In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institution shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

A REFINED INVESTIGATION OF THE VISCOSITY OF SUPERCRITICAL STEAM

A THESIS

Presented to

The Faculty of the Graduate Division

by

Charles E. Willbanks

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

Georgia Institute of Technology

December, 1964

A REFINED INVESTIGATION OF THE

VISCOSITY OF SUPERCRITICAL STEAM

APPROVED:

____
Chairman

____

____

Date approved by Chairman: 8 March 1965

ACKNOWLEDGMENTS

Space does not permit a full and proper recognition of all those individuals who made this work possible; consequently, formal acknowledgments are extended only to those individuals with whom the author worked closely. The moral support and technical advice of Dr. T. W. Jackson the author's advisor, were invaluable. Appreciation is extended to Mr. J. W. Hodgson with whom is shared the credit for the experimental data recorded. The author thanks Professor W. A. Hinton and Dr. H. C. Ward for their helpful comments and suggestions.

The author is indebted to the Callaway Fellowship Foundation and the National Aeronautics and Space Administration for funds making his graduate study possible. The support of the National Science Foundation which provided funds for equipment making this research possible is sincerely appreciated.

Finally, the author expresses his appreciation to his wife Annette and to his parents for their endurance and words of encouragement.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The viscosity of supercritical steam was determined experimentally with an annulus viscosimeter. Isobars of 3750, 5000, 7500 and 10,000 psia were considered with temperatures ranging up to 1000° Fahrenheit. The experimental results agree closely with Barnett's (1) results but are lower than the results of Ray (6).

A semi-empirical equation was derived for the purpose of correlating the experimental data obtained in the present investigation. The approach was essentially the same as used by Yen (8).

The experimentally determined viscosity is estimated to be accurate to within ±4 percent and the semi-empirical equation is estimated to represent the experimentally determined viscosity of the present investigation to within ±3 percent.

CHAPTER I

INTRODUCTION

General

Determination of transport properties for fluids at high temperatures and pressures is a very difficult task. This is evidenced by the scant amount of good data available to date on properties such as thermal conductivity, diffusion coefficients, and viscosity. Unlike other thermodynamic properties, the transport properties cannot be obtained simply from specific heat data and an equation of state for the substance.* Many attempts have been made at measuring, with relatively little success, the viscosity of dense gases at high temperatures and pressures. The success in predicting the viscosity of dense gases wholly from theoretical considerations has been even less. The present study was initiated to obtain accurate experimental values for the viscosity of high temperature and high pressure steam.

The ideal method of measurement of the viscosity of a fluid makes use of an absolute viscometer; however, practically all viscometers which are absolute in principle must actually be calibrated against some standard. Such is the case with the annulus viscosimeter used in the present study.

The fluid mechanics of the flow of a fluid through a perfectly concentric annulus yields the following simple equation relating the

---

*It should be noted that lecture notes of Dr. Lee de Forest tell of Dr. J.W. Gibbs' belief that viscosity is actually a thermodynamic property.

kinematic viscosity to the dimensions of the annulus, mass flow rate, and pressure drop through the annulus,

$$\frac{\nu W}{\Delta p} = \frac{\pi}{8L}\left[b^4 - a^4 - \frac{(b^2 - a^2)^2}{\ln \frac{b}{a}} \right] \tag{1}$$

where

ν = kinematic viscosity

Δp = pressure drop across the annulus

L = length of annulus

W = mass flow rate

b = inside radius of outer annulus tube

a = outside radius of inner annulus tube.

If however, the annulus is non-concentric, has skewed center lines or wavy inside surfaces, the relation among the flow variables is not as simple as the one indicated above. Furthermore, if any of these conditions are present, it may not be possible to separate the variables as was done in equation (1), and specifically, the parameter on the lefthand side of equation (1) may become a function of the Reynolds number as well as geometry. (See Appendix I). It is unfortunate that the dimensions of the annulus are so critical that tolerances of machining and accuracy of measuring techniques restrict the annulus from its role as an absolute viscosimeter; however, advantages of the annulus viscosimeter far outweigh the disadvantage of calibration.

CHAPTER II

APPARATUS

A schematic diagram of the experimental equipment used in the present investigation is shown in Figure 1 and a photograph in Figure 2. Although most of the equipment was the same as that used by Barnett (1)* and Whitesides (2), it was completely rebuilt and a number of refinements made.

Theory of flow through an annulus yields the following relation among the variables governing the flow

$$CC_T = \frac{\mu Q}{\Delta p} \tag{2}$$

or

$$\nu = \frac{CC_T \Delta p}{W} \tag{3}$$

where

μ = dynamic viscosity

Q = volume flow rate

CC_T = annulus constant (See Appendix I).

As mentioned before, it is impractical if not impossible to obtain the dimensional constant from the geometry of the annulus; consequently, CC_T

---

*See references.

is determined experimentally by passing a gas of known viscosity through the annulus and measuring the quantities that appear on the right hand side of equation (2) above. After obtaining CC_T by calibrating the annulus, measurement of the quantities on the right hand side of equation (3), as steam flows through the annulus, allows calculation of the viscosity of steam.

Viscosity Measuring Equipment

Pump

High pressure water was supplied to the system by an American Instrument Co. variable stroke positive displacement pump having a maximum flow capacity of 0.8 gallons per hour and a maximum working pressure of 30,000 psi.

Surge Chambers

Two surge chambers placed in series were used to smooth out the pulsations resulting from each pump stroke. Water from the pump passed through an unheated surge chamber and then into a heated surge chamber before entering the preheater. A secondary function of the heated surge chamber was to supply some degree of preheating to the water.

Preheater

The preheater was made from approximately fifteen feet of 1/4 inch type 304 stainless steel tubing coiled into a closely wound helix about two inches in diameter. Heating was accomplished by inserting the coil into a small metallurgical furnace. Temperature control was by a continuous controller. The controller strived to maintain a null emf output of an iron-constantan thermocouple having one junction in the annulus assembly and one in the preheater by varying the power input to the preheating furnace. This setup allowed the steam from the preheater to reach essentially annulus temperature before entering the annulus.

On two occasions preheating coils made from tubing having an inside diameter of 1/16 inch became plugged with scale and had to be discarded. It was suspected that high residual stresses remaining in the small tubing after cold-winding the coil on a lathe were responsible for the scaling of the inside wall. Using tubing with a 1/8 inch inside diameter corrected the problem.

Annulus and Furnace

A detailed discussion on the construction of the annulus may be found in the theses of Barnett (1) and Whitesides (2). A schematic drawing of the annulus is shown in Figure 3. The annulus assembly was welded into a high pressure autoclave, which eliminated the need for obtaining the system constant as a function of pressure. Although the furnace was capable of maintaining the annulus at 1200° Fahrenheit, the melting temperature of the silver solder used in its construction limited the temperature range to 1000° Fahrenheit. An off-on type controller was used to control the furnace temperature. The large thermal inertia of the autoclave-annulus assembly and furnace resulted in a very nearly constant temperature of the fluid flowing in the annulus.

Static Pressure Gages

The system pressure was continuously monitored with a Bourdon-tube-type test gage while actual pressure measurements were made with an American Instrument Co. dead-weight tester.

Thermometry

The annulus temperature was measured with an iron-constantan thermocouple calibrated by the supplier and certified to be accurate to within $1/2^{\circ}$ Fahrenheit. The emf output from the thermocouple was measured with a Leeds and Northrup Co. potentiometer reading to the nearest 0.001 millivolt. The continuous controller used to control the preheater was also a recorder and allowed continuous monitoring of the difference in preheater and annulus temperatures.

Throttling Capillary

For pressures above 3750 psia, throttling of the high pressure steam to atmospheric pressure with valves was found to be wholly unsatisfactory. Above 3750 psia a length of capillary tube having an inside diameter of 0.007 inches was used as a throttle.

Condenser

In order to collect the steam leaving the annulus it was neccessary to condense it to the liquid state. This was done by a simple single pass counter-flow heat exchanger using tap water as the coolant. The steam was condensed before being throttled.

Micromanometer

Details of the unique high pressure micromanometer used in the investigation may be found in reference (11). The valve system shown schematically in Figure 4 allowed a null reading of the manometer to be made at the system pressure without disrupting the flow in the annulus. This arrangement was necessary due to expansion of the volume of the micromanometer lines with pressure.

Filters

Before entering the pump, the water was passed through an automotive fuel filter. After leaving the heated surge chamber and before entering the preheating coil, the water was passed through a five micron filter to remove any minute foreign particles. The condensed steam leaving the annulus was filtered by a five micron filter to protect against the plugging of the small passage in the throttling capillary.

Miscellany

The time of each run was recorded with an electric timer reading to one tenth second. The condensate was collected in small bottles having caps and then weighed on analytical scales.

Auxiliary Equipment For Calibration

Flow Meter

The flow rate of the calibrating gas was measured with a wet test meter calibrated by a local gas company.

Static Pressure Gage

The static pressure in the annulus during calibration was measured with a 40 inch mercury U-tube manometer.

Micromanometer

A sixteen inch null type water micromanometer was used to measure the pressure drop across the annulus during calibration.

CHAPTER III

EXPERIMENTAL PROCEDURE

Calibration

General

Inability to calculate an accurate annulus constant from geometrical dimensions requires that the annulus be calibrated. It can be seen from the equation

$$CC_T = \frac{\mu Q}{\Delta p} \tag{2}$$

that CC_T can be experimentally determined by allowing a gas for which the viscosity is known very accurately to flow through the annulus while the volume flow rate and pressure drop are measured. Nitrogen was chosen as the calibrating gas in the present study. Viscosity data for dry nitrogen was obtained from reference (7).

The product CC_T was determined as a function of the temperature by calibrating at several temperatures from 70 to 1000° Fahrenheit. The temperature correction factor C_T was defined to be unity at 70° Fahrenheit; thus defining C to be numerically equal to CC_T at 70° Fahrenheit. After the product CC_T had been experimentally determined as a function of the temperature, a temperature correction curve was constructed by plotting C_T versus temperature (See Figure 5). The temperature correction factor was also found theoretically and agreed closely with the experimentally determined values.

The constant C was always determined before and after each isobar of data was taken. There was usually slight disagreement between the before and after values so an average of the two values was used to calculate the viscosity of steam. The average values used to calculate the viscosity of each isobar are listed in Table 2 of Appendix II.

Procedure

After removing all moisture from the system, dry nitrogen was passed through the annulus at a regulated static pressure between 10 and 36 inches of mercury gage. The static pressure was measured with a U-tube mercury manometer at the upstream pressure tap of the annulus. The desired flow rates were obtained by throttling the flow of nitrogen from the annulus. Actual annulus pressure was taken to be the average of upstream and downstream values by subtraction of one half of the pressure drop from the upstream static pressure. The pressure drop across the annulus test section was usually between 4 and 16 inches of water and measured with a water manometer. The volume flow rate leaving the annulus was measured with a wet test meter by noting the time for a given amount of gas to pass through the meter. For a given temperature the product CC_T was computed from the formula

$$CC_T = \frac{\mu Q_m \left(\frac{T_a}{T_m} \frac{p_b - p_m}{p_s - 1/2\Delta p} \right)}{\Delta p} \tag{4}$$

where

Q_m = volume flow rate through the wet test meter

p_b = barametric pressure

p_m = saturation pressure of water in wet test meter

p_s = system pressure at upstream pressure tap

T_a = absolute temperature in annulus

T_m = absolute temperature in wet test meter.

Viscosity Measurement

General

It can be seen from the equation

$$\nu = CC_T \frac{\Delta p}{W} \tag{3}$$

that the kinematic viscosity of steam can be calculated from the pressure drop across the annulus test section, the mass rate of flow through the annulus and the value of CC_T as determined by calibration.

Procedure

Preliminary warmup procedure of the equipment was as follows:

1. The dial on the furnace controller was set to the desired temperature.
2. Weights to give the desired static system pressure were placed in the pan of the dead-weight tester.
3. The proper length throttling capillary tube was installed.
4. The pump was started and allowed to increase the system

pressure to the desired value. A microswitch affixed to the dead-weight tester pan was actuated as a function of the system pressure by the up and down motion of the pan. If the system pressure rose to a value higher than desired, the pan moved up and actuated the microswitch which cut off the pump motor. When the system pressure fell below the value, the falling pan actuated the microswitch which cut on the pump motor. Thus the system pressure was allowed to oscillate around the desired pressure until the temperature of the system was stable.

5. After the temperature became stable, the pump stroke was adjusted until the dead-weight tester pan floated in one position. Pan elevation was indicated by a dial indicator.

Before and after each run the following readings were recorded:

1. emf of annulus thermocouple
2. emf of preheater thermocouple
3. system pressure
4. pressure drop across annulus test section

The mass flow rate W was computed from the amount of condensate collected during a measured time interval. The electric timer was started simultanuously with placement of the collection bottle under the discharge from the system and stopped simultanuously with the bottle's removal. The collected sample was weighed on an analytical balance.

The kinematic viscosity of steam was then computed from equation (3).

CHAPTER IV

THEORY

Recent investigations on the viscosity of fluids indicate that the coefficient of viscosity can be expressed as the sum of two contributions; one due to momentum exchange by molecular collisions and the second due to the action of intermolecular forces. Kinetic theory of gases predicts that the contribution due to molecular collisions in a dilute gas is a function only of the absolute temperature. By assuming that the intermolecular force potential between two molecules depends only on the absolute temperature, Reingamun (9) and Sutherland (9) were able to treat theoretically the contribution due to intermolecular attraction in a dilute gas. However, the equations derived by Reingamun and Sutherland are not satisfactory for dense gases. Enskog (5) found an approximate solution to the Boltzmann equations of change and propounded the best theory to date for the calculation of transport coefficients of dense gases. Although Enskog's theory is not directly applicable to dense steam since the water molecule is unsymmetrical and polar as well, it does give some insight into the type of equation one might attempt to use in correlating experimental data.

The viscosity for high density gases can be written

$$\mu(\rho, T) = \mu_K(T, \rho) + \mu_{\varphi R}(T, \rho) + \mu_{\varphi A}(T, \rho) \tag{5}$$

where:

$\mu_K(T, \rho)$ = contribution due to molecular collisions

$\mu_{\varphi R}(T, \rho)$ = contribution due to repulsive forces

$\mu_{\varphi A}(T, \rho)$ = contribution due to attractive forces.

T = absolute temperature

ρ = density

It is an experimental fact that μ_K is essentially a function of temperature only. Examination of standard intermolecular force potential functions, such as the Lennard-Jones and Stockmayer potentials, indicates that for a dilute gas the contribution to viscosity due to attraction is much much greater than the contribution due to repulsion. Therefore, the viscosity of dilute gases can be represented by

$$\mu_o(T) = \mu_K(T) + \mu_{\varphi A}(T, \rho) \tag{6}$$

where μ_o is the total viscosity in dilute gases. The accuracy of the equations of Sutherland and Reingamun indicates that the intermolecular forces in a dilute gas may be assumed to be a function of the temperature only. Thus

$$\mu_o(T) = \mu_K(T) + \mu_{\varphi A}(T) \tag{7}$$

Enskog's solution to the Boltzmann equations for the viscosity of dense gases suggested the form

$$\mu(\rho, T) = \mu_o(T) \left[1 + \sum_{k=1}^{\infty} a_k (b\rho)^k \right] \tag{8}$$

where

$$a_k \; = \; \text{set of constants}$$

$$b \; = \; \text{function of temperature and density.}$$

For simple molecules the constants a_k and the function b may be calculated; however, the assumptions necessary to make the water molecule amenable to solution destroy the accuracy of the analysis.

Yen (8) assumed that the series in Enskog's solution could be replaced by the exponential function

$$\exp \frac{B\rho}{T}$$

where B is a constant which can be determined from the relation

$$B = \frac{T}{\rho} \ln \frac{\mu}{\mu_{\circ}} \tag{9}$$

In Yen's analysis, μ_o was taken as Reingamun's equation which can be stated

$$\mu_o = A \sqrt{T} \;/\; \exp \frac{K}{T} \tag{10}$$

where A and K are constants to be determined from existing data at low pressures. Thus Yen's equation for the viscosity of steam is

$$\mu = A \sqrt{T} \exp \frac{B\rho - K}{T}$$

In an attempt to fit Yen's equation to the experimental data obtained in the present study, it was found that it predicted the data too high by an average of about ten percent over the temperature and pressure range considered.

In the present investigation it was assumed that the series in Enskog's equation could be replaced by the function

$$C_1 \exp\left(C_2 \frac{\rho}{T}\right) + C_3 \exp\left(C_4 \frac{\rho}{T}\right) \tag{11}$$

where C_1, C_2, C_3 and C_4 are constants. For convenience in the analysis, it was assumed that

$$C_1 = C_3 = 1/2$$

and

$$C_2 = C_4 = D$$

The resulting expression for viscosity is

$$\mu = \mu_o \cosh \frac{D\rho}{T} \tag{12}$$

Obviously D can be obtained from the expression

$$D = \frac{T}{\rho} \cosh^{-1} \frac{\mu}{\mu_o} \tag{13}$$

In the present investigation μ_o is taken to be Sutherland's equation which can be written

$$\mu_o = \frac{A\sqrt{T}}{1 + \frac{K}{T}} \tag{14}$$

where the constants were evaluated from Latto's atmospheric data (4) and found to be

$$A = 1.28 \times 10^{-5} \ \frac{\text{pounds mass}}{\text{sec FT} \sqrt{{}^{o}R}}$$

$$K = 2070 \ {}^{o}R$$

In the process of fitting the experimental data obtained in the present investigation to equation (9), it was found that D was not actually a constant but depended slightly on temperature and to a lesser degree on pressure. The variation of D with temperature was found to be practically a linear one and was accounted for by plotting the values of D for several temperatures against temperature and fitting a linear equation through the points. That is

$$D = C' \ T + C'' \qquad (15)$$

where C' and C'' were found to be

$$C' = 0.0564 \text{ FT}^{3}/\text{pounds mass}$$

and

$$C'' = -17.35 \text{ FT}^{3} \ {}^{o}R/\text{pounds mass}$$

The resulting expression for viscosity is

$$\mu = \mu_o \cosh \rho \left(0.0564 - \frac{17.35}{T}\right) \qquad (16)$$

or using the Sutherland expression for μ_o

$$\mu = \frac{1.28 \times 10^{-5}}{1 + \frac{2070}{T}} \sqrt{T} \cosh \rho \left(0.0564 - \frac{17.35}{T}\right) \tag{17}$$

where:

T is in °Rankine

ρ is in pounds mass per cubic foot

μ is in pounds mass per foot second.

The expression appears cumbersome; however since the hyperbolic cosine function is tabulated and even appears as a scale on some slide rules, it is relatively easy to use in calculations. Furthermore, as shown in Figure 6, it represents the data obtained in the present investigation with a maximum deviation of ten percent from a smooth curve drawn through the experimental data points.

It should be noted that the equation behaves as it should in the limit as the temperature approaches infinity. Since all gases obey the perfect gas equation of state at extremely high temperatures for a given pressure, one may replace ρ in the hyperbolic cosine factor of equation (12) by

$$\rho = \frac{p}{RT} \tag{18}$$

where R is the gas constant and p the absolute pressure, and observe that

$$\lim_{T \to \infty} \cosh \frac{p}{RT^2} (C'T + C'') = 1$$

Obviously this implies that at high temperatures, the viscosity of a gas having a high density approaches the viscosity of that same gas at low density and at the same temperature -- regardless of its pressure.

It should be further noted that the equation shows the correct trend in the compressed and saturated liquid regions. It predicts the correct order of magnitude for the viscosity of liquid water; however, the predicted value is off from the accepted value by about a factor of four.

CHAPTER V

EXPERIMENTAL ERROR

Temperature Measurement

The temperature of the steam flowing in the annulus was measured by an iron-constantan thermocouple supplied and calibrated by the Thermoelectric Company. The thermocouple was certified to be accurate to the nearest one half degree Fahrenheit.

Since variation of the temperature during any run was less than one half degree Fahrenheit, it is felt that the actual steam temperature was measured accurately to within one degree Fahrenheit.

Pressure Measurement

Actual system pressure was measured with a dead-weight tester manufactured by the American Instrument Company. The tester was certified to be accurate to within ±10 psi for a perfectly static situation; however, since there were small pulsations from the pump, there was usually slight movement of the tester's pan during each run.

Since a variation in the weights on the tester's pan equivalent to 10 psi would move the pan from its extreme down position to its extreme up position, it is felt that the system pressure was measured accurately to within ±20 psi.

Mass Flow Rate

The condensate collected was weighed on analytical balance scales graduated to read accurately to the nearest 0.0001 gram. Considering that two readings were necessary to compute the mass of the condensate, the mass was measured accurately to within ±0.0005 grams. Since the total mass of condensate was always greater than five grams, the percent error in mass measurement was very small in comparison to the error in measuring the time period for collection and consequently can be neglected.

The time period for each run was measured with an electric timer reading to one tenth second. Since the time period was from 100 to 300 seconds, it is estimated that the time was measured accurately to within ±1/2 percent.

Annulus Pressure Drop Measurement

Although the least count of the manometer was 0.0001 inches of mercury, it was found that a change in elevation of only 0.0005 inches of mercury could be detected consistently. Since two readings were necessary to obtain the pressure drop, the mercury column could be read accurately to within 0.001 inches of mercury.

Considering that the pressure drop was never less than 0.1 inches of mercury the pressure drop should be accurate to at least ±1 percent.

Annulus Constant

After the initial calibration of the annulus, the thirty two data points obtained were plotted versus the Reynolds number. The result was random scatter around the average value for all Reynolds numbers. The maximum percent deviation from the average was less than one half percent.

On the basis of the number of experimental data points taken, it is felt that the annulus constant should be in error no more than one percent.

Accuracy of Results

The errors in individual measurements are as follows:

temperature measurement	order zero
pressure measurement	order zero
pressure drop	one percent
mass flow rate	one half percent
annulus constant	one percent.

Considering that

$$\nu = \frac{CC_T \Delta p}{W} \qquad (3)$$

the kinematic viscosity can be in error by 2 1/2 percent.

All factors considered, for a given temperature and pressure, the experimentally determined kinematic viscosity as obtained in the present investigation should be within a conservative ± 4 percent of the actual value.

CHAPTER VI

DISCUSSION OF RESULTS

The results of the present investigation are presented in Appendix II and III in both tabular and graphical form. Figure 6 is a plot of the dynamic viscosity obtained in the present investigation and also of the semi-empirical equation (17) given in Chapter IV. Density data for the conversion of the measured kinematic viscosity to dynamic viscosity were obtained from the VDI Steam Tables, 5th Edition (10). Figure 7 shows the comparison of Ray's (6) and Barnett's (1) data with that of the present investigation.

Since Ray recorded no data below 300 kg_f/cm^2 pressure, and since his data must be interpolated, it is difficult to make a comparison of his results to the 3750 psia isobar of data obtained in the present investigation. Plotted in Figure 7 are the data for the 350 and 700 kg_f/cm^2 isobars obtained by Ray. These were plotted since no interpolation was necessary (350 kg_f/cm^2 = 4,980 psia and 700 kg_f/cm^2 = 9,960 psia). It is noted that Ray's data are consistently higher than that of the present investigation and there is no explanation for this.

For temperature above 788° Fahrenheit, Barnett's isobars of 3750 and 5000 psia are lower than those of Ray and of the present investigation. It is felt that Barnett's equipment provided insufficient preheating of the steam entering the annulus which may have caused this discrepancy. Insufficient preheating results in the measuring of the viscosity of steam that is at a lower temperature and consequently lower viscosity than the recorded temperature of the annulus. In the present investigation this problem was eliminated by using a longer preheating coil and controlling it with an electronic temperature controller.

It is questionable that the shape of the actual viscosity curve for the 5000 psia isobar below 788° Fahrenheit is as shown in Figure 6. At present there is no theoretical explanation for the shape; however, equation (17) does predict such a trend for the 3750 psia isobar. Furthermore, a smooth curve can be drawn through all of the experimental data points and no reason to eliminate any of the points was found.

Equation (17) of Chapter IV correlated the 3750 and 10,000 psia data very well; however, it predicts the 5000 and 7500 psia data too low by an average of about five percent.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

After careful analysis, it is felt that the experimental results of the present investigation are accurate to within ±4 percent. For engineering calculations, the following equation is recommended:

$$\mu = \frac{1.25 \times 10^{-5}\sqrt{T}}{1 + \frac{2070}{T}} \cosh \rho \left(0.0564 - \frac{17.35}{T}\right) \qquad (17)$$

This semi-empirical equation is estimated to represent the data obtained in the present investigation to within ±3 percent.

Concerning further study on the viscosity of steam, it is recommended that considerable effort be devoted to resolving the apparent discrepancy between data taken with the Rankine viscosimeter (Ray, et al.) and that taken with the annulus viscosimeter (Barnett's and the present investigation). For the annulus viscosimeter it is recommended that a study be initiated to determine optimum dimensions for the annulus.

APPENDIX I

The solution to the equations of motion for laminar flow in a perfectly concentric annulus yields the following equation for the annulus constant:

$$CC_T = \frac{\pi b^4}{8L} \left[1 - (1 + \epsilon/b)^4 - \frac{\left(1 - (1 + \frac{\epsilon}{b})^2\right)^2}{\ln (1 - \epsilon/b)}\right]$$

where ϵ is the radial clearance between the tubes forming the annulus and b is the inside radius of the outer tube of the annulus (See reference (3)). To obtain a measurable pressure drop, the annular clearance must be very small; thus the ratio ϵ/b is very much smaller than unity in absolute value and

$$\ln (1 - \frac{\epsilon}{b}) \simeq -\frac{\epsilon}{b}$$

Using this approximation and neglecting terms of order higher than unity results in the expression

$$CC_T = \frac{\pi}{L} \; b^3 \; \epsilon$$

At 70° Fahrenheit the radial clearance of the annulus used in the present study was of the order of 0.005 inches. For C to be accurate to one percent obviously would require that ϵ be accurate to at least one percent. This implies that ϵ would have to be machined and measured accurate to

0.00005 inches -- an impractical task.

Since the annulus constant is so sensitive to small variations in the radii of the annulus, critics of the annulus viscosimeter are quick to point out the possibility of the annulus constant obtained from the nitrogen calibration being different from the annulus constant for steam. This appendix is devoted to showing that the annulus constant for an annulus at a given temperature under the condition of laminar flow is at most a function of the Reynolds number. The condition of a fixed temperature must be imposed because the geometry is a function of temperature. If the annulus constant is found to be independent of the Reynolds number, one can conclude that the annulus constant for a given temperature is not a function of the fluid flowing or the flow situation present in the annulus.

Consider steady flow of an incompressible fluid with constant viscosity through an annulus. These conditions are justified because of the small pressure drop across the annulus compared with the large static pressure on the system. Actual solution of the equations of motion is not attempted here, so for the heuristic argument to be presented, writing the equations of motion in rectangular cartesian coordinates produces a symmetrical form which is a distinct advantage. The equations of motion in rectangular cartesian coordinates are:

<u>Continuity</u>

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] - \frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] - \frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \nu \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] - \frac{1}{\rho} \frac{\partial p}{\partial z}.$$

where

x, y, and z = position coordinates

u = x component of velocity

v = y component of velocity

w = z component of velocity

p = pressure

ρ = density

It is advantageous to nondimensionalize the equations of motion by introducing the following nondimensional variables:

$$x' = x/D_H$$

$$y' = y/D_H$$

$$z' = z/D_H$$

$$u' = u/V$$

$$v' = v/V$$

$$w' = w/V$$

$$p' = p/_{\rho v^2}$$

$$R_e = \frac{\rho V D_H}{\mu}$$

where D_H is the hydraulic diameter, V the average velocity, and R_e the Reynolds number based on the hydraulic diameter and the average velocity. Utilizing the nondimensional variables the equations of motion become:

Continuity

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0$$

Momentum

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} + w' \frac{\partial u'}{\partial z'} = \frac{1}{R_e}\left[\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right] - \frac{\partial p'}{\partial x'}$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} + w' \frac{\partial v'}{\partial z'} = \frac{1}{R_e}\left[\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2} + \frac{\partial^2 v'}{\partial z'^2}\right] - \frac{\partial p'}{\partial y'}$$

$$u' \frac{\partial w'}{\partial x'} + v' \frac{\partial w'}{\partial y'} + w' \frac{\partial w'}{\partial z'} = \frac{1}{R_e}\left[\frac{\partial^2 w'}{\partial x'^2} + \frac{\partial^2 w'}{\partial y'^2} + \frac{\partial^2 w'}{\partial z'^2}\right] - \frac{\partial p'}{\partial z'}$$

Given the geometry of an annulus, one can solve, in principle, for u', v' and w' and p' in terms of x', y', z' and R_e only. Let the solution for p' be represented by

$$p' = F(x', y', z', R_e)$$

Then

$$p = \rho v^2 F(x', y', z', R_e)$$

The pressure difference between two fixed points in the annulus is

$$\Delta p = \rho V^2 (F(a', b', c', R_e) - F(a_1', b_1', c_1', R_e)) = \rho V^2 g(R_e)$$

where $g(R_e)$ represents a function of the Reynolds number only. The annulus constant CC_T is given by

$$CC_T = \frac{\nu m}{\Delta p} = \frac{\nu \rho A V}{\Delta p}$$

where A is the cross sectional flow area. Combining the last two expressions results in the following expression for CC_T:

$$CC_T = \frac{\nu A}{V g(R_e)} = \frac{A D_H}{R_e \, g(R_e)}$$

This shows that for any given annulus at a single fixed temperature, CC_T is at most a function of the Reynolds number. Note that the above analysis is not restricted to a perfectly "true" annulus but applies equally well to one that is eccentric or has skewed centerlines.

The annulus constant for the present study was plotted against the Reynolds number and found to be independent of it. Although this does not guarantee that the annulus was concentric, it does suggest the possibility that it was. Since the annulus constant can be at most a function of the Reynolds number, the geometrical configuration was of no real consequence.

APPENDIX II

TABLE I

and

TABLE 2

Table I

Viscosity of Steam - Experimental Data

P (Psia)	T (Deg. Farh)	Reynolds No.	ν (ft^2/sec)x10^6	μ (lbm/ft sec)x10^5
3750	732.6	76.5	1.191	.200
3750	733.6	73.4	1.209	.203
3750	734.6	74.0	1.222	.193
3750	734.6	71.6	1.229	.194
3750	735.6	70.3	1.231	.195
3750	755.6	60.3	1.702	.191
3750	755.6	61.6	1.692	.190
3750	755.6	97.9	1.654	.185
3750	755.6	57.2	1.702	.191
3750	756.6	50.8	1.793	.199
3750	756.6	56.6	1.683	.187
3750	756.6	59.9	1.712	.190
3750	756.6	59.4	1.711	.190
3750	757.6	58.9	1.721	.189
3750	757.6	55.8	1.707	.187
3750	759.6	54.8	1.705	.184
3750	759.6	52.9	1.734	.187
3750	759.6	87.0	1.736	.187
3750	760.6	53.4	1.719	.184
3750	760.6	52.3	1.745	.187
3750	762.6	81.2	1.777	.187
3750	766.6	75.5	1.822	.184
3750	770.6	67.4	1.871	.184
3750	786.5	55.3	2.089	.186
3750	786.5	54.5	2.093	.186
3750	786.5	53.7	2.099	.187
3750	786.5	52.9	2.106	.188
3750	787.5	51.8	2.116	.188
3750	817.5	58.9	2.544	.198
3750	818.5	60.5	2.565	.200
3750	818.5	62.2	2.557	.199
3750	818.5	61.5	2.558	.199
3750	842.5	42.6	2.745	.197
3750	842.5	42.1	2.735	.196
3750	842.5	41.7	2.733	.196
3750	842.5	40.9	2.751	.197
3750	842.5	40.0	2.771	.198
3750	867.5	33.9	2.974	.198
3750	868.5	40.0	2.912	.194

Table I (Continued)

Viscosity of Steam - Experimental Data

P (Psia)	T (Deg. Farh)	Reynolds No.	ν (ft^2/sec)x10^6	μ (lbm/ft sec)x10^5
3750	868.5	39.6	2.873	.191
3750	889.4	54.9	3.126	.199
3750	889.4	53.8	3.147	.200
3750	889.4	53.0	3.165	.201
3750	889.4	53.6	3.146	.200
3750	889.4	52.9	3.153	.200
3750	903.4	95.1	3.103	.191
3750	904.4	90.9	3.130	.193
3750	904.4	85.9	3.177	.196
3750	904.4	86.4	3.138	.193
3750	904.4	85.8	3.113	.192
3750	908.4	44.5	3.362	.205
3750	909.4	46.3	3.321	.203
3750	909.4	45.6	3.349	.204
3750	909.4	45.7	3.343	.204
3750	909.4	45.4	3.347	.204
3750	915.4	72.2	3.189	.193
3750	915.4	71.7	3.192	.193
3750	915.4	71.1	3.225	.195
3750	915.4	70.5	3.225	.195
3750	916.4	70.9	3.233	.195
3750	940.4	57.6	3.548	.203
3750	940.4	57.1	3.543	.203
3750	940.4	56.2	3.553	.204
3750	940.4	55.7	3.554	.204
3750	941.4	55.7	3.637	.208
3750	972.4	43.8	3.797	.208
3750	973.4	44.2	3.811	.209
3750	973.4	42.9	3.847	.211
3750	974.4	43.8	3.825	.210
3750	974.4	43.3	3.820	.209
3750	1014.0	55.2	4.251	.220
3750	1014.0	54.8	4.246	.220
3750	1014.0	53.5	4.290	.222
3750	1014.0	53.0	4.285	.222
3750	1014.0	51.8	4.313	.223
5000	792.5	98.6	1.390	.256
5000	754.6	72.8	1.227	.349
5000	754.6	73.3	1.222	.348
5000	754.6	70.7	1.268	.361
5000	754.6	73.6	1.220	.347
5000	767.6	76.3	1.293	.331

Table I (Continued)

Viscosity of Steam - Experimental Data

P (Psia)	T (Deg. Farh)	Reynolds No.	ν (ft^2/sec)x10^6	μ (lbm/ft sec)x10^5
5000	767.6	76.8	1.281	.327
5000	767.6	78.4	1.258	.322
5000	767.6	73.3	1.338	.342
5000	774.5	31.8	1.271	.300
5000	781.5	112.1	1.293	.268
5000	783.5	110.6	1.344	.273
5000	791.5	96.6	1.403	.261
5000	791.5	96.0	1.412	.262
5000	791.5	96.0	1.412	.262
5000	791.5	95.2	1.425	.265
5000	791.5	98.0	1.384	.257
5000	815.5	106.6	1.647	.239
5000	816.5	109.1	1.605	.233
5000	816.5	111.3	1.571	.228
5000	816.5	109.8	1.599	.232
5000	847.5	115.0	1.898	.225
5000	847.5	117.2	1.865	.221
5000	846.5	115.9	1.868	.223
5000	846.5	114.6	1.887	.226
5000	845.5	115.8	1.873	.224
5000	874.5	122.1	2.070	.218
5000	872.5	120.8	2.073	.221
5000	875.5	121.3	2.063	.220
5000	876.5	121.7	2.096	.219
5000	876.5	120.4	2.072	.217
5000	876.5	118.8	2.108	.221
5000	876.5	119.4	2.094	.219
5000	876.5	119.9	2.092	.219
5000	876.5	119.8	2.092	.219
5000	916.4	121.1	2.397	.220
5000	915.4	121.2	2.400	.220
5000	915.4	121.2	2.399	.220
5000	915.4	122.1	2.386	.218
5000	949.4	119.9	2.654	.222
5000	952.4	121.5	2.622	.219
5000	953.4	119.1	2.661	.222
5000	954.4	120.5	2.642	.221
5000	956.4	120.9	2.652	.220
7500	728.6	76.8	1.257	.482
7500	729.6	76.5	1.261	.482
7500	730.6	77.0	1.259	.479
7500	749.6	80.1	1.260	.459

Table I (Continued)

Viscosity of Steam - Experimental Data

P (Psia)	T (Deg. Farh)	Reynolds No.	ν (ft^2/sec)x10^6	μ (lbm/ft sec)x10^5
7500	749.6	80.4	1.258	.458
7500	757.6	82.5	1.243	.444
7500	758.6	82.7	1.245	.443
7500	758.6	82.9	1.244	.442
7500	760.6	83.0	1.246	.441
7500	765.6	83.8	1.253	.437
7500	783.5	86.8	1.266	.419
7500	785.5	86.6	1.275	.420
7500	785.5	87.8	1.266	.417
7500	786.5	87.8	1.262	.416
7500	786.5	87.1	1.267	.417
7500	823.5	101.5	1.241	.358
7500	823.5	100.4	1.255	.362
7500	824.5	105.6	1.215	.344
7500	825.5	102.0	1.257	.356
7500	826.5	108.1	1.185	.336
7500	850.5	113.2	1.283	.324
7500	854.5	111.5	1.327	.326
7500	857.5	115.1	1.305	.317
7500	860.5	115.5	1.324	.316
7500	862.5	116.7	1.323	.313
7500	872.5	120.4	1.372	.306
7500	873.5	119.3	1.383	.309
7500	874.5	123.1	1.346	.298
7500	875.5	119.6	1.383	.306
7500	876.5	120.0	1.393	.306
7500	920.4	128.3	1.611	.285
7500	921.4	133.3	1.552	.275
7500	922.4	130.5	1.590	.281
7500	922.4	129.4	1.604	.284
7500	925.4	130.4	1.609	.281
7500	1018.2	97.2	2.024	.264
7500	1022.0	109.5	2.010	.259
10000	758.6	62.9	1.203	.470
10000	759.6	63.3	1.198	.467
10000	759.6	63.4	1.201	.468
10000	834.5	70.1	1.266	.421
10000	837.5	73.9	1.203	.398
10000	837.5	74.2	1.200	.398
10000	886.5	81.2	1.253	.363
10000	885.5	82.6	1.239	.359

Table I (Continued)

Viscosity of Steam - Experimental Data

P (Psia)	T (Deg. Farh)	Reynolds No.	ν (ft^2/sec)x10^6	μ (lbm/ft sec)x10^5
10000	886.5	82.5	1.241	.359
10000	911.4	86.0	1.290	.344
10000	913.4	86.0	1.297	.343
10000	962.4	94.2	1.390	.314
10000	965.4	94.8	1.388	.312
10000	964.4	94.7	1.389	.312
10000	966.4	95.1	1.390	.310

Table 2

Average Annulus Constants Used in Computing Viscosity

Pressure - Psia	Annulus Constant - $ft^3 \times 10^{13}$
3750	17.78
5000	17.72
7500	17.49
10,000	17.51

APPENDIX III

FIGURES

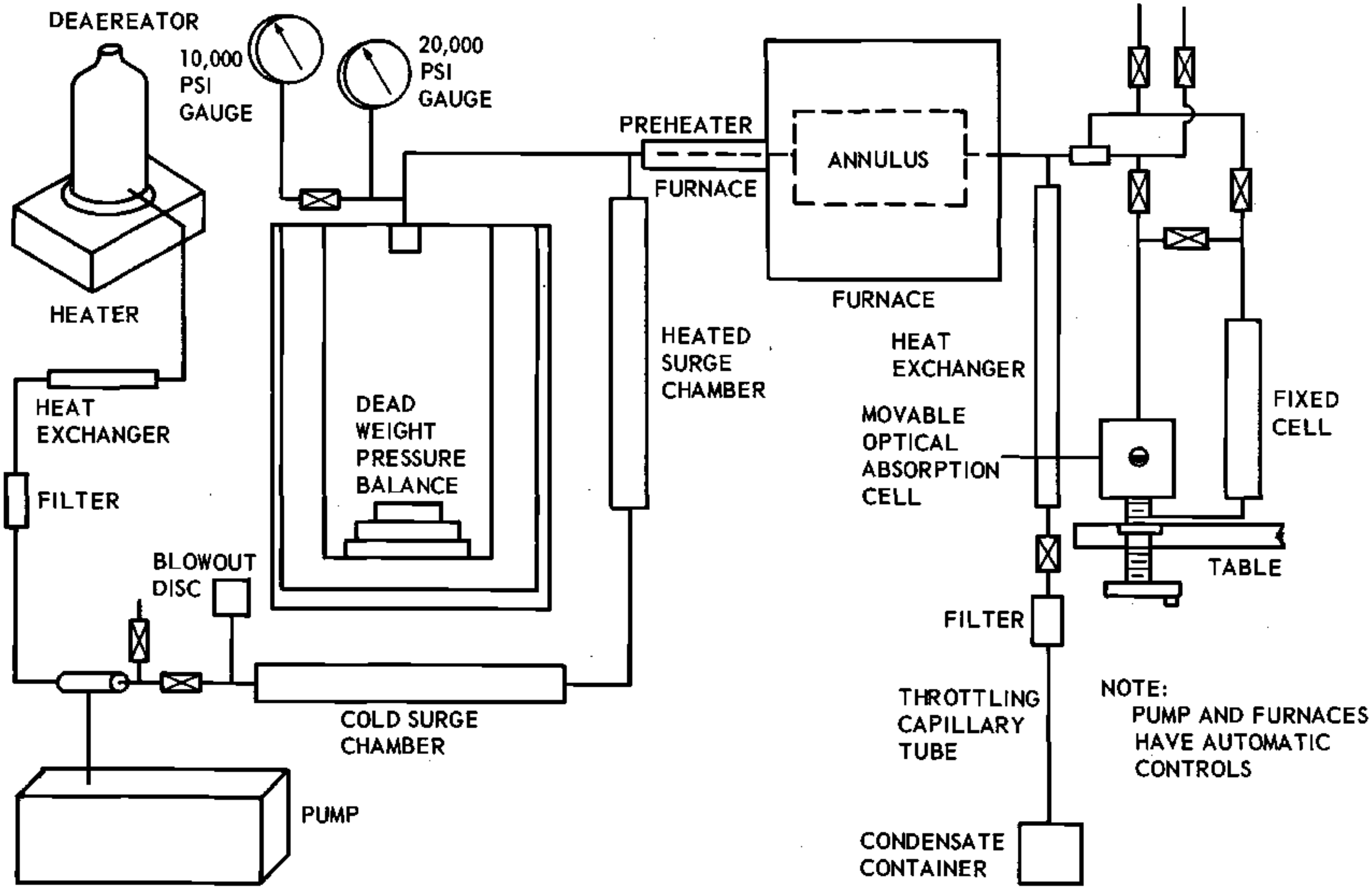

Figure 1. Schematic Diagram of Experimental Apparatus.

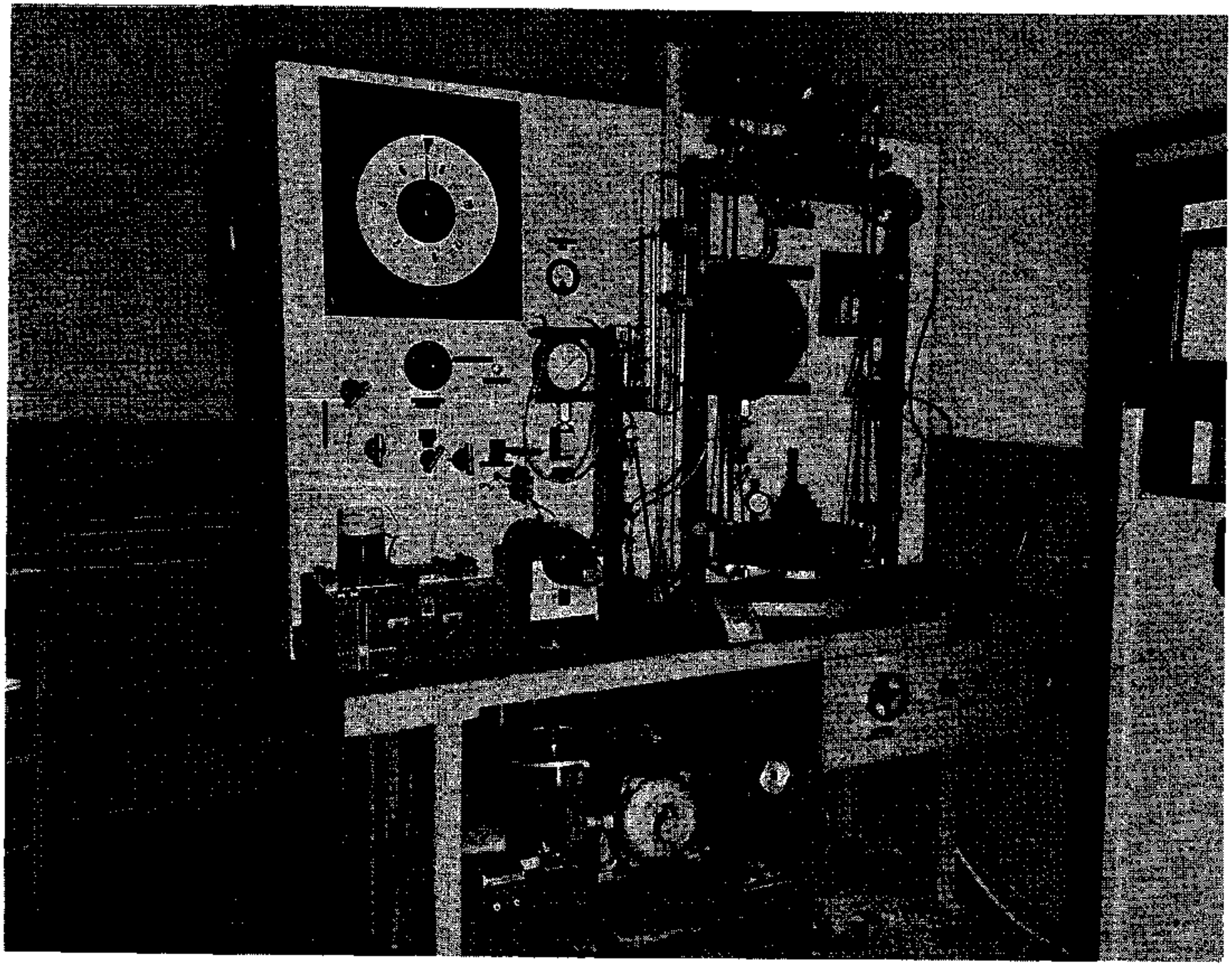

Figure 2. Photograph of Experimental Apparatus.

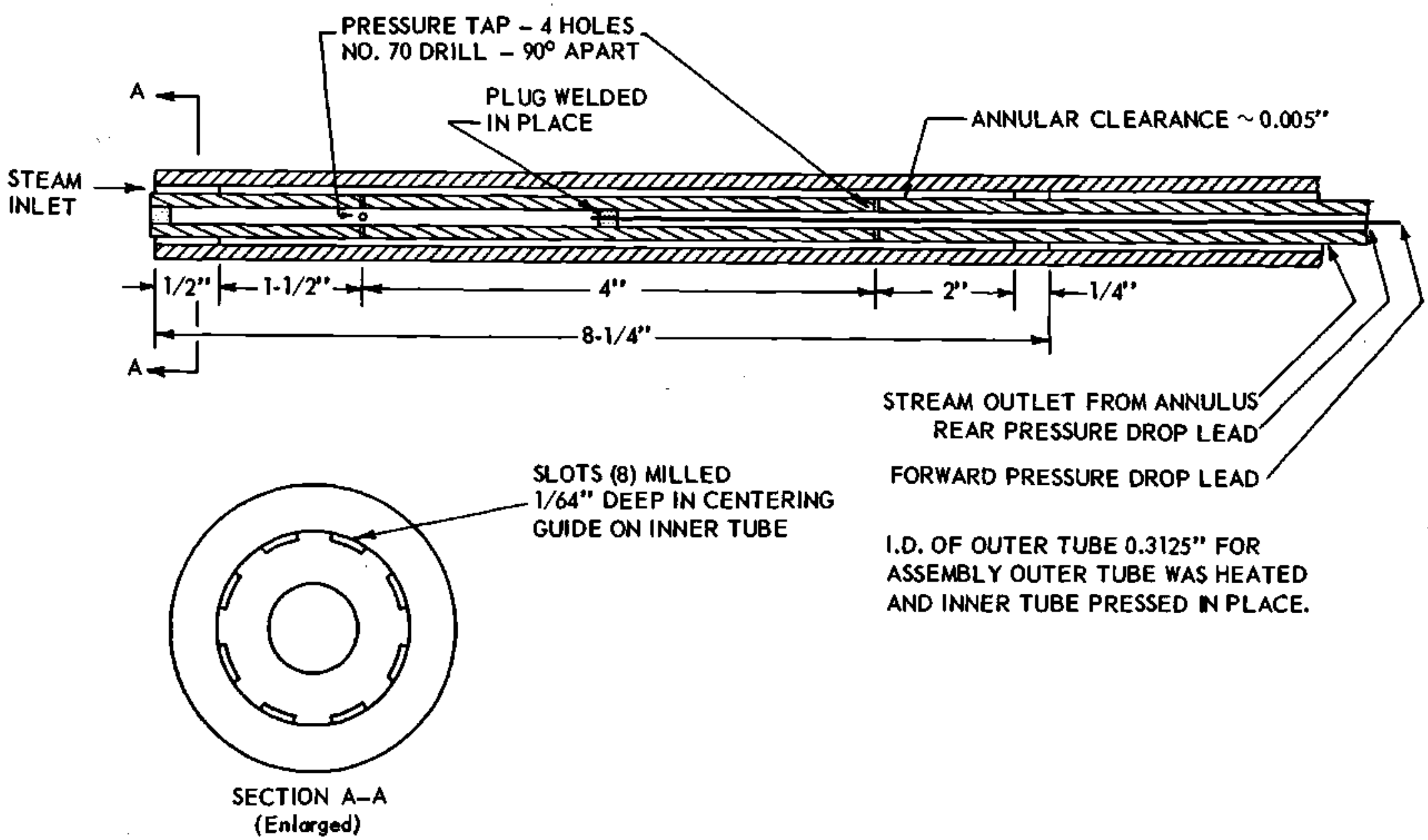

Figure 3. Schematic Diagram of Annulus Assembly.

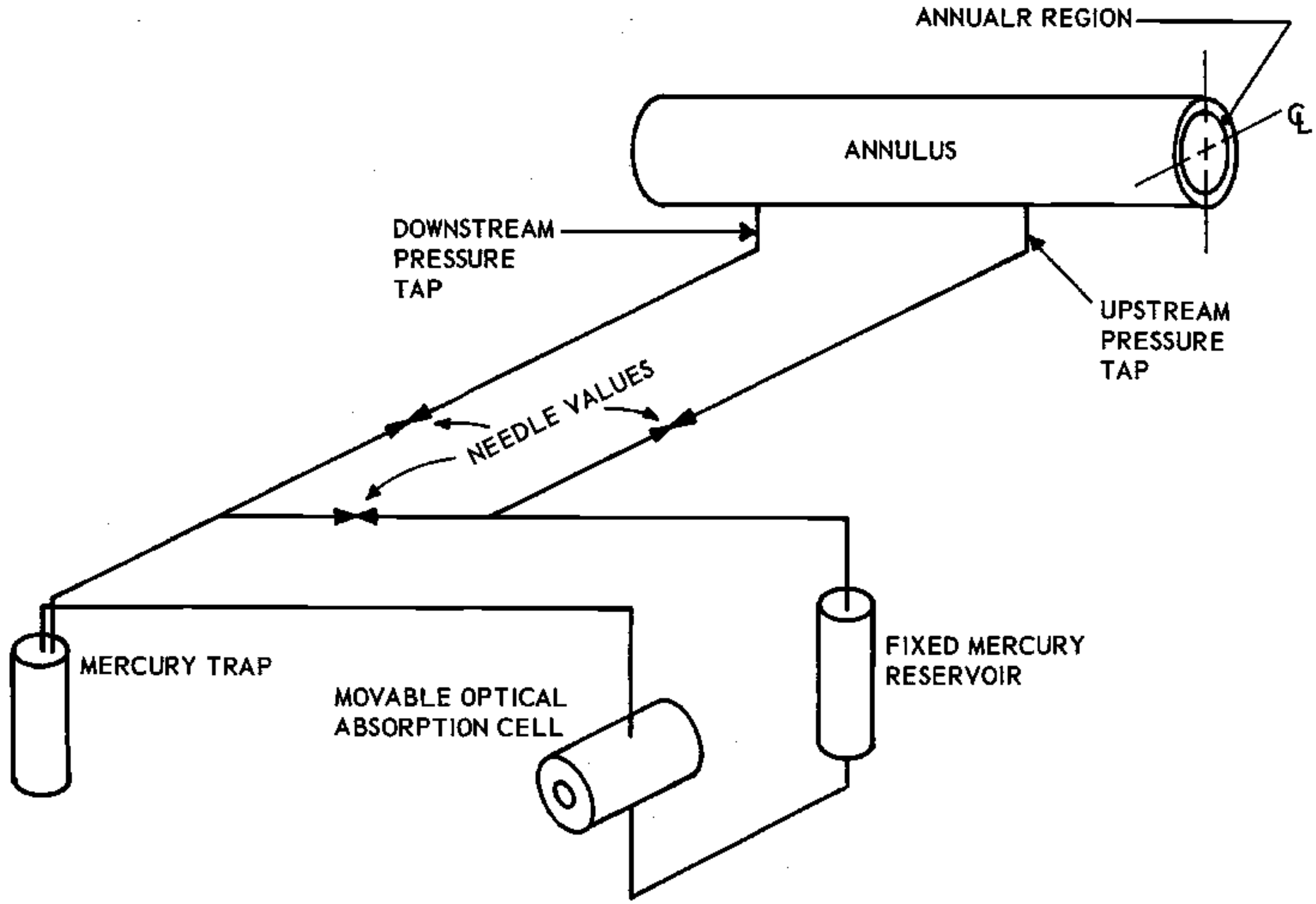

Figure 4. Schematic Diagram of Valve System.

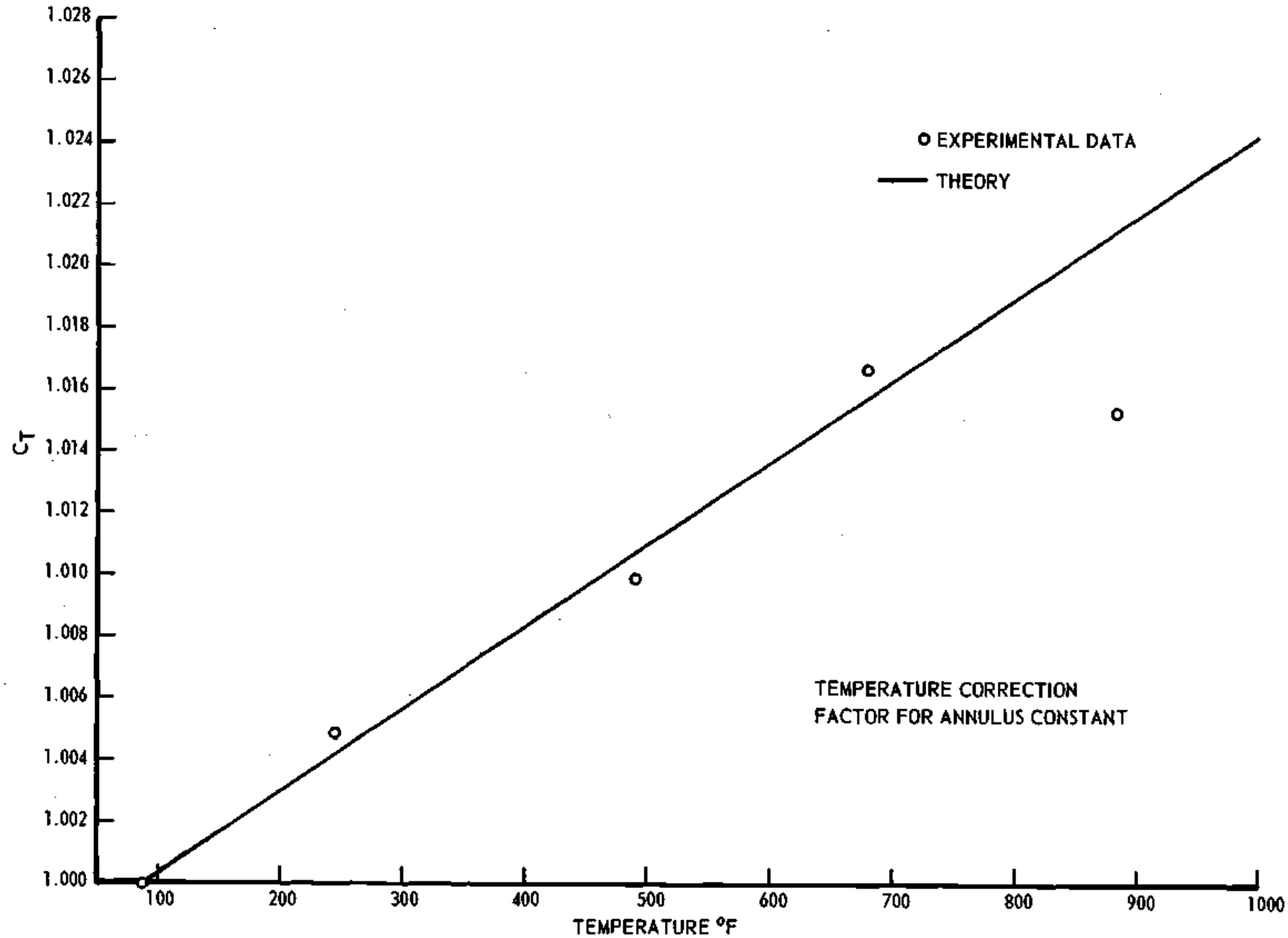

Figure 5. Temperature Correction Curve.

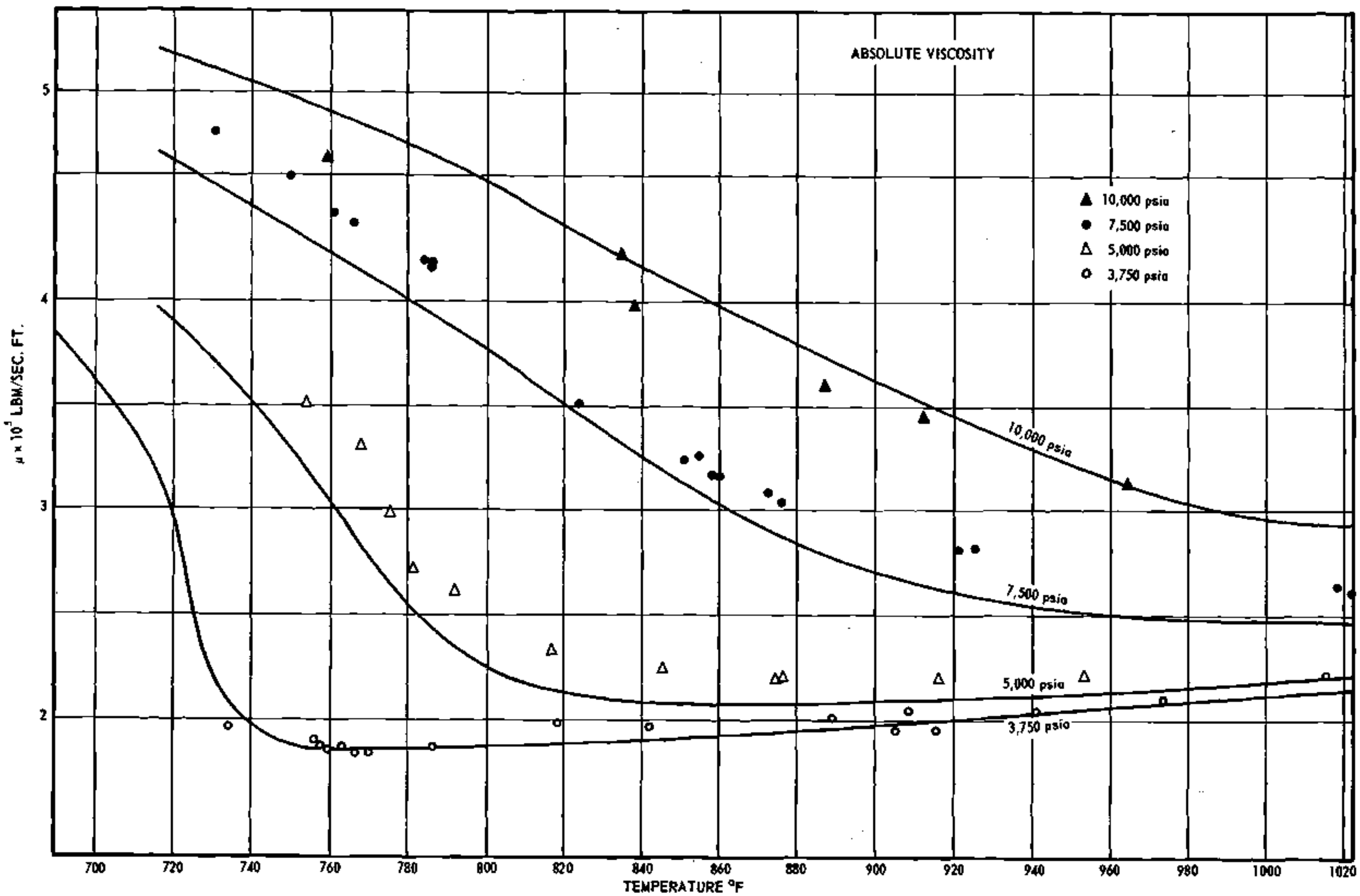

Figure 6. Comparison of Data of Present Investigation with Equation [17] of Chapter 4.

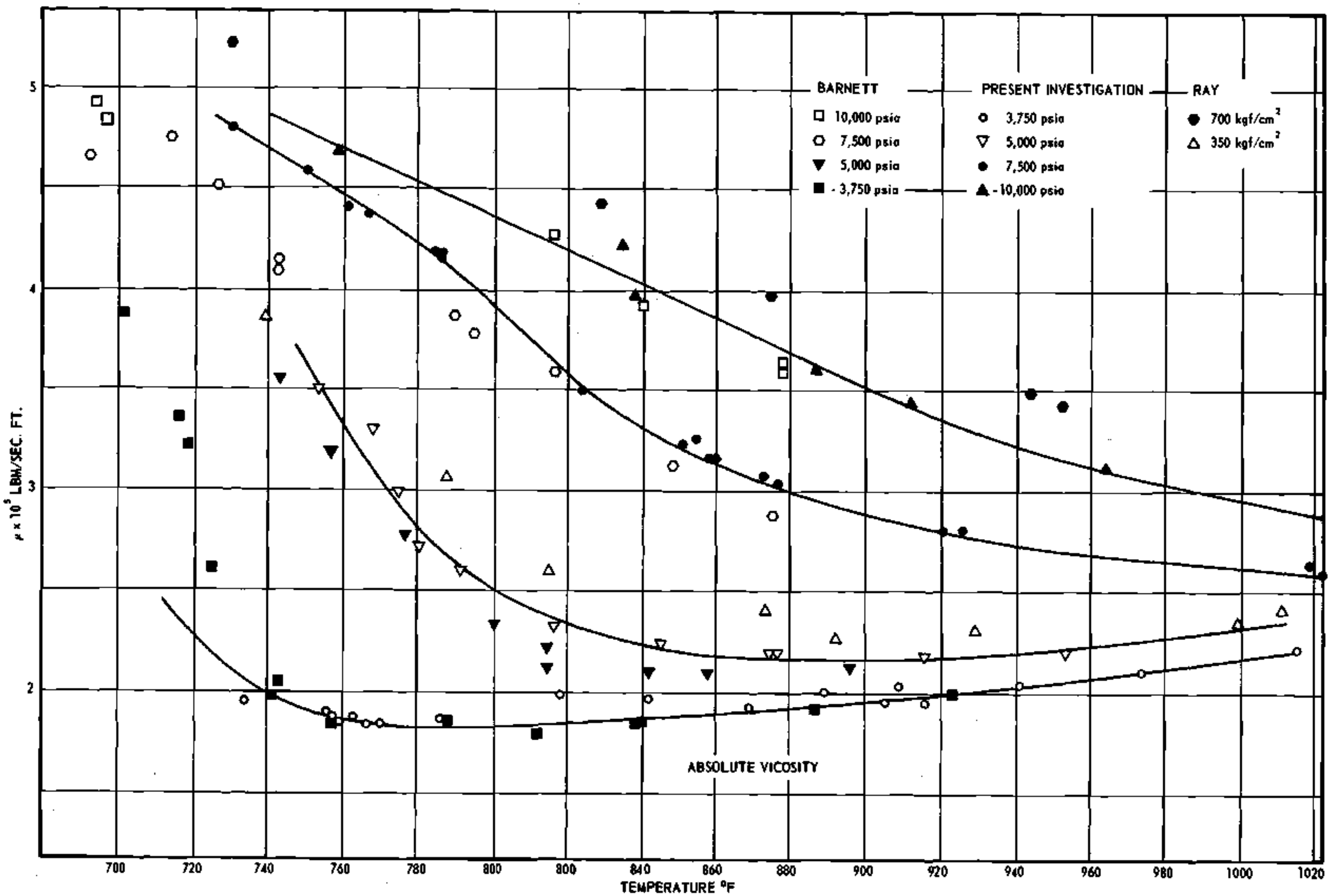

Figure 7. Comparison of Data of Present Investigation with Ray's and Barnett's Results.

BIBLIOGRAPHY

(1) Barnett, S. C., An Investigation of the Viscosity of Steam at High Pressure, unpublished Ph.D. Thesis, Georgia Institute of Technology, 1962. (Also paper No. 63-WA-240 at 1963 Annual Meeting of ASME, New York.)

(2) Whitesides, R. H., The Experimental Determination of Steam Viscosity at a Pressure of 5000 psia and 7500 psia, unpublished M.S. Thesis, Georgia Institute of Technology, 1962.

(3) Schlichting, H., Boundary Layer Theory, New York; McGraw Hill Co., Inc., 1955 pp. 586-587.

(4) Latto, B., Determination of Viscosity of Steam, unpublished Ph.D. Thesis, The University of Glasgow, 1964.

(5) Starling, K. E., Viscosity of Dense Fluids, unpublished Ph.D. Thesis, Illinois Institute of Technology, 1962.

(6) Ray, A. K, "A New Determination of the Kinematic Viscosity of Steam at Supercritical Pressures and Temperatures," Journal of Mechanical Engineering Science, Vol. 6, No. 2, 1964.

(7) Bureau of Standards N_2 Data, Tables of Thermal Properties of Gases, U. S. Bureau of Standards, NBS Circular 564, 1955.

(8) Yen, H. H. Y., "A Comparison of the Available Viscosity Data of High Pressure and High Temperature Steam with a Modified Reingamun's Equation for Viscosity," unpublished M.S. Thesis, Georgia Institute of Technology, 1964.

(9) Loeb, L. B., The Kinetic Theory of Gases, McGraw Hill Book Co., Inc., New York, pp. 59-234, 1934.

(10) Verein Deutscher Ingenieure, Steam Tables, fifth edition, 1960.

(11) Barnett, S. C., Jackson, T. W., and Whitesides, R. H., "A High Pressure Differential Manometer," Journal of Heat Transfer, TRANSACTIONS OF ASME Series C, Vol. 85, No. 2, May, 1963, p. 180.