Modeling Economic Effects of International Retirement Migration within The European Union

by

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Declaration

The contents of the thesis are the author's own work. This Thesis has not been submitted for a degree to another University.

Abstract

International retirement migration (IRM) is a growing and significant feature of the European Union. It has important economic implications in terms of the redistribution of social costs, factors reward and incomes. Using overlapping generations models and simulation techniques this thesis focuses on the economic effects of International Retirement Migration (IRM) within the European Union (EU). Three main parts make up this thesis.

The first part summaries the legal and the social framework within the European Union where IRM takes place. Access to European welfare system is based on the principle of non-discrimination. However, the European Comunity law regulates the possibility of free riding through the resource requirement.

In the second part, after a brief literature review in social security, the thesis develops a quantitative model that tries to explain some reasons why IRM may take place. Starting with a difference between "environment" of European countries, some people may opt for a better life in another country when they retire. We also focus on the capital accumulation effect for home and host countries.

The presence of large populations of retired foreign residents in European countries raises fundamental questions with respect to the right of access to health and welfare services. In the third part, bearing in mind the principle of free movement of capital and the non-discrimination principle in accessing public service within the EU, we focus on the economic effects of IRM for the host country, for the individual migrants themselves, for the host communities and for public policy.

CHAPTER 1

General Introduction

1. The problem of old age

All European Union member states are or have been engaged in reforms designed to contain or reduce the future cost of public pension provision. Several factors have contributed towards this[1].

A fall in birth rates and an increase in life expectancy mean considerable changes in the age structure of Europe's population, with significant increases in the number of people aged 65 plus and 80 plus forecast in the next 30 years. This situation has been viewed by the EU as problematic because it represents a challenge to the feasibility of established national welfare systems.

Another relevant development has been a shift in policy with regards to early retirement. There has been an increase in the number of people aged 50 plus who are no longer active in the paid labour market. In fact, many European countries actively encourage early retirement despite the general health and life expectancy of these individuals being much greater. In the recent years this approach has been reversed with member states now being urged by the Commission to raise official retirement ages and discourage early exit from paid employment. Table 1 shows

[1] OECD, 2000

Country	Greece		Italy		Portugal		UK		Ireland	
Year	1960	2030	1960	2030	1960	2030	1960	2030	1960	2030
Life expect at birth										
Male	68	78	67	79	61	77	68	78	68	79
Female	71	84	72	84	67	83	74	83	72	84
Perc. of population										
aged 65+	8	25	10	30	8	23	12	23	11	18
aged 80+	1	7	1	8	1	6	2	6	2	5

TABLE 1. Projected demographic trends in six member states

the old age dependency ratio, comparing people aged 16-64 with those aged 65 plus[2].

1.1. EU and International Retirement Migration (IRM). Many changes have taken place in western European society which have contributed to the way many of us experience old age and retirement[3]. Income improvements, increased educational opportunities and changes in the job market (e.g. increase of professional rather than manual jobs) have all had an impact upon the aspirations and

[2]Figures from OECD (2000)

[3]for example, Laslett, 1989; Warnes, 1993.

Country	Greece		Italy		Portugal		UK		Ireland	
Year	2000	2030	2000	2030	2000	2030	2000	2030	2000	2030
Male	23	33	33	27	39	39	38	34	38	34
Female	21	32	19	27	39	33	32	38	24	38
Old age Dependency Ratio	2	1.5	2	1	3	2	3	2	3.5	2.5

TABLE 2. Lifetime years spent in work

choices available to people in later life. Overseas travel, longer holidays and early retirement options have increasingly become a feature of many peoples lives in recent decades. This said, old age is not always characterised by positive experience and many people still experience poverty, lack of opportunities and debilitating illness. The argument is, however, that in contemporary Europe we can now differentiate between a 'third age' of "well resourced and healthy retirement" and a 'fourth age' of later life starting in the late seventies in which the onset of old age-related illness and need for care become important considerations (cf Laslett, 1989).[4]

When discussing the movement of retired people abroad, most of the literature defines this phenomenon as "International Retirement Migration" (IRM). Much

[4]See Warnes, 1993, p 451

of existing literature has focused on the relocation of UK pensioners towards the warmer regions of southern Europe[5].

From a statistical point of view, it appears that an accurate picture cannot be painted with regards to IRM. There is little other statistical data available and most of it is unreliable[6].

The problems can be seen in Table 3[7]. A number of countries fail to differentiate by age at all, and, whilst the figures give an indication of the number of older EU nationals officially resident in another member state, there is no way of assessing how many of them moved following retirement. In addition, the figures do not record those who have failed to inform the official authority of their permanent relocation. O'Reilly's comment (made in relation to British retirees relocating to Spain) that, "existing statistics are both difficult to obtain and to trust because of the fluidity, undocumented and unofficial nature of this form of migration" (2000b, p 481) holds true when considering IRM across the whole of Europe.

[5] see for example, Williams et al, 1997; King et al, 2000; O'Reilly, 1999.

[6] see William et al, 1997; O'Reilly,1999;Warners 1999.

[7] Source: Figure adapted from Eurostat, 1999.

[a] Numbers differetiated by age not available.

		Age			
Country	**50-54**	**55-59**		**60-64**	**65+**
Austria	3,900	2,900		1,900	6,600
Belgium	11,441	33,373		29,563	71,807
Denmark	4,078	2,974		1,709	2,582
Finland	656	513		434	1,586
France	99,729	88,973		73,642	58,622
Germany			1,850,032[a]		
Greece	3,344	2,301		1,391	2,573
Italy			133,512[a]		
Ireland		Non	Available		
Luxemburg			131,410[a]		
Netherlands	14,066	10,971		6,542	11,077
Portugal		Non	Available		
Spain			219,790[a]		
Sweden	18,163	12,893		8,875	13,782
UK	65,000	66,100		56,400	137,900

TABLE 3. Number of EU national resident in another member state, by age

2. Motivations to migration: an overview

Generally, according to Williams et al (1997), there are several motivating factors for people to relocate on retirement, which could be assembled into five loose clusters[8]:

- for work, with work, lower living costs
- domestic care, proximity to family, marriage effect, loss of partner (divorce, separation, bereavement), for children.
- healthcare services as a factor in initial and, especially, return movements.
- the wish to be buried 'at home', enforced unemployment, retirement plan
- region appeal, holidays, climate, a desire to return to one's roots/homeland.

The above five clusters are not mutually exclusive of one another. A combination of different issues and events such as geographical location, age and economic and familial relationships are significant in influencing the movements and preferred locations of retired EU migrants. Various factors working in combination with each other are often the key in migration decision making process.

Discussing IRM across Europe more specifically, Williams et all (1997) offer three main explanations as to why an increasing number of nationals from northern European states relocate to southern Europe on retirement.

[8]see Williams et al, 1997; King et al, 1998.

1. Relocating south is a sensible economic decision because of lower cost of living, heating and cheaper house prices. In addition, certain southern countries have more favourable fiscal and tax policies. This, in turn, means that northern migrants, who, it is argued are mostly "either retired or 'active young elderly' [sic] persons with above average wealth and incomes" (Williams et al, 1997, p 116), can build on and benefit even further from their already advantaged economic position.
2. A warmer climate is an obvious pull for those who wish to escape from the colder northern regions of Europe.
3. Certain retirement movements are brought about by the search for an idealised middle class myth, characterised by certain lifestyles, cultures and landscapes. According to King et all (1998), other influential decisions to migrate internationally have included previous holiday visits and prior occupations.

Taking the above issues into account, Williams et al (1997) distinguish between discrete groups within a more general category of post-retirement migrants. They include seasonal migrants (snowbirds), who relocate for temporary periods and have a base in their home country. Another significant classification, lifetime expats, consists of people who were previously employed abroad and have experienced continuous movement and relocation throughout their working life. This group includes the military, international companies or high ranking civil servants. Williams et al (1997) label this group 'tax dodgers' who on retirement choose southern European locations in order to store their accumulated wealth in offshore havens, thus avoiding the higher tax regimes in their northern European countries of origin.

It is important to note that retirement migration is not a simple or quick move. It often involves a series of moves, including extended vacations, followed by seasonal moves. This is especially true for European Union countries where the geographical distances are relatively short. People may retire from their main occupation and then take up part-time, voluntary or self-employed work. Indeed, such movements into and out of work may become increasingly common in the context of the labour market and increasing policy emphasis on the importance of delaying retirement and active encouragement of retired people to resume paid work.

In addition to the above factors, the provision of welfare benefits and services in different countries should not be overlooked. This is specifically important with regard to return migration decisions linked to serious illness or increased frailty in old age.

The geography of retirement migration raises some interesting questions in terms of material entitlement:

1. The movements generally take place at a time when peoples lives are associated with the need for increased dependency in terms of healthcare

2. People move away from generous welfare systems to coastal and rural regions in southern Europe. These locations typically lack a comprehensive infrastructure of community care and support services.

3. With the exception of some returnees, the majority of retired migrants are also moving away from potential sources of informal care (their families). Despite the diversity of national welfare systems, European Community law clearly provides an important basis for welfare claims in the host state for retirement migrants.

3. Organisation of the thesis

This introduction is followed by three parts that try to analyse three different aspects of IRM.

In the first part (Chapter 2) we look at the European legal aspects of IRM and deal with the complex and controversial issues that it brings about. On one hand, the European Union is based on non-discriminatory principle with regards to the free movement between countries, while at the same time, certain European Directives put a resource requirement limit which stops migrants from free riding the host country's social security system.

In the second part after a brief literature review on social security (Chapter 3), the thesis develops a quantitative model which explains why IRM may take place between two countries, taking into account the environmental variable (Chapter 4-5). This phenomenon may result in a modified capital accumulation for both countries.

In part three, after a brief technical discussion of the algorithm used to find the equilibrium of the simulation model, we analyse the economic effect of social security and IRM through simulation techniques. In Chapter 6, we deal primarily with the impact of social security on the economy and then we also analyse the effect of social security on savings, labour supply and the welfare of different generations. Chapter 7 considers the economic effect of IRM, focusing on two aspects. The first one is that international retirement migrants can access free of charge part of the social security system of the host country. The second one is that in order to support themselves, the migrants need to bring a certain amount of savings with them to the host country.

Chapter 8 provides a summary of the thesis findings and suggestion for further studies.

The simulation results should not be mistaken for empirical estimates, which they are not. Simulation analysis is certainly no substitute for empirical research. Rather, it provides a method of exploring the full implications of economic relations and empirical findings.

Matlab encoding of the numerical simulation models are included in the appendices.

Part 1

Legal Framework

CHAPTER 2

The legal Status of Retirement Migrants

1. The legal status of retired migrants

1.1. Citizenship of the Union and Mobility rights. At the present, Citizenship of the Union, is on a national basis and applies to all those persons resident within the EU who are nationals of one of the EU member states. In the context of EU, the development of citizenship since the Treaty of Rome has taken place in close connection to the evolution of mobility rights. This relationship between mobility and citizenship was given formal constitutional recognition in the Treaty on European Union (TEU) with the insertion of a Article declaring the existence of "Citizenship of the Union".

1.2. Citizenship of the Union. From the Treaty of the European Union we have:

Article 17

1. Citizenship of the Union is hereby established. Every person holding the nationality of a Member State shall be a citizen of the Union. Citizenship of the Union shall complement and not replace national citizenship.
2. Citizens of the Union shall enjoy the rights conferred by this Treaty and shall be subject to the duties imposed thereby.

Article 18

1. Every citizen of the Union shall have the right to move and reside freely within the territory of the Member States, subject to the limitations and conditions laid down in this Treaty and by the measures adopted to give it effect.

Article 19

1 Every citizen of the Union residing in a Member State of which he is not a national shall have the right to vote and stand as a candidate at municipal elections in the Member State in which he resides, under the same conditions as nationals of that State.

2. .....every citizen of the Union residing in a Member State of which he is not a national shall have the right to vote and stand as a candidate in elections to the European Parliament in the Member State in which he resides, under the same conditions as nationals of that State.

Article 17 with the rights attached to mobility in Articles 18 and 19 reaffirms the close relationship between citizenship and mobility in Community law. Mobility is thus not only a right in itself but also constitutes the trigger to other forms of social entitlement.

To the extent that the relationship between citizenship and migration has been subject to analysis, it is usually in the context of drawing a distinction between those migrants who hold community nationality and those who do not (in other words third country nationals outside EU).

Article 18 (1) infers a broad equality of status among Community nationals. The precise wording, however, suggests some caveats, rendering entitlement "subject to the limitations and conditions laid down in this Treaty and by the measures adopted to give it effect". The full implications of this provision and the extent to which the inclusion of Article 18 effectively replaces pre-existing law has not yet been resolved. At present residency rights are provided for under a cluster of Directives specific to different groups of migrants.

1.3. Freedom of movements in community law: the provisions . 'Citizenship of the Union', as Article 17 suggests, does not replace national citizenship but rather 'complements it'. In that context, formal citizenship status reflects not only Community law but also the specific benefits deriving from national citizenship (which vary considerably between member states, not only in a substantive sense but also, importantly, in terms of their transportability). The contribution of national citizenship as a fundamental source of social entitlement is of particular importance given the diversity of domestic welfare systems and the fact that non-discrimination remains the basis of welfare claims in the host state[1].

[1]Community law, in this context, is not concerned to promote harmonization in the social policy field but rather recognizes diversity.

Title III of the Treaty establishing the European Community provides for the 'free movement of persons, services and capital' throughout the Community. Individuals who wish to enter, work and reside in another member state can do so on the basis of Article 39 EC, which provides that

> **Article 39:** Freedom of movement of workers shall be secured within the Community. Such freedom of movement shall entail the abolition of any discrimination based on nationality between workers of the Member States as regards employment, remuneration and other conditions of work and employment...

Article 43 details the freedom of establishment of nationals to include, the right to "take up and pursue activities as self employed persons".

The constitutional right to free movement has been substantiated by secondary legislation that enables the migrant worker to overcome certain financial and social obstacles, which arise as a result of exercising free movement[2]. This has been achieved through the development of two principal mechanisms:

1. **Regulation 1612/68,** implements Articles 39-43 of the Treaty and is the main source of secondary legislation governing the free movement rights of workers. Most significantly, Article 7 (2) of Regulation 1612/68 entitles Community migrant workers to the "same social and tax advantages" as nationals in the host state.

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[2]See , Steiner, 1994, p 201; O'Leary, S., 1999, p 68

2. **Regulation 1408/71**, on the other hand, seeks to coordinate rather than harmonise domestic social security systems.

1.4. Mobility rights and Post-retirement migrants(PRM). In relation to retired EU migrants, there are essentially three different groups

1. Persons who move to another member state as workers and then retire **(retired community workers).**
2. Those who move to another member state for work and then return home on retirement.**(returning community workers).**
3. Persons who retire in the home state and move after retirement. **(post-retirement migrants).**

Whilst the first two groups have full access to host welfare systems, the third group (referred to as post retirement migrants) are effectively caught by the resources requirement stated in the Council Directive 90/365 in what became known as the 'playboy directive'. Even though this Directive extends residency rights to those Community nationals who have ceased their occupational activity in their home state and who wish to move to another member state on retirement, it does so under one important condition the persons concerned as well as their families:

... they themselves and the members of their families are covered by sickness insurance in respect of all risks in the host Member States and have sufficient financial resources to ensure that they will **not become a burden on the public** purse and social security system of the host Member State during their period of residence.

This 'resources requirement' rigorously limits the formal welfare claims that this group of retired people can make against the host state.

1.4.1. *The legal status of returnees.* The social status of returnees (those people who either return home after finishing work in another member state or return after spending a period of retirement abroad) is to a large extent a function of national law and policy. In recent years, however, the European Court has had to consider the legality of certain national laws potentially restricting the social status of returnees (on the grounds that their economic contribution has taken place in another state). A case in point is the UK's habitual residence test. This test, first introduced in 1994, requires that an EU citizen moving to the UK should satisfy certain criteria in determining their access to a range of social security benefits[3]. These criteria relate to the number of years the individual has lived outside the UK. As such, the habitual residence test operates to withhold benefits to EU citizens and returning nationals until residence is considered to have been (re) established.

2. International retirement migration (IRM) and the importance of location

The EU itself is not a welfare provider, instead it regulates access to domestic welfare systems. In order to understand the consequences of a move in retirement,

[3]Including Housing Benefit, Income Support, and Council Tax Benefit.

we need to specify that the right to freedom of movement is based on a principle of non-discrimination rather than social harmonization.

In the past the European welfare systems have been variously classified in relation to a number of criteria by theorists engaged in comparative analysis[4]. Esping-Andersen's (1990) influential study was an attempt to construct a typology of welfare states based on the different ways in which they are organised in relation to market forces, social structures and political interests. He classified a number of states according to three basic "ideal types" of welfare regime:

- 1) "Liberal/Anglo-Saxon" countries developed their welfare systems against a historic backdrop of strong class antagonism. This often resulted in basic social welfare schemes reserved for the working class, whilst private and occupational welfare was available to the middle class.
- 2) The "conservative/corporatist" welfare states of continental Europe are characterised by strong occupational, contribution based, social insurance welfare schemes for paid workers and lower rate social assistance benefits for those outside the paid labour market.
- 3) The "social democratic/Scandinavian" states are distinguished from the others by welfare systems based very much on universalistic services, full employment and equal opportunity, in which access to welfare is less dependent upon activity in the paid labour market.

[4]For example, Esping-Andersen, 1990; Lewis, 1992; Giarchi, 1996.

A fourth category, variously described as a “southern” (Ferrera, 1996) has since been added to Esping-Andersen’s original classification. The key elements that distinguish southern European welfare states have been laid out by Ferrera (1996), Katrougalos, 1966, Guillen and Matsaganis, 2000. Typically, southern welfare states have a “fragmented income maintenance system: generous retirement benefits for 'protected categories', but modest benefits for the rest, plus a low social pension for those with insufficient contributions” (Guillen and Matsaganis, 2000, p 123). In southern welfare states, the role of informal, family based care, is seen as crucial for plugging gaps in formal provision for senior citizens (Symeonidou, 1996).

The importance of a basic awareness of the diversity of welfare provision across the EU becomes clearer when one understands the legal basis of citizenship entitlement under the free movement provisions and the implications of the non-discrimination principle. In other words, *location has a major influence on access to social welfare.* Welfare status thus reflects the wider geography of mobility and the welfare mix of both sending and receiving countries. It is also life course sensitive because social welfare, in the narrow sense of access to services and so on, may figure little in the priorities shaping the initial migration decision of both economic migrants and retirement migrants. The decision of whether or when to return, on the other hand, suggests a much higher concern with access to welfare.

3. Conclusion

This chapter has examined the status of retired migrants within the European Union. It is apparent that this population is not a homogenous group, but it comprises of a number of subgroups with different legal rights.

Access to the European welfare system is based on the principle of non-discrimination. At the same time, the European Directive regulates the possibility of free riding through the resource requirement. This represents an interesting controversy for international retirement migration, because, in reality, the movement between countries is restricted to those with substantial resources even though in theory the free access applies to everyone. These two issues represent an interesting controversy for international retirement migration.

This thesis builds upon the above theories to suggest a couple of economics models to study the economics effects of IRM.. First of all, the second part will discuss a quantitative model which tries to explain the reasons why retired people move from one country to another, including the economic effect of IRM with regards to capital accumulation and private savings.

The third part of the thesis goes on to further explore the economics effects of IRM on the host country. There are two aspects which will be analysed here. First of all, once in the host country and based on the non discrimination principle, international retirement migrants can have a free access to parts of the social security of that country. Secondly, in order to support their living in the host country, these migrants will bring some savings (e.g. with which to buy a house).

The thesis analyses only some aspects of IRM, but leaves some open questions in terms of public policy: How to deal with International Retirement Migrants when they access public services in the host county? Does IRM need to be restricted only to wealthy people that can afford to move to a different country? How to tax pension income of migrants who spend most of their time in a different country where their pension is paid?

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Part 2

International Retirement Migration, Social Security a Quantitative Model

CHAPTER 3

Introduction and Literature review in Social Security

1. Introduction

Social Security systems have played an important role in the economic life of European countries. They not only provide security for the elderly, but can also fulfill a role of automatic stabilizers. They provide a means of income redistribution, as well as impacting upon capital accumulation, and labour supply.

The essential thrust of economic integration in its various forms, from trade liberalization to enhanced labour and capital mobility, can be summarized thus: economic integration, while generally welcomed, usually has adverse consequences for relatively inefficient producers. The interaction between social security and economic integration then becomes particularly interesting whenever it induces some gain or losses between countries. The aim of this part is to provide a theoretical investigation in relation to specific aspects of social security systems, using various overlapping generation models. The main dimensions of this analysis relate to the efficiency of public pensions, in particular their impact upon economic growth in the light of the European economic integration. Our aim is to contribute to the literature on the economics of social security systems, by considering new elements such as mobility of factors of production (capital and labor), and the mobility of retired people.

During this part of the thesis We are going to consider a simple model of resource allocation using a two-country overlapping generation model with free migration of workers, and the retired. We assume that the only role played by the government is to provide Social Security by a capital reserve system. The model defines households characterized by utility functions with the usual formal properties. One new element introduced will be that agents, in making their saving decisions (for pensions) will also consider an environmental incentive. We will look at the determinants of migration choices for workers or pensioners, and examine the effects of wages (workers) or different level of environment (pensioners) on these choices. Different wage conditions and different environment levels, between the two countries will impact upon migration. We will produce a model illustrating the implications of both being with or without trade. Some preliminary results from this model are discussed in the next chapter of this part.

The aim of this model is to set the basis for future analysis considering a two-country, infinite overlapping generations model with different saving preferences. We are going to consider a fully-funded pension system in an overlapping generation model *à la Diamond*[1]. As we mentioned before, a new element introduced will be that agents in making their saving decisions may consider an environmental incentive. When they make a saving decision, agents will also consider the value of the environment during their retirement period. The value of the environment in

[1]Diamond (1965)

the next period (retirement) is one argument within the consumers' utility function. We assume that the value of the environment is a function of an original exogenous endowment of the environment, which is assumed to have an identical value across periods in present value terms. The quality value of the environment is assumed to be negatively related to the presence of negative externalities and hence positively related to the presence of positive externalities. We also assume that the value of the environment is a negative function of the expected level of capital deployed during the next period. Implicit in this assumption is the view that capital investment undermines the sustainability of the environment. We assume that individuals ignore the impact of individual decision making on the overall level of capital accumulation. In other words, individuals know that their action is insignificant compared to environment. They simply maximize their utility function subject to a budget constraint, disregarding the impact on the overall level of capital accumulation. Because the individual can by himself have only a minuscule impact upon the overall level of capital accumulation, it is assumed that such an impact is disregarded when consumers make their decisions. In effect an individual consumer's optimum may lead to a sub-optimal level of capital accumulation at a social level. [Therefore we are assuming that dynamics implicit in learning-by-doing models do not operate.]

In chapter 4 we are going to show the effect on saving decisions (and also capital accumulation) of the presence of an environmental motive in a closed economy.

We will show how saving decisions change if agents consider also the effect of their decisions on the environment when they are retired.

In chapter 5 we are going to consider resource allocation in a model with two countries. The model is an overlapping generation model with free migration, which is otherwise symmetric except for the assumption of different endowments of environmental resources between the two countries, and the impact of the different levels of development in the two countries. We will allow individuals (workers and the retired) and capital, to migrate freely across national borders.

2. Literature review on Social Security

A social security scheme either may rely on pay-as-you-go principles or take the form of a capital reserve system. With a pay-as-you-go system (PAYG), the younger households' contributions are immediately distributed to the old; with a capital reserve system (CR), a capital stock, which is built up for each young generation, will be consumed when this generation becomes old. Most Western European pension systems are funded on a Pay-as-you-go basis.

When pension provision is introduced, there is often little consideration and knowledge of the effects on macroeconomic variables, and little thought about long term sustainability. After the second world war this PAYG approach to providing for old age was adopted in most Western European Countries.

Nowadays, many economists would raise severe concerns about the introduction of an old-age security system based upon a Pay-As-You-Go approach. The wide interest in economic theory relating to public pension systems in Europe, came as a consequence of three facts:

1. In no other parts of the world is provision for old-age so predominantly provided by the state.

2. Most Western European Countries have lower fertility rates than the rest of the world. Without immigration, their populations will already have substantially decreased.

3. The introduction of the Single Market (1992) and the Euro (2002), have increased the degree of economic integration and mobility of factors of production between European Countries.

Following the seminal work of Samuelson (1975), a theoretical literature has grown examining the macroeconomic relationships between social security systems, aggregate savings and the allocation of resources within an overlapping generations framework[2]. An important aspect of the analysis of the social security system involves analysis of the relationship between social security, and dynamic efficiency in terms of an optimal capital-labour ratio. This analysis is generally conducted within a simple two-sector, two-period overlapping generations (closed) economy model with emphasis being placed upon the steady state. In the literature, the term efficiency has also been related to the "Golden Rule" concept, which broadly defines the optimal level of capital for a dynamic economy as that which maximizes the steady state lifetime utility for each generation.

[2]The theoretical literature is reviewed by Felder B. (1992)

The theory of inefficiency of intergenerational transfers provided by Pay-as-you-go schemes is based on two different sets of models. The first set of models assumes a "small open economy". In this case the economies are so small in relation to the size of the world economy that the market rate of interest is assumed to be given. A number of important papers [P.A. Samuelson (1958), H.Aaron (1966), K.Spremann (1984), St.Homburg (1990)] use these kind of models. The second set of models assumes a "closed economy". The models are based upon a life-cycle theory and assume factor prices are determined endogenously. [P.A. Diamond (1965) P.A. Samuelson (1975), F.Breyer/M.Straub (1991)]

It is interesting to note that both groups of hypotheses lead to the same conclusions in terms of the welfare implications of Pay-as-you-go systems (given that an infinite sequence of overlapping generations are assumed): Steady state welfare is improved by introducing a system of transfers from younger to older generations, so long as the interest rate is lower than the growth rate of the population. That is because the rate of return in social security is assumed to be the growth rate of the population, rather than the interest rate as in a capital reserve system.

Samuelson (1975) employs a simple two-sector (private and government) two period overlapping generations model, to characterize the optimal steady state social security programme. In his work he establishes that a less than fully funded state pension scheme can be used to manipulate the level of aggregate saving and hence capital accumulation, in the economy, thereby securing attainment of the Golden Rule capital-labor ratio, where it would otherwise not prevail. This result arises because the introduction of a social security programme reduces the need for individuals to save for their retirement. It can therefore be used to reduce aggregate saving to the Golden Rule level. This Pareto improvement can only be achieved if the initial state is one of dynamic inefficiency with too much (and not too little) capital.

The Samuelson (1975) paper has provided the foundations for much subsequent work, with later authors investigating the impact of relaxing various assumptions. Among the most noteworthy of these are:

1. Presence or absence of "altruism" that can motivate intergenerational transfers such as gifts and bequests[3].

2. Uncertainty with respect to output [4]and length of lifetimes[5].

3. Myopia in the saving decision[6].

4. Inelastic or exogenous supply of labour.

5. Retirement behavior[7].

6. Public choice voting systems [8].

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[3]Barro (1975), Bernheim at al (1984, 1985), Feldstein (1988), Abel (1987), and Weil (1987).

[4]W. Richter and W. Enders (1982), and H.E Lapan. As a starting point in both papers it is assumed that there is uncertainty with respect to labour productivity.

[5]Sheshinski and Weiss(1981) consider uncertainty with respect to mortality.

[6]Feldstein (1985)

[7]Danziger (1981) at all, Aaron (1982), Lazear (1986), Hu(1979)

[8]Browning (1975)

Although these theories have provided a useful insight into various phenomena surrounding social security, in the later models we will only focus on the issue of retirement behaviour.

From the above works, it is possible to extract some themes of particular relevance:

⟹The presence of some of these factors, such a bequest, uncertainty and myopia, may themselves justify the introduction of a compulsory social security scheme.

⟹ Social Security with a Pay as You Go scheme is a substitute for private saving; consequently its availability reduces the need to save for retirement.

⟹Social Security reduces the effective cost of retirement, and may therefore encourage earlier retirement. This in turn may induce additional saving to provide for the longer than expected periods of retirement.

⟹Under the assumption of a single majority voting system, the contributions to the public pension systems are too large, and such a level generally does not converge to the one which maximizes individual utility in the long-run equilibrium.

⟹Empirically the quantitative effect of the saving rate is still a controversial point, because of the impact of several variables[9].

⟹ The introduction of uncertainty seems to confirm the inefficiency of a pay-as-you-go scheme. However, research on this type of model is still being carried out.

[9]Aaron, H. J. (1996)

$\Longrightarrow$ There is a concentrated discussion on the programs of reform in Western Europe. Due to demographic factors, all the Western European countries are facing the prospect of higher and higher contribution rates to retain the same level of pension-income[10]. The possible increase in women's employment will offer only partial relief. Simulation exercises indicate that in addition to a rise in the retirement age, further measures will be necessary.

[10]See Kieran Mc Morrow, Werner Roeger (2002).

CHAPTER 4

A model

1. The basic Model

We consider an overlapping-generations economy in which individuals live for two periods, so that at any point in time, the economy is composed of two cohorts, or generations, the young (Y) and the old (O). At any one time individuals of different generations are alive and may engage in trading with one another, each generation trades with different generations in different periods of their life. Individuals work only when young , he/she can transfer consumption between periods t, and $t+1$, by saving an amount s_t,and receiving an interest rate r_{t+1}. The economy has a fixed endowment of environmental resources $\overline{e}$. The only role played by the government is to provide Social Security by a capital reserve system in which in period t the government raises d_t in contributions from the young, and invests the contributions in capital and returns in period $t+1$, to the old generation. The savings of the young in period t and the pension contributions collected by the government in period t generate the capital stock that is used to produce output in period $t+1$, in combination with the labor supplied by the younger generation during period $t+1$.

1.1. The Decentralized Equilibrium. The economy is composed of individuals, government and firms. We assume that the total population of the economy P_t, grows at a constant relative rate n and is composed of L_t young workers and R_t retired people. Each person born at t lives for two periods and is capable of providing one unit of labour in the first period $L_t = (1+n)R_t$. Let the life time utility $U(t)$ of a representative individual born at t be dependent upon his consumption in both periods, and the value of the environmental variable in the second period:

$$U_t = U[c_t^Y, c_{t+1}^O, e_{t+1}] \tag{1.1}$$

where c_t^Y denotes his/her consumption when he/she is young, $c_{t+1}^O$is consumption when retired , e_{t+1} is the value at time t of the environment function at time $t+1$. It need to be noted that in this simple model there is no uncertenty, so all variables in are known at time t. It is assumed that the life time utility function is increasing and quasi concave with respect to all its arguments.

$$U'(\cdot) > 0, U''(\cdot) < 0$$

1.2. The Environment function. In our model the environment variable, broadly interpreted, refers to the standard of living that a country can offer to retired people i.e. natural resources. The key idea of the model is that elderly people consider this environment variable to be a substitute for consumption over the retirement time. The higher the level of this environment variable in a specific country, the smaller is the need for saving when people are young. The environment variable enters into the overall utility function (U_t). In making a consumption decision consumers also consider the value of e_{t+1} in the next period i.e. when they are old.

$$e_{t+1} = \bar{e} - \varpi \left(\frac{R_{t+1} \pm M_{t+1}}{R_{t+1}} \right) - \beta k_t \tag{1.2}$$

We assume in (1.6) that the value of the environment, is a function of the environmental endowment ($\bar{e}$) i.e. environmental capital stock, minus the sum of the congestion coefficient (ϖ) multiplied by the term in brackets adjacent to ϖ that represents the value of the congestion in period $t + 1$. The numerator of this expression contains the term R_{t+1} (retired people) plus or minus M_{t+1} which relates to migration of retired people in the country. Whilst the denominator contains only the variable R_{t+1}. The idea about the congestion coefficient is that the higher the number of retired people resident in one country, the smaller is the value of the environment variable for a single retired person (i.e. a congestion effect). We can think here of an attractive country with a lot of sun and natural resources and perhaps an asthetically pleasing environment. The presence of many residents in that country can reduce the value of its natural resources.

The final expression (βk_t) involves the coefficient β multiplied by the value of (k_t), which is the ratio of capital to labour. We assume that capital intensive production produces more pollution that a labour intensive production. The more capital intensive production is, the more productive sector in the economy is environmental unfriendly. The pollution produced by highly capital intensive production, reduces the value of the environment variable by the coefficient β.

Note that the value for a single individual of the environment function at time $t+1$ is a function of the capital at time t. That is because we assume that young individuals observe the level of the capital-labour ratio at period t, and they ignore the impact of the individuals decision making, upon the overall level capital accumulation. Thus for single individuals:

$$\frac{dk_{t+1}}{ds_t} = 0 \tag{1.3}$$

In doing so a young individual will think that $k_{t+1} = k_t$, and they will evaluate the value of e_{t+1} as a function of the observed value of the capital k_t.

1.3. The Role of the Government. Without further complications we assume that the only role played by the government is to provide Social Security by a capital reserve system which in period t raises d_t in contributions from the young, invests the contributions as capital, and pays back b_{t+1} to the same generation when they are old.

$$b_{t+1} = (1 + r_{t+1})d_t \tag{1.4}$$

The returns to the contributions to public pensions are the same as returns to private savings. The role of a Social Security system of this kind can be interpreted as involving compulsory savings imposed by the government on the young i.e. to prevent people living miserably in old age because they do not realize when they are young the future consequences of their private saving habits. The governments use of public pension contributions facilitates capital formation for the next period. We assume that the government does not consider the impact upon the environment variable given in (1.6) by its saving decision.

Here we have made an assumption that the government is not interested in affecting the environment for the simple reason that the short term advantages (e.g profit from car & oil industries, implications for inflation etc.) outweigh the long term disadvantages.

1.4. The Household Sector. At this stage in order to develop intuition from the model, we need to specify a utility function that will allow us to obtain full and analytical results. We assume that the typical household has a log-linear utility function over consumption level (c_t^Y, c_{t+1}^O) and over the level of the environment (e_{t+1}).

$$U = \ln(c_t^Y) + \alpha \ln[c_{t+1}^O + e_{t+1}] \tag{1.5}$$

The lifetime allocation problem for the typical household is that of choosing $c_t$and c_{t+1} subject to the lifetime income constraint (1.6) and the value of the environmental function (1.6):

$$
\begin{aligned}
c_t^Y &= w_t - s_t - b_t \\
c_{t+1}^O &= (s_t + b_t)(1 + r_{t+1})
\end{aligned} \tag{1.6}
$$

$$
e_{t+1} = \bar{e} - \varpi \left(\frac{R_{t+1} \pm M_{t+1}}{R_{t+1}} \right) - \beta k_t \tag{1.7}
$$

where w_t is the wage received in period t and r_{t+1} is the interest rate paid on private savings (s_t) and public pension contribution from period t to period $t+1$. Note that having been determined in the period $t-1$ the value of k_t is exogenous in the period t. The value of the environment variable e_{t+1} depends on the value of k_t (exogenous within the period t), because as we mentioned before a single individual will ignore the impact of individual decision making upon the overall level of capital accumulation. By maximizing (1.5) subject to (1.6) and (1.7), we have :

$$
\frac{1 + r_{t+1}}{\alpha} = \frac{c_{t+1}^O + e_{t+1}}{c_t^Y} \tag{1.8}
$$

Equation (1.8) asserts that the marginal rate of substitution of present consumption for future consumption, plus the environmental value, must equal the interest rates discounted by the time preference parameter α.

1.5. The Production Sector. When refering to the production sector, in order to develop some intuition we use a specific production function that allows us to obtain a full and analytical solution. We consider a competitive economy in which firms act competitively and assume log-linear constant returns to technology:

$$Y_t = L_t a \ln(k_t + 1), \;\; a > 0 \tag{1.9}$$

where Y_t is the level of output in period t, L_t is the labour input (the number of young people in period t.), $k_t \equiv K_t/L_t$ the capital-labour ratio and a is an exogenous technology coefficient. We assume that (1.9) is a net production function, with depreciation already accounted for. Output per worker $y_t = Y_t/L_t$, and is given by the production function[1]:

$$y_t = a \ln(k_t + 1) \tag{1.10}$$

Firms maximize profits, taking wage rate w_t, and the rental rate on capital, r_t as given. They act competitively, hiring labour from the young cohort, and renting capital from the government or the private sector. Hence, the equilibrium wage rate and the rate of return on capital, are equal to the respective marginal products of labour and capital:

[1]Note that for $k_t > 0$, the production function respects the usual concavity assumption: $f'(k_t) = \frac{a}{k_t+1} > 0$ and $f''(k_t) = -(\frac{a}{(k_t+1)^2}) < 0$.

$$
\begin{aligned}
r_t &= f'(k_t) = \frac{a}{k_t + 1} \\
w_t &= f(k_t) - k_t f'(k_t) = a\left[\ln(k_t + 1) - \frac{k_t}{k_t + 1}\right]
\end{aligned}
\tag{1.11}
$$

It is helpful to note that y, k, r and w are uniquely and monotonically related by the production function (1.9) and the marginal productivity conditions (1.7). The market clearing condition will require that total investment be equal to total savings including the public pension contribution, so:

$$K_{t+1} - K_t = L_t(s_t + d_t) - K_t$$

Where the left-hand side is the net investment and the right-hand side is the net savings (saving of the young - dissaving of the old). Eliminating K_t from both sides and with $L_{t+1} = (1 + n)L_t$ we can determine the capital accumulation equation in terms of the capital-labour ratio:

$$k_{t+1} = \frac{(s_t + d_t)}{(1 + n)} \tag{1.12}$$

From (12) we can observe that the capital-labour ratio in period $t + 1$ is determined by public and private savings in period t. Equation (1.12) represents the solution to the model in terms of dynamic behavior of the capital stock k_t. In the next section we will derive from (1.12) the capital accumulation equation

$k_{t+1} = f(k_t)$ for a closed economy. The capital accumulation equation is the relationship between k_{t+1} and k_t that describes the path of the capital stock. In the second stage we will analyze the effect of the environment variable on capital accumulation. Finally in order to discuss a full and analytical solution, we will present numerical evaluations of the capital accumulation equation.

2. Dynamics and Steady State in a closed economy

We define as a closed economy an economy where there is no trade between the rest of the world and that economy. The capital stock is generated internally and no capital flows are allowed to enter or exit the economy. Also the labour force is generated internally and there is no migration of workers or retired people between countries. This will imply $M_t = 0$ all the time so we can simplify (1.7) as:

$$e_{t+1} = \overline{e} - \varpi - \beta(k_t) \tag{2.1}$$

Note that the value of k_t is exogenous within the period because it has been predetermined by the savings in period $t-1$. This implies also that the value of the enviroment variable e_{t+1} is exogenous within the period.

2.1. Saving decisions. The lifetime allocation problem for the young person in a closed economy will be to choose the total amount of saving s_t, so as to maximize the lifetime utility function (1.5) subject to the income constraint (1.10), the government budget constraint (1.4), and the value on the environment given by (2.1). The savings as a function of current income is:

$$s_t = \frac{\alpha}{1+\alpha} w_t - \frac{\bar{e} - \varpi - \beta k_t}{(1+r_{t+1})(1+\alpha)} - d_t \tag{2.2}$$

$$1 > s'_{w_t} > 0,\ s'_{r_{t+1}} \lesseqgtr 0 (\text{for } \bar{e} \lesseqgtr \varpi + \beta k_t),\ s'_{\bar{e}} < 0,\ s'_{d_t} = -1, s'_k > 0, s'_{\varpi} > 0$$

Using (2.2) we can easily see that s_t is increasing with respect to the current wage, and with respect to the depletion of the environment given by βk_t and ϖ. It is also decreasing with respect to the environmental endowment. Note that in the case of a fully funded pension system private savings are a perfect substitute for the contribution d_t. Any increase in social security saving, d_t, is exactly offset by a decrease in private savings in such a way that the total $s_t + d_t$ is constant.

$$s_t + d_t = \frac{\alpha}{1+\alpha} w_t - \frac{\bar{e} - \varpi - \beta k_t}{(1+r_{t+1})(1+\alpha)} \tag{2.3}$$

The sum $s_t + d_t$ represents the total savings of the private agents. It is interesting to note that if the government imposes a public pension contribution $d_t \geqslant \frac{\alpha}{1+\alpha} w_t - \frac{\bar{e}-\varpi-\beta k_t}{(1+r_{t+1})(1+\alpha)}$, equation (2.3) does not hold and $s_t = 0$[2]. The economic intuition behind this is straightforward: social security contributions provide a

[2]That is the unusual case where only government provide public saving. The capital accumulation equation in this case is simply determinate exogenously by the government. $k_{t+1} = \frac{(d_t)}{(1+n)}$.

rate of return equal to the private saving ones; this means that the consumer is, therefore, indifferent to who does the saving, they care only about the rate of return; any change in the public pension contributions will be fully balanced out by an equivalent increase/decrease in private savings. If the government imposes a public pension contribution d_t above the optimal level defined in (2.3), then agents will avoid any kind of private saving. In our analysis we assume that governments sets each period $d_t < \frac{\alpha}{1+\alpha}w_t - \frac{\bar{e}-\varpi-\beta k_t}{(1+r_{t+1})(1+\alpha)}$. This means that social security contributions do not exceed the amount of savings that the economy would otherwise have incurred. We need to point it out that the social security system base on a capital reserve does not have any effect on the steady state.

2.2. Capital accumulation. We assume the supply of capital in period t is determined by the sum of total saving decisions of the young, and the government in period $t-1$. We also assume labour is supplied inelastically. The equilibrium in the capital/labour markets is obtained when wage and interest rates are such that firms use the total amount of capital and labour disposable on the markets. The factor market equilibrium is therefore given by equation (1.11). If we combine this equilibrium together with the capital accumulation equation (2.2), and the optimal saving decision equation of (2.3) we can derive the implicit function of the dynamic behavior of capital-labour ratio for a closed economy:

$$k_{t+1} = \frac{\alpha}{(1+\alpha)(1+n)}a\left[\ln(k_t+1) - \frac{k_t}{k_t+1}\right] - \frac{\bar{e}-\varpi-\beta k_t}{\left(1+\frac{a}{k_{t+1}}\right)(1+\alpha)(1+n)} \tag{2.4}$$

Equation (2.4) states the relationship between k_{t+1} and k_t that defines our curve of capital accumulation for the economy. One important property of this function depends on the derivative:

$$\frac{dk_{t+1}}{dk_t} = \frac{\left(\frac{\alpha k_t a}{(k_t+1)^2}(1+\frac{a}{k_{t+1}})^2 + \beta(1+\frac{a}{k_{t+1}})\right)}{(1+\alpha)(1+n)(1+\frac{a}{k_{t+1}})^2 + (\bar{e} - \varpi - \beta k_t)(\frac{a}{k_{t+1}^2})} \tag{2.5}$$

The numerator of equation (2.5) is positive for every $k_t > 0$; it is possible to show also that the denominator is positive even when $\bar{e} < \varpi + \beta k_t$. This means that for $k_t > 0$,$\frac{dk_{t+1}}{dk_t} > 0$ so the capital accumulation equation given by (2.4) is always increasing. At any point at which the capital accumulation (2.4) crosses the 45-degree line at which $k_{t+1} = k_t$ there are steady state points. To define the nature of the steady state (stable/unstable) we need to evaluate (2.5) at the point of steady state; for$\left| \frac{dk_{t+1}}{dk_t} |_{k_{t+1}=k} \right| < 1$, we will have a stable steady state solution otherwise the steady state will be unstable.

Since analytical results in terms of stability of the steady state are hard to obtain given the complexity of (2.4) and (2.5), the future analysis will be carried out using numerical evaluation.

2.3. A numerical evaluation. Since our model is highly simplified it is not our aim to provide precise quantitative estimates; but to derive more realistic results from the overlapping generations model, we need to adopt empirically significant parameter estimates for the simulation analysis.

2.3.1. *The basic case.* Using the famous empirical work of Romer, Mankiw and Weil (1992) we set the technology coefficient of our production function (10) to $a = 350$[3] . We assume a population growth $n = 0.025$[4]. We also assume the rate of intertemporal substitution $\alpha = 0.95$. Setting $\overline{e} = 0$, $\varpi = 0$, $\dot{\beta} = 0$ (14) we get:

$$k_{t+1} = \frac{\alpha}{(1+\alpha)(1+n)} a \left[\ln(k_t + 1) - \frac{k_t}{k_t + 1} \right] \tag{2.6}$$

equation (2.6) represents the dynamic solution for the basic model. In this simple case since the environment variable is $e_{t+1} = 0$ the environment enters with a zero value in the utility function (1.6). The numerical evaluation of (2.6) brings us also to a unique equilibrium in steady state with a positive capital stock. This solution is represented graphically in figure 1 where the ss represents the 45 degree steady state line. The path to the steady state level depends upon the initial level of k_o. The economy starts out at k_o and gradually moves towards the steady state capital stock. For example in the case of k_0 in figure 1, the per-capita level of capital will gradually increase to k^*, the opposite applies to a starting point like k_0'.

---

[3]In their work Romer, Mankiw and Weil using a sample of 22 OECD over the period 1960-80 estimated a value of the logoutput per working age income of $\ln(y) = 7.8$. In our model $a = 350$ is the value that allows our production function (10) to reach a steady state at the same per capita income level.

[4]That is the value estimate in terms of Birth rate-Death rate by the Population Reference Bureau (http://www.prb.org) during the last 20 years in the more developed counties.

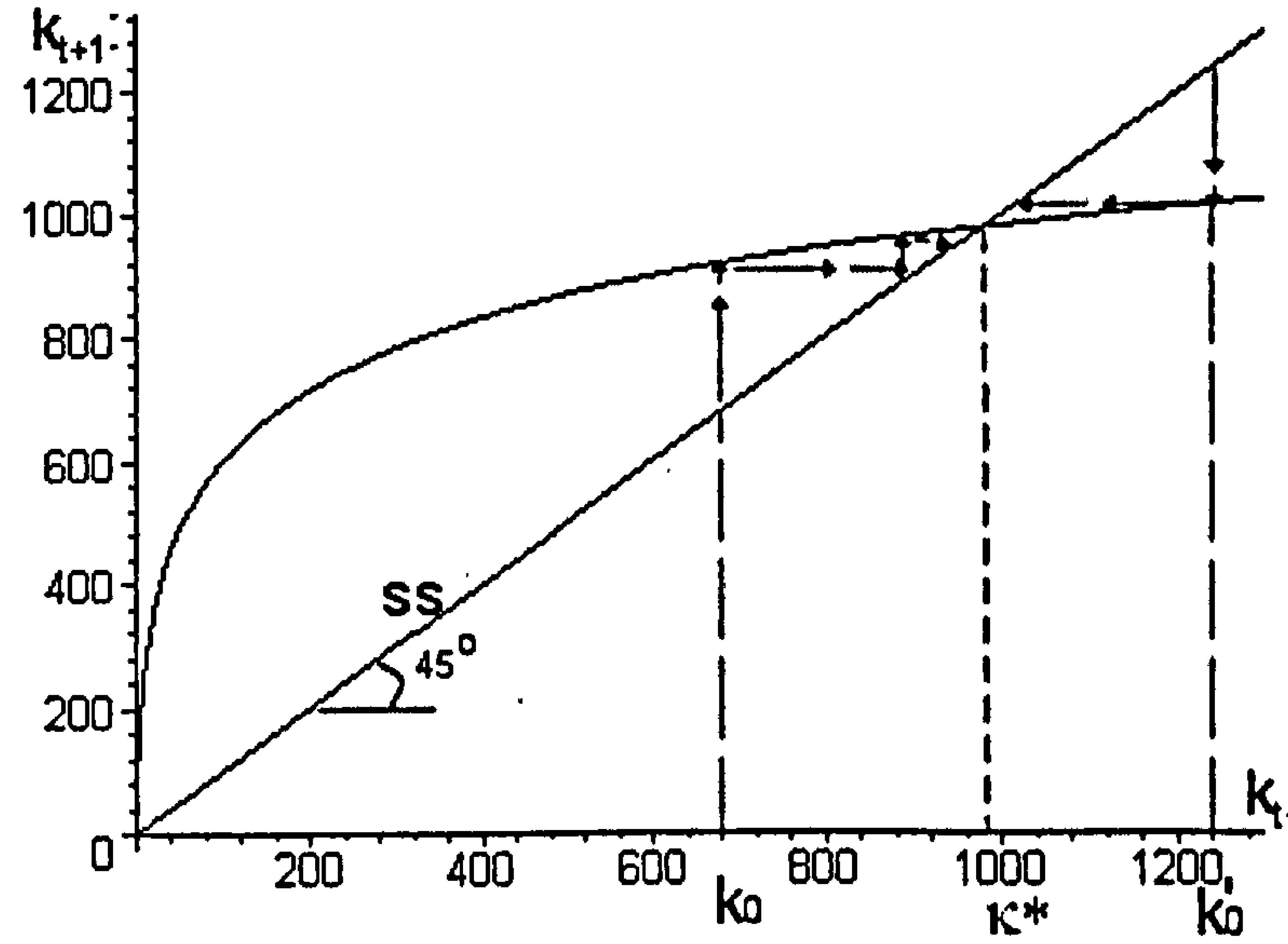

FIGURE 1. Dynamic adjustment for the simplest case $\bar{e} = 0, \varpi = 0, \beta = 0$

We next want to examine how the level of the steady state changes if we specify specific non-zero values for $\bar{e}$, and β.

2.3.2. *The environmental endowment* $\bar{e}$. In figure 2 we show the indications of (2.4) with three positive levels of endowment of the environment $\bar{e}$, and keeping $\varpi = 0, \beta = 0$. Note that in this case the value of the environment function ($e_{t+1} = \bar{e} - \varpi - \beta(k_t)$) is exogenous in the model and is simply given by $e_{t+1} = \bar{e}$. In this case this numerical solution gives us a unique path for the capital accumulation equation (2.4), with two steady states with strictly positive level of capital. The first steady state is not stable and is determined by a very small level of capital ($k = 0.2$)[5]. These different levels are represented by the dotted lines. The continuous line is the same one examined in previous section with $\bar{e} = 0$, $\varpi = 0$, $\beta = 0$. Considering

[5]From picture 2 is not possible to see the unstable steady state.

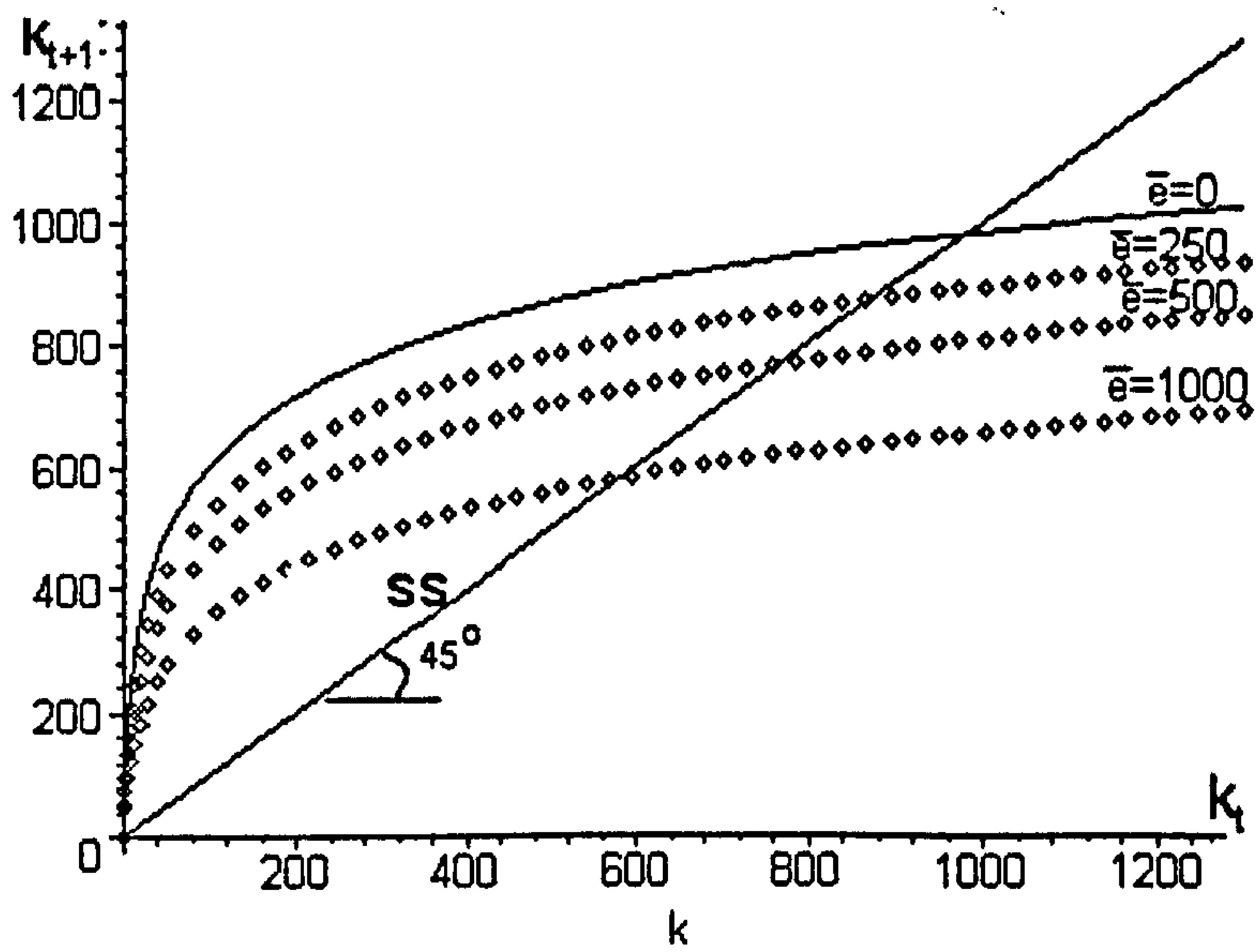

FIGURE 2. Dynamic adjustment for the cases $\overline{e} = 0$, $\overline{e} = 250$, $\overline{e} = 500$, $\overline{e} = 1000$

only the stable stady state it is immediately possible to observe that the larger is the level of the endowment level of environment taken into account in the saving decision, the lower is the level of the Steady State for the economy. The economic intuition behind this is that: the environmental resources are a substitute for the savings of retired people. The higher is the environmental level, the lower is the incentive for each younger generation to save. This obviously is reflected in a lower level of capital in Steady State.

2.3.3. *The environment depreciation coefficient β.* In figure 3 we repeat the same experiment conducted for $\bar{e}$ testing the effect of positive level of β on the steady state solution. Even in this experiment we find that a positive level of β does not compromise the stability of the capital accumulation equation. We tested three different levels of β and we observed that the higher is the level of the environmental depreciation coefficient, then the higher is the level of capital in steady state[6].

The economic reasoning for this arises because of coordination failure arising when the younger generation make their own saving decisions. As we have discussed in a previus section, agents evaluate the variable e_{t+1} as a function of k_t because they think that the level of k_{t+1} will not be affected by their own saving decision. Also the government with a pension system fully funded and with $d_t < \frac{\alpha}{1+\alpha}w_t - \frac{\bar{e}-\varpi-\beta k_t}{(1+r_{t+1})(1+\alpha)}$ is not able to affect the overall saving $(d_t + s_t)$.

If agents evaluate the variable e_{t+1} as a function of k_t, the value of e_{t+1} is exogenous with respect to the period t $(e_{t+1} = \bar{e} - \varpi - \beta(k_t))$. We need to remember from (1.3) that the value of the capital k_t is determined by total savings in period $t-1$. This implies that k_t is exogenous with respect to the period t.

Agents think that they cannot affect the overall level of capital in the next period, and consequently the value of the environmental variable (1.7). Therefore to compensate for the loss arising from the lower level of the environment variable, the typical agents will save more when young. In doing so they will compensate for the loss in the environment with more consumption when old. These factors will obviously result in a higher level of Steady State capital as shown in figure 3.

[6]In this case this numerical solution gives us a unique path for the capital accumulation equation with two steady states with strictly positive capital. The first steady state is not stable and is determined by a very small level of capital (k=0.1). From picture 3 is not possible to see the unstable steady state.

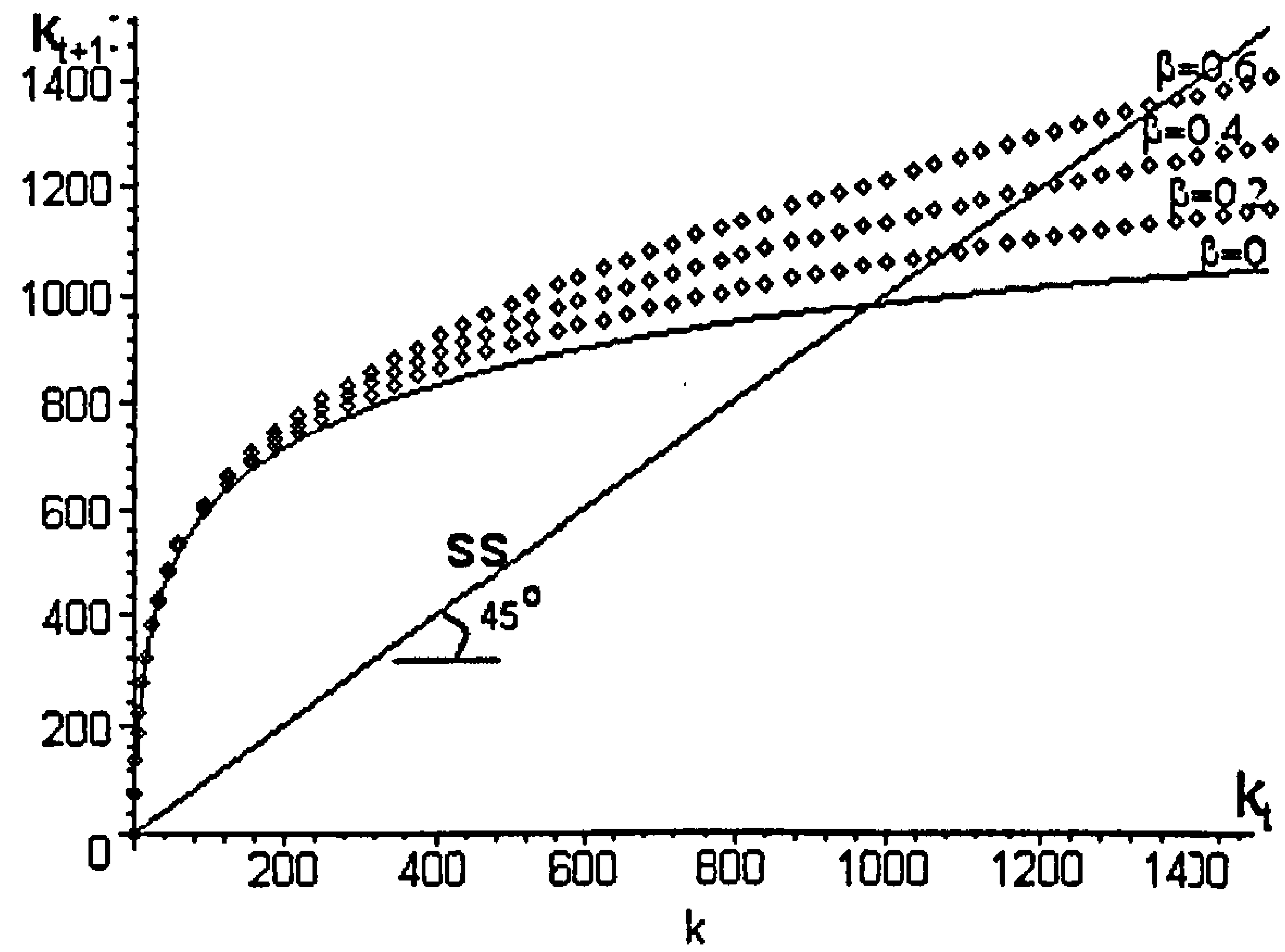

FIGURE 3. Dynamic adjustment for the cases $\beta = 0$, $\beta = 0,2$, $\beta = 0,4$, $\beta = 0,6$

3. Some preliminary results

This chapter has extended the analysis of macroeconomic allocation of resources in a dynamic economy introducing an environmental motive into the utility function. We can draw some stylized results from the case observed.

$\Longrightarrow$ The introduction of an environmental motive can shift down the capital accumulation curve bringing about a lower level of capital stock in Steady State.

$\Longrightarrow$ Two countries with access to the same technology and same utility functions, but with different levels of endowment of environment ($\bar{e}$) have two different capital accumulation curves, and two different Steady States.

$\Longrightarrow$The fact that individuals discount any impact that they individually can have upon capital accumulation can result in a higher level of overall saving. This is in order to compensate for the loss in utility arising from lower environmental levels.

$\Longrightarrow$ If we assume that agents are able to predict correctly the level of capital in the next period, our findings in term of capital accumulation can be reversed.[7]

[7]If we assume $e_{t+1} = \overline{e} - \varpi \left(\frac{R_{t+1} \pm M_{t+1}}{R_{t+1}} \right) - \beta(k_{t+1})$ it is possible to show that for higher levels of β the capital accumulation curve is shifted down to lower a level of capital stock in Steady State.

CHAPTER 5

The two country global-equilibrium model

1. The two economies

In this Chapter we consider resource allocation in a two-country [Home and Foreign (*)] overlapping generations model with free mobility of factors, and free migration of labour and retired people[1].

We assume that these two countries at time $t = 0$ form a Community comprising only of two countries. The community as a whole is characterized in the following way: First, we have free trading of goods and services. Second, when integration takes place the two economies converge to a fully integrated capital market with a unique interest rate r_t and unique wage rate w_t. Thirdly, householders can freely migrate within the community.

Each country has access to the same production technology given in equation (1.9) but may be at a different stage of development. This means that at the time of the integration, the countries can have two different levels of capital stock.

$$k_0 \gtrless k_0^* \tag{1.1}$$

[1]See Obstfeld and Rogoff (1998) chapter 3.

We also assume that residents of the two countries have identical logarithmic preferences given by eq (1.10), and identical population growth rates $n = n^*$. Regarding the environment function we assume that the countries have the same environment function, but different endowments levels. For this specific case we will assume that the foreign country has a higher endownmental.

$$\bar{e}^* > \bar{e} \qquad \varpi^* = \varpi \qquad \beta^* = \beta \tag{1.2}$$

If there were no integration between the two countries we would have two different capital accumulation curves, and also two different levels of capital stock in steady state, $k_{SS} > k^*_{SS}$[2]. This is shown in Figure 1.

1.1. The world competitive equilibrium. In this world comprising of two countries [Home and Foreign (*)], a competitive equilibrium requires that each household resides in one of the two countries, that markets clear, no household can make itself better off by selecting a different consumption bundle, or by locating in another jurisdiction, and no firms can make higher profits.

1.1.1. *Capital and labour market.* With the Home and Foreign markets integrated by a world capital and labour market, if labour and capital move in opposite directions this will equate at time $t = 0$ the interest rate $(r_1^W = r_1^* = r_1)$ and the wage $(w_0^W = w_0^* = w_0)$ in both countries, so that $k_0^W = k_0 = k_0^*$.

[2] This is the case discussed in previous chapter where we show that the higher the endowment $\bar{e}$ the lower the Steady State k_{SS}.

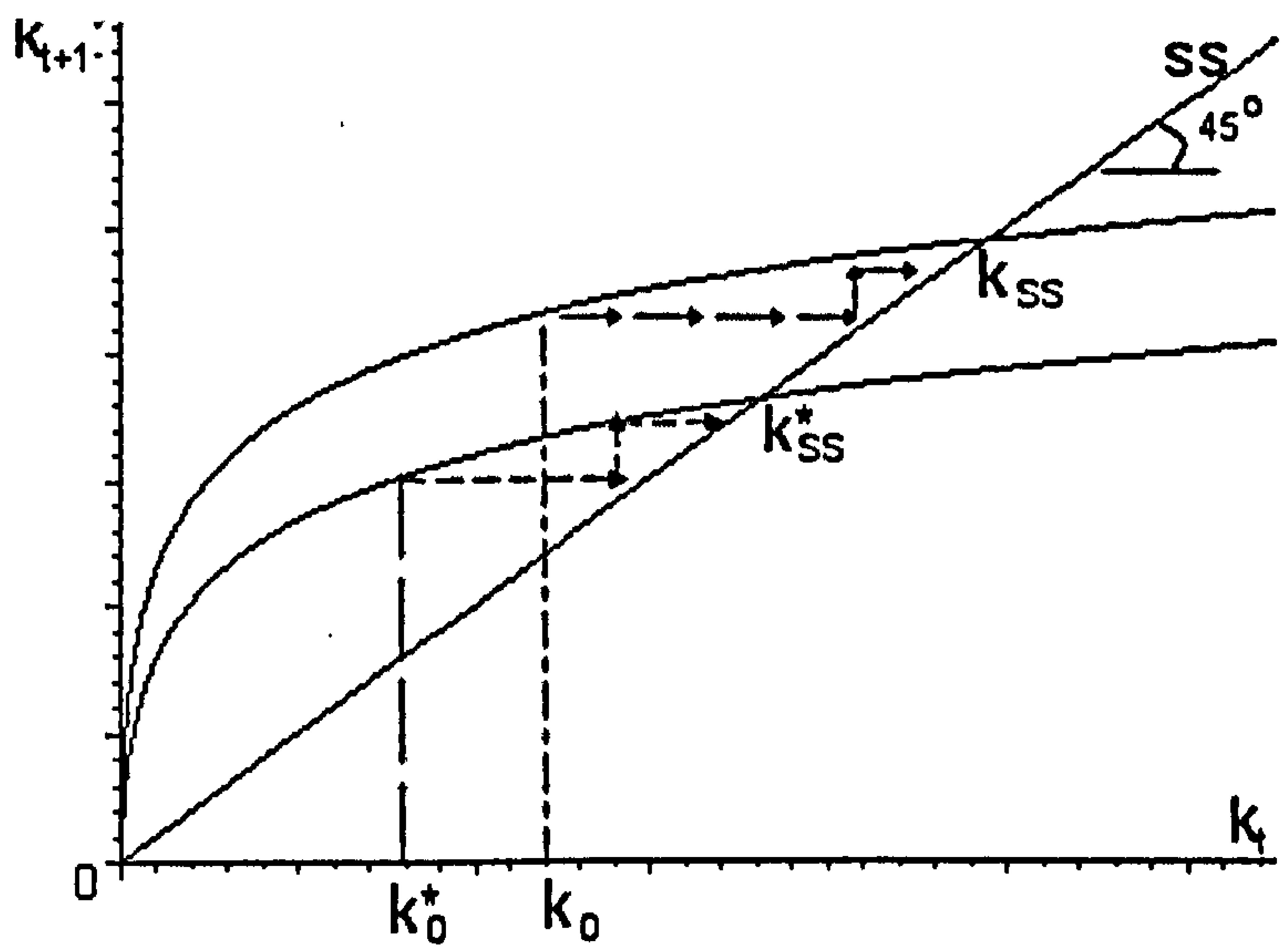

FIGURE 1. Dynamic adjustment for the two counties without integration

$$r_1^W = r_1^* = r_1 = \frac{a}{k_0^W + 1} = \frac{a}{k_0 + 1} = \frac{a}{k_0^* + 1} \tag{1.3}$$

$$\begin{aligned} w_0^W &= w_0^* = w_0 = a\left[\ln(k_0^W + 1) - \frac{k_0^W}{k_0^W + 1}\right] = a\left[\ln(k_0^* + 1) - \frac{k_0^*}{k_0^* + 1}\right] = \\ &= a\left[\ln(k_0 + 1) - \frac{k_0}{k_0 + 1}\right] \end{aligned} \tag{1.4}$$

k_0^W is the unique common capital-labour ratio across the two economies after the integration has taken place. This implies identical production technologies, and is given by the endowment of the labour force and capital in the two economies:

$$k_0^W = k_0 = k_0^* = \frac{K_0 + K_0^*}{L_0 + L_0^*} \tag{1.5}$$

1.1.2. *The environment function.* In the context of open economies retired people can migrate across national borders looking for better living conditions. As stated in part one refering to the European Union, we assume that governments allow retired people to collect their full pension even if they move abroad. According to our model economic integration implies that young agents will also consider the possibility of moving abroad for the retirement period. This possibility is implied in equation (1.2) when $M \neq 0$. If integration takes place in period 0 and $\bar{e}^* > \bar{e}$ a competitive equilibrium requires that in period 1 we observe migration of retired people from home to abroad up to the point where:

$$e_1^* = e_1 = e_1^W = \bar{e} - \varpi\left(\frac{R_1 - M_1}{R_1}\right) - \beta k_0^W = \bar{e}^* - \varpi\left(\frac{R_1^* + M_1}{R_1^*}\right) - \beta k_0^W \tag{1.6}$$

Solving (1.6) with respect to M_1 we can calculate the value of the migration flow between the two countries:

$$M_1 = \frac{\bar{e}^* - \bar{e}}{R_1^* + R_1}\frac{R_1 R_1^*}{\varpi} \tag{1.7}$$

Substituting (1.7) in (1.6) we can calculate the value of the global environmental function that is given by:

$$e_1^W = \overline{e}^* \frac{R_t^*}{R_1 + R_t^*} + \overline{e} \frac{R_t}{R_t + R_t^*} - \varpi - \beta(k_0^W) \tag{1.8}$$

Note that in the new world equilibrium the congestion coefficent ϖ does not change. The new world equilibrium is a function of the population size and the endowment of the environment of the two economies. At this moment we can define the endowment of environment of the world economy such that:

$$\begin{aligned} \overline{e}^W &= \overline{e}^* \frac{R_t^*}{R_1 + R_t^*} + \overline{e} \frac{R_t}{R_t + R_t^*} \\ \text{for } \overline{e}^* &> \overline{e} \implies \overline{e}^* > \overline{e}^W > \overline{e} \end{aligned} \tag{1.9}$$

We assume now that at time 0, when economic integration takes place, young people will have perfect expectations about future migration of retired people, and will make saving decisions considering the new world level of environment given in (1.8). At this point it is possible to define the world capital accumulation function, that is represented in figure 3. Since $\overline{e}^* > \overline{e}^W > \overline{e}$ the new accumulation function lies between the home and the overseas function. It need to be noted that after integration take place, it will occurs every period and this will equalise the environmental variable in the two countries all the time.

1.1.3. *The Dynamics of equilibrium and Steady State in the global economy.* We are now in a position to characterize world equilibrium. In period 0 when the integration takes place movement of capital from home to abroad and/or an opposite movement of labour will take place in order to balance the capital-labour ratio according to equation (1.8)[3], that is represented in Figure 2. The home economy will move from point C up to B, and the overseas one will move from A to B. Note that the bigger the gap between the two economies in terms of the capital-labor ratio, the bigger the jump that the two economies will make in period 0 to reach the world capital accumulation curve. After period 0, the two economies will move across the common World Capital accumulation curve up to the new Steady State k_{SS}^W.

The world capital accumulation equation will be given by:

(1.10)

$$k_{t+1}^W = \frac{\alpha}{(1+\alpha)(1+n)}a\left[\ln(k_t^W+1) - \frac{k_t^W}{k_t^W+1}\right] - \frac{\overline{e}^*\frac{R_t^*}{R_t+R_t^*} + \overline{e}\frac{R_t}{R_t+R_t^*} - \varpi - \beta k_t^W}{\left(1+\frac{a}{k_{t+1}^W}\right)(1+\alpha)(1+n)}$$

It is interesting to note that the new equilibrium in terms of the capital-labour ratio will be smaller for the home economy and bigger for the foreign one[4]. Note also that the world equilibrium k_{SS}^W does not depend upon the starting points of the two economies. We would have the same result even if $k_0 < k_0^*$, but it would depend upon the endowment of the environment $\overline{e}$ that affects the saving decisions of the agents. The path of the capital-labour ratio is also represented in the time-capital axis in figure 3.

[3]No assumptions are made to specify how much capital, and how much labour needs to move in order to reach the equilibrium.

[4]Comparing to the capital-labour ratio before the integration take place.

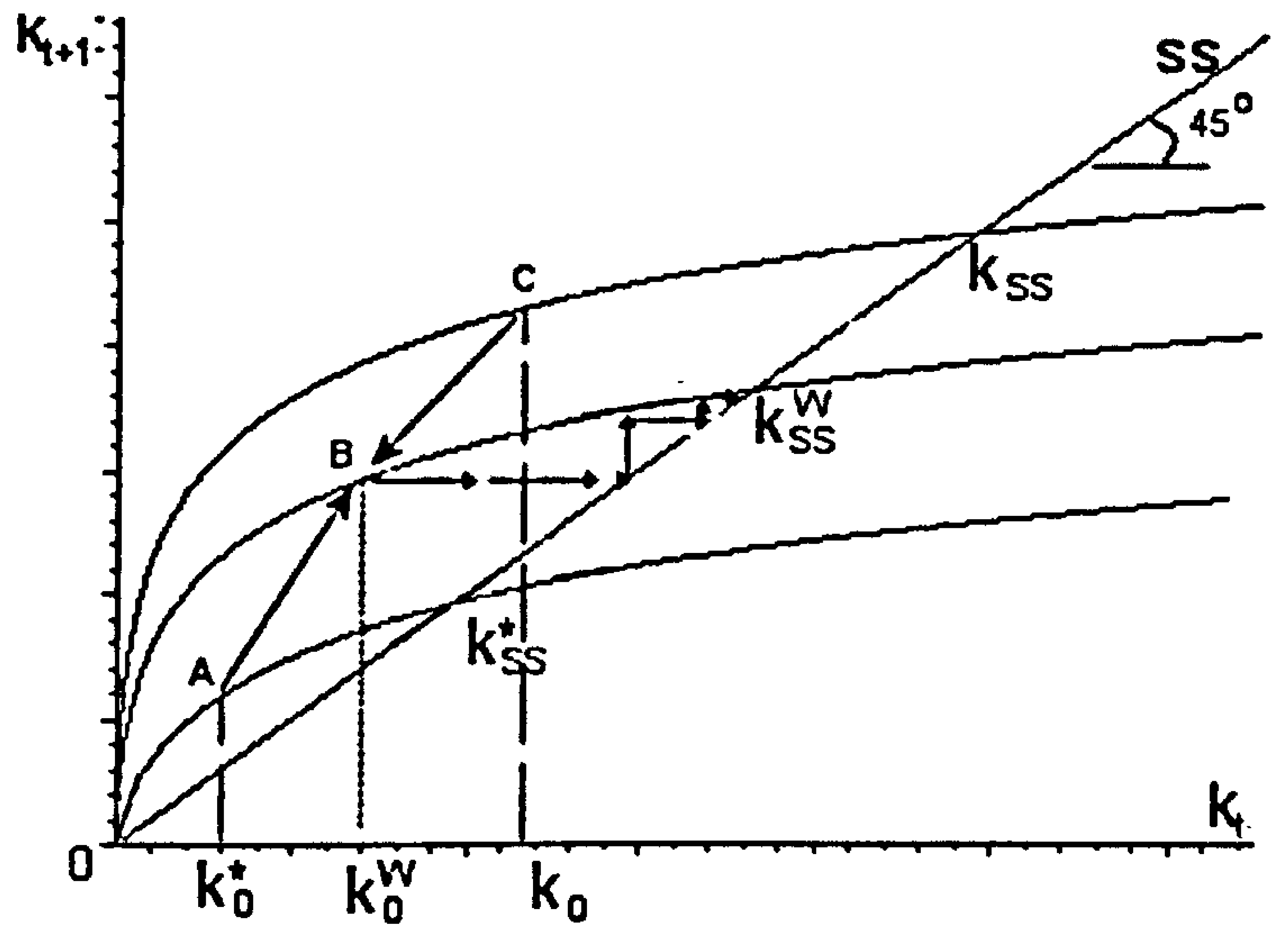

FIGURE 2. Dynamic and Steady State of the world capital-ratio.

One criticism that can be made of this kind of solution is that at the time of the integration ($t = 0$) the model implies a big shock in terms of the capital stock for the two economies. Many people may think that even with full mobility of capital and labour, it is unrealistic in the short run to have that kind of shock. Actually the basic idea that we need to keep in mind is that in an overlapping generation fashion, each period corresponds to the half life of an individual, so one period in our model corresponds to many years in the life of individuals.

2. Conclusions

In this chapter we have analyzed the allocation effects of an environmental motive in a two country overlapping generation model. Our principal findings have been the following:

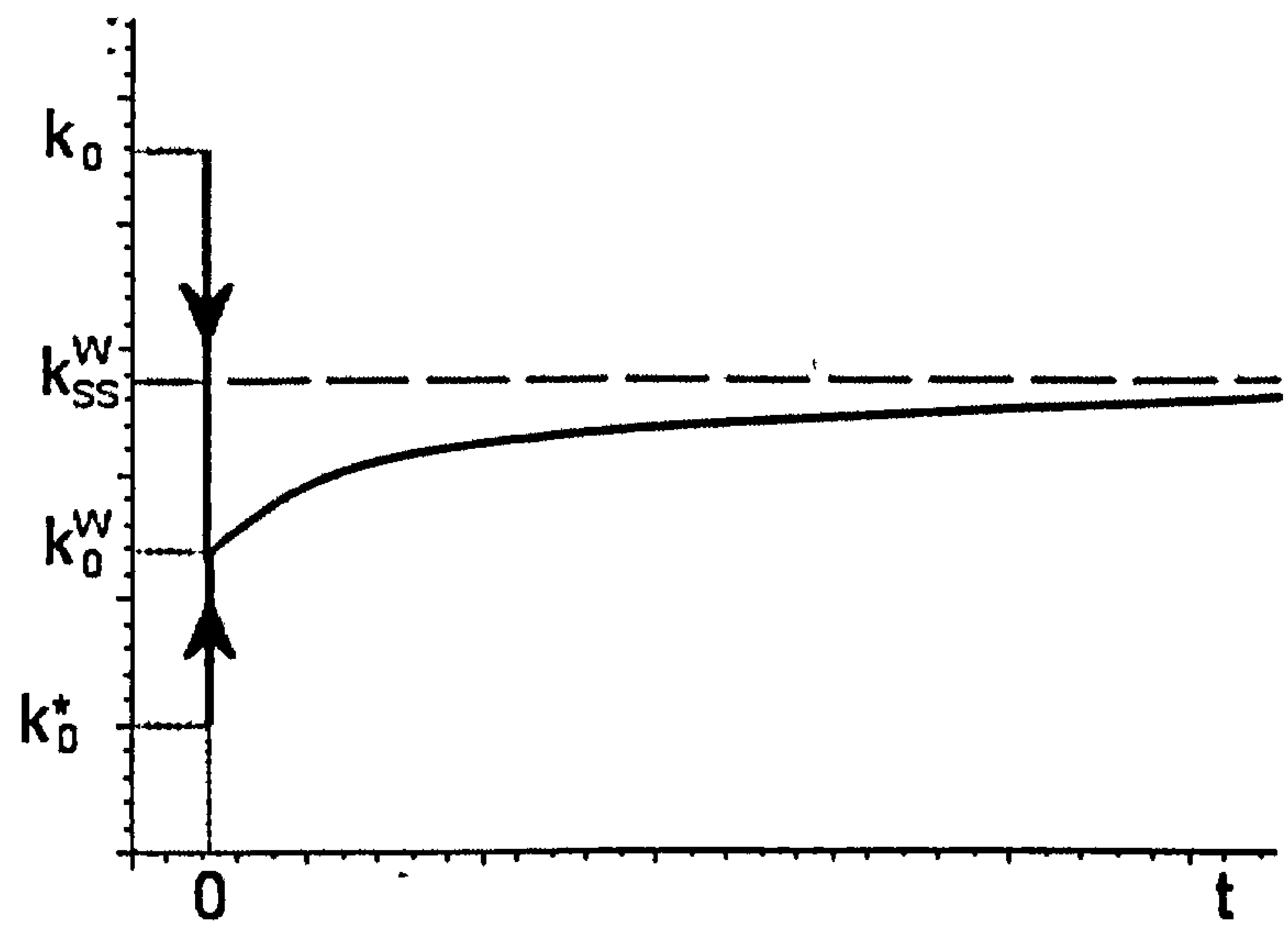

FIGURE 3. Dynamic path of the world capital-ratio.

1) Using a logarithmic production function and a log linear utility function we find a unique stable path for the capital accumulation equation. Given the shape of the capital accumulation curve, we also have a unique and stable equilibrium for positive levels of the capital-labour ratio in Steady State.

2) An environmental motive in the saving decision shifts the capital accumulation curve, and also the steady state capital down.

3) If young people think that they are no longer able to affect the capital level in the next period, they will save more in order to balance with consumption the loss in environment when they are old. This will raise the Steady State capital level. If the young are conscious of the effect of their own saving decisions on capital, then the previous result can be reversed.

4) In a two-country integrated model different endowments of environmental resources will activate flows of migration between the two countries such that the value of the environment function is the same in both countries. This phenomenon will also modify the capital accumulation curve (and the Steady State), for both countries.

5) The bigger the difference in terms of the stage of development between the two countries that are going to integrate, the higher the shock that economic integration will cause to the economies.

We now want to point out in an informal manner some generalizations as well as limitations of our analysis. In the above idealized model, households respond quickly to wage differentials and immediately move to the country offering the highest salary. A natural objection to this would be that in reality many people are immobile, i.e. they will stay at their original location irrespective of wage differentials. We admit, of course that a Spanish worker is unlikely to move to Germany at once if he finds wages to be infinitesimally higher there[5]. Yet, as the 21st century will probably become a "century of migration", we also feel that this point should not be pushed too far. For our results to hold qualitatively, it suffices if there are always some people who consider moving location due to higher wages. It seems to be clear that the development of the Common Market,

[5]This is not least because there are transaction costs associated with moving to another location.

the single currency, and the formation of a political union, will make EC residents even more mobile.

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Part 3

International Retirement Migration, Social Security, Dynamic Aspects a Simulation Approach

CHAPTER 6

Two-period life cycle model

1. Introduction

This part of the thesis examines some effect of International retirement Migration and social security on the economy. As its title suggests, this part is concerned with dynamic aspects. These include the effects of social security on capital formation, economic growth, and intergenerational equity. Dynamic analysis has recently gained favor over static analysis in various fields of economics. A dynamic perspective is crucial in weighing the short-run benefits of particular policies against long run losses and in evaluating the economic efficiency of alternative policies. Dynamic analysis considers both current and future generations and permits one to distinguish policies that truly improve economic efficiency from those that simply redistribute resources across generations. Studying a dynamic general equilibrium model involves a number of issues that are not present in a static model. These include aggregation of behavior of overlapping generations, and solving for the equilibrium transition path of the economy. The difficulties in obtaining qualitative or quantitative analytical results influenced our decision to use a computer simulation model to study the dynamics of the models.

The numerical simulation technique is required because of the complexity of the problems studied here. As a consequences, the simulation results are highly intuitive and most easily understood by tracing the effects of policy change through the differents parts of the models.

In this model, one young and one old generation exist at any point in time. We assume that neither the population or productivity grows and, for the moment, that there is no government.

Since the old in this life cycle model are assumed to spend their old age resources (their saving plus income earned on their saving) entirely on their old age consumption, there are no bequests, gifts, or other forms of net intergenerational transfer to the young. As a consequence, the life time earning of the young correspond to the labour earnings they receive when young.

If we adopt the convention that output is produced, income is received, and consumption occurs at the end of each period, the tangible wealth of the economy at the beginning of any period consists of private asset held by the elderly. Since the elderly consume all available resources in their possession at the end of their last period of life, the capital stock to the economy in the next period consists of savings by the current young that they bring into the next period.

The supplies of productive factors to the economy thus consist of the labour supply of the current young and the old, plus the capital supplied by the elderly. These factors are supplied to the production sector of the economy. The output of the production sector in turn is paid out to the productive factors as returns to capital and labor.

Since the production sector is competitive, factors are hired to the point where marginal revenue products equal factor payments. For the economy to be in equilibrium, the time path of factor demands must equal the time path of factor supplies.

2. Modeling the economy

This model consists of two sectors: a household sector, and a production sector. For each sector, there is a system of nonlinear equations relating endogenous behaviour variable such as consumption and labour supply to predetermined economic variables and the technological parameters.

By jointly solving the equations of the sectors, we can obtain a solution for the equilibrium of the economy.

2.1. Household behaviour. At any given time the household sector comprises 2 overlapping generations (Young and Old).In each period one generation dies and another take its place.

Individual tastes are assumed to be identical, with differences in behavior being generated entirely by differences in economic opportunities. Since all individuals in a group (Young or Old) are assumed to be identical, all differences in economic opportunities are cross-group differences. The assumption that a single member is representative of each generation, makes it possible to describe the aggregate behaviour of members of each generation by the behaviour of a single member.

The model does not include population growth.

Households in the model make lifetime decisions about **consumption** and **leisure** on the basis of the life cycle model of behaviour, leaving no bequests and receiving no inheritances.

2.1.1. *Preferences.* Each household is assumed to have preferences that can be represented by a utility function with current and future values of consumption and leisure as arguments. Leisure is measured as a fraction of the maximum amount of time an individual could work in a given year, taking values between zero and one.

We restrict preferences by requiring the utility function to be time-separable and with constant elasticity of substitution (CES) form.

Time separability means that lifetime utility can be expressed as a function of the individual function of leisure and consumption in each period:

$$U(c,l) = U[u(c_Y, l_Y), u(c_O, l_O)] \tag{2.1}$$

where c, l are consumption and leisure for the period. It also assumed here that the function $u(\cdot)$ does not vary over time, so that $u_t(\cdot) \equiv u(\cdot)$. The nested CES takes the form:

$$u_t = \left[c_t^{(1-\frac{1}{\rho})} + \alpha l_t^{(1-\frac{1}{\rho})}\right]^{\frac{1}{1-\frac{1}{\rho}}} \tag{2.2}$$

while the life time function is written as:

$$U = \frac{1}{1-\frac{1}{\gamma}} \sum_{t=1}^{2} (1+\delta)^{(1-t)} u_t^{(1-\frac{1}{\gamma})} \tag{2.3}$$

or in our specific case (Young and Old):

$$U = \frac{\left[\left(C_Y^{(1-\frac{1}{\rho})} + \alpha L_Y^{(1-\frac{1}{\rho})} \right)^{\left(\frac{\rho(\gamma-1)}{\gamma(\rho-1)}\right)} + \frac{\left(C_O^{(1-\frac{1}{\rho})} + \alpha L_O^{(1-\frac{1}{\rho})} \right)^{\left(\frac{\rho(\gamma-1)}{\gamma(\rho-1)}\right)}}{1+\delta} \right]}{1-\frac{1}{\gamma}} \tag{2.4}$$

where ρ, α, γ, and δ are taste parameters that permit a wide range of individual behaviour to be represented by this general specification of preferences. Each parameter is associated with a different aspect of individual tastes.

Elasticity of substitution (ρ). The parameter ρ determine how responsive is the labour supply to consumption. The elasticity of substitution between c_t, and l_t in expression (2.2) is constant and and equal to ρ.

Leisure preference parameter (α). The term α represents the intensity of household preferences for leisure relative to consumption. The greater is α, the less labour will be supplied in order to obtain consumption goods, since the individual will prefer a greater amount of leisure instead. If $\alpha = 0$ household would choose not to have leisure and the result will be a fixed labor supply assumption.

Pure rate of time preference (δ). In expression (2.3) and (2.4) δ is a discount rate, often referred as the "pure" rate of time preference. It indicates the degree to which the household would prefer leisure and consumption earlier rather than later. The larger is δ the more lifetime resources will be spent early and the less will be saved.

Intertemporal elasticity of substitution (γ). It represents the intertemporal elasticity of substitution between consumption and leisure in different periods. The size of γ governs the responsiveness of households to the incentive to save.

Time separability means that individual decisions at any time depend only on the future; past level of consumption and leisure will not affect future decisions.

The values used for the different parameters of household utility function will be discussed later.

2.1.2. *The household budget constraint.* At each period, the household decides how much to work and how much to consume. Because the household has a life time horizon, it makes its current choice as part of a life time plan for consumption and leisure, deciding on the path for labor and consumption over time that maximizes its utility function (2.4) subject to the lifetime budget constaint that leaves no debts.

The household chooses only its current level of consumption and leisure in each year, along with its planned consumption and leisure in future years. Given that households are assumed to have perfect foresight, however, each year's current decision will be consistent with previously made plans. Therefore we can consider the entire path of consumption and leisure as having resulted from a single optimization decision at the date of the household's birth, when it has no previously accumulated assets.

In the absence of social security, the household budget constraint depends only on current and future values of interest rates and wage rates. In this case:

$$\sum_{t=1}^{2}\prod_{s=2}^{t}[1+r_s]^{-1}[w_t e_t(1-l_t)-c_t]=0 \tag{2.5}$$

or in our specific case of young and old:

$$[W_Y E_Y(1-L_Y)-C_Y]+\left[\frac{1}{1+R_O}\right][W_O E_O(1-L_O)-C_O]=0 \tag{2.6}$$

where R_Y and R_O are the interest rate when young (Y) and old (O), W_Y and W_O are the wage rate. E_Y and E_O is an adjustment factor to allow for the fact that the young household may earn more or less per unit of work because of difference in skill in different ages. The value E can been seen as the human capital profile reflecting the change in earning capacity over time.

In addition to this overall budget constraint, it is reasonable to impose the requirement that labor supply can never be negative; that is :

$$l_t \leqslant 1 \text{ for all } t \tag{2.7}$$

$$L_Y < 1 \tag{2.8}$$

$$L_O \leqslant 1 \tag{2.9}$$

In case $L_O = 1$, the household has decided not to provide any units of work and has completely "retired". Considering that young households do not hold any kind of assets we must exclude the possibility that $L_Y = 1$[1].

2.1.3. *Choice of consumption and leisure.* Let us consider first how households behave in their consumption and labour supply decision in the absence of government policy.

Maximization of the utility function (2.4), subject to the budget constraint (2.6) and retirement constraint (2.9) yields first-order conditions for consumption and leisure in each period that must be satisfied by the optimum values of consumption and leisure[2]:

$$\left(C_Y^{(1-\frac{1}{\rho})} + \alpha L_Y^{(1-\frac{1}{\rho})}\right)^{\left(-\frac{\rho-\gamma}{(\rho-1)\gamma}\right)} C_Y^{-\frac{1}{\rho}} = \lambda \tag{2.10}$$

$$\frac{\left(C_O^{(1-\frac{1}{\rho})} + \alpha L_O^{(1-\frac{1}{\rho})}\right)^{\left(-\frac{\rho-\gamma}{(\rho-1)\gamma}\right)} C_O^{-\frac{1}{\rho}}}{1+\delta} = \lambda \left[\frac{1}{(1+R_O)}\right] \tag{2.11}$$

[1]This will lead us to a solution where $L_Y = 1$ and $C_Y = 0$.We exclude the possibility that young might borrow.

[2]We use the Lagrange multiplier method where:

$$\pounds = \frac{\left[\left(C_Y^{(1-\frac{1}{\rho})}+\alpha L_Y^{(1-\frac{1}{\rho})}\right)^{\left(\frac{\rho(\gamma-1)}{\gamma(\rho-1)}\right)} + \frac{\left(C_O^{(1-\frac{1}{\rho})}+\alpha L_O^{(1-\frac{1}{\rho})}\right)^{\left(\frac{\rho(\gamma-1)}{\gamma(\rho-1)}\right)}}{1+\delta}\right]}{1-\frac{1}{\gamma}} +$$

$$+\lambda\left[\left[\frac{1}{1+R_y}\right][W_Y E_Y(1-L_Y)-C_Y]+\left[\frac{1}{1+R_y}\right]\left[\frac{1}{1+R_O}\right][W_O E_O(1-L_O)-C_O]\right]+\nu(1-L_O)$$

$$\left(C_Y^{(1-\frac{1}{\rho})} + \alpha L_Y^{(1-\frac{1}{\rho})}\right)^{\left(-\frac{\rho-\gamma}{(\rho-1)\gamma}\right)} \alpha L_Y^{-\frac{1}{\rho}} = -\lambda\left[-W_Y E_Y\right] \tag{2.12}$$

$$\left(C_O^{(1-\frac{1}{\rho})} + \alpha L_O^{(1-\frac{1}{\rho})}\right)^{\left(-\frac{\rho-\gamma}{(\rho-1)\gamma}\right)} \alpha L_O^{-\frac{1}{\rho}} = -\lambda\left[\frac{-W_O E_O}{(1+R_O)}\right] + \nu \tag{2.13}$$

using (2.13): $\mu = \frac{\nu(1+R_Y)(1+R_O)}{\lambda}$ we can arrange (2.13) as:

$$\left(C_O^{(1-\frac{1}{\rho})} + \alpha L_O^{(1-\frac{1}{\rho})}\right)^{\left(-\frac{\rho-\gamma}{(\rho-1)\gamma}\right)} \alpha L_O^{-\frac{1}{\rho}} = +\lambda\left[\frac{1}{(1+R_O)}\right]\left[W_O E_0 + \mu\right] \tag{2.14}$$

The term λ is the shadow price of life time budget constraint and represents the utility value of an additional unit of income in present value.

The term μ differs from zero if the individual chooses to fully retire in the second period.

The combination of conditions (2.10) and (2.12) yields an expression relating consumption and leisure for the young generation:

$$L_Y = \left(\frac{W_Y E_Y}{\alpha}\right)^{(-\rho)} C_Y \tag{2.15}$$

while the combination of (2.11) and (2.14) yields an expression relating consumption and leisure for old generation:

$$L_O = \left(\frac{W_O E_O + \mu}{\alpha}\right)^{(-\rho)} C_O \tag{2.16}$$

It is evident how the terms ρ and α influence the labour-leisure trade off. Keeping α constant a percentage change in $\frac{L_Y}{C_Y}$ with respect to a percentage change in effective price of leisure equals ρ; parameter ρ determines how responsive individual labour supply to the wage rate. An increase in α increase the ratio $\frac{L_Y}{C_Y}$.

Substitution of (2.15) into (2.10) and (2.11) takes us to an equation expressing the evolution of consumption over time for the household:

$$C_Y = \left(\frac{1+R_O}{1+\delta}\right)^{-\gamma} \left[\frac{[1+\alpha^{\rho}(W_Y E_Y)^{(1-\rho)}]^{\frac{\rho-\gamma}{1-\rho}}}{[1+\alpha^{\rho}(W_O E_O+\mu)^{(1-\rho)}]^{\frac{\rho-\gamma}{1-\rho}}}\right] C_O \tag{2.17}$$

The interpretation of (2.17) is determined by the wage in both periods.

In a simple model with fixed labor supply ($\alpha = 0$) the wages will not have any effect on the consumption path. In this case from (2.17) consumption will grow over time if the interest rate exceeds the pure rate of time preference δ.

The transition equation for leisure follows from (2.16) and (2.17):

$$L_Y = \left(\frac{1+R_O}{1+\delta}\right)^{-\gamma} \left[\frac{[1+\alpha^{\rho}(W_Y E_Y)^{(1-\rho)}]^{\frac{\rho-\gamma}{1-\rho}}}{[1+\alpha^{\rho}(W_O E_O+\mu)^{(1-\rho)}]^{\frac{\rho-\gamma}{1-\rho}}}\right]^{\rho} \left(\frac{W_Y E_Y}{W_O E_O+\mu}\right)^{\rho} L_O \tag{2.18}$$

from which it can be shown that leisure grows more slowly than wages grow.

It is important to remember that equations (2.17) and (2.18) determine the shape of the consumption and leisure path, but their absolute levels cannot be determined

To attempt such solution, in this type of model, we would apply (2.17) to obtain an expressions of C_Y and C_O and (2.15) to obtain an expression of C_Y and

L_Y. Combining these expression with the budget constraint (2.6) we can obtain an equation in C_Y in terms of a fixed parameter. However, when retirement is present, there are other endogenous variables in this expression such as the multiplier μ, therefore we cannot have a closed form solution for C_Y.

2.2. Production Sector. We assume a single production sector that is assumed to behave competitively, using capital and labor subject to a constant returns to scale production function. Capital is assumed to be homogeneous and non-depreciating, while labor differs only in its efficiency. We assume that all forms of labor are perfect substitutes, but individuals in different periods of their life supply different amounts of some standard measure of labor input per unit of leisure. This amount is the term E discussed in the budget constraint section.

We assume a simple Cobb-Douglas production function:

$$Y_t = AK_t^{\varepsilon} L_t^{1-\varepsilon} \tag{2.19}$$

Where Y_t,K_t, L_t are output capital and labor at time t, A is a scaling parameter and ε is a parameter measuring the intensity of use of capital in production.

In the simulations we are going to present in the next sections, we are going to assume A to be constant over time, in doing so we will rule out the possibility of technological change.

2.2.1. *Wages and demand for labor.* We assume that firms can adjust the amount of labor employed costlessy and they act competitively. This leads us to the standard result that, in equilibrium, the wage W_t must be equal to the marginal product of labor.

$$W_t = (1-\varepsilon)AK_t^{\varepsilon}L_t^{-\varepsilon} \tag{2.20}$$

hence (2.20) expressed the wage as a function of the capital stock and labor.

2.2.2. *Interest rate.* We apply the same logic to determinete the interest rate.

$$R_t = \varepsilon AK_t^{\varepsilon-1}L_t^{1-\varepsilon} \tag{2.21}$$

Equation (2.20) and (2.21) give the wage rate and interest rate as a function of the stocks of capital and labor.

2.3. Government behaviour. The government in this model runs an "unfunded" social security system. Let $\bar{B}$ stand for the social security benefit paid to a member of the generation $t-1$.

The life time budget constraint for individuals can be written as

$$[W_Y E_Y(1-L_Y) - \bar{B} - C_Y] + \left[\frac{1}{1+R_O}\right][W_O E_O(1-L_O) - C_O + \bar{B}] = 0 \tag{2.22}$$

2.4. Equilibrium under perfect foresight. A general equilibrium solution is one in which the behaviour of each sector of the economy is consistent with the prices that are established in markets. The behaviour of households, firms, and the government must be consistent not only with current prices, but also with future ones. Household labor supply and consumption must be optimal, given the entire future path of interest rates, wage rates, and tax rates. Firm investment decisions must adequately reflect the future behavior of interest rates and the stock market. Given the behavior of each sector, markets for labor and capital must clear. Because of the assumption of perfect foresight the behavior of the economy today depends on conditions in the future. One cannot compute a "separate" equilibrium for a given year without a complete characterization of future economic developments. Hence, the solution method must treat the present and future together. The exact methodology used is the subject of the next section.

2.4.1. *Capital and Labor level.* Supply of capital in period t is determined by the sum of total saving decisions of the young in period $t-1$. So in our case:

$$K_{t+1} = W_Y E_Y(1 - L_Y) - B_Y - C_Y \tag{2.23}$$

Labour is supplied by young and old people according to their productivity E :

$$N_t = E_Y(1 - L_Y) + E_O(1 - L_O) \tag{2.24}$$

3. Simulation

The general equilibrium economic model described above forms the basis for the simulation results presented in this part. In this section we are going to examine the choice of parameter values and the method of solving for the quantities that characterize the perfect foresight equilibrium.

3.1. Method. The calculation of the equilibrium path of the economy, given a particular parameterizations, typically proceeds in three stages:

1. Long-run steady state of the economy before any change applies.
2. Solving for the long-run steady state to which the economy eventually converges after the change takes place.
3. Solving for the transition path that the economy takes between these two steady states. The transition begins when information about the change becomes available.

Households and firms have perfect foresight in both old and new regimes, but do not anticipate the change. The policy change may take the form of an immediate change and/or a time dependent change of some variables.

The iteration techniques used in each of the three stages of the solution are very similar although the actual procedure is more complicated when one is solving for the transition path, because economic variables are changing over time.

3.1.1. *The initial and final steady state.* The solution for the equilibrium of the economy in the initial and final steady state is computed solving a difficult system of nonlinear equations based on the behavior of households and firms. The solution is obtained using an iterative technique often referred in the literature as the Gauss-Seidel method[3].

[3]The software Matlab 7.0 is used in order to compute the simulation.

The Gauss-Seidel method is a technique for solving the equations of a system one at a time in sequence, and uses previously computed results as soon as they are available.

The algorithm starts with guesses of a subset of the endogenous variables and initially treats these variables as exogenous in some of the equations of the system in which they appear. This simplification makes the resulting system easier to solve for the endogenous variables, including the variables for which guesses were made. When the solution for these "guessed" variables equals the guesses themselves, a true solution to the full system has been found. Otherwise, the "solution" is not consistent with the values of the guessed endogenous variables, and new guesses are tried, typically a combination of the two sets of values from the previous iteration.

A schematic representation of the algorithm is given in figure 1.

We begin with guesses *ex − ante* about the aggregate capital stock, K, total labor supply, N, the shadow wages μ, and the steady state SS. We thus have:

$$(K^0, N^0, \mu^0, SS^0) \tag{3.1}$$

We use (2.21) and (2.24) to calculate the wage and interest rates consistent with the factor guessed in (3.1). Then we combine with the guessed shadow wage, and social security. This allows us to solve for optimal household behavior using (2.17),(2.15), and the budget constraint(2.6). The individual labor supply decisions that result tell us whether our shadow price μ guesses were accurate or not. Once

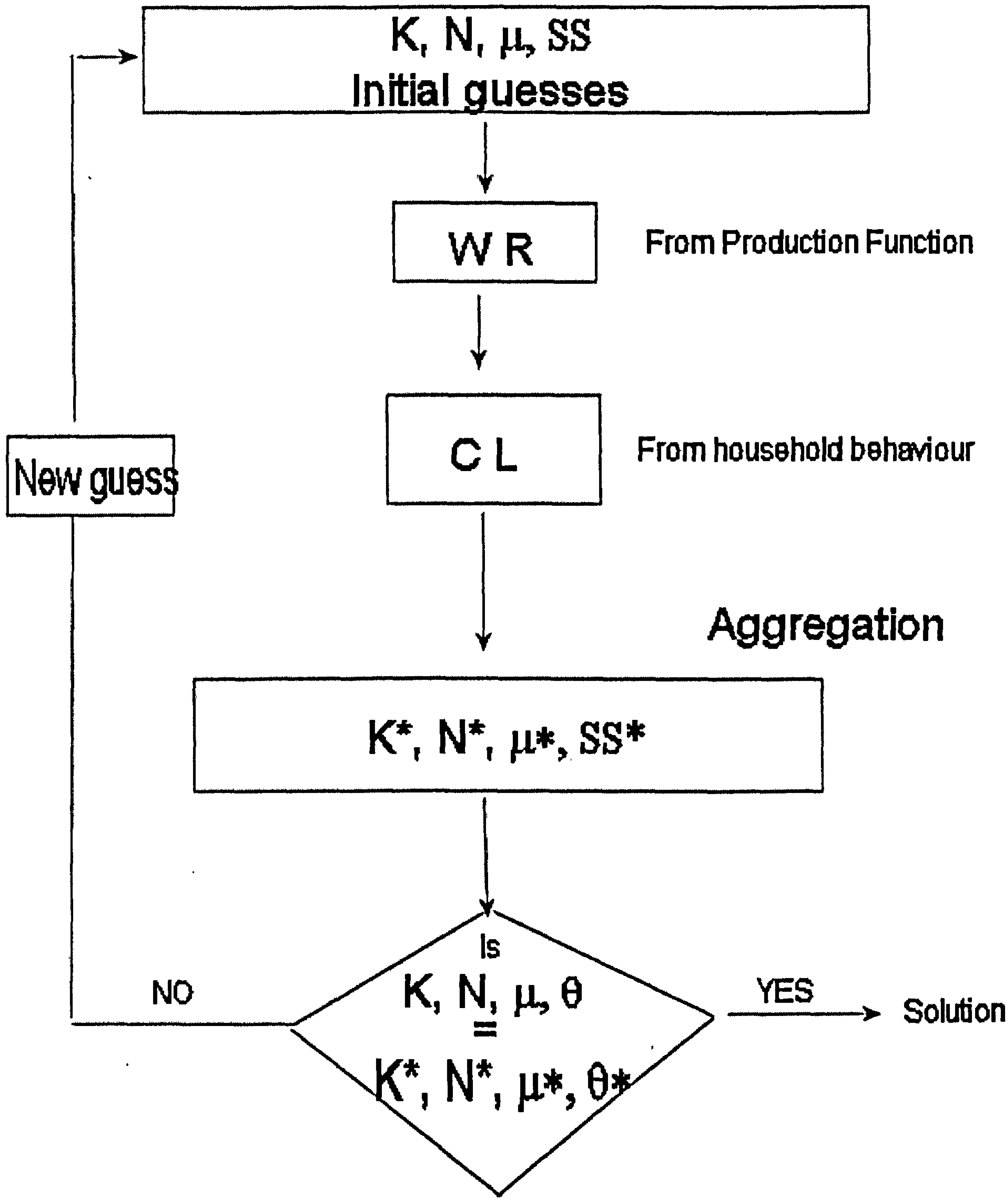


FIGURE 1. Solution by guessing and iteration

we have a vector of possible value of consumption, leisure and shadow price, we aggregate them using (2.23) and (2.24) we obtain a vector of aggregate $ex-post$, say:

$$(K^{*}, N^{*}, \mu^{*}, SS^{*}) \tag{3.2}$$

if $(K, N, \mu, SS) \neq (K^{*}, N^{*}, \mu^{*}, SS^{*})$ we start the algorithm again using a new vector combination of the two sets of values. If $(K^{0}, N^{0}, \mu^{0}, SS^{0}) \simeq (K^{*}, N^{*}, \mu^{*}, SS^{*})$[4] we consider the solution consistent with the value guesses $ex-ante$. Typically, 20-30 iterations are required to achieve convergence to a solution for the initial steady state.

When a change in the model occurs we solve the new steady state without knowledge of the precise transition path.

3.1.2. *The transition path.* The approach used to solve for the economy's equilibrium transition path is similar to that used to calculate the initial and final steady states. There are several complications, however. First, because the economy undergoes a transition in which conditions change over time, it is necessary to solve explicitly for behaviour in each year. Moreover, because households and firms are assumed to take into account future prices in determining their behavior, it is necessary to solve simultaneously for equilibrium in all transition years. This is done in the following way. The simulation model provides the economy with

[4]We consider: $(K^{0}, N^{0}, \mu^{0}, SS^{0}) \simeq (K^{*}, N^{*}, \mu^{*}, SS^{*})$ when the difference of each term is smaller than 1%.

20 periods to reach a new steady state. After 20 periods the model constrains all prices, and shadow wages to be constant to the value of the final steady state. The choice of 20 periods is arbitrary, but is intended to provide enough time so that the economy will settle down by itself well before it is "forced" to in period 20.

3.2. Parameters of the model. To solve the model, we must choose values for the preference parameters, δ,α,ρ and γ, the production elasticity, ε, the production scaling constant, A, and the human capital vector, E. Some of these parameters have been precisely estimated in several empirical studies. In this section we will try to summarize the relevant literature. Unless otherwise stated we are going to use these parameter value.

3.2.1. *Household preferences.*

1.: (γ) intertemporal elasticity of substitution. Weber (1970) estimated γ to lie between 0.13 and 0.41, but in a later study (Weber,1975) found a higher range, between 0.56 and 0.75. Several studies have derived their estimates from models of optimal household portfolio behavior under uncertainty. Grossman and Shiller (1981) found γ to range from 0.07 to 0.35, Mankiw (1981, 1985) recorded values of 0.25 and 0.37, respectively, Summers (1982) reports about 0.33, and Hall (1981) found values generally below 0.1. In the light of this evidence, we choose a value of $\gamma = 0.2$ for our baseline simulations.

2.: (ρ) consumption and leisure elasticity of substitution. Ghez and Becker (1975) found an aggregate value of ρ=0.8.

3.: (δ) pure rate of time preference. There is not much evidence of the appropriate value of δ. We choose a value of 0.015 according to Auerbach and Kotlikoff, (1983).

4.: (α) leisure preference parameter. Auerbach, and Kotlikoff, (1983) choose a value $\alpha = 1.5$ which results in realistic levels of labor supply[5].

3.2.2. *Production parameter.*

1.: (ε)Capital intensity. In our Cobb-Douglas production function we set $\varepsilon =$ 0.25.

2.: (A)Production function constant: This parameter depends on the units chosen for output. Thus, we are free to choose A. It is convenient to choose a value that leads to a wage rate in steady state equal to 1. In our base model this requires a value of $A = 2.4753$, which has been used throughout[6].

4. Simulation results, a change in earning capacity

4.1. The initial steady state. Table 1 presents the simulated initial steady state values of the base case economy. In the base case we assume that there is no difference in earning capacity between young and old so $E_Y = E_O = 1$. Then the base case wage is 1, reflecting our choice of the coefficient A in the production function in (2.19). Return to capital is 3,61% per cent. The economy's interes rate

[5]In their 55 period simulation model a value of α =1.5 lead prime age workers to a 40% labour vs 60% leisure. In an enviroment of 5,000 hours per year this is the equivalent of 2000 hours at year or 40 hours a week.

[6]This is the case also where $Ey = 1$, and $E_O = 1$.

Capital stock	0.0593	Income	0.9392
Labor supply	0.7037	Capital-Output ratio	6,31%
Wage	1	Interest rate	3,95%
Consumption Young	0.3949	Labor supply Young	45.38%
Consumption Old	0.5423	Labor supply Old	24.99%
Private consumption	0.9372	Social security rate	0%

TABLE 1. The base case steady state, $E_Y = E_O = 1$

function in (2.19). Return to capital is 3,61% per cent. The economy's interes rate is 3,95%. Partial retirement occurs in the life cycle. Labor supply is 45.38% (out of a time endowment of 1) for young people and it falls to 24,99% for old people.

4.2. A structural change in earning capacity. Table 2 displays the large impact of a change in youngs earning capacity ($E_Y = 3$) can have on labour supply for old people. This can be justified from the fact that full retirement occurs in the second period if there is this jump in earning capacity. One other interesting result can be observed in Figure 2. Here we plot the labor supply for old people as a function of the earning capacity of the young generation in steady state.

4.3. Sensitivity analysis. The sensitivity of these results to alternative choices of parameter values is examined in Table 3 and Table 4. They presents the level of the capital stock, labor supply, wage rate, interest rate, consumption and leisure choices for a range of parameters values. As Table 3 indicates, the size of the steady state stock of capital is quite sensitive to the choice of certain preference

Capital stock	0.5147	Income	2.5104
Labor supply	1.2715	Capital-Output ratio	20.5%
Wage	1.4808	Interest rate	1.22%
Consumption Young	1.3734	Labor supply Young	42.38%
Consumption Old	1.1299	Labor supply Old	0%
Private consumption	2.5034	Social security rate	0%

TABLE 2. The steady state, $E_Y = 3, E_O = 1$

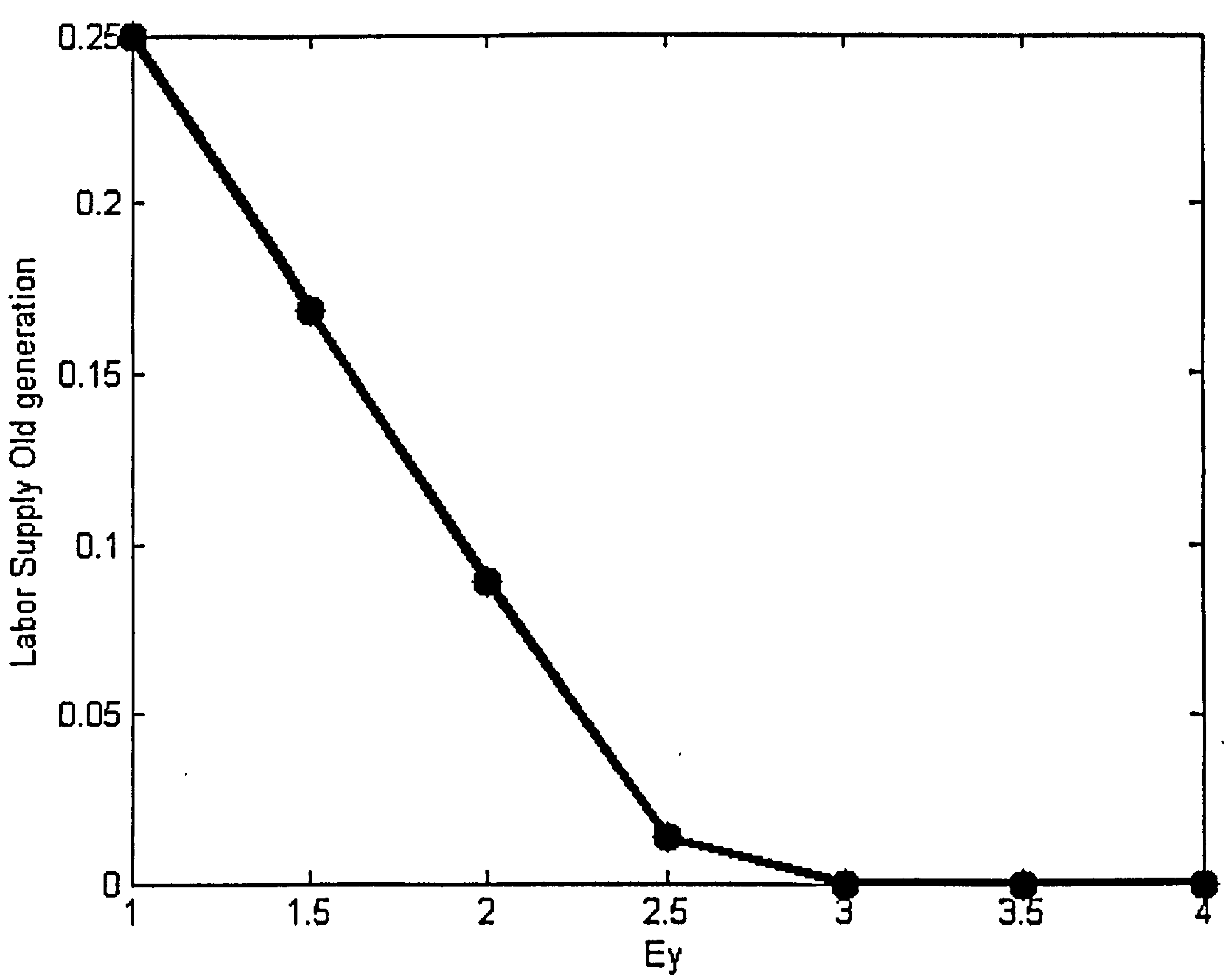


FIGURE 2. Labor Supply old generation $(1 - L_O)$ as a function of earning capacity (E_Y) of young generation in steady state.

parameters. For example, raising γ from 0.10 to 0.50 generates more than a ten times increase in the stock of capital.

Variations in the time preference rate, δ, do not significantly affect the stock of savings: lowering the time-preference rate from 0.03 to -0.015 implies a very small reduction in capital steady state. In contrast to the supply of capital, the steady state aggregate labor supply is relatively unresponsive to these changes in γ. It is, however, quite sensitive to the choice of ρ, the static elasticity of substitution between consumption and leisure, and the choice of the term α, the leisure utility share. Reducing ρ from 0.8 to 0.3 implies, an increase in aggregate labor supply of 15%, while raising α from 0.5 to 4 reduces aggregate labor supply by more than 66%.

From Table 4 and Figure 2 we need to point out that labor supply of the old generation very sensitive to the difference in earning capacity between young and old. In most of the cases we have analyzed the old generation opt for full retirement.

4.4. Transition path. In this section we are going to focus on the transition path that occurs after a change in the earning capacity for young people. In Figure 3 we show the capital path if the change in earning capacity for young people occurs in one period. It is interesting to note how it takes 10 periods to reach 90% level of the new steady state.

In Figure 4 we can see the path for labor supply for old people. It is interesting to note how fast is the impact in retirement incentive after an instantaneous change

Parameters												
γ	ρ	δ	α	C_Y	L_Y	C_O	L_O	K	N	Y	W	R
0.2	0.8	0.015	1.5	0.394	0.546	0.542	0.750	0.0593	0.704	0.939	1	3.95
0.1	0.8	0.015	1.5	0.295	0.550	0.382	0.711	0.014	0.738	0.679	0.69	12.0
0.5	0.8	0.015	1.5	0.471	0.510	0.751	0.813	0.194	0.676	1.22	1.35	1.57
0.2	0.3	0.015	1.5	0.427	0.490	0.602	0.691	0.0552	0.817	1.031	0.94	4.66
0.2	0.8	0.03	1.5	0.367	0.588	0.494	0.791	0.0623	0.620	0.865	1.04	3.46
0.2	0.8	-0.015	1.5	0.394	0.546	0.542	0.750	0.0593	0.704	0.939	1	3.95
0.2	0.8	0.015	0.5	0.501	0.336	0.759	0.510	0.0443	1.153	1.263	0.821	7.13
0.2	0.8	0.015	4	0.275	0.715	0.347	0.899	0.0710	0.385	0.625	1.21	2.19

TABLE 3. Steady state sensitivity analysis, $E_Y = 1, E_O = 1$

in earning capacity for young people. It takes only 2 periods to move from a partial retirement (Labor supply of old people 25%) to a full retirement.[7].

In Figure 5 and 6 we have reproduced the experiment, assuming the change in earning capacity for young people takes 5, 10 and 20 periods to move from the initial value of $E_Y = 1$ to $E_Y = 3$. We assume that E_Y grows at the rate of 24%, 11.6% and 5.6% every period up to the value of Ey=3[8].

The paths for capital and leisure for old people change considerably.

[7]See table 1

[8]Note these growth rate will allow us to reach the new steady state value in 5, 10 and 20 years.

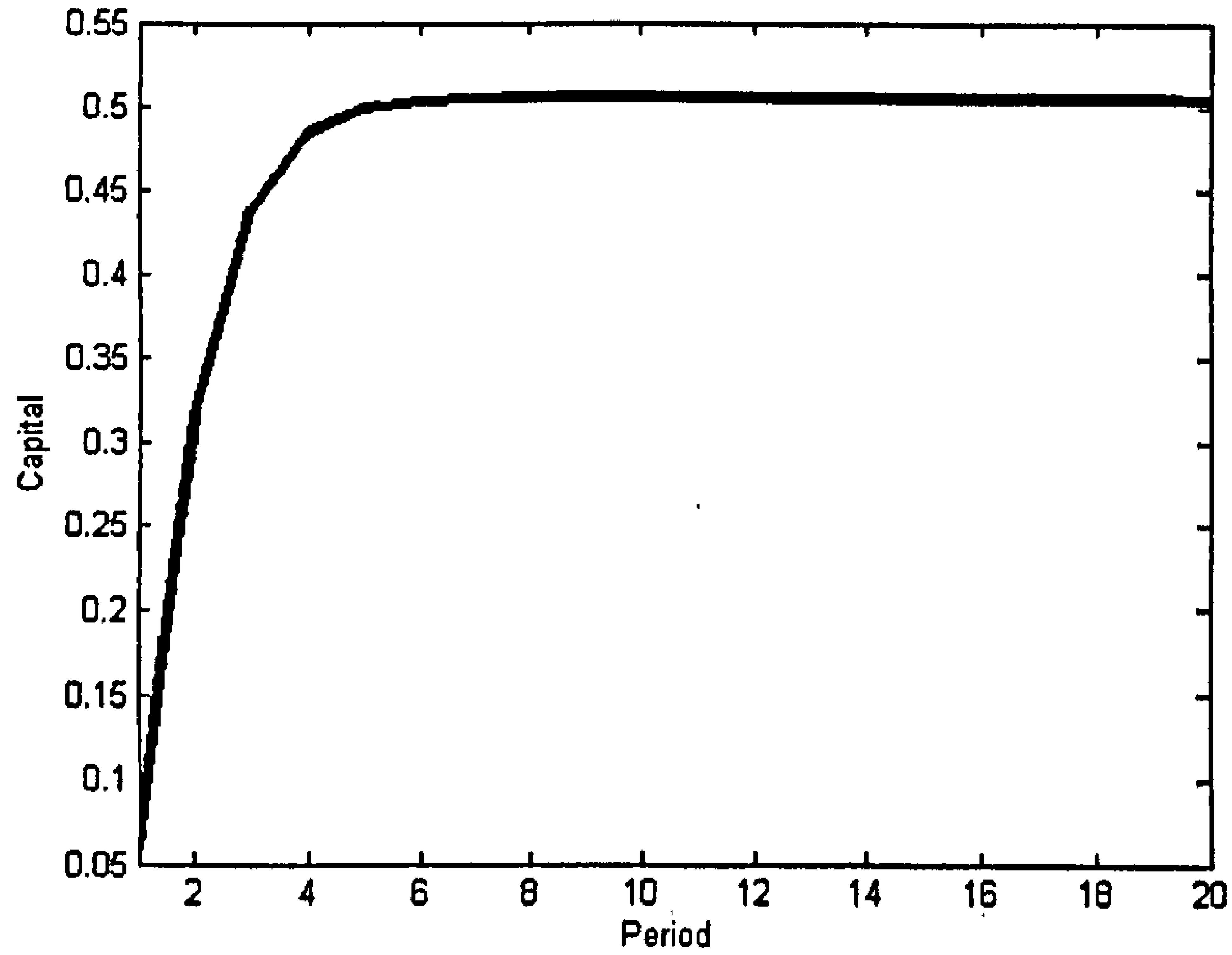


FIGURE 3. Capital path

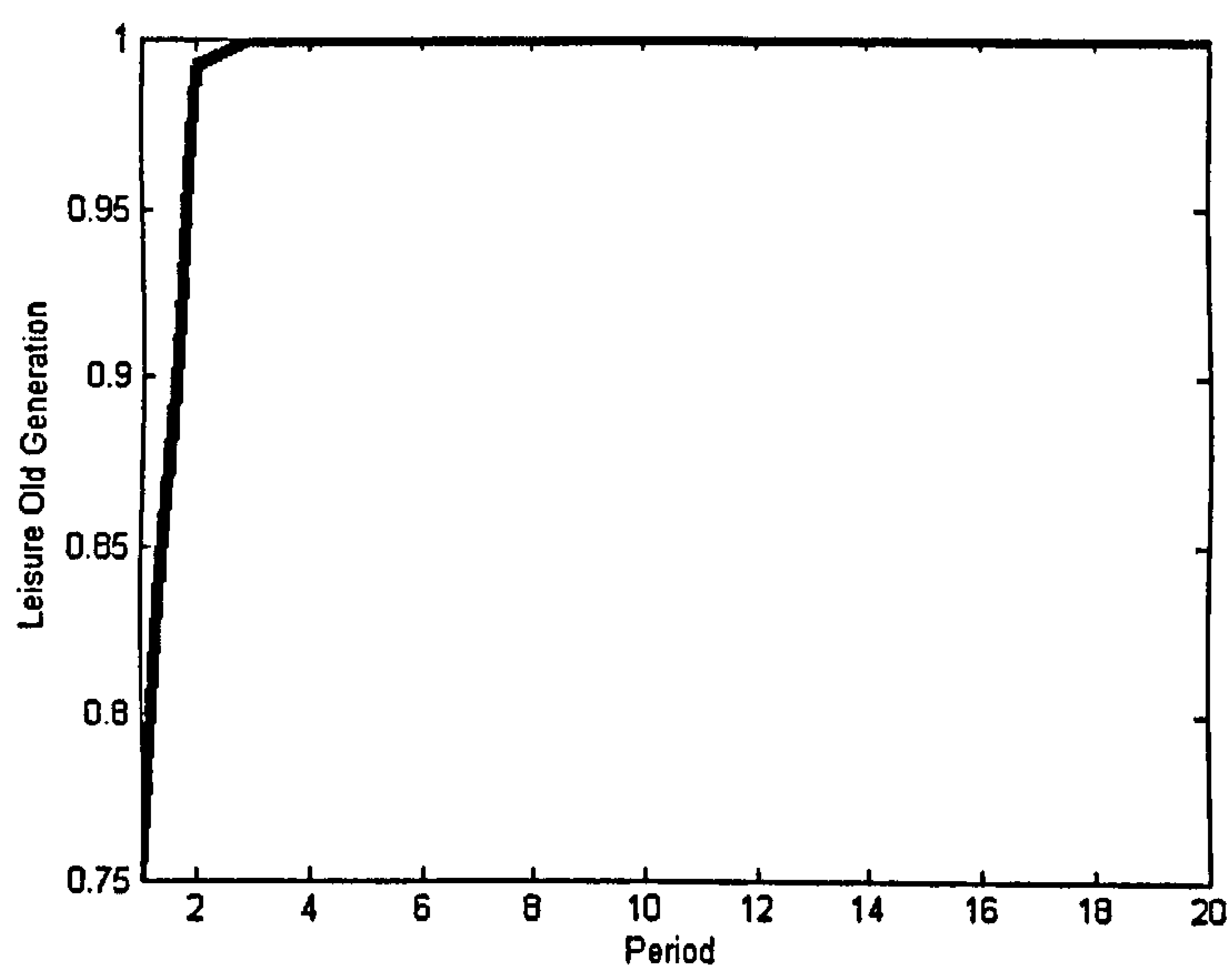


FIGURE 4. Leisure path

Parameters												
γ	ρ	δ	α	C_Y	L_Y	C_O	L_O	K	N	Y	W	R
0.2	0.8	0.015	1.5	1.373	0.576	1.130	1.000	0.515	1.272	2.510	1.481	1.220
0.1	0.8	0.015	1.5	1.295	0.594	0.887	0.979	0.321	1.239	2.188	1.324	1.704
0.5	0.8	0.015	1.5	1.412	0.535	1.714	1.000	0.941	1.395	3.129	1.682	0.832
0.2	0.3	0.015	1.5	0.978	0.688	1.034	1.000	0.536	0.935	2.014	1.615	0.940
0.2	0.8	0.03	1.5	1.373	0.576	1.130	1.000	0.515	1.272	2.510	1.481	1.220
0.2	0.8	-0.015	1.5	1.373	0.576	1.130	1.000	0.515	1.272	2.510	1.481	1.220
0.2	0.8	0.015	0.5	1.985	0.368	1.824	0.813	0.623	2.085	3.816	1.373	1.532
0.2	0.8	0.015	4	0.832	0.741	0.763	1.000	0.369	0.778	1.598	1.541	1.082

TABLE 4. Steady state sensitivity analysis, $E_Y = 3, E_O = 1$

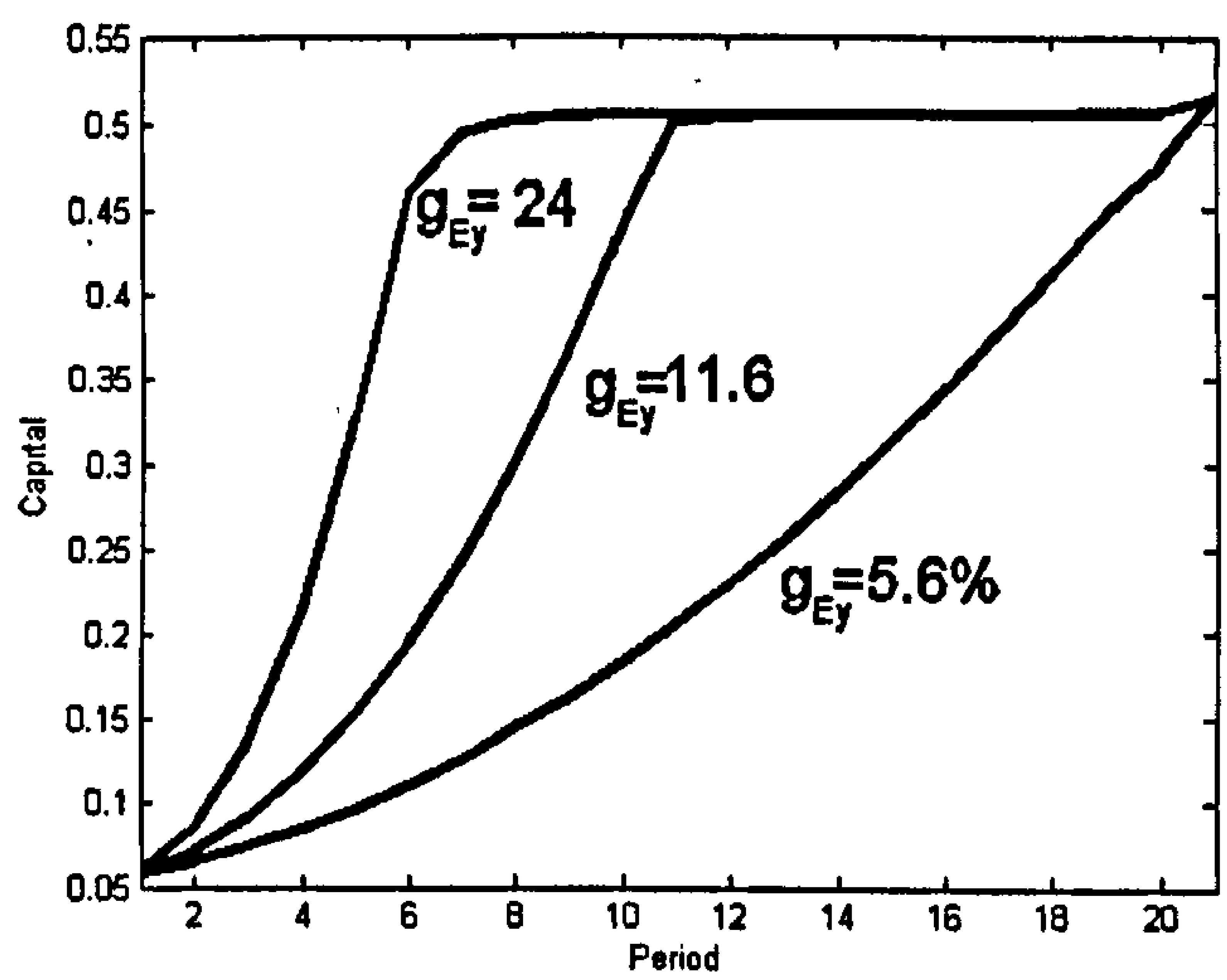


FIGURE 5. Capital path to the new steady state, $g_{Ey} = \frac{\Delta Ey}{Ey}$

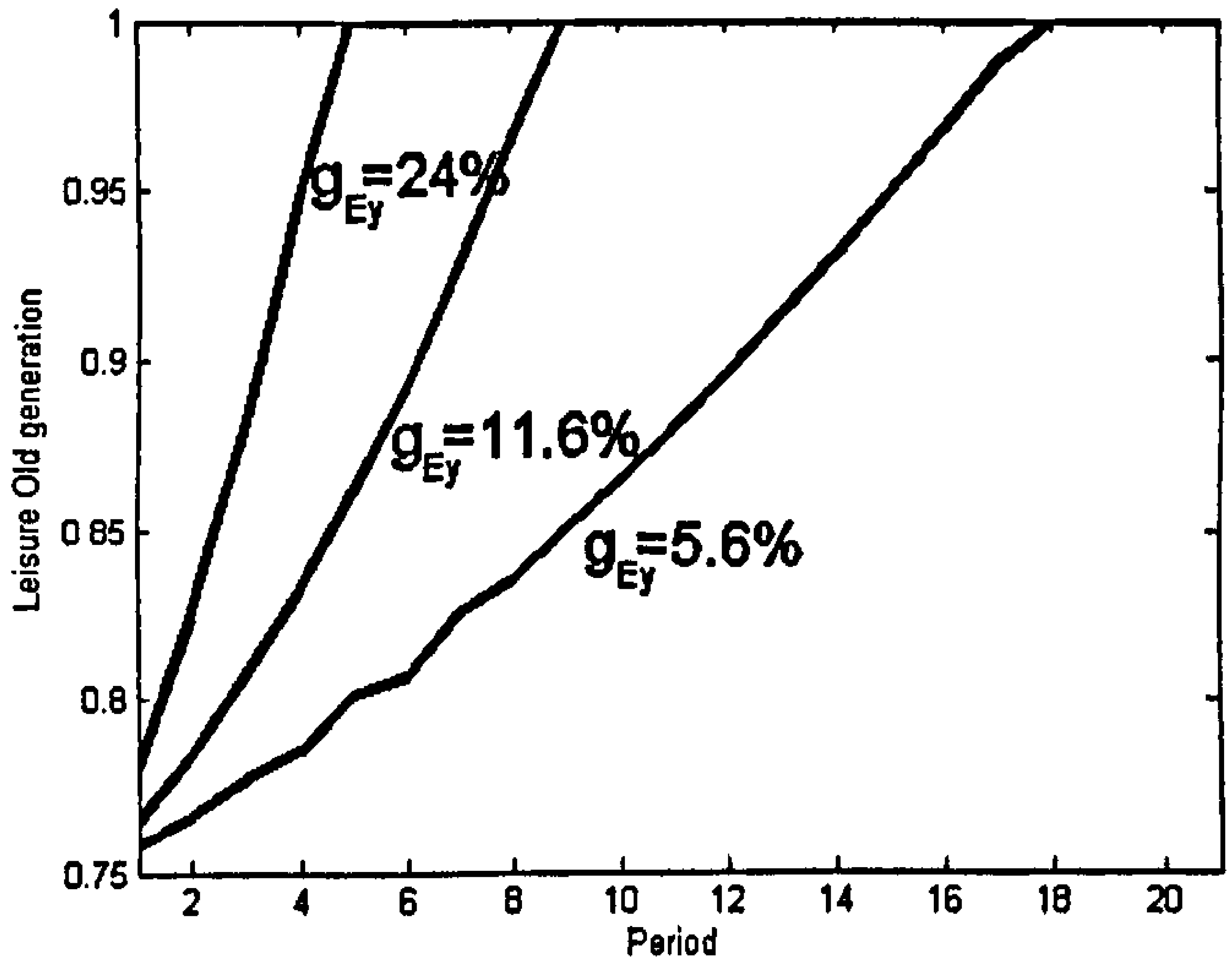


FIGURE 6. Leisure Old generation path to the new steady state, $g_{Ey} = \frac{\Delta Ey}{Ey}$

4.5. Earning capacity VS Retirement incentive. The main finding of this section, based on the evidence of the simulation and the sensitivity analysis are:

1. Without any kind of social security only a big difference in earning capacity (young vs old) is able to justify a full retirement for the old generation. This is evident in our basic model (Table 1) and from the sensitivity analysis of the basic model (Table 3). With a high leisure preference parameter ($\alpha = 4$) the old generation will opt-out for full retirement. The retirement decision seems to be also affected by the intertemporal elasticity of substitution (ρ)
2. On the other end it is quite interesting to note that only a high difference in earning capacity ($E_Y >> E_O$) between young and old is able to justify a full retirement for the older generation. In our simulation results (Figure 2

and Table 2) only if earning capacity for young people is three times bigger than old one we can have a full retirement. As we have seen in Table 4 this result is not affected so much by changing the parameters of the model.

3. Transition paths are relatively fast. It takes only six generations to reach a close (90%) value to the new steady state, (Figure 3 and 4). If the change in productivity does not occur in one period then the transiction path will change and the new steady state will be reached in a longer period (Figure 5 and 4).

In the next section we will focus our attention on a social security pension system and how this affects results of our model.

5. Social Security

This section considers the impact of social security on savings and its potential distortionary impact on labor supply. The impact of social security on retirement is considered using simulation techniques here due to the difficulty of including nonlinear budget constraints in what is already a fairly complex model. We focus on the unfunded, "pay-as-you-go" method of financing social security. This is an implicit form of deficit finance that has transfer sums going from current younger to current older generation.

The principal findings of this section are as follows:

: Unfunded social security can crowd out savings

: In bringing labour supply distortion Social Security could substantially swap labor supply from old to young.

5.1. Conceptual issues. The impact of introducing an unfunded social security system in a life cycle model can be easily understood by adding social security to the simple two-period model of section 1. Let $\bar{B}$, stand for a fixed social security benefit paid to a member of generation old who is old at time t. At time t the budget constraint for each elderly individual is:

$$C_o = WoEo(1 - Lo) + \bar{B} + [W_Y E_Y(1 - L_Y) - C_Y](1 + Ro) \tag{5.1}$$

for young individuals born at the beginning of period t, the lifetime budget constraint is:

$$[W_Y E_Y(1 - L_Y) - \bar{B} - C_Y] + \left[\frac{1}{(1 + Ro)}\right][WoEo(1 - Lo) + \bar{B} - Co] = 0 \tag{5.2}$$

Since the social security system is financed on a pay-as-you-go basis, social security tax revenues per young worker must equal benefit payments to old worker. The economy's capital stock is now:

$$K_t = W_Y E_Y(1 - L_Y) - \bar{B} - C_Y \tag{5.3}$$

From (5.2) and ignoring for the moment potential general equilibrium effects on the values of W and R, one can see that introducing unfunded social security lowers the steady state level of lifetime resources. The consumption of the initial elderly generation increases because this generation receives benefits without having to pay taxes. Equation (5.1) makes this clear; if, starting at time t, $\bar{B}$, is raised from zero to a positive value, the consumption of the elderly at time t, rises by $\bar{B}$ since the marginal consumption propensity of the elderly is unity. If we ignore for the moment changes through time in benefit levels and factor rewards, the young, whose marginal propensity to consume is less than unity, will reduce their consumption by a fraction of $\bar{B}$. Hence, in the initial period in which social security is introduced each elderly individual increases his consumption by $\bar{B}$ while each young person reduces his consumption by a fraction of $\bar{B}$. Total private consumption in the initial period therefore increases, and saving is crowded out. Adding general equilibrium effects to this partial equilibrium story only reinforces the intergenerational transfer away from future generations. As the capital stock is crowded out, the wage falls and the interest rate rises. Those generations that are elderly when interest rates rise benefit from the greater return on their savings, while the corresponding young and future generations are worse off because the concomitant fall in their wages is more detrimental to their economic welfare than the reduced price of old age consumption reflected in the higher interest rate.

In our case of a two-period model in which the social security tax rate is levied at time t and kept constant thereafter, the first generation of young workers benefits from the general equilibrium changes in factor returns; since the crowding out takes one period to get under way, the interest rate is high when they are old,

Capital stock	0.29021	Income	1.7676
Labor supply	0.5261	Life time utility	-3,677
Wage	1.3751	Interes rate	1.522
Consumption Young	0.9140	Labor supply Young	43.73%
Consumption Old	0.8500	Labor supply Old	8.88%
Private consumption	1.7640	Social security $\bar{B}$	0

TABLE 5. The base case steady state,$\bar{B}$=0 $E_Y = 2E_O = 1$

but the wage of this first set of young workers receive when young is unaffected by the introduction of social security. In contrast, generations born after the first generation, while benefiting from higher interest rates, receive lower wages during their initial working period.

5.2. Including social security in the simulation model. In the simulation model, social security $\bar{B}$ are received in a lump sum in the second period. The value of $\bar{B}$ is specified to be a fixed amount. Table 5 presents the simulated initial steady state values of the base case economy[9]. We assume double earning capacity between young and old.

Table 6 display the large impact a change in social security $\bar{B}$ can have on the economy.

Relative to the initial situation introducing a "pay as you go" social security system bring full retirement in the life cycle for old people. It is interesting to

[9]Base case, $E_Y = 2\ E_O = 1$

Capital stock	0.0312	Income	1.0829
Labor supply	0.5280	Life time utility	-0.11346
Wage	0.7696	Interes rate	8.6855
Consumption Young	0.4819	Labor supply Young	52.8%
Consumption Old	0.5987	Labor supply Old	0
Private consumption	1.0805	Social security $\bar{B}$	0.3

TABLE 6. Steady state,$\bar{B}$=0.3 $E_Y = 2E_O = 1$

note that on the one hand the labor supply for young people increases while the labor supply for old generation decreases. The overall labor supply does not reduce much. From a welfare point of view the second situation is worse than the first one. The effect on capital stock can be observed in figure 7 the higher the value of $\bar{B}$ the lower is the steady state level of capital. It is interesting to note that for value $\bar{B} > 0.3$ the economy collapse on a 0 level of capital. Figure 8 plot the leisure for old generation as function of social security $\bar{B}$. It is quite intuitive to see that the higher is the level of $\bar{B}$ the lower (up to full retirement) is the labor supply for old generation. It is also interesting to note that the introduction of social security on Pay As You Go scheme increases the interest rate. If we relate this result to the attainment of the Golden rule[10] (see chapter 3), we get the same result as Samuelson (1975), where the introduction of a social security system based on a PAYG scheme can bees seen as an attempt to reach the golden rule when the

[10] In our case where the growth rate of the population is zero ($n = 0$).

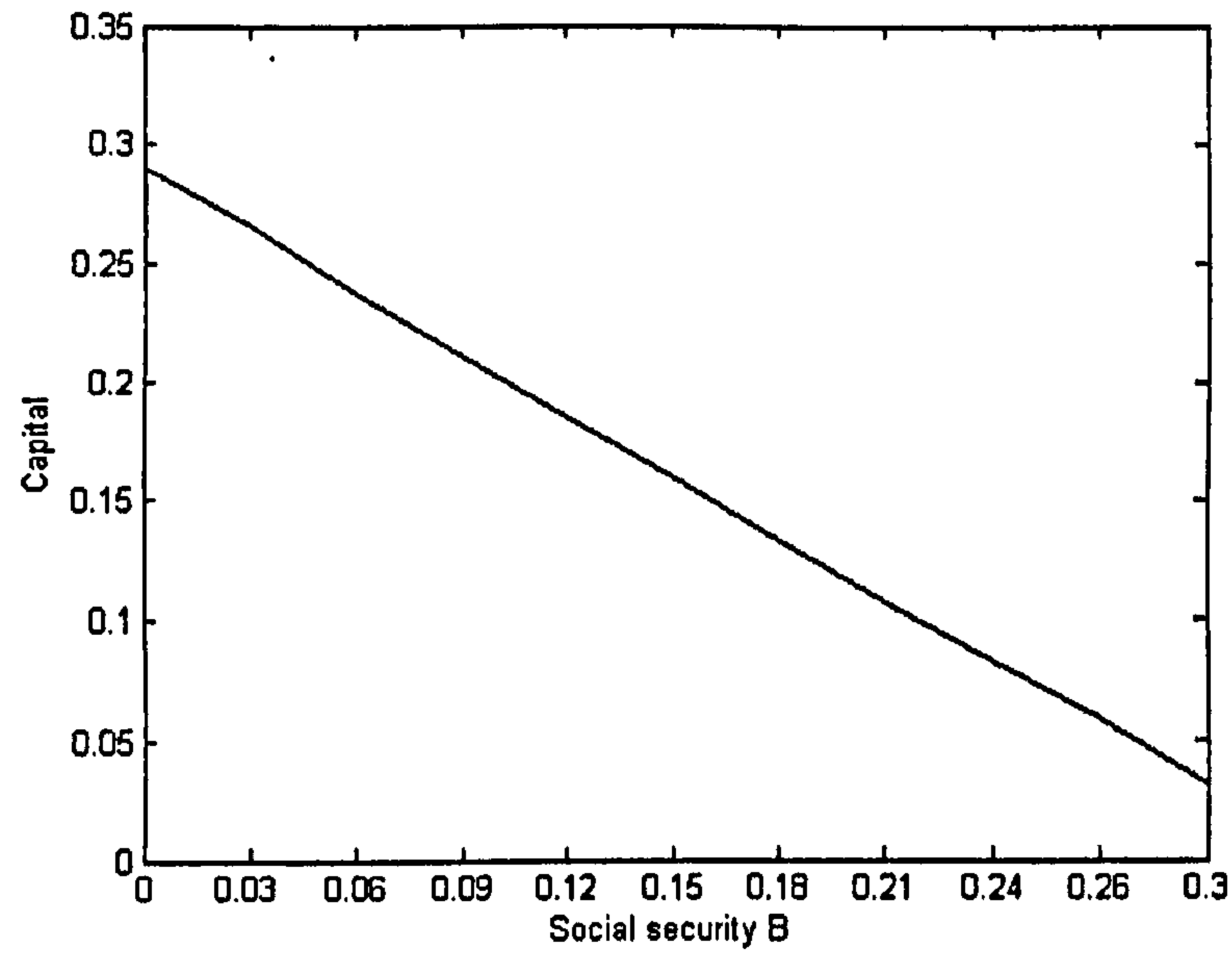


FIGURE 7. Crowing out of Capital vs Social Security $\bar{B}$

interest rate is smaller than the growth rate of the population. Obviously in our case where $n = 0$ the introduction of the PAYG social security takes us away from optimal level of capital that maximizes the steady state life time utility for each generation.

5.3. Simulating the transition to and from an unfunded social security. In this section we present simulations of three transitions to an unfunded social security system. The transaction differs with respect to the timing in which the social security is introduced. We assume that a fixed social security B=0.3 is introduced in the system :

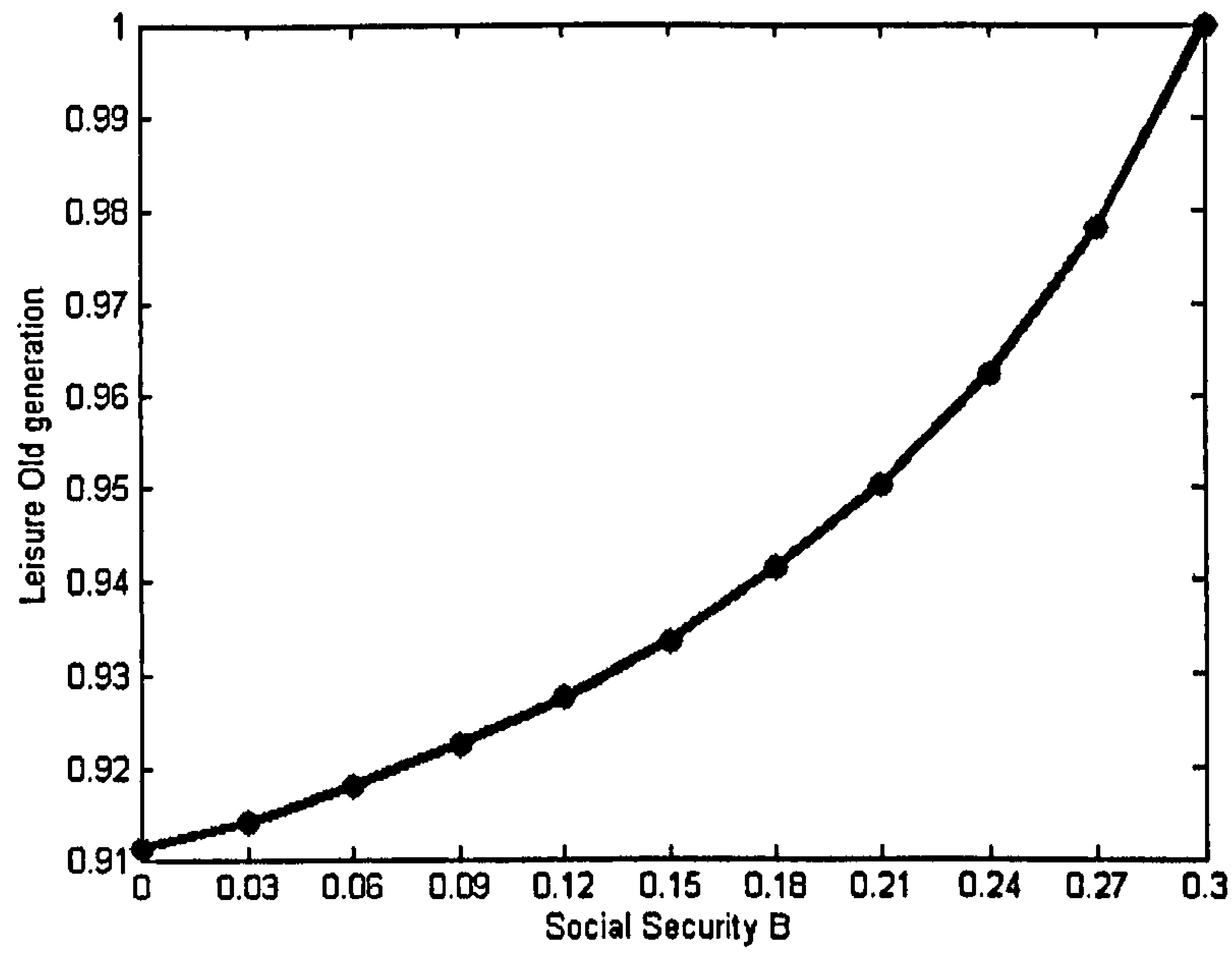


FIGURE 8. Leisure of Old generation vs Social Security $\bar{B}$

1. In the first experiment the transition to an unfunded social security occurs in one period.
2. In the second experiment we assume a smooth transition to B=0.3 taking place in 10 periods[11].
3. In the third experiment we assume a one period transition to B=0.3 then a smooth transaction back to the funded social security taking place in 20 periods[12].

Figure 9 shows the three paths of the variable $\bar{B}$.

[11] A $\bar{B} = 0.03$ is introduced in the first period therefore we assume a constant growth of $\bar{B}$ at 29.20% for the next 9 periods.

[12] We assume $\bar{B}$ decrease at 20% rate during this phase.

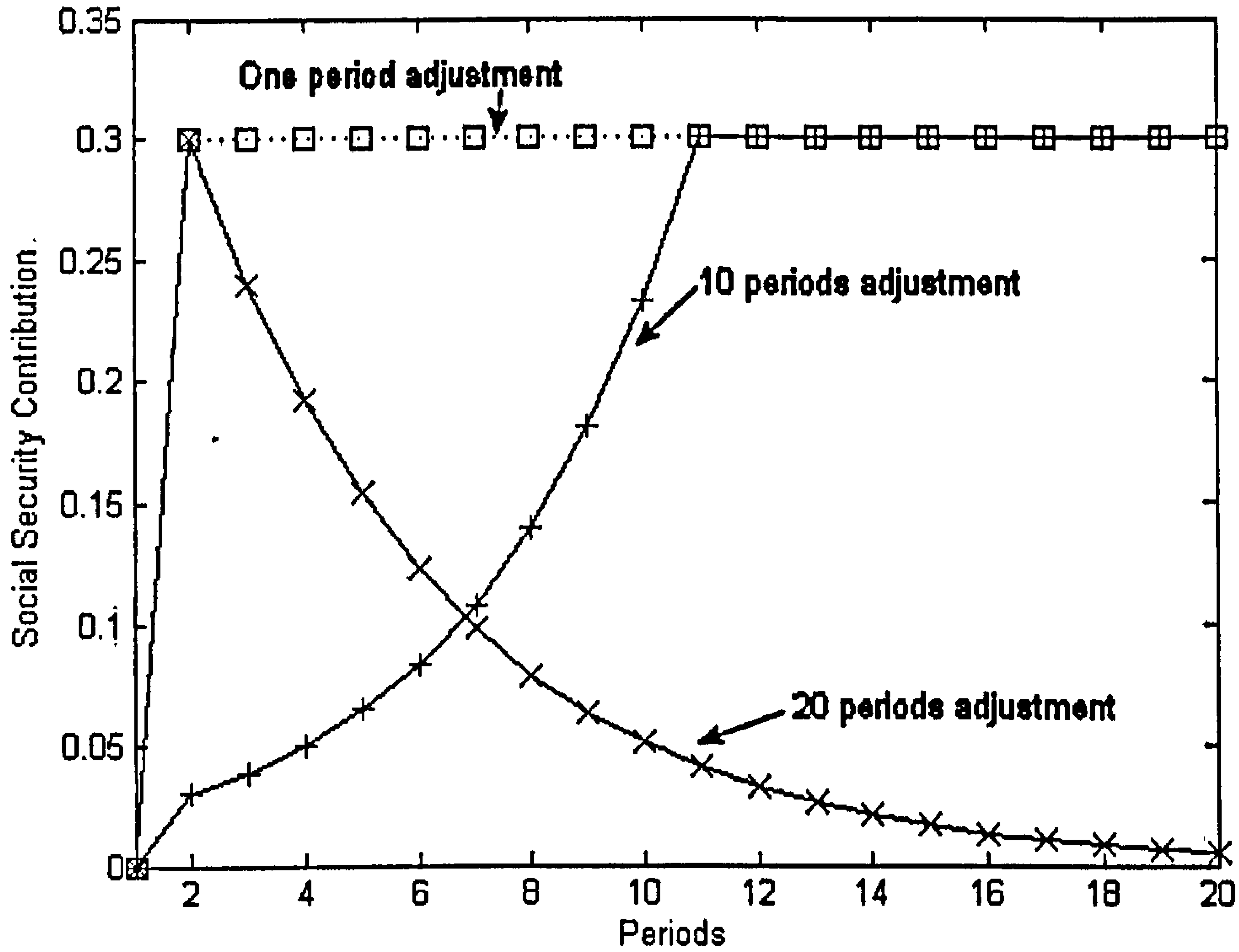


FIGURE 9. Three transitions to an unfunded social security system.

5.3.1. *One period adjustment.* Table 7 presents the simulation results of a transition to an unfunded social security system. We assume that in period 1 the economy is running in steady state[13]. In period 2 an unfunded social security is introduced where the young generation has to pay a $B = 0.3$ to the old generation.

Capital stock falls very rapidly to the new steady state value which is 1/10 of the initial capital stock. Although the crowding out process is somewhat faster in

[13]Where earning capacity are $Ey = 2, Eo = 1$.

	Capital Stock		Labor Supply		
Period of transition	K	W	Young	Old	Aggregate
1 Steady State	0.29	1.351	43%	8.8%	51.8%
2	0.13	1.13	47%	1.5%	48.5%
3	0.088	1.013	49.7%	1%	48.7%
4	0.067	0.941	50.4%	1%	51.4%
5	0.055	0.897	51%	1%	52%
10	0.036	0.830	52.1%	0%	52.1%
15	0.0316	0.772	52.6%	0%	52.6%
20	0.0311	0.769	52.8%	0%	52.6%

TABLE 7. Simulation transition to unfunded social security system in one period

this case, more than half of the reduction in the capital stock occurs within the first period of introducing unfunded social security.

The aggregate supply of labor falls rapidly in the first period. Over time, however, as capital is crowded out, income effects appear to outweight substitution effects, and the supply of labor begins to increase. In the final steady state aggregate labor supply ends up somewhat larger than the initial value. The supply of labor for young generation seems to increase all the time while the old generation are opting for full retirement after five period.

5.3.2. *Impact on welfare.* The effects on welfare of introducing unfunded social security are presented in Table 8. The initial old generation (the second row in

	Life Consumption	Life Income	Life time Utility
Generation			
1 Steady State	1.76	1.32	0
2	1.94	1.37	+2.72%
3	1.44	0.98	-6.11%
4	1.32	0.93	-17.79%
5	1.25	0.90	-27.84%
10	1.12	0.83	-54.72%
15	1.089	0.81	-57.89%
20	1.08	0.81	-71.72%

TABLE 8. Simulation transition to unfunded social security system in one period, impact on welfare

the table) are better off because of the social security, whereas the initial young generation (the third row) suffer welfare losses because of the program.

Although the welfare gains to the initial generation, measured as wealth equivalents, are small, the welfare losses to generations in the long run under social security are substantial.

5.3.3. *Slow adjustment.* Table 9 presents simulation results of a transition to un unfunded social security system in a slow motion. We assume that in period 1 the economy is running in steady state[14]. In period 2 an unfunded social security

[14]Where earning capacity are $Ey = 2, Eo = 1$.

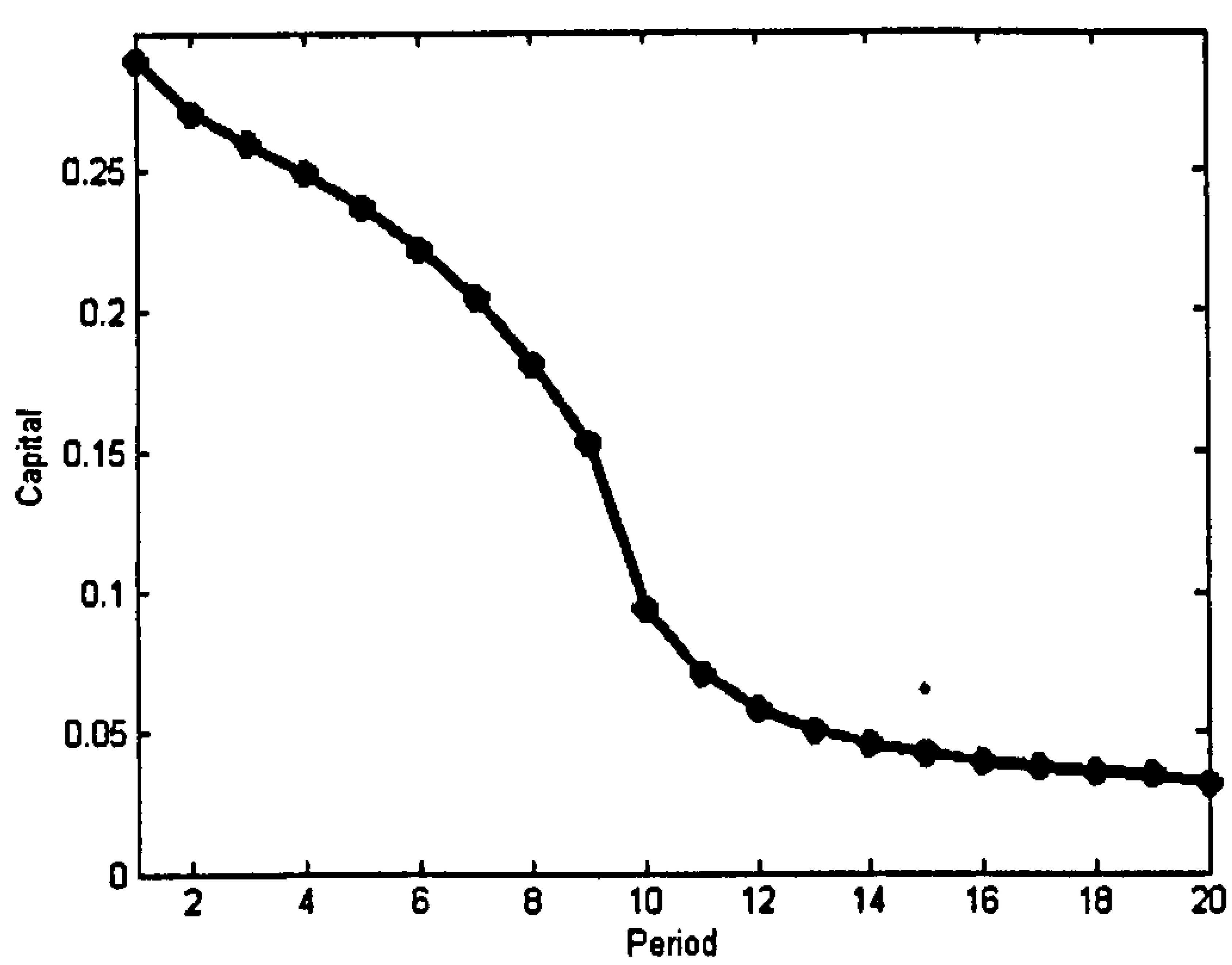
is introduced where young generation has to pay a $\bar{B} = 0.03$ to old generation. In this case the social security contribution is ten times smaller than the one used previously (Table 7). From period 2 we assume a constant growth of $\bar{B}$ at 29.20% for the next 9 periods, after period 10 the social security contribution is kept constant at 0.3.

Capital stock falls, not very fast in the transition period. Although the crowding out process is not very fast, in this case, most of the reduction in the capital stock occurs after period 10 when the social security contribution $\bar{B}$ is kept constant at 0.3 level.

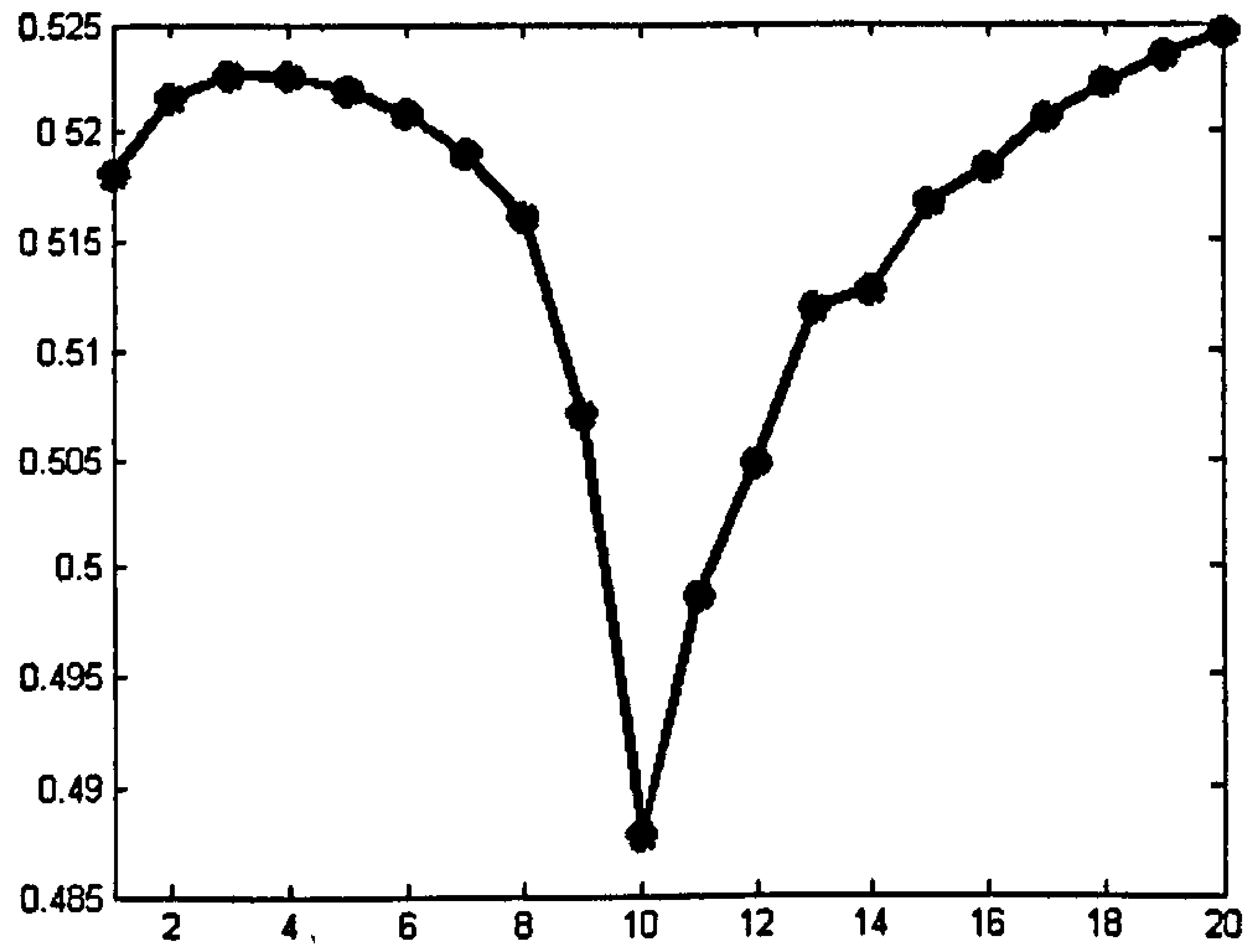
The aggregate supply of labor increases over the first 3 periods, to falls rapidly in the next 7. However, as capital is crowded out, income effect outweight substitution effects, and the supply of labor begins to increase once the social security contribution $\bar{B}$ is stable at 0.3 level. In the final steady state aggregate labor supply ends up somewhat larger than initial value.

	Capital Stock	Social Security		Labor Supply		
Period of transition	K	W	$\bar{B}$	Young	Old	Aggregate
1 Steady State	0.29	1.351	0	43%	8.8%	51.8%
2	0.27	1.351	0.03	45.2%	8.8%	51.3%
3	0.25	1.336	0.038	45.4%	8.8%	51.1%%
4	0.24	1.322	0.05	45.8%	8%	53.8%
5	0.23	1.305	0.064	46%	7.5%	53.5%
10	0.093	1.02	0.3	49.5%	1%	50.5%
15	0.041	0.83	0.3	52.6%	0%	52.6%
20	0.0311	0.769	0.3	52.8%	0%	52.6%

TABLE 9. Simulation transition to an unfunded social security system in slow motion



Capital path in the slow ajustment case



Agregate Labour Supply in the slow ajustment case

5.3.4. *Impact on welfare.* The effect on welfare of introducing unfunded social security in a slow motion are presented in Table 10. The initial old generation (the second row in the table) is marginally better off because of the social security, whereas the initial young generation (the third row) suffer a small welfare losses because of the program.

Although the welfare gains to initial generation, measures as wealth equivalents, are very small, the welfare losses to generations in the long run under social security are substantial.

5.3.5. *Back to the funded social security.* Table 11 presents results of a transition from an unfunded social security system to a situation where social security is not present (the initial steady state situation). We assume that in period 1 the economy in running in steady state. In period 2 a huge unfunded social security

Generation	Life Consumption	Social Security $\bar{B}$	Life Income	Life time Utility
1 Steady State	1.76	0	1.32	0
2	1.78	0.03	1.329	+0.42%
3	1.73	0.038	1.28	-0.45%
4	1.71	0.05	1.27	-1.22%
5	1.69	0.064	1.26	-1.86%
10	1.46	0.3	1	-9.68%
15	1.16	0.3	0.85	-50.47%
20	1.08	0.3	0.769	-71.72%

TABLE 10. Simulation transition to un unfunded social security system in ten periods, impact on welfare

is introduced where young generation has to pay $\bar{B} = 0.3$ to the old generation. From period 3 we assume a constant reduction of the Social Security contribution $\bar{B}$ at 20% rate.

5.3.6. *Impact on welfare.* The effect on welfare of coming back from an unfunded social security in a slow motion are presented in Table 12. The initial old generation (the second row in the table) is better off because of the social security, whereas the initial young generation (the third row) suffer a big welfare losses because of the program and also due to the reduction in capital.

	Capital Stock		Social Security		Labor Supply	
Period of transition	K	W	$\bar{B}$	Young	Old	Aggregate
1 Steady State	0.29	1.351	0	43%	8.8%	51.8%
2	0.13	1.134	0.3	48%	1%	49%
3	0.11	1.076	0.24	48%	4%	52%
4	0.13	1.088	0.192	47%	6%	53%
5	0.15	1.128	0.153	47%	8%	54%
10	0.23	1.299	0.05	45%	9%	54%
15	0.27	1.35	0.016	44%	8.5%	52.5%
20	0.29	1.37	0.005	43%	8%	51%

TABLE 11. Simulation transition back to a funded social security system in slow motion

Although the welfare gains to the first generation, measured as wealth equivalents, are very small, the welfare losses for the third and forth generations are substantial. Only in the long term, when the there is no more social security is there no significant loss in welfare.

5.3.7. *Policy Implications.* With regards to public policy, Table 12 shows an interesting and actual result with respect to the European situation. Since 1990 all European countries have been implementing many policies affecting the way social security is financed. In particular with respect to the pension systems, many European countries are slowly swapping from a PAYG system to a fully funded one. On the debate a lot of attention is given to the reason why all European

	Life Consumption	Social Security	Life Income	Life time Utility
Generation		$\bar{B}$		
1 Steady State	1.76	0	1.32	0
2	1.64	0.3	1.37	+2.72%
3	1.46	0.24	1.06	-17.8%
4	1.51	0.192	1.09	-34%
5	1.56	0.153	1.13	-37%
10	1.72	0.05	1.26	-14%
15	1.75	0.016	1.27	-1.5%
20	1.76	0.005	1.31	-1.5%

TABLE 12. Simulation transition back to a funded social security system, impact on welfare

countries have made this choice, like the low fertility rate and the public finance unsustainability of the current systems[15]. No a lot of attention has been given to the loss for the generations that live during the transition. In other words, as stated in the previous paragraph, moving away from a PAYG to a fully funded system has a cost for the young generations, who on one hand have to pay the PAYG contribution for the current old generation, but on the other hand they will have to save for their retirement. The overall result, as stated in Table 12, is that

[15]See Kieran Mc Morrow, Werner Roeger (2002)

the generations in which changes take place suffer a substantial lost in terms of life time consumption and utility.

6. Conclusion

The purpose of this chapter has been to explain and illustrate the dynamic impact of alternative social security system.

The simulation results, although based on a highly simplified economic model, suggest that social security can have powerful effects on the economy. Prolonged and significant social security are policies that can substantially alter the course of saving, investment and factor rewards. Many of these policies are effective primarily because they transfer resources across generations. Such changes in economic incentives can significantly alter the degree of economic efficiency.

The simulation conducted in this part suggests that introducing unfunded social security can substantially crowd out long run capital formation. The effects on welfare of introducing unfunded social security in our model show unique results. All initial older generations are better off because of social security, whereas all younger generations suffer welfare losses because of the introduction of social security.

CHAPTER 7

Modeling International Retirement Migration

This chapter considers the impact of international retirement migration on savings and its potential distortionary impact on labour supply for the host country. As we have seen in the first part of this thesis, the EU itself is not a welfare provider, instead it regulates access to domestic social security systems. In order to understand the economic consequences of a migration in retirement, we need to specify that in the EU the right to freedom of movement is based on a principle of non-discrimination. The impact of international retirement migration is considered again using simulation techniques. We focus on two aspects of IRM:

1. Based on the non-discrimination principle, once in the host country the international retirement migrant can access certain parts of **social security** free of charge. (i.e. health care, public servicies).
2. In order to support their living in the host country international retirement migrants will bring with them all or part of their savings. We will refer to these savings as **foreign savings**. These savings, by affecting the overall capital stock in the host country, may modify the general equilibrium values of wage and interest rate. We also assume that once in the host country the retire migration will not work.

The principal findings of this section are as follows:

: Generally International Retirement Migration increases savings of the host country.

: In accessing part of the **social security** free of charge in the host contry, IRM reduces the welfare of the first generation. This welfare loss is compensated by the welfare gain of the future generations.

: IRM could compensate the crowding out of savings due to social security. In other words IRM induce a displacement effect of social security.

1. IRM and Social security

Keeping in mind the unfunded, "pay-as-you-go" method of financing social security discussed in chapter 6, the impact of introducing international retirement migration in a life cycle model can be understood by reducing the benefit paid to a member of the older generation who is old at time t. Let $\bar{B}$, stand for a fixed social security benefit paid from a young generation. The old generation at time t will receive only a fraction of $\bar{B}$ let's say $(1-\theta)\bar{B}$ due to the presence of international retirement migration, where θ is the fraction of social security that the international retirement migrant enjoys due to the non-discrimination principle. The parameter θ is a function of two elements concerning IRM. It reflects how large is the IRM community in the host country. Also it reflects how many services a retired migrants can access in the host country[1]. At time t the budget constraint for each elderly individual is:

[1]Here we assume a sort of implicit discrimination in accessing social security for foreigner.

$$C_o = WoEo(1 - Lo) + \bar{B}(1 - \theta) + [WyEy(1 - Ly) - Cy - \bar{B}](1 + Ro) \tag{1.1}$$

for young individuals born at the beginning of period t, the lifetime budget constraint is:

(1.2)

$$[WyEy(1 - Ly) - \bar{B} - Cy] + \left[\frac{1}{(1 + Ro)}\right] [WoEo(1 - Lo) + \bar{B}(1 - \theta) - Co] = 0$$

Since the social security system is financed on a pay-as-you-go basis, social security tax revenues per young worker must equal benefit payments to old workers and international retirement migrants . The economy's capital stock is:

$$K_t = WyEy(1 - Ly) - \bar{B} - Cy \tag{1.3}$$

From (1.1) and ignoring for the moment potential general equilibrium effects on the values of W and R, one can see that introducing IRM increases the steady state level of lifetime resources. The consumption of the initial elderly generation will decrease because this generation has to share the social security benefits with IRM. Equation (1.1) makes this clear; if, starting at time t, θ, is raised from zero to a positive value, the consumption of the elderly at time t, decreases by $\bar{B}\theta$. If we ignore for the moment changes through time in factor rewards, the young, whose marginal propensity to consume is less than unity, will reduce their consumption by

Capital stock	0.1162	Income	1.4436
Labor supply	0.998	Capital-Output ratio	8,04%
Wage	1.08	θ, IRM coefficent	0
Consumption Young	0.70	Labor supply Young	0.52
Consumption Old	0.73	Labor supply Old	0.94
Private consumption	1.43	Social Security $\bar{B}$	0.2

TABLE 1. The base case steadystate for International Retirement Migration,B=0.2, $E_Y = 2E_O = 1$

a fraction of $\bar{B}\theta$. Hence, in the initial period in which IRM takes place each elderly individual reduces their consumption by $\bar{B}\theta$ while each young person reduces their consumption by a fraction of $\bar{B}(1-\theta)$. Total private consumption in the initial period therefore decreases and savings increase.

Adding general equilibrium effects, the capital stock due to the high saving will increase, the wage will increase and the interest rate will decrease.

1.1. Simulation results, social security and IRM.

1.1.1. *The base case.* Table 1 presents the simulated initial steady state values of the base case economy from which we can study the economic effects of IRM. In the base case we assume double earning capacity between young and old so $E_Y = 2$, $E_O = 1$. Let $\bar{B} = 0.2$, stand for a fixed social security benefit paid to a member of old generation.

1.2. Steady state effects of social security and International Retirement Migration. Starting from the base case discussed in table 1, table 2 displays the impact of various IRM coefficents (θ) on capital stock income and aggregate labor supply. In our case of a two-phase model in which the IRM is introduced at time t and kept constant thereafter, the first generation of old workers will be worse off and the first generation of young will save more. In the long run from the general equilibrium changes in factor returns; since the increase of saving takes one period to get under way, the interest rate is lower when they are old, but the wage this first set of young workers receive when young is unaffected by the introduction of IRM. In contrast, generations born after the first generation, while benefiting from lower interest rates, receive higher wages during their working period.

1.3. Transition path with social security and International Retirement Migration. In this section we present simulations of two possible scenario of IRM:

1. In the first experiment we assume a small value of IRM coefficent $\theta = 0.05$.
2. In the second experiment we assume a large value, $\theta = 0.3$.

1.3.1. *A low level of IRM (*$\theta = 0.05$*).* Table 3 presents the simulation results of a transition from one steady state without IRM to another one with IRM. We assume that in period 1 the economy is running in steady state with social security $\bar{B} = 0.2$ and no IRM ($\theta = 0$). In period 2 we introduce IRM with a small value of $\theta = 0.05$. This is the fraction of social security that the international retirement migrant enjoys due to the non-discrimination principle. This means that from

IRM coefficent	Capital Stock	Income	Social Security
θ	K	Y	$\bar{B}$
0	0.1162	1.4436	0.2
0.01	0.11628	1.444	0.2
0.05	0.11984	1.4584	0.2
0.1	0.1235	1.4733	0.2
0.15	0.1272	1.488	0.2
0.2	0.1311	1.503	0.2
0.3	0.1379	1.532	0.2
0.5	0.1540	1.589	0.2

TABLE 2. International Retirement Migration, effect on Steady State

the second generation, old natives will have to share part of the social security entitlement with IRM. It is interesting to note that $\theta = 0.05$ increases the capital level in steady state by 3.1%. Labour supply changes only slightly.

1.3.2. *Impact on welfare.* The effects on welfare of introducing IRM are presented in Table 4. The initial old generation (the second row in the table 4) are worse off by a very small amount because of the IRM, whereas the initial young generation (the third row in table 4) suffers also very small welfare losses because of the IRM.

The welfare lost to the initial generation, measured as wealth equivalents, is very small and is compensated by the welfare gain to generations in the long run.

	Capital Stock		Labour Supply		
Period of transition	K	θ	Young	Old	Aggregate
1 Steady State	0.1162	0	47.21%	5.3%	52.56%
2	0.117	0.05	47.27%	5.4%	52.67%
3	0.1183	0.05	47.26%	5.4%	52.66%
4	0.1194	0.05	47.26%	5.4%	52.66%
5	0.1195	0.05	47.25%	5.5%	52.75%
10	0.1197	0.05	47.24%	5.5%	52.74%
15	0.1198	0.05	47.25%	5.5%	52.75%
20	0.11984	0.05	47.25%	5.6%	52.85%

TABLE 3. Simulation transition with International Retirement Migration, $\theta = 0.05$

1.3.3. *A high level of IRM ($\theta = 0.3$).* Table 5 presents the simulation results of a transition from one steady state without IRM to another one with IRM with a high level of $\theta = 0.3$

1.3.4. *Impact on welfare.* The effects on welfare of introducing IRM are presented in Table 6. The initial old generation (the second one in the table) is worse off because of the IRM, whereas the initial young generation (the third one) suffers welfare losses because of the IRM.

Although the welfare lost to the initial generation, measured as wealth equivalents, is small, it is compensated by the welfare gain to generations in the long run.

	Life Consumption	Life time Utility
Generation		
1 Steady State	1.43928	0
2	1.4375	-0.50%
3	1.436	-0.32%
4	1.4355	-0.11%
5	1.4393	+0.01%
10	1.444	+0.41%
15	1.4445	+0.46%
20	1.44424	+0.47%

TABLE 4. Simulation transition with International Retirement Migration, $\theta = 0.05$, impact on welfare

2. Foreign savings and IRM

Let's assume $\bar{K}^*$ is a fixed amount of savings brought by the international retirement migrants in to the host country. The economy's capital stock will be:

$$K_t = WyEy(1 - Ly) - \bar{B} - Cy + \bar{K}^* \tag{2.1}$$

We will express $\bar{K}^*$ as percentage of K_t :

$$\bar{K}^* = \tau K_0 \tag{2.2}$$

	Capital Stock		Labor Supply		
Period of transition	K	θ	Young	Old	Aggregate
1 Steady State	0.1162	0	47.21%	5.3%	52.56%
2	0.12684	0.3	47.57%	7.3%	54.56%
3	0.1312	0.3	47.46%	7.2%	54.48%
4	0.13353	0.3	47.44%	7.1%	54.48%
5	0.13476	0.3	47.30%	7.1%	54.4%
10	0.13604	0.3	47.3%	7.1%	54.4%
15	0.13608	0.3	47.3%	7.1%	54.4%
20	0.1379	0.3	47.33%	7.1%	54.43%

TABLE 5. Simulation transition with International Retirement Migration, $\theta = 0.3$

and we assume $1 \geqq \tau \geqq 0$. Where we call τ the IRM savings coefficent. From (2.1) it is clear that once IRM takes place the level of capital will increase with consequent reduction in R and an increase in W. The consumption of the initial elderly generation will decrease because this generation will receive less interest on their savings. In order to discuss the long term effects we will use a simulation technique. In our transition path the IRM foreign savings will be kept constant over time.

2.1. Simulation results, foreign savings and IRM.

	Life Consumption	Life time Utility
Generation		
1 Steady State	1.43928	0
2	1.42697	-3.675%
3	1.44444	-0.918%
4	1.45366	0.487%
5	1.45847	1.209%
10	1.46349	1.965%
15	1.46368	1.979%
20	1.46369	1.981%

TABLE 6. Simulation transition with International Retirement Migration, $\theta = 0.3$, impact on welfare

2.1.1. *The base case.* Table 7 presents the simulated initial steady state values of the base case economy we choose to study the economic effects of foreign saving. Also in this situation we assume double earning capacity between young and old so $E_Y = 2$, $E_O = 1$ and social security contributions $\bar{B} = 0.2$.

2.2. Steady state effects of foreign savings. Starting from the base case shown in table 7, table 8 displays the impact of various IRM saving coefficents on capital stock interest rates and wages in steady state. For simplicity in our analysis we keep $\theta = 0$. It is clear that the introduction of foreign savings will increase the level of capital in steady state, thereby modifying the levels of interest rates and

Capital stock	0.1162	Income	1.4436
Labor supply	0.998	Capital-Output ratio	8,04%
Wage	1.08	τ, IRM Saving coefficent	0
Consumption Young	0.70	Labor supply Young	0.52
Consumption Old	0.73	Labor supply Old	0.94
Private consumption	1.43	Social Security $\bar{B}$	0.2

TABLE 7. The base case steady state to study International Retirement Migration and foreign saving, θ=0, B=0.2, $E_Y = 2, E_O = 1$

wages. It is interesting to note how the IRM saving coefficent also increases savings in the host country. The economic intuition behind this is that in the presence of foreign savings home capital increases and consequently interest rates decrease. In the new scenario young people will face lower interests rates and so they will have to save more in order to keep the same consumption when old. In this case we find the that income effect is bigger than substitution effect.

2.3. Transition path with foreign savings. We assume that in period 1 the economy is running in steady state with social security $\bar{B} = 0.2$ and no foreign savings ($\tau = 0$). In period 2 we introduce foreign savings and we keep it constant in nominal terms for the future. In having foreign savings the old generation in period 2 will be worse off because they will receive less interest for their savings. In this section we present simulations of the transition of two possible scenarios of IRM savings coefficient:

IRM saving coefficent	Capital Stock	Interest Rate	Wage
τ	K	R	W
0	0.1162	3.1036	1.08
0.05	0.1272	2.909	1.108
0.1	0.13795	2.7473	1.129
0.2	0.1572	2.502	1.165
0.3	0.1755	2.311	1.196
0.5	0.2104	2.02	1.249
0.8	0.2598	1.7414	1.314
1	0.2931	1.595	1.353

TABLE 8. International Retiremnt Migration and foreign saving, effect on Steady State

1. In the first experiment we assume a small value of IRM savings coefficient $\tau = 0.1$.
2. In the second we assume a large value, $\tau = 1$

2.3.1. *A low level or foreign savings ($\tau = 0.1$).* Table 9 presents the simulation results of a transition from one steady state without foreign savings to another one with foreign savings. The immediate effect is a shift of labour supply for old people while the supply of labour for young stays constant and in the long term slightly reduces. On the capital side foreign savings will make home people save more, in

	Capital Stock	Foreign saving	Labour Supply		
Period of transition	K	$\bar{K}^*$	Young	Old	Aggregate
1 Steady State	0.1162	0	47.21%	5.3%	52.56%
2	0.1266	0.0116	47.21%	6.878%	54.08%
3	0.13101	0.0116	47.09%	6.718%	53.81%
4	0.13326	0.0116	47.04%	6.637%	53.68%
5	0.1344	0.0116	47.01%	6.596%	53.61%
10	0.13553	0.0116	46.98%	6.556%	53.54%
15	0.13556	0.0116	46.98%	6.554%	53.54%
20	0.13795	0.0116	46.96%	6.685%	53.64%

TABLE 9. Simulation transition with International Retirement Migration, $\tau = 0.1$

fact a 10% injection of foreign savings into the economy increase the overall capital by 18.7%.

2.3.2. *Impact on welfare.* The effects on welfare of IRM foreign savings are presented in table 10 . The initial old generation (the second one in the table) are slightly worse off by a very small amount because of the IRM. The welfare loss to the initial generation, measured as wealth equivalents, is very small and it is more than compensated by the substantial welfare gain to the future generations.

2.3.3. *A high level or foreign savings ($\tau = 1$).* We are assuming now that the level of foreign savings is the same amount of the initial capital of the home economy. Table 11 presents the simulation results. The effects we have are the

	Life Consumption	Life time Utility
Generation		
1 Steady State	1.43928	0
2	1.43817	-0.750%
3	1.45665	2.065%
4	1.46605	3.448%
5	1.4708	4.136%
10	1.47546	4.799%
15	1.47561	4.820%
20	1.4797	4.837%

TABLE 10. Simulation transition with International Retirement Migration, $\tau = 0.1$, impact on welfare

same as the ones we find in the previous section ($\tau = 0.1$) with higher magnitude. We have an immediate shift of labour supply for old people that will smoothly reduce in the long term. On the capital side foreign savings will make home people save much more.

2.3.4. *Impact on welfare.* Welfare effects are presented in table 12 . The initial old generation (the second one in the table) is worse off for a small amount because of the lost in interest rate they get from lower interest rate. The welfare lost to the initial generation, measured as wealth equivalents, is small and it is more than compensated by the substiantial welfare gains of the future generations.

	Capital Stock	Foreign saving	Labor Supply		
Period of transition	K	$\bar{K}^*$	Young	Old	Aggregate
1 Steady State	0.1162	0	47.21%	5.3%	52.56%
2	0.22375	0.116	46.98%	16.05%	63.04%
3	0.2642	0.116	46.26%	13.25%	59.50%
4	0.28022	0.116	46.02%	12.29%	58.31%
5	0.28614	0.116	45.94%	11.95%	57.89%
10	0.28944	0.116	45.90%	11.83%	57.66%
15	0.28944	0.116	45.90%	11.79%	57.66%
20	0.29315	0.116	45.89%	11.96%	57.85%

TABLE 11. Simulation transition with International Retirement Migration, $\tau = 1$

3. Conclusion

The purpose of this chapter has been to explain and illustrate the dynamic impact of International Retirement Migration. IRM has a displacement effect on social security, in terms of saving, investment and factor rewards. We have focused on free riding of IRM in the host country and foreign savings. In both cases IRM increases the savings of the host country. In contrast to social security, all initial old generations are worse off because of IRM, whereas all young generations have a welfare gain. The simulation methodology has proved useful not only for tracing

	Life Consumption	Life time Utility
Generation		
1 Steady State	1.43928	0
2	1.43883	-4.829%
3	1.58954	15.760%
4	1.64537	21.768%
5	1.66555	23.768%
10	1.6767	24.839%
15	1.67674	24.844%
20	1.67675	24.777%

TABLE 12. Simulation transition with International Retirement Migration, $\tau = 1$, impact on welfare

the channels through which IRM operates, but also for obtaining a quantitative sense of the relative impacts of alternative actions.

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CHAPTER 8

General conclusions and suggestions for further studies

1. Conclusion

The purpose of this thesis has been to explain and illustrate the static and dynamic economic impact of international retirement migration. Divided into three parts, the thesis considers different aspects of international retirement migration.

The first part introduced the problem and examined the legal framework in which it exists. Based on the principle of non-discrimination, the access to the welfare system is contradicted by various European Directives which regulate the possibility of free riding through the resource requirement. This represents a debatable issue for international retirement migration. Another obstacle to examining the issue of IRM has been the lack of reliable statistics. Although figures are unknown, we can conclude that the IRM is not a homogeneous group and it can be divided into three different subgroups. From a legal perspective, these groups have different rights with respect to the access to the social security system. The most obvious economic implication of IRM within the European Union is twofold. Migrants can access part or all of social security services in the host country. Also, they are required to bring into the host country some source of income in order to sustain themselves.

The second part of the thesis examines the reasons for IRM from one country to another. Using a quantitative model, we have identified an environmental variable which can explain one reason why IRM may take place. Different endowments of environmental resources may activate flows of migration between two countries. We have assumed that people in their younger years live in countries with a high capital per capita and on retirement move to a country where this rate is lower but the environmental variable is high in order to enjoy quality of life. This phenomenon may result in a modified capital accumulation for both countries.

In the third and the most important part of the thesis, we analyze the economic effect of social security and IRM through simulation techniques. The simulation results, although based on a highly simplified economic model, suggest that IRM can have powerful effects on the economy. Prolonged and significant presence of IRM can substantially alter the course of savings, investment and factor rewards. Many of these outcomes are effective primarily because they transfer resources across generations. Others are effective because they change economic incentives. Such changes can significantly alter the degree of economic efficiency, in a direction which may not necessarily correspond to the direction of change in long-run welfare. The simulation methodology has proved useful not only for tracing the channels through which IRM and social security operate and interact, but also for obtaining a quantitative sense of the relative impacts. The simulations suggest that introducing unfunded social security can substantially crowd out long-run capital formation. Furthermore, IRM may have a displacement effect on social security. It can be seen as a mechanism for disguising a shift to a non funded social security system. In some countries however IRM and unfunded social security interplay together diminishing the negative effects of unfunded social security.

2. Suggestions for further studies

There are many points to be considered for future studies. Here we want to summarise some:

- In chapter 7 while we were analysing foreign savings and IRM we have assumed that there are no capital flows without flows of people. In other words, we assume that capital moves only with IRM. This is a strong limitation from a globalisation perspective.
- From a public point of view if K also includes public infrastructures, then K^*(foreign saving brought by IRM) could be viewed as an alternative tax revenue turned into capital.
- If public policy can affect the level of θ (the fraction of social security that the international retirement migrants enjoys) for example in making benefits more available for migrants, this would be a way to attract foreign capital from abroad.
- An unsolved issue of this thesis is the way to compensate the initial old generation which is worse off from IRM.

Some of the suggestions do not require extensive modeling but different interpretation of the models. For instance tax on foreign saving can be a way to compensate the initial old generation that is worse off from IRM.

2.1. Limitations of the models. The present models are just a stereotype and differ from the economic reality in a number of obvious ways. There is no involuntary unemployment, only a single asset, only a single homogeneous labor input, no international trade, no uncertainty, no differences across individuals in tastes, no market imperfections, no disequilibrium, and no money. For these reasons and many others, this model cannot be used for economic predictions or for providing explicit policy recommendations. However, the model can greatly expand one's intuition about the ways in which international retirement migration may operate.

APPENDIX A

Matlab Codes

In this appendix we are going to provide all the codes used to evaluate the results of the third part of this thesis

1. The Long run Steady State before and after any change

1.1. Two period model. These files simulate the steady state in section 3.1

with respect to Table 1 and 2

1.1.1. *File master.*

```
Ly=0.8;

Lo=0.5;
K=1.15;
Kguess=1;
Nguess=0.1;
N=2;
parameter
%set all parameter
while K>1.011*Kguess | K<0.989*Kguess | N>1.011*Nguess| N<0.989*Nguess
if K>1.01*Kguess
```

```
K=0.99*K;
end
if K<0.99*Kguess
K=1.001*K;
end
if N>1.01*Nguess
Ly=0.999*Ly;
end
if N<0.99*Nguess
Ly=1.001*Ly;
end
%calculate N as average of Eo Ey Ly Lo
N=Ey*(1-Ly)+Eo*(1-Lo);
%N=2-Ly-Lo;
% income,interest rate and wage
production
%consumption and leisure guessing
consumptionleisure
agregation
P=[K,R,W,Ly,Lo];
end
con_leysur;
```

1.1.2. *Subprograms.* .

1. Parameters; this sets the parameters

```
%parameter

% production function constant A

A=2.4753;

Eo=1;

Ey=1;

alpha=1.5;

rho=0.8;

gamma=0.2;

% intensity of use of capital epsilon

e=0.25;

delta=0.015;
```

2. Production; this sets the value of the production function

```
Y=A*(K^e)*(N^(1-e));

R=A*K^(-1+e)*e*N^(1-e);

W=A*K^e*N^(-e)*(1-e);

prod=[Y,R,W];

prod.Y=Y;

prod.R=R;

prod.W=W;
```

3. consumptionleisure; this calculate a vector of consumption and Leisure for young and old.

```
%consumption and leisure guessing
%tempt
mu=0;
%soltion for young people in period one
Ry=prod.R;
Wy=prod.W;
Ro=prod.R;
Wo=prod.W;
%sistem
AA=zeros(4);
AA(1,1)=(Wy*Ey/alpha)^(-rho);
AA(1,2)=-1;
AA(2,3)=((Wo*Eo+mu)/alpha)^(-rho);
AA(2,4)=-1;
AA(3,1)=-1;
AA(3,3)=((1+Ro)/(1+delta))^(-gamma)*[[1+alpha^rho*(Wy*Ey)^(1-rho)]^((rho-
gamma)/(1-rho))/
([1+alpha^rho*(Wo*Eo+mu)^(1-rho)]^((rho-gamma)/(1-rho)))];
AA(4,1)=1/(1+Ry);
AA(4,2)=1/(1+Ry)*Wy*Ey*[1];
```

```
AA(4,3)=1/(1+Ry)/(1+Ro)*[1];
AA(4,4)=1/(1+Ry)/(1+Ro)*Wo*Eo*[1];
bb(4,1)=(Wo*Eo+Wy*Ey+Wy*Ey*Ro)/(1+Ry)/(1+Ro);
x=inv(AA)*bb;
Cy=x(1,1);
Ly=x(2,1);
Co=x(3,1);
Lo=x(4,1);
' Cy Ly Co Lo';
con_leysur=[Cy,Ly,Co,Lo];
% retirement constrain
if Lo>=1
Lo=1;
mu=0.01;
AB=zeros (3);
AB(1,1)=(Wy*Ey/alpha)^(-rho);
AB(1,2)=-1;
AB(2,3)=((Wo*Eo+mu)/alpha)^(-rho);
AB(3,1)=1/(1+Ry);
AB(3,2)=1/(1+Ry)*Wy*Ey*[1];
AB(3,3)=1/(1+Ry)/(1+Ro)*[1];
ab=zeros(3,1);
```

```
ab(2,1)=1;
ab(3,1)=(Wo*Eo+Wy*Ey+Wy*Ey*Ro)/(1+Ry)/(1+Ro)-1/
(1+Ry)/(1+Ro)*Wo*Eo*[1];
y=inv(AB)*ab;
y;
Cy=y(1,1);
Ly=y(2,1);
Co=y(3,1);
' Cy Ly Co Lo';
con_leysur=[Cy,Ly,Co,Lo];
while Cy >((1+Ro)/(1+delta))^(-gamma)*
[[1+alpha^rho*(Wy*Ey)^(1-rho)]^((rho-gamma)/(1-rho))/
([1+alpha^rho*(Wo*Eo+mu)^(1-rho)]^((rho-gamma)/(1-rho)))]*Co
mu=mu*1.01;
AB=zeros (3);
AB(1,1)=(Wy*Ey/alpha)^(-rho);
AB(1,2)=-1;
AB(2,3)=((Wo*Eo+mu)/alpha)^(-rho);
AB(3,1)=1/(1+Ry);
AB(3,2)=1/(1+Ry)*Wy*Ey*[1];
AB(3,3)=1/(1+Ry)/(1+Ro)*[1];
ab=zeros(3,1);
```

```
ab(2,1)=1;
ab(3,1)=(Wo*Eo+Wy*Ey+Wy*Ey*Ro)/(1+Ry)/
(1+Ro)-1/(1+Ry)/(1+Ro)*Wo*Eo*[1];
y=inv(AB)*ab;
y;
Cy=y(1,1);
Ly=y(2,1);
Co=y(3,1);
end
' Cy Ly Co Lo';
con_leysur=[Cy,Ly,Co,Lo];
end
' Cy Ly Co Lo';
con_leysur=[Cy,Ly,Co,Lo];
```

4. agregation; as figure 1

```
%agregation of N
Nguess=Ey*(1-Ly)+Eo*(1-Lo);
%agregation of kapital
Kguess=Wy*Ey*(1-Ly)-Cy;
```

5. cange in young old productivity

```
%some experiments on E
clear all
```

```
parameter
jx=0;
for Ey=1:0.5:4
jx=jx+1;
twoperiodmodel2
con_leysur(1,5)=K;
gra(jx,:)=con_leysur;
end
%plot (gra, 'DisplayName','gra', 'YDataSource', 'gra'); figure(gcf)
```

2. Solving the trasition path

This file simulate the transition path in part 3, section 3

2.0.3. *File master.* .

```
%dynamic 20 period ajustment
clear all
%steady syate before change
parameter
consumptionleisureSS
SSvalueante
%steady state after change
```

```
%input here the change in the sistem
%change in young productivity
Ey=3;
consumptionleisureSS
SSvaluepost
%vector of K and N linear approximation
%change in K
DeltaK=(SSpost.K-SSante.K)/20;
DeltaN=(SSpost.N-SSante.N)/20;
K=SSante.K-DeltaK;
N=SSante.N-DeltaN;
for i=1:21
K=K+DeltaK;
factor(i,1)=K;
N=N+DeltaN;
factor(i,2)=N;
end
for p=1:100
%vector consumption-leisure for generation
for i=1:21
% production function
K=factor(i,1);
```

```
N=factor(i,2);
production
factor(i,3)=R;
factor(i,4)=W;
factor(i,5)=Y;
kapitalevolution(i,1)=K;
Labourevolution(i,1)=N;
end
for i=1:20
%consumption leisure
Ry=factor(i,3);
Ro=factor(i+1,3);
Wy=factor(i,4);
Wo=factor(i+1,4);
consumptionleisuredy
consleisure(i,:)=con_leysur;
consleisure(21,:)=con_leysur;
end
%agregation
for i=1:20
%kapital 6
factor(1,6)=SSante.K;
```

```
factor(i+1,6)=factor(i,4)*Ey*(1-consleisure(i,2))-consleisure(i,1);

factor(21,6)=SSpost.K;

%Labour 7

factor(1,7)=SSante.N;

factor(i+1,7)=Ey*(1-consleisure(i+1,2))+Eo*(1-consleisure(i,4));

factor(21,7)=SSpost.N;

kapitalevolution(i,p+1)=factor(i,6);

kapitalevolution(21,p+1)=SSpost.K;

Labourevolution(i,p+1)=factor(i,7);

Labourevolution(21,p+1)=SSpost.N;

end

factor (:,1)=factor(:,6);

factor (:,2)=factor(:,7);

end

plot (kapitalevolution, 'DisplayName','kapitalevolution', 'YDataSource', 'kapi-
talevolution'); figure(gcf)

%
```

3. Social security

This file simulate the transition path in part 3, section 4

3.1. File master. .

```
%clear all
```

```
Ly=0.8;
Lo=0.5;
K=1.15;
Kguess=1;
Nguess=0.1;
N=2;
parameter
%set all parameter
while K>1.011*Kguess | K<0.989*Kguess | N>1.011*Nguess| N<0.989*Nguess
if K>1.01*Kguess
K=0.99*K;
end
if K<0.99*Kguess
K=1.001*K;
end
if N>1.01*Nguess
Ly=0.999*Ly;
end
if N<0.99*Nguess
Ly=1.001*Ly;
end
%calculate N as average of Eo Ey Ly Lo
```

```
N=Ey*(1-Ly)+Eo*(1-Lo);

%N=2-Ly-Lo;

% income,interest rate and wage

production

%consumption and leisure guessing

consumptionleisure

agregation

P=[K,R,W,Ly,Lo];

end

con_leysur;

%welfare analisis

U=-[(Cy^(1-1/rho)+alpha*(Ly)^(1-1/rho))^(rho*(gamma-1)/(rho-1)/gamma)+1/(1+delta)*
(Co)^(1-1/rho)+(alpha*(Lo)^(1-1/rho))^(rho*(gamma-1)/(rho-1)/gamma)]/(1-
1/gamma)
```

3.1.1. *Subprograms.* .

1. consumptionleisure; this calculate a vector of consumption and Leisure for young and old with social security

```
%parameter

%consumption and leisure guessing

%tempt

mu=0;

%soltion for young people in period one
```

```
Ry=prod.R;
Wy=prod.W;
Ro=prod.R;
Wo=prod.W;
%sistem
AA=zeros(4);
AA(1,1)=(Wy*Ey/alpha)^(-rho);
AA(1,2)=-1;
AA(2,3)=((Wo*Eo+mu)/alpha)^(-rho);
AA(2,4)=-1;
AA(3,1)=-1;
AA(3,3)=((1+Ro)/(1+delta))^(-gamma)*[[1+alpha^rho*(Wy*Ey)^(1-rho)]
^((rho-gamma)/(1-rho))/([1+alpha^rho*(Wo*Eo+mu)^(1-rho)]^((rho-gamma)/(1-
rho)))];
AA(4,1)=1/(1+Ry);
AA(4,2)=1/(1+Ry)*Wy*Ey*[1];
AA(4,3)=1/(1+Ry)/(1+Ro)*[1];
AA(4,4)=1/(1+Ry)/(1+Ro)*Wo*Eo*[1];
bb(4,1)=(Wo*Eo+Wy*Ey+Wy*Ey*Ro-B*Ro)/(1+Ry)/(1+Ro);
x=inv(AA)*bb;
Cy=x(1,1);
Ly=x(2,1);
```

```
Co=x(3,1);

Lo=x(4,1);

' Cy Ly Co Lo';

con_leysur=[Cy,Ly,Co,Lo];

% retirement constrain

if Lo>=1

Lo=1;

mu=0.01;

AB=zeros (3);

AB(1,1)=(Wy*Ey/alpha)^(-rho);

AB(1,2)=-1;

AB(2,3)=((Wo*Eo+mu)/alpha)^(-rho);

AB(3,1)=1/(1+Ry);

AB(3,2)=1/(1+Ry)*Wy*Ey*[1];

AB(3,3)=1/(1+Ry)/(1+Ro)*[1];

ab=zeros(3,1);

ab(2,1)=1;

ab(3,1)=(Wy*Ey+Wy*Ey*Ro-B*Ro+Wo*Eo)/(1+Ry)/

(1+Ro)-1/(1+Ry)/(1+Ro)*Wo*Eo*[1];

y=inv(AB)*ab;

y;

Cy=y(1,1);
```

```
Ly=y(2,1);

Co=y(3,1);

' Cy Ly Co Lo';

con_leysur=[Cy,Ly,Co,Lo];

while Cy >((1+Ro)/(1+delta))^(-gamma)*

[[1+alpha^rho*(Wy*Ey)^(1-rho)]^((rho-gamma)/(1-rho))/

([1+alpha^rho*(Wo*Eo+mu)^(1-rho)]^((rho-gamma)/(1-rho)))]*Co

mu=mu*1.01;

AB=zeros (3);

AB(1,1)=(Wy*Ey/alpha)^(-rho);

AB(1,2)=-1;

AB(2,3)=((Wo*Eo+mu)/alpha)^(-rho);

AB(3,1)=1/(1+Ry);

AB(3,2)=1/(1+Ry)*Wy*Ey*[1];

AB(3,3)=1/(1+Ry)/(1+Ro)*[1];

ab=zeros(3,1);

ab(2,1)=1;

ab(3,1)=(Wy*Ey+Wy*Ey*Ro-B*Ro+Wo*Eo)/(1+Ry)/(1+Ro)-1/

(1+Ry)/(1+Ro)*Wo*Eo*[1];

y=inv(AB)*ab;

y;

Cy=y(1,1);
```

```
Ly=y(2,1);
Co=y(3,1);
end
' Cy Ly Co Lo';
con_leysur=[Cy,Ly,Co,Lo];
end
' Cy Ly Co Lo';
con_leysur=[Cy,Ly,Co,Lo];
```

2. this evaluate the dynamic of the model with social security changing over time

```
%dynamic 20 period ajustment
clear all
%steady syate before change
parameter
%social security parameter
B=0;
twoperiodmodelSS
SSvalueante
%steady state after change
%input here the change in the sistem
%change in young productivity
B=0.3;
```

```
twoperiodmodelSS

SSvaluepost

%dynamic social security

for t=1:20

if t==1

Bt(t,1)=0;

end

if t>1 & t<11

Bt(t,1)=(0.03)*(1.292)^(t-2);

end

if t>9 & t<20

Bt(t,1)=0.3;

end

Bt(20,1)=0;

if t>11

Bt(t,1)=0.3;

end

end

%vector of K and N linear approximation

%change in K

DeltaK=(SSpost.K-SSante.K)/20;

DeltaN=(SSpost.N-SSante.N)/20;
```

```
K=SSante.K-DeltaK;
N=SSante.N-DeltaN;
for i=1:21
K=K+DeltaK;
factor(i,1)=K;
N=N+DeltaN;
factor(i,2)=N;
end
for p=1:100
%vector consumption-leisure for generation
factor(1,1)=SSante.K;
factor(20,1)=SSpost.K;
factor(21,1)=SSpost.K;
for i=1:21
% production function per period
K=factor(i,1);
N=factor(i,2);
production
factor(i,3)=R;
factor(i,4)=W;
factor(i,5)=Y;
kapitalevolution(i,1)=K;
```

```
Labourevolution(i,1)=N;

end

for i=2:20

%consumption leisure

Ry=factor(i-1,3);

Ro=factor(i,3);

Wy=factor(i-1,4);

Wo=factor(i,4);

B=Bt(i,1);

consumptionleisuredy

consleisure(i,:)=con_leysur;

end

%conumption leisure first generation SS and last generation

consleisure(1,:)=SSante.CL;

consleisure(21,:)=SSpost.CL;

%agregation

for i=2:20

B=Bt(i,1);

%kapital 6

factor(1,6)=SSante.K;

factor(i,6)=factor(i-1,4)*Ey*(1-consleisure(i,2))-consleisure(i,1)-B;

factor(20,6)=SSpost.K; %Labour 7
```

```
factor(1,7)=Ey*(1-conleisure(2,2))+Eo*(1-SSante.Lo);

factor(i,7)=Ey*(1-conleisure(i+1,2))+Eo*(1-consleisure(i,4));

factor(21,7)=SSpost.N;

kapitalevolution(i,p+1)=factor(i,6);

kapitalevolution(21,p+1)=SSpost.K;

Labourevolution(i,p+1)=factor(i,7);

Labourevolution(21,p+1)=SSpost.N;

kapitalevolution(1,p+1)=SSante.K;

Labourevolution(1,p+1)=SSante.N;

end

factor (:,1)=factor(:,6);

factor (:,2)=factor(:,7);

end

plot (kapitalevolution, 'DisplayName','kapitalevolution', 'YDataSource',
'kapitalevolution'); figure(gcf)

%

%welfare analisys

for i=2:20

U(i+1,1)=1/(1-1/gamma)*[((consleisure(i,1))^(1-1/rho)+alpha*(consleisure(i,2))^(1-
1/rho))^
(rho*(gamma-1)/(rho-1)/gamma)+1/(1+delta)*((consleisure(i,3))^(1-1/rho)+alpha*
(consleisure(i,4))^(1-1/rho))^(rho*(gamma-1)/(rho-1)/gamma)];
```

```
end
U(1,1)=1/(1-1/gamma)*[(SSante.Cy^(1-1/rho)+alpha*SSante.Ly^(1-1/rho))^
(rho*(gamma-1)/(rho-1)/gamma)+1/(1+delta)*((SSante.Co)^
(1-1/rho)+alpha*SSante.Lo^(1-1/rho))^(rho*(gamma-1)/(rho-1)/gamma)];
U(2,1)=1/(1-1/gamma)*[(SSante.Cy^(1-1/rho)+alpha*SSante.Ly^(1-1/rho))
^(rho*(gamma-1)/(rho-1)/gamma)+1/(1+delta)*((0.03+SSante.Co)^(1-
1/rho)+
alpha*SSante.Lo^(1-1/rho))^(rho*(gamma-1)/(rho-1)/gamma)];%U(21,1)=1/(1-
1/gamma)*
[(SSpost.Cy^(1-1/rho)+alpha*SSpost.Ly^(1-1/rho))^(rho*(gamma-1)/(rho-
1)/gamma)+1/(1+delta)
*(SSpost.Co^(1-1/rho)+alpha*SSpost.Lo^(1-1/rho))^(rho*(gamma-1)/(rho-
1)/gamma)];
```

4. International Retirement Migration

4.1. File master. This file has been use to find the results of chapter 7

```
%dynamic 20 period ajustment I
clear all
%steady syate before change love
parameter
%social security parameter almina
B=0.2;
```

```
theta=0;

Kstar=0;

twoperiodmodelSS

SSvalueante

%steady state after change

%input here the change in the system

%change in young productivity

B=0.2;

theta=0;

Kstar=0.1162;;

twoperiodmodelSS

SSvaluepost

%change in K

DeltaK=(SSpost.K-SSante.K)/20;

DeltaN=(SSpost.N-SSante.N)/20;

K=SSante.K-DeltaK;

N=SSante.N-DeltaN;

for i=1:21

K=K+DeltaK;

factor(i,1)=K;

N=N+DeltaN;

factor(i,2)=N;
```

```
end
for p=1:100
%vector consumption-leisure for generation
factor(1,1)=SSante.K;
factor(20,1)=SSpost.K;
factor(21,1)=SSpost.K;
for i=1:21
% production function per period
K=factor(i,1);
N=factor(i,2);
production
factor(i,3)=R;
factor(i,4)=W;
factor(i,5)=Y;
kapitalevolution(i,1)=K;
Labourevolution(i,1)=N;
end
for i=2:20
%consumption leisure
Ry=factor(i-1,3);
Ro=factor(i,3);
Wy=factor(i-1,4);
```

```
Wo=factor(i,4);

%B=Bt(i,1);

consumptionleisuredy

consleisure(i,:)=con_leysur;

%conumption leisure first generation SS and last generation

consleisure(1,:)=SSante.CL;

consleisure(21,:)=SSpost.CL;

end

%aggregation

for i=2:20

%B=Bt(i,1);

%kapital 6

factor(1,6)=SSante.K;

factor(i,6)=factor(i-1,4)*Ey*(1-consleisure(i,2))-consleisure(i,1)-B+Kstar;

factor(20,6)=SSpost.K;

%Labour 7

factor(1,7)=Ey*(1-consleisure(2,2))+Eo*(1-SSante.Lo);

factor(i,7)=Ey*(1-consleisure(i+1,2))+Eo*(1-consleisure(i,4));

factor(21,7)=SSpost.N;

kapitalevolution(i,p+1)=factor(i,6);

kapitalevolution(21,p+1)=SSpost.K;

Labourevolution(i,p+1)=factor(i,7);
```

```
Labourevolution(21,p+1)=SSpost.N;
kapitalevolution(1,p+1)=SSante.K;
Labourevolution(1,p+1)=SSante.N;
end
factor (:,1)=factor(:,6);
factor (:,2)=factor(:,7);
end
plot (kapitalevolution, 'DisplayName','kapitalevolution', '
YDataSource', 'kapitalevolution'); figure(gcf)
%plot (kapitalevolution(1:21,101), 'DisplayName',
'kapitalevolution(1:21,101)', 'YDataSource', 'kapitalevolution(1:21,101)'); fig-
ure(gcf)
%
%welfare analisys
for i=1:21
U(i,1)=1/(1-1/gamma)*[((consleisure(i,1))^(1-1/rho)+alpha*(consleisure(i,2))^(1-
1/rho))^
(rho*(gamma-1)/(rho-1)/gamma)+1/(1+delta)*((consleisure(i,3))^(1-1/rho)+alpha*
(consleisure(i,4))^(1-1/rho))^(rho*(gamma-1)/(rho-1)/gamma)];
end
U(1,1)=1/(1-1/gamma)*[(SSante.Cy^(1-1/rho)+alpha*SSante.Ly^(1-1/rho))^
```

```
(rho*(gamma-1)/(rho-1)/gamma)+1/(1+delta)*((SSante.Co)^(1-1/rho)+alpha*SSante.Lo^(1-
1/rho))
^(rho*(gamma-1)/(rho-1)/gamma)];
U(21,1)=1/(1-1/gamma)*[(SSpost.Cy^(1-1/rho)+alpha*SSpost.Ly^(1-1/rho))^(rho*(gamma-
1)
/(rho-1)/gamma)+1/(1+delta)*((SSpost.Co)^(1-1/rho)+alpha*SSpost.Lo^(1-
1/rho))
^(rho*(gamma-1)/(rho-1)/gamma)];
```

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