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AN EXPERIMENTAL INVESTIGATION OF LAMINAR FREE CONVECTION HEAT TRANSFER FROM THE SURFACE OF A VERTICAL CIRCULAR CYLINDER

A THESIS

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the Faculty of the Graduate Division

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AN EXPERIMENTAL INVESTIGATION OF

LAMINAR FREE CONVECTION HEAT TRANSFER

FROM THE SURFACE OF A VERTICAL CIRCULAR CYLINDER

Approved:

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Thesis Advisor

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Date Approved By Chairman: 9/2/60

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SUMMARY

An experimental investigation was made of laminar free convection to air from the outer surfaces of vertical cylinders with constant surface temperatures. The object of the experiment was to obtain data which would be compared to existing analytical solutions and to obtain an empirical correlation for this data.

The data were first compared to the solution of Sparrow and Gregg (1) which gives the ratio $\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$ as a function of $\xi = \frac{2^{3/2}}{Gr_L^{\frac{1}{4}}} \frac{L}{r_o}$ for Prandtl numbers of 0.72 and 1.0. This solution predicted lower values than those experimentally obtained.

The data were also compared to the solution of LeFevre and Ede (2), which gives the ratio $\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$ as a function of Pr and $\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$. This solution also predicted lower values than those actually obtained.

Further investigation of these two analytical solutions provided an expression for the Nusselt number of the form:

$$Nu_{L\ cyl} = C_1 Gr_L^{\frac{1}{4}} + C_2 \frac{L}{r_o} \quad .$$

where C_1 and C_2 are functions of the Prandtl number. It was found, however, that C_2 was not a function of the Prandtl number only but that it depended in some manner, which could not be determined from the experimental data, upon the cylinder. Therefore, it was suggested that further experiments be conducted to investigate this dependence.

Two empirical correlations were obtained which described the experimental results with a mean deviation of five per cent. For the range of

$$1.6 \times 10^5 \leq Gr_L \leq 3.84 \times 10^8 ,$$

$$Nu_{L\ cyl} = 1.666\ (Gr_L Pr)^{0.195}$$

$$Nu_{L\ cyl} = 1.08 \left[\frac{D}{L} (Gr_L Pr) \right]^{0.242} .$$

CHAPTER I.

INTRODUCTION

Most of the analytical and experimental work in the field of free convection has been conducted for vertical flat plates and horizontal cylinders. Most of the vertical cylinders which have been experimentally studied have had large enough diameters that the results could be expressed by flat plate solutions (see Part D of Reference 3). This creates a problem for those who must make practical calculations for free convection from cylinders of smaller diameters.

Analytical solutions for the problem of natural convection heat transfer from the outer surface of a vertical cylinder with an uniform surface temperature have been published by Sparrow and Gregg (1)* and LeFevre and Ede (2). These authors mentioned that sufficient experimental data were not available to check their solutions. The purpose of the present research was to obtain experimental values of the heat transfer coefficient for natural convection from the outer surfaces of vertical cylinders with uniform surface temperatures to compare with the solutions of Sparrow and Gregg as well as LeFevre and Ede and to obtain an empirical correlation which will describe these experimental data.

Sparrow and Gregg have recently solved the partial differential equations governing the boundary layer around a vertical isothermal cylinder in free convection. They first transformed the boundary layer

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*Numbers in parentheses refer to references in the Bibliography.

equations to dimensionless form. Then by assuming series solutions for the dependent variables, they obtained a set of ordinary differential equations which were numerically solved on an IBM Card Programmed Calculator. There is a disadvantage here in that this computer operation must be performed for each value of the Prandtl number. The resulting solution compared the average Nusselt number for the cylinder to that for the flat plate; it was of the form:

$$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}} = f(Pr, \xi)^{*} .$$

This is graphically presented as a function of ξ for $Pr = 0.72$ in Figure 1 in Appendix B. The authors have limited this solution to the region in which $10^4 \leq Gr_L \leq 10^9$, and to values of $\xi \leq 1.0$. In his discussion of the solution in Reference 1, S. I. Pai pointed out that the boundary layer equations, which are valid for large values of distance along the cylinder, were solved with a series, which should be valid for small values of distance along the cylinder. Thus there is some question concerning the range of cylinder lengths for which the solution is valid.

Soon after Sparrow and Gregg published their solution, LeFevre and Ede presented a solution for the problem of natural convection from the outer surface of a vertical cylinder with a uniform surface temperature. They began with the integrated boundary layer equations. Then by substituting into these equations assumed velocity and temperature

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*Symbols are defined in the Nomenclature in Appendix C.

profiles and by assuming series solutions for the dependent variables, they obtained ordinary differential equations which were easily solved. The solution was given in the form:

$$\frac{Nu_{L\ cyl}}{(Gr\ Pr)^{\frac{1}{4}}} = f\left[Pr,\ \frac{D}{L}(Gr\ Pr)^{\frac{1}{4}} \right] .$$

This solution is shown in Figure 2 in Appendix B. It is an approximate solution, but it has an advantage over that of Sparrow and Gregg in that it is easier to obtain values of Nusselt number for many different values of the Prandtl number.

The first significant experimental work on this problem was conducted by Carne (4) in 1937. He obtained his data by allowing steam to condense on the inside of hollow cylinders to maintain the cylinders at a constant temperature. However, very little of his data lay in the laminar region; and when those data which are in the laminar region are plotted together with the solution of Sparrow and Gregg or LeFevre and Ede, the points are scattered and the majority of them lie above the analytical solutions (see Figures 1 and 2).

In the present experiment, data will be obtained from electrically heated aluminum cylinders. An attempt will be made to cover the laminar region so that there will be sufficient data from which to draw reasonable conclusions.

CHAPTER II.

INSTRUMENTATION AND EQUIPMENT

Five polished* aluminum cylinders fitted with thermocouples to measure the surface temperature were vertically suspended, one at a time, inside a cylindrical cardboard shield and were heated from within by wound resistance heaters. A wattmeter was used to measure the power dissipated in the heater and a potentiometer was used with the thermocouples to measure the temperatures of the air, the inside surface of the shield, and the surface of the cylinder. Schematic diagrams and electrical circuits may be seen in Figures 3 and 4 in Appendix B.

Several problems were considered before constructing the cylinders. These cylinders had to be made such that they would have uniform surface temperatures and so that the heat dissipated from them by convection could be easily measured. The use of electrical heaters would allow the power dissipation to be easily and accurately measured. Also, the use of thick-walled cylinders should cause any axial temperature variation of the heater to be damped out at the outer surface of the cylinder. In practice this worked quite well, for the difference in the cylinder-to-air temperature differentials for the top and the bottom of the cylinder, which were always the maximum and minimum differentials, was always below five per cent. Convection from the heater to the inner wall of the

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*The term "polished" was used to designate the surface for the purpose of determining the emissivity as listed in Reference 5. Actually, the surfaces were shiny with light steel wool scratches and some tool marks.

cylinder could be a problem, but according to Article 24-4 of Reference 3 this problem could be avoided by making the inside diameter of the cylinder no more than one quarter inch greater than the heater diameter. Two problems could occur in measuring the surface temperature of the cylinders; heat could be conducted away from the cylinder along the thermocouple wires, and a thermocouple wire projecting from the surface would disrupt the boundary layer flow around the cylinder. According to Article 33-3 of Reference 3, the first problem can be avoided by leading the wires along the surface before applying the junction to the surface. By imbedding the wires underneath the surface both of these problems were eliminated. The final problem was that of insulating the ends of the cylinder so that almost all of the heat would leave through the cylindrical surface. This was done with balsa wood, which has a low thermal conductivity of 0.03 Btu/hr ft ^{O}F and yet is strong enough to be used structurally.

The above features were incorporated into a cylinder in the following manner: The cylinder was machined from a soft aluminum alloy and an axial hole approximately one quarter inch in diameter was drilled through the center to accommodate a wound resistance heater. An axial slot was milled the full length of the outside of the cylinder and holes were drilled normal to the slot from the outer surface of the cylinder. The axial slot acted as a channel down which the thermoucouple wires were laid, and it worked best when the width of the slot was the same as the diameter of the thermocouple wires. The holes which were drilled into the slot were Number 42 drill (0.0935 inch), and they allowed the thermocouple junction to be led from the slot to the surface of the cylinder.

Each thermocouple was then placed in the cylinder with its junction at the surface and the wires were run through the hole and up the slot to the top of the cylinder. The holes and the slot were then filled with plastic aluminum. The heater was then inserted into the cylinder and the balsa wood insulation was joined to the ends with the heater leads and thermocouple wires passing through the insulation. On the three smaller cylinders the balsa wood insulation was glued to both ends; and on the two larger cylinders it was glued to the bottom end and bolted to the upper end with two $1/4 - 20 \times 2\frac{1}{2}$ stove bolts, which were capped with insulation. The bottom piece of balsa wood insulation was sanded to a conical shape so that the boundary layer would be more likely to start from the bottom edge of the cylinder than from some point on the insulation. Two screw eyes were then screwed into the balsa wood insulation at the top of the cylinder, and the cylinder was suspended by a wire which was tied to the two screw eyes and passed over a rod. A drawing of an assembled cylinder may be seen in Figure 5.

The presence of the stove bolts necessitated an end correction for the two larger cylinders. This was made by assuming heat was conducted away from the cylinder by the bolts according to the equation $q = 2K_s A \Delta ti/\Delta x$. Here K_s is the thermal conductivity of steel, A is the cross sectional area of one bolt, Δt_i is the difference in temperature between the cylinder and the top of the insulation, and Δx is the length of the bolt in the insulation, i.e. two inches.

It should be mentioned that the balsa wood insulation had a tendency to char when the cylinder was left at a temperature of 300 $^{\circ}$F or more for several hours.

The wound resistance heater was made by winding Number 28 B & S Gage Nichrome Wire around a 0.15 inch diameter brass rod. The coil was slid off the rod and onto a glass tube with the same outer diameter as the rod. Two pieces of plastic-insulated Number 16 A.W.G. copper wire were inserted into the ends of the glass tube, and their insulations were broken at the point where they entered the tube. The ends of the Nichrome wire were then wrapped around the copper wire at these breaks. The coil was then coated with Sauereisen Heater Cement for insulation and allowed to dry. This made a sturdy, compact heater which worked well at temperatures below 300 $^{\circ}F$. Above 300 $^{\circ}F$, the insulation burned off the copper wires and the glass tube cracked from the heat. A more careful selection of heat resistant materials could easily overcome these problems.

The cylindrical cardboard shield was sixteen inches in diameter and five feet high. Holes were cut in the bottom to allow air to pass freely up into the shield. The cylinders were suspended from a stiff rod placed across the top of the shield. The temperature of the air inside the shield was measured by a thermocouple suspended from the rod which supported the cylinders. This thermocouple was protected by a radiation shield so that it would read the true air temperature. The inside surface of the shield was painted with aluminum paint to reduce the amount of radiation from the cylinder, and the temperature of this surface was measured by a thermocouple whose leads passed through the shield to the outside.

All the thermocouples used to test the three-quarter inch diameter cylinder were Number 30 B & S Gage chromel- alumel thermocouples with fiberglass insulation. On the other four cylinders iron-constantan wire with the same specifications was used. All the thermocouples were run through a selector switch to a Leeds and Northrup Portable Precision Potentiometer.

Power for the heater was obtained from the building's 115 volt A. C. supply through a Sola Constant Voltage Transformer and a Superior Powerstat. The power to the heater was measured by a Weston A. C., D. C., Model 310 Wattmeter. To obtain accurate power readings below five watts, a General Electric Instrument Potential Transformer was used in the voltage circuit of the wattmeter.

CHAPTER III.

PROCEDURE

To begin a series of runs, a cylinder was suspended in the shield, and the power leads were connected to the heater. The voltage setting on the Powerstat was then varied until the desired value of net power through the heater had been obtained. The net power is the difference between the gross power, which is read when the heater is connected, and the meter losses, which are read when the heater is disconnected. When the desired value of net power had been obtained, the apparatus was left for three to six hours, or until it was operating at steady state. Readings were then taken every ten minutes for forty minutes. The readings that were recorded were: gross power, surface temperatures, air temperature, and shield temperature. At the end of forty minutes the meter losses were again read. This procedure constituted one run.

Each cylinder was used for seven to ten runs, each run being at a different power level. An attempt was made to vary the power levels evenly between the lowest, which corresponded to a temperature difference between the cylinder and the air of about 30 degrees Fahrenheit, and the highest, which corresponded to a temperature difference of about 200 degrees Fahrenheit.

CHAPTER IV.

DISCUSSION OF RESULTS

The experimental data were plotted on a graph with the solution of Sparrow and Gregg. It can be seen in Figure 8 that all but four of the forty points fell above Sparrow's and Gregg's solution, the mean deviation being 14.0 per cent. The data for the large cylinder, which corresponds to small values of the independent variable ξ, had a much smaller deviation from the solution than the data for the smaller cylinders.

The experimental data were also plotted on a graph with the solution of LeFevre and Ede. It can be seen in Figure 9 that all but seven of the forty points fall above LeFevre's and Ede's solution, the mean deviation being 14.6 per cent. Again the data for the large cylinder, which correspondes to larger values of the independent variable $(Gr_L\ Pr)^{\frac{1}{4}}\ \frac{D}{L}$, had a much smaller deviation from the solution than the data for the smaller cylinders.

The closer agreement between the experimental data and the solutions of Sparrow and Gregg and LeFevre and Ede for the large cylinder is supported by the statement that both solutions are based on the boundary layer equations which are valid for large values of the distance along the cylinder. Another possible explanation is that the experimental error for the large cylinder is considerably less than that for the small cylinder (see Error Analysis Appendix A).

Further investigation was made of the solutions of Sparrow and Gregg as well as LeFevre and Ede to see if a better method could be found to correlate the experimental data. The following analysis gave the best results.

The graph of Sparrow's and Gregg's solution for a particular Prandtl number closely approximates a straight line which can be written as

$$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}} = a + b\,\xi$$

or

$$\frac{Nu_{L\ cyl}}{0.476\ Gr_L^{\frac{1}{4}}} = a + \frac{b\ 2^{3/2}}{Gr_L^{\frac{1}{4}}}\ \frac{L}{r_o} \quad .$$

When both sides are multiplied by $0.476\ Gr_L^{\frac{1}{4}}$,

$$Nu_{L\ cyl} = A\ Gr_L^{\frac{1}{4}} + B\,\frac{L}{r_o} \quad . \qquad (1)$$

The solution of Levre and Ede is written as:

$$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}} = f(Pr) + g(Pr)\ \frac{L}{D(Gr_L Pr)^{\frac{1}{4}}} \quad .$$

When both sides are multiplied by $(Gr_L Pr)^{\frac{1}{4}}$,

$$Nu_{L\ cyl} = \Big[f(Pr) \Big] (Gr_L Pr)^{\frac{1}{4}} + g(Pr)\ \frac{L}{D} \quad .$$

This can also be written as:

$$Nu_{L\ cyl} = \left[f_1(Pr) \right] (Gr_L^{\frac{1}{4}}) + g_1(Pr)\, \frac{L}{r_o} \quad . \qquad (2)$$

Since both solutions yielded the Nusselt number as a straight line function of the fourth root of the Grashof number for a constant Prandtl number and $\frac{L}{r_o}$ ratio, and since the Prandtl number was essentially constant throughout the experiment, the Nusselt number was plotted in Figure 10 against the fourth root of the Grashof number for each cylinder. After the data were plotted in this manner, a line was drawn through the data for each cylinder. These lines were drawn parallel even though better agreement would have been obtained if they had not been drawn parallel; however, the greatest mean deviation was still only 6.9 per cent. The following equations and mean deviations were obtained from these lines for the indicated range of Grashof numbers:

$$1.6 \times 10^5 \leq Gr_L \leq 3.84 \times 10^8$$

Cylinder a, $Nu_{L\ cyl} = 0.68\ Gr_L^{\frac{1}{4}} + 0.081\ \frac{L}{r_o}$ 1.8%

Cylinder b, $Nu_{L\ cyl} = 0.68\ Gr_L^{\frac{1}{4}} - 0.088\ \frac{L}{r_o}$ 6.9%

Cylinder c, $Nu_{L\ cyl} = 0.72\ Gr_L^{\frac{1}{4}} - 0.72\ \frac{L}{r_o}$ 4.1%

Cylinder d, $Nu_{L\ cyl} = 0.68\ Gr_L^{\frac{1}{4}} - 1.06\ \frac{L}{r_o}$ 1.9%

Cylinder e, $Nu_{L\ cyl} = 0.73\ Gr_L^{\frac{1}{4}} - 2.25\ \frac{L}{r_o}$ 6.6%

Since the slope for each equation is very nearly the same, and since the Prandtl number for the experiment was essentially constant, it seems that the slopes of equations (1) and (2) are a function of the

Prandtl number only. Since the coefficient of $\frac{L}{r_o}$ is different in each of these equations it is concluded that equation (2) will not correlate the data for all of the cylinders. This fact displays a dependence of the Nusselt number upon the particular cylinder used. The form of this dependence could not be determined from the data; however, it should be studied further. The coefficient of $\frac{L}{r_o}$ is plotted as a function of $\frac{L}{r_o}$ in Figure 11 in Appendix B.

The data were also correlated in the classical coordinates of free convection, ln $Nu_{L\ cyl}$ vs ln $Gr_L Pr$. This resulted in the following equation which described the results with a mean deviation of 4.7 per cent.

$$1.6 \times 10^5 \leq Gr_L \leq 3.84 \times 10^8$$

$$Nu_{L\ cyl} = 1.666\ (Gr_L Pr)^{0.195} \quad .$$

This correlation is shown in Figure 12.

It was thought that even better results might be obtained if ln $Nu_{L\ cyl}$ were plotted against ln $\frac{D}{L}(Gr_L Pr)$, but there was no improvement. The following equation was obtained, and it described the data with a mean deviation of 5.5 per cent.

$$1.6 \times 10^5 \leq Gr_L \leq 3.84 \times 10^8$$

$$Nu_{L\ cyl} = 1.08 \left[\frac{D}{L}(Gr_L Pr)\right]^{0.242} \quad .$$

This correlation is plotted in Figure 13.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The solution of Sparrow and Gregg did not accurately predict the results of this experiment, for as ξ increased from 0.250 to 0.725 the value of the Nusselt number predicted by the solution became increasingly smaller than that obtained in the experiment.

The solution of LeFevre and Ede also failed to predict the results of this experiment accurately, for as $\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$ decreased from 20 to 7.0 the value of $\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$ predicted by the solution became increasingly smaller than that obtained in the experiment.

Two empirical equation were determined which describe the experimental data within a mean deviation of 5.0 per cent. In the range of

$$1.6 \times 10^5 \leq Gr_L \leq 3.84 \times 10^8; \text{ they are:}$$

$$Nu_{L\ cyl} = 1.666\ (Gr_L Pr)^{0.195}$$

$$Nu_{L\ cyl} = 1.08\left[\frac{D}{L}(Gr_L Pr)\right]^{0.242}.$$

It was found that the data for this experiment could be expressed by equations of the form:

$$Nu_{L\ cyl} = C_1 Gr_L^{\frac{1}{4}} + C_2 \frac{L}{r_o}$$

where C_1 is a function of the Prandtl number and C_2 is dependent in some manner upon the particular cylinder. It is recommended that further experiments be conducted to investigate the dependence of C_1 upon the Prandtl number and of C_2 upon the particular cylinder. These experiments should employ a variety of fluids so that a wide range of Prandtl numbers could be covered. It is also recommended that before undertaking future experiments of this type the value of the emissivity of the test cylinders be experimentally determined so that the estimated error due to uncertainty in the emissivity can be reduced.

APPENDIX A

ERROR ANALYSIS

An attempt is made here to estimate the maximum error in the average Nusselt number for the cylinder due to the various errors which could be made in taking measurements and assuming constants. The analysis is conducted for the two extremes of operation, i.e. a small cylinder operating at low power and a large cylinder operating at high power.

The errors which could be expected from measurements are as follows:

The error in mesuring the diameter of the cylinder is $\pm$ 0.001 inch.

The error in measuring the length of the cylinder is $\pm$ 1/64 inch.

The error in measuring the power is $\pm$ 0.25 per cent of the full scale reading. This amounts to $\pm$ 0.3 watts multiplied by the scale factor of the wattmeter.

The error in measuring an absolute temperature, as given by the thermocouple manufacturer, is $\pm$ 5 $^{\circ}$F.

The error in measuring a temperature difference was assumed to be $\pm$ 1 $^{\circ}$F. The assumption was made after observing that all the thermocouples read within 1°F of each other prior to heating the cylinder.

The only error involved in assuming a constant was that incurred in assuming the emissivity of the cylinders. This error was assumed to be $\pm$ 20 per cent.

No error was assumed in determining the properties of air at the mean air temperature. An end correction was made only for the two cylinders whose insulations were attached by stove bolts.

For the purpose of this analysis, the Nusselt number is written as the sum of three terms. The first term is dependent on the net power dissipated by the heater; the second term is a correction for heat transferred by radiation, and the third term is a correction for axial heat transfer from the ends of the cylinders. The magnitude, per cent error, and absolute error are determined for each term. Then the magnitudes and absolute errors are summed and the maximum per cent error is determined from these two sums.

$$Nu_{L\ cyl} = \frac{hL}{K} = \frac{Q_c}{A_c \Delta t}\ \frac{L}{K} = \frac{P - qr - \text{End Effect}}{\pi D L \Delta t}\ \frac{L}{K}$$

$$Nu_{L\ cyl} = \frac{3.36\ p}{\pi D K \Delta t} - \frac{\sigma \epsilon_c (T_c^4 - T_s^4) L}{K \Delta t} - \frac{0.1073(t_c - t_i)}{\pi D K \Delta t}$$

Low Power | Small Cylinder | Run No. 1

First Term

$$\text{Magnitude} = \frac{(3.36)(0.3)(12)}{(3.14)(0.752)(.0153)(21.8)} = 15.3$$

Δp = 0.03 watts = 10 o/o of 0.3

ΔD = 0.001 inch = 1.33 o/o of 0.75

$\Delta\Delta t$ = 1 ^{o}F = 5 o/o of 20

o/o error - 16.3

absolute error = 2.5

Second Term

$$\text{Magnitude} = \frac{(0.1714 \times 10^{-8})(0.1)(132)(10^{8})(2)}{(0.0153)(21.8)(12)} = 1.13$$

$$\left.\begin{array}{l}\Delta\epsilon_c = 0.02 = 20 \text{ o/o of } 0.1 \\ \Delta L = 1/64 \text{ inch} = 0.76 \text{ o/o of } 2 \\ \Delta\Delta t = 1\ ^{\circ}F = 5 \text{ o/o of } 20 \\ \Delta(T_c^{\ 4} - T_s^{\ 4}) = 74.72 \times 10^8\ ^{\circ}R = 57.5 \text{ o/o of } 132 \times 10^8\end{array}\right\} \begin{array}{l}\text{o/o error} = 83.3 \text{ o/o} \\ \text{absolute error} = 0.94\end{array}$$

Third Term Not Applicable

Total Magnitude of 1st and 2nd Terms = 15.30-1.13 = 14.17

Total of Absolute Errors = 2.50 + 0.94 = 3.44

Per Cent Error = $\frac{3.44}{14.17}$ (100) = $\underline{\underline{24.3}}$

High Power | Large Cylinder | Run No. 36

First Term

$$\text{Magnitude} = \frac{(3.36)(50)(12)}{(3.14)(2.4)(0.0171)(163)} = 96$$

$$\left.\begin{array}{l}\Delta p = 0.6 \text{ watts} = 1.2 \text{ o/o of } 50 \\ \Delta D = 0.001 \text{ inch} = 0.04 \text{ o/o of } 2.5 \\ \Delta\Delta t = 1\ ^{\circ}F = 0.61 \text{ o/o of } 163\end{array}\right\} \begin{array}{l}\text{o/o error} = 1.85 \\ \text{absolute error} = 1.78\end{array}$$

Second Term

$$\text{Magnitude} = \frac{(0.1714 \times 10^{-8})(0.1)(1665 \times 10^{8})(16)}{(0.0171)(163)(12)} = 13.7$$

$$\left.\begin{array}{l}\Delta\epsilon_c = 0.02 = 20 \text{ o/o of } 0.10 \\ \Delta(T_c^{\ 4} - T_s^{\ 4}) = 137.8 \times 10^8 = 8.3 \text{ o/o of } 1665 \times 10^8 \\ \Delta\Delta t = 1\ ^{\circ}F = 0.61 \text{ o/o of } 163 \\ \Delta L = 1/64 \text{ inch} = 0.10 \text{ o/o of } 16\end{array}\right\} \begin{array}{l}\text{o/o error} = 29 \\ \text{absolute error} = 3.98\end{array}$$

Third Term

$$\text{Magnitude} = \frac{(0.1073)(113.6)(12)}{(3.14)(2.4)(.0171)(163)} = 6.97$$

$$\left.\begin{array}{l} \Delta(t_c - t_i) = 1\ {}^{\circ}F = 0.74 \text{ o/o of } 113.6 \\ \Delta D = 0.001 \text{ in} = 0.04 \text{ o/o of } 2.5 \\ \Delta\Delta t = 1\ {}^{\circ}F = 0.61 \text{ o/o of } 163 \end{array}\right\} \quad \begin{array}{l} \text{o/o error} = 1.39 \\ \text{absolute error} = 0.10 \end{array}$$

Total of Magnitudes = 96 - 13.7 - 6.97 = 75.33

Total of Absolute Errors = 5.86

$$\text{Per Cent Error} = \frac{5.86}{75.33}(100) = \underline{\underline{7.8}}$$

APPENDIX B

Table 1

Sample Calculation Sheet

Cylinder e

$D = 2.402$ in. $= 0.200$ ft.

$L = 16.00$ in. $= 1.333$ ft.

Surface Area, $A_c = \pi DL = 0.839\ ft^2$

Radiation Constant $= \sigma \epsilon_c A_c$

$= (0.1714 \times 10^{-8})(0.1)(0.839) =$

$0.0144 \times 10^{-8} \frac{Btu}{hr\ ft^2\ R^4}$

$L^3 = 2.369\ ft^3$

$L/r_o = 13.32$

$D/L = 0.150$

Item No.		Run Number 31
1	Top Temperature, t_t , F	114
2	Bottom Temperature, t_b , F	113
3	Air Temperature, t_a, F	86
4	$\Delta t_{top} = t_t - t_a$, F	28
5	$\Delta t_{bottom} = t_b - t_a$, F	27
6	R $(\Delta t_{top} - \Delta t_{bottom})/\Delta t_{top}$, If $R \leq 0.05$, the run is considered to be one of constant surface temperature and will be used.	0.038
7	Average surface temperature, t_c, F	114
8	Average surface temperature, $T_c = t_c + 460$, R	574

Table 1 (Continued)

Sample Calculation Sheet

9	$(T_c)^4$, $R^4 \times 10^{-8}$		1085
10	Inner Surface Temperature of Shield, t_s, F		86
11	Shield Temperature, $T_s = t_s + 460$, R		546
12	$(T_s)^4$, $R^4 \times 10^{-8}$		889
13	Radiation Potential = $(T_c^4 - T_s^4)$, R^4		196
14	Insulation Temperature, t_i, F		93
15	$\Delta t_i = t_c - t_i$, F		21
16	$\Delta t = t_c - t_a$, F		28
17	Mean Air Temperature, $t_m = \frac{t_c + t_a}{2}$, F		100
18	DETERMINED AT MEAN AIR TEMPERATURE	*Thermal Conductivity of air, K, $\frac{Btu}{hr\ ft\ F}$	0.0155
19		*Prandtl No. of air, Pr, dimensionless	0.711
20		**Y for air, $\frac{1}{ft^3\ F} \times 10^{-6}$	1.775
21		Grashof No., $Gr_L = YL^3\Delta t \times 10^{-6}$, dimensionless	117
22		$Gr_L^{\frac{1}{4}}$, dimensionless	102
23		$Gr_L Pr \times 10^{-6}$, dimensionless	90.3
24		$(Gr_L Pr)^{\frac{1}{4}}$, dimensionless	97.2
25	Net Power, p, watts		6.50

*Values of K and Prandtl number for air obtained from Table II - 2, Reference 6.

**Values of Y for air obtained from Table A - 2, Reference 7.

Table I (Continued)

Sample Calculation Sheet

26	Net Power, $P = 3.36\ p$, Btu/hr	21.8
27	Radiation, $q_r = \sigma \epsilon_c A_c (T_c{}^4 - T_s)^4$, Btu/hr	2.82
28	End effects = $2K_s A \dfrac{\Delta t_i}{\Delta X} = 0.1003\ \Delta t_i$, $\frac{Btu}{hr}$	2.25
29	Heat transferred by convection, $q_c = P - q_r$ - End effects, $\frac{Btu}{hr}$	16.8
30	Heat transfer coefficient, $h = \dfrac{q_c}{A_c(t_c - t_a)}$	0.714
31	Average Nusselt number for cylinder, $Nu_{L\ cyl} = hL/K$, dimensionless	61.4
32	*Average Nusselt number for flat plate $Nu_{L\ f.p.} = 0.4757\ Gr_L^{\frac{1}{4}}$, dimensionless	48.5
33	Ratio of Nusselt numbers, $Nu_{L\ cyl}/Nu_{L\ f.p.}$, dimensionless	1.27
34	$Nu_{L\ cyl}/(Gr_L Pr)^{\frac{1}{4}}$, dimensionless	0.632
35	$\xi = (2^{3/2}/Gr_L^{\frac{1}{4}})\ \frac{L}{r_o}$, dimensionless	0.370
36	$D/L\ (Gr_L Pr)^{\frac{1}{4}}$, dimensionless	14.6
37	$D/L\ (Gr_L Pr) \times 10^{-6}$	13.6

*According to Ostrach (8), $Nu_{L\ f.p.} = 0.4757\ Gr_L^{\frac{1}{4}}$ when $Pr = 0.72$.

Table II

Summary of Tabulated Data

Cylinder a, $D = 0.752$ inches, $L = 2.00$ inches

Run No.	1	2	3	4	5	6	7
t_c, F	102	139	161	168	204	286	238
t_a, F	80	83	85	83	84	87	85
t_s, F	83	84	85	84	85	87	86
t_i, F	---	---	---	---	---	---	---
P, Btu/h_r	1.01	3.36	4.87	5.71	8.40	15.6	11.3
Pr	0.712	0.710	0.710	0.710	0.709	0.707	0.708
$Gr_L \times 10^{-6}$	0.192	0.421	0.524	0.584	0.708	0.878	0.797
$Gr_L^{\frac{1}{4}}$	20.9	25.5	26.9	27.6	29.0	30.6	29.9
$Gr_L Pr \times 10^{-6}$	0.137	0.299	0.372	0.415	0.502	0.620	0.564
$D/L(Gr_L Pr)^{\frac{1}{4}}$	7.22	8.80	9.29	9.55	10.0	10.6	10.3
$Nu_{L\ cyl}$	14.2	18.0	19.1	19.6	20.0	20.8	20.4
$Nu_{L\ f.p.}$	9.94	12.1	12.8	13.1	13.8	14.6	14.2
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.43	1.48	1.49	1.50	1.45	1.43	1.43
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.741	0.769	0.774	0.773	0.753	0.741	0.743
ξ	0.720	0.590	0.560	0.545	0.519	0.492	0.503
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	0.052	0.113	0.140	0.156	0.189	0.234	0.212

Table II (Continued)

Summary of Tabulated Data

Cylinder b, D = 1.003 inches, L = 4.00 inches

Run No.	8	9	10	11	12	13	14
t_c, F	114	157	213	180	223	139	286
t_a, F	83	86	83	88	85	85	84
t_s, F	83	86	83	88	85	85	85
t_i, F	---	---	---	---	---	---	---
P, Btu/hr	3.36	9.25	23.5	14.5	22.6	6.22	35.3
Pr	0.711	0.710	0.709	0.709	0.708	0.714	0.707
$Gr_L \times 10^{-6}$	2.24	3.98	5.78	4.74	6.14	3.24	7.23
$Gr_L^{\frac{1}{4}}$	38.7	44.6	49.0	46.7	49.8	42.5	51.9
$Gr_L Pr \times 10^{-6}$	1.59	2.83	4.10	3.36	4.35	2.32	5.11
$\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$	8.90	10.3	11.3	10.7	11.4	9.80	11.9
$Nu_{L\ cyl}$	26.2	28.1	39.5	33.3	33.8	25.1	34.7
$Nu_{L\ f.p.}$	18.4	21.2	23.3	22.2	23.7	20.2	24.7
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.42	1.32	1.69	1.50	1.43	1.24	1.41
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.738	0.683	0.878	0.778	0.741	0.641	0.731
ξ	0.584	0.506	0.461	0.484	0.453	0.531	0.435
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	0.400	0.734	1.03	0.842	1.09	0.582	1.28

Table II (Continued)

Summary of Tabulated Data

Cylinder c, $D = 1.484$ inches, $L = 8.00$ inches

Run No.	15	16	17	18	19	20	21
t_c, F	119	171	218	252	293	192	148
t_a, F	85	88	86	86	90	88	85
t_s, F	85	88	87	86	90	88	86
t_i, F	---	---	---	---	---	---	---
P, $\frac{Btu}{hr}$	8.40	27.6	51.2	67.2	86.5	37.8	18.1
Pr	0.711	0.710	0.708	0.708	0.706	0.709	0.710
$Gr_L \times 10^{-6}$	17.8	35.2	46.5	52.9	55.7	41.0	29.1
$Gr_L^{\frac{1}{4}}$	65.0	76.9	82.3	85.2	86.8	79.9	73.1
$Gr_L Pr \times 10^{-6}$	12.7	25.0	32.9	37.4	39.4	29.0	20.7
$\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$	11.1	13.2	14.1	14.5	14.7	13.6	12.5
$Nu_{L\ cyl}$	35.3	46.8	54.1	54.2	54.9	52.3	41.7
$Nu_{L\ f.p.}$	30.9	36.6	39.2	40.5	41.3	38.0	34.8
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.14	1.28	1.38	1.34	1.33	1.38	1.20
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.592	0.661	0.714	0.694	0.695	0.715	0.619
ξ	0.469	0.397	0.371	0.358	0.352	0.382	0.417
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	2.35	4.65	6.12	6.96	7.33	5.40	3.84

Table II (Continued)

Summary of Tabulated Data

Cylinder d,	D = 1.960 inches,			L = 12.00 inches	
Run No.	22	23	24	25	26
t_c, F	108	140	161	185	205
t_a, F	78	85	83	84	83
t_s, F	79	85	84	84	84
t_i, F	87	102	107	114	122
P, $\frac{Btu}{hr}$	15.1	33.6	50.4	67.2	83.2
Pr	0.711	0.710	0.710	0.709	0.709
$Gr_L \times 10^{-6}$	57.1	90.5	119	142	155
$Gr_L^{\frac{1}{4}}$	86.9	97.3	104	109	112
$Gr_L Pr \times 10^{-6}$	40.6	64.3	84.2	101	110
$\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$	13.0	14.5	15.6	16.3	16.6
$Nu_{L\ cyl}$	46.1	57.3	61.4	61.4	63.4
$Nu_{L\ f.p.}$	41.3	46.3	49.5	51.9	53.3
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.12	1.24	1.24	1.18	1.19
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.577	0.642	0.641	0.614	0.622
ξ	0.398	0.356	0.333	0.318	0.309
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	6.62	10.5	13.7	16.4	17.9

Table II (Continued)

Summary of Tabulated Data

Cylinder d, D = 1.960 inches, L = 12.00 inches

Run No.	27	28	29	30
t_c, F	230	246	266	300
t_a, F	86	83	86	89
t_s, F	87	84	86	89
t_i, F	129	137	142	143
P, $\frac{Btu}{hr}$	101	118	138	168
Pr	0.708	0.708	0.707	0.706
$Gr_L \times 10^{-6}$	168	179	185	190
$Gr_L^{\frac{1}{4}}$	114	116	117	119
$Gr_L Pr \times 10^{-6}$	119	126	131	134
$\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$	17.0	17.3	17.4	17.6
$Nu_{L\ cyl}$	63.1	64.7	67.4	68.2
$Nu_{L\ f.p.}$	54.2	55.2	55.4	56.6
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.16	1.17	1.22	1.20
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.607	0.610	0.630	0.631
ξ	0.304	0.299	0.297	0.291
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	19.4	20.6	21.4	21.8

Table II (Continued)

Summary of Tabulated Data

Cylinder e	D = 2.402 inches			L = 16.00 inches	
Run No.	31	32	33	34	35
t_c, F	114	136	162	197	227
t_a, F	86	89	87	86	89
t_s, F	86	89	88	86	88
t_i, F	93	101	111	120	128
P, $\frac{Btu}{hr}$	21.8	38.6	66.4	102	134
Pr	0.711	0.710	0.710	0.709	0.708
$Gr_L \times 10^{-6}$	117	181	267	341	378
$Gr_L^{\frac{1}{4}}$	102	116	128	136	139
$Gr_L Pr \times 10^{-6}$	90.3	129	189	241	268
$\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$	14.6	15.9	17.6	18.6	19.2
$Nu_{L\ cyl}$	61.4	63.5	68.9	69.9	71.3
$Nu_{L\ f.p.}$	48.5	55.2	60.9	64.7	66.1
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.27	1.15	1.13	1.08	1.08
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.632	0.599	0.589	0.564	0.557
ξ	0.370	0.325	0.295	0.277	0.271
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	13.6	19.4	28.3	36.2	40.2

Table II (Continued)

Summary of Tabulated Data

Cylinder e,	D = 2.402 inches			L = 16.00 inches	
Run No.	36	37	38	39	40
t_c, F	251	280	304	327	350
t_a, F	88	90	88	88	87
t_s, F	87	89	88	87	88
t_i, F	137	147	154	162	170
P, $\frac{Btu}{hr}$	168	202	237	267	302
Pr	0.708	0.707	0.706	0.706	0.705
$Gr_L \times 10^{-6}$	414	440	461	468	482
$Gr_L^{\frac{1}{4}}$	142	144	146	147	148
$Gr_L Pr \times 10^{-6}$	293	311	326	330	340
$\frac{D}{L}(Gr_L Pr)^{\frac{1}{4}}$	19.5	19.8	20.1	20.3	20.4
$Nu_{L\ cyl}$	75.4	74.8	76.7	77.0	77.9
$Nu_{L\ f.p.}$	67.6	68.5	69.5	69.9	70.4
$\frac{Nu_{L\ cyl}}{Nu_{L\ f.p.}}$	1.12	1.09	1.10	1.10	1.11
$\frac{Nu_{L\ cyl}}{(Gr_L Pr)^{\frac{1}{4}}}$	0.580	0.567	0.572	0.570	0.573
ξ	0.266	0.262	0.258	0.256	0.255
$\frac{D}{L}(Gr_L Pr) \times 10^{-6}$	43.9	46.7	48.9	49.5	51.0

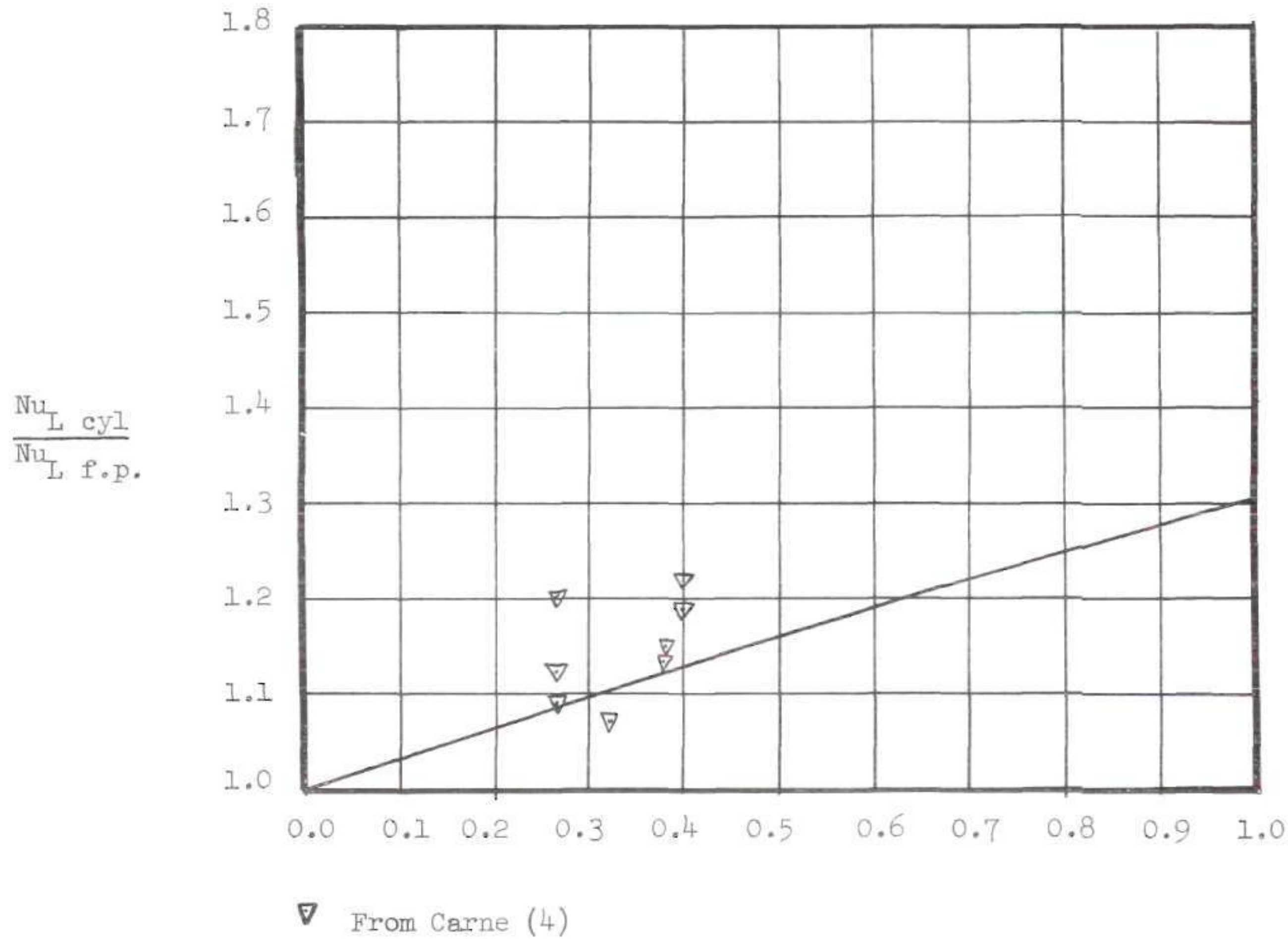


Figure 1. The Solution of Sparrow and Gregg for Pr = 0.72

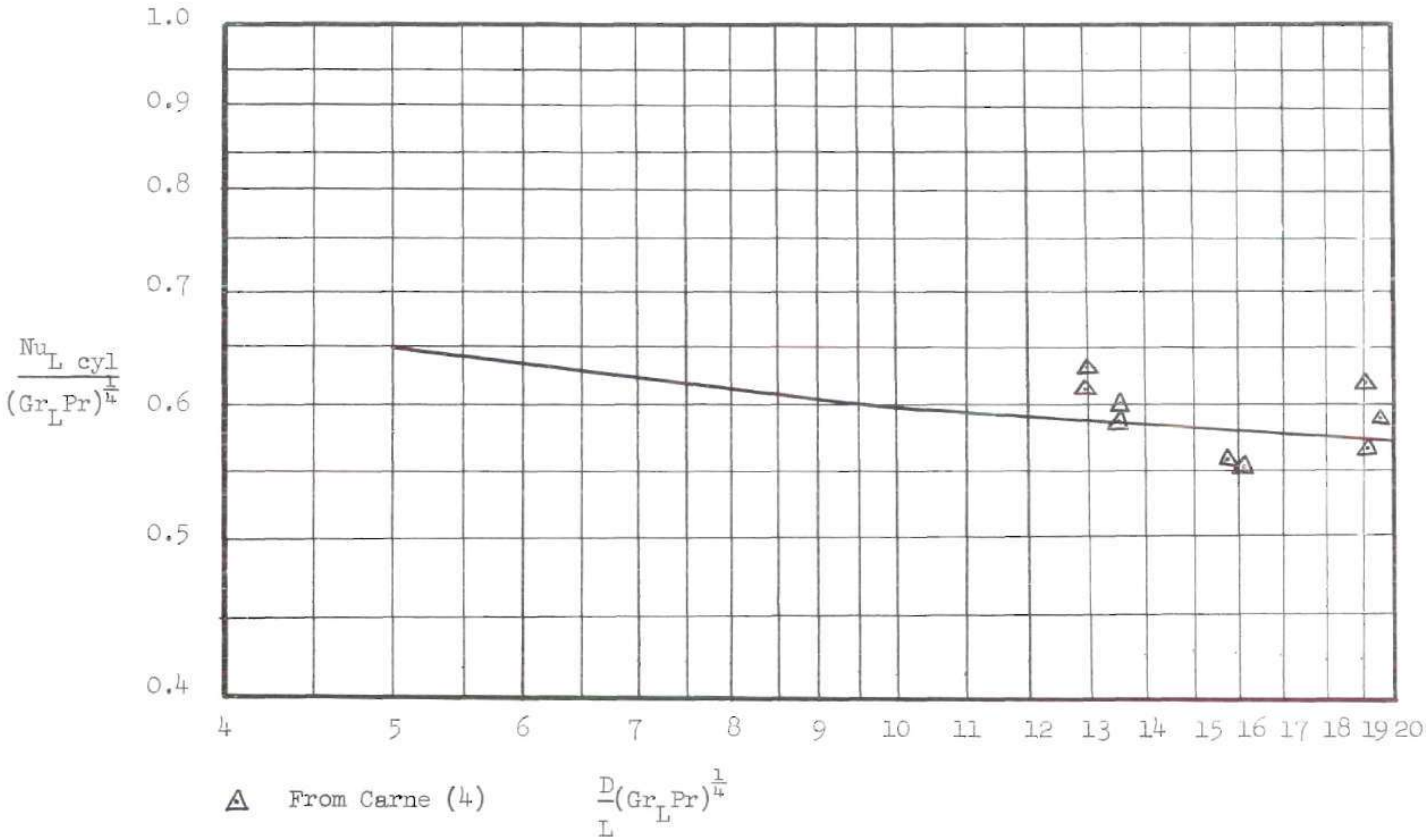


Figure 2. The Solution of LeFevre and Ede for Gases

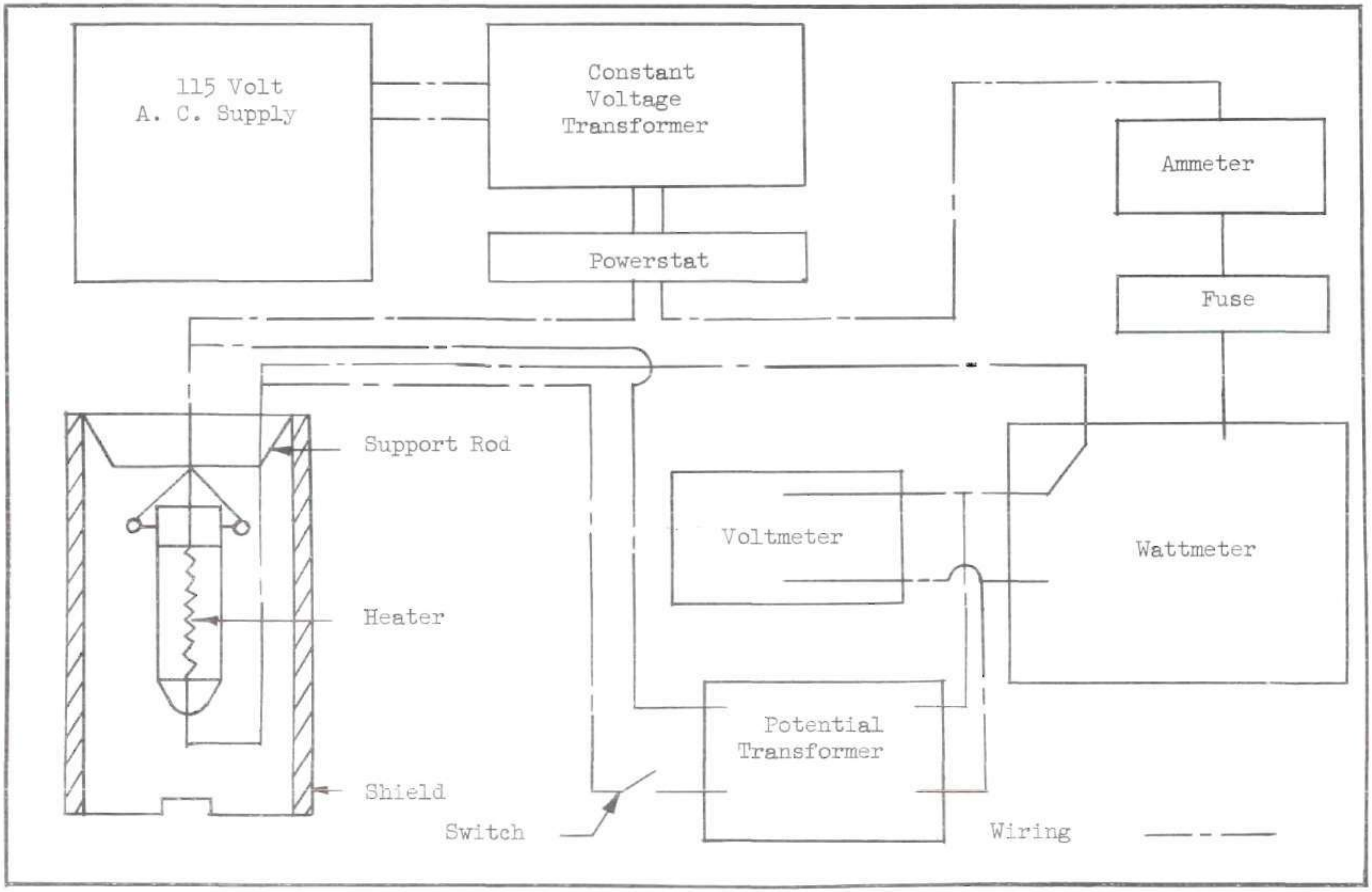


Figure 3. Schematic Diagram of Power Equipment.

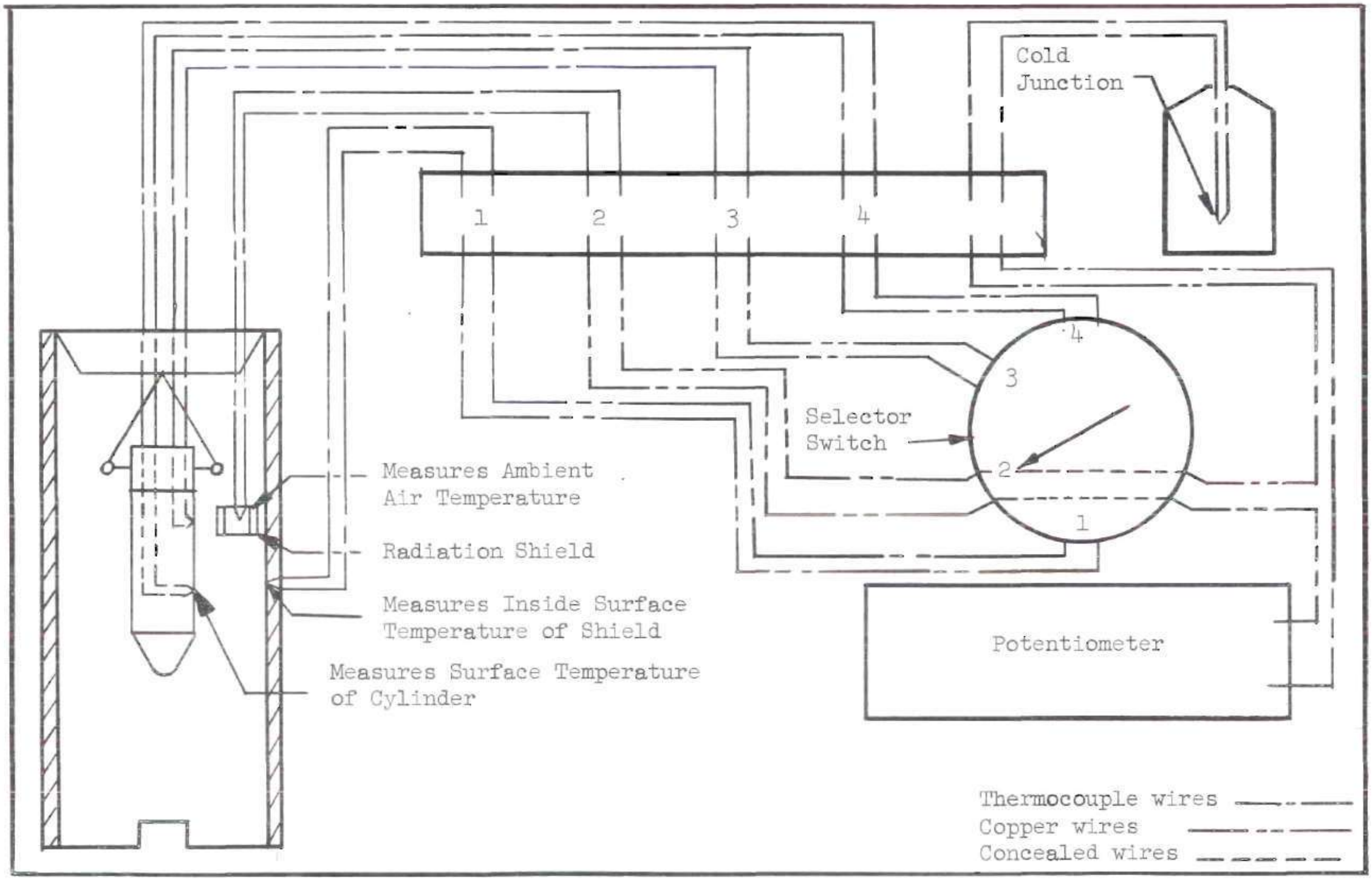


Figure 4. Schematic Diagram of Thermocouple Circuit

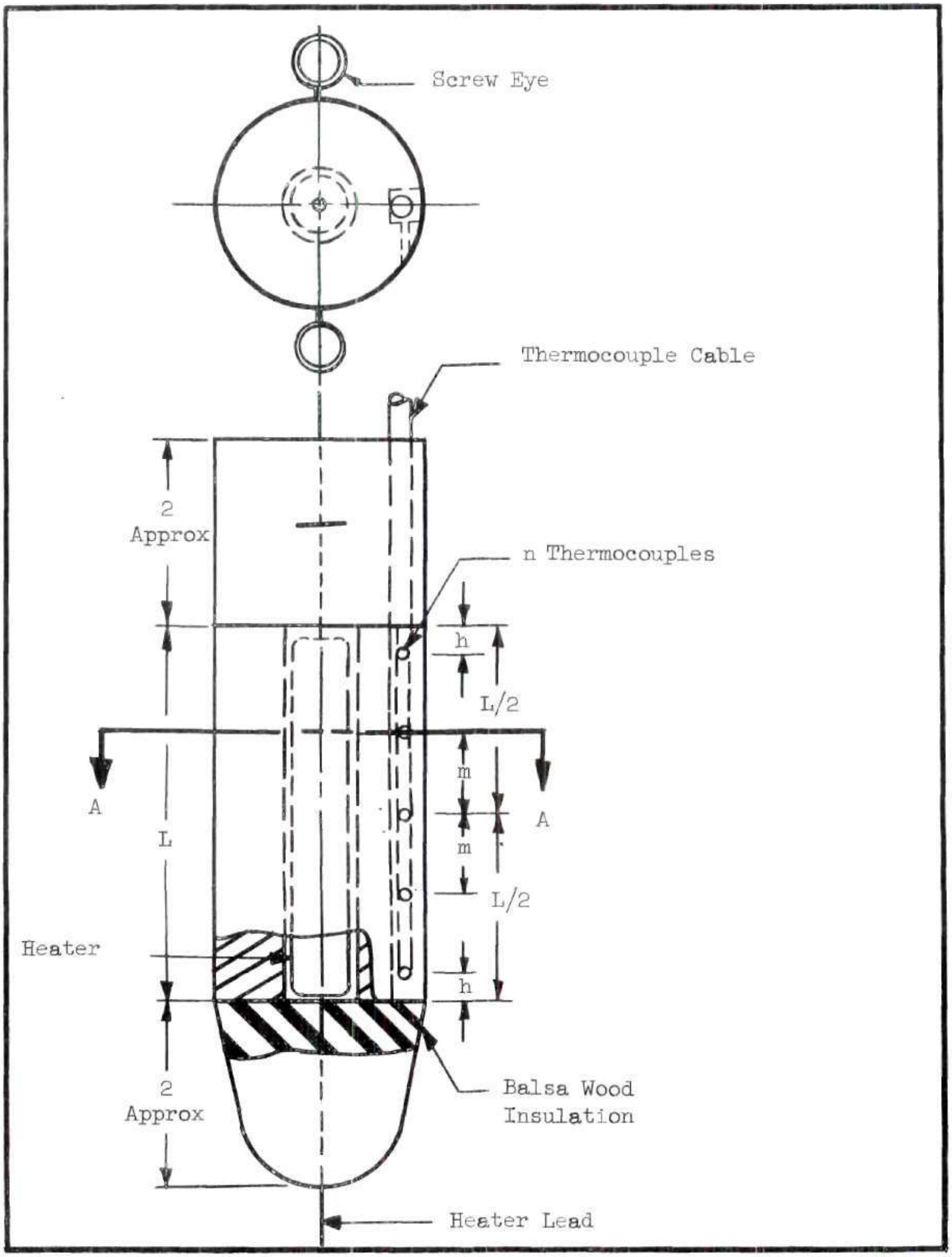
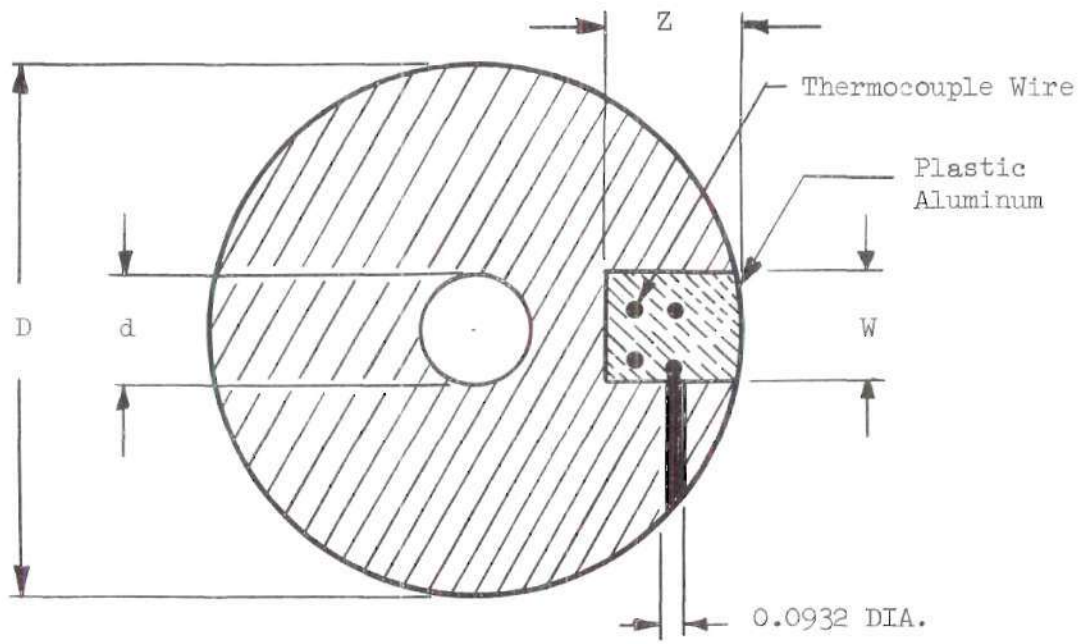


Figure 5. Cylinder Construction



Section A - A

Schedule of Dimensions (inches)

CYL	L	D	d	n	h	m	w	z
a	2.00	0.752	0.250	3	0.250	-	0.250	0.200
b	4.00	1.003	0.250	3	0.500	-	0.250	0.250
c	8.00	1.484	0.375	5	0.500	1.75	0.250	0.300
d	12.00	1.960	0.375	5	0.500	2.75	0.250	0.300
e	16.00	2.402	0.375	5	0.500	3.75	0.163	0.500

Figure 5. Cylinder Construction (Continued).

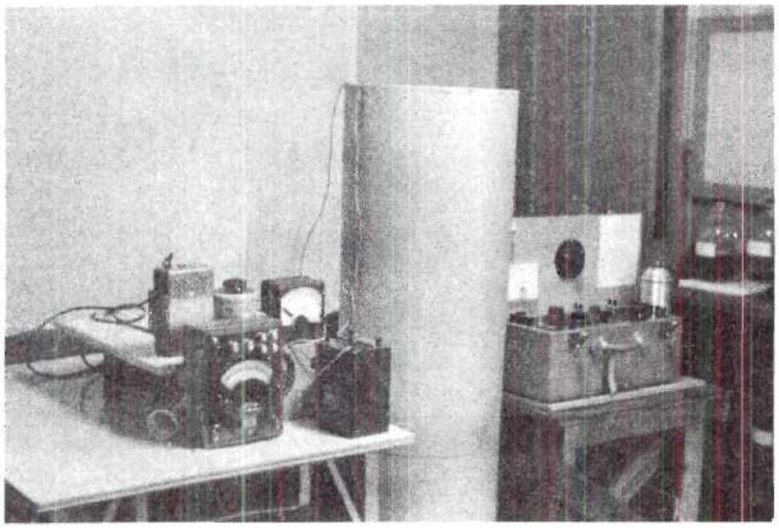


Figure 6. Photograph of Equipment.

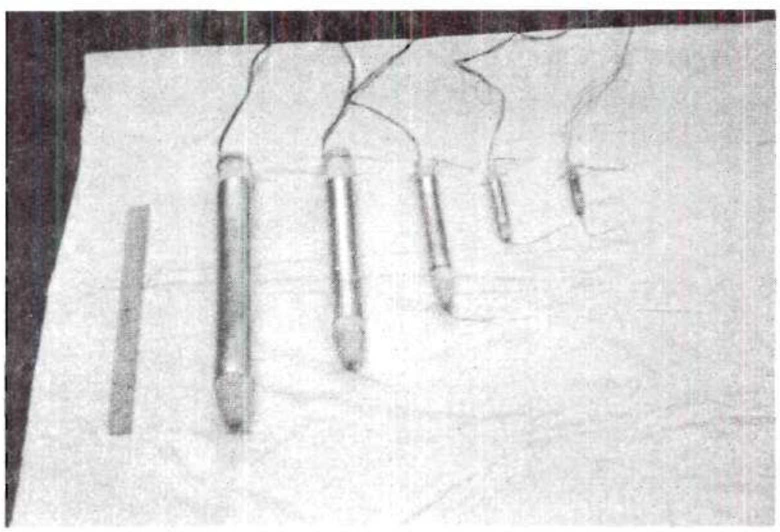


Figure 7. Photograph of Cylinders.

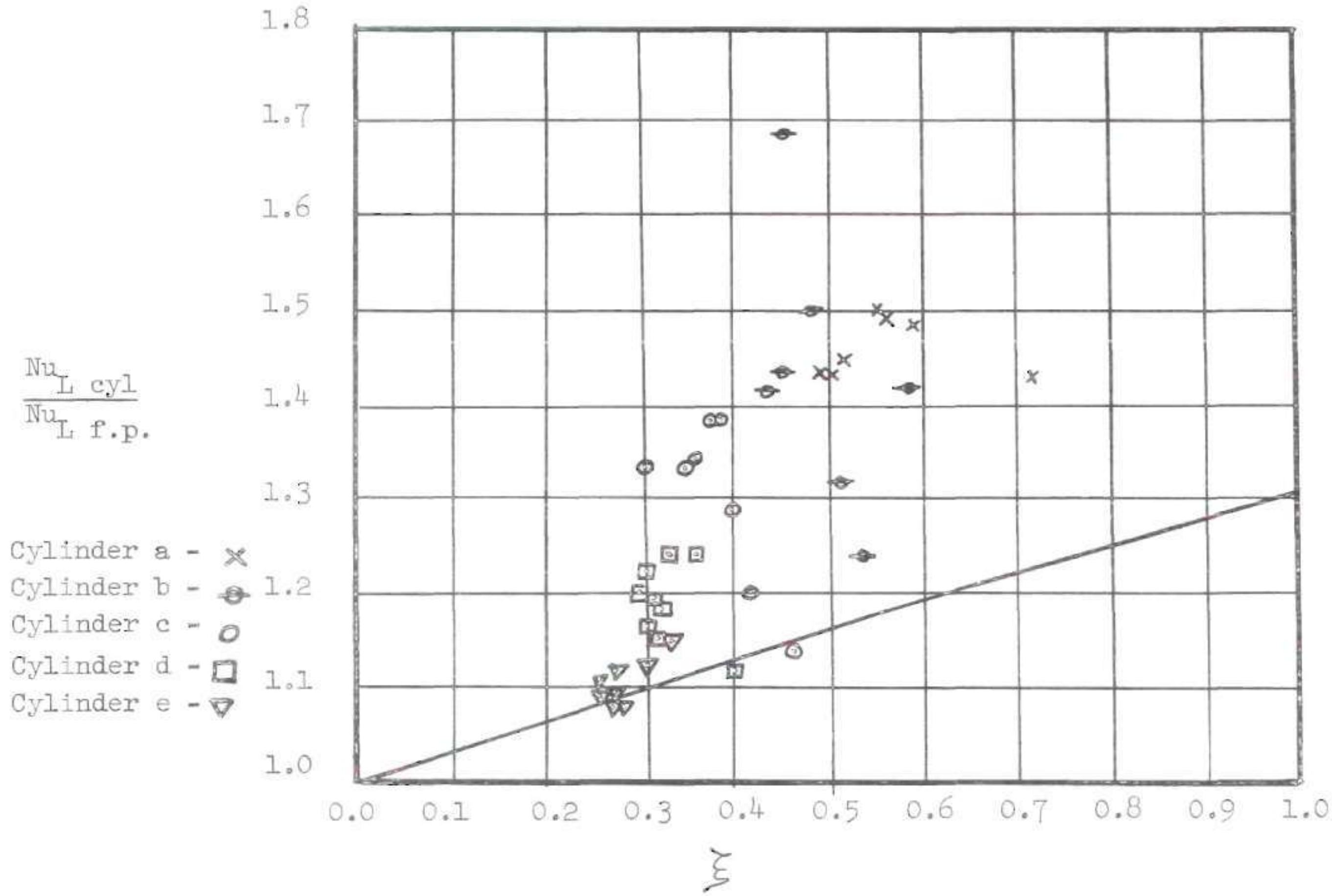


Figure 8. Experimental Data Compared to Solution of Sparrow and Gregg.

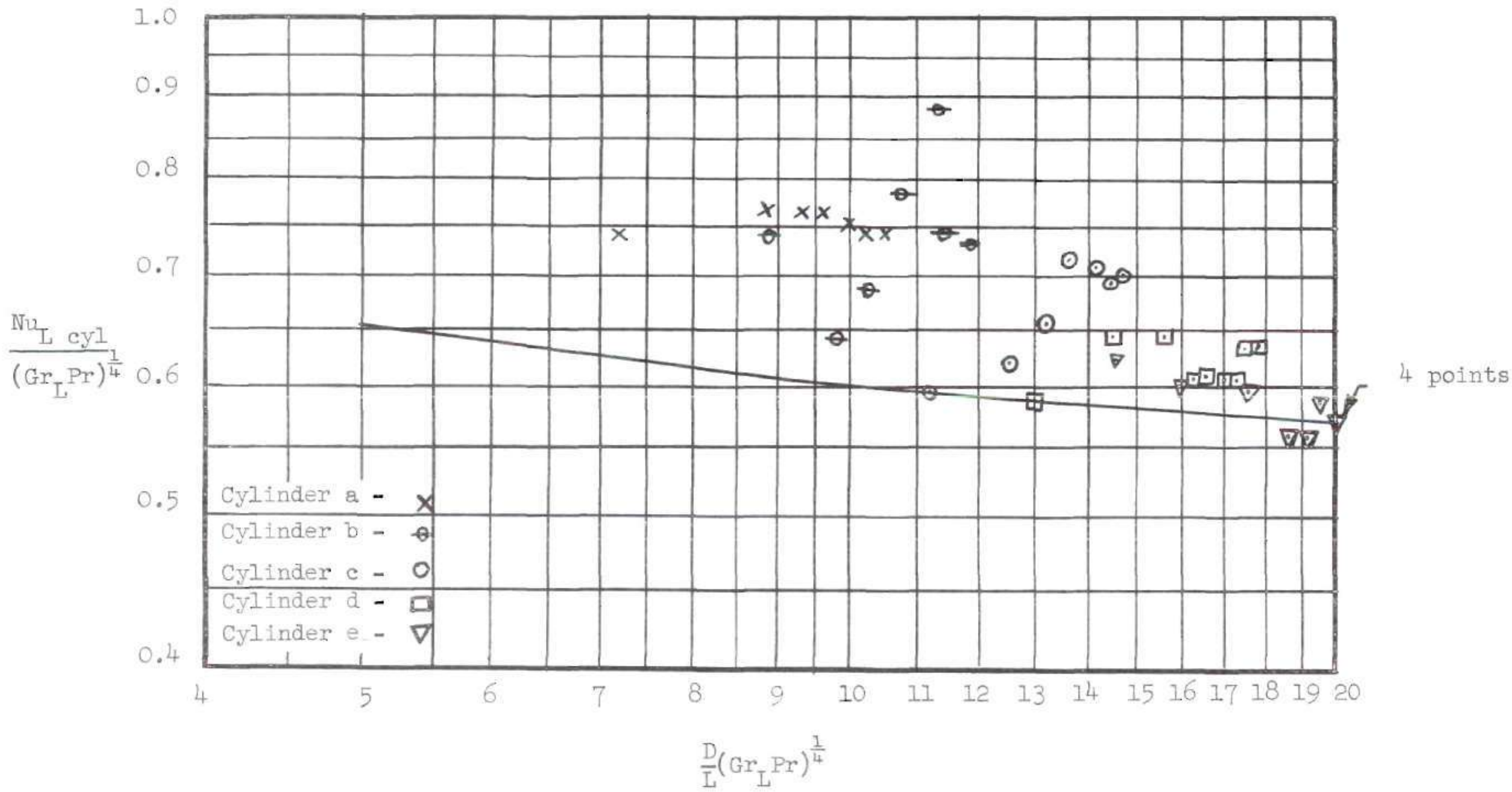


Figure 9. Experimental Data Compared with Solution of LeFevre and Ede.

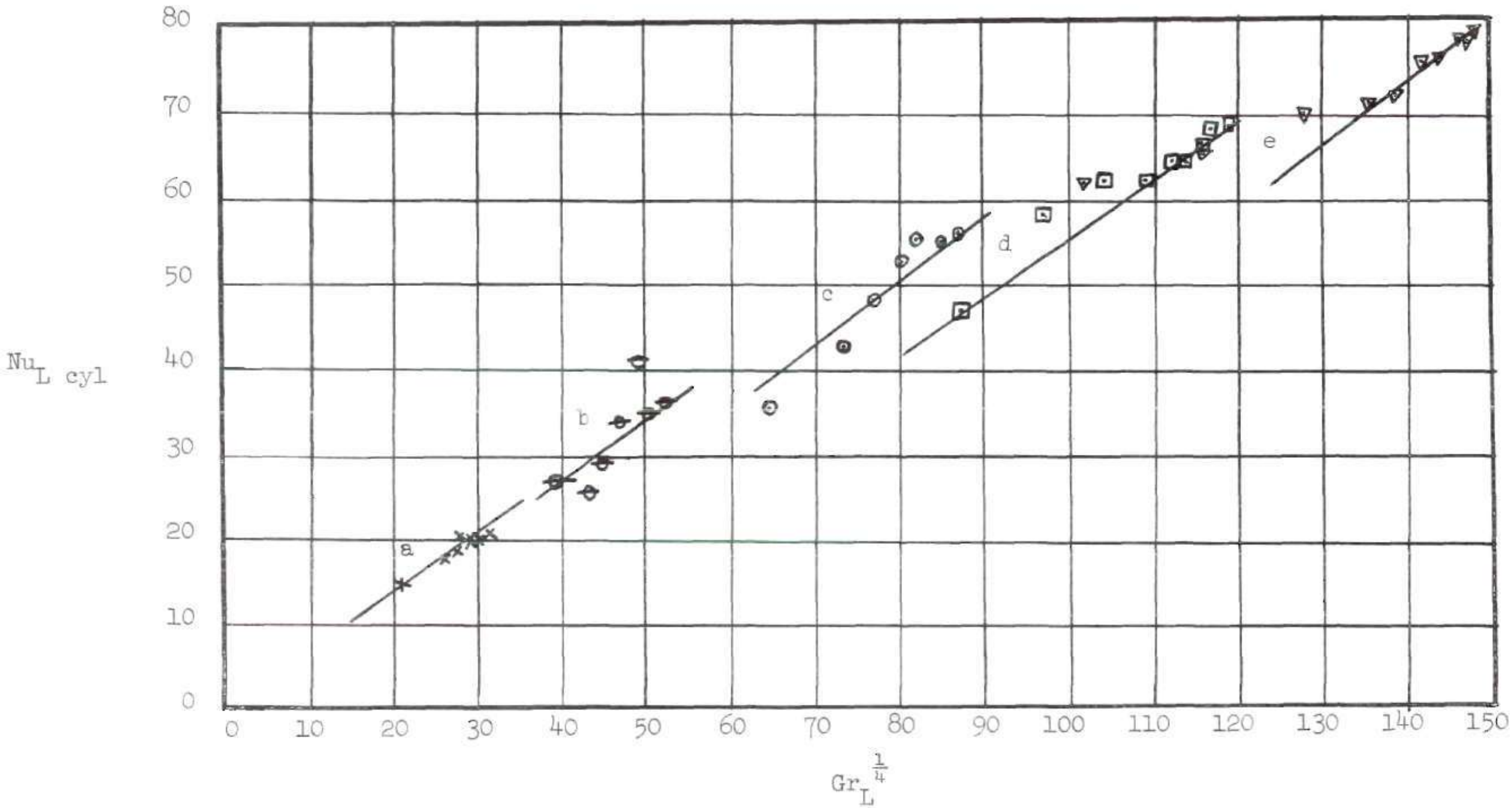


Figure 10. Experimental Data $Nu_{L\ cyl}$ vs $Gr_L^{\frac{1}{4}}$

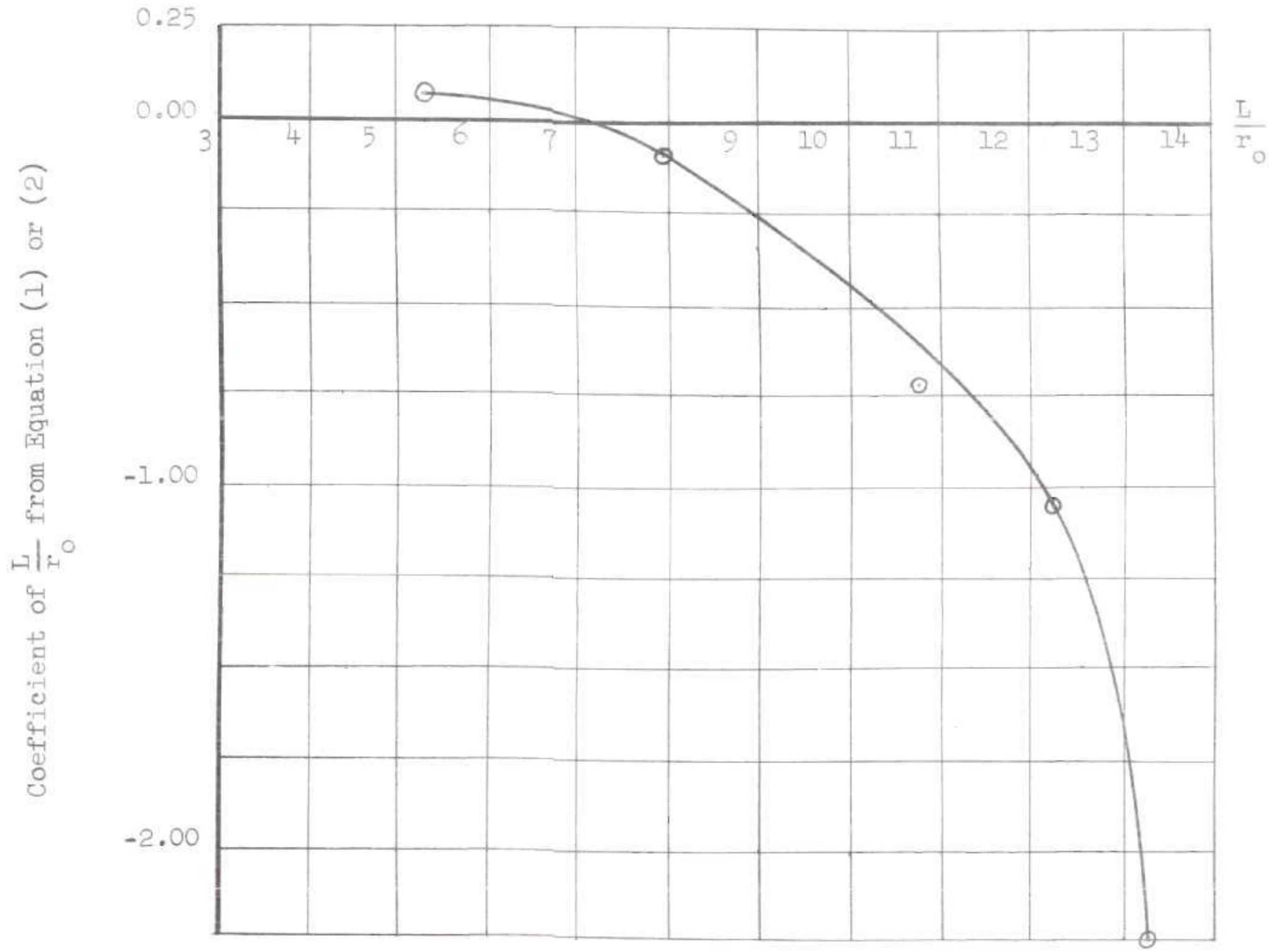


Figure 11. Coefficient of $\frac{L}{r_o}$ from Equation (1) or (2) vs $\frac{L}{r_o}$

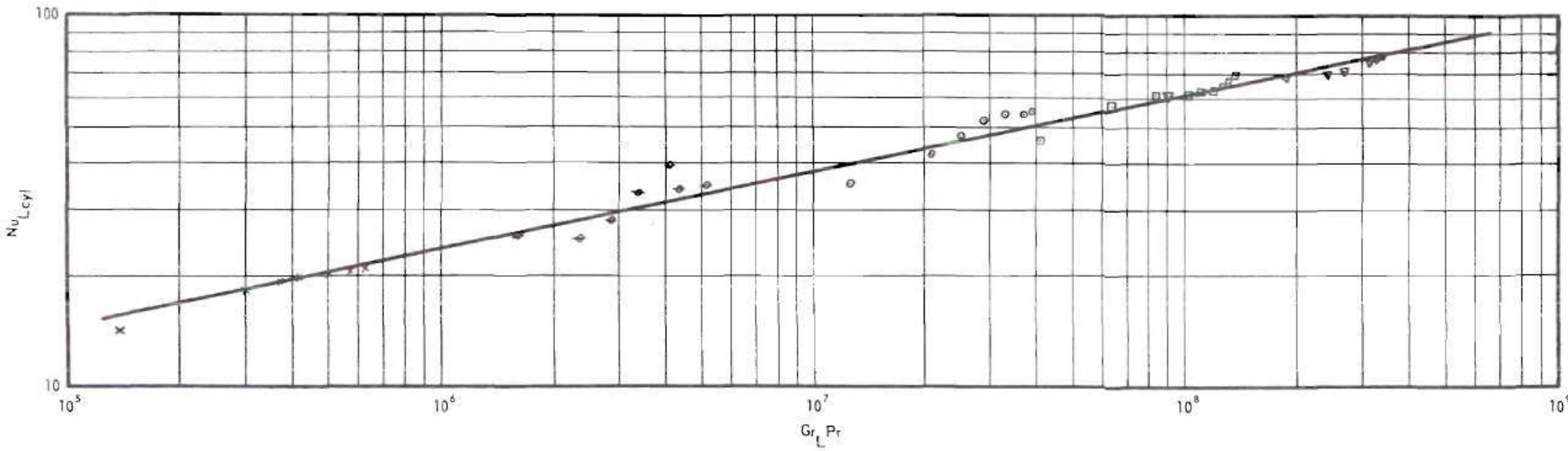


Figure 12. Experimental Data Nu_{Lcyl} Versus $Gr_L Pr$.

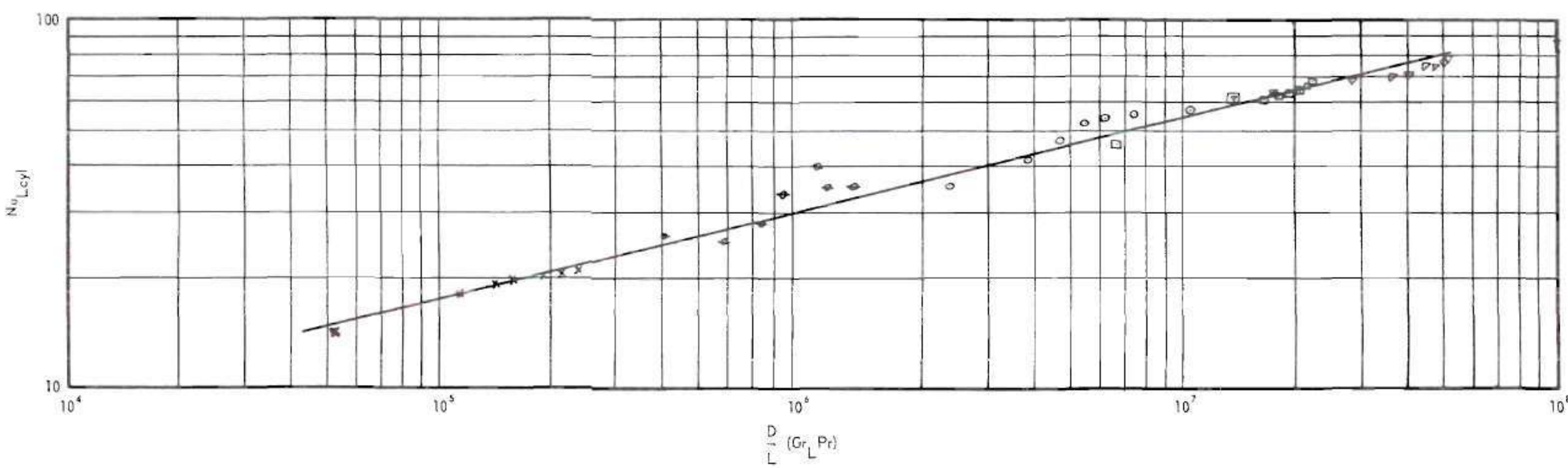


Figure 13. Experimental Data Nu_{Lcyl} Versus D/L $(Gr_L Pr)$.

APPENDIX C

NOMENCLATURE

A_c	Area of cylinder surface	sq. ft.
D	Diameter of cylinder	ft.
Gr_L	$YL^3(t_c - t_a)$, Grashof number based on cylinder length.	dimensionless
h	Heat transfer coefficient	$\frac{Btu}{hr\ ft^2\ {}^oF}$
K	Thermal conductivity of air at mean air temperature	$\frac{Btu}{hr\ ft\ {}^oF}$
L	Length of cylinder	ft.
$Nu_{L\ cyl}$	$\frac{hL}{K}$, Average Nusselt number for cylinder	dimensionless
$Nu_{L\ f.p.}$	Average Nusselt number for flat plate	dimensionless
p	Power dissipated in heater	watts
Pr	Prandtl number of air determined at mean air temperature	dimensionless
q_c	Heat transferred by convection	Btu/hr
t	Temperature	^oF
t_a	Temperature of Ambient air	^oF
t_c	Temperature of cylinder surface	^oF
t_i	Temperature at top of insulation	^oF
t_m	$\frac{t_a + t_c}{2}$, Mean air temperature	^oF
t_s	Temperature at inside surface of shield	^oF

NOMENCLATURE

(concluded)

γ	$\frac{g\beta}{\nu^2}$, property of air determined at mean air temperature	$\frac{1}{ft^3\ {}^oF}$
ϵ_c	Emissivity of cylinder surface	dimensionless
ξ	$\frac{2\ 3/2}{Gr_x^{\frac{1}{4}}}\ \frac{L}{r_o}$ independent variable occurring in the solution of Sparrow and Gregg	dimensionless
σ	Stefan-Boltzman constant	$\frac{Btu}{hr\ ft^2\ {}^oR^4}$

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