# Relicensing as a Secondary Market Strategy

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## Abstract

Secondary markets in the Information Technology (IT) industry, where used or refurbished equipment is traded, have been growing steadily. For Original Equipment Manufacturers (OEMs) in this industry, the importance of secondary markets has grown in parallel, not only as a source of revenue, but also because of their impact on these firms' competitive advantage and market strategy. Recent articles in the press have severely criticized some OEMs who are perceived to be actively trying to eliminate the secondary market for their products. Others have policies that enhance their secondary markets. The goal of this paper is to understand how an OEM's incentives and optimal strategies vis-à-vis the secondary market are shaped contingent on her relative competitive advantage, product characteristics and consumer preferences. The critical tradeoff that we examine is whether the indirect benefit from maintaining an active secondary market (the resale value effect) can outweigh the potentially negative effect of the sales of used products at the expense of new product sales (the cannibalization effect). To that end, we develop a model where the OEM can directly affect the resale value of her product through a relicensing fee charged to the buyer of the refurbished equipment. Moreover, we introduce a measure of the consumers' willingness to return their used products to account for the fact that the higher the price offered by a third-party entrant, the higher the ratio of returned products at their end-of-use. We analyze the OEM's decision in both the monopoly and the duopoly cases, characterize the optimal relicensing fee set by the OEM, and draw conclusions on the conditions that favor stimulating or deterring the secondary market.

Keywords: Cannibalization, Secondary Market, Relicensing Fee, Remanufacturing, Closed-Loop Supply Chain

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# 1    Introduction

Today, Original Equipment Manufacturers (OEMs) in the Information Technology (IT) industry often face difficult decisions when forming strategies involving secondary markets for their products. In the years before the dot-com bubble of the late 1990s, there was a limited secondary IT market. Some reasons for this lack of demand for refurbished IT equipment included: 1) IT OEMs focused on their primary sales channels and discouraged customers from considering refurbished equipment; 2) buyers of IT equipment were leery of the quality level of a refurbished product; and 3) there was a lack of independent secondary market firms to refurbish, resell, and support IT equipment. Shortages of higher-end IT equipment such as servers and routers during the late 1990s however, led to unmet demand that was often satisfied by a new market of third-party IT equipment brokers and refurbishers[1]. In the years following, the dot-com bust resulted in a large surplus of barely used IT equipment for sale from companies who failed when the bubble burst. The availability of so much inexpensive used IT equipment led to significant price discounts compared to the price of new equipment and even more brokers and refurbishers entering the secondary market (Berinato 2002).

One of the lasting effects from the dot-com era is that major customers of IT equipment have started accepting refurbished IT equipment as a viable alternative to new equipment and a new body of IT refurbishers has entered the market to meet this demand. According to a 2002 survey of 187 IT executives in CIO magazine, 77 percent said they were purchasing secondary market equipment and 46 percent expected to increase their spending on refurbished equipment in the next year by an average of 15 percent (Berinato 2002). In another article, Computer Business Review highlights that "third-party companies have built $100+ million per year businesses in buying used computer equipment, refurbishing it, and selling or leasing it out to someone else" (CBRonline.com 2005). Given the size and growth of the secondary market, the days of ignoring it and only focusing on the sale of new products are over for all major IT OEMs. OEMs may either embrace the secondary market or try to eliminate it, but one thing is now evident, they must form strategies to respond to it.

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[1]Third-party refurbishers do not manufacture their own products, but instead rebuild and reconfigure used OEM products that they buy from IT users who upgrade or no longer need those products. Unlike other markets such as the automotive market, potential customers in the used IT equipment market typically expect the equipment to be refurbished before purchasing; thus the vast majority of the sales in the IT secondary market are between refurbishers and the end-users rather than between the end-users themselves. Following industry usage, we will use the terms "refurbished" and "remanufactured" interchangeably in this paper; for a detailed definition of these terms, see Thierry et al. (1995).

Some of the major OEMs in the IT industry have not only embraced the existence of a secondary market, but also deploy it to obtain competitive advantage over their rivals. IBM and Hewlett Packard, for instance, create high resale values for their used equipment by facilitating the resale process and secondary use (e.g. charging small relicensing fees, offering maintenance and inspection) so that the original customers gain a higher net benefit from their new product purchases. Such a proactive, and in a sense cooperative, relationship with third-party brokers and refurbishers, however, is not a standard policy among all IT OEMs. An alternative strategy is to institute policies and fees that attempt to eliminate the secondary market. For example, Sun Microsystems (Sun), one of the leading firms in the IT server business, was "under fire for deliberately attempting to eliminate the secondary market for its machines worldwide through their new pricing and licensing schemes" (Marion 2004). Cisco is another company that requires each buyer of its refurbished equipment to pay high relicensing fees for the proprietary software that makes the equipment run.

The following excerpts, typical of the IT industry, shed some light on how the relicensing mechanism works. "Cisco adopts a policy of non-transferability of its software to protect its intellectual property rights." What this means is that owners of Cisco products are only allowed to transfer, resell, or re-lease used Cisco hardware and not the embedded software that runs on it. This practice, in effect, eliminates the secondary market and creates customer dissatisfaction. Cisco's response to this criticism was to institute relicensing fees, albeit significant: "As Cisco's installed base of equipment has grown to such large numbers over the years, our customers have become more interested in selling and leasing used Cisco equipment on the secondary market. In order to provide our valued customers and partners with this capability, Cisco is now setting up a program where companies who are interested in buying used equipment, may now purchase a new software license to do so" (Cisco.com 2007).

Despite such statements that a relicensing fee mechanism allows reselling refurbished equipment on the secondary market, many industry observers argue that some OEMs use unreasonably high relicensing fees as a means of limiting the secondary market. In the case of Sun, Marion (2004) highlights the fact that the relicensing fee is deliberately set so high that the overall cost of a unit of refurbished equipment, including hardware and software, reaches that of a new one: "In the end, the potential buyer for the refurbished equipment may have no choice but to return to Sun for a new product." He concludes by stressing another interesting facet of the problem: "End users need to know this and take action to adjust the Sun hardware values reflected on their respective balance sheets to account for the impact that Sun's actions, described above, will have on resale

and residual values." In other words, users should be aware that Sun's practices result in very low resale values of used equipment and this information should be factored into their original purchase decision. In fact, many IT consulting companies (e.g. www.computereconomics.com) offer detailed forecasts regarding future resale values of used IT equipment, underlining the critical role of the resale value in the initial IT purchase decision.

From a research perspective, the discussion above raises the fundamental question addressed in this paper. Given the OEM's ability to interfere with the IT secondary market through pricing and relicensing schemes, is limiting this market or, conversely, encouraging its existence, a more profitable strategy? If one strategy is dominant over the other, the winner is currently not clear based on anecdotal evidence alone. Our goal is to understand how the OEM's incentives and optimal strategies are shaped contingent on costs, consumer preferences and the intensity of remanufacturing competition. Motivated by the industry articles concerning Sun, a company that has historically been considered the premium brand in the server market (Sun.com 2007), we also examine whether such a brand premium could justify an aggressive strategy vis-à-vis the secondary market.

We begin our analysis by studying the optimal strategy of an OEM that has a monopoly on the new product market, but faces future competition from a third-party entrant who purchases the used products from the OEM's customers, refurbishes them, and resells them in competition with the OEM's new products. The OEM collects a relicensing fee on every product sold by the entrant; and can effectively "shut down" the secondary market by charging a high fee. Our key finding is that whether or not consumers take the resale value into account is a key determinant of the OEM strategy. When consumers act strategically (taking into account the resale value effect in their purchase of new products), and when the refurbishing cost is low, it is suboptimal for the OEM to shut down the secondary market. Note that the latter condition is counter-intuitive since it suggests that as the entrant becomes more competitive, the OEM is more willing to support the secondary market. This is because a low refurbishing cost allows for a larger secondary market, and therefore a significant resale value. In contrast, when consumers act myopically, the resale value effect vanishes, the benefit from supporting the secondary market is significantly weakened and shutting down the secondary market becomes an optimal strategy under a much wider range of conditions (e.g. even at a low refurbishing cost).

We examine how the OEM's strategy changes as the number of the independent entrants increases, i.e. the secondary market becomes more competitive. We find that both the OEM's profits as well as the size of the secondary market grow with an increase in the number of entrants. Inter-

estingly, the OEM decreases her relicensing fee even as the sales volume of refurbished equipment grows, and the cannibalization of new products increases. This is because an increasing network of resellers strengthens the marginal impact of the relicensing fee on the resale value effect relative to the corresponding impact on the cannibalization effect. As a result, the OEM chooses to lower the relicensing fee, further stimulating the procurement competition among the entrants, and benefiting from the higher resale value of its used product.

We conclude by analyzing OEM strategies in a differentiated new product duopoly setting. Our numerical results show the high-end OEM always charges a higher relicensing fee than the low-end OEM and the difference between relicensing fees can be significant. Thus, a high relicensing fee need not be indicative of an attempt to shut down the secondary market, but rather reflect the brand premium the high-end OEM commands. This result may help explain the significantly different relicensing fees observed in practice. Overall, our research highlights the strategic importance of supporting an active secondary market under a wide range of circumstances, particularly in the presence of strategic consumers and low refurbishing costs.

## 2    Literature Review

A rapidly growing stream of literature on remanufacturing has focused on the competition between the OEM and independent refurbishers/remanufacturers. Debo et al. (2005) determine the optimal pricing and remanufacturability level decisions of a firm competing with independent remanufacturers. They find that an increase in the competitive intensity reduces the OEM's incentive to invest in the remanufacturability of its products. Ferrer and Swaminathan (2006) study the optimal pricing schemes for an OEM and a single entrant in a multiperiod setting where consumers show a higher preference for the OEM's product over the entrant's product. They show an OEM may forgo some of the first-period profits by making additional units to increase the number of cores available for remanufacturing in subsequent periods. Majumder and Groenevelt (2001) derive the Nash equilibrium quantity solutions between an OEM and an entrant contingent on the availability of used products. Ferguson and Toktay (2006) analyze two common entry-deterrent strategies: remanufacturing and preemptive collection. They find a firm may choose to remanufacture or preemptively collect its used products to deter entry, even when the firm would not have chosen to do so under a pure monopoly environment. Atasu et al. (2007) identify the conditions under which remanufacturing can be used as a strategic marketing tool in the presence of a green segment, and

4

show that competition in the primary market is a reason for an OEM to remanufacture its own products; this works best against a low-cost competitor with a strong brand.

Although the above models provide a theoretical framework for analyzing the competition between the OEM and potential entrants that refurbish and sell the OEM's product, with the exception of an extension in Debo et al. (2005), they do not incorporate the effect of the resale value on the consumers' net utility from purchasing a new product. As a result, they focus only on the cannibalization effect, and therefore, the existence of independent remanufacturers is always detrimental for the OEM's profit. We contribute to the existing literature on remanufacturing by endogenizing the resale value, and more importantly, by linking it to the consumers' willingness to pay for a new product. Thus, competition from an independent refurbisher has both a positive (resale value effect) and a negative (cannibalization of new product sales) impact on the OEM's profit. Debo et al. (2005) find that as the number of remanufacturers increases (cannibalization increases), the OEM's profit decreases despite the positive resale value effect. With the relicensing fee mechanism, we show the resale value effect can dominate, and a higher competitive intensity in the secondary market can benefit the OEM. This happens because the relicensing fee allows the OEM to directly impact the secondary market: The OEM increases its profits by reducing the relicensing fee and increasing the product's resale value as remanufacturing competition increases.

While the idea that a secondary market can benefit the OEM is relatively new in the remanufacturing literature, it is well established in the durable goods literature, a thorough review of which can be found in Waldman (2003). Until the early 1970s, the main conclusion regarding the impact of secondary markets on a monopolist's profitability was due to the cannibalization effect between new and used products. In the words of Gaskins (1974), "conventional economic wisdom...contends that the existence of a competitive secondhand market constitutes a major long-run restraint on monopoly power in a primary market." Motivated by the market for diamonds, however, Miller (1974) argues that "the buyer of a newly produced diamond pays a price consistent with what the diamond can be sold for to others including members of later generations" and thus "the initial price captures the present value of all subsequent transactions." In essence, he points out the "resale value effect," arguing that a secondary market might increase the value derived by the consumer, and in turn, the price that the monopolist can charge for it. This argument is also stressed by Benjamin and Kormendi (1974), Liebowitz (1982), Rust (1986), and Levinthal and Purohit (1989), who all argue that whether or not a monopolist has the incentive to eliminate the secondary market is not clear-cut. A limitation of these papers is the assumption that the demand side is modeled by

a representative consumer (homogeneous consumer preferences). Anderson and Ginsburgh (1994) argue that in those models, the size of the second-hand market is indeterminate since the representative consumer buys both new goods and used goods each period and essentially sells the used good to herself. By introducing a model in which consumers have heterogeneous tastes, they show that the existence of a secondary market enables the monopolist to achieve price discrimination between high and low valuation consumers who buy new and used products, respectively.

Models allowing consumers to have heterogeneous tastes are refined in further research by Waldman (1996, 1997), Desai and Purohit (1998), Hendel and Lizzeri (1999) and Desai et al. (2004, 2007). Waldman (1996) employs the seminal Mussa and Rosen (1978) analysis of market segmentation and product-line pricing to allow customers to vary in their valuations of quality. His main result is that because of the substitution effect between new and used products, the price at which old units trade on the secondary market constrains the price that the monopolist can charge for the new units. Therefore, he demonstrates that the monopolist may have an incentive to "shut down" the market by reducing durability to "sufficiently low" values. In a follow-up paper, Waldman (1997) demonstrates that leasing versus selling can be used to eliminate the secondary market, and argues that this motivation might have been the primary reason for many prominent anti-trust leasing cases (United Shoe, IBM, Xerox). Hendel and Lizzeri (1999) study leasing and selling strategies under secondary markets when durability is endogenous and the OEM can either allow a fully functioning secondary market (perfectly competitive with no restrictions) or shut down the secondary market completely. They show conditions where the OEM would not want to shut down the secondary market but prefers reducing the durability instead. Finally, Desai and Purohit (1998) and Desai et al. (2004, 2007) include the discounted resale price (resulting from perfect competition in the second period) in the customer's first-period valuation of the new product, but their primary focus is on evaluating leasing versus selling, solving the time-consistency problem, or evaluating the impact of demand uncertainty, respectively.

We extend the above literature on the economics of the secondary markets in three directions. First, we relax the assumption of perfect competition in the secondary market and allow for a profit-maximizing entrant to collect and refurbish the used products (in the durable goods literature, consumers are allowed to sell the used product to each other, creating a perfectly competitive secondary market). The value offered to the consumers for the used product by the entrant is determined as his optimal response to the OEM's decisions. Thus, the purchase price for used units and the prices charged to consumers for new and refurbished products arise as the Nash

equilibrium of the game between the OEM and entrant. This allows us to examine the impact of the refurbishing cost on the size of the secondary market and on the OEM's strategy (there is no refurbishing cost in the durable goods literature). In addition, we study how the above strategy changes with respect to the number of entrants. Second, by explicitly incorporating the relicensing fee component in our decision framework, we are the first to capture the strategic implications of this widespread mechanism. By treating the relicensing fee as a continuous decision variable, we avoid restricting the OEM to either fully supporting or completely shutting down the secondary market as in Hendel and Lizzeri (1999). Third, in an extension, we relax the assumption of a monopolist OEM by allowing vertically differentiated products to compete in the primary market. We find that the high-end OEM always charges a higher relicensing fee than the low-end OEM and that the difference between relicensing fees can be significant. Yet, whether a high-end or a low-end OEM has a greater secondary market depends on the market conditions and the relative brand differential between the two OEMs. Our results indicate that even with competition in the primary market, it remains rare for either OEM to eliminate the secondary market, although the total size of the secondary market decreases as the brand premium of the high-end OEM decreases. To our knowledge, we are the first to model differentiated new and refurbished products competing in both the primary and secondary markets.

## 3    Key Assumptions and Notation

Our base-line analysis assumes the OEM holds a monopoly in the new product market, but potential third-party entrants may create a secondary market by refurbishing and reselling used products collected from the OEM's customers. Our goal is to examine the OEM's relicensing fee strategy in the face of future competition in the secondary market. The link between current and future sales is captured by a two-period model with a one-period useful product life; refurbishing extends the product's life to two-periods. Other papers that use a two-period model with a one-period useful product life include Majumder and Groenevelt (2001), Ray et al. (2005), Ferrer and Swaminathan (2006), Ferguson and Toktay (2006), and Atasu et al. (2007).

The sequence of events is as follows: In the first period, the OEM sells the new product as a monopolist. Used IT equipment, before it can be reused, requires some costly refurbishing effort that the original consumers do not have the technical capability to perform. Thus, we assume that consumers cannot sell their used products directly to each other. Instead, a third-party refurbisher

buys used products from first-period consumers (the volume depends on the price offered by the entrant), and enters the market in the second period by refurbishing and reselling these products. This assumption reflects the current practice in the used IT market where most used equipment, before it can be resold, requires software updates and the replacement of wearable parts that the original consumers do not have the technical capability to perform. Note that consumers who do not resell their used product cannot continue to use it in the second period because the product's useful life (in the absence of being refurbished by the entrant) is only one period. These customers re-enter the market in the second period and can buy another new product or a refurbished product. Consumers purchasing the refurbished product from the entrant must also pay a relicensing fee to the OEM to operate the product. Thus, in the second period, the OEM's new product sales face competition from the refurbished products offered by the entrant. At the same time, the OEM generates relicensing fee revenues from the refurbished products.

In this competitive setting, the OEM has a significant advantage over the entrant: she controls the relicensing fee that consumers of refurbished products need to pay on top of the purchase price charged by the entrant. As the relicensing fee increases, the cost to consumers of the refurbished product increases, which in turn reduces demand and shifts consumers to the new product. At first sight, a high value for the relicensing fee may seem like a good idea for the OEM, since it eliminates the competition from the refurbished product. Eliminating the secondary market, however, has an important impact on first-period profits. Since consumers can no longer sell their used products to an entrant, the net utility they obtain from the new product decreases. Consequently, the price charged by the monopolist OEM, along with her first-period profits, is lower than it would have been had the consumers foreseen a positive resale value for their used products. Hence, the OEM needs to balance the impact of two opposite forces: A lower relicensing fee leads to competition in the second period, but allows the OEM to charge a premium in the first period that reflects the consumer's ability to resell the product in the second period. We now discuss our key assumptions.

**Assumption 1**. *Consumer willingness-to-pay is heterogeneous and uniformly distributed in the interval* $[0, 1]$.

We assume that consumers' types are distributed uniformly in the interval $[0, 1]$ where a consumer of type $\theta \in [0, 1]$ has a willingness-to-pay of $\theta$ for a new product. In any period, each consumer uses at most one unit. The market size is normalized to 1. In the first period when only the new product is available, if no secondary market existed for the product (zero resale value), the consumer utility function would be $U_1 = \theta - p_1$, where $U_1$ represents the consumer's utility in the

first period and $p_1$ is the price paid for the new product. This would lead to the familiar inverse demand function $p_1 = 1 - q_1$, where $q_1$ is the quantity of new product sold in the first period.

**Assumption 2**. *Each consumer's willingness-to-pay for the refurbished product is a fraction $\delta$ of their willingness-to pay for the new product.*

Under this assumption, a consumer with a willingness-to-pay $\theta$ for the new product has a willingness-to-pay $\delta\theta$ for the refurbished one. The nature of competition between new and refurbished units is thus one of vertical differentiation. That is, for the same price consumers prefer a new product to a refurbished one. This assumption is driven by the evidence that consumers are concerned about the quality of a refurbished product and this is reflected in their willingness to pay for it. Guide and Li (2007) show empirical evidence of this phenomenon by recording the selling prices of both new and refurbished versions of the same product on eBay auctions. This perspective is also reflected in a number of articles in the practitioner and academic literature (Lund and Skeels 1983, Hauser and Lund 2003, Kandra 2002, Debo et al. 2005, Vorasayan and Ryan 2006, Jin et al. 2007). Note that if $\delta = 0$, consumers are not willing to pay anything for the refurbished product; this eliminates the option of maintaining a secondary market. If $\delta = 1$, consumers view the new and refurbished units as being identical and are willing to pay the same amount for either product. Most products fall between the two extremes; we assume $0 < \delta < 1$.

**Assumption 3**. *The disutility to a customer of reselling a used product is a fraction of his original willingness-to-pay for the new product.*

We assume that a consumer with a willingness-to-pay $\theta$ for a new product will incur a perceived transactional disutility (hereafter disutility) of $\gamma\theta$ (where $0 < \gamma < \delta$) to sell his used product to the entrant (e.g. perceived disutility of searching for IT resellers, removing sensitive data, etc.). Therefore, he will sell only if the purchase price $s$ offered to him is more than $\gamma\theta$. Hence, a higher incentive is needed to induce a higher willingness-to-pay customer to resell his used product. This behavioral characteristic forms the basis behind the common use of product rebates that allow price discrimination between consumers who will take the time to send in the rebate and those who will not (Gerstner and Hess 1991, 1995). Obviously, the higher the rebate, the higher the percentage of customers that claim it. Similarly, with this assumption, the higher the price offered by the entrant, the higher the percentage of customers who will sell their used products to the entrant. In line with previous research on reverse logistics and remanufacturing, this assumption ensures that the average cost of acquisition increases in the quantity of the products collected (Guide 2000, Guide and Van Wassenhove 2001, Galbreth and Blackburn 2006, Ferguson and Toktay 2006).

**Assumption 4**. *Customers are strategic.*

There is empirical evidence that IT customers are strategic in their purchasing behavior (Song and Chintagunta 2003, Nair 2004, Plambeck and Wang 2006). Accordingly, we assume that customers take into account the future resale value of the product in making their purchase decisions. This is facilitated in practice by the existence of IT consulting companies that offer resale value forecasts. In our model, consumer $\theta$ chooses to sell the used product to the entrant for a price $s$, as long as this value is greater than the disutility $\gamma\theta$. Therefore, a strategic consumer of type $\theta$ derives a net utility of $U_1(\theta) = \theta - p_1 + (s - \gamma\theta)I_{(s \geq \gamma\theta)}$ from purchasing a new product in period 1, where $I_{(s \geq \gamma\theta)} = 1$ when $s \geq \gamma\theta$ and 0 otherwise. The reason for the indicator variable in the utility expression is that the consumers with a large disutility for selling their old unit to the entrant choose not to do so.
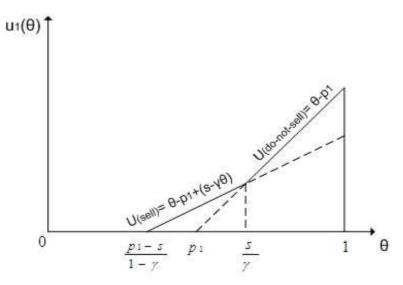


Figure 1: Consumer state space and corresponding utilities from selling versus not selling the used product,

As shown in Figure 1, contingent on their type, first-period consumers fall in one of three segments. If $\theta \leq \frac{p_1 - s}{1 - \gamma}$, consumers do not purchase the new product, while for $\frac{p_1 - s}{1 - \gamma} < \theta \leq \frac{s}{\gamma}$, consumers purchase the new product and subsequently resell it. Finally, for $\frac{s}{\gamma} < \theta \leq 1$, consumers purchase the new product and do not resell it. Therefore, the total sales quantity in period 1 is $q_1 = 1 - \frac{p_1 - s}{1 - \gamma}$, or, $p_1 = (1 - \gamma)(1 - q_1) + s$, and the total number of units acquired by the entrant is given by $q_u = \frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma}$. Note that the entrant would never set $s > \gamma$, as $s = \gamma$ is sufficient to ensure all consumers sell their used products ($q_u = q_1$).

**Assumption 5**. *The OEM charges a relicensing fee $h$ in the second period to any consumer who*

*purchases a refurbished product.*

The establishment of a relicensing fee, typically called a Digital License Agreement (DLA), has been widely employed by OEMs as a means of protecting their intellectual property rights. A DLA allows a consumer to re-install the necessary software for the equipment to operate and thus, a refurbished product is of no use without it. OEMs publish list prices for new equipment (that implicitly includes both hardware and software cost) and most publish a separate list where their relicensing policies are explicitly laid out. The relicensing fee, declared in the first period, constitutes an important element of our model, since it affects the resale value, which is taken into account by strategic consumers of new products.

The utility that each consumer derives from purchasing a refurbished product is given by the difference of their willingness-to-pay and the price plus the relicensing fee. Let $p_2$ and $p_r$ denote the second-period prices of new and refurbished products, respectively. The corresponding consumer utilities obtained by purchasing each type of product in the second-period are $U_2 = \theta - p_2$ for the new product and $U_r = \delta\theta - p_r - h$ for the refurbished product. From these utility functions, and letting $q_2$ and $q_r$ represent the second-period quantities of new and refurbished product respectively, the inverse demand functions are

$$p_2 = 1 - q_2 - \delta q_r$$
$$p_r = \delta(1 - q_r - q_2) - h.$$

# 4    Analysis

In this section, we present our baseline analysis of a single OEM who sells a new product in the first period and charges a relicensing fee for refurbished products that are acquired, refurbished and resold by a single entrant in the second period. Under this scenario, we find that unless the per unit refurbishing cost of the entrant is above a threshold, it is not optimal for the OEM to charge a relicensing fee that is so high that it effectively eliminates the secondary market. Recall that the timeline is as follows. In the first period, the OEM is the only firm in the market and only sells the new product. At the end of the first period, the entrant buys used products from a portion of the consumers who purchased the new product and resells them as refurbished products in the second period to lower willingness-to-pay consumers. The OEM also sells new products in the second period and faces cannibalization from the refurbished units. We solve the problem by backward induction, starting with the second period.

Let $\Pi_2$ and $\Pi_e$ denote the OEM's and the entrant's second-period profit, respectively. At this stage, the OEM decides the quantity of new products that she will sell in the market, while the entrant decides both the price $s$ that he will offer to the consumers to obtain their used products, as well as the quantity of refurbished products that he will make available in the market, denoted by $q_r$. The unit production cost is $c < 1$, and the unit refurbishing cost is $c_r < c$.

The OEM's second-period objective given the entrant's choice of $q_r$ is

$$Max_{q_2} \ \Pi_2(q_2|q_r) = (p_2 - c)q_2 + hq_r = (1 - q_2 - \delta q_r - c)\, q_2 + hq_r \quad s.t. \ q_2 \geq 0. \tag{1}$$

The first part of (1) captures the profit obtained from selling $q_2$ units of new products while the second part represents the profit from the relicensing fee ($h$), obtained from the $q_r$ customers who purchase the refurbished units from the entrant. The quantity of new products to sell is the only decision variable for the OEM in the second period as the relicensing fee is set in the first period.

The entrant's corresponding objective given the OEM's choice of $q_2$ is

$$Max_{q_r,s} \ \Pi_e(q_r, s|q_2) = (p_r - c_r)q_r - sq_u \quad s.t. \ 0 \leq q_r \leq q_u \tag{2}$$

$$\text{where } q_u = \frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma}.$$

The constraint in (2) ensures the quantity of refurbished product is no greater than the number of units collected from the consumers at a resale price of $s$, given by $q_u = \frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma}$ (see Figure 1). In practice, the amount collected falls far short of the volume of existing used products, so we do not explicitly model the constraint $q_u \leq q_1$ and limit the analysis to parameters where $q_u^* \leq q_1^*$ in equilibrium. Where appropriate, the potential effect of this constraint is discussed. The following lemmas characterize the price the entrant will pay for the used units.

**Lemma 1** *At optimality, the entrant has no incentive to collect more units than the ones he intends to sell in the market. That is, the constraint $q_r \leq q_u$ is binding and the optimal resale price offered by the entrant satisfies*

$$s^*(q_r) = \gamma(1 - \gamma)q_r + \gamma p_1. \tag{3}$$

**Proof** All proofs are provided in Appendix A. ∎

**Lemma 2** *For equilibria where both new and refurbished products co-exist in period 2, the equilibrium resale value is given by $s^*(q_1, h) = \frac{\gamma\delta c - \gamma[2\gamma(1-\gamma) + \delta(2-\delta)]q_1 - 2\gamma(h - c_r)}{2\gamma(2-\gamma) + \delta(4-\delta)}$ while the corresponding*

12

*second-period quantities are* $q_2^*(q_1, h) = \frac{\delta h - \gamma \delta q_1 - \delta(\delta - \gamma) + \delta c_r - (1-c)[\gamma(\gamma-2) - 2\delta]}{2\gamma(2-\gamma) + \delta(4-\delta)}$ *and*
$q_r^*(q_1, h) = \frac{2\gamma q_1 - 2h - 2(\gamma + c_r) + \delta(1+c)}{2\gamma(2-\gamma) + \delta(4-\delta)}.$

The second lemma reveals two interesting properties of the equilibrium resale value. First, $s^*$ decreases in the quantity of new products sold in the first period. This observation is consistent with the resale values we observe in practice; whenever a large supply of a specific used model becomes available, its resale value drops dramatically. Second, $s^*$ increases as the relicensing fee $h$ decreases: A low value of $h$ means a higher profit potential from the secondary market, thus the entrant is willing to offer a higher resale price to first-period consumers. In addition, the entrant's decision of whether to enter the market or not is directly related to the relicensing fee $h$, since the latter affects the profitability of refurbished products. Therefore, the OEM acts as a Stackelberg leader who decides between allowing the existence of a secondary market or not by her choice of $h$. To characterize the optimal OEM strategy, we need to examine the total profit across both periods. Thus, we now move to the OEM's first-period decisions.

In the first period, the OEM's decisions include the quantity of new units to sell as well as the relicensing fee to announce. More specifically, the OEM's problem is

$$Max_{q_1, h} \ \Pi(q_1, h) = \Pi_1(q_1, h) + \Pi_2^*(q_1, h) \ \ s.t. \ q_1 \geq 0, \ h \geq 0, \tag{4}$$

where $\Pi_1(q_1, h)$ denotes the profit from the sales of new products in the first period.

$$\Pi_1(q_1, h) = [p_1(q_1, h) - c] \, q_1 = [(1 - \gamma)(1 - q_1) + s^*(q_1, h) - c] \, q_1,$$

where $s^*(q_1, h)$ is characterized in Lemma 2. Although we ignore discounting in our formulation, the addition of a discount factor to the second-period profit does not fundamentally change our results, but reinforces the resale value effect, as the OEM cares more about first-period profits.

We are now ready to state our main result for this section. The following proposition states that as long as the refurbishing cost is below a threshold value, the OEM is always better off by maintaining a secondary market for her products.

**Proposition 1** *For $c_r < c(\delta - \gamma)$, A) It is not optimal for the OEM to eliminate the secondary market: $q_r^* > 0$; B) The OEM charges a positive relicensing fee: $h^*(\delta, \gamma, c, c_r) > 0$; C) For $c \geq \widetilde{c}(\delta, \gamma, c_r) \doteq \frac{\delta(8 - 3\delta + 2c_r) + 8\gamma(1-\gamma)}{\delta(8 - \delta - 2\gamma) + 8\gamma(1-\gamma)}$, the OEM does not sell any new products in the second period ($q_2^* = 0$); and for $c_r \geq c(\delta - \gamma)$, there is no market for refurbished products ($q_r^* = 0$).*

Part A in Proposition 1 may appear counter-intuitive at first glance; as the entrant becomes more competitive in relation to the OEM ($c_r$ decreases in relation to $c$), the OEM chooses not to eliminate the secondary market. In fact, this result is counter to the previous results in the remanufacturing competition paper of Ferguson and Toktay (2006) who find that as the entrant becomes more competitive ($c_r$ becomes lower) and the cannibalization threat increases, the OEM should increase her efforts to deter the secondary market. The difference in these findings is driven by the inclusion of the positive effect of a strong secondary market on the utility of the first-period customers (the resale value effect) in the current model. This new result demonstrates the importance of including the resale value effect in an OEM's secondary market strategy and warns against the common perception of many OEMs that competition from an outside firm through the secondary market is always detrimental to their profits.

The other interesting observation about part A of proposition 1 is the condition $\frac{c_r}{c} \leq (\delta - \gamma)$. Ferguson and Koenigsberg (2007) explore when an OEM should sell a depreciated used product that incurs holding cost $c_r$, but no acquisition cost, in competition with new units of the same product (no external competition). They show that offering both versions of the product is profitable when $\frac{c_r}{c} \leq \delta$. Note that both conditions compare the relative cost advantage ($\frac{c_r}{c}$) with the consumers' relative willingness-to-pay. In our model, however, this relative willingness-to-pay for the refurbished product versus the new product is discounted by the consumer's disutility from returning the product, which acts as an acquisition cost for the entrant and therefore a barrier to the profitability of the secondary market. Thus, when $\gamma = 0$ (all customers return the product for free) the OEM's decision to facilitate a secondary market is the same as if she were selling the refurbished product herself. At the other extreme, when $\gamma = \delta$ (the entrant must pay an amount equal to the consumer's willingness-to-pay for a refurbished product to acquire the old units), there is no secondary market because it is not profitable for the entrant at any value of $c_r$.

Part B states that even though the OEM allows the secondary market to exist, she does so while charging a positive relicensing fee. Thus, the common practice of IT companies charging relicensing fees has some theoretical merit. Part C states the optimal policy depends on the unit production cost. If the production cost is above a certain threshold ($c \geq \widetilde{c}(\delta, \gamma, c_r)$), the OEM is better off completely abstaining from the second period ($q_2^* = 0$), and therefore, fully promoting the secondary market and reaping the benefit of the resale value effect. The latter result has some similarities with the seminal paper of Bulow (1982) on the incentives of a durable good monopolist to invest in reducing his unit production cost. He shows that as the production cost increases, the

monopolist sells fewer new products in the second period, which in turn, raises the prices of new products sold in the first period. In contrast, for low values of the production cost ($c < \widetilde{c}(\delta, \gamma, c_r)$), the OEM is in a stronger competitive position vis-à-vis the entrant and maximizes her profits by a more aggressive competition strategy in the second period. The gain in market share outweighs the loss in first-period profits.

Lastly, the condition $c_r \geq c(\delta - \gamma)$ merits further clarification. In particular, for $c(\delta - \gamma) \leq c_r \leq \frac{1}{2}(\delta - \gamma)(1 + c)$, the OEM sets $h^* > 0$ so as to eliminate the secondary market ($q_r^* = 0$) since the high refurbishing cost prevents the entrant from offering a high enough resale price. Hence, the resale value benefit from maintaining an active secondary market does not outweigh the detrimental effect of cannibalization. For even higher values of the refurbishing cost, $c_r > \frac{1}{2}(\delta - \gamma)(1 + c)$, the secondary market is not viable: $q_r^* = 0$ even if the relicensing fee was set to zero. We now focus on the most interesting case where the refurbishing cost is low, $c_r < c(\delta - \gamma)$, and examine how the optimal relicensing fee changes with respect to the consumer and product characteristics.

**Corollary 1** *When $c_r < c(\delta - \gamma)$, the optimal relicensing fee $h^*$ increases in $\delta$ and decreases in $\gamma$. If the OEM participates in the market in the second period, $c < \widetilde{c}(\delta, \gamma, c_r)$, then $h^*$ decreases in $c$, otherwise it does not depend on $c$.*

The relationship between $\delta$ and $h^*$ is to be expected: All else being equal, when consumers value the refurbished product more, the OEM can charge a higher fee for licensing the software to use it. Similarly, the more willing they are to return their products (lower $\gamma$), the lower the procurement cost for the entrant, and therefore the OEM can charge more for the relicensing fee without affecting the positive impact of the secondary market on her profits. Interestingly, when the OEM produces new products in the second period, a higher production cost leads to lower marginal profits from selling new products and shifts the OEM's focus to exploiting the resale value effect by decreasing $h^*$. The OEM produces fewer new products since production is more expensive, but she charges a higher premium for them in the first period. In contrast, when the OEM abstains from the market in the second period, the optimal relicensing fee does not depend on $c$.

## 5  Extensions

Our key finding from our baseline model was that unless the refurbishing cost is high enough, the IT OEM is better off by maintaining an active secondary market. Through this secondary market,

consumers of new products enjoy a higher net utility, and the OEM is able to charge a higher price for her new products. This result clearly highlights the importance of an active secondary market on the OEM's profitability and questions the validity of strategies aiming to shut down the secondary market.

To further explore how specific market conditions and customer characteristics affect the OEM's optimal strategy, we relax three key assumptions of our baseline model. First, we examine the case of non-strategic customers who do not take into account the resale value when they purchase a new product. We find that when consumers are non-strategic, it is optimal for the OEM to eliminate the secondary market under a much wider range of conditions (e.g. even for zero refurbishing cost). Second, we increase the competitive intensity within the secondary market by allowing more than one entrant to collect and resell the used products. Interestingly, we find that the OEM lowers her relicensing fee to further stimulate procurement competition among the entrants. This, in turn, leads to higher resale values for the consumers and, as a result, the OEM's profits are concave increasing in the number of entrants. Thus, competition among the entrants intensifies the positive impact of the secondary market and makes the OEM more willing to support it. Finally, we introduce competition in both the primary and secondary markets by examining the case where both new and refurbished products are differentiated by quality (one OEM has a brand premium over the other OEM). By doing so, we can directly compare the optimal pricing and relicensing schemes of different OEMs as well as the corresponding equilibrium quantities for each market segment. We find that the high-end OEM always charges a significantly higher relicensing fee than the low-end OEM. Thus, a high relicensing fee need not be indicative of an attempt to shut down the secondary market, but could reflect the brand premium of the high-end OEM instead. This result may help explain the large range of relicensing fees observed in practice.

## 5.1 Non-strategic consumers

In this subsection, we examine the case where the first-period consumers act non-strategically (they do not take into account the potential resale value of the product during their first-period purchase decisions). In terms of the utility of first-period consumers, acting myopically means the derived utility is given by $U_1 = \theta - p_1$ and only consumers with $\theta \geq p_1$ will buy the product. At the end of the first period, consumers will resell their products as long as $s \geq \theta\gamma$, or, $\theta \leq \frac{s}{\gamma}$. Therefore, the total supply of used products for a given resale price $s$ is $q_{u,non-strategic} = \frac{s}{\gamma} - p_1 < \frac{s}{\gamma} - \frac{p_1 - s}{1-\gamma} = q_{u,strategic}$ (see Figure 1). The following proposition provides the condition for when the OEM will eliminate

the secondary market.

**Proposition 2** *For $c_r < c(\delta - \gamma) - \frac{1}{2}\gamma(1 - c)$, A) It is not optimal for the OEM to eliminate the secondary market: $q_r^* > 0$; B) The OEM charges a positive relicensing fee $h^*(\delta, \gamma, c, c_r) > 0$; C) For $c \geq \widetilde{c}(\delta, \gamma, c_r) \doteq \frac{\delta(8 - 3\delta + 2c_r - \gamma) + \gamma(8 - \gamma)}{\delta(8 - \delta - \gamma) + \gamma(8 - \gamma)}$, the OEM does not sell any new products in the second period ($q_2^* = 0$), and for $c_r \geq c(\delta - \gamma) - \frac{1}{2}\gamma(1 - c)$, there is no market for refurbished products ($q_r^* = 0$).*

Note that the refurbishing cost threshold below which an active secondary market exists is lower than the corresponding threshold of Proposition 1. With non-strategic customers, the benefit from supporting a secondary market is significantly weakened and shutting down the secondary market becomes an optimal strategy under a much wider range of conditions. According to Proposition 2, the OEM will eliminate the secondary market even for zero refurbishing cost ($c_r = 0$); as long as the production of new products is relatively inexpensive ($c < \frac{\gamma}{2\delta - \gamma}$). In addition, the production cost threshold $\widetilde{c}(\delta, \gamma, c_r)$ above which the OEM abstains from the market in the second period is higher than the corresponding threshold of Proposition 1. That is, a non-strategic customer base does not allow the OEM to exploit the resale value effect and forces her to make her profits by producing new products in the second period, despite the relatively high production cost. Moreover, it can easily be shown that although $h_{non-strategic}^* > h_{strategic}^*$, the OEM's profit is lower since she can no longer charge a premium to the first-period consumers. Thus, the OEM is worse off under non-strategic customers.

The above analysis demonstrates that a forward-looking consumer base can influence the OEM's secondary market strategy. The common perception in the IT industry is that historically, customers of IT products did not take into account the future resale value in their initial purchases. This could explain why some IT OEMs have historically deployed policies to deter the secondary market for their products. As mentioned in the introduction however, there are indications that customers of IT equipment are becoming increasingly concerned about resale values during their initial purchase decisions. Our results suggest that this is not necessarily a bad trend for the OEM but her secondary market strategies need to evolve with the market.

## 5.2 Competition in the secondary market ($N$ entrants)

The significant profit opportunities to be made in the secondary market has given rise to a number of firms founded with the sole purpose of buying and refurbishing used IT equipment (CBRonline.com 2005). To deepen our understanding of the optimal OEM strategy, we next study the

impact of competition within the secondary market. In this setting, while the OEM maintains her monopolistic position in the market for new products, $N$ independent entrants compete on both acquiring the used units from the customers as well as refurbishing and reselling them. The critical question is: How are the OEM's profit and relicensing fee affected by an increasing number of entrants?

To answer this question, we assume $N$ symmetric entrants who are in Cournot competition with each other (similar to Debo et al. 2005), and at the same time face competition from new products under the vertical differentiation model outlined in the previous section. Thus, if we let $q_r^i$ denote the quantity of refurbished products offered by entrant $i$, the total quantity of refurbished products in the second period will be $\sum_{i=1}^{N} q_r^i$. Paralleling our analysis in Section 4, the inverse demand functions are given by

$$p_2 = 1 - q_2 - \delta \sum_{i=1}^{N} q_r^i \quad \text{and} \quad p_r = \delta - q_2 \delta \ - h - \delta \sum_{i=1}^{N} q_r^i \ ,$$

and the OEM's second-period objective is

$$Max_{q_2} \ \Pi_2 = \left( 1 - q_2 - \delta \sum_{i=1}^{N} q_r^i - c \right) q_2 + h \sum_{i=1}^{N} q_r^i \quad s.t. \quad q_2 \geq 0 \ ,$$

while the i$^{th}$ entrant's objective is

$$Max_{q_r^i} \ \Pi_e^i = (p_r - s - c_r) q_r^i$$
$$s.t. \quad \sum_{i=1}^{N} q_r^i \leq \frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma}, \quad \text{and} \quad q_r^i \geq 0.$$

As in the baseline case, the OEM's optimization problem is

$$Max_{q_1, h} \ \Pi(q_1, h) = \Pi_1(q_1, h) + \Pi_2^*(q_1, h) \quad s.t. \ q_1 \geq 0, \ h \geq 0,$$

where $\Pi_1(q_1, h) = [p_1(q_1, h) - c] \, q_1$ denotes the profit from the sales of new products in the first period. Because the entrants are symmetric, at equilibrium we have $Q_r^* \doteq \sum_{i=1}^{N} q_r^{i\,*} = N q_r^{1\,*}$. As in the single entrant case, the equilibrium resale price $s^*$ satisfies $N q_r^{1\,*} = \frac{s^*}{\gamma} - \frac{p_1 - s^*}{1 - \gamma}$. Solving the problem in a similar way as the baseline case (presented in Appendix A), we obtain the following result:

**Proposition 3** *If $c_r < c(\delta - \gamma)$, A) The OEM allows for an active secondary market ($q_r^{i\,*} > 0$), B) The OEM charges a positive relicensing fee $h^* > 0$. The OEM's total profit ($\Pi_{OEM}^*$) and the total quantity of refurbished products ($Q_r^*$) are concave increasing in $N$; C) For $c < \widetilde{c}(\delta, \gamma, c_r, N) \doteq \frac{[\delta(4-3\delta+2c_r)+8\gamma(1-\gamma)]N+4\delta}{[-\delta(\delta-4+2\gamma)-8\gamma(\gamma-1)]N-4\delta}$, both new and refurbished products compete in the second period, and the optimal relicensing fee is convex decreasing in $N$; D) For $c \geq \widetilde{c}(\delta, \gamma, c_r, N)$, the OEM does not sell any new products in the second period ($q_2^* = 0$), and for $c_r \geq c(\delta - \gamma)$, there is no market for refurbished products ($q_r^{i\,*} = 0 \ \forall \ i$).*

Proposition 3 reveals some interesting insights about the internal competition among the entrants and how this competition affects the consumer purchasing behavior along with the OEM's profit. First, note that Proposition 3 essentially has the same structure as Proposition 1. Moreover, similar to Debo et al. (2005), the necessary condition for the OEM to allow an active secondary market does not depend on the number of entrants. One may expect that as the number of entrants increases, the OEM employs a more aggressive strategy vis-à-vis the secondary market and her profit decreases. Interestingly, however, we show the OEM's relicensing fee is decreasing and her profit is concave increasing in the number of entrants. Consistent with standard economic theory, as the number of entrants increases, internal competition drives the prices of the refurbished units down and the secondary market attracts more consumers (the overall quantity of refurbished products increases). This leads to higher cannibalization of new units in the second period, but also to a higher resale value. In fact, adding an additional entrant increases the marginal impact of the relicensing fee on the resale value more than it increases the detrimental cannibalization effect. As a result, the OEM charges a lower relicensing fee, providing greater support to the secondary market. This result differs from Debo et al. (2005) who find that an increase in the competitive intensity of the secondary market reduces both the OEM's incentive to invest in remanufacturability and her profit. This difference can be explained through the strategic as well as the economic role of the relicensing fee: The OEM not only has a more powerful mechanism of controlling the demand for refurbished products, she also derives revenues from the relicensed equipment. Finally, the production cost threshold above which the OEM does not sell new units in the second period is decreasing in the number of entrants: A more competitive secondary market strengthens the impact of the resale value effect on the OEM's profit, causing her to produce fewer units in the second period but charging a higher price for the new units in the first period.

## 5.3 Competition in both the primary and secondary markets with quality differentiation

Thus far, we have assumed a monopolist setting in the primary market with the competition being restricted to the secondary market. We now relax the monopolistic primary market assumption and develop a differentiated duopoly model where consumers place a higher value on firm A's product than on firm B's product. This assumption allows us to address two critical questions: What are the pricing and relicensing strategies of each OEM and how do they differ? What is the impact of the brand differential on those strategies?

We capture the difference in the perceived utility between firms as follows: A consumer who derives utility $\theta$ from a new product by firm A derives utility $(1 - \alpha)\theta$ from a new product by firm B. Without loss of generality, we assume that $\alpha > 0$ so that firm B represents the low-end firm. The relative difference in customers' valuations, $\alpha$, is called the brand differential or the brand premium of the high-end OEM. We also assume an equal rate of perceived utility depreciation for both firms. That is, a consumer derives utility $\delta\theta$ from firm A's refurbished product, while he derives utility $(1 - \alpha)\delta\theta$ from firm B's. This assumption allows us to maintain the same relative brand differential between OEMs on the secondary market. We assume that $\delta < (1 - \alpha)$ so that a given consumer values the low-end firm's new product strictly more than the high-end firm's refurbished product. This is a reasonable assumption based on observations of the current state of the IT industry and eliminates the trivial case where one firm dominates both the primary and secondary markets. In addition, we normalize the cost of refurbishing to zero for both products. This rules out refurbishing cost disparity from explaining the differences in the OEMs' strategies and corresponds to the more interesting cases in Propositions 1 and 3 where the existence of a secondary market is beneficial for the OEM. Finally, we assume a perfectly competitive secondary markets for each type of refurbished product. This implies that for any given used product purchase prices, $s^A$ and $s^B$, $p_{2,r}^A = s^A$ and $p_{2,r}^B = s^B$. While we do this for tractability, Propositions 1 and 3 suggest the structure of the optimal policy is essentially the same for any level of competitive intensity on the secondary market.

Similar to our baseline model, we solve the problem by backward induction, starting with the second period (Appendix B). Unlike our previous analysis, however, deriving the Nash equilibrium $(q_{1A}^*, h_A^*, q_{1B}^*, h_B^*)$ for any arbitrary set of parameters is much more complex for two reasons. First, the profit expressions are long and do not allow easy algebraic handling. Second, and more

importantly, the resulting game of two OEMs with two-dimensional action spaces contains joint constraints. In general, even for relatively simple settings (unidimensional action space), constrained games in which the constraints for each player, as well as his payoff function, depends on the strategy of the other player, are difficult to solve analytically (Rosen 1965) and have received limited attention in the literature. Rather, our approach is to solve the unconstrained game and subsequently identify the range of parameter values in which the results are meaningful (e.g. Desai 2001). Therefore, hereafter, we focus on those parameter values for which all non-negativity constraints are satisfied, namely, all market segments have positive quantities in equilibrium. For those parameters, we conduct an extensive numerical investigation and explore how the optimal OEM strategies (relicensing fee and quantity decisions) change as a function of the brand differential. In the numerical study, we calculate the equilibrium quantity and relicensing fee decisions for every combination of the parameter values $\delta \in [0.3, 0.8]$, $\gamma \in [0.01, 0.15]$, and $c \in [0.01, 0.5]$ (discretized in increments of 0.1, 0.03, and 0.05, respectively). We find that as long as all non-negativity constraints are satisfied, the insights remain the same across all the parameter combinations. These insights are described in Observations 1-3 below. Figure 2 provides an illustrative example while Table 1 summarizes the impact of $\delta$, $\gamma$, $c$ on the equilibrium decisions.
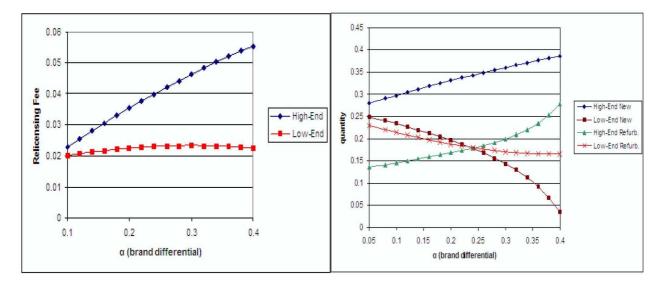


Figure 2: Relicensing Fees (left) and Equilibrium Quantities in Second Period (right) as a function of $\alpha$ for $\delta$=0.5, $\gamma$=0.05, and c=0.15,

| | $h_A^*$ | $h_B^*$ | $q_{2A}^*$ | $q_{2B}^*$ | $q_{UA}^*$ | $q_{UB}^*$ |
|---|---|---|---|---|---|---|
| $\delta \nearrow$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ |
| $\gamma \nearrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ | $\searrow$ | $\searrow$ |
| $c \nearrow$ | $\searrow$ | $\searrow$ | $\searrow$ | $\searrow$ | $\nearrow$ | $\nearrow$ |

Table 1: Comparative Statics when all Market Segments Exist.

**Observation 1:** *The high-end OEM always charges a higher relicensing fee than the low-end OEM and the difference between the relicensing fees can be large.*

This is because the high-end OEM's relative brand differential exists in the secondary market as well, which she capitalizes on by charging a higher relicensing fee. Note that despite the higher relicensing fee $h_A^*$, the high-end OEM maintains an active secondary market. Thus, a high relicensing fee need not be indicative of an attempt to shut down the secondary market, but rather reflect the brand premium a particular OEM commands. As reported in Table 1, and consistent with Corollary 1, our comparative statics analysis suggests that for a fixed brand differential between the two OEMs, both relicensing fees increase in $\delta$, and decrease in $\gamma$ and $c$.

**Observation 2:** *The high-end OEM's relicensing fee increases in the brand differential ($\alpha$). For the low-end OEM, there is a non-monotonic relationship between the relicensing fee and the brand differential: $h_B^*$ first increases and then decreases in $\alpha$.*

To understand this relationship, we must look at how a marginal change in the brand differential affects the equilibrium decisions of each OEM. A marginal increase in $\alpha$ increases both the primary and the secondary markets for the high-end OEM's products. An increase in the brand differential $\alpha$ is translated to a higher relicensing fee at any $\alpha$ value since consumers have higher willingness-to-pay for her refurbished products. In contrast, the low-end OEM increases $h_B^*$ only at low values of $\alpha$. For low values of $\alpha$, the low-end OEM has a considerable presence in both the primary and secondary markets. An increase in the brand differential hurts both the primary and secondary markets, the former to a larger extent. The low-end OEM attempts to maintain his primary market presence by increasing his relicensing fee and limiting the cannibalization effect. On the contrary, for high values of $\alpha$, where the high-end OEM dominates, the low-end OEM's primary market has significantly shrunk, and the impact of a marginal increase in $\alpha$ on cannibalization is much less significant. As a result, we observe a decrease in the relicensing fee as an attempt to strengthen the resale value effect.

The effect of $\delta$, $\gamma$ and $c$ on the equilibrium quantities can be observed in Table 1. A higher

$\delta$ makes the secondary market more profitable, so the secondary market grows at the expense of the primary. A higher $\gamma$ makes the secondary market less profitable, so the opposite effect is seen. Finally, a higher $c$ lowers the profitability of new products, so the primary market shrinks and the secondary market grows.

**Observation 3:** *There is a threshold value for the brand differential $\alpha$ below which the low-end OEM's product makes up a larger share of the secondary market. This threshold increases as $\delta$ decreases, $\gamma$ increases, or $c$ decreases.*

Observation 3 suggests that although a positive brand differential always translates to a larger market share in the primary market (under symmetric production costs), the same is not true for the corresponding secondary markets. This result could explain the strategy of some high-end OEMs who choose not to have large secondary markets for their refurbished products despite the brand premium they command. Note also that a lower $c$ makes the primary market more profitable, while a lower $\delta$ or a higher $\gamma$ reduces the margins of the secondary market. Thus, the above conditions make the primary market more attractive to the high-end OEM, who has a leadership advantage, leaving the low-end OEM to focus on the secondary market (via relicensing fees).

In our analysis, we assume an equal unit production cost for both the high-end and low-end OEM; thus the differentiation is along the brand differential dimension. This is a reasonable assumption for many IT products since they can be characterized as development-intensive-products, i.e. products whose fixed costs of development far outweigh the unit variable costs (Krishnan and Zhu 2006). Because our focus is on a firm's decisions for a given product line, we do not consider these initial fixed costs. If the assumption of equal production costs is relaxed and the high-end OEM has a higher production cost, we expect her to decrease her relicensing fee to increase the resale value of her primary product.

# 6    Conclusions

Secondary markets in the IT industry have grown steadily, forcing OEMs to form strategies to respond to them. For products such as servers and storage devices, OEMs have a powerful mechanism at their disposal: instituting a software relicensing fee charged to secondary users. A high relicensing fee can virtually shut down the secondary market, while a low relicensing fee can allow it to thrive. The optimal strategy is not obvious: An active secondary market has an indirect positive benefit for the OEM by increasing the product's resale value, which in turn, increases the price

that can be charged for the new product (resale value effect). At the same time, it has a direct detrimental effect as the refurbished product competes with the OEM's new product (cannibalization effect). In practice, comparable OEMs have surprisingly different relicensing fee strategies. The existing literature on secondary markets does not provide guidance concerning this widespread mechanism. Our paper fills this gap by contributing to the theory of secondary markets and by providing managerial guidelines on the use of relicensing fees.

Our research makes several theoretical contributions. First, we extend the literature on the economics of secondary markets by studying the relicensing fee mechanism. By treating the relicensing fee as a continuous decision variable, we avoid restricting the OEM to either fully supporting or completely shutting down the secondary market as is the case with other forms of secondary market intervention such as leasing. Second, we model the incentives of independent entrants to purchase, refurbish and sell the used equipment contingent on product characteristics and consumer behavior. A novel feature is the inclusion of the consumers' propensity to sell a used product such that the supply of used products is determined endogenously. Third, we capture the equilibrium secondary market strategies of competing OEMs and compare how they evolve as the brand differential between them increases. To our knowledge, our model is the first to study differentiated new and refurbished products competing in both the primary and secondary markets. In addition, we complement the rapidly growing literature on remanufacturing by linking consumers' willingness to pay for a new product to the potential resale value of the product at the end of use. By doing so, we show that a market for refurbished products can benefit the OEM even if it is operated by independent entrants.

Our results help IT OEMs think more comprehensively about their relicensing fee strategies, along the dimensions of consumers' awareness, refurbishing cost, production cost, attractiveness of refurbished products, consumers' inertia in returning products, secondary market competitive intensity and brand differential. We find that consumers' awareness of the resale value of the product is a key determinant of the OEM strategy. An OEM operating in a market with strategic customers has an incentive to support the existence of a secondary market since significant profits can be obtained through the resale value effect and the revenues from relicensing. If, however, customers act myopically, the OEM's support of the secondary market should decrease since the resale value effect may no longer outweigh the threat of cannibalization. This suggests that OEMs should be active in promoting their products' resale values, an approach adopted for example by IBM (Johnson 2006).

A second critical factor in the OEM's decision is the refurbishing cost. Interestingly, a low refurbishing cost should make an OEM more willing to support her secondary market, even though this means the entrant is more competitive. This is because the OEM can then exploit the high margin of the secondary market through the resale value effect and the relicensing revenues. This is especially important for an OEM with high production costs: The right combination of price and relicensing fee allows the OEM to mitigate the low margins of new products by producing fewer units but charging a price premium for them due to the resale value effect. Our experience is that OEMs are very concerned with cannibalization and tend to overlook the resale value effect. It is precisely in cases where cannibalization is a strong threat that the OEMs should embrace the secondary market to benefit from the strong resale value effect and relicensing fee revenues.

These results highlight the strategic and economic value of an active secondary market, but how should an OEM respond to the entry of more refurbishers? Despite the increased size of the secondary market and competitive intensity, we show the OEM can still use the secondary market to her advantage and further increase her profits by lowering the relicensing fee and strengthening the resale value effect.

In practice, IT OEMs often operate in a competitive primary market. Our differentiated duopoly model offers insights for the strategies of low-end and high-end OEMs. As we would expect, the high-end OEM always charges a higher relicensing fee since the brand differential is maintained in the secondary markets. In fact, the high-end OEM should monotonically increase her relicensing fee as her brand differential is strengthened. One might expect the opposite effect for the low-end OEM - that his relicensing fee should decrease as the brand differential increases. We find that due to the interplay of the resale value and cannibalization effects, there is a non-monotonic relationship between the relicensing fee of the low-end OEM and the brand differential. When the brand differential is low, the optimal response of the low-end OEM to an increase in the brand differential is to increase his relicensing fee and further decrease the demand for his refurbished products. By doing so, he attempts to mitigate the losses in the primary market at the expense of increasing his losses in the secondary market.

To conclude, our paper highlights the strategic importance of supporting an active secondary market under a wide range of circumstances, particularly in the presence of strategic consumers and low refurbishing costs. These conditions are valid in the IT industry today: There exist a large number of industry analyst firms who specialize in forecasting the resale value of IT equipment and who offer comprehensive cost/benefit analyses over the life cycle of the IT equipment. The

modularity of IT solutions makes refurbishment a cost-effective proposition for many products. Thus, charging very high relicensing fees with the purpose of shutting down the secondary market, a strategy attributed to some IT OEMs, appears to be myopic and suboptimal in the presence of strategic consumers. At the same time, we demonstrate that charging higher relicensing fees than its competitors need not mean a firm is doing so with the sole purpose of eliminating the secondary market, but rather that it is capitalizing on its brand premium.

Although we believe our model to be representative of the key trade-offs in IT secondary markets, we recognize that relicensing fee strategies may be moderated by factors beyond the scope of the model, such as OEMs deploying a service versus product oriented strategy, or targeting specific niche markets. We assume that the secondary market is exploited only by independent entrants while the OEM only focuses on new product sales. In practice, however, some OEMs have developed a strong infrastructure to collect, refurbish, and resell IT equipment. Allowing the OEM to enter the secondary market would strengthen the case for supporting the secondary market. Finally, we assumed there is no technological improvement in the product between the two time periods. In the reality of the IT industry, we often observe significant improvements in quality (product innovation) or reductions in the production cost (process innovation) between successive generations of a product. Future research should aim to understand the interactions of balancing new product introductions and support for the secondary market.

# References

Anderson, S. and V. Ginsburgh. 1994. Price Discrimination via Second-Hand Markets. *European Economic Review.* 38, 23-44.

Atasu A., M. Sarvary, and L. N. Van Wassenhove. 2007. Remanufacturing as a Marketing Strategy. INSEAD Working Paper. 2007/58/TOM/MKT.

Benjamin, D. and R. Kormendi. 1974. The Interrelationship Between the Markets for New and Used Durable Goods. *Journal of Law and Economics.* 17, 381-401.

Berinato, S. 2002. Good Stuff Cheap. *CIO Magazine.* Oct. 15. Retrieved June 14, 2007, http://www.cio.com/archive/101502/cheap.html

Bulow, J. 1982. Durable Goods Monopolists. *Journal of Political Economy.* 90, 314-332.

CBRonline.com. 2005. Big Players Emerge in Fragmented Brokerage Market. Retrieved June 14, 2007, http://www.cbronline.com/news_archives.asp?show= 2005-09.

Cisco.com. 2007. Software Transfer and Licensing. Retrieved October 4, 2007, http://www.cisco.com/warp/public/csc/refurb_equipment/swlicense.html.

Debo, L.G., L.B. Toktay, and L. N. Van Wassenhove. 2005. Market Segmentation and Production Technology Selection for Remanufacturable Products. *Management Science.* 51(8), 1193-1205.

Desai, P. 2001. Quality Segmentation in Spatial Markets: When Does Cannibalization Affect Product Line Design? *Marketing Science.* 20(3), 265-283.

Desai, P and D. Purohit. 1998. Leasing and Selling: Optimal Marketing Strategies for a Durable Goods Firm. *Management Science.* 44:11(Part 2), 19-34.

Desai, P., O. Koenigsberg, and D. Purohit. 2004. Strategic Decentralization and Channel Coordination. *Quantitative Marketing and Economics.* 2(1), 5-22.

Desai, P., O. Koenigsberg, and D. Purohit. 2007. A Research Note on the Role of Production Lead Time and Demand Uncertainty in Marketing Durable Goods. *Management Science.* 53(1), 150-158.

Ferguson, M. and O. Koenigsberg. 2007. Production and Pricing Decisions: How Should a Firm Manage a Product With Deteriorating Quality, *Production and Operations Management.* 16(3), 306-321.

Ferguson, M. and L.B. Toktay. 2006. The Effect of Competition on Recovery Strategies. *Production and Operations Management.* 15(3), 351-368.

Ferrer, G. and J. Swaminathan. 2006. Managing New and Remanufactured Products. *Management Science.* 52(1), 15-26.

Galbreth, M.R., and J.D. Blackburn. 2006. Optimal Acquisition and Sorting Policies for Remanufacturing. *Production and Operations Management.* 15(3), 384-392.

Gaskins, D. W. 1974. Alcoa Revisited: The Welfare Implications of a Second Hand Market. *Journal of Economic Theory.* 7, 254-71.

Gerstner, E. and J. D. Hess. 1991. A Theory of Channel Price Promotions. *The American Economic Review* 81, 872-886.

Gerstner, E. and J. D. Hess. 1995. Pull Promotions and Channel Coordination. *Marketing Science.* 14, 43-60.

Guide, Jr., V.D.R. 2000. Production Planning and Control for Remanufacturing: Industry Practice and Research Needs. *Journal of Operations Management.* 18(4), 467 -483.

Guide Jr., V.D.R. and L.N. Van Wassenhove. 2001. Managing Product Returns for Remanufacturing. *Production and Operations Management.* 10(2), 142-155.

Guide, Jr. V.D.R. and K. Li. 2007. Market Cannibalization of New Product Sales by Remanufactured Products. Working Paper, Penn State University.

Hauser, W. and R. T. Lund. 2003. Remanufacturing: An American Resource. Presentation available at http://www.bu.edu/reman/RemanSlides.pdf, Boston University, Boston, MA.

Hendel, I. and A. Lizzeri. 1999. Interfering with Secondary Markets. *Rand Journal of Economics.* 30, 1-21.

Jin, Y., A. Muriel, and Y. Lu. 2007. On the Profitability of Remanufactured Products. Working Paper, Department of Mechanical and Industrial Engineering, University of Massachusetts, Amherst.

Johnson, S. 2006. Director of Global Assets Recovery Systems, IBM. Personal Interview.

Kandra A. 2002. Refurbished PCs: Sweet Deals of Lemons? PC World Magazine.

Krishnan, V. and W. Zhu. 2006. Designing a Family of Development-Intensive Products. *Management Science*. 52(6), 813-825.

Levinthal, D. and D. Purohit. 1989. Durable Goods and Product Obsolescence. *Marketing Science* 8(1), 35-56.

Liebowitz, S. 1982. Durability, Market Structure, and New-Used Goods Models. *American Economic Review*. 72, 816-824.

Lund, R. T., F.D. Skeels. 1983. Start-up Guidelines for the Independent Remanufacturer. Center for Policy Alternatives, Massachusetts Institute of Technology.

Majumder, P. and H. Groenevelt. 2001. Competition in Remanufacturing. *Production and Operations Management*. 10(2), 125-141.

Marion, J. 2004. Sun Under Fire - for Fixing Solaris OS Costs to Reduce Competition in Used Sun Market. Association of Service and Computer Dealers International (ASCDI), June 8, Retrieved October 07, 2007, http://www.sparcproductdirectory.com/view56.html.

Miller, H.L. 1974. On Killing the Market for Used Textbooks and the Relationship Between Markets for New and Secondhand Goods. *Journal of Political Economy*. 82, 612-619.

Mussa, M. and S. Rosen. 1978. Monopoly and Product Quality. *Journal of Economic Theory*. 18, 301-317.

Nair, H. 2004. Intertemporal Price Discrimination with Forward-looking Consumers: Application to the US market for Console Video-games. Working Paper, Stanford Graduate School of Business.

Plambeck, E.L. and Q. Wang. 2007. Effects of E-Waste Regulation on New Product Introduction. Working Paper, Stanford Graduate School of Business.

Ray, S., T. Boyacı, and N. Aras. 2005. Optimal Prices and Trade-in Rebates for Durable, Remanufacturable Products. *Manufacturing & Service Operations Management*. 7(3), 208-228

Rosen, J.B. 1965. Existence and Uniqueness of Equilibrium Points for Concave N-person Games. *Econometrica*. 33(3), 520-534.

Rust, J. 1986. When is it Optimal to Kill off the Market for Used Durable Goods? *Econometrica*. 54, 65-86.

Song, S.C. and P.K. Chintagunta. 2003. A Micromodel of New Product Adoption with Heterogeneous and Forward-looking Consumers: Application to the digital camera category. *Quantitative Marketing and Economics*. 1, 371-407.

Sun.com. 2007. Sun Fire 15K Server Performance. Retrieved October 4, 2007, http://www.sun.com/servers/highend/sunfire15k/performance.html

Thierry, M, M. Salomon, J. Van Nunen, L.N. Van Wassenhove. Strategic Issues in Product Recovery Management. *California Management Review*. 37(2), 114-135.

Vorasayan, J. and S.M. Ryan. 2006. Optimal Price and Quantity of Refurbished Products. *Production and Operations Management*. 15(3), 369-383.

Waldman, M. 1996. Durable Goods Pricing When Quality Matters. *Journal of Business*. 69, 489-510.

Waldman, M. 1997. Eliminating the Market for Secondhand Goods: An Alternative Explanation for Leasing, *Journal of Law and Economics*. 40, 61-92.

Waldman, M. 2003. Durable Goods Theory for Real World Markets. *Journal of Economic Perspectives*. 17(1), 131-154

# Appendices

# Appendix A: Proofs

**Proof of Lemma 1.** The entrant's optimization problem given the OEM's choice of $q_2$ is

$$Max_{q_r,s}\Pi_e(q_r, s|q_2) = [\delta(1 - q_r - q_2) - h - c_r] q_r - s(\frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma})$$

$$\text{s.t.} \quad 0 \leq q_r \leq \frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma}. \tag{1}$$

The Lagrangian for the entrant's problem is $L(q_r, s, \lambda_1, \lambda_2) = [\delta(1 - q_r - q_2) - h - c_r] q_r - s(\frac{s}{\gamma} - \frac{p_1-s}{1-\gamma}) + \lambda_1(\frac{s}{\gamma} - \frac{p_1-s}{1-\gamma} - q_r) + \mu_1 q_r$.

The Kuhn-Tucker conditions for optimality are: $\frac{\partial L}{\partial q_r} = 0, \frac{\partial L}{\partial s} = 0, \lambda_1(\frac{s}{\gamma} - \frac{p_1-s}{1-\gamma} - q_r) = 0$ and $\mu_1 q_r = 0$, with $0 \leq q_r \leq \frac{s}{\gamma} - \frac{p_1-s}{1-\gamma}, \lambda_1 \geq 0, \mu_1 \geq 0$.

Assume $\frac{s}{\gamma} - \frac{p_1-s}{1-\gamma} - q_r > 0$. Then, at optimality, $\lambda_1 = 0$. Solving $\frac{\partial L}{\partial s} = 0$, we get $s^* = \frac{\gamma p_1}{2}$, which gives $\frac{s^*}{\gamma} - \frac{p_1-s^*}{1-\gamma} = -\frac{p_1}{2(1-\gamma)} < 0$, which violates the original condition $\frac{s}{\gamma} - \frac{p_1-s}{1-\gamma} - q_r > 0$.

Since this case cannot meet the KT conditions, we hereafter assume that the right constraint in (1) is binding. Intuitively, the entrant would not be willing to acquire more used units than the quantity she would sell in the secondary market. Rewriting this equality, we obtain $s^*(q_r) = \gamma(1 - \gamma)q_r + \gamma p_1$, where we suppress dependence on $p_1$ determined in period 1.

**Proof of Lemma 2.** Based on Lemma 1 we can reduce the entrant's problem to a single decision variable optimization problem in $q_r$:

$$Max_{q_r}\Pi_e = [p_r - s^*(q_r) - c_r] q_r = [p_r - \gamma(1 - \gamma)q_r - \gamma p_1 - c_r] q_r \quad s.t. \ q_r \geq 0. \tag{2}$$

We also know the profit function of the OEM

$$Max_{q_2}\Pi_2(q_2|q_r) = (p_2 - c)q_2 + hq_r = (1 - q_2 - \delta q_r - c)q_2 + hq_r \quad s.t.\ q_2 \geq 0. \tag{3}$$

Here, $\Pi_e$ and $\Pi_2$ are concave in $q_r$ and $q_2$, respectively. By solving the first-order conditions simultaneously, we can obtain the following Nash equilibrium:

$$q_2^*(p_1, h) = \frac{2(\delta + \gamma - \gamma^2)(1 - c) - \delta^2 + \delta h + \delta\gamma p_1 + \delta c_r}{4\gamma(1 - \gamma) + \delta(4 - \delta)} \tag{4}$$

$$q_r^*(p_1, h) = \frac{\delta(1 + c) - 2c_r - 2h - 2\gamma p_1}{4\gamma(1 - \gamma) + \delta(4 - \delta)}. \tag{5}$$

Substituting $q_r^*$ from (5) into the expression derived in Lemma 1 gives

$$s^*(p_1, h) = \frac{\gamma[(2\gamma(1 - \gamma) + \delta(4 - \delta)]p_1 - 2\gamma(1 - \gamma)(h - c_r) + \gamma\delta(1 - \gamma)c}{4\gamma(1 - \gamma) + \delta(4 - \delta)}. \tag{6}$$

Recall that the quantity of new units sold in the first period by the OEM can be expressed as

$$q_1 = 1 - \frac{p_1 - s}{1 - \gamma}, \text{ or, } p_1 = (1 - \gamma)(1 - q_1) + s. \tag{7}$$

Substituting $p_1$ from (7) into (6), we obtain the equilibrium price $s^*$ that the entrant pays the first-period consumers to collect used products as a function of $q_1$ and $h$:

$$s^*(q_1, h) = \frac{\gamma\delta c - \gamma[2\gamma(1 - \gamma) + \delta(2 - \delta)]q_1 - 2\gamma(h - c_r)}{2\gamma(2 - \gamma) + \delta(4 - \delta)}. \tag{8}$$

Moreover, from (6) and (7) we can rewrite (4) and (5) in terms of $q_1$ and $h$:

$$q_2^*(q_1, h) = \frac{\delta h - \gamma\delta q_1 - \delta(\delta - \gamma) + \delta c_r - (1 - c)[\gamma(\gamma - 2) - 2\delta]}{2\gamma(2 - \gamma) + \delta(4 - \delta)} \tag{9}$$

$$q_r^*(q_1, h) = \frac{2\gamma q_1 - 2h - 2(\gamma + c_r) + \delta(1 + c)}{2\gamma(2 - \gamma) + \delta(4 - \delta)}. \tag{10}$$

This Nash equilibrium is valid as long as the right-hand sides of (9) and (10) are non-negative, respectively, which can be written as $h - \gamma q_1 \geq A$ and $h - \gamma q_1 \leq B$, where $A \doteq (\delta - \gamma) - c_r + \frac{(1-c)\gamma(\gamma-2)}{\delta} - 2(1 - c)$ and $B \doteq -(\gamma + c_r) + \frac{1}{2}\delta(1 + c)$.

**Proof of Proposition 1.** In period 1, the OEM chooses $q_1 \geq 0$ and $h \geq 0$ so as to maximize the sum of first- and second-period profits. The OEM's second-period profit can be obtained using

(9 - 10) as long as $q_1$ and $h$ satisfy $h - \gamma q_1 \geq A$ and $h - \gamma q_1 \leq B$. For completeness, we need to characterize the OEM's second-period profit outside this range, or argue that the optimal solution will satisfy the two conditions. For a given $q_1$, it is in fact sufficient to restrict the domain of $h$ to values yielding a non-negative quantity in (10), $h - \gamma q_1 \leq B$, since once the secondary market has been eliminated, increasing $h$ does not improve the OEM's profits. The same need not be true however for (9); even when the OEM abstains from the primary market in the second period, he can improve his profits by decreasing $h$ and increasing first-period resale value, and we cannot use the expressions in Lemma 2 to calculate second-period profits in this range ($h - \gamma q_1 < A$). We proceed by enforcing $h - \gamma q_1 \leq B$, but determining the optimal OEM strategy for those values of $q_1$ and $h$ yielding $h - \gamma q_1 \geq A$ (Case A) and $h - \gamma q_1 \leq A$ (Case B), separately, and then combining the results.

**Case A** ($h - \gamma q_1 \geq A$). The OEM's optimization problem is

$$Max_{q_1, h} \; \Pi(q_1, h) = \Pi_1(q_1, h) + \Pi_2^*(q_1, h)$$

$$\text{s.t.} \quad A \leq h - \gamma q_1 \leq B$$

$$q_1 \geq 0, \quad h \geq 0,$$

where $\Pi_1(q_1, h) = (p_1(q_1, h) - c)q_1$ denotes the profit from the sales of new products in the first period and $\Pi_2^*(q_1, h)$ is calculated using (9) and (10).

The determinant of the Hessian of the objective function $\Pi(q_1, h)$ is $\frac{4(8\delta - \delta^2 + 8\gamma(1-\gamma))}{[2\gamma(\gamma-2) + \delta(\delta-4)]^2} > 0$ with $\frac{\partial^2 \Pi(q_1, h)}{\partial q_1^2} < 0$. Thus, the Hessian is negative definite and the profit function is concave in $(q_1, h)$.

Define the Lagrangean $L(q_1, h, \lambda_1, \lambda_2) = \Pi(q_1, h) + \lambda_1(h - \gamma q_1 - A) + \lambda_2(B - h + \gamma q_1) + \mu_1 h$. The Kuhn-Tucker conditions for optimality are:

$$\frac{\partial L}{\partial q_1} = 0 \tag{11}$$

$$\frac{\partial L}{\partial h} = 0 \tag{12}$$

$$\lambda_1(h - \gamma q_1 - A) = 0 \tag{13}$$

$$\lambda_2(B - h + \gamma q_1) = 0 \tag{14}$$

$$\mu_1 h = 0 \tag{15}$$

and $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\mu_1 \geq 0$. The constraint $q_1 \geq 0$ will be checked separately. Note that $\lambda_1 \lambda_2 = 0$, since otherwise both constaints (13) and (14) would be binding, which is not possible.

**Case A.I** : $\lambda_1 = 0$, $\lambda_2 \neq 0$, $\mu_1 = 0$.

$\lambda_2 \neq 0$ implies $B - h^* + \gamma q_1^* = 0$. Solving the KT conditions, we obtain $h^* = \frac{1}{2}(\delta - \gamma)(1+c) - c_r$, $q_1^* = \frac{1}{2}(1-c) > 0$, and $\lambda_2 = 2\frac{c(\delta-\gamma)-c_r}{2\gamma(\gamma-2)+\delta(\delta-4)}$ with corresponding profit $\frac{(1-c)^2}{2}$. Case I is valid for $\lambda_2 > 0$ and $h^* \geq 0$, or, $c(\delta - \gamma) < c_r \leq \frac{1}{2}(\delta - \gamma)(1 + c)$ and represents the case of having no refurbished products in the second period due to the high remanufacturing cost and the positive relicensing fee.

**Case A.II** : $\lambda_1 = 0$, $\lambda_2 \neq 0$, $\mu_1 \neq 0$.

$\lambda_2 \neq 0$ implies $B - h^* + \gamma q_1^* = 0$. Moreover, $\mu_1 \neq 0$ implies $h^* = 0$. Solving the KT conditions, we obtain $q_1^* = \frac{1}{2}\frac{2(\gamma+c_r)-\delta(1+c)}{\gamma}$ and $\mu_1 = \frac{(1+c)(\gamma-\delta)+2c_r}{\gamma^2}$. This case is valid for $\lambda_2 > 0$. From the expression for $\lambda_2$ (omitted for brevity), we have $\frac{\partial \lambda_2}{\partial c_r} = \frac{2\delta^2-8\delta+6\gamma^2-8\gamma}{(2\gamma^2-4\gamma-4\delta+\delta^2)} > 0$, so $\lambda_2$ is increasing in $c_r$. Therefore it is sufficient to show that $\lambda_2(c_r = 0) > 0$. But $\lambda_2(c_r = 0) = \frac{(\delta-\gamma)[(2\gamma^2-4\gamma-4\delta+\delta^2)c+4\gamma^2-4\gamma-4\delta+\delta^2)]}{\gamma^2(2\gamma^2-4\gamma-4\delta+\delta^2)} > 0$, so $\lambda_2 > 0$.

We also need $\mu_1 > 0$, which is true for $c_r > \frac{1}{2}(\delta - \gamma)(1 + c)$ and represents the case of having no refurbished products in the second period due to the high remanufacturing cost even if the OEM sets the relicensing fee to zero. This condition also ensures that $q_1^* > 0$.

**Case A.III** : $\lambda_1 \neq 0$, $\lambda_2 = 0$, $\mu_1 = 0$.

$\lambda_1 \neq 0$ implies $h^* - \gamma q_1^* = A$. Solving the KT conditions, we obtain $q_1^* = \frac{1}{2}\frac{(1-c)(\delta+2\gamma)}{\delta} > 0$, $h^* = \frac{1}{2}\frac{[4\gamma(1-\gamma)+\delta(4-\gamma)]c+4\gamma(\gamma-1)+2\delta(\delta-c_r)-\delta(4+\gamma)}{\delta}$, and $\lambda_1 = \frac{[\delta(\delta-8+2\gamma)+8\gamma(\gamma-1)]c+\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(2\gamma^2-4\gamma-4\delta+\delta^2)}$. Case III is valid for $\lambda_1 > 0$ and $h^* \geq 0$ or, $c \geq \max\{\frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{-\delta(\delta-8+2\gamma)-8\gamma(\gamma-1)}, \frac{4\gamma(\gamma-1)+2\delta(\delta-c_r)-\delta(4+\gamma)}{4\gamma(\gamma-1)+\delta(\gamma-4]}\} = \max\{c_{\lambda_1}, c_{h^*}\}$, where $c_{\lambda_1} = \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$ and $c_{h^*} = \frac{4\gamma(\gamma-1)+2\delta(\delta-c_r)-\delta(4+\gamma)}{4\gamma(\gamma-1)+\delta(\gamma-4]}$ are the values of $c$ that satisfy $\lambda_1(c) = 0$ and $h^*(c) = 0$, respectively. But $c_{\lambda_1} - c_{h^*} = -\frac{2\delta(4\gamma^2-4\gamma+\delta\gamma-4\delta+\delta^2)(\delta-\gamma-c_r)}{(8\gamma^2-8\gamma+2\delta\gamma-8\delta+\delta^2)(4\gamma^2-4\gamma-4\delta+\delta\gamma)} > 0$ because $c(\delta - \gamma) > c_r$ (for if we assume that $c(\delta - \gamma) \leq c_r$, $\lambda_1 < 0$ and this case becomes impossible) and $c < 1$. Therefore, $\max\{c_{\lambda_1}, c_{h^*}\} = c_{\lambda_1}$ and $h^* > 0$.

Case III represents the case of having no new products in the second period due to the high unit production cost, but charging a positive relicensing fee, and is valid for $c > \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$.

**Case A.IV** : $\lambda_1 \neq 0$, $\lambda_2 = 0$, $\mu_1 \neq 0$.

$\lambda_1 \neq 0$ implies $h^* - \gamma q_1^* = A$. Moreover, $\mu_1 \neq 0$ implies $h^* = 0$. Solving the KT conditions, we obtain $q_1^* = \frac{-\delta^2+[2(1-c)+(\gamma+c_r)]\delta+\gamma(2-\gamma)(1-c)}{\delta}$ and $\mu_1 = -\frac{[4\gamma(1-\gamma)+\delta(4-\gamma)]c+4\gamma(\gamma-1)+2\delta(\delta-c_r)-\delta(4+\gamma)}{\delta\gamma^2}$. Case IV is valid for $\lambda_1 > 0$ and $\mu_1 > 0$. However, $\mu_1$ is linearly decreasing in $c$, $\lambda_1$ is linearly increasing in $c$, and $c_{\mu_1} < c_{\lambda_1}$. Therefore, $\lambda_1$ and $\mu_1$ can never be positive at the same time, and

this case is impossible.

**Case A.V** : $\lambda_1 = 0, \lambda_2 = 0, \mu_1 = 0$.

Solving the KT conditions we obtain $h^* = \frac{1}{2}\frac{\delta^2(\delta-\gamma)c - 8\gamma^2(\gamma-1+c_r-\delta) + 4\delta^2(2+c_r) - 8c_r(\gamma+\delta) + 3\delta^2(\gamma-\delta)}{8\gamma(\gamma-1)+3\delta^2-8\delta}$,
$q_1^* = \frac{1}{2}\frac{[-3\delta^2+8\delta-4\gamma(\gamma+\delta-2)]c+4\gamma c_r+8(\gamma^2-\gamma-\delta)+3\delta^2}{8\gamma(\gamma-1)+3\delta^2-8\delta}$, $q_2^* = \frac{1}{2}\frac{[\delta(\delta-8+2\gamma)+8\gamma(\gamma-1)]c+\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{8\gamma(\gamma-1)+3\delta^2-8\delta}$, and
$q_r^* = \frac{2(c_r-c(\delta-\gamma))}{8\gamma(\gamma-1)+3\delta^2-8\delta}$. We can see that $q_2^* \geq 0$ for $c \leq c_{q_2^*} = \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$, while $q_r^* \geq 0$
for $c_r \leq c(\delta - \gamma)$. Moreover, $h^* \geq 0$ for $c \leq c_{h^*} = \frac{8\gamma^2(\gamma-1+c_r-\delta) + 4\delta^2(2+c_r) - 8c_r(\gamma+\delta) + 3\delta^2(\gamma-\delta)}{\delta^2(\delta-\gamma)}$.
But $c_{h^*} - c_{q_2^*} = -\frac{(3\delta^2-8\delta+8\gamma^2)(\delta^2-4\delta+\delta\gamma-4\gamma+4\gamma^2)(\delta-\gamma-c_r)}{\delta^2(\delta-\gamma)(\delta^2-8\delta+2\delta\gamma-8\gamma+8\gamma^2)} > 0$ and therefore this case is valid for
$c \leq \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$. Case V represents the case where both new and refurbished products exist
in the second period with a positive relicensing fee.

**Case A.VI** : $\lambda_1 = 0, \lambda_2 = 0, \mu_1 \neq 0$.

Here $q_2^* > 0$ and $h^* = 0$. This case was also found to be impossible because $q_2^* \mu_1 < 0$. Another
of way of seeing this is to note that both $q_2^*$ and $h^*$ decrease in $c$, but as $c$ increases, it is always
$q_2^*$ that becomes zero first $(c_{h^*} > c_{q_2^*})$. Therefore the case of $q_2^* > 0$ and $h^* = 0$ is not possible.

**Case B** $(h - \gamma q_1 \leq A)$. Solving for the Nash equilibrium in the second period under this condition,
we obtain $q_2^*(q_1, h) = 0$ and $q_r^*(q_1, h) = \frac{\gamma q_1 - h + \delta - \gamma - c_r}{2(\delta+\gamma) - \gamma^2}$. The OEM's optimization problem is:

$$Max_{q_1,h} \Pi(q_1, h) = \Pi_1(q_1, h) + hq_r^*(q_1, h) = (p_1(q_1, h) - c)q_1 + h\frac{\gamma q_1 - h + \delta - \gamma - c_r}{2(\delta + \gamma) - \gamma^2} \tag{16}$$

$$s.t. \quad h - \gamma q_1 \leq A \tag{17}$$

$$h - \gamma q_1 \leq B \tag{18}$$

$$q_1 \geq 0, \quad h \geq 0.$$

Note that since $A < B$, constraint (18) will never be binding at the optimal solution, and
therefore can be eliminated. Solving the constrained maximization problem, we have the following
cases:

**Case B.I** : For $c \geq \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$, constraint (17) is non-binding and the optimal values are
$q_1^* = \frac{1}{4}\frac{\delta(2+\gamma)+2\gamma(1-\gamma)-\gamma c_r-(2\gamma+2\delta-\gamma^2)c}{\delta+\gamma-\gamma^2}$ and $h^* = \frac{1}{2}(\delta - \gamma - c_r)$, yielding $q_r^* = \frac{1}{4}\frac{\delta-\gamma c-c_r}{\delta+\gamma-\gamma^2}$. In this
parameter range, $c_r < c(\delta - \gamma)$, which is in turn less than $\delta - \gamma$, so $h^* > 0$, $q_r^* > 0$ and $q_1^* > 0$.

**Case B.II** : For $c \leq \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$, constraint (17) is binding and the optimal values are
$q_1^* = \frac{1}{2}\frac{(1-c)(\delta+2\gamma)}{\delta}$, $h^* = \frac{1}{2}\frac{[4\gamma(1-\gamma)+\delta(4-\gamma]c+4\gamma(\gamma-1)+2\delta(\delta-c_r)-\delta(4+\gamma)}{\delta}$, yielding $q_r^* = \frac{1-c}{\delta}$. Note that
this case yields the same optimal solution and objective function value with Case A.III.

We illustrate the structure of the optimal solution subject to the conditions of Cases A and B in Figure 3, where we use the observation that $c \geq \tilde{c}(\delta, \gamma, c_r)$ implies $c \geq \frac{c_r}{\delta - \gamma}$, or, $c_r \leq c(\delta - \gamma)$.
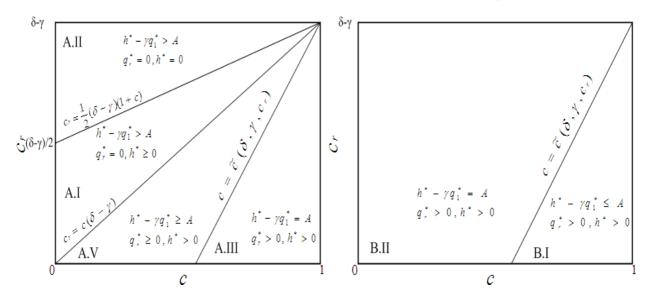


Figure 3: Structure of Optimal Solution subject to constraints $h - \gamma q_1 \geq A$ (left panel) and $h - \gamma q_1 \leq A$ (right panel).

We now compare the optimal constrained solutions of cases A and B to find the global optimal solution structure.

For $c \geq \tilde{c}(\delta, \gamma, c_r) \doteq \frac{\delta(8 - 3\delta + 2c_r) + 8\gamma(1 - \gamma)}{\delta(8 - \delta - 2\gamma) + 8\gamma(1 - \gamma)}$, Cases A.III and Case B.I need to be compared to find $q_1^*$ and $h^*$ in this parameter range. Since both cases A and B include the boundary $h - \gamma q_1 = A$, but the optimal solution in case B satisfies $h^* - \gamma q_1^* < A$, while that in case A.III satisfies $h^* - \gamma q_1^* = A$, we conclude that case B.I gives the global optimum in this range.

For $c < \tilde{c}(\delta <, \gamma, c_r)$, Case B.II needs to be compared with Cases A.I, A.II and A.V to find $q_1^*$ and $h^*$ in their respective parameter ranges. Since both cases A and B include the boundary $h - \gamma q_1 = A$, but the optimal solutions in case A satisfy $h^* - \gamma q_1^* > A$, while that in case B.II satisfies $h^* - \gamma q_1^* = A$, we conclude that cases A.I, A.II and A.V give the global optimum in their respective parameter ranges. The structure of the optimal solution is summarized in the following table.

| Condition | Equilibrium Outcome in the Second Period |
|---|---|
| $c > \widetilde{c}(\delta, \gamma, c_r) \doteq \frac{\delta(8-3\delta+2c_r)+8\gamma(1-\gamma)}{\delta(8-\delta-2\gamma)+8\gamma(1-\gamma)}$ | Only refurbished products |
| $c \leq \widetilde{c}(\delta, \gamma, c_r)$ and $c_r \leq c(\delta - \gamma)$ | Both new and refurbished products |
| $c \leq \widetilde{c}(\delta, \gamma, c_r)$ and $c(\delta - \gamma) < c_r \leq \frac{1}{2}(\delta - \gamma)(1 + c)$ | Only new products. ($q_r^* = 0$ due to $h^* > 0$) |
| $c \leq \widetilde{c}(\delta, \gamma, c_r)$ and $c_r > \frac{1}{2}(\delta - \gamma)(1 + c)$ | Only new products. ($q_r^* = 0$ even if $h^* = 0$) |

**Proof of Corollary 1.** When $c_r < c(\delta - \gamma)$, the optimal solution is given by the expressions in Case A.V when $c \leq \tilde{c}(\delta, \gamma, c_r)$, and by those in Case B.I when $c > \tilde{c}(\delta, \gamma, c_r)$. We begin with Case A.V where we have both new and refurbished products in the second period. The expression for $h^*$ is given by

$$h^* = \frac{1}{2} \frac{\delta^2(\delta - \gamma)c - 8\gamma^2(\gamma - 1 + c_r - \delta) + 4\delta^2(2 + c_r) - 8c_r(\gamma + \delta) + 3\delta^2(\gamma - \delta)}{8\gamma(\gamma - 1) + 3\delta^2 - 8\delta}, \qquad (19)$$

for which $\frac{\partial^2 h^*}{\partial c_r \partial \delta} = \frac{1}{2} \frac{16\delta\gamma(1-\gamma)+8\delta^2}{3\delta^2 - 8\delta + 8\gamma^2 - 8\gamma} < 0$ and $\frac{\partial^2 h^*}{\partial c \partial \delta} = \frac{1}{2} \frac{-16\gamma\delta^2 + 16\delta\gamma^2 - 16\delta^3 + 24\delta^2\gamma^2 - 16\delta\gamma^3 + 3\delta^4}{(8\gamma(\gamma-1)+3\delta^2 - 8\delta)^2} < 0$ (because the numerator is decreasing in $\gamma$ and is negative for $\gamma = 0$). Therefore, in order to show that $\frac{\partial h^*}{\partial \delta} > 0$, it is sufficient to show that $\left.\frac{\partial h^*}{\partial \delta}\right|_{c_r = c(\delta - \gamma), c=1} > 0$ since $c_r \leq c(\delta - \gamma)$ in this range and $c < 1$ by assumption. Indeed, $\left.\frac{\partial h^*}{\partial \delta}\right|_{c_r = c(\delta - \gamma), c=1} = \frac{2(\delta^2 - 2\delta - 2\gamma + 2\gamma^2)}{(3\delta^2 - 8\delta + 8\gamma^2 - 8\gamma)} > 0$. We now show that $\frac{\partial h^*}{\partial \gamma} < 0$. First, note that $\frac{\partial^2 h^*}{\partial c \partial \gamma} = \frac{1}{2} \frac{16\delta - 11\delta^2 + 8(\gamma - \delta)^2}{(3\delta^2 - 8\delta + 8\gamma^2 - 8\gamma)^2} > 0$. Therefore, it is sufficient to show that $\left.\frac{\partial h^*}{\partial \gamma}\right|_{c=1} < 0$. The latter can be easily shown with some algebraic manipulation, but is omitted for brevity. Thus, $\frac{\partial h^*}{\partial \gamma} < 0$. Finally, $\frac{\partial h^*}{\partial c} = \frac{1}{2} \frac{\delta^2(\delta - \gamma)}{(3\delta^2 - 8\delta + 8\gamma^2 - 8\gamma)} < 0$.

For Case B.I, where we have only refurbished products in the second period, the expression for $h^*$ is $h^* = \frac{1}{2}(\delta - \gamma - c_r)$, which is increasing in $\delta$, decreasing in $\gamma$, but does not depend on $c$.

**Proof of Proposition 2**

Following an analysis similar to that in Proposition 1, we derive the following table:

| Condition | Equilibrium Outcome in the Second Period |
|---|---|
| $c > \widetilde{c}(\delta, \gamma, c_r) \doteq \frac{\delta(8-3\delta+2c_r-\gamma)+\gamma(8-\gamma)}{\delta(8-\delta-\gamma)+\gamma(8-\gamma)}$ | Only refurbished products |
| $c \leq \widetilde{c}(\delta, \gamma, c_r)$ and $c_r \leq c\delta - \frac{1}{2}\gamma(1 + c)$ | Both new and refurbished products |
| $c \leq \widetilde{c}(\delta, \gamma, c_r)$ and $c\delta - \frac{1}{2}\gamma(1 + c) < c_r \leq \frac{1}{2}(\delta - \gamma)(1 + c)$ | Only new products. $q_r^* = 0$ due to $h^* > 0$ |
| $c \leq \widetilde{c}(\delta, \gamma, c_r)$ and $c_r > \frac{1}{2}(\delta - \gamma)(1 + c)$ | Only new products. ($q_r^* = 0$ even if $h^* = 0$) |

**Proof of Proposition 3.** Let $Q_r^{-i} = \sum_{j=1, j \neq i}^{N} q_r^j$ . Then we can rewrite the OEM's and entrants' problems as :

$$Max_{q_2} \Pi_2 = \left(1 - q_2 - \delta q_r^i - \delta Q_r^{-i} - c\right) q_2 + h(q_r^i + Q_r^{-i})$$

$$\text{s.t.} \quad q_2 \geq 0$$

$$Max_{q_r^i} \Pi_e = (\delta - q_2\delta - h - \delta q_r^i - \delta Q_r^{-i} - s - c_r)q_r^i$$

$$\text{s.t.} \quad q_r^i + Q_r^{-i} \leq \frac{s}{\gamma} - \frac{p_1 - s}{1 - \gamma}$$

$$q_r^i \geq 0$$

The FOC with respect to $q_2$ and $q_r^i$ give

$$-2q_2^* + 1 - \delta q_r^{i*} - \delta Q_r^{-i*} - c = 0$$

$$\delta - q_2^*\delta - h - 2\delta q_r^{i*} - \delta Q_r^{-i*} - s - c_r = 0.$$

Since we assume $N$ symmetric entrants, $Q_r^{-i*} = (N - 1)q_r^{i*}$ and the FOC can be rewritten as

$$-2q_2^* + 1 - \delta N q_r^{i*} - c = 0 \tag{20}$$

$$\delta - q_2^*\delta - h - \delta(N + 1)q_r^{i*} - s - c_r = 0. \tag{21}$$

In addition, the market clearing price $s^*$ satisfies

$$N q_r^{i*} = \frac{s^*}{\gamma} - \frac{p_1 - s^*}{1 - \gamma}. \tag{22}$$

Solving (20), (21), and (22) simultaneously, we obtain the equilibrium quantities for the second period for the case where both new and refurbished products exists in the market.

From hereafter we proceed as in the case of one entrant since from the OEM's standpoint, what determines the optimal policy is the aggregate demand for refurbished products $(N q_r^{i*})$ in the second period. Following an analysis similar to that in Proposition 1, we derive the following table:

| Condition | Equilibrium Outcome in the Second Period |
|---|---|
| $c > \widetilde{c}(\delta, \gamma, c_r, N) \doteq \frac{[\delta(4 - 3\delta + 2c_r) + 8\gamma(1-\gamma)]N + 4\delta}{[\delta(4 - \delta - 2\gamma) + 8\gamma(1-\gamma)]N + 4\delta}$ | Only refurbished products |
| $c \leq \widetilde{c}(\delta, \gamma, c_r, N)$ and $c_r \leq c(\delta - \gamma)$ | Both new and refurbished products |
| $c \leq \widetilde{c}(\delta, \gamma, c_r, N)$ and $c(\delta - \gamma) < c_r \leq \frac{1}{2}(\delta - \gamma)(1 + c)$ | Only new products. ($q_r^* = 0$ due to $h^* > 0$) |
| $c \leq \widetilde{c}(\delta, \gamma, c_r, N)$ and $c_r > \frac{1}{2}(\delta - \gamma)(1 + c)$ | Only new products. ($q_r^* = 0$ even if $h^* = 0$) |

Note that the threshold value for the remanufacturing cost below which the secondary market exists remains the same as in the case of one entrant $(c_r < c(\delta - \gamma))$. The production cost threshold $\widetilde{c} \doteq \frac{[\delta(4-3\delta+2c_r)+8\gamma(1-\gamma)]N+4\delta}{[\delta\,(4-\delta-2\gamma)+8\gamma(1-\gamma)]N+4\delta}$ above which the OEM does not sell new units in the second period now decreases in $N$: $\frac{\partial \widetilde{c}}{\partial N} = -\frac{8\delta^2(\delta-\gamma-c_r)}{[(\delta\,(4-\delta-2\gamma)+8\gamma(1-\gamma))N+4\delta]^2} < 0$.

For $c < \widetilde{c}$, the optimal values are:

$$q_2^* = \frac{1}{2} \frac{[(\delta\,(4-\delta-2\gamma)+8\gamma(1-\gamma))c - \delta(4-3\delta+2c_r) - 8\gamma(1-\gamma)]N - 4\delta(1-c)}{[8\gamma(1-\gamma)+4\,\delta-3\,\delta^2]N + 4\,\delta}$$

$$q_r^{i*} = \frac{2(c(\delta-\gamma)-c_r)}{[8\gamma(1-\gamma)+4\,\delta-3\,\delta^2]N+4\,\delta}$$

$$\frac{\partial h^*}{\partial N} = -2 \frac{\delta^3\,((\delta-\gamma)\,c-c_r)}{[4\,N\gamma(1-\gamma)+3\,\delta N(1-\delta)+\delta\,(N+4)\,]^2} < 0$$

$$\frac{\partial^2 h^*}{\partial N^2} = 4 \frac{\delta^3\,((\delta-\gamma)\,c-c_r)\,[4\,\gamma(1-\gamma)+3\,\delta(1-\delta)+\delta]}{[4\,N\gamma(1-\gamma)+3\,\delta N(1-\delta)+\delta\,(N+4)\,]^3} > 0$$

$$\frac{\partial \Pi_{OEM}^*}{\partial N} = \frac{4\delta[(\delta-\gamma)\,c-c_r]}{[4\,N\gamma(1-\gamma)+3\,\delta N(1-\delta)+\delta\,(N+4)\,]^2} > 0$$

$$\frac{\partial^2 \Pi_{OEM}^*}{\partial N^2} = -8 \frac{\delta[(\delta-\gamma)\,c-c_r][4\,\gamma(1-\gamma)+3\,\delta(1-\delta)+\delta]}{[4\,N\gamma(1-\gamma)+3\,\delta N(1-\delta)+\delta\,(N+4)\,]^3} < 0$$

For $c \geq \widetilde{c}$, the optimal values are:

$$q_2^* = 0$$

$$q_r^{i*} = \frac{1}{2} \frac{(\delta-\gamma c-c_r)}{\delta(N+1)+N\gamma(1-\gamma)}$$

$$h^* = \frac{1}{2}(\delta-\gamma-c_r)$$

$$\frac{\partial \Pi_{OEM}^*}{\partial N} = \frac{1}{4} \frac{\delta(\delta-\gamma c-c_r)^2}{[\delta(N+1)+N\gamma(1-\gamma)\,]^2} > 0$$

$$\frac{\partial^2 \Pi_{OEM}^*}{\partial N^2} = -\frac{1}{2} \frac{\delta(\delta+\gamma-\gamma^2))(\delta-\gamma c-c_r)^2}{[\delta(N+1)+N\gamma(1-\gamma)\,]^3} < 0$$

# Appendix B: Competition in both the primary and secondary markets with brand differentiation.

**Second-Period Analysis**

The net utility consumer $\theta$ derives from purchasing firm $A$'s new product is $U_2^A = \theta - p_2^A$, firm B's new product $U_2^B = (1 - \alpha)\theta - p_2^B$, firm A's refurbished product $U_{2,r}^A = \delta\theta - p_{2,r}^A - h^A$, and firm B's refurbished product $U_{2,r}^B = (1 - \alpha)\delta\theta - p_{2,r}^B - h^B$. Solving for the marginal consumers, we get

$$\theta_1 = \frac{p_2^A - p_2^B}{\alpha}, \theta_2 = \frac{p_2^B - p_{2,r}^A - h^A}{1 - \alpha - \delta}, \theta_3 = \frac{p_{2,r}^A - p_{2,r}^B + h^A - h^B}{\alpha\delta}, \theta_4 = \frac{p_{2,r}^B + h^B}{(1 - \alpha)\delta}$$

with respective demand for each product of $q_2^A = 1 - \theta_1$, $q_2^B = \theta_1 - \theta_2$, $q_{2,r}^A = \theta_2 - \theta_3$, and $q_{2,r}^A = \theta_3 - \theta_4$. Figure 4 illustrates the four market segments.
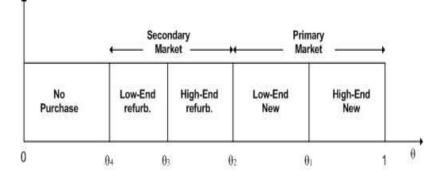


Figure 4: Consumer State Space in the Second Period

Under perfect competition in the secondary markets and no refurbishing cost, the refurbished products are available at a price equal to the resale value of used products ($p_{2,r}^A = s^A$ and $p_{2,r}^B = s^B$) with corresponding inverse demand functions

$$p_2^A = (\delta - 1 + \alpha)q_2^B + h^A - (1 - \delta)q_2^A + 1 - \delta + s^A$$
$$p_2^B = (\delta - 1 + \alpha)q_2^A + h^A - (1 - \alpha - \delta)q_2^B + 1 - \alpha - \delta + s^A.$$

Finally, the second-stage optimization problems for firms A and B are

$$Max_{q_2^A}\Pi_2^A(q_2^A|q_2^B) = (p_2^A - c)q_2^A + h^A q_{2,r}^A$$
$$Max_{q_2^B}\Pi_2^B(q_2^B|q_2^A) = (p_2^B - c)q_2^B + h^B q_{2,r}^B.$$

By solving the first-order conditions simultaneously, we derive the N.E. of this game, $q_2^{A*}(h^A, h^B, s^A, s^B)$

and $q_2^{B*}(h^A, h^B, s^A, s^B)$, and subsequently the quantities $q_{2,r}^{A*}(h^A, h^B, s^A, s^B)$ and $q_{2,r}^{B*}(h^A, h^B, s^A, s^B)$, from the demand equations corresponding to the market segmentation presented in Figure 4.
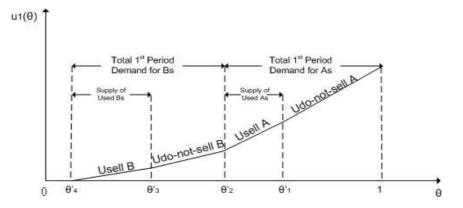


Figure 5: Consumer State Space in the First Period

**First-period analysis.**

Similar to our analysis for the monopolistic OEM, if $s^j$ denotes the resale value of firm $j$'s new product ($j = A, B$) at the end of period 1, then customers of firms A and B will derive the corresponding utilities in period 1:

$$U_1^A = \theta - p_1^A + (s^A - \gamma\theta)I_{(s^A \geq \gamma\theta)}$$
$$U_1^B = (1 - \alpha)\theta - p_1^B + (s^B - (1 - \alpha)\gamma\theta)I_{(s^B \geq (1-\alpha)\gamma\theta)}.$$

Figure 5 illustrates the total demand in the first period as well as the segment of consumers who decide to sell their used products. The marginal consumers are $\theta'_1 = \frac{s^A}{\gamma}$, $\theta'_2 = \frac{p_1^A - p_1^B - s^A}{\alpha - \gamma}$, $\theta'_3 = \frac{s^B}{(1-\alpha)\gamma}$ and $\theta'_4 = \frac{p_1^B - s^B}{(1-\alpha)(1-\gamma)}$, with respective demand for new products of $q_1^A = 1 - \theta'_2$, and $q_1^B = \theta'_2 - \theta'_4$, and respective supply of used products of $q_{1,r}^A = \theta'_1 - \theta'_2$ and $q_{1,r}^B = \theta'_3 - \theta'_4$. By setting these quantities equal to the equilibrium secondary market sizes of the second period $q_{2,r}^{A*}(h^A, h^B, s^A, s^B)$ and $q_{2,r}^{B*}(h^A, h^B, s^A, s^B)$, we can express the resale values in terms of the prices of new products and the relicensing fees: $s^A(h^A, h^B, p_1^A, p_1^B)$ and $s^B(h^A, h^B, p_1^A, p_1^B)$. The first-period profits are given by $\Pi_1^A(q_1^A | q_1^B) = (p_1^A - c)q_1^A$ and $\Pi_1^B(q_1^B | q_1^A) = (p_1^B - c)q_1^B$, while the total optimal profits over the two-period horizon are:

$$Max_{q_1^A, h^A} \Pi_A(q_1^A, h^A | q_1^B, h^B) = (p_{1A} - c)q_1^A + \Pi_{2A}^*(q_1^A, h^A | q_1^B, h^B)$$
$$Max_{q_1^B, h^B} \Pi_B(q_1^B, h^B | q_1^A, h^A) = (p_{1B} - c)q_1^B + \Pi_{2B}^*(q_1^B, h^B | q_1^A, h^A)$$

We verify that the conditions for a unique unconstrained Nash Equilibrium are met (convex strategy set, Hessian negative definite) and solve the first-order conditions simultaneously for all

the decision variables to derive the values $q_1^{A*}, h^{A*}, q_1^{B*}, h^{B*}$. The equilibrium is valid only for parameters yielding positive quantities, thus, the analysis in the paper is reflective of this set. For example, Figure 2 in the paper is plotted for $\alpha \in [0.1, 0.4]$. The upper threshold $\overline{\alpha}$ is the highest value of $\alpha \in (0, 1 - \delta)$ for which the low-end OEM produces new products in the second period. That is, for values of $\alpha$ above that point, the low-end OEM is priced out of the primary market in the second period (this constraint is always the first to be violated). On the other hand, the lower threshold $\underline{\alpha} = \gamma$ denotes the lowest value of $\alpha$ for which the ordering of the consumer state space in Figure 5 is valid (low-end OEM's new product above high-end OEM's refurbished product).