

## 11 Appendix

For this journal submission, we have expanded our workshop version of the paper. Below is a list of the additional contributions that are not presented in the workshop version:

1. We have added one more dataset (synthetic): The description of the dataset can be found in section 6.1. We show some extra result based on this dataset in section 6.5 and section 6.4.
2. We have added Theoretical analysis of the algorithm: FS<sup>3</sup> ranks the subgraph patterns based on the expected support ( $support_a$ ). In this section 5.6, we analyze the expected value of  $support_a$  for a  $p$ -subgraph pattern  $g$ .
3. We have added proof of the following claims:
  - FS<sup>3</sup>'s random walk is ergodic: Proof can be found in section 5.2.
  - $s_1(g) \geq support(g)$  and  $s_2(g) \geq support(g)$ : Proof can be found in section 5.1.
  - The random walk of FS<sup>3</sup> achieves the target probability distribution, which is proportional to the chosen scoring function ( $s_i$ ): Proof can be found in section 5.2
4. Added more experiments:
  - Correlation between actual support and scores: Here in this version, we analyze the experimental results of this experiment in detail. In this experiment, we use PS and Mutagen dataset and mine a collection of frequent patterns for a suitable size value using GTrieScanner. For each of the frequent patterns, we also compute their score value,  $s_1$  and  $s_2$ , which we have used for constructing the target distribution of MCMC sampling. Our objective is to analyze how good our scoring functions are as a proxy of actual support of a graph. Detail analysis can be found in section 6.3
  - Scalability with size  $p$ : The execution time of FS<sup>3</sup> has three components: sampling time, canonical code generation time, and queue insertion time. In this experiment, we check how these times vary as we vary the desired size of the subgraphs to be sampled( $p$  value). Results can be found in 6.5.
  - Impact of target distribution and queue size: FS<sup>3</sup>'s MCMC sampling uses  $s_1$  or  $s_2$  score to construct its target distribution. In this experiment, we validate the impact of these choices by comparing their performance with a case, where the target distribution is uniform, i.e., each of the  $p$ -subgraphs of a database graph  $G_i$  has equal likelihood to be visited, that is the score of any  $p$ -subgraph is 1, a constant. Experimental result can be found in section 6.6
  - Impact of  $k$  on the algorithm: We also study the performance of FS<sup>3</sup> for different choices of  $k$  value in top- $k$ . Here we show that, for the entire range of  $k$  values, the performance remains almost constant. Experimental result can be found in section 6.7

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- Choosing iteration count: In this experiment, we show how we know how many iterations would yield a representative set of frequent pattern? The detailed experimental description and result can be found in section 9.
  - Mixing rate of random walk: In this experiment, we present our result on spectral gap experiment. Details can be found in Section 7.
  - Percentage of Acceptance: Here, we show our proposal distribution has a high likelihood of acceptance. Details of this experiment can be found in Section 8.