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LAMINAR BOUNDARY LAYER MOTION
OF A GAS WITH SOLID PARTICLE INJECTION

A THESIS

Presented to

The Faculty of the Graduate Division

by

James Burwell Urquhart III

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

Georgia Institute of Technology

January 1968

LAMINAR BOUNDARY LAYER MOTION

OF A GAS WITH SOLID PARTICLE INJECTION

Approved: ____________________

Chairman ____________________

Date approved by Chairman: 2/15/1968

ACKNOWLEDGMENTS

The author is indebted to the individuals who have contributed to the success of this work. In particular, he would like to express his appreciation to his Thesis Advisor, Dr. P. Durbetaki, for his valuable assistance. He would also like to thank Dr. C.W. Gorton and Dr. P.V. Desai, who served as members of the thesis committee. An extreme debt of gratitude is owed to the Mechanical Engineering Department at Duke University, especially Prof. E.Elsevier and Prof. J. N. Macduff, for their assistance during the author's undergraduate career.

The author would like to express his sincere appreciation to his parents for their loving guidance and to his wife for her love, patience, and understanding.

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NOMENCLATURE

c = volume of the particles per unit volume of mixture

c_p = constant pressure specific heat of the gas

c_s = specific heat of the solid particles

C_D = drag coefficient

d = diameter of a particle

D = drag force per particle

F = frictional parameter between the phases

F_p = force per unit volume of mixture acting on the gas

h = convective heat transfer coefficient

injangle = injection angle

injvel = injection velocity

k = thermal conductivity

L = non-dimensionalizing length

m = mass of a particle

M = Mach number

n_p = number of particles per unit volume of mixture

Nu = Nusselt number

P = pressure

Pr = Prandtl number

q_p = heat transfer per unit volume of mixture

R = gas constant

Re = Reynolds number

Re_d = particle Reynolds number

t = time

t_m = momentum equilibrium time

T = temperature of the gas

T_p = temperature of the particles

T_w = temperature of the wall

T = temperature of the gas in free stream

u = fluid velocity component parallel to the plate

u_p = particle velocity component parallel to the plate

u_∞ = free-stream velocity of the gas

v = fluid velocity component normal to the plate

v_p = particle velocity component normal to the plate

$\bar{w}_p - \bar{w}$ = relative particle velocity with respect to the fluid in any direction

x = Cartesian coordinate parallel to the plate

y = Cartesian coordinate normal to the plate

δ = boundary layer thickness

δ_p = particle boundary layer thickness

ϕ = viscous dissipation function

μ = viscosity of the mixture of gas and solid

μ_o = viscosity of the gas

ρ = density of the gas

ρ_p = density of the particle phase, apparent particle density

ρ_s = density of the solid material of which the particles are composed

λ_m = momentum equilibrium length

σ = radius of a particle

θ = non-dimensional temperature of the gas

θ_p = non-dimensional temperature of the particles

SUMMARY

This thesis discusses the problem of laminar flow of a viscous gas over a semi-infinite flat plate with wall-slot injection of solid particles. The boundary layer effects are studied.

The analysis treats the particles as a continuum. Thus, a continuity equation and two conservation of momentum equations are used to represent the particle flow. These equations together with the continuity and momentum equations for the gas phase and the appropriate boundary conditions describe the problem mathematically.

The coupling of the momentum equations by the shear between the two phases results in the necessity of solving simultaneous partial differential equations. The finite difference technique is employed to handle this difficulty. With this method of solution the Burroughs 5500 digital computer is programmed to solve for the velocities and densities necessary to describe the flow of the two phases.

The injection velocity and angle are varied to study various flow situations. Flow situations with the parallel component of the injection velocity both less than and greater than the free-stream velocity are investigated.

CHAPTER I

INTRODUCTION

Gas-solid particle flows have been the object of research for many years. Early studies were designed to obtain information for the design of pneumatic conveying systems, sedimentation systems, dust collection systems, and various transport systems. Lumped design parameters were usually adequate for these applications. More recently, such fields as reactor technology, metallized propellant rockets, and air purification (Clean Air Bill of 1963) have stimulated new interest in the flow of a gas-solids suspension. In addition to lumped design parameters, these applications demand knowledge of distribution within the systems. Thus, it has become necessary to understand the basic transport phenomena, represent them mathematically, and develop methods of solution.

Many publications have been written on the subject of gas-solid particle systems, but very few deal with two-phase flow in boundary layers. The most important of these are papers by S. L. Soo (1,2,3,4), F. E. Marble (5), and R. E. Singleton (6). Both Soo and Marble discuss various aspects of two-phase flow, including boundary layer effects. Singleton deals only with two-phase boundary layer flow.

S. L. Soo thoroughly discusses the transport phenomena of two-phase flow. In a paper presented in 1962 (1), he develops the differential equations of conservation of mass, momentum, and energy for the two-phase boundary layer. Flow over a flat plate is investigated. In this analysis Soo uses the integral form of the conservation equations. He concludes that the solid particles could result in an increase or decrease in the fluid boundary layer thickness, depending on the fluid Reynolds number and the particle diameter.

In 1965 Soo presented a paper that deals with many aspects of two-phase flow (2). The two-phase flow conservation equations are again developed. Soo applies these equations to boundary layer motion over a flat plate. As in the 1962 paper Soo uses the integral approach. The boundary layer analysis in this paper is more extensive but is basically identical to that of the earlier paper.

Two other papers by Soo merit a brief consideration. One on laminar and separated flow(3) examines suspensions of particles so small that Brownian motion becomes important. This difficulty is resolved with the introduction of a Brownian diffusivity equation. The second paper discusses particle size distribution (4). In this paper Soo develops a method of solution to deal with particles of varying size. The approach is an extension of the continuum concept.

Like Soo, F.E. Marble (5) develops the boundary layer differential equations of conservation. In his analysis of laminar flow over a flat plate these equations are combined and rewritten in terms of the velocity differences between the two phases. With the assumption that the y-component of the gas velocity is equal to the y-component of the particle velocity, Marble expresses the equations in terms of a stream function. This enables a power series solution to be used.

In his thesis Singleton (6) studies two-phase boundary layer motion in detail. The conversation equations are used to solve the problems of flow over a flat plate and flow perpendicular to the axis of a cylinder. Singleton reduces these partial differential equations to ordinary differential equations with use of the stream function. This enabled him to solve the equations numerically by power series expansion.

The problem considered in this thesis follows logically from the gas-solid particle flow over a flat plate. This problem is the laminar flow of a gas over a flat plate with wall-slot injection of solid particles. This situation is investigated to determine the boundary layer behavior of the two-phase system. The velocity distributions are of primary concern. To study these distributions various flow situations are created by varying the injection velocity and angle.

CHAPTER II

THEORY OF TWO-DIMENSIONAL GAS-PARTICLE BOUNDARY LAYERS

The situation to be examined is the laminar flow of a viscous gas over a flat plate. Solid particles are injected into the gas at the plate's leading edge. Since the particles are not injected parallel to the plate and are not injected at the free-stream velocity, the particles must slip with respect to the gas. The magnitude of the particle slip velocity (velocity of the particles with respect to the gas) is dependent upon the region of the boundary layer under consideration. Near the plate the slip velocity is large, but it decreases to zero in the free stream.

The apparent particle density, defined as the mass of the particles per unit volume of mixture of both phases, is assumed to be sufficiently low and the particles are assumed to move at so nearly the same speed that they do not collide with each other. The interaction of the flow fields around the individual particles is neglected. The particles are assumed to have no random motion and exert no pressure. Since their individual behavior is of no interest, the particles are considered as a continuum.

With the no collision assumption, the behavior of the two-phase system is entirely dependent upon the interaction between fluid and particles. The particle Reynolds number and the molecular mean free path of the fluid are considered to be small enough that Stokes drag law for spheres is a reasonable approximation. From Stokes law the drag force of the fluid on the particle phase, and the force of the particles on the fluid phase, may be evaluated (see APPENDIX A). The force per unit volume of mixture acting on the gas is

$$F_p = n_p \, (6\,\pi)\, \mu\sigma \, (\bar{w}_p - \bar{w}) \qquad \text{(II-1)}$$

where $\bar{w}_p - \bar{w}$ is the slip velocity in any direction and n_p is the number of particles per unit volume of mixture

$$n_p = \frac{\rho_p}{m} = \frac{\rho_p}{\frac{4}{3}\,\pi\sigma^2\rho_s} \qquad \text{(II-2)}$$

By substitution

$$F_p = \frac{\rho_p}{\frac{4}{3}\,\pi\sigma^3\rho_s}\, 6\,\pi\,\mu\sigma\,(\bar{w}_p - \bar{w}) \qquad \text{(II-3)}$$

$$F_p = \frac{\rho_p \mu_\infty \, (\bar{w}_p - \bar{w})}{\frac{2}{9}\left(\frac{\rho_s}{\rho}\right)\left(\frac{\rho\mu_\infty\sigma}{\mu}\right)\sigma} \qquad \text{(II-4)}$$

$$F_p = \frac{\rho_p \mu_\infty}{\lambda_m}\,(\bar{w}_p - \bar{w}) \qquad \text{(II-5)}$$

where λ_m = momentum equilibrium length

$$\lambda_m = t_m\,\mu_\infty = \frac{2}{9}\left(\frac{\rho_s}{\rho}\right)\left(\frac{\rho u_\infty\,\sigma}{\mu}\right)\sigma \qquad \text{(II-6)}$$

and where t_m = momentum equilibrium time

$$t_m = \frac{2}{9} \frac{\rho_s \sigma^2}{\mu} \qquad \text{(II-7)}$$

Singleton (6) discusses lift forces on the particles in addition to the Stokes drag forces. The lift results from particle spin, and the particle spin is caused by shear. Singleton concludes that the lift forces are negligible if $\left(\frac{\rho u_\infty \sigma}{\mu}\right)\left(\frac{\sigma}{L}\right)$ is small enough. In this thesis the lift is neglected since $\left(\frac{\rho u_\infty \sigma}{\mu}\right)\left(\frac{\sigma}{L}\right)$ is on the order of 10^{-4}.

In Marble's paper (5) the importance of the momentum equilibrium parameter, $\frac{\lambda_m}{x}$, is considered. Physically, the momentum equilibrium parameter is the ratio of the distance required for the particle velocity to reach that of the fluid to the characteristic length of the flow field. Singleton (6) investigates the magnitude of this parameter and states that if

$$\frac{\lambda_m}{x} \gg 1 \qquad \text{(II-8)}$$

then the particles have not had time to adjust to the gas flow and consequently have large slip velocities. Because of these large slips the particle motion is determined by the injection conditions. If

$$\frac{\lambda_m}{x} \ll 1 \qquad \text{(II-9)}$$

then the particles have the required length to reduce their injection velocities. Hence, the slip velocities in this regime are small, and the particle motion is influenced by the gas flow to a greater extent.

The x-injection ratio, the ratio of the x-component of the particle injection velocity to the free-stream velocity, is an important factor in this two-phase flow system. For small x-injection ratios the injection velocities exert relatively little influence downstream as their effects are damped out in short distances. However, at larger x-injection ratios the effect of the particle injection velocities is important farther downstream.

CHAPTER III

CONSERVATION EQUATIONS FOR TWO-PHASE FLOW

To develop equations that represent the two-phase flow system, the particle phase is treated as a continuum. This treatment enables equations of conservation of mass and momentum to be written for the particle phase. Conservation equations of mass and momentum may also be written for the gas.

These conservation laws are written for a two-dimensional system (1). The continuity equation for the gas is,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad \text{(III-1)}$$

The conservation of momentum equations for the gas are

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x} + \mu\frac{\partial u}{\partial y}\right) + \rho_p F(u_p - u) \qquad \text{(III-2)}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)$$

$$+ \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x} + \mu\frac{\partial u}{\partial y}\right) + \rho_p F(v_p - v) \qquad \text{(III-3)}$$

where $F = \frac{F_p}{\rho_p(\bar{w}_p - \bar{w})} = \frac{9}{2} \frac{\mu}{\rho_s \sigma^2}$.

The continuity equation of the particle phase is

$$\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0 \qquad \text{(III-4)}$$

The conservation of momentum equations for the particle phase are

$$\rho_p\left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y}\right) = - \rho_p F(u_p - u) \qquad \text{(III-5)}$$

$$\rho_p\left(\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y}\right) = - \rho_p F(v_p - v) \qquad \text{(III-6)}$$

Using the boundary layer simplification, fluid-phase incompressibility, and a steady state analysis (7), the non-dimensional forms of equations (III-1) through (III-6) are written as follows (see APPENDIX B):

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad \text{(III-7)}$$

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = \frac{1}{Re} \frac{\partial^2 u'}{\partial y'^2} + \frac{FL}{u_\infty} \frac{\rho_p}{\rho} (u'_p - u') \qquad \text{(III-8)}$$

$$\frac{\partial}{\partial x'}(\rho_p u'_p) + \frac{\partial}{\partial y'}(\rho_p v'_p) = 0 \qquad \text{(III-9)}$$

$$u'_p \frac{\partial u'_p}{\partial x'} + v'_p \frac{\partial u'_p}{\partial y'} = - \frac{FL}{u_\infty} (u'_p - u') \qquad \text{(III-10)}$$

$$u'_p \frac{\partial v'_p}{\partial x'} + v'_p \frac{\partial v'_p}{\partial y'} = - \frac{FL}{u_\infty} (v'_p - v') \qquad \text{(III-11)}$$

where $Re = \frac{\rho u_\infty L}{\mu}$. The primes indicate non-dimensional variables. They will be used throughout this thesis except in the next chapter. In CHAPTER IV the non-dimensional primes are omitted to avoid confusion with the primes denoting derivatives. All velocities and distances in CHAPTER IV are non-dimensional.

The viscosity, μ, is the viscosity of the mixture of gas and particles and not of the gas alone. There is disagreement as to whether the viscosity increases (8) or decreases (9) when particles are suspended in the gas. According to Einstein (8)

$$\mu = \mu_o [1 + 2.5c] \qquad \text{(III-12)}$$

where μ_o is the viscosity of the gas and c is the volume of the spheres per unit volume of mixture,

$$c = \frac{\rho_p}{\rho_s} = \frac{n_p \rho}{\rho_s} \qquad \text{(III-13)}$$

c is of the order of 10^{-3} or smaller for any particulate material. This leads to

$$\mu = \mu_o [1 + 2.5 \cdot 0(10^{-3})] \approx \mu_o \qquad \text{(III-14)}$$

Thus, the viscosity is taken to be the gas-phase viscosity.

The establishment of the appropriate boundary conditions along with equations (III-7) through (III-11) completes the mathematical description of the problem. For the gas phase the boundary conditions are:

(i) potential flow at the plate's leading edge

$$u'[0,y] = 1 \qquad \text{(III-15)}$$

$$v'[0,y] = 0 \qquad \text{(III-16)}$$

(ii) gas velocity vanishes at the plate

$$u'[x,0] = 0 \qquad \text{(III-17)}$$

$$v'[x,0] = 0 \qquad \text{(III-18)}$$

(iii) gas velocity (x-component) must approach free-stream value as y approaches infinity

$$\lim_{y \to \infty} u'[x,y] = 1. \qquad \text{(III-19)}$$

For the particle phase the boundary conditions are:

(i) constant injection conditions

$$u'_p\,[0,0] = (\text{injvel}) \cdot [\cos(\text{injangle})] \qquad \text{(III-20)}$$

$$v'_p\,[0,0] = (\text{injvel}) \cdot [\sin(\text{injangle})] \qquad \text{(III-21)}$$

$$\rho_p\,[0,0] = \rho_s(0.1) \qquad \text{(III-22)}$$

(ii) particle velocity (y-component) vanishes at the plate

$$v_p' [x,0] = 0 \tag{III-23}$$

(iii) particle velocity (x-component) must approach free-stream value as y approaches infinity

$$\lim_{y\to\infty} u_p' [x,y] = 1 \tag{III-24}$$

(iv) particle density vanishes at the plate

$$\rho_p [x,0] = 0. \tag{III-25}$$

The condition that $\rho_p [0,0] = \rho_s(0.1)$ is arbitrary, and any similar condition could be used. However, at all times the no collision assumption must be reasonable.

The above mathematical description of the problem is correct only when Stokes drag law is reasonably accurate. If the drag law is valid then

$$Re_d = \frac{\rho|\bar{w}_p - \bar{w}|\ d}{\mu} < 1 \tag{III-26}$$

For a given fluid and a given particle diameter equation (III-26) restricts the slip velocity

$$|\bar{w}_p - \bar{w}| < \frac{\mu}{d\rho} . \tag{III-27}$$

Hence, it is apparent that the extent of the regime of small slip depends on the flow field parameters.

The x-injection ratio influences the slip velocities. Large ratios cause large slip velocities. Thus, there is a limit on the x-injection ratio for which the small slip velocity requirement is satisfied.

CHAPTER IV

FINITE DIFFERENCE TECHNIQUE

The finite difference technique was developed to numerically solve differential equations. The technique is valuable when the solution of complicated ordinary differential equations, partial differential equations, and simultaneous differential equations is needed.

The method is dependent on the replacement of derivatives by approximations derived from the Taylor series (10). When a function u and its derivatives are single-valued, finite, and continuous functions of x, then by the Taylor series,

$$u(x + h) = u(x) + hu'(x) + \tfrac{1}{2} h^2 u''(x) + \frac{1}{6} h^3 u'''(x) + \ldots \quad \text{(IV-1)}$$

and

$$u(x - h) = u(x) - hu'(x) + \tfrac{1}{2} h^2 u''(x) - \frac{1}{6} h^3 u'''(x) + \ldots \quad \text{(IV-2)}$$

The addition of these expansions yields

$$u(x + h) + u(x - h) = 2u(x) + h^2 u''(x) + O(h^4), \quad \text{(IV-3)}$$

where $O(h^4)$ denotes terms containing fourth and higher powers of h. If these higer order terms are neglected, it follows that

$$u''(x) = \frac{d^2u}{dx^2} = \frac{1}{h^2}\left[u(x + h) - 2u(x) + u(x - h)\right] \quad \text{(IV-4)}$$

Equation (IV-2) is subtracted from equation (IV-1), and terms of order h^3 and higher are neglected

$$u'(x) = \frac{du}{dx} = \frac{1}{2h}\left[u(x + h) - u(x - h)\right] . \qquad \text{(IV-5)}$$

The equation for $u'(x)$ approximates the slope of the tangent at P by the slope of chord AB (see Figure 1), and is called a central difference approximation. The slope of the tangent can also be approximated by the slope of chord PB (forward difference)

$$u'(x) = \frac{1}{h}\left[u(x + h) - u(x)\right] \qquad \text{(IV-6)}$$

or the slope of chord AP (backward difference)

$$u'(x) = \frac{1}{h}\left[u(x) - u(x - h)\right] \qquad \text{(IV-7)}$$

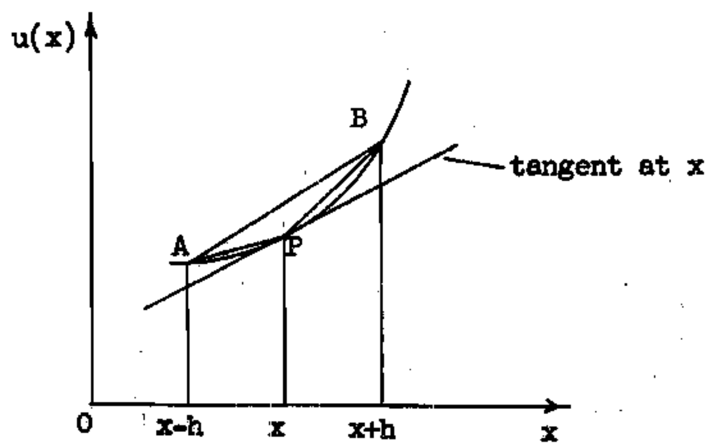

Figure 1. Finite Differences

With the use of equation (IV-4), (IV-6), and (IV-7) the non-dimensional equations (III-7) through (III-11) are approximated as follows:

$$\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \frac{v_{i,j+1} - v_{i,j}}{\Delta y} = 0 \qquad \text{(IV-8)}$$

$$u_{i,j}\,\frac{u_{i+1,j}-u_{i,j}}{\Delta x} + v_{i,j}\,\frac{u_{i,j}-u_{i,j-1}}{\Delta y} = \frac{1}{Re}\,\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta y^2}$$

$$+ \frac{\rho_{p_{i,j}}}{\rho}\,\frac{FL}{u_\infty}\,(u_{p_{i,j}}-u_{i,j}) \qquad \text{(IV-9)}$$

$$\rho_{p_{i,j}}\,\frac{u_{p_{i,j}}-u_{p_{i-1,j}}}{\Delta x} + u_{p_{i,j}}\,\frac{\rho_{p_{i+1,j}}-\rho_{p_{i,j}}}{\Delta x} + v_{p_{i,j}}\,\frac{v_{p_{i,j}}-v_{p_{i,j-1}}}{\Delta y}$$

$$+v_{p_{i,j}}\,\frac{\rho_{p_{i,j}}-\rho_{p_{i,j-1}}}{\Delta y} = 0 \qquad \text{(IV-10)}$$

$$u_{p_{i,j}}\,\frac{u_{p_{i+1,j}}-u_{p_{i,j}}}{\Delta x} + v_{p_{i,j}}\,\frac{u_{p_{i,j}}-u_{p_{i,j-1}}}{\Delta y} = -\frac{FL}{u_\infty}\,(u_{p_{i,j}}-u_{i,j}) \qquad \text{(IV-11)}$$

$$u_{p_{i,j}}\,\frac{v_{p_{i+1,j}}-v_{p_{i,j}}}{\Delta x} + v_{p_{i,j}}\,\frac{v_{p_{i,j}}-v_{p_{i,j-1}}}{\Delta y} = -\frac{FL}{u_\infty}\,(v_{p_{i,j}}-v_{i,j}) \qquad \text{(IV-12)}$$

From equation (IV-9)

$$u_{i+1,j} = u_{i,j} - \frac{\Delta x}{\Delta y}\,v_{i,j} + \frac{\Delta x}{\Delta y}\,v_{i,j}\,\frac{u_{i,j-1}}{u_{i,j}} + \frac{\rho_{p_{i,j}}}{\rho}\,\frac{FL}{u_\infty}\,\frac{u_{p_{i,j}}}{u_{i,j}}\,\Delta x$$

$$-\frac{\rho_{p_{i,j}}}{\rho}\,\frac{FL}{u_\infty}\,\Delta x + \frac{\Delta x}{\Delta y^2}\,\frac{1}{Re}\,\frac{u_{i,j+1}}{u_{i,j}} - \frac{\Delta x}{\Delta y^2}\,\frac{2}{Re} + \frac{\Delta x}{\Delta y^2}\,\frac{u_{i,j-1}}{u_{i,j}} \qquad \text{(IV-13)}$$

From equation (IV-8)

$$v_{i,j+1} = v_{i,j} - \frac{\Delta y}{\Delta x} u_{i,j} + \frac{\Delta y}{\Delta x} u_{i-1,j} \qquad \text{(IV-14)}$$

From equation (IV-11)

$$u_{p_{i+1,j}} = u_{p_{i,j}} - \frac{\Delta x}{\Delta y} v_{p_{i,j}} + \frac{\Delta x}{\Delta y} v_{p_{i,j}} \frac{u_{p_{i,j-1}}}{u_{p_{i,j}}} - \frac{FL}{u_\infty} \Delta x + \frac{FL}{u_\infty} \frac{u_{i,j}}{u_{p_{i,j}}} \Delta x \qquad \text{(IV-15)}$$

From equation (IV-12)

$$v_{p_{i+1,j}} = v_{p_{i,j}} - \frac{\Delta x}{\Delta y} \frac{v_{p_{i,j}}}{u_{p_{i,j}}} v_{p_{i,j}} + \frac{\Delta x}{\Delta y} \frac{v_{p_{i,j}}}{u_{p_{i,j}}} v_{p_{i,j-1}} - \frac{FL}{u_\infty} \frac{v_{p_{i,j}}}{u_{p_{i,j}}} \Delta x$$

$$+ \frac{FL}{u_\infty} \frac{v_{i,j}}{u_{p_{i,j}}} \Delta x \qquad \text{(IV-16)}$$

From equation (IV-10)

$$\rho_{p_{i+1,j}} = \rho_{p_{i,j}} - \rho_{p_{i,j}} + \rho_{p_{i,j}} \frac{u_{p_{i-1,j}}}{u_{p_{i,j}}} - \frac{\Delta x}{\Delta y} \frac{\rho_{p_{i,j}}}{u_{p_{i,j}}} v_{p_{i,j}} + \frac{\Delta x}{\Delta y} \frac{\rho_{p_{i,j}}}{u_{p_{i,j}}} v_{p_{i,j-1}}$$

$$- \frac{\Delta x}{\Delta y} \frac{v_{p_{i,j}}}{u_{p_{i,j}}} \rho_{p_{i,j}} + \frac{\Delta x}{\Delta y} \frac{v_{p_{i,j}}}{u_{p_{i,j}}} \rho_{p_{i,j-1}} \qquad \text{(IV-17)}$$

The criteria of convergence and stability must be satisfied if the solution of these five equations is to be a reasonable approximation to the solution of the partial differential equations describing the two-phase flow, equations (III-7) through (III-11). Both of these conditions are concerned with the total error that results from the approximation. The first condition is concerned with the convergence of the solution of the difference equations to the solution of the differential equations; the second deals with the unstable growth or stable decay of the errors produced by the arithmetic operations needed to solve the finite difference equations.

Let D represent the exact solution of a partial differential equation, Δ represent the exact solution of the corresponding equation, and η represent the numerical solution. The discretization error for convergence, $(D - \Delta)$, is due to the increment size. The conditions under which the discretization error approaches zero is the convergence analysis. The error $(\Delta - \eta)$ is the result of round-off in the arithmetic operations. The finite difference equation is stable if the cumulative effect of all of the round-off errors is negligible. Hence, in order to make

$$(D - \eta) \equiv (D - \Delta) + (\Delta - \eta) \qquad \text{(IV-18)}$$

small, the numerical solution must be both convergent and stable (10).

The problem of convergence and stability of the numerical solution for non-linear partial differential equations with variable coefficients can be handled in only a few particular cases. However, if the coefficients of the derivative terms are always at least one order lower than the derivatives themselves, then the non-linear equations are considered quasi-linear. The variable coefficients are treated as constants throughout the analysis. They take on their most adverse values in order to determine the restriction on the increment size (10).

Rewriting equations (III-7) through (III-11) in quasi-linear form yields:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{IV-19}$$

$$\frac{\partial u}{\partial x} + \frac{v}{u}\frac{\partial u}{\partial y} = \frac{1}{uRe}\frac{\partial^2 u}{\partial y^2} + \frac{\rho_p}{\rho}\frac{FL}{u_\infty}\left(\frac{u_p}{u} - 1\right) \tag{IV-20}$$

$$\frac{\partial \rho_p}{\partial x} + \frac{\rho_p}{u_p}\frac{\partial u_p}{\partial x} + \frac{v_p}{u_p}\frac{\partial \rho_p}{\partial y} + \frac{\rho_p}{u_p}\frac{\partial v_p}{\partial y} = 0 \tag{IV-21}$$

$$\frac{\partial u_p}{\partial x} + \frac{v_p}{u_p}\frac{\partial u_p}{\partial y} = -\frac{FL}{u_\infty}\left(1 - \frac{u}{u_p}\right) \tag{IV-22}$$

$$\frac{\partial v_p}{\partial x} + \frac{v_p}{u_p}\frac{\partial v_p}{\partial y} = -\frac{FL}{u_\infty}\left(\frac{v_p}{u_p} - \frac{v}{u_p}\right) \tag{IV-23}$$

The von Neumann stability analysis uses the quasi-linear form of the partial differential equations. This method expresses the solution in terms of finite Fourier series. A complete description of the von Neumann method can be found in O'Brien, Hyman, and Kaplan (11). In essence, the method depends on expressing the solution as

$$u = \sum_{k_1}\sum_{k_2} e^{i(k_1\Delta x + k_2\Delta y)} = \sum_{k_1}\sum_{k_2} \xi^n e^{ik_2\Delta y} \tag{IV-24}$$

where $\xi^n = e^{ik_1\Delta x}$ and $i = \sqrt{-1}$. This form is substituted into the equation, and if a solution is found, a necessary and sufficient condition for stability is that

$$|\xi| \leq 1 \qquad \text{(IV-25)}$$

for all values of k_1 and k_2.

Richtmyer (12) has extended the von Nurmann stability analysis to include systems of equations. The system to be analyzed here consists of five independent variables

$$u_{i,j},\ v_{i,j},\ u_{p_{i,j}},\ v_{p_{i,j}},\ \rho_{p_{i,j}} \qquad \text{(IV-26)}$$

and a set of five linear equations

$$u_{i+1,j} = u_{i,j} - C_1 u_{i,j} + C_1 u_{i,j-1} + C_2 u_{i,j+1} - 2C_2 u_{i,j} + C_2 u_{i,j-1} + C_3 \qquad \text{(IV-27)}$$

$$v_{i,j+1} = v_{i,j} - C_4 u_{i,j} + C_4 u_{i-1,j} \qquad \text{(IV-28)}$$

$$u_{p_{i+1,j}} = u_{p_{i,j}} - C_5 u_{p_{i,j}} + C_5 u_{p_{i,j-1}} + C_6 \qquad \text{(IV-29)}$$

$$v_{p_{i+1,j}} = v_{p_{i,j}} - C_5 v_{p_{i,j}} + C_5 v_{p_{i,j-1}} + C_7 \qquad \text{(IV-30)}$$

$$\rho_{p_{i+1,j}} = \rho_{p_{i,j}} - C_8 u_{p_{i,j}} + C_8 u_{p_{i-1,j}} - C_9 \rho_{p_{i,j}} + C_9 \rho_{p_{i,j-1}} - C_{10} v_{p_{i,j-1}}$$

$$+ C_{10} v_{p_{i,j-1}} \qquad \text{(IV-31)}$$

where

$$C_1 = \frac{\Delta x}{\Delta y} \frac{v}{u}$$

$$C_2 = \frac{\Delta x}{\Delta y^2} \; \frac{1}{uRe}$$

$$C_3 = \frac{\rho_p}{\rho} \frac{FL}{u_\infty}\left(\frac{u_p}{u} - 1\right) \Delta x$$

$$C_4 = \frac{\Delta y}{\Delta x}$$

$$C_5 = \frac{\Delta x}{\Delta y} \frac{v_p}{u_p}$$

$$C_6 = - \frac{FL}{u_\infty}\left(1 - \frac{u}{u_p}\right) \Delta x$$

$$C_7 = - \frac{FL}{u_\infty}\left(\frac{v_p}{u_p} - \frac{v}{u_p}\right) \Delta y$$

$$C_8 = \frac{\rho_p}{u_p}$$

$$C_9 = C_5$$

$$C_{10} = \frac{\Delta x}{\Delta y} \frac{\rho_p}{u_p}$$

The following solutions are assumed and substituted into equation (IV-27) through (IV-31).

$$u_{i,j} = \sum_{k_1} \sum_{k_2} e^{i(k_1 \Delta k + k_2 \Delta y)} = \sum_{k_1} \sum_{k_2} \xi_1^n \, e^{ik_2 \Delta y} \qquad \text{(IV-32)}$$

$$v_{i,j} = \sum_{k_1} \sum_{k_2} e^{i(k_1 \Delta x + k_2 \Delta y)} = \sum_{k_1} \sum_{k_2} \xi_2^n \, e^{ik_1 \Delta x} \qquad \text{(IV-33)}$$

$$u_{p_{i,j}} = \sum_{k_1} \sum_{k_2} e^{i(k_1 \Delta x + k_2 \Delta y)} = \sum_{k_1} \sum_{k_2} \xi_3^n \, e^{ik_2 \Delta y} \qquad \text{(IV-34)}$$

$$v_{p_{i,j}} = \sum_{k_1} \sum_{k_2} e^{i(k_1 \Delta x + k_2 \Delta y)} = \sum_{k_1} \sum_{k_2} \xi_4^n \, e^{ik_2 \Delta y} \qquad \text{(IV-35)}$$

$$\rho_{p_{i,j}} = \sum_{k_1} \sum_{k_2} e^{i(k_1 \Delta x + k_2 \Delta y)} = \sum_{k_1} \sum_{k_2} \xi_5^n \, e^{ik_2 \Delta y} \qquad \text{(IV-36)}$$

The substitution of equations (IV-32) through (IV-36) into equations (IV-27) through (IV-31) results in the relation

$$\vec{g}_1 = \underline{G} \, \vec{g}_0 \qquad \text{(IV-37)}$$

where $\underline{G}$ is the amplification matrix, and $\vec{g}_1$ and $\vec{g}_0$ are vectors.

$$\vec{g}_1 = \begin{bmatrix} \xi_1^{n+1} \\ \xi_2^{n+1} \\ \xi_3^{n+1} \\ \xi_4^{n+1} \\ \xi_5^{n+1} \\ 1 \end{bmatrix} \quad ; \quad \vec{g}_o = \begin{bmatrix} \xi_1^{n} \\ \xi_2^{n} \\ \xi_3^{n} \\ \xi_4^{n} \\ \xi_5^{n} \\ 1 \end{bmatrix} \qquad \text{(IV- 38)}$$

Lax (12) has shown that the von Neumann necessary condition for stability is

$$| \lambda_n | \leq 1 \qquad \text{(IV-39)}$$

where λ_n are the eigenvalues of the amplification matrix. If all of the elements in $\underline{G}$ are bounded for all k_1 and k_2 and if all of the eigenvalues with the possible exception of one satisfy relation (IV-39), then the condition is necessary and sufficient.

The amplification matrix of equations (IV-32) through (IV-36) is

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & a_{16} \\ a_{21} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & a_{36} \\ 0 & 0 & 0 & a_{44} & 0 & a_{46} \\ 0 & 0 & a_{53} & a_{54} & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(IV-40)}$$

where

$$a_{11} = 1 - C_1(1-e^{-i\Delta y}) + C_2(e^{i\Delta y}-2+e^{-i\Delta y})$$

$$a_{16} = C_3 e^{-ik_2\Delta y}$$

$$a_{21} = C_4(\xi_1^{-1}-1)(e^{ik_2\Delta y-ik_1\Delta x})$$

$$a_{33} = 1 - C_5(1-e^{-i\Delta y})$$

$$a_{36} = C_6 e^{-ik_2\Delta y}$$

$$a_{44} = 1 - C_5(1-e^{-i\Delta y})$$

$$a_{46} = C_7 e^{-ik_2\Delta y}$$

$$a_{53} = C_8(\xi_3^{-1} - 1)$$

$$a_{54} = C_{10}(e^{-i\Delta y}-1)$$

$$a_{55} = 1 - C_9(1-e^{-i\Delta y}).$$

From this matrix the six eigenvlaues are determined

$$\lambda_1 = 1 - C_1(1-e^{-i\Delta y}) + C_2(e^{i\Delta y} - 2 + e^{-i\Delta y})$$

$$\lambda_1 = 1 - \frac{\Delta x}{\Delta y}\frac{v}{u}(1 - e^{-i\Delta y}) + \frac{\Delta x}{\Delta y^2}\ \frac{1}{uRe}(e^{i\Delta y} - 2 + e^{-i\Delta y}) \qquad \text{(IV-41)}$$

$$\lambda_2 = 1 \qquad \text{(IV-42)}$$

$$\lambda_3 = 1 - C_5(1 - e^{i\Delta y}) = 1 - \frac{\Delta x}{\Delta y}\frac{v_p}{u_p}(1 - e^{i\Delta y}) \qquad \text{(IV-43)}$$

$$\lambda_4 = \lambda_3 \qquad \text{(IV-44)}$$

$$\lambda_5 = \lambda_3 \qquad \text{(IV-45)}$$

$$\lambda_6 = 1 \qquad \text{(IV-46)}$$

λ_2 and λ_6 immediately satisfy the stability criteria. From an examination of λ_3, λ_4, and λ_5 for the most adverse C_5, it is apparent that $|\lambda_3| \leq 1$ if

$$0 \leq \frac{\Delta x}{\Delta y}\frac{v_p}{u_p} \leq 1. \qquad \text{(IV-47)}$$

Since u_p is always greater than or equal to v_p for injection angles of 45° or less, and v_p is always positive, $|\lambda_3| \leq |$ if

$$\Delta x \leq \Delta y. \qquad \text{(IV-48)}$$

The von Neumann method allows one eigenvalue to violate the λ_n relationship. Thus, the difference equations of the system are stable since

$\Delta x = \Delta y$ and injangle $\leq 45°$ in this thesis. λ_1 also obeys the stability criteria in most flow situations; however, with large injection velocities λ_1 is slightly greater than one.

Lax (12) has shown the equivalence of convergence and stability for problems which satisfy the consistency conditions. For linear equations the consistency condition states that the truncation error, E_t, must vanish as Δx and Δy approach zero.

The truncation error is defined by

E_t = (Finite Difference Equation) - (Partial Differential Equation).

This error can be determined by expanding the point values in the difference equations in a Taylor series. These terms are substituted into the finite difference equations. The truncation error is obtained from the remainder terms. Hence, the error in the continuity equations is

$$E_t = 0\,[(\Delta x)^2, (\Delta y)^2] \tag{IV-49}$$

and that in the momentum equations is

$$E_t = 0\,[\Delta x, (\Delta y)^2]. \tag{IV-50}$$

The truncation error vanishes in all cases as Δx and Δy approach zero, and the difference equations converge.

In conclusion, the finite difference equations have been shown to converge and to be stable. The von Neumann method is used to establish stability, and the Lax consistency condition is used in the convergence analysis. With the establishment of these conditions the Burroughs 5500 digital computer is used to solve the difference equations (see APPENDIX E).

CHAPTER V

DISCUSSION OF RESULTS

The Burroughs 5500 digital computer was used to solve numerically the problem of wall-slot particle injection into a gas flowing over a flat plate. The data, displayed in APPENDIX D, consist of the velocity and density distributions of the two phases. This data is based on

$$Re = 5.5253 \times 10^5 \qquad (V\text{-}1)$$

$$\frac{FL}{u_\infty} = 14.827 \qquad (V\text{-}2)$$

$$\rho_p[0,0] = 14.5 \frac{lbm}{ft^3} \ . \qquad (V\text{-}3)$$

These values result if:

(i) air is the viscous gas

$$\rho = 0.071 \frac{lbm}{ft^3}$$

$$\mu_o = 1.2850 \times 10^{-5} \frac{lbm}{ft\text{-}sec} \ ,$$

(ii) each particle is 10.0 microns in diameter and is composed of a material such that

$$\rho_s = 145.0 \frac{lbm}{ft^3} \ ,$$

(iii) the free-stream velocity is 100 $\frac{ft}{sec}$,

(iv) the non-dimensionalizing length, L, is 1 ft.

For a system described by these conditions the momentum equilibrium length, λ_m , is 0.0678 ft.

The above set of conditions is not unique. The data in this thesis represents any two-phase flow system in which equations (V-1), (V-2), and (V-3) are valid.

The results of primary concern are the x-velocity profiles and the boundary layer thicknesses. In order to discuss these results it is necessary to define the fluid boundary layer thickness, δ, and the particle boundary layer thickness, δ_p. In this thesis the boundary layer thicknesses are defined as the distances for which

$$| u - u_\infty | = 0.00001\, u_\infty, \text{for } \delta \qquad \text{(V-4)}$$

$$| u_p - u_\infty | = 0.00001\, u_\infty, \text{ for } \delta_p \qquad \text{(V-5)}$$

This definition enables similar treatment of situations when the x-component of injection velocity is less than or greater than the free-stream velocity.

Figures 2 and 3 show a comparison of the boundary layer thicknesses at different injection conditions. From an observation of these graphs it is apparent that for any injection conditions both thicknesses are considerably thicker than the boundary layer thickness with no injection (the Blasius problem). Figures 7 through 14 demonstrate that the particle boundary layer thickness is always thicker than the fluid thickness for a given injection velocity and angle.

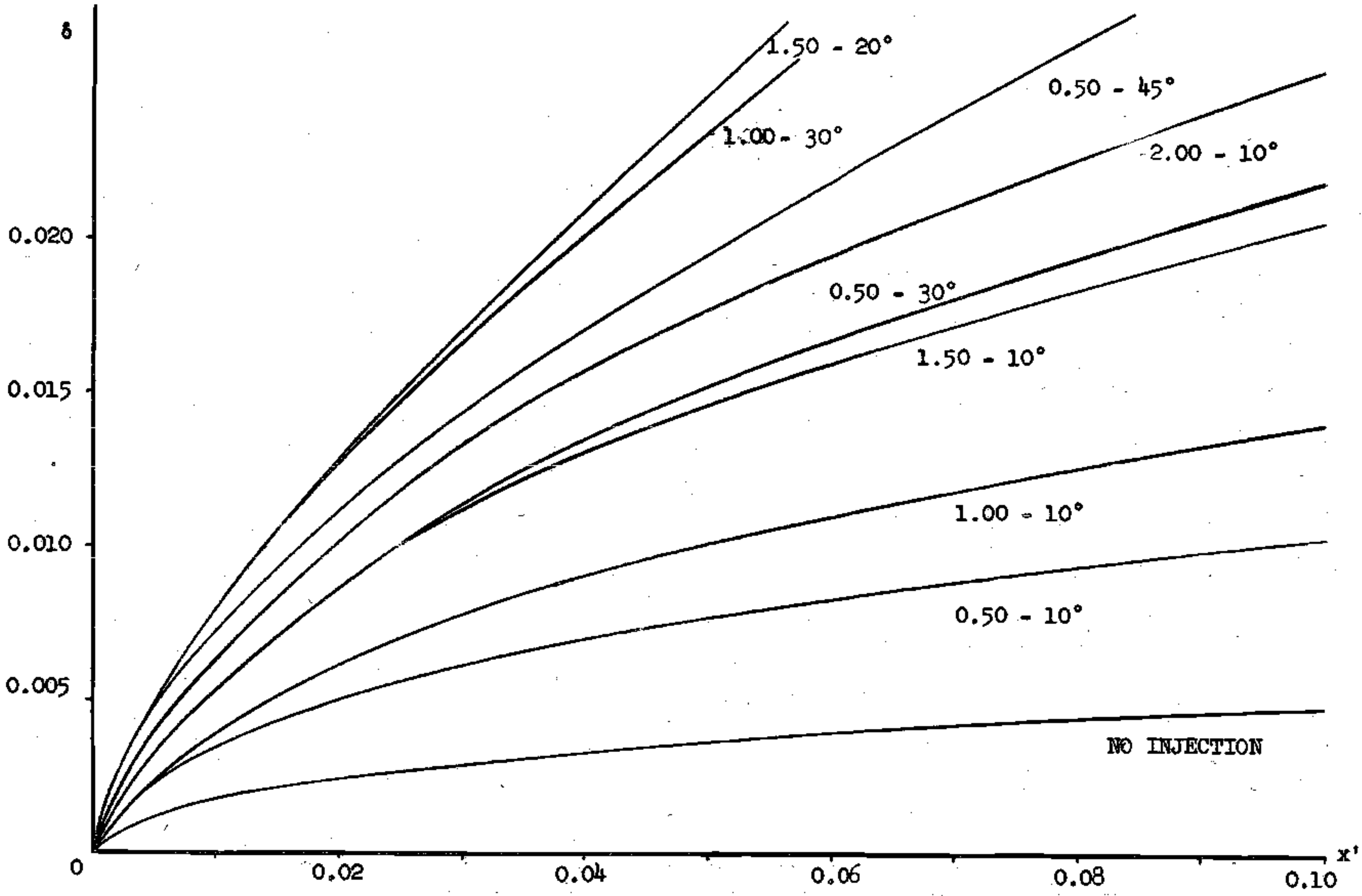

Figure 2. Fluid Boundary Layer Thickness versus Distance Along Plate

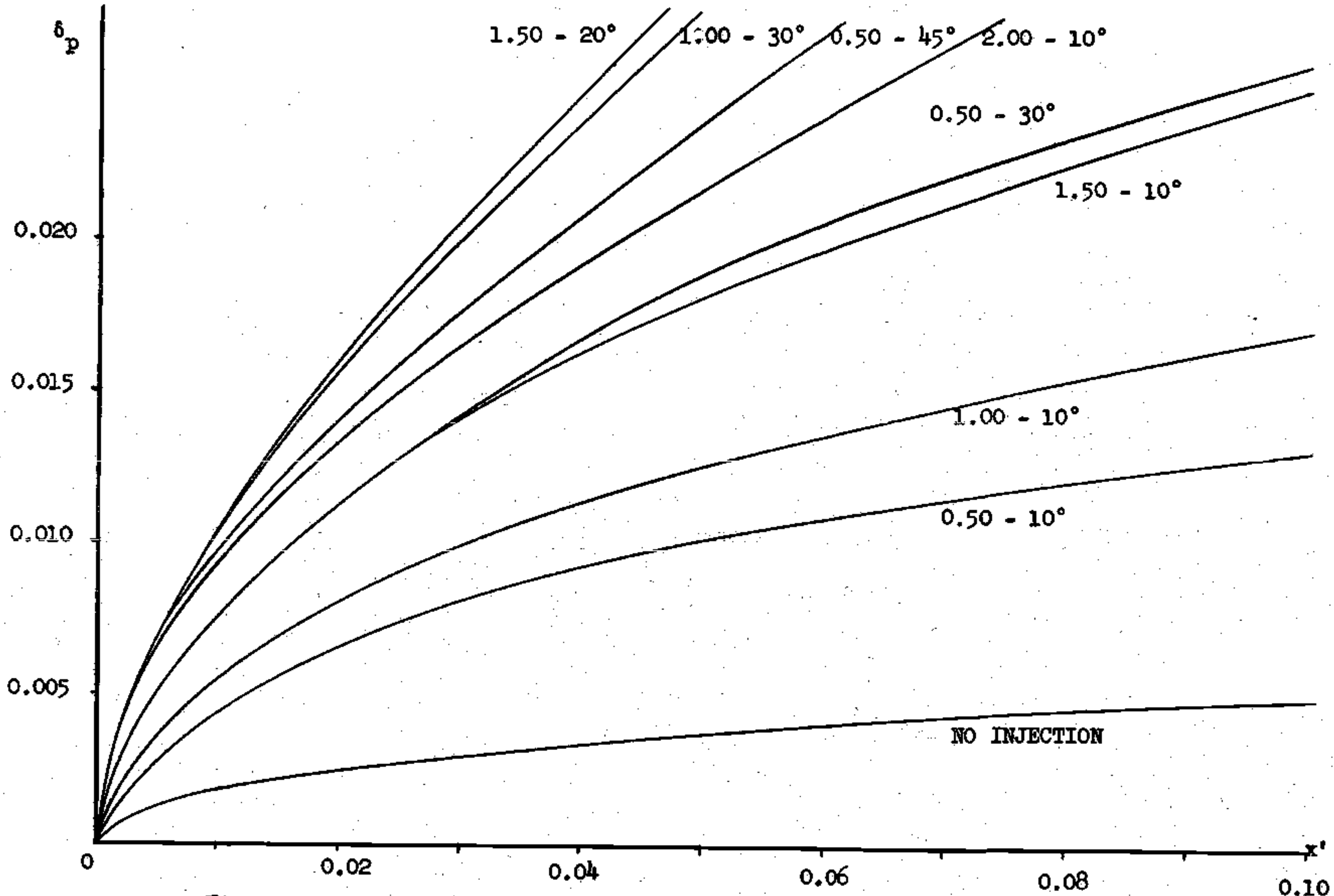

Figure 3. Particle Boundary Layer Thickness versus Distance Along Plate

An examination of Figures 2 and 3 and the y-component of injection velocity in Table 1 reveals a definite dependency of the boundary layer thicknesses on the y-injection velocity. Figures 4 and 5 show this dependency. These graphs demonstrate that both thicknesses are approximately proportional to the y-injection velocity.

Schlichting (7) demonstrates that for flow over a flat plate with no particle injection the boundary layer thickness is

$$\delta_{\text{no injection}} = A \sqrt{\frac{\mu}{\rho u_\infty}}\ x^{\frac{1}{2}} \qquad \text{(V-6)}$$

where A is a constant. The fluid and particle boundary layer thicknesses are also proportional to some power of x (see Figures 2 and 3). By analogy with the no-injection case,

$$\delta = B \sqrt{\frac{\mu}{\rho u_\infty}}\ x^{a} \qquad \text{(V-7)}$$

$$\delta_p = C \sqrt{\frac{\mu}{\rho u_\infty}}\ x^{b} \qquad \text{(V-8)}$$

where a and b are fractional, positive constants. B and C are proportional to the y-injection velocity.

An examination of the x-velocity profiles of Figures 7 through 14 reveals that the fluid x-velocity and the particle x-velocity approach each other as the flow proceeds downstream. This characteristics results from the momentum transfer between the two phases.

The x-velocity profiles with the x-injection velocities less than free stream (see Figures 7 through 11) are of the same form as the

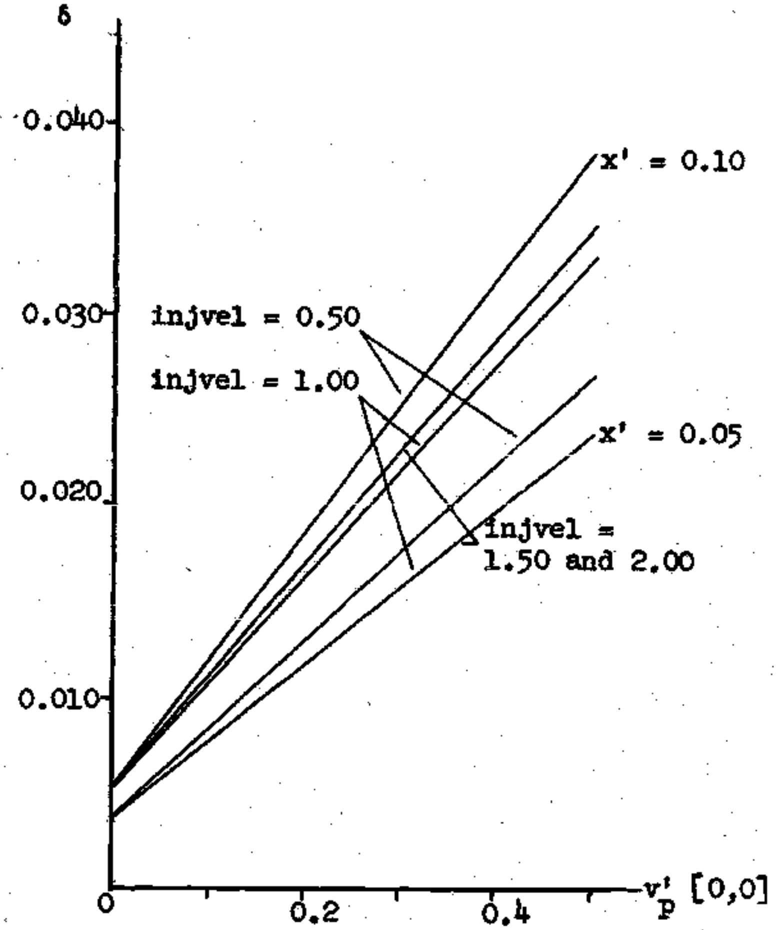

Figure 4. δ versus v'_p [0.0]

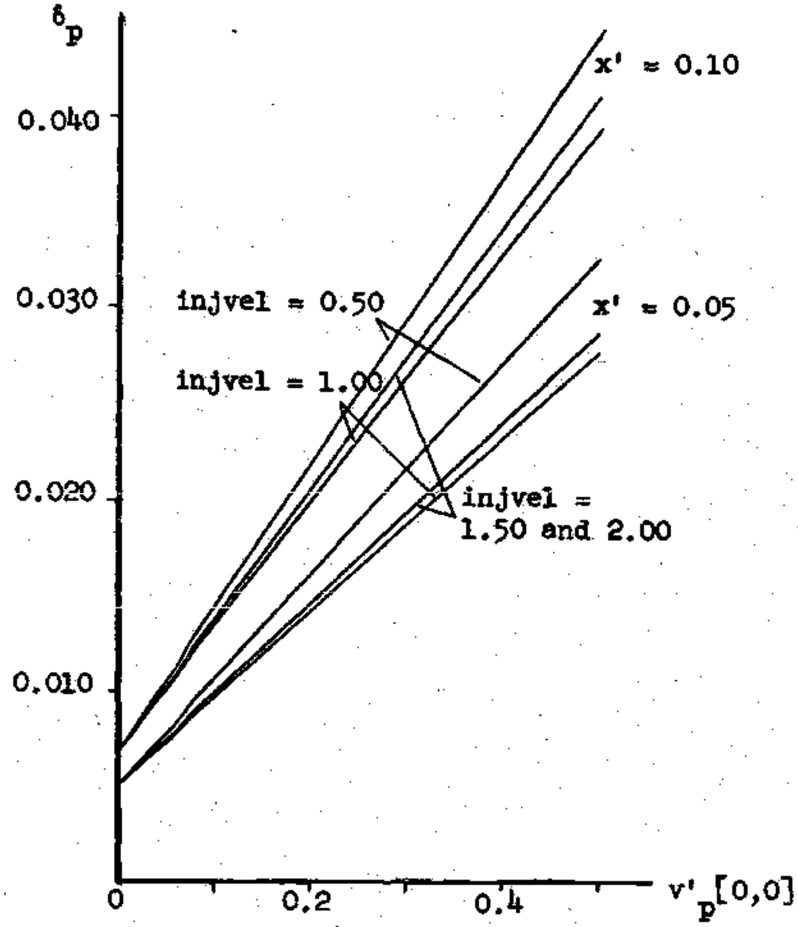

Figure 5. δ_p versus v'_p [0,0]

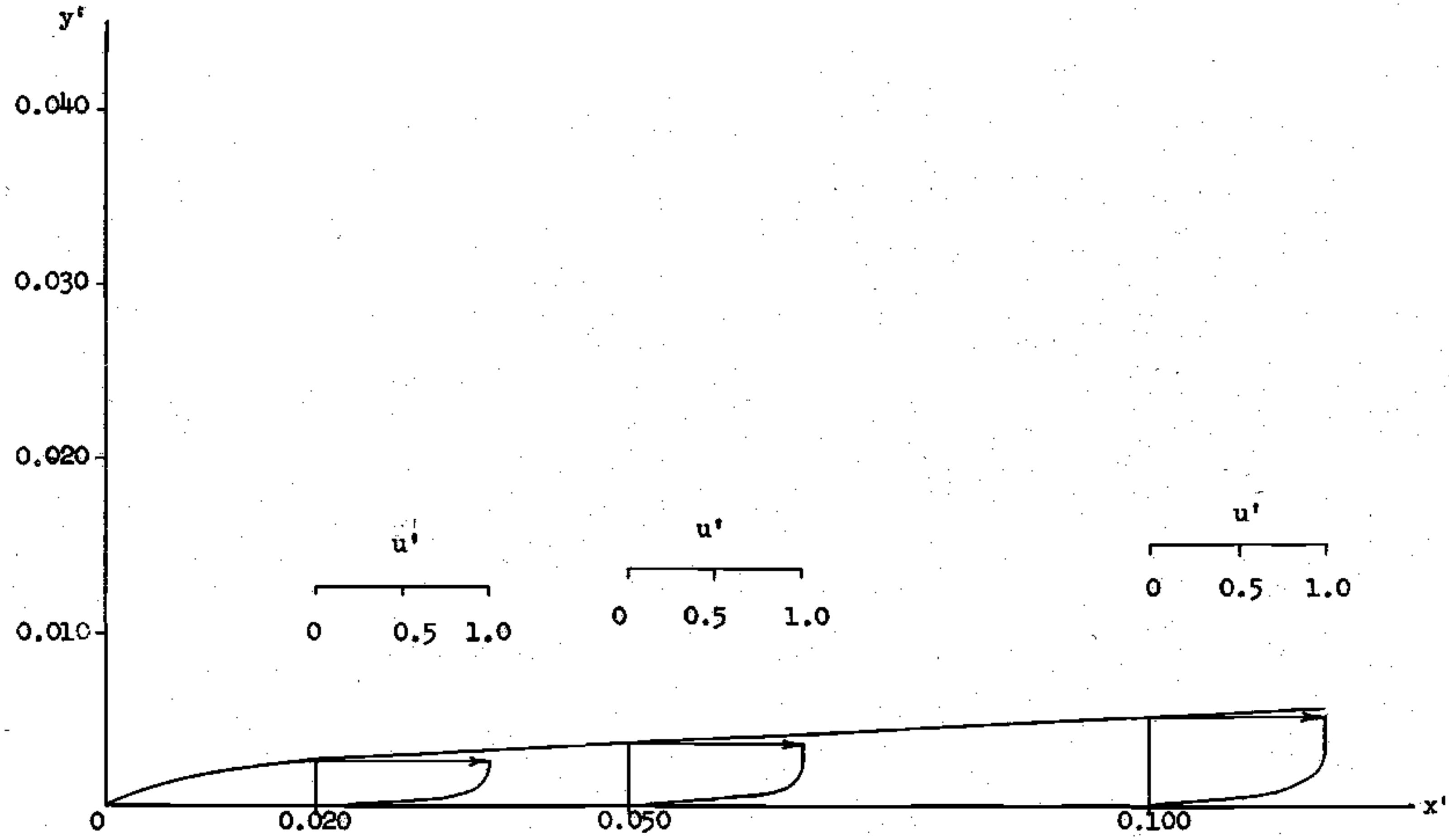

Figure 6. x-Velocity Profiles with No Injection (Blasius Flat Plate Solution)

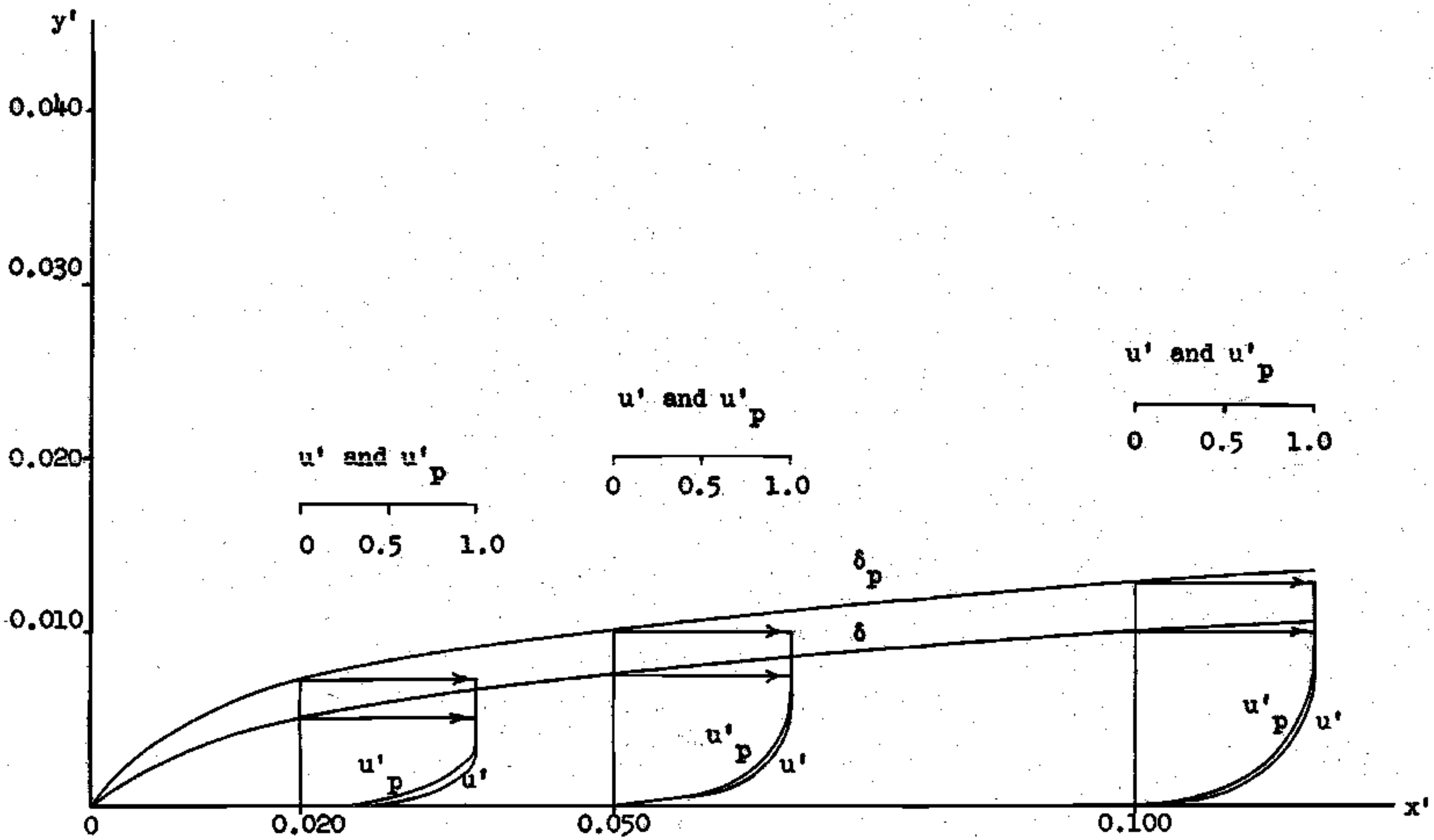

Figure 7. x-Velocity Profiles for injvel = 0.50 and injangle = 10°

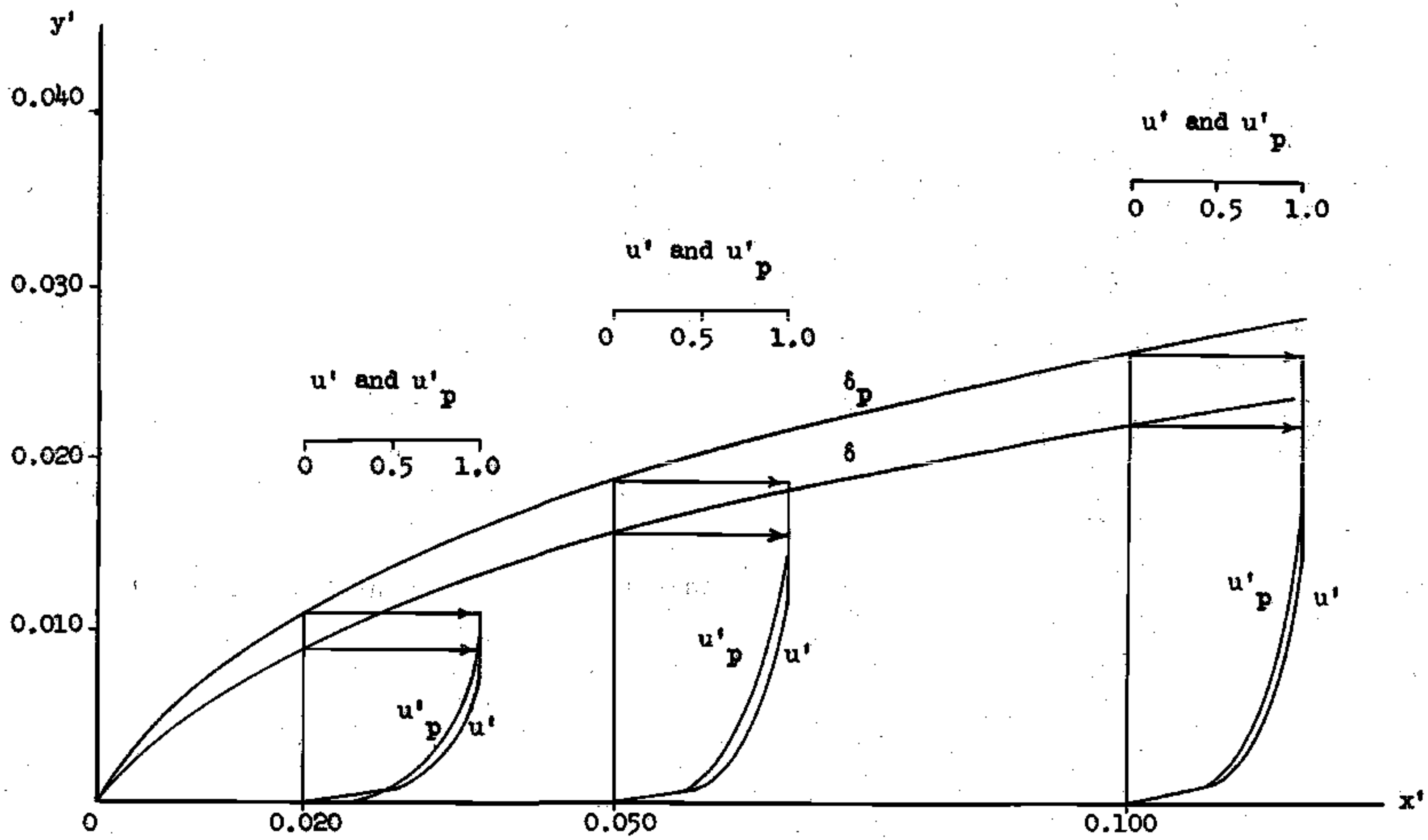

Figure 8. x-Velocity Profiles for injvel = 0.50 and injangle = 30°

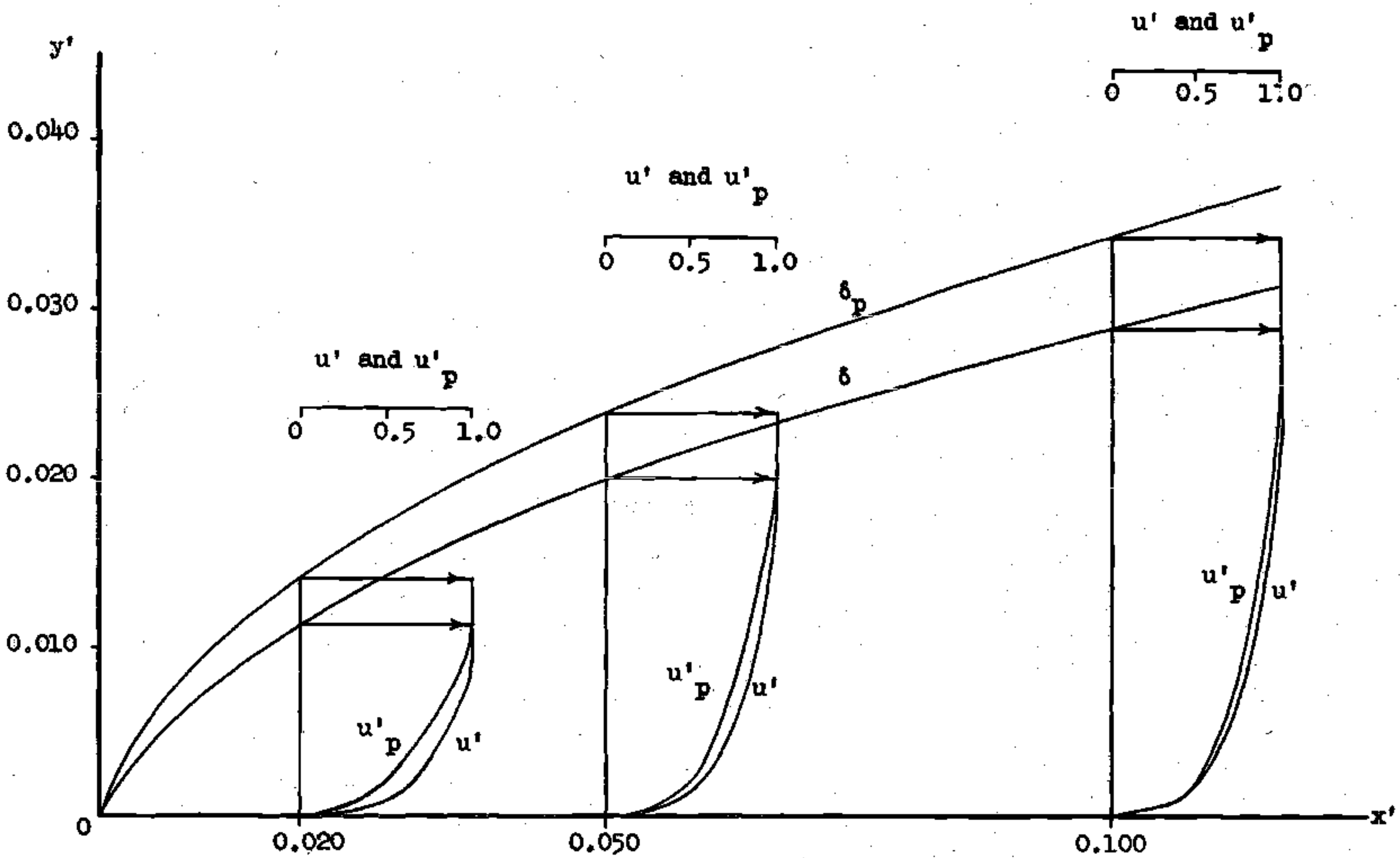

Figure 9. x-Velocity Profiles for injvel = 0.50 and injangle = 45°

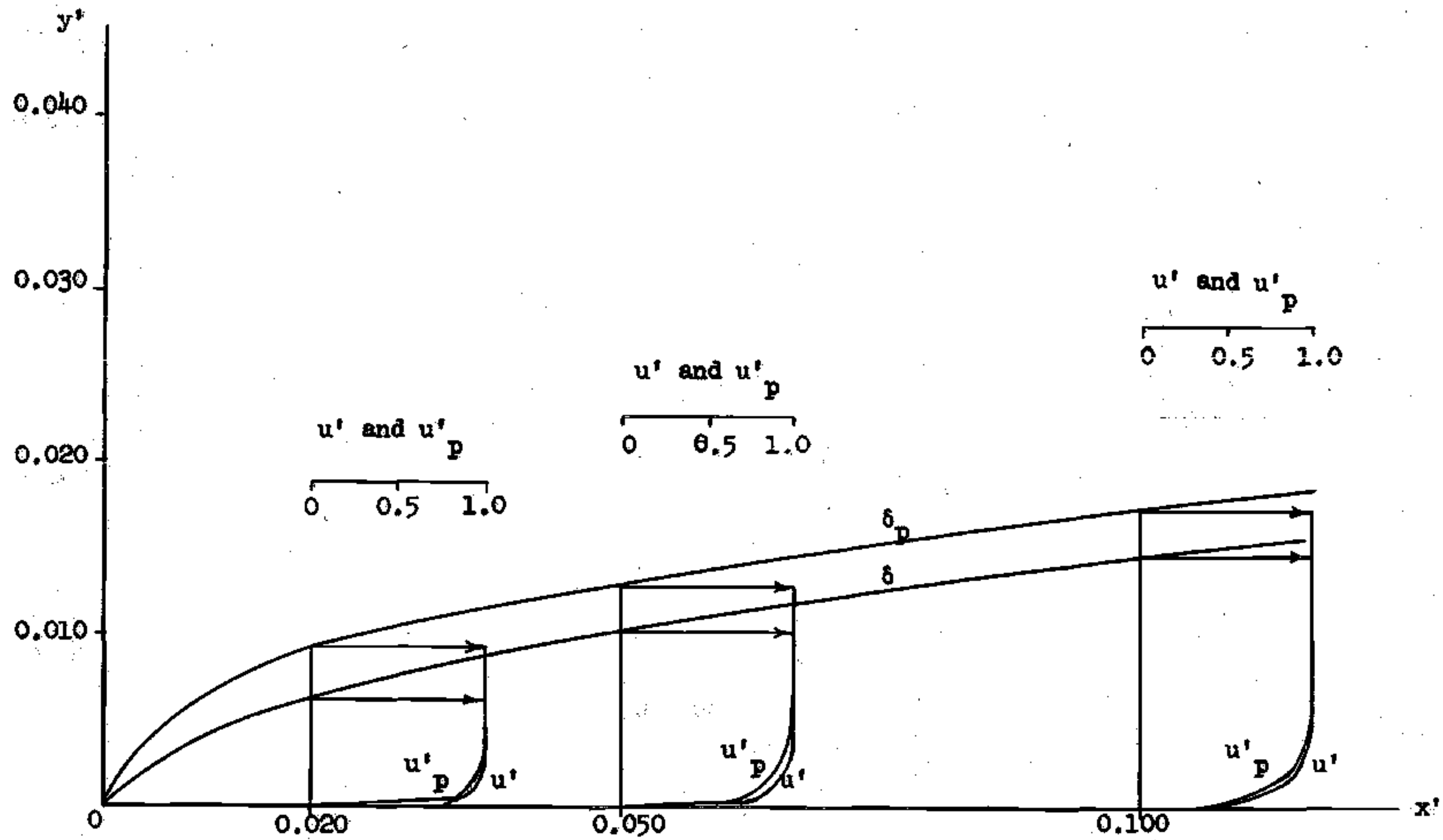

Figure 10. x-Velocity Profiles for injvel = 1.00 and injangle = 10°

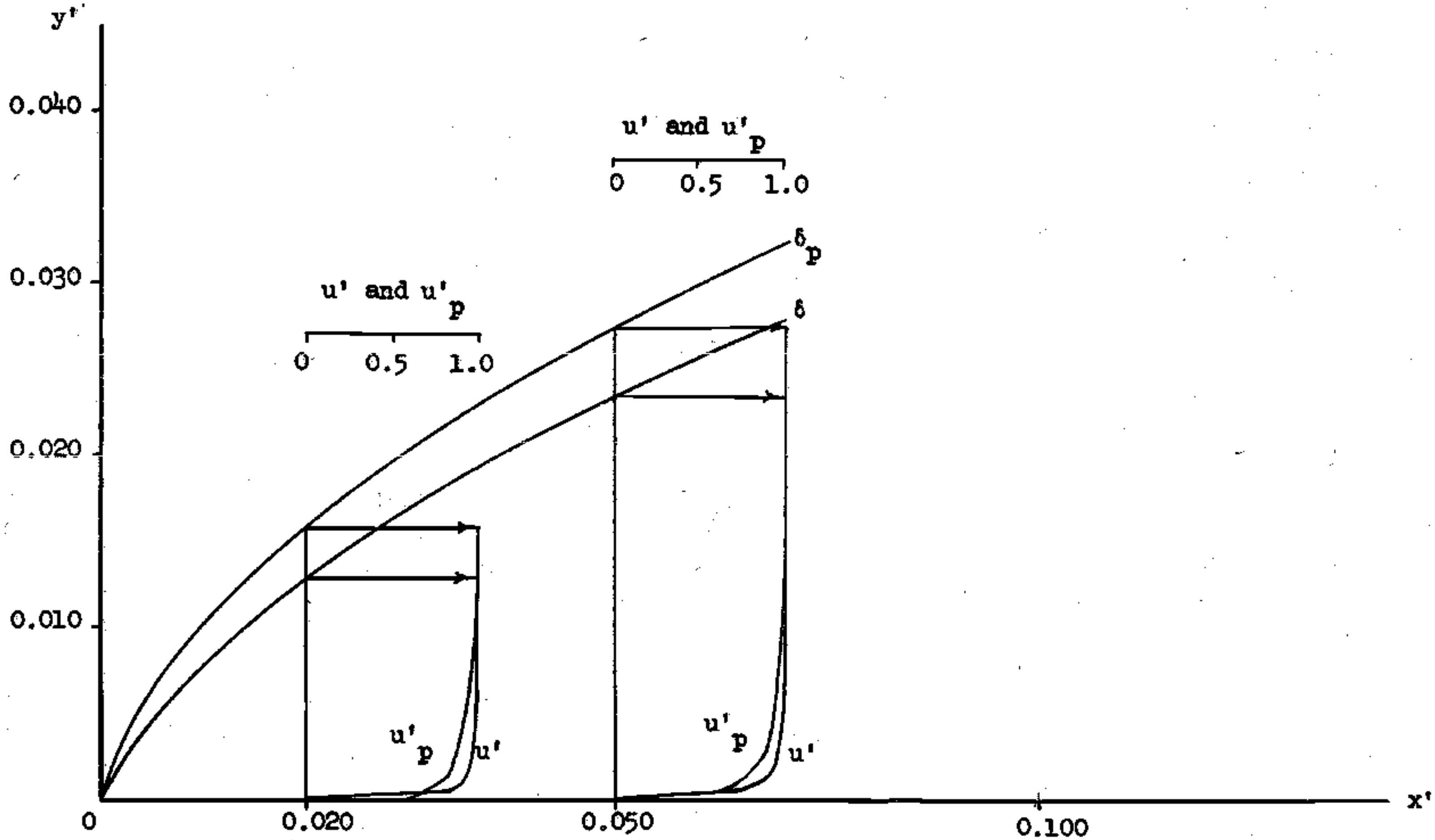

Figure 11. x-Velocity Profiles for injvel = 1.00 and injangle = 30°

no-injection profiles of Figure 6. The x-velocities increase from their wall values to the free-stream velocity. Since the particles slip at the plate, their wall velocities near the leading edge are greater than the fluid's. In spite of this they are slower to accelerate to the free-stream velocity because the solid material is so much denser than the gas. Therefore, the x-velocity profiles of the particles and the fluid must cross until the flow is far enough downstream that the shear has reduced the particle wall velocity to zero. Figure 8 is a good example to show how the profiles cross. At $x' = 0.02$ the two profiles cross; but by $x' = 0.05$ the particle wall velocity has been reduced to zero and there is no crossing.

For greater than free-stream x-injection velocities the profiles increase to a maximum and then decrease to the free-stream value. Figures 12 through 14 display this behavior. In these graphs there is no crossing of the profiles since the particle velocity can never be decreased fast enough to become less than the fluid velocity. An examination of these figures reveals that the maximum of both the fluid and particle x-velocities occur at approximately the same distance above the plate. This distance is always small compared to the boundary layer thicknesses. Figure 15 shows these maximum velocities as a function of the distance down the plate. This graph clearly demonstrates that the particle x-velocity and the fluid x-velocity approach each other.

A comparison of the velocity profiles of different injection conditions, Figures 7 through 14, demonstrates that at constant injection angle and at a given distance downstream the x-velocity profiles become flatter as the x-injection velocity approaches the free-stream velocity. This behavior results from the decrease in slip velocity as

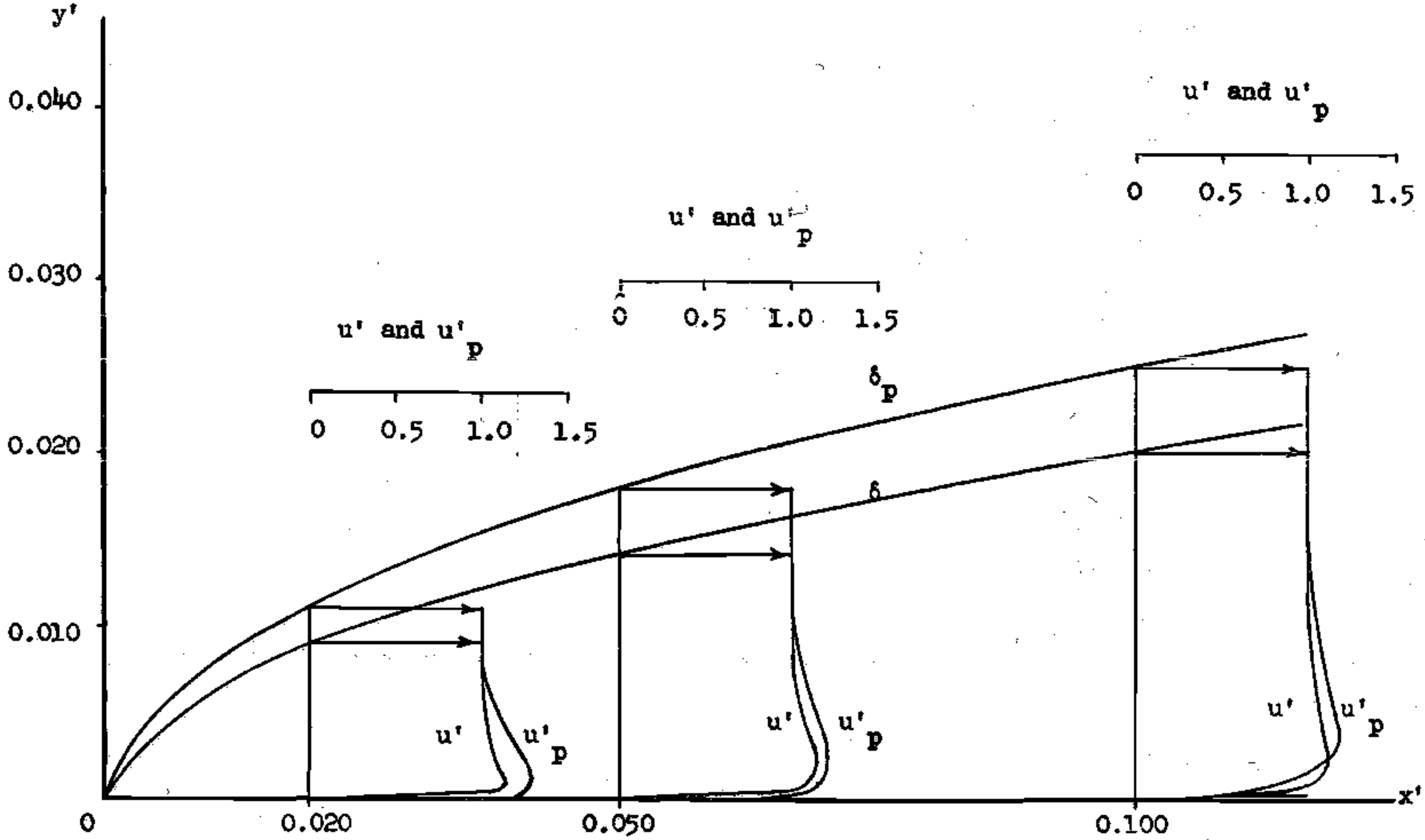

Figure 12. x-Velocity Profiles for injvel = 1.50 and injangle = 10°

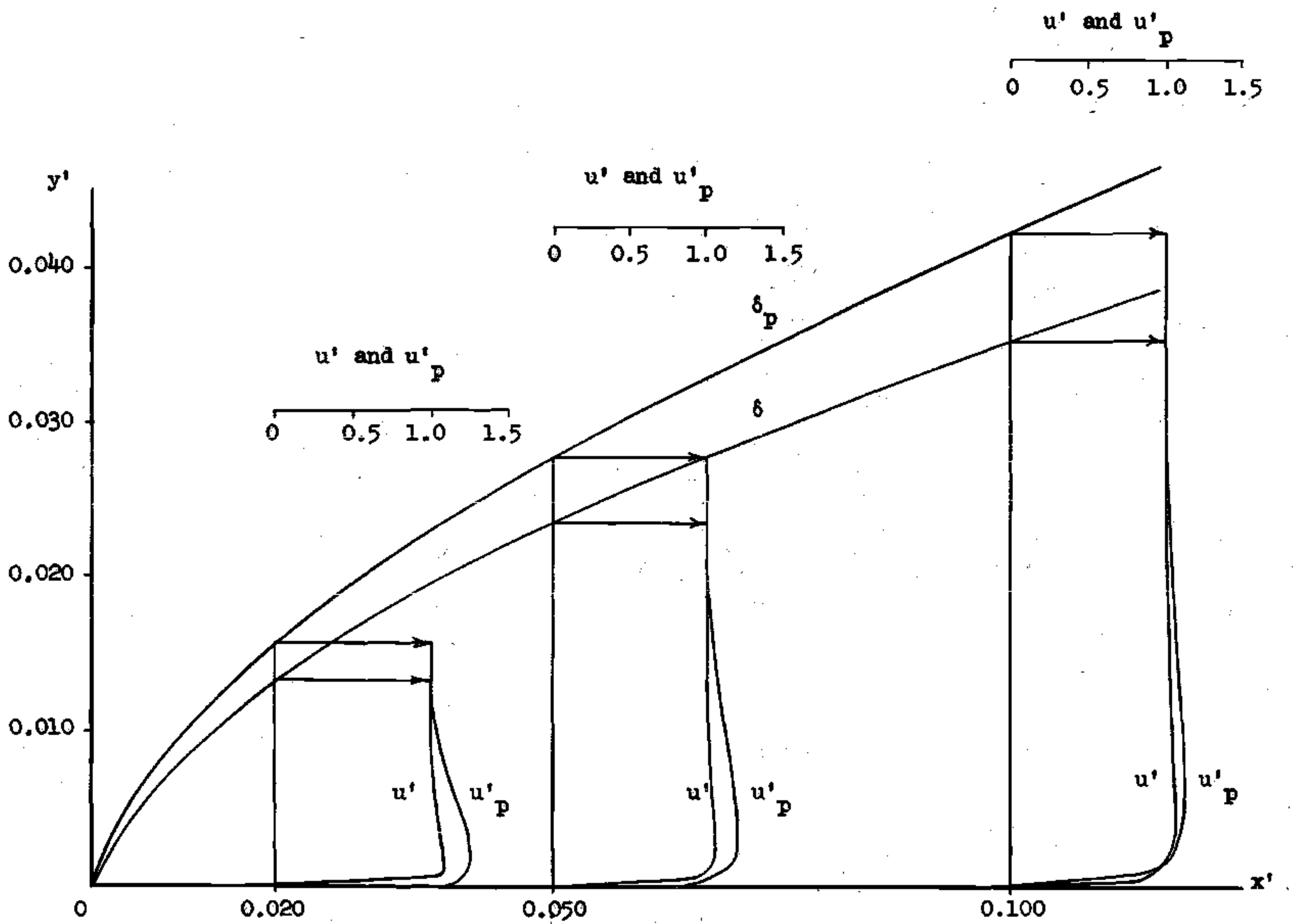

Figure 13. x-Velocity Profiles for injvel = 1.50 and injangle = 20°

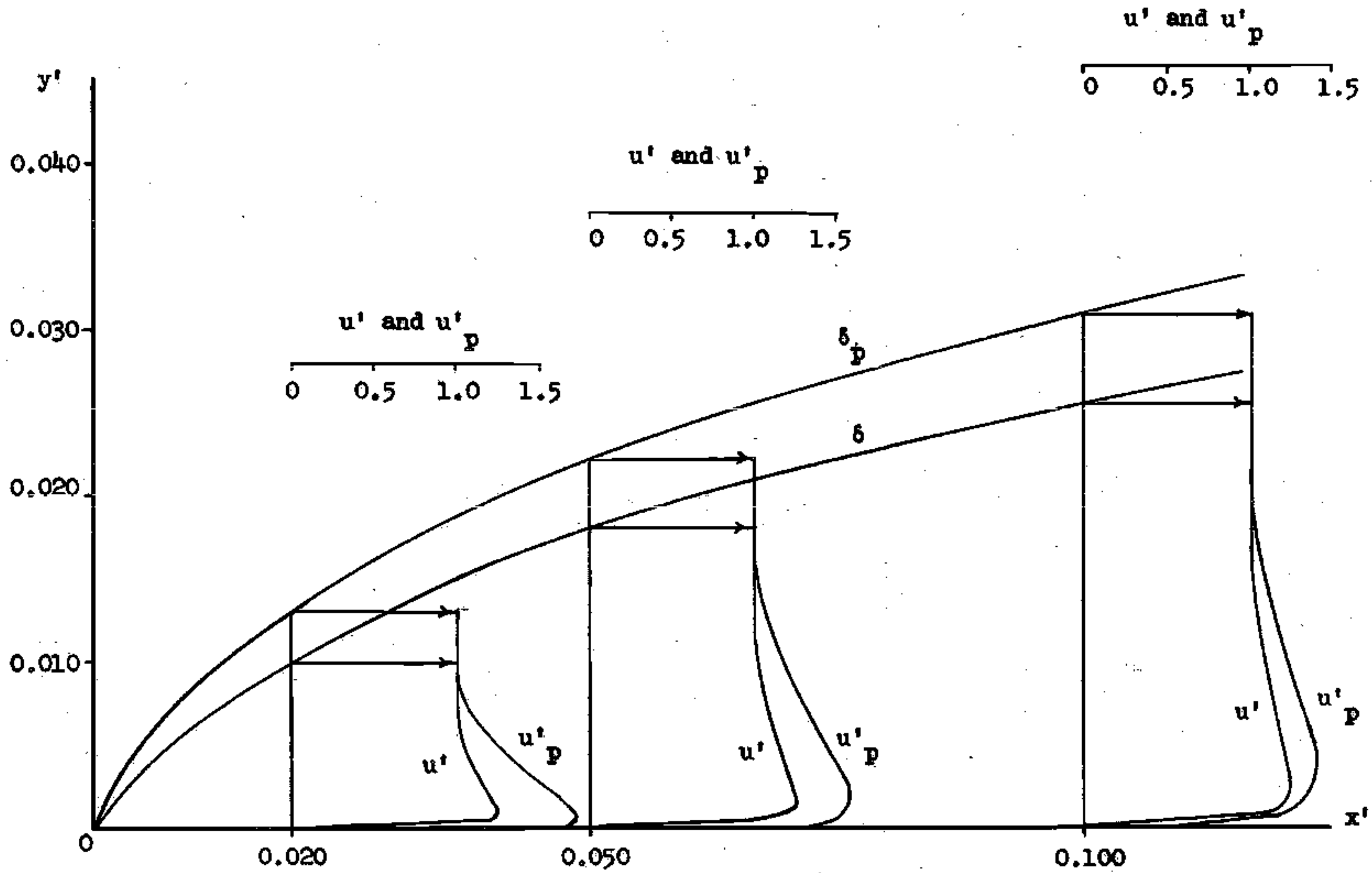

Figure 14. x-Velocity Profiles for injvel = 2.00 and injangle = 10°

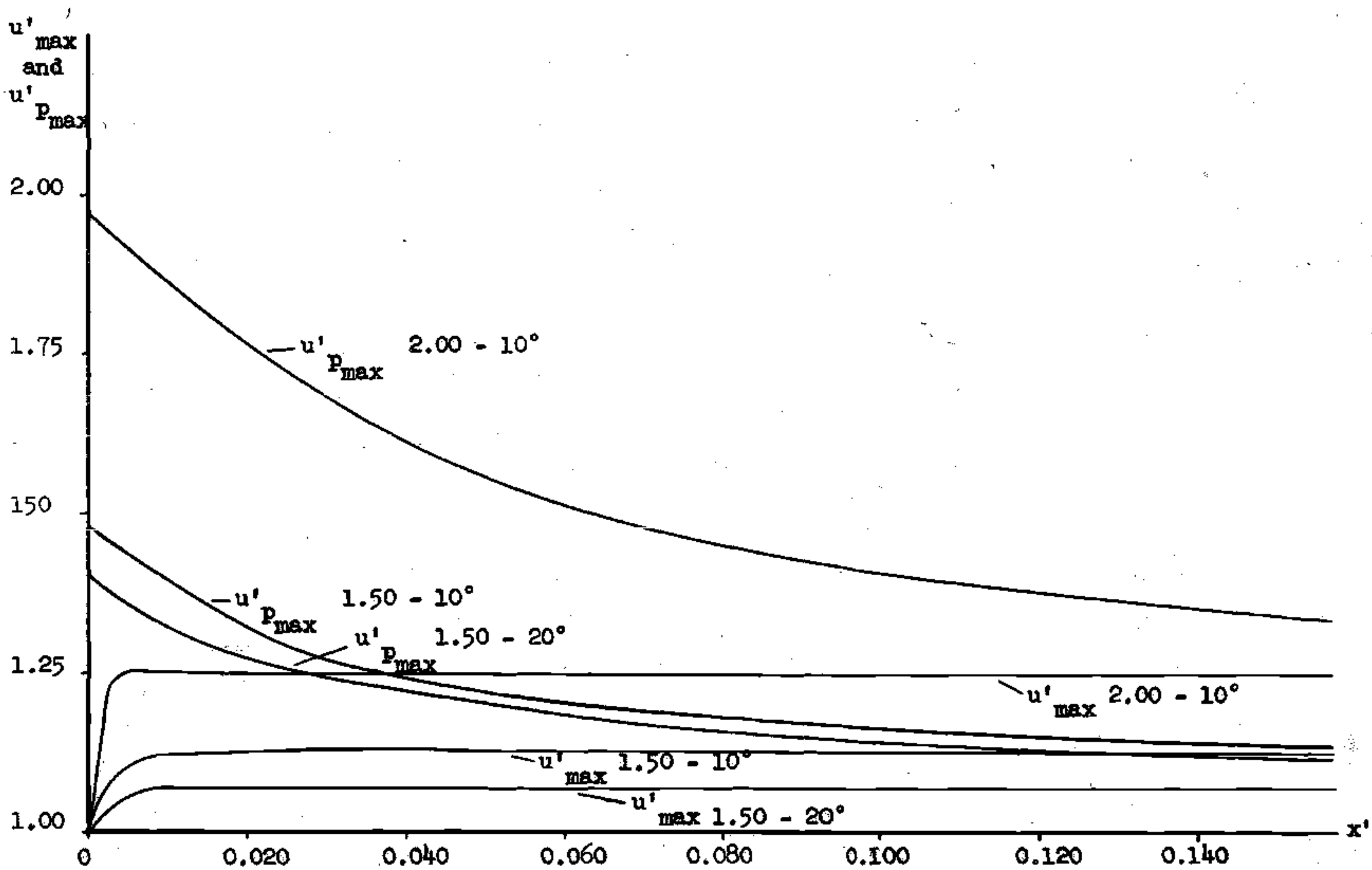

Figure 15. Maximum x-Velocities versus Distance Along Plate

the free-stream velocity is approached.

The particle density profiles are all of the same form once the flow has moved downstream. The density is zero at the plate, increases to a maximum, and then decreases to zero at the particle boundary layer limit. Tables 3 through 10 (APPENDIX D) show that the particles disperse as the flow proceeds downstream. The particle density decreases as the particles disperse and is always considerably less than the injection density.

If the injected particles are at a different temperature than the viscous gas, then heat will be transferred. This heat transfer can be described in differential equation form as follows (see APPENDIX C):

$$u' \frac{\partial \theta}{\partial x'} + v' \frac{\partial \theta}{\partial y'} = \frac{1}{Pr} \frac{1}{Re} \frac{\partial^2 \theta}{\partial y'^2} + \frac{1}{3} \frac{\rho_p}{\rho} \frac{Nu}{Pr} \frac{FL}{u_\infty} (\theta_p - \theta) \qquad \text{(V-9)}$$

$$u'_p \frac{\partial \theta_p}{\partial x'} + v'_p \frac{\partial \theta_p}{\partial y'} = - \frac{1}{3} \frac{Nu}{Pr} \frac{c_p}{c_s} \frac{FL}{u_\infty} (\theta_p - \theta) \qquad \text{(V-10)}$$

These equations become similar to equations (III-8) and (III-10) if

$$Pr = 1 \qquad \text{(V-11)}$$

$$Nu = 3 \qquad \text{(V-12)}$$

$$c_s = c_p. \qquad \text{(V-13)}$$

The Nusselt number can be shown to be about three by applying the various formulae for gas flow over a sphere. Kreith (13) gives one such formula that yields Nu = 2.68.

Since equations (V-9) and (V-10) and equations (III-8) and (III-10) have similar boundary conditions, the temperature profiles are similar to the x-velocity profiles. The profiles become identical if

$$u'_p [0,0] = \theta_p [0,0] \qquad \text{(V-14)}$$

Figures 7 through 14 are x-velocity profile plots. If equation (V-14) holds true, then the figures are also plots of the temperature profiles. The fluid and particle x-velocity data of APPENDIX D are also the fluid and particle temperature data.

At large injection velocities or large injection angles the results become quantitatively less accurate than for small injection parameters. However, they are still qualitatively correct. For all injection conditions the slip velocities are large near the leading edge ($x < \lambda_m$). Large x-injection velocities result in slip velocities that are so large that Stokes drag law is not strictly valid (see APPENDIX A).

For either large injection velocities or large injection angles the boundary layer simplification may not be entirely valid. This approximation is based on the Blasius no-injection boundary layer thickness. The boundary layer thicknesses involved here are much thicker which may render the simplified equations somewhat inaccurate.

The fluid in this analysis has been considered to be incompressible. This assumption introduces very little error as long as the Mach number is small (7)

$$\tfrac{1}{2} M^2 \ll 1. \qquad \text{(V-15)}$$

If this condition is not met, the equations to be solved would be equations (III-1) through (III-6). These equations could be simplified and non-dimensionalized to:

$$\frac{\partial}{\partial x'} (\rho u') + \frac{\partial}{\partial y'} (\rho v') = 0 \qquad \text{(V-16)}$$

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = - \frac{1}{\rho u_\infty^2} \frac{\partial P}{\partial x'} + \frac{1}{Re} \frac{\partial^2 u'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (u_p' - u) \qquad \text{(V-17)}$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = - \frac{1}{\rho u_\infty^2} \frac{\partial P}{\partial y'} + \frac{1}{Re} \frac{\partial^2 v'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (v_p' - v') \qquad \text{(V-18)}$$

$$\frac{\partial}{\partial x'} (\rho_p u_p') + \frac{\partial}{\partial y'} (\rho_p v_p') = 0 \qquad \text{(V-19)}$$

$$u_p' \frac{\partial u_p'}{\partial x'} + v_p' \frac{\partial u_p'}{\partial y'} = - \frac{FL}{u_\infty} (u_p' - u') \qquad \text{(V-20)}$$

$$u_p' \frac{\partial v_p'}{\partial x'} + v_p' \frac{\partial v_p'}{\partial y'} = - \frac{FL}{u_\infty} (v_p' - v') \qquad \text{(V-21)}$$

This thesis has dealt with the analytical problem of wall-slot particle injection into a gas flowing over a flat plate. The system can be simulated by channel flow of a viscous gas in a rectangular duct. The duct must be large enough that the interaction of the boundary layers can be neglected. Particles alone cannot be injected into the channel. There must be a medium to transport them into the main flow. This difficulty is overcome by suspension of the particles in the same gas that is in the channel. The injection gas should be at the same density as the gas in the channel. This method is approximately equivalent to pure particle injection if the injection velocity is not too large relative to the free-stream velocity and if the injection angle is small. Figure 16 is a drawing of this channel flow system.

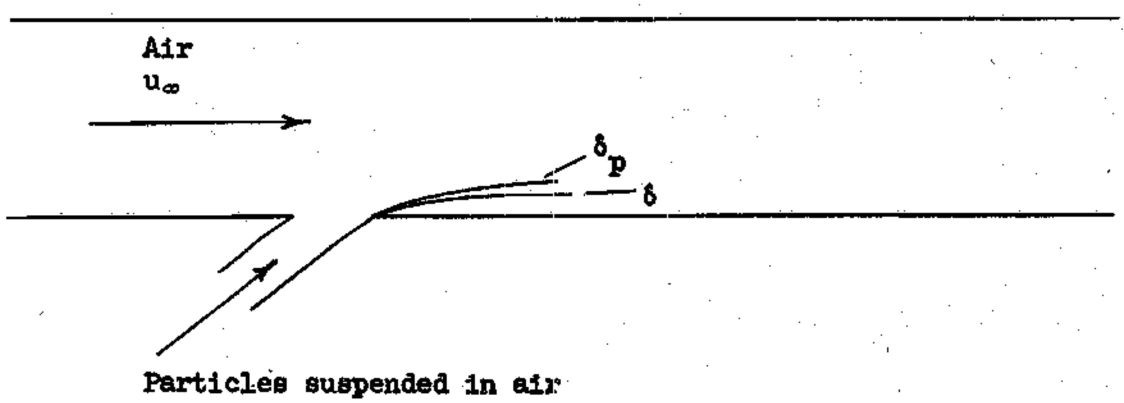

Figure 16. Channel Flow

APPENDIX A

FORCE OF INTERACTION BETWEEN THE TWO PHASES

The drag force of the fluid on the particle phase can be evaluated with the use of Stokes drag law for spheres. The drag coefficient is formed with reference to the dynamic head $\frac{1}{2}\rho(\bar{w} - \bar{w}_p)^2$ and the frontal area,

$$D = C_D \pi \sigma^2 \left[\tfrac{1}{2}\rho(\bar{w} - \bar{w}_p)^2\right] \qquad \text{(A-1)}$$

Stokes drag law (7) states

$$C_D = \frac{24}{Re_d} \qquad Re_d = \frac{\rho(\bar{w} - \bar{w}_p)d}{\mu} < 1 \qquad \text{(A-2)}$$

The substitution of C_D into equation (A-1) yields the drag force per particle

$$D = \frac{24\mu}{\rho(\bar{w} - \bar{w}_p)d} \pi \sigma^2 \left[\tfrac{1}{2}\rho(\bar{w} - \bar{w}_p)^2\right] \qquad \text{(A-3)}$$

where d is the diameter of one particle and $(\bar{w} - \bar{w}_p)$ is the relative velocity of the fluid with respect to the particles. The above drag equation is simplified to

$$D = 6\pi\mu\sigma(\bar{w} - \bar{w}_p) \qquad \text{(A-4)}$$

The total drag per unit volume of mixture is

$$Dn_p = n_p\ (6\ \pi)\ \mu\sigma(\bar{w} - \bar{w}_p) = -\ F_p \qquad \text{(A-5)}$$

where n_p is the number of particles per unit volume of mixture.

Oseen's equation (7) could be used in place of Stokes'

$$C_D = \frac{24}{Re_d} + \frac{9}{2} \qquad Re_d < 5 \qquad \text{(A-6)}$$

It is a slightly better approximation and is valid for particle Reynolds numbers up to five. However, for small slip velocities Stokes law is quite accurate and is used in this thesis. If air is the viscous gas (ρ = 0.071 lbm/ft^3, μ_o = 1.2850 x 10^{-5} lbm/ft-sec) and the particle diameter is 10 microns (3.28 x 10^{-5} ft), then Stokes drag law is valid for

$$|\bar{w} - \bar{w}_p| < 5.5\ \frac{ft}{sec}\ . \qquad \text{(A-7)}$$

This condition is satisfied except near the leading edge of the plate for most injection conditions.

APPENDIX B

SIMPLIFICATION OF CONSERVATION EQUATIONS

The equations to be simplified are eqations (III-1) through (III-6).

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial v}{\partial y} + v \frac{\partial \rho}{\partial y} = 0 \qquad \text{(B-1)}$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = - \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left(2\mu \frac{\partial u}{\partial x} - \frac{2}{3} \mu \frac{\partial u}{\partial x} - \frac{2}{3} \mu \frac{\partial v}{\partial y}\right)$$

$$+ \frac{\partial}{\partial x}(\mu \frac{\partial v}{\partial x} + \mu \frac{\partial u}{\partial y}) + \rho_p F(u_p - u) \qquad \text{(B-2)}$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}\right) = - \frac{\partial P}{\partial y} + \frac{\partial}{\partial y} \left(2 \mu \frac{\partial v}{\partial y} - \frac{2}{3} \mu \frac{\partial u}{\partial x} - \frac{2}{3} \mu \frac{\partial v}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y} (\mu \frac{\partial v}{\partial x} + \mu \frac{\partial u}{\partial y}) + \rho_p F(v_p - v) \qquad \text{(B-3)}$$

$$\frac{\partial \rho_p}{\partial t} + \rho_p \frac{\partial u_p}{\partial x} + u_p \frac{\partial \rho_p}{\partial x} + \rho_p \frac{\partial v_p}{\partial y} + v_p \frac{\partial \rho_p}{\partial y} = 0 \qquad \text{(B-4)}$$

$$\rho_p\left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y}\right) = - \rho_p F(u_p - u) \qquad \text{(B-5)}$$

$$\rho_p\left(\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y}\right) = - \rho_p F(v_p - v) \qquad \text{(B-6)}$$

Schlichting (7) has shown that if the Mach number is small compared with unity, fluid compressibility can be neglected. With the use of fluid incompressibility and the assumption of steady state, the equations reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{B-7}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial x^2} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\rho_p}{\rho} F(u_p - u) \tag{B-8}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 v}{\partial x^2} + \frac{\mu}{\rho} \frac{\partial^2 v}{\partial y^2} + \frac{\rho_p}{\rho} F(v_p - v) \tag{B-9}$$

$$\frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0 \tag{B-10}$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = - F(u_p - u) \tag{B-11}$$

$$u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = - F(v_p - v) \tag{B-12}$$

For convenience, the conservation equations are put in non-dimensional form by using the following relations:

$$x' = \frac{x}{L} \tag{B-13}$$

$$y' = \frac{y}{L} \tag{B-14}$$

$$u' = \frac{u}{u_\infty} \tag{B-15}$$

$$v' = \frac{v}{u_\infty} \tag{B-16}$$

$$u'_p = \frac{u_p}{u_\infty} \tag{B-17}$$

$$v'_p = \frac{v_p}{u_\infty} \tag{B-18}$$

$$Re = \frac{\rho u_\infty L}{\mu} \tag{B-19}$$

The non-dimensionalization leads to

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{B-20}$$

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = \frac{1}{Re} \frac{\partial u'}{\partial x'^2} + \frac{1}{Re} \frac{\partial^2 u'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (u'_p - u') \tag{B-21}$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = \frac{1}{Re} \frac{\partial^2 v'}{\partial x'^2} + \frac{1}{Re} \frac{\partial^2 v'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (v'_p - v') \tag{B-22}$$

$$\frac{\partial}{\partial x} (\rho_p u'_p) + \frac{\partial}{\partial y} (\rho_p v'_p) = 0 \tag{B-23}$$

$$u'_p \frac{\partial u'_p}{\partial x'} + v'_p \frac{\partial u'_p}{\partial y'} = -\frac{FL}{u_\infty} (u'_p - u') \tag{B-24}$$

$$u'_p \frac{\partial v'_p}{\partial x'} + v'_p \frac{\partial v'_p}{\partial y'} = -\frac{FL}{u_\infty} (v'_p - v') \tag{B-25}$$

The final simplification is obtained from the boundary layer approximations (6). The terms of the fluid momentum equations are examined on an order of magnitude basis to determine which are negligible in the boundary layer. Equations (B-21) and (B-22) are rewritten and examined term by term

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = \frac{1}{Re} \frac{\partial^2 u'}{\partial x'^2} + \frac{1}{Re} \frac{\partial^2 u'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (u'_p - u') \tag{B-26}$$

$$1 \quad 1 \qquad \delta \quad \frac{1}{\delta} \qquad \delta^2 \quad 1 \qquad \delta^2 \quad \frac{1}{\delta^2} \qquad\qquad 1 \quad 1$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = \frac{1}{Re} \frac{\partial^2 v'}{\partial x'^2} + \frac{1}{Re} \frac{\partial^2 v'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (v'_p - v') \tag{B-27}$$

$$1 \quad \delta \qquad \delta \quad \frac{\delta}{\delta} \qquad \delta^2 \quad \delta \qquad \delta^2 \quad \frac{\delta}{\delta^2} \qquad\qquad \delta \quad \delta$$

In the first equation $\frac{1}{Re} \frac{\partial^2 u'}{\partial x'^2}$ may be neglected, and the second equation can be discarded completely.

Therefore, the governing equations of the two-phase flow are:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{B-28}$$

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = \frac{1}{Re} \frac{\partial^2 u'}{\partial y'^2} + \frac{\rho_p}{\rho} \frac{FL}{u_\infty} (u'_p - u') \tag{B-29}$$

$$\frac{\partial}{\partial x'} (\rho_p u'_p) + \frac{\partial}{\partial y'} (\rho_p v'_p) = 0 \tag{B-30}$$

$$u'_p \frac{\partial u'_p}{\partial x'} + v'_p \frac{\partial v'_p}{\partial y'} = - \frac{FL}{u_\infty} (u'_p - u') \tag{B-31}$$

$$u_p' \frac{\partial v_p'}{\partial x'} + v_p' \frac{\partial v_p'}{\partial y'} = - \frac{FL}{u_\infty} (v_p' - v') \tag{B-32}$$

APPENDIX C

HEAT TRANSFER BETWEEN THE TWO PHASES

When the particles and the viscous gas are at different temperatures, there is heat transfer between the two phases. The convective heat transfer from the particles to the gas per unit volume of mixture is (6)

$$q_p = n_p h(4\ \pi\sigma^2)\ (T_p - T). \tag{C-1}$$

This equation can be rewritten in terms of the Nusselt number

$$Nu = \frac{hd}{k} = \frac{2h\sigma}{k} \tag{C-2}$$

$$q_p = n_p \frac{kNu}{2\sigma}\ (4\ \pi\sigma^2)\ (T_p - T) \tag{C-3}$$

where q_p = heat transfer per unit volume of mixture,

h = convective heat transfer coefficient,

k = themal conductivity,

T = temperature gas,

T_p = temperature of the particles,

n_p = number of particles per unit volume of mixture = $\dfrac{\rho_p}{\frac{4}{3}\ \pi\sigma^2 \rho_s}$.

n_p is substituted into equation (C-3)

$$q_p = \frac{3}{2} \frac{\rho_p}{\rho_s} \frac{kNu}{\sigma^2} (T_p - T) \qquad (C-4)$$

The two-dimensional conservation of energy equation for the gas is

$$\rho \; c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)$$

$$+ \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \phi + q_p \qquad (C-5)$$

and for the particles is

$$\rho_p c_s \left(\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right) = - q_p \qquad (C-6)$$

where c_p = constant pressure specific heat of the gas,

c_s = specific heat of the solid particles,

ϕ = viscous dissipation function.

With the use of the simplifications of APPENDIX B plus the assumption that k and μ are independent of temperature, equations (C-5) and (C-6) in non-dimensional form become

$$u' \frac{\partial \theta}{\partial x'} + v' \frac{\partial \theta}{\partial y'} = \frac{k}{\rho c_p} \frac{1}{u_\infty L} \frac{\partial^2 \theta}{\partial y'^2} + \frac{3}{2} \frac{kNu}{\rho_s c_p \sigma^2} \frac{\rho_p}{\rho} \frac{L}{u_\infty} (\theta_p - \theta) \qquad (C-7)$$

$$u'_p \frac{\partial \theta_p}{\partial x'} + v'_p \frac{\partial \theta_p}{\partial y'} = - \frac{3}{2} \frac{kNu}{\rho_s c_p \sigma^2} \frac{c_p}{c_s} \frac{L}{u_\infty} (\theta_p - \theta) \qquad (C-8)$$

where T_w = temperature of the wall,

T_∞ = temperature of the gas in free stream,

$$\theta = \frac{T - T_w}{T_\infty - T_w} ,$$

$$\theta_p = \frac{T_p - T_w}{T_\infty - T_w} ,$$

By substitution of

$$Pr = \frac{\mu c_p}{k} \tag{C-9}$$

$$F = \frac{9}{2} \frac{\mu}{\sigma^2 \rho_s} \tag{C-10}$$

equations (C-7) and (C-8) may be written as

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{1}{Re} \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{3} \frac{\rho_p}{\rho} \frac{Nu}{Pr} \frac{FL}{u_\infty} (\theta_p - \theta) \tag{C-11}$$

$$u_p \frac{\partial \theta_p}{\partial x} + v \frac{\partial \theta_p}{\partial y} = - \frac{1}{3} \frac{Nu}{Pr} \frac{c_p}{c_s} \frac{FL}{u_\infty} (\theta_p - \theta) \tag{C-12}$$

The boundary conditions for these two equations are as follows:

(i) temperature of the gas at the leading edge of the plate is constant

$$\theta [0,y] = 1 \tag{C-13}$$

(ii) temperature of the gas at the plate is that of the plate

$$\theta [x,0] = 0 \tag{C-14}$$

(iii) temperature of the gas must approach free stream value as y approaches infinity

$$\lim_{y\to\infty} \theta\,[x,y] = 1 \qquad \text{(C-15)}$$

(iv) particle temperature at the injection point is constant

$$\theta_p\,[0,0] = \text{constant} \qquad \text{(C-16)}$$

(v) particle temperature must approach free stream value as y approaches infinity

$$\lim_{y\to\infty} \theta_p\,[x,y] = 1. \qquad \text{(C-17)}$$

APPENDIX D

DATA

APPENDIX D contains complete data for the various injection conditions. Table 1 displays the injection conditions investigated. Tables 2 through 10 give the data separated according to the injection parameters. Tables 11 through 14 compare the x-velocity profiles for the different injection parameters considered.

All quantities in this appendix are non-dimensional except the angles which are in degrees and the densities which are in pounds-mass per cubic foot.

Table 1. Injection Parameters Investigated

Injection Velocity	Injection Angle	x-Injection Velocity	y-Injection Velocity
0.50	10	0.49240	0.08682
0.50	30	0.43301	0.25000
0.50	45	0.35355	0.35355
1.00	10	0.98481	0.17365
1.00	30	0.86603	0.50000
1.50	10	1.47721	0.26047
1.50	20	1.40954	0.51303
2.00	10	1.96962	0.34730

Table 2. Blasius Flat Plate Solution
(no injection)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	0.00000	0.00000	0.00000
0.0200	0.0000	0.00000	0.00000			
	0.0010	0.98421	0.00315			
	0.0020	0.99991	0.00398			
	0.0030	1.00000	0.00399			
0.0400	0.0000	0.00000	0.00000			
	0.0010	0.94605	0.00269			
	0.0020	0.99871	0.00411			
	0.0030	0.99999	0.00418			
	0.0040	1.00000	0.00418			
0.0600	0.0000	0.00000	0.00000			
	0.0010	0.89718	0.00227			
	0.0020	0.99484	0.00406			
	0.0030	0.99988	0.00422			
	0.0040	1.00000	0.00423			
0.0800	0.0000	0.00000	0.00000			
	0.0010	0.84439	0.00188			
	0.0020	0.98741	0.00385			
	0.0030	0.99948	0.00414			
	0.0040	0.99999	0.00416			
	0.0050	1.00000	0.00416			
0.1000	0.0000	0.00000	0.00000			
	0.0010	0.79192	0.00153			
	0.0020	0.97642	0.00354			
	0.0030	0.99854	0.00396			
	0.0040	0.99995	0.00400			
	0.0050	1.00000	0.00401			
0.1500	0.0000	0.00000	0.00000			
	0.0010	0.67617	0.00088			
	0.0020	0.93776	0.00264			
	0.0030	0.99259	0.00329			
	0.0040	0.99945	0.00341			
	0.0050	0.99997	0.00342			
	0.0060	1.00000	0.00342			
0.2000	0.0000	0.00000	0.00000			
	0.0010	0.58881	0.00052			
	0.0020	0.89229	0.00193			
	0.0030	0.98119	0.00266			
	0.0040	0.99782	0.00286			
	0.0050	0.99983	0.00289			
	0.0060	0.99999	0.00289			
	0.0070	1.00000	0.00289			

Table 3. Solution for injvel = 0.50 and injangle = 10°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	0.49240	0.08682	14.50000
0.0200	0.0000	0.00000	0.00000	0.19586	0.00000	0.00000
	0.0010	0.82085	0.00490	0.66449	0.02314	0.14632
	0.0020	0.96186	0.01284	0.87160	0.04948	0.17911
	0.0030	0.99722	0.01659	0.97000	0.06413	0.08745
	0.0040	0.99994	0.01706	0.99610	0.06808	0.01852
	0.0050	1.00000	0.01708	0.99971	0.06861	0.00193
	0.0060	1.00000	0.01708	0.99999	0.06865	0.00011
	0.0070	1.00000	0.01708	1.00000	0.06865	0.00000
0.0400	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.69702	0.00312	0.57848	0.00902	0.06217
	0.0020	0.86478	0.00764	0.76363	0.02327	0.10486
	0.0030	0.95426	0.01140	0.88247	0.03750	0.11572
	0.0040	0.99114	0.01375	0.95475	0.04792	0.08387
	0.0050	0.99918	0.01449	0.98751	0.05306	0.03774
	0.0060	0.99997	0.01459	0.99760	0.05468	0.01042
	0.0070	1.00000	0.01459	0.99968	0.05502	0.00184
	0.0080	1.00000	0.01459	0.99997	0.05507	0.00022
	0.0090	1.00000	0.01459	1.00000	0.05507	0.00000
0.0600	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.61639	0.00203	0.54877	0.00469	0.03976
	0.0020	0.80246	0.00509	0.72627	0.01317	0.06729
	0.0030	0.90319	0.00750	0.83601	0.02253	0.08672
	0.0040	0.96328	0.00955	0.91306	0.03141	0.08797
	0.0050	0.99074	0.01085	0.96189	0.03817	0.06770
	0.0060	0.99864	0.01136	0.98686	0.04205	0.03723
	0.0070	0.99989	0.01146	0.99655	0.04365	0.00411
	0.0080	1.00000	0.01147	0.99931	0.04411	0.00372
	0.0090	1.00000	0.01147	0.99990	0.04421	0.00070
	0.0100	1.00000	0.01147	0.99999	0.04423	0.00009
	0.0110	1.00000	0.01147	1.00000	0.04423	0.00000
0.0800	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.55798	0.00131	0.52872	0.00279	0.03098
	0.0020	0.76197	0.00357	0.70914	0.00841	0.05008
	0.0030	0.86701	0.00520	0.81435	0.01468	0.06702
	0.0040	0.93499	0.00665	0.88910	0.02114	0.07561
	0.0050	0.97511	0.00782	0.94138	0.02695	0.07136
	0.0060	0.99332	0.00852	0.97374	0.03127	0.05391
	0.0070	0.99886	0.00880	0.99037	0.03381	0.03090
	0.0080	0.99989	0.00886	0.99717	0.03494	0.01298
	0.0090	0.99999	0.00887	0.99934	0.03532	0.00399
	0.0100	1.00000	0.00887	0.99988	0.03542	0.00092
	0.0110	1.00000	0.00887	0.99998	0.03544	0.00016
	0.0120	1.00000	0.00887	1.00000	0.03544	0.00000

Table 3. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.1000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.51409	0.00087	0.50768	0.00178	0.02708
	0.0020	0.73280	0.00262	0.69757	0.00577	0.04101
	0.0030	0.84195	0.00382	0.80254	0.01016	0.05533
	0.0040	0.91299	0.00486	0.87568	0.01482	0.06498
	0.0050	0.95929	0.00576	0.92836	0.01933	0.06683
	0.0060	0.98514	0.00642	0.96361	0.02314	0.05870
	0.0070	0.99614	0.00679	0.98419	0.02583	0.04204
	0.0080	0.99934	0.00692	0.99425	0.02735	0.02343
	0.0090	0.99993	0.00695	0.99827	0.02802	0.00988
	0.0100	1.00000	0.00695	0.99958	0.02826	0.00316
	0.0110	1.00000	0.00695	0.99991	0.02832	0.00078
	0.0120	1.00000	0.00695	0.99999	0.02833	0.00015
	0.0130	1.00000	0.00695	1.00000	0.02833	0.00000

Table 4. Solution for injvel = 0.50 and injangle = 30°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	0.43301	0.25000	14.50000
0.0200	0.0000	0.00000	0.00000	0.13647	0.00000	0.00000
	0.0010	0.66438	0.00378	0.49138	0.01782	0.05578
	0.0020	0.76252	0.00901	0.60073	0.04382	0.11854
	0.0030	0.84791	0.01631	0.69380	0.07631	0.18673
	0.0040	0.92210	0.02569	0.78793	0.11342	0.23957
	0.0050	0.97249	0.03454	0.87693	0.14965	0.24029
	0.0060	0.99447	0.03972	0.94513	0.17707	0.17176
	0.0070	0.99946	0.04125	0.98261	0.19156	0.08002
	0.0080	0.99998	0.04145	0.99625	0.19656	0.02332
	0.0090	1.00000	0.04146	0.99946	0.19766	0.00428
	0.0100	1.00000	0.04146	0.99995	0.19782	0.00051
	0.0110	1.00000	0.04146	1.00000	0.19783	0.00000
0.0400	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.58615	0.00257	0.47757	0.00654	0.02176
	0.0020	0.69221	0.00540	0.58562	0.01671	0.04158
	0.0030	0.75339	0.00754	0.64848	0.02864	0.06434
	0.0040	0.80865	0.01020	0.70238	0.04266	0.08627
	0.0050	0.85965	0.01348	0.75391	0.05871	0.10905
	0.0060	0.90513	0.01725	0.80442	0.07644	0.12946
	0.0070	0.94300	0.02116	0.85331	0.09508	0.14329
	0.0080	0.97107	0.02470	0.89859	0.11335	0.14465
	0.0090	0.98841	0.02732	0.93729	0.12951	0.12844
	0.0100	0.99661	0.02880	0.96650	0.14189	0.09581
	0.110	0.99933	0.02938	0.98506	0.14974	0.05735
	0.0120	0.99991	0.02952	0.99458	0.15371	0.02667
	0.0130	0.99999	0.02955	0.99842	0.15528	0.00953
	0.0140	1.00000	0.02955	0.99963	0.15576	0.00263
	0.0150	1.00000	0.02955	0.99993	0.15587	0.00057
	0.0160	1.00000	0.02955	0.99999	0.15589	0.00010
	0.0170	1.00000	0.02955	1.00000	0.15590	0.00000
0.0600	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.52470	0.00175	0.47335	0.00345	0.01426
	0.0020	0.66033	0.00407	0.59233	0.00931	0.02553
	0.0030	0.71947	0.00530	0.65126	0.01565	0.03807
	0.0040	0.76711	0.00654	0.69653	0.02292	0.05077
	0.0050	0.81052	0.00801	0.73719	0.03123	0.06346
	0.0060	0.85043	0.00972	0.77551	0.04057	0.07597
	0.0070	0.88666	0.01165	0.81216	0.05084	0.08784
	0.0080	0.91869	0.01372	0.84718	0.06183	0.09823
	0.0090	0.94581	0.01580	0.88021	0.07323	0.10573
	0.0100	0.96730	0.01773	0.91059	0.08458	0.10837
	0.0110	0.98273	0.01934	0.93739	0.09525	0.10397
	0.0120	0.99235	0.02049	0.95962	0.10455	0.09110
	0.0130	0.99730	0.02117	0.97651	0.11187	0.07075
	0.0140	0.99927	0.02148	0.98794	0.11692	0.04721

Table 4. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0150	0.99985	0.02159	0.99464	0.11990	0.02638
	0.0160	0.99998	0.02161	0.99796	0.12137	0.01217
	0.0170	1.00000	0.02162	0.99934	0.12198	0.00463
	0.0180	1.00000	0.02162	0.99982	0.12218	0.00145
	0.0190	1.00000	0.02162	0.99996	0.12224	0.00038
	0.0200	1.00000	0.02162	0.99999	0.12225	0.00008
	0.0210	1.00000	0.02162	1.00000	0.12226	0.00000
0.0800	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.47388	0.00117	0.45949	0.00211	0.01152
	0.0020	0.63679	0.00312	0.59522	0.00608	0.01920
	0.0030	0.70092	0.00407	0.65628	0.01003	0.02840
	0.0040	0.74616	0.00482	0.69925	0.01440	0.03753
	0.0050	0.78608	0.00564	0.73631	0.01937	0.04641
	0.0060	0.82260	0.00659	0.77032	0.02498	0.05507
	0.0070	0.85607	0.00765	0.80218	0.03120	0.06347
	0.0080	0.88650	0.00882	0.83223	0.03797	0.07145
	0.0090	0.91372	0.01007	0.86050	0.04519	0.07868
	0.0100	0.93743	0.01133	0.88690	0.05273	0.08463
	0.0110	0.95732	0.01256	0.91118	0.06039	0.08848
	0.1200	0.97312	0.01368	0.93297	0.06792	0.08919
	0.0130	0.98475	0.01461	0.95190	0.07499	0.08560
	0.0140	0.99244	0.01531	0.96758	0.08127	0.07688
	0.0150	0.99683	0.01576	0.97977	0.08644	0.06323
	0.0160	0.99892	0.01600	0.98849	0.09030	0.04649
	0.0170	0.99971	0.01610	0.99413	0.09287	0.02988
	0.0180	0.99994	0.01613	0.99734	0.09463	0.01651
	0.0190	0.99999	0.01614	0.99894	0.09510	0.00777
	0.0200	1.00000	0.01614	0.99963	0.09542	0.00312
	0.0210	1.00000	0.01614	0.99989	0.09553	0.00107
	0.0220	1.00000	0.01614	0.99997	0.09557	0.00032
	0.0230	1.00000	0.01614	0.99999	0.09558	0.00008
	0.0240	1.00000	0.01614	1.00000	0.09558	0.00000
0.1000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.43289	0.00079	0.43779	0.00138	0.01044
	0.0020	0.61641	0.00242	0.59256	0.00431	0.01598
	0.0030	0.68824	0.00324	0.65888	0.00702	0.02352
	0.0040	0.73344	0.00377	0.70189	0.00987	0.03102
	0.0050	0.77170	0.00430	0.73768	0.01308	0.03813
	0.0060	0.80635	0.00489	0.76995	0.01671	0.04493
	0.0070	0.83803	0.00555	0.79978	0.02076	0.05148
	0.0080	0.86710	0.00628	0.82758	0.02521	0.05777
	0.0090	0.89345	0.00706	0.85355	0.03000	0.06370
	0.0100	0.91704	0.00788	0.87770	0.03508	0.06908
	0.0110	0.93776	0.00871	0.90000	0.04036	0.07362
	0.0120	0.95545	0.00951	0.92031	0.04575	0.07688
	0.0130	0.96996	0.01027	0.93848	0.05109	0.07823

Table 4. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0140	0.98124	0.01092	0.95430	0.05622	0.07694
	0.0150	0.98938	0.01145	0.96762	0.06094	0.07230
	0.0160	0.99468	0.01184	0.97833	0.06507	0.06394
	0.0170	0.99772	0.01208	0.98646	0.06843	0.05223
	0.0180	0.99918	0.01221	0.99219	0.07095	0.03863
	0.0190	0.99976	0.01227	0.99589	0.07265	0.02539
	0.0200	0.99995	0.01229	0.99805	0.07366	0.01463
	0.0210	0.99999	0.01230	0.99917	0.07420	0.00733
	0.0220	1.00000	0.01230	0.99969	0.07445	0.00319
	0.0230	1.00000	0.01230	0.99989	0.07455	0.00121
	0.0240	1.00000	0.01230	0.99997	0.07458	0.00040
	0.0250	1.00000	0.01230	0.99999	0.07459	0.00012
	0.0260	1.00000	0.01230	1.00000	0.07460	0.00000

Table 5. Solution for injvel = 0.50 and injangle = 45°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	0.35355	0.35355	14.50000
0.0200	0.0000	0.00000	0.00000	0.05701	0.00000	0.00000
	0.0010	0.57057	0.00352	0.41330	0.01364	0.03771
	0.0020	0.65057	0.00765	0.49693	0.03371	0.07784
	0.0030	0.72249	0.01319	0.56102	0.05921	0.12522
	0.0040	0.79590	0.02148	0.62725	0.09104	0.17933
	0.0050	0.86736	0.03256	0.70084	0.12943	0.23436
	0.0060	0.92962	0.04523	0.78186	0.17283	0.27282
	0.0070	0.97307	0.05650	0.86369	0.21624	0.26546
	0.0080	0.99369	0.06320	0.93235	0.25137	0.19694
	0.0090	0.99923	0.06541	0.97531	0.27222	0.10159
	0.0100	0.99996	0.06575	0.99371	0.28060	0.03436
	0.0110	1.00000	0.06578	0.99891	0.28281	0.00750
	0.0120	1.00000	0.06578	0.99987	0.28318	0.00106
	0.0130	1.00000	0.06578	0.99999	0.28323	0.00010
	0.0140	1.00000	0.06578	1.00000	0.28323	0.00000
0.0400	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.50094	0.00231	0.41798	0.00506	0.01571
	0.0020	0.59844	0.00484	0.50935	0.01301	0.02911
	0.0030	0.65043	0.00645	0.55847	0.02206	0.04361
	0.0040	0.69815	0.00841	0.59925	0.03272	0.05863
	0.0050	0.74420	0.01092	0.63789	0.04519	0.07431
	0.0060	0.78868	0.01405	0.67620	0.05955	0.09071
	0.0070	0.83127	0.01778	0.71497	0.07586	0.10764
	0.0080	0.87130	0.02206	0.75454	0.09406	0.12447
	0.0090	0.90777	0.02672	0.79489	0.11395	0.13980
	0.0100	0.93935	0.03147	0.83556	0.13506	0.15114
	0.0110	0.96461	0.03581	0.87547	0.15649	0.15476
	0.0120	0.98249	0.03950	0.91275	0.17690	0.14639
	0.0130	0.99306	0.04194	0.94493	0.19460	0.12365
	0.0140	0.99793	0.04323	0.96960	0.20808	0.08961
	0.0150	0.99956	0.04372	0.98575	0.21675	0.05361
	0.0160	0.99994	0.04384	0.99445	0.22131	0.02575
	0.0170	0.99999	0.04386	0.99823	0.22323	0.00981
	0.0180	1.00000	0.04387	0.99954	0.22387	0.00296
	0.0190	1.00000	0.04387	0.99990	0.22404	0.00071
	0.0210	1.00000	0.04387	1.00000	0.22409	0.00000
0.0600	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.44421	0.00150	0.41459	0.00271	0.01084
	0.0020	0.57259	0.00362	0.52114	0.00738	0.01872
	0.0030	0.62541	0.00464	0.57057	0.01216	0.02758
	0.0040	0.66717	0.00556	0.60758	0.01759	0.03638
	0.0050	0.70604	0.00665	0.64072	0.02385	0.04481
	0.0060	0.74295	0.00797	0.67200	0.03101	0.05332
	0.0070	0.77811	0.00951	0.70219	0.03908	0.06188
	0.0080	0.81153	0.01129	0.73163	0.04808	0.07051

Table 5. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0090	0.84310	0.01329	0.76051	0.05800	0.07919
	0.0100	0.87262	0.01549	0.78894	0.06879	0.08776
	0.0110	0.89982	0.01783	0.81690	0.08038	0.09592
	0.0120	0.92434	0.02026	0.84430	0.09266	0.10318
	0.0130	0.94576	0.02267	0.87094	0.10540	0.10876
	0.0140	0.96367	0.02493	0.89647	0.11833	0.11155
	0.0150	0.97773	0.02693	0.92038	0.13100	0.11015
	0.0160	0.98784	0.02853	0.94199	0.14287	0.10316
	0.0170	0.99428	0.02966	0.96055	0.15334	0.08985
	0.0180	0.99777	0.03034	0.97540	0.16183	0.07107
	0.0190	0.99930	0.03067	0.98618	0.16802	0.04978
	0.0200	0.99983	0.03080	0.99312	0.17199	0.03020
	0.0210	0.99997	0.03083	0.99700	0.17419	0.01562
	0.0220	1.00000	0.03084	0.99887	0.17523	0.00684
	0.0230	1.00000	0.03084	0.99963	0.17565	0.00254
	0.0240	1.00000	0.03084	0.99990	0.17579	0.00080
	0.0250	1.00000	0.03084	0.99997	0.17583	0.00022
	0.0260	1.00000	0.03084	0.99999	0.17585	0.00005
	0.0270	1.00000	0.03084	1.00000	0.17585	0.00000
0.0800	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.39819	0.00095	0.39697	0.00164	0.00919
	0.0020	0.55172	0.00272	0.52358	0.00487	0.01453
	0.0030	0.61119	0.00356	0.57773	0.00786	0.02133
	0.0040	0.65159	0.00414	0.61457	0.01107	0.02790
	0.0050	0.68762	0.00476	0.64636	0.01474	0.03408
	0.0060	0.72135	0.00547	0.67573	0.01894	0.04000
	0.0070	0.75323	0.00630	0.70348	0.02368	0.04578
	0.0080	0.78340	0.00723	0.72998	0.02897	0.05148
	0.0090	0.81194	0.00829	0.75543	0.03480	0.05713
	0.0100	0.83883	0.00945	0.77996	0.04116	0.06275
	0.0110	0.86403	0.01071	0.80364	0.04803	0.06830
	0.0120	0.88744	0.01206	0.82650	0.05538	0.07369
	0.0130	0.90896	0.01346	0.84853	0.06416	0.07880
	0.0140	0.92846	0.01489	0.86968	0.07130	0.08341
	0.0150	0.94568	0.01631	0.88986	0.07971	0.08719
	0.0160	0.96056	0.01767	0.90894	0.08825	0.08968
	0.0170	0.97292	0.01891	0.92673	0.09675	0.09027
	0.0180	0.98268	0.02000	0.94299	0.10500	0.08826
	0.0190	0.98987	0.02088	0.95745	0.11271	0.08294
	0.0200	0.99470	0.02152	0.96985	0.11961	0.07392
	0.0210	0.99759	0.02194	0.97994	0.12542	0.06144
	0.0220	0.99907	0.02218	0.98763	0.12995	0.04676
	0.0230	0.99970	0.02229	0.99303	0.13316	0.03201
	0.0240	0.99992	0.02233	0.99644	0.13521	0.01941
	0.0250	0.99998	0.02234	0.99837	0.13636	0.01033
	0.0260	1.00000	0.02235	0.99933	0.13694	0.00480

Table 5. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0270	1.00000	0.02235	0.99976	0.13718	0.00195
	0.0280	1.00000	0.02235	0.99992	0.13728	0.00070
	0.0290	1.00000	0.02235	0.99998	0.13731	0.00022
	0.0300	1.00000	0.02235	0.99999	0.13732	0.00006
	0.0310	1.00000	0.02235	1.00000	0.13732	0.00000
0.1000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.36203	0.00063	0.37274	0.00105	0.00868
	0.0020	0.53284	0.00209	0.51914	0.00345	0.01238
	0.0030	0.60088	0.00285	0.58033	0.00553	0.01811
	0.0040	0.64197	0.00329	0.61845	0.00760	0.02372
	0.0050	0.67678	0.00369	0.65014	0.00993	0.02885
	0.0060	0.70892	0.00414	0.67898	0.01259	0.03362
	0.0070	0.73909	0.00464	0.70594	0.01561	0.03815
	0.0080	0.76754	0.00521	0.73141	0.01899	0.04253
	0.0090	0.79439	0.00585	0.75560	0.02273	0.04680
	0.0100	0.81970	0.00655	0.77865	0.02683	0.05100
	0.0110	0.84350	0.00731	0.80065	0.03126	0.05514
	0.0120	0.86577	0.00812	0.82167	0.03602	0.05920
	0.0130	0.88650	0.00898	0.84172	0.04109	0.06316
	0.0140	0.90563	0.00987	0.86083	0.04644	0.06695
	0.0150	0.92311	0.01078	0.87898	0.05202	0.07048
	0.0160	0.93887	0.01170	0.89613	0.05780	0.07360
	0.0170	0.95282	0.01259	0.91224	0.06371	0.07610
	0.0180	0.96490	0.01344	0.92723	0.06968	0.07769
	0.0190	0.97504	0.01423	0.94101	0.07559	0.07800
	0.0200	0.98322	0.01492	0.95346	0.08133	0.07660
	0.0210	0.98948	0.01549	0.96448	0.08676	0.07304
	0.0220	0.99395	0.01594	0.97395	0.09172	0.06697
	0.0230	0.99688	0.01625	0.98179	0.09604	0.05832
	0.0240	0.99858	0.01645	0.98798	0.09961	0.04757
	0.0250	0.99944	0.01656	0.99258	0.10236	0.03580
	0.0260	0.99982	0.01661	0.99577	0.10430	0.02451
	0.0270	0.99995	0.01663	0.99778	0.10555	0.01509
	0.0280	0.99999	0.01663	0.99894	0.10627	0.00829
	0.0290	1.00000	0.01664	0.99954	0.10664	0.00406
	0.0300	1.00000	0.01664	0.99982	0.10681	0.00177
	0.0310	1.00000	0.01664	0.99994	0.10688	0.00069
	0.0320	1.00000	0.01664	0.99998	0.10691	0.00024
	0.0330	1.00000	0.01664	0.99999	0.10692	0.00007
	0.0340	1.00000	0.01664	1.00000	0.10692	0.00000
	0.0140	0.90563	0.00987	0.86083	0.04644	0.06695
	0.0150	0.92311	0.0.078	0.7898	0.05202	0.07048
	0.0160	0.93887	0.01170	0.89613	0.05780	0.07360
	0.0170	0.95282	0.01259	0.91224	0.06371	0.07610
	0.0180	0.96490	0.01344	0.92723	0.06968	0.07769
	0.0190	0.97504	0.01423	0.94101	0.07559	0.07800

Table 6. Solution for injvel = 1.00 and injangle = 10°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	0.98481	0.17365	14.50000
0.0200	0.0000	0.00000	0.00000	0.98826	0.00000	0.00000
	0.0010	0.96895	0.00321	0.92565	0.03387	0.08381
	0.0020	0.99169	0.00477	0.97035	0.06691	0.15116
	0.0030	0.99753	0.00543	0.98756	0.09511	0.17876
	0.0040	0.99956	0.00575	0.99553	0.11520	0.14515
	0.0050	0.99996	0.00584	0.99881	0.12573	0.07395
	0.0060	1.00000	0.00585	0.99978	0.12933	0.02202
	0.0070	1.00000	0.00585	0.99997	0.13009	0.00385
	0.0080	1.00000	0.00585	0.99999	0.13018	0.00041
	0.0090	1.00000	0.00585	1.00000	0.13019	0.00000
0.0400	0.0000	0.00000	0.00000	0.39172	0.00000	0.00000
	0.0010	0.92173	0.00280	0.85700	0.01482	0.03257
	0.0020	0.97732	0.00466	0.93874	0.03110	0.06223
	0.0030	0.98832	0.00527	0.96738	0.04673	0.08893
	0.0040	0.99430	0.00569	0.98177	0.06132	0.10815
	0.0050	0.99764	0.00597	0.99015	0.07421	0.11402
	0.0060	0.99925	0.00614	0.99515	0.08457	0.10154
	0.0070	0.99983	0.00622	0.99794	0.09176	0.07256
	0.0080	0.99998	0.00624	0.99929	0.09580	0.03934
	0.0090	1.00000	0.00625	0.99980	0.09753	0.01561
	0.0100	1.00000	0.00625	0.99996	0.09809	0.00451
	0.0110	1.00000	0.00625	0.99999	0.09822	0.00097
	0.0120	1.00000	0.00625	1.00000	0.09825	0.00000
0.0600	0.0000	0.00000	0.00000	0.09518	0.00000	0.00000
	0.0010	0.86715	0.00240	0.80201	0.00826	0.02014
	0.0020	0.96365	0.00448	0.91824	0.01824	0.03781
	0.0030	0.98012	0.00506	0.95513	0.02778	0.05553
	0.0040	0.98823	0.00541	0.97273	0.03695	0.07154
	0.0050	0.99337	0.00567	0.98308	0.04566	0.08400
	0.0060	0.99661	0.00587	0.98973	0.05367	0.09041
	0.0070	0.99852	0.00600	0.99410	0.06065	0.08806
	0.0080	0.99948	0.00608	0.99690	0.06627	0.07544
	0.0090	0.99986	0.00612	0.99856	0.07027	0.05468
	0.0100	0.99998	0.00613	0.99942	0.07269	0.03225
	0.0110	1.00000	0.00614	0.99981	0.07389	0.01507
	0.0120	1.00000	0.00614	0.99994	0.07436	0.00553
	0.0130	1.00000	0.00614	0.99999	0.07451	0.00160
	0.0140	1.00000	0.00614	1.00000	0.07455	0.00000
0.0800	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.81007	0.00199	0.75799	0.00516	0.01410
	0.0020	0.94898	0.00416	0.90457	0.01209	0.02745
	0.0030	0.97322	0.00478	0.94819	0.01847	0.04073
	0.0040	0.98318	0.00507	0.96771	0.02457	0.05351
	0.0050	0.98946	0.00528	0.97896	0.03046	0.06487
	0.0060	0.99369	0.00545	0.98627	0.03606	0.07368

Table 6. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0070	0.99651	0.00558	0.99127	0.04127	0.07842
	0.0080	0.99829	0.00567	0.99571	0.04591	0.07738
	0.0090	0.99929	0.00573	0.99701	0.04981	0.06933
	0.0100	0.99976	0.00577	0.99846	0.05280	0.05481
	0.0110	0.99994	0.00578	0.99929	0.05482	0.03703
	0.0120	0.99999	0.00578	0.99972	0.05600	0.02079
	0.0130	1.00000	0.00579	0.99990	0.05657	0.00955
	0.0140	1.00000	0.00579	0.99997	0.05680	0.00358
	0.0150	1.00000	0.00579	0.99999	0.05687	0.00110
	0.0160	1.00000	0.00579	1.00000	0.05690	0.00000
0.1000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.75494	0.00158	0.72368	0.00347	0.01248
	0.0020	0.93279	0.00370	0.89421	0.00867	0.02192
	0.0030	0.96706	0.00435	0.94411	0.01321	0.03276
	0.0040	0.97906	0.00461	0.96499	0.01746	0.04355
	0.0050	0.98624	0.00479	0.97669	0.02156	0.05360
	0.0060	0.99114	0.00492	0.98427	0.02552	0.06227
	0.0070	0.99457	0.00503	0.98952	0.02927	0.06868
	0.0080	0.99691	0.00512	0.99324	0.03276	0.07167
	0.0090	0.99842	0.00518	0.99585	0.03587	0.07001
	0.0100	0.99931	0.00522	0.99762	0.03851	0.06284
	0.0110	0.99975	0.00524	0.99874	0.04057	0.05059
	0.0120	0.99993	0.00526	0.99940	0.04201	0.03553
	0.0130	0.99998	0.00526	0.99974	0.04290	0.02124
	0.0140	1.00000	0.00526	0.99990	0.04336	0.01064
	0.0150	1.00000	0.00526	0.99997	0.04357	0.00445
	0.0160	1.00000	0.00526	0.99999	0.04364	0.00155
	0.0170	1.00000	0.00526	1.00000	0.04366	0.00000

Table 7. Solution for injvel = 1.00 and injangle = 30°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	0.86603	0.50000	14.50000
0.0200	0.0000	0.00000	0.00000	0.56948	0.00000	0.00000
	0.0010	0.94717	0.00307	0.81969	0.03146	0.00210
	0.0020	0.97133	0.00412	0.86962	0.06532	0.00893
	0.0030	0.97427	0.00440	0.89182	0.10007	0.02090
	0.0040	0.97780	0.00486	0.90726	0.13569	0.03823
	0.0050	0.98194	0.00554	0.92076	0.17214	0.06093
	0.0060	0.98640	0.00645	0.93390	0.20923	0.08824
	0.0070	0.99081	0.00754	0.94726	0.24647	0.11774
	0.0080	0.99471	0.00868	0.96079	0.28275	0.14362
	0.0090	0.99761	0.00968	0.97389	0.31602	0.15507
	0.0100	0.99926	0.01033	0.98529	0.34322	0.13914
	0.0110	0.99986	0.01062	0.99349	0.36147	0.09457
	0.0120	0.99999	0.01069	0.99791	0.37057	0.04415
	0.0130	1.00000	0.01069	0.99954	0.37365	0.01320
	0.0140	1.00000	0.01069	0.99993	0.37432	0.00244
	0.0150	1.00000	0.01069	0.99999	0.37441	0.00027
	0.0160	1.00000	0.01069	1.00000	0.37442	0.00000
0.0400	0.0000	0.00000	0.00000	0.27294	0.00000	0.00000
	0.0010	0.90594	0.00261	0.77324	0.01306	0.00060
	0.0020	0.96750	0.00415	0.86028	0.02815	0.00218
	0.0030	0.97049	0.00430	0.88899	0.04328	0.00484
	0.0040	0.97205	0.00440	0.90382	0.05859	0.00859
	0.0050	0.97397	0.00455	0.91386	0.07413	0.01346
	0.0060	0.97618	0.00475	0.92189	0.08991	0.01948
	0.0070	0.97861	0.00501	0.92899	0.10597	0.02662
	0.0080	0.98121	0.00533	0.93569	0.12228	0.03486
	0.0090	0.98392	0.00570	0.94223	0.13884	0.04411
	0.0100	0.98667	0.00613	0.94877	0.15562	0.05418
	0.0110	0.98937	0.00660	0.95535	0.17253	0.06472
	0.0120	0.99195	0.00710	0.96197	0.18945	0.07512
	0.0130	0.99429	0.00760	0.96860	0.20617	0.08441
	0.0130	0.99429	0.00760	0.96860	0.20617	0.08441
	0.0140	0.99629	0.00806	0.97510	0.22234	0.09107
	0.0150	0.99785	0.00846	0.98129	0.23749	0.09309
	0.0160	0.99893	0.00876	0.98692	0.25099	0.08833
	0.0170	0.99957	0.00895	0.99168	0.26215	0.07565
	0.0180	0.99987	0.00905	0.99531	0.27044	0.05650
	0.0190	0.99997	0.00909	0.99772	0.27578	0.03555
	0.0200	1.00000	0.00910	0.99906	0.27866	0.01830
	0.0210	1.00000	0.00910	0.99968	0.27994	0.00758
	0.0220	1.00000	0.00910	0.99991	0.28040	0.00251
	0.0230	1.00000	0.00910	0.99998	0.28053	0.00066
	0.0240	1.00000	0.00910	1.00000	0.28056	0.00000

Table 8. Solution for injvel = 1.50 and injangle = 10°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	1.47721	0.26047	14.50000
0.0200	0.0000	0.00000	0.00000	1.18067	0.00000	0.00000
	0.0010	1.12644	0.00270	1.28978	0.04820	0.05436
	0.0020	1.08616	0.00102	1.24175	0.08981	0.10273
	0.0030	1.04803	-0.00240	1.17736	0.12383	0.14066
	0.0040	1.02155	-0.00593	1.11568	0.15038	0.15970
	0.0050	1.00692	-0.00856	1.06444	0.16954	0.14909
	0.0060	1.00136	-0.00986	1.02865	0.18150	0.10623
	0.0070	1.00014	-0.01022	1.00940	0.18735	0.05212
	0.0080	1.00001	-0.01027	1.00214	0.18936	0.01636
	0.0090	1.00000	-0.01027	1.00033	0.18982	0.00323
	0.0100	1.00000	-0.01027	1.00003	0.18989	0.00041
	0.0110	1.00000	-0.01027	1.00000	0.18990	0.00000
0.0400	0.0000	0.00000	0.00000	0.88412	0.00000	0.00000
	0.0010	1.11416	0.00272	1.21672	0.02278	0.02011
	0.0020	1.12154	0.00329	1.23875	0.04454	0.03933
	0.0030	1.09442	0.00222	1.21520	0.06383	0.05802
	0.0040	1.06919	0.00071	1.18104	0.08073	0.07537
	0.0050	1.04715	-0.00101	1.14463	0.09529	0.09022
	0.0060	1.02913	-0.00275	1.10935	0.10760	0.10073
	0.0070	1.01575	-0.00429	1.07730	0.11771	0.10430
	0.0080	1.00708	-0.00545	1.05007	0.12567	0.09811
	0.0090	1.00247	-0.00617	1.02893	0.13151	0.08098
	0.0100	1.00063	-0.00650	1.01443	0.13533	0.05609
	0.0110	1.00011	-0.00661	1.00602	0.13745	0.03121
	0.0120	1.00001	-0.00663	1.00205	0.13841	0.01353
	0.0130	1.00000	-0.00663	1.00056	0.13876	0.00453
	0.0140	1.00000	-0.00663	1.00012	0.13885	0.00117
	0.0150	1.00000	-0.00663	1.00002	0.13888	0.00024
	0.0160	1.00000	-0.00663	1.00000	0.13888	0.00000
0.0600	0.0000	0.00000	0.00000	0.58758	0.00000	0.00000
	0.0010	1.08095	0.00000	1.10456	0.01316	0.01208
	0.0020	1.13083	0.00166	1.19578	0.02676	0.02316
	0.0030	1.11347	0.00154	1.20133	0.03904	0.03455
	0.0040	1.09335	0.00093	1.18645	0.05003	0.04597
	0.0050	1.07392	0.00008	1.16099	0.05978	0.05707
	0.0060	1.05591	-0.00092	1.13522	0.06836	0.06735
	0.0070	1.03990	-0.00199	1.10950	0.07581	0.07607
	0.0080	1.02640	-0.00304	1.08512	0.08218	0.08213
	0.0090	1.01579	-0.00400	1.06304	0.08754	0.08412
	0.0100	1.00828	-0.00478	1.04400	0.09192	0.08059
	0.0110	1.00366	-0.00533	1.02858	0.09535	0.07086
	0.0120	1.00132	-0.00567	1.01703	0.09786	0.05591
	0.0130	1.00037	-0.00583	1.00916	0.09955	0.03874

Table 8. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0140	1.00008	-0.00589	1.00440	0.10057	0.02319
	0.0150	1.00001	-0.00590	1.00186	0.10110	0.01188
	0.0160	1.00000	-0.00591	1.00070	0.10134	0.00519
	0.0170	1.00000	-0.00591	1.00023	0.10143	0.00194
	0.0180	1.00000	-0.00591	1.00006	0.10146	0.00062
	0.0190	1.00000	-0.00591	1.00001	0.10147	0.00017
	0.0200	1.00000	-0.00591	1.00000	0.10147	0.00000
0.0800	0.0000	0.00000	0.00000	0.29103	0.00000	0.00000
	0.0010	1.04638	0.00000	0.99071	0.00807	0.00903
	0.0020	1.12922	0.00176	1.14141	0.01738	0.01673
	0.0030	1.12155	0.00199	1.17385	0.02584	0.02483
	0.0040	1.10568	0.00174	1.17168	0.03342	0.03342
	0.0050	1.08908	0.00128	1.15742	0.04019	0.04177
	0.0060	1.07277	0.00070	1.13860	0.04619	0.05021
	0.0070	1.05730	0.00003	1.11824	0.05146	0.05821
	0.0080	1.04313	-0.00069	1.09783	0.05606	0.06528
	0.0090	1.03064	-0.00141	1.07833	0.06002	0.07070
	0.0100	1.02026	-0.00209	1.06042	0.06338	0.07355
	0.0110	1.01223	-0.00268	1.04465	0.06620	0.07287
	0.0120	1.00662	-0.00315	1.03139	0.06851	0.06798
	0.0130	1.00314	-0.00348	1.02085	0.07033	0.05894
	0.0140	1.00127	-0.00368	1.01298	0.07171	0.04689
	0.0150	1.00044	-0.00379	1.00751	0.07267	0.03384
	0.0160	1.00013	-0.00383	1.00402	0.07330	0.02196
	0.0170	1.00003	-0.00385	1.00197	0.07366	0.01276
	0.0180	1.00001	-0.00385	1.00089	0.07386	0.00662
	0.0190	1.00000	-0.00385	1.00036	0.07395	0.00308
	0.0200	1.00000	-0.00385	1.00014	0.07399	0.00128
	0.0210	1.00000	-0.00385	1.00005	0.07401	0.00048
	0.0220	1.00000	-0.00385	1.00001	0.07401	0.00016
	0.0230	1.00000	-0.00385	1.00000	0.07401	0.00000
0.1000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	1.01091	0.00000	0.90716	0.00512	0.00749
	0.0020	1.12264	0.00177	1.09893	0.01190	0.01339
	0.0030	1.12451	0.00218	1.14875	0.01805	0.01979
	0.0040	1.11236	0.00211	1.15638	0.02351	0.02659
	0.0050	1.09812	0.00186	1.14866	0.02837	0.03364
	0.0060	1.08348	0.00151	1.13467	0.03266	0.04083
	0.0070	1.06904	0.00107	1.11810	0.03644	0.04796
	0.0080	1.05523	0.00059	1.10070	0.03973	0.05474
	0.0090	1.04242	0.00008	1.08348	0.04258	0.06075
	0.0100	1.03101	-0.00043	1.06713	0.04501	0.06539
	0.0110	1.02133	-0.00092	1.05219	0.04707	0.06794
	0.0120	1.01362	-0.00134	1.03905	0.04880	0.06764

Table 8. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0130	1.00795	-0.00169	1.02801	0.05023	0.06398
	0.0140	1.00417	-0.00195	1.01016	0.05138	0.05694
	0.0150	1.00193	-0.00212	1.01245	0.05229	0.04723
	0.0160	1.00078	-0.00222	1.00764	0.05296	0.03620
	0.0170	1.00027	-0.00227	1.00441	0.05344	0.02546
	0.0180	1.00008	-0.00229	1.00239	0.05374	0.01636
	0.0190	1.00002	-0.00230	1.00120	0.05393	0.00959
	0.0200	1.00000	-0.00230	1.00056	0.05403	0.00512
	0.0210	1.00000	-0.00230	1.00025	0.05408	0.00249
	0.0220	1.00000	-0.00230	1.00010	0.05410	0.00111
	0.0230	1.00000	-0.00230	1.00004	0.05411	0.00045
	0.0240	1.00000	-0.00230	1.00001	0.05411	0.00017
	0.0250	1.00000	-0.00230	1.00000	0.05411	0.00000

Table 9. Solution for injvel = 1.50 and injangle = 20°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	1.40954	0.51303	14.50000
0.0200	0.0000	0.00000	0.00000	1.11299	0.00000	0.00000
	0.0010	1.06882	0.00345	1.27034	0.05120	0.00143
	0.0020	1.06767	0.00366	1.27390	0.10086	0.00864
	0.0030	1.06081	0.00315	1.25575	0.14762	0.02165
	0.0040	1.05220	0.00209	1.22959	0.19101	0.04024
	0.0050	1.04213	0.00042	1.19902	0.23062	0.06378
	0.0060	1.03141	-0.00182	1.16572	0.26612	0.09091
	0.0070	1.02104	-0.00443	1.13104	0.29717	0.11884
	0.0080	1.01209	-0.00708	1.09649	0.32342	0.14224
	0.0090	1.00555	-0.00934	1.06413	0.34450	0.15216
	0.0100	1.00181	-0.01083	1.03665	0.35997	0.13788
	0.0110	1.00036	-0.01149	1.01683	0.36970	0.09715
	0.0120	1.00004	-0.01167	1.00575	0.37447	0.04813
	0.0130	1.00000	-0.01169	1.00136	0.37613	0.01546
	0.0140	1.00000	-0.01169	1.00021	0.37651	0.00308
	0.0150	1.00000	-0.01169	1.00002	0.37656	0.00037
	0.0160	1.00000	-0.01169	1.00000	0.37657	0.00000
0.0400	0.0000	0.00000	0.00000	0.95352	0.00000	0.00000
	0.0010	1.04831	0.00000	1.18111	0.02495	0.00032
	0.0020	1.07044	0.00401	1.22655	0.05038	0.00194
	0.0030	1.06783	0.00393	1.23223	0.07492	0.00493
	0.0040	1.06450	0.00372	1.22653	0.09852	0.00931
	0.0050	1.06023	0.00336	1.21573	0.12107	0.01510
	0.0060	1.05510	0.00282	1.20197	0.14248	0.02231
	0.0070	1.04924	0.00209	1.19620	0.16268	0.03089
	0.0080	1.04283	0.00116	1.16879	0.18157	0.04078
	0.0090	1.03608	0.00006	1.15065	0.19908	0.05179
	0.0100	1.02923	-0.00120	1.13157	0.21517	0.06359
	0.0110	1.02254	-0.00256	1.11206	0.22976	0.07560
	0.0120	1.01631	-0.00394	1.09251	0.24280	0.08683
	0.0130	1.01087	-0.00526	1.07339	0.25424	0.09576
	0.0140	1.00648	-0.00642	1.05529	0.26402	0.10021
	0.0150	1.00334	-0.00731	1.03892	0.27205	0.09766
	0.0160	1.00141	-0.00791	1.02506	0.27828	0.08630
	0.0170	1.00047	-0.00823	1.01439	0.28270	0.06683
	0.0180	1.00011	-0.00837	1.00717	0.28548	0.04371
	0.0190	1.00002	-0.00840	1.00301	0.28697	0.02336
	0.0200	1.00000	-0.00841	1.00105	0.28762	0.00998
	0.0210	1.00000	-0.00841	1.00030	0.28786	0.00337
	0.0220	1.00000	-0.00841	1.00007	0.28793	0.00090
	0.0230	1.00000	-0.00841	1.00001	0.28794	0.00019
	0.0240	1.00000	-0.00841	1.00000	0.28794	0.00000
0.0600	0.0000	0.00000	0.00000	0.55698	0.00000	0.00000
	0.0010	1.01353	0.00000	1.03088	0.01349	0.00018

Table 9. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0020	1.06847	0.00171	1.14601	0.02833	0.00087
	0.0030	1.06963	0.00187	1.18047	0.04280	0.00209
	0.0040	1.06805	0.00183	1.19001	0.05681	0.00387
	0.0050	1.06585	0.00173	1.18994	0.07035	0.00623
	0.0060	1.06308	0.00157	1.18538	0.08341	0.00919
	0.0070	1.05976	0.00134	1.17829	0.09596	0.01276
	0.0080	1.05592	0.00102	1.16957	0.10796	0.01696
	0.0090	1.05164	0.00062	1.15968	0.11939	0.02180
	0.0100	1.04697	0.00013	1.14887	0.13022	0.02727
	0.0110	1.04199	-0.00044	1.13736	0.14042	0.03333
	0.0120	1.03677	-0.00110	1.12528	0.14997	0.03993
	0.0130	1.03143	-0.00183	1.11279	0.15885	0.04694
	0.0140	1.02608	-0.00262	1.10003	0.16704	0.05415
	0.0150	1.02088	-0.00344	1.08717	0.17453	0.06123
	0.0160	1.01600	-0.00428	1.07441	0.18131	0.06766
	0.0170	1.01164	-0.00508	1.06197	0.18737	0.07279
	0.0180	1.00794	-0.00582	1.05012	0.19270	0.07574
	0.0190	1.00502	-0.00645	1.03914	0.19730	0.07566
	0.0200	1.00289	-0.00695	1.02932	0.20116	0.07186
	0.0210	1.00149	-0.00732	1.02090	0.20429	0.06421
	0.0220	1.00067	-0.00755	1.01407	0.20671	0.05340
	0.0230	1.00026	-0.00767	1.00887	0.20847	0.04090
	0.0240	1.00009	-0.00773	1.00520	0.20967	0.02858
	0.0250	1.00002	-0.00776	1.00281	0.21042	0.01810
	0.0260	1.00001	-0.00776	1.00140	0.21085	0.01034
	0.0270	1.00000	-0.00777	1.00063	0.21107	0.00531
	0.0280	1.00000	-0.00777	1.00026	0.21118	0.00245
	0.0290	1.00000	-0.00777	1.00010	0.21122	0.00102
	0.0300	1.00000	-0.00777	1.00003	0.21124	0.00038
	0.0310	1.00000	-0.00777	1.00001	0.21124	0.00013
	0.0320	1.00000	-0.00777	1.00000	0.21125	0.00000
0.0800	0.0000	0.00000	0.00000	0.26043	0.00000	0.00000
	0.0010	0.97983	0.00000	0.91865	0.00796	0.00013
	0.0020	1.06418	0.00166	1.07847	0.01770	0.00054
	0.0030	1.06993	0.00192	1.13213	0.02723	0.00125
	0.0040	1.06929	0.00192	1.15240	0.03645	0.00228
	0.0050	1.06795	0.00188	1.15940	0.04539	0.00364
	0.0060	1.06618	0.00181	1.16021	0.05404	0.00535
	0.0070	1.06397	0.00171	1.15773	0.06239	0.00742
	0.0080	1.06135	0.00157	1.15327	0.07044	0.00987
	0.0090	1.05834	0.00138	1.14751	0.07818	0.01271
	0.0100	1.05497	0.00114	1.14079	0.08559	0.01595
	0.0110	1.05127	0.00086	1.13334	0.09265	0.01961
	0.0120	1.04726	0.00053	1.12529	0.09935	0.02369
	0.0130	1.04301	0.00015	1.11675	0.10569	0.02817

Table 9. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0140	1.03854	-0.00028	1.10782	0.11164	0.03302
	0.0150	1.03395	-0.00075	1.09858	0.11720	0.03820
	0.0160	1.02931	-0.00126	1.08912	0.12235	0.04359
	0.0170	1.02473	-0.00180	1.07954	0.12711	0.04903
	0.0180	1.02031	-0.00235	1.06997	0.13145	0.05429
	0.0190	1.01616	-0.00290	1.06053	0.13539	0.05904
	0.0200	1.01237	-0.00343	1.05137	0.13893	0.06283
	0.0210	1.00904	-0.00393	1.04264	0.14207	0.06520
	0.0220	1.00625	-0.00437	1.03451	0.14482	0.06563
	0.0230	1.00405	-0.00474	1.02712	1.14719	0.06373
	0.0240	1.00243	-0.00503	1.02062	0.14919	0.05935
	0.0250	1.00134	-0.00524	1.01510	0.15082	0.05266
	0.0260	1.00067	-0.00538	1.01060	0.15212	0.04424
	0.0270	1.00030	-0.00546	1.00710	0.15310	0.03500
	0.0280	1.00012	-0.00551	1.00452	0.15380	0.02593
	0.0290	1.00004	-0.00553	1.00273	0.15429	0.01793
	0.0300	1.00001	-0.00553	1.00155	0.15459	0.01154
	0.0310	1.00000	-0.00554	1.00083	0.15478	0.00690
	0.0320	1.00000	-0.00554	1.00042	0.15488	0.00383
	0.0330	1.00000	-0.00554	1.00020	0.15494	0.00197
	0.0340	1.00000	-0.00554	1.00009	0.15496	0.00094
	0.0350	1.00000	-0.00554	1.00004	0.15497	0.00042
	0.0360	1.00000	-0.00554	1.00001	0.15498	0.00017
	0.0370	1.00000	-0.00554	1.00001	0.15498	0.00007
	0.0380	1.00000	-0.00554	1.00000	0.15498	0.00000
0.1000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
	0.0010	0.94724	0.00000	0.83939	0.00489	0.00011
	0.0020	1.05815	0.00160	1.03191	0.01175	0.00039
	0.0030	1.06952	0.00194	1.09638	0.01841	0.00089
	0.0040	1.06978	0.00197	1.12234	0.02482	0.00160
	0.0050	1.06892	0.00196	1.13328	0.03103	0.00254
	0.0060	1.06767	0.00192	1.13721	0.03704	0.00373
	0.0070	1.06606	0.00187	1.13741	0.04285	0.00517
	0.0080	1.06409	0.00179	1.13542	0.04847	0.00687
	0.0090	1.06179	0.00169	1.13202	0.05388	0.00885
	0.0100	1.05916	0.00156	1.12762	0.05909	0.01112
	0.0110	1.05622	0.00141	1.12249	0.06408	0.01370
	0.0120	1.05299	0.00122	1.11676	0.06885	0.01659
	0.0130	1.04950	0.00100	1.11055	0.07338	0.01980
	0.0140	1.04576	0.00075	1.10393	0.07766	0.02335
	0.0150	1.04183	0.00047	1.09696	0.08170	0.02721
	0.0160	1.03776	0.00016	1.08972	0.08547	0.03136
	0.0170	1.03361	-0.00018	1.08226	0.08901	0.03576
	0.0180	1.02943	-0.00053	1.07465	0.09227	0.04033
	0.0190	1.02529	-0.00091	1.06698	0.09526	0.04494
	0.0200	1.02124	-0.00129	1.05933	0.09800	0.04942

Table 9. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0210	1.01737	-0.00168	1.05179	0.10047	0.05352
	0.0220	1.01376	-0.00206	1.04447	0.10269	0.05693
	0.0230	1.01051	-0.00242	1.03748	0.10467	0.05931
	0.0240	1.00768	-0.00274	1.03094	0.10642	0.06030
	0.0250	1.00534	-0.00302	1.02495	0.10794	0.05958
	0.0260	1.00351	-0.00326	1.01959	0.10925	0.05696
	0.0270	1.00216	-0.00344	1.01495	0.11035	0.05246
	0.0280	1.00123	-0.00358	1.01106	0.11126	0.04633
	0.0290	1.00065	-0.00366	1.00790	0.11199	0.03906
	0.0300	1.00031	-0.00372	1.00543	0.11255	0.03132
	0.0310	1.00014	-0.00375	1.00358	0.11297	0.02380
	0.0320	1.00005	-0.00376	1.00227	0.11327	0.01709
	0.0330	1.00002	-0.00377	1.00137	0.11347	0.01158
	0.0340	1.00001	-0.00377	1.00079	0.11359	0.00739
	0.0350	1.00000	-0.00378	1.00043	0.11367	0.00445
	0.0360	1.00000	-0.00378	1.00022	0.11372	0.00252
	0.0370	1.00000	-0.00378	1.00011	0.11374	0.00134
	0.0380	1.00000	-0.00378	1.00005	0.11375	0.00068
	0.0390	1.00000	-0.00378	1.00002	0.11376	0.00032
	0.0400	1.00000	-0.00378	1.00001	0.11376	0.00014
	0.0410	1.00000	-0.00378	1.00001	0.11376	0.00006
	0.0420	1.00000	-0.00378	1.00000	0.11376	0.00000

Table 10. Solution for injvel = 2.00 and injangle = 10°

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.0000	0.0000	1.00000	0.00000	1.96962	0.34730	14.50000
0.0200	0.0000	0.00000	0.00000	1.67307	0.00000	0.00000
	0.0010	1.23579	0.00273	1.69159	0.06433	0.03383
	0.0020	1.17385	-0.00036	1.57861	0.11751	0.06730
	0.0030	1.11546	-0.00574	1.45539	0.15953	0.09833
	0.0040	1.06892	-0.01198	1.33805	0.19151	0.12400
	0.0050	1.03526	-0.01793	1.23284	0.21476	0.13995
	0.0060	1.01432	-0.02255	1.14427	0.23070	0.13983
	0.0070	1.00411	-0.02530	1.07657	0.24069	0.11759
	0.0080	1.00072	-0.02641	1.03253	0.24611	0.07627
	0.0090	1.00007	-0.02667	1.01028	0.24843	0.03460
	0.0100	1.00000	-0.02670	1.00229	0.24915	0.01029
	0.0110	1.00000	-0.02670	1.00035	0.24930	0.00196
	0.0120	1.00000	-0.02670	1.00004	0.24932	0.00024
	0.0130	1.00000	-0.02670	1.00000	0.24932	0.00000
0.0400	0.0000	0.00000	0.00000	1.37652	0.00000	0.00000
	0.0010	1.24257	0.00283	1.61351	0.03173	0.01192
	0.0020	1.23021	0.00276	1.59142	0.06073	0.02404
	0.0030	1.19011	0.00099	1.53180	0.08593	0.03644
	0.0040	1.15287	-0.00142	1.46376	0.10744	0.04887
	0.0050	1.11881	-0.00427	1.39447	0.12545	0.06106
	0.0060	1.08842	-0.00738	1.32686	0.14018	0.07255
	0.0070	1.06217	-0.01053	1.26264	0.15189	0.08268
	0.0080	1.04055	-0.01350	1.20315	0.16091	0.09037
	0.0090	1.02392	-0.01607	1.14965	0.16757	0.09401
	0.0100	1.01230	-0.01807	1.10345	0.17226	0.09163
	0.0110	1.00526	-0.01942	1.06577	0.17537	0.08158
	0.0120	1.00177	-0.02016	1.03752	0.17727	0.06411
	0.0130	1.00044	-0.02048	1.01867	0.17832	0.04276
	0.0140	1.00008	-0.02057	1.00789	0.17882	0.02339
	0.0150	1.00001	-0.02060	1.00278	0.17902	0.01025
	0.0160	1.00000	-0.02060	1.00080	0.17909	0.00356
	0.0170	1.00000	-0.02060	1.00019	0.17911	0.00098
	0.0180	1.00000	-0.02060	1.00004	0.17911	0.00021
	0.0190	1.00000	-0.02060	1.00001	0.17911	0.00004
	0.0200	1.00000	-0.02060	1.00000	0.17911	0.00000
0.0600	0.0000	0.00000	0.00000	1.07998	0.00000	0.00000
	0.0010	1.21737	0.00000	1.47918	0.01911	0.00681
	0.0020	1.24986	0.00144	1.53163	0.03766	0.01354
	0.0030	1.22139	0.00090	1.51066	0.05424	0.02063
	0.0040	1.19217	-0.00017	1.46987	0.06887	0.02802
	0.0050	1.16401	-0.00157	1.42271	0.08160	0.03565
	0.0060	1.13714	-0.00324	1.37359	0.09250	0.04347
	0.0070	1.11183	-0.00510	1.32445	0.10164	0.05134
	0.0080	1.08838	-0.00708	1.27639	0.10912	0.05905

Table 10. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
	0.0090	1.06714	-0.00910	1.23026	0.11505	0.06627
	0.0100	1.04846	-0.01107	1.18677	0.11957	0.07243
	0.0110	1.03273	-0.01289	1.14667	0.12286	0.07669
	0.0120	1.02025	-0.01446	1.11069	0.12512	0.07798
	0.0130	1.01120	-0.01572	1.07958	0.12655	0.07515
	0.0140	1.00538	-0.01662	1.05392	0.12738	0.06752
	0.0150	1.00218	-0.01718	1.03404	0.12780	0.05554
	0.0160	1.00073	-0.01747	1.01977	0.12798	0.04109
	0.0170	1.00020	-0.01759	1.01045	0.12804	0.02693
	0.0180	1.00004	-0.01763	1.00498	0.12805	0.01549
	0.0190	1.00001	-0.01764	1.00212	0.12804	0.00777
	0.0200	1.00000	-0.01765	1.00081	0.12804	0.00339
	0.0210	1.00000	-0.01765	1.00027	0.12803	0.00129
	0.0220	1.00000	-0.01765	1.00008	0.12803	0.00043
	0.0230	1.00000	-0.01765	1.00002	0.12803	0.00012
	0.0240	1.00000	-0.01765	1.00000	0.12803	0.00000
0.0800	0.0000	0.00000	0.00000	0.78343	0.00000	0.00000
	0.0010	1.18684	0.00000	1.33349	0.01236	0.00485
	0.0020	1.25564	0.00158	1.44968	0.02521	0.00943
	0.0030	1.23703	0.00147	1.46224	0.03688	0.01429
	0.0040	1.21322	0.00089	1.44246	0.04728	0.01945
	0.0050	1.18949	0.00009	1.41076	0.05646	0.02490
	0.0060	1.16616	-0.00090	1.37425	0.06446	0.03064
	0.0070	1.14342	-0.00205	1.33586	0.07130	0.03663
	0.0080	1.12146	-0.00333	1.29700	0.07702	0.04284
	0.0090	1.10049	-0.00469	1.25853	0.08168	0.04915
	0.0100	1.08077	-0.00610	1.22107	0.08533	0.05537
	0.0110	1.06259	-0.00752	1.18515	0.08805	0.06119
	0.0120	1.04632	-0.00888	1.15133	0.08995	0.06611
	0.0130	1.03235	-0.01014	1.12018	0.09114	0.06941
	0.0140	1.02104	-0.01125	1.09227	0.09176	0.07028
	0.0150	1.01256	-0.01216	1.06810	0.09197	0.06793
	0.0160	1.00675	-0.01282	1.04803	0.09192	0.06199
	0.0170	1.00322	-0.01327	1.03218	0.09173	0.05282
	0.0180	1.00134	-0.01353	1.02034	0.09150	0.04162
	0.0190	1.00048	-0.01366	1.01207	0.09130	0.03008
	0.0200	1.00015	-0.01372	1.00668	0.09115	0.01984
	0.0210	1.00004	-0.01374	1.00344	0.09105	0.01189
	0.0220	1.00001	-0.01375	1.00164	0.09098	0.00648
	0.0230	1.00000	-0.01375	1.00072	0.09095	0.00320
	0.0240	1.00000	-0.01375	1.00029	0.09094	0.00144
	0.0250	1.00000	-0.01375	1.00011	0.09093	0.00059
	0.0260	1.00000	-0.01375	1.00004	0.09093	0.00022
	0.0270	1.00000	-0.01375	1.00001	0.09093	0.00007
	0.0280	1.00000	-0.01375	1.00000	0.09093	0.00000

Table 10. (cont.)

x'	y'	u'	v'	u'_p	v'_p	ρ_p
0.1000	0.0000	0.00000	0.00000	0.48689	0.00000	0.00000
	0.0010	1.15482	0.00000	1.20934	0.00830	0.00387
	0.0020	1.25507	0.00161	1.37473	0.01770	0.00734
	0.0030	1.24556	0.00175	1.41129	0.02628	0.01108
	0.0040	1.22580	0.00143	1.40673	0.03396	0.01509
	0.0050	1.20526	0.00093	1.38621	0.04076	0.01938
	0.0060	1.18466	0.00030	1.35872	0.04670	0.02397
	0.0070	1.16416	-0.00046	1.32801	0.05182	0.02885
	0.0080	1.14393	-0.00132	1.29587	0.05614	0.03402
	0.0090	1.12412	-0.00225	1.26327	0.05968	0.03947
	0.0100	1.10489	-0.00325	1.23085	0.06247	0.04510
	0.0110	1.08686	-0.00429	1.19908	0.06555	0.05080
	0.0120	1.06910	-0.00533	1.16842	0.06599	0.05630
	0.0130	1.05316	-0.00635	1.13933	0.06684	0.06120
	0.0140	1.03903	-0.00731	1.11231	0.06720	0.06493
	0.0150	1.02706	-0.00818	1.08787	0.06717	0.06678
	0.0160	1.01749	-0.00892	1.06644	0.06687	0.06604
	0.0170	1.01040	-0.00950	1.04836	0.06643	0.06224
	0.0180	1.00560	-0.00992	1.03375	0.06594	0.05644
	0.0190	1.00271	-0.01020	1.02251	0.06548	0.04632
	0.0200	1.00116	-0.01036	1.01429	0.06510	0.03607
	0.0210	1.00044	-0.01044	1.00861	0.06481	0.02608
	0.0220	1.00014	-0.01047	1.00490	0.06461	0.01745
	0.0230	1.00004	-0.01049	1.00263	0.06448	0.01079
	0.0240	1.00001	-0.01049	1.00133	0.06441	0.00617
	0.0250	1.00000	-0.01049	1.00063	0.06436	0.00326
	0.0260	1.00000	-0.01049	1.00028	0.06434	0.00159
	0.0270	1.00000	-0.01049	1.00012	0.06433	0.00072
	0.0280	1.00000	-0.01049	1.00005	0.06433	0.00030
	0.0290	1.00000	-0.01049	1.00002	0.06433	0.00012
	0.0300	1.00000	-0.01049	1.00001	0.06433	0.00004
	0.0310	1.00000	-0.01049	1.00000	0.06433	0.00000

Table 11. Fluid Velocity Comparison at x' = 0.05
(x-Component)

y'	u'				
	Injection Conditions				
	No Injection	0.50* 10°	0.50 30°	0.50 45°	1.00 10°
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0010	0.92245	0.65320	0.55404	0.47126	0.89505
0.0020	0.99719	0.82988	0.67451	0.58444	0.97052
0.0030	0.99995	0.92699	0.73333	0.63571	0.98402
0.0040	1.00000	0.97837	0.78377	0.67954	0.99116
0.0050		0.99629	0.83024	0.72109	0.99553
0.0060		0.99967	0.87276	0.76088	0.99806
0.0070		0.99999	0.91054	0.79893	0.99933
0.0080		1.00000	0.94244	0.83509	0.99983
0.0090			0.96724	0.86900	0.99997
0.0100			0.98424	0.90019	1.00000
0.0110			0.99394	0.92801	
0.0120			0.99825	0.95173	
0.0130			0.99964	0.97065	
0.0140			0.99995	0.98434	
0.0150			0.99999	0.99296	
0.0160			1.00000	0.99746	
0.0170				0.99929	
0.0180				0.99986	
0.0190				0.99998	
0.0200				1.00000	

*The upper number is injvel and the lower is injangle

Table 11. (cont.)

y'	u'			
	Injection Conditions			
	1:00 30°	1.50 10°	1:50 20°	2:00 10°
0.0000	0.00000	0.00000	0.00000	0.00000
0.0010	0.88145	1.09701	1.03079	1.23120
0.0020	0.96513	1.12845	1.06980	1.24289
0.0030	0.96972	1.10598	1.06903	1.20781
0.0040	0.97099	1.08432	1.06678	1.17589
0.0050	0.97252	1.06250	1.06381	1.14492
0.0060	0.97430	1.04399	1.06015	1.11609
0.0070	0.97628	1.02852	1.05584	1.08978
0.0080	0.97841	1.01657	1.05097	1.06637
0.0090	0.98067	1.00829	1.04565	1.04629
0.0100	0.98300	1.00341	1.03999	1.02990
0.0110	0.98538	1.00109	1.03412	1.01745
0.0120	0.98775	1.00025	1.02818	1.00890
0.0130	0.99006	1.00004	1.02237	1.00381
0.0140	0.99225	1.00000	1.01688	1.00131
0.0150	0.99426		1.01194	1.00035
0.0160	0.99602		1.00778	1.00007
0.0170	0.99746		1.00458	1.00001
0.0180	0.99855		1.00237	1.00000
0.0190	0.99928		1.00106	
0.0200	0.99970		1.00039	
0.0210	0.99990		1.00012	
0.0220	0.99997		1.00003	
0.0230	0.99999		1.00001	
0.0240	1.00000		1.00000	
0.0250				
0.0260				
0.0270				
0.0280				
0.0290				
0.0300				

Table 12. Particle Velocity Comparison at x' = 0.05 (x-Component)

y'	u_p'				
	Injection Conditions				
	No In- jection	0.50* 10°	0.50 30°	0.50 45°	1.00 10°
0.0000	0.00000	0.00000	0.00000	0.00000	0.24345
0.0010	0.92245	0.56070	0.47625	0.41839	0.82798
0.0020	0.99719	0.74062	0.58900	0.51644	0.92737
0.0030	0.99995	0.85464	0.64877	0.56497	0.96041
0.0040	1.00000	0.93150	0.69701	0.60312	0.97663
0.0050		0.97484	0.74152	0.63806	0.98618
0.0060		0.99313	0.78427	0.67167	0.99221
0.0070		0.99863	0.82562	0.70471	0.99598
0.0080		0.99980	0.86514	0.73753	0.99818
0.0090		0.99998	0.90177	0.77031	0.99930
0.0100		1.00000	0.93395	0.80305	0.99978
0.0110			0.95999	0.83557	0.99994
0.0120			0.97875	0.86748	0.99999
0.0130			0.99035	0.89806	1.00000
0.0140			0.99632	0.92626	
0.0150			0.99884	0.95076	
0.0160			0.99969	0.97031	
0.0170			0.99993	0.98420	
0.0180			0.99999	0.99272	
0.0190			1.00000	0.99714	
0.0200				0.99905	
0.0210				0.99973	
0.0220				0.99994	
0.0230				0.99999	
0.0240				1.00000	

* The upper number is injvel and the lower is injangle

Table 12. (cont.)

y'	u'_p			
	Injection Conditions			
	1.00 30°	1.50 10°	1.50 20°	2.00 10°
0.0000	0.12466	0.73585	0.70525	1.22825
0.0010	0.75586	1.17425	1.10068	1.55989
0.0020	0.86054	1.22295	1.18636	1.56878
0.0030	0.89227	1.21195	1.20720	1.52742
0.0040	0.90735	1.18657	1.20946	1.47376
0.0050	0.91692	1.15691	1.20460	1.41657
0.0060	0.92415	1.12659	1.19619	1.35904
0.0070	0.93028	1.09752	1.18559	1.30281
0.0080	0.93584	1.07103	1.17348	1.24900
0.0090	0.94114	1.04821	1.16022	1.19854
0.0100	0.94632	1.02988	1.14609	1.15235
0.0110	0.95147	1.01652	1.13130	1.11137
0.0120	0.95663	1.00794	1.11604	1.07652
0.0130	0.96182	1.00325	1.10055	1.04859
0.0140	0.96702	1.00111	1.08507	1.02797
0.0150	0.97218	1.00032	1.06991	1.01429
0.0160	0.97725	1.00008	1.05543	1.00636
0.0170	0.98212	1.00001	1.04204	1.00243
0.0180	0.98664	1.00000	1.03019	1.00079
0.0190	0.99065		1.02029	1.00022
0.0200	0.99398		1.01259	1.00005
0.0210	0.99650		1.00713	1.00001
0.0220	0.99820		1.00364	1.00000
0.0230	0.99919		1.00166	
0.0240	0.99969		1.00067	
0.0250	0.99990		1.00024	
0.0260	0.99997		1.00007	
0.0270	0.99999		1.00002	
0.0280	1.00000		1.00000	
0.0290				
0.0300				

Table 13. Fluid Velocity Comparison at x' = 0.10 (x-Component)

y'	u'				
	Injection Conditions				
	No Injection	0.50* 10°	0.50 30°	0.50 45°	1.00 10°
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0010	0.79192	0.51409	0.43289	0.36203	0.75494
0.0020	0.97642	0.73280	0.61641	0.53284	0.93279
0.0030	0.99854	0.84195	0.68824	0.60088	0.96706
0.0040	0.99995	0.91299	0.73344	0.64197	0.97906
0.0050	1.00000	0.95929	0.77170	0.67789	0.98624
0.0060		0.98514	0.80635	0.70892	0.99114
0.0070		0.99614	0.83808	0.73909	0.99457
0.0080		0.99934	0.86710	0.76754	0.99691
0.0090		0.99993	0.89345	0.79439	0.99842
0.0100		1.00000	0.91704	0.81970	0.99931
0.0110			0.93776	0.84350	0.99975
0.0120			0.95545	0.86577	0.99993
0.0130			0.96996	0.88650	0.99998
0.0140			0.98124	0.90563	1.00000
0.0150			0.98958	0.92311	
0.0160			0.99468	0.93887	
0.0170			0.99772	0.95282	
0.0180			0.99918	0.96490	
0.0190			0.99976	0.97504	
0.0200			0.99995	0.98322	
0.0210			0.99999	0.98948	
0.0220			1.00000	0.99395	
0.0230				0.99688	
0.0240				0.99858	
0.0250				0.99944	
0.0260				0.99982	
0.0270				0.99995	
0.0280				0.99999	
0.0290				1.00000	

*The upper number is injvel and the lower is injangle

Table 13. (cont.)

y'	u'			
	Injection Conditions			
	1.00 30°	1.50 10°	1.50 20°	2.00 10°
0.0000	0.00000	0.00000	0.00000	0.00000
0.0010		1.01091	0.94724	1.15482
0.0020		1.12264	1.05815	1.25507
0.0030		1.12451	1.06952	1.24556
0.0040		1.11236	1.06972	1.22580
0.0050		1.09812	1.06892	1.20526
0.0060		1.08348	1.06767	1.18466
0.0070		1.06904	1.06606	1.16416
0.0080		1.05523	1.06409	1.14393
0.0090		1.04242	1.06179	1.12412
0.0100		1.03101	1.05916	1.10489
0.0110		1.02133	1.05622	1.08646
0.0120		1.01362	1.05299	1.06910
0.0130		1.00795	1.04950	1.05316
0.0140		1.00417	1.04576	1.03903
0.0150		1.00193	1.04183	1.02706
0.0160		1.00078	1.03766	1.01749
0.0170		1.00027	1.03361	1.01040
0.0180		1.00008	1.02943	1.00560
0.0190		1.00002	1.02529	1.00271
0.0200		1.00000	1.02124	1.00116
0.0210			1.01737	1.00044
0.0220			1.01376	1.00014
0.0230			1.01051	1.00004
0.0240			1.00768	1.00001
0.0250			1.00534	1.00000
0.0260			1.00351	
0.0270			1.00216	
0.0280			1.00123	
0.0290			1.00065	
0.0300			1.00031	
0.0310			1.00014	
0.0320			1.00005	
0.0330			1.00002	
0.0340			1.00001	
0.0350			1.00000	

Table 14. Particle Velocity Comparison at $x' = 0.10$ (x-Component)

y'	u'_p Injection Conditions: No Injection	0.50* 10°	0.50 30°	0.50 45°	1.00 10°
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0010	0.79192	0.50768	0.43779	0.37274	0.72368
0.0020	0.97642	0.69757	0.59256	0.51914	0.89421
0.0030	0.99854	0.80254	0.65888	0.58033	0.94411
0.0040	0.99995	0.87568	0.70189	0.61855	0.96499
0.0050	1.00000	0.92836	0.73768	0.65014	0.97669
0.0060		0.96361	0.76995	0.67898	0.98427
0.0070		0.98419	0.79978	0.70594	0.98952
0.0080		0.99425	0.82758	0.73141	0.99324
0.0090		0.99827	0.85355	0.75560	0.99585
0.0100		0.99958	0.87770	0.77865	0.99762
0.0110		0.99991	0.90000	0.80065	0.99874
0.0120		0.99999	0.92031	0.82167	0.99940
0.0130		1.00000	0.93849	0.84172	0.99974
0.0140			0.95430	0.86083	0.99990
0.0150			0.96762	0.87898	0.99997
0.0160			0.97833	0.89613	0.99999
0.0170			0.98646	0.91224	1.00000
0.0180			0.99219	0.92723	
0.0190			0.99589	0.94101	
0.0200			0.99805	0.95346	
0.0210			0.99917	0.96448	
0.0220			0.99969	0.97395	
0.0230			0.99989	0.98179	
0.0240			0.99997	0.98798	
0.0250			0.99999	0.99258	
0.0260			1.00000	0.99577	
0.0270				0.99778	
0.0290				0.99964	
0.0300				0.99982	
0.0310				0.99994	
0.0320				0.99998	
0.0330				0.99999	
0.0340				1.00000	

*The upper number is injvel and the lower is injangle

Table 14. (cont.)

y'	u_p' Injection Conditions 1.00 30°	1.50 10°	1.50 20°	2.00 10°
0.0000	0.00000	0.00000	0.00000	0.48689
0.0010		0.90716	0.83939	1.20934
0.0020		1.09893	1.03191	1.37473
0.0030		1.14875	1.09638	1.41129
0.0040		1.15638	1.12234	1.40673
0.0050		1.14866	1.13328	1.38621
0.0060		1.13467	1.13721	1.35872
0.0070		1.11810	1.13741	1.32801
0.0080		1.10070	1.13542	1.29587
0.0090		1.08348	1.13202	1.26327
0.0100		1.06713	1.12762	1.23085
0.0110		1.05219	1.12249	1.19908
0.0120		1.03905	1.11676	1.16842
0.0130		1.02801	1.11055	1.13933
0.0140		1.01916	1.10393	1.11231
0.0150		1.01245	1.09696	1.08787
0.0160		1.00764	1.08972	1.06644
0.0170		1.00441	1.08226	1.04836
0.0180		1.00239	1.07465	1.03375
0.0190		1.00120	1.06698	1.02251
0.0200		1.00056	1.05933	1.01429
0.0210		1.00025	1.05179	1.00861
0.0220		1.00010	1.04447	1.00490
0.0230		1.00004	1.03748	1.00263
0.0240		1.00001	1.03094	1.00133
0.0250		1.00000	1.02495	1.00063
0.0260			1.01959	1.00028
0.0270			1.01495	1.00012
0.0280			1.01106	1.00005
0.0290			1.00790	1.00002
0.0300			1.00543	1.00001
0.0310			1.00358	1.00000
0.0320			1.00227	
0.0330			1.00137	
0.0340			1.00079	
0.0350			1.00043	
0.0360			1.00022	
0.0370			1.00011	
0.0380			1.00005	
0.0390			1.00002	
0.0400			1.00001	
0.0410			1.00001	
0.0420			1.00000	

APPENDIX E

COMPUTER PROGRAM

The Burroughs 5500 digital computer was programed to solve the finite difference equations. The computer language that was used is ALGOL.

The program is set up so that it will start computations at the plate's leading edge or at any distance downstream. The program presented here starts at the leading edge. To start at a downstream location it is necessary to know all of the data at this location from the wall to the limit of the particle boundary layer. In addition, the data at one increment upstream must be known. This data is punched into cards and added to the end of the card deck. Cards 133, 002, and 168 must be changed. The zeros on cards 133 and 168 are replaced by the starting x-coordinate. The zero on card 002 is changed to the number of y-increments necessary to get to the particle boundary layer limit.

To use this program for injection conditions with the x-injection ratios greater than one, some changes must be made. Cards 208 through 211, 215 through 218, 223 through 226, 228, and 229 are removed from the deck. Duplicates of cards 220 and 221 are placed after card 227. These alterations enable the program to be used for large injection velocities.

COMPUTER NOMENCLATURE

A = particle radius = σ

B = $\frac{\Delta x}{\Delta y}$

C = $\frac{\Delta y}{\Delta x}$

DX = increment in x-direction = Δx

DY = increment in y-direction = Δy

F = $\frac{FL}{u_\infty}$

FF = F

G = $\frac{\Delta x}{\Delta y^2}$

H = $\frac{FL\Delta x}{u_\infty}$

I = increment counter in x-direction = i

INJANGLE = injection angle = injangle

INJVEL = injection velocity = injvel

J = increment counter in y-direction = j

L = non-dimensionalizing length = L

MU = viscosity of the mixture = μ

RE = Reynolds number = Re

RHO = density of the gas = ρ

RHOP = apparent particle density = ρ_p

RHOS = density of the solid material = ρ_s

U = fluid velocity in x-direction = u'

UINFIN = fluid free-stream velocity = u_∞

UP	= particle velocity in x-direction = u'_p
UPINFIN	= particle free-stream velocity = u_{p_∞}
V	= fluid velocity in y-direction = v'
VP	= particle velocity in y-direction = v'_p
X	= distance parallel to the plate = x'
Y	= distance normal to the plate = y'

Computer Program

```
?COMPILE  T013912    ALGOL    .03S800015   *0000  URQUHART  J  B
?DATA.
BEGIN
COMMENT    URQUHART-TWO PHASE FLOW-NUMBER 1;
INTEGER    I,J,M,N,P,L,R,Q;
REAL       X,Y,DX,DY,UINFIN,MU,RHO,RE;
REAL       A,RHOS,UPINFIN,F,FF,INJVEL,INJANGLE,PI;
REAL       B,C,G,H;
FILE IN    F1(2,10);
FILE OUT   F2 6(2,15);
FORMAT IN  FM1(E11.4,3(F10.4),I5,I5,F7.1;
FORMAT IN  FM2(E11.4,F7.2,F7.2,F7.2);
FORMAT OUT FM3("TWO PHASE FLOW-NUMBER 1"////);
FORMAT OUT FM4("MU=",E11.4,X4,"RHO=",F6.4,X4,"DX=",F6.4,X4,"DY
           =",F6.5,X4,"M=",I4,X4,"UNINFIN=",F6.1/);
FORMAT OUT FM5("A=",E11.4,X4,"RHOS=",F7.1,X4,"UPINFIN=",F7.2/);
FORMAT OUT FM6("REYNOLDS NUMBER =",E11.4/);
FORMAT OUT FM7("F=",E11.4,///);
FORMAT OUT FM8(/,"FLOW OVER FLAT PLATE WITH PARTICLE INJECTION"/);
FORMAT OUT FM9("INJECTION VEL=",F6.2,X5,"INJECTION ANGLE=",F6.2/);
FORMAT OUT FM10(X1,"I",X8,"J",X10,"X",X11,"Y",X12,"U",X13,"V",
           X13,"UP",X12,"VP",X11,"RHOP"/);
FORMAT OUT FM11(I4,X5,I4,X5,F7.4,X5,F7.4,X5,F9.5,X5,F9.5,X5,
           F9.5,X5,F9.5,X5,F10.5);
```

```
WRITE       (F2,FM3);
READ        (F1,FM1,MU,RHO,DX,DY,M,N,UINFIN);
READ        (F1,FM2,A,RHOS,UPINFIN,INJVEL,INJANGLE);
P:=N;L:=1;PI:=3.14159;
F:=9xMUxL/(2xA*2xRHOSxUNIFIN);FF:=FxUNIFIN/L;
RE:=RHOxUNIFINxL/MU;
B:=DX/DY:C:=DY/DX;G=DX/DY*2,H:=FxDX/RHO;
WRITE       (F2,FM4,MU,RHO,DX,DY,M,N,UINFIN);
WRITE       (F2,FM5,A,RHOS,UPINFIN);
WRITE       (F2,FM6,RE);
WRITE       (F2,FM7,FF);
X:=0;Y:=0;
      BEGIN
      REAL ARRAY [U,V,UP,VP,RHOP -2:M+1,-2:N+1];
      R:=-10;
      WRITE (F2,FM8);
      WRITE (F2,FM9,INJVEL,INJANGLE);
      U [0,0]:=1;V [0,0]:=0;
      UP [0,0]:=INJVELxCOS(INJANGLExPI/180)/UNIFIN;
      VP [0,0]:=INJVELxSIN(INJANGLExPI/180)/UNIFIN;
      RHOP [0,0]:=RHOSx0.1;
      U [-1,0]:=1;V[-1,0];=0;UP[0,0];VP[-1,0]:=VP[0,0];
      RHOP[-1,0]:=RHOP[0,0];
      U[0,-1];=1;V[0,-1]:=0;UP[0,-1];=UP[0,0];VP[0,-1]:=VP[0,0];
      RHOP[0,-1]:=RHOP[0,0];
```

```
U[0,-2]:=1;V[0,-2]:=0;UP[0,-2]:=UP[0,0];VP[0,-2]:=VP[0,0];
RHOP[0,-2]:=RHOP[0,0];
FOR I:=1 STEP 1 UNTIL M+1 DO
BEGIN
U[I,0]:=V[I,0]:=VP[I,0]:=U[I,-1]:=V[I,-1]:=VP[I,-1]:=0;
U[I,-2]:=V[I,-2]:=VP[I,-2]:=0;
END;
FOR J:=1 STEP 1 UNTIL N+1 DO
BEGIN
U[0,J]:=1;V[0,J]:=0;VP[0,J]:=VP[0,0];
U[-1,J]:=1;V[-1,J]:=0;VP[-1,J]:=VP[0,0];
UP[0,J]:=UPINFIN/UINFIN;RHOP[0,J]:=0;
UP[-1,J]:=UPINFIN/UINFIN;RHOP[0,J]:=0;
END;
IF X>0 THEN BEGIN
FORMAT IN FM12(5(F10.5));
Q:=0;
FOR I:=-1 STEP 1 UNTIL 0 DO BEGIN
FOR J:=-2 STEP 1 UNTIL Q DO
READ (F1,FM12,U[I,J],V[I,J],UP[I,J]VP[I,J]RHOP[I,J]);
FOR J:=Q STEP 1 UNTIL N+1 DO
BEGIN
U[I,J]:=U[I,Q];V[I,J]:=V[I,Q];UP[I,J]:=UP[I,Q];
VP[I,J]:=VP[I,Q];RHOP[I,J]:=0;
END;END;
FOR I:=1 STEP 1 UNTIL M+1 DO
```

```
     BEGIN                                                      012
     U[I,0]:=V[I,0];=VP[I,0];=U[I,-1]:=VP[I,-1]:=0             ;013
     U[I,-2]:=V[I,-2]:=VP[I,-2]:=0                             ;014
     END;END                                                   ;015
     WRITE (F2,FM10)                                           ;161
     P:=N                                                      ;162
     FOR I:=0 STEP 1 UNTIL M DO                                 163
     BEGIN                                                      164
     FOR J:=0 STEP 1 UNTIL P DO                                 165
     BEGIN                                                      166
     LABEL VT,UPT,VPT,RT,RHOPT,CONT                            ;167
     X:=0+IxDX;Y:=JxDY                                         ;168
     IF J=0 THEN U[I+1,J]:=0 ELSE                               169
     IF U[I+1,J-1]=1 THEN U[I+1,J]:=1 ELSE                      170
     U[I+1,J]: =U[I,J]+G/REx(U[I,J+1]/U[I,J]-2+U[I,J-1])        171
     -BxV[I,J]+BxV[I,J]xU[I,J-1]/U[I,J]                         172
     +HxRHOP[I,J]xUP[I,J]/U[I,J]-HxRHOP[I,J]                   ;173
     IF U[I+1,J]> THEN U[I+1,J]:=1 ELSE GO TO VT               ;174
VT:  IF I=0 THEN V[I,J+1]:=0 ELSE                               175
     IF J=0 THEN V[I,J+1]:= 0 ELSE                              176
     V[I,J+1]:=V[I,J]+CxU[I-1,J]-CxU[I,J]                      ;177
     IF UP[I,J]=0 THEN BEGIN UP [I+1,J]:=0;VP[I+1,J]:=VP[I,J]  ;178
     RHOP[I+1,J]:= RHOP[I,J]; GO TO CONT; END ELSE              179
     IF J>0 THEN BEGIN                                          180
     IF UP [I+1,J-1] = UPINFIN/UINFIN THEN UP [I+1,J]:=         181
     UPINFIN/UINFIN ELSE                                        182
```

```
       UP[I+1,J]:=UP[I,J]-BxVP[I,J]xUP[I,J-1]/UP[I,J]
       -FxDX+FxDXxU[I,J]/UP[I,J]                                   ;
       GO TO UPT;END ELSE
       IF UP [I,J]-FxDX>0 THEN UP [I+1,J]:=UP[I,J]-FxDX
       ELSE UP [I+1,J]:=0                                          ;
UPT:   IF UP [I+1,J] UPINFIN/UINFIN THEN UP [I+1,J]:=UPINFIN/UINFIN
       ELSE TO TO VPT                                              ;
VPT:   IF J=0 THEN VP [I,J]:=0 ELSE
       VP[I+1,J]:=VP[I,J]-BxVP[I,J]xVP[I,J]/UP[I,J]+BxVP[I,J]
       VP[I,J-1]/UP[I,J]-FxDXxVP[I,J]/UP[I,J]+FxDXxV[I,J]/
       UP[I,J]                                                     ;
RT:    IF J>0 THEN RHOP[I+1,J]:=RHOP[I,J]xUP[I-1,J]/UP[I,J]-
       BxVP[I,J]xRHOP[I,J]/UP[I,J]+BxVP[I,J]xRHOP[I,J-1]/UP[I,J]-
       BxVP[I,J]xRHOP[I,J]/UP[I,J]+BxVP[I,J-1]xRHOP[I,J]/UP[I,J]
       ELSE RHOP [I+1,J]:=0                                        ;
RHOPT:IF RHOP [I+1,J]>RHOS THEN RHOP [I+1,J]: = RHOS
       ELSE GO TO CONT                                             ;
CONT: UP [I+1,-1]:=UP[I+1,0]                                       ;
       UP [I+1,-2]:=UP[I+1,0]                                      ;
       VP [I+1,-1]:=VP[I+1,0]                                      ;
       VP [I+1,-2]:=VP[I+1,0]                                      ;
       RHOP[I+1,-1]:=RHOP[I+1,0]                                   ;
       RHOP [I+1,-2]:=RHOP[I+1,0]                                  ;
       IF I≤1OTHEN
       BEGIN
       IF U[I,J-2]<1 THEN
```

```
WRITE (F2,FM11,I,J,X,Y,U[I,J],V[I,J],UP[I,J],VP[I,J],
RHOP [I,J]
ELSE IF UP [I,J-2]<UPINFIN/UINFIN THEN
WRITE (F2,FM11,I,J,X,Y,U[I,J],V[I,J],UP[I,J],VP[I,J]
RHOP [I,J]);END
ELSE IF R=5 THEN BEGIN
IF U[I,J-2]< THEN
WRITE (F2,FM11,I,J,X,Y,U[I,J],V[I,J],UP[I,J],VP[I,J],
RHOP [I,J]
ELSE IF UP [I,J-2]<UPINFIN/UINFIN THEN
WRITE (F2,FM11,I,J,X,Y,U[I,J],V[I,J],UP[I,J],VP[I,J],
RHOP [I,J]                                              ;
END ELSE
IF I=M-1 THEN BEGIN
IF U [I,J-2]<1 THEN
Write (F2,FM11,I,J,X,Y,U[I,J],V[I,J],UP[I,J],VP[I,J],
RHOP [I,J]
ELSE IF UP [I,J-2]<UPINFIN/UINFIN THEN
WRITE (F2,FM11,I,J,X,Y,U[I,J],V[I,J],UP[I,J],VP[I,J],
RHOP [I,J]
ELSE END ELSE END                                       ;
IF R=5 THEN R:=1 ELSE R:=R+1;P:=P-1;END                 ;
END.                                                    ;
?DATA F1
1.2850@-05    0.0710    0.0005  0.0005  100   200   100.00
1.6400@-05  145.0  100.00    50.00   10.00
```

BIBLIOGRAPHY

1. Soo, S.L., "Boundary Layer Motion of a Gas-Solids Suspension," Proc. of Symposium on Interaction between Fluids and Particles (London), Institution of Chemical Engineers, 1962, pp. 50-63.

2. Soo, S. L., "Gas-Solid Flow," Single and Multi-Component Flow Processes, Rutgers University, New Brunswick, N. J., 1965, pp. 1-52.

3. Soo, S. L., "Laminar and Separated Flow of a Particulate Suspension," Astronautica Acta, volume II, November-December 1965, pp. 422-431.

4. Soo, S. L., "Dynamics of Multiphase Flow Systems, " I & EC Fundamentals, volume 4, November 1965, pp. 426-433.

5. Marble, Frank E., "Dynamics of a Gas Containing Small Solid Particles,"Fifth AGARD Combustion and Propulsion Colloquim, London, 1962, pp. 175-215.

6. Singleton, Robert E., "Fluid Mechanics of Gas-Solid Particle Flow in Boundary Layers," Ph.D. Thesis, California Institute of Technology, Pasadena, California, 1964.

7. Schlichting, Hermann, Boundary Layer Theory, McGraw-Hill Book Co., Inc., New York, 1960.

8. Maude, A. D., "The Viscosity of a Suspension of Spheres," Journal of Fluid Mechanics, volume 7, 1959, pp. 230-236.

9. Sproull, Wayne T., "The Viscosity of Dusty Gases", Nature, volume 190, 1961, pp. 976-978.

10. Smith, G. D., Numerical Solution of Partial Differential Equations, Oxford University Press, London, 1965.

11. O'Brien, G. G., M. A. Hyman, and S. Kaplan, "A Study of the Numerical Solution of Partial Differential Equations," Journal of Mathematics and Physics, volume 29, 1950, pp. 223-251.

12. Richtmyer, R. D., Difference Methods for Initial Value Problems, Interscience Publisher Inc., New York, 1964.

13. Kreith, Frank, Principles of Heat Transfer, International Textbook Company, Scranton, Penn., 1963, pp. 379-380.