

Sub-megahertz frequency stabilization of a diode laser by digital laser current modulation

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Digital laser current modulation (DLCM) is a convenient laser stabilization scheme whose major advantages are simplicity and inexpensiveness of implementation. However, there is a tradeoff between the SNR of the error signal and the laser linewidth due to the direct laser frequency modulation. In this paper, we demonstrated that DLCM can reduce the FWHM linewidth of a tunable diode laser down to 500 kHz using the modulation transfer spectrum of D_2 line of a ^6Li atomic vapor. For this purpose, a theoretical model is provided to analyze the DLCM-based modulation transfer spectrum. From the analysis, we experimentally explored the modulation effect on the DLCM spectrum to minimize the laser linewidth. Our result shows the optimized DLCM can stabilize a diode laser into the sub-megahertz regime without requiring acousto-optic and electro-optic modulators. © 2015 Optical Society of America

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1. INTRODUCTION

Active frequency stabilization of a tunable single-frequency diode laser plays an important role in many atomic physics experiments, such as magneto-optical trap and absorption imaging [1,2]. A broad range of experimental setups employ an atomic vapor cell for locking the laser frequency to a Doppler-free spectrum, where the modulated pump and/or probe beams are applied to generate the error signals for locking. The modulation techniques are quite varied, including laser current modulation spectroscopy (LCMS) [3], electro-optic frequency modulation spectroscopy (EOFMS) [4–6], acousto-optic modulation transfer spectroscopy (AOMTS) [7–9], and their recent variants such as acousto-optic frequency modulation spectroscopy (AOFMS) [10], piezo frequency modulation spectroscopy (PFMS) [11], and so on.

EOFMS is one of the most widely used methods to stabilize the laser frequency, requiring expensive EOMs and radio-frequency (RF) devices to generate the sideband frequencies of the probe beam. AOMTS modulates the frequency of the pump beam instead of the probe beam to reduce the noise in the detected signal. It usually stabilizes the laser linewidth at the megahertz (MHz) regime due to the stability of the voltage-controlled oscillator (VCO) of the acousto-optic modulation (AOM) driver [10]. To improve the locking stability of AOMTS down to the sub-MHz level, a customized wideband

VCO is implemented [9]. For a tunable diode laser, both EOFMS and AOMTS can be replaced by diode current modulation for generating sideband frequencies [12]. Comparing to AOMTS and EOFMS, digital laser current modulation (DLCM) is a simpler method, but it is challenging to obtain sub-MHz laser frequency stability because of two difficulties: first the laser frequency noise will be increased by directly modulating the laser current, and second the laser linewidth will be broadened by sideband frequencies. To address these problems, we simulated the DLCM-based modulation transfer spectrum to estimate the modulation effect, which helps us to narrow the laser linewidth down to 500 kHz by optimizing the modulation frequency, the modulation amplitude, and the servo bandwidth.

The paper is organized as follows. In Section 2, we describe the experimental setup that stabilizes a Toptica Photonic TA laser with the DLCM-based Doppler-free spectroscopy of a hot lithium vapor cell. In Section 3, we present a theoretical model to simulate the DLCM spectrum. In Section 4, we present the measured DLCM spectrum and laser linewidth to determine the optimized modulation parameters. Finally, we summarize the results and discuss the relevant applications.

2. EXPERIMENTAL SETUP

DLCM is applied to an external cavity diode laser with a tapered amplifier (ECDL1, Toptica Photonic TA Pro) as shown

in Fig. 1. A 1.6 mW beam from the ECDL1 is divided by a polarizing beamsplitter: one beam for the Doppler-free spectroscopy of a ${}^6\text{Li}$ vapor cell and the other for generating the beating signals with an offset-locking laser (ECDL2, Toptica Photonic DL 100 Pro) to measure the laser linewidth. For the DLCM spectrum from the vapor, the retro-reflected pump beam is used as the probe beam and detected by a homemade photodetector (PD), which has 10^5 gain and 6 MHz bandwidth with electronic noise of $300 \text{ nV} \cdot \text{Hz}^{-1/2}$ at 100 kHz. The lithium vapor cell is heated to $340 \pm 1^\circ\text{C}$, and the resonant absorption ratio of the ${}^6\text{Li}$ D₂ line is about 0.5. In order to reduce the magnetic field generated by the heating coil, a single-core heating element tape is uniformly wrapped and divided into two layers with opposite current directions. The measured magnetic field inside the cell is $100 \pm 10 \text{ mG}$. The fluctuation of the magnetic field corresponds to about 30 kHz Zeeman shift of the ground state $2^2S_{1/2}$, $F = 3/2$ of ${}^6\text{Li}$, which cause negligible effects for the locking at hundreds of kilohertz (kHz) level. To compare the laser locking performance between DLCM and AOFMS, the laser beam can also be forwarded to a double-passed AOM for AOFMS. The digital laser current modulation is realized by modulating the diode current through a DigiLock 110 module (Toptica Photonic). The clock rate of this module is 100 MHz, providing the maximum modulation frequency of 25 MHz and the maximum servo loop bandwidth of 10 MHz. For DLCM, the modulation sideband frequencies exist in both the pump and probe beams, generating the DLCM spectrum by four-wave mixing in the atomic vapor [13,14]. The spectrum is then electronically demodulated by the DigiLock module for locking error signal. A digital low-pass filter (LPF) with the bandwidth of the half of the modulation frequency is put before the proportional–integral–derivative (PID) controllers to remove the high-frequency components. Such high-frequency components cannot be suppressed by the servo loop according to the Nyquist–Shannon sampling theorem [15]. After the LPF, the error signal is sent to two PID modules separately, where

PID1 controls the piezoelectric transducer (PZT) of the ECDL1 with the cut-off frequency of 1 kHz, and PID2 adjusts the current of the diode.

3. SPECTRUM MODELING

The sideband frequencies of the DLCM are generated by modulating the diode current with frequency ω_m , giving the electric field of the laser output as

$$\begin{aligned} E(t) &= \frac{E_0}{2} \exp[i(\omega_0 t + \beta \sin(\omega_m t))] + \text{c.c.} \\ &= \frac{E_0}{2} \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{i(\omega_0 + n\omega_m)t} + \text{c.c.}, \end{aligned} \quad (1)$$

where $\beta = \Delta\omega/\omega_m$ is the modulation index, $\Delta\omega$ is the modulation amplitude, and $J_n(\beta)$ is the n th-order Bessel function. DLCM modulates both the pump and probe beams for generating modulation transfer spectrum through a four-wave mixing process in an atomic vapor cell. This is similar to AOMTS [7,14], where the amplitude modulation of the probe beam is generated from the frequency modulation of the pump beam through a four-wave mixing process in the atomic vapor [8,9]. For efficient four-wave mixing, ω_m must be within the frequency range of the Doppler-free feature.

When both the pump and probe beams are modulated, the four-wave mixing process involves multiple frequency sidebands. The amplitude-modulated probe signal recorded by the photodetector is described by [14]

$$\begin{aligned} S &= C \sum_{m,m',n,n'=-\infty}^{+\infty} J_m(\beta) J_{m'}(\beta) J_n(\beta) J_{n'}(\beta) \\ &\times \text{Re} \left\{ \frac{\exp[i(m-m'+n-n')\omega_m t]}{\Gamma + i(n-n')\omega_m} \right. \\ &\left. \times \frac{1}{2\Gamma - 2i\Delta - i(m'+2n'-n)\omega_m} \right\}, \end{aligned} \quad (2)$$

where Δ is the laser frequency detuning from the resonant frequency, Γ is the spectrum linewidth, $J_n(\beta)$ and $J_{n'}(\beta)$ are the amplitude coefficients of two sidebands of the pump beam, $J_{m'}(\beta)$ and $J_m(\beta)$ are the coefficients of the probe beam, and J_n , $J_{n'}$, and $J_{m'}$ represent the incoming waves of the four-wave mixing process. $J_m(\beta)$ represents the beating wave with the outgoing wave. When the modulation index is much larger than the unity, Eq. (2) includes many terms. For example, a modulation index of 10 makes 29 Bessel terms with more than 1% contribution to the DLCM signal.

In practice, only the first-order of the modulation frequency in the DLCM signal is electronically demodulated for locking, so we restrict $(m-m') - (n-n') = \pm 1$. The first-order signal of Eq. (2) can be simplified as a combination of Lorentzian and dissipative profile [7,16] as follows

$$\begin{aligned} S &= C \sum_{m',n,n'=-\infty}^{+\infty} \frac{J_{m'}(\beta) J_n(\beta) J_{n'}(\beta)}{\Gamma^2 + ((n-n')\omega_m)^2} \\ &\times \left[(J_{m_+}(\beta) + J_{m_-}(\beta)) L_{\frac{m'}{2} + n' - \frac{n}{2}}(\Delta) \cos(\omega_m t + \phi) \right. \\ &\left. + (J_{m_+}(\beta) - J_{m_-}(\beta)) D_{\frac{m'}{2} + n' - \frac{n}{2}}(\Delta) \sin(\omega_m t + \phi) \right], \end{aligned} \quad (3)$$

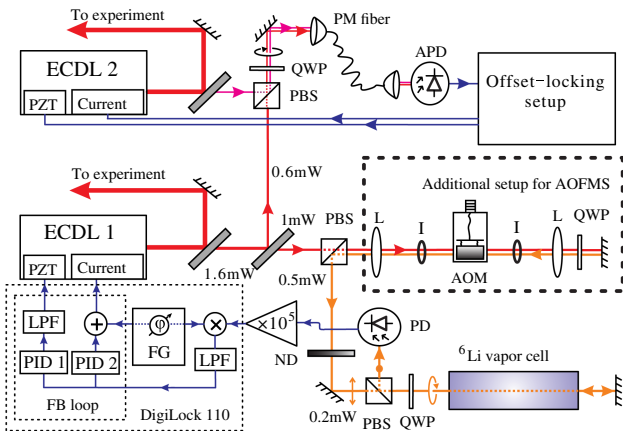


Fig. 1. Experimental setup of the DLCM laser locking. APD, avalanche photodetector; FB loop, feedback loop; FG, function generator; I, iris; L, lens; ND, neutral density filter; PM fiber, polarization-maintaining single-mode fiber; PBS, polarizing beamsplitter; PD, photodetector; PZT, piezoelectric transducer; QWP, quarter waveplate.

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142 where

$$L_n(\Delta) = \frac{\Gamma^2}{\Gamma^2 + (\Delta - n\omega_m)^2}, \quad (4)$$

$$D_n(\Delta) = \frac{\Gamma(\Delta - n\omega_m)}{\Gamma^2 + (\Delta - n\omega_m)^2},$$

143 where $m_{\pm} = m' + n' - n \pm 1$ for the $\pm\omega_m$ sidebands of the
 144 demodulated signal, and ϕ is the phase difference between
 145 the DLCM signal and the local oscillation.

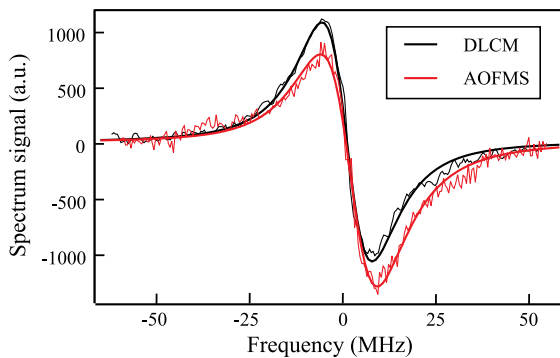
146 **4. MEASUREMENT AND RESULT**

147 To optimize the DLCM locking, we first optimize the modu-
 148 lation parameters, including the modulation index and the
 149 modulation frequency, to improve the SNR of the DLCM
 150 spectrum. Then we minimize the laser linewidth by tuning
 151 these modulation parameters. Finally we studied long-term sta-
 152 bility of the locking.

153 **A. DLCM Spectrum Measurement**

154 The DLCM spectrum is obtained on the $2^2S_{1/2}, F = 3/2$ to 2
 155 $^2P_{3/2}$ transition of ^6Li . The pump beam is 200 μW with
 156 0.8 mm beam waist. The retro-reflected beam is used as the
 157 probe beam, which has 100 μW due to the vapor absorption.
 158 In Fig. 2, a typical spectrum with $\Delta\omega = 0.8$ MHz and
 159 $\omega_m = 100$ kHz is represented by the thin black curve. The
 160 laser current modulation amplitude $\Delta\omega$ is calibrated by addi-
 161 tional AOFMS with known frequency modulation amplitudes.
 162 The spectrum curve is taken at 4 kHz sampling rate by scan-
 163 ning the PZT of ECDL1 in 50 ms to reduce the PZT thermal
 164 drift. The thick black curve is the fitting using Eq. (3), where
 165 the fitted linewidth $\Gamma = 2\pi \cdot 10.9$ MHz, in agreement with
 166 the combination of the 5.9 MHz natural linewidth of ^6Li D_2
 167 transition and the 4.4 MHz separation between three hyperfine
 168 states of the excited levels.

169 We compared the DLCM spectrum with the AOFMS
 170 spectrum by implementing an independent acousto-optic
 171 (AO) frequency modulation. The thin red curve in Fig. 2
 172 shows the AOFMS spectrum with the same modulation param-
 173 eters as the DLCM one. We found that the SNR of the DLCM
 174 spectrum is about 3 times higher than the AOFMS one. It
 175 is also shown that the DLCM spectrum has much lower

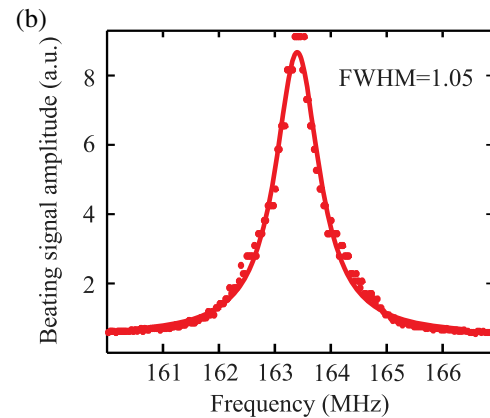
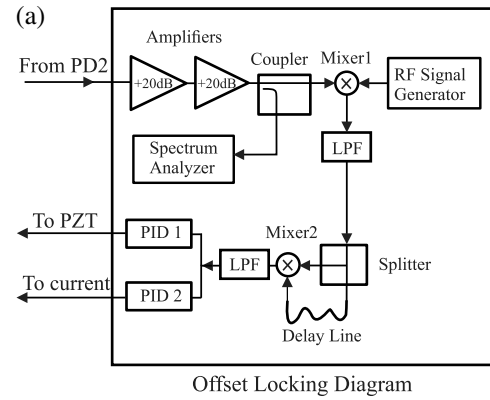


F2:1 **Fig. 2.** DLCM and AOFMS spectra. Thin black (red) curves, mea-
 F2:2 sured spectrum of DLCM (AOFMS). Thick black (red) lines, fitting
 F2:3 results with the fixed $\beta = 8$, $\omega_m = 100$ kHz, and $\phi = 0.91\pi$.

residual amplitude modulation (RAM) effect than the 176
 AOFMS one, indicated by the frequency asymmetry of the 177
 spectrum. These findings imply that DLCM, even though 178
 the laser itself is frequency-modulated, may result in a better 179
 locking performance than AOFMS in certain modulation 180
 conditions. 181

182 **B. Laser Linewidth Measurement**

183 To measure the laser linewidth, the DLCM-stabilized laser 183
 beats with the other diode laser ECDL2 locked by the offset- 184
 locking [18], as shown in Fig. 3(a). Two laser beams are 185
 coupled into a PM fiber to improve the SNR of the beating 186
 signal. A high-speed APD (Newport 877) with two low-noise 187
 RF amplifiers (Minicircuit ZX60-3018G+) is applied to probe 188
 the beating signal, where 1 of the beating signal is used for 189
 the linewidth measurement and the rest signal for the offset- 190
 locking. The cut-off frequency of the LPF between Mixer 2 191
 and the two feedback PIDs is set to 270 kHz which determines 192
 the bandwidth of the offset-locking loop. The offset-locking 193
 beating method is a variant of the delayed self-heterodyne 194
 scheme for measuring the laser linewidth [21]. The FWHM 195
 linewidth of the beating signal reflects residual high-frequency 196
 fluctuations of the individual laser beyond the bandwidth of the 197
 offset-locking loop [18]. In our case the linewidth broadening 198
 due to frequency modulation is small compared to the initial 199



F3:1 **Fig. 3.** Offset locking setup and the power spectrum of the beating
 F3:2 signal; (a) offset-locking scheme; (b) typical power spectrum of the
 F3:3 beating signal with the fitted FWHM is 1.05 MHz, showing the indi-
 F3:4 vidual laser linewidth 525 kHz. Beating signal is taken by a GW Instek
 F3:5 GSP-730 spectrum analyzer with 300 kHz resolution bandwidth and
 F3:6 averaged over 200 sweeps of 500 ms each.

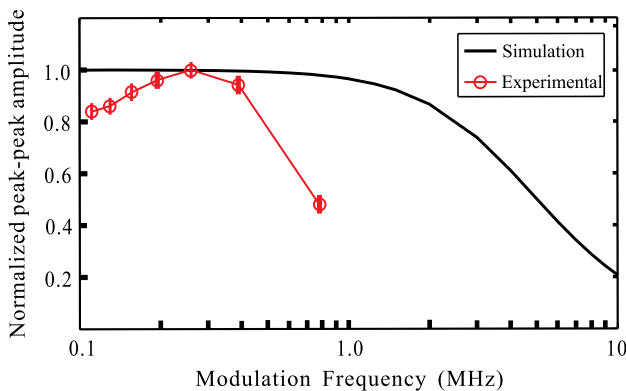
200 laser linewidth; the beating signal can be well-described by a
 201 Lorentzian profile as shown in Fig. 3. When the profile is
 202 Lorentzian, the laser FWHM linewidth is then given by the
 203 half of the beating FWHM linewidth [22].

204 C. Optimization of DLCM

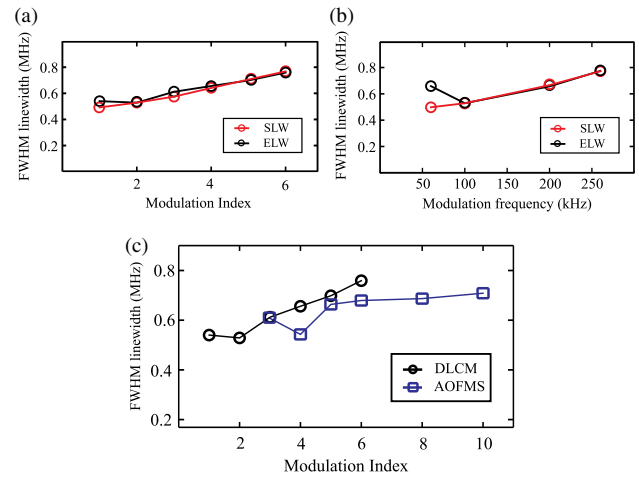
205 When the modulation index decreases, the strength of the
 206 higher-orders' sideband decreases, resulting in a fast decay of
 207 the amplitude of the spectrum. The dependence of the ampli-
 208 tude of the DLCM spectrum on the modulation frequency ω_m
 209 is simulated and measured as shown in Fig. 4 by fixing $\Delta\omega$
 210 at 1 MHz.

211 The simulation predicts a cut-off frequency of the amplitude
 212 at around 3.0 MHz, while our measurement shows that the
 213 decay appears at 500 kHz. The difference between the simu-
 214 lation and the measurement arises from the fact that the theo-
 215 retical model does not include gas dynamics of atomic vapor
 216 [7,9,17]. Our experiment is implemented with 340°C lithium
 217 vapor, and the most probable speed of lithium atoms is about
 218 1300 m/s. The gas dynamic effect significantly changes the
 219 atom-light interaction region when the beam waists of the
 220 pump and probe beams are small. With our beam diameter
 221 of 1.6 mm, based on a heuristic argument that the cut-off fre-
 222 quency is determined by the timescale where excited atoms es-
 223 cape the optical field of the pumped beam, the cut-off frequency
 224 will approximately at 800 kHz [9]. This estimation qualitatively
 225 explains the observed fast decay in our experiments. To search
 226 the optimal modulation index, we limit the modulation fre-
 227 quency lower than the decay threshold of 250 kHz.

228 We choose the modulation frequency $\omega_m = 100$ kHz to
 229 study the dependence of laser linewidth on the modulation in-
 230 dex as shown in Fig. 5(a). For this small modulation frequen-
 231 cy compared to the laser linewidth, we find that all the beating
 232 signal can be well-fitted by the Lorentzian profile when the
 233 modulation index β is less than 6. The laser FWHM linewidth
 234 shows a minimum value of 525 kHz at $\beta = 2$. In principle,
 235 the laser linewidth should decrease monotonically with the
 236 decrease of the modulation index β for a fixed modulation fre-
 237 quency. However, when β decreases, the amplitude of the
 238 DLCM spectrum also decreases. For $\beta = 2$, we measured the
 239 peak-peak amplitude of the spectrum decreases to 575 (a.u. in
 240 DigitLock) corresponding to the 10.9 MHz absorption line-
 241 width. To stabilize the laser down to 500 kHz, the residue error



F4:1 **Fig. 4.** Dependence of the amplitude of the DLCM signal on the
 F4:2 modulation frequency with $\Delta\omega = 1$ MHz.



F5:1 **Fig. 5.** Laser FWHM linewidth in terms of different modulation
 F5:2 parameters; (a) dependence of the linewidth on the modulation
 F5:3 index β for $\omega_m = 100$ kHz; (b) dependence of the linewidth on the
 F5:4 modulation frequency ω_m for $\beta = 2$; (c) linewidth comparison between
 F5:5 DLCM and AOFMS for $\omega_m = 100$ kHz. Black (red) open circles,
 F5:6 experimental (simulated) linewidth for DLCM, where the simulation
 F5:7 is calculated by the standard frequency modulated linewidth. Blue
 F5:8 square, measured linewidth for AOFMS. Curves, simple joint lines.

242 signal is around 26 (a.u.), which approaches to the system noise
 243 limit of 12 (a.u., root-mean square value) in the DLCM spec-
 244 trum. This analysis explains our observation that the further
 245 decreasing of β below 2 will increase the laser linewidth.

246 The modulation index $\beta = 2$ is then used to search the op-
 247 timal value of the modulation frequency below the 250 kHz
 248 region. The dependence of the FWHM linewidth on the
 249 modulation frequency shows a minimum value at 100 kHz in
 250 Fig. 5(b). When the modulation frequency is below 100 kHz,
 251 the increase of the laser linewidth is induced by the decreased
 252 bandwidth of the servo loop.

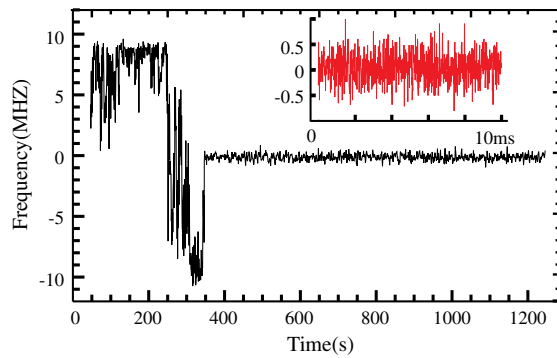
253 We also compared the locking performance between the
 254 DLCM and the AOFMS as shown in Fig. 5(c). While the laser
 255 linewidth by the AOFMS locking almost has no dependence
 256 on the modulation index, it is limited the SNR of the spectrum
 257 as discussed in Section 4, Subsection A. The minimized laser
 258 linewidth is around 550 kHz, which shows that the DLCM
 259 method is comparable with the AOFMS one.

260 D. Long-Term Stability

261 In order to measure the frequency stability of DLCM in the
 262 long-term, we record the error signal before and after switching
 263 on the servo loop as shown in Fig. 6. The fluctuation of the
 264 laser frequency relative to the $2^2S_{1/2}$, $F = 3/2$ to $2^2P_{3/2}$ tran-
 265 sition of ^6Li is estimated to be ± 10 MHz in the free-run stage.
 266 With the servo loop on, the RMS fluctuation linewidth is re-
 267 duced to 0.5 MHz over more than 20 min. It demonstrates that
 268 the laser frequency noise is suppressed significantly in the long-
 269 term by the DLCM locking.

270 5. SUMMARY

271 A convenient and cost-efficient method is presented to stabilize
 272 the frequency of a tunable diode laser using DLCM-based



F6:1 **Fig. 6.** Error signal of locking over the long-term. Inset, error signal
F6:2 in the short timescale after locking.

273 modulation transfer spectroscopy of an atomic vapor cell. The
274 modulation transfer spectrum is simulated in detail by a theo-
275 retical model, and the high SNR of the spectrum is obtained
276 by optimizing the modulation amplitude and frequency. By
277 frequency-beating the stabilized laser and the offset-locking
278 laser, we confirm that the laser linewidth can be reduced down
279 to 500 kHz. This work realizes a digital current modulation
280 scheme to stabilize the frequency of a tunable diode laser in
281 sub-MHz regime without requiring additional optical modula-
282 tors and can be easily adopted by many atomic physics and
283 spectroscopy experiments.

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Supplementary Material

This article has the following supplementary material items associated with it.