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A THESIS
Presented to
The Faculty of the Graduate Division
by
Steven Lawrence Haas
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In Partial Fulfillment
of the Requirements for the Degree Master of Science in Mechanical Engineering
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Georgia Institute of Technology
April, 1966

# STRUCTURE SYNTHESIS AND PRELIMINARY DESIGN OF A FOUR-INPUT DECODER MECHANISM 

Approved:


## ACKNOWLEDGMENTS

I wish to take this opportunity to thank those who have helped make this thesis possible. I am especially indebted to Dr. F. R. E. Crossley for serving as my thesis advisor, and for the many hours of assistance he gave. His enthusiasm for the subject left a permanent impression upon me. I would also like to thank Dr. H. L. Johnson and Dr. C. E. Stoneking for serving on the reading committee and for reviewing the manuscript. Additionally, I would like to thank Mr . Andrew M. Agoos for his work in preparing many of the illustrations.

This research was sponsored by the Olivetti Corporation of Ivrea, Italy.

I wish to dedicate this work to my wife, Helen Joy, for the love, patience, and understanding which she has so unselfishly given, and to my Mother and Father, for the wisdom and guidance they have shown me over the years.

With gratitude, I thank God for the blessings he has bestowed upon me and my family.

TABLE OF CONTENTS
Page
ACKNOWLEDGMENTS ..... ii
LIST OF TABLES ..... v
LIST OF ILLUSTRATIONS. ..... vi
SUMMARY. ..... viii
Chapter
I. INTRODUCTION ..... 1
Preliminary Design ConsiderationsTerminologyPermutation Synthesis as a Design Tool
II. GRUEBLER'S CRITERIA OF CONSTRAINMENT ..... 5
Basic Structural ConceptsDistinct Configurations
III. THE DERIVATION OF THE KINEMATIC CHAINS OF CLASS I, II, AND III USING MANOLESCU'S METHOD ..... 12
Manolescu's MethodA Collection of Kinematic Chains of Class I, II, and III
IV. APPLICATION OF FRANKE'S METHOD ..... 40
Initial Stipulations
Thirteen-Bar Chains
Fifteen-Bar Chains
v. SELECTION OE SUITABLE MECHANISMS ..... 73
Basic Concepts
Linkage Number OneLinkage Number TwoLinkage Number ThreeAn Inertia Study of Linkage Number One and Number Two
VI. CONCLUSIONS ..... 107
Chapter Page
VI. CONCLUSIONS (Continued)Application of Manolescu's MethodApplication of Franke's MethodPreliminary Design of Three Working Mechanisms
VII. RECOMMENDATIONS ..... 111
APPENDICES
A. DETERMINATION OF THOSE SUBGROUPS WHICH WILL
YIELD ONLY KINEMATIC CHAINS OF FRACTIONATED MOBILITY ..... 113
B. TABULATED INERTIA CONSTANTS FOR LINKAGE
NUMBER ONE AND LINKAGE NUMBER TWO. ..... 117
LITERATURE CITED ..... 120
OTHER REFERENCES ..... 121

## LIST OF TABLE:S

Table Page1. Classification of Kinematic Chains with Four
Degrees of Mobility ..... 9
2. The Method of Formation of the Kinematic Chain Having $L=7, X=4, N=7$, and $J=7$ ..... 24
3. The Method of Formation of the Kinematic Chains
Having $L=7, X=4, N=9$, and $J=10$. ..... 25
4. The Method of Formation of the Kinematic ChainsHaving $L=7, X=4, N=11$, and $J=13$29
5. A Collection of the Suitable Kinematic Chains of Class IV ..... 48
6. A Representative Collection of the Kinematic Chains of Class V ..... 56
7. Input-Output Inversions of the Kinematic Chains of Class IV (d) 9-3-0-1 ..... 75
8. Inertia Constants for Linkage No. I ..... 118
9. Inertia Constants for Linkage No. 2 ..... 119

## LIST OF ILLUSTRATIONS

Figure Page

1. (a) A Kinematic Chain Described by an Incidence Matrix
(b) An Incidence Matrix
(c) Two Examples of Isomorphic Pairs that Appear Unlike. ..... 11
2. A Representation of a Kinematic Chain Restricted to Planar Motion ..... 13
3. A Collection of Characteristic Assur Groups ..... 16
4. An Example of the Derivation of MechanismsHaving One Degree of Mobility by (a) Amplification,and (b) Enlargement18
5. An Example of the Construction of Kinematic Chains by the Addition of Basic Assur Groups ..... 20
6. The Formation of an Unsuitable Varient
from Two Characteristic Assur Groups. ..... 21
7. A Collection of Several Acceptable Varients of Basic Assur Groups. ..... 22
8. The Formation of a Kinematic Chain ( $X=2$ ) HavingFractionated Mobility, Building Twice from theBase Link No. 3 . . . . . . . . . . . . . . .23
9. Two Thirteen-Bar Kinematic Chains and a Fifteen-Bar Kinematic Chain Having Fractionated Mobility ..... 41
10. Permutations of Class IV (d) 9-3-0-1 ..... 44
11. An Example of Applying Franke's Method. ..... 45
12. An Example of Applying Franke's Method. ..... 46
13. Permutations of Class IV (g) 10-1-1-1 ..... 4714. Suitable and Unsuitable Kinematic ChainsDerived from Permutation 1 of Figure 1347
14. Permutation 1 of the Subgroup 9-5-0-1 ..... 49
15. Four Distinct Arrangements of Permutation 1 ..... 50
Figure Page
16. The Resulting Kinematic Chains from Arrangement (a) of Figure 16 ..... 51
17. The Formation of a "Hidden" Assur Group ..... 53
18. The Formation of an Assur Group and Six-Link Kinematic Chain from a Fifteen-Bar Chain with Three Inputs Fixed ..... 54
19. The Formation of Linkage No. I from Its Kinematic Chain ..... 78
20. A Linkage Similar in Behavior to Linkage No. 1 ..... 79
21. Location of the Leverage Points $x^{\prime}, x^{\prime \prime}$, and $x^{\prime \prime \prime}$ on Linkage No. 1. ..... 81
22. A Working Model of Linkage No. l as Shown with (a) No Inputs Actuated, and (b) Inputs 1 and 4 Actuated ..... 86
23. Location of the Leverage Point $x^{\prime}$ on Linkage No. 2 ..... 87
24. Several Design Alternatives of Linkage No. 2 ..... 89
25. A Working Model of Linkage No. 2 as Shown with (a) No Inputs Actuated, and (b) Inputs 2 and 4 Actuated ..... 91
26. A Modification of Linkage No. 2 Incorporating Slider Substitutions ..... 92
27. The Formation of Linkage No. 3 from its Kinematic Chain ..... 94
28. Applying a Four-Bar Linkage Analogy to a Complex Fifteen-Bar Linkage ..... 96
29. A Working Model of Linkage No. 3 as Shown with (a) Inputs 1,2 , and 4 Actuated, and (b) All Inputs Actuated ..... 98
30. Determining the Resultant Center of Rotation of Link 4 Upon Operation from Output Position One to Output Position Six ..... 101
31. Another Configuration of Linkage No. 2 with the Same Relative Link Sizes and Output Displacement as that of Linkage No. 1 ..... 105

## SUMMARY

In the design of any linkage mechanism, there is usually more than one linkage, having the same mobility, which can perform the same basic function. Unless the existence of all these linkages is first determined, it can never be concluded which linkage will perform this function in the most satisfactory manner. By applying the methods of permutation synthesis, such a collection of linkages may be made.

Thus, the objective of this study is twofold. First, planar kinematic chains having four degrees of mobility are studied to provide a basis for the optimum design of a mechanical decoder having a fourbit binary input and a sixteen-position output. Secondly, this procedure illustrates an application of the methods of permutation synthesis in the design of any linkage mechanism.

Using the Gruebler Criterion, the kinematic chains having four degrees of mobility are initially classified by the number of links and joints in each chain. The presentation of all those chains having seven, nine, eleven, thirteen, and fifteen members then follows.

Two different approaches are used in collecting these kinematic chains. A method described by N. I. Manolescu, in which the construction of linkages is proposed by the systematic addition of Assur Groups, is used for collecting those chains having seven, nine and eleven members. In this collection, those chains are omitted which could yield mechanisms having fractionated mobility, as such chains would be unsuitable for the purpose in mind.

An alternative method of determining all the varieties of kinematic chains of a given class is that described by Franke. For the derivations of thirteen-bar and fifteen-bar chains, this approach is utilized. The kinematic chains in these classes are further classified into subgroups according to the number of each type of link (binary, ternary, etc.) contained by the chain. The subgroups are then analyzed to determine which yield workable mechanisms; for example, a thirteenbar chain having twelve binary links and one octagonal link is always most certainly unsatisfactory. An equation is derived to determine which subgroups would provide practical mechanisms (not having fractionated mobility).

The kinematic chains are then analyzed to determine which can yield promising designs. Three mechanisms are selected for further study. Working models of these mechanisms have been constructed to observe the dynamic action of each of these mechanisms.

The collection of those chains belonging to the suitable subgroups are included in this study, so that further investigation of those linkages not selected at this time may be undertaken at a later date.

## CHAPTER I

## INTRODUCTION

A study in the permutation or number synthesis of planar linkages with four degrees of mobility will be used as the preliminary stage in the design of a mechanical decoder. The decoder is to have a four-bit binary input with a sixteen-position rotary or translational output, and will be expected to perform at high speeds with relatively small input motions.

## Preliminary Design Considerations

The primary consideration in this design is the need for small size and rapid action of the elements in the mechanism. Due to the high operating speeds, inertia forces can cause an output to lag behind the input during acceleration, and overshoot it during deceleration. These effects are caused by the storing of strain energy during the acceleration phase, and the release of this energy during the deceleration.

Based upon the preceding considerations, it was decided that the mechanism should be a linkage. Linkages are extremely suitable for highspeed operation. They are light and yet rigid enough to resist excessive elastic deformations caused by the inertia forces. Additionally, linkage mechanisms are relatively free of friction, and, when compared with other similar mechanisms, are inexpensive to manufacture.

Though this study concerns itself entirely with linkages, the use of other constructional elements such as cams, pulleys, or gears will not
be precluded from the final design, and the eventual incorporation of these elements may in fact be found desirable based on the analysis of the linkage synthesis.

## Terminology

Link
A rigid body with two or more kinematic elements is called a link. Each element represents half of a kinematic "pair" forming a joint with another link. A binary link has two elements; a ternary link has three elements; a quaternary link has four elements; and so on. Kinematic Chain

A kinematic chain is an assemblage of links connected by kinematic pairs (joints). If each link in the chain is connected to at least two other links, the chain is closed; otherwise, it is open. A kinematic chain will always be considered closed unless otherwise stated.

Mechanism
A mechanism is a kinematic chain having at least one degree of mobility with one of its links considered stationary. Degree of Mobility

Mobility is the number of independent variables needed to define the relative positions of the links in the kinematic chain. Isokinetic Kinematic Chain

An isokinetic chain (1) of $X$ degrees of mobility is defined as one in which there exists no subassembly of links, which, when considered alone, would form a kinetic chain with less than $X$ degrees of mobility.

## Permutation Synthesis as a Design Tool

The synthesis of mechanisms may be classified into three phases. The first phase is called type synthesis, in which the form or type of mechanism is determined. For example, this design will utilize a linkage mechanism. The second phase, known as permutation or number synthesis, may be broadly defined as the method of determining the number of different arrangements (permutations) of kinematic chains having a given degree of mobility. The third phase is known as dimensional synthesis, which is the determination of the geometric proportions of the members necessary to accomplish the desired motion requirements.

Most of the work done in the study of linkages has been concerned with the dimensional synthesis, while much less research has been done in the field of permutation synthesis. For example, it is known that there exists one four-bar, two six-bar, and sixteen eight-bar linkages with one degree of mobility. Only recently, Crossley (2) published a collection of 222 distinct varieties of ten-bar chains (with $X=1$ ). It remains unknown, however, how many varieties of one degree of mobility linkages with twelve members or more exist. Even less is known of the varieties of kinematic chains with two degrees of mobility. Both Crossley (3) and Manolescu (4) have published collections of those linkages (with $X=2$ ) belonging to the first three classes, i.e., of five, seven, and nine members, respectively. But it is apparent that as this factor of mobility increases, so does the degree of difficulty in analyzing these linkages.

In spite of the difficulty involved, the advantages to be gained from permutation synthesis are invaluable. By having at hand a complete
collection of a particular class of linkage, careful study will reveal those mechanisms which best fulfill the given design requirements.

By using kinematic inversion, each chain in the collection may offer several different mechanisms based on which link is taken as the reference or frame. The substitution of a slider for a turning joint may also be incorporated to modify the mechanism. Additionally, input and output links may reverse their roles creating new mechanisms.

The design requirements call for a mechanism with four binary inputs. This study therefore concerns itself with the collection and analysis of those kinematic chains having four degrees of mobility.

The kinematic chains are classified by the number of links in the chain. The kinematic chains in each class can be further subdivided according to the number of each type of link (binary, ternary, etc.) contained by the chain. Thereafter, linkages are analyzed to determine which are best suited for a decoder mechanism.

## Basic Structural Concepts

When the motion of a rigid body is confined to a plane, it has three degrees of freedom: one of rotation and two of translation. Considering a system of ( $N$ ) independent disconnected members in the plane, the system then has (3N) degrees of freedom. By connecting any two members with a hinge or joint, the number of degrees of freedom of the system is reduced by (2J). Since we are only concerned with the relative motions of the members, one of the links must be considered as the frame or reference link. Therefore, three degrees of freedom may be reduced from the system; the result is finally:

$$
X=3 N-2 J-3
$$

or,

$$
X=3(N-1)-2 J
$$

where

```
X = number of degrees of mobility (freedom)
N = number of links
J = number of kinematic joints
```

This equation is commonly known as the Gruebler Criterion for planar kinematic chains. Although usually ascribed to Gruebler in 1883, an equivalent relation was derived by Chebyshev in 1869. The present derivation, however, has followed the simplified method of Bottema (5).

For a system with four degrees of mobility, equation (1) becomes

$$
4=3(N-1)-2 J
$$

therefore,

$$
\begin{equation*}
N=\frac{7+2 J}{3} \quad(J \geq 7) \tag{2}
\end{equation*}
$$

The possible combinations of N and J for a kinematic chain with four degrees of mobility are therefore

| N | 7 | 9 | 11 | 13 | 15 | $7+2 i$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| J | 7 | 10 | 13 | 16 | 19 | $7+3 \mathrm{i}$. |$(i=0,1,2, \ldots)$

These kinematic chains may consist of binary, ternary, quaternary, etc., members. The total number of links and joints may be expressed as

$$
\begin{equation*}
N=n_{2}+n_{3}+n_{4}+n_{5}+\ldots+n_{i}=\sum_{i=2} n_{i} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
2 J=2 n_{2}+3 n_{3}+4 n_{4}+\ldots+(i) n_{i}=\sum_{i=2}(i) n_{i} \tag{4}
\end{equation*}
$$

Substituting Equations (3) and (4) into Equation (1)

$$
\begin{equation*}
x=n_{2}-3-\sum_{i=4}(i-3) n_{i} \tag{5}
\end{equation*}
$$

The significance of Equations (3), (4), and (5) may be determined by following the reasoning discussed by Crossley (3), who has outlined an algorithm for determining the combinations of component members of linkages based on these equations.

In any linkage (of mobility $X$ ) having $N$ members consisting of $n_{2}$ binary links, $n_{3}$ ternary links, ..., $n_{p}$ polygonal links of $p$ sides, there exists another linkage having the same mobility with ( $n_{3}+2$ ) ternary links. In other words, a linkage with a specified mobility is independent of the number of ternary links providing that the additions or deletions of ternary members is of an even number.

Additionally, in any given assortment of links $n_{2}, n_{3}, n_{4}, n_{5}$,
..., $n_{p}$, the quantity of three polygonal links

$$
n_{p-i}, n_{p}, n_{p+i} \quad(p-i \geq 2)
$$

may be changed to

$$
\left(n_{p-i} \pm 1\right),\left(n_{p} \mp 2\right),\left(n_{p+i} \pm 1\right)
$$

where $i$ is any positive integer.
A grouping of the different classes of kinematic chains with four degrees of mobility may now be tabulated based upon the preceding considerations. This classification is represented in Table 1.

## Distinct Configurations

The schematic representation of kinematic chajns may lead to the collection of certain isomorphic pairs that appear unlike. In order to eliminate these discrepancies, distinct configurations will be defined using a method described by Crossley (1,2).

Every kinematic chain may be considered to consist of a set of incident links and joints. To every chain or configuration there exists a related incidence matrix in which, for example, the columns may represent joints, and the lines represent links. Each element of the matrix is either 1 or 0 , depending on whether the designated link and joint are, or are not, incident. For example, in the configurationof Figure 1(a), if the links are numbered and the joints are designated by letters in some arbitrary manner, the incidence matrix appears as in Figure l(b). Now, since the choice of designating letters and numbers"was arbitrary, any column may be interchanged with any other column, or any line with any other line, and all such matrices will correspond identically to the same configuration.

A configuration is then distinct (unique) when its incidence matrix cannot be made to agree with the incidence matrix of any other pattern by interchanging lines or columns.


Table l. Classification of Kinematic Chains With Four Degrees of Mobility (Cont inued)

| CLASS | $N_{t}$ | $n_{2}$ | $n_{3}$ | $n_{4}$ | $n_{5}$ | $n_{6}$ | $n_{7}$ | $n_{8}$ | $n_{9}$ | $n_{10}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $V(n)$ |  | 12 | 1 | 0 | 1 | 1 |  |  |  |  |
| (o) |  | 12 | 0 | 1 | 2 |  |  |  |  |  |
| (p) | 12 | 1 | 1 | 0 | 0 | 1 |  |  |  |  |
| (q) | 12 | 0 | 2 | 0 | 1 |  |  |  |  |  |
| (r) | 13 | 0 | 0 | 0 | 2 |  |  |  |  |  |
| (s) | 13 | 0 | 0 | 1 | 0 | 1 |  |  |  |  |
| (t) | 13 | 0 | 1 | 0 | 0 | 0 | 1 |  |  |  |
| (u) | 13 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |
| (v) | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |

The following conclusions may then be made:

1. Mirror images do not represent a new configuration.
2. The choice of a joint connecting a link to a neighboring link is arbitrary.
3. Every configunation may be considered elastic and may be turned inside out.

Two examples of iscrorphic pairs which appear unlike are shown in Figure $1(\mathrm{c})$.
(a)

(b)

JOINTS

(c)


Figure 1. (a) A Kinematic Chain Described by an Incidence Matrix, (b) An Incidence Matrix, and (c) Two Examples of Isomorphic Pairs That Appear Unlike.

THE DERIVATION OF THE KINEMATIC CHAINS OF
CLASS I, II, AND III USING MANOLESCU'S METHOD

## Manolescu's Method

Kinematic chains with four degrees of mobility will be studied using a method proposed by Manolescu (4). Those chains of Class I, II, and III, that is to say those having seven, nine, and eleven members, respectively, will be discussed in this chapter.

The degree of mobility of any planar kinematic chain may be expressed as

$$
\begin{equation*}
X=L-3 \tag{6}
\end{equation*}
$$

where $L$ is defined as the number of degrees of freedom of the kinematic chain before the selection of the reference link. It may then be noted that Manolescu utilizes the notation "degree of freedom" to indicate the number of independent variables in a chain before the selection of the reference link, whereas this same term is often taken to have the same meaning as mobility, which is, of course, evaluated after the selection of a reference link.

Thus, the five-bar open chain of Figure 2 has seven degrees of freedom (before selection of the reference link) and four degrees of mobility (after the selection of a reference link).


Figure 2. A Representation of a Kinematic Chain Restricted to Planar Motion.

In forming kinematic chains, Manolescu makes use of Assur Groups. These groups were introduced by the Russian Scientist L. V. Assur (6), and may be broadly defined as combinations of kinematic links and joints which, when added to (or subtracted from) a kinematic chain, do not modify the value of the initial mobility.

This combination of links and joints may be considered as an (open) chain having both "interior" joints (coupled elements) which connect the links of the Assur Group in some configuration, and "exterior" or free joints (uncoupled elements) which may attach themselves to another chain; for example, a free joint of an Assur Group may attach itself to a link $n_{p}$ (having $p$ sides) of some kinematic chain, forming a new link $n_{p+1}$.

> Applying the Gruebler equation for planar kinematic chains to any Assur Group, take

$$
\begin{equation*}
X=3(N-1)-2 J \tag{1}
\end{equation*}
$$

and then set $X=0$, and $N^{\prime}=(N-1)$, to obtain

$$
\begin{equation*}
3 N^{\prime}-2 J=0 \tag{7}
\end{equation*}
$$

Such a configuration of $N^{\prime}$ links and $J$ joints may be arrived at by taking a kinematic chain with mobility $X=0$, and removing one link $n_{j}$ (leaving $N^{\prime}$ links on the chain), but retaining the $J$ joints (as uncoupled elements) on the adjacent links. This configuration meets the conditions of equation (7) and may therefore be attached (by using methods to be described later in the chapter) to any kinematic chain without altering the mobility of that chain. However, not every assemblage or configuration of links and joints meeting these conditions will be considered as a characteristic Assur Group, due to certain non-isokinetic linkages which they may yield, as will also be explained later in this chapter.

From Equation (7), it follows that an Assur group may be considered as an assemblage of links and joints which, when connected to a frame, will have a mobility of $X=0$. Hence, it is evident that an Assur Group must always contain an even number of links. They may therefore be classified by the total number of links and joints in the configuration as follows:

| $N^{\prime}$ | 2 | 4 | 6 | 8 | $2 i$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $J$ | 3 | 6 | 9 | 12 | $3 i$ |$\quad(i=1,2,3, \ldots$.

where J includes both interior and free joints. Assur Groups may also be classified by the number of free joints they will yield. For example, a dyad (consisting of two binary links) has two free joints, while Assur Groups having four members will yield a maximum of three and a minimum of two free joints; Assur Groups having six members will yield a maximum of four and a minimum of two free joints. They may also be classified by the number of interior loops or circuits they possess. For example, this number may range from zero to one for those groups with four links, and from zero to two for those groups with six links. A collection of characteristic Assur Groups is shown in Figure 3.

Therefore, a distinct kinematic chain may be constructed by the joining of an Assur Group to another kinematic chain. It is evident then, that the use of Assur Groups can be an extremely useful tool for the collection of kinematic chains or mechanisms with any given degree of mobility.

In a paper by Manolescu, Erceanu, and Antonescu (7), the seventyone eight-bar mechanisms and five six-bar mechanisms of one degree of mobility are constructed utilizing the inclusion of Assur Groups. This particular procedure does not concentrate on the kinematic chain, but rather on the mechanisms that are formed from simpler mechanisms as the result of adding Assur Groups.





Figure 3. A Collection of Characteristic Assur Groups.

Their paper defines this particular derivation of mechanisms as being either of "amplification" or "enlargement." The amplification of a mechanism is achieved by joining all the exterior or free joints of the Assur Group to only mobile elements of the mechanism; the initial mechanism and the final mechanism will therefore have an identical reference link. On the other hand, enlargement of a mechanism is affected by joining the free joints of the Assur Group with one or more of the mobile elements, except that at least one joint is attached to the frame. In both cases, however, all free joints must not be connected to the same link. It follows then, that if these two operations are utilized, all the varieties of mechanisms with a certain number of links may be obtained. An example of some mechanisms formed by amplification is shown in Figure 4(a), and some of those mechanisms formed by enlargement may be seen in Figure 4(b).

Since the collection of mechanisms can be accomplished most easily by kinematic inversion of a kinematic chain, the purpose here will be to utilize the Assur Groups in the collection of kinematic chains rather than mechanisms. Manolescu has outlined such a procedure (4) in the collection of kinematic chains having one and two degrees of mobility.

The procedure is initiated by starting with a kinematic chain of a given mobility having the smallest possible number of links. Then, Assur Groups may be added to this chain forming new chains. Therefore, if the simplest chain of a given mobility has (m) members, the next larger chain will have ( $m+2$ ) members, due to the addition of a dyad having two links. The next class of chains will have ( $\mathrm{m}+4$ ) members, and may be formed either by adding another dyad to different links of the


Figure 4. An Example of the Derivation of Mechanisms having One Degree of Mobility by (a) Amplification, and (b) Enlargement.
$(m+2)$ chain or by adding an Assur Group of the next larger size (four members) to the simple chain of ( m ) members.

An example of this process pertaining to chains with one degree of mobility is shown in Figure 5. It may be noted that when adding the dyad to the four-bar linkage in Figure 5(c), it may attach itself in one of two ways, that is, either on adjacent or opposite links. As the chains grow larger, there will be larger numbers of distinct positions for the dyad (or any Assur Group) to attach itself.

It follows that the same or different Assur Groups may be connected to each other forming variants which also meet the conditions of Equation (7). Some of these variants, however, are undesirable. For example, the addition of two dyads may produce the resulting variant as shown in Figure 6. As there are two free joints (a and b) on adjacent links (l and 2), the connection of these joints to ancther link would result in a rigid triangular loop. Therefore, any variant which contains free joints on adjacent links will not be utilized in this collection. A collection of several acceptable variants of characteristic Assur Groups is shown in Figure 7.

In many cases, there is no single characteristic Assur Group or suitable variant which can be used in the construction of a particular chain. In such cases, two or more Assur Groups must be added independently to a lesser chain. Linkage $9 / 4$ of Table 3 is such a case. It is interesting to note that the variant shown in Figure 6 could actually be used in this case, since the free joints a and b would not be attached to the same link. However, rather than consider special cases, such unsuitable variants will not be utilized in this collection, inasmuch as
(a)

$$
2_{1}+\sqrt[3]{4}=
$$



(c)


Figure 5. An Example of the Construction of Kinematic Chains by the Addition of Basic Assur Groups.


Figure 6. The Formation of an Unsuitable Variant from Two Characteristic Assur Groups.
their inclusion is not essential.

A collection may now be made of the kinematic chains with four degrees of mobility. Before starting, however, it should be noted that those chains having fractionated mobility will not be included in this collection.

The condition of fractionated mobility arises when a kinematic chain consists of two independent linkages sharing the same reference link. An example of a kinematic chain having fractionated mobility is shown in Figure 8. This chain has an overall mobility of two, but it is made up of two individual linkages, each with one degree of mobility. For, with link 3 as the frame, it would consist of two mechanisms able to move quite independently.

A Collection of Kinematic Chains of Class I, II, and IJ.I

The collection may now be initiated by starting with the basic kinematic chain having four degrees of mobility. Thus, the collection will begin by utilizing the five-bar open chain, which, as mentioned earlier in this chapter, contains the smallest number of links (four input members and a frame.) This procedure is illustrated in Table 2.


Figure 7. A Collection of Several Acceptable Variants of Basic Assur Groups.


Figure 8. The Formation of a Kinematic Chain ( $X=2$ ) having Fractionated Mobility, Building Twice from the Base Link No. 3.

Table 2. The Method of Formation of the Kinematic Chain Having $L=7, X=4, N=7$, and $J=7$

| Initial Chair: + Assur Group | Resulving Ches: No. |
| :---: | :---: |
|  |  |

There results one, and only one chain with seven members and four degrees of mobility. Now that the basic closed chain has been derived, the collection will only consider those kinematic chains which are closed, though the five-bar open chain will be used considerably in the derivation of such chains.

The kinematic chains having nine members may now be formed by adding a dyad to the seven-bar chain just found, or by the addition of Assur Groups having four members to the five-bar open chain. This procedure is performed in Table 3.

There are then six distinct nine-bar linkages that exist with four degrees of mobility. However, it should be noted that three isomorphic chains were formed during this process. As the chains continue

Table 3. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=9$, and $J=10$
initial he:

Table 3. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=9$, and $J=10$
(Continued)

|  |  |
| :---: | :---: |
|  | identical to $9 / 3$ |
|  |  |
|  |  |

Table 3. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=9$, and $J=10$ (Continued)
Initial Chain + Assur Group Resulting Chain
to grow in size, it is inevitable that this duplication will increase. Unfortunately, the duplication of effort appears to be unavoidable, and care must be taken to identify any isomorphic chains in the collection.

The collection of kinematic chains in Class III may now be determined. A collection of eleven-bar chains is shown in Table 4. The derivation of duplicate chains will not be illustrated in this collection.

Examination of Table 4 reveals that there are at least 128 distinct eleven-bar chains with four degrees of mobility. Though the differences in the construction of many of these chains may appear trivial, their inclusion in the collection is warranted if this method is to be used to derive kinematic chains having more than eleven members. As the chains grow larger, an increasing difficulty in applying this method is apparent. The number of duplications increase considerably as do the number of possible combinations of joining Assur Groups to the lesser chains.

In most cases, only those kinematic chains which may yield practical mechanisms are desired. A complete collection of such chains could not practically be achieved by use of this method, since it does not differentiate between subgroups of a particular class.

Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$


Table 4. The Method of Formation of the Kinematic Shains Having $L=7, X=4, N=11$, and $J=13$ (Continued)

| initial Chain ${ }^{+}$Assur Grour | Resulting Chains |
| :---: | :---: |
|  |  |

Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$ (Continued)

| Initial Chain <br> Assur Group |
| :---: |

Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$
(Continued)


Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$
(Continued)


Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$


Table 4. The Method of Formation of the Kinematic Chains Having $\mathrm{L}=7, X=4, N=11$, and $\mathrm{J}=13$
(Continued)
Initial Chain
Assur Group

Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$ (Continued)

| ${ }_{\text {In }}$ Inimat Ciain | Reemulurs ciatas |
| :---: | :---: |
| $\begin{gathered} \square \\ +\AA+\AA \end{gathered}$ |  |
|  |  |
| 约 |  |
| $\begin{aligned} & \dot{8} \\ & +8 \end{aligned}$ |  |

Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$
(Continued)


Table 4. The Method of Eormation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$
(Continued)
Assur Group

Table 4. The Method of Formation of the Kinematic Chains Having $L=7, X=4, N=11$, and $J=13$

| Initial Chain <br> Assur Group |
| :---: |

## APPLICATION OF FRANKE'S METHOD

## Initial Stipulations

An alternative method of determining all the varieties of kinematic chains of a given class is that described by Franke (8). The following derivations of thirteen-bar and fifteen-bar chains will make use of this approach.

Before beginning this study, the following requirements particular to the decoder mechanism must be considered:

1. The design requirements call for four inputs and one output. The collection of suitable kinematic chains will therefore be based upon those chains having a reference link with at least five kinematic joints.
2. Kinematic chains having fractionated mobility are obviously unsuitable for the decoder mechanism and will be disregarded. For any particular subgroup (such as $10-2-0-0-1$ ), if the number of joints $J_{f}$ on the frame meets the condition

$$
\begin{equation*}
J_{f}>\sum_{i=3}\left(\frac{i-2}{2}\right) n_{i}+2 \tag{8}
\end{equation*}
$$

then the subgroup may only produce chains of fractionated mobility, and will be omitted from the collection. Three examples of kinematic chains of this type are shown in Figure 9.


Figure 9. Two Thirteen-Bar Kinematic Chains and a Fifteen-Bar Kinematic Chain having Fractionated Mobility.
3. Any binary link, not connected to the frame, may not be connected to any other binary link unless that link is connected to the Irame. In other words, the inclusion of a binary pair (dyad) into a kinematic chain is of no value unless one of the binary members is an input or output member. Any binary pair located on the girth or located between the girth and input or output members cannot generally transmit motion, but rather will "absorb" the motion.
4. No loop (circuit) having the frame as a member may consist of less than five members with the following exception: one loop of four members will be permitted for each kinematic joint on the frame in excess of five. The inclusion of a loop of four members (including the frame) means that if one of the movable members is held stationary, all other members in the loop will be immovable. Thus, if there were only five kinematic joints on the frame, and there existed a loop of four members including the frame, fixing one input belonging to this loop would cause the other input or output member of the loop to be needlessly fixed.

The preceding stipulations are, unfortunately, not able to prevent the inclusion of all inoperable linkages into the initial collection. Certain inoperable mechanisms may only be detected by careful examination, as they contain irregularities which are less obvious than those listed in the preceding stipulations. In general, however, these irregularities may be defined as follows:

5, All loops which contain an input or output member must be considered independent of other similar loops or circuits. Thus, no such loop containing an input or output member may be held fixed by the action of fixing other input links belonging to the same and/or other loops.

## Thirteen-Bar Chains

Of the eleven groups in Class IV, only (d) and (g) are feasible for the decoder mechanism. The other nine groups in this class are eliminated because of the initial stipulations given in the preceding section. Class IV (d) 9-3-0-1

In Figure 10, all members except the binary links are arranged in all of the possible configurations (permutations). Each of the twelve permutations may be further subdivided into different paths which the binary members may take.

For example, considering permutation 1 , it can be seen from Figure li(a) that there are seven paths open for the nine binary members. Now, considering the loops I, II, III, and IV, it is necessary that each loop (considered by itself) have at least two degrees of mobility. Each loop must therefore have a minimum of five members. This imposes the condition that paths $a, b, c, d$, and $e$ must be so arranged.

There is, therefore, only one possible kinematic chain that may arise from this permutation, and it is as shown in Figure ll(b).

Permutation 2 yields three kinematic chains; however, one is unsuitable as may be seen in Figure 12. Both chains (a) and (b) are acceptable; close examination of (c) reveals that it will be inoperable due to stipulation $N$. 5 stated at the beginning of the chapter. For example, if links 1 and 2 are fixed, link 3 will be fixed also. Likewise, if links 1 and 3 , or 2 and 3 are fixed, then the same result would happen to links 2 and 1 , respectively. Therefore, chain (c) will be dropped from the collection.
(2)

Figure 10. Permutations of Class IV (d) 9-3-0-1.

(a)

(b)

Figure ll. An Example of Applying Franke's Method.

Permutation 3 produces two distinct kinematic chains, both of which are suitable. No further kinematic chains may be derived from this group as the initial stipulations will prevent the use of the remaining permutations.

Class IV (g) 10-1-1-1
There are two basic configurations in this group, as shown in Figure 13. From permutation l.of Figare l3, there-are two resulting chains, as shown in Figure 14. Linkage (b), however, is inoperable, due to the inevitable formation of an Assur Group involving the quaternary member. In other words, if any two of the binary input links leading to the quaternary link are held stationary, the third link, whether it be an input or an output member would be unnecessarily fixed.

From permutation 2 there is only one resulting kinematic chain. A complete collection of all the chains in class IV is shown in Table 5.


(a)

(b)


Figure 12. An Example of Applying Franke's Method.


1


2

Figure 13. Permutations of Class IV (g) 10-1-1-1.


Figure 14. Suitable and Unsuitable Kinematic Chains Derived from Permutation 1 of Figure 13.

Table 5. A Collection of the Suitable Kinematic Chains of Class IV

Class IV (d) 9-3-0-1


Class IV (g) 10-1-1-1


Of the twenty-two groups in Class $V$, there are initially only nine which may yield satisfactory mechanisms. The thirteen other groups were eliminated based upon stipulations No. 1 and No. 2.

The suitable groups in Class $V$ are as follows:

| V (d) $9-5-0-1$ | $\mathrm{~V}(\mathrm{k})$ |
| ---: | :--- |
|  | $11-2-0-2$ |
| (e) $10-4-0-0-1$ | (n) $12-1-0-1-1$ |
| (g) $10-3-1-1$ | (o) $12-0-1-2$ |
| (i) $11-1-2-1$ | (q) $12-0-2-0-1$ |
| (j) $11-2-1-0-1$ |  |



Eigure 15. Permutation 1 of the Group 9-5-0-1.

The analysis of the permutations of the links in Class $V$ is of much greater complexity than those of Class IV. As a result, the determination of the existing linkages in this class by a systematic analysis such as that of Franke is one of enormous proportions.

As an example, consider the permutation of group $V(d)$ as shown in Figure 15 above. From this one configuration alone, there are four different paths which the binary members may occupy. These different options may be seen in Figure 16. Option (a) alone yields six suitable kinematic chains which are shown in Eigure 17.


Figure 16. Four Distinct Arrangements of Permutation 1.


Figure 17. The Resulting Kinematic Chains from Arrangement (a).

In many cases with linkages in this class, the formation of "hidden" Assur Groups is more difficult to find than in the Class IV chains. For example, the linkage shown in Figure 18 (a) may easily be taken as a workable mechanism; however, close examination will reveal otherwise.

A detailed examination of the linkage in Figure 18 (a) will reveal that whichever link is chosen as the output, a situation of overconstraint will arise. For example, if link 2 is considered as the output, fixing links 1,3 , and 4 will cause output link 2 to be fixed. By holding these three links stationary, an Assur Group has been formed in which output link 2 is a member (see Figure 18 (b)). Since link 2 is immovable, the chain is unsuitable as a decoder mechanism. No matter which link is chosen as the output, a condition of overconstraint will arise, causing an output or an input link to be unnecessarily fixed.

Unfortunately, there is currently no set rule for quickly determining which kinematic chains will be unsuitable. In general, however, one may search for the formation of Assun Groups (or any combination of members with $\mathrm{X}=0$ ) which contain an input or output member.

In many cases, however, the formation of Assur Groups are extremely advantageous providing that an input or output member does not belong to the group. For example, considering the linkage in Figure 19 (a), if input members 1,2 , and 3 are held stationary, then an Assur Group is formed as shown in Figure 19 (b). However, no input or output members belong to this group, so the chain has now been reduced to a six-link Watts mechanism. This characteristic can be very desirable due to the reduction of moving parts and the subsequent reduction of inertia forces, and will be further discussed in Chapter V.


Figure 18. The Formation of a "Hidden" Assur Group.

(a)

(c)

Figure 19. The Formation of an Assur Group and Six-Link Kinematic Chain from a Fifteen-Bar Chain with Three Inputs Fixed.

The process of determining those suitable kinematic chains in Class V then continues. Each distinct permutation must be analyzed to determine the different paths which the binary members may take. The chains may then be sketched and inspected to determine which of those are suitable for the collection. A representative sample of the kinematic chains in Class $V$ is shown in Table 6.

Table 6. A Representative Collection of the Kinematic Chains of Class V


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class V (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)



Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collevtion of the Kinematic Chains of Ciass $v$ (Continued)



Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)


Table 6. A Representative Collection of the Kinematic Chains of Class V (Continued)


## Table 6. A Representative Collection of the Kinematic Chains of Class V (Continued)



Table 6. A Representative Collection of the Kinematic Chains of Class $V$ (Continued)

Class V (q) 12-0-2-0-1


## Basic Concepts

Now that a collection of thirteen-bar and fifteen-bar kinematic chains is at hand, it may be studied to determine which linkages will be best suited for use as decoder mechanisms. However, before initiating this search, the basic concepts of the decoder mechanism will be discussed.

As mentioned previously, the decoder is to have four binary inputs. This immediately leads to the conclusion that there will be sixteen distinct output positions.

By simple application of the binary number theory, if each of the four inputs adds one, two, four, and eight output steps, respectively, then the decoder may be considered essentially a mechanical adder, and will operate by adding (or subtracting) one or more of the input displacements.

No requirements are given as to the spacing between output positions. However, nearly equal spacing between successive output positions is desired.

By selecting a member of the kinematic chain as the frame (a quinary link or larger), the chain may now be defined as a mechanism. By alternately choosing different links as the output member, the mechanism may have as many variations as it has distinct output links.

Though the relative motion of the links in the chain is the same, the input-output relations will vary based on the selection of the output link.

Table 7 shows the different input-output variations of the kinematic chains of Class IV. It should be noted that these mechanisms are generally represented schematically with sliding pairs denoting the input and output members; the purpose of this is to aid in identifying the input and output members. However, unless otherwise indicated, this is not meant specifically to call for the use of a sliding pair in the linkage. A two-position crank would be equally satisfactory.

From the study in the number synthesis of mechanisms with four degrees of mobility, three mechanisms have been selected which appear to have desirable attributes for the decoder design. Two of them are of Class IV (thirteen members) while one is of Class $V$ (fifteen members). A working model of each of the three mechanisms plus a modification of one of these has been designed and built. The properties of each of the three basic mechanisms (which will be denoted as linkage numbers one, two and three) will be discussed in the following sections.

Linkage Number One
This mechanism is derived from a kinematic chain in Class IV (d) $9-3-0-1$, and is shown in part $1(a)$ of Table 7 and in Figure 20. This arrangement appears to be basically a modified linkage differential with the inputs attached in series. It has the advantage that it readily lends itself to analytical derivations of its behavior.

Table 7. Input-Output Inversions of the Kinematic Chains of Class IV (d) 9-3-0-1


Table 7. Input-Output Inversions of the Kinematic Chains of Class IV (d) 9-3-0-1 (Continued)


Table 7. Input-Output Inversions of the Kinematic Chains of Class IV (d) 9-3-0-1 (Continued)


From Figure 20 (b), it may be seen that members two, four, and six need not be triangular shaped, and may take the form of a straight bar (having the advantage of minimum mass). It is interesting to note that the linkage shown in Figure 21 (linkage $5(\mathrm{a})$ of Table 7) has nearly the same basic behavior as linkage No. 1; however, in this case links $\mathrm{a}, \mathrm{b}$, and c cannot be straight bars unless a multi-planar or spacing arrangement is used.

(a)

(b)

Figure 20. The Formation of Linkage No. 1 From Its Kinematic Chain.


Figure 21. A Linkage Similar in Behavior to Linkage No. 1.

In order to study the basic geometry of linkage number one, it may be drawn as shown in Figure 22. It is apparent that the longer the links are in relation to the inputs, the more uniform will be the output. However, if the input displacement is relatively small, the effect of the rotation of links one, three, five, and seven may be neglected.

Now, the values of $x^{\prime}, x^{\prime \prime}$, and $x^{\prime \prime \prime}$ which locate the leverage poilts on links two, four, and six, respectively, may be found as follows:

Letting $A B$ and $C D$ represent arbitrary parallel input displacements bisected by and perpendicular to a common center line, the trapazoid $A B C D$ may be constructed as shown in Figure 22 (b). If $h$ is the vertical distance between lines $A C$ and $B D$ at any value of $x$, then

$$
h=a+(b-a)(x / L)
$$

and

$$
y=h /(n-1)
$$

where $n$ is equal to four, eight, or sixteen equally spaced points on $h$, $y$ is the length of the ( $n-1$ ) equal segments of $h$, and $a, b, L$, and $x$ are dimensions defined by Figure 22 (b). This implies that

$$
\begin{equation*}
y=\frac{a+\frac{b-a}{L}(x)}{n-1} \tag{9}
\end{equation*}
$$


(a)

(b)

Figure 22. Location of the Leverage Points $x^{\prime}, x^{\prime \prime}$, and $x^{\prime \prime \prime}$ on Linkage No. 1.

Letting $z$ denote the value of $x$ where the diagonals $A D$ and $B C$ intersect, the following proportion may be determined from the properties of similar triangles:

$$
\frac{z}{a}=\frac{L-z}{b}
$$

or

$$
\begin{equation*}
z=\frac{a L}{(a+b)} \tag{10}
\end{equation*}
$$

Also, from similar triangles

$$
\frac{z}{a}=\frac{x-z}{y}
$$

or

$$
\begin{equation*}
z=\frac{a x}{a+y} \tag{13}
\end{equation*}
$$

Eliminating 2 from Equations (10) and (11)

$$
\begin{equation*}
\frac{x}{y+a}=\frac{L}{a+b} \tag{1}
\end{equation*}
$$

Substituting the value of $y$ from Equation (9) into Equation (12) and simplifying gives the result

$$
\begin{equation*}
x=\frac{a L}{a+b(n-2) / n} \tag{13}
\end{equation*}
$$

Therefore, for $\mathrm{n}=4$

$$
\begin{equation*}
x^{\prime}=a L /(a+\nu / 2) \tag{14}
\end{equation*}
$$

for $n=8$

$$
\begin{equation*}
x^{\prime \prime}=a L /(a+3 b / 4) \tag{15}
\end{equation*}
$$

for $n=16$

$$
\begin{equation*}
x^{\prime \prime \prime}=a L /(a+7 b / 8) \tag{16}
\end{equation*}
$$

The ratio of the horizontal to vertical links may be taken as two to one with very good results. Thus, if members one, three, five, and seven are taken as one unit length, links two, four, and six can be of two unit lengths and the linkage will have only two basic link sizes with the exception of the input and output members and the varying locations of points $x^{\prime}, x^{\prime \prime}$ and $x^{\prime \prime \prime}$. For an input displacement of one-half of a unit length ( $a=b=\frac{1}{2}$ ), the values of $x^{\prime}$, $x^{\prime \prime}$ and $x^{\prime \prime \prime}$ are 4/3, $8 / 7$ and $16 / 15$ units, respectively.

Summarizing then, it may be seen that linkage number one has the desirable property that its output displacement, due to the series configuration, is relatively easy to determine. This property can become
very important with mechanisms having a large number of members, as the output response may become exceedingly difficult to determine due to the many parameters involved. Additionally, there need be only two basic link sizes in the mechanism, exclusive of input and output members, thus allowing for a compact design with reduced manufacturing costs.

A notable disadvantage, however, is that the output displacement between the sixteen positions is small compared to the input displacements. Since a relatively small input is desirable, the output would most likely need to be amplified.

In papers by Okcuoglu (9) and Voit (10), the operation of the selection system on the IBM Selectric Typewriter is discussed. It is interesting to note that a linkage, similar to linkage number one, is used to rotate the spherical typehead to the proper printing position. The inputs consist of $+1,+2,+2$, and -5 adders, and can rotate the sphere horizontally from the "home" position to one of five positions to the right or left of the home position. Thus, for any row of eleven characters (there are four rows on the sphere), the typehead will be rotated to the correct position. A smaller but similar linkage is used to tilt the sphere to the proper row. To reach the 44 upper case characters, a shift mechanism rotates the typehead 180 degrees to the other side of the sphere.

A working model of linkage number one, shown in Figure 23, was designed and built using relative link sizes of two units for the horizontal members (links two, four and six of Figure 22 (a)), one unit for the vertical members (links one, three, five and seven), and one-half a unit for the input displacement. The input side of link four was
rearranged as shown in Figure 23 to provide a more compact arrangement. The output link was designed as a pointer for demonstrational purposes.

## Linkage Number Two

Linkage number two, as shown in Figure 24 (a) is derived from the same kinematic chain as was linkage number one. However, this is essentially a parallel arrangement whereas linkage number one was considered as a series arrangement.

Referring to Figure 24 (a), if the rotation of members one, three, five and seven is neglected (again assuming relatively small input motion), then the determination of the output response is fairly straightforward. It must be kept in mind, however, that while these assumptions may be acceptable for a preliminary model, a more detailed analysis will be necessary for final design purposes. The basic geometry of linkage number two may now be determined.

Input links A, B, C and D are taken to be the one, two, four, and eight adders, respectively. The value of $x^{\prime}$ may be determined from Equation (14). The values of $a$ and $b$ on link six may be determined as follows:

By similar triangles

$$
\begin{equation*}
\frac{a}{s}=\frac{b}{m} \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{a}{s}=\frac{a+b}{n} \tag{18}
\end{equation*}
$$



Figure 23. A Working Model of Linkage Number One as Shown with
(a) No Inputs Actuated, and
(b) Inputs 1 and 4 Actuated

(a)

(b)

Figure 24. Location of the Leverage Point $x^{\prime}$ on Linkage No. 2.
where $s$ is the input displacement, and $m$ and $n$ are the resultant output displacements when inputs $A$ and $B$, or inputs $C$ and $D$, respectively, have equal displacements s. Combining Equations (17) and (18)

$$
\begin{equation*}
a=b\left(\frac{n-m}{m}\right) \tag{19}
\end{equation*}
$$

Since $m=3 / 15$ and $n=12 / 15$, then

$$
\begin{equation*}
a=3 b \tag{20}
\end{equation*}
$$

When all four inputs are held fixed, and output link E is assumed to be unrestrained, then it may be seen that point $P$, common to link six and the output link, will trace a particular coupler curve depending on the positions of the input links. Thus, for a more exhaustive study of this linkage, the path of the coupler curve of point $P$ must be known for each of the sixteen input combinations. Then the output displacements may be found by determining where the coupler curves intersect the path of the output link $E$.

It is significant to note that during the operation of this mechanism, when inputs $A$ and $B$, or inputs $C$ and $D$ are held stationary, then links one and two, or links three and four, respectively, will also be fixed, lowering the total number of links in motion. If the total number of links in motion going from output position one to each of the other fifteen output positions are added together, and an average taken, linkage number two has an average of 7.20 links in motion--per displacement from output position one to another output position--


Figure 25. Several Design Alternatives of Linkage Number Two.
while linkage number one has an average of 7.47 links in motion. This average considers all links which are not held absolutely rigid by the fixing of certain inputs; however, it does not take into consideration the relative sizes of the members and the differing effects of translation and rotation.

The symmetric character of linkage number two is very desirable and can lead to a neat and compact arrangement of input and output members. For example, the output link could be tucked underneath link six resulting in the symmetric arrangement of Figure 25 (a).

A working model of linkage number two, as shown in Figure 26 was constructed. This model utilized a two-to-one ratio between horizontal links two and four, and vertical links one and three, respectively. Though the following dimensions differ somewhat from the model, making links five and seven one unit length (reducing the number of different link sizes) and using input displacements of one-half a unit length will give excellent results. As is true with the other members, the relative size of link six is arbitrary and may be given a wide range of values providing the proportions established in Equation (20) are maintained.

Slider substitutions for sevenal of the intermediate links could be utilized in this mechanism. For example, members five and seven could be incorporated as sliders, with their respective joints changed into sliding pairs, as shown in Figure 25 (b). Additionally, links one and three may also become sliding members. A model of such a modification was built and is shown in Figure 27.

(a)


Figure 26. A Working Model of Linkage Number Two as Shown with
(a) No Inputs Actuated, and
(b) Inputs 2 and 4 Actuated


Figure 27. A Modification of Linkage Number Two Incorporating Slider Substitutions.

Though the use of sliding pairs appears to result in a more compact arrangement, the use of such members may mean increased friction and wear in the system, backlash, and higher manufacturing costs. On the other hand, efficient design could reduce the inertia of the system justifying some of the shortcomings of sliding pairs in this mechanism.

## Linkage Number Three

This particular mechanism is derived from a kinetic chain in Class V (d) 9-5-0-1, and is represented in Figure 28. This linkage is a marked departure from the series or parallel arrangements of linkages numbers one and two, and has the advantage of an extremely symmetric configuration well suited for purposes of a practical design. Additionally, it appears from initial experimentation to have an excellent response to relatively small input displacements.

On the other hand, the output response is extremely difficult to predict intuitively. The output motion cannot be traced directly from the input displacements as it was in the series configuration of linkage number one. For example, close examination will reveal that there is no partial constraint or formation of Assur Groups with any members of the chain when any three inputs are held stationary. Additionally, there does not exist any loop or circuit in the chain with less than five members, even when all inputs are held stationary.

Because of the large number of parameters involved, an analytical determination of the output motion would necessarily be quite involved. Therefore, for the purposes of a preliminary design, a trial-and-error procedure is warranted.


Fizure 28. The Formetion of Linkage No. 3 from Its Kinematic Chain.

One of the most rewarding methods of determining the behavior of any complex linkage is to build a model to olvan some idea of its behavior. First, initial assumptions must be made concerning: (a) the relative size of the links, (b) selection of the output link, and (c) the relative locations of the input and output members.

To obtain a basis for these assumptions, analogies may be made to simpler mechanisms that might resemble the behavior of the one in question. For example, linkage number one and number two were considered as series and parallel modifications of a linkage differential.

In order to provide a starting point for the mechanism in question, its output displacement may be considered analogous to à four-bar linkage. For example, referring to Eigure 29 , the linkage may be constructed so that the actuation of input link fourteen causes negligible motion in link eleven, and that the actuation of input link thirteen causes negligible motion of link eight. Then points $O_{b}$ and ${ }^{0}$ c, respectively, may be considered stationary. In both cases, an equivalent four-bar linkage is formed involving output link two, the motion of which may be considered a function of input links thirteen and fourteen, respectively.

Now, if the linkage is constructed so that the points $0_{a}$ and ${ }^{0} \mathrm{~d}$ have relatively small motion from the action of input links twelve and fifteen, respectively, two more equivalent four-bar linkages may be considered formed, with points $0_{a}$ and $O_{\epsilon}$ or points $0_{d}$ anc $o_{e}$ ?s reference points, with the output a function of inputs twelve and fifteen, respectively.


Figure 29. Applying a Four-Bar Linkage Analogy to a Complex FifteenBar Linkage.

Then, by constructing a model that will hold closely to the preceding analogy, a suitable mechanism may be designed. The location of the leverage points on links four and seven and the displacements of the input links may be determined by a similar analytical procedure as that of linkage number two. It must be kept in mind, however, that this analogy is approximate at best, and that certain modifications will inevitably have to be made based on the response of the mechanism to the initial values of the design parameters.

A mechanism, which was built on this basis, is shown in Figure 30. It is, of course, one solution only, but was built to provide a valid basis for determining initial values of the design parameters which must be assumed for the more explicit analytical solution which must follow, although it was not attempted during the present study.

The outstanding feature of the operation of this model was its response to relatively sma.ll input displacements. Joints $0_{a}$ and $O_{b}$ and joints $O_{c}$ and $O_{d}$ on links four and seven, respectively, may be considered as leverage points. By locating the corresponding joints close to each other, small input displacements will be amplified, resulting in comparatively large output displacements.

It should be noted, however, that care must be taken to avoid the condition of dead centers between links three and four, and between links three and seven. Thus, the output response is apparently limited if this condition is to be avoided.

In summary, this model may be considered as having a large output response compared to relatively small input displacements, which, in comparison to the small output displacements of linkages number one and


Figure 30. A Working Model of Linkage Number Three as Shown with
(a) Inputs 1, 2, and 4 Actuated, and
(b) All Inputs Actuated.
number two, could offset the fact that it has two additional members. Its configuration is essentially symmetric, and therefore well suited for purposes of a practical design. The exact behavior of this linkage is, however, extremely complex, and can only be precisely determined by a detailed analytical approach.

An Inertia Study of Linkage Number One and Number Two
Because of the similarity of linkage number one and number two, it was decided that a comparative study of the energy required to overcome the inertia of these linkages might provide another basis for deciding which is the better of the two for the purpose in mind. The mechanisms will be considered to receive step inputs of velocity (v) at each of the four input positions, depending upon which output position is desired. It must be kept in mind, however, that this is a purely hypothetical case for comparative purposes only, since a step velocity input may never be actually realized.

For any linkage nember (i) in translation, the energy required is

$$
E_{t}=\frac{1}{2} m_{i}\left(\nabla_{i}\right)^{2}
$$

or

$$
\begin{equation*}
E_{t}=\frac{1}{2} m_{i}\left(c_{i} v\right)^{2} \tag{21}
\end{equation*}
$$

where $c_{i}$ is a constant based upon that portion of the input velocity ( $v$ ) which is transmitted to link (i).

In addition, the energy required for a member in rotation is

$$
E_{r}=\frac{1}{2} I_{i} \omega_{i}{ }^{2}=\frac{1}{2} I_{i}\left(\frac{v_{i}}{E_{i}}\right)^{2}
$$

or

$$
\begin{equation*}
E_{r}=\frac{1}{2} I_{i}\left(\frac{c_{i}}{d_{i}}\right)^{2} \tag{22}
\end{equation*}
$$

where ( $d_{i}$ ) is the distance from the axis of wotation (or instant center) to the point of application of velocity $\left(\mathrm{v}_{\mathrm{i}}\right)$.

Linkage Number One
Referring to Figure 22, it will again be assumed that members one, three, five and seven move in pure translation, so that the small rotational motions of these links may be neglected. Members two, four, and six will move either in translation or rotation. The energy required to overcome the inertias of the four imput members and the output member will be disregarded as they will be approximately the same for both linkage number one and linkage number two.

The moment of inertia of those links subject to rotation may be determined by first finding the instant center of rotation. To get from output position one to output position six for example, link four receives two distinct velocity inputs as shown in Figure 31. The value of ( $r$ ), as defined by Figure 31 , and the subsequent location of the instant center, may be found using the properties of


Figure 31. Determining the Instant Center of Rotation of Link 4 upon Operation from Output Position One to Output. Position Six.
similar triangles.
Assuming the link is a homogeneous rod of length $L$

$$
\begin{aligned}
I_{i} & \left.=\frac{i}{12} m_{i}\left(L_{i}\right)^{2}+m_{i} \cdot \frac{L_{i}}{2}+r\right)^{2} \\
& =m_{i}\left[\frac{L^{2}}{12}+\left(\frac{L_{i}}{2}+r\right)^{2}\right]
\end{aligned}
$$

letting $\mathrm{L}=2$ unit iengths

$$
\begin{equation*}
I_{i}=m_{i}\left[\frac{1}{3}+(1+r)^{2}\right] \tag{24}
\end{equation*}
$$

setting $q_{i}=\left[\frac{1}{3}+(1+r)^{2}\right]$, and $\frac{c_{i}}{d_{i}}=a_{i}$, then

$$
\begin{equation*}
E_{r}=\frac{\pi_{i} v^{2}}{2}\left(\bar{q}_{i} \bar{a}_{i}^{2}\right) \tag{25}
\end{equation*}
$$

Now, the value of ( $r$ ) for any horizontal link (two, four, or six) may vary from zero to infinity (translation) depending upon which inputs are actuated. Thus, the energy required to actuate a horizontal link in rotation may be expressed by Equation (25). The ?ink will be in translation when it receives inputs of equal magnitude, and the energy required for such motion is expressed by Equation (21). Since vertical links one, three, five, and seven will only be considered to move in translation, the energy required will also be expressed by Equation (21).

The total energy involved in the operation from output position one to any other output position may be expressed as

$$
\begin{equation*}
E_{\text {total }}=\frac{v^{2}}{2} \sum_{i=1}^{7} m_{i} k_{i} \tag{26}
\end{equation*}
$$

where $k_{i}$ is equal to $\left(c_{i}{ }^{2}\right)$ if the link (i) is in translation, and is equal to $\left(q_{i} a_{i}{ }^{2}\right)$ if the link is in rotation.

If the energy required to operate the linkage from position one to each of the other fifteen positions is calculated, a comparison between the two linkages may be determined. The energy equations for linkage number one, neglecting input and output members, may be written as follows:

$$
\begin{aligned}
& E_{1-2}=\frac{v^{2}}{2}\left[m_{1}+\frac{m_{2}}{3}+\frac{m_{3}}{9}+\frac{m_{4}}{27}+\frac{m_{5}}{49}+\frac{m_{6}}{147}+\frac{m_{7}}{225}\right] \\
& E_{1-3}=\frac{v^{2}}{2}\left[\frac{m_{2}}{3}+\frac{4 m_{3}}{9}+\frac{4 m_{4}}{27}+\frac{4 m_{5}}{49}+\frac{4 m_{6}}{147}+\frac{4 m_{7}}{225}\right] \\
& \text { •. } \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
& E_{1-15}=\frac{v^{2}}{2}\left[\frac{m_{2}}{3}+\frac{4 m_{3}}{9}+\frac{19 m_{4}}{27}+\frac{36 m_{5}}{49}+\frac{127 m_{6}}{147}+\frac{196 m_{7}}{225}\right] \\
& E_{1-16}=\frac{v^{2}}{2}\left[m_{1}+m_{2}+m_{3}+m_{4}+m_{5}+m_{6}+m_{7}\right]
\end{aligned}
$$

and, the total energy required to move from position one to each of the other fifteen output positions may be expressed as

$$
\begin{aligned}
E_{\text {total }} & =E_{1-2}+E_{1-3}+\ldots+E_{1-15}+E_{1-16} \\
& =31.3 \mathrm{mv}^{2}
\end{aligned}
$$

where (m) is the mass per unit length, and links one, three, five, and seven are of one unit length, while links two, four, and six are of two unit lengths. The value of the constants $\left(c_{i}\right)$ and $\left(q_{i} a_{i}{ }^{2}\right)$ for each distinct output position may be found in Table 8.

## Linkage Number Two

This linkage will be modified slightly as shown in Figure 32 so that the output displacement and the relative sizes of the links will be similar to that of linkage number one. The same basic assumptions will hold as with linkage number one. That is, the rotation of links one, three, five, and seven will be neglected, and they will each be of one unit length. Links two, four, and six will each be of two unit lengths. Again, the inertias of the input and output members will not be considered. The energy equations may then be written as follows:

$$
\begin{aligned}
& E_{1-2}=\frac{v^{2}}{2}\left[m_{1}+\frac{m_{2}}{3}+\frac{m_{5}}{9}+\frac{m_{6}}{27}\right] \\
& E_{1-3}=\frac{v^{2}}{2}\left[\frac{m_{2}}{3}+\frac{4 m_{5}}{9}+\frac{4 m_{6}}{27}\right] \\
& \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
& E_{1-15}=\frac{v^{2}}{2}\left[\frac{m_{2}}{3}+m_{3}+m_{4}+\frac{4 m_{5}}{9}+\frac{19 m_{6}}{27}+m_{7}\right]
\end{aligned}
$$



Figure 32. Another Configuration of Linkage No. 2 with the Same Relative Iink Sizes and Output Displacement as that of Linkage No. 1.

$$
E_{1-16}=\frac{v^{2}}{2}\left[m_{1}+m_{2}+m_{3}+m_{4}+m_{5}+m_{6}+m_{7}\right]
$$

The total energy required to move from output position one to each of the other fifteen positions may now be expressed as follows:

$$
\begin{aligned}
E_{\text {total }} & =E_{1-2}+E_{1-3}+\ldots+E_{1-15}+E_{1-16} \\
& =33.0 \mathrm{mv}^{2}
\end{aligned}
$$

The value of the constants $\left(c_{i}\right)$ and $\left(q_{i} a_{i}{ }^{2}\right)$ may be found in Table 9.
From the results of the previous calculations, it appears that under the given conditions, slightly less energy is required to operate linkage number one. This may seem surprising in that linkage number two has a smaller average number of links in motion. However, it may be seen that the members of linkage number two are subject to more pure translation than those members of linkage number one, and it may readily be determined that it requires more energy to displace one of the smaller links in translation $\left(m v^{2}\right)$ than one of the larger links in rotation about one end $\left(\frac{2}{3} \mathrm{mv}^{2}\right)$. Thus, reducing the mass of the vertical members could significantly improve the output response of both mechanisms. It must be kept in mind, however, that reducing the length of these vertical links could affect the linearity of the output displacement. Thus, it is up to the designer to find the proper balance between the two factors, based on which is the more essential.

## CHAPTER VI

CONCLUSIONS

## Application of Manolescu's Method

A method proposed by N. I. Manolescu has been used in the collection of kinematic chains of Class I, II, and III, having seven, nine, and eleven members, respectively, and four degrees of mobility. The results of applying this procedure to those chains are given as follows:

1. There exists one seven-bar kinematic chain. It is the single loop chain and consists of seven binary members.
2. There are six distinct nine-bar chains, each containing seven binary members and two ternary members.
3. There exist at least 128 eleven-bar chains exclusive of those with fractionated mobility. They may be classified in the three subgroups (a) 7-4, (b) 8-2-1, and (c) 9-0-2.
4. As the chains grow larger, it becomes increasingly difficult to apply this method. Duplications increase considerably, as do the numbers of possible ways of attaching Assur Groups to the lesser chains. Additionally, the method does not concentrate on the derivation of any particular subgroup, but rather on the whole class of chains. Thus, this method appears to be best suited for the collection of those chains belonging to the first two or three classes which contain the smaller number of members.

## Application of Franke's Method

By analyzing the design requirements, it was determined that the minimum number of links for a practical mechanism would be thirteen. A collection was made of the thirteen-bar and fifteen-bar kinematic chains of Class IV and $V$, respectively, utilizing the method of Franke. However, only those subgroups were derived which were not of fractionated mobility, since chains belonging to such subgroups would not yield mechanisms applicable to our purpose. Equation (8) was derived which indicated the conditions for a subgroup to be of fractionated mobility.

The results of applying this method indicated the following:

1. There are two subgroups in Class IV which will yield suitable mechanisms. They are (a) 9-3-0-1 which contains five suitable chains, and (b) 10-1-1-1 which contains two suitable chains.
2. There are nine subgroups in Class $V$ which will yield suitable mechanisms. These subgroups contain a combined total of several hundred kinematic chains, each of which may yield workable mechanisms.

Preliminary Design of Three Working Mechanisms
From the collection of the kinematic chains of Class IV and $V$, three mechanisms were selected for further study. From initial observations, these mechanisms appeared to have desirable attributes for the decoder design. However, it is certainly quite possible that further study will reveal other promising mechanisms in the collection.

The details for calculating the dimensions of linkage number one and linkage number two have been given. The determination of exact dimensions for linkage number three was not attempted at this time;
however, a trial-and-error procedure was worked out which allowed a working model of this linkage to be constructed. Working models of the three mechanisms were designed and built.

The following observations regarding the characteristics of these mechanisms were made:

1. Linkage number one has an output displacement that may be considered essentially linear. However, the spacing between the sixteen output positions is relatively small compared to the input displacements. The mechanism contains only two basic link sizes exclusive of the input and output members, thus permitting a design with reduced manufacturing costs. An analytical study of the energy required to overcome the inertia of its members indicated that it required slightly less energy than did linkage number two, which actually contains a smaller average number of moving parts. Such a study, however, is not conclusive, and final conclusions cannot be made until the performance of the operational prototypes is analyzed.
2. A geometric approach was successfully used in the determination of the dimensions of linkage number two, and the resulting behavior of the model indicated an essentially linear output displacement. The symmetric character of linkage number two is very desirable, and can lead to a neat and compact arrangement of input and output members. Slider substitutions for several of the intermediate links might profitably be utilized in this mechanism.
3. Linkage number three may be considered as having a large output response compared to relatively small input displacements, which, in comparison to the small output displacements of linkages number one and
number two, could offset the fact that it has two additional members. Its configuration is essentially symmetric and well suited for the purpose of a practical design. The exact behavior of this linkage is, however, extremely complex, and can only be precisely determined by a detailed analytical appnoach. Additionally, it was found to have many dead center positions which complicated the task of design.

## CHAPTER VII

## RECOMMENDATIONS

Based upon the experience gained from this study, the following recommendations will be made for further work to be done:

1. It is certainly most probable that further study of the tabulated collection of kinematic chains of Class IV and V will reveal other mechanisms well suited for use as a decoder. Thus, the collection of kinematic chains may be used as a reference source from which selected chains may be analyzed for further study.
2. From the observations noted from the performance of the model of linkage number three, it is recommended that further study be attempted to determine, in more detail, the behavior of this mechanism. The approach may be wholly analytical in nature; or, a trial-and-error procedure, programmed for a digital computer, could determine behavior patterns of the mechanism based upon various values of the design parameters.
3. It is evident that the permutation synthesis of linkages is limited by the enormously large collections of kinematic chains that inevitably occur when the number of links in the chain becomes large. A solution to this problem would be to program the methods of permutation synthesis for a digital computer. This is, of course, not a simple task. For example, it would be most difficult to efficiently program
the definition of a distinct configuration. However, the benefits to be gained would certainly justify such a study.

## APPENDIX A

DETERMINATION OF THOSE SUBGROUPS WHICH WILL YIELD ONLY KINEMATIC CHAINS OF ERACTIONATED MOBILITY

In general, it may be stated that the condition of fractionated mobility arises when a kinematic chain consists of two independent linkages sharing the same reference link. If a linkage is not to have fractionated mobility, all links, exclusive of the frame and binary members, must be considered to contribute or share one or more joints so that they will be tied together in some configuration.

The minimum amount of joints contributed by these links may be expressed as

$$
j_{c}=2\left(\sum_{i=3} n_{i}-2\right)
$$

where the term $\sum_{i=3} n_{i}$ includes all members except binary links, and the subscript (i) identifies the type of link. Thus, three ternary links of the subgroup $9-3-0-1$ will contribute a minimum of

$$
j_{m}=2\left(\sum_{i=3} n_{i}-2\right)=2(4-2)=4 \text { joints }
$$

in order to prevent the condition of frectionated mobility.
The total number of joints available on all members of the chain exclusive of the frame and binary links may be expressed as

$$
j_{a}=\sum_{i=3} i n_{i}-j_{f}
$$

where $j_{f}$ is the number of joints on the frame and $\sum_{i=3} i n_{i}$ again includes all members except binary links.

Now, if the number of joints on the frame is greater than the total number of available joints ( $j_{a}$ ) less the joints that must be shared ( $j_{c}$ ), the resulting chain will be of fractionated mobility. This condition may be expressed as

$$
j_{f}>j_{a}-j_{c} \quad \text { (for fractionated mobility) }
$$

or

$$
\begin{align*}
& j_{f}>\sum_{i=3} i n_{i}-j_{f}-2\left(\sum_{i=3} n_{i}-2\right)  \tag{8}\\
& 2 j_{f}>\sum_{i=3}(i-2) n_{i}+4 \\
& j_{f}>\sum_{i=3}\left(\frac{i-2}{2}\right) n_{i}+2
\end{align*}
$$

Thus, for the subgroup $10-2-0-0-1$

$$
j_{f}=6>(1+2+2)=5
$$

and will therefore only yield chains of fractionated mobility, since $j_{f}>j_{a}-j_{c}$. The subgroup 9-3-0-1 shows

$$
j_{f}=5=(3 / 2+3 / 2+2)=5
$$

and may therefore yield chains not having fractionated mobility, although such chains might be so arranged intentionally if desired.

APPENDIX B

## TABULATED INERTIA CONSTANTS FOR <br> LINKAGE NUMBER ONE AND LINKAGE NUMBER TWO

Table 8. Inertia Constants for Linkage Number One

| Output <br> Position | LINKAGE MEMBER |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1 / 3$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=1 / 7$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 14 \end{aligned}$ | $c=1 / 15$ |
| 3 | $c=0$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=2 / 3$ | $\begin{aligned} & q=4 / 3 \\ & q=1 / 3 \end{aligned}$ | $c=2 / 7$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 7 \end{aligned}$ | $c=2 / 15$ |
| 4 | $c=1$ | $c=1{ }^{\text {T }}$ | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=3 / 7$ | $\begin{aligned} & q=4 / 3 \\ & a=3 / 14 \end{aligned}$ | $c=1 / 5$ |
| 5 | $\mathrm{c}=0$ | 0 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=4,7$ | $\begin{aligned} & q=4 / 3 \\ & a=2 / 7 \end{aligned}$ | $c=4 / 15$ |
| 6 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1 / 3$ | $\begin{aligned} & q=41 / 3 \\ & a=1 / 3 \end{aligned}$ | $c=5 / 7$ | $\begin{aligned} & q=4 / 3 \\ & a=5 / 14 \end{aligned}$ | $c=1 / 3$ |
| 7 | $c=0$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=2 / 3$ | $\begin{aligned} & q=25 \cdot 1 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=6 / 7$ | $\begin{aligned} & q=4 / 3 \\ & a=3 / 7 \end{aligned}$ | $c=2 / 5$ |
| 8 | $c=1$ | $c=1^{T}$ | $\mathrm{c}=1$ | $c=1{ }^{T}$ | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=7 / 15$ |
| 9 | $c=0$ | 0 | 0 | 0 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=8 / 15$ |
| 10 | $\mathrm{c}=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1 / 3$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 6 \end{aligned}$ | $\mathrm{c}=1,7$ | $\begin{aligned} & q=21 / 9 \\ & a=3 / 7 \end{aligned}$ | $c=3 / 5$ |
| 11 | $c=0$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=2 / 3$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 3 \end{aligned}$ | $c=2 / 7$ | $\begin{aligned} & q=343 / 75 \\ & a=5 / 14 \end{aligned}$ | $c=2 / 3$ |
| 12 | $c=1$ | $c=1{ }^{\text {T }}$ | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=3 / 7$ | $\begin{aligned} & q=6 \quad 7 / 12 \\ & a=2 / 7 \end{aligned}$ | c $=11 / 15$ |
| 13 | $c=0$ | 0 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=4 / 7$ | $\begin{aligned} & q=13 \\ & \hline \end{aligned}$ | $c=4 / 5$ |
| 14 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & \mathrm{a}=1 / 2 \end{aligned}$ | $c=1 / 3$ | $\begin{aligned} & q=41 / 3 \\ & a=1 / 3 \end{aligned}$ | $c=5 / 7$ | $\begin{aligned} & q=36 \quad 1 / 3 \\ & a=1 / 7 \end{aligned}$ | c $=13 / 15$ |
| 15 | $\mathrm{c}=0$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=2 / 3$ | $\begin{aligned} & q=25 \quad 1 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=6 / 7$ | $\begin{aligned} & \mathrm{q}=169 \quad 1 / 3 \\ & \mathrm{a}=1 / 14 \end{aligned}$ | c $=14 / 15$ |
| 16 | $c=1$ | $c=I^{T}$ | $\mathrm{c}=1$ | $c=1{ }^{\text {T }}$ | $\mathrm{c}=1$ | $c=1{ }^{\text {T }}$ | $c=1$ |

NOTE: The superscript $T$ denotes a horizontal member in translation.

Table 9. Inertia Constants for Linkage Number Two

| Output Position | LINKAGE MEMBER |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | $c=1$ | $\mathrm{q}=4 / 3$ $\mathrm{a}=1 / 2$ | 0 | 0 | $c=1 / 3$ | $q=4 / 3$ $a=1 / 6$ | 0 |
| 3 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | 0 | 0 | $c=2 / 3$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 3 \end{aligned}$ | 0 |
| 4 | $c=1$ | $c=1^{T}$ | 0 | 0 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | 0 |
| 5 | 0 | 0 | $c=1$ | $q=4 / 3$ $a=1 / 2$ | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=1 / 3$ |
| 6 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1 / 3$ | $c=1 / 3^{T}$ | $c=1 / 3$ |
| 7 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ | $\begin{aligned} & a=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=2 / 3$ | $\begin{aligned} & q=91 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=1 / 3$ |
| 8 | $c=1$ | $c=1^{T}$ | $\mathrm{c}=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ | $\begin{aligned} & a=41 / 3 \\ & a=1 / 3 \end{aligned}$ | $c=1 / 3$ |
| 9 | 0 | 0 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 3 \\ & \hline \end{aligned}$ | $c=2 / 3$ |
| 10 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1 / 3$ | $\begin{aligned} & q=91 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=2 / 3$ |
| 11 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | 0 | $\begin{aligned} & a=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=2 / 3$ | $c=2 / 3^{T}$ | $c=2 / 3$ |
| 12 | $c=1$ | $c=1^{T}$ | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ | $\begin{aligned} & q=25 \quad 1 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=2 / 3$ |
| 13 | 0 | 0 | $c=1$ | $c=1^{T}$ | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ |
| 14 | $c=1$ | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ | $c=1^{T}$ | $c=1 / 3$ | $\begin{aligned} & q=41 / 3 \\ & a=1 / 3 \end{aligned}$ | $c=1$ |
| 15 | 0 | $\begin{aligned} & q=4 / 3 \\ & a=1 / 2 \end{aligned}$ | $c=1$ | $c=1{ }^{T}$ | $c=2 / 3$ | $\begin{aligned} & q=25 \quad 1 / 3 \\ & a=1 / 6 \end{aligned}$ | $c=1$ |
| 16 | $c=1$ | $c=1{ }^{\text {T }}$ | $c=1$ | $c=1^{T}$ | $c=1$ | $c=1^{T}$ | $c=1$ |

NOTE: The superscript $T$ denotes a horizontal member in translation.

## LITERATURE CITED

1. F. R. E. Crossley, "The Permutations of Kinematic Chains of Eight Members or Less from the Graph-Theoretic Viewpoint," Developments in Theoretical and Applied Mechanics (edited by W. A. Shaw), Pergamon Press, Oxford, England, vol. 2, pp. 467-486, 1965.
2. F. R. E. Crossley, "The Permutations of Ten-Link Plane Kinematic Chains," Antriebstechnik, vol. 3, 1964, pp. 181-185.
3. F. R. E. Crossley, "A Contribution to Gruebler's Theory in the Number Synthesis of Plane Mechanisms," Transactions American Society of Mechanical Engineers, Journal of Engineering for Industry, vol. 86, 1964, pp. 1-8.
4. N. I. Manolescu, "Une Méthode Unitaire Pour La Formation des Chains Cinématiques et des Méchanismes Plans Articulés avec Differents Degrés de Liberté et Mobilité," Mechanique Appliquee, Tome 9, 1964, pp. 1263-1313.
5. O. Bottema, "On Gruebler's Formulae for Mechanisms," Applied Scientific Research, vol. 2, 1960, pp. 162-164.
6. L. V. Assur, "An Investigation of Coplanar Linkage Mechanisms with the Least Joints from the Viewpoint of Structure and Classification," (in Russian), Izdat. Akad. Nauk SSSR, 1952.
7. N. I. Manolescu, I. Erceanu, and P. Antonescu, "Die Kinematischen Untersucungsmethoden Ebner Gelenkgetriebe II," Mechanique Appliquee, Tome 9, 1964, pp. 341-363.
8. R. Franke, Vom Aufbau der Getriebe, I. Band, Dusseldorf, VDI-Verlag, 3rd edition, 1958.
9. S. A. Okcuoglu, "Input Motion Synthesis and Analysis for the IBM Selectric Selection System," ASME Paper No. 63-MD-11b.
10. W. F. Voit, Jr., "Machine Operation and Some Major Design Crossroads of the Selectric Typewriter," ASME Paper No. 63-MD-llc.
11. Hartenberg, R. S., and Denavit, J., Kinematic Synthesis of Linkages, New York, McGraw-Hill, 1964.
12. Hinkle, R. T., Kinematics of Machines, 2nd ed., Englewood Cliffs, N. J., Prentice-Hall, 1960.
13. Manolescu, N. I., and Manafu, V., "Sur La Determination Du Degree De Mobilite Des Mechanismes," Buletinul Institului Politehnic Bucuresti, tomul XXV, pp. 45-66, 1963.
