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Branch and Price Solution Approach for Order Acceptance and Capacity Planning in Make-to-Order Operations

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FLORIDA INTERNATIONAL UNIVERSITY

Miami, Florida

BRANCH AND PRICE SOLUTION APPROACH FOR ORDER ACCEPTANCE AND CAPACITY PLANNING IN MAKE-TO-ORDER OPERATIONS

A dissertation submitted in partial fulfillment of the

requirements for the degree of

DOCTOR OF PHILOSOPHY

in

INDUSTRIAL AND SYSTEMS ENGINEERING

by

Siddharth D. Mestry

2010

To: Dean Amir Mirmiran
College of Engineering and Computing

This dissertation, written by Siddharth D. Mestry, and entitled Branch and Price Solution Approach for Order Acceptance and Capacity Planning in Make-to-Order Operations, having been approved in respect to style and intellectual content, is referred to you for judgment.

We have read this dissertation and recommend that it be approved.

Martha A. Centeno

José A. Faria

Purushothaman Damodaran, Co-Major Professor

Chin-Sheng Chen, Co-Major Professor

Date of Defense: March 25, 2010

The dissertation of Siddharth D. Mestry is approved.

Dean Amir Mirmiran
College of Engineering and Computing

Interim Dean Kevin O'Shea
University Graduate School

Florida International University, 2010

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DEDICATION

I dedicate this dissertation to my wife Vrunda and to my parents for their tremendous patience and understanding. Without their support, the completion of this work would not have been possible.

And, to my son Arav for giving me the strength to push myself to the limits of my capabilities and potential, to help me see the light at the end of the tunnel, to believe in the future with all our dreams fulfilled.

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I am greatly indebted to Dr. Purush Damodaran for being an excellent academic advisor, and a tremendous mentor. He has not only helped me in my scholastic pursuits but has also given me valuable guidance in personal matters. I owe him this dissertation and much more.

I would like to thank Dr. Chin-Sheng Chen for the confidence he showed in my capabilities, for his practical insights and for sharing his limitless knowledge with me. I will forever remember the discussions which we had in his office.

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ABSTRACT OF THE DISSERTATION

BRANCH AND PRICE SOLUTION APPROACH FOR ORDER ACCEPTANCE AND CAPACITY PLANNING IN MAKE-TO-ORDER OPERATIONS

by

Siddharth D. Mestry

Florida International University, 2009

Miami, Florida

Professor Chin-Sheng Chen, Co-Major Professor

Professor Purushothaman Damodaran, Co-Major Professor

The increasing emphasis on mass customization, shortened product lifecycles, synchronized supply chains, when coupled with advances in information system, is driving most firms towards make-to-order (MTO) operations. Increasing global competition, lower profit margins, and higher customer expectations force the MTO firms to plan its capacity by managing the effective demand. The goal of this research was to maximize the operational profits of a make-to-order operation by selectively accepting incoming customer orders and simultaneously allocating capacity for them at the sales stage.

For integrating the two decisions, a Mixed-Integer Linear Program (MILP) was formulated which can aid an operations manager in an MTO environment to select a set of potential customer orders such that all the selected orders are fulfilled by their deadline. The proposed model combines order acceptance/rejection decision with detailed scheduling. Experiments with the formulation indicate that for larger problem sizes, the computational time required to determine an optimal solution is prohibitive. This

formulation inherits a block diagonal structure, and can be decomposed into one or more sub-problems (i.e. one sub-problem for each customer order) and a master problem by applying Dantzig-Wolfe's decomposition principles. To efficiently solve the original MILP, an exact Branch-and-Price algorithm was successfully developed. Various approximation algorithms were developed to further improve the runtime. Experiments conducted unequivocally show the efficiency of these algorithms compared to a commercial optimization solver.

The existing literature addresses the static order acceptance problem for a single machine environment having regular capacity with an objective to maximize profits and a penalty for tardiness. This dissertation has solved the order acceptance and capacity planning problem for a job shop environment with multiple resources. Both regular and overtime resources is considered.

The Branch-and-Price algorithms developed in this dissertation are faster and can be incorporated in a decision support system which can be used on a daily basis to help make intelligent decisions in a MTO operation.

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α_j	:	Dual value of j^{th} convexity constraint representing job j
b_{rts}	:	Number of hours resource r is available in source s of time period t
c_{rs}	:	Processing cost per hour of resource r in source s
d_j	:	Due-date for job j
ε_p^o	:	Precedence error length between operation o and o+1
γ	:	Optimality tolerance used to prune branch and bound tree
J	:	Set of jobs
l_{ts}	:	Length of source s in time period t
λ_j^k	:	Binary variable in the restricted master problem
O_j	:	Set of operations in job j
p_{jor}	:	Processing time needed for completing operation o of job j on resource r
q_j	:	Selling price of job j
R	:	Set of resources
S	:	Set of sources
T	:	Set of time periods
τ	:	smallest time unit for which a operation has to be processed on a resource, if processed at all
θ_j	:	Convex combination of columns for job j
U_j	:	Binary decision variable indicating acceptance or rejection of job j
w_{rts}	:	Capacity constraint dual value
X_{jorts}	:	Continuous decision variable for the number of hours operation o of job j is

to be processed on resource *r* in source *s* of time period *t*

Y_{jorts} : Binary decision variable indicating if operation *o* of job *j* is processed on resource *r* in source *s* of time period *t*

Z_{RMP}^{BUB} : Best upper bound of all the unexplored nodes in the branch and bound tree for the restricted master problem

Z_{SP}^{j} : Objective function value of the j^{th} sub-problem

Z_{IP} : Objective function value of the best known integer solution

Z_{RMP}^{LP} : Objective function value of the linear relaxation of the restricted master problem

LIST OF ACRONYMS

BeFS	:	Best First Search
BPS1	:	Branch and Price Strategy 1
BPS2	:	Branch and Price Strategy 2
DC Ratio	:	Demand-to-Capacity Ratio
DFS	:	Depth First Search
dtu	:	Discretized time unit
MTO	:	Make-to-Order
MTS	:	Make-to-Stock
RMP	:	Restricted Master Problem
SP	:	Sub-Problem

1. INTRODUCTION

1.1. Background

Manufacturing systems are usually classified along two dimensions: 1) the internal organization of the production system and 2) logistic product/market relations [1]. The former is concerned with the internal structure of the manufacturing and assembly system. The three basic structures are dedicated flow lines, job shops, and on-site manufacturing. In dedicated flow lines, a number of operations have to be carried out on every customer order following the same process route. Job shops on the other hand can manufacture a broad spectrum of products, each product having its own process route. On-site (or fixed location) manufacturing is one in which all the equipments have to be moved to the product manufacturing location. Examples include bridge construction, ship building, etc.

The second classification dimension (i.e. logistic product/market relations) is operational policies for fulfilling customer orders. The three basic operational modes are: 1) make-to-stock, 2) make-to-order, and 3) hybrid [2]. Make-to-stock (MTS) is a philosophy to fill customer orders by stocking finished goods for immediate delivery. MTS is characterized by high-volume production and is normally followed when the firms are product-focused with relatively less component level customization. The second operational mode is make-to-order (MTO). A MTO firm starts working on an order only after it has been placed by the customer. This policy is advantageous when the end product is customer specific, with high level of customization. This policy allows a high degree of flexibility and the products manufactured are one-of-a-kind or in small batches. Typical examples of MTO system include print shop, semiconductor manufacturing, engineering tooling, special equipments, and large hydraulic pumps, DNA sequencing, laundry service, etc. Most of the MTO firms are process-focused, as the products manufactured share the same kind of operations but differ in the design details. Firms can also have a hybrid of MTS and MTO system for different products that are manufactured at the same production facility. Flow shops and job shops can be operated under MTS or MTO policies or the combination of the above two.

In the current business scenario, customers demand products with a high level of customization. The focus is on innovation and customer satisfaction, leading to shortened product development life cycle. These trends compel the manufacturers to remain agile and flexible, leading to an increase in the appeal and popularity of make-to-order or a hybrid operational philosophy [3]. MTO systems are not only used for unique product manufacturing but are also very efficient in producing greater product variety at lower cost [4].

The major difference between MTO and MTS is the way in which customer demand is handled. By definition, MTS holds finished goods inventory to meet the customer demand. The focus is on anticipating the demand and planning to meet the demand. The main issues that need to be addressed are inventory planning, lot size determination and demand forecasting. Since MTO is characterized by back orders with zero inventories as each customer order is unique and cannot be manufactured in advance, the only way for managing the effective demand is by holding its capacity in inventory. The production planning focus is on order execution and the competitive priority is shorter delivery lead-time, and adherence to due-dates. Hence the most important operating issue in MTO is the effective and efficient use of available capacity to meet customer demands.

Capacity planning determines the resource requirements of an organization to sustain a given demand over a planning horizon. There are three tiers of capacity planning based on their planning horizon. Long term capacity planning relates primarily to strategic issues involving the firm's major production facilities with a planning horizon anywhere between three to five years. It focuses on determining facility locations, plant capacities, division of new and existing product lines, technology and transferability of process to other products, subject to demand forecast and availability of investment funds. It is used to determine major supplier's plans and their vertical integration; principal operation modes and production methods. The fundamentals of long-term capacity planning are mostly the same for both MTO and MTS operations.

Medium term capacity planning or aggregate planning focuses on setting monthly or quarterly resource requirements for each plant for typically a one-year planning horizon. The process includes developing, analyzing and maintaining a preliminary, approximate schedule of the overall operations of an organization. It decides on workforce level, raw materials and inventory policy by product group and department. Based on sales forecast, it generates production capacity plans for (1) labor-employment level (i.e. lay-offs, hiring, recalls, overtime, and part-timer), (2) inventory policy, (3) utility requirements,

(4) facility modifications, (5) outsourcing, and (6) major material supply contracts. Capacity requirements may vary from period to period in their regular time labor, overtime labor, inventory and subcontracting.

Two conventional aggregate planning approaches for MTS are: (1) matching demand and (2) level capacity. With the matching demand approach, production capacity in each time period varies to exactly match the aggregate demand as forecasted for that time period, by hiring and laying-off workers. With the level capacity approach, production capacity is held constant over the planning horizon; the difference between the constant production rate and the varying demand rate is made up by inventory, backlog, overtime labor, part time labor, temporary labor, and/or sub-contracting. An MTO operation usually adopts a hybrid approach of both. On one hand, it needs to maintain a certain level of production capacity for its core competency. On the other, it cannot leverage on inventory, as every order is a backorder and it requires customization. The common practice is to maintain a minimum level of production capacity, and liberally rely on overtime and subcontracting to adjust capacity and to accommodate demand fluctuation. Aggregate plans serve as foundation for future short term capacity plans.

Short term capacity planning sets a daily or weekly capacity plan for a planning horizon long enough to accommodate each order's lead time. The objective of short-term capacity planning is to ensure an appropriate match between the resources availability and the capacity requirement for a production plan at the work center level [5]. For a MTO operation, it has to specify resources requirement of each labor and machine type for each customer order at its component level. Each customer order first is translated into internal orders and detailed work orders, which are then summarized into a load schedule (time-phased capacity requirements) by labor and/or equipment, in coordination with materials arrival. A typical MTO operation routinely considers the use of alternative sources such as overtime and outsourcing, in order to meet work order's due date, which is a critical issue for MTO operations.

In most MTO operations, meeting due-dates is considered a hard constraint. With multiple orders and bids competing for common resources, meeting these deadlines is the first criterion used to reject or accept a backorder as due-date feasibility depends on the availability of the time-phased resource capacity. Reliable due-dates require a continuous coordination between marketing/sales and manufacturing departments during the bidding phase. Zijm [1] mentions that it is important to integrate these decisions.

In practice, decisions on order acceptance and production planning are often functionally separated. The objective of the sales department is to bring as much revenue as possible. The sales department will tend to accept all orders, regardless of the available capacity, because their goal is turnover. Production is concerned with limited capacity and it tries to maximize utilization and minimize the number of tardy deliveries. Given these conflicting goals of turnover and tardiness, order acceptance decisions are often made without involving production department or with incomplete information on the available capacity in production department [6]. Accepting too many orders, which is the objective of the sales department, leads to an over-loaded production system, in which lead times increase and orders are increasingly delivered late. To deal with this short term capacity problems, management may try to use additional non-regular capacity like overtime and outsourcing, thereby increasing the costs significantly. This may lead to lower profit margins or even negative profits. Tardy deliveries may also lead to higher penalty costs, and possibly lead to loss of customer goodwill [6, 7].

While negotiating contracts in MTO environment, the company can either adjust the price or lead time for an order. If the order has tighter deadlines, the MTO enterprise can charge a premium for processing that order as it might have to be expedited with use of non-regular capacity. Recent experiences of firms, such as Amazon.com, indicate that customers may be unwilling to accept dynamic pricing as fair [8]. An alternative to dynamic pricing would be to view the issue as one of allocating capacity between competing orders, making it a capacity allocation problem. When multiple orders, each providing a different profit contribution is present, the capacity allocation problem becomes an order acceptance or refusal problem [9, 10, 11].

1.2. Research Problem

A job shop environment is considered to model the make-to-order operation. A MTO firm receives a set of bids or customer orders. A customer order is referred to as jobs in the context of this research. The decision to be made is which customer order is to be accepted and how to schedule it to maximize profit. The decisions should be made simultaneously; otherwise, an order may be accepted but the residual capacity available may not permit timely delivery.

Each customer order has a set of operations to be processed with linear precedence constraint and deterministic processing times, a fixed due-date and a known sales price. No tardy deliveries are allowed. There are multiple types of resources each having one or more machines. Furthermore, job recirculation is allowed, which means that the jobs can visit the same machine more than once. The cost of using a resource in each source is known and is represented in unit cost per hour. The planning horizon is discretized into time buckets of equal length know as a time period. Without loss of generality we assume that each time period is one day. Furthermore each day is divided into sources viz. regular time and overtime. Overtime is usually considered more expensive. The decision of accepting or rejecting the orders is done on day zero.

1.3. Research Objective

The goal of this dissertation was to maximize the operational profit of a make-to-order operation by combining the order acceptance decision with capacity planning at the sales stage. The objectives were to mathematically model the problem and solve it within the bounds of practicality.

A mixed-integer linear program was formulated to model the order acceptance and capacity planning problem. Large-scale mixed integer linear programs are difficult to solve because of their combinatorial nature and solving industry-sized problems would take prohibitively long runtimes. Over the last decade, column generation has proven to be one of the most successful approaches for solving large-scale integer programs [12]. Column generation, also known as the Dantzig-Wolfe (DW) decomposition, is implemented along with branch-and-bound scheme which is collectively known as branch-and-price. The original formulation has to be decomposed into a restricted master problem (RMP) and one or more pricing or sub-problems (SPs). The major components of a branch-and-price scheme are generating columns by solving the sub-problems and branching to restore the integrality constraints for integer variables, as the RMP is solved as a linear relaxation. To solve the mixed-integer linear program in reasonable time we have developed a branch and price algorithm.

Figure 1-1 shows a schematic representation of a typical order acceptance problem in a job shop environment of a MTO operation. For illustration purposes the job shop used has three resources. Resource 1 has two machines of the same type, while resources 2 and 3 each have a single machine. Each resource has fixed operating costs in regular time and overtime. There are three orders, each having a known sales price and a fixed due-date. Each order has a different process route with deterministic processing times. For example, the process route for customer order 1 is Resource 1 → Resource 2 → Resource 3. The model takes these as input parameters and gives a solution which comprises of the orders that have been accepted so as to maximize the operational profits and their corresponding schedules.

For the example shown in Figure 1-1, customer orders 1 and 3 are accepted. Figure 1-2 shows the schedule generated by the model for these orders.

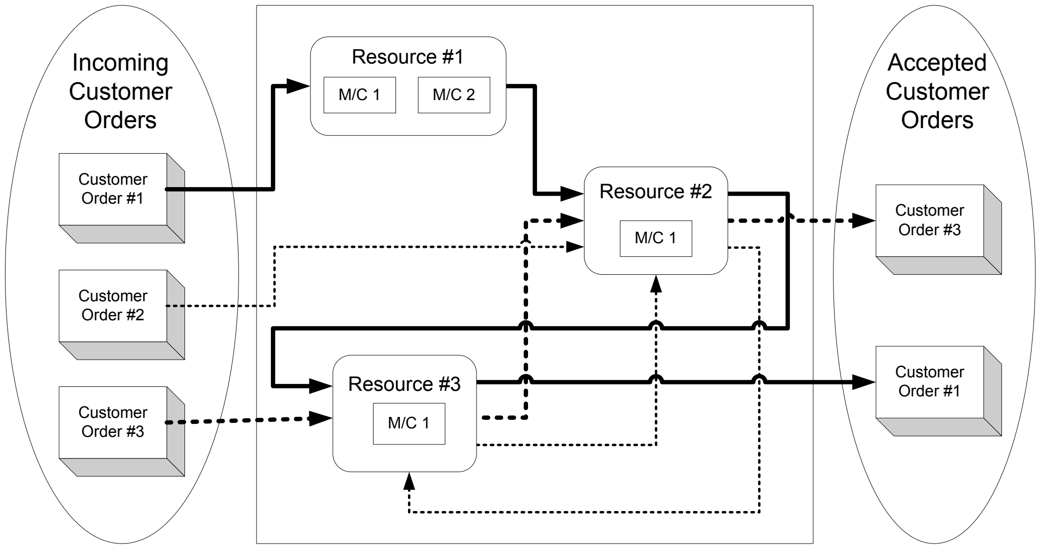


Figure 1-1. Customer order process route for jobshop-MTO operation

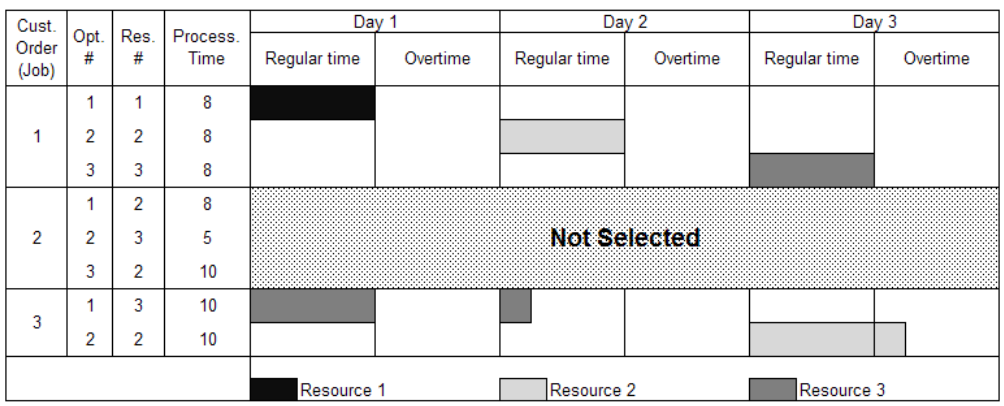


Figure 1-2. Optimal production plan generated by the mathematical model

For simplicity, a common due-date of three days was assumed for this example. Each day is divided into regular time and overtime, each eight hours long. All the operations for order 1 are processed in the regular time of days 1, 2 and 3. Operation 2 of customer order 3 uses two additional hours of overtime on day 3 in order to meet its due-date.

1.4. Significance of the Research Problem

The ever increasing emphasis on mass customization, shortened product lifecycles, synchronized supply chains, when coupled with advances in information system, is driving most firms towards make-to-order operations. Increasing global competition, lower profit margins, and higher customer expectations force the MTO firms to effectively manage the capacity to make sustainable profits. Because the main driver in MTO operations is customer orders, coordination of operations and marketing functions for effectively managing capacity by managing the demand placed on the system has been long recognized as vital [10]. This research integrates these two important decisions, by coordinating the order acceptance at the sales stage with a detailed capacity plan at the production level.

The policy of acceptance or rejection of an order based on capacity available is referred to as available-to-promise (ATP), which is common in an Enterprise Resource Planning (ERP) system to search and check resource availability. With most ERP systems, it is a simple search in a database, accompanied by a simple heuristic rule such as first-come-first-serve (FCFS). When an order is accepted, it is usually inserted into the existing master production schedule (MPS). The acceptance/rejection decision in this case is to check whether the available capacity is sufficient to meet the order due-date. ERP systems lack intelligent planning [1] tools in order to maximize the profits based on selectively accepting incoming orders. Concepts like manufacturing resources planning (MRP), and enterprise resource planning (ERP) were embraced almost immediately after their introduction because of their simplicity and the available computing power. But there are many implementation failures because there are a great number of conditions to be fulfilled before the apparently simple logic can work successfully. Many managers recognized these major drawbacks and are demanding more intelligent solutions. Furthermore, with the recent increase in the computing power Operations Research models have gained a lot of importance.

1.5. Dissertation Structure

The rest of the dissertation is organized as follows. In Chapter 2, past and current closely related research is explored. In Chapter 3 the mathematical model is presented along with the assumptions and limitations of this research. Chapter 4 explains the Dantzig-Wolfe decomposition, and an exact branch and price technique. The solution quality from the exact branch and price technique is compared to the results from an optimization solver. In Chapter 5, we present approximation algorithms and comparison between the various solution methodologies. We conclude this dissertation with Chapter 6 by summarizing the contributions of this research and possible extensions.

2. LITERATURE REVIEW

Order acceptance in manufacturing is closely related to the principles of revenue management (RM) which is a commonly used in service industry for order acceptance and refusal process, with differential pricing, capacity reallocation and overbooking [9]. There has been a recent interest in applying RM to manufacturing industry in both MTS and MTO operation modes. In MTO, the decisions of order acceptance, lead-time or due-date quotation, pricing and capacity planning are closely related. In the absence of differential pricing, RM becomes a capacity allocation and order acceptance problem. For the purpose of this research we focus on applications of RM to MTO and literature closely related to order acceptance in MTO. Order acceptance in make-to-order can be broadly classified based on static [6, 13, 14, 15, 16, 17] and dynamic arrivals [4, 7, 10, 11, 11, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27] of customer orders. Section 2.1 discusses research done in the area of order acceptance with dynamic customer arrivals while Section 2.2 focuses on the static arrivals. Section 2.3 is dedicated to the applications of column generation technique, especially in the area of scheduling.

2.1. Order acceptance with dynamic arrivals

The earliest research in order acceptance was done by Miller [28]. He considered an n-server queuing system with m customer classes distinguished by the reward associated with serving customers of that class. The objective was to accept or reject customers so as to maximize the expected value of the rewards received over an infinite planning horizon. Poisson arrivals with common exponential service time is considered and formulated as an infinite horizon continuous time Markov decision problem. Lippman and Ross [29] considered the problem of maximizing the long-run average return in a single server traffic reward system with a customer's offer is a joint distribution of reward and of service time required to earn this reward which is independent of the renewal process which governs customer orders. They formulate it as a semi-Markov decision process with the characterization of accepting the customer if and only if the ratio of his expected reward to his expected service time is larger than g, the long-run average return.

Recently, the concepts of revenue management have been applied to manufacturing for tackling the order acceptance problem. Carr and Duenyas [30] address the problem of admission control and sequencing in a production system that produces two classes of products in a hybrid MTO/MTS operation. The first class of products is made-to-stock and the second is make-to-order. The firm has the option to accept or reject a particular MTO customer order. They model the joint admission control / sequencing decision in the context of a single M/M/1 queue with two classes of products.

Webster [31] studies a single stage make-to-order production system to examine policies for adjusting price and capacity in response to periodic and unpredictable shifts in the importance placed by the market on the price and lead-time. He suggests that maintaining a fixed capacity while using lead-time and/or price to absorb changes in the market will be most attractive when stability in throughput and profits are highly valued, but in volatile markets, this stability comes at a cost of low profits.

Balakrishnan et al. [18, 19] and Sridharan and Balakrishnan [32] described how to ration the available capacity when the forecasted demand is higher than the available capacity over a planning horizon, using a decision-theory based approach. Barut and Sridharan [10] extended this research for investigating the effectiveness of a tactical demand-capacity management policy to maximize profit by selectively accepting or rejecting customer orders for multiple product classes. Tardiness is not allowed but earliness is permissible without penalty. They propose a dynamic capacity apportionment procedure (DCAP) to prescribe a static acceptance policy. Ebadian et al. [24] also proposed a decision-making structure for the order entry stage in MTO environments. The aim of the proposed structure is to manage the arriving orders so that the MTO system just proceeds to produce those arriving orders which are feasible and profitable for the system. The appropriate decisions on the arriving orders are taken based on two criteria including price and delivery time. The arriving orders have either fixed or negotiable delivery times. During the first two steps, the new arriving orders either are rejected or appropriate decisions to meet their delivery time are made. At the next step, the optimal prices along with delivery times (if negotiable) of non-rejected orders are determined by a mixed-integer programming model. In the case of the final approval by the customers at the fourth step, another mixed-integer programming model is launched to select a set of suppliers and subcontractors that are able to provide required raw material and workload for the newly accepted orders. They consider a job shop with sub-contracting but without overtime capacity.

Defregger and Heinrich [33] considered the application of revenue management in make-to-order manufacturing company with limited inventory capacity. Orders with different profit margins arrive stochastically over an infinite time horizon and the company has to decide which orders to accept and which to reject. They model the problem as a discrete Markov decision process and propose a heuristic procedure. In numerical tests, they showed the potential benefits of using revenue management compared to a FCFS policy.

Herbots et al. [25] examine the simultaneous dynamic order-acceptance and capacity planning decision. They consider only one resource type, which represents the bottleneck of the company. They consider regular capacity and non-regular capacity.

Charnsirisakskul et al. [20] studied integrated order selection and scheduling decision, where the manufacturer has the flexibility to choose lead-times. They provide a mechanism for coordinating order selection, lead-time and scheduling decisions and to determine under what conditions lead-time flexibility is most useful for increasing the profits. They consider a single machine production system with a bottleneck and no buffers between stations capable of producing multiple products with negligible setup times and preemptions. If the manufacturer cannot complete the order by the latest acceptable due-date tardiness costs are incurred. As an extension to this research, Charnsirisakskul et al. [21] study the simultaneous pricing, order acceptance, scheduling and lead-time decisions, both in the case where the manufacturer has and does not have the flexibility to charge different prices for different customers. They present decision models that can be solved by commercial optimization software and present simple rounding heuristics that provide initial solution with the objective functions values within 87% of the optimal solution.

Ebben et al. [7] investigate the importance of order acceptance and the benefits of cooperation between the sales and planning function. They develop several workload based order acceptance methods and develop a simulation model of generic MTO job shop, which enables them to simulate the order arrival and production process to test the proposed methods. They use utilization rate and the service level as the performance measures. Nandi and Rogers [34] using simulation demonstrate how to optimally control a manufacturing system under different environments and how main performance measures of a system are affected by using a parameterized order acceptance rule. The order acceptance rule is similar to the path load order review introduced in Philipoom and Fry [35].

Akkan [27] develops heuristics to insert a new work order in the existing schedule in order to minimize the holding costs and the total contribution lost due to rejected work orders in the case where one has to reject the incoming work order because it cannot be fit in the current schedule.

Modarres et al. [36] formulate stochastic capacity allocation problem in a manufacturing system with two classes of “frequent” and “occasional” customers demanding its capacity. The stochastic nature of capacity is caused by machine failures, stops or breakdowns during the operation; and the maintenance duration is random. The price rate as well as the penalty for order cancellation caused by overbooking is different for each class.

2.2. Order acceptance with static arrivals

Within the operational domain of job shop planning with static customer arrivals, job selection has been a topic of growing interest. The problem of selection and ordering of elements from a given set so as to optimize a given objective function was considered by Bahram et al. [17]. They present a generalization of the best-in rule that in many cases can solve the problem while the best-in rule does not. A characterization of such a greedy algorithm has been presented.

Slotnick and Morton [13] examine a set of trade-offs that can arise if a manufacturing facility has more potential work than it can handle easily. They formulate a one-machine model with static arrivals, fixed processing times, due-dates and profits. The objective function maximizes total net profit, which is the sum of the revenues of all jobs minus weighted lateness penalties, by selecting a subset of jobs. Ghosh [16] proves that the Slotnick-Morton [13] version of the job selection problem is NP-Hard. He proposes two dynamic programs that produce the exact solution to the problem.

In an extension to [13] Lewis and Slotnick [14] examine the profitability of job selection decisions over a number of periods when current orders exceed capacity with the objective of maximizing profit and when rejecting a job will result in no future jobs from that customer. The firm processes jobs, over a set number of time periods (stages) within a given time horizon. The firm has m customers at the beginning of the first period; each customer submits one job at each stage, until one of the jobs is rejected. Each job has pre-determined revenue, and the firms pay back a discount to customers whose jobs are completed past a pre-determined due-date; customers are willing to pay a premium for early delivery. Each job has a known processing time and importance. The importance of the job is the weight assigned to it for calculating the lateness penalty. This weight allows the firm to indicate that certain jobs may have importance beyond their immediate profit. The firm has the option of rejecting any job. If a job is rejected, the customer is lost (i.e. never sends another job to be processed within the planning horizon).

In [6], Slotnick and Morton model a manufacturing facility that considers a pool of orders, and chooses for processing the subset that results in the highest profit. In addition to the problem characteristics in [13] they consider customer weight. The objective is to maximize profit, which is the sum of per-job revenues minus total weighted tardiness. They propose two approaches: separation of sequencing and job acceptance decisions, utilizing a property of the problem that is exploited to good advantage in the analogous problem with weighted lateness; and a joint consideration of sequencing and acceptance, using relaxation. They state that the joint approach is far superior to the first. Rom and Slotnick [15] also propose a GA to solve the order acceptance problem with tardiness penalties.

2.3. Applications of column generation in scheduling

Decomposition techniques like column generation have been widely used to solve large-scale optimization problems [12, 37]. Column generation has been successfully used in job scheduling for common due date [38], parallel machines [39], and single machines [40, 41]. For a detailed taxonomy of the column generation literature we refer to [12]. Hans [42] developed a branch-and-price loading method that is an exact approach for solving the pre-emptive resource loading problem. The objective is to generate an order schedule for each order, such that the total costs of the required non-regular capacity and the order tardiness penalties are minimized.

2.4. Summary

This research considers an order acceptance problem in multi-resource job shop environment with regular and non-regular capacity and static customer arrivals. The only research which tackles a multi-resource job shop problem is by Ebben et al. [7]; but they do not consider non-regular capacity (overtime) and the customer arrivals are dynamic. We solve the problem under study using a branch-and-price algorithm. To the best of our knowledge this approach has never been used for order acceptance; although Hans [42] has developed a branch-and-price resource loading (BPRL) approach for scheduling orders which have already been selected. Ebben et al. [7] use the BPRL technique in their simulation for scheduling the already accepted orders.

Table 2-1 summarizes the literature related to the proposed problem under study. The table compares and contrasts the literature reported on problems similar to the problem under study. It is evident from this table that the proposed problem and the solution approach are different from what is reported in the literature so far.

Table 2-1. Summary of relevant research

Research	Objective	Order Acceptance	Multiple Resources	Non-regular Capacity	Fixed Due-dates	Solution Approach
[42]	Minimize non-regular capacity costs and tardiness penalties	No	Yes	Yes	No	Branch and Price
[13]	Maximize Profit	Yes	No	No	No	Heuristic
[6]	Maximize Profit	Yes	No	No	No	Branch and Bound
[14]	Maximize Profit	Yes	No	No	No	DP, Heuristic
Proposed Research	Maximize Profit	Yes	Yes	Yes	Yes	Branch and Price

3. MATHEMATICAL FORMULATION

This chapter describes the problem environment, proposes a mathematical formulation for the short-term capacity planning problem for the MTO operation environment. The outline of the chapter is as follows. We formally present the problem in Section 3.1. Section 3.3 describes the model developed for solving the MTO order acceptance and scheduling problem. Preliminary results are presented in Section 3.4 that illustrates the usefulness of the model proposed and Section 3.5 provides the motivation for solving this problem by the proposed branch and price technique.

3.1. Problem characteristics

The MTO production system is modeled as a job shop with multiple resources $\{r \in R\}$. Each resource type r can have multiple machines. A finite planning horizon is considered which is discretized into equal interval time periods $\{t \in T\}$. Without loss of generality we assume that each time period is one day. Furthermore each day is divided into sources $\{s \in S\}$ viz. regular time and overtime. The length of each source s in time period t is given by l_{ts} and is eight hours, but can be varied according to the need. The available capacity of resource r in source s of time period t is denoted by b_{rts}. The MTO firm receives a set of customer orders or jobs $\{j \in J\}$. Each job j has a set of operations $\{o \in O_j\}$ with a processing time p_{jor} on resource r, a fixed due date d_j and a sales price q_j. Each job can follow different processing route and the operations have a linear precedence relationship. The cost of using a resource r in each source s is represented in unit cost per hour c_{rs}. Overtime is usually considered more expensive. The objective is to maximize the profit of the MTO operation by selectively accepting the customer orders and planning for their capacity within the planning horizon, such that the accepted orders are completed before their due dates.

3.2. Assumptions

For the problem under consideration we assume a single deliverable job without any sub-assemblies. Raw material costs are not considered. The cost of operating machines is same during regular and overtime hours. The operational profit is a function of the sales price of a job and labor costs incurred to process that job in regular time and overtime. The sales price is dictated by the market and hence is considered to be fixed. Preemptions are allowed in scheduling the jobs. Processing times are additive and the amount of processing a preempted job already has received is not lost. Machine failure is not considered. The problem addresses in this research is deterministic.

3.3. Mathematical model

The problem is modeled as a mixed-integer linear program. The decision variables used in the model are:

X_{jorts}= hours operation o of job j is processed on resource r in source s of period t

$$Y_{jorts}=\begin{cases}1, \text{ if operation } o \text{ of job } j \text{ is processed on resource } r \text{ in source } s \text{ of period } t\\ 0, \text{ otherwise}\end{cases}$$

$$U_j=\begin{cases}1, \text{ if job } j \text{ is selected}\\ 0, \text{ otherwise}\end{cases}$$

The mathematical formulation for the problem under study is presented below.

$$\text{Maximize } Z = \sum_{j \in J} q_j U_j - \sum_{j \in J} \sum_{o \in O_j} \sum_{r \in R} \sum_{t \in T} \sum_{s \in S} c_{rs} X_{jorts} \tag{3-1}$$

subject to

$$\sum_{j \in J} \sum_{o \in O_j} X_{jorts} \leq b_{rts} \qquad \forall r \in R, t \in T, s \in S \tag{3-2}$$

$$\sum_{s \in S} \sum_{t \in T} X_{jorts} = p_{jor} U_j \qquad \forall j \in J, o \in O_j, r \in R \tag{3-3}$$

$$\sum_{o \in O_j} \sum_{r \in R} X_{jorts} \leq l_{ts} \qquad \forall j \in J, t \in T, s \in S \tag{3-4}$$

$$X_{jorts} \geq \tau Y_{jorts} \qquad \forall j \in J, o \in O_j, r \in R, t \in T, s \in S \tag{3-5}$$

$$X_{jorts} \leq p_{jor} Y_{jorts} \qquad \forall j \in J, o \in O_j, r \in R, t \in T, s \in S \tag{3-6}$$

$$\sum_{r \in R} t Y_{j|O_j|rts} \leq d_j U_j \qquad \forall j \in J, t \in T, s \in S \tag{3-7}$$

$$\sum_{s' \in S} \sum_{t'=1}^{t-1} X_{j(o-1)rt's'} + \sum_{s'=1}^{s} X_{j(o-1)rts'} \geq p_{j(o-1)r} \sum_{r' \in R} Y_{jor'ts} \qquad \forall j \in J, o \in O_j \backslash \{1\}, r \in R, t \in T, s \in S \backslash \{|S|\} \tag{3-8}$$

$$\sum_{s \in S} \sum_{t'=1}^{t} X_{j(o-1)rt's} \geq p_{j(o-1)r} \sum_{r' \in R} Y_{jor'|S|t} \qquad \forall j \in J, o \in O_j \backslash \{1\}, r \in R, t \in T \tag{3-9}$$

$$X_{jorts} \geq 0 \qquad \forall j \in J, o \in O_j, r \in R, t \in T, s \in S \tag{3-10}$$

$$Y_{jorts} \in \{0,1\} \qquad \forall j \in J, o \in O_j, r \in R, t \in T, s \in S \tag{3-11}$$

$$U_j \in \{0,1\} \qquad \forall j \in J \tag{3-12}$$

The objective (3-1) is formulated to maximize the total net profit over the planning horizon. The first term in the objective function is the total revenue and the second term is the total processing or labor cost. The constraint set (3-2) ensures that the capacity of resource r of source s in time period t is not violated. Constraint set (3-3) ensures that adequate resources are allocated to process operation o of job j. The total number of hours allocated to process an operation should be equal to its processing time. The equality constraint set (3-3) can be replaced with an inequality ($\geq$) constraint. The second term in the objective function will prevent allocating more resources than what is required.

The constraint set (3-4) ensures that each operation of a job is processed for no more than l_{ts} hours in each source during each time period. If the processing time of operation o is less than l_{ts}, then it is possible to start processing the next operation (o+1) in the same time period. Since operation (o+1) cannot be started before operation o, the remaining time available for operation (o+1) in period t is only (l_{ts}-p_{jor}). Consequently, the total time allocated to process job j in any time period cannot exceed l_{ts} hours. The constraint sets (3-5) and (3-6) set the Y_{jorts} decision variables to either 1 or 0. The Y_{jorts} variable is an indicator variable. It takes a value of 1 when $X_{jorts} > 0$, indicating that operation o of job j is scheduled for processing on resource r of source s in time period t; otherwise it takes a value of 0. The parameter "τ" in constraint (3-5) indicates that whenever an operation is processed on a resource it should be processed for at least "τ" units of time. The Y_{jorts} variables are used to ensure the precedence relationship. The constraint set (3-7) ensures that when an order for a job is accepted, the completion time of the last operation of that order does not exceed the order due date.

The next two constraints impose precedence restrictions. The constraint set (3-8) ensures that operation o of job j can be processed in period t during regular hours only after completing operation (o-1). The first term in constraint (3-8) represents the total number of hours allotted to process operation (o-1) in time periods 1,…,(t-1). It includes both the regular time and overtime hours allocated to process operation (o-1) in each time period up to and including (t-1). The second term in constraint (3-8) represents the number of hours allocated to process operation (o-1) in time period t during regular hours. Figure 3-1 illustrates how the precedence relationship (i.e., constraint 3-8) will take effect when operation (o-1) is completed in time period t during regular hours. The processing of operation (o-1) begins during the regular hours of production in time period (t-1), continues during the overtime and then to regular hours of production in time period t. The processing of operation o can begin either during the regular hours of production or during overtime in time period $t' \geq t$. For illustration purposes, it is assumed that operation (o-1) and o require same resource r in Figure 3-1. Figure 3-2 illustrates the case when both the operations require different resources. Operation o can be processed on resource r' only after operation (o-1) is completely processed on resource r ($r' \neq r$).

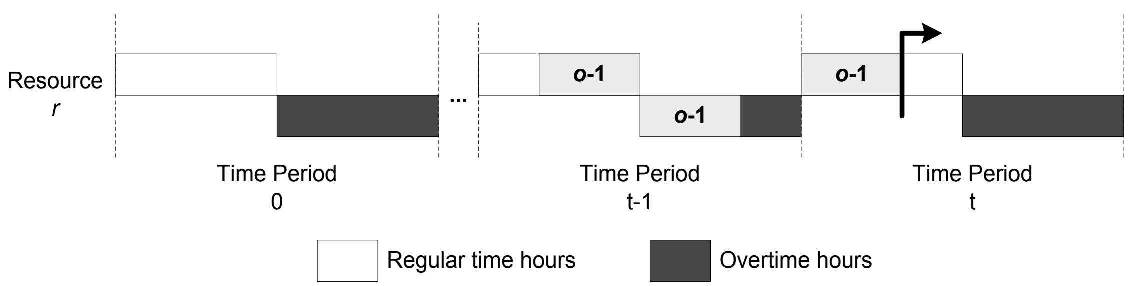


Figure 3-1. Constraint (3-8) with successive operations on the same resource

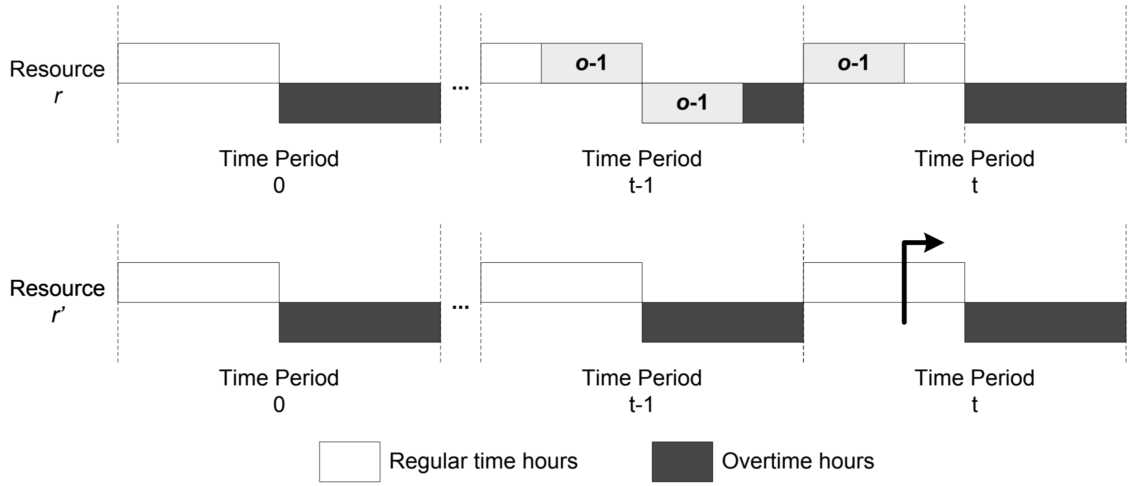


Figure 3-2. Constraint (3-8) with successive operations on different resources

The constraint set (3-9) ensures that operation o of job j can be processed in period t during overtime only after completing operation (o-1). Figure 3-3 illustrates how the precedence relationship (i.e., constraint 3-9) will take effect when operation (o-1) is completed in time period t during overtime. The processing of operation o can begin during the remaining overtime hours in time period (t-1) or any source (either regular hours or overtime) in time period $t' > t$.

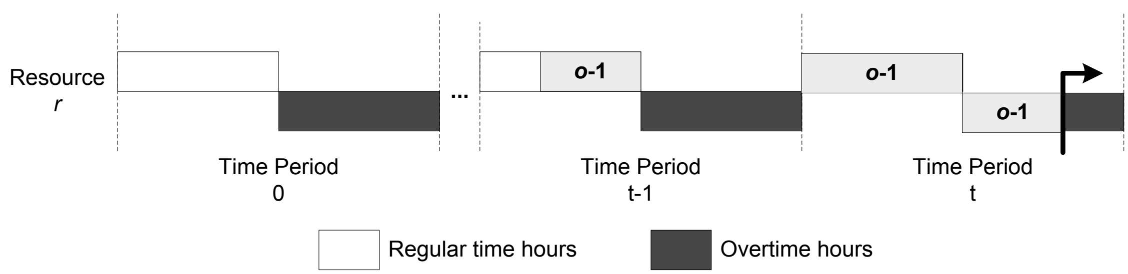


Figure 3-3. Precedence constraint (3-9)

The constraint sets (3-10) – (3-12) impose the non-negativity restrictions on the decision variables. In particular, the constraint sets (3-11) and (3-12) impose the binary restrictions on the decision variables *Y* and *U*.

This model can help the operations manager to determine which subset of incoming customer orders should be selected to maximize profits. The model presents a detailed capacity plan for the accepted orders such that they are completed before their due-dates. This is useful to carefully plan for the resources used in overtime hours. The model can be run at the beginning of each decision period, such that the operations manager can reserve capacity for already accepted orders and determine which new orders to accept. In situation where a particular order or orders have to be selected for strategic reasons, a corresponding subset of orders that will maximize the profits can be known. The model is also useful to reschedule the already accepted orders when new orders have to be accepted. Section 3.3 presents some examples to illustrate the above scenarios that the end user might encounter and to show how the formulation when solved can be used to one's advantage. The mathematical model was solved using ILOG CPLEX 10.1, which is a tool for solving linear optimization problems. ILOG CPLEX can also solve several extensions of linear programs like network flow, quadratic problems, and mixed-integer programs.

3.4. "What-if" scenario analysis using the MTO formulation

This section presents some illustrations to show the usefulness of the formulation for making decisions to maximize the revenue in different situations. Table 3-1 through

Table 3-3 give the data for Experiment A for illustrating how a decision maker could use the mathematical formulation proposed. Two kinds of sources are considered, namely regular production time and overtime. It is assumed that the regular production time and overtime is 8 hours each. In each source, three resources are considered. The planning horizon includes three time periods. The decision maker has to decide which jobs to accept and how to schedule the accepted jobs such that they are processed before their due date. The objective is to maximize total profit. At the beginning of the planning horizon, three jobs are available to choose from. The selling price and due date for each job is given.

Table 3-1. Experiment A to show the usefulness of the model

$j \in J=\{1,2,3\}$	Three jobs
$o \in O_j=\{1,2,3\}$	Each job comprises of 3 operations
$r \in R=\{1,2,3\}$	Three resources
$t \in T=\{1,2,3\}$	A planning horizon of 3 periods
$s \in S=\{1,2\}$	1: regular hours of production; 2: overtime

Table 3-2. Customer order data for Experiment A

Job #	# of operations	Due Date (d_j)	Selling Price (q_j)	Operation #	Resource #	Process Time (p_{jor})
1	3	3	10000	1	3	14
				2	1	12
				3	2	14
2	3	3	9800	1	3	14
				2	1	10
				3	2	14
3	3	3	6500	1	1	8
				2	3	6
				3	2	8

Table 3-3. Data of resource cost for Experiment A

Processing Cost/hr (c_{rs})		
Resource (r)	Source (s)	
	1	2
1	200	300
2	200	250
3	200	200

Figure 3-4 shows the Gantt chart for the example instance under consideration. The total profit is \$3800 and the orders for jobs 2 and 3 are accepted. The detailed schedule for each job is also shown. Since the cost of using resource 1 during overtime is higher than resources 2 and 3, resource 1 is not used during the overtime hours. The utilization of each resource can be easily computed. The utilization of resources 1, 2, and 3 is 37.5%, 45.83%, and 41.67%, respectively.

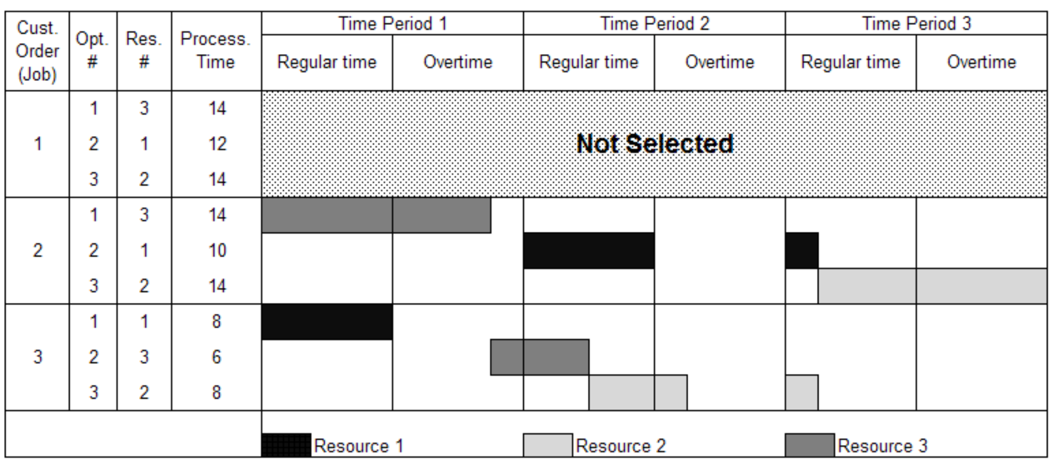


Figure 3-4. Gantt chart for Experiment A

Suppose two new jobs (say jobs 4 and 5) become available at the end of the first time period. The decision maker would like to know whether or not to accept these orders as some of the resources have already been reserved to process orders for jobs 2 and 3. Table 3-4 gives the data associated with jobs 4 and 5.

Table 3-4. Data for Experiment A extension

Job #	# of Operations	Due Date (d_j)	Selling Price (q_j)	Opt #	Resource #	Process Time (p_{jor})
4	2	4	5800	1	1	12
				2	3	14
5	3	4	6200	1	3	10
				2	2	8
				3	1	10

When the mathematical model was solved with the new information (fixing the resources already committed to process jobs 2 and 3), job 4 was chosen and the total profit increases to $4000. The model recommends to select job 4 and not to select job 5. The Gantt chart for the new schedule is shown in Figure 3-5.

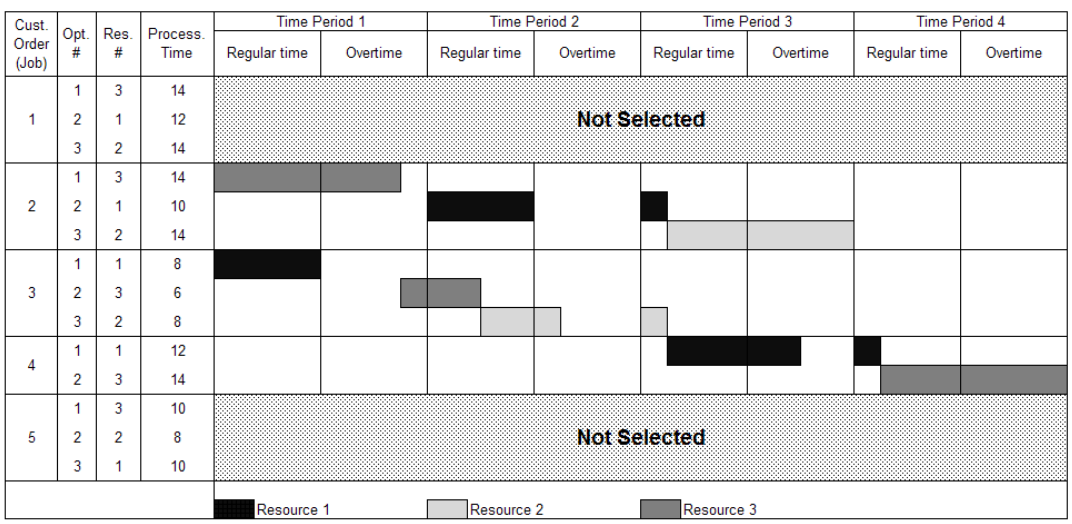


Figure 3-5. Gantt chart for extension of Experiment A

In the case where the company policy would have been to maximize capacity utilization both jobs 4 and 5 would have been accepted. The total profit in such a scenario would have been $3900 which is less than the optimal found by solving the model.

Another interesting illustration is the analysis of the effect of the change in the overtime cost with respect to the regular time cost. Four jobs were assumed to be available at the beginning of the planning horizon. Other job specific information is given in Table 3-5. The processing times of the operations were randomly generated from a discrete uniform distribution [3, 12]. The unit cost of all the resources was considered the same and the ratio of the unit cost of the resource in regular time to unit cost of the resource in overtime was varied. The different ratios under which the experiment was run were 1:1, 1:1.2, 1:1.5 and 1:2. The planning horizon was assumed to be five time periods. In this experiment, the combined regular time and overtime hours available is more than the time required to complete all the orders. Consequently, when the ratio was 1:1, the orders for all the jobs were accepted. The results obtained for the various cost structure considered are summarized in Figure 3-6. Even when the ratio was 1:1.2, the orders for all the jobs were accepted. However, the profit reduced due to the additional cost incurred for processing some jobs during the overtime. It can be seen that with the increase in the overtime cost, the number of hours the resources are allocated in overtime is reduced. At a certain point the increase in overtime cost was so high that it prohibited accepting orders for some jobs. For example, when the ratio was 1:1.5, job 1 was not selected. When the ratio was further increased to 1:2, job 4 was not selected.

Table 3-5. Data for Experiment B

Job #	No. of Operations (O_j)	Due-date (d_j in days)	Selling Price (q_j in \$)
1	3	4	3375
2	4	4	3335
3	3	5	2990
4	5	5	4750

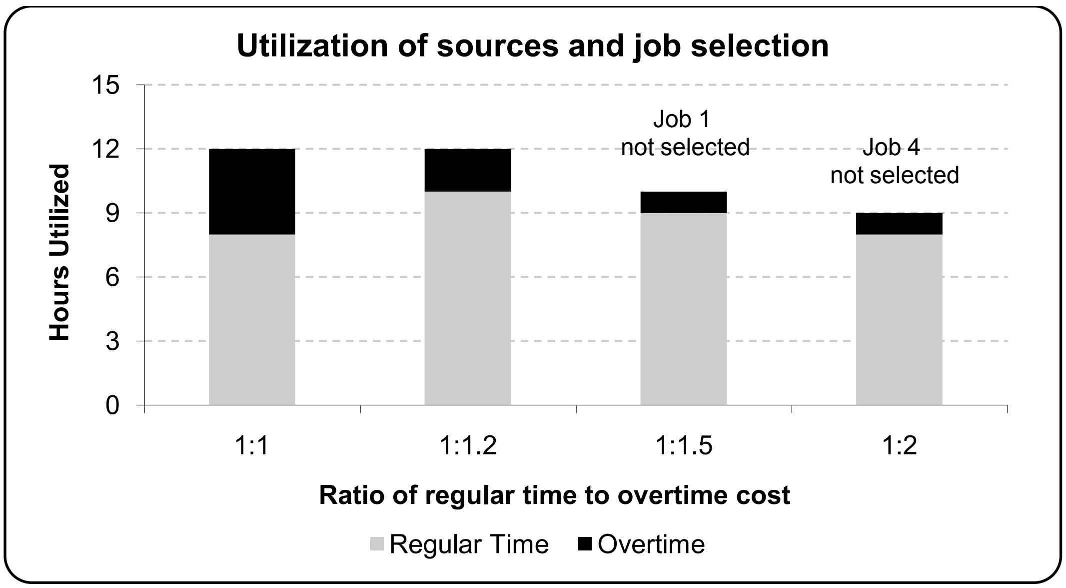


Figure 3-6. Source utilization and selection of jobs

In practice, the MTO firms would generally accept an order whenever the capacity is available to fulfill an order. By adopting this policy, the utilization of the resources is maximized. However the next set of experiment (Experiment B) shows that this policy may not be profitable. In addition, by committing all the resources available in the current time period, the more profitable orders which may arise in the future may not be accepted due to lack of capacity. When the cost ratio was 1:1.5, job 1 was not selected. Suppose the firm decides to accept the orders for all the jobs (note the regular time + overtime capacity available is more than the demand), the profit reduces to \$1450. When the cost ratio was 1:2, job 4 was not selected. But when the manager accepts all the orders, the profit reduces to \$450. These experiments clearly indicate that even when the available capacity is more than the demand, the cost incurred to operate the resources during the overtime may prohibit accepting all the orders.

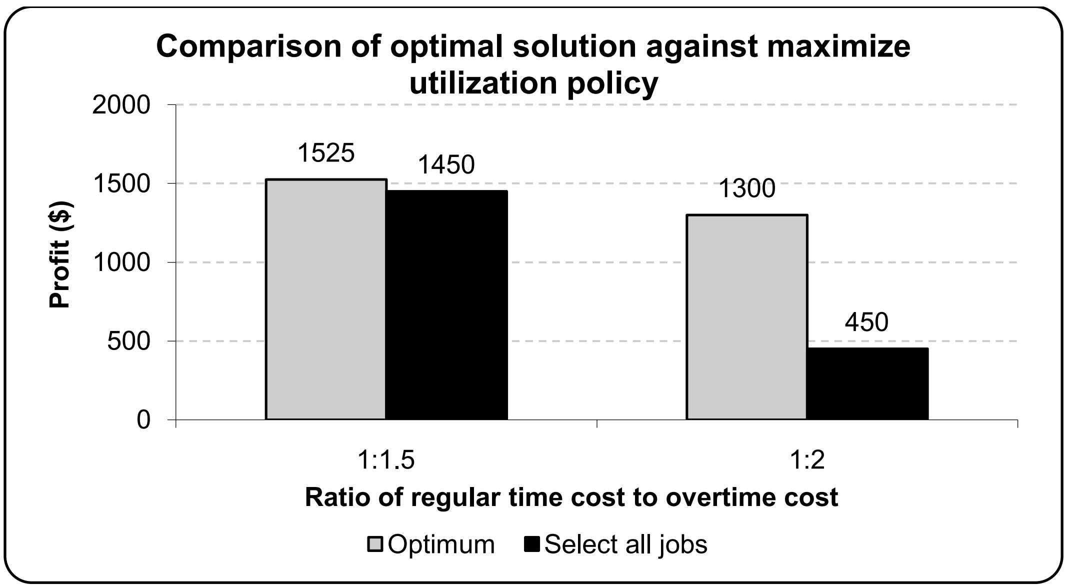


Figure 3-7. Optimal solution against utilization maximizing policy

Figure 3-7 shows the reduction in profit when the orders for all the jobs were accepted. These experiments clearly indicate the usefulness of the model proposed. Without the model, the decision maker would incline to accept orders based on the residual resource capacity alone.

3.5. Computational runtime analysis

In order to solve a mixed-integer problem, CPLEX uses a branch-and-bound approach to fix the fractional variables to integer values. Consequently, it may not be able to solve problem instances with large number of integer variables in reasonable time. An experimental study (Experiment C) was conducted to determine the effect of problem size on the run-time (computation time) required to find an optimal solution. Various factors determine the size of the problem, namely, the number of customer orders or jobs, number of operations for each job, the number of resources, due-dates for each job and the planning horizon. We introduce a demand-to-capacity ratio (DC ratio) to control the load on the MTO shop-floor. The DC ratio is the ratio of the demand to the regular time capacity available in the MTO operation given by equation (3-13), over the planning horizon with $|T|$ time periods. If we know the total demand and the available resources, we can generate problem instances by computing the number of time periods required for a fixed DC ratio using Equation (3-14).

$$\text{DC Ratio}= \frac{\sum_{j\in J}\sum_{o\in O_j}\sum_{r} p_{jor}}{\sum_{r\in R}\sum_{t\in T} b_{rt,s=1}} \tag{3-13}$$

$$\text{Number of Time Period } (|T|)= \left\lceil \frac{\sum_{j\in J}\sum_{o\in O_j}\sum_{r\in R} p_{jor}}{|R|*l_{t,s=1}*(\text{DC ratio})} \right\rceil \tag{3-14}$$

Table 3-6 presents the data used for Experiment C. Number of jobs and numbers of operations for each job are the two factors which are varied. The levels for the two factors are presented in Table 3-6. The length of each source was fixed to 8 hours. For a

DC ratio of 1.0 with different levels for jobs and operations, the planning horizon varied from 3 to 17 time periods. For each combination of the factor and level, three instances were randomly generated. The due-date for each job was equal to the planning horizon computed for that problem instance. The ratio of regular time to overtime cost was kept constant at 1:1.5. The runtime to solve the mathematical model to optimality was reported.

Table 3-6. Data for Experiment C (computational runtime analysis)

Factors	Levels	
Number of jobs	3	5
Number of Operations	3,5, and 8	
Number of resources	3	
Number of sources	2 (Regular Time, and overtime)	
Processing time	Discrete Uniform (4,16) hours	
Demand-to-Capacity Ratio	1.0	

Figure 3-8 shows the runtime in seconds against the number of operations per job for 3 job and 5 job instances. In two instances for 5 jobs with 8 operation problem, CPLEX was not able to find an optimal solution even after a runtime of more than 16 hours (>62000 seconds), hence for those we report the optimality gap in Figure 3-8.

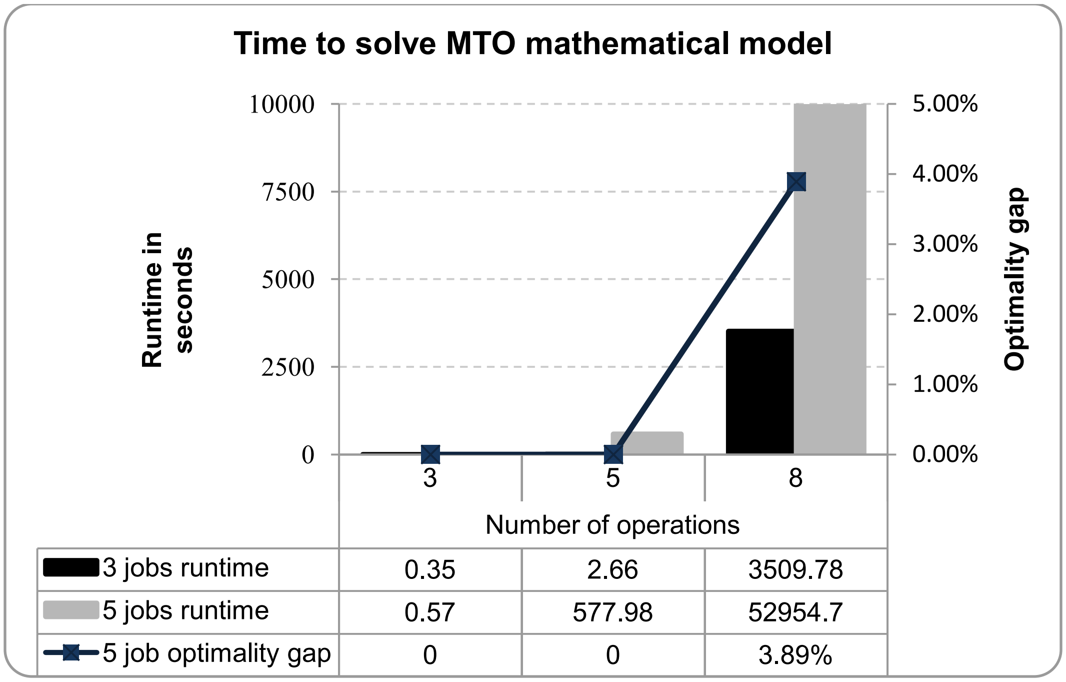


Figure 3-8. Computation runtime to solve MTO mathematical model to optimality

For the above problems, the planning horizon was anywhere between 3 to 17 days. We aspire to solve short-term capacity planning problems with a planning horizon up to a month (30 days) and a set of 8 to 10 customer orders, each having more than 5 operations to bid for each day. The bidding process is an iterative negotiation process and hence the sales department may have to consider different scenarios before accepting an order as illustrated in Section 3.4. The decision process in a bidding negotiation is highly time sensitive. Considering the above factors, there is a need to generate solutions to the order acceptance and capacity planning in MTO operations relatively quickly for large problem sizes.

To effectively solve large-scale integer programs column generation has proven to be one of the most successful approaches [12]. Column generation is based on the Dantzig-Wolfe decomposition principle for linear programs. For solving integer programs the column generation procedure is combined with branch-and-bound procedure, which is commonly referred to as branch-and-price.

4. BRANCH AND PRICE ALGORITHM

This chapter explains the Dantzig-Wolfe decomposition, column generation and the branch-and-price algorithm. In Section 4.2, the decomposition of the model proposed in Chapter 3 is described.

4.1. Theory of Branch-and-Price Algorithm

The decomposition algorithm has an interesting economic interpretation. Consider the case of a large system that is composed of smaller subsystems. Each subsystem has its own objective and constraints, and the objective function of the overall system is the sum of the objective functions of the subsystems. In addition, all the subsystems share a few common resources, and hence, the consumption of these resources by all the subsystems must not exceed the availability. With this in mind the decomposition algorithm can be interpreted as follows. With the current proposals of the subsystems, the total system obtains a set of optimal weights for these proposals and announces a set of prices for using the common resources. These prices are passed down to the subsystem, which modify their proposals according to these new prices [43].

The problem under consideration is well-suited for applying column generation. Each job or customer order can be considered as a subsystem, independent of other job, and a schedule can be generated. Later the individual subsystems should be merged together to obtain a feasible schedule for the entire problem under study.

The decomposition principle is a systematic procedure for solving large-scale linear programs or linear programs that contain specially structured constraints. The constraints are partitioned into two sets: general constraints (or complicating constraints) and constraints having a special structure. Special structure, when available, enhances the efficiency of the procedure.

The strategy of the decomposition procedure is to operate on two separate linear programs: one over the set of general constraints and one over the set of special constraints. Information is passed back and forth between the two linear programs until an optimal solution to the original problem is achieved. The linear program over the general constraints is called the master problem, and the linear program over the special constraints is called the sub-problem. The master problem passes down a continually revised set of cost coefficients to the sub-problem, and receives from the sub-problem a new column (or columns) based on these cost coefficients. For this reason, such a procedure is also known as a column generation technique.

Consider the following linear program, where X is a polyhedral set representing specially structured constraints, A is an m x n matrix, and b is an m vector:

Minimize $\mathbf{cx}$

Subject to $\mathbf{Ax}=\mathbf{b}$

$\mathbf{x} \in X$

To simplify the presentation, assume that X is nonempty and bounded. Since X is a bounded polyhedral set any feasible point $x \in X$ can be represented as a convex combination of the finite number of extreme points of X. Denoting these extreme points by $x_1, x_2, \ldots, x_t$ any $x \in X$ can be represented as:

$$x = \sum_{j=1}^{t} \lambda_j x_j$$

$$\sum_{j=1}^{t} \lambda_j = 1$$

$$\lambda_j \geq 0, \qquad j = 1, 2, \ldots, t$$

Substituting for x, the foregoing optimization problem can be transformed into the following so-called master problem in the variables $\lambda_1, \lambda_2, \ldots, \lambda_t$.

$$\text{Minimize } \sum_{j=1}^{t} (cx_j)\lambda_j \tag{4-1}$$

Subject to

$$\sum_{j=1}^{t} (\boldsymbol{A}x_j)\lambda_j = \boldsymbol{b} \tag{4-2}$$

$$\sum_{j=1}^{t} \lambda_j = 1 \tag{4-3}$$

$$\lambda_j \geq 0, \qquad j = 1, 2, \ldots, t \tag{4-4}$$

Since t, the number of extreme points of the set X, is usually very large, attempting to explicitly enumerate all the extreme points $x_1, x_2, \ldots, x_t$ and explicitly solving this problem is a very difficult task. In column generation, an extreme point or column is generated as and when required by solving a sub-problem known as the pricing problem. After adding the column to the master problem the LP is re-optimized. This is done iteratively until no new column can price out of the sub-problem (SP); this being the check for optimality of the LP master problem. The pricing problem is given by,

$$\text{Maximize } (\boldsymbol{wA}\text{-}\boldsymbol{c})\boldsymbol{x} + \alpha \tag{4-5}$$

$$\text{Subject to } \boldsymbol{x} \in X \tag{4-6}$$

Where, $\boldsymbol{w}$ and α denote the dual variables for equations (4-2) and (4-3), respectively [43]. The master problem is restricted in the sense that all the columns are not known explicitly and hence it is called as a Restricted Master Problem (RMP).

To start the column generation scheme, an initial feasible solution to the restricted master problem has to be determined. The initial RMP could then pass proper dual information to the pricing problem. The pricing problem or sub-problem plays a critical role through their structure, symmetry, complexity and whether or not they exhibit the Integrality Property. The integer variables in an IP typically become decision variables in one or more SPs. Formulations that have block diagonal structure (i.e. each SP is separable) are attractive because they result in small, independent SPs that are typically more effectively solved. Integer SPs with no special structure are NP-hard and should be avoided. The original IP is NP-hard but need to be solved only once. Solving one or more NP-hard SPs repetitively offers no worst-case advantage and is typically computationally prohibitive. Thus, the ideal SP should have a structure that can be solved effectively since it must be solved repetitively. Experience has shown that NP-hard SPs that can be solved in pseudo-polynomial time satisfy the criterion of being solvable relatively effectively; in addition they avoid the Integrality Property as well [12].

In a maximization linear program, any column with positive reduced cost is a candidate to enter the basis. The pricing problem is to find a column with highest reduced cost. Therefore, if a column with positive reduced cost exists the pricing problem will always identify it. This guarantees that the optimal solution to the linear program will be found. However, it is not necessary to select the column with the highest reduced cost - any column with a positive reduced cost will do. Using this observation can improve the overall efficiency when the pricing problem is computationally intensive. Depending on the pricing problem, it may even be possible to generate more than one column with positive reduced cost per iteration with a large increase in computation time. Such a scheme increases the time per iteration, since a larger RMP has to be solved, but it may decrease the number of iterations.

The second important point to consider while generating columns is the fashion in which the columns are generated and added to the RMP. One strategy would be to solve all sub-problems and select the best improving column to enter the RMP. Instead, all improving columns could be made available to the RMP through column management procedure. A third strategy would be to solve SPs in a round robin fashion, entering each improving column identified and re-optimizing the RMP, solving SPs in a random order. Another method would be to solve SP using a heuristic to generate good solution quickly and in case it fails an optimizing algorithm which takes more runtime can be used to identify an improving column. Preliminary tests may be used to identify the best way to add columns to the RMP [12].

In solving integer programs (a program incorporating integer, and / or binary, or mixed-integers) there is a binary restriction on the λ variable in the RMP. The RMP is optimized as a LP but when we have an optimal solution of the LP if those variables do not satisfy the integrality restriction then a branch-and-bound procedure is implemented. Branch-and-price, which is a generalization of branch-and-bound with LP relaxations, allows column generation to be applied throughout the branch-and-bound tree [44]. This is known as Branch & Price (B&P). However, this implementation is not straight forward and there are fundamental difficulties in applying column generation techniques for linear programming in integer programming solution methods [45]. These problems arise because conventional integer programming branching may not be effective as fixing variables can destroy the structure of the sub-problem and solving the relaxed master problem may be inefficient. Furthermore the branching should result in child nodes that represent balanced set of solutions. Balancing is important because it can be expected to result in a tighter bound at each sibling node, facilitating solution. A branching that does not balance solutions defines one sibling that represents just a few of the solutions associated with the parent node and another that represents all remaining solutions. The bound associated with the former node is not likely to be as good as that of the latter. Devising an effective branching strategy may present one of the most difficult challenges to composing an effective B&P algorithm [12]. The three major aspects of implementing branch-and-price algorithm are decomposing the original model, formulating and efficiently solving the sub-problem and lastly determining the branching strategy. The B&P procedure is illustrated in Figure 4-1.

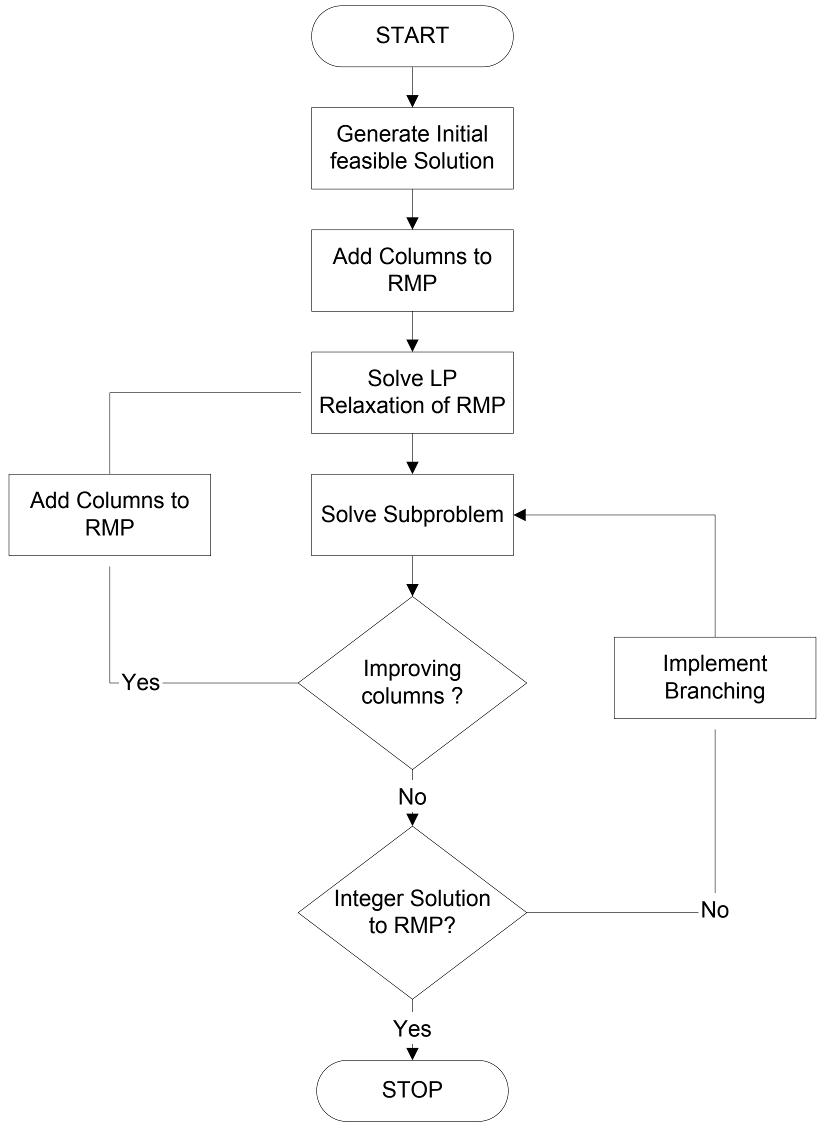


Figure 4-1. Flowchart for the Branch and Price procedure

The next sections discuss the implementation of the above with respect to the problem under consideration.

4.2. Decomposition of the MTO model

The proposed MTO model has a block diagonal or angular structure as shown in Figure 4-2. This special structure is well suited for applying the decomposition principle. The capacity constraint (3-2) is the binding or complicating constraint. The rest of the constraints can be decomposed into sets of constraints for each job that can go in the sub-problem. The sub-problem solution will generate the schedule for the corresponding job that can be added as a column to the RMP.

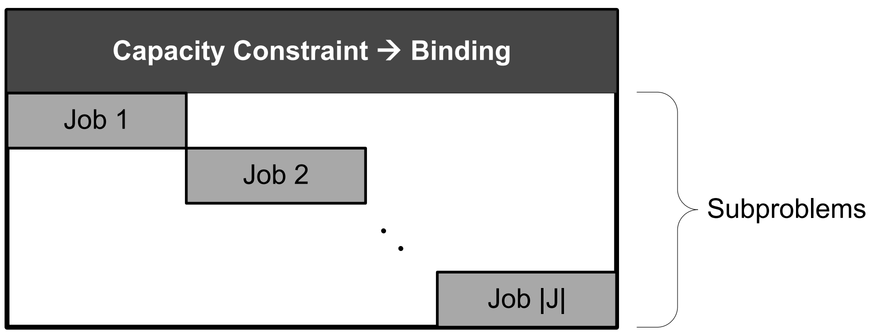


Figure 4-2. Decomposition of the MTO model (Block-Diagonal Structure)

The MTO restricted master problem is formulated as follows,

$$\text{Maximize } Z_{RMP}^{LP} = \sum_{j \in J} q_j U_j - \sum_{j \in J} \sum_{o \in O_j} \sum_{r \in R} \sum_{t \in T} \sum_{s \in S} \sum_{k \in K_j} (c_{rs} x_{jorts}^{k}) \lambda_j^k \tag{4-7}$$

Subject to

$$\sum_{j \in J} \sum_{o \in O_j} \sum_{k \in K_j} x_{jorts}^{k} \lambda_j^k \leq b_{rts} \qquad \forall r \in R, t \in T, s \in S \tag{4-8}$$

$$\sum_{k \in K_j} \lambda_j^k = U_j \qquad \forall j \in J \tag{4-9}$$

$$\lambda_j^k \geq 0 \ binary \qquad \forall j \in J, k \in K_j \qquad (4\text{-}10)$$

$$U_j \geq 0 \ binary \qquad \forall j \in J \qquad (4\text{-}11)$$

Where, K_j is the set of columns generated by the sub-problem for job j that are added to the RMP. The column generated by the sub-problem is a feasible schedule for the corresponding job. An initial feasible solution to the RMP is provided by a greedy heuristic presented in Section 4.4.

4.3. Solution approach for solving sub-problem

A feasible schedule for job j should satisfy the processing time constraint (3-3), the physical constraint of processing job j for not more than l_{ts} hours in source s of time period t, the due-date constraint (3-7) and the precedence constraints (3-8) and (3-9). The corresponding formulation for the sub-problem or pricing problem of job j will consist of the constraint set (3-3) to (3-11) with an objective of minimizing the cost of processing job j by its due-date. The objective function for the pricing problem is formulated as,

$$\text{Minimize } Z_{sp}^j = \sum_{o \in O_j} \sum_{r \in R} \sum_{t \in T} \sum_{s \in S} (c_{rs} + w_{rts}) x_{orts} + \alpha_j \qquad (4\text{-}12)$$

Where, w_{rts} and α is the dual variables of constraints (4-8) and (4-9) respectively.

Ideally, the sub-problem should be able to generate columns very quickly as the sub-problem has to be solved many times during the B&P procedure. Solving the sub-problem formulation to optimality using a commercial solver to generate columns is not an efficient way. Figure 4-3 and Figure 4-4 show the average computational time (or runtime) to solve a sub-problem with 3 jobs and 5 jobs instances to optimality using a commercial solver. It can be seen that the time taken to solve each sub-problem increases as the size of the sub-problem increases. Thus, there is a need to efficiently solve the sub-problem. The data used for this experiment was the same as used in Experiment C (Refer to Section 3.5). The sub-problem size is defined as the product of the number of operations per job and the time periods in the planning horizon, given that the number of resources and sources is kept constant.

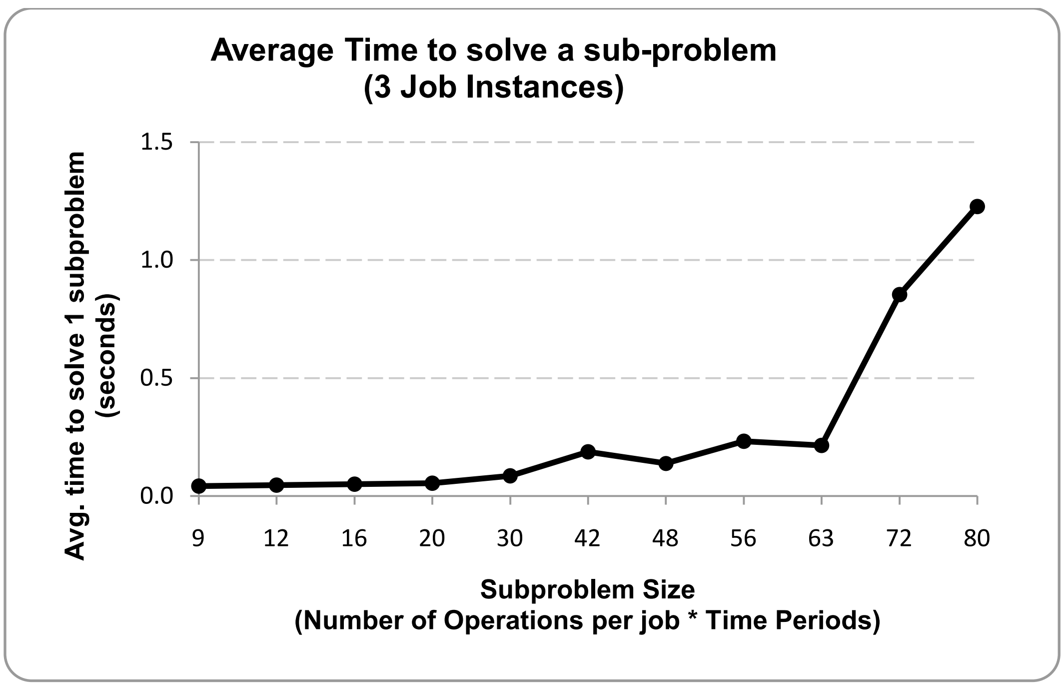


Figure 4-3. Average time to solve sub-problem formulation for 3 job instances

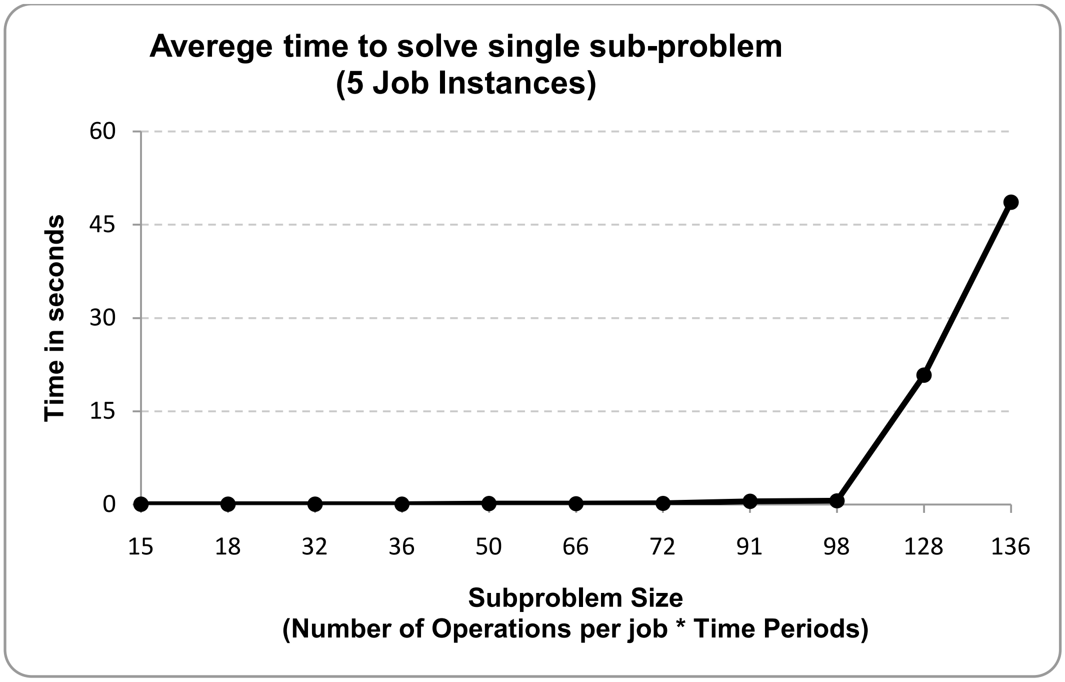


Figure 4-4. Average time to solve sub-problem formulation for 5 job instances

We propose an exact procedure to solve the sub-problem to optimality. We represent the sub-problem for job j as a Directed Acyclic Graph (DAG) $G^j=\{N^j,A^j\}$ where N^j denotes the set of nodes and A^j denotes the set of arcs. Each time period is discretized into smaller intervals with equal length denoted by *dtu*. Let the set of discretized time instants for job j from time period one till its due-date d_j be $H^j=\left\{1,2,\ldots,\sum_{t=1}^{d_j}\sum_{s\in S}\frac{l_{ts}}{dtu}\right\}$. Each operation o of job j is split into *dtu* sized operations. Let the set of split operations for all the operations in j be $E^j=\left\{1,\ldots,\frac{\sum_{o\in O_j}p_{jor}}{dtu}\right\}$ where r is the resource type on which operation o of job j needs to be processed and let the set $I_o^j=\left\{1,\ldots,{}^{p_{jor}}/_{dtu}\right\}$ be the set of split operations for operation o of job j. The set of nodes consists of three types, an artificial source node, an artificial sink node, and *OperationTimeNodes*. The nodes in *OperationTimeNodes* set are denoted by a 2-tuple $N\{e\in E^j, h\in H^j\}$ such that we have $|H^j|$ nodes corresponding to each element in E^j. We have l_{ts}/dtu nodes in H^j corresponding to each source s in time period t. There is a set of secondary attribute for each node represented by a 4-tuple $\mathrm{SA}_{e,h}\{o\in O_j, i\in I_o^j r\in R, t\in T, s\in S\}$. The set of arcs consists of two distinct types, set of idle arcs $\{\mathrm{IArcs}\subseteq A^j\}$ and set of processing arcs $\{\mathrm{PArcs}\subseteq A^j\}$. An arc is represented by the notation $A_{e,h}^{e'h'}$, where (e,h) and (e',h') is the tail node and head node respectively. There is a cost associated with each arc denoted by $C_{eh}^{e'h'}$. Idle arcs are connected between two consecutive nodes of the same split operation starting at node $\{e,h\}$ and ending at $\{e,h+1\}$. The processing arc starting from node $\{e,h\}$ goes to node $\{e+1,h+1\}$. This ensures that each discretized operation e is completed before starting discretized operation e+1. This structure captures the precedence constraint of the sub-problem. All arc capacities are set to one. A unit flow in the processing arc implies that the split operation e is processed for *dtu* time units in time instance h. A unit flow in the idle arc implies that the split operation e will not be processed for *dtu* time units in time instance h. A unit flow sent from the source node reaching the sink node ensures that all the operations in job j are processed by the due-date d_j. The cost of idle arc is zero while the cost of the processing arc is given by $c_{rs}+w_{rts}$, where r is the resource on which operation o of job j needs to be processed in source s of time period t. The arc connecting the source node to the first node in the *operationTimeNodes* $N\{1,1\}$ is denoted by $A_{\mathrm{source}}^{1,1}$ and cost of that is fixed to zero. All the arcs to the sink node are denoted $A_{e,h}^{\mathrm{sink}}$. The shortest path from the source node to the sink node gives us the schedule for job j at the minimum processing cost. Figure 4-5 shows a general DAG representation of the sub-problem. In the general DAG, it is apparent that there exist nodes which cannot be reached from the source node or nodes whose outbound flow can never reach the sink node and as such they can never be part of the shortest path. Hence we can eliminate such nodes.

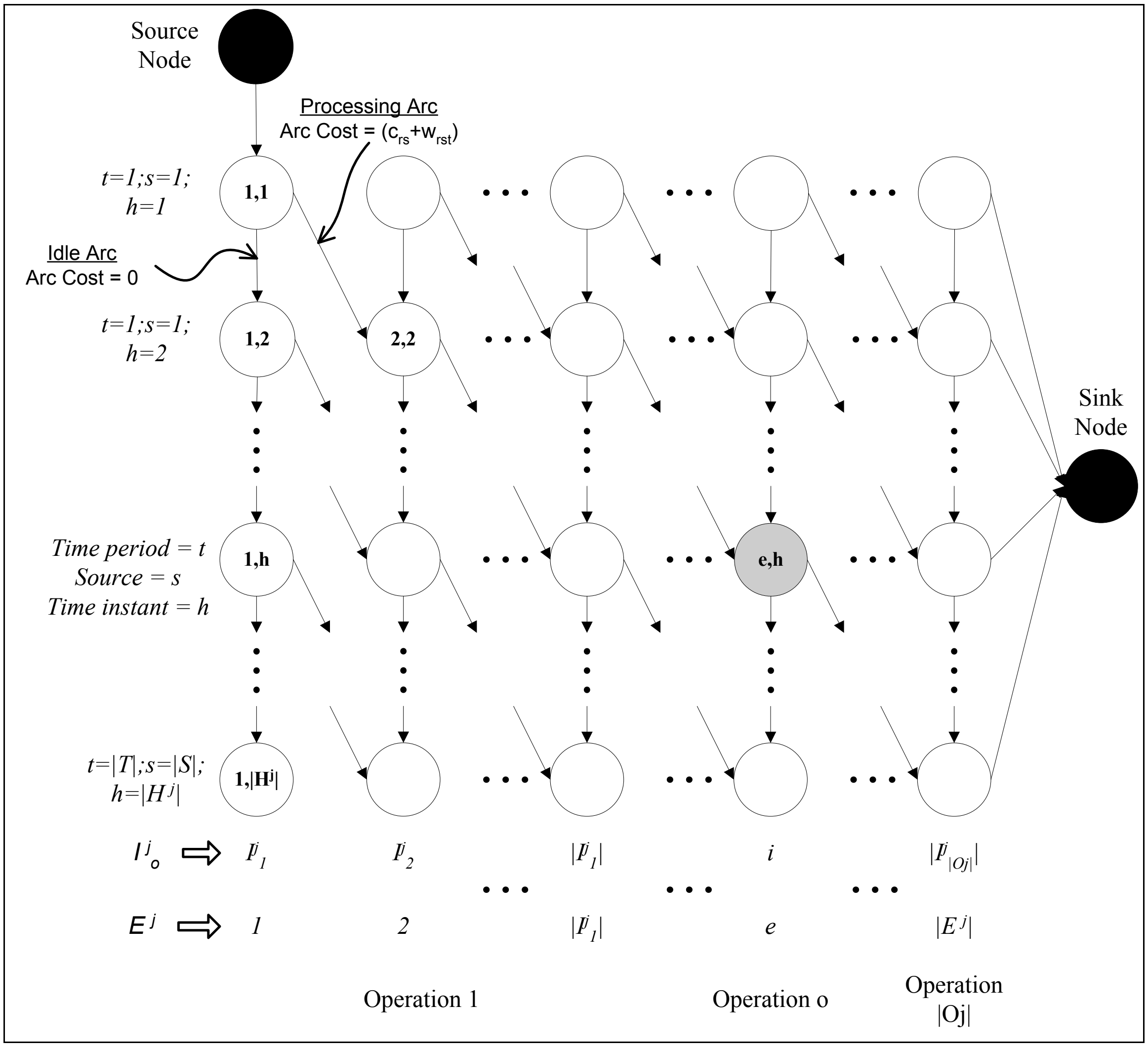


Figure 4-5. General Directed Acyclic Graph representation of the sub-problem

To further understand this concept, consider a sub-problem for job j with three operations having processing times 5, 2, and 3 hours, respectively. For simplicity consider that they need to be processed on the same resource. Let the due-date for job j be d_j = 1 day and we have two sources, regular time and over time of 8 hours each. We discretize time in units of one hour. The corresponding graph for the sub-problem is shown in Figure 4-6. The earliest we can process split operation e=1 is in time instance 1 corresponding to h=1, which implies that the earliest we can process split operation e'=e+1=2 is in h'=h+1=2, and thus all the nodes for e'=2 before time instance h'=2 can be ignored in G^j. For processing job j by its due-date, the latest we can process the split operation e=1 is in time instance 7 corresponding to h=7, thus the flow from all the nodes {h∈8,...,16 ,e=1} cannot reach the sink node and thus the corresponding nodes can be ignored in G^j. We can extend this logic for all the split operations and time instances $\left\{e \in E^j,\ h \in H^j\right\}$ and eliminate the corresponding nodes.

Figure 4-6 shows the shortest path from the source node to the sink node. The nodes visited in the shortest path are shaded in black and the path is represented by thick arrows. In each time period and source we can count for each operation how many processing arcs have been traversed which will give us the number of hours of processing of that operation. For example, for regular time in time period 1, we are processing operation 1 for four hours. Then the processing of operation 1 is continued in overtime for one hour.

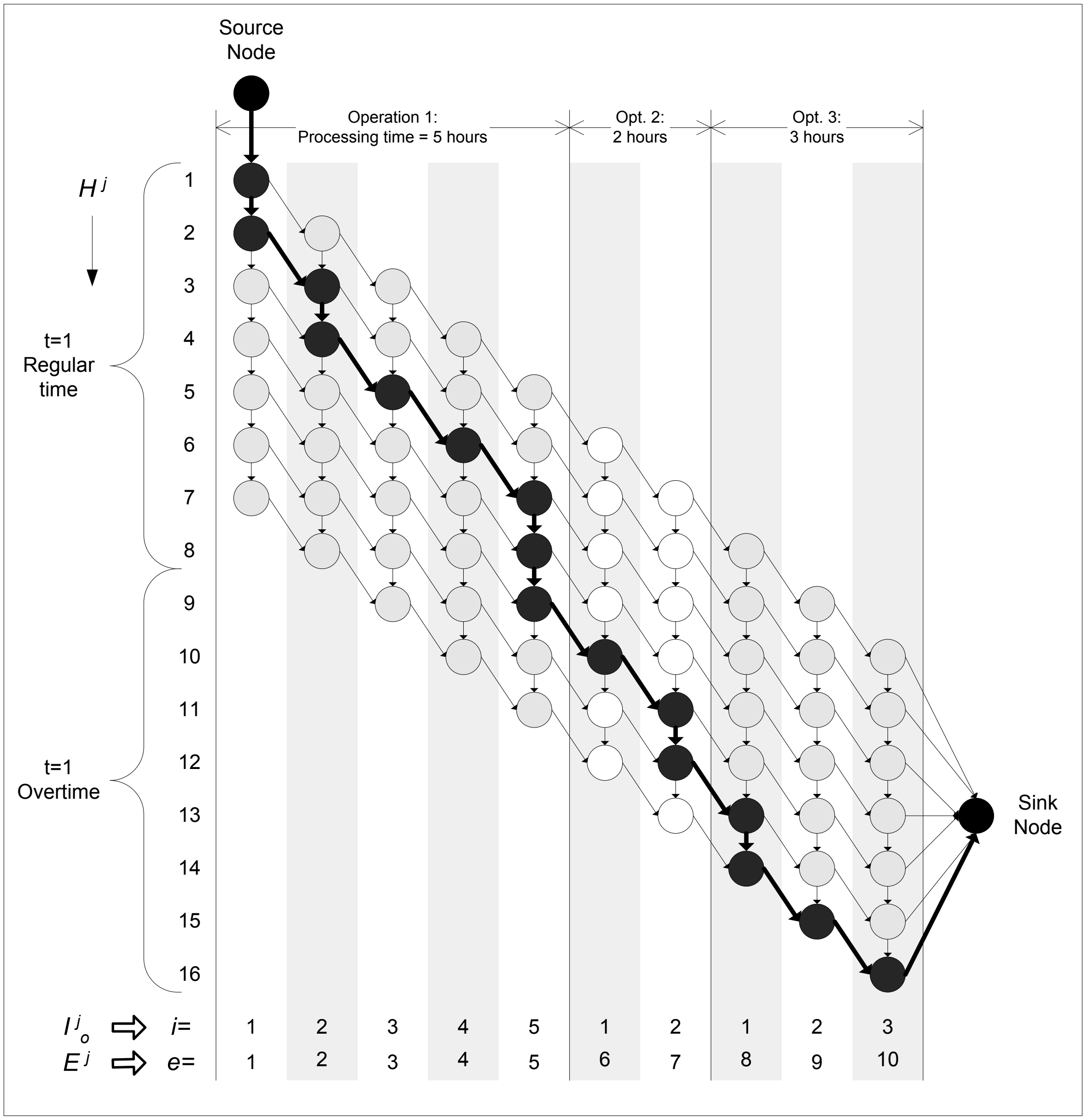


Figure 4-6. Directed acyclic graph for sub-problem representation

The pseudo code for finding the shortest path for acyclic digraph [46] and extracting the schedule from the shortest path solution are given below:

Algorithm for finding the shortest path:

BEGIN

$v[N\{1,1\}] \leftarrow 0$

$optPathTo[N\{1,1\}] \leftarrow source\ node$

For $N\{e',h'\} \in OperationTimeNodes\ \{e \in E^j, h \in H^j\} \backslash N\{1,1\}$

If $N\{e',h'\}$ *exists then*

For $N\{e,h\} \in \{N\{e',h'-1\}, N\{e'-1,h'1-\}\}$

$v[N\{e',h'\}] \leftarrow \min\{v[N\{e,h\}] + C_{eh}^{e'h'} : (A_{eh}^{e'h'}\ exists)\}$

$optPathTo[N\{e',h'\}] \leftarrow$ *node* $N\{e,h\}$ *achieving the minimum cost*

End for

End if

End for

Let $v[sink\ node] \leftarrow \min\{v[N\{|E^j|,h\}] + C_{eh}^{sink} : (A_{|E^j|h}^{sink} exists, \forall h \in H^j)\}$

Let $optPathTo[sink\ node] \leftarrow$ *node* $N\{e,h\}$ *achieving the minimum cost*

END

Algorithm to extract schedule from the shortest path solution for sub-problem *j*:

BEGIN

Initialize $x_{jorts}=0\ (\forall o \in O_j, r \in R, t \in T, s \in S)$

Let $N\{e,h\} \leftarrow optPathTo[sink\ node]$

If $(A_{e,h}^{sink} \in PArcs)$ *then*

Let $x_{jorts} \leftarrow x_{jorts}+1\ (\forall o,r,t,s \in SA_{e,h})$

End if

While (N{e,h} ≠ source node)

Let N{e',h'}←optPathTo[N{e,h}]

If ($A^{e,h}_{e',h'}$ *∈PArcs) then*

Let $x_{jorts} \leftarrow x_{jorts}+1$ *(∀o,r,t,s ∈* $SA_{e',h'}$*)*

End if

Let N{e,h}←N{e',h'}

End while

END

4.4. Greedy heuristic for initial solution to RMP

In order to start the column generation procedure, a basic feasible solution to the RMP has to be provided. We propose a greedy heuristic to obtain this solution. The set of available jobs *J* are sorted in a non-increasing order of their profit margins, where profit margin is the ratio of the sales price to the cost of processing the job in regular time, and stored in a list. Each job in this list is scheduled one at a time with an objective of minimizing their processing costs. If scheduling a job improves the objective function value, the total profit, then the job is accepted and the residual capacities for the resources are updated along with the total profit yielded by accepting the current job, else the job is rejected. The schedule obtained for the job is added as a column to the RMP. For scheduling each job so that we can minimize its processing costs, we make use of the sub-problem solution approach described in Section 4.3. The dual prices are set to zero for the capacity constraints and the convexity constraints. The costs of the processing arcs which have been already utilized by previously scheduled jobs are set to infinity, to take care of the residual capacities of the resources in their respective time periods and sources.

4.5. Branching in Branch and Price algorithm

In Section 4.5.1 we define an integer feasible solution to the original problem. In Section 4.5.2 we discuss the proposed branching strategies. In Sections 4.5.3 and 4.5.4 we derive the Lagrangean bounds for fathoming the nodes and node selection for exploring the branch and bound tree respectively.

4.5.1. Definition of an integer feasible solution to RMP

We formally define a feasible integer solution to the RMP.

Definition 4.1: Consider a set of columns $k \in K_j$ for job j represented by the basic variables $\lambda_j^{k \in K_j}$ in the RMP, such that $\sum_{k \in K_j} \lambda_j^k = 1$, which implies U_j=1 (from constraint (4-9)). The convex combination $\theta_j = \sum_{k \in K_j} x_{jorts}^k \lambda_j^k$ is a feasible integer solution to the RMP for job j if for any pair of operations $(o_i, o_{i+1})\{i=1,\ldots,|O_j|-1\}$ in θ_j, there is no precedence violation. Since there is no restriction over x_{jorts} to be integer in the original problem, θ_j is a feasible solution for the original problem.

Consider job j with 3 operations having processing times 6, 10 and 4 hours, respectively. In the RMP, suppose we have two schedules corresponding to the basic variables,

λ_j^1=0.45 and λ_j^2=0.55. For an intuitive representation of a schedule we represent it as a matrix, where the rows denote the time period t and source s while the columns denote the operations. Then the schedules corresponding to the basic variables are as follows:

$$\lambda_j^1 \Rightarrow x_{jorts}^1 \Rightarrow \begin{array}{cl} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} t=1, s=1 \\ t=1, s=2 \\ t=2, s=1 \\ t=2, s=2 \\ \\ \vdots \\ \\ t=4, s=2 \end{array} \end{array}$$

$$\lambda_j^2 \Rightarrow x_{jorts}^2 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The convex combination θ_j is given by,

$$\theta_j = \sum_{k=1}^{2} x_{jorts}^k \lambda_j^k \Rightarrow \begin{bmatrix} 1.8 & 0.0 & 0.0 \\ 3.3 & 0.0 & 0.0 \\ 0.9 & 2.7 & 0.0 \\ 0.0 & 4.4 & 0.0 \\ 0.0 & 2.9 & 2.9 \\ 0.0 & 0.0 & 1.1 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

In θ_j, none of the adjacent operation pairs have a precedence violation; the processing time constraint (3-3), physical constraint (3-4) and due-date constraint (3-7) are satisfied and hence θ_j is an integer feasible solution to RMP for job j.

Now consider another basic column λ_j^3 with a corresponding schedule given by x_{jorts}^3 and the new solution to RMP is λ_j^1=0.15, λ_j^2=0.35, and λ_j^3=0.5.

$$\lambda_j^3 \Rightarrow x_{jorts}^3 \Rightarrow \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The convex combination θ'_j for the above RMP for job j is,

$$\theta'_j = \sum_{k=1}^{3} x_{jorts}^k \lambda_j^k \Rightarrow \begin{bmatrix} 3.6 & 0.0 & 0.0 \\ 2.1 & 3.0 & 0.0 \\ 0.3 & 2.9 & 2.0 \\ 0.0 & 2.8 & 0.0 \\ 0.0 & 1.3 & 1.3 \\ 0.0 & 0.0 & 0.7 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

In θ'_j, operation 1 ends in time period 2, source 1, while operation 2 starts in time period 1, source 2. Thus for operation pair (1,2) there is a precedence violation. Similarly for operation pair (2,3) the precedence constraint is violated. Thus θ'_j, is an infeasible integer solution to the RMP for job j.

4.5.2. Branching strategies

In our original formulation, we have two binary variables Y and U. While branching in B&P algorithm the literature suggests to branch on the original variables, instead of branching on the variable λ in the RMP. From the Definition 4.1, we know that, to get an integer feasible solution to the RMP for job j, U_j should be exactly equal to 1, implying that job j is selected and the processing time requirements of it are satisfied. Hence at any node in the branch and bound tree if we find U_j to be fractional, we branch on U_j, setting U_j to 0 in the first (left) child node and U_j to1 in its twin (right) node. If at any node in the branch and bound tree, if we find U_j $\{j \in J\}$ to be either 0 or 1, and all the corresponding θ_j $\{j \in J\}$ to be integer feasible solutions as per Definition 4.1, then we report the corresponding integer solution.

In the case that we find U to be binary, but θ_j $\{j \in J\}$ to be integer infeasible, then we have to fix the original variable Y. Fixing Y variables is not as straight forward as fixing U, since we do not have them as explicit variables in the RMP. In the original formulation Y is an indicator variable used for ensuring that the linear precedence amongst operations is maintained. Whenever we have θ_j as fractional, all the constraints except the precedence constraint are satisfied. In such a case we try to restore the precedence between the pair of adjacent operations violating this constraint such that the solution subsets are equally divided so that we get a balanced branch and bound tree. We propose a branching strategy to accomplish this as explained in the rest of the section.

A pair of operations (o,o+1) has a precedence violation if operation o+1 starts before the completion of operation o. We define this violation in absolute terms as the precedence error length ε_p^o for job pair (o,o+1) given by the difference in the end time of operation o and start time of operation o+1. Equation (4-13) computes the precedence error length between the violating adjacent pair of operations.

$$\varepsilon_p^o=2\left(et_o-st_{o+1}\right)+(es_o-ss_{o+1}) \tag{4-13}$$

Where, et_o and es_o is the time period and source respectively in which operation o is completed, while st_{o+1} and ss_{o+1} is respectively the time period and source in which operation o+1 starts. For the purposes of this research we refer to a particular time period and source combination as time-source instance.

Let θ_j' be an integer infeasible solution at node n in the branch and bound tree and all U_j $\{\forall j\in J\}$ be binary. We try to restore the precedence amongst the violating operation pair with maximum precedence error length. Let this pair be denoted by (o,o+1).

Consider the illustration in Section 4.5.1., with schedule θ_j' being an integer infeasible solution at some node n in the branch and bound tree and all U_j $\{\forall j\in J\}$ are binary. Figure

4-7 shows the Gantt chart for the schedule generated from this convex combination. Operation pair (2,3) has the maximum precedence error length (ε_p^2) of 2 units. Hence, we select this pair to restore precedence feasibility.

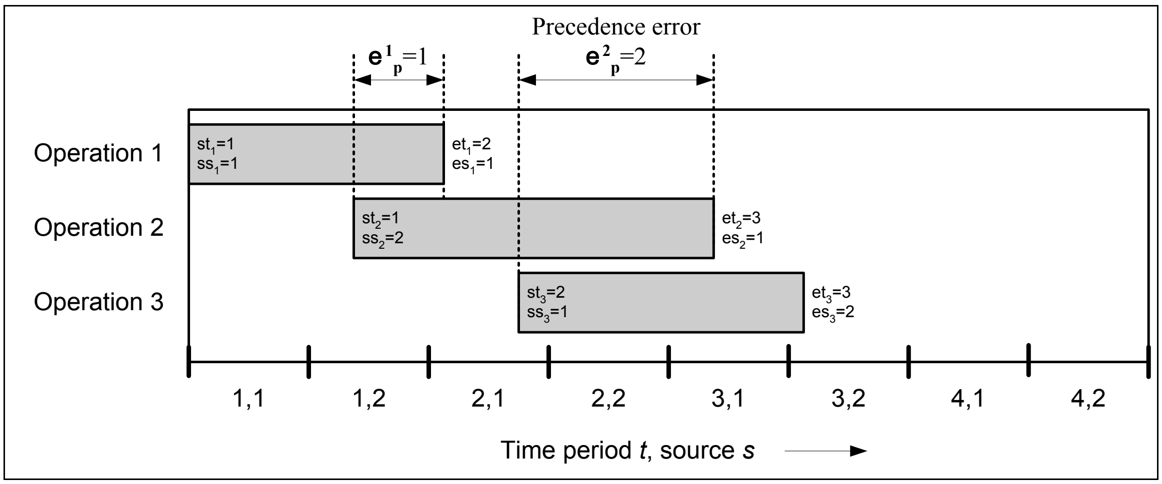


Figure 4-7. Gantt chart for the schedule obtained from the convex combination

In the B&B tree at node n, we form two child nodes. In branching strategy 1 (BPS1) for the first child node we place the restriction that operation o cannot be scheduled in the time period (et_o) and source (es_o) in which it had finished its processing in schedule $\theta_j^{'}$. In the second child node we place the restriction that operation o+1 cannot be scheduled in the time period (st_{o+1}) and source (ss_{o+1}) when it began its processing in schedule $\theta_j^{'}$. There are no other restrictions on scheduling either these operations or other operations.

Continuing the discussion on the illustration from Section 4.5.1, if we want to implement BPS1 for this example then in child node 1 we place the restriction that operation 2 is not allowed to be scheduled in regular time (s=1) of the third time period, while in child node 2 we do not allow operation 3 to be scheduled during the regular time (s=1) of the second time period. Figure 4-8 shows the branching with the restrictions on each node. The black colored box shows that scheduling during those time periods and sources is not allowed.

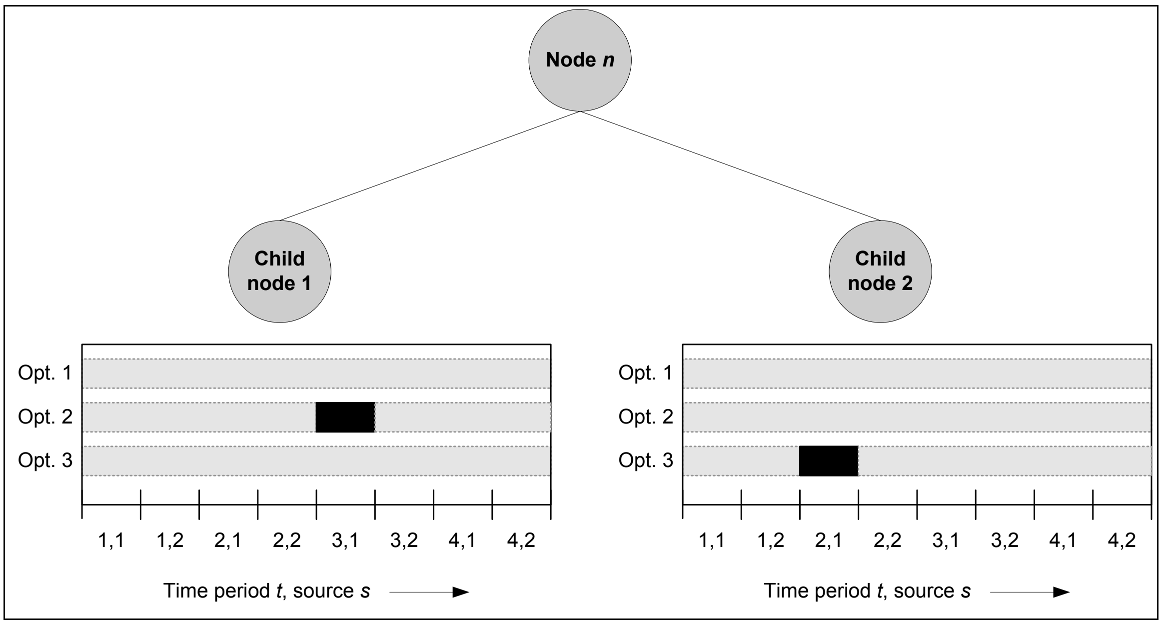


Figure 4-8. Branching strategy 1 (BPS1)

4.5.3. Lagrangian bounds

Column generation process carries out many iterations with very small improvements to objective function value of the RMP. Thus it takes relatively longer times to prove optimality of the current solution. This is called the "tailing-off" effect. We can reduce this effect by stopping the column generation procedure earlier by proving optimality of the current solution. To achieve this we provide an upper bound (since the original problem is a maximization problem). If the upper bound is less than the best known integer solution value then we can terminate the column generation procedure at the node and fathom the corresponding node without risk of missing the optimum.

Lasdon [47] provides a lower bound calculation for the master problem from the current objective value and the reduced costs obtained by solving the sub-problems. We follow a similar method, but unlike Lasdon our RMP is a maximization problem and hence the bound which we get is in fact an upper bound to the Master Problem (MP). Also, we have an additional variable U_j which is non-decomposable; as such it is not a part of the sub-problem solution. We now discuss the computation of the upper bound.

Proposition 4-1: Given that Z_{RMP}^{LP} is the current objective function value of the RMP at optimality, then the upper bound to the optimal objective value for the MP is given by Z_{RMP}^{LP}- $\left[min\left(\sum_j\left(min\ Z_{SP}^{j}\right),\ 0\right)\right]$

Proof:

The proof is presented in matrix notations. We use bold-face capital letters to represent variables and bold-face lower-case letters to represent parameters and dual values.

$$\boldsymbol{qU} - \boldsymbol{cx\lambda} - \boldsymbol{wb} = \boldsymbol{qU} - \boldsymbol{cx\lambda} - \boldsymbol{wx\lambda} - \boldsymbol{\alpha}(\boldsymbol{\lambda} - \boldsymbol{U}) \tag{4-14}$$

$$\boldsymbol{qU} - \boldsymbol{cx\lambda} - \boldsymbol{wb} = \boldsymbol{qU} - \boldsymbol{\lambda}[(\boldsymbol{c} + \boldsymbol{w})\boldsymbol{x} + \boldsymbol{\alpha}\ ] + \boldsymbol{\alpha U} \tag{4-15}$$

From Equation (4-12) we know $(\boldsymbol{c}+\boldsymbol{w})\boldsymbol{x}+\boldsymbol{\alpha} = \sum_j (min\ Z_{SP}^j)$. Since $min\ Z_{SP}^j$ is the reduced cost of j^{th} sub-problem, we consider only those that will improve the objective function value of RMP, hence $(\boldsymbol{c}+\boldsymbol{w})\boldsymbol{x}+\boldsymbol{\alpha}$ can be replaced by $min\left(\sum_j (min\ Z_{SP}^j), 0\right)$.

$$\boldsymbol{qU} - \boldsymbol{cx\lambda} - \boldsymbol{wb} \leq \boldsymbol{qU} - \sum_{j \in J} \sum_{k \in K_j} \lambda_j^k \left[min\left(\sum_j \left(min\ Z_{SP}^j\right), 0\right)\right] + \boldsymbol{\alpha U} \tag{4-16}$$

Rearranging the terms in Equation (4-16), we get,

$$\boldsymbol{qU} - \boldsymbol{cx\lambda} \leq \boldsymbol{qU} + \boldsymbol{wb} + \boldsymbol{\alpha U} - \sum_{j \in J} \sum_{k \in K_j} \lambda_j^k \left[min\left(\sum_j \left(min\ Z_{SP}^j\right), 0\right)\right] \tag{4-17}$$

The dual objective function value of the RMP is given by $\boldsymbol{qU} + \boldsymbol{wb} + \boldsymbol{\alpha U}$, which is equal to the objective function value of the primal RMP at optimality. We can re-write equation (4-17) as,

$$\mathbf{qU} - \mathbf{cx}\boldsymbol{\lambda} \leq Z_{RMP}^{LP} - \sum_{j \in J} \sum_{k \in K_j} \lambda_j^k \left[min\left(\sum_j \left(min\ Z_{SP}^j\right), 0\right)\right] \tag{4-18}$$

4.5.4. Node Selection

For searching the branch and bound tree we use three strategies, namely, Depth first Search (DFS), Best First Search (BeFS) and a combination of depth first and best first strategy which we denote by (DFS+BeFS). In DFS strategy when exploring a particular node, we form two child nodes and select the node with the best bound for exploration. We continue this till we find an integer solution and then backtrack to the nodes which are unexplored. In BeFS strategy we search for the node with the best bound in the complete B&B tree for exploration. In DFS+BeFS strategy we try to combine the first and second strategy. We begin with DFS strategy and after finding an integer solution we implement BeFS so as to select an unexplored node having the best bound in the B&B tree and thereafter continue with DFS.

4.6. Experimentation

The research problem under consideration does not have benchmark problems and solutions. Hence, we compare the results from Branch & Price algorithm to the best known integer solution provided by a commercial solver for the original formulation. For it is seen from some initial experimentation with the original formulation that the solution time depends on the number of jobs, number of operations in each job, and number of time periods considered in the planning horizon. The length of the planning horizon and the number of jobs and operations determine the complexity of the problem. As mentioned earlier, if we consider a very long planning horizon and relatively few jobs, the problem is trivial to solve, because we will have enough capacity to accept and schedule all the available jobs in regular time. Thus, we control the length of the planning horizon indirectly by introducing the demand-to-capacity ratio (DC ratio) explained in Section 3.5. The objective is to obtain an experimental range for the DC ratio for which the commercial solver yields relatively good results in the stipulated time. The pilot experiment which we conducted for this purpose is explained in Section 4.6.1.

4.6.1. Pilot experiment

The experimental setup is provided in Table 4-1. We consider a full factorial experimental design. The number of operations per job, and the number of resources is kept constant at 5 and 3 respectively. The processing times are generated randomly for each replication from a discrete uniform distribution. We have two replications for each factor-level combination. The DC ratio is varied from 0.2 to 2.2 with a step size of 0.4 as shown in the experimental setup. The time periods obtained by using these DC ratios varied from 2 to 144. The due-date for each job is randomly generated within 60% to 100% of the planning horizon. The regular time cost for each resource is randomly generated from a uniform distribution between 20 and 80. The ratio of regular time to over time cost is 1:1.5. The sales price for each job is decided using Equation 4-19.

Table 4-1. Pilot Experiment

Factors	Levels
Number of jobs	3, 10 and 15
Number of operations	5
DC ratio	0.2, 0.6, 1.0, 1.4, 1.8, and 2.2
Number of resources	3
Processing time distribution	DU[2,8]

$$q_j = \sum_{o \in O_j} p_{jor} c_{r,s=1} * (1 + (0.7 * U[0,1])) \quad (4\text{-}19)$$

The problem instances are solved for the original formulation by using a commercial solver CPLEX 10.1. Each problem instance is allowed to be run either till the optimality of the solution is proved or for 1800 seconds, whichever is the first. We report the time taken to solve the problem, and the absolute and relative gap. The relative gap is defined by CPLEX as given in Equation (4-20), where Z_{IP} is the best known integer solution and the Bound is the best bound in the branch and bound tree.

$$\text{Relative gap in \%} = \left|\frac{Z_{IP}\text{-}LP\ Bound}{Z_{IP}}\right| * 100 \qquad (4\text{-}20)$$

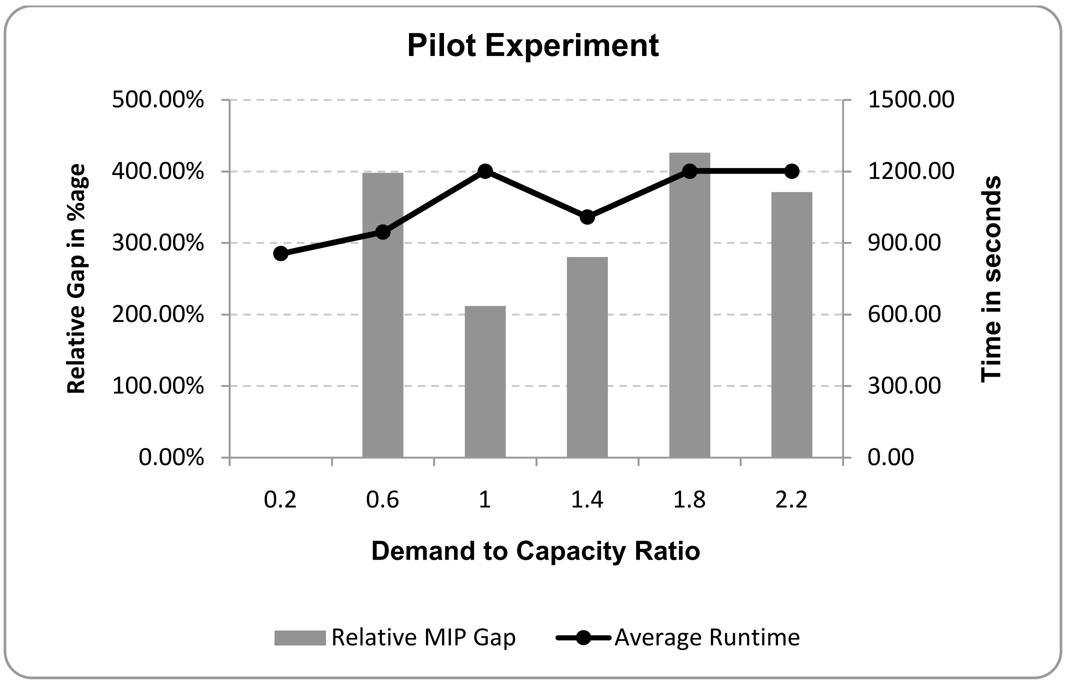


Figure 4-9. Summary of results for Pilot Experiment

When DC ratio is 0.2, we have enough capacity to accept all the orders and schedule them in regular time, hence the problem is trivial to solve. It is seen that the solution quality of CPLEX is relatively better for problem instances with a DC ratio of 1.0. Hence for further experimentation to assess the solution quality of the B&P algorithm we generate a set of problem instances with a DC ratio as 0.8, 1.0 and 1.2, by comparing them with CPLEX.

4.6.2. Experimental setup

As discussed in Section 4.6.1, we have three levels for the DC ratio, namely 0.8, 1.0 and 1.2. The complete experimental setup (Experiment D) to assess the solution quality of the B&P algorithm is presented in Table 4-1. We conduct a full factorial experiment for the different factors and their respective levels with three replications. The number of resources is fixed to 3 for problem instances with 3 and 5 operations per job, and to 5 for instances with 8 and 10 operations per job. The due-dates for each job, the regular time and overtime costs and the sales price are generated as discussed in Section 4.6.1.

Table 4-2. Experiment D setup for assessing the quality of B&P Algorithm

Factors	Levels
Number of jobs	3, 5, 8 and 10
Number of operations	3, 5, 8 and 10
DC ratio	0.8, 1.0 and 1.2
Processing time distribution (hours)	DU[4,16]

4.6.3. Solution quality of Branch and Price Algorithm

In this section we first show that column generation provides tighter bounds than LP relaxation of the original problem at root node. Then we discuss the relative performance of the three node selection strategies implemented in BPS1 with CPLEX results as the benchmark. CPLEX was not able to provide feasible solutions to all the problem instances; hence we report the number of instances that CPELX could provide a feasible solution and the number instances solved to optimality. We also present the relative gap reported by CPLEX at the end of 1800 seconds and the average runtime in seconds. We empirically show that BPS1 with DFS+BeFS as the node selection strategy proves to be the best solution approach with solution quality and runtime as the measures of performance. Finally we show the efficiency of the sub-problem solution strategy, which is a very important factor in successfully implementing a branch-and-price algorithm.

Figure 4-10 graphically shows the percentage gap between the bounds obtained by LP relaxation of the original problem and bounds obtained from column generation at the root node of the branch and bound tree. The X-axis consists of two rows, the first row gives the number of operations for the number of jobs in the second row. The Y-axis gives the percentage gap between the two bounds. A positive gap means that the objective value obtained from the column generation was lesser than LP relaxation. For a maximization problem this means that we found tighter bounds using column generation. On an average we find that the percentage gap between the two bounds is anywhere from 2% to 10%. In some instances especially with 3 and 5 jobs having 8 and 10 operations each, the gaps are relatively larger. This can be attributed to the length of the planning horizon where very few jobs or none could be accepted in reality because of the due-date constraint which is implicit in column generation but yields a highly fractional solution in LP relaxation. A specific example is for the instance with 3 jobs, 10 operations and planning horizon of 6 days. The LP relaxation yields an objective function value of

1109.797 with all jobs being selected, but in the integer solution we find that none of the jobs could be processed because they violated their due-dates and the integer solution had an objective function value of 0.

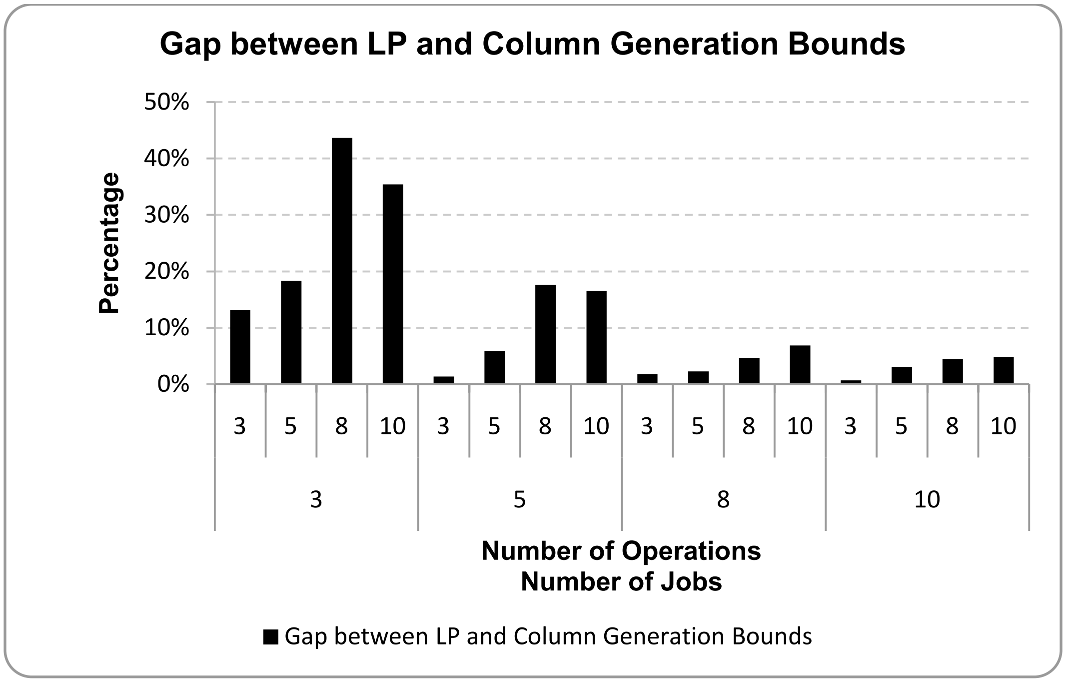


Figure 4-10. Percentage gap between LP and column generation bounds

Figure 4-11 shows the time taken to reach the root node solution for the LP relaxation of the original problem and column generation. We can easily verify from this graph that column generation takes comparatively lesser time to determine the root node solution of the branch and bound tree as compared to the LP relaxation of the original problem, as the problem size increases.

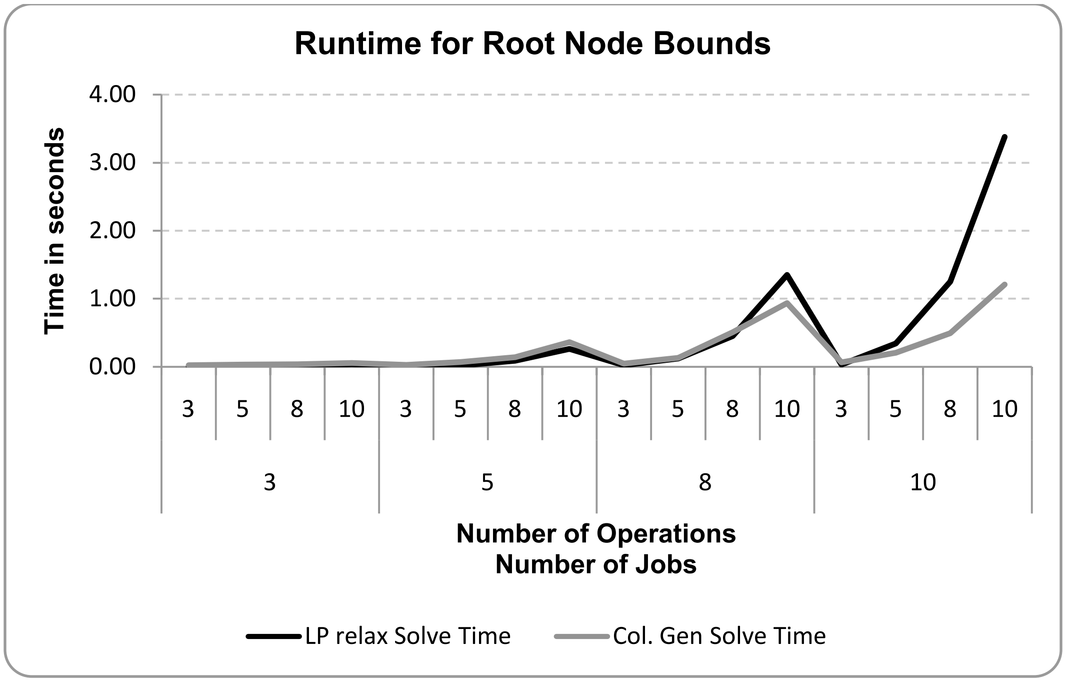


Figure 4-11. Time to reach root node in LP relaxation and column generation

For BPS1 procedure we experimented with three different node selection strategies as explained in Section 4.5.4. In Table 4-3, we report the number of optimal solutions (A) obtained by CPLEX and B&P. We also report the relative gap in percent (B) computed using Equation 4-21 between the best known integer solution Z_{IP} provided by the solution method and the best bound computed in the branch and bound tree Z_{RMP}^{BUB}. We allow BPS1 to run till it proves optimality or a maximum of 900 seconds, whichever comes first. The improvement made by BPS1 over CPLEX is shown in Table 4-4. For problem instances with less than or equal to 5 jobs CPLEX yields marginally better results than BPS1. But as the number of jobs increase, BPS1 shows huge improvements over CPLEX. From these results we can see that CPLEX cannot provide feasible solutions for some problem instances with high number of jobs and operations. As the number of operations increase, the problem complexity increases, and hence CPLEX cannot provide optimal solutions in the stipulated 1800 seconds. Also the relative gap increases with this increase in number of operations. A side-by-side comparison between the relative gap of the four solution strategies shows that BPS1 with DFS and DFS+BeFS consistently provides low relative gap and relatively more number of optimal solutions as compared to CPLEX. BPS1(BeFS) yields poor quality solutions for problems with more than 3 jobs as compared to the other node selection strategies of BPS1. On an average the relative gap for BPS1 with DFS and DFS+BeFS is always less than 4%; which means that the solutions provided by these two strategies is within 4% of the optimum. On the other hand, the solutions obtained from CPLEX can be on an average as far as 200% and up to 1200% away from the optimum.

$$\text{Relative gap in \%} = \frac{Z_{IP} - Z_{RMP}^{BUB}}{Z_{IP}} * 100 \tag{4-21}$$

The improvements made by BPS1 over CPLEX are shown in Table 4-4. For problem instances with less than or equal to 5 jobs CPLEX yields marginally better results than BPS1. But as the number of jobs increase, BPS1 shows huge improvements over CPLEX.

Table 4-3. Number of optimal solutions obtained and relative gap

			(A) Number of Optimal Solutions (B) Relative gap in %							
Jobs	Opts.	# of Inst. solved by CPLEX	CPLEX		BPS1					
					DFS		BeFS		DFS +BeFS	
			A	B	A	B	A	B	A	B
3	3	9	9	0.00	9	0.00	9	0.00	9	0.00
	5	9	9	0.01	8	0.22	9	0.00	7	0.27
	8	9	9	0.00	9	0.00	9	0.00	8	0.14
	10	9	7	3.94	8	2.31	9	7.83	6	0.63
Average				0.99		0.63		1.96		0.26
5	3	9	9	0.00	9	0.00	9	0.00	8	0.02
	5	9	4	4.39	6	0.60	6	4.25	6	0.41
	8	9	0	14.51	2	10.3	4	20.89	0	9.03
	10	9	0	22.86	1	3.06	1	77.71	0	2.69
Average				10.44		3.42		25.71		3.04
8	3	9	5	1.52	7	0.48	5	6.00	9	0.00
	5	9	0	5.24	3	0.96	0	35.66	4	0.58
	8	9	0	29.62	0	4.51	0	29.81	0	3.96
	10	8	0	176.86	0	4.71	0	39.27	0	2.73
Average				49.76		2.61		27.35		1.79
10	3	9	6	0.52	8	0.05	1	21.63	8	0.05
	5	9	1	5.02	5	0.37	0	22.31	5	0.16
	8	9	0	60.79	0	2.37	0	30.61	0	1.54
	10	5	0	1269.25	0	3.82	0	25.8	0	3.40
Average				216.98		1.38		24.89		1.02
Grand Average				65.44		2.02		19.78		1.54

It can be seen from Table 4-3 and Table 4-4 that for problems with less number of jobs, DFS and BeFS performs better than DFS+BeFS. But as the number of jobs increase there is no significant difference between DFS and DFS+BeFS, while BeFS provides no optimal solutions and on an average performs very poorly as compared to the other two strategies.

Table 4-4. BPS1 solution improvement over CPLEX

		Improvement over CPLEX in %		
		BPS1		
Jobs	# of Opts.	DFS	BeFS	DFS +BeFS
3	3	0.00	0.00	0.00
	5	-0.05	0.00	0.00
	8	0.00	0.00	0.00
	10	-1.51	-4.44	-0.04
Average		-0.39	-1.11	-0.01
5	3	0.00	0.00	0.00
	5	-0.34	-3.56	-0.13
	8	-4.18	-11.82	-3.24
	10	4.16	-29.56	4.43
Average		-0.09	-11.23	0.26
8	3	-0.46	-4.89	0.00
	5	0.80	-20.73	1.18
	8	20.54	-0.73	20.99
	10	139.84	84.59	144.34
Average		37.33	12.56	38.69
10	3	0.00	-16.36	0.00
	5	1.13	-14.85	1.35
	8	50.79	23.46	52.13
	10	1175.60	1032.22	1178.07
Average		198.29	159.10	199.11
Grand Avg.		54.93	36.59	55.65

MILP problems are hard to solve using conventional branch and bound procedures. This is evident from our previous results in Sections 3.5, 4.6.1 and also from Table 4-5 where we show the average runtimes for different problem instances. We can observe that BPS1 is comparatively faster than CPLEX and at the same time provides better quality solutions. The DFS and DFS+BeFS strategies are comparable in terms of improvement over CPLEX, but on an average DFS+BeFS terminates earlier than DFS.

It is typical for a decomposition solution approach to find good quality feasible solutions early on in the solution process. This is true for the problem under consideration where the best integer solution was found much earlier than it took to terminate the B&P procedure. Table 4-6 summarizes the average time taken to find the best solution, after which no improvement was made. These results also suggest that BPS1(DFS+BeFS) finds solutions of comparable quality as BPS1(DFS) but quite early in the B&P process. Furthermore, BPS1(BeFS) makes no improvement over the initial solution for larger problem instances. Since BPS1(DFS+BeFS) performs the best with respect to the solution quality and runtime, for the rest of the chapter we will restrict our discussion to this strategy.

Table 4-5. Average runtime for CPLEX and BPS1

		Average runtime in seconds			
			BPS1		
Jobs	Opts.	CPLEX	DFS	BeFS	DFS +BeFS
3	3	0.04	0.03	0.03	0.33
	5	8.34	100.44	1.00	4.33
	8	81.79	1.43	1.51	0.54
	10	537.74	133.12	105.39	102.08
Average		156.98	58.75	26.98	26.82
5	3	0.69	23.52	8.68	0.32
	5	1181.86	319.15	458.18	210.50
	8	1800.59	708.87	542.98	483.20
	10	1800.23	805.60	801.41	800.67
Average		1195.84	464.28	452.81	373.67
8	3	955.33	200.63	443.33	0.89
	5	1800.15	602.08	839.96	502.71
	8	1800.14	900.46	900.35	900.45
	10	1800.05	900.38	900.33	900.38
Average		1582.88	643.76	767.30	566.84
10	3	665.43	103.84	792.90	101.87
	5	1783.29	408.87	892.04	408.61
	8	1800.04	900.40	900.19	900.43
	10	1800.12	900.23	900.44	900.76
Average		1476.23	538.10	867.76	537.56
Grand Avg.		1088.79	421.44	512.24	370.21

Table 4-6. Time to Best Integer Solution for BPS1 (Experiment)

		(A)Avg. time to Best Integer Solution for BPS1 in seconds (B) Improvement over Initial Solution in %					
		DFS		BeFS		DFS+BeFS	
Jobs	Opts.	A	B	A	B	A	B
3	3	0.03	31.67	0.02	31.67	0.32	31.67
	5	19.64	16.39	0.89	16.44	4.19	16.44
	8	1.23	137.52	1.51	137.52	0.45	137.52
	10	38.07	46.05	2.79	41.18	2.09	48.49
Average		14.74	57.91	1.3	56.7	1.76	58.53
5	3	22.55	21.3	8.67	21.3	0.15	21.3
	5	44.12	26.65	156.69	22.91	9.52	26.86
	8	112.51	52.16	23.67	40.74	131.57	53.51
	10	158.62	84.49	0.89	10.36	126.63	84.84
Average		84.45	46.15	47.48	23.83	66.97	46.63
8	3	5.25	19.81	43.28	14.36	0.89	20.37
	5	184.11	34.51	0	0	83.44	34.96
	8	333.64	24.59	0	0	142.51	25.16
	10	210.79	33.04	0	0	211.23	35.59
Average		182.66	27.84	11.13	3.69	106.61	28.83
10	3	60.57	22.53	71.19	14.36	2.09	22.53
	5	70.36	21.88	0	0	36.25	22.12
	8	183.91	27.58	0	0	258.33	28.15
	10	463.43	20.56	0	0	492.07	21.17
Average		160.96	23.46	20.02	0.27	160.32	23.88
Grand Average		108.74	39.36	20.05	21.85	81.55	39.99

Figure 4-12 graphically presents the comparison of total time taken to the time taken to find the best solution for BPS1 with DFS+BeFS strategy.

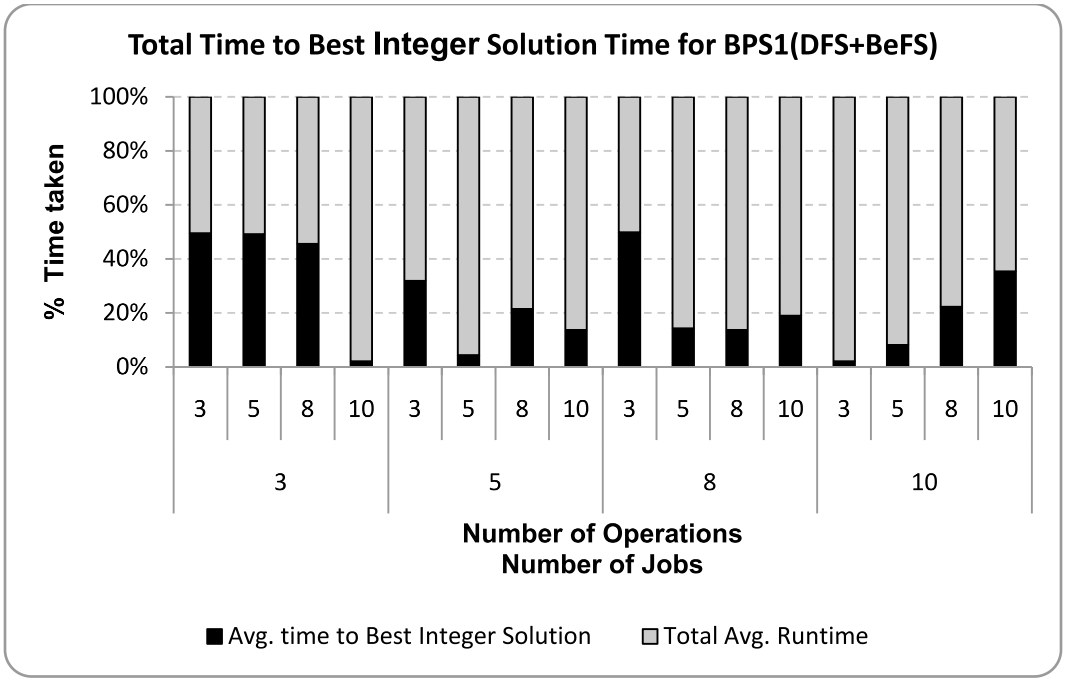


Figure 4-12. Comparison of total time and time taken to find best integer solution

These results are summarized graphically in Figure 4-13. We can see that for problems with lesser number of jobs and operations, CPLEX can provide optimal results and hence the improvements are zero. In some instance CPLEX actually performs better than BPS1 in solution quality, as BPS1 is unable to find optimal solution in the allotted 900 seconds, although the average runtime of CPLEX is much higher that BPS1 as we terminate CPLEX after 1800 seconds. Typically, CPLEX can solve most of the 3 operation instances with higher number of jobs to optimality. As the number of jobs and number of operations per job increase BPS1 consistently provides better results at a much lesser computational time. This is corroborated by the main effects plot and the interaction plots. In 10 job-10 operation instance BPS1 shows an improvement over CPLEX of close to 1178%. The improvement made by BPS1 decreases as the DC ratio increases from 0.8

to 1.2. This corroborates the results from Section 4.6.1 where we found that it was more difficult to solve problems with DC ratio of 0.6 than those with 1.0.

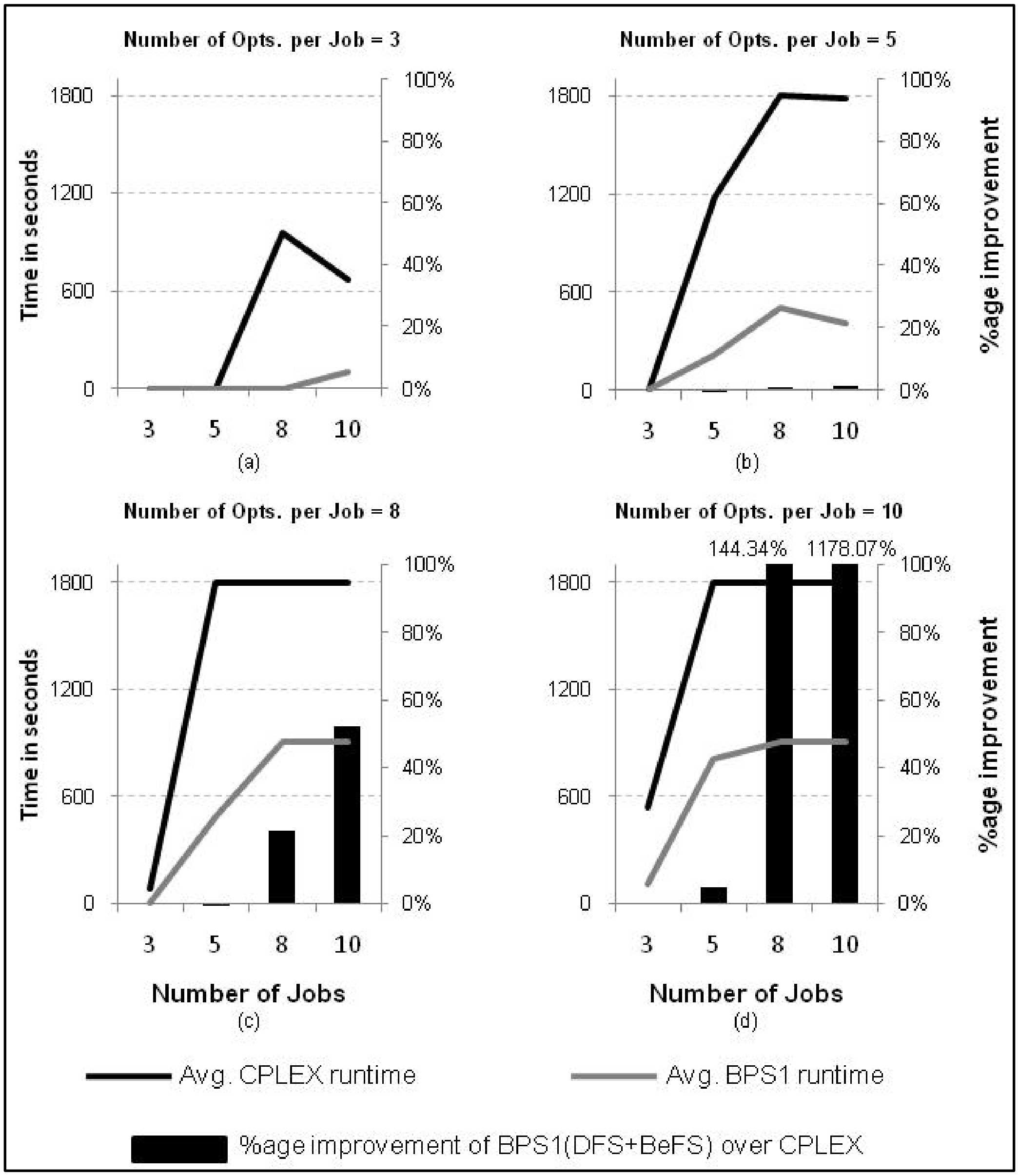


Figure 4-13. Comparative graphs of BPS1 with CPLEX

Another important aspect of column generation is the number of times a subproblem needs to be solved and the efficiency in solving it. Table 4-7 shows maximum times a sub-problem was solved for each combination of job and operation. We see that for a sub-problem size of 10 operations with a planning horizon of 30 days and on an average 100 hours of processing time per job take at most 0.00395 seconds to solve the sub-problem. Table 4-8 shows the average maximum time to solve a sub-problem and average percentage time spent in solving the sub-problems for a problem instance with respect to the number of operations and the DC ratio.

Table 4-7. Maximum number of times sub-problem solved

	Number of Operations per job			
Number of Jobs	3	5	8	10
3	94	108580	7389	95042
5	80828	136190	156880	138331
8	203608	209408	157189	142188
10	234384	260582	180230	124804

Table 4-8. Runtime analysis for sub-problem solution

	Average Maximum time to solve a single sub-problem (sec.)			% time spent in solving sub-problems		
	DC Ratio			DC Ratio		
Operations	0.8	1	1.2	0.8	1	1.2
3	0.00046	0.00042	0.00034	44.41	47.75	28.84
5	0.00192	0.00116	0.00097	56.32	38.83	30.34
8	0.00255	0.00192	0.00178	34.89	27.57	25.12
10	0.00395	0.00321	0.00246	36.04	33.70	29.11

5. BRANCH AND PRICE HEURISTIC & APPROXIMATION ALGORITHMS

BPS1 guarantees an optimal solution. However, it takes a long time to prove optimality, which is common to decomposition procedures. Furthermore, since we are fixing one original variable at a time in BPS1, the branch and bound tree can grow exponentially. To overcome this problem we propose a branch and price heuristic in Section 5.1. In Section 5.2 we propose approximation algorithms for the proposed branch and price algorithms. In Section 5.3 we present comparative analysis for the various branch and price strategies discussed.

5.1. Branch and Price Strategy 2 (BPS2)

For BPS2, unlike in BPS1 instead of fixing a single time period and source in each child node, we introduce time windows, during which operations o and o+1 are not allowed to be scheduled. This proposed method for fixing original variables gives us an approximate solution; but is intended to reduce the computational time. In $\theta_j^{'}$, we can look at the violation as two mutually exclusive events. The first is keeping the start time of operation o+1 as is. In that case, operation o has to be completely processed by the time period and source in which operation o+1 has started in $\theta_j^{'}$. The second event is that we keep the end time of operation o as is, so in such a case the earliest we can start processing operation o+1 is from the time operation o ends. Thus we can create the time windows during which we cannot schedule the two operations. In the first child node we place the restriction that operation o cannot be processed after source (ss_{o+1}) in time period (st_{o+1}), since this is the start time of operation o+1. Also, since we want to keep this start time as is, we can have an additional restriction that we cannot process operation o+1 before this time/source instance. In the other child node we place a restriction that operation o cannot be scheduled after time period (et_o) and source (es_o) onwards along with operation o+1 not to be scheduled before this time period and source. Figure 5-1 shows the way in which we can branch for $\theta_j^{'}$ in the previous example (Section 4.5.1) using BPS2.

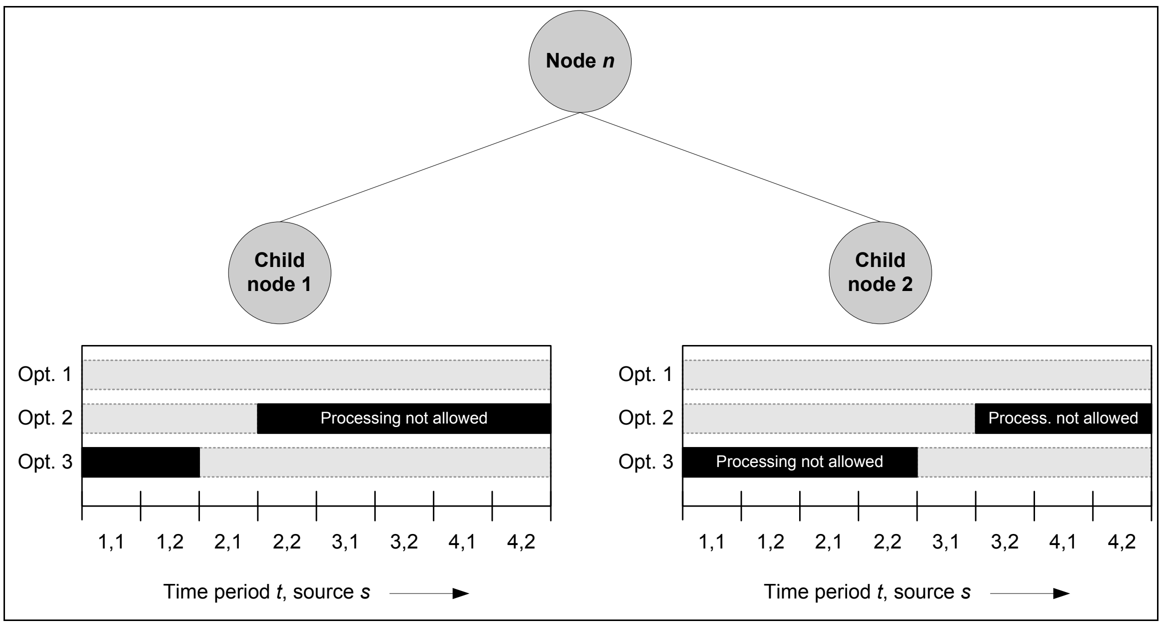


Figure 5-1. Branching in BPS2

We implement these restrictions of disallowing operations to be processed during certain time periods and sources in the sub-problem network by fixing the processing arc costs for the corresponding time periods and sources to +∞.

5.2. Approximation Algorithms for Branch & Price

In decomposition algorithms, we have tighter linear programming bounds as compared to the linear relaxations of the aggregated or non-decomposed formulation, and we usually find good feasible solutions early on in the solution process. Due to this reason, truncated tree search algorithms may provide very good approximation algorithms. In truncated tree search algorithms the number of nodes evaluated in the solution process is reduced according to some pre-specified scheme [48].

In the approximation algorithm, which we propose, we introduce a optimality tolerance γ, such that a node is fathomed if $Z_{RMP}^{LP} \leq (1+\gamma) Z_{IP}$, where Z_{IP} is the value of the best known integer solution.

We implement the approximation algorithms for both BPS1 and BPS2 with γ value of 0.01 and 0.05. For γ value of 0.0 we get the original BPS1 and BPS2 strategies. For being concise, we represent the name of the branching strategy followed by the optimality tolerance within round brackets. For example, BPS2(0.01) represents branch strategy BPS2 with an optimality tolerance $\gamma = 0.01$. We follow this convention in the following sections. Section 5.3 presents a comprehensive analysis of the different branch and price strategies along with the results for the approximation algorithms.

5.3. Comparative Analysis

In this section we present a comprehensive analysis of the different branch and price strategies along with the results for the approximation algorithms. In this section we show a paired t-test analysis between BPS1 and BPS2 with γ value of 0.00, and prove that there is no statistical difference between the solution qualities of the two strategies. Then we summarize the improvements made over CPLEX by the various B&P strategies in Table 5-3. In Table 5-4 we present the computational runtime required to solve the problem instances and in Table 5-5 we summarize the reductions in runtime by using BPS2 and the various approximation algorithms instead of BPS1(0.0). Finally we give some statistics pertaining to the size of the branch and bound tree, and number of columns generated for the various strategies. For all the above implementations we use Lagrangean bounds discussed in Section 4.5.3 to fathom the nodes in the branch and bound tree and DFS+BeFS strategy for node selection.

5.3.1. Comparing solution quality of BPS2 against BPS1

A paired t-test is conducted to conclude whether there is any statistical difference between the solution quality from BPS1 and BPS2. The null hypothesis is that there is no statistical difference in the results obtained by the two strategies at a 95% significance level. The null and alternate hypothesis can be states as follows:

Null Hypothesis H_0: $\mu_{BPS1(0.0)}=\mu_{BPS2(0.0)}$

Alternate Hypothesis H_1: $\mu_{BPS1(0.0)}\neq\mu_{BPS2(0.0)}$

Table 5-1. Results of the paired-t test

	N	Mean	St. Dev	SE Mean
BPS1(0.0)	139	5408	4171	354
BPS2(0.0)	139	5415	4164	353
Difference	139	-7.4	259.9	22.0
95% CI for mean difference: (-51.0, 36.2)				
T-Test of mean difference = 0 (vs not = 0): T-Value = -0.34 P-Value = 0.738				

Since p-value is 0.738, we fail to reject the null hypothesis with a 95% confidence level. A test to check for equal variances was conducted. The 95% Bonferroni confidence intervals for standard deviations and the result from the Levene's Test is shown in Table 5-2. The null hypothesis is testing for equal variances are that they are the same. In the Levene's Test we get a p-value of 0.971, and hence we fail to reject the null hypothesis at a 0.05 significance level. We can infer the variances for BPS1(0.0) and BPS2(0.0) are equal

Table 5-2. Test for Equal Variances

	N	Lower	St. Dev	Upper
BPS1(0.0)	139	3674.03	4171	4171.30
BPS2(0.0)	139	3667.18	4164	4808.32
Levene's Test	Test statistic = 0.00			
(Any Continuous Distribution)	p-value = 0.971			

From the above two tests we can conclude that there is no statistical difference between the objective function values of BPS1(0.0) and BPS2(0.0). This implies that the solution quality obtained from the two strategies is the same.

In Table 5-3, we show the percentage improvement made by the various B&P strategies over CPLEX.

Table 5-3. Improvement over CPLEX

Jobs	Opts.	%age improvement over CPLEX					
		BPS1			BPS2		
		$\gamma = 0.00$	0.01	0.05	0.00	0.01	0.05
3	3	0.00	0.00	0.00	-0.12	-0.12	-0.12
	5	0.00	-0.05	-0.81	-0.28	-0.36	-0.52
	8	0.00	-0.04	-0.83	-0.13	-0.13	-0.92
	10	-0.04	-0.10	-0.14	-0.09	-0.14	-0.23
Average		-0.01	-0.05	-0.44	-0.15	-0.19	-0.45
5	3	0.00	0.00	-0.24	-0.07	-0.16	-0.34
	5	-0.13	-0.35	-1.24	-0.18	-0.38	-0.80
	8	-3.24	-3.28	-3.28	0.36	0.31	-0.26
	10	4.43	4.16	3.96	2.36	2.22	1.70
Average		0.26	0.13	-0.20	0.62	0.50	0.07
8	3	0.00	-0.04	-1.69	0.00	-0.07	-1.65
	5	1.18	1.09	-0.36	1.48	1.27	-0.23
	8	20.99	20.94	20.51	21.65	21.14	19.52
	10	144.34	140.65	139.81	141.76	141.03	139.44
Average		38.69	37.80	36.70	38.35	37.98	36.41
10	3	0.00	-0.07	-0.69	0.00	-0.05	-0.76
	5	1.35	1.28	0.12	1.47	1.46	-0.22
	8	52.13	51.70	50.58	51.78	51.64	50.91
	10	1178.07	1178.07	1175.39	1200.37	1199.62	1170.61
Average		199.11	198.95	197.72	202.53	202.36	196.95

For instances with 3 jobs we see that CPLEX performs marginally better than the B&P strategies. We see marginal improvement in the solution quality of B&P as the number of jobs increase. For 8 job and 10 job instances we see an average improvement of 35% and 200% respectively.

In Table 5-4 we show the computation runtime to solve the various problem instances by the different B&P strategies.

Table 5-4. Runtime analysis of Branch and Price

		Runtime in seconds					
Jobs	Opts.	BPS1			BPS2		
		$\gamma = 0.00$	0.01	0.05	0.00	0.01	0.05
3	3	0.33	0.03	0.03	0.03	0.03	0.03
	5	4.33	0.87	0.16	0.15	0.13	0.10
	8	0.54	0.34	0.30	0.18	0.14	0.09
	10	102.08	100.68	0.90	0.94	0.71	0.60
Average		26.82	25.48	0.35	0.32	0.25	0.20
5	3	0.32	0.16	0.16	0.10	0.09	0.08
	5	210.50	102.33	0.70	0.56	0.41	0.31
	8	483.20	403.18	102.94	14.02	5.83	1.69
	10	800.67	633.56	205.96	360.15	308.88	110.18
Average		373.67	284.81	77.44	93.71	78.80	28.07
8	3	0.89	0.82	0.31	0.34	0.29	0.16
	5	502.71	204.83	4.32	122.79	102.27	1.74
	8	900.45	802.12	145.02	807.37	711.64	155.66
	10	900.38	900.94	301.34	900.64	755.10	254.47
Average		566.84	465.07	107.36	445.13	381.96	98.68
10	3	101.87	101.88	1.62	0.94	0.65	0.41
	5	408.61	33.88	16.63	21.73	7.40	4.28
	8	900.43	639.28	180.69	900.34	805.50	81.14
	10	900.76	780.15	487.35	900.81	900.95	391.71
Average		537.56	339.88	132.10	400.34	369.59	85.35

BPS1(0.0) takes the most time to solve the problems. When the number of operations are less (3 and 5) BPS2(0.0) is faster than BPS1(0.0) and BPS1(0.01), but as the number of operations increase BPS1(0.01) is faster than BPS1 (0.0). The approximation algorithms with optimality tolerance of 0.05 are much faster than any other, while BPS2(0.05) performs the best in terms of runtime. Since BPS1(0.0) is slowest, we compute the reduction in runtime achieved by using the other B&P strategies. The results are shown in Table 5-5.

Table 5-5. Reduction in runtime

		%age reduction in runtime				
Jobs	Opts.	BPS1		BPS2		
		0.01	0.05	0.00	0.01	0.05
3	3	22.74	22.81	30.35	25.17	13.27
	5	13.68	28.77	24.52	29.80	40.39
	8	-40.30	7.29	10.64	15.22	14.33
	10	21.74	34.38	31.52	32.51	35.52
Average		4.46	23.31	24.26	25.67	25.88
5	3	6.04	1.92	37.03	41.05	49.65
	5	25.25	40.97	62.46	66.31	66.00
	8	30.43	70.58	88.68	90.31	91.33
	10	24.34	71.89	50.86	59.82	84.45
Average		21.52	46.34	59.76	64.37	72.86
8	3	6.81	47.95	48.31	54.39	67.85
	5	32.78	69.73	67.94	70.33	87.30
	8	10.92	83.89	10.34	20.97	82.71
	10	-0.06	66.53	-0.03	16.13	71.74
Average		12.98	67.04	32.55	41.15	77.56
10	3	-0.13	26.91	70.54	66.53	81.05
	5	41.58	54.30	79.57	81.27	87.19
	8	29.00	79.93	0.01	10.54	90.99
	10	13.39	45.90	-0.01	-0.02	56.50
Average		21.91	52.49	42.22	44.53	81.74

For 10 job problems BPS2(0.05) can show on an average 81% reduction in runtime over BPS1(0.0). As seen from the improvements made over CPLEX BPS2(0.05) makes 196% improvement as opposed to BPS2(0.0) which makes 202.53% but at a much lesser computation overhead. This shows that the approximation algorithms are a viable alternative to the exact procedure.

Finally we present the maximum number of columns generated, maximum nodes formed and number of times the sub-problem is solved for the different number of jobs and operations. This information is helpful to analyze the size of the branch and bound tree, the effectiveness of the sub-problem solution approach and the memory requirements for the solution approach. It can be seen from Table 5-6 that there is no clear pattern as regards to the columns generated and the problem size. But for high number of jobs and operations per each job, the number of columns generated by BPS2 are more than BPS1. The maximum number of columns generated were for a 5 job 8 operation problem by BPS2(0.01).

The number of nodes in the branch and bound tree, as shown in Table 5-7 are lesser in BPS2 as compared to BPS1, which makes intuitive sense because in BPS2, we are setting a set of original variables to zero based on the concept of time windows, as compared to BPS1, where we are setting the value of a single original variable to zero. Hence, BPS1 takes more time because the search space is much bigger. Also the overhead of traversing the branch and bound tree to set the columns at each node in the branch to zero which violates the current restrictions is much more when the number of nodes in the branch is more and hence the increase in computational time.

Table 5-6. Number of columns generated in B&P

		Maximum number of columns generated					
Jobs	Opts.	BPS			BPS2		
		$\gamma = 0.0$	0.01	0.05	0	0.01	0.05
3	3	41	41	41	81	81	81
	5	1713	396	348	316	231	268
	8	851	851	376	498	447	253
	10	50546	49085	1064	2631	1350	400
5	3	9370	2972	388	437	379	191
	5	61797	68180	808	1197	500	412
	8	64480	62893	2904	61562	83342	2423
	10	64747	15366	7588	20085	6646	3851
8	3	1891	1172	1083	642	447	287
	5	64786	66269	48435	16900	7873	1614
	8	67009	65031	51689	74524	77517	71755
	10	63420	55750	44805	73850	76483	47446
10	3	51436	53432	1560	2321	1874	1536
	5	61202	63623	65702	67678	68219	70360
	8	59340	62094	54651	66367	65360	67664
	10	50688	52075	27938	61918	65165	63451

Table 5-7. Size of branch and bound tree in B&P

		Maximum nodes formed in the branch and bund tree					
Jobs	Opts.	BPS			BPS2		
		$\gamma = 0.0$	0.01	0.05	0	0.01	0.05
3	3	15	15	15	19	19	19
	5	2327	691	85	49	37	19
	8	213	111	103	67	43	23
	10	6239	6097	135	161	127	99
5	3	325	75	95	43	33	35
	5	14627	6541	109	89	43	27
	8	9111	9057	9105	809	353	97
	10	6589	5245	4283	3557	3065	2027
8	3	261	261	161	67	57	31
	5	10845	11225	389	4161	4699	143
	8	5455	4179	2797	4725	2861	1541
	10	3939	3081	3047	3055	2865	1665
10	3	13921	13969	319	253	141	41
	5	9411	1771	849	959	321	209
	8	4915	5163	2555	2997	2903	1253
	10	3377	2233	2211	1981	1649	1061

6. CONCLUSIONS AND FUTURE WORK

This dissertation is concluded by summarizing the problem and the solution approach in Section 6.1. Section 6.2 states the contributions of this research followed by their significance. Lastly in Section 6.3 possible extensions of this research are outlined.

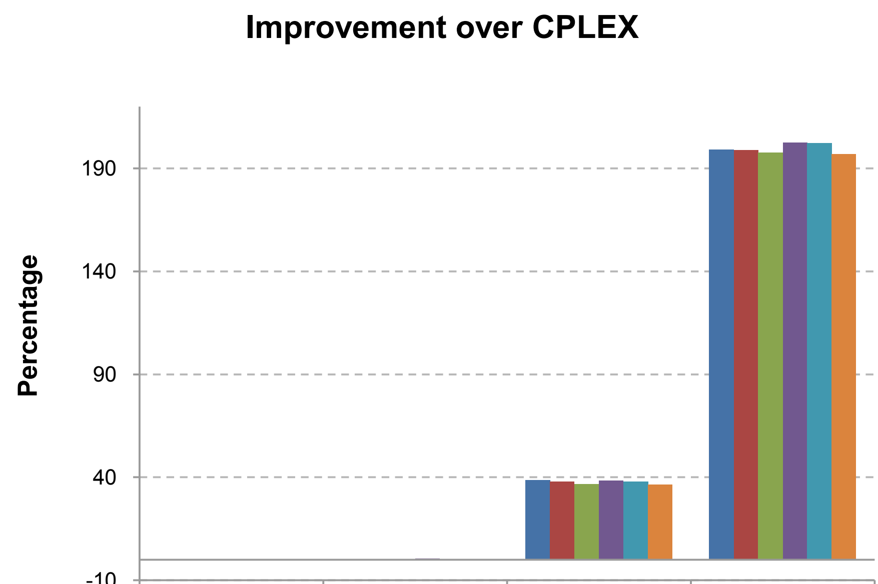
6.1. Summary

Integrating order acceptance and capacity planning provides tremendous opportunities to maximize the operational profits of make-to-order operations. This is done by selectively accepting jobs from the available pool of customer orders and simultaneously planning for their capacity. This integrated problem is difficult to solve and many researchers have tried to simplify the problem by planning for the bottleneck machines and solving the problem as a single machine problem. But in reality, the bottleneck is floating as it depends on the orders which are selected. Furthermore, capacity is not fixed since you can extend your capacities by adding overtime and outsourcing, which might be beneficial for improving the profits. Non-regular capacity has not been considered in any of the previous work done in the area of MTO order acceptance problem. In this dissertation we model the MTO operation as a job shop with multiple resources and recirculation. We consider regular capacity (regular shift) and non-regular capacity (overtime shift). The MTO operation receives customer orders or jobs each with a number of operations having linear precedence relationship. Typically order acceptance problems are solved on daily basis for short term capacity planning with a rolling planning horizon of 3 to 4 weeks. Hence the solution approach to this integrated problem should be quick such that the decision maker can use it recursively not only to find the optimal set of orders and to allocate capacity but also to explore various other scenarios that would help in negotiating order due-dates, prices and be aligned with the organization's long-term business strategy. The goal of this dissertation is to develop such a tool.

In order to achieve this goal we first proposed a Mixed-Integer Linear Program (MILP) to model the research problem under consideration. Using the model we illustrate that integrating the two decisions of order acceptance and capacity planning can achieve our goal to maximize the operational profits. Since the proposed MILP had a block diagonal structure, and column generation can solve models with this structure efficiently, we proposed an exact branch and price algorithm (BPS1). We also develop approximate branching scheme (BPS2) and various approximation algorithms for the exact and approximate branching schemes.

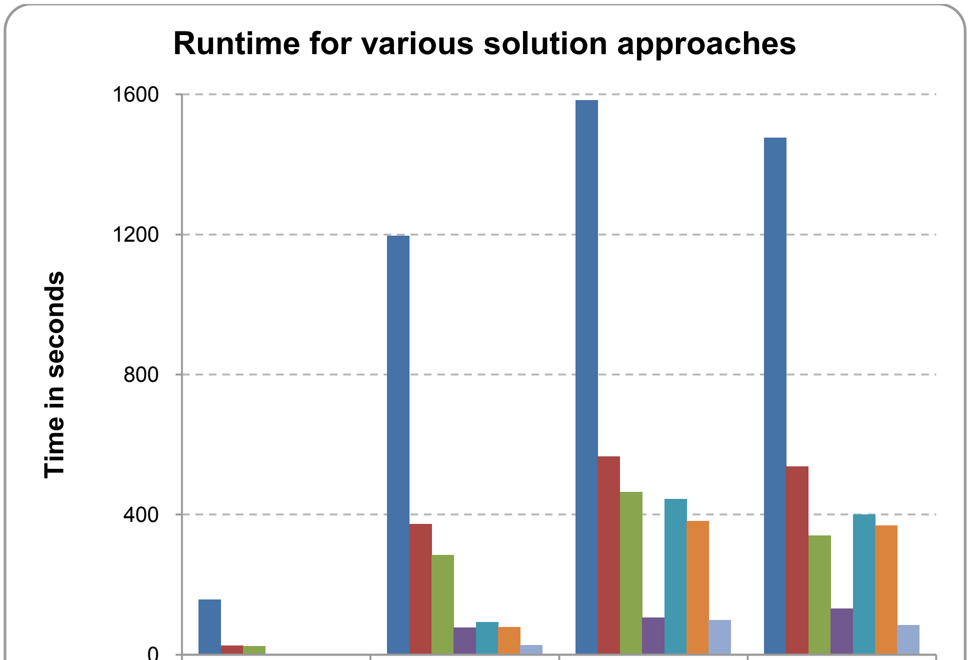
We show through experiments that the BPS1 and other approximation schemes perform better than the solution provided by the commercial solver, and can solve problems of sizes which can be typically found in the real-life applications. Figure 6-1 and Figure 6-2 graphically summarize the improvements made by B&P algorithms and the computational runtime of various solution approaches discussed in this dissertation. We observe that B&P performs 200% better than the results obtained from solving the MILP at a much lesser computational overhead as compared to a commercial solver CPLEX. BPS2(0.05) can solve, on an average, 10 jobs problems in 85 seconds and making 196%

improvements over CPLEX. B&P algorithms are significantly faster and solve problems in reasonable time, and thus can be utilized in a decision support system used on a daily basis to help make intelligent decisions in a MTO operation.



	3 jobs	5 jobs	8 jobs	10 jobs
BPS1	-0.01	0.26	38.69	199.11
BPS1 0.01	-0.05	0.03	37.80	198.95
BPS1 0.05	-0.44	-0.20	36.70	197.72
BPS2	-0.15	0.62	38.35	202.53
BPS2 0.01	-0.19	0.50	37.98	202.36
BPS2 0.05	-0.45	0.07	36.41	196.95

Figure 6-1. Average improvement in solution quality



	3 jobs	5 jobs	8 jobs	10 jobs
CPLEX	156.98	1195.84	1582.88	1476.23
BPS1	26.82	373.67	566.84	537.56
BPS1 0.01	25.48	284.81	465.07	339.88
BPS1 0.05	0.35	77.44	107.36	132.1
BPS2	0.32	93.71	445.13	400.34
BPS2 0.01	0.25	78.8	381.96	369.59
BPS2 0.05	0.2	28.07	98.68	85.35

Figure 6-2. Runtime for various solution approaches

6.2. Contributions & Significance

The contributions of this dissertation are:

- Development of a mathematical model for maximizing the operational profits of a make-to-order operation by combining order acceptance with capacity planning at the sales stage in job-shop environment having multiple resources with regular time and overtime capacity.

- Development of a Branch and Price solution approach along with various approximation algorithms were proposed for solving this model within practical considerations of time-limit.

Existing literature on MTO with static job arrivals only considers order acceptance for single resource cases with regular capacity. However, most MTO operations have multiple resources in a job shop environment with regular and non-regular capacity like overtime and outsourcing available to them. These have not been addressed in the past research efforts. Hans [42] has developed a B&P solution approach for capacity planning in job shop with multiple resources and non-regular capacity, but he does not consider order acceptance. Ebben [7] has studied resource loading based order acceptance using simulation. In one of the approaches they used BPRL approach proposed by Hans [42] to schedule already accepted orders. This dissertation has made significant contribution to the scientific community by studying and modeling the MTO production system as a job shop with multiple resources having regular and overtime capacity. The branch and algorithm developed and implemented in this dissertation can solve larger problem sizes, typically found in MTO operations within practical time limits.

6.3. Future Work

Column generation is a decomposition approach and is efficient when the formulation exhibits a block diagonal structure. The MILP proposed in this dissertation exhibited this special structure and hence column generation was adopted to solve the MTO problem. There are other decomposition techniques in literature like Lagrangian relaxation and Benders' decomposition which are commonly used in solving large-scale optimization problems. It could be worthwhile to explore the Lagrangian relaxation approach to solve the MTO order acceptance problem. Since the capacity constraint is the complicating or binding constraint, it could be relaxed and put in the objective function. Once this constraint is relaxed, a solution to the problem can be found out by generating schedules for individual jobs by using the sub-problem solution approach proposed in this dissertation. The Lagrange multipliers can be updated by looking at the capacity violations, and these multipliers can be used to find the new schedules from the sub-problem.

The problem under consideration is typically found in many practical applications. It lays the foundation for other complex problems of practical interest having variations to this basic problem. For example, we consider non-regular capacity as overtime, but many make-to-order operations have outsourcing options. Integrating outsourcing is another important variation to the problem we have considered. The solution approach proposed in this dissertation can be extended to cases where pre-emption is not allowed. In many other operations, instead of a single deliverable job, we may have a bigger assemblies made of smaller sub-assemblies. Hence, the precedence relations are much more complex although each sub-assembly will have linear precedence amongst their sub-operations or tasks. The solution approach proposed in this research forms the foundation for solving such complex problems.

One of the assumptions made here in this research is that all the parameters are known with certainty, however, in real-life there may be some uncertainties. In such case a stochastic formulation is necessary, and hence, the solution approaches may have to be modified. There are B&P implementations for stochastic MILP's for other applications. Consequently, a stochastic version of the proposed B&P algorithm may have to be implemented. A scenario tree approach is commonly applied in the literature to tackle uncertainties. The size of the model (constraints, and decision variables) will increase as a result. A goal programming approach can be devised to relax the due date constraints to capture the capability to negotiate due dates for different customer orders.

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APPENDICES

APPENDIX A

Number of Jobs	Number Of Operations per job	Time Period (Days)	Random Instance	CPLEX Runtime (seconds)	Absolute MIP Gap	Relative MIP Gap
3	3	3	1	0.047		
3	3	4	2	0.063		
3	3	4	3	0.938		
3	4	4	1	0.938		
3	4	4	2	0.141		
3	4	5	3	0.156		
3	5	6	1	0.266		
3	5	6	2	0.578		
3	5	6	3	7.141		
3	6	7	1	66.063		
3	6	7	2	0.859		
3	6	8	3	0.625		
3	7	8	1	9.250		
3	7	9	2	17.391		
3	7	9	3	16.484		
3	8	9	1	385.516		
3	8	10	2	10135.500		
3	8	10	3	8.328		
5	3	6	1	0.313		
5	3	5	2	1.219		
5	3	5	3	0.172		
5	4	8	1	0.641		
5	4	8	2	1.469		
5	4	9	3	0.859		
5	5	10	1	9.109		
5	5	10	2	1234.860		
5	5	10	3	489.969		
5	6	12	1	346.938		
5	6	12	2	47896.600	68	2.81%
5	6	11	3	11.188		
5	7	13	1	24.563		
5	7	14	2	23.234		
5	7	14	3	1917.980		
5	8	17	1	62186.400	22	0.42%
5	8	16	2	61971.800	28	0.03%
5	8	16	3	34705.900	552	11.22%

APPENDIX B

Number Of Jobs	Number of Operations per job	Time Period (Days)	Random Instance	No. of Sub-Problems Solved	Total Sub-Problem Solve Time (seconds)	Average Sub-Problem Solve Time (seconds)
3	3	3	1	18	0.766	0.043
3	3	4	2	15	0.750	0.050
3	3	4	3	21	0.922	0.044
3	4	4	1	21	1.031	0.049
3	4	4	2	18	0.953	0.053
3	4	5	3	21	1.156	0.055
3	5	6	1	33	2.625	0.080
3	5	6	2	30	2.250	0.075
3	5	6	3	54	5.625	0.104
3	6	7	1	60	11.047	0.184
3	6	7	2	54	10.359	0.192
3	6	8	3	30	4.156	0.139
3	7	8	1	51	11.875	0.233
3	7	9	2	36	6.797	0.189
3	7	9	3	27	6.531	0.242
3	8	9	1	42	35.875	0.854
3	8	10	2	63	109.938	1.745
3	8	10	3	21	14.922	0.711
5	3	6	1	40	1.859	0.046
5	3	5	2	30	1.453	0.048
5	3	5	3	30	1.406	0.047
5	4	8	1	45	2.938	0.065
5	4	8	2	35	2.297	0.066
5	4	9	3	35	2.422	0.069
5	5	10	1	60	8.016	0.134
5	5	10	2	120	15.031	0.125
5	5	10	3	50	11.016	0.220
5	6	12	1	45	9.109	0.202
5	6	12	2	40	6.813	0.170
5	6	11	3	45	5.900	0.131
5	7	13	1	35	18.000	0.514
5	7	14	2	55	43.297	0.787
5	7	14	3	60	25.078	0.418
5	8	17	1	150	7294.660	48.631
5	8	16	2	80	770.562	9.632
5	8	16	3	115	3675.120	31.958

APPENDIX C

Instance	LP Relaxation	Time to Root Node	CPLEX Integer Solution	CPLEX Best Bound	CPLEX Runtime	Absolute MIP Gap	Relative MIP Gap (%)
j3o3r3lr0.8t4_i1	906.00	0	794	794	0.06	0.000	0.000
j3o3r3lr0.8t5_i2	1163.06	0	1135	1135	0.03	0.000	0.000
j3o3r3lr0.8t5_i3	1890.00	0	1807	1807	0.14	0.000	0.000
j3o3r3lr1t4_i1	1161.06	0	1120	1120	0.02	0.000	0.000
j3o3r3lr1t4_i2	1169.00	0	1169	1169	0.02	0.000	0.000
j3o3r3lr1t3_i3	787.51	0.02	729	729	0.06	0.000	0.000
j3o3r3lr1.2t2_i1	257.13	0	92	92	0.02	0.000	0.000
j3o3r3lr1.2t3_i2	377.95	0	274	274	0.02	0.000	0.000
j3o3r3lr1.2t3_i3	1333.88	0.02	1202	1202	0.03	0.000	0.000
j3o5r3lr0.8t8_i1	3009.00	0	2781	2781.225	15.53	0.225	0.010
j3o5r3lr0.8t6_i2	3546.00	0	3416	3416.294	6.34	0.294	0.010
j3o5r3lr0.8t7_i3	2271.73	0	2157	2157	1.53	0.000	0.000
j3o5r3lr1t6_i1	2474.00	0.01	2134	2134.138	11.06	0.138	0.010
j3o5r3lr1t6_i2	4090.85	0	3661	3661.327	12.27	0.327	0.010
j3o5r3lr1t6_i3	2651.00	0.01	2377	2377.2	27.19	0.204	0.010
j3o5r3lr1.2t5_i1	2949.61	0	2282	2282	0.39	0.000	0.000
j3o5r3lr1.2t6_i2	834.64	0.02	164	164	0.67	0.000	0.000
j3o5r3lr1.2t5_i3	1335.12	0	988	988	0.08	0.000	0.000
j3o8r5lr0.8t8_i1	3180.41	0.01	2235	2235.214	57.36	0.214	0.010
j3o8r5lr0.8t7_i2	2589.25	0.02	1889	1889	1.27	0.000	0.000
j3o8r5lr0.8t8_i3	3605.25	0.03	3034	3034.3	673.94	0.300	0.010
j3o8r5lr1t6_i1	1158.91	0.02	266	266	0.27	0.000	0.000
j3o8r5lr1t5_i2	1842.81	0	1175	1175	0.77	0.000	0.000
j3o8r5lr1t6_i3	3216.30	0.02	1610	1610	1.92	0.000	0.000
j3o8r5lr1.2t4_i1	2010.64	0.02	649	649	0.08	0.000	0.000
j3o8r5lr1.2t4_i2	2043.22	0.02	1102	1102	0.05	0.000	0.000
j3o8r5lr1.2t5_i3	4064.47	0.02	1806	1806	0.45	0.000	0.000
j3o10r5lr0.8t9_i1	2433.76	0.03	1620	1620.147	133.39	0.147	0.010
j3o10r5lr0.8t9_i2	4628.15	0.06	3954	4420.984	1800	466.984	11.810
j3o10r5lr0.8t9_i3	3693.22	0.06	2563	3169.34	1800	606.340	23.660
j3o10r5lr1t8_i1	7709.53	0.06	6645	6645.637	1092.74	0.637	0.010
j3o10r5lr1t7_i2	1020.29	0.03	861	861	0.58	0.000	0.000
j3o10r5lr1t7_i3	3717.05	0.01	3171	3171	12.34	0.000	0.000
j3o10r5lr1.2t6_i1	1109.80	0.03	1E-07	0	0.09	0.000	0.000
j3o10r5lr1.2t5_i2	3583.72	0.02	3004	3004	0.2	0.000	0.000
j3o10r5lr1.2t6_i3	1109.90	0.03	137	137	0.28	0.000	0.000
j5o3r3lr0.8t7_i1	1775.46	0	1729	1729	0.16	0.000	0.000
j5o3r3lr0.8t8_i2	2259.00	0.01	2259	2259	3.31	0.000	0.000
j5o3r3lr0.8t8_i3	2793.23	0.02	2627	2627	1.2	0.000	0.000
j5o3r3lr1t6_i1	3691.67	0.02	3621	3621	0.25	0.000	0.000
j5o3r3lr1t6_i2	3187.28	0.02	3031	3031	0.39	0.000	0.000
j5o3r3lr1t5_i3	3041.00	0	3041	3041	0.44	0.000	0.000
j5o3r3lr1.2t4_i1	1319.17	0	1312	1312	0.05	0.000	0.000
j5o3r3lr1.2t5_i2	1470.32	0	1321	1321	0.11	0.000	0.000
j5o3r3lr1.2t5_i3	2751.36	0	2670	2670	0.34	0.000	0.000
j5o5r3lr0.8t10_i1	2905.00	0.01	2657	2781.752	1800.28	124.752	4.700
j5o5r3lr0.8t14_i2	3566.74	0.03	3404	3543	1800.02	139.000	4.080
j5o5r3lr0.8t12_i3	7187.00	0.03	6891	7187	1801.24	296.000	4.300
j5o5r3lr1t10_i1	3989.59	0.02	3862	3862	3.63	0.000	0.000
j5o5r3lr1t10_i2	4003.40	0.02	3188	3790	1800.01	602.000	18.880
j5o5r3lr1t11_i3	6443.00	0.03	5992	6443	1800.58	451.000	7.530

Instance	LP Relaxation	Time to Root Node	CPLEX Integer Solution	CPLEX Best Bound	CPLEX Runtime	Absolute MIP Gap	Relative MIP Gap (%)
j5o5r3lr1.2t8_i1	1538.37	0.02	1182	1182.111	1377.17	0.111	0.010
j5o5r3lr1.2t9_i2	3264.17	0.01	3114	3114.275	251.77	0.275	0.010
j5o5r3lr1.2t8_i3	1906.20	0	1700	1700	2	0.000	0.000
j5o8r5lr0.8t11_i1	3057.00	0.09	2640	2982.154	1800.19	342.154	12.960
j5o8r5lr0.8t13_i2	9445.00	0.09	8657.5	9414	1800	756.500	8.740
j5o8r5lr0.8t12_i3	11458.83	0.06	11091.5	11444	1803.53	352.500	3.180
j5o8r5lr1t9_i1	2920.34	0.09	1559	2039.745	1800.8	480.745	30.840
j5o8r5lr1t9_i2	4275.43	0.08	3667	4062.136	1800.03	395.136	10.780
j5o8r5lr1t9_i3	3907.58	0.09	2626	3310	1800.44	684.000	26.050
j5o8r5lr1.2t9_i1	4487.86	0.11	3752	3964.069	1800	212.069	5.650
j5o8r5lr1.2t8_i2	3417.08	0.13	2018	2332.823	1800.03	314.823	15.600
j5o8r5lr1.2t8_i3	8365.71	0.08	6833	7982.636	1800.3	1149.640	16.820
j5o10r5lr0.8t15_i1	7765.44	0.37	6664	7695	1800.05	1031.000	15.470
j5o10r5lr0.8t14_i2	3795.69	0.24	2967	3611.158	1800.03	644.158	21.710
j5o10r5lr0.8t16_i3	11333.81	0.3	7667	11056	1800.02	3389.000	44.200
j5o10r5lr1t12_i1	7116.60	0.25	5661	6964.841	1800.05	1303.840	23.030
j5o10r5lr1t12_i2	7509.24	0.23	6274	7313	1800	1039.000	16.560
j5o10r5lr1t12_i3	7930.32	0.3	5438	7596.897	1800.02	2158.900	39.700
j5o10r5lr1.2t10_i1	5238.88	0.25	3615	4511.766	1801.88	896.766	24.810
j5o10r5lr1.2t10_i2	6544.65	0.23	5676	6246.1	1800.02	570.100	10.040
j5o10r5lr1.2t10_i3	5508.28	0.2	4401	4851.961	1800.02	450.961	10.250
j8o3r3lr0.8t13_i1	2391.00	0.03	2295	2391	1801.34	96.000	4.180
j8o3r3lr0.8t13_i2	4341.00	0.03	4341	4341	3.89	0.000	0.000
j8o3r3lr0.8t11_i3	4365.53	0.02	4193	4299	1800.19	106.000	2.530
j8o3r3lr1t9_i1	2953.62	0.02	2889	2889	1.02	0.000	0.000
j8o3r3lr1t9_i2	2082.87	0.02	2069	2069	0.27	0.000	0.000
j8o3r3lr1t10_i3	3285.82	0.01	3218	3266	1800.02	48.000	1.490
j8o3r3lr1.2t8_i1	3109.27	0.02	2963	2963.295	1389.91	0.295	0.010
j8o3r3lr1.2t8_i2	5439.52	0.02	5436	5436	1.31	0.000	0.000
j8o3r3lr1.2t8_i3	3492.07	0.02	3202	3377	1800.01	175.000	5.470
j8o5r3lr0.8t20_i1	6205.00	0.17	5340	6205	1800.03	865.000	16.200
j8o5r3lr0.8t20_i2	6039.13	0.14	5848	6010	1800	162.000	2.770
j8o5r3lr0.8t20_i3	7039.50	0.13	6924	7028	1801.03	104.000	1.500
j8o5r3lr1t17_i1	7072.25	0.14	6838	7072	1800.05	234.000	3.420
j8o5r3lr1t14_i2	6045.00	0.11	5724	6034	1800.17	310.000	5.420
j8o5r3lr1t16_i3	4183.03	0.14	4009	4177	1800.03	168.000	4.190
j8o5r3lr1.2t14_i1	5739.75	0.09	5369	5685	1800.03	316.000	5.890
j8o5r3lr1.2t14_i2	5966.38	0.11	5807	5887	1800.03	80.000	1.380
j8o5r3lr1.2t12_i3	4336.10	0.08	4003	4259	1800.02	256.000	6.400
j8o8r5lr0.8t21_i1	7346.00	0.63	4129	7346	1800.05	3217.000	77.910
j8o8r5lr0.8t18_i2	6931.50	0.2	3305	6803	1800.06	3498.000	105.840
j8o8r5lr0.8t19_i3	15558.67	0.74	13522	15452	1800.01	1930.000	14.270
j8o8r5lr1t15_i1	17591.00	0.47	15757	17549.47	1800.03	1792.470	11.380
j8o8r5lr1t16_i2	11858.60	0.31	10583	11713	1800.06	1130.000	10.680
j8o8r5lr1t16_i3	11831.00	0.56	9902	11831	1800.03	1929.000	19.480
j8o8r5lr1.2t13_i1	11489.89	0.5	10102	11090.85	1800.88	988.854	9.790
j8o8r5lr1.2t13_i2	7388.77	0.33	6889	7357	1800.09	468.000	6.790
j8o8r5lr1.2t13_i3	4850.94	0.31	4309	4757	1800.01	448.000	10.400
j8o10r5lr0.8t26_i1	18082.60	2.45	0	18001	1800.05	18001.000	infinity
j8o10r5lr0.8t23_i2	11581.00	1.84	6441	11581	1800.02	5140.000	79.800
j8o10r5lr0.8t23_i3	18384.00	1.56	11813	18384	1800.03	6571.000	55.630
j8o10r5lr1t19_i1	13919.71	1.23	8399	13528	1800.08	5129.000	61.070
j8o10r5lr1t17_i2	11238.00	0.94	2985	11238	1800.08	8253.000	276.480
j8o10r5lr1t19_i3	9858.08	1.09	2836	9850	1800.05	7014.000	247.320

Instance	LP Relaxation	Time to Root Node	CPLEX Integer Solution	CPLEX Best Bound	CPLEX Runtime	Absolute MIP Gap	Relative MIP Gap (%)
j8o10r5lr1.2t15_i1	9478.00	0.66	3457	9478	1800.03	6021.000	174.170
j8o10r5lr1.2t16_i2	16972.46	1.23	7713	16737	1800.02	9024.000	117.000
j8o10r5lr1.2t16_i3	12323.25	1.13	2430	12220.57	1800.05	9790.570	402.900
j10o3r3lr0.8t15_i1	7366.00	0.03	7366	7366	2.17	0.000	0.000
j10o3r3lr0.8t15_i2	3593.58	0.05	3529	3592	2.63	0.000	0.000
j10o3r3lr0.8t14_i3	4795.27	0.03	4676	4676	1.53	0.000	0.000
j10o3r3lr1t12_i1	3719.00	0.03	3639	3719	1800	90.000	2.200
j10o3r3lr1t12_i2	5296.00	0.05	5218	5266	1800.14	48.000	0.920
j10o3r3lr1t11_i3	4669.00	0.02	4658	4658	5.11	0.000	0.000
j10o3r3lr1.2t10_i1	4683.00	0.03	4683	4683	2.53	0.000	0.000
j10o3r3lr1.2t9_i2	4464.71	0.03	4373	4373.433	574.72	0.433	0.010
j10o3r3lr1.2t10_i3	4476.02	0.03	4306	4372	1800.02	66.000	1.530
j10o5r3lr0.8t26_i1	9833.00	0.34	9833	9833	1648.53	0.000	0.000
j10o5r3lr0.8t28_i2	12311.29	0.59	11425	12049	1800	624.000	5.460
j10o5r3lr0.8t25_i3	9519.00	0.34	9169	9519	1800.03	350.000	3.820
j10o5r3lr1t20_i1	4709.46	0.34	4672	4703	1800.01	31.000	0.660
j10o5r3lr1t19_i2	6191.70	0.41	6156	6178	1800.02	22.000	0.360
j10o5r3lr1t19_i3	7237.36	0.25	6967	7215	1800.03	248.000	3.560
j10o5r3lr1.2t16_i1	4071.03	0.22	3896	4018	1800.01	122.000	3.130
j10o5r3lr1.2t18_i2	6851.82	0.31	6123	6782	1800.92	659.000	10.760
j10o5r3lr1.2t18_i3	8083.24	0.3	6880	8082	1800.05	1202.000	17.470
j10o8r5lr0.8t24_i1	13565.00	1.58	7206	13565	1800.03	6359.000	88.250
j10o8r5lr0.8t24_i2	10602.00	1.31	8169	10602	1800.06	2433.000	29.780
j10o8r5lr0.8t22_i3	11857.94	1.55	4377	11804	1800.06	7427.000	169.680
j10o8r5lr1t17_i1	12802.59	0.81	10782	12750	1800.05	1968.000	18.250
j10o8r5lr1t18_i2	9899.41	1.39	6798	9783	1800.03	2985.000	43.910
j10o8r5lr1t19_i3	9671.39	1.22	5408	9581	1800.03	4173.000	77.160
j10o8r5lr1.2t16_i1	15029.52	1.01	9231	14717	1800.02	5486.000	59.430
j10o8r5lr1.2t16_i2	13955.67	1.2	11403	13785	1800.08	2382.000	20.890
j10o8r5lr1.2t16_i3	10523.54	1.17	7518	10510	1800.03	2992.000	39.800
j10o10r5lr0.8t29_i1	18526.50	3.23	4.97E-14	18491	1800.09	18491.000	infinity
j10o10r5lr0.8t29_i2	12322.00	4.06	3474	12322	1800.09	8848.000	254.690
j10o10r5lr0.8t30_i3	17822.00	4.23	358	17822	1800.22	17464.000	4900.000
j10o10r5lr1t24_i1	11457.00	4.02	1737	11397	1800.17	9660.000	556.130
j10o10r5lr1t23_i2	18395.00	3.14	0	18395	1800.03	18395.000	infinity
j10o10r5lr1t25_i3	17716.31	3.72	4.75E-13	17643	1800.13	17643.000	infinity
j10o10r5lr1.2t19_i1	19037.58	2.66	7304	18841	1800.08	11537.000	157.950
j10o10r5lr1.2t21_i2	14230.88	3.06	-3.2E-14	14201	1800.13	14201.000	infinity
j10o10r5lr1.2t19_i3	16799.00	2.28	2909	16799	1800.03	13890.000	477.480

APPENDIX D

Instance	Fractional U_j	Fractional λ_j	Initial Solution	Max. Initial Integer Profit	Root Node Solution	Columns Added @ Root Node	Time to Root Node (seconds)
j3o3r3lr0.8t4_i1	1	5	716	716	902	17	0.031
j3o3r3lr0.8t5_i2	0	9	960	960	1135	18	0.031
j3o3r3lr0.8t5_i3	0	2	1181	1181	1807	19	0.031
j3o3r3lr1t4_i1	0	7	1070	1070	1120	14	0.031
j3o3r3lr1t4_i2	0	0	420	420	1169	19	0.016
j3o3r3lr1t3_i3	1	7	655	655	742.818	16	0.016
j3o3r3lr1.2t2_i1	0	0	92	92	92	1	0.031
j3o3r3lr1.2t3_i2	0	0	274	274	274	3	0.016
j3o3r3lr1.2t3_i3	0	5	1107	1107	1202	12	0.016
j3o5r3lr0.8t8_i1	0	13	2518	2518	2839.54	46	0.047
j3o5r3lr0.8t6_i2	0	9	3366	3366	3416	33	0.031
j3o5r3lr0.8t7_i3	0	9	1624	1624	2170	34	0.047
j3o5r3lr1t6_i1	1	10	1679	1679	2145.4	29	0.031
j3o5r3lr1t6_i2	1	9	3223	3223	3670.27	45	0.047
j3o5r3lr1t6_i3	0	11	2225	2225	2413.7	36	0.031
j3o5r3lr1.2t5_i1	1	7	1466	1466	2399	30	0.031
j3o5r3lr1.2t6_i2	1	1	164	164	217.5	5	0.031
j3o5r3lr1.2t5_i3	0	0	988	988	988	1	0.016
j3o8r5lr0.8t8_i1	0	3	2195	2195	2258.4	14	0.046
j3o8r5lr0.8t7_i2	1	5	284	284	1900.18	11	0.046
j3o8r5lr0.8t8_i3	1	14	1324	1324	3125.5	49	0.078
j3o8r5lr1t6_i1	0	0	203	203	266	7	0.031
j3o8r5lr1t5_i2	2	3	1148	1148	1294.21	8	0.031
j3o8r5lr1t6_i3	2	8	1550	1550	1675.68	19	0.031
j3o8r5lr1.2t4_i1	0	0	649	649	649	1	0.031
j3o8r5lr1.2t4_i2	0	0	284	284	1102	3	0.031
j3o8r5lr1.2t5_i3	0	2	571	571	1806	16	0.031
j3o10r5lr0.8t9_i1	0	6	1208	1208	1621.71	18	0.063
j3o10r5lr0.8t9_i2	0	16	2375	2375	4154.04	49	0.094
j3o10r5lr0.8t9_i3	0	7	2297	2297	2634.36	25	0.063
j3o10r5lr1t8_i1	0	18	3686	3686	6706.5	73	0.125
j3o10r5lr1t7_i2	0	0	861	861	861	1	0.016
j3o10r5lr1t7_i3	0	5	1795	1795	3197.31	32	0.078
j3o10r5lr1.2t6_i1	0	0	0	0	0	0	0.016
j3o10r5lr1.2t5_i2	0	0	1121	1121	3004	10	0.046
j3o10r5lr1.2t6_i3	0	0	137	137	137	1	0.016
j5o3r3lr0.8t7_i1	1	11	1449	1449	1775.46	27	0.031
j5o3r3lr0.8t8_i2	0	7	2113	2113	2259	31	0.031
j5o3r3lr0.8t8_i3	2	18	2341	2341	2786.08	37	0.031
j5o3r3lr1t6_i1	0	12	2883	2883	3621	31	0.031
j5o3r3lr1t6_i2	2	14	2555	2555	3081.58	35	0.031
j5o3r3lr1t5_i3	0	14	2843	2843	3041	35	0.031
j5o3r3lr1.2t4_i1	1	9	773	773	1319.17	18	0.016
j5o3r3lr1.2t5_i2	2	12	1206	1206	1408.43	33	0.031
j5o3r3lr1.2t5_i3	0	16	2174	2174	2674.13	44	0.031
j5o5r3lr0.8t10_i1	1	18	2119	2119	2688.8	68	0.063
j5o5r3lr0.8t14_i2	1	25	2882	2882	3470.11	60	0.094
j5o5r3lr0.8t12_i3	0	17	6440	6440	6891	30	0.047
j5o5r3lr1t10_i1	1	14	3411	3411	3989.59	51	0.063
j5o5r3lr1t10_i2	1	21	2658	2658	3424.76	84	0.078
j5o5r3lr1t11_i3	0	20	5767	5767	5992	60	0.078
j5o5r3lr1.2t8_i1	1	11	647	647	1380.03	45	0.063

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j5o5r3lr1.2t9_i2	2	18	2503	2503	3170.5	65	0.078
j5o5r3lr1.2t8_i3	2	16	1144	1144	1833.44	46	0.062
j5o8r5lr0.8t11_i1	1	16	1995	1995	2802.2	54	0.11
j5o8r5lr0.8t13_i2	1	29	7905	7905	8788.49	136	0.25
j5o8r5lr0.8t12_i3	0	23	10094	10094	11234	85	0.188
j5o8r5lr1t9_i1	3	18	1146	1146	1705.34	63	0.11
j5o8r5lr1t9_i2	1	18	2127	2127	3809.49	55	0.11
j5o8r5lr1t9_i3	0	14	1057	1057	2818.31	50	0.11
j5o8r5lr1.2t9_i1	1	16	3091	3091	4026.43	58	0.141
j5o8r5lr1.2t8_i2	1	18	869	869	2161.05	55	0.094
j5o8r5lr1.2t8_i3	1	28	4229	4229	7212.8	119	0.156
j5o10r5lr0.8t15_i1	1	30	5419	5419	6965.13	160	0.469
j5o10r5lr0.8t14_i2	2	25	755	755	3074.46	115	0.391
j5o10r5lr0.8t16_i3	2	34	7241	7241	10019.8	172	0.5
j5o10r5lr1t12_i1	0	44	4123	4123	6025.39	223	0.5
j5o10r5lr1t12_i2	1	25	2828	2828	6388.75	102	0.328
j5o10r5lr1t12_i3	1	35	4177	4177	6327.77	173	0.391
j5o10r5lr1.2t10_i1	2	20	1871	1871	3677.49	75	0.219
j5o10r5lr1.2t10_i2	1	29	5019	5019	5877.61	113	0.266
j5o10r5lr1.2t10_i3	2	24	2185	2185	4556.57	91	0.188
j8o3r3lr0.8t13_i1	1	23	2263	2263	2349.3	48	0.063
j8o3r3lr0.8t13_i2	0	27	4181	4181	4341	53	0.063
j8o3r3lr0.8t11_i3	1	25	4057	4057	4240.05	48	0.046
j8o3r3lr1t9_i1	1	22	2305	2305	2953.62	53	0.046
j8o3r3lr1t9_i2	1	14	1616	1616	2082.87	52	0.046
j8o3r3lr1t10_i3	1	25	2473	2473	3248.93	56	0.047
j8o3r3lr1.2t8_i1	2	21	2133	2133	2996.59	69	0.046
j8o3r3lr1.2t8_i2	1	24	4147	4147	5439.24	69	0.047
j8o3r3lr1.2t8_i3	2	21	2641	2641	3269.25	56	0.047
j8o5r3lr0.8t20_i1	1	31	3993	3993	5964.56	77	0.125
j8o5r3lr0.8t20_i2	1	44	5681	5681	6039.13	74	0.125
j8o5r3lr0.8t20_i3	1	34	6630	6630	6962.5	81	0.141
j8o5r3lr1t17_i1	1	36	5833	5833	6946	93	0.156
j8o5r3lr1t14_i2	2	38	4989	4989	5903.37	80	0.094
j8o5r3lr1t16_i3	1	27	1698	1698	4082.04	61	0.11
j8o5r3lr1.2t14_i1	3	44	4338	4338	5567.58	128	0.156
j8o5r3lr1.2t14_i2	2	38	4040	4040	5835.14	106	0.156
j8o5r3lr1.2t12_i3	2	34	3224	3224	4166.66	137	0.141
j8o8r5lr0.8t21_i1	1	48	6360	6360	7280.75	112	0.375
j8o8r5lr0.8t18_i2	1	34	5514	5514	6857.5	97	0.281
j8o8r5lr0.8t19_i3	0	49	13046	13046	15035.8	293	0.922
j8o8r5lr1t15_i1	1	56	14347	14347	16285.8	328	0.75
j8o8r5lr1t16_i2	2	54	8145	8145	11080.2	184	0.453
j8o8r5lr1t16_i3	0	51	9819	9819	10996	160	0.375
j8o8r5lr1.2t13_i1	1	43	6443	6443	10429.2	218	0.469
j8o8r5lr1.2t13_i2	1	47	5481	5481	7350.56	243	0.547
j8o8r5lr1.2t13_i3	1	31	2824	2824	4564.68	142	0.39
j8o10r5lr0.8t26_i1	1	52	15038	15038	17460.8	184	0.843
j8o10r5lr0.8t23_i2	2	57	8724	8724	11333.7	168	0.594
j8o10r5lr0.8t23_i3	1	47	16541	16541	18279.8	162	0.688
j8o10r5lr1t19_i1	1	51	7217	7217	13069.8	326	1.266
j8o10r5lr1t17_i2	1	54	7382	7382	9356.48	280	0.844
j8o10r5lr1t19_i3	1	44	5850	5850	8981.05	202	0.735
j8o10r5lr1.2t15_i1	1	56	5440	5440	9242.17	339	1.032

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j8o10r5lr1.2t16_i2	4	67	13092	13092	15463.8	490	1.438
j8o10r5lr1.2t16_i3	2	57	8131	8131	10754.3	328	1
j10o3r3lr0.8t15_i1	0	35	6757	6757	7366	70	0.078
j10o3r3lr0.8t15_i2	1	34	3247	3247	3593.58	86	0.078
j10o3r3lr0.8t14_i3	1	34	4532	4532	4795.27	70	0.078
j10o3r3lr1t12_i1	0	35	2813	2813	3639	86	0.078
j10o3r3lr1t12_i2	1	38	4781	4781	5280	89	0.078
j10o3r3lr1t11_i3	1	25	3663	3663	4669	65	0.047
j10o3r3lr1.2t10_i1	0	31	3542	3542	4683	79	0.063
j10o3r3lr1.2t9_i2	2	26	3125	3125	4384.69	63	0.046
j10o3r3lr1.2t10_i3	1	34	2989	2989	4382.75	89	0.062
j10o5r3lr0.8t26_i1	0	46	9405	9405	9833	110	0.25
j10o5r3lr0.8t28_i2	1	50	10474	10474	12037	109	0.312
j10o5r3lr0.8t25_i3	1	45	8585	8585	9245.9	110	0.234
j10o5r3lr1t20_i1	1	43	4215	4215	4684.4	109	0.188
j10o5r3lr1t19_i2	0	44	5903	5903	6156	108	0.172
j10o5r3lr1t19_i3	2	47	6083	6083	7184.96	100	0.187
j10o5r3lr1.2t16_i1	2	38	3062	3062	3970.99	105	0.14
j10o5r3lr1.2t18_i2	1	28	4564	4564	6226.5	97	0.172
j10o5r3lr1.2t18_i3	1	34	4050	4050	7350	136	0.234
j10o8r5lr0.8t24_i1	0	62	12568	12568	13293	126	0.438
j10o8r5lr0.8t24_i2	0	70	9930	9930	10589.1	141	0.5
j10o8r5lr0.8t22_i3	1	58	10337	10337	11319	118	0.344
j10o8r5lr1t17_i1	1	66	10408	10408	12341.1	188	0.454
j10o8r5lr1t18_i2	3	45	6189	6189	8824.09	205	0.454
j10o8r5lr1t19_i3	1	42	6268	6268	9378.57	95	0.297
j10o8r5lr1.2t16_i1	0	44	7276	7276	13790	154	0.391
j10o8r5lr1.2t16_i2	2	54	10204	10204	13313.9	265	0.563
j10o8r5lr1.2t16_i3	1	72	8061	8061	10207	398	1
j10o10r5lr0.8t29_i1	1	70	15839	15839	18500.4	196	0.985
j10o10r5lr0.8t29_i2	1	83	10919	10919	12169.9	161	0.766
j10o10r5lr0.8t30_i3	1	73	15273	15273	17592.9	291	1.438
j10o10r5lr1t24_i1	0	91	9028	9028	11023.9	332	1.437
j10o10r5lr1t23_i2	2	59	7497	7497	16441	223	0.828
j10o10r5lr1t25_i3	2	87	11700	11700	16344.9	541	2.531
j10o10r5lr1.2t19_i1	1	72	13871	13871	18232.6	336	1.218
j10o10r5lr1.2t21_i2	2	47	6483	6483	12910.3	219	0.938
j10o10r5lr1.2t19_i3	1	69	10791	10791	15929.2	205	0.734

APPENDIX E

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	14	0.046	41	794	0.046	15
j3o3r3lr0.8t5_i2	1135	3	0.031	24	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.031	24	1807	0.031	3
j3o3r3lr1t4_i1	1120	1	0.031	23	1120	0.031	3
j3o3r3lr1t4_i2	1169	3	0.031	26	1169	0.031	5
j3o3r3lr1t3_i3	729	13	0.031	55	729	0.031	15
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.031	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.016	3
j3o3r3lr1.2t3_i3	1202	1	0.031	24	1202	0.031	3
j3o5r3lr0.8t8_i1	2781	23	0.297	2113	2781	3.157	247
j3o5r3lr0.8t6_i2	3416	15	0.094	92	3416	0.094	17
j3o5r3lr0.8t7_i3	2157	7	0.094	150	2157.89	0.156	17
j3o5r3lr1t6_i1	2134	3	0.063	73	2134	0.063	5
j3o5r3lr1t6_i2	3661	7	0.078	79	3661	0.078	9
j3o5r3lr1t6_i3	2367	5649	175.87	50259	2413.7	900.046	13167
j3o5r3lr1.2t5_i1	2282	39	0.25	275	2282	0.328	55
j3o5r3lr1.2t6_i2	164	0	0	20	164	0.046	5
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.016	3
j3o8r5lr0.8t8_i1	2235	4	0.062	24	2235	0.078	7
j3o8r5lr0.8t7_i2	1889	1	0.046	19	1889	0.046	3
j3o8r5lr0.8t8_i3	3034	498	10.704	4268	3034.12	12.423	563
j3o8r5lr1t6_i1	266	1	0.031	7	266	0.031	3
j3o8r5lr1t5_i2	1175	6	0.047	22	1175	0.047	7
j3o8r5lr1t6_i3	1610	9	0.11	79	1610	0.125	11
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.031	3
j3o8r5lr1.2t4_i2	1102	1	0.031	3	1102	0.031	3
j3o8r5lr1.2t5_i3	1806	0	0.031	16	1806	0.031	1
j3o10r5lr0.8t9_i1	1620	3	0.094	39	1620	0.125	5
j3o10r5lr0.8t9_i2	3417	2587	130.891	51012	4126.9	900.183	8073
j3o10r5lr0.8t9_i3	2563	409	5.735	2006	2563.49	6.141	435
j3o10r5lr1t8_i1	6645	5101	205.615	24045	6645	291.264	5879
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	8	0.234	104	3171	0.234	9
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.046	10	3004	0.046	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.016	3
j5o3r3lr0.8t7_i1	1729	10	0.062	58	1729	0.062	11
j5o3r3lr0.8t8_i2	2259	30	0.141	98	2259	0.141	31
j5o3r3lr0.8t8_i3	2627	71	0.234	155	2627	0.234	73
j5o3r3lr1t6_i1	3621	15	0.094	122	3621	0.094	17
j5o3r3lr1t6_i2	3031	55	0.187	197	3031	0.187	57
j5o3r3lr1t5_i3	3041	8412	201.895	23426	3041	201.911	8413
j5o3r3lr1.2t4_i1	1312	22	0.063	97	1312	0.063	23
j5o3r3lr1.2t5_i2	1321	25	0.093	105	1321	0.109	29
j5o3r3lr1.2t5_i3	2670	49	0.172	5016	2670	8.845	1001
j5o5r3lr0.8t10_i1	2635	8051	250.322	52274	2684.97	900.079	14773
j5o5r3lr0.8t14_i2	3404	87	0.969	450	3404	0.985	89
j5o5r3lr0.8t12_i3	6891	107	0.797	326	6891	0.813	109
j5o5r3lr1t10_i1	3862	62	0.578	334	3862	0.578	63
j5o5r3lr1t10_i2	3176	2117	86.46	64930	3226.57	900.016	7971
j5o5r3lr1t11_i3	5881	791	19.392	64901	5992	900.129	8167

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j5o5r3lr1.2t8_i1	1182	65	0.516	342	1182	0.531	67
j5o5r3lr1.2t9_i2	3114	1294	37.94	28121	3114	169.149	3059
j5o5r3lr1.2t8_i3	1700	14	0.109	71	1700	0.109	15
j5o8r5lr0.8t11_i1	2725	19	0.422	666	2725	1.735	89
j5o8r5lr0.8t13_i2	8600	1044	64.597	64339	8776.36	900.011	5469
j5o8r5lr0.8t12_i3	11074	2991	257.855	62414	11227.6	900.318	6119
j5o8r5lr1t9_i1	1565	104	1.891	55125	1604.94	900.422	8725
j5o8r5lr1t9_i2	3653	268	7.453	58247	3756.73	900.088	7523
j5o8r5lr1t9_i3	2710	6895	657.063	54945	2805.72	900.13	8167
j5o8r5lr1.2t9_i1	3484.5	928	20.469	51105	3817.4	900.187	9271
j5o8r5lr1.2t8_i2	2059	143	2.828	14025	2059	76.845	1851
j5o8r5lr1.2t8_i3	4229	0	0	48497	7121.13	900.09	9109
j5o10r5lr0.8t15_i1	6785	316	23.188	51005	6922.13	900.527	5647
j5o10r5lr0.8t14_i2	2967	2865	308.488	61307	3047.61	900.479	5287
j5o10r5lr0.8t16_i3	9715	201	19.672	56125	9967.47	901.318	3431
j5o10r5lr1t12_i1	5693	3465	570.098	51686	6024.62	900.834	4607
j5o10r5lr1t12_i2	6277.33	72	4.719	61371	6355.08	900.334	3983
j5o10r5lr1t12_i3	5995	1617	233.861	55398	6326.13	901.068	3799
j5o10r5lr1.2t10_i1	3615	889	37.578	8815	3615.53	44.5	1007
j5o10r5lr1.2t10_i2	5628	2075	221.892	58146	5858.59	900.756	4933
j5o10r5lr1.2t10_i3	4351	194	8.063	51928	4504.98	900.579	6879
j8o3r3lr0.8t13_i1	2295	231	2.172	851	2295	2.188	233
j8o3r3lr0.8t13_i2	4341	109	0.813	312	4341	0.828	111
j8o3r3lr0.8t11_i3	4193	57	0.343	217	4193	0.359	59
j8o3r3lr1t9_i1	2889	65	0.296	167	2889	0.312	69
j8o3r3lr1t9_i2	2069	276	1.343	594	2069	1.343	277
j8o3r3lr1t10_i3	3218	50	0.25	149	3218	0.25	51
j8o3r3lr1.2t8_i1	2963	44	0.296	248	2963	0.296	45
j8o3r3lr1.2t8_i2	5417	75	0.438	54257	5436	900	13913
j8o3r3lr1.2t8_i3	3081	2402	41.266	53880	3202	900.094	13375
j8o5r3lr0.8t20_i1	5940	155	2.484	613	5940	2.5	157
j8o5r3lr0.8t20_i2	5941	624	13.734	1885	5941	13.734	625
j8o5r3lr0.8t20_i3	6942	9562	322.906	32755	6954	900.203	18503
j8o5r3lr1t17_i1	6739	4033	356.578	65076	6915.17	900.515	6897
j8o5r3lr1t14_i2	5652	5070	390.344	62914	5724	900.219	7945
j8o5r3lr1t16_i3	4009	128	1.328	429	4009	1.328	129
j8o5r3lr1.2t14_i1	5369	2622	119.641	62272	5405.5	900	8201
j8o5r3lr1.2t14_i2	5775	8591	430.859	47246	5807	900.234	12819
j8o5r3lr1.2t12_i3	3899	731	19.11	58654	4029	900.031	9127
j8o8r5lr0.8t21_i1	7046	852	191.563	52036	7252	900.313	2857
j8o8r5lr0.8t18_i2	6748	599	37.172	63811	6802	900.531	4921
j8o8r5lr0.8t19_i3	14660	3768	865.219	55034	15033.3	900.187	3885
j8o8r5lr1t15_i1	15751	354	49.531	60840	16230.2	900.031	3659
j8o8r5lr1t16_i2	9146	3139	761	52870	11032.9	900.515	3497
j8o8r5lr1t16_i3	10741	404	47.406	54057	10996	900.203	4047
j8o8r5lr1.2t13_i1	9986	338	35.359	61866	10375.1	900.609	4223
j8o8r5lr1.2t13_i2	7154	3311	611.25	64948	7347.76	900.203	4155
j8o8r5lr1.2t13_i3	4484	2146	404.25	70409	4557.27	901.515	3305
j8o10r5lr0.8t26_i1	17003	1639	463.922	38406	17432	900.578	2999
j8o10r5lr0.8t23_i2	11073	1840	537.61	48132	11258	900.547	2673
j8o10r5lr0.8t23_i3	18021	753	127.391	55007	18213	900.344	3051
j8o10r5lr1t19_i1	12758	388	77.704	51462	13067.4	900.219	3663
j8o10r5lr1t17_i2	9056	469	66.172	65335	9354.93	900.359	3093

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j8o10r5lr1t19_i3	8645	312	44.844	62333	8870.84	900.657	3475
j8o10r5lr1.2t15_i1	7899	441	76.813	53938	9229.92	900.047	3619
j8o10r5lr1.2t16_i2	14621	876	237.25	56144	15248.3	900.672	2729
j8o10r5lr1.2t16_i3	10111.3	2278	518.516	52221	10667.7	900.172	3389
j10o3r3lr0.8t15_i1	7366	205	2.281	595	7366	2.297	207
j10o3r3lr0.8t15_i2	3529	317	3.797	1064	3529	3.828	319
j10o3r3lr0.8t14_i3	4676	303	2.766	644	4676	2.781	305
j10o3r3lr1t12_i1	3639	186	1.421	388	3639	1.437	187
j10o3r3lr1t12_i2	5218	198	2.453	933	5218	2.453	199
j10o3r3lr1t11_i3	4658	166	1.297	556	4658	1.297	167
j10o3r3lr1.2t10_i1	4683	197	1.469	569	4683	1.485	199
j10o3r3lr1.2t9_i2	4373	1546	19	5423	4373	19	1547
j10o3r3lr1.2t10_i3	4306	8542	510.687	56346	4323.5	900	11481
j10o5r3lr0.8t26_i1	9833	744	29.938	2439	9833	29.938	745
j10o5r3lr0.8t28_i2	11643	847	85.656	47689	11817	901.094	5199
j10o5r3lr0.8t25_i3	9189	499	15.797	1575	9189	15.922	501
j10o5r3lr1t20_i1	4672	481	10.407	1284	4672	10.438	483
j10o5r3lr1t19_i2	6104	5354	445.625	56265	6156	900.235	8235
j10o5r3lr1t19_i3	7072	399	12.922	2446	7072	15.172	465
j10o5r3lr1.2t16_i1	3874	794	20.812	61149	3896	900.047	8421
j10o5r3lr1.2t18_i2	6193	200	5.187	40741	6221	900.046	11637
j10o5r3lr1.2t18_i3	7350	272	6.906	1379	7350	6.906	273
j10o8r5lr0.8t24_i1	13219	900	125.579	47423	13293	900.594	3271
j10o8r5lr0.8t24_i2	10424	1063	213.109	45801	10589.1	900.656	2865
j10o8r5lr0.8t22_i3	11032	1151	154.375	47225	11289	900.329	4117
j10o8r5lr1t17_i1	11909	785	130.266	55772	12294	900	3785
j10o8r5lr1t18_i2	8552	1601	315.797	57191	8749.66	900.282	2969
j10o8r5lr1t19_i3	8978	640	70.75	54967	9346	900.281	4349
j10o8r5lr1.2t16_i1	13490	2199	421.938	56042	13790	900.219	3635
j10o8r5lr1.2t16_i2	12987.5	1150	125.813	58582	13293	900.453	4397
j10o8r5lr1.2t16_i3	9903	628	97.594	51449	10160.5	900.829	4195
j10o10r5lr0.8t29_i1	18435	884	146.438	31466	18491	900.907	3503
j10o10r5lr0.8t29_i2	11507	1767	781.094	30290	12091	900.266	1983
j10o10r5lr0.8t30_i3	16690.5	1447	661.907	31413	17464	900.407	1915
j10o10r5lr1t24_i1	10682	1127	546.922	33994	11023.9	900.343	1761
j10o10r5lr1t23_i2	15578	1731	458.781	45250	16190.3	900.297	3123
j10o10r5lr1t25_i3	15369	754	204.937	45808	16095.9	901.047	2277
j10o10r5lr1.2t19_i1	17710	637	171.484	49363	18177.4	900.031	2707
j10o10r5lr1.2t21_i2	12650	590	220.703	54019	12867.1	900.875	1671
j10o10r5lr1.2t19_i3	15336	679	155.765	54988	15882.9	900.078	2765

APPENDIX F

Instance	Best Integer Solution	Node of Best Integer Solution	Time to Best Integer Solution (seconds)	Columns Generated	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	14	2.719	41	794	2.719	15
j3o3r3lr0.8t5_i2	1135	3	0.031	24	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.031	24	1807	0.046	3
j3o3r3lr1t4_i1	1120	1	0.031	23	1120	0.031	3
j3o3r3lr1t4_i2	1169	3	0.031	26	1169	0.031	5
j3o3r3lr1t3_i3	729	8	0.046	40	729	0.046	9
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.015	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.031	3
j3o3r3lr1.2t3_i3	1202	1	0.031	24	1202	0.031	3
j3o5r3lr0.8t8_i1	2781	23	0.312	976	2825.36	1.25	105
j3o5r3lr0.8t6_i2	3416	15	0.078	92	3416	0.078	17
j3o5r3lr0.8t7_i3	2157	7	0.109	150	2157.89	0.156	17
j3o5r3lr1t6_i1	2134	3	0.078	73	2134	0.078	5
j3o5r3lr1t6_i2	3661	7	0.078	79	3661	0.093	9
j3o5r3lr1t6_i3	2377	2321	36.796	9370	2395.4	36.953	2327
j3o5r3lr1.2t5_i1	2282	41	0.265	258	2282	0.312	51
j3o5r3lr1.2t6_i2	164	0	0	20	164	0.046	5
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.031	3
j3o8r5lr0.8t8_i1	2235	4	0.063	24	2235	0.078	7
j3o8r5lr0.8t7_i2	1889	1	0.047	19	1889	0.047	3
j3o8r5lr0.8t8_i3	3034	174	3.687	1891	3071.97	4.421	213
j3o8r5lr1t6_i1	266	1	0.031	7	266	0.031	3
j3o8r5lr1t5_i2	1175	6	0.046	22	1175	0.046	7
j3o8r5lr1t6_i3	1610	9	0.11	79	1610	0.125	11
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.016	3
j3o8r5lr1.2t4_i2	1102	1	0.031	3	1102	0.031	3
j3o8r5lr1.2t5_i3	1806	0	0.031	16	1806	0.031	1
j3o10r5lr0.8t9_i1	1620	3	0.094	39	1620	0.11	5
j3o10r5lr0.8t9_i2	3938	133	4.625	51436	4107.27	900.39	6239
j3o10r5lr0.8t9_i3	2563	24	0.375	1216	2582.52	3.406	259
j3o10r5lr1t8_i1	6645	427	13.469	3910	6686.4	14.422	453
j3o10r5lr1t7_i2	861	0	0	1	861	0.062	3
j3o10r5lr1t7_i3	3171	8	0.234	104	3171	0.234	9
j3o10r5lr1.2t6_i1	0	0	0	0	0	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.031	10	3004	0.047	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	29	0.109	103	1729	0.109	31
j5o3r3lr0.8t8_i2	2259	30	0.141	98	2259	0.141	31
j5o3r3lr0.8t8_i3	2627	71	0.235	155	2627	0.25	73
j5o3r3lr1t6_i1	3621	15	0.094	122	3621	0.094	17
j5o3r3lr1t6_i2	3031	55	0.172	197	3031	0.172	57
j5o3r3lr1t5_i3	3041	73	0.266	396	3041	0.266	75
j5o3r3lr1.2t4_i1	1312	22	0.063	97	1312	0.063	23
j5o3r3lr1.2t5_i2	1321	25	0.094	105	1321	0.094	29
j5o3r3lr1.2t5_i3	2670	49	0.187	1713	2674.13	1.718	325
j5o5r3lr0.8t10_i1	2635	138	1.203	50357	2675.87	900.015	14627
j5o5r3lr0.8t14_i2	3404	87	0.953	450	3404	0.985	89
j5o5r3lr0.8t12_i3	6891	107	0.797	326	6891	0.813	109
j5o5r3lr1t10_i1	3862	62	0.578	334	3862	0.578	63
j5o5r3lr1t10_i2	3176	171	2.625	61797	3221.05	900.11	8681
j5o5r3lr1t11_i3	5992	307	4.078	1633	5992	4.094	309
j5o5r3lr1.2t8_i1	1182	85	0.641	409	1182	0.656	87

Instance	Best Integer Solution	Node of Best Integer Solution	Time to Best Integer Solution (seconds)	Columns Generated	Best Bound	Runtime (seconds)	Total Nodes
j5o5r3lr1.2t9_i2	3114	2024	74.672	18289	3136	87.172	2201
j5o5r3lr1.2t8_i3	1700	14	0.109	71	1700	0.109	15
j5o8r5lr0.8t11_i1	2725	19	0.422	439	2735.96	1.11	59
j5o8r5lr0.8t13_i2	8630.5	3562	544.39	64786	8773.76	900.343	4615
j5o8r5lr0.8t12_i3	11059	149	5.656	61079	11209.7	900.406	6341
j5o8r5lr1t9_i1	1559	6429	622.36	50076	1588.72	711.875	6915
j5o8r5lr1t9_i2	3650	77	2.156	59277	3736.14	900.516	7323
j5o8r5lr1t9_i3	2751	101	2.813	3835	2802.8	13.141	499
j5o8r5lr1.2t9_i1	3752	164	4.625	1790	3764.19	5.672	199
j5o8r5lr1.2t8_i2	2059	83	1.703	4518	2120.61	15.188	627
j5o8r5lr1.2t8_i3	4229	0	0	48534	7121.13	900.531	9111
j5o10r5lr0.8t15_i1	6774	1195	109.094	56508	6918.58	900.531	4719
j5o10r5lr0.8t14_i2	2970	442	22.031	59252	3041.38	900.14	5843
j5o10r5lr0.8t16_i3	9826	3908	813.172	55019	9967.47	900.172	4123
j5o10r5lr1t12_i1	5693	259	25.578	57217	6023.01	900.61	4753
j5o10r5lr1t12_i2	6277.33	450	31.047	52202	6334.43	900.281	6589
j5o10r5lr1t12_i3	6011	413	44.344	61202	6317.32	900.313	3545
j5o10r5lr1.2t10_i1	3615	51	2	1028	3616.29	3.5	103
j5o10r5lr1.2t10_i2	5659	313	22.906	53155	5842.08	900.343	6117
j5o10r5lr1.2t10_i3	4361	1098	69.516	56294	4497.4	900.141	5753
j8o3r3lr0.8t13_i1	2295	231	2.188	851	2295	2.188	233
j8o3r3lr0.8t13_i2	4341	109	0.812	312	4341	0.828	111
j8o3r3lr0.8t11_i3	4193	57	0.344	217	4193	0.36	59
j8o3r3lr1t9_i1	2889	65	0.281	167	2889	0.297	69
j8o3r3lr1t9_i2	2069	260	1.281	547	2069	1.281	261
j8o3r3lr1t10_i3	3218	50	0.25	149	3218	0.25	51
j8o3r3lr1.2t8_i1	2963	44	0.281	248	2963	0.281	45
j8o3r3lr1.2t8_i2	5436	193	1.156	777	5436	1.156	195
j8o3r3lr1.2t8_i3	3202	241	1.406	766	3202	1.406	243
j8o5r3lr0.8t20_i1	5940	155	2.469	613	5940	2.5	157
j8o5r3lr0.8t20_i2	5941	566	12.203	1773	5941	12.203	567
j8o5r3lr0.8t20_i3	6942	9293	679.485	53960	6954	900.172	10845
j8o5r3lr1t17_i1	6739	388	14.235	64480	6915.17	900.203	5817
j8o5r3lr1t14_i2	5724	493	7.64	2123	5724	7.656	495
j8o5r3lr1t16_i3	4009	128	1.313	429	4009	1.313	129
j8o5r3lr1.2t14_i1	5373	583	16.094	64322	5405.5	900.016	7217
j8o5r3lr1.2t14_i2	5759	431	7.875	56914	5807	900.234	9699
j8o5r3lr1.2t12_i3	3991	464	9.688	64058	4029	900.063	7377
j8o8r5lr0.8t21_i1	7046	719	155.563	49896	7252	900.641	3553
j8o8r5lr0.8t18_i2	6748	568	29.781	57211	6802	900.156	5455
j8o8r5lr0.8t19_i3	14641	428	66.672	58312	15031.8	900.641	3713
j8o8r5lr1t15_i1	15751	354	49.5	57209	16230.2	900.844	3441
j8o8r5lr1t16_i2	9331	974	178.297	50851	11032.9	900.125	3841
j8o8r5lr1t16_i3	10889	1020	133.062	52694	10996	900.109	4179
j8o8r5lr1.2t13_i1	10082	2381	439.156	61897	10369.6	900.735	3883
j8o8r5lr1.2t13_i2	7154	318	33.141	60971	7347.76	900.438	3921
j8o8r5lr1.2t13_i3	4484	1634	197.438	67009	4546.92	900.36	4003
j8o10r5lr0.8t26_i1	17087	1408	465.718	43904	17432	900.453	2327
j8o10r5lr0.8t23_i2	11107	1334	368.563	47942	11258	900.797	2373
j8o10r5lr0.8t23_i3	18045	1465	284.531	48316	18213	900.219	3083
j8o10r5lr1t19_i1	12758	388	77.516	57787	13067.4	900.125	2839
j8o10r5lr1t17_i2	9056	422	59.125	59340	9354.86	900.188	3939
j8o10r5lr1t19_i3	8721	554	85.922	57969	8870.84	900.594	3351
j8o10r5lr1.2t15_i1	8803	1977	522.125	54429	9225.48	900.235	2911

Instance	Best Integer Solution	Node of Best Integer Solution	Time to Best Integer Solution (seconds)	Columns Generated	Best Bound	Runtime (seconds)	Total Nodes
j8o10r5lr1.2t16_i2	14616	482	140.094	50381	15121	900.532	1851
j8o10r5lr1.2t16_i3	10269	721	151.938	52648	10665.6	900.329	3263
j10o3r3lr0.8t15_i1	7366	205	2.281	595	7366	2.297	207
j10o3r3lr0.8t15_i2	3529	317	3.812	1064	3529	3.843	319
j10o3r3lr0.8t14_i3	4676	303	2.766	644	4676	2.766	305
j10o3r3lr1t12_i1	3639	186	1.422	388	3639	1.422	187
j10o3r3lr1t12_i2	5218	198	2.438	933	5218	2.438	199
j10o3r3lr1t11_i3	4658	166	1.297	556	4658	1.297	167
j10o3r3lr1.2t10_i1	4683	197	1.469	569	4683	1.485	199
j10o3r3lr1.2t9_i2	4373	218	1.296	621	4373	1.296	219
j10o3r3lr1.2t10_i3	4306	221	2.046	50546	4323.5	900.015	13921
j10o5r3lr0.8t26_i1	9833	744	29.922	2439	9833	29.922	745
j10o5r3lr0.8t28_i2	11760	1770	180.312	47298	11817	900.437	5643
j10o5r3lr0.8t25_i3	9189	499	15.812	1575	9189	15.953	501
j10o5r3lr1t20_i1	4672	481	10.39	1284	4672	10.422	483
j10o5r3lr1t19_i2	6145	1175	45.641	56551	6156	900.172	8419
j10o5r3lr1t19_i3	7072	399	12.906	2070	7072	13.015	405
j10o5r3lr1.2t16_i1	3884	747	19.141	64747	3896	900.453	8487
j10o5r3lr1.2t18_i2	6193	200	5.203	55699	6221	900.203	9411
j10o5r3lr1.2t18_i3	7350	272	6.906	1379	7350	6.906	273
j10o8r5lr0.8t24_i1	13218.5	877	120.672	44386	13293	900.829	3333
j10o8r5lr0.8t24_i2	10518	1849	396.969	37037	10589.1	900.234	3921
j10o8r5lr0.8t22_i3	11262	1556	202.953	47638	11289	900.656	4641
j10o8r5lr1t17_i1	12053	2140	507.344	52869	12294	900.484	3277
j10o8r5lr1t18_i2	8552	227	16.5	63420	8716.18	900.14	3879
j10o8r5lr1t19_i3	9162	983	119.672	57238	9346	900.579	3847
j10o8r5lr1.2t16_i1	13469	365	37.469	52620	13790	900.579	4915
j10o8r5lr1.2t16_i2	13040	997	118.219	57951	13293	900.063	4237
j10o8r5lr1.2t16_i3	9951	3383	805.156	54541	10160.5	900.281	3649
j10o10r5lr0.8t29_i1	18435	884	146.593	30370	18491	900.828	3361
j10o10r5lr0.8t29_i2	11507	1767	780.703	27938	12091	900.297	2211
j10o10r5lr0.8t30_i3	16690.5	1447	662.328	29949	17464	902.219	1919
j10o10r5lr1t24_i1	10682	1127	547.016	37154	11023.9	900.406	1741
j10o10r5lr1t23_i2	15578	773	219.25	47741	16092.2	900.719	2983
j10o10r5lr1t25_i3	15369	754	204.968	41056	15980.6	900.562	2435
j10o10r5lr1.2t19_i1	17632	636	170.969	50688	18177.4	900.172	2417
j10o10r5lr1.2t21_i2	12637	951	361.156	49398	12867	901.719	1933
j10o10r5lr1.2t19_i3	15726	1229	299.328	43617	15882.9	900.688	3377

APPENDIX G

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	15	0.031	51	794	0.031	17
j3o3r3lr0.8t5_i2	1135	3	0.016	24	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.016	24	1807	0.031	3
j3o3r3lr1t4_i1	1120	1	0.031	23	1120	0.031	3
j3o3r3lr1t4_i2	1169	3	0.031	26	1169	0.031	5
j3o3r3lr1t3_i3	729	8	0.031	40	729	0.031	9
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.016	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.016	3
j3o3r3lr1.2t3_i3	1202	1	0.031	24	1202	0.031	3
j3o5r3lr0.8t8_i1	2781	165	2.484	2110	2782	3.343	233
j3o5r3lr0.8t6_i2	3416	11	0.078	82	3416	0.078	13
j3o5r3lr0.8t7_i3	2157	19	0.203	183	2157	0.203	21
j3o5r3lr1t6_i1	2134	3	0.063	73	2134	0.063	5
j3o5r3lr1t6_i2	3661	7	0.063	79	3661	0.078	9
j3o5r3lr1t6_i3	2377	565	4.875	2709	2377	4.891	567
j3o5r3lr1.2t5_i1	2282	35	0.281	258	2282	0.297	41
j3o5r3lr1.2t6_i2	164	0	0	20	164	0.047	5
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.016	3
j3o8r5lr0.8t8_i1	2235	4	0.063	24	2235	0.078	7
j3o8r5lr0.8t7_i2	1889	1	0.046	19	1889	0.062	3
j3o8r5lr0.8t8_i3	3034	442	13.171	4244	3034.5	13.171	443
j3o8r5lr1t6_i1	266	1	0.046	7	266	0.046	3
j3o8r5lr1t5_i2	1175	6	0.046	22	1175	0.046	7
j3o8r5lr1t6_i3	1610	9	0.11	79	1610	0.125	11
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.015	3
j3o8r5lr1.2t4_i2	1102	1	0.031	3	1102	0.031	3
j3o8r5lr1.2t5_i3	1806	0	0.046	16	1806	0.046	1
j3o10r5lr0.8t9_i1	1620	3	0.094	39	1620	0.125	5
j3o10r5lr0.8t9_i2	2375	0	0	50377	4047.08	900.016	6177
j3o10r5lr0.8t9_i3	2563	367	5.578	1959	2563.49	6.156	403
j3o10r5lr1t8_i1	6645	621	19.203	8330	6645.86	41.875	1115
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	8	0.219	104	3171	0.219	9
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.031	10	3004	0.031	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	23	0.078	75	1729	0.078	25
j5o3r3lr0.8t8_i2	2259	179	0.641	343	2259	0.641	181
j5o3r3lr0.8t8_i3	2627	986	4.047	1780	2627	4.047	987
j5o3r3lr1t6_i1	3621	50	0.25	291	3621	0.25	51
j5o3r3lr1t6_i2	3031	3650	44.235	10508	3031	44.235	3651
j5o3r3lr1t5_i3	3041	1601	17.485	7889	3041	17.5	1603
j5o3r3lr1.2t4_i1	1312	23	0.062	100	1312	0.078	25
j5o3r3lr1.2t5_i2	1321	41	0.172	193	1321	0.188	43
j5o3r3lr1.2t5_i3	2670	1073	11.046	5603	2670	11.062	1075
j5o5r3lr0.8t10_i1	2657	10314	506.281	36648	2657	506.313	10315
j5o5r3lr0.8t14_i2	3404	2893	34.891	6135	3404	34.922	2895
j5o5r3lr0.8t12_i3	6891	16196	783.297	40542	6891	783.328	16197
j5o5r3lr1t10_i1	3411	0	0	50117	3862	900.141	14813
j5o5r3lr1t10_i2	2658	0	0	69198	3218.78	900.218	6775
j5o5r3lr1t11_i3	5767	0	0	48036	5992	900.141	15391

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j5o5r3lr1.2t8_i1	1182	735	7.281	3054	1182	7.297	737
j5o5r3lr1.2t9_i2	3114	1678	78.359	19339	3114.88	91.125	1859
j5o5r3lr1.2t8_i3	1700	16	0.125	76	1700	0.125	17
j5o8r5lr0.8t11_i1	2725	26	0.485	667	2725	1.656	87
j5o8r5lr0.8t13_i2	7905	0	0	58321	8748.8	900.063	5301
j5o8r5lr0.8t12_i3	10094	0	0	58946	11184.8	900.203	6247
j5o8r5lr1t9_i1	1259	213	3.531	54681	1571.88	900.485	7447
j5o8r5lr1t9_i2	2127	0	0	58608	3700.03	900.297	6981
j5o8r5lr1t9_i3	2767.8	2742	186.656	32011	2768.71	328.469	3947
j5o8r5lr1.2t9_i1	3752	214	6.766	1962	3752	6.781	215
j5o8r5lr1.2t8_i2	2059	525	15.61	10551	2060	48.391	1313
j5o8r5lr1.2t8_i3	4229	0	0	67428	7090.16	900.438	5813
j5o10r5lr0.8t15_i1	5419	0	0	52996	6902.17	900.328	5267
j5o10r5lr0.8t14_i2	755	0	0	59933	2985.11	900.5	4389
j5o10r5lr0.8t16_i3	7241	0	0	61672	9955.26	900.844	3495
j5o10r5lr1t12_i1	4123	0	0	66217	6001.42	900.219	2713
j5o10r5lr1t12_i2	2828	0	0	52691	6307.99	900.156	5863
j5o10r5lr1t12_i3	4177	0	0	66364	6309.76	900.75	3267
j5o10r5lr1.2t10_i1	3615	236	8.047	2087	3615	8.063	237
j5o10r5lr1.2t10_i2	5019	0	0	61266	5772.75	901.063	3559
j5o10r5lr1.2t10_i3	2185	0	0	56904	4468.03	900.766	5445
j8o3r3lr0.8t13_i1	2263	0	0	53845	2295	900.203	15907
j8o3r3lr0.8t13_i2	4341	43	0.36	250	4341	0.375	45
j8o3r3lr0.8t11_i3	4057	0	0	45159	4193	900.031	19415
j8o3r3lr1t9_i1	2889	2475	17.906	4528	2889	17.906	2477
j8o3r3lr1t9_i2	1616	0	0	43764	2069	900.11	20159
j8o3r3lr1t10_i3	3218	1376	9.141	2797	3218	9.141	1377
j8o3r3lr1.2t8_i1	2963	50	0.312	306	2963	0.312	51
j8o3r3lr1.2t8_i2	5436	10210	361.781	33668	5436	361.796	10211
j8o3r3lr1.2t8_i3	2641	0	0	51307	3202	900.063	17485
j8o5r3lr0.8t20_i1	3993	0	0	41417	5940	900.016	16305
j8o5r3lr0.8t20_i2	5681	0	0	42904	5941	900.141	14067
j8o5r3lr0.8t20_i3	6630	0	0	41107	6954	900.062	14283
j8o5r3lr1t17_i1	5833	0	0	40237	6915.17	900.015	13483
j8o5r3lr1t14_i2	4989	0	0	42307	5724	900.031	17739
j8o5r3lr1t16_i3	1698	0	0	43244	4009	900.125	16919
j8o5r3lr1.2t14_i1	4338	0	0	53583	5405.5	900.172	13445
j8o5r3lr1.2t14_i2	4040	0	0	53823	5807	900.125	12943
j8o5r3lr1.2t12_i3	3224	0	0	4834	4029	358.985	30001
j8o8r5lr0.8t21_i1	6360	0	0	56119	7252	900.156	4865
j8o8r5lr0.8t18_i2	5514	0	0	47939	6802	900.094	10717
j8o8r5lr0.8t19_i3	13046	0	0	54221	15023.6	900.078	3171
j8o8r5lr1t15_i1	14347	0	0	63849	16213.5	900.328	3297
j8o8r5lr1t16_i2	8145	0	0	54627	11032.9	900.187	5791
j8o8r5lr1t16_i3	9819	0	0	59293	10996	900.437	6825
j8o8r5lr1.2t13_i1	6443	0	0	60277	10350.5	900.453	3763
j8o8r5lr1.2t13_i2	5481	0	0	68924	7344.64	900.984	2729
j8o8r5lr1.2t13_i3	2824	0	0	61133	4535.7	900.453	3663
j8o10r5lr0.8t26_i1	15038	0	0	48761	17432	900.437	5053
j8o10r5lr0.8t23_i2	8724	0	0	49889	11258	900.484	5537
j8o10r5lr0.8t23_i3	16541	0	0	50786	18213	900.188	6993
j8o10r5lr1t19_i1	7217	0	0	49206	13065.4	900.109	2717
j8o10r5lr1t17_i2	7382	0	0	55177	9343.65	900.703	3721

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j8o10r5lr1t19_i3	5850	0	0	55114	8867.88	900.25	4031
j8o10r5lr1.2t15_i1	5440	0	0	56454	9210.53	900.125	2759
j8o10r5lr1.2t16_i2	13092	0	0	49455	15121	900.125	3771
j8o10r5lr1.2t16_i3	8131	0	0	58395	10652.4	900.625	2725
j10o3r3lr0.8t15_i1	6757	0	0	50850	7366	900.125	16497
j10o3r3lr0.8t15_i2	3529	14375	640.75	37731	3529	640.906	14377
j10o3r3lr0.8t14_i3	4532	0	0	48726	4676	900.016	17423
j10o3r3lr1t12_i1	2813	0	0	39861	3639	900.047	20659
j10o3r3lr1t12_i2	4781	0	0	50621	5218	900.125	15111
j10o3r3lr1t11_i3	3663	0	0	51481	4658	900.031	16873
j10o3r3lr1.2t10_i1	3542	0	0	1701	4683	194.64	30001
j10o3r3lr1.2t9_i2	3125	0	0	47009	4373	900.015	19059
j10o3r3lr1.2t10_i3	2989	0	0	50292	4323.5	900.188	17097
j10o5r3lr0.8t26_i1	9405	0	0	5649	9833	827.657	30001
j10o5r3lr0.8t28_i2	10474	0	0	49126	11817	900	9115
j10o5r3lr0.8t25_i3	8585	0	0	43950	9189	900.125	12283
j10o5r3lr1t20_i1	4215	0	0	36826	4672	900.094	17207
j10o5r3lr1t19_i2	5903	0	0	57187	6156	900.094	10189
j10o5r3lr1t19_i3	6083	0	0	53297	7072	900.25	12093
j10o5r3lr1.2t16_i1	3062	0	0	48248	3896	900.047	13917
j10o5r3lr1.2t18_i2	4564	0	0	45725	6221	900.094	14467
j10o5r3lr1.2t18_i3	4050	0	0	43211	7350	900	14153
j10o8r5lr0.8t24_i1	12568	0	0	39774	13293	900.172	6353
j10o8r5lr0.8t24_i2	9930	0	0	48398	10589.1	900.109	6271
j10o8r5lr0.8t22_i3	10337	0	0	54217	11289	900.188	6987
j10o8r5lr1t17_i1	10408	0	0	55672	12294	900.141	7205
j10o8r5lr1t18_i2	6189	0	0	61227	8713.3	900.297	5383
j10o8r5lr1t19_i3	6268	0	0	46532	9346	900.157	7299
j10o8r5lr1.2t16_i1	7276	0	0	61194	13790	900.594	7185
j10o8r5lr1.2t16_i2	10204	0	0	59153	13293	900.047	7371
j10o8r5lr1.2t16_i3	8061	0	0	53898	10160.5	900.047	4593
j10o10r5lr0.8t29_i1	15839	0	0	30209	18491	900.359	6155
j10o10r5lr0.8t29_i2	10919	0	0	29526	12091	900.015	4593
j10o10r5lr0.8t30_i3	15273	0	0	32658	17464	900.234	6347
j10o10r5lr1t24_i1	9028	0	0	51289	11023.9	901.453	2175
j10o10r5lr1t23_i2	7497	0	0	47829	16092.2	900.344	3811
j10o10r5lr1t25_i3	11700	0	0	27377	15980.6	900.015	7251
j10o10r5lr1.2t19_i1	13871	0	0	43390	18177.4	900.469	5379
j10o10r5lr1.2t21_i2	6483	0	0	49759	12864.4	901.047	1957
j10o10r5lr1.2t19_i3	10791	0	0	47793	15882.9	900.016	5003

APPENDIX H

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	14	0.063	41	794	0.063	15
j3o3r3lr0.8t5_i2	1135	3	0.031	24	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.031	24	1807	0.031	3
j3o3r3lr1t4_i1	1120	1	0.016	23	1120	0.031	3
j3o3r3lr1t4_i2	1169	3	0.016	26	1169	0.031	5
j3o3r3lr1t3_i3	729	8	0.031	40	729	0.031	9
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.016	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.016	3
j3o3r3lr1.2t3_i3	1202	1	0.031	24	1202	0.031	3
j3o5r3lr0.8t8_i1	2781	23	0.312	711	2825.36	0.89	71
j3o5r3lr0.8t6_i2	3416	15	0.093	92	3416	0.093	17
j3o5r3lr0.8t7_i3	2157	7	0.11	109	2157	0.125	9
j3o5r3lr1t6_i1	2134	3	0.063	73	2134	0.063	5
j3o5r3lr1t6_i2	3661	7	0.078	79	3661	0.078	9
j3o5r3lr1t6_i3	2367	649	5.687	2972	2413.7	6.234	691
j3o5r3lr1.2t5_i1	2282	41	0.281	228	2282	0.281	43
j3o5r3lr1.2t6_i2	164	0	0	20	164	0.062	5
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.031	3
j3o8r5lr0.8t8_i1	2235	4	0.063	20	2235	0.063	5
j3o8r5lr0.8t7_i2	1889	1	0.047	19	1889	0.047	3
j3o8r5lr0.8t8_i3	3023	101	2.359	1172	3077.92	2.562	111
j3o8r5lr1t6_i1	266	1	0.047	7	266	0.047	3
j3o8r5lr1t5_i2	1175	6	0.047	22	1175	0.047	7
j3o8r5lr1t6_i3	1610	9	0.14	79	1610	0.156	11
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.031	3
j3o8r5lr1.2t4_i2	1102	1	0.063	3	1102	0.078	3
j3o8r5lr1.2t5_i3	1806	0	0.062	16	1806	0.062	1
j3o10r5lr0.8t9_i1	1620	3	0.11	39	1620	0.125	5
j3o10r5lr0.8t9_i2	3951.5	374	15.219	53432	4117.38	900.11	6097
j3o10r5lr0.8t9_i3	2563	24	0.375	197	2632.48	0.5	31
j3o10r5lr1t8_i1	6589	83	2.484	1651	6691.83	4.984	197
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	8	0.219	104	3171	0.219	9
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.046	10	3004	0.046	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	29	0.109	103	1729	0.109	31
j5o3r3lr0.8t8_i2	2259	30	0.141	98	2259	0.141	31
j5o3r3lr0.8t8_i3	2627	71	0.25	155	2627	0.25	73
j5o3r3lr1t6_i1	3621	15	0.094	122	3621	0.094	17
j5o3r3lr1t6_i2	3031	55	0.171	197	3031	0.187	57
j5o3r3lr1t5_i3	3041	73	0.281	396	3041	0.297	75
j5o3r3lr1.2t4_i1	1312	22	0.062	97	1312	0.062	23
j5o3r3lr1.2t5_i2	1321	25	0.093	105	1321	0.109	29
j5o3r3lr1.2t5_i3	2670	49	0.171	223	2670	0.187	51
j5o5r3lr0.8t10_i1	2635	138	1.203	3847	2684.62	8.985	765
j5o5r3lr0.8t14_i2	3404	87	0.953	450	3404	0.969	89
j5o5r3lr0.8t12_i3	6891	107	0.797	326	6891	0.797	109
j5o5r3lr1t10_i1	3862	62	0.578	334	3862	0.578	63
j5o5r3lr1t10_i2	3167	103	1.703	68180	3221.25	900	6541
j5o5r3lr1t11_i3	5992	307	4.078	1633	5992	4.078	309

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j5o5r3lr1.2t8_i1	1171	58	0.453	288	1171	0.453	59
j5o5r3lr1.2t9_i2	3092	59	0.875	2393	3136	5.016	281
j5o5r3lr1.2t8_i3	1700	14	0.11	71	1700	0.11	15
j5o8r5lr0.8t11_i1	2725	19	0.421	181	2725	0.437	21
j5o8r5lr0.8t13_i2	8600	149	7.969	66269	8775.25	900.281	3943
j5o8r5lr0.8t12_i3	11059	149	6.079	57856	11227.4	900.172	5463
j5o8r5lr1t9_i1	1559	45	0.843	2972	1599.36	8.234	429
j5o8r5lr1t9_i2	3650	77	2.203	58081	3749.83	900.203	6323
j5o8r5lr1t9_i3	2751	101	2.812	2534	2803.45	8.078	305
j5o8r5lr1.2t9_i1	3752	164	4.672	1441	3752	4.688	165
j5o8r5lr1.2t8_i2	2059	78	1.75	2320	2122.91	6.313	259
j5o8r5lr1.2t8_i3	4229	0	0	48234	7121.13	900.234	9057
j5o10r5lr0.8t15_i1	6759	114	9.141	50345	6920.54	900.156	5245
j5o10r5lr0.8t14_i2	2967	118	6.391	30280	3026.17	286.313	2289
j5o10r5lr0.8t16_i3	9810	551	65.094	55001	9967.47	900.812	2923
j5o10r5lr1t12_i1	5693	259	25.687	59400	6023.01	901.062	4157
j5o10r5lr1t12_i2	6269	63	4.219	2158	6348.84	10.437	155
j5o10r5lr1t12_i3	5993	205	21.141	63623	6325.47	900.531	3215
j5o10r5lr1.2t10_i1	3593	40	1.672	530	3593	1.672	41
j5o10r5lr1.2t10_i2	5628	122	8.36	55920	5842.08	900.735	4729
j5o10r5lr1.2t10_i3	4348	3420	453.422	54911	4499.46	900.344	5113
j8o3r3lr0.8t13_i1	2295	231	2.171	851	2295	2.187	233
j8o3r3lr0.8t13_i2	4341	109	0.828	312	4341	0.828	111
j8o3r3lr0.8t11_i3	4193	57	0.343	217	4193	0.359	59
j8o3r3lr1t9_i1	2889	65	0.281	167	2889	0.297	67
j8o3r3lr1t9_i2	2069	260	1.281	547	2069	1.281	261
j8o3r3lr1t10_i3	3218	50	0.25	149	3218	0.25	51
j8o3r3lr1.2t8_i1	2963	44	0.281	248	2963	0.281	45
j8o3r3lr1.2t8_i2	5417	75	0.438	329	5417	0.438	77
j8o3r3lr1.2t8_i3	3202	241	1.406	766	3202	1.422	243
j8o5r3lr0.8t20_i1	5940	155	2.5	613	5940	2.516	157
j8o5r3lr0.8t20_i2	5941	566	12.281	1773	5941	12.281	567
j8o5r3lr0.8t20_i3	6942	243	4.438	821	6942	4.453	245
j8o5r3lr1t17_i1	6739	388	14.313	62893	6915.17	900.344	6647
j8o5r3lr1t14_i2	5724	493	7.672	2123	5724	7.688	495
j8o5r3lr1t16_i3	4009	128	1.328	429	4009	1.328	129
j8o5r3lr1.2t14_i1	5369	238	5.188	1754	5369	5.188	239
j8o5r3lr1.2t14_i2	5719	259	4.485	46766	5807	900.016	11225
j8o5r3lr1.2t12_i3	3991	464	9.687	3144	3991	9.687	465
j8o8r5lr0.8t21_i1	7046	719	156.359	52623	7252	900.593	2819
j8o8r5lr0.8t18_i2	6748	277	13.813	2036	6748	13.891	279
j8o8r5lr0.8t19_i3	14641	428	66.625	56358	15031.8	900.187	3165
j8o8r5lr1t15_i1	15751	354	49.531	56903	16230.2	900.984	2975
j8o8r5lr1t16_i2	9331	974	178.281	52344	11032.9	900.312	3391
j8o8r5lr1t16_i3	10858	645	84.359	54052	10996	900.859	4179
j8o8r5lr1.2t13_i1	10067	1966	355.062	61010	10373.8	900.187	3371
j8o8r5lr1.2t13_i2	7154	318	33.156	56363	7347.76	900.984	3933
j8o8r5lr1.2t13_i3	4484	120	8.578	65031	4556.04	901.047	3991
j8o10r5lr0.8t26_i1	17003	793	236.328	42222	17432	900.672	2469
j8o10r5lr0.8t23_i2	11073	686	164.891	45519	11258	901.438	2281
j8o10r5lr0.8t23_i3	18021	753	127.172	45769	18213	901.156	2723
j8o10r5lr1t19_i1	12758	388	77.625	51867	13067.4	900	3081
j8o10r5lr1t17_i2	9056	422	59.188	62094	9354.86	900.407	2915

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j8o10r5lr1t19_i3	8645	295	41.656	60511	8870.84	901.406	2769
j8o10r5lr1.2t15_i1	7899	441	76.829	54134	9229.78	900.25	2717
j8o10r5lr1.2t16_i2	14616	482	140.532	56460	15121	901.813	1809
j8o10r5lr1.2t16_i3	10269	721	151.984	53186	10666.8	901.016	2563
j10o3r3lr0.8t15_i1	7366	205	2.281	595	7366	2.312	207
j10o3r3lr0.8t15_i2	3529	317	3.812	1064	3529	3.843	319
j10o3r3lr0.8t14_i3	4676	303	2.765	644	4676	2.781	305
j10o3r3lr1t12_i1	3639	186	1.422	388	3639	1.422	187
j10o3r3lr1t12_i2	5218	198	2.438	933	5218	2.438	199
j10o3r3lr1t11_i3	4658	166	1.296	556	4658	1.296	167
j10o3r3lr1.2t10_i1	4683	197	1.468	569	4683	1.484	199
j10o3r3lr1.2t9_i2	4373	218	1.297	621	4373	1.297	219
j10o3r3lr1.2t10_i3	4278	112	1.031	49085	4323.5	900.078	13969
j10o5r3lr0.8t26_i1	9833	744	29.907	2439	9833	29.907	745
j10o5r3lr0.8t28_i2	11760	1770	180.437	15366	11760	180.469	1771
j10o5r3lr0.8t25_i3	9189	499	15.875	1575	9189	16.015	501
j10o5r3lr1t20_i1	4672	481	10.469	1284	4672	10.5	483
j10o5r3lr1t19_i2	6104	649	23.688	3980	6104	23.797	651
j10o5r3lr1t19_i3	7072	399	12.906	2061	7072	12.922	401
j10o5r3lr1.2t16_i1	3884	747	19.172	3156	3884	19.219	749
j10o5r3lr1.2t18_i2	6193	200	5.188	1006	6193	5.188	201
j10o5r3lr1.2t18_i3	7350	272	6.906	1379	7350	6.906	273
j10o8r5lr0.8t24_i1	13218.5	877	120.391	7107	13218.5	120.454	879
j10o8r5lr0.8t24_i2	10424	1043	207.625	42640	10589.1	901.188	3111
j10o8r5lr0.8t22_i3	11262	1556	203.125	13643	11262	203.156	1557
j10o8r5lr1t17_i1	11909	785	129.454	46238	12294	900.5	3465
j10o8r5lr1t18_i2	8552	227	16.516	4519	8713.3	26.703	319
j10o8r5lr1t19_i3	9162	983	119.765	55750	9346	900.75	3303
j10o8r5lr1.2t16_i1	13469	365	37.484	51499	13790	900.375	3303
j10o8r5lr1.2t16_i2	12961.5	491	51.297	50673	13293	900.031	5163
j10o8r5lr1.2t16_i3	9903	628	97.672	54366	10160.5	900.359	3699
j10o10r5lr0.8t29_i1	18435	884	146.531	5435	18435	146.547	885
j10o10r5lr0.8t29_i2	11507	1767	781.234	27917	12091	900.016	2209
j10o10r5lr0.8t30_i3	16690.5	1447	662.407	29892	17464	900.328	1917
j10o10r5lr1t24_i1	10682	1127	547.515	37143	11023.9	900.422	1739
j10o10r5lr1t23_i2	15578	773	218.984	43074	16092.2	900.703	2779
j10o10r5lr1t25_i3	15369	754	205.031	42278	15980.6	900.672	2263
j10o10r5lr1.2t19_i1	17632	636	171.25	52075	18177.4	900.031	2233
j10o10r5lr1.2t21_i2	12593	480	166.984	52139	12867	900	1645
j10o10r5lr1.2t19_i3	15726	1229	299.328	22335	15726	299.969	1231

APPENDIX I

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	14	0.046	41	794	0.046	15
j3o3r3lr0.8t5_i2	1135	3	0.031	24	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.016	24	1807	0.031	3
j3o3r3lr1t4_i1	1120	1	0.016	23	1120	0.016	3
j3o3r3lr1t4_i2	1169	3	0.031	26	1169	0.031	5
j3o3r3lr1t3_i3	729	8	0.031	40	729	0.031	9
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.016	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.031	3
j3o3r3lr1.2t3_i3	1202	1	0.016	24	1202	0.031	3
j3o5r3lr0.8t8_i1	2781	23	0.328	257	2781	0.343	25
j3o5r3lr0.8t6_i2	3366	0	0	38	3366	0.047	3
j3o5r3lr0.8t7_i3	2157	7	0.109	109	2157	0.109	9
j3o5r3lr1t6_i1	2134	3	0.062	73	2134	0.078	5
j3o5r3lr1t6_i2	3661	7	0.078	79	3661	0.093	9
j3o5r3lr1t6_i3	2339	84	0.453	388	2339	0.453	85
j3o5r3lr1.2t5_i1	2186	26	0.172	223	2296.4	0.25	41
j3o5r3lr1.2t6_i2	164	0	0	19	164	0.047	3
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.031	3
j3o8r5lr0.8t8_i1	2195	0	0	17	2195	0.063	3
j3o8r5lr0.8t7_i2	1889	1	0.046	19	1889	0.062	3
j3o8r5lr0.8t8_i3	3023	101	2.297	1083	3023	2.313	103
j3o8r5lr1t6_i1	266	1	0.031	7	266	0.031	3
j3o8r5lr1t5_i2	1156	3	0.031	18	1156	0.031	5
j3o8r5lr1t6_i3	1550	0	0	30	1550	0.063	3
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.016	3
j3o8r5lr1.2t4_i2	1102	1	0.031	3	1102	0.031	3
j3o8r5lr1.2t5_i3	1806	0	0.047	16	1806	0.047	1
j3o10r5lr0.8t9_i1	1620	3	0.109	39	1620	0.125	5
j3o10r5lr0.8t9_i2	3938	133	4.625	1560	3938	4.672	135
j3o10r5lr0.8t9_i3	2563	24	0.375	142	2563	0.375	25
j3o10r5lr1t8_i1	6589	83	2.5	943	6589	2.531	85
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	8	0.234	104	3171	0.234	9
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.046	10	3004	0.046	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	29	0.109	103	1729	0.125	31
j5o3r3lr0.8t8_i2	2259	30	0.141	98	2259	0.141	31
j5o3r3lr0.8t8_i3	2627	71	0.235	155	2627	0.235	73
j5o3r3lr1t6_i1	3621	15	0.109	122	3621	0.109	17
j5o3r3lr1t6_i2	3031	55	0.172	197	3031	0.172	57
j5o3r3lr1t5_i3	2975	94	0.328	348	2975	0.328	95
j5o3r3lr1.2t4_i1	1312	22	0.078	97	1312	0.078	23
j5o3r3lr1.2t5_i2	1321	25	0.094	99	1321	0.094	27
j5o3r3lr1.2t5_i3	2670	49	0.172	223	2670	0.188	51
j5o5r3lr0.8t10_i1	2577	66	0.656	397	2577	0.656	67
j5o5r3lr0.8t14_i2	3404	87	1.016	450	3404	1.032	89
j5o5r3lr0.8t12_i3	6891	107	0.797	326	6891	0.797	109
j5o5r3lr1t10_i1	3862	62	0.687	334	3862	0.687	63
j5o5r3lr1t10_i2	3100	78	1.312	808	3100	1.312	79
j5o5r3lr1t11_i3	5767	0	0	60	5767	0.109	3

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j5o5r3lr1.2t8_i1	1171	58	0.5	288	1171	0.5	59
j5o5r3lr1.2t9_i2	3092	59	1	489	3092	1.032	61
j5o5r3lr1.2t8_i3	1700	14	0.141	71	1700	0.141	15
j5o8r5lr0.8t11_i1	2725	19	0.547	181	2725	0.578	21
j5o8r5lr0.8t13_i2	8600	149	8.266	1987	8600	8.282	151
j5o8r5lr0.8t12_i3	11059	149	5.735	1634	11059	5.797	151
j5o8r5lr1t9_i1	1559	45	0.843	427	1559	0.875	47
j5o8r5lr1t9_i2	3650	77	2.156	867	3650	2.187	79
j5o8r5lr1t9_i3	2751	101	2.782	1011	2751	2.829	103
j5o8r5lr1.2t9_i1	3752	164	4.75	1441	3752	4.75	165
j5o8r5lr1.2t8_i2	2059	39	0.828	375	2059	0.844	41
j5o8r5lr1.2t8_i3	4229	0	0	48435	7121.13	900.33	9105
j5o10r5lr0.8t15_i1	6759	114	9.064	1793	6759	9.064	115
j5o10r5lr0.8t14_i2	2967	118	6.298	1857	2967	6.313	119
j5o10r5lr0.8t16_i3	9715	201	19.612	3412	9715	19.752	203
j5o10r5lr1t12_i1	5693	259	25.55	53865	6023.01	900.418	4283
j5o10r5lr1t12_i2	6269	63	4.188	998	6269	4.219	65
j5o10r5lr1t12_i3	5993	205	21.408	65702	6325.47	900.356	3111
j5o10r5lr1.2t10_i1	3593	40	1.656	530	3593	1.656	41
j5o10r5lr1.2t10_i2	5628	122	7.954	1869	5628	7.954	123
j5o10r5lr1.2t10_i3	4325	90	3.922	1189	4325	3.938	91
j8o3r3lr0.8t13_i1	2263	0	0	56	2263	0.078	3
j8o3r3lr0.8t13_i2	4181	0	0	53	4181	0.063	3
j8o3r3lr0.8t11_i3	4057	0	0	56	4057	0.062	3
j8o3r3lr1t9_i1	2889	65	0.281	167	2889	0.296	67
j8o3r3lr1t9_i2	2012	159	0.75	376	2012	0.75	161
j8o3r3lr1t10_i3	3218	50	0.265	149	3218	0.265	51
j8o3r3lr1.2t8_i1	2963	44	0.281	248	2963	0.281	45
j8o3r3lr1.2t8_i2	5417	75	0.437	329	5417	0.453	77
j8o3r3lr1.2t8_i3	3081	80	0.516	334	3081	0.516	81
j8o5r3lr0.8t20_i1	5940	155	2.485	613	5940	2.5	157
j8o5r3lr0.8t20_i2	5681	0	0	74	5681	0.172	5
j8o5r3lr0.8t20_i3	6630	0	0	88	6630	0.172	3
j8o5r3lr1t17_i1	6739	388	14.204	2904	6739	14.204	389
j8o5r3lr1t14_i2	5652	322	5.188	1494	5652	5.188	323
j8o5r3lr1t16_i3	4009	128	1.313	429	4009	1.313	129
j8o5r3lr1.2t14_i1	5369	238	5.141	1754	5369	5.141	239
j8o5r3lr1.2t14_i2	5719	259	4.438	1234	5719	4.454	261
j8o5r3lr1.2t12_i3	3875	275	5.687	1965	3875	5.719	277
j8o8r5lr0.8t21_i1	7046	719	155.714	13467	7046	155.948	721
j8o8r5lr0.8t18_i2	6748	277	13.844	2036	6748	13.923	279
j8o8r5lr0.8t19_i3	14641	428	66.63	7435	14641	66.645	429
j8o8r5lr1t15_i1	15751	354	49.457	6850	15751	49.472	355
j8o8r5lr1t16_i2	9146	448	78.505	51689	11032.9	900.199	2797
j8o8r5lr1t16_i3	10732	362	42.565	5494	10732	42.565	363
j8o8r5lr1.2t13_i1	9986	338	34.783	5918	9986	34.799	339
j8o8r5lr1.2t13_i2	7154	318	33.111	5847	7154	33.111	319
j8o8r5lr1.2t13_i3	4484	120	8.547	2261	4484	8.547	121
j8o10r5lr0.8t26_i1	17003	793	236.342	14141	17003	236.467	795
j8o10r5lr0.8t23_i2	11073	686	164.697	12599	11073	164.728	687
j8o10r5lr0.8t23_i3	18021	753	127.117	10120	18021	127.492	755
j8o10r5lr1t19_i1	12758	388	77.441	8003	12758	77.457	389
j8o10r5lr1t17_i2	9056	422	59.096	7415	9056	59.112	423

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Number of Columns	Best Bound	Runtime (seconds)	Total Nodes
j8o10r5lr1t19_i3	8645	295	41.58	5412	8645	41.721	297
j8o10r5lr1.2t15_i1	7899	441	76.723	49190	9229.78	900.103	3047
j8o10r5lr1.2t16_i2	14616	482	140.021	12795	14616	140.052	483
j8o10r5lr1.2t16_i3	10107	380	71.472	54651	10666.9	900.081	2577
j10o3r3lr0.8t15_i1	7366	205	2.265	595	7366	2.281	207
j10o3r3lr0.8t15_i2	3529	317	3.813	1064	3529	3.844	319
j10o3r3lr0.8t14_i3	4532	0	0	70	4532	0.078	3
j10o3r3lr1t12_i1	3639	186	1.406	388	3639	1.406	187
j10o3r3lr1t12_i2	5218	198	2.438	933	5218	2.438	199
j10o3r3lr1t11_i3	4658	166	1.296	556	4658	1.296	167
j10o3r3lr1.2t10_i1	4683	197	1.469	569	4683	1.485	199
j10o3r3lr1.2t9_i2	4263	119	0.719	380	4263	0.735	121
j10o3r3lr1.2t10_i3	4278	112	1.046	603	4278	1.046	113
j10o5r3lr0.8t26_i1	9405	0	0	120	9405	0.312	3
j10o5r3lr0.8t28_i2	11643	847	85.503	7588	11643	85.566	849
j10o5r3lr0.8t25_i3	9189	499	15.782	1575	9189	15.907	501
j10o5r3lr1t20_i1	4672	481	10.406	1284	4672	10.438	483
j10o5r3lr1t19_i2	5903	0	0	118	5903	0.204	3
j10o5r3lr1t19_i3	7072	399	12.876	2061	7072	12.907	401
j10o5r3lr1.2t16_i1	3815	457	12.266	1954	3815	12.297	459
j10o5r3lr1.2t18_i2	6193	200	5.188	1006	6193	5.188	201
j10o5r3lr1.2t18_i3	7350	272	6.891	1379	7350	6.891	273
j10o8r5lr0.8t24_i1	13218.5	877	120.348	7107	13218.5	120.426	879
j10o8r5lr0.8t24_i2	9930	0	0	44805	10589.1	900.496	2555
j10o8r5lr0.8t22_i3	11262	1556	202.974	13643	11262	203.005	1557
j10o8r5lr1t17_i1	11909	785	129.41	13296	11909	129.613	787
j10o8r5lr1t18_i2	8552	227	16.626	2679	8552	16.782	229
j10o8r5lr1t19_i3	8942	629	69.455	6937	8942	69.549	631
j10o8r5lr1.2t16_i1	13469	365	37.611	4767	13469	37.72	367
j10o8r5lr1.2t16_i2	12961.5	491	50.736	6559	12961.5	50.97	493
j10o8r5lr1.2t16_i3	9903	628	97.659	9811	9903	97.675	629
j10o10r5lr0.8t29_i1	18435	884	146.409	5435	18435	146.425	885
j10o10r5lr0.8t29_i2	11507	1767	780.421	27938	12091	900.126	2211
j10o10r5lr0.8t30_i3	16690.5	1447	661.887	22628	16690.5	662.184	1449
j10o10r5lr1t24_i1	10682	1127	546.806	23433	10682	547.322	1129
j10o10r5lr1t23_i2	15578	773	218.502	15248	15578	218.737	775
j10o10r5lr1t25_i3	15369	754	204.924	13562	15369	204.955	755
j10o10r5lr1.2t19_i1	17632	636	171.049	14011	17632	171.08	637
j10o10r5lr1.2t21_i2	12593	480	166.846	12687	12593	166.877	481
j10o10r5lr1.2t19_i3	15336	679	155.736	12888	15336	156.018	681

APPENDIX J

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Columns Added	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	7	0.031	28	794	0.031	9
j3o3r3lr0.8t5_i2	1135	3	0.031	30	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.031	26	1807	0.031	3
j3o3r3lr1t4_i1	1120	2	0.016	20	1120	0.016	3
j3o3r3lr1t4_i2	1169	3	0.031	25	1169	0.031	5
j3o3r3lr1t3_i3	721	17	0.046	81	721	0.046	19
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.016	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.015	3
j3o3r3lr1.2t3_i3	1202	3	0.016	30	1202	0.016	5
j3o5r3lr0.8t8_i1	2723	46	0.531	437	2737.6	0.547	49
j3o5r3lr0.8t6_i2	3416	10	0.093	107	3416	0.093	11
j3o5r3lr0.8t7_i3	2157	10	0.125	130	2157	0.14	13
j3o5r3lr1t6_i1	2134	3	0.062	77	2134	0.078	5
j3o5r3lr1t6_i2	3661	8	0.078	80	3661	0.078	9
j3o5r3lr1t6_i3	2367	12	0.094	208	2377	0.172	27
j3o5r3lr1.2t5_i1	2282	18	0.125	123	2282	0.125	19
j3o5r3lr1.2t6_i2	164	0	0	20	164	0.047	5
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.031	3
j3o8r5lr0.8t8_i1	2235	3	0.047	22	2235	0.063	7
j3o8r5lr0.8t7_i2	1889	1	0.046	19	1889	0.062	3
j3o8r5lr0.8t8_i3	2998	22	0.531	642	3086.86	1.234	67
j3o8r5lr1t6_i1	266	1	0.031	7	266	0.031	3
j3o8r5lr1t5_i2	1175	5	0.046	21	1175	0.046	7
j3o8r5lr1t6_i3	1610	9	0.125	86	1610	0.14	11
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.016	3
j3o8r5lr1.2t4_i2	1102	1	0.016	3	1102	0.016	3
j3o8r5lr1.2t5_i3	1806	0	0.031	16	1806	0.031	1
j3o10r5lr0.8t9_i1	1620	3	0.156	57	1620	0.171	5
j3o10r5lr0.8t9_i2	3923	110	4.531	2321	3996.5	6.343	161
j3o10r5lr0.8t9_i3	2563	18	0.36	163	2563	0.36	19
j3o10r5lr1t8_i1	6645	38	0.844	476	6659.33	1.235	59
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	6	0.218	105	3171	0.218	7
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.046	10	3004	0.046	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	13	0.078	107	1729	0.078	15
j5o3r3lr0.8t8_i2	2259	11	0.078	66	2259	0.093	13
j5o3r3lr0.8t8_i3	2627	13	0.063	68	2627	0.078	15
j5o3r3lr1t6_i1	3621	9	0.063	82	3621	0.078	11
j5o3r3lr1t6_i2	3031	27	0.11	179	3031	0.125	29
j5o3r3lr1t5_i3	3041	11	0.063	114	3041	0.063	13
j5o3r3lr1.2t4_i1	1312	8	0.046	58	1312	0.046	9
j5o3r3lr1.2t5_i2	1321	28	0.125	130	1321	0.125	29
j5o3r3lr1.2t5_i3	2652	32	0.171	316	2670	0.218	43
j5o5r3lr0.8t10_i1	2657	21	0.266	253	2657	0.281	23
j5o5r3lr0.8t14_i2	3404	23	0.421	224	3404	0.421	25
j5o5r3lr0.8t12_i3	6891	26	0.359	237	6891	0.359	27
j5o5r3lr1t10_i1	3862	22	0.375	246	3862	0.375	23
j5o5r3lr1t10_i2	3182	72	1.265	1197	3194	1.625	89
j5o5r3lr1t11_i3	5992	27	0.515	352	5992	0.531	29

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j5o5r3lr1.2t8_i1	1182	31	0.375	339	1182	0.375	33
j5o5r3lr1.2t9_i2	3070	38	0.641	606	3116.63	0.922	51
j5o5r3lr1.2t8_i3	1700	9	0.109	85	1700	0.125	11
j5o8r5lr0.8t11_i1	2725	13	0.344	191	2725	0.422	19
j5o8r5lr0.8t13_i2	8527	69	4.969	16900	8666.37	83.188	809
j5o8r5lr0.8t12_i3	11024	328	20.953	8617	11158.6	30.312	437
j5o8r5lr1t9_i1	1502	72	1.531	826	1502	1.531	73
j5o8r5lr1t9_i2	3667	39	1.281	577	3667	1.313	41
j5o8r5lr1t9_i3	2751	36	1.187	564	2788.88	1.359	43
j5o8r5lr1.2t9_i1	3733	43	1.719	670	3733	1.719	45
j5o8r5lr1.2t8_i2	2059	26	0.64	665	2119.44	1.468	77
j5o8r5lr1.2t8_i3	6796.5	120	3.985	2255	6928.87	4.828	141
j5o10r5lr0.8t15_i1	6769	213	22.782	61553	6915.16	901.1	3557
j5o10r5lr0.8t14_i2	2784	89	6.86	2417	2899.8	8.11	103
j5o10r5lr0.8t16_i3	9682	182	30.078	46728	9935.32	482.269	1777
j5o10r5lr1t12_i1	5534	2935	765.895	67678	5954.96	901.037	3247
j5o10r5lr1t12_i2	6175	34	3.063	1733	6237.16	6.485	67
j5o10r5lr1t12_i3	5879	2717	599.598	66885	6286.69	900.74	3447
j5o10r5lr1.2t10_i1	3589	67	2.828	925	3589	2.891	71
j5o10r5lr1.2t10_i2	5471	249	21.032	6453	5692.76	26.078	291
j5o10r5lr1.2t10_i3	4383	202	10.625	3795	4441.17	12.641	243
j8o3r3lr0.8t13_i1	2295	37	0.437	311	2295	0.468	39
j8o3r3lr0.8t13_i2	4341	65	0.766	498	4341	0.797	67
j8o3r3lr0.8t11_i3	4193	19	0.172	154	4193	0.188	21
j8o3r3lr1t9_i1	2889	27	0.203	168	2889	0.219	31
j8o3r3lr1t9_i2	2069	43	0.297	282	2069	0.297	45
j8o3r3lr1t10_i3	3218	25	0.218	174	3218	0.234	27
j8o3r3lr1.2t8_i1	2963	15	0.14	160	2963	0.14	17
j8o3r3lr1.2t8_i2	5436	17	0.234	253	5436	0.25	19
j8o3r3lr1.2t8_i3	3202	56	0.469	447	3202	0.469	57
j8o5r3lr0.8t20_i1	5940	60	1.765	715	5940	1.765	61
j8o5r3lr0.8t20_i2	5941	74	3	924	5941	3	75
j8o5r3lr0.8t20_i3	6942	1503	60.969	33282	6954	270.595	4161
j8o5r3lr1t17_i1	6787	2192	223.064	61562	6852.04	515.956	3613
j8o5r3lr1t14_i2	5724	90	2	945	5724	2	91
j8o5r3lr1t16_i3	4009	30	0.594	289	4009	0.594	31
j8o5r3lr1.2t14_i1	5385	418	17.313	37494	5405.5	226.595	2293
j8o5r3lr1.2t14_i2	5807	39	1.36	742	5807	1.406	41
j8o5r3lr1.2t12_i3	4029	1566	83.173	22585	4029	83.204	1567
j8o8r5lr0.8t21_i1	6976	189	47.719	63373	7252	900.771	2261
j8o8r5lr0.8t18_i2	6766	3705	598.217	63682	6802	900.316	4725
j8o8r5lr0.8t19_i3	14456	1145	278.127	68958	14855.9	900.412	2821
j8o8r5lr1t15_i1	15748	1969	627.973	69665	16200.2	900.537	2465
j8o8r5lr1t16_i2	10686	1589	512.816	69088	10965.7	901.6	2345
j8o8r5lr1t16_i3	10780	86	12.609	65103	10996	900.252	2879
j8o8r5lr1.2t13_i1	9914	541	75.876	74524	10187.3	900.287	2859
j8o8r5lr1.2t13_i2	7185	2666	745.692	69986	7325.49	901.381	2997
j8o8r5lr1.2t13_i3	4397	352	44.516	11631	4428.51	60.735	455
j8o10r5lr0.8t26_i1	16940	446	257.424	50640	17432	902.459	1557
j8o10r5lr0.8t23_i2	10866	104	32.016	54375	11258	901.091	1611
j8o10r5lr0.8t23_i3	17926	90	21.578	62888	18213	902.005	2031
j8o10r5lr1t19_i1	12560	131	31.282	66367	12863.2	900.162	2347
j8o10r5lr1t17_i2	8914	1099	221.767	60610	9314.21	900.069	3055

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j8o10r5lr1t19_i3	8709	610	148.142	65446	8801.83	900.124	2283
j8o10r5lr1.2t15_i1	8661	307	89.032	64537	9183.11	901.162	2287
j8o10r5lr1.2t16_i2	14370	1409	643.458	59671	15103.6	900.475	1733
j8o10r5lr1.2t16_i3	10243	1199	404.55	62480	10590.5	900.069	2155
j10o3r3lr0.8t15_i1	7366	33	0.547	290	7366	0.563	35
j10o3r3lr0.8t15_i2	3529	35	0.578	376	3529	0.594	37
j10o3r3lr0.8t14_i3	4676	37	0.438	282	4676	0.453	39
j10o3r3lr1t12_i1	3639	30	0.438	323	3639	0.438	31
j10o3r3lr1t12_i2	5218	55	0.985	649	5218	1.016	57
j10o3r3lr1t11_i3	4658	99	1.11	821	4658	1.11	101
j10o3r3lr1.2t10_i1	4683	40	0.515	355	4683	0.515	41
j10o3r3lr1.2t9_i2	4373	24	0.203	166	4373	0.203	25
j10o3r3lr1.2t10_i3	4306	54	0.703	2631	4323.5	3.531	253
j10o5r3lr0.8t26_i1	9833	76	4.922	928	9833	4.922	77
j10o5r3lr0.8t28_i2	11817	958	135.594	20085	11817	135.641	959
j10o5r3lr0.8t25_i3	9189	58	2.953	691	9189	2.953	59
j10o5r3lr1t20_i1	4672	144	6.781	1740	4672	6.781	145
j10o5r3lr1t19_i2	6156	54	2.75	972	6156	2.75	55
j10o5r3lr1t19_i3	7072	61	3.484	1110	7072	3.64	67
j10o5r3lr1.2t16_i1	3896	93	3.547	1376	3896	3.578	95
j10o5r3lr1.2t18_i2	6200	447	21.766	7530	6221	32.75	617
j10o5r3lr1.2t18_i3	7350	39	2.438	951	7350	2.516	41
j10o8r5lr0.8t24_i1	13264	2324	875.532	64685	13293	901.11	2361
j10o8r5lr0.8t24_i2	10475	590	138.579	58375	10589.1	900.005	2243
j10o8r5lr0.8t22_i3	11251	2599	846.943	69720	11289	900.334	2709
j10o8r5lr1t17_i1	11999	352	94.11	68135	12294	900.225	2129
j10o8r5lr1t18_i2	8518	58	6.375	72622	8716.19	900.093	2759
j10o8r5lr1t19_i3	9089	85	12.906	73850	9346	900.865	2469
j10o8r5lr1.2t16_i1	13436	1224	309.862	70151	13790	900.084	2645
j10o8r5lr1.2t16_i2	12954	359	62.954	67727	13293	900.178	2997
j10o8r5lr1.2t16_i3	9986	170	29.422	68425	10160.4	900.132	2591
j10o10r5lr0.8t29_i1	18323	128	29.36	50941	18491	900.6	2151
j10o10r5lr0.8t29_i2	11760	143	48.219	47956	12091	901.537	1545
j10o10r5lr0.8t30_i3	17129	391	247.126	42764	17464	900.365	1113
j10o10r5lr1t24_i1	10490	621	536.831	50884	11023.9	901.146	1067
j10o10r5lr1t23_i2	15561	190	68.141	57537	16083.5	900.162	2069
j10o10r5lr1t25_i3	15390	218	88.829	56452	15980.6	900.74	1683
j10o10r5lr1.2t19_i1	17323	289	131.438	57506	18176.6	900.568	1981
j10o10r5lr1.2t21_i2	12694	177	64.25	60206	12818.8	901.43	1687
j10o10r5lr1.2t19_i3	15639	82	24.672	61918	15882.9	900.443	1699

APPENDIX K

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Columns Added	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	7	0.078	28	794	0.078	9
j3o3r3lr0.8t5_i2	1135	3	0.031	30	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.031	26	1807	0.031	3
j3o3r3lr1t4_i1	1120	2	0.031	20	1120	0.031	3
j3o3r3lr1t4_i2	1169	3	0.016	25	1169	0.016	5
j3o3r3lr1t3_i3	721	17	0.046	81	721	0.046	19
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.015	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.016	3
j3o3r3lr1.2t3_i3	1202	3	0.031	30	1202	0.031	5
j3o5r3lr0.8t8_i1	2703	16	0.219	379	2749.54	0.453	37
j3o5r3lr0.8t6_i2	3416	10	0.093	107	3416	0.093	11
j3o5r3lr0.8t7_i3	2157	10	0.125	121	2157	0.125	11
j3o5r3lr1t6_i1	2134	3	0.063	77	2134	0.063	5
j3o5r3lr1t6_i2	3661	8	0.078	80	3661	0.078	9
j3o5r3lr1t6_i3	2367	12	0.094	119	2367	0.11	17
j3o5r3lr1.2t5_i1	2282	18	0.125	123	2282	0.14	19
j3o5r3lr1.2t6_i2	164	0	0	20	164	0.062	5
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.016	3
j3o8r5lr0.8t8_i1	2235	3	0.062	20	2235	0.062	5
j3o8r5lr0.8t7_i2	1889	1	0.031	19	1889	0.047	3
j3o8r5lr0.8t8_i3	2998	22	0.531	447	3086.86	0.844	43
j3o8r5lr1t6_i1	266	1	0.031	7	266	0.031	3
j3o8r5lr1t5_i2	1175	5	0.046	21	1175	0.046	7
j3o8r5lr1t6_i3	1610	9	0.125	86	1610	0.141	11
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.016	3
j3o8r5lr1.2t4_i2	1102	1	0.016	3	1102	0.016	3
j3o8r5lr1.2t5_i3	1806	0	0.031	16	1806	0.031	1
j3o10r5lr0.8t9_i1	1620	3	0.156	57	1620	0.171	5
j3o10r5lr0.8t9_i2	3911	98	4.016	1874	4013.17	5	127
j3o10r5lr0.8t9_i3	2563	18	0.375	163	2563	0.375	19
j3o10r5lr1t8_i1	6634	20	0.531	231	6634	0.531	21
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	6	0.203	105	3171	0.203	7
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.047	10	3004	0.047	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	13	0.079	107	1729	0.079	15
j5o3r3lr0.8t8_i2	2259	11	0.078	66	2259	0.078	13
j5o3r3lr0.8t8_i3	2627	13	0.078	68	2627	0.078	15
j5o3r3lr1t6_i1	3621	9	0.078	82	3621	0.078	11
j5o3r3lr1t6_i2	3031	27	0.125	179	3031	0.14	29
j5o3r3lr1t5_i3	3041	11	0.063	114	3041	0.063	13
j5o3r3lr1.2t4_i1	1312	8	0.047	58	1312	0.047	9
j5o3r3lr1.2t5_i2	1311	17	0.093	94	1311	0.093	21
j5o3r3lr1.2t5_i3	2652	32	0.171	231	2652	0.171	33
j5o5r3lr0.8t10_i1	2657	21	0.281	253	2657	0.281	23
j5o5r3lr0.8t14_i2	3404	23	0.422	224	3404	0.422	25
j5o5r3lr0.8t12_i3	6891	26	0.36	237	6891	0.36	27
j5o5r3lr1t10_i1	3862	22	0.39	246	3862	0.39	23
j5o5r3lr1t10_i2	3176	23	0.485	453	3217.71	0.563	27
j5o5r3lr1t11_i3	5992	27	0.516	352	5992	0.547	29

j5o5r3lr1.2t8_i1	1171	15	0.203	184	1171	0.219	17
j5o5r3lr1.2t9_i2	3048	17	0.375	500	3136	0.781	43
j5o5r3lr1.2t8_i3	1700	9	0.11	85	1700	0.11	11
j5o8r5lr0.8t11_i1	2725	13	0.359	166	2725	0.375	15
j5o8r5lr0.8t13_i2	8512	46	3.406	7873	8666.37	31.031	353
j5o8r5lr0.8t12_i3	10999	46	2.672	3320	11166.4	9.563	147
j5o8r5lr1t9_i1	1502	72	1.531	826	1502	1.531	73
j5o8r5lr1t9_i2	3667	39	1.266	577	3667	1.297	41
j5o8r5lr1t9_i3	2751	36	1.171	564	2788.88	1.359	43
j5o8r5lr1.2t9_i1	3733	43	1.688	670	3733	1.703	45
j5o8r5lr1.2t8_i2	2059	26	0.64	545	2120.61	1.171	53
j5o8r5lr1.2t8_i3	6796.5	116	3.922	2052	6928.87	4.422	129
j5o10r5lr0.8t15_i1	6713	65	6.844	61873	6916.71	900.312	2983
j5o10r5lr0.8t14_i2	2784	98	7.782	2648	2897.65	8.719	109
j5o10r5lr0.8t16_i3	9682	54	6.703	8381	9944.86	41.922	203
j5o10r5lr1t12_i1	5526	48	6.656	68219	5956.73	901.047	2675
j5o10r5lr1t12_i2	6175	34	3.047	1116	6237.16	4.219	43
j5o10r5lr1t12_i3	5937	556	61.063	66812	6283.92	900.781	3065
j5o10r5lr1.2t10_i1	3559	23	1.359	614	3615.09	1.859	39
j5o10r5lr1.2t10_i2	5461	39	3.766	3222	5707.14	11.797	119
j5o10r5lr1.2t10_i3	4370	111	6.125	2875	4463.52	9.219	177
j8o3r3lr0.8t13_i1	2295	35	0.422	311	2295	0.453	37
j8o3r3lr0.8t13_i2	4313	26	0.313	184	4313	0.313	27
j8o3r3lr0.8t11_i3	4193	19	0.187	154	4193	0.203	21
j8o3r3lr1t9_i1	2889	27	0.188	168	2889	0.203	29
j8o3r3lr1t9_i2	2069	43	0.297	282	2069	0.297	45
j8o3r3lr1t10_i3	3218	25	0.218	174	3218	0.234	27
j8o3r3lr1.2t8_i1	2963	15	0.14	160	2963	0.156	17
j8o3r3lr1.2t8_i2	5436	17	0.234	253	5436	0.25	19
j8o3r3lr1.2t8_i3	3202	56	0.469	447	3202	0.469	57
j8o5r3lr0.8t20_i1	5940	60	1.766	715	5940	1.766	61
j8o5r3lr0.8t20_i2	5941	74	3	924	5941	3	75
j8o5r3lr0.8t20_i3	6924	37	1.125	423	6924	1.172	39
j8o5r3lr1t17_i1	6739	133	6.375	83342	6915.17	900.094	4699
j8o5r3lr1t14_i2	5724	90	2	945	5724	2.016	91
j8o5r3lr1t16_i3	4009	30	0.578	289	4009	0.578	31
j8o5r3lr1.2t14_i1	5371	193	7.265	3178	5371	7.296	195
j8o5r3lr1.2t14_i2	5807	39	1.344	742	5807	1.406	41
j8o5r3lr1.2t12_i3	4004	128	3.063	1770	4004	3.063	129
j8o8r5lr0.8t21_i1	6942	177	45.094	60409	7252	900.515	2077
j8o8r5lr0.8t18_i2	6784	697	59.5	9563	6784	59.64	699
j8o8r5lr0.8t19_i3	14415	55	12.156	69131	14865.7	900.14	2861
j8o8r5lr1t15_i1	15741	131	30.672	77517	16218.5	902.672	1645
j8o8r5lr1t16_i2	10644	771	198.703	68139	10965.7	900.386	2285
j8o8r5lr1t16_i3	10780	86	12.453	68735	10996	900.906	2297
j8o8r5lr1.2t13_i1	9776	941	151.141	72692	10187.3	900.125	2763
j8o8r5lr1.2t13_i2	7055	197	31.39	69320	7342.7	900.719	2709
j8o8r5lr1.2t13_i3	4382	245	36.062	8255	4441.61	39.687	283
j8o10r5lr0.8t26_i1	16650	97	38.484	52009	17432	901.083	1199
j8o10r5lr0.8t23_i2	10866	104	32	55694	11258	900.266	1259
j8o10r5lr0.8t23_i3	18116	1147	531.765	43534	18116	531.984	1149
j8o10r5lr1t19_i1	12560	131	31.375	65360	12863.2	901.594	1621
j8o10r5lr1t17_i2	8884	94	19.297	64462	9314.21	900.706	2865
j8o10r5lr1t19_i3	8704	360	89.953	13987	8809.95	101.297	395
j8o10r5lr1.2t15_i1	8599	411	92.079	64714	9200.57	902.188	2083
j8o10r5lr1.2t16_i2	14060	101	44.782	59747	15110.5	901.485	1909
j8o10r5lr1.2t16_i3	10233	287	84.672	61304	10591.5	901.281	1905
j10o3r3lr0.8t15_i1	7366	33	0.532	290	7366	0.547	35

j10o3r3lr0.8t15_i2	3529	35	0.563	376	3529	0.578	37
j10o3r3lr0.8t14_i3	4676	37	0.437	282	4676	0.453	39
j10o3r3lr1t12_i1	3639	30	0.422	323	3639	0.422	31
j10o3r3lr1t12_i2	5195	32	0.594	400	5195	0.61	33
j10o3r3lr1t11_i3	4658	139	1.813	1350	4658	1.828	141
j10o3r3lr1.2t10_i1	4683	40	0.531	355	4683	0.531	41
j10o3r3lr1.2t9_i2	4373	24	0.188	166	4373	0.188	25
j10o3r3lr1.2t10_i3	4306	54	0.703	555	4306	0.703	55
j10o5r3lr0.8t26_i1	9833	76	4.906	928	9833	4.906	77
j10o5r3lr0.8t28_i2	11817	319	37.579	6646	11817	37.844	321
j10o5r3lr0.8t25_i3	9189	58	2.937	691	9189	2.937	59
j10o5r3lr1t20_i1	4672	144	6.781	1740	4672	6.781	145
j10o5r3lr1t19_i2	6156	54	2.766	972	6156	2.766	55
j10o5r3lr1t19_i3	7072	61	3.485	1098	7072	3.563	63
j10o5r3lr1.2t16_i1	3896	93	3.562	1376	3896	3.578	95
j10o5r3lr1.2t18_i2	6193	37	1.735	569	6193	1.766	39
j10o5r3lr1.2t18_i3	7350	39	2.437	951	7350	2.5	41
j10o8r5lr0.8t24_i1	13191	224	43.579	5239	13191	43.594	225
j10o8r5lr0.8t24_i2	10399	109	22.204	58867	10589.1	900.69	2105
j10o8r5lr0.8t22_i3	11163	101	11.406	71622	11289	900.281	2365
j10o8r5lr1t17_i1	12070	1404	574.188	68225	12294	901	1835
j10o8r5lr1t18_i2	8518	58	6.375	76483	8716.19	901.047	2241
j10o8r5lr1t19_i3	9181	2847	878.093	63098	9346	901.656	2903
j10o8r5lr1.2t16_i1	13412	214	42.516	69163	13790	900.672	2577
j10o8r5lr1.2t16_i2	13027	457	95.063	72173	13293	900.094	2271
j10o8r5lr1.2t16_i3	9986	170	29.391	69556	10160.4	900.454	2249
j10o10r5lr0.8t29_i1	18323	128	29.282	2307	18323	29.282	129
j10o10r5lr0.8t29_i2	11760	143	48	48338	12091	901.847	1489
j10o10r5lr0.8t30_i3	17129	391	246.844	47893	17464	901	1031
j10o10r5lr1t24_i1	10422	241	188.75	52179	11023.9	901.282	933
j10o10r5lr1t23_i2	15561	190	68.312	62622	16083.5	900.234	1585
j10o10r5lr1t25_i3	15390	218	88.813	56203	15980.6	900.505	1289
j10o10r5lr1.2t19_i1	17337	336	140.75	57634	18177.1	900.547	1349
j10o10r5lr1.2t21_i2	12694	177	64.172	8026	12694	69.578	187
j10o10r5lr1.2t19_i3	15639	82	24.625	65165	15882.9	900.063	1649

APPENDIX L

Instance	Best Integer Solution	Best Integer @ node	Time to Best Integer Solution (seconds)	Columns Added	Best Bound	Runtime (seconds)	Total Nodes
j3o3r3lr0.8t4_i1	794	7	0.031	28	794	0.031	9
j3o3r3lr0.8t5_i2	1135	3	0.031	30	1135	0.031	5
j3o3r3lr0.8t5_i3	1807	1	0.031	26	1807	0.031	3
j3o3r3lr1t4_i1	1120	2	0.031	20	1120	0.031	3
j3o3r3lr1t4_i2	1169	3	0.016	25	1169	0.016	5
j3o3r3lr1t3_i3	721	17	0.031	81	721	0.047	19
j3o3r3lr1.2t2_i1	92	0	0	1	92	0.031	3
j3o3r3lr1.2t3_i2	274	0	0	3	274	0.016	3
j3o3r3lr1.2t3_i3	1202	3	0.031	30	1202	0.031	5
j3o5r3lr0.8t8_i1	2703	16	0.234	191	2703	0.234	17
j3o5r3lr0.8t6_i2	3366	0	0	49	3366	0.047	3
j3o5r3lr0.8t7_i3	2157	10	0.125	121	2157	0.125	11
j3o5r3lr1t6_i1	2134	3	0.062	77	2134	0.078	5
j3o5r3lr1t6_i2	3661	8	0.078	80	3661	0.078	9
j3o5r3lr1t6_i3	2367	12	0.093	107	2367	0.093	13
j3o5r3lr1.2t5_i1	2282	18	0.14	123	2282	0.14	19
j3o5r3lr1.2t6_i2	164	0	0	19	164	0.046	3
j3o5r3lr1.2t5_i3	988	0	0	1	988	0.015	3
j3o8r5lr0.8t8_i1	2195	0	0	17	2195	0.047	3
j3o8r5lr0.8t7_i2	1889	1	0.031	19	1889	0.047	3
j3o8r5lr0.8t8_i3	2998	22	0.531	287	2998	0.531	23
j3o8r5lr1t6_i1	266	1	0.046	7	266	0.046	3
j3o8r5lr1t5_i2	1156	3	0.031	18	1156	0.031	5
j3o8r5lr1t6_i3	1550	0	0	29	1550	0.047	3
j3o8r5lr1.2t4_i1	649	0	0	1	649	0.031	3
j3o8r5lr1.2t4_i2	1102	1	0.016	3	1102	0.016	3
j3o8r5lr1.2t5_i3	1806	0	0.031	16	1806	0.031	1
j3o10r5lr0.8t9_i1	1620	3	0.156	57	1620	0.171	5
j3o10r5lr0.8t9_i2	3911	98	4.016	1536	3911	4.031	99
j3o10r5lr0.8t9_i3	2541	14	0.328	144	2541	0.328	15
j3o10r5lr1t8_i1	6634	20	0.516	231	6634	0.516	21
j3o10r5lr1t7_i2	861	0	0	1	861	0.031	3
j3o10r5lr1t7_i3	3171	6	0.218	105	3171	0.218	7
j3o10r5lr1.2t6_i1	1E-07	0	0	0	1E-07	0.031	3
j3o10r5lr1.2t5_i2	3004	1	0.032	10	3004	0.032	3
j3o10r5lr1.2t6_i3	137	0	0	1	137	0.031	3
j5o3r3lr0.8t7_i1	1729	13	0.078	107	1729	0.078	15
j5o3r3lr0.8t8_i2	2259	11	0.078	66	2259	0.078	13
j5o3r3lr0.8t8_i3	2627	13	0.078	68	2627	0.078	15
j5o3r3lr1t6_i1	3621	9	0.078	82	3621	0.078	11
j5o3r3lr1t6_i2	2961	18	0.078	123	2961	0.078	19
j5o3r3lr1t5_i3	3041	11	0.063	114	3041	0.063	13
j5o3r3lr1.2t4_i1	1312	8	0.031	58	1312	0.031	9
j5o3r3lr1.2t5_i2	1311	17	0.078	93	1311	0.078	19
j5o3r3lr1.2t5_i3	2670	33	0.187	268	2670	0.187	35
j5o5r3lr0.8t10_i1	2657	21	0.281	253	2657	0.281	23
j5o5r3lr0.8t14_i2	3404	23	0.453	224	3404	0.453	25
j5o5r3lr0.8t12_i3	6891	26	0.36	237	6891	0.36	27
j5o5r3lr1t10_i1	3862	22	0.375	246	3862	0.375	23
j5o5r3lr1t10_i2	3176	23	0.485	412	3176	0.516	25
j5o5r3lr1t11_i3	5767	0	0	75	5767	0.094	3

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j5o5r3lr1.2t8_i1	1171	15	0.203	184	1171	0.219	17
j5o5r3lr1.2t9_i2	3048	17	0.375	245	3048	0.375	19
j5o5r3lr1.2t8_i3	1700	9	0.109	85	1700	0.125	11
j5o8r5lr0.8t11_i1	2725	13	0.344	166	2725	0.375	15
j5o8r5lr0.8t13_i2	8512	46	3.406	1120	8512	3.406	47
j5o8r5lr0.8t12_i3	10839	23	1.453	612	10839	1.531	25
j5o8r5lr1t9_i1	1502	78	1.796	976	1502	1.796	79
j5o8r5lr1t9_i2	3530	34	1.187	515	3530	1.187	35
j5o8r5lr1t9_i3	2751	36	1.156	481	2751	1.156	37
j5o8r5lr1.2t9_i1	3733	43	1.688	670	3733	1.703	45
j5o8r5lr1.2t8_i2	2059	26	0.625	303	2059	0.625	27
j5o8r5lr1.2t8_i3	6796.5	96	3.453	1614	6796.5	3.453	97
j5o10r5lr0.8t15_i1	6628	40	4.515	1048	6628	4.531	41
j5o10r5lr0.8t14_i2	2719	35	2.937	1267	2925.25	3.953	47
j5o10r5lr0.8t16_i3	9682	54	6.672	1631	9682	6.672	55
j5o10r5lr1t12_i1	5526	48	6.609	70360	5961.02	900.484	2027
j5o10r5lr1t12_i2	6175	34	3.032	780	6175	3.032	35
j5o10r5lr1t12_i3	5985	433	58.938	11937	6287.74	64.156	465
j5o10r5lr1.2t10_i1	3501	16	1.047	368	3501	1.047	17
j5o10r5lr1.2t10_i2	5461	39	3.719	1048	5461	3.75	41
j5o10r5lr1.2t10_i3	4346	72	4	1337	4346	4	73
j8o3r3lr0.8t13_i1	2263	0	0	56	2263	0.063	3
j8o3r3lr0.8t13_i2	4181	0	0	64	4181	0.078	3
j8o3r3lr0.8t11_i3	4057	0	0	56	4057	0.063	3
j8o3r3lr1t9_i1	2889	27	0.188	168	2889	0.203	29
j8o3r3lr1t9_i2	2012	24	0.171	147	2012	0.171	25
j8o3r3lr1t10_i3	3218	25	0.219	174	3218	0.219	27
j8o3r3lr1.2t8_i1	2963	15	0.125	160	2963	0.14	17
j8o3r3lr1.2t8_i2	5436	17	0.235	253	5436	0.25	19
j8o3r3lr1.2t8_i3	3081	29	0.25	236	3081	0.265	31
j8o5r3lr0.8t20_i1	5859	34	0.985	381	5859	0.985	35
j8o5r3lr0.8t20_i2	5681	0	0	78	5681	0.172	5
j8o5r3lr0.8t20_i3	6630	0	0	88	6630	0.172	3
j8o5r3lr1t17_i1	6820	142	7.109	2423	6820	7.109	143
j8o5r3lr1t14_i2	5724	90	1.968	945	5724	1.968	91
j8o5r3lr1t16_i3	4009	30	0.61	289	4009	0.61	31
j8o5r3lr1.2t14_i1	5199	65	2.25	1022	5199	2.281	67
j8o5r3lr1.2t14_i2	5807	39	1.344	742	5807	1.391	41
j8o5r3lr1.2t12_i3	3951	41	1	688	3951	1	43
j8o8r5lr0.8t21_i1	6930	87	24.297	3360	6930	24.437	89
j8o8r5lr0.8t18_i2	6565	80	5.359	1176	6565	5.359	81
j8o8r5lr0.8t19_i3	14415	55	12.016	2267	14415	12.25	57
j8o8r5lr1t15_i1	15617	83	18.609	3936	15617	18.687	85
j8o8r5lr1t16_i2	10303	89	18.578	71755	10965.7	901.234	1479
j8o8r5lr1t16_i3	10780	86	12.282	2430	10780	12.297	87
j8o8r5lr1.2t13_i1	9643	1480	385.281	44536	10211.6	408.828	1541
j8o8r5lr1.2t13_i2	6984	60	10.031	3349	6984	14.203	83
j8o8r5lr1.2t13_i3	4329	32	3.64	1173	4329	3.64	33
j8o10r5lr0.8t26_i1	16650	97	38.25	4117	16650	38.516	99
j8o10r5lr0.8t23_i2	10866	104	31.844	3891	10866	31.86	105
j8o10r5lr0.8t23_i3	17926	90	21.344	2854	17926	21.36	91
j8o10r5lr1t19_i1	12339	74	16.032	2624	12339	16.047	75
j8o10r5lr1t17_i2	8884	94	18.953	3434	8884	18.953	95

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j8o10r5lr1t19_i3	8530	63	15.672	2501	8530	15.735	65
j8o10r5lr1.2t15_i1	8466	79	24.219	67664	9203.25	900.373	1665
j8o10r5lr1.2t16_i2	14060	101	44.422	66222	15117	900.688	1047
j8o10r5lr1.2t16_i3	10269	419	130.406	15683	10269	130.765	421
j10o3r3lr0.8t15_i1	7366	33	0.516	290	7366	0.516	35
j10o3r3lr0.8t15_i2	3529	35	0.563	376	3529	0.578	37
j10o3r3lr0.8t14_i3	4532	0	0	70	4532	0.078	3
j10o3r3lr1t12_i1	3639	30	0.422	323	3639	0.422	31
j10o3r3lr1t12_i2	5195	32	0.609	400	5195	0.609	33
j10o3r3lr1t11_i3	4574	26	0.328	285	4574	0.328	27
j10o3r3lr1.2t10_i1	4683	40	0.531	355	4683	0.531	41
j10o3r3lr1.2t9_i2	4373	24	0.187	166	4373	0.187	25
j10o3r3lr1.2t10_i3	4240	31	0.438	358	4240	0.453	33
j10o5r3lr0.8t26_i1	9405	0	0	121	9405	0.344	3
j10o5r3lr0.8t28_i2	11709	208	21.969	3851	11709	21.969	209
j10o5r3lr0.8t25_i3	9189	58	2.937	691	9189	2.937	59
j10o5r3lr1t20_i1	4511	61	3.016	820	4511	3.063	63
j10o5r3lr1t19_i2	5903	0	0	138	5903	0.235	3
j10o5r3lr1t19_i3	7072	61	3.469	1098	7072	3.547	63
j10o5r3lr1.2t16_i1	3809	56	2.172	923	3809	2.172	57
j10o5r3lr1.2t18_i2	6193	37	1.719	569	6193	1.765	39
j10o5r3lr1.2t18_i3	7350	39	2.438	951	7350	2.5	41
j10o8r5lr0.8t24_i1	13197	1251	566.344	47446	13197	567.391	1253
j10o8r5lr0.8t24_i2	10458	203	41.047	4370	10458	41.25	205
j10o8r5lr0.8t22_i3	11163	101	11.359	1619	11163	11.641	103
j10o8r5lr1t17_i1	11929	193	48.735	8196	11929	48.969	195
j10o8r5lr1t18_i2	8518	58	6.359	1523	8518	6.359	59
j10o8r5lr1t19_i3	9089	85	13.016	2243	9089	13.141	87
j10o8r5lr1.2t16_i1	13274	68	14.688	3096	13274	14.704	69
j10o8r5lr1.2t16_i2	12875	81	12.844	3024	12875	13	83
j10o8r5lr1.2t16_i3	9873	77	13.657	2681	9873	13.829	79
j10o10r5lr0.8t29_i1	18323	128	29.547	2307	18323	29.562	129
j10o10r5lr0.8t29_i2	11760	143	48	3875	11760	49.031	145
j10o10r5lr0.8t30_i3	16664	139	82.781	5932	16664	83.156	141
j10o10r5lr1t24_i1	10200	125	93.766	51936	11023.9	900.857	839
j10o10r5lr1t23_i2	15561	190	68.124	7955	15561	68.14	191
j10o10r5lr1t25_i3	14936	104	41.078	55796	15980.6	900.438	1157
j10o10r5lr1.2t19_i1	17161	99	38.297	63451	18177.1	900.86	1061
j10o10r5lr1.2t21_i2	12370.5	90	33.859	4285	12370.5	33.859	91
j10o10r5lr1.2t19_i3	15639	82	24.61	3424	15639	24.625	83

VITA

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PUBLICATIONS & PRESENTATIONS

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