View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by DigitalCommons@Florida International University

Florida International University
FIU Digital Commons

HWCOM Faculty Publications | Herbert Wertheim College of Medicine

7-2-2015

Fibonacci Series, Golden Proportions, and the Human Biology

Dharam Persaud
Herbert Wertheim College of Medicine, Florida International University, Dpers001@fiu.edu

James P. O'Leary
Herbert Wertheim College of Medicine, Florida International University, olearyp@fiu.edu

Follow this and additional works at: http://digitalcommons.fiu.edu/com_facpub

Part of the Medicine and Health Sciences Commons

Recommended Citation
Persaud, Dharam and O'Leary, James P., "Fibonacci Series, Golden Proportions, and the Human Biology" (2015). *HWCOM Faculty Publications.* Paper 27.
http://digitalcommons.fiu.edu/com_facpub/27

This work is brought to you for free and open access by the Herbert Wertheim College of Medicine at FIU Digital Commons. It has been accepted for inclusion in HWCOM Faculty Publications by an authorized administrator of FIU Digital Commons. For more information, please contact dcc@fiu.edu.

Austin Journal of Surgery

Open Access

Austin Publishing Group

Review Article

Fibonacci Series, Golden Proportions, and the Human Biology

Dharam Persaud-Sharma* and James P O'Leary
Florida International University, Herbert Wertheim College of Medicine

***Corresponding author:** Dharam Persaud-Sharma, Florida International University, Herbert Wertheim College of Medicine, Miami, FL, 33199, USA, Email: dpers001@fiu.edu

Received: April 01, 2015; **Accepted:** June 25, 2015; **Published:** July 02, 2015

Abstract

Pythagoras, Plato and Euclid's paved the way for Classical Geometry. The idea of shapes that can be mathematically defined by equations led to the creation of great structures of modern and ancient civilizations, and milestones in mathematics and science. However, classical geometry fails to explain the complexity of non-linear shapes replete in nature such as the curvature of a flower or the wings of a Butterfly. Such non-linearity can be explained by fractal geometry which creates shapes that emulate those found in nature with remarkable accuracy. Such phenomenon begs the question of architectural origin for biological existence within the universe. While the concept of a unifying equation of life has yet to be discovered, the Fibonacci sequence may establish an origin for such a development. The observation of the Fibonacci sequence is existent in almost all aspects of life ranging from the leaves of a fern tree, architecture, and even paintings, makes it highly unlikely to be a stochastic phenomenon. Despite its wide-spread occurrence and existence, the Fibonacci series and the Rule of Golden Proportions has not been widely documented in the human body. This paper serves to review the observed documentation of the Fibonacci sequence in the human body.

Keywords: Fibonacci; Surgery; Medicine; Anatomy; Golden proportions; Math; Biology

Introduction

Classical geometry includes the traditional shapes of triangles, squares, and rectangles that have been well established by the brilliance of Pythagoras, Plato, and Euclid. These are fundamental shapes that have forged the great structures of ancient and modern day civilization. However, classical geometry fails to translate into complex non-linear forms that are observed in nature. To explain such non-linear shapes, the idea of fractal geometry is proposed which states that such fractal shapes/images possess self-similarity and can be of non-integer or non-whole number dimensions. Whereas classical geometric shapes are defined by equations and mostly whole number (integer) values, the shapes of fractal geometry can be created by iterations of independent functions. Observing the patterns created by repeating 'fractal images' closely emulate those observed in nature, thus questioning whether this truly is the mechanism of origination of life on the planet. Two important properties of fractals include self-similarity and non-integer or non-whole number values. The idea of 'self-identity' is that its basic pattern or fractal is the same at all dimensions and that its repetition can theoretically continue to infinity. Examples of such repeating patterns at microscopic and macroscopic scales can be seen in all aspects of nature from the florescence of a sunflower, the snow-capped peaks of the Himalayan Mountains, to the bones of the human body.

The observation of self-similarity belonging to fractal geometry found in multiple aspects of nature, leads to the question of whether such an occurrence is merely stochastic or whether it has a functional purpose. While a singular unifying equation to define the creation and design of life has yet to be determined, an un-coincidental phenomenon known as the Fibonacci sequence and the rule of 'Golden Proportions' may serve as a starting point for uncovering the methods of a universal architect, should one exist.

Fibonacci and rule of golden proportions

Fibonacci sequences: The Fibonacci sequence was first recognized by the Indian Mathematician, Pingala (300-200 B.C.E.) in his published book called the Chandaśāstra, in which he studied grammar and the combination of long and short sounding vowels [1,2]. This was originally known as mātrāmeru, although it is now known as the Gopala-Hemachandra Number in the East, and the Fibonacci sequence in the West [1,2]. Leonardo Pisano developed the groundwork for what is now known as "Fibonacci sequences" to the Western world during his studies of the Hindu-Arab numerical system. He published the groundwork of Fibonacci sequences in his book called Liber Abaci (1202) in Italian, which translates into English as the "Book of Calculations". However, it should be noted that the actual term "Fibonacci Sequences" was a tributary to Leonardo Pisano, by French Mathematician, Edouard Lucas in 1877 [3]. The Fibonacci sequence itself is simple to follow. It proposes that for the integer sequence starting with 0 or 1, the sequential number is the sum of the two preceding numbers as in Figure 1.

A) 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…. ∞

Or

B) 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144… ∞

Figure 1: Illustration of Fibonacci Sequences. A) Fibonacci Series starting with 1 B) Fibonacci Series starting with 0.

Persaud-Sharma et al. © All rights reserved

Citation: Persaud-Sharma D and O'Leary JP. Fibonacci Series, Golden Proportions, and the Human Biology. Austin J Surg. 2015;2(5): 1066.

Table 1: Ratios of Fibonacci Numbers for the sequences starting with 1. Ratios approximate Fibonacci value of Phi (φ) 1.618.

F_{N+1}	F_N	$(F_{N+1})/F_N$
2	1	2
3	2	1.5000
5	3	1.6666
8	5	1.6000
13	8	1.6250
21	13	1.6154
34	21	1.6190
55	34	1.6176
89	55	1.6182
144	**89**	**1.6179** (Nearest approximation of φ)

As shown in Figure 1, Fibonacci sequences can either begin with 0 or 1. Although the sequence, originally proposed in Liber Abaci by Leonardo (follows the sequence in A) begins with 1, this sequence essentially reduces to the recurrence relationship of: $F_N = F_{n-1} + F_{n-2}$

Given that the starting numbers of the sequence can be either: A) $F_1 = 1$ $F_2 = 1$ or B) $F_1 = 0$ $F_2 = 1$.In this sequence it can be seen that each additional number after the given conditions of 1 or 0, is a direct result of the sum of the two preceding numbers. For example, in series (A), 1+ 1 = 2, where 2 is the third number in the sequence. To find the third number (F_4), simply add $F_3 = 2$ and $F_2 = 1$, which equals 3 (F_4).

Golden proportion, rectangle, and spiral: Another observation that can be made from the series of the Fibonacci numbers includes the rule of golden proportions. In essence, this is an observation that the ratio of any two sequential Fibonacci numbers approximates to the value of 1.618, which is most commonly represented by the Greek Letter Phi (φ). The larger the consecutive numbers in the sequence, the more accurate the approximation of 1.618. A summary of these results illustrating this concept can be seen in Table 1.

Furthering this observation, shapes can be created based upon length measurements of the Fibonacci numbers in sequence. For example, creating a rectangle with the values of any of the two successive Fibonacci numbers form what is known as the “Golden Rectangle”. These rectangles can be divided into squares that are equally sided and are of smaller values from the Fibonacci sequence Figure 2.

Rectangles created using consecutive numbers from the Fibonacci sequence can be divided into equally sided squares of such numbers as well. The length of a single side can be divided into smaller values of the Fibonacci sequence. Connecting the diagonals of each of the squares creates a spiral, known as the Golden Spiral.

This image compares the spiral of the human ear, the shell of a nautilus, and the golden spiral. Both the human ear and the shell of a nautilus approximate the dimensions of the golden spiral.

Connecting the corners of each of the squares by an arc reveals a spiral pattern. This spiral pattern is known as the ‘golden spiral’. This pattern is seen in several different forms in nature including, but not limited to the human ear and the shell of a nautilus as seen in Figure 3.

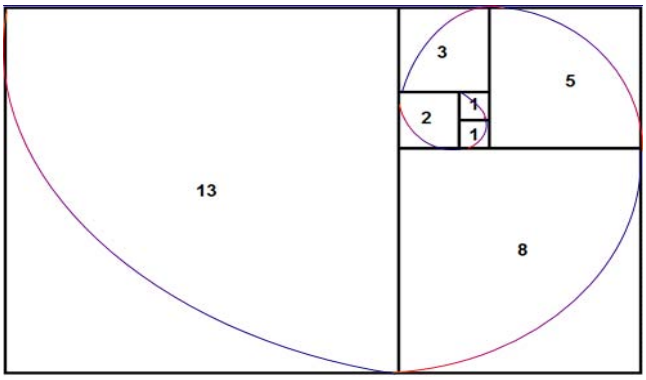

Figure 2: Golden Rectangle and Golden Spiral.

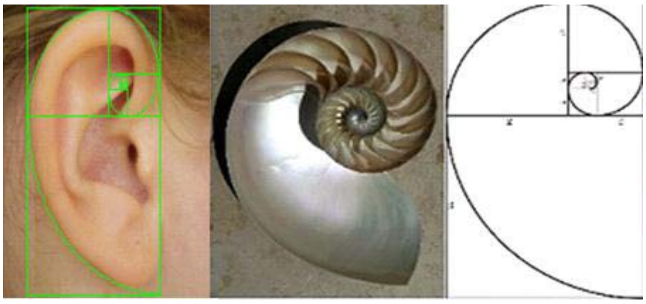

Figure 3: Golden Spiral in Nature.

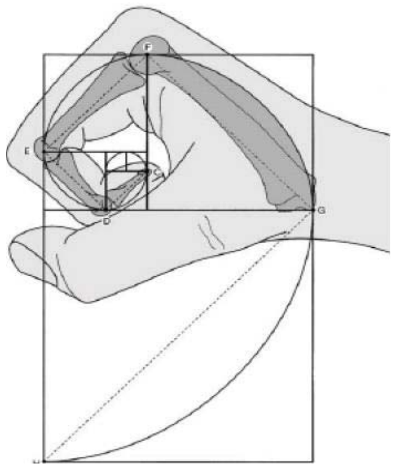

Figure 4: The human hand superimposed on the Golden Rectangle, with approximations to the Golden Spiral [4].

Human anatomy and Fibonacci

Proportion of human bones: In the human body, there are instances of the Fibonacci Phi (φ) although it has not been widely discussed. In 1973, Dr. William Littler proposed that by making a clenched fist, Fibonacci’s spiral can be approximated in Figure 4 [4].

In this paper, Litter proposed that such geometry is observed based upon ratio of the lengths of the phalanges, and that the flexor and extensor movement of the primary fingers approximate the golden spiral. Such a spiral would have to be created based upon the relationship between the metacarpophalangeal and interphalanges of the digits [4]. However, due a lack of statistical and well documented

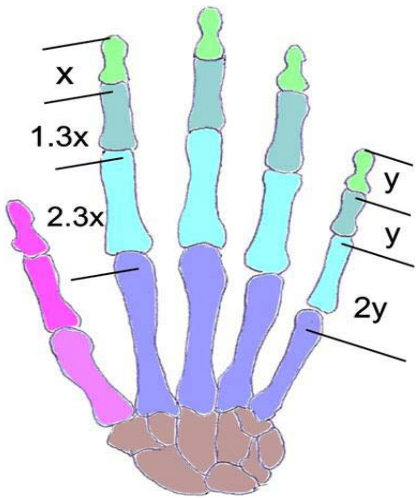

Figure 5: Phalangeal lengths as reported by Hamilton et al. [6]. For the index, second and third digits: ratios of the DIP-tip/PIP-DIP/MCP-PIP distances were 1:1.3:2.3, and 1:1:2 for the little finger. DIP: Distal interphalangeal; PIP: Proximal Interphalangeal; MCP: Metacarpophalangeal Joint.

empirical data, accurate representations of the golden spiral could not be readily ascertained. In 1998, Gupta et al., confirmed that the motion of the phalanges do approximate the pattern of the golden spiral, excluding for the fifth digit because of abduction of the digit during extension [5]. Through empirical evidence obtained through radiographic analysis of the hands of 197 individuals, Hamilton and Dunsmuirin 2002 concluded that the ratios between the phalange lengths of the digits do not approximate the golden ratio value of (φ) 1.618 [6]. However, Hutchison and Hutchison (2010) showed that the data collected by Hamilton et al. (2002) overlooked the relationship that Littler actually proposed. Hamilton and Dunsmuir (2010) confirmed that the phalangeal length ratio data obtained from their subjects compared to those that were almost arbitrarily listed by Littler in 1973, were in fact comparable and approximated the Fibonacci value of (φ) of 1.618. What Hamilton et al. failed to notice in their own data was that the values they obtained from their research approximated a Lucas series, which essentially underscores a Fibonacci sequence in that the sum of the first two lengths equaled the third length [7].

Hamilton and Dunsmuir's findings highlighted that the index, second, and third digits follow a series of 1x, 1.3x, 2.3x [7]. Hamilton et al. provided length measurement data for the fourth digit (little finger) that follows the initial values of a Fibonacci sequence of 0,1,1,2, represented by y, y, and 2y in Figure 5 [7]. Hence, the data collected by Hamilton et al., supported Littler's initial proposal of a clenched fist approximating the dimensions of the golden spiral. Additionally, Hamilton's data showed that the ratios for the length of the phalanges follows the additive rule of a Lucas series for the index, second and third digits, while approximating Fibonacci values for the fourth digit (little finger).

The aesthetics of a smile and beauty: The rule of golden proportions has been proposed in an attempt to define anatomical beauty. It is commonly accepted that facial beauty is correlates with anatomical symmetry [8]. One of the main features of the human face is the mouth and teeth. Thus, professionals within the field of dentistry have attempted to quantitatively characterize the parameters of an aesthetically appealing smile. The most logical starting point is that of the rule of golden proportions. In 1973, Lombardi was the first to officially propose the existence of having proportionate teeth, but dismissed the idea of using the rule of golden proportions to create aesthetic teeth [9]. He did suggest an underlying repeated ratio of the maxillary anterior teeth for aesthetics, but stated the inappropriate use of the rule of golden proportions for such purposes. In 1978, Levin was the first to observe that: 1) the width of the maxillary central incisor is in golden proportion to the width of the lateral incisor. 2) The width of the maxillary lateral incisor is in golden proportion to the width of the canine [10]. Based upon this observation, Levin later developed a tooth caliper to gauge whether an individual's teeth are in golden proportion to one another, and diagnostic grid to verify if the teeth are appropriately spaced. Further research into the relationship between the golden proportion, dental arrangement, and an aesthetic smile was developed by Rickets in 1982. He implemented the use of the golden proportions in the treatment of patients [11,12]. While Levin's observation of the golden ratio existing in dentition is undeniable, its application to developing an aesthetically appealing smile has recently been questioned and often dismissed [9,13-15].

In 2007, Dio et al. attempted to empirically define the basis for determining/judging beauty [16]. Dio et al., utilized fMRI imaging of the brain to formulate an association between brain stimulation from the interpretation of visual models with different physical proportions-using a model constructed according to golden proportions as an independent variable. The results of the study indicate that defining beauty is a joint process of cortical neurons, where Dio et al. identified that objective beauty stimulates the insula, whereas subjective beauty based upon the judgment of the individual perceiving the experience shows functional excitation of the Amygdala. Dio et al. echo the sentiments referenced by Gombrich [17] and Ramachandran [18], in that the criteria for making overall assessments of beauty seem to be related to our biological heritage, although it cannot be explained by a conscious explanation [16]. In our opinion, while the existence of the golden ratio continues to be revealed in several different instances of the human anatomy, attempting to use its proportions as a measure of beauty is not something that should be standardized. It is influenced by many different factors including cultural influences. This is something that may have been overlooked by Dio et al. study, in that in the demographic information about the 14 participants in the study was not included. The idea of 'beauty' itself is a subjective term and not one that can be universally held constant. The diversity of biology in its essence is what constitutes its beauty, while the observation of the golden proportion in nature highlights an architectural design, proportions, and symmetry.

Phyllotaxis, coronary arteries, and the human heart beat: Although the blood vessels of the human body and a plant have different physiological components, they are very similar in their basic function and visual appearance Figure 6. The vascular network in its basic structure is nearly identical to shapes that can be created by fractal geometry, as seen in Figure 6C. Mathematically, the properties of these structures are created by the self-similarity of the fractal. This means that the whole image is the result of multiple smaller copies of itself that share the same statistical properties at different scales of measurement.

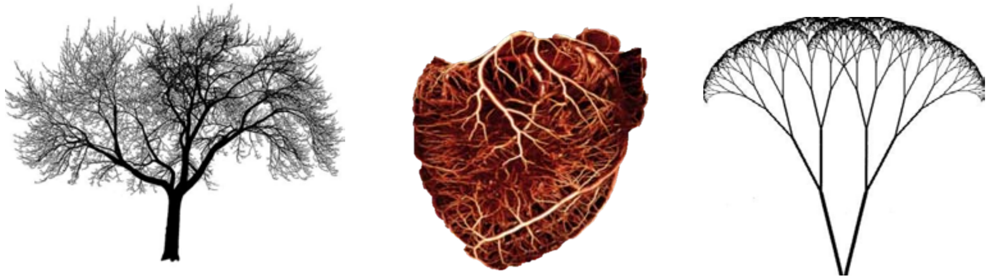

Figure 6: Comparison between the silhouettes of a tree (A), the blood vessels of a heart (B), and a fractal image (C).

Table 2: Table summarizing the number of branching from blood vessels of the Heart showing that branching numbers follow a Fibonacci Series [19].

Number of branches													
2	LCA	RCA											
3	LAD	LCx	RCA										
5	LAD	LCx	RI	AM	PDA								
8	LAD	Ldiag1	LCx	OM1	Sep1	PDA	Rdiag1	AM					
13	LAD	PLV1	PRV1	Sep3	Sep4	OM2	LDiag2	Sep2	OM3	PLV2	PLV3	PRV2	PRV3

Sequence branding of the coronary arterial tree. LCA: Left Coronary Artery; RCA: Right Coronary Artery; LAD: Left Anterior Descending; LCX: Left Circumflex; OM: Obtuse Marginal; AM: Acute Marginal; PDA: Posterior Descenting Artery; PRV: Posterior Right Ventricular (1) = lateral; (2) = intermediate; (3) = medial, Sep: Septal; RI: Ramus Intermedius (authors interpretation).

Of importance here is that in 1977, Mitchison revealed that the phyllotaxis of plants follows the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21… as seen in Figure 1. Likewise in 2011, Ashrafian and Atasiounoted that the number of branches from vessels in the heart follows the Fibonacci sequences Table 2 [19]. Ashrafian and Atanasiou, supported their observation by correlating studies which showed that atherosclerotic lesions in coronary arteries follow a Fibonacci distribution [20].

In 2013, Yetkin et al. showed that the golden ratio of Phi (φ) 1.618, exists within the cardiac cycle of the human heart beat. It is known that the time periods for the systolic and diastolic phases vary with the method of measurement. The three most commonly used techniques for determining systolic and diastolic phases of the cardiac cycleinclude: electrographical, echocardiogaphical, and phonocardiographical techniques. Yetkin et al. defined systole as the time between the 'R wave' and the end of the 'T wave' using an electrocardiogram [21]. The data from their research included an assessment of 162 healthy subjects, and concluded that the ratio of the Diastolic time interval/Systolic time interval was 1.611, while the R-R interval/Diastolic time interval was 1.618 which is the value of Phi (φ) [21].

Fibonacci, Fractals, and the Human Genome: The supposition that the structure of DNA and its organization pattern is a fractal was first proposed by Benoit Mandelbot in 1982 [22]. Mandelbot, for the most part, is responsible for first coining the term "fractal" and providing mathematical explanation non-euclidean geometries observed in nature. In the early 1990's, based upon the supposition made by Mandelbot, it was proven that the Human genome contains fractal behavior - Fibonacci series, and the golden proportion. For the past 25 years, this work has mainly been led by French independent-researcher Jean-Claude Perez. In 1991, Perez proposed that the DNA gene-coding region sequences were strongly related to the Golden Ratio and Fibonacci/Lucas integer numbers [23]. His work was bolstered by Yamagishi et al who found consistency of a Fibonacci series level of organization across the whole human genome [24]. In 2010, Perez provided evidence that the genetic code in Table 3 not only maps codons to amino acids, but also serves as a global error-detection scheme [25].

In the simplest explanation of his research, Perez analyzed the entirety of the human genome yet only evaluated a single strand considering that 1 strand is a mere complement of the other. As outlined by Perez in 2010, the cumulative number of each of the 64 genetic codons for the three different codon reading frames was obtained [25]. Then analysis on 24 human chromosomes, using the same method, was performed and the results were totaled for the 3 codon reading frames and the 24 chromosomes. The 64 codon population was arranged according the Genetic Code as seen in Table 3. The table was then separated according to the 6 binary splits of a Dragon Curve. A dragon curve is an irregularly shaped curve with a self-similar pattern, hence a fractal image as shown in Figure 7.

Conceptually, a Dragon Curve can be created by folding of a piece of paper into a long strip. Then as the strip of paper is unfolded, the adjacent segments are rotated and formed at right angles [26,27]. Computer modeling of this concept is shown in Figure 7. Perez's research in 2010makes several conclusions regarding the fractal nature of DNA. First, the universal genetic code serves as a macro-level structural matrix [25]. This matrix serves as a regulatory mechanism, controlling and balancing the codon population within

Table 3: The 64 codon populations of the whole human genome for the 3 codon reading frames of single stranded DNA (2843411612 codons). In this table, the 3 values in each cell are: the codon label, the codon's total population, the "Codon Frequency Ratio" (CFR). CFR is computed as: codon population x 64 / 2.843.411.612. (Where 2.843.411.612 is the whole genome cumulated codons). Then, if CFR < 1, the codon is rare, If CFR>1, the codon is frequent.

		SECOND NUCLEOTIDE					
FIRST NUCLEOTIDE		T	C	A	G		THIRD NUCLEOTIDE
	T	TTT 109591342 2.4667	TCT 62964964 1.4172	TAT 58718182 1.3216	TGT 57468177 1.2935	T	
		TTC 56120623 1.2632	TCC 43850042 0.9870	TAC 32272009 0.7264	TGC 40949883 0.9217	C	
		TTA 59263408 1.3339	TCA 55697529 1.2536	TAA 59167883 1.3318	TGA 55709222 1.2539	A	
		TTG 54004116 1.2133	TCG 6263386 0.1410	TAG 36718434 0.8265	TGG 52453369 1.1806	G	
	C	CTT 36828780 1.2791	CTT 50494519 1.1365	CAT 52236743 1.1758	CGT 7137644 0.1607	T	
		CTC 47838959 1.0768	CCC 37290873 0.8393	CAC 42634617 0.9596	CGC 6737724 0.1517	C	
		CTA 36671812 0.8254	CCA 52352507 1.1784	CAA 53776608 1.2104	CGA 6251611 0.1407	A	
		CTG 57598215 1.2964	CCG 7815619 1.1759	CAG 57544367 1.2952	CGG 7815677 0.1759	G	
	A	ATT 71001746 1.5981	ACT 45731927 1.0293	AAT 70880610 1.5954	AGT 45794017 1.0307	T	
		ATC 37952376 0.8542	ACC 33024323 0.7433	AAC 41380831 0.9314	AGC 39724813 0.8941	C	
		ATA 58649060 1.3201	ACA 57234565 0.7433	AAA 109143641 2.4566	AGA 62837294 1.4144	A	
		ATG 52222957 1.1754	ACG 7117535 0.1602	AAG 56701727 1.2763	AGG 50430220 1.1351	G	
	G	GTT 415577671 0.9354	GCT 39746348 0.8946	GAT 37990593 0.8551	GGT 33071650 0.7444	T	
		GTC 26866216 0.6047	GCC 33788267 0.7605	GAC 26820898 0.6037	GGC 33774099 0.7602	C	
		GTA 32292235 0.7268	GCA 40907730 0.9208	GAA 56018645 1.2609	GGA 43853584 0.9871	A	
		GTG 42755364 0.9623	GCG 6744112 0.1518	GAG 47821818 1.0764	GGG 37333942 0.8203	G	

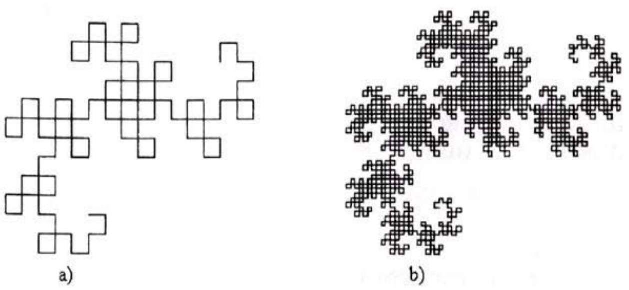

Figure 7: Dragon curve after 7 folds (a), and after 11 folds (b) [27].

the whole human genome. By modeling of the arrangement of the nitrogenous bases, Adenine (A), Guanine (G), Cytosine (C), and Thymine (T), using both the Universal Genetic Code Table Figure 3 and the Dragon-Fractal paradigm, symmetry in the data was revealed. Such symmetry showed two attractors towards values of "1" and that of Phi (φ) 1.618 [25]. According to Chaos theory of dynamic systems, attractors can be thought of 'magnetic' points to which the initial state of a complex system evolves towards in its final conditions. Attractors are important in mathematical modeling and systems because they serve as the link between Chaos theory and Fractal geometry [28]. Perez concludes that the ratios between 3-base pair codons sorted by A or G in the second base positions and those by T and C in the second base position tend to an attractor value of "1" [25]. The ratios between codons C or G and T or A in the second base position tend to a cluster value of "3(φ)/2" [25]. Furthermore, the distance that separates these two attractors of "1" and "3(φ)/2", is ½(φ). In a 2009 follow-up study by Perez, he analyzed the codon populations of 20 various species like Eukaryotes, bacteria and viruses. The results showed that 3 parameters (1, 2, and Phi (φ)) define codon populations within their genomes to a precision of 99% and often 99.999% [29]. For the human and chimpanzee genomes, codon frequencies are 99.99% correlated [29]. This exemplifies the widespread occurrence of Phi (φ) and Fibonacci series not only within the human genome but in other species as well.

Conclusion

Fractal geometry lays the foundation to understanding the complexity of the shapes in nature. In the exploration of the origins of life through mathematics, the occurrence of the Golden Ratio, Fibonacci Series and the underlying Lucas series are observed in several aspects of life on planet earth and within the cosmos. Although widely identified in non-biological fields such as architecture and art, it has not been well explored in the human biology. Recent work has begun to explore the understanding of such phenomenon documented at several different scales and systems in the human anatomy and physiology ranging from orthopedics, dentistry, the spiral of the human ear, the cardiovascular system and the human genome. The observance of such a seemingly universal concept begs the question of the origin of life; however, more research needs to be performed to explore its physiological role in biology. Understanding its functional role may be a keystone to making quantum advances in several fields such as artificial intelligence, biomedical engineering designs, and human regeneration, amongst others.

References

1. Goonatilake S. Toward a Global Science. Indiana University Press. 1998; 126.
2. Shah JA. History of Pingala's Combinatorics. GanitaBharati.
3. Dunlap RA. The Golden Ratio and Fibonacci Numbers. World Scientific Publishing, Singapore. Chapter 6.Lucas number and generalized Fibonacci numbers. 1997.
4. Littler JW. On the adaptability of man's hand (with reference to the equiangular curve). Hand. 1973; 5: 187-191.
5. Gupta A, Rash GS, Somia NN, Wachowiak MP, Jones J, Desoky A. The motion path of the digits. J Hand Surg Am. 1998; 23: 1038-1042.
6. Hamilton R, Dunsmuir RA. Radiographic assessment of the relative lengths of the bones of the fingers of the human hand. J Hand Surg Br. 2002; 27: 546-548.
7. Hutchison AL, Hutchison RL. Fibonacci, littler, and the hand: a brief review. Hand (NY). 2010; 5: 364-368.
8. Rhodes G, Proffitt F, Grady H, Sumich A. Facial symmetry and the perception of beauty. Psychonomic Bulletin &Review. 1998; 5: 659-669.
9. Lombardi RE. The principles of visual perception and their clinical application to denture esthetics. J Prosthet Dent. 1973; 29: 358-382.
10. Levin E. Dental esthetics and the golden proportion. J Prosthet Dent. 1978; 40: 244-252.
11. Ricketts RM. Divine proportion in facial esthetics. Clin Plast Surg. 1982; 9: 401-422.
12. Ricketts RM. The biologic significance of the divine proportion and Fibonacci series. Am J Orthod. 1982; 81: 351-370.
13. Mahshid M, Khoshvaghti A, Varshosaz M, Vallaei N. Evaluation of "golden proportion" in individuals with an esthetic smile. J Esthet Restor Dent. 2004; 16: 185-192.
14. Gillen RJ, Schwartz RS, Hilton TJ, Evans DB. An analysis of selected normative tooth proportions. Int J Prosthodont. 1994; 7: 410-417.
15. Ward DH. Proportional smile design using the recurring esthetic dental (red) proportion. Dent Clin North Am. 2001; 45: 143-154.
16. Di Dio C, Macaluso E, Rizzolatti G. The golden beauty: brain response to classical and renaissance sculptures. PLoS One. 2007; 2: e1201.
17. Gombrich EH. Tributes. Interpreters of our cultural tradition. Oxford: Phaidon Press. 1984.
18. Ramachandran VS. A brief tour of human consciousness. New York: Pearson Education. 2004.
19. Mitchison GJ. Phyllotaxis and the fibonacci series. Science. 1977; 196: 270-275.
20. Ashrafian H, Athanasiou T. Fibonacci series and coronary anatomy. Heart Lung Circ. 2011; 20: 483-484.
21. Yetkin G, Sivri N, Yalta K, Yetkin E. Golden Ratio is beating in our heart. Int J Cardiol. 2013; 168: 4926-4927.
22. Mandelbot B. The fractal geometry of nature. WH Freeman and Company. New York, USA. 1982.
23. Perez JC. Chaos, DNA, and Neuro-computers: A golden link: The hidden language of genes, global language and order in the human genome. Speculations in Science and Technology. 1991; 14: 336-346.
24. Yamagishi ME, Shimabukuro AI. Nucleotide frequencies in human genome and fibonacci numbers. Bull Math Biol. 2008; 70: 643-653.
25. Perez JC. Codon populations in single-stranded whole human genome DNA Are fractal and fine-tuned by the Golden Ratio 1.618. Interdiscip Sci. 2010; 2: 228-240.
26. Crilly AJ, Earnshaw RA, Jones H. Fractals and Chaos, Springer-Verlag. New York, USA. 1991.
27. Chang A, Zhang T. The fractal geometry of the boundary of the dragon curves. Journal of recreational mathematics. 2000; 30: 9-22.
28. Robert LVT, Advised by Dr. John David, "Attractors: Nonstrange to Chaotic".
29. Perez JC. Codex Biogenesis. Resurgence, Li`ege Belgium. 2009.

Austin J Surg - Volume 2 Issue 5 - 2015
ISSN : 2381-9030 | www.austinpublishinggroup.com
Persaud-Sharma et al. © All rights reserved

Citation: Persaud-Sharma D and O'Leary JP. Fibonacci Series, Golden Proportions, and the Human Biology. Austin J Surg. 2015;2(5): 1066.