

DESIGN OF A DEVICE
FOR DRILLING LONG, SMALL-DIAMETER TUNNELS

A THESIS

Presented to
The Faculty of the Division of Graduate
Studies and Research

By
Baparao Uppaluri

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mechanical Engineering

Georgia Institute of Technology

December, 1972

DESIGN OF A DEVICE FOR DRILLING LONG, SMALL-DIAMETER TUNNELS

Approved:

Robert B. Evans, Chairman

George J. Simitzes

B. B. Mazanti

H. L. Johnson

Date Approved by Chairman: 7-6-73

ACKNOWLEDGMENTS

I owe heartfelt gratitude to Dr. Robert B. Evans who has pioneered the research project and been a constant source of inspiration, Dr. George J. Simitzes for his invaluable suggestions in setting up the mathematical model, solving and verifying it for various physical phenomena, Dr. Harold Johnson for his guidance in the design process, and Dr. Billy B. Mazanti for furnishing pertinent information. I appreciate the help rendered by the fellow graduate students in computer programming. Finally, I thank Miss Vicki L. Terrell for her verification of the English.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS.....	ii
LIST OF ILLUSTRATIONS.....	v
Chapter	
I. INTRODUCTION.....	1
II. STATEMENT OF THE PROBLEM AND PLAN OF ATTACK.....	2
III. LITERATURE AND PATENT SURVEY.....	4
IV. PRESENT TUNNELLING DEVICES.....	6
V. DETERMINATION OF PERFORMANCE SPECIFICATIONS.....	14
VI. PRELIMINARY PROPOSAL OF THE DEVICE.....	15
VII. MATHEMATICAL MODEL	
a) Mathematical Model.....	23
b) Solution of the Mathematical Model for	
i. Deflection.....	23
ii. Buckling.....	29
iii. Dynamic Stability.....	29
VIII. STRENGTH ANALYSIS OF THE DESIGN	
a) Thrust Required.....	36
b) Stress Analysis.....	37
IX. DETAILED DESIGN OF THE DRILL BIT	
i. Bit Penetration.....	42
ii. Cutter Profile.....	44
iii. Rate of Tunnelling.....	45

	Page
a) Air Flow Analysis.....	44
b) Thread Joint.....	52
X. CONCLUSION.....	54
APPENDIX.....	57
BIBLIOGRAPHY.....	58

LIST OF ILLUSTRATIONS

Figure	Page
1. a) Pipe Pushing	11
b) Spoil Augering.....	11
c) Water Boring.....	11
d) Impact Penetrating.....	11
e) Compacting Augering.....	11
2. Bore Specifications and Cost of Drilling.....	12
3. Rate and Accuracy of Drilling.....	13
4. Shaft Joint.....	17
5. Drill Bit.....	19
6. Mathematical Model.....	23
7. Friction Factors for Flow in Pipes.....	48
8. Pipe Roughness Factors.....	49
9. Turbo Compressor Minimum Capacity at Supply Pressure	51
10. Set-up of the Proposed Device.....	56

CHAPTER I

INTRODUCTION

Although underground tunnelling dates far back into history, much of the progress in tunnelling has been made during the past decade. A new era is blossoming out for tunnelling and an exciting new technology is emerging. Underground tunnelling is playing a historic role in the advance of civilization. For example, subsurface transportation is helping to solve many urbanization problems. Development of tunnelling technology enhances the scope of expansion of the mining industry. World mineral consumption will increase fivefold by the year 2000. To meet this requirement, new underground mines will have to be opened, thus necessitating modern tunnelling techniques^{1*}.

Research has been under way on the other uses of underground tunnels, such as laying cables for power transmission, power distribution, and telephone service. As a result of this research improved devices for laying power or telephone lines under ground have been developed. The intention of this thesis is to develop a suitable tunnelling device for use in intricate situations where the existing devices are not feasible.

* All numerical superscripts refer to the Bibliography.

CHAPTER II

STATEMENT OF THE PROBLEM AND PLAN OF ATTACK

Statement of the Problem

An economical device, capable of drilling a long, small-diameter tunnel, would be advantageous for tunnelling under metropolitan areas, thus saving the cost of repairing the damaged streets, sidewalks and other paved surfaces. It, therefore, seems desirable to have a device which could economically tunnel a horizontal bore of small diameter (say less than six inches), between manholes 200 feet or more apart with reasonable accuracy (say within six inches of the mark in 200 feet). Economical tunnelling will also require a reasonable speed of operation, at relatively low initial and maintenance costs. The development of such a Long, Small-diameter Tunnelling Device (LSTD) would be of tremendous help to many industries. Power companies pay a huge amount of money for breaking open and repaving the ground after laying utility lines*.

With the increasing extent of modernization, telephone companies find it astronomically difficult to cope with the overhead telephone lines.

* The problem is based on material obtained from Mr. Glen P. Robinson, Chairman of Board, Scientific Atlanta, Incorporated, Atlanta.

Plan of Approach

The research is to be conducted in the following steps:

First, a thorough survey of existing models and their specifications is to be conducted.

Second, an extensive patent search is to be performed.

Third, performance specifications which represent a substantial improvement are to be determined, based on the preceding steps.

Finally, a device which meets these specifications is to be designed.

CHAPTER III

LITERATURE AND PATENT SURVEY

Advantages of Underground Lines

Underground lines are less subject to storms and accidents than are overhead lines and, being hidden, they do not detract from the landscape. They also ensure improved reliability of service and less maintenance. In large cities, as demand for telephone service grows, overhead lines become impractical. They have, therefore, been largely replaced by cables in protective underground ducts. However, with today's technology, constructing ducts in very densely populated areas is extremely costly².

In some cases it is virtually impossible to bury transmission lines by the conventional methods of digging a trench or plowing the cable into the ground. These methods become impractical in heavily populated areas as they require breaking up streets or pavements and disrupting traffic. The only alternative, then, is to tunnel underneath the obstructions because this:

1. minimises restoration costs,
2. lowers the amount of security bond to be posted with the road repair authorities,
3. avoids legal restrictions on cutting through

major roads,

4. creates only a minimum of traffic disturbance,
- and 5. increases public safety.

The great difference in unit costs of underground and overhead transmission lines narrows as right-of-way costs for overhead transmission lines in congested areas increase. Improved methods of underground emplacement will certainly reduce this difference in costs and even further favor underground lines. Also, in the future, overhead transmission lines probably will not be permitted by metropolitan authorities in highly built-up areas. A few states and many municipalities have already passed ordinances that require all utilities to be buried³.

Tunnelling Methods and Devices

Many methods have been employed for underground tunnelling for power lines. Each of them will be explained briefly in Chapter IV. None of these devices can tunnel between manholes like the proposed LSTD. They all necessitate a long trench for installation and operation.

A thorough search of patents on the existing devices has been conducted. As far back as 1961, there were not any devices similar to the proposed LSTD. The patent numbers of the related devices are included in the Appendix.

CHAPTER IV

PRESENT TUNNELLING DEVICES

Pipe Pushing

This method consists of forcing sections of pipe through the soil either by mechanical leverage or by hydraulic ram. The hydraulic system of a back-hoe (a small power shovel) or a separate engine-pump supplies the power. Although Pipe Pushing is fairly reliable it is slow and requires extensive preparation. A large pit must be dug so that the Pipe Pusher can be lowered in and levelled. Then the rear wall of the pit must be lined with heavy bracing blocks to resist the pushing reaction. After one pipe section has been driven into the earth the ram is retracted so that an additional section can be added. Pipe Pushing can be used in most soils although soils containing large rocks or boulders or close packed sandy soils may present problems with this method. (Fig.1-a)

A high degree of accuracy can be achieved by this method with two feet or larger diameter pipes that have been carefully aligned and deviations of one foot or less in 100 feet bores are not uncommon. Penetration rates are a function of soil, thickness of pipe wall and the available power unit.²

Equipment

A hydraulic pump powered by a gasoline engine generally supplies the pushing force. The hydraulic cylinders supplied by these pumps can be arranged singly or in groups and often provide forces in excess of 800 tons.¹

Spoil Augering

The system is essentially a three component unit used to bore holes in soil or weak rock. An auger consists of a drag bit (which cuts the earth and conveys it back), an attached spiral conveyor and a power source. The drag bit, pushed and rotated simultaneously, penetrates the soil while the spirals convey the cuttings to a discard point. An air motor, gasoline engine or hydraulic motor supplies torque and a hydraulic ram or manually operated lever supplies forward thrust. Sections of auger shaft must be added as the bit progresses. Reactions are resisted by either the floor or rear wall of the pit.² (Fig. 1-b)

Equipment

A typical small auger unit is powered by a 9.2 HP engine which can bore and case a 2.75 inch hole about 120 feet long.

A larger auger unit is powered by a 63 HP engine for a 42 inch diameter hole, 300-400 feet long.²

Water Boring

Another form of spiral augering makes use of water to

remove soil. The water is supplied through a hollow shaft connected to a short fish tail bit. As the rotating bit drills through the earth the water flushes the crumbled soil out through the tunnel. An air motor usually supplies the torque. Forward thrust, supplied by working a system of levers or by pushing on the air motor handle, requires considerable labor. Because, the bit tends to wander off course, access holes must be dug about every 35 feet so that the shafts can be realigned.² (Fig. 1-c)

Impact Penetrating

This method uses some form of rapidly oscillating piston enclosed in a pointed cylinder. The piston alternately bounces against an air cushion at the rear of the cylinder and strikes a steel anvil at the front. The sharp steel to steel impact drives the cylinder through the ground. A pneumatic hammer working on high pressure air can also be employed in this principle (Pneuma Gopher). Air for the hammer is supplied by a trailing air hose connected to an air compressor, out side of the hole. As the hammer rapidly impacts against the anvil, the tool is driven forward. It can bore 3.75 inch diameter hole up to 100 feet long.² (Fig. 1-d)

Equipment

The mechanical mole uses 45-50 cfm of air at 90 psi. The mole is 45 inches long, its diameter is 3.75 inches and weighs 64 lbs.¹

Compacting Augering

This process is similar to turning of a screw through wood. As the auger bit rotates spiral threads along its conical surface react against soil forcing the auger forward. Thus compacting auger requires no external thrust. It compresses the soil into the tunnel wall. The auger bit is turned by a series of connected steel shafts. These are flexible enough so that the auger can be aimed horizontally from the bottom of a narrow trench three or four feet deep. A hydraulic motor, gasoline engine or air motor mounted on a cart that rolls along the ground supplies torque. As the auger bit progresses through the earth additional shaft sections are connected quickly² (Fig.1-e)

Equipment

The auger is rotated by a mechanical or hydraulic drive unit which is usually powered by a small gasoline engine with control equipment consisting of a sighting guide and positioning stakes. The power unit and auger drive unit can be mounted as a complete package on a small two wheeled cart.¹

Vibratory Conduit Driving

The vibratory method of driving rigid metal conduit through the soil utilizes the random motion response of soils to forced vibrations and the corresponding decrease in the soil resistance which these vibrations induce. The vibrations are set up in the pipe to be emplaced by a vari-

able frequency mechanical exciter operating at frequencies ranging from 0-200 cps, with a power range up to 1000 H.P. These waves are reinforced and amplified in the pipe by the resonance effect and so much energy becomes available for reducing the resistance. The force necessary is therefore, reduced as much as 98 %. The penetration rate by this method is one foot per second¹.

Equipment

Resonant vibrator, 500 H.P gasoline engine.

The bore specifications and cost of drilling for various existing devices described above, are compared in Figure 2,

The rate and accuracy of drilling for these systems are as given in Figure 3.¹

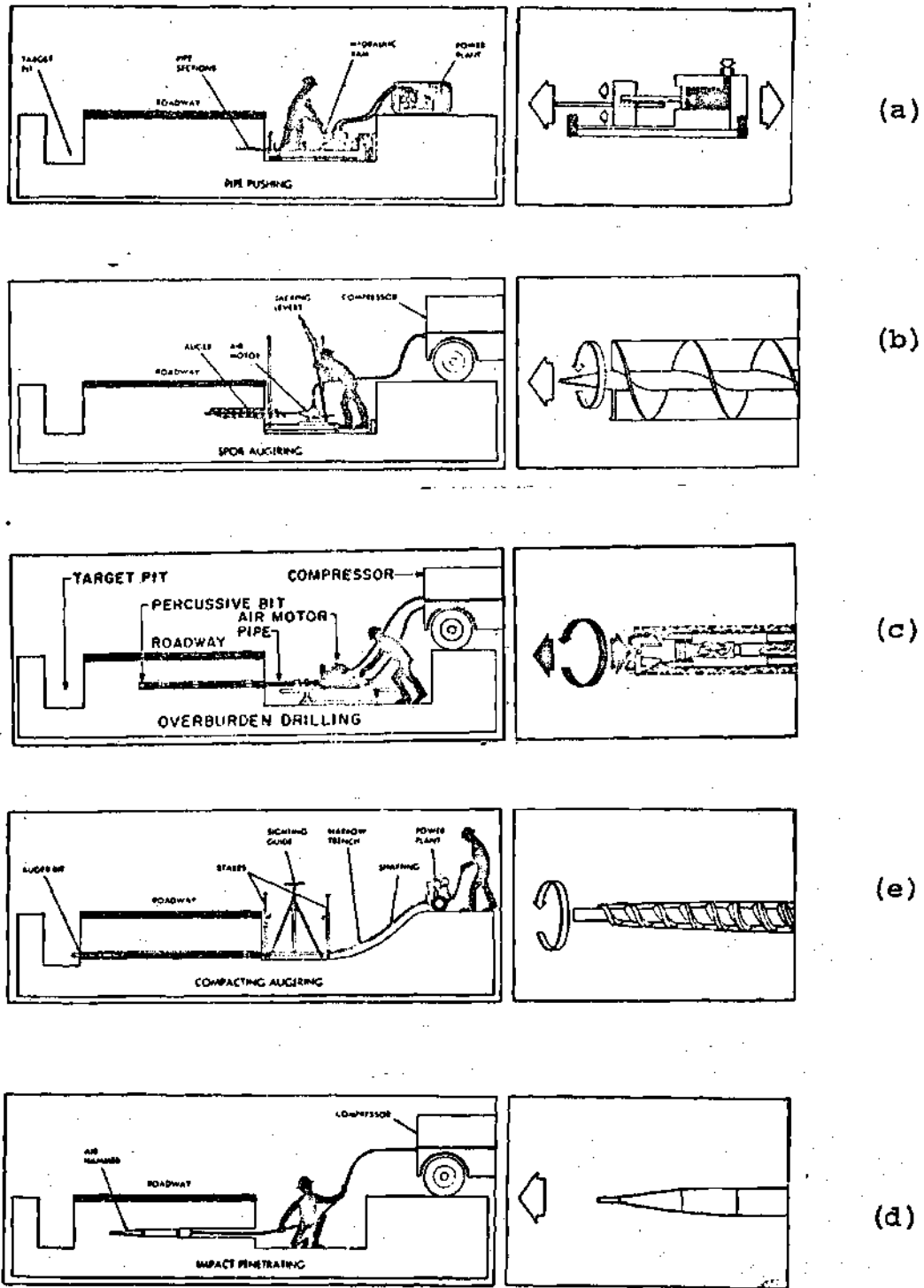


Figure 1. Various Tunneling Devices

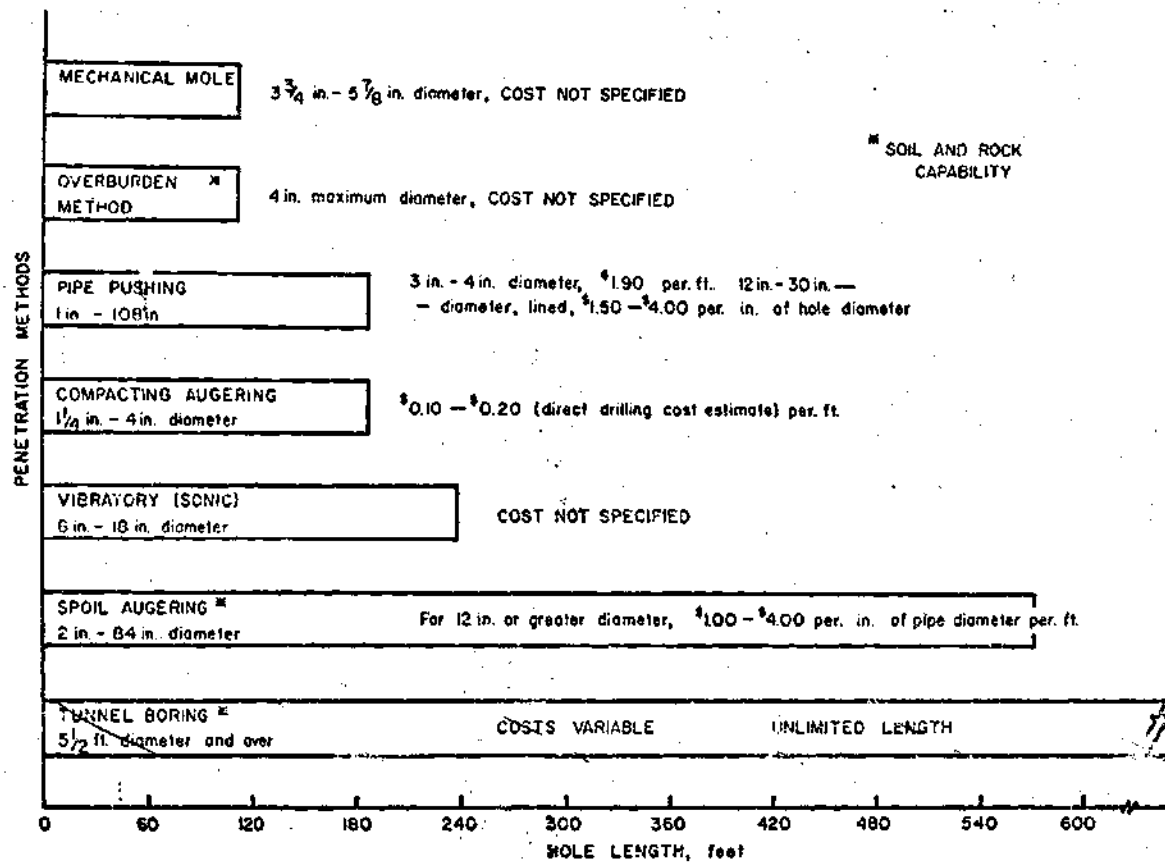


Figure 2. Bore Specifications and Cost of Drilling
(After 'Yardley')

Method	Material Bored	Maximum Hole Length, Ft	Range of Hole Diameter, In.	Accuracy	Penetration Rates, Fpm	Cost, \$ per Ft of Hole
Spoil augering	Soils, soft rock	570	2 to 84	Not specified	½ to 6	For 12-in. or greater diameter: \$1 to \$4 per in. of pipe diameter
Compacting augering	Soils	200	1¼ to 4 (reamed to 8 in.)	About 1° error	2 to 8	\$0.10 to 0.20 (direct drilling cost estimate)
Mechanical mole	Soils	100	3¾ to 5¾	Not specified	1 to 4	Not specified
Pipe pushing	Soils	200	1 to 108	Error about 1% of hole length for large diameter holes	0.1 to 0.2 and over	(3 to 4-in.-diam) \$1.90 (12 to 30-in.-diam lined) \$1.50 to \$4 per in. of hole diameter
Overburden drilling	Any material soils and/or rock	100	4	Error about 1% of hole length	0.44 in broken rock and gravel	Not specified
Vibratory (sonic)	Soils	240	Up to 18	Less than 1% error in some cases	60	Not specified
Machine tunneling	Soils	Unlimited	66 to 450*	Excellent	Up to 14 or more	Costs variable

*Present information shows that 50-ft-diam tunneling machines are in the design stage.

Figure 3. Rate and Accuracy of Boring
(After 'Yardley')

CHAPTER V

DETERMINATION OF PERFORMANCE SPECIFICATIONS

The major criterion in determining the bore specifications is the practical purpose for which the tunnel is going to be used. Laying power transmission cables or telephone cables or gas pipes requires different size tunnels. Keeping these points in view the bore diameter is specified to be less than six inches.

If the length of the tunnel is too small, it necessitates too many manholes. On the other hand if the tunnel is too long the driving mechanism becomes bulky and a bigger manhole may be required. From this point of view, the tunnel length is specified to be 200 feet.

The speed and accuracy of operation should be comparable to those of the existing devices. The speed and accuracy of various existing devices are compared in Figs.2 and 3. The speed of operation of the proposed LSTD is specified to be five feet per minute and accuracy to be six inches in 200 feet.

Specifications

Diameter of tunnel.....	less than 6 in.
Length of tunnel.....	200 ft.
Speed of tunnelling.....	5 fpm.
Accuracy.....	6 in. in 200 ft.

CHAPTER VI

PRELIMINARY PROPOSAL OF THE DEVICE

The proposed LSTD utilizes the principle of drilling through ground from one manhole to the next, each of about five feet diameter and 200 ft. apart. It should consist of:

1. Hollow Drive Shaft
2. Drill Bit
3. Liner (Casing)
4. Motor
5. Gear Box and Clutch
6. Air Blower
7. Hydraulic Ram

Operation

The drill bit is fixed to the leading end of the hollow drive shaft. The electric motor drives the shaft through gear box provided for varying the torque during the operation. The shaft rotates inside the casing. There is an annular clearance between the liner casing and the drive shaft. As the shaft rotates, the drill bit cuts the earth, and the removed earth falls in to the hollow of the shaft in the form of small particles. High velocity air is blown through the annular space between the liner and the shaft to

carry away the debris through the hollow shaft. As the drilling progresses, more and more sections are added. The thrust to push the casing and the shaft is provided by the hydraulic ram.

Description of Parts

Shaft

The shaft transmits the torque to the drill bit. In the case of cased tunnels it serves as an exit passage for the earth particles. In the case of caseless tunnels it also serves as the housing for the inlet air hose. The size of the shaft in the latter case has to be greater than in the former case because the hollowness of the shaft has to serve the dual purpose of entrance and exit.

When there is no guide casing, the shaft has to be rigid enough to serve as a self guiding system to prevent wandering off course during drilling.

The various forces acting on the shaft are:

1. Torque:

- a) To shear the earth
- b) To overcome friction while rotating in the casing (ground if no casing)

2. Thrust:

- a) To create adequate normal stress in the ground to induce shear failure of the ground,

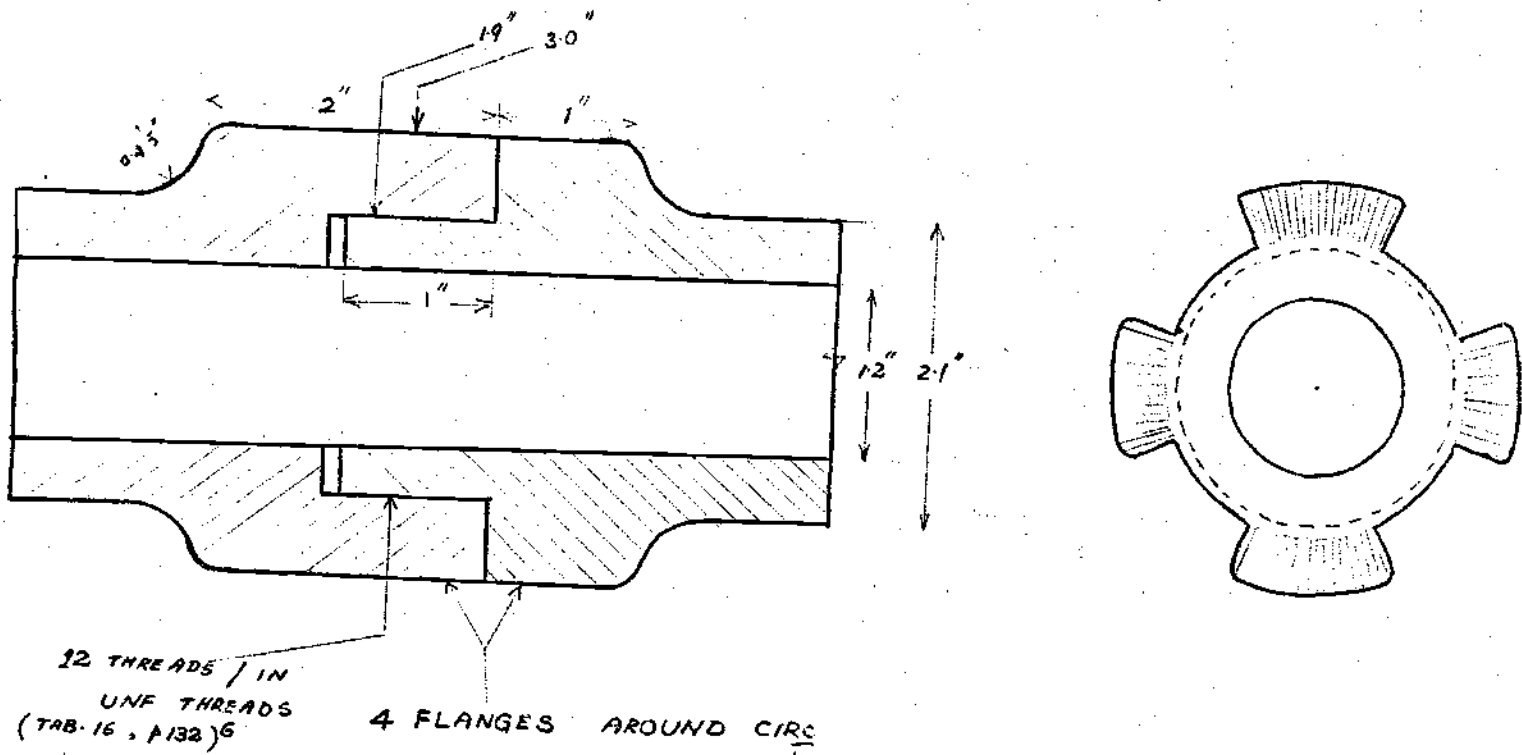


Figure 4 Shaft Joint

b) To overcome friction to the axial motion.

3. Bending Moment:

- a) Due to the resistance of the earth which could be axial or inclined if the earth is formed of inclined strata,
- b) Due to the uniformly distributed ground load,
- d) Due to the self weight.

Design Considerations:-

- a) The deflection of the end of the shaft
- b) Fatigue Considerations
- c) Maximum Principal Stress Theory of Failure
- d) Maximum Shear Stress Theory of Failure

The material selected for the shaft is wrought iron, C 1020, in annealed condition. It is cheaper than steel, easy to manufacture as tubes, ductile, corrosion resistant. It is made of two feet long sections joined by thread connection. Flanges are provided at the joints to increase bending rigidity (Fig. 4). (E of the material = 30×10^6 psi)

Drill Bit

The teeth of the drill bit first penetrate into the soil and then shear the earth off in the form of small grains.

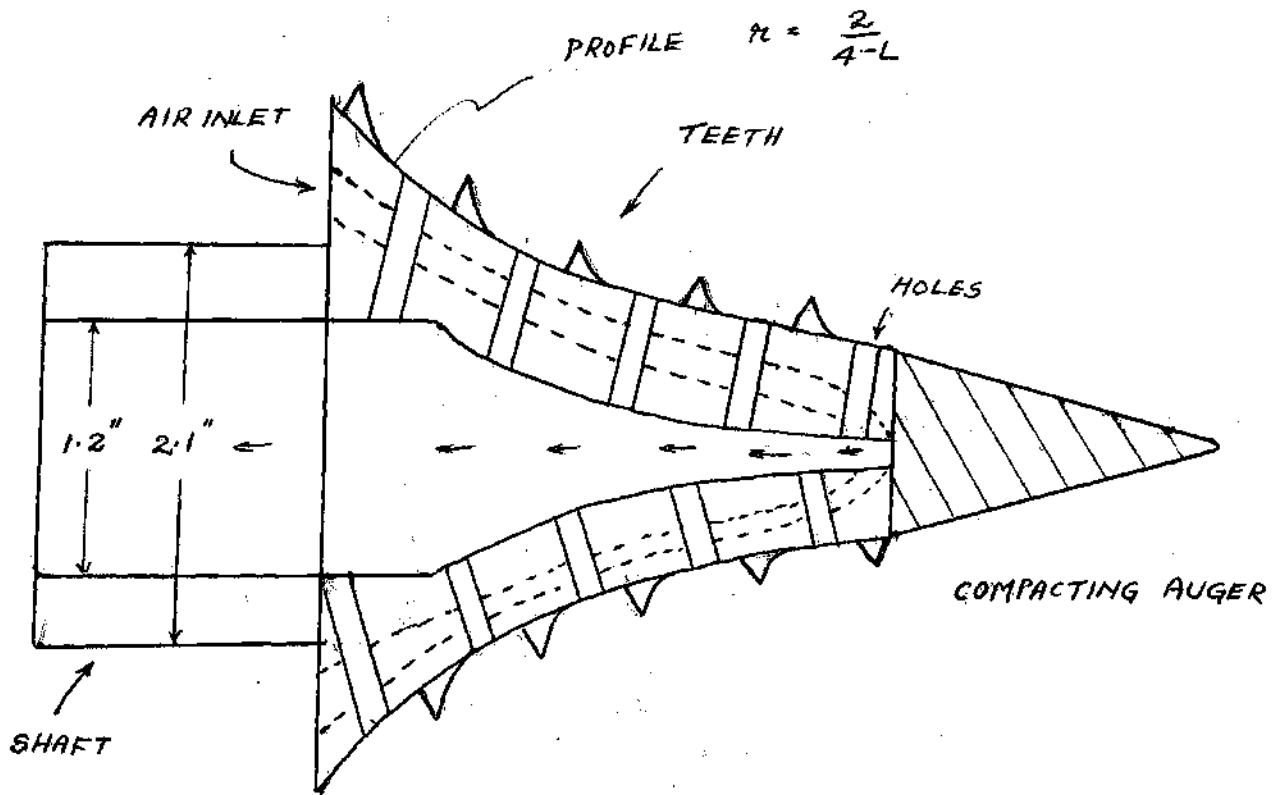


FIGURE 5. DRILL BIT
Figure 5. Drill Bit

The sheared off earth particles fall into the hollow of the shaft and receive adequate kinetic energy from the air current to travel the length of the shaft.

The tooth size and the rate of progress of the shaft are interdependent. Because, the size of the tooth determines the extent of earth removed per revolution and consequently the progress of the drill bit or shaft.

The teeth are made of a hard, abrasion resistant metal (such as Tungsten Carbide) and are arranged in helical rows with alternate rows of holes. The helical array causes axial motion of the sheared earth particles directly into the holes.

There is a pointed compacting auger attached to the leading tip of the bit. This initially creates a pilot hole by compacting the laterally. The teeth on the surface of the bit then ream this pilot hole.

Forces acting on the drill bit are:

- a) axial or inclined earth resistance
- b) torque
- c) bending moment

Liner (Casing)

The liner serves to

1. prevent collapsing of the tunnel wall,
2. increase the rigidity of shaft,
3. act as inlet passage to air.

It is composed of two feet sections. It is pushed in simultaneously with the shaft by means of the hydraulic ram. When casing is not used, the shaft hollowness acts both as inlet and exit. A flexible air hose may be used to carry air in through the hollow. The earth particles and exit air come out through the remaining space surrounding the air hose.

Differences:- a) The joint between sections of the shaft can not have collars as before.
b) The shaft wall thickness may have to be greater.
c) Frictional torque will be greater than in the former case, because the shaft has to rotate in the ground. In the former case the shaft has to rotate in the liner casing and lubrication minimises friction.

Motor, Gear Box, and Clutch

An electric motor developing 10 HP at 330 rpm is used as the power source. The natural frequency of the shaft is found to be about 60 cpm. Hence, the speed of rotation can not be any integer multiple (1,2,3 etc.) of 60. At 330 rpm the rate of drilling is 5.5 fpm which is comparable with other existing systems.

For harder soils, higher torques at lower speeds are obtained by means of the gear box.

A hydraulic coupling can be employed to connect the shaft with the gear box. It is a fluid clutch in which there is 100% slip as long as the motor is running and the

shaft is still. As soon as the gear is engaged, the shaft starts rotating. A mechanical clutch may also be used instead.

Blower

The purpose of the blower is to blow in high velocity air into the hollow shaft.

Hydraulic Ram

The purpose of the hydraulic ram is to push the shaft and the casing axially into the tunnel. A maximum thrust of 5400 lbs is required.

CHAPTER VII

MATHEMATICAL MODEL

a) Mathematical Model

The proposed mathematical model is a, "one end fixed, one end free, and elastically supported beam-column"* as shown below:

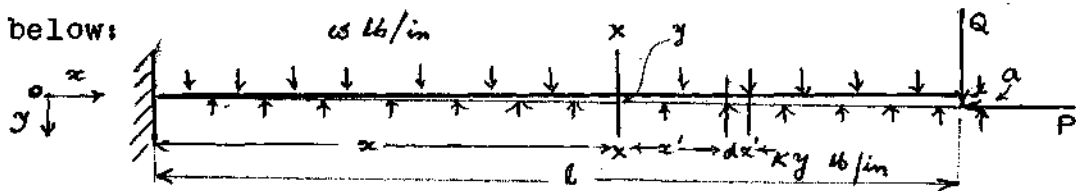


Figure 6. Mathematical Model

K = Modulus of the ground (500 psi); y = Deflection

w = Uniformly distributed load

P, Q = Components of earth's resistance

b) i. Solution of the Mathematical Model for Deflection

Summing up the moments acting on the section X-X, the following integro-differential equation of the model is obtained^{4,5}.

$$EI \frac{d^2 y}{dx^2} = w \int_0^{l-x} x' dx' - \int_0^{l-x} k y x' dx' + P(a-y) + Q(l-x) \quad (1)$$

In order to avoid the complicated integro-differential

* The formation of the mathematical model is as a result of discussions held with Dr. George J. Simitzes.

equation, differentiation is performed. The first differentiation yields:

$$EI \frac{d^3 y}{dx^3} + P \frac{dy}{dx} - \int_0^{l-x} ky dx' = \omega(x-l) - Q \quad (2)$$

Differentiating again, the following fourth order linear, ordinary differential equation is obtained:

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + ky = \omega \quad (3)$$

Particular Solution

$$\text{Rearranging (3), } (D^4 + \alpha^2 D^2 + \beta^2) y_p = \gamma$$

$$\left[\alpha^2 = \frac{P}{EI}; \beta^2 = \frac{K}{EI}; \gamma = \frac{\omega}{EI} \right]$$

$$y_p = (D^4 + \alpha^2 D^2 + \beta^2)^{-1} \gamma$$

$$= (1 - \dots) \frac{\gamma}{\beta^2} \quad \text{if } \beta^2 \neq 0$$

$$= \frac{\gamma}{\beta^2} = \frac{\omega}{K}$$

Complementary Solution

The auxiliary equation is: $D^4 + \alpha^2 D^2 + \beta^2 = 0$

$$\therefore D^2 = \frac{1}{2} [-\alpha^2 \pm \sqrt{\alpha^4 - 4\beta^2}]$$

$$\text{WHEN } \alpha^4 > 4\beta^2 \rightarrow \left(\frac{P^2}{E^2 I^2} > \frac{4K}{EI} \right) \rightarrow \left(P > (4KEI)^{\frac{1}{2}} \right)$$

$$\text{WHEN } \alpha^4 < 4\beta^2 \rightarrow \left(P < (4KEI)^{\frac{1}{2}} \right)$$

There are two solutions corresponding to whether the discriminant is positive or negative. As the latter case is the one encountered in practice (since P has an upper limit, unlike in the former case, see p.29), solution for this case only is found. $D^2 = A + Bi$

$$A = -\frac{\alpha^2}{2}, \quad B = \frac{1}{2} (4\beta^2 - \alpha^4)^{\frac{1}{2}}$$

$$D^2 = \pi (\cos \theta + i \sin \theta)$$

$$\pi^2 = \frac{\alpha^4}{4} + \frac{4\beta^2 - \alpha^4}{4} = \beta^2 \quad \therefore \pi = \beta$$

$$\cos \theta = \cos(\theta + 2k\pi) \Big|_{k=0,1} = \frac{-\alpha^2}{2\beta}$$

$$\therefore \cos \frac{\theta}{2} = \left(\frac{2\beta - \alpha^2}{4\beta} \right)^{\frac{1}{2}}$$

$$\sin \theta = \sin(\theta + 2k\pi) \Big|_{k=0,1} = \frac{(4\beta^2 - \alpha^4)^{\frac{1}{2}}}{2\beta}$$

$$\therefore \sin \frac{\theta}{2} = \left(\frac{2\beta + \alpha^2}{4\beta} \right)^{\frac{1}{2}}$$

$$\begin{aligned} \text{Now } D &= \sqrt{\pi} \left(\cos \frac{\theta + 2k\pi}{2} \pm i \sin \frac{\theta + 2k\pi}{2} \right) \\ &= \pm \frac{1}{2} \left[\sqrt{2\beta - \alpha^2} \pm i \sqrt{2\beta + \alpha^2} \right] \\ &= \pm k_1 \pm i k_2 \end{aligned}$$

Thus, the auxiliary equation has complex, conjugate roots, for which case the form of solution is:

$$y_c = e^{-k_1 x} (C \cos k_2 x + D \sin k_2 x) + e^{k_1 x} (A \cos k_2 x + B \sin k_2 x)$$

General Solution

$$y = y_p + y_c$$

$$y = \frac{W}{K} + e^{k_1 x} (A \cos k_2 x + B \sin k_2 x) + e^{-k_1 x} (C \cos k_2 x + D \sin k_2 x) \quad (4)$$

Boundary Conditions are obvious from (1), (2), and (3) as

$$x=0, \quad y=0 \quad x=L, \quad EI \frac{d^2 y}{dx^2} = 0$$

$$x=0, \quad \frac{dy}{dx} = 0 \quad x=L, \quad EI \frac{d^3 y}{dx^3} + P \frac{dy}{dx} = -P$$

$$\begin{aligned}
 K_1 &= \frac{1}{2} \sqrt{(2\beta - \alpha^2)} \\
 &= \frac{1}{2} \sqrt{\frac{2\sqrt{K}}{EI} - \frac{P}{EI}} \\
 &= \frac{1}{2} \left[\frac{\sqrt{4KEI - P}}{EI} \right]^{\frac{1}{2}} \\
 K_2 &= \frac{1}{2} \left[\frac{\sqrt{4KEI + P}}{EI} \right]^{\frac{1}{2}}
 \end{aligned}$$

$$\frac{dy}{dx} = K_1 e^{K_1 x} (A \cos K_2 x + B \sin K_2 x) + K_2 e^{K_1 x} (-A \sin K_2 x + B \cos K_2 x) - K_1 e^{-K_1 x} (C \cos K_2 x + D \sin K_2 x) + K_2 e^{-K_1 x} (-C \sin K_2 x + D \cos K_2 x)$$

$$\begin{aligned}
 \frac{d^2 y}{dx^2} &= 2K_1 K_2 e^{K_1 x} (-A \sin K_2 x + B \cos K_2 x) + (K_1^2 - K_2^2) e^{K_1 x} (A \cos K_2 x + B \sin K_2 x) - 2K_1 K_2 e^{-K_1 x} (-C \sin K_2 x + D \cos K_2 x) \\
 &\quad + (K_1^2 - K_2^2) e^{-K_1 x} (C \cos K_2 x + D \sin K_2 x)
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^3 y}{dx^3} &= 2K_2 K_1^2 e^{K_1 x} (-A \sin K_2 x + B \cos K_2 x) - 2K_2^2 K_1 e^{K_1 x} (A \cos K_2 x + B \sin K_2 x) + K_1 (K_1^2 - K_2^2) e^{K_1 x} (A \cos K_2 x + B \sin K_2 x) + \\
 &\quad K_2 (K_1^2 - K_2^2) e^{K_1 x} (-A \sin K_2 x + B \cos K_2 x) + 2K_1^2 K_2 e^{-K_1 x} (-C \sin K_2 x + D \cos K_2 x) + 2K_1 K_2^2 e^{-K_1 x} (C \cos K_2 x + D \sin K_2 x) - K_1 (K_1^2 - K_2^2) \\
 &\quad e^{-K_1 x} (C \cos K_2 x + D \sin K_2 x) + K_2 (K_1^2 - K_2^2) e^{-K_1 x} (-C \sin K_2 x + D \cos K_2 x)
 \end{aligned}$$

Substituting the Boundary Conditions,

$$0 = \frac{w_0}{K} + A + C \quad (5)$$

$$0 = K_1 A + K_2 B - K_1 C + K_2 D \quad (6)$$

$$\begin{aligned}
 0 &= 2K_1 K_2 e^{K_1 l} (-A \sin K_2 l + B \cos K_2 l) + (K_1^2 - K_2^2) e^{K_1 l} (A \cos K_2 l + B \sin K_2 l) - 2K_1 K_2 e^{-K_1 l} (-C \sin K_2 l + D \cos K_2 l) + \\
 &\quad (K_1^2 - K_2^2) e^{-K_1 l} (C \cos K_2 l + D \sin K_2 l)
 \end{aligned} \quad (7)$$

$$\begin{aligned}
 -Q &= A \left[(-2K_1^2 K_2 \sin K_2 l - 2K_1 K_2^2 \cos K_2 l) e^{K_1 l} EI - \frac{P}{2} e^{K_1 l} (K_2 \sin K_2 l - K_1 \cos K_2 l) \right] \\
 &+ B \left[(2K_1^2 K_2 \cos K_2 l - 2K_1 K_2^2 \sin K_2 l) e^{K_1 l} EI + \frac{P}{2} e^{K_1 l} (K_2 \cos K_2 l + K_1 \sin K_2 l) \right] \\
 &+ C \left[(-2K_1^2 K_2 \sin K_2 l + 2K_1 K_2^2 \cos K_2 l) e^{-K_1 l} EI - \frac{P}{2} e^{-K_1 l} (K_2 \sin K_2 l + K_1 \cos K_2 l) \right] \\
 &+ D \left[(2K_1^2 K_2 \cos K_2 l + 2K_1 K_2^2 \sin K_2 l) e^{-K_1 l} EI + \frac{P}{2} e^{-K_1 l} (K_2 \cos K_2 l - K_1 \sin K_2 l) \right]
 \end{aligned}$$

Simplifying,

$$\begin{aligned}
 -Q &= \sqrt{KEI} \left[A e^{K_1 l} (-K_2 \sin K_2 l - K_1 \cos K_2 l) + B e^{K_1 l} (K_2 \cos K_2 l - \right. \\
 &K_1 \sin K_2 l) + C e^{-K_1 l} (K_1 \cos K_2 l - K_2 \sin K_2 l) + D e^{-K_1 l} \\
 &\left. (K_1 \sin K_2 l + K_2 \cos K_2 l) \right] \quad (8)
 \end{aligned}$$

From (2) $A = -\frac{W}{K} - C$

Substituting for A in (8),

$$0 = \left(-\frac{W}{K} - C\right) K_1 + K_2 B - K_1 C + K_2 D$$

$$\therefore B = \frac{WK_1}{KK_2} + \frac{2K_1 C}{K_2} - D$$

Substituting for A and then for B in (8),

$$\begin{aligned}
 \frac{W}{K} \left[(-K_1^2 - K_2^2) e^{K_1 l} \cos K_2 l - K_1 K_2 e^{K_1 l} \sin K_2 l - \frac{K_1^3}{K_2} e^{K_1 l} \sin K_2 l \right] = \\
 e \left[4K_1^2 e^{K_1 l} \cos K_2 l + \frac{2K_1}{K_2} (K_1^2 - K_2^2) e^{K_1 l} \sin K_2 l + 2K_1 K_2 e^{K_1 l} \sin K_2 l - \right. \\
 \left. (K_1^2 - K_2^2) e^{K_1 l} \cos K_2 l + 2K_1 K_2 e^{-K_1 l} \sin K_2 l + (K_1^2 - K_2^2) e^{-K_1 l} \cos K_2 l \right] + \\
 D \left[-2K_1 K_2 e^{K_1 l} \cos K_2 l - (K_1^2 - K_2^2) e^{K_1 l} \sin K_2 l - 2K_1 K_2 e^{-K_1 l} \cos K_2 l + \right. \\
 \left. (K_1^2 - K_2^2) e^{-K_1 l} \sin K_2 l \right] \quad (9)
 \end{aligned}$$

Substituting for A and then for B in (8),

$$-\frac{Q}{\sqrt{KEI}} - \frac{W}{K} e^{K_1 l} \left[K_2 \sin K_2 l + 3K_1 \cos K_2 l - 2\frac{K_1^2}{K_2} \sin K_2 l \right] =$$

$$C \left[e^{K_1 l} (K_2 \sin K_2 l + 3K_1 \cos K_2 l - \frac{2K_1^2}{K_2} \sin K_2 l) + e^{-K_1 l} (K_1 \cos K_2 l - K_2 \sin K_2 l) \right] + D \left[e^{K_1 l} (K_1 \sin K_2 l - K_2 \cos K_2 l) + e^{-K_1 l} (K_1 \sin K_2 l + K_2 \cos K_2 l) \right] \quad (10)$$

Eliminating D from (9) and (10),

$$\begin{aligned} & \left[-\frac{Q}{EI} - \frac{W}{K} e^{K_1 l} (K_2 \sin K_2 l + 3K_1 \cos K_2 l - \frac{2K_1^2}{K_2} \sin K_2 l) \right] \times \\ & \left[(-2K_1 K_2 \cos K_2 l + \frac{P}{2EI} \sin K_2 l) e^{K_1 l} - (2K_1 K_2 \cos K_2 l + \frac{P}{2EI} \sin K_2 l) e^{-K_1 l} \right] \\ & + \left[\frac{W}{K} (K_1^2 + K_2^2) e^{K_1 l} (\cos K_2 l + \frac{K_1}{K_2} \sin K_2 l) \right] \times \\ & \left[e^{K_1 l} (K_1 \sin K_2 l - K_2 \cos K_2 l) + e^{-K_1 l} (K_1 \sin K_2 l + K_2 \cos K_2 l) \right] \end{aligned}$$

$$\begin{aligned} C = & \frac{\left[e^{K_1 l} (K_2 \sin K_2 l + 3K_1 \cos K_2 l - \frac{2K_1^2}{K_2} \sin K_2 l) + e^{-K_1 l} (K_1 \cos K_2 l - K_2 \sin K_2 l) \right] \times \left[(-2K_1 K_2 \cos K_2 l + \frac{P}{2EI} \sin K_2 l) e^{K_1 l} - (2K_1 K_2 \cos K_2 l + \frac{P}{2EI} \sin K_2 l) e^{-K_1 l} \right] - \left[\{(3K_1^2 + K_2^2) \cos K_2 l + \frac{2K_1^3}{K_2} \sin K_2 l\} e^{K_1 l} + (2K_1 K_2 \sin K_2 l - \frac{P}{2EI} \cos K_2 l) e^{-K_1 l} \right] \times \left[e^{K_1 l} (K_1 \sin K_2 l - K_2 \cos K_2 l) + e^{-K_1 l} (K_1 \sin K_2 l + K_2 \cos K_2 l) \right]}{-0.008 + 0.25 e^{-K_1 l}} \end{aligned}$$

Then the other values are calculated as,

$$D = 0.712 e^{-K_1 l} - 0.008$$

$$B = -0.215 e^{-K_1 l}$$

$$A = -0.25 e^{-K_1 l}$$

Substituting these values in the general solution the deflection of the free end is calculated as,

$$y_{x=1} = 0.33 \text{ in. (approx.)}$$

ii. Buckling

The condition for buckling is obtained by setting the determinant of the coefficients of the four arbitrary constants in the four simultaneous equations (after substituting zero for 'w' and 'Q') to zero. The determinant is as follows:

A	B	C	D
1	0	1	0
K_1	K_2	$-K_1$	K_2
$-2K_1K_2 e^{K_1 l} \sin K_2 l$	$2K_1K_2 e^{K_1 l} \cos K_2 l$	$2K_1K_2 e^{-K_1 l} \sin K_2 l +$	$-2K_1K_2 e^{-K_1 l} \cos K_2 l$
$+(K_1^2 - K_2^2) e^{K_1 l} \cos K_2 l$	$+(K_1^2 - K_2^2) e^{K_1 l} \sin K_2 l$	$(K_1^2 - K_2^2) e^{-K_1 l} \cos K_2 l$	$+(K_1^2 - K_2^2) e^{-K_1 l} \sin K_2 l$
$-K_2 \sin K_2 l e^{K_1 l}$	$K_2 \cos K_2 l e^{K_1 l}$	$K_1 \cos K_2 l e^{-K_1 l}$	$K_1 \sin K_2 l e^{-K_1 l}$
$-K_1 \cos K_2 l e^{K_1 l}$	$-K_1 \sin K_2 l e^{K_1 l}$	$-K_2 \sin K_2 l e^{-K_1 l}$	$+K_2 \cos K_2 l e^{-K_1 l}$

Simplifying this determinant:

$$\begin{aligned}
 & 4P(\sqrt{4KEI} - P) \sin^2 K_2 l - 4(4KEI - P^2) \cos^2 K_2 l \\
 & + P(\sqrt{4KEI} + P) (e^{K_1 l} - e^{-K_1 l})^2 - (4KEI - P^2) \\
 & (e^{K_1 l} + e^{-K_1 l})^2 = 0
 \end{aligned}$$

Solving this transcendental equation for P after substituting the values for k, EI, and l, by means of computer,

$$P_{cr} = 173,200 \text{ lb.}$$

iii. Dynamic Stability

For the system to be dynamically stable it should be

free from resonance. The natural frequency of the shaft can be found out by equating the kinetic energy during vibration to the total potential energy stored in the system.¹¹

The total potential energy is given by,¹²

$$P.E = \frac{1}{2} \int_0^l (EI \frac{d^2y}{dx^2} - Py) \frac{dy}{dx} dx + \frac{K}{2} \int_0^l y^2 dx$$

Neglecting the effect of pre-stress, since it is only about one % of the buckling load, the total potential energy is given by,

$$P.E = \frac{1}{2} \int_0^l EI \left(\frac{dy}{dx} \right)^2 dx + \frac{K}{2} \int_0^l y^2 dx$$

The kinetic energy is given by,¹²

$$K.E = \frac{1}{2} \frac{\omega}{g} \omega^2 \int_0^l y^2 dx$$

Equating both,

$$\frac{1}{2} \frac{\omega}{g} \omega^2 \int_0^l y^2 dx = \frac{1}{2} \int_0^l EI \left(\frac{dy}{dx} \right)^2 dx + \frac{K}{2} \int_0^l y^2 dx$$

$$\text{ie } \frac{\omega}{g} \omega^2 \int_0^l y^2 dx = \int_0^l EI \left(\frac{dy}{dx} \right)^2 dx + K \int_0^l y^2 dx$$

Solving for ω^2 ,

$$\omega^2 = \frac{\int_0^l EI \left(\frac{d^2 y}{dx^2}\right)^2 dx + \int_0^l Ky^2 dx}{\int_0^l \frac{\omega}{g} y^2 dx}$$

$$\begin{aligned} \int_0^l EI \left(\frac{d^2 y}{dx^2}\right)^2 dx &= EI \int_0^l \left[2K_1 K_2 e^{K_1 x} (-A \sin K_2 x + B \cos K_2 x) \right. \\ &\quad + (K_1^2 - K_2^2) e^{K_1 x} (A \cos K_2 x + B \sin K_2 x) \\ &\quad - 2K_1 K_2 e^{-K_1 x} (-C \sin K_2 x + D \cos K_2 x) \\ &\quad \left. + (K_1^2 - K_2^2) e^{-K_1 x} (C \cos K_2 x + D \sin K_2 x) \right]^2 dx \\ &= EI \int_0^l \left[4K_1^2 K_2^2 e^{2K_1 x} (-A \sin K_2 x + B \cos K_2 x)^2 + 4K_1^2 K_2^2 \right. \\ &\quad \left. e^{-2K_1 x} (-C \sin K_2 x + D \cos K_2 x)^2 - 8K_1^2 K_2^2 (-A \sin K_2 x + \right. \\ &\quad \left. B \cos K_2 x)(-C \sin K_2 x + D \cos K_2 x) \right] dx \end{aligned}$$

consider $\int_0^l e^{2K_1 x} \sin 2K_2 x dx = \frac{K_2^2 e^{2K_1 l}}{K_1^2 + K_2^2} \left[\frac{-\cos 2K_2 l}{2K_2} + \frac{K_1 \sin 2K_2 l}{2K_2^2} \right.$

and $\int_0^l e^{2K_1 x} \cos 2K_2 x dx = \frac{K_2^2 e^{2K_1 l}}{K_1^2 + K_2^2} \left[\frac{\sin 2K_2 l}{2K_2} + \frac{K_1 \cos 2K_2 l - e^{-2K_1 l}}{2K_2^2} \right]$

$$\begin{aligned} \int_0^l EI \left(\frac{d^2 y}{dx^2}\right)^2 dx &= EI 4K_1^2 K_2^2 A^2 \left[-\frac{1}{2} \frac{K_2^2 e^{2K_1 l}}{K_1^2 + K_2^2} \left\{ \frac{\sin 2K_2 l}{2K_2} + \frac{K_1}{2K_2^2} \right. \right. \\ &\quad \left. \left. (\cos 2K_2 l - e^{-2K_1 l}) \right\} + \frac{e^{2K_1 l} - 1}{4K_1} \right] + \end{aligned}$$

$$EI 4 K_1^2 K_2^2 B^2 \left[\frac{1}{2} \frac{K_2^2 e^{2K_1 l}}{K_1^2 + K_2^2} \left\{ \frac{\sin 2K_2 l}{2K_2} + \frac{K_1}{2K_2^2} \right. \right.$$

$$\left. \left. (\cos 2K_2 l - e^{-2K_1 l}) \right\} + \frac{e^{2K_1 l} - 1}{4K_1} \right] -$$

$$EI 4 K_1^2 K_2^2 AB \left[\frac{K_2^2 e^{2K_1 l}}{K_1^2 + K_2^2} \left\{ \frac{K_1}{2K_2^2} \sin 2K_2 l - \right. \right.$$

$$\left. \frac{1}{2K_2} (\cos 2K_2 l + e^{-2K_1 l}) \right\} \right] + EI 4 K_1^2 K_2^2 C^2$$

$$\left[-\frac{1}{2} \frac{K_2^2 e^{-2K_1 l}}{K_1^2 + K_2^2} \left\{ \frac{\sin 2K_2 l}{2K_2} - \frac{K_1}{2K_2^2} (\cos 2K_2 l - e^{2K_1 l}) \right\} \right.$$

$$\left. + \frac{e^{-2K_1 l} - 1}{4K_1} \right] + EI 4 K_1^2 K_2^2 D^2 \left[\frac{1}{2} \frac{K_2^2 e^{-2K_1 l}}{K_1^2 + K_2^2} \right.$$

$$\left. \left\{ \frac{\sin 2K_2 l}{2K_2} + \frac{K_1}{2K_2^2} (\cos 2K_2 l - e^{-2K_1 l}) \right\} + \frac{e^{-2K_1 l} - 1}{4K_1} \right]$$

$$-EI 4 K_1^2 K_2^2 CD \left[\frac{1}{2} \frac{K_2^2}{K_1^2 + K_2^2} \left\{ -\frac{K_1 \sin 2K_2 l}{2K_2^2} - \frac{1}{2K_2} \right. \right.$$

$$\left. (\cos 2K_2 l + e^{2K_1 l}) - 4 K_1^2 K_2^2 \left[A c \left(l - \frac{\sin 2K_2 l}{2K_2} \right) \right. \right.$$

$$\left. - (AD + BC) \left(\frac{1 - \cos 2K_2 l}{4K_2} \right) + BD \left(l + \frac{\sin 2K_2 l}{2K_2} \right) \right]$$

$$= 133.5$$

$$\frac{\omega}{g} \int_0^l y^2 dx = \omega \int_0^l \left[\frac{\omega}{K} + e^{k_1 x} (A \cos k_2 x + B \sin k_2 x) + e^{-k_1 x} (C \cos k_2 x + D \sin k_2 x) \right]^2 dx$$

$$= \omega \left[\left(\frac{\omega}{K} \right)^2 l + A^2 \left\{ \frac{k_2^2 e^{2k_1 l}}{2(k_1^2 + k_2^2)} \left(\frac{\sin 2k_2 l}{2k_2} + \frac{k_1}{2k_2^2} \frac{\cos 2k_2 l - e^{-2k_1 l}}{2k_2} \right) + \frac{e^{2k_1 l} - 1}{4k_1} \right\} + B^2 \left\{ -\frac{1}{2} \frac{k_2^2 e^{2k_1 l}}{k_1^2 + k_2^2} \left(\frac{\sin 2k_2 l}{2k_2} + \frac{k_1}{2k_2^2} \frac{\cos 2k_2 l - e^{-2k_1 l}}{2k_2} \right) + \frac{e^{2k_1 l} - 1}{4k_1} \right\} + AB \left\{ \frac{k_2^2 e^{2k_1 l}}{k_1^2 + k_2^2} \left(\frac{k_1}{2k_2^2} \sin 2k_2 l - \frac{\cos 2k_2 l + e^{-2k_1 l}}{2k_2} \right) + \frac{2\omega A}{K} \left\{ \frac{k_2^2}{k_1^2 + k_2^2} e^{k_1 l} \left(\frac{\sin k_2 l}{k_2} + \frac{k_1}{k_2^2} \frac{\cos k_2 l - e^{-k_1 l}}{2k_2} \right) \right\} + \frac{2\omega B}{K} \left\{ \frac{k_2^2}{k_1^2 + k_2^2} e^{k_1 l} \left(\frac{k_1}{k_2^2} \sin k_2 l - \frac{\cos k_2 l + e^{-k_1 l}}{k_2} \right) \right\} + \frac{2\omega}{K} \left\{ C \frac{\sin k_2 l}{k_2} - D \frac{\cos k_2 l - 1}{k_2} \right\} \right] = \frac{3.48}{32.2 \times 12}$$

$$K \int_0^l y^2 dx = K \cdot \frac{g}{\omega} \left[\frac{\omega}{g} \int_0^l y^2 dx \right] = \frac{500 \times 32.2 \times 12 \times 3.48}{4 \times 32.2 \times 12}$$

$$= 3.48 / 4 \times 500 = 435$$

$$\omega^2 = \frac{133.5 + 435}{3.48 / 32.2 \times 12}$$

$$= 63100$$

$$f = 63100^2 / 2 \times 3.14$$

$$= 40 \text{ cps.}$$

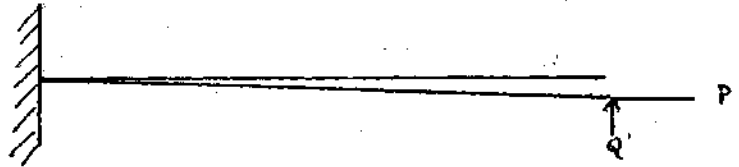
$\omega = 6.2 \text{ RAD/SEC}$
Natural frequency = 40 cps.

Hence for the system to be free from resonance, the shaft speed should be other than 40 rps.

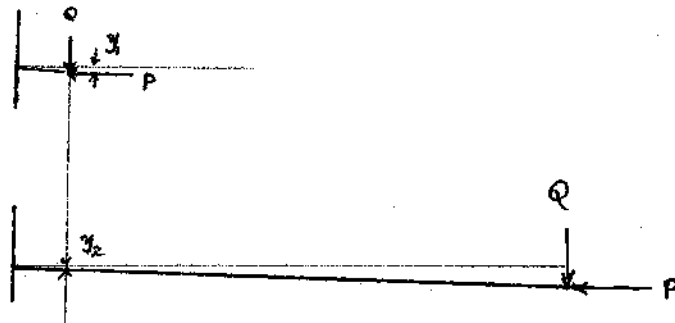
Discussion of the Mathematical Model

One end of the shaft is considered as rigidly fixed in the frame and the other is free to deflect. The shaft is resting on the inside of the liner which in turn is being elastically supported by the ground. There is lateral load uniformly distributed throughout the length due to any earth that might have collapsed on the lining during tunnelling. There is an axial loading on the shaft because of the resistance of the earth to the applied thrust. In view of all these factors, "an elastically supported beam-column with one end fixed and the other free" is considered as the mathematical model.

The axial load P and the lateral load Q are components of resistance of ground due to the applied thrust. The resistance is inclined if the earth is formed of inclined strata. The deflection due to the uniformly distributed lateral load is ~~increased when~~ the inclined ground resistance is directed downwards. As this is the worst case, it is considered in the mathematical model. If the resistance is inclined upwards, it tends to reduce the downward deflection. When the ground is formed of vertical strata, the resistance is purely axial, i.e. Q vanishes.



Due to the lateral deflection of the leading end, the ground resistance continuously changes direction. However, the vertical component of this resistance acts like a correcting force, tending to reduce the deflection.



In any situation if y_1 is less than y_2 , the model safely represents the final configuration.

CHAPTER VIII

STRENGTH ANALYSIS OF THE DESIGN

The following two analyses are performed to show that the system meets the strength requirements;

1. Stress Analysis
2. Fatigue Analysis

(a) Thrust Required

To force the cone into soil to embed the teeth, analyze as a bearing capacity problem. If q_0 is the bearing capacity of deep foundation, force required is given by

$q_0 \times \text{Projected Area}$

$q_0 = b/2 N + cN_c + qN_q$ where N , N_c , N_q are bearing capacity factors for weight, cohesion, and surcharge respectively, b is width of foundation, c is cohesion of soil, and q is surcharge. For an internal friction angle of 20 degrees the bearing capacity factors are as follows:¹³

$$N = 5; N_c = 17; N_q = 7; c = 1000 \text{ psf}; q = 100 \times 10^3; b = 3.6'$$

$$\begin{aligned} \text{Therefore } q_0 &= 100/2 (3.6/12) 5 + 1000(17) + 100(10) 7 \\ &= 24,000 \text{ psf} \end{aligned}$$

$$\begin{aligned} \text{Force } Q_0 &= 24,000 \times 3.14 \times 3.6^2 / (4 \times 144) \\ &= 1700 \text{ lbs} \end{aligned}$$

This is the force required to just force the cone through the soil, without rotating it. Since the cutter in the LSTD rotates this force must be adequate conservatively.

b) Stress Analysis

The design is an integral process in which many parameters are unknown to begin with and so have to be assumed. When calculations are carried out with these assumed parameters, they should be compatible with each other i.e. there should not be any incoherence between the assumed values. It is a tedious trial and error procedure.

First, depending the nature of loading (mild shock loading), a safe design factor (about 4 based on yield strength) is assumed. Based on the bore specifications fixed earlier, the outer diameter and inner diameter of the shaft are arbitrarily assumed. Knowing the thrust and torque during operation, then, the principal stresses and hence the design stress are found out. A suitable material is then selected to withstand the design stress. While determining the design stress, stress concentration factor and fatigue factor (and surface factor) are taken into account.

Weight of Shaft and Lining

1. Inner diameter of shaft

Outer diameter = 1.9 in. (trial and error procedure)

Moment of inertia of the weakest section (for shaft c/s alone i.e. without considering the casing) = 0.5 in^4 (trial and error)

$$\text{Therefore } I = 3.14 (1.9^4 - D_{in}^4) / 64$$

$$\text{i.e. } D_{in} = 1.2 \text{ in.}$$

2. Weight of the shaft

$$L \times A \times \gamma = 200 \times 12 \times 3.14 (2.1^2 - 1.2^2) / 4 \times 0.3$$

$$= 1000 \text{ lb.}$$

3. Weight of lining

$$= 200 \times 12 \times 3.14 (3.5^2 - 3^2) / 4 \times 0.27$$

$$= 1600 \text{ lb.}$$

Let the weight of the collapsed ground be assumed as two pounds per inch which acts on the lining. The total weight due to this earth

$$= 200 \times 12 \times 2$$

$$= 4800 \text{ lb.}$$

4. Total load

$$= 1000 + 1600 + 4800$$

$$= 7400 \text{ lb.}$$

5. Frictional force

$$= 7400 \times 0.5$$

$$= 3700 \text{ lb.}$$

The maximum thrust to overcome friction when the full length of the shaft is in the tunnel is 3700 lb.

Minimum thrust required for the earth to be sheared off by the drill bit is 218 lb.

Maximum thrust required for harder soils 1700 lb.

Therefore maximum possible thrust on the shaft in the worst case is

$$= 3700 + 1700$$

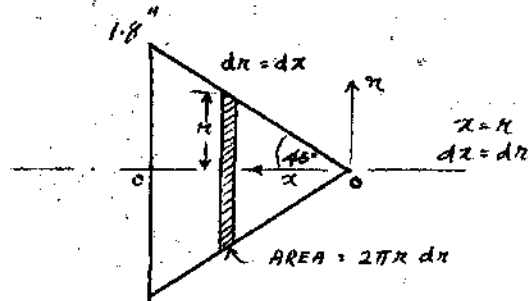
$$= 5400 \text{ lb.}$$

Torque Requirements

Let the applied shear stress = 2000 psf.

= 14 psi.

Torque to shear the earth $T_1 = \tau \int_0^{1.8} 2\pi x r^2 dr$



$$= 14 \times 2\pi (r^3/3) \Big|_0^{1.8}$$

$$= 14 \times 2\pi 1.8^3/3$$

$$= 171 \text{ lb.in.}$$

Frictional Torque:-

Weight of the shaft = 980 lb.

Frictional torque $T_2 = 980 \times 0.5 \times 1.5$

$$= 775 \text{ lb.in.}$$

Total Torque:- Total torque is the sum of the torque to shear the earth and the frictional torque.

$$T = T_1 + T_2$$

$$= 171 + 775$$

$$= 946 \text{ lb.in.}$$

Horse power of motor to supply this torque at 330 rpm

$$= 2\pi \times 330 \times 946 / 33,000 \times 12$$

$$= 5$$

With an overload factor of two the motor H.P is 10.

Stress calculations

$$EI \frac{d^2y}{dx^2} \quad x=0 = 0.7 \times 10^{+2}$$

$$\frac{1}{b} = M_{max} c / I$$

$$= 0.7 \times 10^{+2} \times 1.05 / 0.5$$

$$= \pm 144 \text{ psi.}$$

$$\text{Axial Thrust} = 3700 + 1700$$

$$= 5400 \text{ lb.}$$

$$\text{c/s Area} = \pi/4 (2.1^2 - 1.2^2 + 3.5^2 - 3^2)$$

$$= 3.46 - 1.13 + 9.6 - 7.06$$

$$= 4.86 \text{ in}^2$$

$$\text{Direct Stress} = - 5400/4.86$$

$$= - 1100 \text{ lb./in}^2$$

$$\text{Maximum Stress} = - 1100 - 144$$

$$= -1244 \text{ psi.}$$

$$\text{Minimum Stress} = -1100 + 144$$

$$= - 956 \text{ psi.}$$

Torsional

$$\text{Stress} = T.r/J$$

$$\text{Torque} = 1000 \text{ lb.in.}$$

$$\text{Polar M.I} = \pi/32 (1.9^4 - 1.2^4)$$

$$= 1.0 \text{ in}^4$$

$$\text{Outer Radius} = 1.9/2$$

$$= 0.95 \text{ in}$$

Max. Torsional

$$\text{Stress} = 1000 \times 0.95/1.0$$

$$= 1000 \text{ psi.}$$

$$\begin{aligned}\text{Max. Principal Stress}^7 &= -1244/2 - \sqrt{(1244/2)^2 + 1000^2} \\ &= -622 - 1180 \\ &= -1802 \text{ psi.}\end{aligned}$$

Stress Concentration

$$\begin{aligned}\text{Factor}^6 K_f &= 1 + q(K_t - 1) \quad \text{where } q=1, K_t=K_f \\ &= 1.38 \text{ from p-p 117} \\ &\quad \text{to 119 Faires.}\end{aligned}$$

For minor shock loading the design factor should be four based on yield strength.

$$\text{Size Factor}^6 = 0.85$$

$$\text{Surface Factor}^6 = 0.90 \text{ (for machined surface)}$$

From Table 2, p 34, Design of Machine Elements, by Faires, C 1020 (annealed wrought iron) gives a tensile ultimate strength of 57,000 psi.

$$\begin{aligned}\text{Therefore Compressive Yield Strength} \\ &= 0.8 \times 57,000 \\ &= 33,600 \text{ psi.}\end{aligned}$$

$$\begin{aligned}\text{From the same table Endurance Limit} \\ &= 57,000/2 \\ &= 28,500 \text{ psi.}\end{aligned}$$

Design Factor based on Endurance Strength

$$\begin{aligned}F &= 0.9 \times 28,500 \times 0.85 / 1.38 \times 1800 \\ &= 4.0\end{aligned}$$

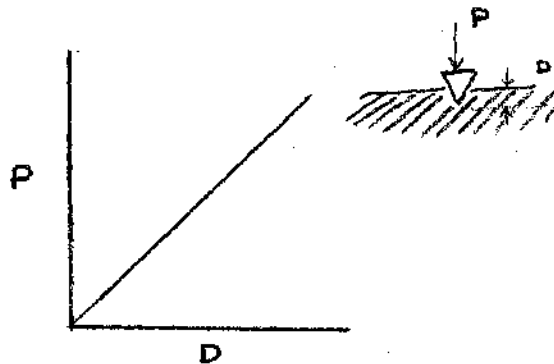
$$\begin{aligned}\text{Angular Twist:- } T x l / N x J &= 946 \times 2400 / 11.5 \times 10^6 \times 1 \\ &= 0.1974 \text{ rad.} \\ &= 11.3 \text{ deg.}\end{aligned}$$

CHAPTER IX

DETAILED DESIGN OF THE DRILL BIT

i. Bit Penetration

Consider the penetration of the flat surface of a homogeneous, elastic-plastic material by a sharp two-dimensional chisel bit. The pattern of stress and strain generated beneath the bit depends only on the properties of the material, angle of the chisel, being similar for all depths of operation. It follows from this similitude that the force of penetration P , is proportional to the area of contact between the bit and the material and hence is linearly related to the depth of penetration D , as shown in the figure below.



Let the tooth size = 0.2 in.

In one rotation earth sheared = $3.14xD^2/4 \times 0.2 \text{ in}^3$, where $3.14xD^2/4$ is the projected area of the drill bit.

When the teeth are uniform all around, the bit removes a uniform layer of 0.2 inch thickness all around. Hence the minimum extent of progress of the shaft so as to maintain contact with earth is 0.2 inch per revolution.

Bending moment on the drill bit may be neglected since it is small in length and located near the end of the shaft ($x=1, M=0$), and it is elastically supported.

Tip :-

Let the area at the nose be one square inch.

Maximum thrust that might be applied on the bit

$$= 1700 \text{ lb.}$$

Direct compressive stress = $1700/1.0$

$$= 1700 \text{ psi.}$$

Maximum torque applied to the bit

$$= 171 \text{ lb.in.}$$

Polar moment of inertia = $\pi \times 1.13^4 / 32$

$$= 0.16 \text{ in}^4$$

Maximum twisting stress = $T.r/J$

$$= 171 \times 1.13 / 0.16 \times 2$$

$$= 604 \text{ psi.}$$

Maximum principal stress =

$$= -850 - 1042$$

$$= -1892 \text{ psi.}$$

For C 1020 (annealed wrought iron) compressive yield

strength = 33,600 psi.

Design factor = $33,600 \times 0.9 \times 0.9 / 1892$
 = 14.3

End of the Bit

Outer diameter = 3.6 in.

Thickness = 0.6 in.

Inner diameter = 2.4 in.

C/s area = 5.58 in^2 .

Let the effective area (since there are holes) be

= 0.5×5.58

= 2.79 in^2 .

Polar M.I. = 13.22 in^4 .

Polar M.I. of holes = 11.75 in^4 .

Effective polar M.I. = 1.47 in^4 .

Twisting stress = $171 \times 3.6 / 1.47 \times 2$

= 209 psi.

Direct compressive stress

= $1700 / 2.79$

= 610 psi.

Maximum principal stress

= $-\frac{610}{2} - \sqrt{\frac{610^2}{4} + 209^2}$

= -675 psi.

Design factor

= $33,600 \times 0.9 \times 0.9 / 675$

= 40.4

ii. Cutter Profile

While evaluating the cutting torque as 171 lb.in.,

shape of drill bit is assumed to be a straight cone. However, with this shape the torque on the surface of the drill bit is not uniform- it increases with the radius of the cutter. Hence, the wear on the cutter at the bigger end is more than at the leading end. In order to have a uniform torque, and thus a uniform wear the profile may be determined as follows.

Torque on any elemental section = $r.2\pi r.dl.\tau$

Here r and l should vary such that the torque is constant.

$$2\pi r^2 \tau .dl = 171$$

$$r^2 .dl = 171/2\pi.14$$

$$= 2$$

$$dl = 2/r^2$$

$$l = -2/r + k$$

To evaluate k , substitute $r=0.5$, $l=0$

Therefore $k = 2/0.5$

$$= 4$$

$$l = -2/r + 4$$

$$r = 2/(4-l)$$

The cutter profile is determined by the above expression.

iii. Determination of the speed of tunnelling

The speed of rotation and rate of axial motion are inter-related. Rotating at a given speed, the drill removes only a certain quantity of earth and the axial movement of the drill should cope with this. If it is moved at

a greater rate than this the drill may stall due to increased resistance. If it is rotated at a lesser rate there will not be any contact between the bit and earth and no drilling will be performed.

If the drill bit has teeth of 0.2 in. all around, in one revolution its axial movement can be figured out to be 0.2 in. per revolution.

Hence, for N rpm the rate of progress = $0.2N$ in./min.

Rate of drilling at 210 rpm	= 42 ipm.
	= 3.5 fpm.
at 330 rpm	= 5.5 fpm.
at 570 rpm	= 9.5 fpm.

a) Flow Analysis of Air and Earth

Rate of earth removal at 210 rpm.

$$= 3.5 \times 12 \times \pi \times 3.6^2 / 4$$

$$= 426 \text{ in}^3.$$

This is in loosened form, so let it occupy $3 \times 426 \text{ in}^3$.

$$= 0.2464 \text{ ft}^3 / \text{min.}$$

$$= 0.2618 \text{ lb. / sec.}$$

Minimum average velocity required.

$$= 3 \times 426 / \pi \times 1.5^2 / 4$$

$$= 1.005 \text{ ft. / sec.}$$

Therefore let the average velocity be

$$= 5 \text{ fpm}$$

Initial velocity

$$= 10 \text{ fpm}$$

Final velocity $= 0$

Let the weight rate of air be W lb./sec. at a velocity V .

Kinetic Energy of the air $= WV^2/2g$

Kinetic Energy of the mixture of air and earth
 $= (0.2618+W)10^2/2g$

Equating both, $\frac{1}{2} \frac{WV^2}{g} = (0.2618+W)100/2g$

$$W(V^2-100) = 13.09 \times 2 = 26.18$$

$$L \times V = 11 \text{ ft/sec}$$

$$W \times 21 = 26.18$$

Flow rate of air $= 26.18/21$
 $= 1.246 \text{ lb./sec.}$
 $= 997.2 \text{ ft}^3/\text{min.}$

Pneumatic Action When the Shaft is Clogged

In the worst case let the earth fill all the 200 ft. length of the tube.

Weight of earth $= 200 \times 12 \times \pi d^2 \times 80 / 4 \times 3$

Frictional force between this earth and tube wall
 $= 0.5 \times 200 \times 12 \times \pi d^2 \times 80 / 4 \times 3$

To remove this the air pressure required is given by
 $= 0.5 \times 200 \times 12 \times \pi d^2 \times 80 / 4 \times 3 \times 12^3$
 $= 18.5 \text{ psi.}$

Frictional Head Loss in Annular Air Passage

To find the Hydraulic Diameter:-

C/s area of flow for the air-in $= 1.97 \text{ in}^2$.

Wetted perimeter $= 11.85 \text{ in.}$

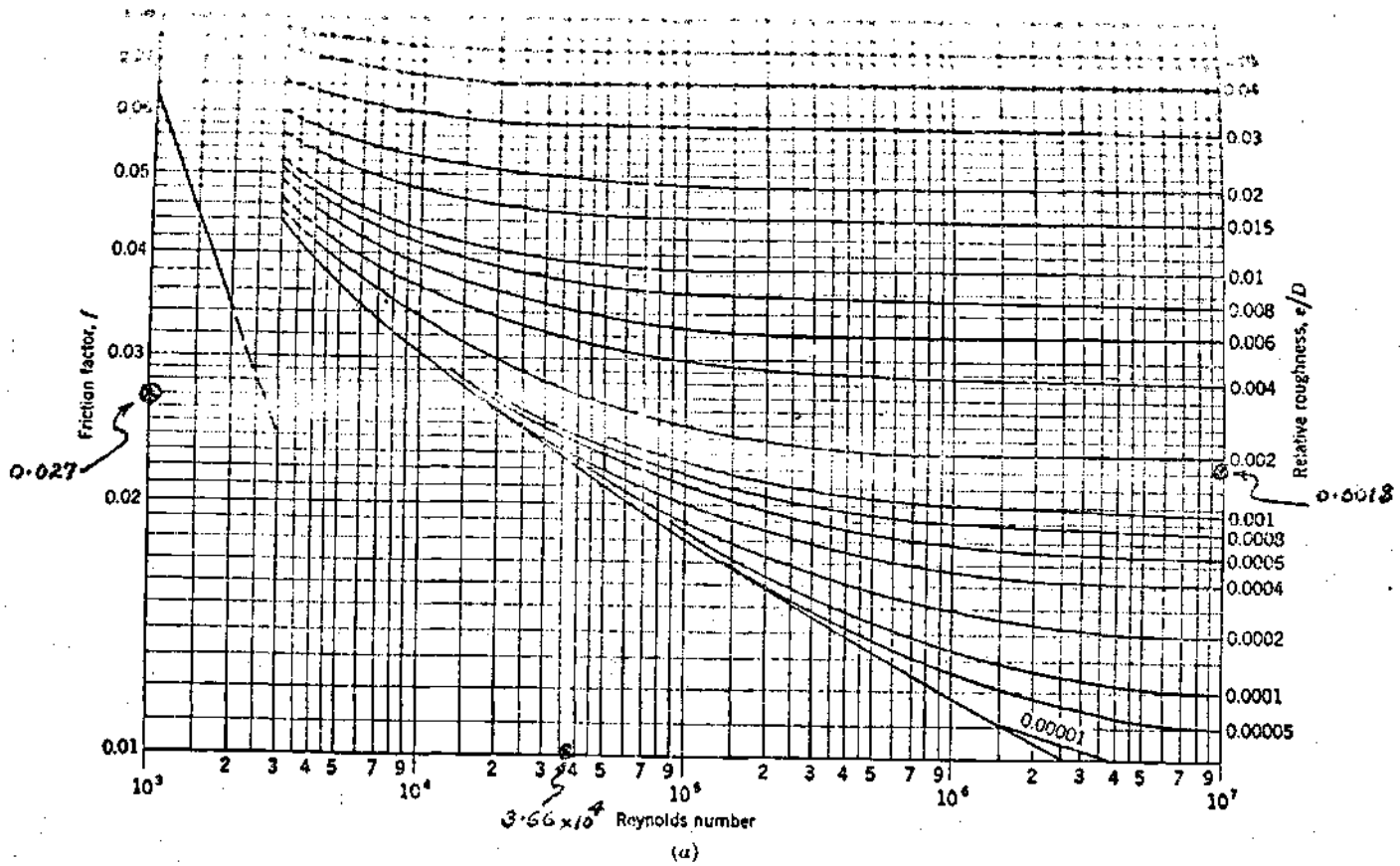


Fig. 7. Friction factors for flow in pipes.

(After 'Shames')

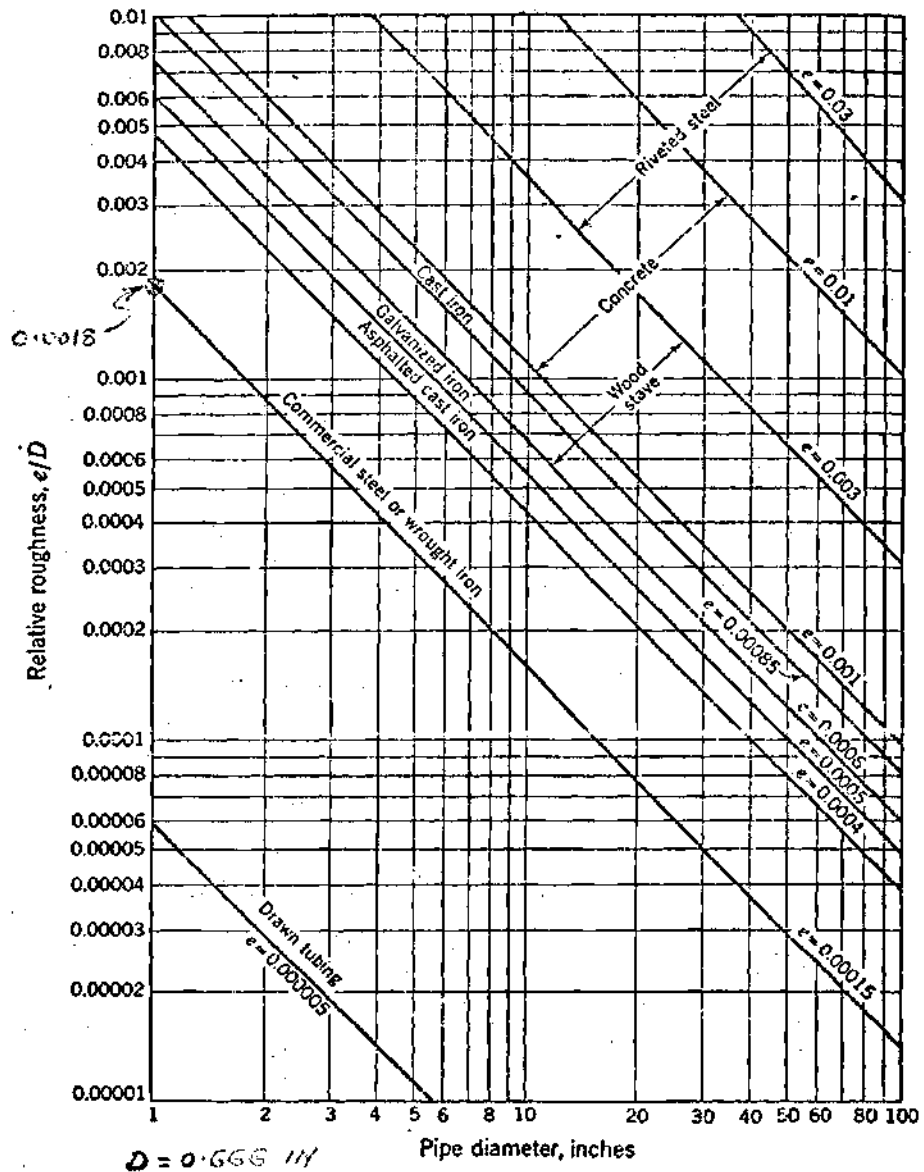


Figure 8 - Pipe Roughness Factors (After 'Shames')

$$\begin{aligned}
 \text{Hydraulic diameter } H &= 4 \times 1.97 / 11.85 \\
 &= 0.666 \text{ in.} \\
 \text{Velocity} &= 11 \text{ fps.} \\
 \text{At } 100^\circ \text{F kinematic viscosity} &= 2 \times 10^{-4} \text{ ft}^2/\text{sec} \\
 \text{Reynold's No.} &= 11 \times 0.666 / 2 \times 10^{-4} \\
 &= 3.66 \times 10^4
 \end{aligned}$$

From Moody diagram (Fig. 10.22 (b) p300 Shames) for wrought iron pipes of 1 inch diameter the relative roughness is

$$e/D = 0.0018$$

From Moody diagram (Fig. 10.22 (a) p299 Shames) friction factor, f

$$= 0.027$$

Frictional Head

$$\begin{aligned}
 &= 0.027 \times 200 \times 12 \times 11^2 / 0.666 \times 2 \\
 &= 5860 \text{ ft.lb./slug}
 \end{aligned}$$

$$\frac{P_b \times 144}{0.075} = \frac{5860}{32.2}$$

$$= 5860 \times 0.075 / 144 \times 32.2$$

$$= 0.0949 \text{ psi.}$$

$$P_s - P_f = 18.5 \text{ psi.}$$

$$P_s = 18.4 \text{ psi.}$$

When the pneumatic action is not required the blower can supply the air at a slightly above atmospheric pressure, say one psig. Then, the blower H.P. is

$$\text{H.P.} = 1.246 \times 144 \times 2 / 0.8 \times 550 \times 0.075$$

$$= 11$$

With overload capacity H.P. = 15

When pneumatic action is required the compressor H.P. is 40.

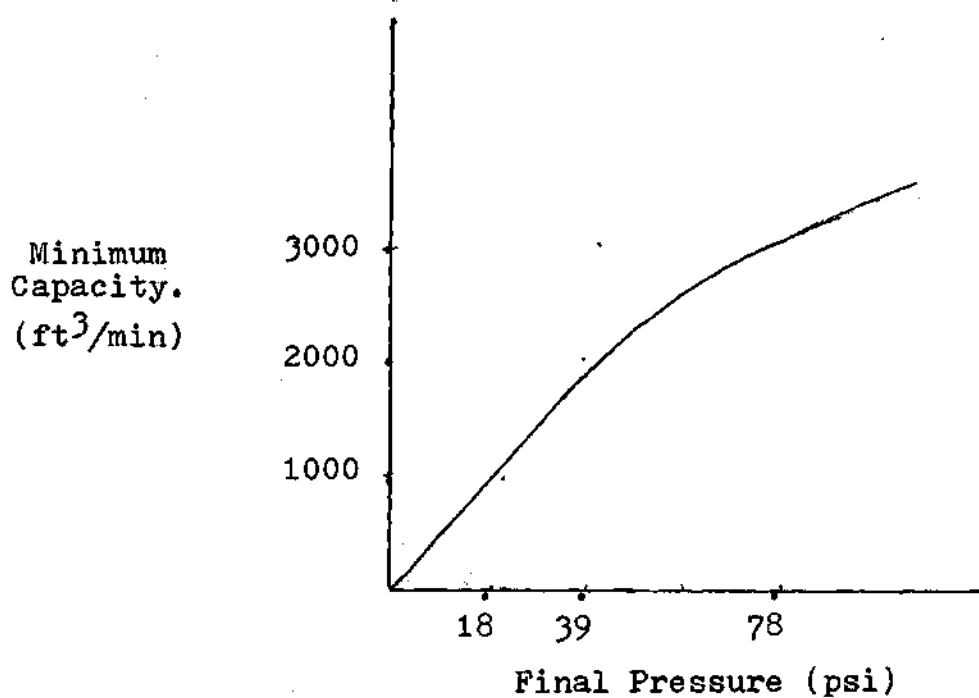


Figure 9. Turbo Compressor Minimum Capacity
at Supply Pressure.*

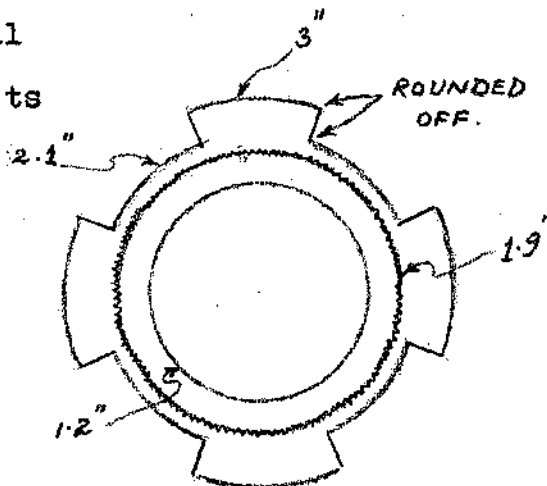
From the graph above, the minimum capacity of the compressor at 24 psi. is about 1400 cubic feet per minute. But the required flow is only 1000 cubic feet per minute, at 22.4 psi and 11 feet per second velocity (thus making a total head of about 24 psi). This means that a turbo blower if used, supplies more than the required flow, thus increasing the cost of operation. Hence, a positive displacement type compressor has to be used.

* Turbo Blowers and Compressors- p.7, By W.J.Kearton.

b) Design of the Thread Joint

The c/s taking the total thrust is as shown beside. Its area is summed as follows.

$$\begin{aligned} \text{Area} &= (2.89 - 1.13) + 1.96 \\ &\quad (3.46 - 2.83) \\ &= 1.76 + 1.96 + 1.63 \\ &= 5.35 \text{ in}^2. \end{aligned}$$



C/s area at the joint	= 5.35 in ²
Design stress	= 5400/5.35
	= 1010 psi.
Compressive yield strength of the material	= 33,600 psi.
Design factor	= 33,600/1010
	= 33.3

Length of the Threaded Portion

Shear area at thread section	= $\pi \times 1.9 \times 1 \text{ in}^2$
Torque	= 946 lb.in.
Therefore, shear force	= 946/1.9/2
	= 1000
Design shear stress	= 1000/ $\pi \times 1.9 \times 1$
Shear yield strength	= 42,000 x 0.5
	= 21,000 psi.

For a design factor of four,

$$21,000/4 = 1000/ x1.9x1$$

$$\text{Length of the joint, } l = 0.0325 \text{ in.}$$

Therefore, let the length be taken as
= 1 in.

Bearing Strength of the Flange

$$\text{Bearing force} = S_c (\text{area of flange})$$

$$= S_c \times 1.06$$

$$= 1.6 \times 11.2 \times 1000 \times 1.06$$

$$= 19,000 \text{ lb.}$$

Shearing Strength of the Flange

Let the width of the flange = w in.

$$\text{Shear area} = w \times x1.9/8$$

$$= 0.745 w$$

$$\text{Shear force} = S_s (0.745 w)$$

$$\text{Therefore } \frac{5400}{4} = S_s (0.745 w)$$

$$w = 0.0863 \text{ in.}$$

Therefore, let the width be taken as two inches.

CHAPTER X

CONCLUSION

Description of the Set-up

The various components can be arranged as shown in Fig. 10. The hydraulic unit and compressor (or air blower) unit are established outside the manhole. The motor, gear box, clutch, hydraulic ram, and other gears are conveniently fixed in a framed structure which can be introduced into the manhole as a single unit. There is a telescope attached to the top of the frame which is useful for initial alignment.

The clutch -flywheel unit drives a splined shaft (or a long gear) which in turn drives the thrust plate. The thrust plate is a thick metallic disc with gears around its periphery. The hydraulic ram pushes the thrust plate fixed to the hollow shaft and resting against the liner. Thus the thrust plate not only drives the shaft but also pushes it axially, as it slides along the splined shaft while being driven by it.

The in-let air hose is connected to the hole in the liner right behind the thrust plate as shown in the figure. The out-let air along with debris is drawn out through a hose connected to the hole in the middle of the thrust plate.

Conclusion

Conclusions

The overall impression is that the device satisfactorily performs in homogeneous soft soils. The accuracy of tunnelling depends on the rigidity of the shaft which is made up of two feet sections. By minimizing the mechanical imperfections while machining the joints, the rigidity and hence the accuracy can be maintained.

Water might be substituted for air for the purpose of removing the debris from the tunnel. However, handling water might be more troublesome.

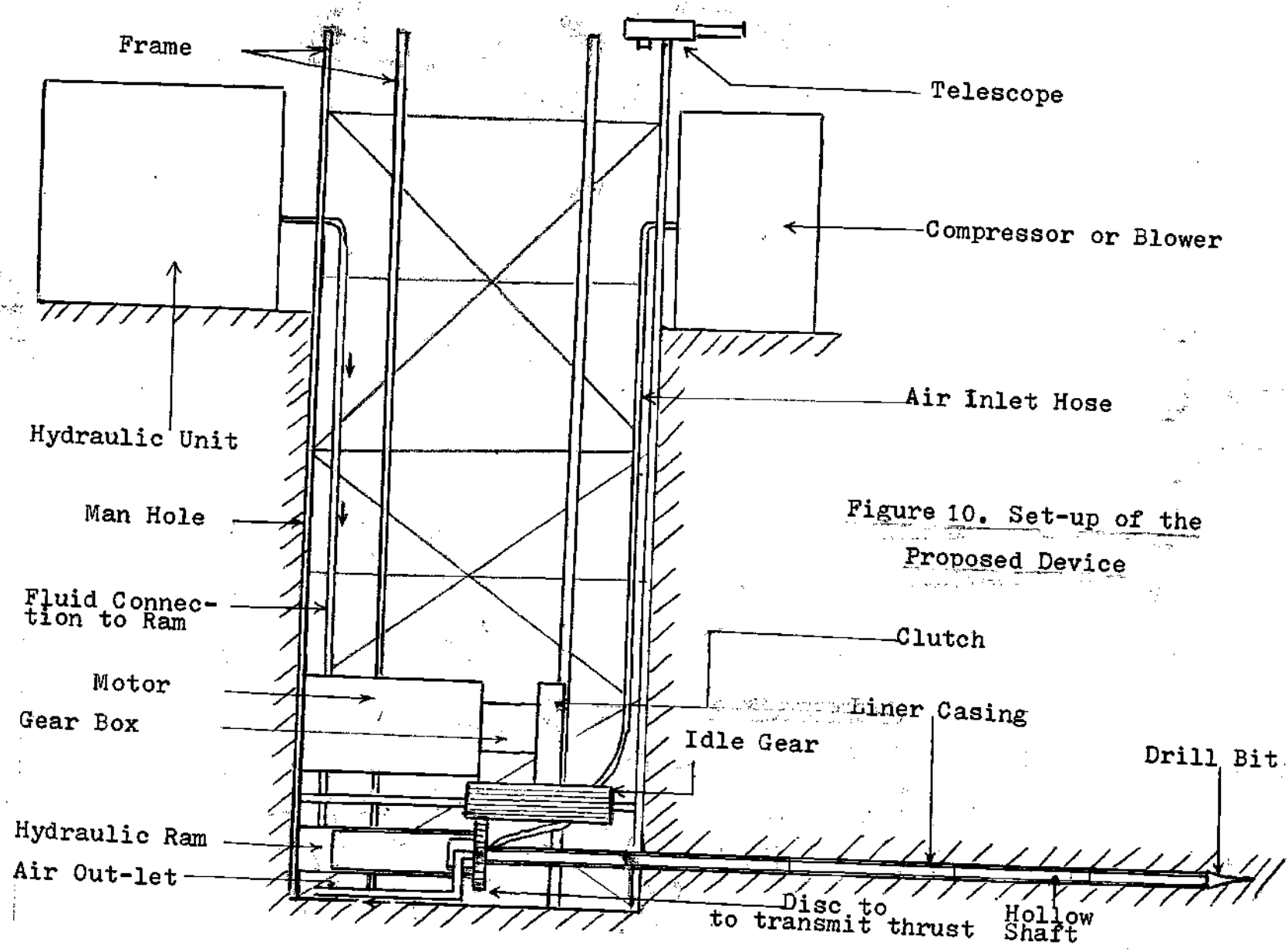


Figure 10. Set-up of the Proposed Device

APPENDIX

PATENT SEARCH

From Index to Classification:-

Class 61,	
Sub-class 84	"Tunnels-Devices"
Class 175	"Boring or Penetrating Earth"
Sub-class 19,	"Boring without earth removal"
20,	"Combined with earth removal"
21,	"Fluid passage to outside of drive point"
22,	"Drive point detached from shaft to form cased boring or with installation of casing"
23,	"Drive point retracted through shaft or casing"

Each class has patent numbers listed in the Gazette. Listing the individual patent numbers is avoided due to space limitation.

BIBLIOGRAPHY

1. Donald H. Yardley, Editor, "Rapid Excavation Problems and Progress", proceedings of the Tunnel and Shaft conference, Minneapolis, Minnesota, May 15-17, 1968.
2. "RECORD", March, 1967, pp 70-73
3. "RECORD", January, 1971, pp 13-21
4. S. Timoshenko, Strength of Materials, Part II
5. S. Timoshenko, Applied Elasticity.
6. V.M.Faires, Design of Machine Elements, third edition, 1957
7. S.B.Junarkar, Mechanics of Structures, third edition, 1961.
8. W.J.Kearton, Turbo Blowers and Compressors.
9. S.R.Beitler and E.J.Lindahl, Hydraulic Machinery
10. Irving H. Shames, Mechanics of Fluids, 1962.
11. M.E 485 Class Notes, Mechanics of Machines, Winter 1972
12. Leonard Meirovitch, Analytical Methods in Vibrations 1967, Macmillan Co, pp 440-41.
13. Sowers and Sowers, Introductory Soil Mechanics and Foundations, third edition, Macmillan Co., 1970.