1958	2298
BKST	0	400	400	400	400	400	400	400	400	400
	20	1849	1728	1836	1841	1720	1835	1712	1613	2171
	40	4417	4070	4600	4556	6168	4644	4222	3703	5798
PINC	0	353	353	353	353	353	353	353	353	353
	20	969	893	950	953	876	941	863	903	1019
	40	1682	1535	1790	1771	1592	1858	1645	1637	1937
CPI	0	1967	1967	1967	1967	1967	1967	1967	1967	1967
	20	4367	4027	3953	3968	3648	3695	3389	3759	4243
	40	6732	6144	5424	5369	4825	4604	4075	4960	5870

NOTE: Time T=0 represents 1960.

To facilitate the comparison of the nine runs, the absolute values of GNP, PINC, and CPI have been transformed into growth factors which are presented in Table 7. These growth factors show the ratio of the variable at time T to its value at time T=0.

Table 7. Growth Factors for the Gross National Product (GNP), the Private Income (PINC), and the Per Capita Income (CPI), as a Function of Selected Test Inputs

APC		0.93						0.96	0.88	
ABR		0.015	0.0236			0.030		0.0236		
PCA		0.55	0.75			0.55		0.55		
APR		0.6	0.55	0.6	0.6	0.55	0.6	0.55	0.6	
Run No.		1	2	3	4	5	6	7	8	9
Time										
GNP	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	20	2.72	2.54	2.74	2.74	2.55	2.76	2.57	2.62	2.90
		(-1%)	(-5.2)	(0.0)		(-5.3)	(+1)	(-6)	(-4.4)	(+6)
	40	4.83	4.45	5.43	5.36	4.90	5.85	5.30	4.96	5.86
		(-10%)	(-17)	(+1.2)		(-9.6)	(+9.2)	(-1)	(-7.4)	(+9.3)
PINC	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	20	2.74	2.53	2.69	2.70	2.48	2.67	2.45	2.56	2.89
		(+1.5%)	(-6)	(-0.3)		(-8)	(-1)	(-9.4)	(-5.5)	(+7)
	40	4.76	4.45	5.06	5.02	4.51	5.26	4.46	4.63	5.48
		(-5%)	(-11)	(+0.9)		(-10)	(+5)	(-11)	(-8)	(+9)
CPI	0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	20	2.22	2.05	2.01	2.02	1.86	1.88	1.68	1.91	2.16
		(+10%)	(+1.5)	(-0.3)		(-8)	(-7)	(-17)	(-5.5)	(+7)
	40	3.42	3.12	2.76	2.74	2.45	2.34	2.07	2.52	2.99
		(+25%)	(+14)	(0.9)		(-10)	(-14)	(-24)	(-8)	(+9)

NOTE: Time T=0 represents 1960.

In addition, the growth factors of all nine runs are compared with their corresponding values in run number four. The term in parentheses below each growth factor in Table 7 shows the difference between the particular growth factor and the corresponding value of run number four expressed as per cent of the value of run number four. The reason

for the selection of run number four as basis for the comparison is the fact that this run best reflects the probable future behavior of the economy.

In run number four the average propensity to consume is at a middle value $APC=0.93$, the birthrate is constant at $BR=0.0236$, participation in college education shows a slow increase to $PCA=0.55$, and the labor force participation is constant at $APR=0.6$. With these test inputs, the gross national product (GNP) increases by a factor 2.74 in 20 years and by a factor 5.36 in 40 years. The increase of the personal disposable income (PINC) is slightly lower since its share of the gross national product decreases in favor of the public and business sector. Per capita disposable income (CPI) grows considerably less than absolute disposable income due to the increase in the total population. Its growth factors are 2.02 after 20 years and 2.74 after 40 years.

Effects of the Birthrate

Comparison of runs one, four, and six permits to evaluate the effects of changes in the birthrate when all other test inputs are unchanged. Lowering the birthrate from the constant value $BR=0.0236$ (Run 4) to $BR=0.015$ (Run 1) results in a smaller gross national product because the labor force grows slower when the birthrate is low. Due to the time lag of about 18 years between birth and the entry into the labor force the difference in gross national product is only 1 per cent after 20 years. The difference rises to 10 per cent after 40 years.

Lowering the birthrate increases personal disposable income by 1.5 per cent after 20 years and decreases this variable by 5 per cent

after 40 years, compared to the case of the constant birthrate. These figures can be explained by the fact that the share of the personal disposable income increases when the birthrate falls because the expenditures of the public sector for education and other services depend directly on the size of the population.

The small growth of the population at the low birthrate increases per capita disposable income considerably when compared to the case of the constant birthrate. The difference is 10 per cent after 20 years and 25 per cent after 40 years.

Run six uses a high birthrate $BR=0.030$. The effects of increasing the birthrate from its constant value $BR=0.0236$ in run four are exactly opposite to the effects of decreasing the birthrate that have been discussed above.

Effects of the Participation in College Education

Comparison of run three and run four permits to evaluate the effects of the participation in college education when all other test inputs are unchanged.

The differences caused by the two test inputs for the participation in college education are very small. Gross national product (GNP) is 1.2 per cent higher after 40 years when the college attendance follows the high test input $PCA=0.75$. Personal disposable (PINC) income is lower in the case of the high participation rate for the first 25 years of the run. After this breakeven point, the previous investments in education begin to pay off. After 40 years, personal disposable income corresponding to the high participation rate in

college education exceeds its value at the low participation rate by 0.9 per cent. The changes in per capita disposable income (CPI) are the same as those in absolute personal disposable income (PINC).

Considering the small relative size of the differences in the results and the simplifications in the assumptions on which they are based, it seems that the results provide no basis for definite quantitative conclusions.

Effects of the Participation in the Labor Force

Comparison of run four and run five in Table 7 permits to evaluate the effects of the participation in the labor force when all other test inputs are kept constant. In run four the labor force participation is at a constant level $APR=0.6$, whereas in run six the participation rate gradually decreases to $APR=0.55$.

The decrease in the size of the labor force that results from a lower participation rate causes a difference in gross national product (GNP) of -5.3 per cent after 20 years and of -9.6 per cent after 40 years. Personal disposable income (PINC) and per capita disposable income (CPI) differ from their value at constant participation in the labor force by -8 per cent after 20 years and by -10 per cent after 40 years.

It is worth noting that personal disposable income falls stronger than gross national product because the absolute amount of spending of the public sector remains unchanged, regardless of the lower total output of the economy.

Effects of the Average Propensity to Consume

Comparison of run five and run eight with run four permits the evaluation of the effects of an upward or downward change in the average propensity to consume when all other test inputs are unchanged.

Since the economy is assumed to be closed, the value of the average propensity to consume determines the amount of capital available for investments. If APC increases, then savings are reduced and less investments are possible. Run eight shows the case of a high average propensity to consume with $APC=0.96$. Because of the low investments the business capital stock grows at a slower rate than at $APC=0.93$ and the gross national product (GNP) is 4.4 per cent lower after 20 years and 7.4 per cent lower after 40 years when compared to run four. The corresponding differences in the personal disposable income (PINC) and the per capita income (CPI) are 5.5 per cent after 20 years and 8 per cent after 40 years.

If the average propensity to consume is lowered to $APC=0.89$, as shown in run nine, then the growth of the gross national product (GNP) increases by 6 per cent in 20 years and by 9.3 per cent in 40 years. The difference in the growth of the personal disposable income (PINC) and the per capita income (CPI) is 7 per cent and 9 per cent, respectively.

The Behavior of the Model Variables over Time

DYNAMO permits plotting the behavior of the model variables over time, a feature which considerably facilitates the understanding of the model.

BR=B, PTL=P, PSTUD=S, PWRK=W, EDUC=E, PINC=I, BKST=K, GNP=G 0

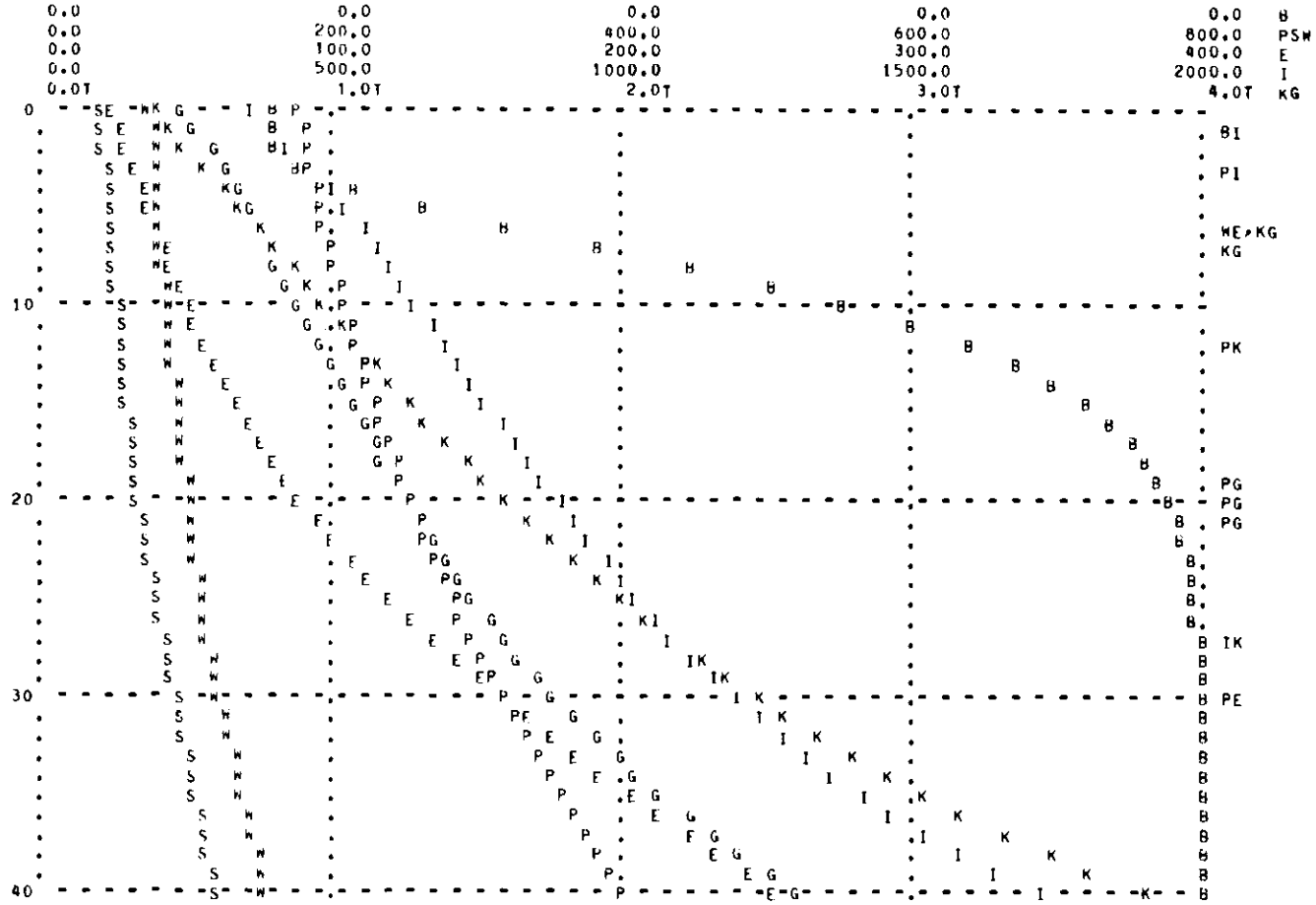
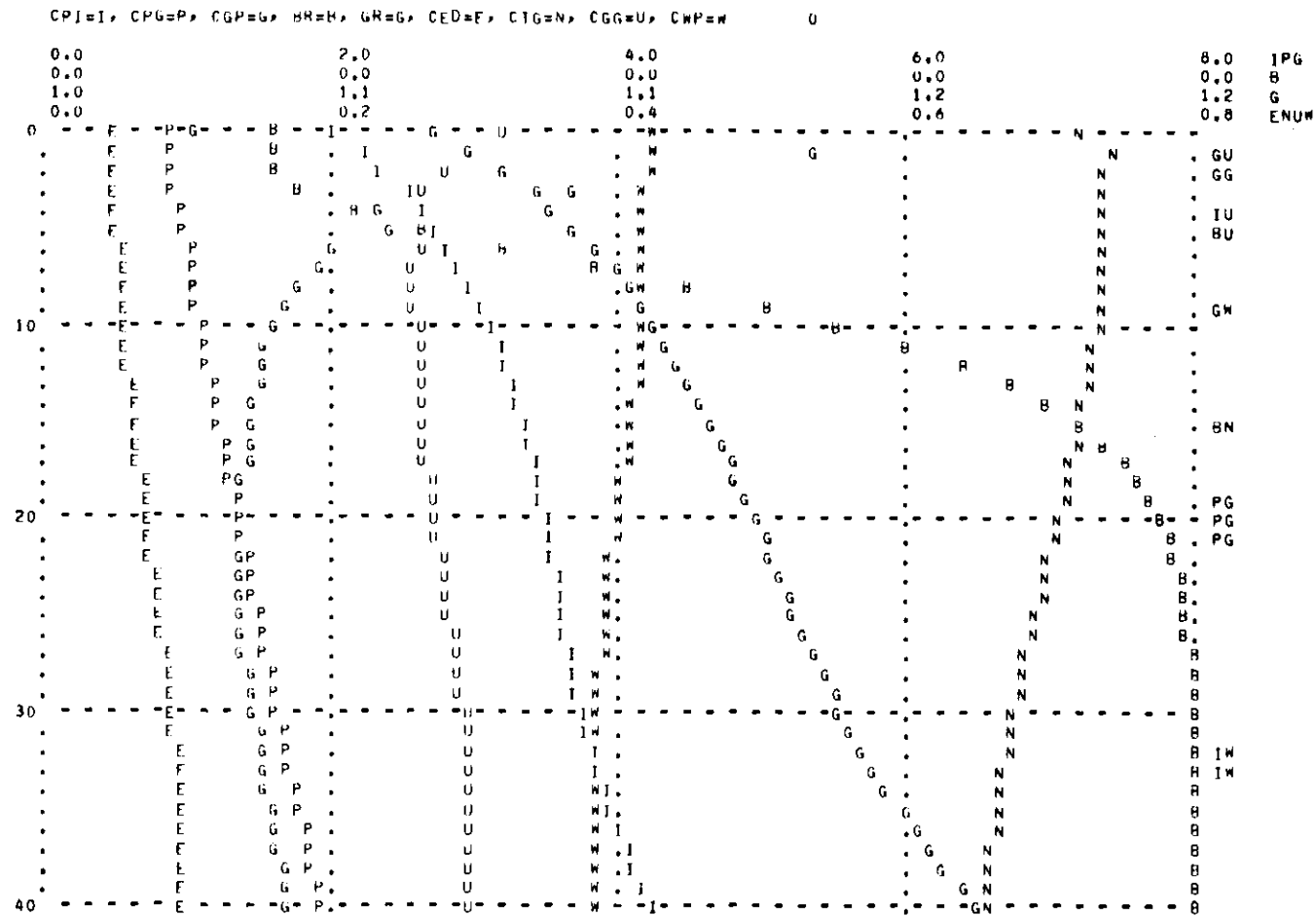


Figure 12. Time Behavior of Some Variables of Rising Birthrate.



FINISHED RUN NUMBER AT 05:52.6314 24 NOVEMBER 1968

Figure 13. Time Behavior of Four Variable Ratios of Rising Birthrate.

The plots of two runs are presented here to show the pattern of growth of the economy under extreme test inputs. The first run, plotted in Figure 12 and 13 has the following test inputs:

$$\text{APC}=0.96; \text{ABR}=0.30; \text{APR}=0.6; \text{PCA}=0.55$$

Figure 12 shows the absolute growth of eight variables. Due to a strong increase of the birthrate ($\text{BR}=\text{B}$), total population ($\text{PTL}=\text{P}$), labor force ($\text{PWRK}=\text{W}$), and total school enrollment ($\text{PSTUD}=\text{S}$) grow fast and at exponential rates.

Gross national product ($\text{GNP}=\text{G}$) and private disposable income ($\text{PINC}=\text{J}$) grow in a linear way for about 20 years. Then the strong population growth becomes effective and causes an increasing growth of the two variables. Spending for education ($\text{EDUC}=\text{E}$) is the fastest growing variable on the plot, caused by the strong growth of school enrollment and the rising rates of spending per student. The increase of the business capital stock ($\text{BKST}=\text{K}$), which is a level variable, cannot be compared with the spending for education ($\text{EDUC}=\text{E}$), which is a rate variable.

The plot of the index values in Figure 13 gives more information about the nature of the growth of the economy.

The fraction of the population that is in the labor force ($\text{CWP}=\text{W}$) falls steadily from 42 to 38.6 per cent. As a result, gross national product per capita ($\text{CGP}=\text{G}$), although steadily growing, show a slowdown at about $T=20$ in the run. The slowdown is repeated in the per capita

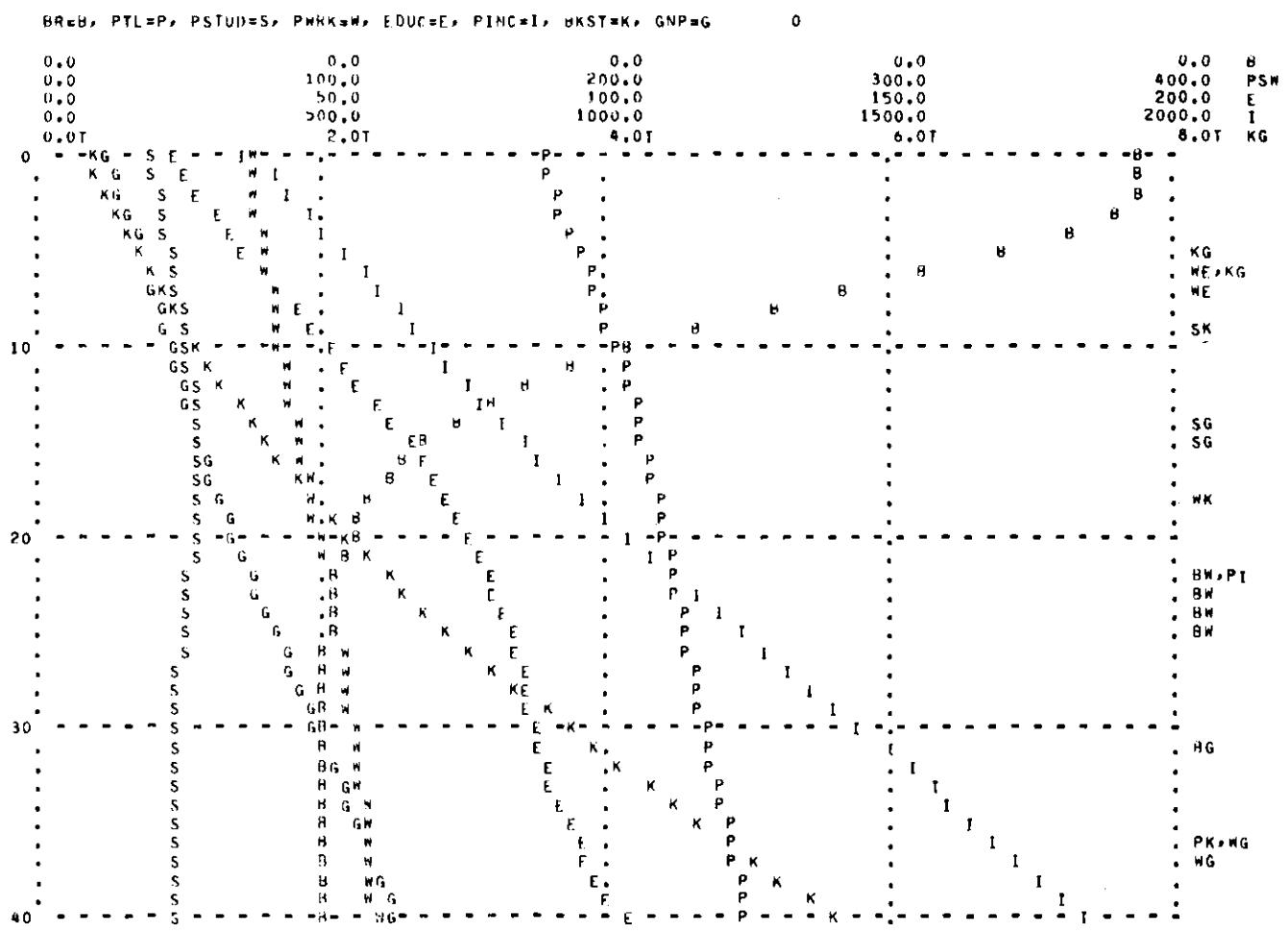
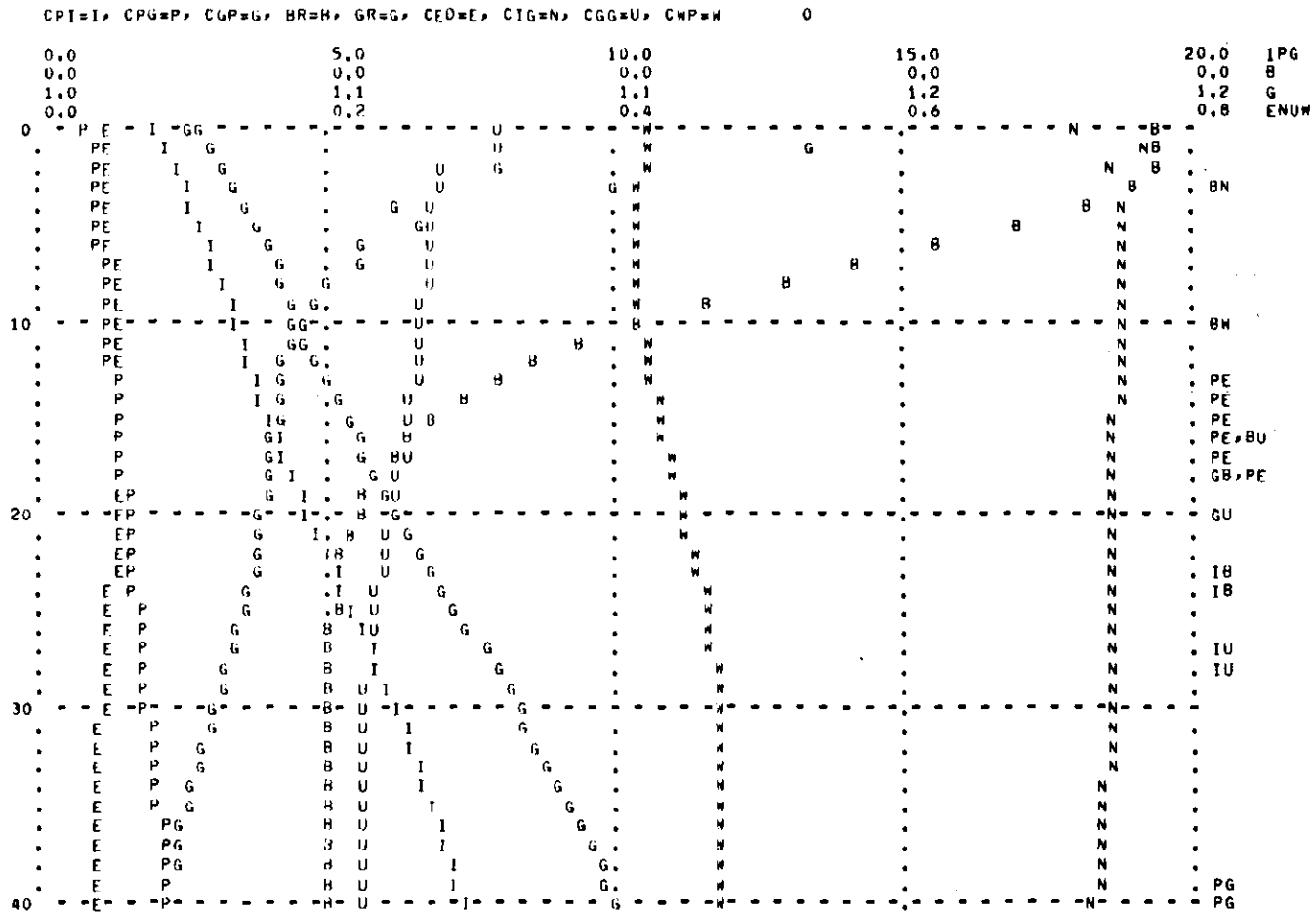


Figure 14. Time Behavior of Some Variables of Falling Birthrate.



FINISHED RUN NUMBER AT 05:52.9525 24 NOVEMBER 1968

Figure 15. Time Behavior of Some Variable - Ratios of Falling Birthrate.

disposable income (CPI=I). Due to the strong population growth, the share of the gross national product that is spent for education, i.e., (CED=E), grows steadily. The same growth can be observed in the share of government spending (CGG=U), after an initial decline in the first five years. As a consequence, the share of private disposable income in the gross national product (CIG=N) declines from 72 to 65.4 per cent in 40 years.

The growth rate of the gross national product (GR=G) shows an initial decline and a subsequent increase after $T=25$ in the run. The decline occurs because the initial slow growth of the labor force is not fully offset by high investments, since the high average propensity to consume limits the available amount of capital. After about 25 years the increase of the birthrate leads to an increase of the labor force big enough to accelerate the growth rates of gross national product again.

In contrast to the first run, a run with test inputs reflecting the opposite extreme is discussed below. The test inputs are:

$$APC=0.89; ABR=0.015; APR=0.6; PCA=0.55.$$

The plot in Figure 14 shows that the decreasing birthrate (BR=B) results in a very slow growth of the total population (PTL=P) and, with the typical delay of about 18 years, it leads to a slowdown in the growth of the labor force (PWRK=W). Total school enrollment reaches a peak after 20 years and then falls back to a slightly lower

level for the remainder of the run. The expenditures for education, although growing steadily, reflect this decrease in enrollment by a decreasing growth between $T=20$ and $T=35$. The low average propensity to consume leads to high investments and, as a consequence, to a strong growth of the business capital stock ($BKST=K$). Gross national product ($GNP=G$) and personal disposable income ($PINC=I$) show a steady absolute growth, since the decrease of population growth is counter-balanced by the strong growth of the business capital stock.

The second plot of the run, shown in Figure 15, shows the steady and almost linear growth of the per capita income ($CPI=I$) and the per capita gross national product ($CGP=P$). This strong growth of the per capita values is mostly due to the slow population growth. In addition, the share of the total population that is in the labor force, i.e. ($CWP=W$), rises during the run from 42 to 47.5 per cent. Although government expenditures per capita ($CPG=P$) grow permanently, the fraction of gross national product spent by government ($CGG=U$) declines during the first 30 years. The share of the gross national product received as private income ($CIG=N$) decreases very slowly due to a gradual increase of the share of business investments. The slow growth of the labor force, in spite of high investments, is not able to sustain a constant rate of growth of the gross national product ($GR=G$). The steady decline of the growth rate is mainly caused by the slow increase of the labor force.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Considerable efforts have been made in recent years in the field of economic model building and econometric research. The intention of this study, to put it into perspective, was not to compete with such endeavors as "The Brookings SSRC Quarterly Econometric Model of the United States." Instead, it has been attempted to build a model that is able to provide some numerical answers to questions in the limited area of the interaction between population and the economy.

The model in its present form is able to generate a great number of numerical results. These results, presented in the previous chapter, should not be viewed as precise predictions but rather as orders of magnitude.

Keeping in mind the limitations of the assumptions of the model, the following conclusions can be summarized from the results of the experiments:

1. The size of the population reacts immediately to a change in the birthrate. School enrollment reacts sensibly after about eight years and the size of the labor force after 18 years. Demand, depending on the size of the population, is affected immediately by a change in the birthrate while production and income, depending on the size of the labor force, are affected with a delay of about 18 years. This

time lag creates an "artificial" change in per capita income and spending whenever the birthrate is not steady. For example, if the birthrate falls, population growth falls 18 years earlier than the growth of economic output and income. Per capita income is "artificially" increased during these 18 years when compared to a constant birthrate.

2. Higher participation in education pays off in higher output. But the possible gain is likely to be very limited, since the present level of participation in education is already high.

3. A lower average propensity to consume permits more investments which lead to an increased growth of the business output and, as a consequence, increased private income. Due to the faster growth of disposable income, private spending at a low average propensity to consume exceeds spending at a high APC after a limited period of time.

As it is the case with probably any economic model, there is considerable room for qualitative improvements and quantitative refinements. The model could be extended or disaggregated in many directions. However, a definite limit is set by the capabilities of the DYNAMO compiler for the Burroughs B5500 computer. Some of the planned extensions of the model had to be abandoned for this reason.

The results presented in this study are only a small part of the results obtainable from experiments for which the model could be adapted without major changes. An interesting experiment of this kind would be to feed the model with the initial values of an underdeveloped

country, such as India, or a country with a different population structure, such as Germany, and compare the results.

A further field where the model is believed to be of great practical value is the testing of different policies of fiscal stabilization. The results of such a study could contribute to the theory of feedback stabilization and demonstrate the usefulness of various policies.

The author feels that aside from the actual output, the process of building an economic model provides the model builder with an excellent understanding of the underlying economic theory and enables him to penetrate deeper into the subject.

APPENDIX

THE COMPUTER PROGRAM

RUN			
NOTE			
NOTE	** SUPPLY-DEMAND ECONOMY **		
NOTE			
NOTE	POPULATION INCREASE EQUATIONS		
52L	$P0.K = P0.J + (DT)(RBR.JK - R0.JK - RDT0.JK + 0)$		P1
37R	$BR.KL = DELAY3(ABR, 10)$	BIRTHRATE	P2-1
N	$BR = 0.0236$	INITIAL VALUE	P2-2
C	$ABR = 0.0236$	ASS VALUE OF BIRTHRATE	P2-3
12R	$RBK.KL = (PTL.K)(BR.K)$	BIRTH	B2-4
12R	$RDT0.KL = (P0.K)(DR0)$		P3
6R	$R0.KL = P0.K$		P4
1L	$P1.K = P1.J + (DT)(R0.JK - R1.JK)$		P5
6R	$R1.KL = P1.K$		P6
1L	$P2.K = P2.J + (DT)(R1.JK - R2.JK)$		P7
6R	$R2.KL = P2.K$		P8
1L	$P3.K = P3.J + (DT)(R2.JK - R3.JK)$		P9
6R	$R3.KL = P3.K$		P10
52L	$P4.K = P4.J + (DT)(R3.JK - R4.JK - RDT4.JK + 0)$		P11
19R	$RDT4.KL = (DR4)(P1.K + P2.K + P3.K + P4.K)$		P12
6R	$R4.KL = P4.K$		P13
37B	$BX1 = BOXLIN(10, 1.0)$		P14
6A	$BX1*1.K = P4.K$		P15
6A	$P13.K = BX1*10.K$		P16
6R	$R13.KL = P13.K$		P17
52L	$P14.K = P14.J + (DT)(R13.JK - R14.JK - RDT14.JK + 0)$		P18
6R	$R14.KL = P14.K$		P19
10A	$PAX1.K = BX1*2.K + BX1*3.K + BX1*4.K + BX1*5.K + BX1*6.K + BX1*7.K$		P20
10A	$P514.K = PAX1.K + BX1*8.K + BX1*9.K + BX1*10.K + P14.K + 0$		P21
12R	$RDT14.KL = (P514.K)(DR14)$		P22
37B	$BX2 = BOXLIN(10, 1.0)$		P23
6A	$BX2*1.K = P14.K$		P24
6A	$P23.K = BX2*10.K$		P25
12R	$RDT24.KL = (P1524.K)(DR24)$		P26
6R	$R23.KL = P23.K$		P27
52L	$P24.K = P24.J + (DT)(R23.JK - R24.JK - RDT24.JK + 0)$		P28
6R	$R24.KL = P24.K$		P29

10A	$PAX2.K = BX2*2.K + BX2*3.K + BX2*4.K + BX2*5.K + BX2*6.K + BX2*7.K$	P30
10A	$P1524.K = PAX2.K + BX2*8.K + BX2*9.K + BX2*10.K + P24.K$	P31
37B	$BX3 = BOXLIN(10,1.0)$	P32
6A	$BX3*1.K = P24.K$	P33
6A	$P33.K = BX3*10.K$	P34
6R	$R33.KL = P33.K$	P35
52L	$P34.K = P34.J + (DT)(R33.JK - R34.JK - RDT34.JK + 0)$	P36
6R	$R34.KL = P34.K$	P37
10A	$PAX3.K = BX3*2.K + BX3*3.K + BX3*4.K + BX3*5.K + BX3*6.K + BX3*7.K$	P37
10A	$P2534.K = PAX3.K + BX3*8.K + BX3*9.K + BX3*10.K + P34.K$	P38
12R	$RDT34.KL = (P2534.K)(DR34)$	P39
37B	$BX4 = BOXLIN(10,1.0)$	P40
6A	$BX4*1.K = P34.K$	P41
6A	$P43.K = BX4*10.K$	P42
6R	$R43.KL = P43.K$	P43
52L	$P44.K = P44.J + (DT)(R43.JK - R44.JK - RDT44.JK + 0)$	P44
6R	$R44.KL = P44.K$	P45
10A	$PAX4.K = BX4*2.K + BX4*3.K + BX4*4.K + BX4*5.K + BX4*6.K + BX4*7.K$	P46
10A	$P3544.K = PAX4.K + BX4*8.K + BX4*9.K + BX4*10.K + P44.K$	P47
12R	$RDT44.KL = (P3544.K)(DR44)$	P48
1L	$P45.K = P45.J + (DT)(R44.JK - R45.JK)$	P49
6R	$R45.KL = P45.K$	P50
1L	$P46.K = P46.J + (DT)(R45.JK - R46.JK)$	P51
6R	$R46.KL = P46.K$	P52
1L	$P47.K = P47.J + (DT)(R46.JK - R47.JK)$	P53
6R	$R47.KL = P47.K$	P54
1L	$P48.K = P48.J + (DT)(R47.JK - R48.JK)$	P55
6R	$R48.KL = P48.K$	P56
52L	$P49.K = P49.J + (DT)(R48.JK - R49.JK - RDT49.JK + 0)$	P57
6R	$R49.KL = P49.K$	P58
10A	$P4549.K = P45.K + P46.K + P47.K + P48.K + P49.K + 0$	P59
12R	$RDT49.KL = (P4549.K)(DR49)$	P60
1L	$P50.K = P50.J + (DT)(R49.JK - R50.JK)$	P61
6R	$R50.KL = P50.K$	P62
1L	$P51.K = P51.J + (DT)(R50.JK - R51.JK)$	P63
6R	$R51.KL = P51.K$	P64
1L	$P52.K = P52.J + (DT)(R51.JK - R52.JK)$	P65

6R	$R52.KL=P52.K$	P66
1L	$P53.K=P53.J+(DT)(R52.JK-R53.JK)$	P67
6R	$R53.KL=P53.K$	P68
52L	$P54.K=P54.J+(DT)(R53.JK-R54.JK-RDT54.JK+0)$	P69
6R	$R54.KL=P54.K$	P70
10A	$P5054.K=P50.K+P51.K+P52.K+P53.K+P54.K+0$	P71
12R	$RDT54.KL=(P5054.K)(DR54)$	P72
1L	$P55.K=P55.J+(DT)(R54.JK-R55.JK)$	P73
6R	$R55.KL=P55.K$	P74
1L	$P56.K=P56.J+(DT)(R55.JK-R56.JK)$	P75
6R	$R56.KL=P56.K$	P76
1L	$P57.K=P57.J+(DT)(R56.JK-R57.JK)$	P77
6R	$R57.KL=P57.K$	P78
1L	$P58.K=P58.J+(DT)(R57.JK-R58.JK)$	P79
6R	$R58.KL=P58.K$	P80
52L	$P59.K=P59.J+(DT)(R58.JK-R59.JK-RDT59.JK+0)$	P81
6R	$R59.KL=P59.K$	P82
12R	$RDT59.KL=(P5559.K)(DR59)$	P83
10A	$P5559.K=P55.K+P56.K+P57.K+P58.K+P59.J+0$	P84
1L	$P60.K=P60.J+(DT)(R59.JK-R60.JK)$	P85
6R	$R60.KL=P60.K$	P86
1L	$P61.K=P61.J+(DT)(R60.JK-R61.JK)$	P87
6R	$R61.KL=P61.K$	P88
1L	$P62.K=P62.J+(DT)(R61.JK-R62.JK)$	P89
6R	$R62.KL=P62.K$	P90
1L	$P63.K=P63.J+(DT)(R62.JK-R63.JK)$	P91
6R	$R63.KL=P63.K$	P92
52L	$P64.K=P64.J+(DT)(R63.JK-R64.JK-RDT64.JK+0)$	P93
6R	$R64.KL=P64.K$	P94
12R	$RDT64.KL=(P6064.K)(DR64)$	P95
10A	$P6064.K=P60.K+P61.K+P62.K+P63.K+P64.K+0$	P96
12R	$RDT65.KL=(P65PL.K)(DR65P.K)$	P97
1L	$P65PL.K=P65PL.J+(DT)(R64.JK-RDT65.JK)$	P98
NOTE		
NOTE	CONSTANTS AND INITIAL CONDITIONS FOR POP.INC.EQUATIONS	
NOTE		
C	$DR0=0.0240/DR4=0.0010/DR14=0.0004/DR24=0.0011/DR34=0.0015$	

C DR44=0.0030/DR49=0.0060/DR54=0.007/DR59=0.014/DR64=0.019
 14A DR65P.K=0.060-(0.0005)(T.K)
 6N P0=4.11/P1=4.10/P2=4.10/P3=3.98/P4=3.95
 C BX1*=3.95/3.82/3.78/3.65/3.48/3.48/3.47/3.57/3.50/2.74
 6N P14=2.75
 C BX2*=2.75/2.79/2.86/2.53/2.27/2.19/2.20/2.14/2.10/2.14
 6N P24=2.17
 C BX3*=2.17/2.07/2.19/2.16/2.28/2.33/2.33/2.42/2.39/2.47
 6N P34=2.55
 C BX4*=2.55/2.49/2.47/2.52/2.47/2.40/2.39/2.29/2.23/2.25
 6N P44=2.23/P45=2.21/P46=2.21/P47=2.08/P48=2.12/P49=2.03/P50=1.98
 6N P51=1.95/P52=1.86/P53=1.87/P54=1.80/P55=1.70/P56=1.69/P57=1.54
 6N P58=1.86/P59=1.50/P60=1.44/P61=1.38/P62=1.37/P63=1.41/P64=1.35
 6N P65PL=16.65

NOTE

NOTE AUXILIARY POPULATION EQUATIONS

NOTE

10A	PKID.K=P0.K+P1.K+P2.K+P3.K+P4.K+BX1*2.K	P99
8A	PPUP.K=P514.K-BX1*2.K+BX2*2.K	P100
10A	P16PL.K=PTL.K-P1.K-P2.K-P3.K-P4.K-P514.K-BX2*2.K	P101
10A	P2564.K=P2534.K+P3544.K+P4549.K+P5054.K+P5559.K+P6064.K	P102
10A	PTL.K=P65PL.K+P2564.K+P1524.K+PPUP.K+PKID.K+0	P103
20A	RR.K=P65PL.K/PTL.K	P104

NOTE

NOTE EDUCATION SECTOR

NOTE

7A	PSTUD.K=PES.K+SCL.K	ALL STUDENTS	E1
NOTE	*** ELEMENTARY SCHOOL		
19A	SES.K=(1.02)(P514.K-BX1*2.K-P14.K-0)	STUD ELEM SCHOOL	E2
12A	GES.K=(0.98)(P13.K)	GRAD ELEM SCHOOL	E3
NOTE	***HIGHSCHOOL		
12A	HIN.KL=(PH.K)(P13.K)	ENTER HIGHSCHOOL	E4
3L	PH.K=PH.J+(DT)(1/10)(0.99-PH.J)	PART RATE HIGHSCHOOL	P5-1
N	PH=0.96	INITIAL VALUE	P5-2
1L	HS1.K=HS1.J+(DT)(HIN.JK-HR1.JK)	1ST YR HIGHSCHOOL	E6
6R	HR1.KL=HS1.K		E7
52L	HS2.K=HS2.J+(DT)(HR1.JK-HR2.JK-0)	2ND YR HIGHSCHOOL	E8

6R	$HR2.KL=HS2.K$		E9
52L	$HS3.K=HS3.J+(DT)(HR2.JK-HR3.JK-DP2.JK-0)$	3RD YR HIGHSCDOL	E10
6R	$HR3.KL=HS3.K$		E11
52L	$HS4.K=HS4.J+(DT)(HR3.JK-HR4.JK-DP3.JK-0)$	4TH YR HIGHSCHOOL	E12
6R	$HR4.KL=HS4.K$		E13
14A	$D1.K=0.024-(0.0005)(T.K)$	DROPOUT RATE	E14
12R	$DP1.KL=(D1.K)(HS1.K)$	DROPOUT 1ST YR	E15
13R	$DP2.KL=(D1.K)(HS2.K)(3.0)$	DROPOUT 2ND YR	E16
13R	$DP3.KL=(2.5)(D1.K)(HS3.K)$	DROPOUT 3RD YR	E17
6A	$GHS.K=HS4.K$	GRAD HIGHSCHOOL	E19
9A	$SHS.K=HS1.K+HS2.K+HS3.K+HS4.K$	ENROLLM HIGHSCHOOL	E20
9A	$HDP.K=DP1.K+DP2.K+DP3.K$	DROPOUT HIGHSCHOOL	E21
6N	$HS1=2.5/HS2=2.51/HS3=2.54/HS4=2.1$	INITIAL VALUES	
7A	$PES.K=SHS.K+SES.K$	ENROLLM CLASS 1-12	E22
13A	$KPS.K=(PES.K)(KE.K)(0.001)$	COST ELEM + HIGHSCH	E23
14A	$KE.K=489+(29.5)(T.K)$	COST PER STUD	E24
NOTE	*** COLLEGE		
12A	$ECL.K=(PC.K)(BX2*4.K)$	ENTER COLL	E25
14A	$PC.KL=DELAY3(PCA,20)$	PART RATE IN COLLEGE	E26-1
N	$PC=0.45$	INITIAL VALUE	E26-2
C	$PCA=0.55$	ASYMPT VALUE	E26-3
1L	$CL1.K=CL1.J+(DT)(ECL.JK-RC1.JK)$	1ST YR COLL	E27
6R	$RC1.KL=CL1.K$		E28
1L	$CL2.K=CL2.J+(DT)(RC1.JK-RC2.JK)$	2ND YR COLL	E29
6R	$RC2.KL=CL2.K$		E30
52L	$CL3.K=CL3.J+(DT)(RC2.JK-RC3.JK-GPS.JK-0)$	3RD YR COLL	E31
6A	$RC3.KL=CL3.K$		E32
1IR	$GPS.KL=(PP.K)(CL2.K)$	GRAD 2 YR INSTIT	E33
14A	$PP.K=0.40-(0.002)(T.K)$	GRAD RATE 2 YR INST	E34
1L	$CL4.K=CL4.J+(DT)(RC3.JK-RC4.JK)$	4TH YR COLL	E35
6R	$RC4.KL=CL4.K$		E35
52L	$CL5.K=CL5.J+(DT)(RC4.JK-RC5.JK-GBS.JK-0)$	5TH YR COLL	E36
6R	$RC5.KL=CL5.K$		E37
12R	$GBS.KL=(PBS.K)(CL4.K)$	GRAD 4 YR INSTIT	E38
14A	$PBS.K=0.80-(0.004)(T.K)$	GRAD RATE 4 YR INST	E39
1L	$CL6.K=CL6.J+(DT)(RC5.JK-GMS.JK)$	6TH YR COLL	E40
6R	$GMS.KL=CL6.K$	GRAD 6 YR INSTIT	E41

10A	$SCL.K = CL1.K + CL2.K + CL3.K + CL4.K + CL5.K + CL6.K$	ENROLLM COLLEGE	E42
6N	$CL1 = 1.11 / CL2 = 1.0 / CL3 = 0.55 / CL4 = 0.42 / CL5 = 0.08 / CL6 = 0.076$		
13A	$KCL.K = (SCL.K)(KC.K)(0.001)$	COST COLLEGE	E43
14A	$KC.K = 1750 + (82)(T.K)$	COST PER STUDENT	E44
7R	$EDUC.K = KPS.K + KCL.K$	TOTAL EXPEND EDUC	E45
12A	$INST.K = (TR.K)(PES.K + (SCL.K)(2))$	INSTUCTORS	E46
11A	$TR.K = 0.042 + (0.0011)(T.K)$	INST-STUD RATIO	E46-1
20A	$RS.K = PSTUD.K / PTL.K$	STUDENT RATE	E47
NOTE			
1L	$T.K = T.J + (DT)(1.0 + 0)$	TIME COUNT	E48-1
N	$T = 0$	INITIAL VALUE	E48-2
NOTE			
NOTE	LABOR-FURCE		
NOTE			
12A	$PWRK.K = (P16PL.K)(PR.K)$	AVAIL LABOR FORCE	L1
11A	$PR.KL = DELAY3(APR, 10)$	PART RATE IN LABOR FORCE	L2-1
N	$PR = 0.6$	INIT VLUE OF PART RATE	L2-2
C	$APR = 0.6$	ASSYMT VAL OF PART RATE	L2-3
14A	$GESL.K = GES.K - (PH.K)(P13.K)$	ENTER FROM EL SCHOOL	L3
11A	$GSL.K = GESL.K + GHS.K - ECL.K + GPS.K + GBS.K + GMS.K + HDP.K$		
NOTE		ENTER FROM ALL SCOOLS	L4
12A	$GLF.K = (GSL.K)(0.94)$	ADJUST ENTRIES	L5
8A	$GRT.K = GLF.K - PWRK.K + BXL8*2.K$	RETIREMENTS	L6
37B	$BXL8 = BUXLIN(2, 1)$	PAST LABOR FORCE	L7
C	$BXL8* = 71.5 / 70.8$	INITIAL VALUE	L38
6A	$BXL8*1.K = PWRK.K$		L9
1L	$SYLF.K = SYLF.J + (DT)(SYIN.K - SYOUT.K)$	TOT EDUC OF LF	L10
12N	$SYLF = (PWRK)(11.2)$	INITIAL VALUE	L11
18A	$SYIN.K = (0.94)((GESL.K)(8) + (GHS.K)(12) + (GPS.K)(14) + (GBS.K)(16) + (GMS.K)(18) + (HDP.K)(9) - (ECL.K)(12))$	EDUC OF ENTRIES	L12
X1			
12A	$SYOUT.K = (GRT.K)(YSRT.K)$	EDUC OF RETIREM	L13
52L	$YSRT.K = YSRT.J + (DT)(1/40)(11.5 - YSRT.J)$	AVER EDUC UF RETIR	L14-1
N	$YSRT = 8.5$	INITIAL VALUE	L14-2
20A	$YSGL.K = SYIN.K / GLF.K$	AVER EDUC OF ENTRIES	L15
20A	$AYS.K = SYLF.K / PWRK.K$	AVER EDUC OF LF	L16
21A	$AJ.K = ((AYS.K)(0.9) / 11.2) + 0.1$	ADJUSTM COEFF	L17
NOTE			

NOTE PRIVATE SECTOR

NOTE

10A	$PINC.K = GNP.K + GWLF.K - RTE.K - BDPR.K - TTAX.K + GINT.K$	PERS DISP INCOME	PR1
1L	$PPCH.K = PPCH.J + (DT)(APC)(PINC.K) - PPCH.J$	PRIVATE SPENDING	PR2-1
N	PPCH=316	INITIAL VALUE	PR2-2
C	APC=0.93	AVERAGE PROP TO CONSUME	PR3
22A	$PPCR.K = (PPCH.K) / IC.K$	DEFLATED PRIV SPENDING	PR4
7A	$PSAV.K = PINC.K - PPCR.K$	PRIVATE SAVINGS	PR5
1L	$PMST.K = PMST.J + (DT)(PSAV.K)$	PRIV SAVINGS ACCOUNT	PR6-1
N	PMST=210	INITIAL VALUE	PR6-2
12A	$PINV.K = (0.15)(PPCR.K)$	INVESTM IN HOUSING	PR7
12A	$PCONS.K = (0.85)(PPCR.K)$	PRIV NONINVESTM SPENDING	PR8
1L	$HST.K = HST.J + (DT)(PINV.K - HDPR.K)$	STOCK OF RES HOUSING	PR9-1
6N	HST=750	INITIAL VALUE	PR9-2
12R	$HDPR.KL = (0.035)(HST.K)$	DEPREC ON HOUSING	PR10

NOTE

NOTE GOVERNMENT SECTOR

NOTE

10A	$GEXP.K = GSRV.K + GINV.K + GINT.K + GWLF.K + GCTR.K + EDUC.K / 2$	PUBL EXPEND	G1
12R	$GINT.KL = (GMST.K)(-0.038)$	INT ON PUBLIC DEBT	G2
1L	$GMST.K = GMST.J + (DT)(TTAX.K - GEXP.K)$	PUBLIC DEBT	G3
N	GMST=-320	INIT VALUE	G4
11A	$GINV.K = (-21 + (0.268)(PTL.K)(GRDW.K + 1) / 2)$	PUBLIC INVESTM	G5
11A	$GSRV.K = (-40 + (0.600)(PTL.K))$	PUBLIC SERVICES	G6
7A	$GWLF.K = WLFP.K - PINS.K$	PUBL WELF EXPEND	G7
14	$WLFP.KL = -90 + (7.8)(P65PL.K)$	WELF AND SO SEC PAYM	G8
14R	$PINS.KL = -60 + (4.1)(P65PL.K)$	PRIVAT PAYM TO SO SEC	G9
11A	$GCTR.K = \max(BSPS.K - BSDS.K, 0)$	CONTROL OF AGG DEMAND	G10
11R	$TTAX.K = GEXP.K - (GCTR.K)(2)$	TOTAL TAXES	G11
18A	$GEPL.K = (INST.K + (C62)(GSRV.K + GINV.K)) / AJ.K$	PUBLIC EMPLOYMENT	G12
22A	$GWAG.K = (GEPL.K + PMIL)(4.7)(GRDW.K + 1) / 2$	PUBLIC WAGE BILL	G13
8A	$GPCH.K = EDUC.K / 2 + GSRV.K + GINV.K - GWAG.K$	PUBLIC PURCHASES	G14
C	PMIL=2.5	MILIT PERSONNEL	G15

C

C62=0.066 BUSINESS SECTOR

NOTE

NOTE

8A	$BEAJ.K = PWRK.K - GEPL.K - PMIL$	BUS EMPLOYMENT	B1
----	-----------------------------------	----------------	----

1L	$PPRD.K = PPRD.J + (DT)(PPIN.K - (0.980)(PPRD.J))$	POSS BUS PRODUCTION	B2-1
N	$PPRD = 440$	INITIAL VALUE	B2-2
28A	$PPIN.K = (2.39)EXP((0.71)LOGN(BEAS.K) + (0.39)LOGN(BKST.K))$	PROD FKT	B2-3
12A	$BSPS.K = (0.96)(PPRD.K) + STOCK.K$	POSSIBLE SALES	B3
54A	$BLSL.K = BSPS.K$	ACTUAL SALES	B4
11A	$BSDS.K = PPCH.K + GPCH.K + BINV.K$	DESIRED SALES	B4-2
11A	$BPRD.K = PPRD.K$	BUSINESS PRODUCTION	B4-3
1L	$STOCK.K = STOCK.J + (DT)(BPRD.JK - BLSL.K)$	INVENTORY	B5-1
N	$STOCK = (0.04)(PPRD)$	INITIAL VALUE	B5-2
11A	$FPPR.K = PPRD.K + (BL3*1.K - BL3*4.K)/3$	EXPECTED PRODUCTION	B6
37B	$BL3 = BOXLIN(4, 1)$	AUX DATA STORAGE	B7-1
6A	$BL3*1.K = PPRD.K$		B7-2
C	$BL3*4.K = 440/410/385/365$	INITIAL VALUES	B7-3
11A	$BD.K = BDPK.K + (BKST.K)(FPPR.K - PPRD.K)(1.5)/PPRD.K$	INVESTM	B8
11A	$CPAV.K = BSPS.K - PPCH.K - GPCH.K - BD.K$	AVAILABLE CAPACITY	B9
11A	$BINV.K = BU.K + MAX(CPAV.K/2, 0)$	GROSS BUS INVESTMENT	B10
1L	$BKST.K = BKST.J + (DT)(BIVR.JK - BDPK.JK)$	BUS CAPITAL STOCK	B11
N	$BKST = 400$	INITIAL VALUE	B11-2
20A	$BIVR.K = BINV.K/IC.K$	DEFLATED BUS INVESTM	B12
12R	$BDPK.KL = (0.055)(BKST.K)$	BUS DEPRECIATION	B13
11A	$RTE.K = (0.03)(BLSL.K)$	RETAINED EARNINGS	B14
1L	$BMST.K = BMST.J + (DT)(RTE.JK - BINV.JK)$	BUS SAVINGS ACCOUNT	B15-1
N	$BMST = -400$	INITIAL VALUE	B15-2
11A	$IVIN.K = PPRD.K - BLSL.K$	INVENTORY INCREASE	B16
6A	$GNP.K = PPRD.JK + GWAG.K$	GROSS NATIONAL PRODUCT	B17
20A	$GR.K = BXL1*1.K/BXL1*2.K$	GROWTH RATE OF GNP	B18
6A	$BXL1*1.K = GNP.K$	AUX DATA STORAGE	B19-1
37B	$BXL1 = BOXLIN(3, 1.0)$		B19-2
C	$BXL1*2.K = 487/475/447$	INITIAL VALUES	B19-3
1L	$GROW.K = GROW.J + (DT)((GNP.K/490) - GROW.J)$	GROWTH INDEX	B20-1
N	$GROW = 1$	INITIAL VALUE	B20-2
20A	$CI.K = BSDS.K/BSPS.K$	DEMAND-SUPPLY-RATIO	B21
20A	$IC.K = MAX(CI.K, 1)$	INFLATION COEFFICIENT	B22
NOTE			
NOTE	INDEX VALUES		
NOTE			
20A	$C6G.KL = GEXP.K/GNP.K$		

20A CVG.KL=BINV.K/GNP.K
 20A CIG.KL=PINC.K/GNP.K
 20A CED.KL=EDUC.K/GNP.K
 20A CPI.KL=PINC.K/PTL.K
 20A CPG.KL=GEXP.K/PTL.K
 20A CPC.KL=PPCH.K/PTL.K
 20A CHS.KL=HST.K/PTL.K
 20A CWP.KL=PWRK.K/PTL.K
 20A CPS.KL=PSAV.K/PTL.K
 20A CGP.KL=GNP.K/PTL.K
 20A CAS.KL=PMST.K/PTL.K
 20A PIC.K=PINC.K/347.8

NOTE

PRINT 1)PTL,PSTUD,SCL/2)EDUC,SES,SHS/3)PWRK,GEPL,AJ/4)P65PL,RR,RS/5)GNP,
 X1 GEXP,PPRD/6)GWAG,BINV,BKST/7)PINC,PMST,CPI/8)CAS,CGP,CHS/9)CGG,CVG
 X2 ,CIG/10)CWP,CPG,CGP/11)AYS,CI,GR/12)APC,ABR,PCA/13)APR,BR,GROW/14)
 X3 GCTR,PIC
 PLOT BR=B/PTL=P,PSTUD=S,PWRK=W/EDUC=E/PINC=I/BKST=K,GNP=G
 PLOT CPI=I,CPG=P,CGP=G/BR=B/GR=R/CED=E,CIG=N,CGG=U,CWP=W
 SPEC DT=1/LENGTH=40/PRTPER=5/PLTPER=1

A SAMPLE PRINT OUTPUT

TIME	PTL PSTUD SCI	EDUC SES SHS	PMRX GEPL AJ	P65PL RR RS	GNP GEXP PPRD	GWAG BINV BKST	PINC PMST CPI	CAS CGP CHS	CGG CVG CIG	CNP CPG CGP	AYS CI GR	APC ABR PCA	APR BR GRDW	GCTR PIC CED
E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+03 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E+00 E+00	E+00 E-03 E+00	E+00 E-03 E+00	E+00 E+00 E-03
0+000	179.59 41.109 3.2360	24.18 28.223 9.650	75.86 8.175 1.0000	16.650 0.09271 0.22891	489.9 156.18 440.0	49.94 61.53 400.0	353.2 210.0 1.9667	1.169 2.728 4.176	0.31877 0.12558 0.72090	0.42240 0.8696 2.728	11.200 0.98765 1.0253	0.89000 15.000 0.55000	0.60000 23.600 1.0000	5.435 1.0000 49.360
5+000	191.80 47.299 4.2459	36.57 30.948 12.105	80.15 9.188 1.1107	17.978 0.09374 0.24661	726.1 198.99 658.3	67.83 122.15 740.3	547.7 588.9 2.8558	3.070 3.786 4.602	0.27405 0.16822 0.75435	0.41787 1.0375 3.786	12.578 0.98072 1.0667	0.89000 15.000 0.55000	0.60000 22.198 1.4693	12.660 1.5508 50.370
10+000	204.26 52.721 5.4996	51.16 33.608 13.613	85.71 10.863 1.1434	19.768 0.07678 0.25811	928.4 246.80 836.8	91.64 147.84 1141.8	697.1 1063.8 3.4127	5.208 4.545 5.380	0.26583 0.15924 0.75085	0.41961 1.2082 4.545	12.984 0.97854 1.0446	0.89000 15.000 0.55000	0.60000 18.292 1.9183	17.929 1.9736 55.101
15+000	213.94 55.824 6.3978	65.11 34.115 15.311	92.31 12.315 1.1739	21.938 0.10254 0.26093	1141.8 293.07 1024.6	117.21 193.51 1602.9	853.5 1616.4 3.9893	7.556 5.337 6.421	0.25667 0.16947 0.74748	0.43149 1.3699 5.337	13.364 0.98022 1.0413	0.89000 15.000 0.55000	0.60000 16.091 2.3667	20.239 2.4164 57.020
20+000	221.84 54.726 7.3379	76.01 31.307 16.082	99.39 13.580 1.2039	24.486 0.11038 0.24670	1387.4 340.54 1240.7	146.72 251.74 2181.0	1034.7 2281.2 4.6644	10.283 6.254 7.682	0.24544 0.18144 0.74579	0.44804 1.5351 6.254	13.737 0.98253 1.0389	0.89000 15.000 0.55000	0.60000 15.305 2.8827	21.645 2.9296 54.783
25+000	228.54 51.192 7.9766	83.31 28.241 14.974	105.94 14.682 1.2378	26.883 0.11763 0.22400	1605.5 388.68 1485.0	180.56 319.47 2698.9	1240.1 3074.3 5.4263	13.452 7.288 9.176	0.23337 0.19181 0.74457	0.46354 1.7007 7.288	14.160 0.98436 1.0356	0.89000 15.000 0.55000	0.60000 15.077 3.4716	23.189 3.5111 50.023
30+000	235.17 48.152 7.5173	87.48 27.224 13.411	110.84 15.749 1.2731	29.763 0.12656 0.20476	1952.4 442.60 1734.1	218.33 384.59 3740.3	1449.7 3990.0 6.1645	16.967 8.302 10.864	0.22669 0.19698 0.74252	0.47134 1.8821 8.302	14.598 0.98378 1.0301	0.89000 15.000 0.55000	0.60000 15.018 4.0910	26.101 4.1045 44.806
35+000	242.24 47.264 8.9689	93.51 27.386 12.910	114.87 16.994 1.3029	32.315 0.13340 0.19512	2232.0 502.41 1971.0	261.07 446.80 8659.8	1647.5 5010.2 6.8012	20.683 9.214 12.661	0.22509 0.20018 0.73811	0.47419 2.0741 9.214	14.969 0.98215 1.0253	0.89000 15.000 0.55000	0.60000 15.004 4.6988	35.148 4.6645 41.893
40+000	249.79 47.925 6.8903	103.15 27.989 13.046	118.70 16.449 1.3263	34.045 0.13630 0.19187	2503.9 564.96 2194.3	309.54 511.26 5633.2	1831.5 6123.2 7.3325	24.514 10.024 14.480	0.22564 0.20419 0.73149	0.47522 2.2618 10.024	15.261 0.98088 1.0221	0.89000 15.000 0.55000	0.60000 15.001 5.2876	41.924 5.1856 41.195

SOME RESULTS OF THE MODEL VALIDATION

The following table compares the actual values of some of the model variables with the values obtained during the model validation. For each year, the upper line shows the actual value, the second line shows the simulated value, and the third line gives the difference between the actual and the simulated value in percentage of the actual value.

Table 8. Some Results of the Model Validation
(Population Values in Millions, Money
Values in Billions of Dollars)

Year	PTL	P65PL	GEXP	GEPL	GNP	PPCH	BINV	PWRK
60	180	16.53	94.9	8.35	487.7	316	47.1	72.1
	179.6	16.65	102.1	8.30	490.2	316	47.5	71.1
	-0.22%	0.0%	7.6%	-0.0%	0.5%	0.0%	0.1%	-1.4%
62	186	17.1	107.5	8.89	529.8	338	47.9	74.0
	184.9	17.27	115.4	8.7	527.3	340	51.7	72.9
	-0.59	0.0	6.8	-0.2	-0.5	0.6	7.5	-1.5
64	192	17.7	111.2	9.58	581.1	374	57.8	75.8
	191.3	17.7	122.1	9.4	568.3	353	58.8	74.8
	-0.36	0.0	9.8	-0.2	-2.2	-5.0	1.7	-1.5
66	196.6		124.5	10.87	652.6	418	75.0	78.9
	195.6		132.9	9.9	629.9	389.2	69.3	77.5
	-0.5		6.8	-8.9	-3.5	-6.7	-7.6	-1.9
67				11.61	669.2	430		
				10.15	637.5	339		
				-12.5	-4.7	-7.1		

BIBLIOGRAPHY

1. Cohen, K. J., and Cyert, R. M., "Computer Models in Dynamic Economics," *Quarterly Journal of Economics*, LXXV, 1961, pp. 112-127.
2. Denison, Edward F., *The Sources of Economic Growth in the United States and the Alternatives Before Us*, Supplementary Paper No. 13, Committee for Economic Development, 1962.
3. Domar, E. D., "Capital Expansion, Rate of Growth, and Employment," *Econometrica*, 1946, pp. 137-143.
4. Duesenberry, James S., *Business Cycles and Economic Growth*, McGraw-Hill Book Company, New York, 1958.
5. Duesenberry, Fromm, Klein, and Kuh (editors), *The Brookings SSRC Quarterly Economic Model of the United States*, Rand McNally and North Holland Press, Chicago, 1965.
6. Forrester, Jay W., *Industrial Dynamics*, The M.I.T. Press, Cambridge, Massachusetts, 1965.
7. Galbraith, Kenneth J., *The New Industrial State*, Houghton Mifflin & Company, Boston, 1967.
8. Hicks, J. R., *A Contribution to the Theory of the Trade Cycle*, Clarendon Press, Oxford, 1950.
9. Holland, Edward P., and Gillespie, Robert W., *Experiments on a Simulated Underdeveloped Economy: Development Plans and Balance of Payment Policies*, The M.I.T. Press, Cambridge, Mass., 1963.
10. Kalecki, M., "A Macrodynamic Theory of Business Cycles," *Econometrica*, 3, pp. 327-344.
11. Malthus, Thomas R., *An Essay on Population*, E. P. Dutton, New York, 1914.
12. Marx, Karl, *Das Kapital*, International Publishers, New York, 1967.
13. Naylor, Balintfy, Burdick, and Chu, *Computer Simulation Techniques*, John Wiley and Sons, New York, 1966.

14. Orcutt, Greenberger, Corbel, and Rivlin, *Microanalysis of Socio-economic Systems*, Harper and Brothers, New York, 1966.
15. Patterson, Wallace C., *Income, Employment, and Economic Growth*, W. W. Norton Co., New York, 1958.
16. Phillips, A. W., "Stabilization Policy in a Closed Economy," *Economic Journal*, 64, pp. 290-323.
17. Pugh, Alexander L., III, *DYNAMO User's Manual*, The M.I.T. Press, Cambridge, Mass., 1965.
18. Ricardo, David, *The Principle of Political Economy and Taxation*, R. D. Irwin, Homewood, Ill., 1963.
19. Sasser, Earl W., and Naylor, Thomas H., "Computer Simulation of Economic Systems," *Simulation*, Jan. 1967, pp. 21-32.
20. Samuelson, Paul A., "Interactions between the Multiplier Analysis and the Principle of Acceleratio," *Review of Economic Statistics*, XII, 1938, pp. 75-78.
21. Samuelson, Paul A., *Economics: An Introductory Analysis*, McGraw-Hill Book Co., New York, 1964.
22. Solow, R., "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, Vol. 39.
23. Smith, Adam, *An Inquiry into the Nature and Causes of the Wealth of Nations*, E. P. Dutton, New York, 1913.

Other References

- Abramowitz, Moses, "Economic Growth in the United States," *American Economic Review*, 4, 1964, pp. 762-782.
- Allen, R. G. D., *Mathematical Economics*, MacMillan & Co., London, 1965.
- Chorafas, Dimitris, *Systems and Simulation*, Academic Press, New York, 1965.
- Churchman, C. West, "An Analysis of the Concept of Simulation," *Symposium on Simulation Models*, Edited by Austin C. Hogatt and Frederick E. Balderston, Cincinnati, South-Western Publishing Co., 1963.
- Dillard, Dudley, *The Economics of John Maynard Keynes*, Prentice-Hall, New York, 1948.

- Duesenberry, James S., *Income, Saving, and the Theory of Consumer Behavior*, Harvard University Press, Cambridge, 1952.
- Friedman, Milton, *A Theory of the Consumption Function*, Princeton Univ. Press, Princeton, 1957.
- Ginsberg, Allen S., *Simulation Programming and Analysis of Results*, The Rand Corporation, 1965.
- Jarmain, Edwin W., Editor, *Problems in Industrial Dynamics*, The M.I.T. Press, Cambridge, Mass., 1963.
- Keynes, John M., *The General Theory*, Harcourt and Brace, Paperback, 1967.
- Krol, Joseph, "Use of DYNAMO in the Simulation of Dynamic Models," *Record of Proceedings First Annual Simulation Symposium*, Tampa, Florida, January 18-19, 1968.
- Kuznets, Ratner-Miller, Easterlin, Editors, *Population Redistribution and Economic Growth, United States 1870-1950, Part II: Analysis of Economic Change*, The American Philosophical Society, Philadelphia, 1960.
- Smithies, A., "Economic Fluctuations and Growth," *Econometrica*, Vol. XXV, 1957.
- Tocher, K. D., *The Art of Simulation*, D. van Nostrand Co., 1963.
- Zymelman, Manuel, "A Stabilization Policy for the Cotton Textile Cycle," *Management Science*, Vol. XI, 1965.