DESIGNING ALLOCATION MECHANISMS FOR CARRIER ALLIANCES

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Lori M. Houghtalen

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Approved by:

Dr. Özlem Ergun, Advisor
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Joel Sokol, Advisor
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Alan Erera
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Ellis Johnson
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Dr. Beril Toktay
College of Management
Georgia Institute of Technology

Date Approved: 5 July 2007

To Brandon, and to Mom and Dad.

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SUMMARY

The goal of the first part of this thesis is to obtain a high-level theoretical understanding of how some alliances (for example, air cargo alliances) can be managed such that their resources are used in an optimal manner. We propose a pricing mechanism to manage the interactions of carriers, through the allocation of alliance resources and profits, in a manner that encourages individual carriers to make decisions that are optimal for the alliance as a whole. We assume that carriers act to optimize profit, and model this profit-maximizing behavior using multi-commodity flow linear programs. These models are incorporated into a mechanism that manages carrier interactions by setting resource prices such that an appropriate allocation of both alliance resources and profits is attained. Because the behavioral models are used to determine the impact of resource prices on carrier behavior, the allocations of resources and profits achieved by the mechanism are heavily dependent on the underlying model employed. Thus it is important to consider the impact of the model selected on the overall performance of the mechanism. After introducing two distinct behavioral models, the performance of the mechanism using each model is analyzed for its ability to ensure alliance optimal behavior is attained. We find that the behavioral model selected can significantly impact the characteristics of allocations obtained using the mechanism.

In the second part of the thesis, we seek to establish practical insights regarding how the characteristics of potential partners impact the benefit that can be gained by collaborating with these partners. Computational experiments are conducted to evaluate the impact of network size, fleet capacity, demand distribution, and network compatibility on the benefit associated with collaborating. A comprehensive study for simulated two and three-carrier alliances establishes general insights regarding the compatibility of carriers with varying network sizes and fleet capacities. The impact of increasing hub-to-hub connectivity between partnering carriers is then investigated, followed by a study of the effect of market overlap on

alliance success. Finally, a real-world cargo alliance is analyzed, demonstrating the validity of the observations obtained from studying the simulated alliances.

In the third and final part of this thesis, we develop new approaches for determining and inducing fair profit allocations in alliances, providing alternatives to traditional approaches which equate minimum acceptance requirements and satisfaction. The mechanism established in the first part of the thesis is adapted to more precisely control the profit allocations obtained, in particular so that an allocation as close to some predetermined "fair" allocation is obtained. Several measures of fairness are proposed and implemented, and their performance analyzed for each of the behavioral models discussed in the first part of the thesis. The results lead to further practical insights regarding the compatibility of various types of carriers, as well as confirm the importance of pursuing the notion of fairness in allocation.

CHAPTER I

INTRODUCTION

Consider a group of independent cargo carriers (for example, in the air cargo, sea cargo, or trucking industries) who each wish to improve their own profitability. They may choose to integrate some portion of their transportation networks in order to make better use of their capacity by delivering more-valuable cargo loads. A group of carriers working together in such a manner is referred to as an *alliance*. There are a variety of circumstances in which the formation of an alliance among cargo carriers might be preferable to a merger or acquisition; for example, carriers operating in different countries (or even in the same country, depending on the industry) might face significant legal barriers to both merging and acquisition. Second, carriers who operate under significantly different business models might prefer autonomy to merging. Regardless of the motivation, it is reasonable to assume that carriers considering forming an alliance are interested in designing that alliance to function as well as possible, from the standpoint of both profitability and sustainability over time. The challenge in achieving these goals lies in the tradeoff between decisions that are good for the alliance versus decisions that are good for an individual carrier: decisions that are good for the alliance are not always good for an individual carrier within the alliance, and vice versa. In order for an alliance to operate in a manner that achieves maximum profit, this discrepancy must be resolved.

To illustrate the questions that must be addressed when a potential alliance among cargo carriers is considered, examine the simple case demonstrated in Figure 1. In this simplified air cargo system, a time-expanded network with two cities and four time periods is depicted; two flights operate between city 1 and city 2, both of which are operated by carrier A and have one unit of capacity. There are four loads, each of unit size, that need to be accepted or rejected for transport, and we assume that carriers individually make decisions to accept or reject their associated loads. All loads originate in city 1 and are shown at their earliest available departure time. The destination of every load is city 2. The carrier and revenue associated with a load, as well as the delivery deadline for that load, are as shown in the figure. Finally, the *ground edges* are fictitious edges that represent the ability of a load to wait in a location over time.

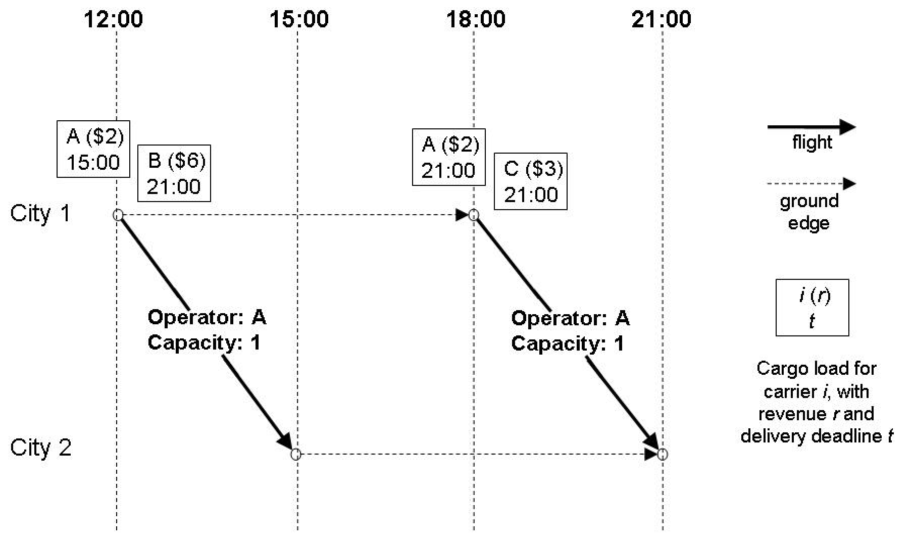

Figure 1: Air Cargo Example with 3 Carriers and 4 Loads

Let us examine the case where each carrier operates independently. Carrier A owns the capacity on the flights, and therefore can deliver both of his loads, for a total revenue of $4. Carriers B and C have no capacity, and therefore must reject their loads and earn no revenue. If carriers A and B choose to collaborate by sharing capacity, then two loads worth a total of $8 can be accepted. Similarly, if carriers A and C collaborate, then two loads worth a total of $5 can be accepted. If all three carriers collaborate, then the two highest value loads can be accepted, for a total revenue of $9.

Throughout this thesis, we are assuming that earning the maximum amount of revenue is the primary goal of the alliance. Clearly there is benefit to be gained by collaborating in the above example; there is $5 in extra revenue that can only be captured by the alliance. Yet, if carriers receive revenue only by delivering their loads, carrier A has no incentive to participate in this collaboration because he can earn more revenue operating alone. One possibility is to arrange for a payment as compensation for the loss of revenue carrier A experiences by joining the collaboration. This could easily be accomplished if we assume a centralized decision-maker exists and can distribute payments. But given that a collaboration is composed of autonomous carriers who make independent operational decisions, influencing the interaction among the carriers in reality is more challenging.

1.1 Contributions and Organization of Thesis

The goals of this thesis are threefold. First, we seek to obtain a high-level theoretical understanding of how an alliance can be managed such that its resources are used in an optimal manner. Second, we seek to establish practical insights regarding how the characteristics of potential partners, in particular network structure and demand distribution, impact the benefit that can be gained by collaborating with these partners. Third, we seek to develop new approaches for determining and inducing fair profit allocations in alliances, providing alternatives to traditional approaches which equate minimum acceptance requirements and satisfaction.

In the remainder of this chapter we introduce air cargo alliances as well as present an overview of literature related to air cargo alliances. Chapter 2 addresses the first major goal of the thesis: we propose a mechanism to manage the interactions of carriers, through the allocation of alliance resources and profits, in a manner that encourages individual carriers to make decisions that are optimal for the alliance as a whole. We assume that carriers act to optimize profit, and model this profit-maximizing behavior using multi-commodity flow linear programs. These models are incorporated into a mechanism that manages carrier interactions by setting resource prices such that an appropriate allocation of both alliance resources and profits is attained. Since the behavioral models are used to determine the impact of resource prices on carrier behavior, the allocations of resources and profits achieved by the mechanism are heavily dependent on the underlying model employed. Thus it is important to consider the impact of the model selected on the overall performance of the mechanism. After introducing two distinct behavioral models, the performance of the mechanism using each model is analyzed for its ability to ensure alliance optimal behavior is attained. A significant portion of the analysis is conducted using concepts from cooperative game theory; a brief discussion of related game theory literature is included in the chapter.

We find that the behavioral model selected can significantly impact the characteristics of allocations obtained using the mechanism.

In pursuit of the second goal of the thesis, Chapter 3 contains results and analysis of computational experiments conducted to evaluate the benefit to be gained by collaborating for alliances comprised of various types of carriers. The chapter begins with a description of how the alliances used in the experiments are generated, and results for two and three-carrier alliances are then presented and analyzed. The chapter concludes with the analysis of a real-world alliance, conducted by simulating an alliance based on the WOW cargo alliance comprised of Lufthansa, SAS, Singapore Airlines, and Japan Airlines.

The experiments conducted in this chapter utilize the mechanism developed in Chapter 2. The results confirm that the mechanism performs as expected, but also demonstrate that the mechanism may allocate alliance benefit among alliance members in an arbitrarily disproportionate manner. Consequently, Chapter 4 focuses on adapting the mechanism to more precisely control the profit allocations obtained. After discussing literature related to fairness in allocation, several measures of fairness are proposed. The measures are then implemented using the adapted allocation mechanism, and their performance analyzed for each of the behavioral models discussed in Chapter 2. The results lead to further practical insights regarding the compatibility of various types of carriers, as well as confirm the importance of pursuing the notion of fairness in allocation. Conclusions are presented in Chapter 5, as well as a description of some additional high level research questions and more technical extensions that are motivated by this work.

1.2 Air Cargo Alliances

As is implied by the previous example, our motivating application is the air cargo industry. Air cargo is assumed to be any freight, excluding mail and passenger baggage, transported using aircraft. More specifically, we focus on combination carriers, which are those carriers transporting cargo using passenger aircraft. As carriers take steps to improve the profitability of their cargo business, they are increasingly considering collaborations for cargo that are independent of those already established for the passenger industry. The first cargo alliance, SkyTeam Cargo, formed in 2000 and was comprised of the cargo components of Aeromexico, Air France, Delta, and Korean Air [27]. These four airlines were already part of the SkyTeam passenger alliance, but SkyTeam Cargo was formed as an independent strategic cargo alliance. Similarly, the WOW Alliance formed in 2002 with the cargo businesses of Lufthansa, Scandinavian Airlines, and Singapore Airlines. Again, these carriers were already partners in the passenger industry, under the Star Alliance. However, a carrier outside the Star Alliance, Japan Airlines, was added later in 2002 [32]. Cargo alliances among carriers that are not already partners in the passenger business are likely to become more common, since carriers compatible for passenger alliances may not be compatible for a cargo alliance. This is due to differences in flow patterns: passengers typically complete a round trip, resulting in balanced flow, while cargo flow follows unbalanced trade patterns [36].

We assume that service network design is determined according to other business considerations (for example, in the airline industry, combination carriers set their schedules and fleet assignments based on passenger demand); the alliance network is comprised of the service networks operated by each participating member, or possibly some portion of each member's network. The key decisions for a cargo alliance therefore include, similar to the passenger setting, how to share space and revenue among members. An additional consideration in the cargo setting, however, is that of route selection. In contrast to passengers, cargo is relatively insensitive to routing decisions; therefore the decision of how to route cargo through the alliance network becomes a relevant factor in considering collaborations among air cargo carriers. Determining the overall most profitable set of cargo to deliver, and how this cargo should be routed through the combined network, requires a centralized perspective. Full centralization is generally not an option, however, given the technical and legal challenges associated with integrating the information systems of autonomous carriers. Thus the maximum benefit will only be attained if the participating carriers can be encouraged to make their own acceptance and routing decisions in accordance with the centralized optimal decision.

1.3 Related Literature

This work unites concepts from optimization, cooperative game theory, and mechanism design, all of which have substantial dedicated bodies of literature. Whereas cooperative game theory studies properties of cost or benefit allocations among players, mechanism design is focused on how to design a system such that a given allocation, typically one that maximizes some system benefit, is achieved even when individual players are acting to maximize their own gain. An introduction to the concepts of optimization and mechanism design can be found in [7] and [18], respectively; cooperative game theory will be discussed in Section 2.5.2. In this section we will focus on literature related to carrier collaboration.

There is very little available in the literature relating to air cargo alliances, most likely since alliances among air cargo carriers are a very recent development. Most literature concerning air cargo is related to dedicated cargo carriers, cargo operations, or the relationship between the cargo and passenger industries. For example, the network design problem for dedicated cargo carriers is addressed by [14] and [17], and short-term capacity planning is studied in [10]. Analysis of airline alliances in the passenger industry is more prevalent, but no existing literature uses a similar methodology or addresses the same questions as in this work. [26] investigates the impact of international alliances on the passenger market by comparing alliances comprised of airlines with complementary and parallel networks; it is predicted that an alliance that joins complementary networks will be more profitable. In response to a concern that alliances would lead to a situation where major carriers would have a monopoly, [21] finds instead that alliances have merely allowed carriers to preserve, not increase, their narrow profit margins through an increase in load factors and productivity. [13] in fact finds that consumers benefit from the formation of passenger alliances; in the two domestic alliances that were studied fares decreased on the markets impacted by the alliance, in part due to increased competition from rivals competing with the alliance. [1] analyzes potential international alliances among carriers, applying non-cooperative game theory to determine the profitability of an alliance under a given level of competition. The primary issue addressed is the selection of international hubs to maximize the profit of merging airlines.

There is also limited research available on the impact that an alliance in one industry (air cargo or passenger) can have on the other. [20] studied a passenger alliance between KLM and Northwest and found that, ultimately, the effects on cargo service were positive. From the other perspective, [35] investigates the effect of an air cargo alliance on the passenger market, finding that cargo service integration can increase outputs in both the cargo and passenger markets.

A widely studied topic in the passenger airline industry that is only recently being applied in the alliance setting is that of revenue management. Literature in this field seeks to maximize revenue through management of seat capacity. [19] provides a review of revenue management literature, but is focused primarily on revenue management implemented by a single carrier. [8] describes the technical challenges associated with alliance revenue management; in addition to addressing challenges, [30] discusses how coordination of seat pricing and capacity planning are currently executed in the alliance setting. [33] provides a more formal analysis of alliance revenue management mechanisms in which a free sale scheme and three types of dynamic trading schemes are discussed. The mechanisms are analyzed to determine their effect on the equilibrium behavior of alliance members and the potential for the mechanism to maximize alliance revenue. Revenue management applied to the air cargo industry is even more limited; differences between the cargo revenue management problem and the passenger yield management problem are discussed in [16], as are complexities in developing additional models to facilitate cargo revenue management.

Outside the airline industry, carrier collaboration has also been studied in the ocean liner shipping industry. [3] addresses issues related to the formation of alliances in the sea cargo industry; in addition to the distribution of alliance revenue, design of the alliance network is of critical importance. [28] demonstrates that alliances among sea cargo carriers lead to increased service frequency and ship size, as well as increased similarity of service routes among carriers. [29] provides a conceptual framework for the application of game theory to alliances in the liner shipping industry. The ability to explain the instability of strategic alliances using cooperative game theory is discussed, as well as the practical limitations of applying game theory to the industry. For an overview of issues related to carrier alliances, including alliances in both the ocean liner and air cargo industries, we refer the reader to [4].

CHAPTER II

DESIGNING A MECHANISM TO MANAGE ALLIANCE BEHAVIOR

Even small introductory examples like that of Chapter 1 give rise to important challenges that must be addressed when considering an alliance among carriers:

- How can resources be utilized such that the overall system profit is maximized?
- Given that this utilization will not necessarily be optimal for individual carriers in the alliance, what incentives are necessary to encourage carriers to not only participate, but make decisions that lead to system optimal performance?
- How can these incentives be distributed to the carriers, without relying on a centralized decision-maker?

To address these challenges, we propose a mechanism that manages carrier interactions by setting resource prices such that an appropriate allocation of both alliance resources and profits is attained. These resource prices are henceforth referred to as *capacity exchange prices.* We assume that carriers act to optimize profit, and model this profit-maximizing behavior using multi-commodity flow linear programs. These models are incorporated into a mechanism that manages carrier interactions by setting resource prices such that an appropriate allocation of both alliance resources and profits is attained. Because the individual carrier models are used to determine the impact of resource prices on carrier behavior, the allocations of resources and profits achieved by the mechanism are heavily dependent on the underlying model employed. Thus it is important to consider the impact of the model selected on the overall performance of the mechanism. In this chapter we analyze the performance of our proposed mechanism when two distinct models are employed, comparing the mechanism output from a practical and theoretical standpoint. The models discussed differ in how the actions of other carriers are acknowledged in the model for an individual carrier within the alliance. Subsequently these models will be referred to as behavioral models, since they represent different ways to model the behavior, or interaction, of carriers in the alliance.

After introducing a centralized model to determine the optimal solution from the centralized, or alliance, perspective, the framework for modeling the perspective of an individual carrier within the alliance is discussed. Two behavioral models are introduced, and a methodology for finding capacity exchange prices using these models is then described. In the remainder of the chapter we analyze the effectiveness of each model with respect to ensuring optimal alliance behavior. This analysis is conducted from three perspectives:

1. Centralized feasibility: does the model yield individual carrier solutions that remain feasible when aggregated? Clearly an infeasible solution (for example, an aggregate solution that utilizes more capacity on a leg than is available) is undesirable, as it cannot be optimal for the alliance as a whole.

2. Cooperative game theory: does the model yield solutions that exhibit desirable game-theoretic properties (i.e. being budget-balanced and stable)? Understanding the potential for each model to yield such solutions is important because these properties are desirable in an alliance setting.

3. Secondary markets: do the capacity exchange prices obtained using the model give carriers incentive to buy capacity on a leg from a carrier who is not the operator of that leg? Secondary markets lead to behavior that is detrimental to some members of the alliance and, consequently, jeopardize alliance optimality.

The contributions of this chapter are both practical and theoretical. First, naturally, is a demonstration that it is possible to develop a mechanism that can influence participating carriers to behave in an alliance-optimal manner, and do so without relying on a centralized distributor for allocation of alliance revenue. Second, the majority of the chapter is devoted to comparing the results obtained using two distinct models for the perspective of an individual carrier within the alliance; that the solutions obtained under the two models have different characteristics leads to an important insight: model selection can significantly impact alliance recommendations. Third, we prove that overestimating the amount of control wielded by individual carriers makes it more difficult to control the aggregate solution obtained. A fourth contribution is the complete characterization of allocations obtained under each model with respect to the core of the carrier alliance game, which is a valuable application of the concepts of cooperative game theory. Finally, limitations of the notion of the core are exploited in exploring the potential for resale of capacity in a secondary market, and the negative impact this behavior has on the sustainability of the alliance.

2.1 Centralized Model

An important motivation for the formation of an alliance among carriers is the recognition by those carriers that the alliance will yield benefit beyond what each carrier can accomplish individually. Given that increasing the revenue earned by the alliance increases the benefit that can be distributed among the participating members, it is reasonable to attempt to determine the set of cargo loads to deliver, and the optimal routing of these loads, that will maximize the alliance profit. This information is obtained by solving a network flow problem from the centralized, or system, perspective; that is, the network and demand from each participating carrier are integrated to create one large pseudo-carrier. The network of a carrier is determined according to the amount of cargo capacity available on each flight leg operated by that carrier; the demand associated with each carrier is presumed to be a set of loads that the carrier must accept or reject for delivery. Note that a freight forwarder can be incorporated into this modeling framework by introducing a carrier with a set of associated loads, but no network capacity.

Because the focus of this work is on developing a methodology to manage the interactions among carriers such that alliance-optimal behavior is achieved, several simplifying assumptions are made to improve tractability. First, it is assumed that both cargo loads and flight capacity have single dimension units and are deterministic. In reality, both cargo and capacity are multi-dimensional, and the actual capacity available for cargo is dependent on several factors including the weight of passengers, baggage, fuel, and mail shipments. Furthermore, shipments may not be known or finalized until very close to departure time, therefore the deterministic demand is a fairly strong assumption. Second, it is assumed that origins and destinations for loads correspond to airports, which implies that we are not considering door-to-door pick-up and delivery services. This assumption is minor, and implies that the responsibility of carriers is limited to transportation by air only. Third, costs incurred by operating the network are ignored. While this assumption is reasonable for combination carriers, as it is assumed that the flight schedule for an individual carrier is motivated by the passenger industry and is therefore fixed, it would be strong in other industries such as the ocean-liner shipping or trucking industries. Finally, load splitting is permitted in order to obtain a standard multi-commodity flow linear program. This assumption is reasonable if the standard unit of measure for a cargo load is small relative to the capacity of a plane, both of which depend on the specific carrier(s) studied.

The length and units of the time horizon considered are intentionally not specified, because these should be determined according to the needs and preferences of the alliance. For example, in order to determine the long-term compatibility of a group of carriers or other strategic decisions, it may be appropriate to consider a longer horizon with larger units of time, as exact flight schedules and fleet assignments are only known for the immediate future. On the other hand, a shorter time horizon with more exact flight information (and hence shorter units of time) is required to effectively determine capacity exchange prices, routing, and other operational decisions; these types of decisions should therefore be made at appropriate intervals on a rolling time horizon. In practice, the frequency with which capacity exchange prices are updated will depend on how robust the prices are with respect to variability in demand; this topic is identified in Chapter 5 as a direction for future research.

Let N denote the set of carriers, and E^i the set of legs operated by each carrier $i \in N$. The set A contains all airports covered by the legs in E. Given a planning horizon of T time periods, let V denote the set of nodes (a, t) for each $a \in A$ and $t = 1..T$. Each leg $e \in E$ has capacity k_e. Each carrier has a load set L^i in which an individual load (o, d, i) is characterized by an origin o and destination d. The size and per unit revenue of load (o, d, i) is $d^{(o,d,i)}$ and $r^{(o,d,i)}$, respectively. The centralized goal is to find a flow of loads f

such that the system revenue is maximized, which is accomplished by solving the following multi-commodity flow problem:

$$(C): \quad \max \sum_{(o,d,i)\in L} r^{(o,d,i)} f^{(o,d,i)}_{(d,o,i)} \tag{1}$$

$$\begin{aligned}
s.t. \quad \sum_{(u,v)\in E} f^{(o,d,i)}_{(u,v)} - \sum_{(v,w)\in E} f^{(o,d,i)}_{(v,w)} &\le 0 && \forall v \in V, \forall (o,d,i) \in L && (2)\\
\sum_{(o,d,i)\in L} f^{(o,d,i)}_{e} &\le k_e && \forall e \in E && (3)\\
f^{(o,d,i)}_{(d,o,i)} &\le d^{(o,d,i)} && \forall (o,d,i) \in L && (4)\\
f^{(o,d,i)}_{e} &\ge 0. &&
\end{aligned}$$

(1) reflects the centralized goal of maximizing the amount of revenue earned from delivering loads; the flow variable $f^{(o,d,i)}_{(d,o,i)}$ represents flow on a fictitious edge from the destination d to the origin o of load i, which is introduced to account for the amount of load (o,d,i) that is delivered. (2) are flow balance constraints, enforcing that every unit accepted for shipment must be appropriately routed through the network. (3) are capacity constraints for each flight, while (4) ensure that the amount of a load delivered does not exceed its size. Let f^* be the optimal solution to C; from f^* we obtain the optimal accept-reject decision for each load, as well as the optimal routing for the set of accepted loads.

2.2 Capacity Exchange Prices and Resulting Allocations

In order for the alliance to realize its maximum profit, carriers must make their accept-reject and routing decisions in accordance with f^*. We seek to provide a structure to encourage the exchange of capacity among carriers, as carriers will clearly need incentive to allow their capacity to be used by other carriers so that f^* can be achieved. A natural way to provide this incentive is by establishing a system in which carriers receive payments in exchange for capacity used by other carriers. We refer to these payments as *capacity exchange prices.* How to determine capacity exchange prices will be discussed later in this section.

If c_e is the capacity exchange price on leg e, the net profit from capacity exchanges for carrier i is then given by:

$$s^i = \sum_{e \in E^i} c_e \Big(\sum_{(o,d,k) \notin L^i} f_e^{(o,d,k)} \Big) - \sum_{e \notin E^i} c_e \Big(\sum_{(o,d,i) \in L^i} f_e^{(o,d,i)} \Big). \tag{5}$$

The term s^i is essentially a side payment provided to carrier i to compensate i for the value of capacity being used by other carriers. If s^i is negative, then carrier i can be thought of as a net consumer of capacity value.

Let q^i be the revenue carrier i earns by delivering loads in accordance with f^*:

$$q^i = \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f_{(d,o,i)}^{*(o,d,i)}. \tag{6}$$

The net profit x^i earned by carrier i is given by $x^i = q^i + s^i$. We say that x^i is carrier i's *allocation.*

2.3 Individual Carrier Behavioral Models

What should the value of the capacity exchange prices be in order to ensure that each carrier's allocation is such that the carrier will willingly participate in the alliance? Given that a carrier chooses to participate in the alliance, how can he be encouraged to abide by the centralized solution f^*?

In this section we discuss two distinct ways to model the perspective of an individual carrier in an alliance. The goal in establishing these models is to understand how capacity exchange prices impact the acceptance and routing decisions of an individual carrier. First, however, we discuss a key component in developing alternative behavioral models.

2.3.1 Recognizing the Use of Capacity by Partner Carriers

A critical consideration in modeling the perspective of an individual carrier within an alliance is the fact that a carrier does not operate in isolation. Rather, a carrier must consider the use of capacity by other carriers when making routing decisions. To demonstrate why a model which ignores the use of capacity by other carriers is invalid, consider the following example.

Figure 2 illustrates the time-expanded network, with two capacitated edges (edges $(1,3)$ and $(2,4)$) that represent flights operated by carrier B. The capacity on each of these edges is two. Edges $(1,2)$ and $(3,4)$ are ground edges, representing the ability of a load to wait in a location over time, and therefore have unlimited capacity. The loads are described in Table 1. For example, load $(1,4,A)$ represents a load associated with carrier A with ready time and origin location corresponding to node 1, and delivery deadline and destination corresponding to node 3. The revenue $r^{(1,3,A)}$ associated with this load is 1, and the size $d^{(1,3,A)}$ of the load is two units.

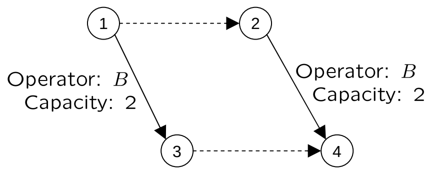

Figure 2: Alliance Network for Example Ignoring Capacity Use by Partner Carriers

Table 1: Loads for Example Ignoring Capacity Use by Partner Carriers

Load	Per-Unit Revenue ($r^{(o,d,k)}$)	Size ($d^{(o,d,k)}$)
$(1,4,A)$	1	2
$(1,3,B)$	1	1
$(2,4,B)$	1	1

Clearly, the centralized optimal solution is to deliver all loads: $f^{*(1,4,A)}_{(4,1,A)} = 2, f^{*(1,3,B)}_{(3,1,B)} = f^{*(2,4,B)}_{(4,2,B)} = 1$. The optimal routing is for load $(1,3,B)$ to travel on leg $(1,3)$, load $(2,4,B)$ to travel on leg $(2,4)$, and for load $(1,4,A)$ to be split, with one unit travelling on leg $(1,3)$ and one unit on $(2,4)$. A behavioral model for carrier A which ignores the use of capacity by carrier B is as follows:

$$(Model^A): \quad \max f^{(1,4,A)}_{(4,1,A)} - c_{(1,3)} f^{(1,4,A)}_{(1,3)} - c_{(2,4)} f^{(1,4,A)}_{(2,4)} \tag{7}$$

$$
\begin{aligned}
s.t.\quad f^{(1,4,A)}_{(4,1,A)} - f^{(1,4,A)}_{(1,2)} - f^{(1,4,A)}_{(1,3)} &\leq 0 \\
f^{(1,4,A)}_{(1,2)} - f^{(1,4,A)}_{(2,4)} &\leq 0 \\
f^{(1,4,A)}_{(1,3)} - f^{(1,4,A)}_{3,4)} &\leq 0 \\
f^{(1,4,A)}_{(2,4)} + f^{(1,4,A)}_{(3,4)} - f^{(1,4,A)}_{(4,1,A)} &\leq 0 \\
f^{(1,4,A)}_{(1,3)} &\leq 2 \qquad (8) \\
f^{(1,4,A)}_{(2,4)} &\leq 2 \qquad (9) \\
f^{(1,4,A)}_{(1,4,A)} &\leq 2 \\
f^{(1,4,A)}_{(4,1,A)}, f^{(1,4,A)}_{e} &\geq 0 \quad \forall e \in E
\end{aligned}
$$

where $c_{(1,3)}$ and $c_{(2,4)}$ in (7) are the capacity exchange prices that carrier A must pay for the use of capacity on legs $(1,3)$ and $(2,4)$, respectively. Notice that the only flow variables in the above model pertain to the load associated with carrier A, and that the capacity constraints (8) and (9) imply that carrier A has full use of the alliance capacity.

Let solution $S1$ be the solution in which $f^{(1,4,A)}_{(1,3)} = f^{(1,4,A)}_{(3,4)} = f^{(1,4,A)}_{(4,1,A)} = 2$, and solution $S2$ be the solution in which $f^{(1,4,A)}_{(1,2)} = f^{(1,4,A)}_{(2,4)} = f^{(1,4,A)}_{(4,1,A)} = 2$. That is, in solution $S1$ both units of load $(1,4,A)$ are travelling on leg $(1,3)$, while in solution $S2$ both units of the load are travelling on leg $(2,4)$. If $c_{(1,3)} < c_{(2,4)}$, then $S1$ is optimal for $Model^A$, while if $c_{(1,3)} > c_{(2,4)}$, then $S2$ is optimal for $Model^A$. If $c_{(1,3)} = c_{(2,4)}$, then both $S1$ and $S2$ are optimal for $Model^A$. The centralized optimal solution, in which load $(1,4,A)$ is delivered using both leg $(1,3)$ and $(2,4)$, does not in fact correspond to a basic solution for $Model^A$, and therefore cannot be obtained using a standard LP solver even when $c_{(1,3)} = c_{(2,4)}$. Given that the goal of establishing a behavioral model is to find a set of capacity exchange prices that will encourage each carrier to behave in an alliance-optimal manner, $Model^A$ is clearly not sufficient: no matter how the exchange prices are set, carrier A will never choose to route load $(1,4,A)$ in accordance with the centralized optimal solution.

Although it is clear in principal that capacity used by other carriers must be acknowledged in the model for an individual carrier, it is not clear mathematically how this can best be accomplished. In the remainder of this section we present two behavioral models, each with a distinct method for incorporating the use of capacity other carriers. In the first model, the capacity utilized by other carriers is protected by introducing appropriate capacity restrictions. In the second model, the use of capacity by other carriers is acknowledged by including flow variables for loads associated with other carriers.

2.3.2 Limited Control Model

We have developed the *Limited Control* model to represent realistic restrictions on the decisions available to an individual carrier participating in an alliance. In the model, the use of capacity by other carriers is acknowledged by limiting carrier i's use of capacity on each flight. Pre-determining capacity allotments is a realistic approach given current industry practice; a carrier typically dedicates space on each flight to specific partnering carriers and freight forwarders. Intuitively, the capacity available on a flight can be partitioned according to the centralized solution f^*; if carrier i uses k_e^i units of capacity on leg e in f^*, then the individual model for carrier i will restrict carrier i to at most k_e^i units of capacity on leg e. As is typically the case in reality, the operator of a leg has use of all capacity that is not set aside for other carriers (or forwarders). More specifically, we use the following rules to determine the amount of capacity allotted to each carrier (or forwarder):

- For each edge e utilized at full capacity in f^* ($\sum_{(o,d,i)\in L} f_e^{*(o,d,i)} = k_e$), allot $\sum_{(o,d,i)\in L^i} f_e^{*(o,d,i)}$ to each carrier i.
- For an edge e that is not utilized at full capacity ($\sum_{(o,d,i)\in L} f_e^{*(o,d,i)} < k_e$), allot $\sum_{(o,d,i)\in L^i} f_e^{*(o,d,i)}$ to each carrier i such that $e \notin E^i$. Allot $k_e - \sum_{(o,d,i)\notin L^k} f_e^{*(o,d,i)}$ to the operating carrier k.
- Ground edges are not subject to capacity allotments, as they are assumed to have infinite capacity.

Given an allotment of capacity k_e^i on every leg $e \in E$, the Limited Control model for carrier i is as follows:

$$(LC^i): \quad \max \sum_{(o,d,i)\in L^i} r^{(o,d,i)} f_{(d,o,i)}^{(o,d,i)} - \sum_{e\notin E^i} c_e \Big(\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} \Big) \tag{10}$$

$$s.t. \quad \sum_{(u,v)\in E} f_{(u,v)}^{(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{(o,d,i)} \leq 0 \qquad \forall v \in V, \forall (o,d,i) \in L^i \tag{11}$$

$$\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} \leq k_e^i \qquad \forall e \in E \tag{12}$$

$$f_{(d,o,i)}^{(o,d,i)} \leq d^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{13}$$

$$f_e^{(o,d,i)} \geq 0. \tag{14}$$

Like the Centralized model C, the Limited Control model is a multi-commodity flow LP. The objective function value (10) is equal to the total revenue earned from delivered loads minus the sum of capacity exchange prices paid. Note that this value is a lower bound on x^i, the actual profit allocated to carrier i, because it excludes exchange prices that will be paid to carrier i.

2.3.3 Strict Control Model

The second model we discuss is based on a model utilized by [3] for one component of their work in the liner shipping industry. In this alternative model, the flow variables for all the loads in the system, including loads associated with other carriers, are included in the model for carrier i. Thus the use of capacity by other carriers is acknowledged explicitly through the flow variables of their associated loads. The Strict Control multi-commodity flow LP is as follows:

$(Strict^i)$:

$$\max \sum_{(o,d,i)\in L^i} r^{(o,d,i)} f_{(d,o,i)}^{(o,d,i)} + \sum_{e\in E^i} (c_e \sum_{(o,d,i)\notin L^i} f_e^{(o,d,i)}) - \sum_{e\notin E^i} (c_e \sum_{(o,d,i)\in L^i} f_e^{(o,d,i)}) \tag{15}$$

$$s.t \quad \sum_{(u,v)\in E} f_{(u,v)}^{(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{(o,d,i)} \leq 0 \qquad \forall v \in V, \forall (o,d,i) \in L \tag{16}$$

$$\sum_{(o,d,i)\in L} f_e^{(o,d,i)} \leq k_e \qquad \forall e \in E \tag{17}$$

$$f_{(d,o,i)}^{(o,d,i)} \leq d^{(o,d,i)} \quad \forall (o,d,i) \in L \tag{18}$$

$$f_e^{(o,d,i)} \geq 0. \tag{19}$$

The second term of the objective function (15) reflects the capacity exchange prices received by carrier i as other carriers use capacity operated by carrier i. Therefore when capacity exchange prices are high enough, carrier i is encouraged to leave capacity open for use by other carriers. We refer to this model as the *Strict Control* model ($Strict^i$) because it implies mathematically that a single carrier has full control over the decisions of other carriers.

2.4 Using Inverse Optimization to Find Capacity Exchange Prices

Given a model to represent the behavior of an individual within the alliance, we seek capacity exchange prices c_e such that the optimal solution to the individual model for carrier i will correspond to the centralized optimal solution f^*. This can be accomplished using inverse optimization. In traditional optimization, optimal values for variables are identified based on a given set of model parameters, whereas in inverse optimization we seek a set of model parameters that will make a particular feasible solution optimal [5]. Because a solution f^* must be optimal when it satisfies primal feasibility, dual feasibility, and complementary slackness conditions, the inverse problem for a carrier is a formulated using the dual of his individual problem, making modifications to the constraints to ensure that a feasible dual solution will satisfy complementary slackness conditions with f^*.

In this section we formulate the inverse optimal problem under each behavioral model, and prove that the problem is feasible for any alliance. The section concludes with an example demonstrating the methodology.

2.4.1 The Inverse Problem Under the Limited Control Model

As described previously, the inverse problem for carrier i under the Limited Control model will be formulated using the dual of LC^i, which is as follows:

$$(DLC^i): \quad \min \sum_{e \in E} k_e^i \alpha_e^i + \sum_{(o,d,i) \in L^i} d^{(o,d,i)} \beta^{i,(o,d,i)}$$

$$
\begin{aligned}
s.t. \quad \pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha_{(u,v)}^i &\geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E^i && (20)\\
\pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha_{(u,v)}^i &\geq -c_{(u,v)} \quad \forall (o,d,i) \in L^i, (u,v) \notin E^i && (21)\\
\pi_o^{i,(o,d,i)} - \pi_d^{i,(o,d,i)} + \beta^{i,(o,d,i)} &\geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i && (22)\\
\pi_v^{i,(o,d,i)} &\geq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i && (23)\\
\alpha_{(u,v)}^i &\geq 0 \quad \forall (u,v) \in E\\
\beta^{i,(o,d,i)} &\geq 0 \quad \forall (o,d,i) \in L^i && (24)
\end{aligned}
$$

where π^i, α^i, and β^i are the dual variables associated with the flow balance constraints, capacity constraints, and demand constraints, respectively, for carrier i. Constraints (20)-(21) correspond to each flow variable $f_{(u,v)}^{(o,d,i)}$. When carrier i operates leg (u,v), he is not required to pay to use capacity for transporting load (o,d,i) on (u,v), hence the right-hand sides of (20) are 0. When carrier i does not operate leg (u,v), he must pay the capacity exchange price $c_{(u,v)}$ for each unit of load (o,d,i) transported on (u,v), therefore the right-hand sides of (21) are $-c_{(u,v)}$. (22) correspond to the variables $f_{(d,o,i)}^{(o,d,i)}$; for each unit of load (o,d,i) delivered, carrier i earns $r^{(o,d,i)}$ in revenue. For this reason, the right-hand sides of (22) are $r^{(o,d,i)}$.

The inverse problem for carrier i based on the Limited Control model, $InvLC^i$, is formed by modifying the constraints of DLC^i in order to ensure that complementary slackness conditions will be satisfied for $(\pi^i, \alpha^i, \beta^i)$ and f^*. For each variable that is positive in the centralized optimal solution f^*, the corresponding dual constraint must hold with equality. In addition, the following constraints are included in $InvLC^i$:

$$
\begin{aligned}
\alpha_e^i &= 0 \quad \forall e \in E : \sum_{(o,d,i) \in L^i} f_e^{*(o,d,i)} < k_e^i && (25)\\
\beta^{i,(o,d,i)} &= 0 \quad \forall (o,d,i) \in L^i : f_{(d,o,i)}^{*(o,d,i)} < d^{(o,d,i)} && (26)\\
c_e &\geq 0 \quad \forall e \in E. && (27)
\end{aligned}
$$

(25) invokes the complementary slackness condition for (12) in LC^i; when carrier i does not use his full allotment of capacity on a leg, the dual variable corresponding to that leg must equal 0. Similarly, (26) enforces the complementary slackness condition for constraint (13) in LC^i; when a load is not fully delivered, the dual variable corresponding to that

load must equal 0. Intuitively, it makes sense to restrict the capacity exchange prices c_e to non-negative values; this assumption is reflected in (27).

Because the parameters of interest, the capacity exchange prices c_e, appear only in the constraints of the inverse problem ($InvLC^i$), it follows that our interest in the inverse problem is in finding a set of prices c_e and dual variables π^i, α^i, and β^i that will make the set of constraints in the inverse problem feasible. Any vector c_e that, together with $(\pi^i, \alpha^i, \beta^i)$, satisfies the constraints of $InvLC^i$ will make f^* optimal for LC^i. Thus, to ensure f^* is optimal for every carrier, we must find one common vector c and dual vectors $(\pi^i, \alpha^i, \beta^i)$ that satisfy $InvLC^i$ for every carrier i. Let $InvLC$ be the constraint set created by combining the constraints of $InvLC^i$ over all carriers i.

Theorem 1. *A feasible solution (π^i, α^i, β^i, and c) to $InvLC$ is guaranteed to exist.*

Proof. Associate dual variables $f_{(u,v)}^{(o,d,i)}$ and $y^{(o,d,i)}$ with constraints (20)-(21) and (22), respectively. Now consider the dual of $InvLC$ when an objective function of $\min 0$ is added:

$$\max \sum_{(o,d,i)\in L} r^{(o,d,i)} y^{(o,d,i)} \tag{28}$$

$$\begin{aligned}
s.t. \quad \sum_{(u,v)\in E} f_{(u,v)}^{(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{(o,d,i)} &\leq 0 \quad \forall (o,d,i) \in L, v \in V : v \notin \{o,d\} && (29)\\
y^{(o,d,i)} - \sum_{(o,w)\in E} f_{(o,w)}^{(o,d,i)} &\leq 0 \quad \forall (o,d,i) \in L && (30)\\
\sum_{(v,d)\in E} f_{(v,d)}^{(o,d,i)} - y^{(o,d,i)} &\leq 0 \quad \forall (o,d,i) \in L && (31)\\
\sum_{(o,d,i)\in L} f_{(u,v)}^{(o,d,i)} &\leq 0 \quad \forall (u,v) \in E, \forall i \in N && (32)\\
y^{(o,d,i)} &\leq 0 \quad \forall (o,d,i) \in L && (33)\\
\sum_{i\in N:(u,v)\notin E^i} \sum_{(o,d,j)\in L^j} f_{(u,v)}^{(o,d,j)} &\leq 0 \quad \forall (u,v) \in E && (34)\\
f_{(u,v)}^{(o,d,i)} &\geq 0 \quad \forall (o,d,i) \in L, (u,v) \in E : f_{(u,v)}^{*(o,d,i)} = 0 \\
f_{(u,v)}^{(o,d,i)} &\; unr. \quad \forall (o,d,i) \in L, (u,v) \in E : f_{(u,v)}^{*(o,d,i)} > 0 \\
y^{(o,d,i)} &\geq 0 \quad \forall (o,d,i) \in L : f_{(d,o,i)}^{*(o,d,i)} = 0 \\
y^{(o,d,i)} &\; unr. \quad \forall (o,d,i) \in L : f_{(d,o,i)}^{*(o,d,i)} > 0.
\end{aligned}$$

Equations (29)-(31) are associated with $\pi_v^{(o,d,i)}$, (32) with $\alpha^i_{(u,v)}$, (33) with $\beta^{(o,d,i)}$, and (34) with $c_{(u,v)}$. Assuming all load revenues $r^{(o,d,i)}$ are non-negative, (33) implies that the objective function (28) is bounded. Furthermore, the solution $\mathbf{y} = \mathbf{f} = \mathbf{0}$ is clearly feasible. Because the dual is bounded and feasible, $InvLC$ must also be feasible. □

2.4.2 The Inverse Problem Under the Strict Control Model

As in the Limited Control model, the inverse problem under the Strict Control model is based on the dual of the Strict Control model for carrier i. The constraints of this dual, $DStrict^i$, can be obtained by adding the following constraints to DLC^i:

$$\pi_v^{i,(o,d,j)} - \pi_u^{i,(o,d,j)} + \alpha^i_{(u,v)} \geq c_{(u,v)} \quad \forall (o,d,j) \notin L^i, (u,v) \in E^i \tag{35}$$

$$\pi_v^{i,(o,d,j)} - \pi_u^{i,(o,d,j)} + \alpha^i_{(u,v)} \geq 0 \quad \forall (o,d,j) \notin L^i, (u,v) \notin E^i \tag{36}$$

$$\pi_o^{i,(o,d,j)} - \pi_d^{i,(o,d,j)} + \beta^{i,(o,d,j)} \geq 0 \quad \forall (o,d,j) \notin L^i. \tag{37}$$

In addition, (23) and (24) are constrained over all $(o,d,i) \in L$. When carrier i operates leg (u,v) he receives capacity exchange price $c_{(u,v)}$ for each unit of load (o,d,j) transported on (u,v), since carrier j must pay for the use of carrier i's capacity. As a result, the right-hand sides of (35) are $c_{(u,v)}$. When carrier i does not operate leg (u,v), he neither receives nor pays for the transportation of load (o,d,j); correspondingly, the right-hand sides of (36) are 0. The right-hand sides for (37) reflect that carrier i receives no direct revenue from delivery of load (o,d,j).

The inverse problem $InvStrict^i$ is formulated as follows:

$$(InvStrict^i):$$

$$\pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha_{(u,v)}^i \begin{Bmatrix} \geq \\ = \end{Bmatrix} 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E^i \tag{38}$$

$$\pi_v^{i,(o,d,j)} - \pi_u^{i,(o,d,j)} + \alpha_{(u,v)}^i \begin{Bmatrix} \geq \\ = \end{Bmatrix} c_{(u,v)} \quad \forall (o,d,j) \notin L^i, (u,v) \in E^i \tag{39}$$

$$\pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha_{(u,v)}^i \begin{Bmatrix} \geq \\ = \end{Bmatrix} -c_{(u,v)} \quad \forall (o,d,i) \in L^i, (u,v) \notin E^i \tag{40}$$

$$\pi_v^{i,(o,d,j)} - \pi_u^{i,(o,d,j)} + \alpha_{(u,v)}^i \begin{Bmatrix} \geq \\ = \end{Bmatrix} 0 \quad \forall (o,d,j) \notin L^i, (u,v) \notin E^i \tag{41}$$

$$\pi_o^{i,(o,d,i)} - \pi_d^{i,(o,d,i)} + \beta^{i,(o,d,i)} \begin{Bmatrix} \geq \\ = \end{Bmatrix} r^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{42}$$

$$\pi_o^{i,(o,d,j)} - \pi_d^{i,(o,d,j)} + \beta^{i,(o,d,j)} \begin{Bmatrix} \geq \\ = \end{Bmatrix} 0 \quad \forall (o,d,j) \notin L^i \tag{43}$$

$$\alpha_{(u,v)}^i = 0 \quad \forall (u,v) \in E : \sum_{(o,d,i) \in L} f_{(u,v)}^{*(o,d,i)} < k_{(u,v)} \tag{44}$$

$$\beta^{i,(o,d,i)} = 0 \quad \forall (o,d,i) \in L : f_{(d,o,i)}^{*(o,d,i)} < d^{(o,d,i)} \tag{45}$$

$$\pi_v^{i,(o,d,i)} \geq 0 \quad \forall v \in V, \forall (o,d,i) \in L \tag{46}$$

$$\alpha_{(u,v)}^i \geq 0 \quad \forall (u,v) \in E \tag{47}$$

$$\beta^{i,(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L \tag{48}$$

$$c_{(u,v)} \geq 0 \quad \forall (u,v) \in E \tag{49}$$

where the constraints (38)-(41) hold with equality for every $(o,d,i) \in L, (u,v) \in E$: $f_{(u,v)}^{*(o,d,i)} > 0$. Similarly, constraints (42)-(43) hold with equality for every $(o,d,i) \in L$: $f_{(d,o,i)}^{*(o,d,i)} > 0$.

In order to ensure f^* is optimal for all individual problems $Strict^i$, we combine the constraints $InvStrict^i$ over all carriers, obtaining $InvStrict$, and search for a feasible set of capacity exchange prices. That a feasible solution to $InvStrict$ must exist can be confirmed by examining a result in [2], in which a multi-commodity flow game with multiple owners on an edge is studied. It is proved that edge prices must exist that satisfy a problem formulated by aggregating the inverse problem of each owner. Because the carrier alliance game is a simplified version of the multi-commodity flow game in [2], the result follows.

2.4.3 Example: Finding Capacity Exchange Prices

In this section we present an example to illustrate feasible exchange prices under each model presented in Section 2.3, and the resulting profit allocations. Characteristics of allocations obtained using the Limited and Strict Control models will be discussed in detail in Section 2.5.

Figure 3 illustrates the alliance network; carrier A operates legs $(1,3)$ and $(2,4)$, each with one unit of capacity. (The capacity on the ground edges $(1,2)$ and $(3,4)$ is unlimited.) The set of loads is described in Table 2. The unique centralized optimal solution is to deliver load $(1,4,B)$ using leg $(1,3)$ and ground edge $(3,4)$, and to deliver load $(2,4,C)$ using let $(2,4)$. Mathematically, $f^{*(1,4,B)}_{(4,1,B)} = f^{*(2,4,C)}_{(4,2,C)} = f^{*(1,4,B)}_{(1,3)} = f^{*(1,4,B)}_{(3,4)} = f^{*(2,4,C)}_{(2,4)} = 1$, and all other variables are zero. The optimality of this solution can be confirmed by solving the centralized problem C for this example.

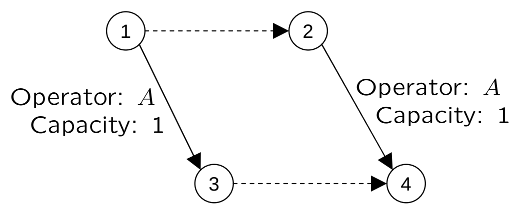

Figure 3: Alliance Network for Allocation Example

Table 2: Loads for Allocation Example

Demand	Per-Unit Revenue ($r^{(o,d,k)}$)	Size ($d^{(o,d,k)}$)
$(1,3,A)$	2	1
$(2,4,A)$	2	1
$(1,4,B)$	6	1
$(2,4,C)$	3	1

The Limited Control model for carrier A is presented below, with the associated dual variables defined next to each constraint. The capacity on legs $(1,3)$ and $(2,4)$ is fully utilized by carriers B and C in the centralized optimal solution; consequently, carrier A is allotted no capacity on these legs.

$$(LC^A): \quad \max 2f^{(1,3,A)}_{(3,1,A)} + 2f^{(2,4,A)}_{(4,2,A)}$$

$$
\begin{array}{llcll}
s.t. & & & & \\
(\pi_v^{A,(o,d,A)}) & \sum_{(u,v)\in E} f^{(o,d,A)}_{(u,v)} - \sum_{(v,w)\in E} f^{(o,d,A)}_{(v,w)} & \leq & 0 & \forall v \in V, \forall (o,d,A) \in L^A \\
(\alpha_e^A) & \sum_{(o,d,A)\in L^A} f^{(o,d,A)}_e & \leq & 0 & \forall e \in \{(1,3),(4,2)\} \\
(\beta^{A,(o,d,A)}) & f^{(o,d,A)}_{(d,o,A)} & \leq & d^{(o,d,A)} & \forall (o,d,A) \in L^A \\
 & f^{(o,d,A)}_e & \geq & 0 & \forall e \in E, \forall (o,d,A) \in L^A.
\end{array}
$$

LC^B and LC^B are formulated in a similar manner. The inverse problem $InvLC$ is formulated using the duals of LC^A, LC^B, and LC^C together with complementary slackness conditions. The constraints comprising $InvLC$ are as follows:

$$
\begin{array}{rrcl}
(InvLC): & \pi_3^{(1,3,A)} - \pi_1^{(1,3,A)} + \alpha_{(1,3)}^A & \geq & 0 \\
& \pi_4^{(2,4,A)} - \pi_2^{(2,4,A)} + \alpha_{(2,4)}^A & \geq & 0 \\
& \pi_2^{(1,4,B)} - \pi_1^{(1,4,B)} & \geq & 0 \\
& \pi_3^{(1,4,B)} - \pi_1^{(1,4,B)} + \alpha_{(1,3)}^B & = & -c_{(1,3)} \\
& \pi_4^{(1,4,B)} - \pi_2^{(1,4,B)} + \alpha_{(2,4)}^B & \geq & -c_{(2,4)} \\
& \pi_4^{(1,4,B)} - \pi_3^{(1,4,B)} & = & 0 \\
& \pi_4^{(2,4,C)} - \pi_2^{(2,4,C)} + \alpha_{(2,4)}^C & = & -c_{(2,4)} \\
& \pi_1^{(1,3,A)} - \pi_3^{(1,3,A)} + \beta^{(1,3,A)} & \geq & 2 \\
& \pi_2^{(2,4,A)} - \pi_4^{(2,4,A)} + \beta^{(2,4,A)} & \geq & 2 \\
& \pi_1^{(1,4,B)} - \pi_4^{(1,4,B)} + \beta^{(1,4,B)} & = & 0 \\
& \pi_2^{(2,4,C)} - \pi_4^{(2,4,C)} + \beta^{(2,4,C)} & = & 0 \\
& \beta^{(1,3,A)} & = & 0 \\
& \beta^{(2,4,A)} & = & 0 \\
& \pi, \alpha, \beta, c & \geq & 0.
\end{array}
$$

Note that the constraints and dual variables associated with ground edges (1,2) and (3,4) have been excluded from LC^i and $InvLC$; since ground edges have infinite capacity, the dual variables associated with these edges must always be zero.

The capacity exchange prices $c_{(1,3)} = 6, c_{(2,4)} = 3$ are one feasible solution to $InvLC$ (this can be verified by solving $InvLC$ with the objective function $\max c_{(1,4)} + c_{(2,4)}$). The side payments s^i resulting from these exchange prices are computed as follows:

$$
\begin{aligned}
s^A &= c_{(1,3)} f_{(1,3)}^{*(1,4,B)} + c_{(2,4)} (f_{(2,4)}^{*(1,4,B)} + f_{(2,4)}^{*(2,4,C)}) && (50) \\
s^B &= -c_{(1,3)} f_{(1,3)}^{(1,4,B)} - c_{(2,4)} f_{(2,4)}^{(1,4,B)} && (51) \\
s^C &= -c_{(2,4)} f_{(2,4)}^{(2,4,C)} && (52)
\end{aligned}
$$

which simplify to $s^A = 9, s^B = -6$, and $s^C = -3$. The carriers also receive direct revenue q^i from delivering loads: $q^A = 0, q^B = 6, q^C = 3$. Recall that the profit x^i allocated to carrier

i is the sum of q^i and s^i. The capacity exchange prices $c_{(1,3)} = 6, c_{(2,4)} = 3$ therefore result in the following allocations: $x^A = 9, x^B = 0, x^C = 0$.

When using the Strict Control model, capacity exchange prices are found by solving the constraints $InvStrict$. $Strict^A$ is presented below, with the associated dual variables defined next to each constraint:

$$(Strict^A): \quad \max 2f^{(1,3,A)}_{(3,1,A)} + 2f^{(2,4,A)}_{(4,2,A)} + c_{(1,3)}f^{(1,4,B)}_{(1,3)} + c_{(2,4)}(f^{(1,4,B)}_{(2,4)} + f^{(2,4,C)}_{(2,4)})$$

$$\begin{array}{llcll}
s.t. & & & & \\
(\pi_v^{A,(o,d,j)}) & \sum_{(u,v)\in E} f^{(o,d,j)}_{(u,v)} - \sum_{(v,w)\in E} f^{(o,d,j)}_{(v,w)} & \leq & 0 & \forall v \in V, \forall (o,d,j) \in L \\
(\alpha_e^A) & \sum_{(o,d,j)\in L} f_e^{(o,d,j)} & \leq & 1 & \forall e \in \{(1,3),(2,4)\} \\
(\beta^{A,(1,3,A)}) & f^{(o,d,i)}_{(d,o,i)} & \leq & d^{(o,d,i)} & \forall (o,d,i) \in L \\
 & f_e^{(o,d,i)} & \geq & 0 & \forall e \in E, \forall (o,d,j) \in L.
\end{array}$$

While the constraints for $Strict^B$ and $Strict^C$ are identical to those of $Strict^A$, the dual variables π, α, and β have superscripts B and C, respectively, in the place of A. The objective functions are as follows:

$$(Strict^B): \quad \max 6f^{(1,4,B)}_{(4,1,B)} - c_{(1,3)}f^{(1,4,B)}_{(1,3)} - c_{(2,4)}f^{(1,4,B)}_{(2,4)}$$

$$(Strict^C): \quad \max 3f^{(2,4,C)}_{(4,2,C)} - c_{(2,4)}f^{(2,4,C)}_{(2,4)}.$$

The inverse problem for carrier A, $InvStrict^A$, is formulated using the dual of $Strict^A$ together with complementary slackness conditions. The constraints comprising $InvStrict^A$ are as follows:

$(InvStrict^A):$

$$
\begin{array}{rcl}
\pi_3^{A,(1,3,A)} - \pi_1^{A,(1,3,A)} + \alpha_{(1,3)}^A & \geq & 0 \\
\pi_4^{A,(2,4,A)} - \pi_2^{A,(2,4,A)} + \alpha_{(2,4)}^A & \geq & 0 \\
\pi_2^{A,(1,4,B)} - \pi_1^{A,(1,4,B)} & \geq & 0 \\
\pi_3^{A,(1,4,B)} - \pi_1^{A,(1,4,B)} + \alpha_{(1,3)}^A & = & c_{(1,3)} \\
\pi_4^{A,(1,4,B)} - \pi_2^{A,(1,4,B)} + \alpha_{(2,4)}^A & \geq & c_{(2,4)} \\
\pi_4^{A,(1,4,B)} - \pi_3^{A,(1,4,B)} & = & 0 \\
\pi_4^{A,(2,4,C)} - \pi_2^{A,(2,4,C)} + \alpha_{(2,4)}^A & = & c_{(2,4)} \\
\pi_1^{A,(1,3,A)} - \pi_3^{A,(1,3,A)} + \beta^{A,(1,3,A)} & \geq & 2 \\
\pi_2^{A,(2,4,A)} - \pi_4^{A,(2,4,A)} + \beta^{A,(2,4,A)} & \geq & 2 \\
\pi_1^{A,(1,4,B)} - \pi_4^{A,(1,4,B)} + \beta^{A,(1,4,B)} & = & 0 \\
\pi_2^{A,(2,4,C)} - \pi_4^{A,(2,4,C)} + \beta^{A,(2,4,C)} & = & 0 \\
\beta^{A,(1,3,A)} & = & 0 \\
\beta^{A,(2,4,A)} & = & 0 \\
\pi, \alpha, \beta, c & \geq & 0.
\end{array}
$$

To obtain $InvStrict^B$, replace $\pi_v^{A,(o,d,i)}$ with $\pi_v^{B,(o,d,i)}$, $\alpha_{(u,v)}^A$ with $\alpha_{(u,v)}^B$, and $\beta^{A,(o,d,i)}$ with $\beta^{B,(o,d,i)}$. Furthermore, the vector of right-hand sides for equations (53)-(53) becomes $[0, 0, 0, -c_{(1,3)}, -c_{(2,4)}, 0, 0, 0, 0, 6, 0, 0, 0]$. Similarly, to obtain $InvStrict^C$ replace $\pi_v^{A,(o,d,i)}$ with $\pi_v^{C,(o,d,i)}$, $\alpha_{(u,v)}^A$ with $\alpha_{(u,v)}^C$, and $\beta^{A,(o,d,i)}$ with $\beta^{C,(o,d,i)}$. The right-hand side vector for $InvStrict^C$ is $[0, 0, 0, 0, 0, 0, -c_{(2,4)}, 0, 0, 0, 3, 0, 0]$.

$InvStrict$ is formed by combining the constraints of $InvStrict^A$, $InvStrict^B$, and $InvStrict^C$. One feasible set of capacity exchange prices is $c_{(1,3)} = c_{(2,4)} = 2$, which can be verified by adding the objective function $\min c_{(1,3)} + c_{(2,4)}$ and solving $InvStrict$ to optimality. The side payments s^i are computed according to equations (50)-(52), which now simplify to $s^A = 4, s^B = -2, s^C = -2$. The direct revenue from delivering loads is not dependent on the behavioral model employed, and therefore remains $q^A = 0, q^B = 6, q^C = 3$.

Thus the allocations resulting from the capacity exchange prices $c_{(1,3)} = c_{(2,4)} = 2$ are as follows: $x^A = 4, x^B = 4, x^C = 1$.

It is an interesting observation that the set of capacity exchange prices in which $c_{(1,3)} = c_{(2,4)} = 2$ is feasible for $InvLC$, which implies that the allocation $x^A = 4, x^B = 4, x^C = 1$ may also be obtained when using the Limited Control model. However, the set in which $c_{(1,3)} = 6, c_{(2,4)} = 3$ is not feasible for $InvStrict$, implying that the allocation $x^A = 9, x^B = 0, x^C = 0$ may only be obtained when using the Limited Control model. Differences in allocations obtained using the two models will be characterized for the general case in the following section.

2.5 Comparison of Models

Having established two distinct models for the behavior of an individual carrier within the system and the methodology for obtaining capacity exchange prices using each of the models, we now focus on the characteristics of allocations obtained using each model. What are the advantages and disadvantages of the Strict and Limited Control models? Qualitatively, it can be argued that the Limited Control model offers a more realistic view of the decisions available to an individual carrier. Quantitatively, this section focuses on analyzing the allocations obtained under each model to determine their respective ability to ensure alliance optimal behavior is attained. This analysis will be conducted from the perspective of (1) centralized feasibility, (2) cooperative game theory, and (3) secondary markets for capacity.

2.5.1 Centralized Feasibility

In this section we demonstrate that when using the Strict Control model, the aggregated individual solutions may be suboptimal or even infeasible from the centralized perspective. In contrast, centralized feasibility is guaranteed under the Limited Control model. The inability to ensure feasibility of the aggregation of the individual solutions obtained when each carrier solves $Strict^i$ for a given set of capacity exchange prices is an obvious limitation of the Strict Control model. It is perhaps counterintuitive, as one might expect that as the level of control represented in a given model increases, the ability to produce the desired action (in this case, behavior consistent with the alliance optimal solution) would also increase. Instead we find that it is exactly this increased control on the part of an individual carrier that leads to behavior inconsistent with the centralized solution.

Theorem 2. *Given a set of capacity exchange prices feasible for InvStrict, it is possible that optimal solutions exist for* $Strict^i$ *that create infeasibility in the centralized setting.*

Proof. This proof is by counterexample. Consider the simple system depicted in Figure 4 in which carrier A operates a leg with origin o, destination d, and capacity 2. Each carrier has one associated load, described in Table 3. For example, load (o, d, A) represents a load associated with carrier A, with ready time and origin location corresponding to node o, and delivery deadline and destination corresponding to node d.

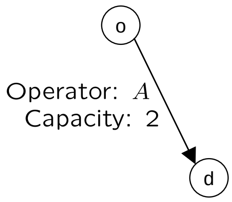

Figure 4: Alliance Network for Feasibility Example

Table 3: Load Descriptions for Feasibility Example

Load	Per-Unit Revenue $(r^{(o,d,k)})$	Size $(d^{(o,d,k)})$
(o, d, A)	2	1
(o, d, B)	1	2

The centralized optimal solution is to deliver one unit of each load. That is, $f^{*(o,d,A)}_{(d,o,A)} = f^{*(o,d,A)}_{(o,d)} = f^{*(o,d,B)}_{(d,o,B)} = f^{*(o,d,B)}_{(o,d)} = 1$. The constraints for the inverse problem under the Strict Control model are written below, with the primal variable corresponding to each constraint indicated to the left of the constraint:

$(InvStrict)$:

$$\begin{array}{rrcl}
(f_{(o,d)}^{(o,d,A)}) & \pi_d^{A,(o,d,A)} - \pi_o^{A,(o,d,A)} + \alpha_{(o,d)}^A & = & 0 \\
(f_{(d,o)}^{(o,d,A)}) & \pi_o^{A,(o,d,A)} - \pi_d^{A,(o,d,A)} + \beta^{A,(o,d,A)} & = & 2 \\
(f_{(o,d)}^B) & \pi_d^{A,(o,d,B)} - \pi_o^{A,(o,d,B)} + \alpha_{(o,d)}^A & = & c_{(o,d)} \\
(f_{(d,o)}^B) & \pi_o^{A,(o,d,B)} - \pi_d^{A,(o,d,B)} & = & 0 \\
(f_{(o,d)}^{(o,d,A)}) & \pi_d^{B,(o,d,A)} - \pi_o^{B,(o,d,A)} + \alpha_{(o,d)}^B & = & 0 \\
(f_{(d,o)}^{(o,d,A)}) & \pi_o^{B,(o,d,A)} - \pi_d^{B,(o,d,A)} + \beta^{B,(o,d,A)} & = & 0 \\
(f_{(o,d)}^B) & \pi_d^{B,(o,d,B)} - \pi_o^{B,(o,d,B)} + \alpha_{(o,d)}^B & = & -c_{(o,d)} \\
(f_{(d,o)}^B) & \pi_o^{B,(o,d,B)} - \pi_d^{B,(o,d,B)} & = & 1.
\end{array}$$

The only value of $c_{(o,d)}$ that is feasible for $InvStrict$ is $c_{(o,d)} = 1$. Now consider the Strict Control model for carrier B, written in standard form:

$$Strict^B : \quad \max f_{(d,o,B)}^{(o,d,B)} - c_{(o,d)} f_{(o,d)}^{(o,d,B)}$$

$$\begin{array}{rrcl}
s.t & f_{(d,o,A)}^{(o,d,A)} - f_{(o,d)}^{(o,d,A)} + s^1 & = & 0 \\
& f_{(o,d)}^{(o,d,A)} - f_{(d,o,A)}^{(o,d,A)} + s^2 & = & 0 \\
& f_{(d,o,B)}^{(o,d,B)} - f_{(o,d)}^{(o,d,B)} + s^3 & = & 0 \qquad (53) \\
& f_{(o,d)}^{(o,d,B)} - f_{(d,o,B)}^{(o,d,B)} + s^4 & = & 0 \qquad (54) \\
& f_{(o,d)}^{(o,d,A)} + f_{(o,d)}^{(o,d,B)} + s^5 & = & 2 \\
& f_{(d,o,A)}^{(o,d,A)} + s^6 & = & 1 \\
& f_{(d,o,B)}^{(o,d,B)} + s^7 & = & 2 \\
& f_e^{(o,d,i)} & \geq & 0 \\
& s^j & \geq & 0 \quad \forall (j = 1..7).
\end{array}$$

From (53), if $f_{(d,o,B)}^{(o,d,B)} > f_{(o,d)}^{(o,d,B)}$ then $s^3 < 0$. Similarly, from (54), if $f_{(d,o,B)}^{(o,d,B)} < f_{(o,d)}^{(o,d,B)}$ then $s^4 < 0$. Because s^j must be nonnegative, it follows that $f_{(d,o,B)}^{(o,d,B)} = f_{(o,d)}^{(o,d,B)}$ in any feasible solution to $Strict^B$, which then implies that all feasible solutions have an objective function value of 0.

Consider the following solutions:

S1: $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 1, s^7 = 1, s^1 = s^2 = s^3 = s^4 = s^5 = s^6 = 0;$

S2: $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = 0, f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 2, s^6 = 1, s^1 = s^2 = s^3 = s^4 = s^5 = s^7 = 0.$

We claim that S1 and S2 are basic feasible solutions, and it follows that each of these solutions is an optimal solution that may be obtained by using a standard LP solver such as CPLEX. Assuming carrier i will implement only those decisions pertaining to his associated loads, the aggregate alliance solution is formed by retaining the optimal value of $f^{(o,d,i)}$ from $Strict^i$, while the optimal value of $f^{(o,d,j)}$ from $Strict^i$ is ignored. The unique optimal solution to $Strict^A$ is $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 1$; therefore the contribution of carrier A to the aggregate solution is $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = 1$. The contribution of carrier B based on S1 is $f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 1$; S1 therefore leads to an aggregate solution which is in fact the centralized optimal solution. The contribution of carrier B based on S2 is $f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 2$, which implies an aggregate solution in which load (o,d,A) and both units of load (o,d,B) are delivered. This aggregate solution is infeasible from the centralized perspective because it requires three units of capacity on leg (o,d).

It remains to prove our claim that S1 and S2 are basic feasible solutions. Given a system of linear equations $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A}$ is m x n, $\mathbf{x}$ is n x 1, $\mathbf{b}$ is m x 1, and $n \geq m$, a basic solution can be obtained by setting $n - m$ variables equal to zero and solving for the remaining m variables, assuming that the columns in $\mathbf{A}$ corresponding to the m variables are linearly independent [31]. Any basic solution in which all variables are nonnegative is a basic feasible solution.

The constraints of $Strict^B$, excluding the nonnegativity constraints, can be written in the form $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} f^{(o,d,A)}_{(d,o,A)} \\ f^{(o,d,A)}_{(o,d)} \\ f^{(o,d,B)}_{(d,o,B)} \\ f^{(o,d,B)}_{(o,d)} \\ s^1 \\ s^2 \\ s^3 \\ s^4 \\ s^5 \\ s^6 \\ s^7 \end{pmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ 2 \end{pmatrix}.$$

Let $\mathbf{B}$ be an m x m matrix made up of m linearly independent columns of $\mathbf{A}$, $\mathbf{x^B}$ be an m x 1 vector containing the variables corresponding to the columns in $\mathbf{B}$, and $\mathbf{x^N}$ be an $(n-m)$ x 1 vector containing the remaining $n-m$ variables. Then $\mathbf{x^B} = \mathbf{B^{-1}b}, \mathbf{x^N} = \mathbf{0}$ is a basic solution.

We will now examine the solutions S1 and S2 and verify that each is a basic solution. For each solution, we state the vector of basic variables ($\mathbf{x^B}$) and the basis matrix $\mathbf{B}$ which contains the columns of $\mathbf{A}$ corresponding to the variables in $\mathbf{x^B}$. To verify that $\mathbf{B}$ has m linearly independent columns, we calculate the determinant of $\mathbf{B}$; $\det(\mathbf{B}) \neq 0$ if and only if the columns of $\mathbf{B}$ are linearly independent [7]. Finally, we calculate the value of the basic variables in $\mathbf{x^B}$ by solving $\mathbf{x^B} = \mathbf{B^{-1}b}$ (the remaining $n-m$ variables are equal to zero).

$$\text{S1:} \quad \mathbf{x^B} = \begin{pmatrix} f^{(o,d,A)}_{(d,o,A)} \\ f^{(o,d,A)}_{(o,d)} \\ f^{(o,d,B)}_{(d,o,B)} \\ f^{(o,d,B)}_{(o,d)} \\ s^1 \\ s^4 \\ s^7 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \det(\mathbf{B}) = -1, \ \mathbf{B^{-1}b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

$$\text{S2:}\quad \mathbf{x^B} = \begin{pmatrix} f^{(o,d,B)}_{(d,o,B)} \\ f^{(o,d,B)}_{(o,d)} \\ s^1 \\ s^2 \\ s^3 \\ s^4 \\ s^6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \det(\mathbf{B}) = 1,\ \mathbf{B^{-1}b} = \begin{pmatrix} 2 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Finally, note that for each solution, the values of the basic variables are nonnegative. Since the values of the non-basic variables are also nonnegative, we conclude that each of the solutions is a basic feasible solution. □

Due to the possibility of multiple optimal solutions for $Strict^i$, there is no guarantee that carrier i will behave in accordance with the centralized solution, even when there is a single set of exchange prices c that is feasible for $InvStrict$. A set of basic feasible solutions must exist that are optimal, and therefore also feasible, from the centralized perspective. However, there is no clear way to ensure that these basic feasible solutions will be obtained, since it is not known in general which objective function can be used. The inability to ensure centralized feasibility is clearly a practical limitation of the Strict Control model; in contrast, we show in the following theorem that the Limited Control model can in fact ensure that centralized feasibility is maintained.

Theorem 3. *Any solution obtained using the Limited Control model is feasible from the centralized perspective.*

Proof. A flow balance constraint for load (o, d, i) and node v is contained in LC^i, $\forall(o, d, i) \in L, v \in V$. Because a solution feasible for $InvLC$ satisfies the flow balance constraints (11) $\forall i \in N$, all flow balance constraints (2) in the Centralized model C are satisfied. Similarly, the demand constraint $f^{(o,d,i)}_{(d,o,i)} \leq d^{(o,d,i)}$ is contained in LC^i, $\forall(o, d, i) \in L$, and a feasible solution for $InvLC$ satisfies the demand constraints (13) $\forall i \in N$. Consequently, the demand constraints (4) are satisfied. Non-negativity must be satisfied by all variables in LC^i, $\forall i \in N$; non-negativity is therefore satisfied in the centralized case as well. Finally, the Limited Control model employs capacity restrictions k_e^i for each carrier i and leg e that are constructed to ensure that $\sum_{i \in N} k_e^i \leq k_e$. We therefore conclude that (3) must also be satisfied, and every aggregate solution obtained using the Limited Control model will be feasible from the centralized perspective. □

We have shown that from the practical standpoint of ensuring feasibility of an aggregate solution, it is advantageous to use the Limited Control model. This conclusion was reached by evaluating the optimal solution for individual carriers, assuming they are already participating in an alliance. In the following discussion we evaluate the potential of each model to ensure that all carriers will in fact choose to participate.

2.5.2 Comparing Allocations Using Cooperative Game Theory

In this section we compare the allocations, rather than acceptance and routing decisions, obtained using each model. It it is important to understand how an allocation is perceived by alliance members. Is the allocation for carrier i enough to convince him to participate in the alliance? Are certain allocations perceived to be better than others by some members of the alliance? The concepts of cooperative game theory provide a framework for measuring and comparing the benefits of various allocations.

In a *cooperative game*, rational agents attempt to maximize their individual benefit in a setting in which cooperation among agents is allowed. An alliance in which carriers make and receive payments for the use of capacity fits into the structure of a cooperative game with *transferrable payoffs*, or a game in which participants are allowed to exchange utility among each other; in the carrier alliance setting payoffs take the form of money and are transferred via capacity exchange prices. An outcome of a cooperative game is described by an allocation of benefits to each participant; in the carrier alliance game the allocation x^i is comprised of direct revenue from delivering loads plus the net sum of capacity exchange prices paid and received. Of particular interest is the notion of the *core*, which is the set of allocations that are (i) budget-balanced, meaning that all benefits are allocated, and (ii) stable, meaning that no subset of participants can benefit by leaving the alliance. Let $v(S)$ be the total profit that a subset of carriers S can earn on their own; that is, $v(S)$ is the optimal objective function value when the centralized problem C is solved for the subset S. The core is defined as follows:

$$\sum_{i \in N} x^i = v(N) \tag{55}$$

$$\sum_{i \in S} x^i \geq v(S) \quad \forall S \subset N \tag{56}$$

where (55) is the budget-balance condition and (56) is the stability condition. We call the subset of stability equations (56) in which $|S| = 1$ *rationality* constraints, as they ensure that each individual carrier will earn at least as much in the alliance as they could earn operating alone.

Basic cost allocation methods are discussed in [34]; a more detailed discussion about allocation methods and the core of a cooperative game is available in [24]. Key observations from these works include that the core of a cooperative game is often empty, and the core of a game may contain many allocations. Production games based on linear programming models were studied in [23]; flow games were later specifically considered in [15] and [12]. In [15], networks with a single commodity and capacitated edges owned by players were studied. It was shown that such problems have a nonempty core. [12] extended these results into the multi-commodity flow arena by showing that the result applied to networks with many commodities, but one common source and sink. [2] considers multiple sources and sinks and multiple owners on an edge.

The properties of a core allocation are clearly desirable for a carrier alliance. We will ultimately prove in the following discussion that while a core allocation can be obtained regardless of the individual behavioral model employed, a greater number of core allocations are feasible under the Limited Control model. We begin by analyzing the relationship of allocations obtained using the Strict Control model with respect to the set of core allocations.

First, any set of capacity exchange prices feasible for $InvStrict$ will define an allocation in the core of the carrier alliance game. A result in [2] proves that for a simple multi-commodity flow game where every edge has a unique owner, edge prices that satisfy the aggregation of the owners' inverse problems yield an allocation in the core of the multi-commodity flow game. We can apply this result to *InvStrict* because the carrier alliance game in which the Strict Control model is employed is equivalent to the simple multi-commodity flow game studied. However, as we prove in the following theorem, every core allocation is not feasible under the Strict Control model.

Theorem 4. *The set of feasible capacity exchange prices for the Strict Control model might exclude some core allocations.*

Proof. This proof is by counterexample. Consider the example discussed in Section 2.4.3. Refer to Figure 3 and Table 2 for the alliance network and load descriptions, respectively.

From equation (51), $s^B = -c_{(1,3)}f_{(1,3)}^{(1,4,B)} - c_{(2,4)}f_{(2,4)}^{(1,4,B)}$. Since $d^{(1,4,B)} = 1$, it must be true that $f_{(1,3)}^{(1,4,B)} + f_{(2,4)}^{(1,4,B)} \leq 1$, which implies $s^B \geq \min\{-c_{(1,3)}, -c_{(2,4)}\} = -\max\{c_{(1,3)}, c_{(2,4)}\}$. Solving *InvStrict* with an objective function of $\max c_{(1,3)}$, an optimal objective function value of 3 is attained. Solving *InvStrict* with an objective function of $\max c_{(2,4)}$ also yields an optimal objective function value of 3. Because the maximum feasible value of either leg's capacity exchange price is 3, the minimum value of s^B under the Strict Control model is -3.

It can be easily verified that the allocation $x^A = 7, x^B = 1, x^C = 1$ is a core allocation, as it satisfies equations (55) and (56). Because $v^A = 0, v^B = 6$, and $v^C = 3$, in order to obtain this allocation the capacity exchange prices must lead to the following side payments: $s^A = 7, s^B = -5$, and $s^C = -2$. Since this contradicts the minimum attainable value of s^B, this core allocation cannot be obtained using the Strict Control model. $\square$

We will now characterize the relationship between the core of the carrier alliance game and allocations obtained using the Limited Control model. An important step in accomplishing this is establishing the relationship of allocations obtained using the Limited Control model to those obtained using the Strict Control model, which is described in the following theorem:

Theorem 5. *The set of allocations that may be obtained using the Strict Control model is a subset of the set of allocations that may be obtained using the Limited Control model.*

Proof. We first show that any set of capacity exchange prices obtained using the Strict Control model can also be obtained using the Limited Control model. Let $|N| = n$, and consider $f^{(o,d,i)}_{(u,v)}$. Assume, without loss of generality, that leg (u, v) is operated by carrier $j \neq i$. In *InvStrict*, there are exactly n constraints corresponding to $f^{(o,d,i)}_{(u,v)}$:

$$\pi^{i,(o,d,i)}_v - \pi^{i,(o,d,i)}_u + \alpha^i_{(u,v)} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad -c_{(u,v)} \tag{57}$$

$$\pi^{j,(o,d,i)}_v - \pi^{j,(o,d,i)}_u + \alpha^j_{(u,v)} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad c_{(u,v)} \tag{58}$$

$$\pi^{k,(o,d,i)}_v - \pi^{k,(o,d,i)}_u + \alpha^k_{(u,v)} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad 0 \tag{59}$$

where each equation holds with equality if $f^{*(o,d,i)}_{(u,v)} > 0$. Constraint (57) is from $InvStrict^i$, constraint (58) is from $InvStrict^j$, and constraint (59) is from $InvStrict^k$, where $k \notin \{i, j\}$. In *InvLC*, there is exactly one constraint corresponding to $f^{(o,d,i)}_{(u,v)}$, which is constraint (57). Similarly, consider $f^{(o,d,i)}_{(d,o,i)}$. In *InvStrict* there are again n constraints corresponding to $f^{(o,d,i)}_{(d,o,i)}$:

$$\pi^{i,(o,d,i)}_o - \pi^{i,(o,d,i)}_d + \beta^{i,(o,d,i)} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad r^{(o,d,i)} \tag{60}$$

$$\pi^{k,(o,d,i)}_o - \pi^{k,(o,d,i)}_d + \beta^{k,(o,d,i)} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad 0 \tag{61}$$

where each equation holds with equality if $f^{*(o,d,i)}_{(d,o,i)} > 0$. Constraint (60) is from $InvStrict^i$, while constraint (61) is from $InvStrict^k$, where $k \neq i$. In *InvLC* there is only one constraint corresponding to $f^{(o,d,i)}_{(d,o,i)}$, which is (60). We conclude that the constraint set *InvLC* is a subset of the constraint set *InvStrict*, which implies that any solution that is feasible for *InvStrict* must also be feasible for *InvLC*. It follows directly that any allocation obtained under *InvStrict* can also be obtained under *InvLC*.

To demonstrate that the set of allocations that may be obtained using the Limited Control model is not equal to the set of allocations that may be obtained using the Strict

Control model, consider once again the example used in Section 2.4.3. The set of capacity exchange prices $c_{(1,3)} = 5, c_{(2,4)} = 2$ is feasible for $InvLC$, and results in the allocation $x^A = 7, x^B = 1, x^C = 1$. However, it was demonstrated in the proof of Theorem 4 that this particular allocation cannot be obtained using the Strict Control model. It follows that the set of allocations obtained using the Strict Control model must be a subset of the set of allocations obtained using the Limited Control model. □

Because of Theorem 5 we know that it is possible to obtain a core allocation using the Limited Control model. But is one guaranteed a core allocation? We in fact show in the next theorem that non-core allocations may be obtained when using the Limited Control model.

Theorem 6. *The set of feasible capacity exchange prices for the Limited Control model define a set of allocations that may contain allocations outside the core.*

Proof. The capacity exchange prices $c_{(1,3)} = c_{(2,4)} = 0$ are a feasible solution for the problem $InvLC$ corresponding to the example in Section 2.4.3. $c_{(1,3)} = c_{(2,4)} = 0$ implies $s^A = 0, s^B = 0, s^C = 0$. The resulting allocation, $x^A = 0, x^B = 6, x^C = 3$, clearly does not satisfy the set of stability equations 56, as $v(A) = 4$, and is therefore not contained in the core. □

That one may obtain an allocation outside the core when employing the Limited Control model is at first disconcerting, as an allocation in which some subset of members is actually receiving less profit than they could earn on their own is clearly undesirable. However, by adding stability constraints to $InvLC$ it can be assured that such an allocation will not be obtained. The number of stability constraints of type (56) required is $2^{|N|} - 1$. Based on the relatively small number of carriers participating in an air cargo alliance (for example, the SkyTeam Cargo alliance is currently comprised of 8 carriers, while the WOW alliance is comprised of 4 carriers), the total number of stability constraints will not be prohibitively large. However, if the enumeration of all stability constraints does become a concern, one possibility is to incorporate stability constraints for subsets of size m or smaller, where $m < |N|$. This is reasonable under the assumption that carriers have limited information about other carriers participating in the alliance.

An important consequence of Theorem 5 is the following corollary, which implies that it is possible to guarantee a core allocation when using the Limited Control model. Let $InvSLC$ be the model formed by adding stability constraints (56) to $InvLC$. We refer to the Limited Control model with this enhancement to the inverse problem as the *Stabilized Limited Control* model.

Corollary 7. *$InvSLC$ is feasible, and all feasible solutions define a core allocation.*

Let $\{S\}$ be the set of allocations that may be obtained using the Strict Control model, $\{L\}$ be the set of allocations that may be obtained using the Limited Control model, and $\{C\}$ be the set of core allocations. It has been established thus far that $\{S\} \subseteq \{C\}$ and $\{S\} \subseteq \{L\}$. Theorem 6 shows that $\{L\} \nsubseteq \{C\}$. It is also true that $\{C\} \nsubseteq \{L\}$, which is demonstrated with the following example.

Consider an example in which a single (o, d) is operated by carrier B and has two units of capacity. The set of loads is described in Table 4. In this example the notation is changed to differentiate among multiple loads between origin o and destination d associated with carrier i.

Table 4: Loads for Infeasible Core Allocation Example

Demand	Per-Unit Revenue ($r^{(o,d,k)}$)	Size ($d^{(o,d,k)}$)
(o, d, A^1)	2	1
(o, d, A^2)	5	1
(o, d, B)	2	1

The centralized optimal solution is to deliver both of the loads associated with carrier A, for a total revenue of 7 units. The local optimal solution for carrier A (that is, the maximum revenue carrier A can earn operating alone) is 0, while carrier B can earn 2 units of revenue by operating alone. Therefore any allocation in which $x^A \geq 0, x^B \geq 2$, and $x^A + x^B = 7$ is a core allocation. Recall that $InvLC$ is constructed to ensure that for any set of capacity exchange costs feasible for $InvLC$, the optimal solution for LC^i is to act in accordance with the alliance optimal solution. It can easily be verified that the maximum feasible value for $c_{o,d} = 2$, since for any $c_{o,d} > 2$ carrier A will choose to reject load $(o, d, A1)$. It follows that

$x^A \geq 3$, and any core allocation in which $x^A < 3$, for example $x^A = 0, x^B = 7$, cannot be obtained using the Limited Control model. (Neither can such an allocation be obtained under the Strict Control model.)

There are some instances, however, for which the set of allocations that may be obtained using the Limited Control model is in fact a subset of the core of the carrier alliance game. Theorem 8 below characterizes conditions that are necessary in order for this to be the case; these conditions are especially interesting because they imply that in order for the Limited Control model to produce an allocation that is guaranteed to be in the core of the carrier alliance game with transferrable payoffs, the carrier alliance game with non-transferrable payoffs must have a non-empty core as well. (A game with non-transferrable payoffs corresponds to an alliance in which the allocation for carrier i is equal to the direct revenue earned by carrier i, or $x^i = q^i \ \forall i \in N$.) This is the case because, as is proven below, a solution in which all capacity exchange prices are zero is always feasible for $InvLC$.

Theorem 8. *Given a set of carriers N, the set of feasible solutions for $InvLC$ defines a set of allocations that is a subset of the core only if $v(S) \leq \sum_{i \in S} v^i \ \forall S \subset N$.*

Proof. Assume that $c = 0$ is a feasible solution to $InvLC$. The allocation x^i received by carrier i is then equal to v^i, the amount of revenue carrier i receives by delivering loads in accordance with the centralized solution f^*. If there exists a subset $S \subset N$ such that $v(S) > \sum_{i \in S} v^i$, then it must also be true that $v(S) > \sum_{i \in S} x^i$ and x cannot be a core allocation since it violates (56).

It remains to show that $c = 0$ is a feasible solution to $InvLC$. Consider the Limited Control model for carrier i when $c = 0$:

$$LC^i_{c=0}: \quad \max \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{(o,d,i)}_{(d,o,i)}$$

$$s.t. \quad \sum_{(u,v)\in E} f_{(u,v)}^{(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{(o,d,i)} \leq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i \tag{62}$$

$$\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} \leq k_e^i \quad \forall e \in E \tag{63}$$

$$f_{(d,o,i)}^{(o,d,i)} \leq d^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{64}$$

$$f_e^{(o,d,i)} \geq 0. \tag{65}$$

Let f^{*i} be the vector of components of f^* pertaining to the loads of carrier i. That is, f^{*i} is comprised of $f_{(d,o,i)}^{*(o,d,i)}$ and $f_e^{*(o,d,i)}$, $\forall e \in E$. We know that f^{*i} is a feasible solution to $LC_{c=0}^i$, since the capacity limits k_e^i were constructed in a manner that ensures $\sum_{(o,d,i)\in L^i} f_e^{*(o,d,i)} \leq k_e^i$. Let $\widehat{f^i}$ be an optimal solution to $LC_{c=0}^i$, and assume f^{*i} is not optimal for $LC_{c=0}^i$. $f = \widehat{f^i} \cup \bigcup_{j\in N, j\neq i} f^{*j}$ must be a feasible solution to the centralized problem C, since the capacity limits k_e^i were also constructed to ensure that $\sum_{i\in N} k_e^i \leq k_e \ \forall e \in E$. Furthermore, $\sum_{(o,d,i)\in L^i} r^{(o,d,i)} \widehat{f}_{(d,o,i)}^{(o,d,i)} > \sum_{(o,d,i)\in L^i} r^{(o,d,i)} f_{(d,o,i)}^{*(o,d,i)}$ (which must be true due to the optimality of $\widehat{f^i}$) implies that $\sum_{(o,d,i)\in L^i} r^{(o,d,i)} f_{(d,o,i)}^{(o,d,i)} + \sum_{j\in N, j\neq i} \sum_{(o,d,j)\in L^j} r^{(o,d,j)} f_{(d,o,j)}^{*(o,d,j)} > \sum_{i\in N} \sum_{(o,d,i)\in L} r^{(o,d,i)} f_{(d,o,i)}^{*(o,d,i)}$, which contradicts the optimality of f^*. We conclude that f^{*i} must be optimal for $LC_{c=0}^i$.

Now consider the dual of $LC_{c=0}^i$:

$$DLC_{c=0}^i: \quad \min \sum_{(u,v)\in E} k_e \alpha_{(u,v)}^i + \sum_{(o,d,i)\in L} d^{(o,d,i)} \beta^{i,(o,d,i)} \tag{66}$$

$$s.t. \quad \pi_v^{(o,d,i)} - \pi_u^{(o,d,i)} + \alpha_{(u,v)}^i \geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E \tag{67}$$

$$\pi_o^{(o,d,i)} - \pi_d^{(o,d,i)} + \beta^{(o,d,i)} \geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{68}$$

$$\pi_v^{(o,d,i)} \geq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i \tag{69}$$

$$\alpha_{(u,v)}^i \geq 0 \quad \forall (u,v) \in E \tag{70}$$

$$\beta^{(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L^i. \tag{71}$$

Because $LC_{c=0}^i$ has an optimal solution, (namely, f^{*i}), $DLC_{c=0}^i$ must also have an optimal solution. Let $(\pi^{*i}, \alpha^{*i}, \beta^{*i})$ be optimal for $DLC_{c=0}^i$. Then f^{*i} and $(\pi^{*i}, \alpha^{*i}, \beta^{*i})$ must satisfy the following complementary slackness conditions:

$$\pi_v^{*(o,d,i)}\left(\sum_{(u,v)\in E} f_{(u,v)}^{*(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{*(o,d,i)}\right) = 0 \quad \forall v \in V, \forall (o,d,i) \in L^i \tag{72}$$

$$\alpha_e^{*i}\left(\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} - k_e^i\right) = 0 \quad \forall e \in E \tag{73}$$

$$\beta^{*(o,d,i)}\left(f_{(d,o,i)}^{(o,d,i)} - d^{(o,d,i)}\right) = 0 \quad \forall (o,d,i) \in L^i \tag{74}$$

$$f_{(u,v)}^{*(o,d,i)}\left(\pi_v^{(o,d,i)} - \pi_u^{(o,d,i)} + \alpha_{(u,v)}^i\right) = 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E \tag{75}$$

$$f_{(d,o,i)}^{*(o,d,i)}\left(\pi_o^{(o,d,i)} - \pi_d^{(o,d,i)} + \beta^{(o,d,i)} - r^{(o,d,i)}\right) = 0 \quad \forall (o,d,i) \in L^i. \tag{76}$$

That f^{*i} and $(\pi^{*i}, \alpha^{*i}, \beta^{*i})$ satisfy (72)-(76) and are feasible for $LC_{c=0}^i$ and $DLC_{c=0}^i$ implies that f^{*i} and $(\pi^{*i}, \alpha^{*i}, \beta^{*i})$ must also satisfy the following set of inequalities:

$$\pi_v^{*(o,d,i)} - \pi_u^{*(o,d,i)} + \alpha_{(u,v)}^{*i} = 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E : f_{(u,v)}^{*(o,d,i)} > 0 \tag{77}$$

$$\pi_v^{*(o,d,i)} - \pi_u^{*(o,d,i)} + \alpha_{(u,v)}^{*i} \geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E : f_{(u,v)}^{*(o,d,i)} = 0 \tag{78}$$

$$\pi_o^{*(o,d,i)} - \pi_d^{*(o,d,i)} + \beta^{*(o,d,i)} = r^{(o,d,i)} \quad \forall (o,d,i) \in L^i : f_{(d,o,i)}^{*(o,d,i)} > 0 \tag{79}$$

$$\pi_o^{*(o,d,i)} - \pi_d^{*(o,d,i)} + \beta^{*(o,d,i)} \geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i : f_{(d,o,i)}^{*(o,d,i)} = 0 \tag{80}$$

$$\pi_v^{*(o,d,i)} = 0 \quad \forall v \in V, \forall (o,d,i) \in L^i : \left(\sum_{(u,v)\in E} f_{(u,v)}^{*(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{*(o,d,i)}\right) \neq 0 \tag{81}$$

$$\pi_v^{*(o,d,i)} \geq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i : \left(\sum_{(u,v)\in E} f_{(u,v)}^{*(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{*(o,d,i)}\right) = 0 \tag{82}$$

$$\alpha_{(u,v)}^{*i} = 0 \quad \forall (u,v) \in E : \left(\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} < k_e^i\right) \tag{83}$$

$$\alpha_{(u,v)}^{*i} \geq 0 \quad \forall (u,v) \in E : \left(\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} = k_e^i\right) \tag{84}$$

$$\beta^{*(o,d,i)} = 0 \quad \forall (o,d,i) \in L^i : \left(f_{(d,o,i)}^{(o,d,i)} < d^{(o,d,i)}\right) \tag{85}$$

$$\beta^{*(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L^i : \left(f_{(d,o,i)}^{(o,d,i)} = d^{(o,d,i)}\right). \tag{86}$$

Inequalities (81) can be eliminated; (62) are flow balance constraints and therefore must all hold at equality, which implies that the condition for (81) will never be met. It then follows that inequalities (77)-(86) are equivalent to the constraints of $InvLC^i$ when $c = 0$. It therefore follows that $(\pi^{*i}, \alpha^{*i}, \beta^{*i}, c = 0)$ must be feasible for $InvLC^i$.

We have shown that $c = 0$ must be feasible for $InvLC^i$. Because $InvLC$ is constructed by combining the constraints of $InvLC^i$ for all $i \in N$, and the exchange prices c are the only components common among the constraints $InvLC^i$ and $InvLC^j$, $\bigcup_{i \in N}(\pi^{*i}, \alpha^{*i}, \beta^{*i}), c = 0$ must be feasible for $InvLC$. $\square$

In addition to those conditions that are necessary for the set of allocations that may be obtained using the Limited Control model to be a subset of the core, we have also identified a set of conditions that are sufficient for this to be the case.

Theorem 9. *Given a set of carriers N, the set of feasible solutions for $InvLC$ defines a set of allocations that is a subset of the core if $v(S) = 0 \; \forall S \subset N$.*

Proof. $InvLC$ is constructed such that for any set of exchange costs c that are feasible for $InvLC$, f^{*i} is optimal for LC^i for all $i \in N$. Assume $r^{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)} < \sum_{e \notin E^i} c_e f_e^{*(o,d,i)}$ for some $(o,d,i) \in L : f^{*(o,d,i)}_{(d,o,i)} > 0$. (This implies that the per unit cost of delivering a load is more expensive than the per unit revenue earned by delivering the load.) But then it cannot be optimal for $f^{*(o,d,i)}_{(d,o,i)} > 0$, since carrier i is better off by not delivering the load at all. Because this is a contradiction, it must be true that $r^{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)} \geq \sum_{e \notin E^i} c_e f_e^{*(o,d,i)} \; \forall (o,d,i) \in L :$ $f^{*(o,d,i)}_{(d,o,i)} > 0$. This implies that $\sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)} - \sum_{e \notin E^i} c_e \Big(\sum_{(o,d,i) \in L^i} f_e^{*(o,d,i)} \Big) \geq 0$.

Consider the allocation for carrier i:

$$x^i = q^i + s^i = \left[\sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)} - \sum_{e \notin E^i} c_e \Big(\sum_{(o,d,i) \in L^i} f_e^{*(o,d,i)} \Big) \right] + \sum_{e \in E^i} c_e \Big(\sum_{(o,d,k) \notin L^i} f_e^{*(o,d,k)} \Big).$$

Since $c \geq 0$, it follows that $x^i \geq 0$.

If $v(S) = 0 \; \forall S \in N$, we must have $\sum_{i \in S} x^i \geq v(S)$. We conclude that if $v(S) = 0 \; \forall S \in N$, then x is a core allocation. ($\sum_{i \in N} x^i = v(N)$ must be satisfied by optimality of f^*.)

$\square$

The relationship among the set of allocations that may be obtained using the Limited and Strict Control models and the core of the carrier alliance game is depicted in Figure 5. In summary, we have shown that a core allocation can be obtained using either behavioral model, but that in general the solution space for the Limited Control model includes more

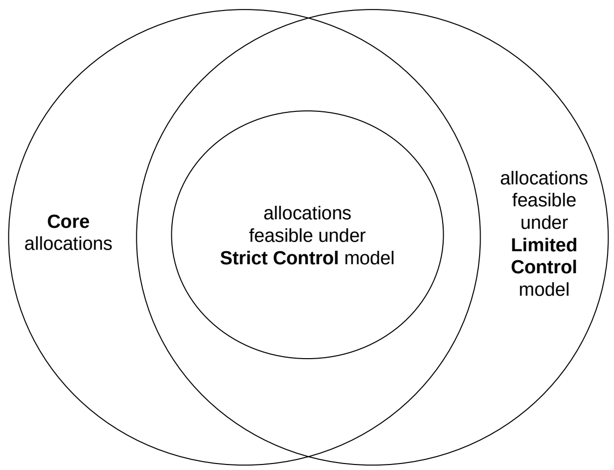

Figure 5: General Relationship of Allocations

core allocations than does the solution space for the Strict Control model. Furthermore, even though it is possible to obtain allocations outside the core when using the Limited Control model, the model can easily be adapted (to the Stabilized Limited Control model) to ensure a core allocation is obtained. Thus the Limited Control, model in addition to offering the practical advantage of centralized feasibility, offers desirable theoretical properties as well.

2.5.3 Secondary Markets for Capacity

In this section we examine the potential for a given set of capacity exchange prices to create a secondary market for capacity. It is assumed that while capacity exchange prices are determined up front (by solving the inverse problem $InvStrict$, $InvLC$, or $InvSLC$, depending on the behavioral model used), carriers do not pay for capacity until it is used. Furthermore, it is assumed that a carrier is allowed to purchase (from the operator of a leg) any amount of capacity on the leg up to the amount of capacity used by the carrier in the centralized optimal solution, but no more. Because carriers are not required to pay for all capacity up front, the time that elapses between when capacity exchange prices are set and when capacity is used gives carriers an opportunity exists to exchange information and determine profitable ways to trade capacity, at the expense of other members of the alliance. Capacity traded in such a manner is referred to as a *secondary market*, since trading occurs after capacity has first been purchased from the leg operator. The initial exchange of capacity, in which capacity is sold only by the operator of a leg, is referred to as the *primary market*.

Traditionally, secondary markets have been studied in the context of markets for used goods, such as automobiles and books. Our interpretation here is slightly different, since the capacity is being sold as "unused" in the secondary market. However, similar to secondary markets in more traditional applications, we find that understanding the impact of secondary markets on the overall system is critical to ensure that the system will operate optimally. After an example illustrating the mechanics of a secondary market and why the existence of the secondary market is detrimental for the alliance, we will show that a secondary market can never occur when capacity exchange prices are determined using the Strict Control model.

Consider an example in which carrier A operates three legs, each with one unit of capacity. The alliance network is depicted in Figure 6, and load descriptions are given in Table 5. In the unique centralized optimal solution, load $(1, 4, B)$ is delivered using leg $(1, 3)$ and ground edge $(3, 4)$, load $(2, 4, C)$ is delivered using ground edge $(1, 2)$ and leg $(2, 4)$, and load $(3, 5, D)$ is delivered using leg $(3, 5)$. The centralized optimal revenue is \$10. The capacity exchange prices $c_{(1,3)} = 6, c_{(2,4)} = 3, c_{(3,5)} = 1$ are feasible for $InvLC$, which can be verified by solving $InvLC$ with the objective function $\max c_{(1,3)} + c_{(2,4)} + c_{(3,5)}$. These exchange prices result in the following allocations: $x^A = 10, x^B = x^C = x^D = 0$.

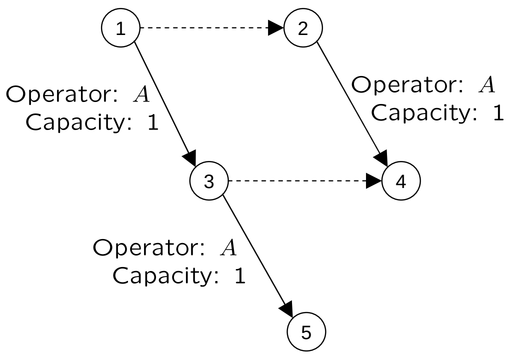

Figure 6: Alliance Network for Secondary Market Example

Assume that carrier B and carrier C exchange information about their respective loads, which leads carrier B to realize that rather than paying carrier A \$6 for a unit of capacity on leg $(1, 3)$, he can instead offer carrier C \$$y$ for a unit of capacity on leg $(2, 4)$, where

Table 5: Loads for Secondary Market Example

Demand	Per-Unit Revenue ($r^{(o,d,k)}$)	Size ($d^{(o,d,k)}$)
$(1,5,A)$	3	1
$(2,4,A)$	2	1
$(1,4,B)$	6	1
$(2,4,C)$	3	1
$(3,5,D)$	1	1

$\$3 < y < \$6$. This sale of capacity from carrier B to carrier C, if executed, occurs in the secondary market, since the capacity in question has already been purchased from carrier A at a price of \$3. The resulting allocations realized by the carriers after this exchange are given in Table 6.

Table 6: Allocations After Secondary Market Exchange

Carrier	Direct Revenue	Capacity Sold/Purchased	Net Value of Exchange Prices (s^i)	Allocation (x^i)
A	0	(2,4) and (3,5) sold	3 + 1	4
B	6	(2,4) purchased from C	-y	6-y
C	0	(2,4) purchased from A, sold to C	-3+y	-3+y
D	1	(3,5) purchased from A	-1	0

Secondary markets are clearly detrimental for the owner of capacity that is unsold because of a trade in the secondary market, in this case the capacity on leg $(1,3)$. In fact, in this example carrier A will earn less revenue than he could earn by operating alone, which implies that the alliance will not be sustainable because carrier A should not continue to participate. Our original analysis, however, which considered the primary market only, predicted the allocations $x^A = 10, x^B = x^C = x^D = 0$, which is in fact a core allocation and therefore stable in the game-theoretic sense.

Observation 1. *Capacity exchange prices may lead to a secondary market even when the allocation predicted by analysis of the primary market is a core allocation.*

The implication of this observation is that traditional game theoretic tools alone are not enough to evaluate the sustainability of an alliance. Because secondary markets lead to behavior that is suboptimal for the alliance, it is valuable to understand how they can be prevented. In the following theorem we show that capacity exchange prices obtained using the Strict Control model never lead to a (profitable) secondary market for capacity.

Theorem 10. *Given a set of capacity exchange prices feasible for InvStrict, a profitable secondary market for capacity will not exist.*

Proof. In order for a capacity exchange in the secondary market to be profitable for the seller of capacity on leg (u, v), he must be able to sell (u, v) for a price $\overline{c}_{(u,v)}$, where $\overline{c}_{(u,v)} > c_{(u,v)}$. Let f' represent the aggregate solution once a secondary capacity exchange has taken place. There must exist some carrier i and leg $(u, v) \notin E^i : \delta = \sum\limits_{(o,d,i)\ inL^i} f'^{(o,d,i)}_{(u,v)} - \sum\limits_{(o,d,i)\ inL^i} f^{*(o,d,i)}_{(u,v)} > 0$. *InvStrict* is constructed such that for any set of exchange costs c feasible for *InvStrict*, f^* is optimal for *Strict*^{i}. Because f' is feasible for the centralized model C, it must also be feasible for *Strict*^{i}, which implies that, for *Strict*^{i}, the objective function value of a solution in which $\sum\limits_{(o,d,i)\ inL^i} f'^{(o,d,i)}_{(u,v)}$ units of capacity are purchased at cost $c_{(u,v)}$ is no greater than the objective function value of a solution in which $\sum\limits_{(o,d,i)\ inL^i} f^{*(o,d,i)}_{(u,v)}$ units of capacity are purchased at cost $c_{(u,v)}$. Since purchasing δ more units of capacity on leg (u, v) at cost $c_{(u,v)}$ is not beneficial for carrier i, neither can it be beneficial for carrier i to purchase δ units of capacity on leg (u, v) at a cost which is greater than $c_{(u,v)}$. Because $\overline{c}_{(u,v)} > c_{(u,v)}$, it therefore cannot be beneficial for carrier i to purchase capacity on the secondary market. Given that it is never beneficial for a carrier to purchase capacity in the secondary market, we conclude that a profitable secondary market will not exist. □

Essentially, secondary markets are prevented under the Strict Control model because any solution feasible for the alliance is feasible for *Strict*^{i}. This is not the case, however, for LC^i; capacity restrictions k^i_e in LC^i render infeasible any solution that uses more capacity on leg e than what is used by carrier i in the centralized optimal solution. The opportunity to purchase capacity in the secondary market effectively increases k^i_e, which creates a larger solution space for LC^i. Because the feasible region for LC^i changes when capacity is purchased on the secondary market, it is possible for these purchases to be profitable.

2.6 Summary

A tradeoff has now been established between the Limited Control and Strict Control models. Not only does the Limited Control model ensure that centralized feasibility is maintained, but it also affords a much larger solution space for capacity exchange prices, and therefore more flexibility in allocations which may be obtained under the model. This flexibility, however, may lead to secondary markets, which create opportunities for behavior that is not optimal for the alliance as a whole. The Strict Control model, meanwhile, is able to prevent the emergence of secondary markets, but in so doing, greatly restricts the range of allocations that are possible. In addition, centralized feasibility cannot be guaranteed.

In the next chapter experimental results are presented for alliances comprised of various types of carriers. For each alliance, the Strict Control model, Limited Control model, and Stabilized Limited Control model are each used to determine capacity exchange prices. Analysis of the resulting allocations demonstrates that the mechanism behaves as predicted with regard to the characteristics of allocations obtained under each model. However, it will also become evident that more influence over the allocations obtained is needed, inspiring an investigation into how fairness may be incorporated into the mechanism, which is the subject of Chapter 4. The tradeoffs between the Limited and Strict Control models will present themselves again in this chapter; depending on how fairness is defined, a fair allocation may lie outside the range of allocations feasible for the Strict Control model. In such a case, it may benefit the alliance to contractually prohibit the selling of capacity by anyone other than the operator of the leg. This effectively eliminates secondary markets while leaving the benefits of the Limited Control model (i.e. centralized feasibility and flexibility of solution space) in tact.

CHAPTER III

COMPUTATIONAL STUDY OF CARRIER COMPATIBILITY

The primary goal of this chapter is to gain an empirical understanding of carrier compatibility. To this end, we examine three sets of experiments designed to investigate the benefit to be gained by collaborating in a variety of alliance circumstances. The first two sets of experiments are a general study of how the network size and fleet capacity of the partnering carriers impact the alliance; in the first set of experiments all carrier hubs are connected, while in the second set the impact of network integration and complementarity among partnering carriers is also investigated. The third set of experiments is an empirical analysis of the compatibility of carriers currently involved in the WOW cargo alliance.

The first section of this chapter describes how data is generated for the first two sets of experiments. The results and insights pertaining to alliances in which the networks of partnering carriers are fully integrated are presented in Section 3.2, while results and analysis for alliances in which network integration varies are discussed in Section 3.3. The experimental procedure, results, and insights pertaining to the WOW alliance are described in Section 3.4.

3.1 Data Generation

Using data publicly available from the Bureau of Transportation Statistics [9], we identified 5 classes of combination carriers. The data set used was the "T-100 Segment (All Carriers)" from July 2006; the description of the data set is as follows:

> This table combines domestic and international T-100 segment data reported by U.S. and foreign air carriers, and contains non-stop segment data by aircraft type and service class for transported passengers, freight and mail, available capacity, scheduled departures, departures performed, aircraft hours, and load factor...[9]

where "freight" excludes all passenger baggage. The data was filtered to select only domestic data for US carriers (that is, flight segments operated by US-owned carriers for which both endpoints of the segment lie within the United States or US-owned territories). Furthermore, only segments of service class "F" (scheduled passenger and/or cargo service) were considered.

Our goal was to identify classes of carriers based on network size and capacity of fleet. To accomplish this, for each carrier the sum over all segments flown by that carrier was calculated for the following statistics: number of scheduled departures for the segment, number of passengers transported on the segment, and the amount of freight (in pounds) transported on the segment. In addition, the number of distinct (origin, destination) pairs was calculated for each carrier. In order to ensure that primarily combination carriers were represented in the sample, the results were filtered to retain only those carriers performing more than 1000 departures, delivering more than 10,000 passengers, delivering more than 100,000 pounds of freight, and maintaining an average of more than 15 passengers per departure. (Average passengers per departure for a carrier = $\frac{\text{sum of passengers flown during month}}{\text{sum of departures performed during month}}$.) The resulting data is presented in Table 7.

Based on the information in Table 7, classifications for network size and fleet capacity were determined. Network size is approximated by the number of (origin, destination) pairs served by a carrier, while fleet capacity is approximated by the average number of passengers per departure. For each classification listed in Tables 8(a) and 8(b), the range (from Table 7) represented by the classification is described in the second column.

Using these classifications, we obtain the 5 general classes of carriers in Table 9. These carrier classes were used to generate carriers for the experiments described in Sections 3.2 and 3.3. In the following discussion, the procedure for generating the network and loads associated with each carrier is described.

Each carrier operates a pure hub-and-spoke network, in which the network size and fleet capacity are scaled to reflect, approximately, the relative size relationships among the classes. The number of hubs depends on the size of the carrier; carriers with a large, medium, and small network size operate 3, 2, and 1 hubs, respectively. The number of spoke

Table 7: Selected Carrier Data Obtained Using BTS Segment Data

Carrier Name	Total (o,d) pairs	Total Departures	Total Passengers	Total Freight	Passengers per Departure
AirTran Airways	100	12,752	1,241,375	1,087,916	97.3
Alaska Airlines	165	14,400	1,556,547	10,168,571	108.1
Aloha Airlines	16	4,082	295,147	439,085	72.3
America West Airlines	187	16,583	1,876,737	3,225,398	113.2
American Airlines	366	47,466	6,286,265	27,795,820	132.4
American Eagle Airlines	278	34,535	1,221,852	130,591	35.4
ATA Airlines	20	1,053	154,963	206,477	147.2
Atlantic Southeast Airlines	292	22,991	997,246	257,137	43.4
Comair	225	15,153	609,590	117,013	40.2
Continental Air Lines	345	26,585	3,223,248	12,657,114	121.2
Delta Air Lines	506	41,679	5,784,421	31,561,559	138.8
Era Aviation	49	2,709	40,763	159,867	15.0
Expressjet Airlines	278	28,259	1,093,912	247,095	38.7
Frontier Airlines	71	7,338	797,671	1,738,511	108.7
Hawaiian Airlines	40	4,797	562,193	5,067,573	117.2
Horizon Air	152	13,452	572,053	813,321	42.5
JetBlue Airways	16	1,896	267,163	116,484	140.9
Mesa Airlines	147	9,335	521,621	164,424	55.9
Mesaba Airlines	190	11,428	322,328	135,748	28.2
Midwest Airline	41	3,576	291,675	600,215	81.6
Northwest Airlines	424	36,079	4,144,176	6,713,743	114.9
PSA Airlines	131	9,944	424,455	177,627	42.7
Southwest Airlines	775	92,766	9,481,851	26,546,173	102.2
United Air Lines	454	41,669	5,330,278	27,239,513	127.9
US Airways	259	23,611	2,696,865	3,923,946	114.2

legs operated by a carrier in class C1 is $12n$, the number of spoke legs operated by a carrier in class C2 or C3 is $5n$, and the number of spoke legs operated by a carrier in class C4 or C5 is n, where $n = 5$. The origins for spoke legs operated by carrier i are approximately equally distributed among the hubs operated by carrier i, and every spoke destination is served by exactly one hub. Spoke legs operated by carriers with small fleet capacity have 2 units of capacity, while carriers with large fleet capacity operate spoke legs with 5 units of capacity.

The level of integration of the networks of partnering carriers is defined by the number of hubs that are connected. Further details of integration will be discussed for each set of experiments. The capacity of inter-hub legs is large enough to ensure that the benefit of collaborating is not restricted. In this pure hub-and-spoke system, it can easily be seen

Table 8: Class Descriptions

(a) Network Size

Network Size	# of (o,d) Pairs
large	400 - 775
medium	131-366
small	16-100

(b) Fleet Capacity

Fleet Capacity	# of Passengers per Departure
large	95-150
small	15-90

Table 9: Carrier Classifications

Class	Network Size	Fleet Capacity	# of Carriers in Class
C1	large	large	4
C2	medium	large	5
C3	medium	small	8
C4	small	large	5
C5	small	small	3

that the benefit associated with collaborating increases as the capacity on inter-hub legs increases, because any load associated with carrier i that has a destination outside the network of carrier i must travel on an inter-hub leg. It is therefore assumed that carriers participating in an alliance will increase inter-hub capacity to a level that ensures sufficient benefit. Furthermore, in order to simplify analysis, the network is generated such that the decisions about whether to accept a load and how to route that load are dependent solely on network geography and capacity, and not on time. This is accomplished by orienting all spoke legs from hub to spoke, and then setting the origin and destination time of every hub-to-spoke leg as 1 and 2, respectively. Every inter-hub leg has an origin time of 0 and destination time of 1, and every load as an origin time of 0 and a destination time of 2, respectively.

The number of loads associated with a carrier is equal to the number of spoke legs operated by that carrier, which approximates a proportional relationship between the size of a carrier's network and the number of cargo loads booked by that carrier. Because any load originating at a spoke must be transported to the hub of that spoke before it can be transported anywhere else, the system is simplified by generating the origin of a load associated with carrier i randomly from the set of carrier i's hubs. For each carrier, the maximum size of a load, S^i, is equal to the capacity of that carrier's legs, and the maximum per-unit revenue of each load is 3. The size and per-unit revenue of each load associated with carrier i are generated according to a uniform distribution over the ranges [1, S^i] and [1, 3], respectively. Two classes of freight forwarders are used in the experiments; a large freight forwarder is represented by class F1, and is associated with $12n$ loads. A small freight forwarder is represented by class F2, and is associated with $5n$ loads. The maximum sizes of loads associated with forwarders of type F1 and F2 are 5 and 2, respectively.

Obtaining accurate demand distributions is very difficult, and for this reason we test our mechanism using two different distributions. In the first distribution (D1), a high proportion of a carrier's loads have destinations within his network, while in the second demand distribution (D2) the proportion of loads a carrier can serve using his own network is low. Let p^i represent the probability that the destination of load (o, d, i) is within the set of destinations reached by spoke legs operated by carrier i. All destinations within the network of carrier i are equally likely with probability $\frac{p}{spokes^i}$ where $spokes^i$ is the number of spoke legs operated by carrier i. For a load associated with carrier i, all destinations outside the network of carrier i have an equal probability of being selected; this probability is $\frac{1-p^i}{\sum_{j \neq i} spokes^j}$. Consequently, loads associated with freight forwarders have destinations that are uniformly distributed throughout the network.

The probability p^i is calculated for each demand distribution as follows:

D1: $p^i = \frac{a^i}{a^i+1}$, where a^i is the size of carrier i's network relative to n ($a^i = \frac{legs^i}{\min_{j \in N} legs^j}$). In alliances where only one carrier operates legs (because the remaining partners are forwarders), $p^i = 1$ for this carrier.

D2: $p^i = \frac{legs^i}{\sum_{j \in N^k} legs^j}$ where N^k is the group of carriers in instance k.

To aid in analysis, information regarding the distribution of loads for a two-carrier alliance is contained in Table 10. Note that under demand distribution D1, the probabilities for Carriers A and B do not sum to 1 across a row; this is because the probability that the destination of a load associated with Carrier A is in the network of Carrier A is independent of the size of Carrier B. On the other hand, under distribution D2, the probability that the destination of a load associated with Carrier A is in the network of Carrier A is in fact dependent on the size of Carrier B, and consequently, the probabilities for Carriers A and B sum to 1.

Table 10: Probability that Load Destination is in Network of Associated Carrier

		Distribution D1		**Distribution D2**	
Instance Class	**Carriers**	p^A	p^B	p^A	p^B
1	C1,C1	12/13	12/13	1/2	1/2
2	C1,C2	12/13	5/6	12/17	5/17
3	C1,C3	12/13	5/6	12/17	5/17
4	C1,C4	12/13	1/2	12/13	1/13
5	C1,C5	12/13	1/2	12/13	1/13
6	C1,F1	1	0	1	0
7	C1,F2	1	0	1	0
8	C2,C2	5/6	5/6	1/2	1/2
9	C2,C3	5/6	5/6	1/2	1/2
10	C2,C4	5/6	1/2	5/6	1/6
11	C2,C5	5/6	1/2	5/6	1/6
12	C2,F1	1	0	1	0
13	C2,F2	1	0	1	0
14	C3,C3	5/6	5/6	1/2	1/2
15	C3,C4	5/6	1/2	5/6	1/6
16	C3,C5	5/6	1/2	5/6	1/6
17	C3,F1	1	0	1	0
18	C3,F2	1	0	1	0
19	C4,C4	1/2	1/2	1/2	1/2
20	C4,C5	1/2	1/2	1/2	1/2
21	C4,F1	1	0	1	0
22	C4,F2	1	0	1	0
23	C5,C5	1/2	1/2	1/2	1/2
24	C5,F1	1	0	1	0
25	C5,F2	1	0	1	0

For each experiment, the results reported for an instance class represent the average from 30 instances generated with the same class parameters. The mechanism was implemented using C and CPLEX (ver 9.0.0) callable libraries. A solution for an instance was obtained by (1) solving the centralized problem, (2) solving *InvStrict*, *InvLC*, and *InvSLC*, and (3) using the resulting capacity exchange prices to calculate the corresponding allocation. In order to obtain the benefit gained by collaborating, the local problem for each carrier was solved as well.

3.2 Analysis of the Impact of Network Size and Fleet Capacity on Alliance Success

The goal of this first set of experiments is to explore the benefit to be gained from collaborating for alliances comprised of carriers with various network sizes and fleet capacities. It is assumed that the alliance is formed by completely integrating the networks of participating carriers. That is, all alliance hubs are directly connected: there is a leg from each hub of carrier i to each of hub of carrier j for all pairs of carriers i and j. Figure 7 depicts the system network for an alliance comprised of one carrier with a large network and one carrier with a medium network. In this example, carrier A operates all inter-hub leg originating from H_1^A, H_2^A or H_3^A, while carrier B operates all inter-hub legs originating from H_1^B or H_2^B.

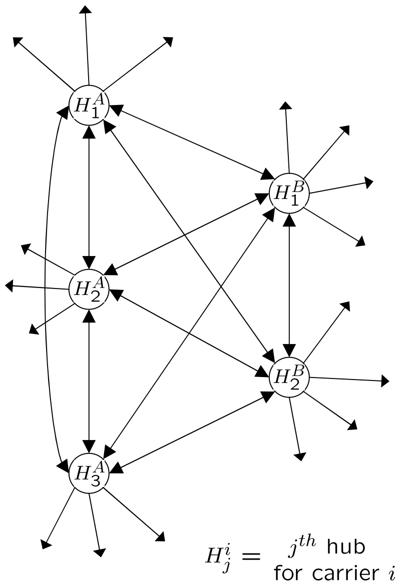

Figure 7: Integrated Hub-and-Spoke Network

3.2.1 Results and Insights from Two-Carrier Alliances

Rounded results pertaining to the alliance optimal solution are contained in Table 11. The "Carriers" column indicates the class from which each carrier is selected; in instance class 1, for example, both carriers in the alliance are carriers from class C1. The increase in system revenue is calculated as the total revenue earned by the alliance minus the sum of the revenue each carrier earns by working independently. The percent increase in accepted loads measures the percent difference in the number of loads that can be completely delivered in the local (independent) solution for a carrier and the number of loads associated with that carrier that are completely delivered in the centralized alliance solution. Table 12 contains rounded results pertaining to the allocations received by each carrier. Specifically, the table shows the benefit each carrier receives by joining the alliance, calculated as the difference between the allocation for carrier i and the revenue carrier i could earn operating alone, or $x^i - v(i)$.

Table 11: System Revenue and Accepted Loads for Two-Carrier Alliances

		Demand Distribution D1				**Demand distribution D2**			
Class	**Carriers** (A,B)	**Chg. in System Revenue**		**Chg. in Loads Accepted**		**Chg. in System Revenue**		**Chg. in Loads Accepted**	
		Actual	**%**	**A**	**B**	**Actual**	**%**	**A**	**B**
1	C1,C1	40.1	7.2%	7.2%	5.9%	266.1	81.6%	72.4%	71.2%
2	C1,C2	33.0	8.4%	6.7%	16.7%	167.2	65.3%	29.7%	242.0%
3	C1,C3	19.6	5.9%	2.2%	12.8%	84.7	34.5%	8.5%	205.0%
4	C1,C4	30.6	10.2%	6.2%	62.7%	42.3	14.7%	5.8%	1018.2%
5	C1,C5	14.6	4.9%	1.9%	44.9%	24.1	8.3%	1.7%	861.5%
6	C1,F1	206.4	68.0%	-24.2%	N/A	206.4	68.0%	-24.2%	N/A
7	C1,F2	55.1	18.2%	-6.5%	N/A	55.1	18.2%	-6.5%	N/A
8	C2,C2	33.6	15.3%	14.5%	14.7%	113.4	79.7%	64.9%	81.0%
9	C2,C3	17.8	11.3%	3.6%	18.1%	60.8	62.8%	18.3%	91.3%
10	C2,C4	33.6	28.9%	14.2%	88.5%	42.3	39.0%	13.5%	568.4%
11	C2,C5	15.3	13.2%	4.4%	86.4%	19.9	17.7%	3.8%	535.0%
12	C2,F1	157.9	124.8%	-44.7%	N/A	157.9	124.8%	-44.7%	N/A
13	C2,F2	54.1	43.9%	-16.2%	N/A	54.1	43.9%	-16.2%	N/A
14	C3,C3	16.3	15.8%	15.8%	14.5%	54.5	82.9%	72.5%	73.7%
15	C3,C4	18.7	29.3%	20.2%	26.8%	22.6	41.1%	18.7%	126.1%
16	C3,C5	13.0	22.3%	15.1%	36.6%	18.6	35.0%	14.1%	409.5%
17	C3,F1	79.6	134.9%	-56.8%	N/A	79.6	134.9%	-56.8%	N/A
18	C3,F2	35.0	60.7%	-28.0%	N/A	35.0	60.7%	-28.0%	N/A
19	C4,C4	23.4	90.1%	58.8%	78.0%	23.4	90.1%	58.8%	78.0%
20	C4,C5	11.1	52.0%	10.4%	55.7%	11.1	52.0%	10.4%	55.7%
21	C4,F1	48.0	185.7%	-95.5%	N/A	48.0	185.7%	-95.5%	N/A
22	C4,F2	33.2	116.9%	-59.8%	N/A	33.2	116.9%	-59.8%	N/A
23	C5,C5	9.8	72.9%	66.7%	65.6%	9.8	72.9%	66.7%	65.6%
24	C5,F1	17.7	147.4%	-98.2%	N/A	17.7	147.4%	-98.2%	N/A
25	C5,F2	16.9	159.4%	-88.2%	N/A	16.9	159.4%	-88.2%	N/A

Table 12: Benefit Experienced by Joining Two-Carrier Alliance

		Demand Distribution D1						**Demand distribution D2**					
Class	**Carriers (A,B)**	**Strict Control**		**Limited Control**		**Stabilized Lim. Control**		**Strict Control**		**Limited Control**		**Stabilized Lim. Control**	
		A	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
1	C1,C1	8%	7%	7%	7%	6%	9%	45%	51%	49%	47%	22%	73%
2	C1,C2	6%	15%	6%	15%	5%	16%	29%	76%	27%	83%	25%	88%
3	C1,C3	3%	24%	4%	15%	2%	29%	10%	109%	15%	83%	16%	81%
4	C1,C4	6%	109%	6%	101%	7%	82%	9%	136%	6%	203%	14%	27%
5	C1,C5	2%	108%	4%	33%	3%	87%	3%	187%	4%	146%	7%	52%
6	C1,F1	35%	N/A	-18%	N/A	68%	N/A	35%	N/A	-18%	N/A	68%	N/A
7	C1,F2	10%	N/A	-3%	N/A	18%	N/A	10%	N/A	-3%	N/A	18%	N/A
8	C2,C2	15%	16%	15%	15%	13%	18%	51%	52%	49%	54%	14%	89%
9	C2,C3	4%	25%	10%	14%	9%	17%	16%	85%	29%	59%	14%	90%
10	C2,C4	17%	131%	18%	125%	17%	129%	22%	162%	17%	207%	28%	112%
11	C2,C5	6%	145%	9%	99%	9%	88%	7%	215%	8%	184%	14%	74%
12	C2,F1	47%	N/A	-34%	N/A	125%	N/A	47%	N/A	-34%	N/A	125%	N/A
13	C2,F2	19%	N/A	-8%	N/A	44%	N/A	19%	N/A	-8%	N/A	44%	N/A
14	C3,C3	16%	16%	15%	16%	12%	19%	43%	63%	52%	54%	11%	96%
15	C3,C4	33%	15%	20%	62%	21%	60%	39%	23%	19%	97%	33%	45%
16	C3,C5	12%	94%	17%	57%	15%	72%	19%	124%	16%	141%	22%	102%
17	C3,F1	43%	N/A	-48%	N/A	135%	N/A	43%	N/A	-48%	N/A	135%	N/A
18	C3,F2	26%	N/A	-20%	N/A	61%	N/A	26%	N/A	-20%	N/A	61%	N/A
19	C4,C4	81%	100%	84%	96%	79%	102%	81%	100%	84%	96%	79%	102%
20	C4,C5	24%	100%	46%	62%	43%	68%	24%	100%	46%	62%	43%	68%
21	C4,F1	49%	N/A	-95%	N/A	186%	N/A	49%	N/A	-95%	N/A	186%	N/A
22	C4,F2	34%	N/A	-49%	N/A	117%	N/A	34%	N/A	-49%	N/A	117%	N/A
23	C5,C5	70%	76%	73%	73%	57%	90%	70%	76%	73%	73%	57%	90%
24	C5,F1	34%	N/A	-98%	N/A	147%	N/A	34%	N/A	-98%	N/A	147%	N/A
25	C5,F2	68%	N/A	-85%	N/A	159%	N/A	68%	N/A	-85%	N/A	159%	N/A

Analyzing the results in Tables 11 and 12, we obtain several observations and insights:

- Not surprisingly, the benefit associated with collaborating increases as the probability that a load can be served by its associated carrier decreases. Note that when the benefit under distribution D2 is not higher than the benefit under distribution D1, it is an instance in which the probability that a load can be served by its associated carrier is the same under both distributions.

- The benefit associated with collaborating, measured by the increase in system revenue, increases with the size of the network and fleet capacity. Under demand distribution D1 there are slightly diminishing returns, as the percentage increase in profit declines as network size and fleet capacity increase, while under distribution D2, the percentage increase in profit increases with network and fleet size. Thus we conclude that the marginal benefit associated with increasing network and fleet sizes in collaborating partners increases as the proportion of loads that a carrier can serve using only his network decreases.

- Under distribution D1, fleet capacity has more impact than network size on the benefit associated with collaborating. Furthermore, a carrier with large fleet capacity does not experience a significant increase in the number of loads completely accepted for delivery when collaborating with a carrier with small fleet capacity. These observations lead to an interesting insight: consider the relationship between a large national carrier and a smaller subsidiary. If the subsidiary carrier can serve a high proportion of its own demand (as is the case under demand distribution 1), the parent carrier stands to benefit more by increasing the fleet size of its subsidiary than by increasing the size of the subsidiary network.

- The benefit associated with collaborating, measured both by the percentage increase in the number of loads completely accepted for delivery as well as by an improvement over $v(i)$, is strictly positive when a carrier collaborates with another carrier. The number of loads completely accepted strictly decreases, however, for a carrier collaborating with a freight forwarder. This result suggests that carriers may want to negotiate rules regarding priority of the carrier's loads relative to the forwarder's loads in order that the carrier's customer service level does not decline as a result of entering into collaboration with a forwarder.

- Given that an allocation in which both carriers receive non-negative benefit is a core allocation, our proposed allocation mechanism is behaving as expected with regard to the results of Section 2.5. Namely, every allocation under the Strict Control model is a core allocation, while for some instances, the Limited Control model yields an allocation outside the core. Adding stability constraints to the Limited Control model results in a core allocation.

Although the results confirm that allocations obtained under the Strict and Limited Control models are as expected with respect to the core, closer inspection reveals that the mechanism can apportion alliance benefit in an arbitrarily disproportionate manner. For example, under the Limited Control model, the second carrier in the alliance always receives more benefit, proportionally, than does the first carrier. This behavior occurs because the mechanism has been designed to be indifferent when selecting among all feasible settings for the capacity exchange prices; when implemented using a standard LP solver, therefore, default pivoting rules will have a large impact on the final solution obtained, in this case resulting in a tendency to favor the second carrier. Clearly, then, it is desirable for an alliance to have more control over the allocations obtained when implementing the mechanism; this can be accomplished by guiding the choice of capacity exchange prices towards those prices that result in desired allocations. Establishing rules for selecting among feasible capacity exchange prices, and the adaptation of the mechanism for incorporating these rules, will be discussed in Chapter 4.

3.2.2 Results and Insights from Three-Carrier Alliances

The results presented for alliances consisting of three carriers pertain only to the system revenue and loads accepted, which are calculated similarly to the two-carrier case. Table 13 contains rounded results obtained when demand distribution D1 is implemented, while Table 14 contains the rounded results obtained when distribution D2 is implemented. For each instance class, the results reported again represent the average from 30 instances generated with the same class parameters. Allocation results are omitted because, as discussed in the previous section, while they demonstrate that the mechanism behaves as predicted, they can arbitrarily favor one carrier over another. This will be addressed in Chapter 4.

Some observations regarding the experiments with three carrier alliances are as follows:

- Under demand distribution D1, the benefit associated with adding a third carrier to an existing (or potential) two-carrier alliance varies greatly. For example, when two carriers of type C5 collaborate, both carriers are helped by the addition of a third carrier with large fleet capacity. For an alliance comprised of C1 and C4, adding a third carrier helps C4, but not C1, if the third carrier has a large fleet capacity. Given an alliance comprised of two C1 carriers, adding a third carrier yields no benefit to the original two C1 carriers. This result suggests that some pairs of carriers are in fact better off (in terms of the number of loads accepted) by not adding a third carrier.
- Under distribution D2, it is in general beneficial to all carriers to grow the alliance.
- As in the two carrier experiments, we observe that under distribution D1, higher fleet capacity seems to yield higher benefits from collaborating than does size of network.
- While the number of loads each carrier accepts does not always decrease when two carriers collaborate with a freight forwarder, we still observe a dramatic decline in the number of carrier loads accepted as compared to when two carriers collaborate with a third carrier.

We also see even more pronouncedly in the three-carrier alliances that collaborating yields much higher benefits as the proportion of loads that can initially be served entirely by their associated carrier decreases. This effect, in addition to the first two observations discussed above, implies that the properties of demand experienced by carriers can greatly impact how much the carriers can benefit by collaborating. Before evaluating the benefits of a potential alliance or additional partner, therefore, it is important to consider the characteristics of the demand associated with each carrier.

Table 13: System Revenue and Accepted Loads for Three-Carrier Alliances (Distribution D1)

Instance Class	Carriers (A, B, C)	Chg. in System Revenue		Chg. in Loads Accepted		
		Actual	%	A	B	C
1	C1,C1,C1	57	7%	6%	6%	7%
2	C1,C1,C2	52	8%	7%	4%	14%
3	C1,C1,C3	42	7%	4%	5%	18%
4	C1,C1,C4	48	8%	4%	6%	74%
5	C1,C1,C5	48	8%	6%	5%	126%
6	C1,C1,F1	270	49%	-9%	-7%	N/A
7	C1,C1,F2	97	17%	4%	2%	N/A
8	C1,C2,C2	55	11%	5%	15%	12%
9	C1,C2,C3	35	8%	5%	11%	11%
10	C1,C2,C4	43	11%	7%	12%	58%
11	C1,C2,C5	40	10%	5%	13%	98%
12	C1,C2,F1	250	63%	-12%	-3%	N/A
13	C1,C2,F2	89	22%	3%	3%	N/A
14	C1,C3,C3	28	7%	2%	16%	12%
15	C1,C3,C4	27	8%	1%	17%	65%
16	C1,C3,C5	25	7%	2%	12%	66%
17	C1,C3,F1	207	60%	-16%	-19%	N/A
18	C1,C3,F2	76	23%	-1%	6%	N/A
19	C1,C4,C4	42	13%	5%	86%	75%
20	C1,C4,C5	31	10%	4%	71%	61%
21	C1,C4,F1	239	82%	-20%	34%	N/A
22	C1,C4,F2	84	28%	-2%	44%	N/A
23	C1,C5,C5	24	8%	1%	83%	89%
24	C1,C5,F1	221	77%	-19%	8%	N/A
25	C1,C5,F2	71	24%	-7%	59%	N/A
26	C1,F1,F1	332	111%	-39%	N/A	N/A
27	C1,F1,F2	243	80%	-28%	N/A	N/A
28	C1,F2,F2	108	35%	-14%	N/A	N/A
29	C2,C2,C2	49	15%	14%	11%	14%
30	C2,C2,C3	33	12%	7%	12%	14%
31	C2,C2,C4	46	20%	11%	13%	95%
32	C2,C2,C5	41	19%	11%	15%	116%
33	C2,C2,F1	234	106%	-23%	-16%	N/A
34	C2,C2,F2	95	42%	3%	5%	N/A
35	C2,C3,C3	28	13%	6%	15%	13%
36	C2,C3,C4	29	17%	4%	15%	57%
37	C2,C3,C5	22	13%	2%	16%	52%
38	C2,C3,F1	175	110%	-26%	-27%	N/A
39	C2,C3,F2	70	46%	-6%	-5%	N/A
40	C2,C4,C4	40	29%	16%	60%	71%

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Table 13 – continued from previous page

Instance Class	**Carriers (A, B, C)**	**Chg. in System Revenue**		**Chg. in Loads Accepted**		
		Actual	**%**	**A**	**B**	**C**
41	C2,C4,C5	28	21%	9%	33%	85%
42	C2,C4,F1	207	174%	-41%	-15%	N/A
43	C2,C4,F2	90	77%	3%	68%	N/A
44	C2,C5,C5	20	16%	5%	105%	76%
45	C2,C5,F1	180	154%	-39%	11%	N/A
46	C2,C5,F2	66	58%	-12%	18%	N/A
47	C2,F1,F1	220	170%	-66%	N/A	N/A
48	C2,F1,F2	179	139%	-54%	N/A	N/A
49	C2,F2,F2	98	80%	-22%	N/A	N/A
50	C3,C3,C3	22	15%	13%	11%	14%
51	C3,C3,C4	21	19%	10%	12%	42%
52	C3,C3,C5	20	18%	7%	13%	86%
53	C3,C3,F1	117	118%	-35%	-30%	N/A
54	C3,C3,F2	57	55%	-9%	-2%	N/A
55	C3,C4,C4	26	31%	19%	27%	35%
56	C3,C4,C5	21	28%	10%	62%	60%
57	C3,C4,F1	126	196%	-42%	-51%	N/A
58	C3,C4,F2	58	92%	-12%	17%	N/A
59	C3,C5,C5	17	25%	16%	49%	59%
60	C3,C5,F1	99	169%	-44%	-23%	N/A
61	C3,C5,F2	52	92%	-19%	28%	N/A
62	C3,F1,F1	110	190%	-79%	N/A	N/A
63	C3,F1,F2	88	151%	-67%	N/A	N/A
64	C3,F2,F2	58	98%	-43%	N/A	N/A
65	C4,C4,C4	36	94%	90%	76%	103%
66	C4,C4,C5	22	66%	31%	53%	63%
67	C4,C4,F1	129	540%	-56%	-39%	N/A
68	C4,C4,F2	66	219%	22%	3%	N/A
69	C4,C5,C5	18	62%	13%	67%	76%
70	C4,C5,F1	94	434%	-66%	-53%	N/A
71	C4,C5,F2	53	240%	-26%	15%	N/A
72	C4,F1,F1	51	212%	-100%	N/A	N/A
73	C4,F1,F2	49	192%	-98%	N/A	N/A
74	C4,F2,F2	45	179%	-88%	N/A	N/A
75	C5,C5,C5	16	81%	70%	90%	61%
76	C5,C5,F1	72	460%	-71%	-44%	N/A
77	C5,C5,F2	43	333%	-29%	-9%	N/A
78	C5,F1,F1	17	134%	-100%	N/A	N/A
79	C5,F1,F2	18	153%	-98%	N/A	N/A
80	C5,F2,F2	18	155%	-96%	N/A	N/A

Table 14: System Revenue and Accepted Loads for Three-Carrier Alliances (Distribution D2)

Instance Class	Carriers (A, B, C)	Chg. in System Revenue		Chg. in Loads Accepted		
		Actual	%	A	B	C
1	C1,C1,C1	558	171%	139%	149%	147%
2	C1,C1,C2	420	139%	111%	100%	342%
3	C1,C1,C3	340	111%	82%	81%	416%
4	C1,C1,C4	316	101%	79%	84%	5400%
5	C1,C1,C5	297	96%	75%	84%	4033%
6	C1,C1,F1	508	160%	51%	51%	N/A
7	C1,C1,F2	332	99%	70%	62%	N/A
8	C1,C2,C2	315	133%	61%	291%	246%
9	C1,C2,C3	225	100%	38%	196%	327%
10	C1,C2,C4	197	77%	34%	190%	1030%
11	C1,C2,C5	178	70%	31%	214%	1333%
12	C1,C2,F1	387	143%	5%	115%	N/A
13	C1,C2,F2	217	83%	24%	173%	N/A
14	C1,C3,C3	164	82%	19%	285%	231%
15	C1,C3,C4	118	50%	15%	213%	889%
16	C1,C3,C5	100	42%	11%	180%	5850%
17	C1,C3,F1	277	115%	-14%	187%	N/A
18	C1,C3,F2	143	59%	5%	165%	N/A
19	C1,C4,C4	84	32%	11%	918%	707%
20	C1,C4,C5	57	21%	8%	746%	1110%
21	C1,C4,F1	251	88%	-18%	388%	N/A
22	C1,C4,F2	94	34%	-3%	800%	N/A
23	C1,C5,C5	45	16%	2%	1786%	1036%
24	C1,C5,F1	227	81%	-23%	790%	N/A
25	C1,C5,F2	76	26%	-5%	762%	N/A
26	C1,F1,F1	347	118%	-40%	N/A	N/A
27	C1,F1,F2	239	79%	-31%	N/A	N/A
28	C1,F2,F2	110	36%	-13%	N/A	N/A
29	C2,C2,C2	227	153%	142%	136%	144%
30	C2,C2,C3	168	141%	95%	91%	177%
31	C2,C2,C4	141	105%	82%	82%	892%
32	C2,C2,C5	131	107%	87%	77%	1036%
33	C2,C2,F1	314	227%	13%	30%	N/A
34	C2,C2,F2	177	127%	60%	51%	N/A
35	C2,C3,C3	121	126%	52%	148%	156%
36	C2,C3,C4	91	95%	26%	113%	691%
37	C2,C3,C5	82	84%	26%	117%	883%
38	C2,C3,F1	222	219%	-23%	27%	N/A
39	C2,C3,F2	116	114%	10%	79%	N/A
40	C2,C4,C4	71	69%	28%	581%	489%

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Table 14 – continued from previous page

Instance Class	Carriers (A, B, C)	Chg. in System Revenue		Chg. in Loads Accepted		
		Actual	%	A	B	C
41	C2,C4,C5	54	51%	17%	220%	443%
42	C2,C4,F1	218	201%	-39%	238%	N/A
43	C2,C4,F2	100	92%	2%	381%	N/A
44	C2,C5,C5	37	35%	8%	656%	467%
45	C2,C5,F1	185	163%	-39%	207%	N/A
46	C2,C5,F2	74	69%	-12%	307%	N/A
47	C2,F1,F1	220	170%	-66%	N/A	N/A
48	C2,F1,F2	179	139%	-54%	N/A	N/A
49	C2,F2,F2	98	80%	-22%	N/A	N/A
50	C3,C3,C3	104	143%	143%	124%	128%
51	C3,C3,C4	68	103%	75%	79%	195%
52	C3,C3,C5	67	107%	79%	87%	1600%
53	C3,C3,F1	150	225%	-2%	9%	N/A
54	C3,C3,F2	90	130%	35%	39%	N/A
55	C3,C4,C4	46	84%	34%	256%	188%
56	C3,C4,C5	39	77%	29%	281%	395%
57	C3,C4,F1	133	241%	-40%	-8%	N/A
58	C3,C4,F2	63	114%	-14%	144%	N/A
59	C3,C5,C5	35	73%	27%	304%	613%
60	C3,C5,F1	105	193%	-43%	136%	N/A
61	C3,C5,F2	56	108%	-20%	228%	N/A
62	C3,F1,F1	110	190%	-79%	N/A	N/A
63	C3,F1,F2	88	151%	-67%	N/A	N/A
64	C3,F2,F2	58	98%	-43%	N/A	N/A
65	C4,C4,C4	50	208%	164%	214%	155%
66	C4,C4,C5	34	150%	75%	117%	216%
67	C4,C4,F1	124	406%	-49%	-62%	N/A
68	C4,C4,F2	72	306%	32%	8%	N/A
69	C4,C5,C5	25	142%	52%	211%	135%
70	C4,C5,F1	97	448%	-65%	-59%	N/A
71	C4,C5,F2	54	257%	-25%	18%	N/A
72	C4,F1,F1	49	188%	-100%	N/A	N/A
73	C4,F1,F2	50	205%	-100%	N/A	N/A
74	C4,F2,F2	46	194%	-89%	N/A	N/A
75	C5,C5,C5	22	162%	122%	149%	211%
76	C5,C5,F1	71	473%	-62%	-62%	N/A
77	C5,C5,F2	42	318%	-20%	-17%	N/A
78	C5,F1,F1	18	151%	-100%	N/A	N/A
79	C5,F1,F2	17	131%	-99%	N/A	N/A
80	C5,F2,F2	17	139%	-96%	N/A	N/A

3.3 Analysis of the Impact of Network Integration and Compatibility on Alliance Success

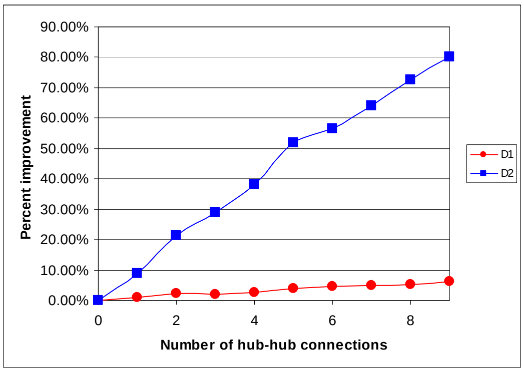
In this second set of experiments we explore how the level of network integration among alliance members impacts the success of the alliance. The goal of one experiment is to gain an understanding of how increasing connectivity between hubs of partnering carriers impacts alliance revenue and accepted loads. In the second experiment the impact of common service points is explored. For example, suppose carrier i is considering two potential partners. One potential partner serves 10% of the same destinations as carrier i, while the other potential carrier has 20% of destinations in common with carrier i. Thus an alliance with the first partner will serve a more diverse set of locations than an alliance with the second partner, but the alliance with the second partner will have more transfer points between the networks of each carrier. Which partner will form a better alliance with carrier i?

Recall that each carrier operates a pure hub-and-spoke network, and that a hub-to-hub route exists between every pair of hubs operated by carrier i. Furthermore, all loads associated with carrier i originate at one of the hubs operated by carrier i; the destinations are randomly generated from the set of spoke destinations according to distribution D1 or D2, as defined in Section 3.1.

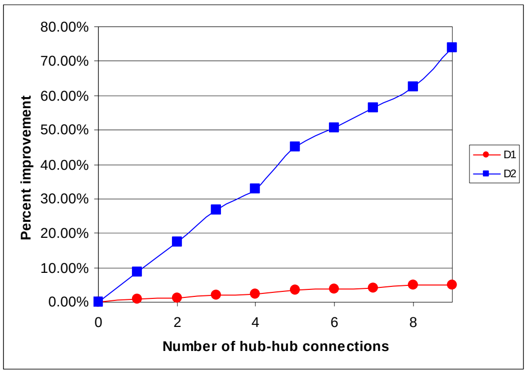
3.3.1 Hub-Hub Connectivity

To investigate the impact of hub-to-hub connectivity, an alliance comprised of two carriers of class C1 is considered. The number of hub-to-hub connections is increased from 0 to 9; as each carrier operates 3 hubs, 9 is the maximum number of connections. The case in which 0 hubs are connected is equivalent to the case in which each carrier is working independently. Let H_1^A, H_2^A, and H_3^A be the hubs operated by carrier A, and H_1^B, H_2^B, and H_3^B be the hubs operated by carrier B. The hub-hub connections are added in the following order: (H_1^A, H_1^B), (H_1^A, H_2^B), (H_1^A, H_3^B), (H_2^A, H_1^B), (H_2^A, H_2^B), (H_2^A, H_3^B), (H_3^A, H_1^B), (H_3^A, H_2^B), (H_3^A, H_3^B). (Changing the order of hub-hub connections does not significantly impact the results.)

For each instance class, where an instance class is defined by the number of hub-to-hub connections, 30 random instances were generated and solved. The results are summarized in Figure 8; for each instance class, the percent improvement in system revenue and number of loads accepted (over the total amount of revenue earned and loads accepted when each carrier works independently) are shown for both demand distributions.



(a) Impact on System Revenue



(b) Impact on Accepted Loads

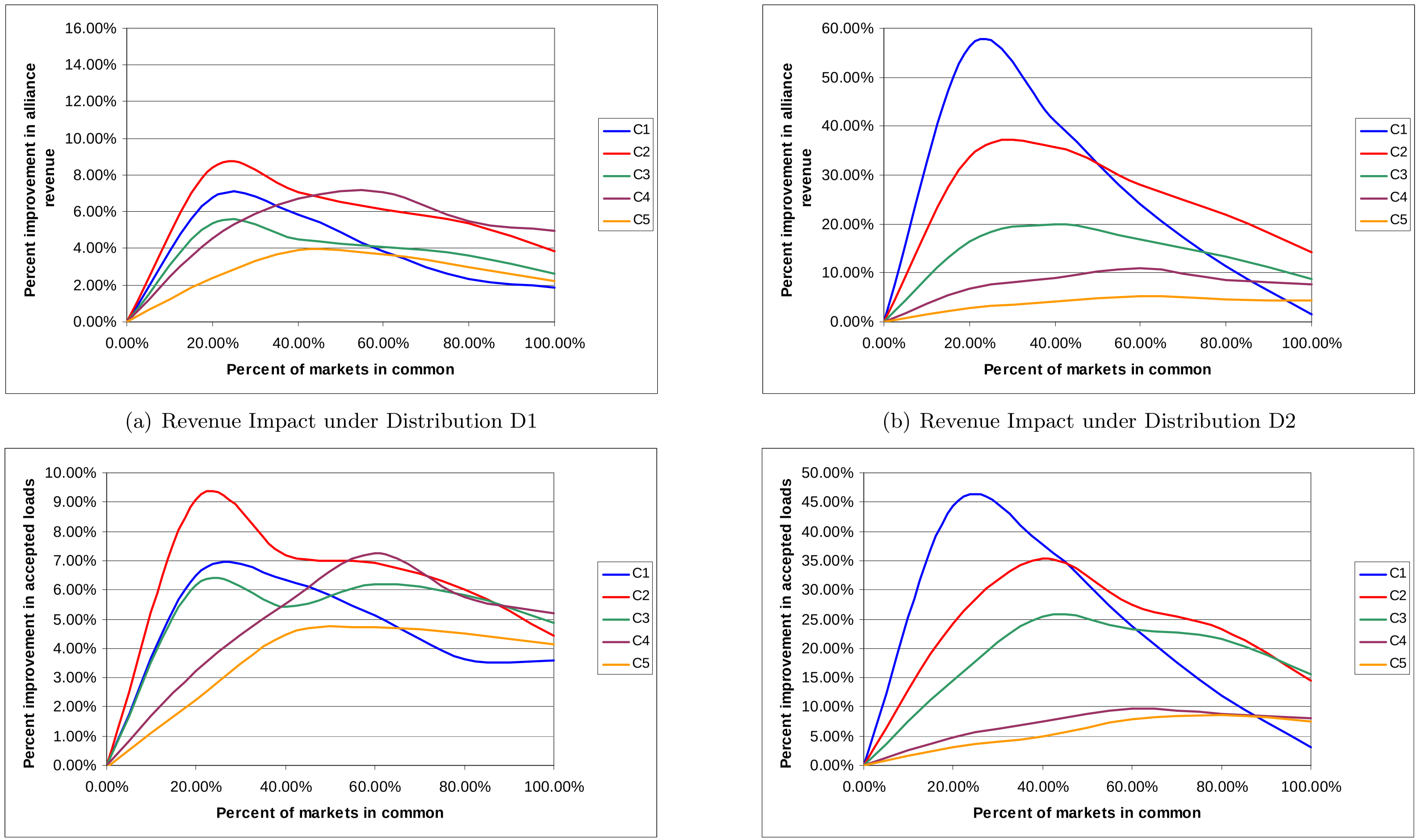
Figure 8: Impact of Hub-Hub Connectivity on {C1,C1} Alliance

As depicted in these graphs, the benefit associated with hub integration increases steadily with the number of interconnected hubs. Not surprisingly, the impact is much more pronounced when carriers must rely on their partner to deliver a high proportion of their associated loads, as is the case in demand distribution D2. The results of this experiment imply that regardless of the distribution of demand, an alliance will be more successful as access between the hubs of partnering carriers is increased.

3.3.2 Complementarity of Markets

In this experiment we investigate how the number of common service points impacts the overall success of the alliance, again in terms of system revenue and loads accepted. In order to allow for loads to be routed from the network of carrier i to carrier j, the set of legs for this experiment is expanded to include legs oriented from spoke-to-hub as well as legs oriented from hub-to-spoke. More specifically, for each pair of hubs h_1, h_2 for which a hub-to-hub route exists, two hub-to-hub legs are generated; one leg originates at time 0 and arrives at time 1, while the second leg originates at time 3 and arrives at time 4. For each (hub, spoke) pair, two hub-to-spoke legs are generated; one leg originates at time 1 and arrives at time 2, while the second leg originates at time 4 and arrives at time 5. Also for each (hub, spoke) pair, one spoke-to-hub leg is generated; this leg originates at time 2 and arrives at time 3. Ultimately, this allows a load to flow along a path comprised of the following legs: $(h_1, h_2), (h_2, s_1), (s_1, h_3), (h_3, s_2)$, where h_1 and h_2 are operated by carrier A, s_2 is a destination in common between both carriers, and h_3 and s_2 are operated by carrier B.

For this experiment, an instance class is defined by the number of spoke destinations in common between two carriers. For all pairs of carriers, the number of common markets was increased from 0 to the maximum number of common markets possible, which is the number of spoke legs operated by the carrier with the smaller network. The results are summarized in Figures 9-13; for a given carrier i, the graphs show the impact of increasing the number of common service points from 0% to 100% when carrier i partners with each of the five classes of carriers. Note that the peak changes as the partner for a given carrier varies–both the optimal percent of market overlap as well as the resulting percent improvement in alliance revenue change with the composition of the alliance. It can be seen in Figure 9 that the impact of common markets is very similar for alliance revenue and accepted loads; this is in fact the case for every carrier. Consequently, only alliance revenue results are summarized here for carriers C2-C5.



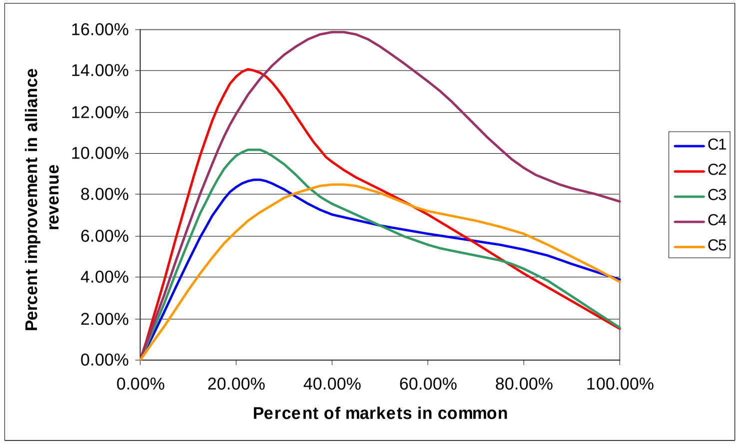
(a) Revenue Impact under Distribution D1

(b) Revenue Impact under Distribution D2

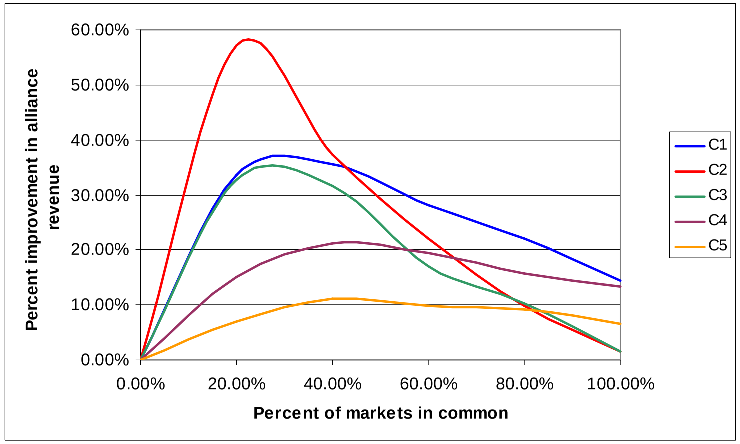
(c) Load Impact under Distribution D1

(d) Load Impact under Distribution D2

Figure 9: Impact of Common Markets for C1 Carrier and Partner

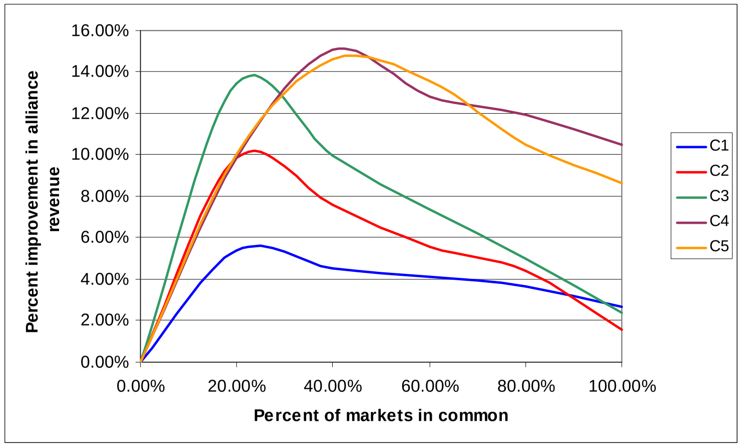


(a) Revenue Impact under Distribution D1

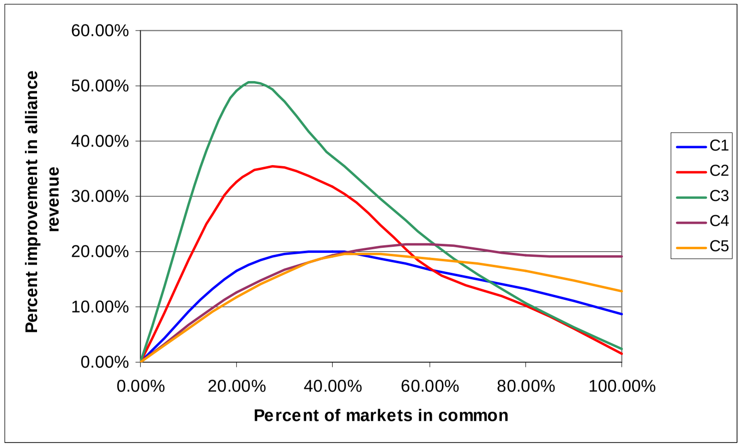


(b) Revenue Impact under Distribution D2

Figure 10: Impact of Common Markets for C2 Carrier and Partner

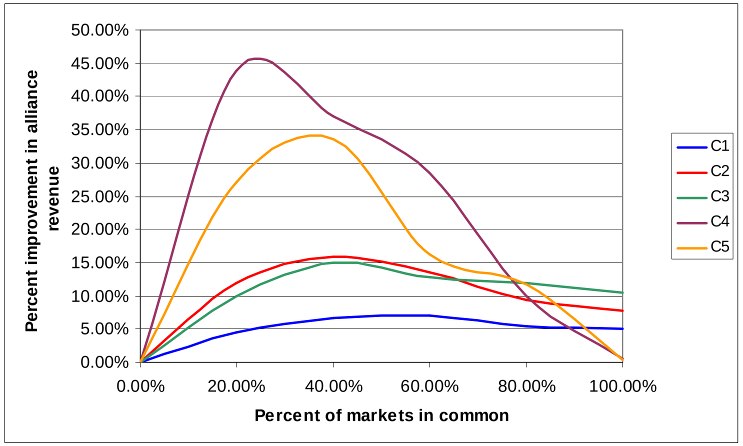


(a) Revenue Impact under Distribution D1

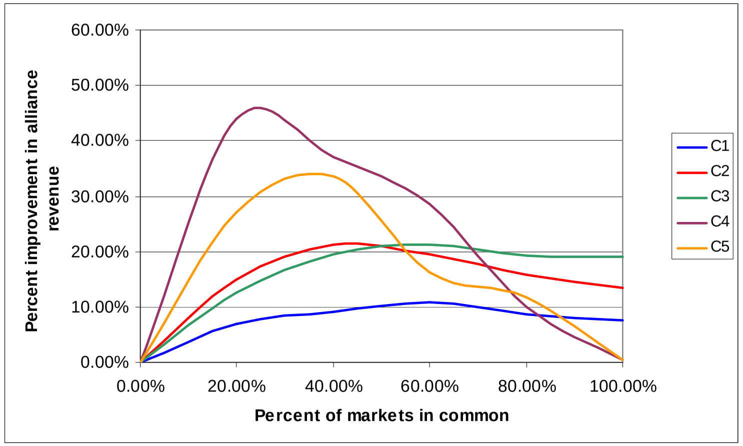


(b) Revenue Impact under Distribution D2

Figure 11: Impact of Common Markets for C3 Carrier and Partner

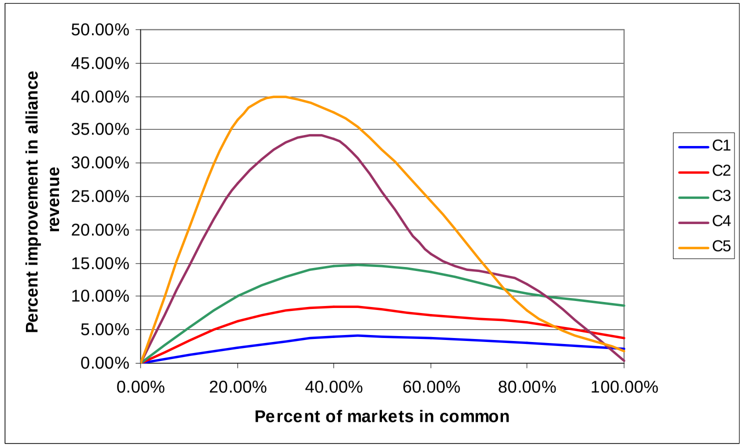


(a) Revenue Impact under Distribution D1

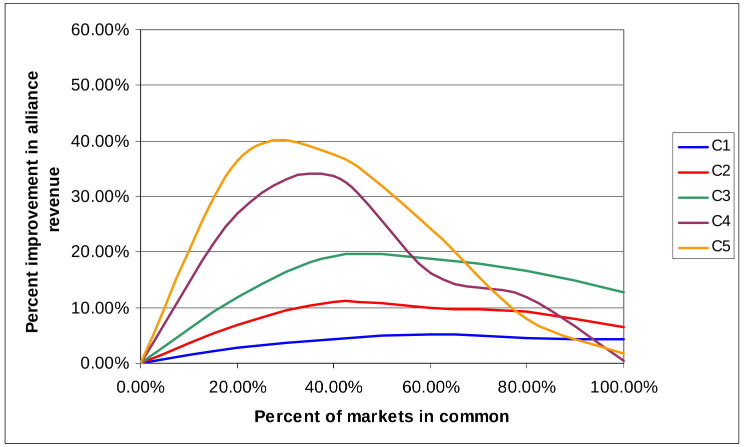


(b) Revenue Impact under Distribution D2

Figure 12: Impact of Common Markets for C4 Carrier and Partner



(a) Revenue Impact under Distribution D1



(b) Revenue Impact under Distribution D2

Figure 13: Impact of Common Markets for C5 Carrier and Partner

Note that these results are based on evaluating the benefit experienced by the alliance as a whole, judged by the percent increase over the total revenue that can be earned when the alliance partners operate independently. We use this measure in favor of measuring the actual increase in alliance revenue because it offers better perspective on the relative gain; a gain that would be considered small for one alliance may be significant for another. Consequently, a partner for which the actual alliance benefit is very high may be considered less attractive than a partner for which the actual alliance benefit is less, but the percent benefit experienced by the alliance is high. This is reasonable, however, given the circumstances under which it occurs. For example, an alliance comprised of a large carrier and a small carrier would experience greater alliance benefit than an alliance comprised of two small carriers. The alliance comprised of two small carriers, however, will experience a higher percent benefit. From the perspective of the small carrier, arguing that another small carrier makes a more attractive partner is not unreasonable, as it is likely that a large alliance partner would command a high proportion of alliance benefit.

Evaluating the impact of increasing the percentage of common markets between alliance partners leads to the following insights pertaining to the optimal level of market overlap, as well characteristics of an attractive alliance partner:

- The results suggest that it is not beneficial for carriers to have more than 60% of their markets in common. This is because increasing the number of common markets essentially increases access to another carrier's network, and as partners' networks more closely resemble each other, the impact of forming an alliance decreases. Consequently, increasing access between the networks is less valuable.

- When a carrier with a large or medium-size network (C1, C2, or C3) collaborates with another carrier with a large or medium-size network, the results suggest that the optimal level of market overlap is approximately 20%. When a carrier with a large or medium-size network collaborates with a carrier with a small network (C4 or C5), the maximum alliance benefit occurs at 40% or 60% market overlap. Because overlap is measured as the percentage of destinations that the smaller carrier has in common with the larger carrier, it is reasonable that the optimal overlap percentage is higher when partnering with smaller carriers.

- For a carrier with a small network size, the maximum percentage alliance benefit occurs at 20% market overlap when collaborating with a carrier with similar network size and fleet capacity, but at 40% or 60% when collaborating with other types of carriers.

- Under demand distribution D1, carriers of type C4 are more attractive partners than carriers of type C3, demonstrating once again that high fleet capacity is a very desirable characteristic in a potential alliance partner.

- Under distribution D2, the most attractive partner for a carrier is always a carrier of similar network size and fleet capacity, yielding an percentage increase in alliance benefit between 40% and 60% for the optimal market overlap of 20%. For other potential partners, the percent alliance benefit decreases with network size and fleet capacity, and the optimal market overlap can vary between 20% and 60%.

The results of this set of experiments demonstrate that for a given carrier, an ideal alliance partner is typically a carrier of similar network size and fleet capacity who serves approximately 20% of the same destinations. The results also imply that for other alliance partners, alliance results can vary greatly according to market overlap, as well as demand, network size, and fleet capacity.

3.4 Analysis of a Real Alliance

In this section a real-world alliance is analyzed; the goal is to evaluate the decisions to both form and grow the alliance. The subject of the analysis is the WOW cargo alliance, which was selected because at the time of this study it is the only cargo alliance for which the set of participating carriers is not a subset of a passenger alliance. This alliance was initially formed in 2002 by Lufthansa, Scandinavian Airlines, and Singapore Airlines, who are currently members of the Star Alliance. Japan Airlines, who is not a member of the

Star Alliance, was later added. After describing how the alliance network was constructed for the experiment, revenue and load acceptance results are presented and discussed.

3.4.1 Data Generation

In order to construct the alliance network, the actual networks operated by the individual members of the alliance were duplicated, subject to the following approximations:

- With the exception of flights between major cities, domestic flights are ignored. For Scandinavian Airlines, intra-Baltic flights (except between major cities) are also ignored. This approximation simplifies the alliance network without significantly compromising the integrity of the study; it is assumed that the formation of an international cargo alliance will not significantly impact strictly domestic (or intra-Baltic) cargo service.

- Networks are approximated as pure hub-and-spoke networks, with only major hubs being considered. Information about the hubs for each carrier is contained in Table 15. While Japan Airlines, Lufthansa, and Scandinavian Airlines each have cities that are the base for around 5-10 flights, the great majority of the destinations served by these "minor" hubs are also served by at least one of the carriers' major hubs. Ignoring minor hubs therefore has minimal impact on the overall study. Furthermore, a few cities are reachable only via a stopover, rather than directly from a hub. For the purposed of this study, these cities (fewer than 15) are included as destinations directly accessible from a hub.

- In general, subsidiary airlines are not included in the study. The exception is Lufthansa; flights operated by both Lufthansa and Lufthansa Regional are considered. Lufthansa Regional operations are considered because they comprise a significant proportion of the overall operations of Lufthanasa. In contrast, the operations of subsidiaries of Japan Airlines (Japan Transocean Air, JAL Express, Japan Air Commuter, J-Air, Hokkaido Air System, JALWays), Singapore Airlines (Silk Air, Tiger Airways), and Scandinavian Airlines (Spanair, Blue1, Air Baltic, Widerøe), as well as other

Table 15: Network Information for Approximated WOW Alliance

Carrier	Hubs	# of Dest. Served from Each Hub	Total # of Dest. Served	(#,%)* of Destinations in Common			
				JAL	LH	SAS	SIA
JAL	Tokyo Osaka	41 17	41	–	23, 56%	22, 54%	25, 61%
LH	Frankfurt Munich	139 79	160	23, 56%	–	39, 62%	38, 63%
SAS	Copenhagen Stockholm	41 17	63	11, 27%	39, 62%	–	16, 27%
SIA	Singapore	60	60	25, 61%	38, 63%	16, 27%	–

* percentage is calculated based on smallest number of destinations, then rounded

Lufthansa subsidiaries (Eurowings, Lufthansa City Line, Air Dolomiti, Augsburg Airways, Contact Air) are relatively insignificant compared to the operations of the parent carriers.

Information regarding hubs, the number of destinations served by each carrier, and the number of destinations in common among the carriers is contained in Table 15. More detailed information regarding the approximated networks can be found in Appendix A. The following abbreviations are used in Table 15, Appendix A, and the remainder of this section: "JAL" for Japan Airlines, "LH" for Lufthansa, "SAS" for Scandinavian Airlines, and "SIA" for Singapore Airlines.

The legs are generated according to the network operated by each carrier, with the origin and destination times being set in a similar manner as in the experiment investigating complementarity of markets (Section 3.3.2). That is, the (origin, destination) time of hub-to-spoke legs is (1,2) and (4,5), the (origin, destination) time of spoke-to-hub legs is (2,3), and the (origin, destination) time of hub-to-hub legs is (0,1) and (3,4). This allows cargo to use the networks of at most two carriers; given that each pair of carriers has at least 27% of destinations in common, it is feasible for cargo to travel between any pair of destinations served by the alliance. Note that generating legs in this manner implies that every city serviced by a carrier is serviced with the same frequency.

In this experiment the hubs operated by a single carrier are connected, but hub H^A, operated by carrier A, and hub H^B, operated by carrier B, are connected only if a route exists between H^A and H^B in the network operated by carrier A or carrier B. Finally, the capacity on each leg is scaled to 5. Each of the four carriers studied has a fleet containing planes with varying capacities. However, all fleets contain a significant number of high-capacity planes, and it is assumed that the higher capacity planes are used on international routes. Therefore, the assigned value reflects the capacity used for carriers with large fleet capacities in Section 3.1.

For this experiment, the loads are generated as described in Section 3.1. The relative size for each carrier, determined according to the number of spoke legs operated by each carrier, is as follows: JAL = 1, LH = 3.5, SAS = 1.5, SIA = 1.

3.4.2 Results and Analysis

We constructed the original three-carrier WOW alliance consisting of LH, SAS, and SIA, as well as the current four-carrier WOW alliance with JAL also included. In addition to evaluating the benefit that the carriers experience by participating in the alliance as compared to working independently, the decision to include JAL in the alliance can also be evaluated. Information pertaining to revenue and load acceptance for each carrier's local (independent) solution is contained in Table 16, along with load acceptance results for the three and four-carrier alliances. The results are the rounded average from 30 instances generated as described in the previous section. Table 17 contains the rounded, averaged results comparing the total revenue for the local solutions and the three and four-carrier alliance solutions.

Based on the alliance revenue and load results, we make the following observations:

- The results for the 3-carrier alliance are as expected when the results of Section 3.2.2 are considered; the 3-carrier alliance of LH, SAS, and SIA, with relative sizes of (3.5, 1.5, 1) is best compared to a {C2,C4,C4} alliance. A {C2,C4,C4} alliance experiences an increase in alliance revenue of 29% over the sum of independent carrier solutions under demand distribution D1, while the increase in revenue is 69% under D2. The

Table 16: Revenue and Load Acceptance Results for Local and Alliance Optimal Solutions

(a) Demand Distribution D1

Carrier	Local Solution		3-Carrier Alliance	4-Carrier Alliance	
	Revenue	Loads Accepted	Chg. in Loads	Chg. in Loads	
				v. local	v. 3-carrier
JAL	126.1	20.4	N/A	140.1%	N/A
LH	917.0	152.1	22.4%	25.9%	2.9%
SAS	245.7	39.8	94.6%	99.7%	2.6%
SIA	264.5	43.7	26.7%	29.1%	1.9%

(b) Demand Distribution D2

Carrier	Local Solution		3-Carrier Alliance	4-Carrier Alliance	
	Revenue	Loads Accepted	Chg. in Loads	Chg. in Loads	
				v. local	v. 3-carrier
JAL	105.9	17.1	N/A	188.3%	N/A
LH	877.0	143.4	29.0%	30.2%	0.9%
SAS	200.9	32.1	137.6%	143.0%	2.3%
SIA	201.9	34.0	55.5%	61.8%	4.0%

Table 17: Change in Revenue for Alliance Before and After JAL

# of Carriers in Alliance	Distribution D1		Distribution D2	
	Change v. Sum of Local	Change v. 3-Carrier	Change v. Sum of Local	Change v. 3-Carrier
3-Carrier alliance	39.2%	–	53.6%	–
4-Carrier alliance	52.1%	19.0%	68.5%	18.7%

3-carrier WOW alliance, meanwhile, experiences a 39% benefit under distribution D1 and a 53.6% benefit under D2. However, as the relative sizes of carriers become closer, the difference between D1 and D2 becomes less pronounced. Consequently, we expect, and do in fact observe, that the gap in benefit between the distributions narrows as well.

- As we would expect based on the results in Sections 3.2.1 and 3.2.2, the number of loads accepted is impacted by the distribution of demand more significantly for the smaller carriers in the alliance. This is not surprising, because the distribution of loads changes more for a small carrier than for a large carrier. This result implies that a small carrier needs to exercise more caution in joining a potential alliance than does a larger carrier, who is impacted less by variation in demand distribution and can therefore be more confident that the expected benefit will be realized.

- Two carriers, JAL and SAS, benefit significantly more from collaborating than do LH and SIA. The reason is different for each carrier, however. SAS has the lowest amount of overlap with the networks of partnering carriers, and therefore experiences significant benefit from gaining access to the other markets. This result makes sense intuitively, as well as in light of the results of Section 3.3.2; note that the network overlap between SAS and JAL, as well as between SAS and SIA, is 27%. JAL, on the other hand, has between 50% and 60% of markets in common with other carriers. However, JAL also serves the fewest number of destinations of any carrier in the alliance, and therefore stands to gain the most (percentage-wise, at least) from access to other markets.

- Based on the percentage increase in accepted loads, LH, SAS, and SIA benefit only marginally from JAL joining the alliance. JAL, on the other hand, experiences a large increase in the number of accepted loads by joining the alliance. This disparity suggests that careful distribution of alliance benefit is necessary to ensure that LH, SAS, and SIA experience measurable gain from the growth of the alliance.

3.5 Summary

In this chapter we have explored different types of alliances, and drawn conclusions pertaining to the potential for success as factors such as demand distribution, network size, fleet capacity, hub integration, and market overlap vary. Implementing the management mechanism developed in Chapter 2 confirms that the mechanism yields allocations within the core of the carrier alliance game when the Strict Control and Stabilized Limited Control behavioral models are employed. However, analysis of allocations obtained for two-carrier alliances demonstrated that refinement was necessary to avoid arbitrary apportionment of alliance benefit.

Several interesting insights were obtained regarding the relationship between demand distribution, network size and fleet capacity. First, the distribution of demand has a large impact on the potential for alliance success. When a high proportion of loads require the use of a partner's network, the benefit associated with collaborating is more substantial, which is to be expected. Alliances must be evaluated more carefully in the case where a greater proportion of demand can be served by a single carrier; in these cases, it may not even be beneficial to grow the alliance. Second, variation in distribution of demand impacts the expected increase in accepted loads for small carriers much more than for large carriers, implying that larger carriers can be more confident regarding the expected benefit (in terms of an increase in accepted loads) associated with collaborating. Third, depending on the distribution of demand, fleet capacity can be a more important factor in determining the benefit associated with collaborating than network size, implying that partners–or subsidiaries–with high fleet capacity may be more valuable than those with smaller fleet capacity but a larger number of markets served.

Regarding the compatibility of member networks, it was demonstrated that increasing hub-to-hub connectivity increases the benefit associated with collaborating in a surprisingly linear fashion. This suggests that carriers in an alliance should take steps to increase access to one another's hubs in order to attain maximum alliance benefit. In a separate study, it was determined that in a majority of cases, ideal partners are those who (1) have similar network size and fleet capacity, and (2) serve 20% of the same destinations.

Analysis of the WOW cargo alliance confirmed that JAL, LH, SAS, and SIA are indeed compatible, experiencing a significant increase in alliance revenue under both demand distributions studied. The addition of JAL to the original alliance of LH, SAS, and SIA increased total alliance revenue around 20%, although the original three carriers experienced only modest gains in the number of loads accepted.

CHAPTER IV

FAIRNESS IN ALLOCATION

In Chapter 2, allocations were examined for their ability to motivate carriers to participate in an alliance. From a game theoretic point of view, any core allocation is desirable, since every subset of participants receives at least as much revenue as it would earn by forming a separate and independent alliance. By examining the allocation results in Table 12 of Section 3.2.1, however, it becomes evident that not all core solutions are equally desirable.

Consider the instance class comprised of carriers C1 (large network size and large fleet capacity) and C5 (small network size and small fleet capacity). The mechanism, when implemented as developed in Chapter 2 and using demand distribution D2, dictates the allocation results contained in Table 18. For each carrier and each behavioral model, the table shows the amount of alliance benefit allocated to each carrier.

Table 18: Allocation Results for {C1,C5} Alliance

Behavioral Model	C1 Benefit	C5 Benefit
Strict Control	8.5	15.6
Limited Control	11.9	12.2
Stabilized Limited Control	19.7	4.3

Carrier C1 is contributing 92.3% of alliance loads and capacity. Yet, under the Strict Control model, C1 is only receiving 35.3% of the total benefit of 24.1 units of revenue gained by the alliance. Under the Limited Control model C1 receives 49.4% of the total benefit, and under the Stabilized Limited Control model C1 receives 81.7% of the total benefit. Because each carrier is receiving non-negative benefit by collaborating, each of these allocations is a core allocation. That the Limited Control model apportions benefit more evenly, or that the Stabilized Limited Control model apportions more benefit to carrier C1, is arbitrary, as any of the above allocations is feasible using any of the behavioral models. (It was shown in Chapter 2 that any solution feasible for the Strict Control model is also feasible for the

Limited Control. Furthermore, because the Stabilized Control model eliminates only non-core allocations from being obtained, any core allocation feasible under the Limited Control model is also feasible under the Stabilized Limited Control model.)

In this chapter we explore how the mechanism developed in Chapter 2 can be adapted to incorporate various notions of fairness. After a brief discussion of literature related to fairness in allocation, we describe a methodology that modifies an inverse problem ($InvStrict$, $InvLC$, or $InvSLC$) to favor an allocation that achieves fairness according to some predetermined measure. Several basic fairness measures are then proposed, and their performance analyzed for the alliances studied in Sections 3.2 and 3.3 of Chapter 3.

4.1 Related Literature

The problem of fairness in allocation has been most widely studied in classic economic applications dealing with the allocation of costs or benefits of publicly owned goods or services. [22] offers a detailed discussion on equity principles that are often applied in theory, but points out that in many applications these principals lead to contradictory or otherwise unsatisfactory solutions. The conflict of selecting from among allocation mechanisms that satisfy some, but not all, desirable properties is also addressed in [34]. In this work several allocation methods are discussed, but two emerge as being particularly suitable when considering principles of equity. The *Shapley value* is shown to be monotonic, which is desirable because allocation schemes exhibiting this property ensure that as a player's contribution to a coalition changes, his allocation will change accordingly. This property is especially important in situations where allocations are periodically reassessed as relevant data changes, such as in the carrier alliance game. However, [34] also demonstrates that monotonicity is contradictory with staying in the core of a game, implying that the Shapley value is not in general a core allocation. An alternative allocation method, which is guaranteed to produce an allocation in the core (when the core is nonempty), is the *nucleolus*. The Shapley and nucleolus allocations will be further discussed in Section 4.3.

Unfortunately there is a disconnect between the theory and practice of fairness in allocation. Allocation schemes with desirable theoretical properties are often computationally or conceptually difficult to implement in practice. Meanwhile, allocation schemes that are commonly used in practice do not often achieve the goals for which they were designed. For example, distribution of revenue based on the relative distance travelled on each carrier's network is a scheme often used among code-sharing partners in the passenger airline industry. It can be easily shown however (see [8], for example), that this scheme leads to very inequitable allocations. [25] addresses both theoretical and practical considerations of cost allocation in the context of transportation procurement networks. In this work, practical limitations of mechanisms that perform well in theory are discussed, and mechanisms are developed that exhibit properties that are desirable in the specific setting addressed. Once again it is shown that no mechanism can satisfy all properties at once.

Outside the realm of traditional cooperative game theory, fairness in assignment of costs or benefits is a relevant consideration in the design of auctions. In classic auction literature, an auction mechanism is designed to maximize the seller's expected revenues. However, auctions are increasingly being used in the public sector, where it is inappropriate to maximize the revenue earned by the seller, or conversely, the prices paid by the bidders. One possibility to increase fairness in the public setting is to design an auction such that outcomes are evaluated with respect to submitted bids, rather than for their ability to simply maximize the seller's revenue. This idea is explored in [6]. [11] proposes a mechanism in which the total payments made by bidders are minimized. The mechanism yields outcomes which are in the core, and can be implemented for auctions dealing with a large number of items.

4.2 Methodology

Before discussing specific fairness measures and how target allocations are computed according to those measures, we introduce notation and discuss the general methodology for adapting the mechanism developed in Chapter 2. Let $\overline{x}$ denote some target allocation, and $z^i = x^i - \overline{x}^i$ be the distance of carrier $i's$ actual allocation x^i from the target allocation for carrier i. $z^i > 0$ indicates that carrier i has been allocated an amount that is strictly greater than targeted allocation amount for him; $z^i < 0$ indicates that carrier i's actual allocation falls short of the target allocation amount. Let $\overline{y}^S$ denote the benefit subset $S \in N$ will experience if the target allocation $\overline{x}$ is attained: $\overline{y}^S = \sum\limits_{i \in S} (\overline{x}^i - v(i))$. Similarly, y^S is the actual benefit experienced by subset S: $y^S = \sum\limits_{i \in S} (x^i - v(i))$. Note that since all mechanism solutions attain the maximum alliance profit, $\sum\limits_{i \in N} x^i = \sum\limits_{i \in N} \overline{x}^i$, implying that $y^N = \overline{y}^N$.

In order to adapt the mechanism to favor a particular target allocation $\overline{x}$ (where $\overline{x}$ is known *a priori*), we introduce for every carrier i the following constraint into $InvStrict$, $InvLC$, or $InvSLC$:

$$\sum_{(o,d,i) \in L^i} f^{*(o,d,k)}_{(d,o,k)} r^{(o,d,k)} + \sum_{e \in E^i} c_e \Big(\sum_{(o,d,j) \notin L^i} f^{*(o,d,j)}_e \Big) - \sum_{e \notin E^i} c_e \Big(\sum_{(o,d,i) \in L^i} f^{*(o,d,i)}_e \Big) - \overline{x}^i \;= z^i. \quad (87)$$

The first three terms of (87) constitute carrier i's actual allocation x^i. By solving the new constraint set using the objective function $\min \sum_i (z^i)^2$, a set of capacity exchange prices will be found that defines an allocation where, overall, carriers are as close as possible to their target allocation. This approach can be easily modified to accommodate alliance needs. For example, if it is more important for some carriers rather than others to achieve their target allocation, which may be the case when some carriers have more bargaining power than others, weights may be incorporated to reflect the relative importance of carriers. In this case one might use the objective function $\min \sum_i w^i |z^i|$, where $\sum_i w^i = 1$ and w^i represents the weight assigned to carrier i.

4.3 Proposed Fairness Rules

In this section, several notions of fairness are proposed. In most cases, the proposed measure is used to allocate the alliance benefit $v(N) - \sum_i v(i)$ among the alliance members. The final notion of fairness discussed, however, enforces a minimum service level for each carrier, where service level is defined as the number of loads accepted. This measure can therefore be used alone or in combination with one of the other measures discussed.

Clearly, the appropriateness of a measure of fairness is dependent on the characteristics of the alliance partners and the underlying network. More sophisticated measures may be defined as necessary, but may require the incorporation of additional data into the model. Of critical importance is understanding the effect that a proposed measure will have on actual allocations; recall for example that the fare proration scheme described in Section 4.1 can lead to allocations that are unstable.

4.3.1 Equal Benefits

A simple definition of fairness might be the following: every carrier in the alliance should receive equal benefit from collaborating. While clearly this would not always be perceived as fair by all participating carriers, the simple *equal benefits* rule is interesting to explore as a base case. In order to achieve fairness according to the equal benefits rule, let $\overline{x}^i = v(i) + \frac{1}{|N|}y^N$. Applying the equal benefits rule to the example in the introduction to this chapter would allocate half of the 24.1 units of alliance benefit to each carrier. Since the amount of revenue carriers C1 and C5 can earn by operating independently are 289.6 and 1.5, respectively, the resulting allocations would be as follows: $x^A = 301.65, x^B = 13.55$. In general, the equal benefits rule distributes the total alliance benefit as equally as possible among all alliance members.

The equal benefits rule is similar in concept to the nucleolus, which maximizes the value of the minimum benefit y^S over all subsets $S \in N$. Intuitively, the nucleolus can be thought of as the "center" of the core. A practical difficulty with the nucleolus is that it cannot in general be efficiently computed. In contrast, the equal benefits allocation can be easily computed and, like all other measures discussed in this section, can be implemented with the addition of $|N|$ constraints.

4.3.2 Value of Contribution: Capacity Value and Load Value

An alternative way to define fairness is in terms of the value each individual member contributes to the alliance. If value was measured only by capacity, then the more valuable the capacity an individual member contributes to the alliance, the more benefit he should receive from participating. This measure might be appropriate when capacity is a scare resource, for example, if airport capacities limit airlines from obtaining landing rights at new locations. Let $R^{(o,d,i)} = r^{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)}$, or the actual amount of revenue earned from load (o, d, i) in the centralized solution f^*. Let $n^{(o,d,i)} = \sum_{(e \in E: k_e < \infty)} f^{*(o,d,i)}$, or the total amount of capacity (over all flights) used by load (o, d, i) in f^*. Then we can allocate the revenue $R^{(o,d,i)}$ over all legs used to deliver load (o,d,i) such that each leg is allocated an amount of revenue proportional to the contribution of that leg's capacity to $n^{(o,d,i)}$. That is, the amount of revenue from (o,d,i) allocated to leg e is $R^{(o,d,i)}\frac{f_e^{*(o,d,i)}}{n^{(o,d,i)}}$. If exactly one path is used to deliver a load, then the load's revenue will be equally allocated among every leg in the path. The total value of capacity on leg e is calculated as $\sum\limits_{(o,d,i)\in L} R^{(o,d,i)}\frac{f_e^{*(o,d,i)}}{n^{(o,d,i)}}$.

Let $value^i$ denote the total value of carrier i's capacity, which is $\sum\limits_{e\in E^i}\sum\limits_{(o,d,i)\in L} R^{(o,d,i)}\frac{f_e^{*(o,d,i)}}{n^{(o,d,i)}}$. The amount of alliance benefit allocated to carrier i is determined by carrier i's proportion of capacity value: $\overline{y}^i = \frac{value^i}{\sum_i value^i} y^N$. Hence the target allocation for carrier i is $\overline{x}^i = v(i) + \frac{value^i}{\sum_i value^i} y^N$.

If instead value was measured only by loads, then the more valuable the loads an individual carrier brings to the alliance, the more benefit he receives. This measure would be appropriate when loads are a scarce resource, for example, in situations where local carriers retain the majority of local business. To implement this measure, we allocate carrier i an amount of benefit according to the proportional value of his loads. The proportional value of carrier i's loads is $P^i = \frac{\sum\limits_{(o,d,i)\in L^i} r^{(o,d,i)} d^{(o,d,i)}}{\sum\limits_{(o,d,i)\in L} r^{(o,d,i)} d^{(o,d,i)}}$, and the target allocation for carrier i is therefore $\overline{x}^i = v(i) + P^i y^N$. Note that in this measure, a carrier is rewarded for all the loads he brings to the alliance; the carrier is not penalized for an alliance decision to reject a load.

For alliances in which some carriers have more local business but do not operate flights to all desired locations, while other carriers operate in-demand flights but do not attract many loads, it makes sense to value both capacity and loads. To this we can assign weights w_c and w_ℓ to the value of capacity and loads, respectively, where $w_c + w_\ell = 1$. Let $\overline{x}_c$ be the target allocation vector computed according to the capacity value rule described above, while $\overline{x}_\ell$ is the target allocation vector computed according to the load value rule. Then the target allocation for carrier i under a fairness measure that values both capacity and loads is $\overline{x}^i = w_c\overline{x}_c^i + w_\ell\overline{x}_\ell^i$.

Computing a target allocation based on the value a carrier brings to an alliance is similar in concept to the Shapley value, since the Shapley value for carrier i is the average marginal contribution of carrier i to each subset of the alliance. However, choosing to define an allocation based on the value of a carrier has two distinct advantages: in addition to the conceptual appeal of defining an allocation based on the setting in which it is applied, the capacity value and load value measures are easy to compute. (Like the nucleolus, the Shapley value in general cannot be efficiently computed.)

4.3.3 Minimum Service Level

The computational experiments performed in Chapter 3 do not indicate that carriers in general suffer a decline in the number of loads accepted, the notable exception being alliances in which one or more partners is a freight forwarder. Even for alliances not containing freight forwarders, however, there is value in exploring the notion of enforcing minimum service levels.

A minimum service level may be defined in two different ways:

(1) a carrier i must deliver at least as many loads as he could deliver by working independently.

(2) every load delivered in carrier i's independent solution must also be delivered in the alliance solution.

The first definition is implemented by adding the following constraints to C: $\sum_{(o,d,i)\in L^i} f^{*(o,d,i)}_{(d,o,i)} \geq \sum_{(o,d,i)\in L^i} f'^{(o,d,i)}_{(d,o,i)} \quad \forall i \in N$, where f' is the optimal solution to carrier i's independent problem. The second definition is implemented by instead adding constraints $f^{*(o,d,i)}_{(d,o,i)} \geq f'^{(o,d,i)}_{(d,o,i)} \quad \forall (o,d,i) \in L$. The latter definition is thus more constraining than the former, but has the appeal of protecting carrier i's existing customers, ensuring that they will not suffer when carrier i joins an alliance.

Because the aggregation of all independent solutions must be feasible from the alliance perspective, enforcing a minimum service level in either of the two ways proposed above does not impact the feasibility of C. The functionality of the mechanism itself is not impacted by changing the alliance optimal solution, since the mechanism is designed to ensure that some alliance solution f^* is attained. As a result, a minimum service level can be implemented (and in fact achieved) in conjunction with one of the revenue allocation methods discussed in Sections 4.3.1 and 4.3.2.

4.4 Computational Analysis of Fairness Measures

In this section we analyze the performance of the equal benefits, capacity value, and load value fairness measures. The goal is to gain a general understanding of (1) how both the targeted and realized allocations change with each measure of fairness, (2) how the appropriateness of the measures changes with the composition of the alliance, and (3) how the choice of behavioral model impacts the success of achieving the targeted allocation. To achieve the first goal, results are presented for two-carrier alliances and the WOW alliance describing in detail the target allocation for each carrier under each rule. (The target allocation is not impacted by the choice of behavioral model). The second and third goals are accomplished by analyzing, for each measure and each model, how close the allocations achieved by the mechanism are to the target allocations.

4.4.1 Two-Carrier Alliance Fairness Results

For each measure of fairness, the target allocation according to that measure, as well as the percent of alliance benefit each carrier will receive if the target allocation is met, are shown in Tables 19 and 20; Table 19 contains the target allocations for instances generated using demand distribution D1, while Table 20 contains the allocations for instances generated using distribution D2. Recall that $\overline{x}^i$ denotes the target allocation for carrier i, and $\overline{y}^i$ denotes the benefit carrier i will receive if he is allocated $\overline{x}^i$. The target benefit under the equal benefits rule is always 50%, and is therefore omitted from the tables.

The target allocations under the capacity value and load value fairness measures behave as we would expect as the network size and fleet capacity of the carriers comprising an alliance change. For example, when two carriers with similar network size and fleet capacity collaborate, the target benefit allocated to each carrier is roughly 50%. When dissimilar carriers collaborate, the larger carrier (in terms of network size or fleet capacity) is targeted to receive more benefit under both measures. We also observe that the distribution of demand has little impact on the percent benefit each carrier will experience if the target allocation is achieved. This is not surprising since changing the distribution of demand does not change the network operated by a carrier or the value of loads associated with a carrier.

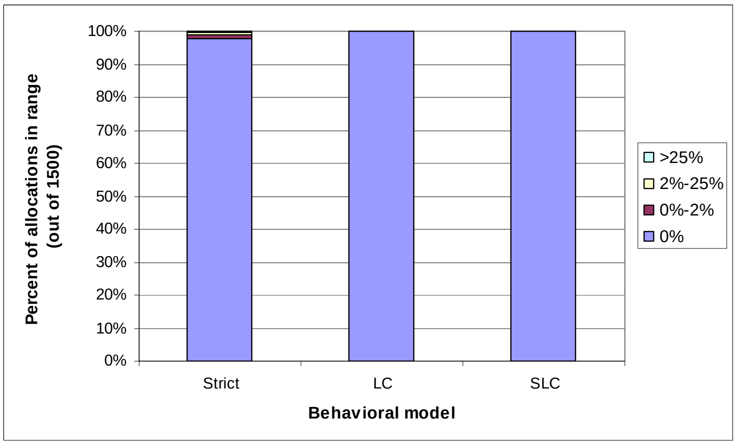
Table 19: Target Allocations for Two-Carrier Alliances (Distribution D1)

Class	Carriers	Equal Benefits		Capacity Value				Load Value			
		$\overline{x}^A$	$\overline{x}^B$	$\overline{x}^A$	$\overline{x}^B$	$\overline{y}^A$ / $\overline{y}^N$	$\overline{y}^B$ / $\overline{y}^N$	$\overline{x}^A$	$\overline{x}^B$	$\overline{y}^A$ / $\overline{y}^N$	$\overline{y}^B$ / $\overline{y}^N$
1	C1,C1	290.6	302.8	290.3	303.1	49.3%	50.7%	290.1	303.3	48.90%	51.10%
2	C1,C2	298.1	127.6	304.6	121.1	69.7%	30.3%	304.9	120.9	70.50%	29.50%
3	C1,C3	289.0	60.6	295.3	54.3	82.2%	17.8%	295.4	54.3	82.44%	17.56%
4	C1,C4	301.1	27.8	313.7	15.2	91.2%	8.8%	314.0	14.9	92.12%	7.88%
5	C1,C5	296.9	15.6	303.5	9.0	95.2%	4.8%	303.6	8.9	96.07%	3.93%
6	C1,F1	406.9	103.2	510.1	0.0	100.0%	0.0%	405.8	104.3	49.47%	50.53%
7	C1,F2	331.0	27.6	358.6	0.0	100.0%	0.0%	348.9	9.6	82.53%	17.47%
8	C2,C2	126.9	127.1	126.7	127.2	49.6%	50.4%	126.8	127.1	49.91%	50.09%
9	C2,C3	115.4	60.3	118.2	57.4	65.9%	34.1%	118.2	57.4	65.92%	34.08%
10	C2,C4	121.2	28.7	131.3	18.6	79.9%	20.1%	132.1	17.8	82.44%	17.56%
11	C2,C5	118.2	13.4	124.2	7.4	89.3%	10.7%	124.5	7.1	91.27%	8.73%
12	C2,F1	205.5	78.9	284.4	0.0	100.0%	0.0%	173.7	110.7	29.85%	70.15%
13	C2,F2	150.1	27.0	177.1	0.0	100.0%	0.0%	158.4	18.7	65.32%	34.68%
14	C3,C3	60.8	58.9	60.8	58.8	50.4%	49.6%	60.8	58.8	50.36%	49.64%
15	C3,C4	59.5	23.1	63.4	19.1	71.2%	28.8%	63.0	19.5	68.98%	31.02%
16	C3,C5	57.6	13.8	61.5	9.9	80.4%	19.6%	61.9	9.5	83.44%	16.56%
17	C3,F1	98.8	39.8	138.6	0.0	100.0%	0.0%	72.9	65.6	17.50%	82.50%
18	C3,F2	75.1	17.5	92.5	0.0	100.0%	0.0%	74.8	17.7	49.37%	50.63%
19	C4,C4	25.2	24.1	25.1	24.2	49.8%	50.2%	25.1	24.2	49.63%	50.37%
20	C4,C5	18.9	13.4	20.0	12.3	59.5%	40.5%	20.7	11.7	65.74%	34.26%
21	C4,F1	49.9	24.0	73.9	0.0	100.0%	0.0%	29.6	44.3	7.84%	92.16%
22	C4,F2	45.0	16.6	61.6	0.0	100.0%	0.0%	38.0	23.6	28.80%	71.20%
23	C5,C5	11.8	11.4	11.9	11.2	51.3%	48.7%	12.0	11.2	52.00%	48.00%
24	C5,F1	20.9	8.9	29.8	0.0	100.0%	0.0%	12.7	17.1	3.84%	96.16%
25	C5,F2	19.1	8.5	27.5	0.0	100.0%	0.0%	13.0	14.5	14.50%	85.50%

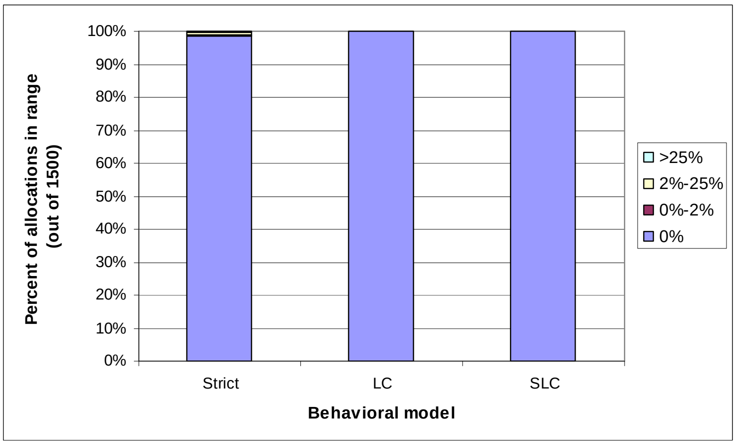
Table 20: Target Allocations for Two-Carrier Alliances (Distribution D2)

Class	Carriers	Equal Benefits		Capacity Value				Load Value			
		$\overline{\mathbf{x}}^{\mathbf{A}}$	$\overline{\mathbf{x}}^{\mathbf{B}}$	$\overline{\mathbf{x}}^{\mathbf{A}}$	$\overline{\mathbf{x}}^{\mathbf{B}}$	$\overline{\mathbf{y}}^{\mathbf{A}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{y}}^{\mathbf{B}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{A}}$	$\overline{\mathbf{x}}^{\mathbf{B}}$	$\overline{\mathbf{y}}^{\mathbf{A}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{y}}^{\mathbf{B}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$
1	C1,C1	289.4	302.6	286.8	305.2	49.0%	51.0%	286.8	305.2	49.01%	50.99%
2	C1,C2	304.9	118.5	339.2	84.1	70.6%	29.4%	338.8	84.6	70.29%	29.71%
3	C1,C3	266.6	63.6	292.2	38.0	80.2%	19.8%	294.3	35.9	82.69%	17.31%
4	C1,C4	307.0	23.5	324.6	5.8	91.6%	8.4%	324.7	5.7	92.00%	8.00%
5	C1,C5	301.6	13.5	312.5	2.6	95.3%	4.7%	312.7	2.5	95.95%	4.05%
6	C1,F1	406.9	103.2	510.1	0.0	100.0%	0.0%	405.8	104.3	49.47%	50.53%
7	C1,F2	331.0	27.6	358.6	0.0	100.0%	0.0%	348.9	9.6	82.53%	17.47%
8	C2,C2	128.8	126.7	128.6	126.9	49.8%	50.2%	129.3	126.3	50.40%	49.60%
9	C2,C3	93.5	64.1	99.0	58.6	59.1%	40.9%	103.2	54.4	66.00%	34.00%
10	C2,C4	125.6	25.0	138.6	12.0	80.9%	19.1%	139.4	11.2	82.69%	17.31%
11	C2,C5	120.5	12.0	128.4	4.1	89.8%	10.2%	128.7	3.8	91.29%	8.71%
12	C2,F1	205.5	78.9	284.4	0.0	100.0%	0.0%	173.7	110.7	29.85%	70.15%
13	C2,F2	150.1	27.0	177.1	0.0	100.0%	0.0%	158.4	18.7	65.32%	34.68%
14	C3,C3	59.6	60.8	59.2	61.1	49.4%	50.6%	59.9	60.4	50.63%	49.37%
15	C3,C4	61.4	16.2	67.5	10.2	76.8%	23.2%	65.7	11.9	69.07%	30.93%
16	C3,C5	60.4	11.3	66.3	5.3	82.2%	17.8%	66.5	5.1	83.38%	16.62%
17	C3,F1	98.8	39.8	138.6	0.0	100.0%	0.0%	72.9	65.6	17.50%	82.50%
18	C3,F2	75.1	17.5	92.5	0.0	100.0%	0.0%	74.8	17.7	49.37%	50.63%
19	C4,C4	25.2	24.1	25.1	24.2	50.0%	50.4%	25.1	24.2	49.85%	50.58%
20	C4,C5	18.9	13.4	20.0	12.3	59.5%	40.5%	20.7	11.7	65.74%	34.26%
21	C4,F1	49.9	24.0	73.9	0.0	100.0%	0.0%	29.6	44.3	7.84%	92.16%
22	C4,F2	45.0	16.6	61.6	0.0	100.0%	0.0%	38.0	23.6	28.80%	71.20%
23	C5,C5	11.8	11.4	11.9	11.2	51.3%	48.7%	12.0	11.2	52.00%	48.00%
24	C5,F1	20.9	8.9	29.8	0.0	100.0%	0.0%	12.7	17.1	3.84%	96.16%
25	C5,F2	19.1	8.5	27.5	0.0	100.0%	0.0%	13.0	14.5	14.50%	85.50%

Figures 14-16 summarize the performance of each fairness rule. For each instance and each carrier, the percentage distance between the actual allocation x^i and the target allocation $\overline{x}^i$ was computed. The graphs depict how many times an allocation missed the target by the given range; the number of total trials is 1500 for each behavioral model because 25 instance classes were tested (all two-carrier alliance combinations), with 30 instances generated for each class, and 2 allocations (one for each carrier) per instance.



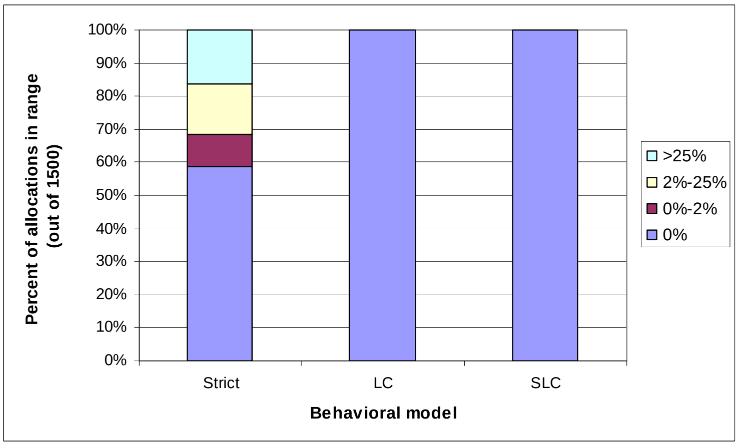
(a) Equal Benefits (D1)



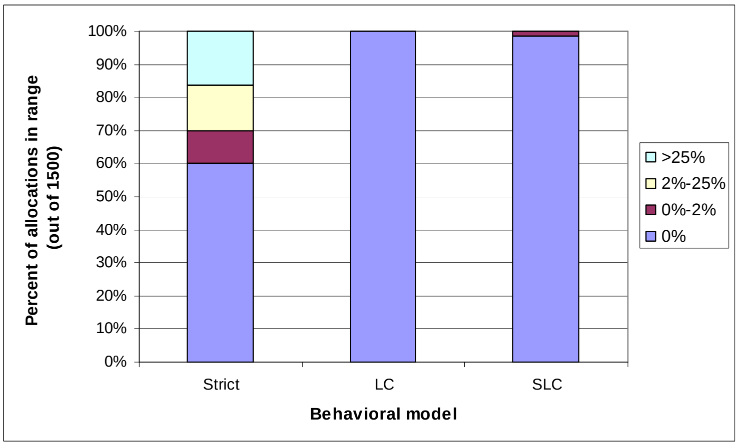
(b) Equal Benefits (D2)

Figure 14: Performance of Equal Benefits Rule for Two Carrier Alliances

In general, the Limited Control and Stabilized Limited Control models are very effective at achieving the target allocation. As expected based on the analysis conducted in Chapter 2, the smaller feasible region of the Strict Control model compromises the performance of the model with regard to fairness. We also observe that the results do not vary significantly

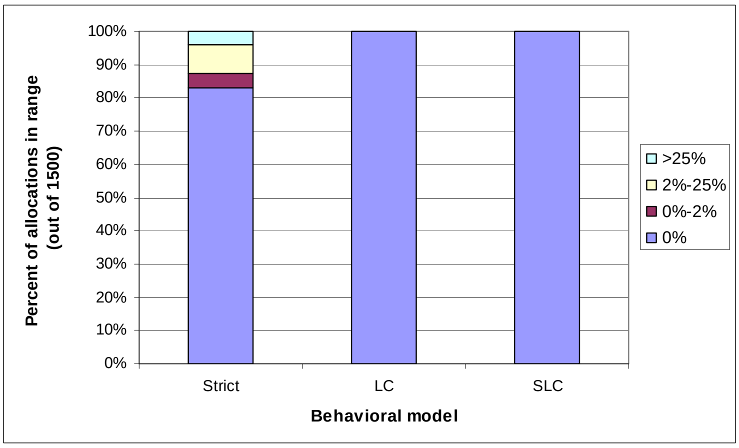


(a) Capacity Value (D1)

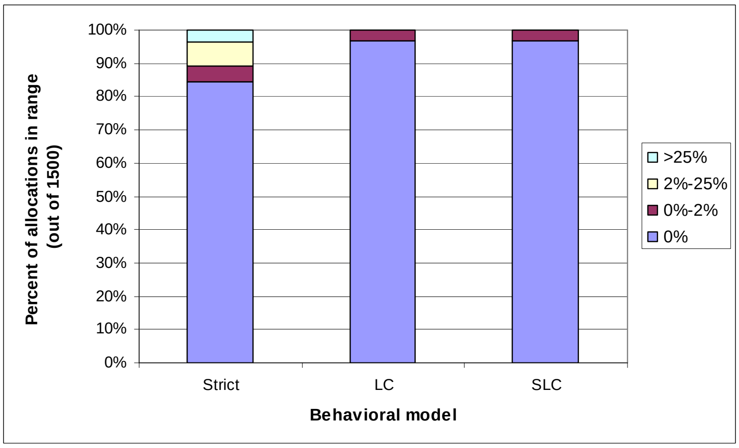


(b) Capacity Value (D2)

Figure 15: Performance of Capacity Value Rule for Two Carrier Alliances



(a) Load Value (D1)



(b) Load Value (D2)

Figure 16: Performance of Load Value Rule for Two Carrier Alliances

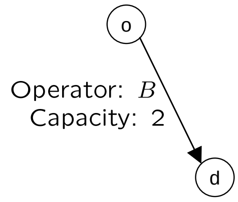


Figure 17: Alliance Network for Fairness Example

Table 21: Load Descriptions for Fairness Example

Load	Per-Unit Revenue ($r^{(o,d,k)}$)	Size ($d^{(o,d,k)}$)
(o,d,A^1)	2	1
(o,d,A^2)	1	1

depending on the distribution of demand. A theoretical analysis of the ability of the mechanism to achieve a target allocation computed according to the Equal Benefits rule using the Strict Control model is conducted for a simple example in Appendix B.

The computational results suggest that the neither the composition of the alliance nor the distribution of demand has significant impact on the performance of the Limited Control and Stabilized Limited Control models, and they have only minor impact on the performance of the Strict Control model. For this reason, we reserve more in depth analysis for three-carrier alliances. However, it is appropriate to note that in this experiment, every equal benefits target allocation was achieved exactly when the Limited Control and Stabilized Limited Control models were implemented. This is not in general always true; consider the simple example depicted in Figure 17 and Table 21. For this example the notation is modified to differentiate between two loads with the same origin and destination, both associated with carrier A.

Leg (o,d) is operated by carrier B and has a capacity of two units. The centralized optimal solution is to deliver both (o,d,A^1) and (o,d,A^2), and the total alliance benefit is 3. Given that neither carrier can earn any revenue by working alone, a target allocation computed according to the equal benefits rule will allocate 1.5 units of revenue to each carrier. However, the maximum feasible value for $c_{(o,d)}$ is 1, since for any $c_{(o,d)} > 1$ carrier A will not deliver (o,d,A^2). The maximum allocation for carrier B is therefore 2, and the equal benefits target cannot be satisfied.

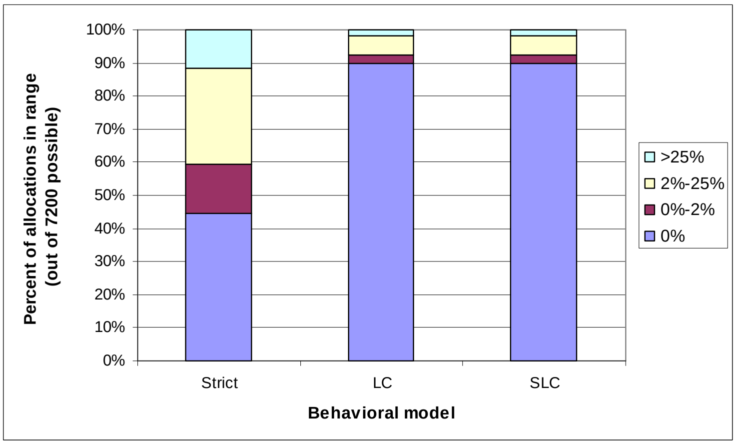
4.4.2 Three-Carrier Fairness Results

The performance of the equal benefits fairness measure for three carrier alliances is summarized for all instances in Figure 18. Similarly to the results for two-carrier alliances, we observe that the behavior of each measure and model combination changes very little with the distribution of demand. For this reason, only the results for distribution D1 are presented in the remainder of the section. Figures 19(a) and 19(b) summarize the performance of the Capacity Value and Load Value fairness measures, respectively, over all three-carrier instances. Over all instances, target allocations are hardest to achieve for the Capacity Value rule, followed by Equal Benefits and then Load Value. Again as expected, the Limited Control and Stabilized Limited Control models are more successful at achieving the target allocations than the Strict Control model.

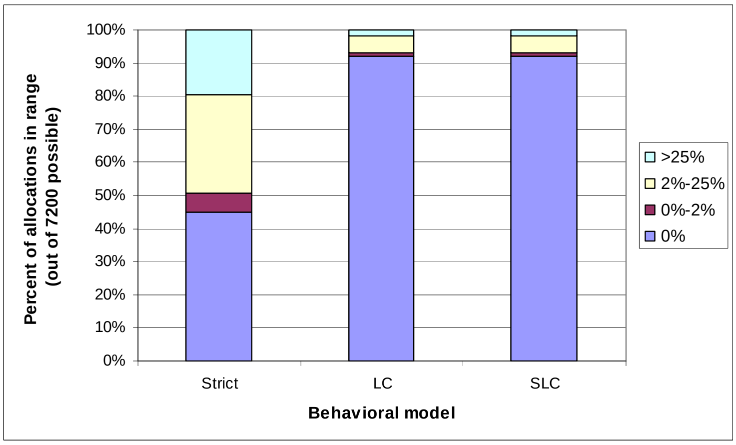
Figures 20 and 21 summarize the performance of fairness measures and behavioral models for alliances comprised of three similar carriers (i.e. three carriers of the same classification) and alliances comprised of three carriers with dissimilar network sizes (i.e. one carrier of type C1, one carrier of type C2 or C3, and one carrier of type C4 or C5). Comparing the results, we observe that relative success in achieving the target allocation does not change significantly between the two sets of instances. This implies that varying network size and fleet capacity of collaborating carriers does not substantially impact the relative performance of a particular fairness measure.

Finally, Figure 22 summarizes the results for each fairness measure and model for alliances containing at least one freight forwarder. The performance of the Strict Control model is markedly worse for these instances, while the Limited Control and Stabilized Limited Control models perform slightly worse for the Equal Benefits and Load Value measures, and marginally better for the Capacity Value measure. (The improved performance for the Capacity Value rule occurs because it is feasible to allocate no benefit to freight forwarders under both the Limited Control model and the Stabilized Limited Control model.)

Overall, the Limited Control and Stabilized Limited Control model perform substantially

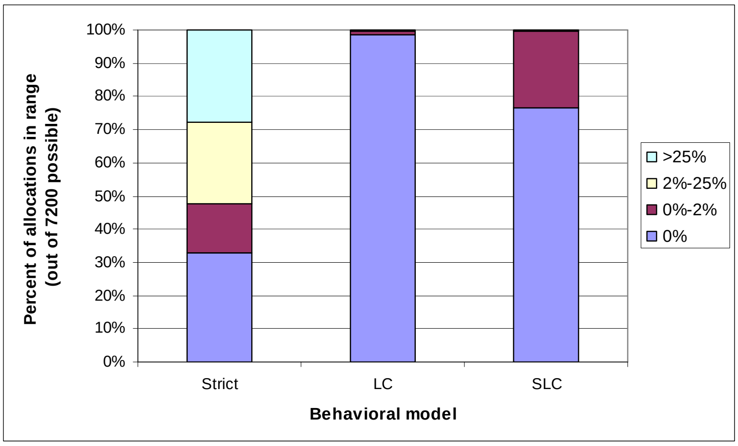


(a) Equal Benefits (D1)

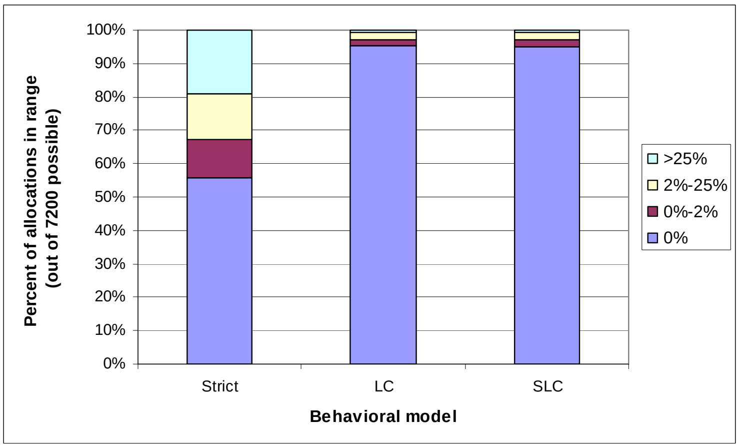


(b) Equal Benefits (D2)

Figure 18: Performance of Equal Benefits Rule for Three Carrier Alliances

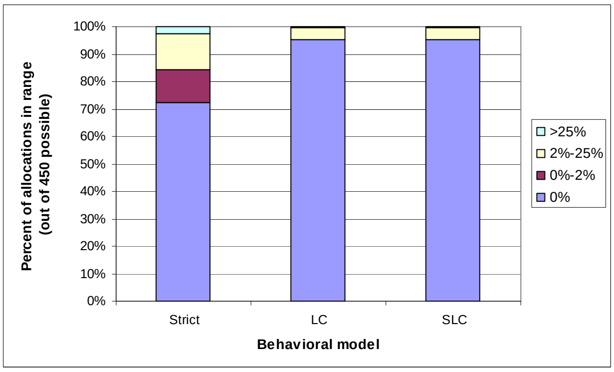


(a) Capacity Value (D1)

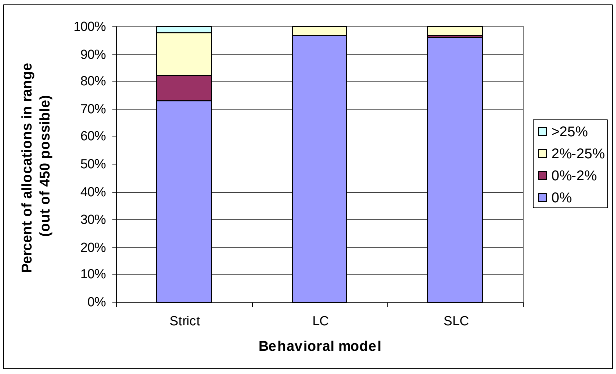


(b) Load Value (D1)

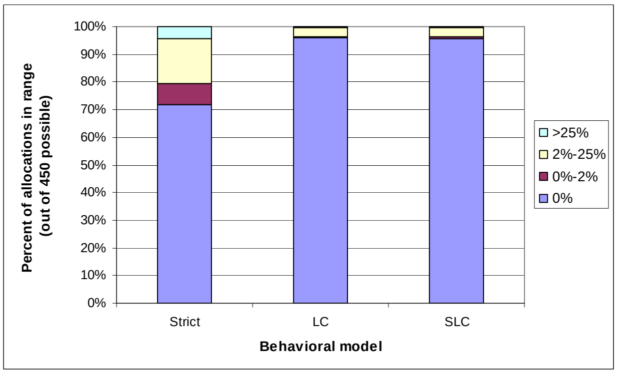
Figure 19: Performance of Capacity Value and Load Value Rules for Three Carrier Alliances



(a) Equal Benefits

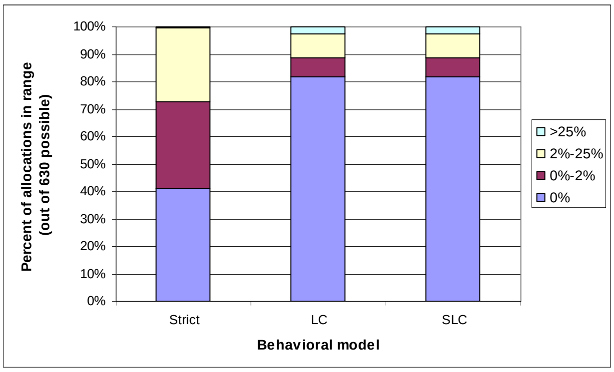


(b) Capacity Value

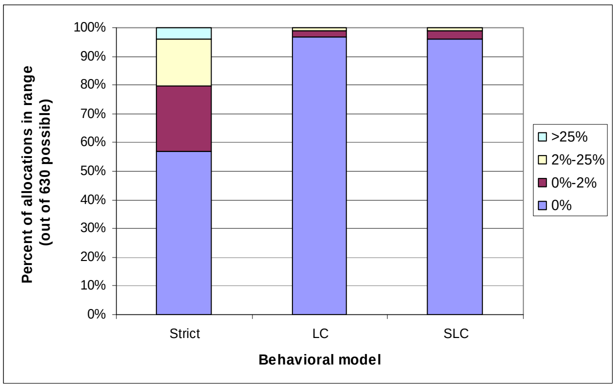


(c) Load Value

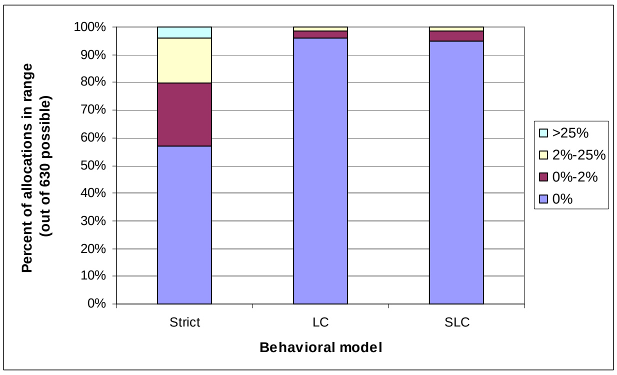
Figure 20: Performance of Fairness Measures for Three Carrier Alliances Containing Similar Carriers



(a) Equal Benefits

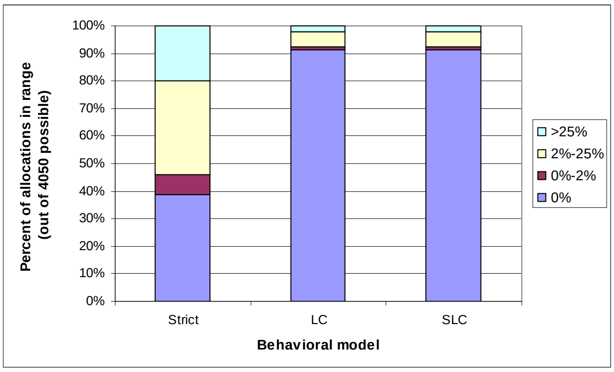


(b) Capacity Value

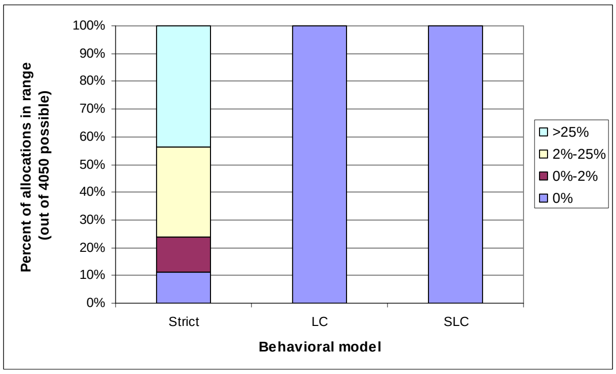


(c) Load Value

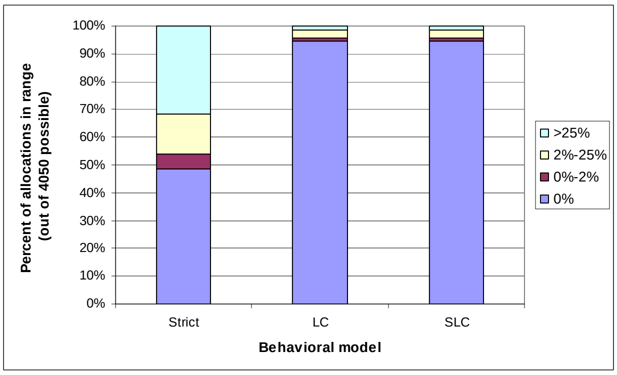
Figure 21: Performance of Fairness Measures for Three Carrier Alliances Containing Carriers with Dissimilar Networks



(a) Equal Benefits



(b) Capacity Value



(c) Load Value

Figure 22: Performance of Fairness Measures for Three Carrier Alliances Containing Forwarders

better for three-carrier instances than does the Strict Control model.

4.4.3 WOW Alliance

When applied to the WOW alliance example, the equal benefits, capacity value, and load value fairness measures perform very well under the Limited Control and Stabilized Limited Control model. In fact, for all 30 instances generated for the four-carrier alliance, the target allocation was exactly matched under these two behavioral models.

The performance of the fairness measures under the Strict Control model was not tested. First, the results of the previous experiments indicate that the Limited Control and Stabilized Limited Control are more appropriate models to use when adapting the mechanism. Second, on even this scaled-down version of a real-world alliance, the Strict Control model yields an inverse problem so large that it cannot be easily solved on a machine with 16G of memory. (Recall that in Section 2.5.2 it was shown that the Strict Control model contains approximately n times the number of constraints as the Limited Control model, where $n = |N|$.)

In order to gain insight into how alliance benefit is allocated under each fairness measure, the target allocations are summarized in Table 22. For each carrier, the target allocation and percent of alliance benefit allocated to the carrier are shown for each fairness measure, where "EB" denotes the Equal Benefits rule, "CV" denotes the Capacity Value rule, and "LV" denotes the Load Value rule. Once again, the results represent the rounded average over 30 generated instances.

As with the two and three-carrier alliances analyzed in the previous section, demand distribution has very little impact on how alliance benefit is apportioned; carriers are allocated a very similar percentage of alliance benefit under both distributions. The capacity value and load value rules in general behave as expected, with LH commanding the largest portion of alliance benefit due to its larger size. (As can be seen in Table 15 in Section 3.4.1, LH operates approximately 55% of all alliances legs, while JAL, SAS, and SIA each operate approximately 15%.)

Table 22: Target Allocations for WOW Alliance

(a) Distribution D1

Measure	JAL		LH		SAS		SIA	
	$\overline{\mathbf{x}}^{\mathbf{JAL}}$	$\overline{\mathbf{y}}^{\mathbf{JAL}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{LH}}$	$\overline{\mathbf{y}}^{\mathbf{LH}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{SAS}}$	$\overline{\mathbf{y}}^{\mathbf{SAS}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{SIA}}$	$\overline{\mathbf{y}}^{\mathbf{SIA}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$
EB	325.5	25.0%	1105.6	25.0%	438.1	25.0%	481.0	25.0%
CV	227.1	12.8%	1324.4	52.0%	388.4	18.9%	410.1	16.3%
LV	230.6	13.3%	1312.9	50.6%	412.6	21.9%	394.0	14.3%

(b) Distribution D2

Measure	JAL		LH		SAS		SIA	
	$\overline{\mathbf{x}}^{\mathbf{JAL}}$	$\overline{\mathbf{y}}^{\mathbf{JAL}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{LH}}$	$\overline{\mathbf{y}}^{\mathbf{LH}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{SAS}}$	$\overline{\mathbf{y}}^{\mathbf{SAS}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$	$\overline{\mathbf{x}}^{\mathbf{SIA}}$	$\overline{\mathbf{y}}^{\mathbf{SIA}}$ / $\overline{\mathbf{y}}^{\mathbf{N}}$
EB	339.7	25.0%	1107.2	25.0%	422.6	25.0%	448.4	25.0%
CV	220.7	12.6%	1369.8	52.4%	363.2	18.8%	364.1	16.2%
LV	227.6	13.3%	1352.1	50.6%	392.5	21.9%	345.7	14.3%

4.5 Summary

It is clear from the performance of the fairness measures under each behavioral model that the Limited Control and Stabilized Limited Control models are more successful at ensuring a target allocation can in fact be obtained. Furthermore, they are more practical from an implementation standpoint; because the inverse problems *InvLC* and *InvSLC* are significantly smaller than *InvStrict*, they require less memory to solve.

The allocations dictated by the fairness measures change appropriately with the characteristics of the carriers, indicating that choosing to allocate revenue using a measure based on characteristics of the alliance setting is a reasonable approach. That the performance of the measures is not significantly impacted by distribution of demand is also promising, implying that the measures may be suitable for a wide variety of alliance circumstances. Finally, the equal benefits, capacity value, and load value fairness measures are all easy to compute, offering another practical advantage over the Shapley and nucleolus allocations.

CHAPTER V

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This chapter summarizes the primary contributions of this thesis and describes both technical extensions and high level research questions motivated by this work.

5.1 Summary

In Chapter 2 a mechanism is proposed that manages the interactions of carriers in an alliance such that the alliance optimal acceptance and routing of loads is attained. The mechanism allocates alliance resources and profits through the use of capacity exchange prices; after appropriate capacity exchange prices are determined, the allocation of revenue is achieved without the need for a centralized manager.

Two distinct ways of modeling the perspective of an individual carrier within an alliance are proposed: a Strict Control model requiring the oversight of a centralized authority, and a Limited Control model that is self-managing. The ability of the mechanism to achieve alliance optimal behavior under the different behavioral models is analyzed, leading to several interesting practical and theoretical insights:

- Surprisingly, the Limited Control model can guarantee centralized feasibility while the Strict Control model cannot; this is a clear practical advantage of the Limited Control model.
- The Strict Control model always defines an allocation in the core, but many core allocations are excluded from the feasible region of the inverse problem defined by this model. On the other hand, the feasible region for the inverse problem defined by the Limited Control model defines more allocations in the core, but may also include non-core allocations. (The Stabilized Limited Control model is proposed to eliminate from the feasible region any allocation outside the core.)
- It is shown that secondary markets will not exist when the Strict Control model is

employed, which is a practical advantage of this model since secondary markets lead to behavior that is detrimental to the alliance as a whole.

- Overall, the discovery that differences in modeling can significantly impact the performance of the mechanism is itself a key insight.

The compatibility of carriers and the potential for alliance success is studied in Chapter 3. In addition to confirming that the management mechanism proposed in Chapter 2 performs as expected, the computational results lead to interesting insights regarding how the characteristics of the associated demand, network, and fleet of collaborating carriers impacts the benefit to be gained by collaborating. In addition to two and three-carrier alliances comprised of various types of carriers, the WOW cargo alliance is also studied. The most notable insights are as follows:

- The benefit to be gained by collaborating increases with the network and fleet size of a partnering carrier, and fleet size is the more important factor.

- Results suggest the benefit associated with collaborating increases in an approximately linear fashion with the number of hub-to-hub routes between hubs of partnering carriers.

- The ideal level of market overlap varies between 20% and 60% depending on the characteristics of the partnering carriers.

The notion of fairness in allocation is the subject of Chapter 4, in response to observing that more control is necessary over the allocations obtained from the mechanism proposed in Chapter 2. Several measures of fairness are proposed, two of which (capacity value and load value) are based on characteristics of the carriers participating in the alliance. In addition to the appeal of allocation methods defined based on the setting in which they are applied, the proposed measures can be efficiently computed, in contrast to traditional allocation schemes studied in the cooperative game theory literature. A methodology for enforcing a minimum service level for each carrier is also proposed.

Key insights from Chapter 4 include the following:

- Computational results suggest that the Limited Control and Stabilized Limited Control model are more suitable than the Strict Control model for adapting the mechanism to incorporate fairness, since these models have a much higher success rate for achieving the desired "fair" allocation.
- Achieving a target allocation using the Strict Control model is especially difficult when freight forwarders are involved in the alliance.
- The equal benefits and load value fairness measures have the highest success rate among the allocations; the capacity value fairness measure is more difficult to achieve.
- The distribution of demand, network size, and fleet capacity of collaborating carriers seem to have minimal impact on the success of a particular fairness measure.

5.2 Future Research Directions

The primary technical extension that follows naturally from this work is the incorporation of time into the centralized model and behavioral models. The decision to represent flight networks through geography only was made in order to simplify analysis; geography was the primary consideration since timing of flights can be more easily adjusted than landing rights can be acquired. The most important impact of including time in the analysis is that frequency of flights between a particular origin and destination can be more easily and accurately be accounted for.

A second technical extension is related to the proposed measures of fairness in Chapter 4. The performance of the measures was explored from a primarily computational perspective; interesting insights may be gained from conducting a thorough theoretical analysis of conditions that restrict the ability of the mechanism to achieve a target allocation. For example, a theoretical analysis of the ability of the Strict Control model to achieve a target allocation computed according to the Equal Benefits fairness measure is conducted for a small example in Appendix B. The conditions impacting whether a target allocation can be achieved will likely vary for various combinations of fairness measure and behavioral model.

Third, there are interesting extensions to explore concerning the definitions of the proposed fairness rules. The proposed rule in which fairness is measured according to the value of loads can be refined to discount loads that are not delivered when all carriers work alone. In addition, using shadow prices for capacity should be explored as a method for defining a measure which values both the capacity and loads associated with a carrier.

There are several interesting high-level research questions motivated by this work. First, it is important to explore how secondary markets for capacity can best be prevented. In Chapter 2 it is shown that the Strict Control model prevents secondary markets, but in Chapter 4 it becomes clear that the Limited Control model (or Stabilized Limited Control model) is a more appropriate modeling choice when fairness in allocation is considered. While trading capacity in the secondary market may be contractually eliminated, a more elegant approach is to identify a way that they can mathematically be prevented. Can the Limited Control model be adapted to prevent a profitable secondary market while maintaining the advantage of a larger feasible region for capacity exchange prices?

Second, this work explores how the characteristics of a carrier's existing network and fleet impact the success of an alliance. While it is important for carriers to be able to identify existing synergies with potential partners, it is also important to understand how a carrier can make strategic decisions to improve the benefit associated with collaborating. For example, what steps can a small carrier take to make himself an attractive partner for a larger carrier? Are there certain markets that the small carrier should add to his network, or on the other hand, markets for which the carrier should discontinue service? Answering questions such as these will not only aid carriers in making business decisions for long-term profitability, but help alliance partners continuously evolve, ensuring sustainability of the alliance.

A third interesting question is the following: how robust are capacity exchange prices with respect to variability in demand? Answering this question will lend insight to how the methodology developed in this thesis can be applied in practice. If capacity exchange prices are shown to be robust, then a stronger argument can be made for the applicability of this work in the real world. If, on the other hand, small changes in demand result in significant changes to the capacity exchange prices, then there is value in exploring whether the methodology can be changed to make it more robust. For example, can the inverse problem be adapted to drive the mechanism towards solutions that are more robust, similar to how the mechanism was driven towards a particular fair allocation in Chapter 4?

Fourth, it is interesting from a theoretical perspective to question if the centralized objective of maximizing alliance revenue is the best choice. Are there other objectives that perform better for some, or perhaps all, secondary considerations such as fairness and prevention of secondary markets? While it can be argued that maximizing revenue is a primary concern for cargo carriers, it may be the case that pursuing other objectives significantly improves alliance performance for secondary considerations with minimal impact on alliance revenue.

Finally, there is value in exploring if the general results or insights from this work can be applied in other contexts. Natural candidates include collaborative ventures that can be modeled using networks or linear programs. Because the methodology is based on collaborative relationships in which decentralized control is desirable (i.e. alliances rather than mergers or acquisitions), applications in which there is are barriers to total integration are the most promising.

APPENDIX A

DESCRIPTION OF APPROXIMATED NETWORKS FOR WOW ALLIANCE

Table 23: Destinations Served by JAL

Destination	From Tokyo	From Osaka	Destination	From Tokyo	From Osaka
Amsterdam	x		London	x	x
Bangkok	x	x	Los Angeles	x	
Beijing	x	x	Manila	x	
Brisbane	x		Mexico City	x	
Busan	x	x	Milan	x	
Chicago	x		Moscow	x	
Dalian	x	x	New York	x	
Delhi	x		Paris	x	
Denpasar	x	x	Qingdao	x	x
Frankfurt	x		Rome	x	
Guam	x	x	San Francisco	x	
Guangzhou	x	x	Sao Paulo	x	
Hangzhou	x	x	Seoul	x	x
Hanoi	x	x	Shanghai	x	x
Ho Chi Minh City	x		Singapore	x	x
Hong Kong	x	x	Sydney	x	
Honolulu	x	x	Taipei	x	x
Jakarta	x		Vancouver	x	
Kaohsiung	x		Xiamen	x	
Kona	x		Xian	x	
Kuala Lumpur	x				

Table 24: Destinations Served by LH

Destination	From Frankfurt	From Munich	Destination	From Frankfurt	From Munich
Abu Dhabi	x		Donetsk		x
Abuja	x		Dubai	x	x
Accra	x		Dublin	x	
Addis Ababa	x		Edinburgh	x	
Alexandria	x		Ekaterinburg	x	
Almaty	x		Faro	x	
Amman	x		Florence	x	x
Amsterdam	x	x	Gdansk	x	x
Ancona		x	Geneva	x	x
Ankara		x	Genoa		x
Ashgabat	x		Gothenburg	x	x
Asmara	x		Graz	x	x
Athens	x	x	Guangzhou	x	
Atlanta	x		Helsinki	x	x
Baku	x		Ho Chi Minh City	x	
Bangalore	x		Hong Kong	x	x
Bangkok	x	x	Houston	x	
Barcelona	x	x	Hyderabad	x	
Bari		x	Istanbul	x	x
Basel	x	x	Izmir		x
Beijing	x	x	Jakarta	x	
Beirut	x		Jeddah	x	
Belgrade	x	x	Johannesburg	x	
Bern		x	Kattowice	x	
Bilbao	x		Kazan	x	
Billund	x	c	Khartoum	x	
Birmingham	x	x	Kiev	x	x
Bologna	x	x	Krakow	x	x
Bordeaux		x	Kuala Lumpur	x	
Boston	x		Kuwait	x	
Bratislava		x	Lagos	x	
Brussels	x	x	Larnaca	x	
Bucharest	x	x	Linz	x	
Budapest	x	x	Lisbon	x	x
Buenos Aires	x		Ljubliana	c	c
Cairo	x		London	x	x
Calgary	x		Los Angeles	x	x
Cape Town	x		Lyon	x	x
Caracas	x		Madrid	x	x
Casablanca	x		Malta	x	
Charlotte		x	Manchester	x	x
Chennai	x		Manila	x	
Chicago	x	x	Marseille	x	x
Copenhagen	x	x	Mexico City	x	
Dallas/Ft. Worth	x		Miami	x	
Dammam	x		Milan	x	x
DC	x	x	Minsk	x	
Delhi	x	x	Montreal		x
Denver	x		Moscow	x	x
Detroit	x		Mumbai	x	x
Dnepropetrovsk	x		Muscat	x	

Continued on next page

Table 24 – continued from previous page

Destination	From Frankfurt	From Munich	Destination	From Frankfurt	From Munich
Nagoya	x		Sofia	x	
Naples		x	St Petersburg	x	
New York	x	x	Stavanger	x	
Nice	x	x	Stockholm	x	
Nizniy Novgorod	x		Strasbourg		x
Osaka	x		Talinn	x	x
Oslo	x	x	Tbilisi		
Paris	x	x	Teheran	x	x
Perm	x		Tel Aviv	x	x
Philadelphia	x		Timisoara		x
Pisa		x	Tokyo	x	x
Port Harcourt	x		Toronto	x	x
Portland	x		Toulouse	x	
Porto	x		Trieste		
Poznan	x	x	Tripoli	x	x
Prague	x	x	Tunis	x	
Riga	x		Turin	x	c
Riyadh	x		Ufa	x	
Rome	x	x	Valencia		x
Rostov	x		Vancouver	x	x
Samara	x		Venice	x	x
San Fancisco	x	x	Verona	x	
Sana'a	x		Vienna	x	x
Santiago de Chile	x		Vilnius	x	x
Sao Paulo	x		Warsaw	x	x
Sarajevo		x	Wroclaw	x	x
Seoul	x		Yerevan		x
Shanghai	x	x	Zagreb	x	
Singapore	x		Zurich	x	

Table 25: Destinations Served by SAS

Destination	From Copenhagen	From Stockholm	Destination	From Copenhagen	From Stockholm
Aberdeen	x		Madrid	x	
Amsterdam	x	x	Malaga		x
Athens	x	x	Manchester	x	x
Bangkok	x		Milan	x	x
Beijing	x	x	Moscow	x	x
Bergen	x	x	Munich	x	x
Berlin	x	x	New York	x	x
Birmingham	x		Nice	x	x
Bologna	x		Nuremberg	x	
Bristol		x	Oslo	x	x
Brussels	x	x	Palanga	x	
Budapest		x	Palma Mallorca		x
Chicago	x	x	Paris	x	x
Cologne	x	x	Poznan	x	
Copenhagen		x	Prague	x	x
DC	x		Pristina	x	
Dublin	x	x	Reykjavik	x	x
Dusseldorf	x	x	Riga		x
Frankfurt	x	x	Rome	x	x
Gdansk	x		Seattle	x	
Geneva	x	x	Split		x
Glasgow		x	St. Petersburg	x	x
Gothenburg	x	x	Stavanger	x	
Hamburg	x	x	Stockholm	x	
Hanover	x		Stuttgart	x	x
Helsinki	x	x	Tallinn		x
Istanbul		x	Tokyo	x	
Kangerlussuaq	x		Venice	x	
Kristiansand	x		Vienna	x	x
London	x	x	Warszaw	x	
Luxembourg	x		Zurich	x	x
Lyon	x				

Table 26: Destinations Served by SIA (from Singapore)

Adelaide	Jakarta
Ahmedabad	Jeddah
Amritsar	Johannesburg
Amsterdam	Kolkata
Athens	Kuala Lumpur
Auckland	London
Bandar Seri Begawan	Los Angeles
Bangalore	Male
Bangkok	Manchester
Barcelona	Manila
Beijing	Melbourne
Brisbane	Milan
Cairo	Moscow
Cape Town	Mumbai
Chennai	Nagoya
Christchurch	Nanjing
Colombo	New York
Copenhagen	Osaka
Delhi	Paris
Denpassar	Penang
Dhaka	Perth
Dubai	Rome
Frankfurt	San Francisco
Fukouka	Seoul
Guangzhou	Shanghai
Hanoi	Sydney
Ho Chi Minh City	Taipei
Hong Kong	Tokyo
Hyderabad	Vancouver
Istanbul	Zurich

Table 27: Common Destinations for Pairs of Carriers in WOW Alliance

JAL, LH	**JAL, SAS**	**JAL, SIA**	**LH, SAS**	**LH, SIA**	**SAS, SIA**
Amsterdam	Amsterdam	Amsterdam	Amsterdam	Amsterdam	Amsterdam
Bangkok	Bangkok	Bangkok	Athens	Athens	Athens
Beijing	Beijing	Beijing	Bangkok	Bangalore	Bangkok
Chicago	Chicago	Brisbane	Beijing	Bangkok	Beijing
Delhi	Frankfurt	Delhi	Birmingham	Barcelona	Copenhagen
Ho Chi Minh City	London	Frankfurt	Bologna	Beijing	Frankfurt
Jakarta	Milan	Guangzhou	Brussels	Cairo	Istanbul
Kuala Lumpur	Moscow	Hanoi	Budapest	Cape Town	London
London	New York	Ho Chi Minh City	Chicago	Chennai	Manchester
Los Angeles	Paris	Jakarta	Copenhagen	Copenhagen	Milan
Manila	Rome	Kuala Lumpur	DC	Delhi	Moscow
Mexico City		London	Dublin	Dubai	New York
Milan		Los Angeles	Gdansk	Ho Chi Minh City	Paris
Moscow		Manila	Geneva	Hong Kong	Rome
New York		Milan	Gothenburg	Hyderabad	Tokyo
Paris		Moscow	Helsinki	Istanbul	Zurich
Rome		New York	Istanbul	Jakarta	
San Fancisco		Paris	London	Jeddah	
Seoul		Rome	Lyon	Johannesburg	
Shanghai		San Francisco	Madrid	Kuala Lumpur	
Singapore		Seoul	Manchester	London	
Toulouse		Shanghai	Milan	Los Angeles	
Vancouver		Sydney	Moscow	Manchester	
		Taipei	New York	Manila	
		Vancouver	Nice	Milan	
			Oslo	Moscow	
			Paris	Mumbai	
			Poznan	Nagoya	
			Prague	New York	
			Riga	Osaka	
			Rome	Paris	
			Stavanger	Rome	
			Stockholm	San Francisco	
			Tallinn	Seoul	
			Tokyo	Shanghai	
			Venice	Tokyo	
			Vienna	Vancouver	
			Warszaw	Zurich	
			Zurich		

APPENDIX B

THE STRICT CONTROL MODEL AND EQUAL BENEFITS FAIRNESS MEASURE: A SMALL FEASIBILITY ANALYSIS

Consider the following simple system, System S, in which carrier A operates a leg with origin o, destination d, and capacity k. There are two loads in the system, also with origin o and destination d; one load is associated with carrier A and one load is associated with carrier B. The revenue and size of each load, as well as the capacity of leg (o, d), are strictly positive.

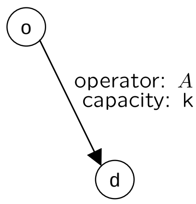


Figure 23: System S

Throughout this analysis we will make use of simplified notation for ease of exposition. The revenue associated with load (o, d, i), $r^{(o,d,i)}$, will be denoted as r^i, size $d^{(o,d,i)}$ as d^i, flow $f^{(o,d,i)}_{(o,d)}$ as $f^i_{(o,d)}$, and fictitious flow $f^{(o,d,i)}_{(d,o,i)}$ as $f^i_{(d,o)}$.

Theorem 11. *Under the Strict Control model, a target allocation for System S computed according to the Equal Benefits rule can be achieved if and only if neither of the following mutually exclusive conditions is satisfied:*

1. $r^B > r^A, d^B \geq k$, $d^A < k$, *and* $r^A > \frac{r^B k}{2k - d^A}$
2. $r^A \geq r^B, d^A < k, d^A + d^B > k.$

Proof. The allocations for each carrier are as follows: $x^A = r^A f^A + cf^B, x^B = r^B f^B - cf^b$. The benefit for carrier B is $y^B = x^B - v(B) = x^B$ since carrier B does not operate any capacity. Under the Equal Benefits rule, $\overline{y}^B = \frac{1}{2}y^N = \frac{1}{2}[x^A + x^B - v(A) - v(B)] = \frac{1}{2}[x^A + x^B - v(A)]$. Because there are only two carriers in this system, $x = \overline{x}$ if and only if $y^B = \overline{y}^B$, which implies $x^B = \frac{1}{2}[x^A + x^B - v(A)]$, or $x^B = x^A - v(A)$. Therefore $x = \overline{x} \Leftrightarrow \exists c : (r^B f^B - cf^B) = (f^A f^A + cf^B) - v(A)$, or $c = \frac{v(A) - r^A f^A + r^B f^B}{2f^B}$. Henceforth the equation

$$c = \frac{v(A) - r^A f^A + r^B f^B}{2f^B} \tag{88}$$

will be referred to as the *key equation*.

The inverse problem constraints under the Strict Control model for System S are written in general form below, with the primal variable corresponding to each constraint indicated to the left of the constraint:

$(InvStrict):$

$$(f^A_{(o,d)}) \quad \pi^{A,A}_d - \pi^{A,A}_o + \alpha^A \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} 0 \tag{89}$$

$$(f^A_{(d,o)}) \quad \pi^{A,A}_o - \pi^{A,A}_d + \beta^{A,A} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} r^A \tag{90}$$

$$(f^B_{(o,d)}) \quad \pi^{A,B}_d - \pi^{A,B}_o + \alpha^A \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} c \tag{91}$$

$$(f^B_{(d,o)}) \quad \pi^{A,B}_o - \pi^{A,B}_d + \beta^{A,B} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} 0 \tag{92}$$

$$(f^A_{(o,d)}) \quad \pi^{B,A}_d - \pi^{B,A}_o + \alpha^B \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} 0 \tag{93}$$

$$(f^A_{(d,o)}) \quad \pi^{B,A}_o - \pi^{B,A}_d + \beta^{B,A} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} 0 \tag{94}$$

$$(f^B_{(o,d)}) \quad \pi^{B,B}_d - \pi^{B,B}_o + \alpha^B \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} -c \tag{95}$$

$$(f^B_{(d,o)}) \quad \pi^{B,B}_o - \pi^{B,B}_d + \beta^{B,B} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} r^B. \tag{96}$$

Inequalities (89)-(92) correspond to carrier A; these inequalities ensure that f^* will be optimal for carrier A. Similarly, inequalities (93)-(96) are associated with carrier B. The variable c represents the capacity exchange price on leg (o,d).

InvStrict simplifies to the following four inequalities:

$$\alpha^A \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad r^A - \beta^{A,A} \tag{97}$$

$$\beta^{A,B} \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad c - \alpha^A \tag{98}$$

$$\alpha^B \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad -\beta^{B,A} \tag{99}$$

$$c + \alpha^B \quad \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \quad r^B - \beta^{B,B} \tag{100}$$

where (97) follows directly from inequalities (89) and (90), (98) follows directly from (91) and (92), (99) follows directly from (93) and (94), and (100) follows directly from (95) and (96).

Note that $f^{*i}_{(o,d)} = f^{*i}_{(d,o)}$, where f^* is the centralized optimal solution for System S. As a result, (97) and (99) must hold with equality when $f^{*A}_{(o,d)} = f^{*A}_{(d,o)} > 0$, and are inequalities otherwise. Similarly, (98) and (100) must hold with equality when $f^{*B}_{(o,d)} = f^{*B}_{(d,o)} > 0$, and are inequalities otherwise. Furthermore, in order to satisfy complementary slackness conditions as described in Section 2.4, $\beta^{i,j} = 0$ when $f^{*j} < d^j$.

The 13 cases depicted in Figure 24 are mutually exclusive and represent all possible relationships among the problem parameters r^i, d^i, and k for System S. Table 28 summarizes, for each case, the values of f^*, $v(A)$, and c in terms of the parameters r^i, d^i, and k, where c is computed according to the key equation (88).

For each of the 13 cases we will further simplify *InvStrict* for System S and find the feasible range for c. We will then analyze if the value of c which satisfies the key equation (88) is within this feasible range. If so, then the target allocation $\overline{x}$ can always be satisfied for that case.

<u>Case 1</u>

In System S, $f^A = 0 < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that (97) and (99) are inequalities. $0 < f^B < d^B$ implies that $\beta^{A,B} = \beta^{B,B} = 0$, and that (98) and (100) must

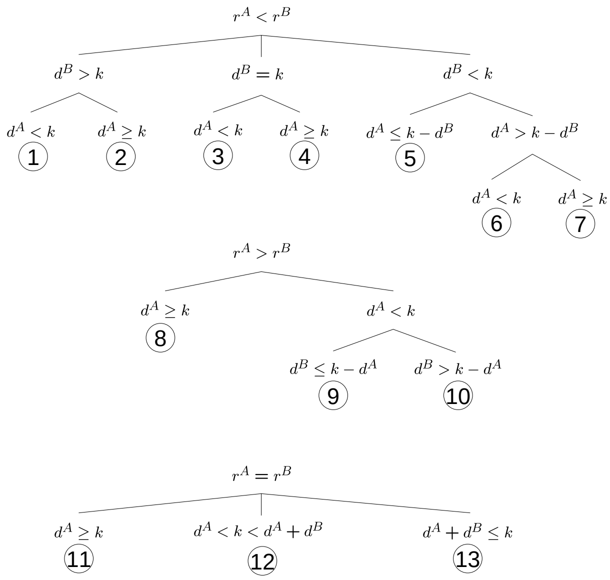


Figure 24: Case Diagram

Table 28: Case Descriptions

Case	f^{*i} †	$v(A)$	$c = \frac{v(A) - r^A f^A + r^B f^B}{2f^B}$
1	$f^A = 0, f^B = k < d^B$	$r^A d^A$	$c = \frac{r^A d^A + r^B k}{2k}$
2	$f^A = 0, f^B = k < d^B$	$r^A k$	$c = \frac{r^A k + r^B k}{2k} = \frac{r^A + r^B}{2}$
3	$f^A = 0, f^B = k = d^B$	$r^A d^A$	$c = \frac{r^A d^A + r^B k}{2k}$
4	$f^A = 0, f^B = k = d^B$	$r^A k$	$c = \frac{r^A k + r^B k}{2k} = \frac{r^A + r^B}{2}$
5	$f^A = d^A \leq k - d^B$ $f^B = d^B < k$	$r^A d^A$	$c = \frac{r^A d^A - r^A d^A + r^B d^B}{2d^B} = \frac{r^B}{2}$
6	$f^A = k - d^B < d^A$ $f^B = d^B < k$	$r^A d^A$	$c = \frac{r^A d^A - r^A (k - d^B) + r^B d^B}{2d^B} = \frac{r^A (d^A + d^B - k) + r^B d^B}{2d^B}$
7	$f^A = k - d^B < d^A$ $f^B = d^B < k$	$r^A k$	$c = \frac{r^A k - r^A (k - d^B) + r^B d^B}{2d^B} = \frac{r^A + r^B}{2}$
8	$f^A = k \leq d^A, f^B = 0$	$r^A k$	$0 = 0$
9	$f^A = d^A < k$ $f^B = d^B \leq k - d^A$	$r^A d^A$	$c = \frac{r^A d^A - r^A d^A + r^B d^B}{2d^B} = \frac{r^B}{2}$
10	$f^A = d^A$ $f^B = k - d^A < d^B$	$r^A d^A$	$c = \frac{r^A d^A - r^A d^A + r^B (k - d^A)}{2(k - d^A)} = \frac{r^B}{2}$
11	$f^A + f^B =$ $k < d^A + d^B$	$r^A k =$ $r^B k$	$c = \frac{r^A (f^A + f^B) - r^A f^A + r^B f^B}{2f^B} = r^A = r^B$
12	$f^A + f^B =$ $k < d^A + d^B$	$r^A d^A =$ $r^B d^A$	$c = \frac{r^A d^A - r^A f^A + r^B f^B}{2f^B} = \frac{r^B (d^A - f^A + f^B)}{2f^B}$
13	$f^A = d^A, f^B = d^B$	$r^A d^A$	$c = \frac{r^A d^A - r^A d^A + r^B f^B}{2f^B} = \frac{r^B}{2}$

† $f^i = f^i_{(o,d)} = f^i(d, o)$

hold with equality (=). Consequently, inequalities (97)-(100) further reduce to $r^A \leq c \leq r^B$.

The key equation (88) for this case simplifies to $c = \frac{r^A d^A + r^B k}{2k}$. We will first show that this value of c is never greater than r^B:

$\frac{r^A d^A + r^B k}{2k} \leq r^B \Rightarrow r^A d^A \leq r^B k \Rightarrow r^B \geq r^A(\frac{d^A}{k})$ which must be true since $r^B > r^A$ and $\frac{d^A}{k} < 1$.

However, $c = \frac{r^A d^A + r^B k}{2k}$ can in fact be less than r^A; this occurs when $r^A > \frac{r^B k}{2k - d^A}$. We have therefore established that in Case 1, the target allocation for System S will be satisfied if and only if $r^A \leq \frac{r^B k}{2k - d^A}$.

Case 2

In System S, $f^A = 0 < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that (97) and (99) are inequalities. $0 < f^B < d^B$ implies that $\beta^{A,B} = \beta^{B,B} = 0$, and that (98) and (100) must hold with equality (=). Consequently, inequalities (97)-(100) further reduce to $r^A \leq c \leq r^B$.

The key equation (88) for this case simplifies to $c = \frac{r^A + r^B}{2}$. We will first show that this value of c is never greater than r^B:

$\frac{r^B + r^A}{2} \leq r^B \Rightarrow r^A \leq r^B$, which must always be true since we are in Case 2.

Next, we show that this value of c is never less than r^A:

$\frac{r^B + r^A}{2} \geq r^A \Rightarrow r^B \geq r^A$, which again must always be true since we are in Case 2.

We have established that in Case 2, there always exists a c feasible for $InvStrict$ that results in an allocation that exactly satisfies the target allocation for System S.

Case 3

In System S, $f^A = 0 < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that (97) and (99) are inequalities. $0 < f^B < d^B$ implies that (98) and (100) must hold with equality. Consequently, inequalities (97)-(100) further reduce to $r^A + \beta^{A,B} \leq c \leq r^B - \beta^{B,B}$, which in turn reduces to $r^A \leq c \leq r^B$ since $\beta^{i,B} \geq 0$.

As in Case 1, the key equation (88) for this case simplifies to $c = \frac{r^A d^A + r^B k}{2k}$. Again, this value of c is never greater than r^B:

$\frac{r^A d^A + r^B k}{2k} \leq r^B \Rightarrow r^A d^A \leq r^B k \Rightarrow r^B \geq r^A(\frac{d^A}{k})$ which must be true since $r^B > r^A$ and $\frac{d^A}{k} < 1$.

However, $c = \frac{r^A d^A + r^B k}{2k}$ is less than r^A when $r^A > \frac{r^B k}{2k - d^A}$. We have therefore established that in Case 3, the target allocation will be satisfied for System S if and only if $r^A \leq \frac{r^B k}{2k - d^A}$.

Case 4

In System S, $f^A = 0 < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that (97) and (99) are inequalities. $0 < f^B < d^B$ implies that (98) and (100) must hold with equality (=). Consequently, inequalities (97)-(100) further reduce to $r^A + \beta^{A,B} \leq c \leq r^B - \beta^{B,B}$, which in turn reduces to $r^A \leq c \leq r^B$ since $\beta^{i,B} \geq 0$.

As in Case 2, the key equation (88) for this case simplifies to $c = \frac{r^A + r^B}{2}$. This value of c is never greater than r^B:

$\frac{r^B + r^A}{2} \leq r^B \Rightarrow r^A \leq r^B$, which must always be true since we are in Case 4.

Furthermore, this value of c is never less than r^A:

$\frac{r^B + r^A}{2} \geq r^A \Rightarrow r^B \geq r^A$, which again must always be true since we are in Case 4.

We have established that in Case 4, there always exists a c feasible for *InvStrict* that results in an allocation that exactly satisfies the target allocation for System S.

Case 5

In System S, $0 < f^A = d^A$ implies that (97) and (99) must hold with equality (=). $0 < f^B = d^B$ implies that (98) and (100) must hold with equality (=). Consequently, *InvStrict* reduces to

$$\begin{aligned}
\alpha^A &= r^A - \beta^{A,A} \\
\alpha^A &= c - \beta^{A,B} \\
\alpha^B &= -\beta^{B,A} = 0 \\
\alpha^B &= r^B - c - \beta^{B,B}
\end{aligned} \tag{101}$$

where the last equality in (102) holds because $\alpha^B \geq 0$ and $\beta^{B,A} \geq 0$. *InvStrict* therefore further reduces to $r^A - \beta^{A,A} + \beta^{A,B} = c = r^B - \beta^{B,B}$. Because $\alpha^A \geq 0$, it must be true that $r^A - \beta^{A,A} \geq 0$, which implies that *InvStrict* ultimately reduces to $0 \leq c \leq r^B$. Note that these bounds on c imply nothing about the relationship between r^A and r^B. This makes intuitive sense because of the conditions of Case 5; because $d^A \leq k - d^B$ and $d^B < k$, it must be true that $d^A + d^B < k$. Therefore, in the Centralized solution for System S, both loads are completely delivered. Because carrier A owns the capacity on leg (o, d), he can also completely deliver his own load when working alone. Carrier A therefore earns the same revenue from delivering loads in both the Centralized and local case. As a result, any solution in which $c \geq 0$ will be feasible for the inverse problem associated with carrier A, which is described by equations (89)-(92). On the other hand, carrier B earns no direct revenue by working alone, and therefore any c for which carrier B's allocation $x^B = r^B f^B_{(d,o)} - c f^B_{(d,o)}$ is non-negative will be feasible for the inverse problem associated with carrier B, described by equations (93)-(96). It follows that any $c \leq r^B$ is feasible for (93)-(96), and we have the desired result that $0 \leq c \leq r^B$.

The key equation (88) for this case simplifies to $c = \frac{r^B}{2}$. Since $r^B > 0$, it follows that $0 \leq c \leq r^B$, and there always exists a c feasible for *InvStrict* that results in an allocation that exactly satisfies the target allocation for System S.

<u>Case 6</u>

In System S, $0 < f^A = k - d^B < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that (97) and (99) must hold with equality (=). $0 < f^B = d^B$ implies that (98) and (100) must hold with equality (=). Consequently, *InvStrict* reduces to $r^A + \beta^{A,B} = c = r^B - \beta^{B,B}$, which in turn reduces to $r^A \leq c \leq r^B$ since $\beta^{i,B} \geq 0$.

The key equation (88) for this case is $c = \frac{r^A(d^A+d^B-k)+r^B d^B}{2d^B}$. We will first show that this value of c is never greater than r^B:

$\frac{r^A(d^A+d^B-k)+r^B d^B}{2d^B} \leq r^B$

$\Rightarrow r^A(d^A + d^B - k) + r^B d^B \leq 2r^B d^B$

$\Rightarrow r^A \leq r^B(\frac{d^B}{d^A+d^B-k})$

Since we are in Case 6, $d^A+d^B-k > 0$. Furthermore, $d^A < k$ implies that $d^A+d^B-k > d^B$. We conclude that $r^A \leq r^B(\frac{d^B}{d^a+d^B-k})$ must always be true since $r^A < r^B$.

Next, we show that this value of c is never less than r^A:

$\frac{r^A(d^A+d^B-k)+r^Bd^B}{2d^B} \geq r^A$

$\Rightarrow r^A(d^A + d^B - k) + r^Bd^B \geq 2r^Ad^B$

$\Rightarrow r^Ad^A - r^Ak + r^Bd^B \geq r^Ad^B$

$\Rightarrow r^A(d^B - d^A + k) \leq r^Bd^B$

$\Rightarrow r^A \leq r^B(\frac{d^B}{d^B-d^A+k})$

$\Rightarrow r^A \leq r^B(\frac{d^B}{d^B-k+k})$ (because $d^A < k$)

$\Rightarrow r^A < r^B$ which must always be true since we are in Case 6.

We have established that in Case 6, there always exists a c feasible for $InvStrict$ that results in an allocation that exactly satisfies the target allocation for System S.

Case 7

In System S, $0 < f^A = k - d^B < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that (97) and (99) must hold with equality (=). $0 < f^B = d^B$ implies that (98) and (100) must hold with equality (=). Consequently, $InvStrict$ reduces to $r^A + \beta^{A,B} = c = r^B - \beta^{B,B}$, which in turn reduces to $r^A \leq c \leq r^B$ since $\beta^{i,B} \geq 0$.

The key equation (88) for this case simplifies to $c = \frac{r^A+r^B}{2}$. Since we are in Case 7, $r^A < r^B$ and it can easily be verified that $r^A < \frac{r^A+r^B}{2} < r^B$. It follows that there always exists a c feasible for $InvStrict$ that results in an allocation that exactly satisfies the target allocation for System S.

Case 8

The key equation reduces to $0 = 0$ because $v(A) = r^Ak, f^A = k$, and $f^B = 0$. The key equation (88) is therefore satisfied by any c which is feasible for $InvStrict$. From Section

2.4.2 we know that a feasible solution to *InvStrict* must exist, so we conclude that the target allocation for System S can always be satisfied in this case.

Case 9

In System S, $0 < f^A = d^A$ implies that (97) and (99) must hold with equality (=). $0 < f^B = d^B$ implies that (98) and (100) must hold with equality (=). Consequently, *InvStrict* reduces to

$$\begin{aligned}\alpha^A &= r^A - \beta^{A,A} \\ \alpha^A &= c - \beta^{A,B} \\ \alpha^B &= -\beta^{B,A} = 0 \\ \alpha^B &= r^B - c - \beta^{B,B}\end{aligned} \tag{102}$$

where the last equality in (102) holds because $\alpha^B \geq 0$ and $\beta^{B,A} \geq 0$. *InvStrict* therefore further reduces to $r^A - \beta^{A,A} + \beta^{A,B} = c = r^B - \beta^{B,B}$. Because $\alpha^A \geq 0$, it must be true that $r^A - \beta^{A,A} \geq 0$, which implies that *InvStrict* ultimately reduces to $0 \leq c \leq r^B$. The intuitive argument made in Case 5 regarding the bounds on c applies in this case as well.

The key equation (88) for this case simplifies to $c = \frac{r^B}{2}$. Since $r^B > 0$, it follows that $0 \leq \frac{r^B}{2} \leq r^B$, and there always exists a c feasible for *InvStrict* that results in an allocation that exactly satisfies the target allocation for System S.

Case 10

In System S, $0 < f^A = d^A$ implies that equations (97) and (99) must hold with equality. $0 < f^B = k - d^A < d^B$ implies that $\beta^{A,B} = \beta^{B,B} = 0$, and that equations (98) and (100) must hold with equality. Consequently, *InvStrict* reduces to

$$\begin{aligned}\alpha^A &= r^A - \beta^{A,A} \\ \alpha^A &= c \\ \alpha^B &= -\beta^{B,A} = 0 \\ \alpha^B &= r^B - c\end{aligned} \tag{103}$$

where the last equality in (103) holds because $\alpha^B \geq 0$ and $\beta^{B,A} \geq 0$. *InvStrict* therefore further reduces to $r^B = c = r^A - \beta^{A,A}$, and finally $c = r^B \leq r^A$. Intuitively, it is reasonable that c cannot be less than r^B, because then it would be optimal for carrier B to deliver more than f^B of load B. (Recall that under the Strict Control model, the capacity available to carrier B in his individual problem is k.) Likewise, if $c > r^B$, carrier B's optimal solution would be $f^B = 0$, since carrier B loses money by delivering load B.

The key equation (88) for this case simplifies to $c = \frac{r^B}{2}$. Because $r^B > 0$, it is not possible for $c = r^B$. We conclude that in this case there cannot exist a c feasible for *InvStrict* that results in an allocation that exactly satisfies the target allocation for System S.

<u>Case 11</u>

For this case, instead of reducing inequalities (97)-(100) based on the characteristics of f^*, we instead propose a solution that satisfies *InvStrict*. The solution $c = r^A = r^B, \alpha^A = \alpha^B = \beta^{B,A} = \beta^{B,B} = 0, \beta^{A,A} = r^A, \beta^{A,B} = r^B$ satisfies (97)-(100) at equality, and is therefore feasible. As $c = r^A = r^B$ is always a feasible solution for *InvStrict*, it follows that in this case the target allocation can always be satisfied for System S.

Intuitively, in this case the centralized profit for System S is the same amount as the profit carrier A can earn alone. Therefore if any capacity is used by carrier B, carrier A must be compensated at least r^A per unit of capacity used by carrier B. That is, $c \geq r^A$. Since $r^A = r^B$, $c \geq r^B$. On the other hand, if $c > r^B$ it cannot be optimal for carrier B to use any capacity, since he would lose money by doing so, so $c \leq r^B$. (Note that in this case, multiple optimal solutions exist for the Centralized problem, so it is possible for $f^B > 0$.)

<u>Case 12</u>

We first analyze five subcases for the relationship of f^i and d^i in System S and demonstrate for each case that the only value of c feasible for *InvStrict* is $c = r^B$.

Case 12a

In System S, $0 < f^A = d^A$ implies that equations (97) and (99) must hold with equality.

$0 < f^B < d^B$ implies that $\beta^{A,B} = \beta^{B,B} = 0$, and that equations (98) and (100) must hold with equality. Consequently, *InvStrict* reduces to

$$\begin{aligned} \alpha^A &= r^A - \beta^{A,A} \\ \alpha^A &= c \\ \alpha^B &= -\beta^{B,A} = 0 \\ \alpha^B &= r^B - c \end{aligned} \tag{104}$$

where the last equality in (104) holds because $\alpha^B \geq 0$ and $\beta^{B,A} \geq 0$. *InvStrict* therefore further reduces to $r^B = c = r^A - \beta^{A,A}$, and finally $c = r^B \leq r^A$.

Case 12b

In System S, $f^A = 0 < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that equations (97) and (99) are inequalities. $0 < f^B = d^B$ implies that equations (98) and (100) must hold with equality. Consequently, inequalities (97)-(100) further reduce to $r^A + \beta^{A,B} \leq c \leq r^B - \beta^{B,B}$, which in turn reduces to $r^A \leq c \leq r^B$ since $\beta^{i,B} \geq 0$. Since we are in Case 12, $r^A = r^B$ and we have that $c = r^B$.

Case 12c

In System S, $0 < f^A < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that equations (97) and (99) must hold with equality. $0 < f^B = d^B$ implies that equations (98), and (100) must hold with equality. Consequently, *InvStrict* reduces to $r^A + \beta^{A,B} = c = r^B - \beta^{B,B}$, which in turn reduces to $r^A \leq c \leq r^B$ since $\beta^{i,B} \geq 0$. Again, since $r^A = r^B, c = r^B$.

Case 12d

In System S, $f^A = 0 < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that equations (97) and (99) are inequalities. $0 < f^B < d^B$ implies that $\beta^{A,B} = \beta^{B,B} = 0$, and that equations (98) and (100) must hold with equality. Consequently, inequalities (97)-(100) further reduce to $r^A \leq c \leq r^B$, and it follows that $c = r^B$ since we are in Case 12.

Case 12e

In System S, $0 < f^A < d^A$ implies that $\beta^{A,A} = \beta^{B,A} = 0$, and that equations (97) and (99) must hold with equality. $0 < f^B < d^B$ implies that $\beta^{A,B} = \beta^{B,B} = 0$, and that equations (98) and (100) must hold with equality. *InvStrict* therefore reduces to $r^A = c = r^B$.

The key equation (88) for Case 12 is $c = \frac{r^B(f^B - f^A + d^A)}{2f^B}$. If $c = r^B$ then we have the following:

$\frac{r^B(d^A - f^A + f^B)}{2f^B} = r^B$

$\Rightarrow r^B(d^A - f^A) = r^B f^B$

$\Rightarrow d^A = f^A + f^B$.

As we are in Case 12, $d^A < k$. Therefore, $d^A = f^A + f^B$ implies $f^A + f^B < k$, which is a contradiction on the optimality of f^A and f^B. We conclude that in this case there cannot exist a c feasible for *InvStrict* that results in an allocation that exactly satisfies the target allocation for System S.

Case 13

In System S,$0 < f^A = d^A$ implies that (97) and (99) must hold with equality (=). $0 < f^B = d^B$ implies that (98) and (100) must hold with equality (=). Consequently, *InvStrict* reduces to

$$\begin{aligned}
\alpha^A &= r^A - \beta^{A,A} \\
\alpha^A &= c - \beta^{A,B} \\
\alpha^B &= -\beta^{B,A} = 0 \qquad (105) \\
\alpha^B &= r^B - c - \beta^{B,B}
\end{aligned}$$

where the last equality in (105) holds because $\alpha^B \geq 0$ and $\beta^{B,A} \geq 0$. *InvStrict* therefore further reduces to $r^A - \beta^{A,A} + \beta^{A,B} = c = r^B - \beta^{B,B}$. Because $\alpha^A \geq 0$, it must be true that $r^A - \beta^{A,A} \geq 0$, which implies that *InvStrict* ultimately reduces to $0 \leq c \leq r^B$. The intuitive argument made in Case 5 regarding the bounds on c applies in this case as well.

The key equation (88) for this case simplifies to $c = \frac{r^B}{2}$. Since $r^B > 0$, it follows that

Table 29: Case Summary

Case	f^{*i} †	Feasible Range for c	Conditions When Key Eqn. (88) Satisfied
1	$f^A = 0, f^B = k < d^B$	$r^A \leq c \leq r^B$	$r^A > \frac{r^B k}{2k-d^a}$
2	$f^A = 0, f^B = k < d^B$	$r^A \leq c \leq r^B$	always satisfied
3	$f^A = 0, f^B = k = d^B$	$r^A \leq c \leq r^B$	$r^A > \frac{r^B k}{2k-d^a}$
4	$f^A = 0, f^B = k = d^B$	$r^A \leq c \leq r^B$	always satisfied
5	$f^A = d^A \leq k - d^B, f^B = d^B < k$	$0 \leq c \leq r^B$	always satisfied
6	$f^A = k - d^B < d^A, f^B = d^B < k$	$r^A \leq c \leq r^B$	always satisfied
7	$f^A = k - d^B < d^A, f^B = d^B < k$	$r^A \leq c \leq r^B$	always satisfied
8	$f^A = k \leq d^A, f^B = 0$	N/A‡	always satisfied
9	$f^A = d^A < k, f^B = d^B \leq k - d^A$	$0 \leq c \leq r^B$	always satisfied
10	$f^A = d^A, f^B = k - d^A < d^B$	$c = r^B \leq r^A$	never satisfied
11	$f^A + f^B = k < d^A + d^B$	$c = r^A = r^B$	always satisfied
12	$f^A + f^B = k < d^A + d^B$	$c = r^B$	never satisfied
13	$f^A = d^A, f^B = d^B$	$0 \leq c \leq r^B$	always satisfied

† $f^i = f^i_{(o,d)} = f^i(d, o)$

‡ A feasible range for c exists, but the specific range is irrelevant since (88) reduces to $0 = 0$.

$0 \leq \frac{r^B}{2} \leq r^B$, and there always exists a c feasible for $InvStrict$ that results in an allocation that exactly satisfies the target allocation for System S.

The results of the thirteen cases are summarized in Table 29. As the table shows, the conditions which prevent the Strict Control model from achieving the Equal Benefits allocation for System S are either (1) $r^B < r^A, d^B \geq k, d^A < k$, and $r^A > \frac{r^B k}{2k-d^A}$ or (2) $r^A \geq r^B$, $d^A < k$, and $d^A + d^B > k$.

□

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VITA

Lori Houghtalen was born in Rochester, New York and was raised in Cincinnati, Ohio. She graduated from the University of Tennessee in Knoxville with a bachelor's degree in Industrial Engineering in the spring of 2002. The following fall she entered the doctoral program in Industrial and Systems Engineering at the Georgia Institute of Technology. Lori received her master's degree concurrent with her Ph.D. studies, and in August of 2007 she will be awarded her Ph.D.

While at Georgia Tech, Lori was the recipient of a President's Fellowship, the National Science Foundation Graduate Research Fellowship, and an ARCS Fellowship. She will begin her career as an Assistant Professor of Operations Research in the Mathematics and Science Division at Babson College in Babson Park, Massachusetts.