DEMAND MANAGEMENT IN DECENTRALIZED LOGISTICS SYSTEMS AND SUPPLY CHAINS

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DEMAND MANAGEMENT IN DECENTRALIZED LOGISTICS SYSTEMS AND SUPPLY CHAINS

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To my husband,

Orkun Kemal Demirag.

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SUMMARY

We analyze issues arising from demand management in decentralized decision-making environments. We consider logistics systems and supply chains, where companies' operations are handled with independent entities whose decisions affect the performance of the overall system. In the first study, we focus on a logistics system in the sea cargo industry, where demand is booked by independent sales agents, and the agents' capacity limits and sales incentives are determined by a central headquarters. We develop models for the central headquarters to analyze and optimize capacity allocation and sales incentives to improve the performance of the decentralized system. We use network flow problems to incorporate agent behavior in our models, and we link these individual problems through an overall optimization problem that determines the capacity limits. We prove a worst-case bound on the decentralized system performance and show that the choice of sales incentive impacts the performance. In the second study, we focus on supply chains in the automotive industry, where decentralization occurs as a result of the non-direct sales channels of the auto manufacturers. Auto manufacturers can affect their demand through sales promotions. We use a game theoretical model to examine the impact of "retailer incentive" and "customer rebate" promotions on the manufacturer's pricing and the retailer's ordering/sales decisions. We consider several models to determine which promotion would benefit the manufacturer under which market conditions. We find that the retailer incentives are preferred when demand is known. On the other hand, when demand is highly uncertain the manufacturer is better off with customer rebates. We extend this research by analyzing a competitive setting with two manufacturers and two retailers, where the manufacturers' promotions vary between retailer incentives and customer rebates. We find an equilibrium outcome where customer rebates reduce the competitor's profits to zero. We observe in numerical examples that the manufacturers are able to increase their sales and profits with retailer incentives, although this can be at the expense of the retailers' profits under some situations.

CHAPTER I

INTRODUCTION

Many companies in different industries operate in decentralized systems where decision-making authority is distributed among the members (agents) of the system instead of collected into a single centralized body. Although in some cases decentralization may seem impractical or inefficient, it is sometimes necessary because of the nature of industry and operations. For example, manufacturers often find it useful to focus on the manufacturing and supply side of the operations as their core competency while managing the sales of their products through independent retailers who service and market to end consumers. The management of decentralized systems is challenging because the individual members' decisions may be conflicting with each other, and their objectives may not be directly aligned with that of the overall system. Designing effective decision support models that account for the independent agent behaviors is of great importance since failing to incorporate those may lead to very poor system performance. Optimization and game theory provide tools to analyze and model decentralized systems. In this dissertation, we study decentralized problems in two application areas: sea cargo shipping and automotive industries. The research problems in both areas were motivated from discussions with our industrial collaborators. We integrate optimization with non-cooperative game theoretical tools to develop models to optimize the performance of the decentralized system.

In the first part of this thesis (Chapter 2), we analyze a decentralized booking/reservation system motivated by practices in some sea cargo companies. Seaborne shipping is the most economical transportation mode for moving large volumes on long distances. Correspondingly, the majority of the world trade volume is carried by ships. Both the world fleet and the world's seaborne trade have experienced continuous growth during the last few decades Christiansen et al. [19] report that 5.625 million tons of goods were estimated to be moved in 2002, representing a 33% increase during the last decade. In spite of its important role on the world economy, sea cargo industry operations and problems have not been investigated by as many researchers as in other areas. The majority of the work done in this area deals with the routing and scheduling of the ships (Christiansen et al. [19]). Besides these operational problems, some carriers are also interested in implementing effective booking/reservation policies in order to manage their demand in the best possible way to match with their capacity. Some sea cargo carriers employ a distributed (decentralized) booking control where the firm (central headquarters) assigns aggregate capacity limits and sales incentives to the decentralized sales agents who then manage cargo bookings from their locations while sharing the system resources. The central headquarters does not directly control the agents' decisions but can influence them through system design and incentives, with the objective of generating high system revenues. We model the firm's problem to determine the best capacity allocation to the agents such that system revenue is maximized. This model incorporates self-optimizing agent behaviors, which we model using network flow problems. In the special case of a single-route, we formulate the capacity allocation problem as a mixed integer program incorporating the optimal agent behavior. For the NP-hard multiple-route case, we propose several heuristics for the problem. Computational experiments show that the heuristics perform reasonably well and that the decentralized system generally performs worse when network capacity is tight. We prove that the decentralized system may perform arbitrarily worse than the centralized system when the number of locations goes to infinity, although the choice of sales incentive impacts the performance. We develop an upper bound for the decentralized system that gives insight on the performance of the heuristics in large systems.

In the second part of the thesis (Chapters 3 and 4), we analyze sales promotions by manufacturers that use non-direct distribution channels to sell their products. The marketing and economics literature is quite rich in analysis of sales promotions with particular focus on empirical investigations of the effects of promotions on the consumers' purchasing behaviors and the firms' profits. The motivation for our research comes from the automotive (auto) industry. In the auto industry, production costs are largely fixed and therefore maximizing revenue is the main objective. Sales promotions such as cash rebate, dealer incentives, and low percent financing programs are frequently offered by the auto manufacturers to increase sales and revenues. The automotive research firm Edmunds.com reports that the aggregate incentives offered by the auto manufacturers in the United States (U.S.) market totaled \$3.39 billion in April 2006. The sales promotions can be directed to the end customers and/or retailers (dealers) in the distribution channels, and the promotional choice may depend on demand characteristics faced by the manufacturers or the characteristics of the manufacturers. We focus on "customer rebate" and "retailer incentive" promotions, and we model the former as a per-unit payment from the manufacturer directly to the end customer and the latter as a lump-sum payment from the manufacturer to the retailer. The main tradeoff is that customer rebates are given to every customer, while the use of retailer incentives are controlled by the retailer. We study a game theoretical model to examine the impact of these promotions on the manufacturer's pricing and the retailer's ordering/sales decisions when the retailer can price discriminate. The auto dealers do not generally announce fixed retail prices for their vehicles, but rather they negotiate with the buyers and price discriminate accordingly. We consider several models with different demand characteristics and information asymmetry between the manufacturer and the retailer, and determine which promotion would benefit the manufacturer under which market conditions by characterizing the subgame-perfect Nash equilibrium decisions. When demand is deterministic, we find that retailer incentives may increase the manufacturer's profits (and sales) while customer rebates do not unless they lead to market expansion. We study several extensions with deterministic demand, and we analyze the sensitivity of our qualitative results to the model assumptions. When the uncertainty in demand ("market potential") is high, we show that a customer rebate can be more profitable than the retailer incentive for the manufacturer. We provide additional insights through numerical examples and empirical analysis of data from domestic auto industry.

Chapter 3 considers a monopolist manufacturer and retailer, and ignores competition, which may be an important factor driving the promotional decisions of the manufacturers. For example, an employee discount program, which was introduced by General Motors in June 2005, was followed by Ford and Chrysler the next month. In Chapter 4, we extend the research on promotions in the auto industry to investigate the effects of competition on the promotional decisions of the manufacturers. We analyze a setting with two competitive manufacturers who sell their products at their exclusive retailers, who are also competitors in the end market. Similar to the analysis in Chapter 3, we adopt a game theoretical framework to model the interactions among the supply chain members, and we find the subgame-perfect Nash equilibrium decisions. We analyze a benchmark case where there are no promotions offered by the manufacturers, and we compare the equilibrium outcomes with those in the cases where the manufacturers offer retailer incentives or customer rebates. We provide several observations using numerical examples. We find that the retailer incentives can be effective in improving the manufacturers' sales and profits, although they can be detrimental to the retailers' profits under some market conditions. We characterize equilibrium decisions where the manufacturers can benefit most from the customer rebates. Unlike the monopolistic setting, we find that customer rebates can be effective in increasing sales and profits of a manufacturer while driving the competitor's profits to zero even when demand is deterministic.

We review the relevant literature in the corresponding chapters. We summarize our findings and present future research directions in Chapter 5.

CHAPTER II

CAPACITY ALLOCATION TO SALES AGENTS IN A DECENTRALIZED LOGISTICS NETWORK

2.1 Introduction

2.1.1 Motivation

The optimization of logistics systems has resulted in many improvements such as reduced costs, shorter lead-times, and better customer service. However, most optimization models have been designed to be used by a centralized planner who makes system-wide decisions, while many organizations actually operate in a decentralized manner with agents making independent decisions. The individual agents may make locally optimal decisions for the part of the system that they manage, but in situations where they share resources, their decisions may have negative consequences for the overall organization.

In practice, many examples of decentralized systems exist due to several reasons. For example, decentralized decision-making can provide responsiveness and flexibility in handling uncertainty in environments such as military operations, where individual units may be authorized to make independent decisions in real-time Lin et al. [55]. Legal barriers to centralization may also exist: franchise laws in the U.S. prohibit auto manufacturers from selling vehicles directly to consumers, leading to decentralized distribution through independent dealerships. For some environments, centralization may be prohibitively expensive, very complex, or the coordination may be too much of a burden. This is especially true for large systems, which would require substantial computational power to store and process large amounts of information for centralized decision-making.

Decentralized systems are sometimes less efficient in terms of the system-wide performance. For example, in a decentralized supply chain, the entire system may incur a revenue loss of 25% due to a phenomenon called "double marginalization" Spengler [73]. Simply applying centralized optimization models to decentralized systems may not be appropriate, but it may be possible to design and optimize the system to achieve good performance. A classic example in the economics literature that analyzes decentralized decision-making is the class of problems called *Principal-Agent*, where the principal contracts with the agent for performing certain acts. (For further details on Principal-Agent problems, see for example Tirole [79] or Varian [82].) In Principal-Agent problems, the utility of the agent may not be directly aligned with that of the principal, thus the principal designs incentives or mechanisms to achieve the desired performance in the decentralized system. The Principal-Agent framework can be applied to decentralized logistics systems by developing optimization models that incorporate individual behaviors and by designing incentives or mechanisms to improve the system performance.

The motivation for our study comes from a decentralized booking practice in some sea cargo companies that transport containerized cargo and provide *liner* service. Liner is one mode of shipping operation in which the companies provide service according to regular schedules and fixed itineraries or service routes Christiansen et al. [19]. In systems we study, a sales agent is located at each port on a network of service routes, and he handles cargo bookings that originate from his port. The sales agent earns revenue according to the incentive determined by the central headquarters, e.g., a fixed proportion of the total revenue generated by the agent's bookings. The central headquarters assigns aggregate capacity to each sales agent for total bookings out of the agent's port, where the firm's objective is to maximize total revenue. This decentralized booking practice can be thought of as a Principal-Agent problem where the principal is the central headquarters and the agents are the sales offices that book cargo. The mechanisms are capacity allocation and sales incentives, where the performance is measured by total system revenue.

In this research, we develop models for the central headquarters to analyze and optimize capacity allocation and sales incentives to improve the performance of the decentralized system. We use network flow problems to incorporate agent behavior in our models, and we link these individual problems through an overall optimization problem that determines the capacity limits to the agents and maximizes system revenue. For the special case of a single route, we formulate a comprehensive model including allocation decisions and agent behavior to solve the problem, and in the NP-hard general case, we develop several heuristics that consider agent behavior. We analyze the worst-case performance of the decentralized system and develop an upper bound on the optimal revenue that provides insight on the performance of the heuristics. Our computational results indicate that decentralized booking control is generally worse when capacity is tight, and the heuristics perform reasonably well and fast. The models that we develop can be used in rough-cut capacity allocation planning and evaluating "what-if" scenarios for the system design. For example, the central headquarters can use these models to determine allocation of capacity to agents, to choose sales incentives, to evaluate investment in an expensive centralized booking system, or to assess alternate designs of the network.

2.1.2 Literature Review

Research relevant to our work has been done in areas such as Principal-Agent analysis, revenue management, sea cargo routing and scheduling, decentralized organizational design, and network equilibria.

Principal-Agent problems are well-studied in the economics and operations management literature. (See, for example, Corbett and De Groot [20], Iyer et al. [40], Tirole [79], and Varian [82].) These problems arise in the field of non-cooperative game theory and include applications to compensation of executives, contracting of workers, and management of supply chains. Our work presents analysis of a Principal-Agent problem in a non-traditional area, with agents booking cargo on a network of service routes.

Revenue management, which is a practice used in industries such as airline, car rental, hotel and entertainment, is concerned with achieving maximum revenue from the sales of perishable assets. The field of revenue management is relevant to the problem that we study since price-differentiated cargo is transported using perishable ship capacity, and the objective is to maximize revenue. For some examples of research in airline and air cargo revenue management see Bertsimas and Popescu [7], Dror et al. [25], Glover et al. [35], Karaesmen and van Ryzin [43], Kasiligam [44], and Kasilingam [45], or see McGill and van

Ryzin [56] for a review. More recently, there has been interest in applying revenue management techniques in the sea cargo industry; Lee et al. [54] analyze a revenue management problem where the carrier decides which demands to satisfy or postpone as they are requested by contract and non-contract shipping customers. While most research in revenue management takes a centralized perspective on booking control, we focus on a decentralized system.

There is also a body of literature that relates specifically to sea cargo beyond the last example, most of which deals with the routing and scheduling of ships or moving empty containers to match supply and demand. (See Christiansen [19] for a review.) Different than the research in this stream, we take the ship routes and schedules as given, and we focus on maximizing revenue in a decentralized system.

Some research related to decentralized systems has been done in the allocation of shared resources in multi-level organizations. The most relevant ones to our study include Burton and Obel [14] and Gazis [30]. In both papers, the authors analyze a problem where a central organizing body distributes the total capacity of common resources to sub-units who make individual decisions regarding these resources. In the formulations, feasibility for each resource is ensured with a constraint that limits its overall usage to the available capacity. Although the problem that we study is similar in concept, our problem is complicated by the fact that the sales agents receive aggregate capacity limits for multiple network resources. An effective distribution of the common resources that achieves high revenue requires us to know the agents' booking decisions, which is difficult to characterize on the network.

A number of papers have studied the loss of efficiency (i.e., the loss in a decentralized system compared to a centralized one) in the context of Nash equilibria or network models. The term "price of anarchy" was first used in Koutsoupias and Papadimitriou [49] and Papadimitriou [65] to quantify the degree of loss in a restricted network. Some examples of research analyzing the price of anarchy in capacitated transportation networks and competitive network environments include Correa et al. [22] and Johari and Tsitsiklis [42]. Perakis [67] generalizes the work in this stream by considering systems with non-separable, asymmetric and nonlinear costs. The author finds that the loss due to decentralization can be unbounded in the worst case. Although the decentralized problem we study is different, we show a similar worst-case result.

This chapter is organized as follows. We describe the problems that we address in Section 2.2 with a focus on capacity allocation in the decentralized system. In Section 2.3, we analyze a special case, present our heuristics and provide further theoretical analysis. We present some observations from our computational study in Section 2.4 and conclude with a summary of our findings in Section 3.4.

2.2 Models

In this section, we introduce some definitions, state our assumptions, and formulate the problems that we analyze.

We are interested in networks that are composed of directed cycles (not necessarily simple), which we call *routes*. A route is a sequence of ports that begins and ends at a specific location in the network. We present an example in Figure 1 where the network consists of a single route with revisitations that originates and terminates at Port 1. A *leg* is identified by a pair of locations on a route that are visited consecutively by the asset, or ship. Demand is in the form of *Origin-Destination (O-D)* pair requests. Ship capacity refers to the number of containers a ship can carry on a particular route. Since liner operators have regularly departing schedules (e.g., weekly) and we are interested in a system with time-stationary parameters, thus we can view a snapshot of the network in time, with all demand forecasts and capacities defined for the time period being studied. In practice, the models we present can be solved in a rolling horizon, where reallocation of capacities to the agents accounts for already confirmed bookings and new information about O-D pair demand.

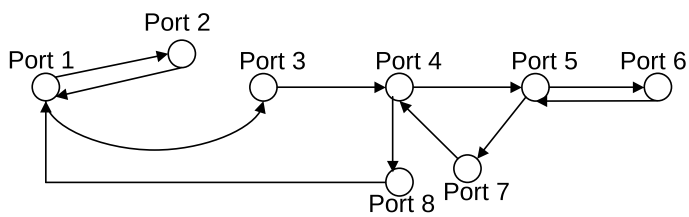


Figure 1: A network with a single route and revisitations.

Associated with each port on the network is a *sales agent*. In the problem we study, an agent at a particular port only books freight leaving from that port; in practice that agent has the most information about the booked cargo from his port and the feeder services into and out of the port. Note that this is different than booking practices for most passenger air travel, where travel agents can book demand to and from any location. The sales agent optimizes his individual objective (independently of the other agents) based on the sales incentive as determined by the central headquarters. Unless otherwise noted, we assume that the sales incentive is based on total revenue, and we do not model sales effort or cost as a function of the incentives. We use the term "agent" to represent the sales office at a port and we assume that the sales office acts as one entity towards the common objective.

The central headquarters assigns capacity on a given ship and route to each agent on that route; the capacity is an aggregate allocation that limits the total demand bookings out of the agent's port rather than a capacity limit for each leg or O-D pair. We assume that the central headquarters has full information on prices and that O-D pair demand is known by both sales agents and central headquarters. In practice, demand is often forecasted by sales agents who give the information to central headquarters.

We assume there is no penalty associated with rejection of demand besides the lost revenue. Since ship operating cost is largely fixed and does not depend on accepted demand, we ignore this cost. We assume that customers are path indifferent, i.e., an O-D pair request may be transported on any ship travelling between its origin and destination ports. We do not allow transshipment and multiple loading/unloading of cargo. Finally, we assume that each unit of demand is equal to one container of the same size.

We identify the following problems:

- ***Capacity Allocation Problem (CAP)***: CAP determines the optimal set of capacity limits to allocate to each sales agent on each route. The central headquarters solves CAP to maximize total revenue, while ensuring that the overall solution determined from the agents' booking decisions is feasible.

- ***Agent Problem (AP)***: AP determines the optimal set of accepted O-D pair demands for an agent. An individual sales agent solves AP to maximize his objective based on the sales incentive, while guaranteeing that the number of accepted demands is within his capacity limit as assigned by central headquarters.

- ***Central Problem (CP)***: CP determines the optimal set of O-D pair demands to accept for the system to maximize total revenue, while ensuring overall feasibility.

Our focus is primarily on CAP. In order to model CAP, we need to understand an agent's behavior, therefore we analyze AP. In CP, all decisions regarding demand acceptance are made in a centralized manner, therefore CP provides benchmark results for CAP.

Table 1 summarizes the main notation used in this chapter. For notational convenience, the price per container for an O-D pair is the same regardless of the route chosen and the type of cargo, and the capacity on each leg of a route is equal to the ship capacity on the route.

We start by analyzing AP. An agent at port p receives the capacity limits $\vec{a}_p$ and solves his individual problem, which we call AP_p. We formulate AP_p as a minimum cost flow problem following the representation in Ahuja et al. [1], and we construct the underlying graph and choose the problem parameters according to our setting. (Minimum cost flow problems are polynomially solvable and can be solved efficiently by specialized network flow algorithms.)

The underlying graph consists of nodes corresponding to the O-D pairs $(n_k, k \in OD_p)$, routes $(m_r, r \in R_p)$ and a sink node (t) for feasibility. We denote the network for agent p with $H_p = (N_p, A_p)$, where N_p is the set of nodes and A_p is the set of arcs. The problem parameters corresponding to the net supply at node i, (b^i), the per unit cost of sending flow on arc (i,j), (c_{ij}), and the upper bound on arc (i,j), (u_{ij}), are defined as follows:

$N_p = \{t \cup (\bigcup_{k \in OD_p} n_k) \cup (\bigcup_{r \in R_p} m_r)\}$

$A_p = \{(\bigcup_{k \in OD_p, r \in RR_k} (n_k, m_r)) \cup (\bigcup_{k \in OD_p} (n_k, t)) \cup (\bigcup_{r \in R_p} (m_r, t))\}$

$$b^i = \begin{cases} d_k & \text{if } i = n_k \\ -\sum_{k \in OD_p} d_k & \text{if } i = t \\ 0 & \text{otherwise} \end{cases} ; \; c_{ij} = \begin{cases} -p_k & \text{if } (i,j) = (n_k, m_r) \\ 0 & \text{otherwise} \end{cases} ;$$

Table 1: Main notation for Chapter 2

P :	Set of ports or agents
R_p :	Set of routes that visit port p ($\cup_{p \in P} R_p = R$, where R denotes the set of all routes)
OD_p :	Set of O-D pairs that originate at port p ($\cup_{p \in P} OD_p = OD$, where OD denotes the set of all O-D pairs)
$\overline{OD}_p$:	Set of O-D pairs for port p ordered by non-increasing prices
$\overline{OD}$:	Set of all O-D pairs ordered by non-increasing prices
op_k, dp_k :	Origin, destination ports of O-D pair k
RR_k :	Set of routes such that a directed path exists from op_k to dp_k
L_r :	Set of legs on route r (denoted as L if $\|R\| = 1$, where $\|R\|$ is the cardinality of the set R)
LL_k^r :	Set of legs O-D pair k traverses on route r (denoted as LL_k if $\|R\| = 1$)
S_l^r :	Set of O-D pairs on route r using leg $l \in L_r$ (denoted as S_l if $\|R\| = 1$)
cap_r :	Ship capacity on route r (denoted as cap if $\|R\| = 1$)
p_k :	Price per unit demand shipped for O-D pair k
d_k :	Total amount of demand for O-D pair k
nv_p^r :	Number of times port p is visited on route r
a_p^r :	Capacity allocated to agent at port p on route r (denoted as a_p if $\|R\| = 1$)
$\vec{a}_p$:	The vector of allocated capacities on all the routes of agent at port p, where $\vec{a}_p = (a_p^1, a_p^2, .., a_p^{\|R_p\|})$
$\vec{a}$:	The vector of allocated capacities of all agents, where $\vec{a} = \{\vec{a}_1 \| \vec{a}_2 \| ... \| \vec{a}_{\|P\|}\}$
e_p^r :	Unit vector for agent p with number of entries = $\|R_p\|$, entry of 1 for route r and 0 otherwise
$y_k^r(\vec{a}_p)$:	Number of demands accepted for O-D pair k on route r when the agent at port p receives $\vec{a}_p$
y_k :	Number of demands accepted for O-D pair k (defined for the special case of a single-route)
z_k^p :	Binary decision variables that equal one if the entire demand of O-D pair $k \in OD_p$ is satisfied and zero otherwise
$Z_p(\vec{a}_p)$:	The objective function value of AP corresponding to the agent at port p
$\mathbb{Z}^+$:	The set of positive integers

$$u_{ij} = \begin{cases} a_p^r & \text{if } (i,j) = (m_r, t) \\ d_k & \text{otherwise.} \end{cases}$$

The decision variables, x_{ij}, $\forall (i,j) \in A_p$ denote the flow on arc (i,j). AP_p is then formulated as follows:

$$\begin{aligned} (AP_p) \qquad Z_p^*(\vec{a}_p) = \quad & \min & & \sum_{(i,j)\in A_p} c_{ij} x_{ij} \\ & \text{subject to} & & \sum_{(i,j)\in A_p} x_{ij} - \sum_{(j,i)\in A_p} x_{ji} = b^i, \ \ \forall i \in N_p \\ & & & 0 \le x_{ij} \le u_{ij}. \end{aligned}$$

The first set of constraints satisfies the flow balance restrictions, and the second set of constraints corresponds to the individual flow bounds. The objective is to maximize the total revenue from accepted demand. From the optimal solution of AP_p, we obtain $y_k^{r*}(\vec{a}_p)$, $\forall k \in OD_p, \forall r \in RR_k$, using the following relation: $y_k^{r*}(\vec{a}_p) = x^*_{(n_k, m_r)}$.

Given the AP models, we present a mathematical formulation for CAP, where the decision variables are the capacity limits to allocate to each agent.

$$(CAP) \quad \max_{\vec{a}\in\mathbb{Z}^+} \quad \left\{ \sum_{p\in P} Z_p^*(\vec{a}_p) \ s.t: \sum_{p\in P} \sum_{k\in(OD_p\cap S_l^r)} y_k^{r*}(\vec{a}_p) \le cap_r, \forall r \in R, \forall l \in L_r \right\}.$$

The objective is to maximize the total revenue generated from the booking decisions of all agents, while ensuring that the total accepted O-D pair demand on each leg is within the capacity limits. We show in Lemma 1 that the multiple-route CAP is NP-hard, since a special case of it is equivalent to the directed multi-commodity integral flow problem, which is NP-hard (Garey and Johnson [29]). However, in general, CAP is not equivalent to a multi-commodity flow problem, which assumes centralized control and does not incorporate the behavior of the individual agents.

Lemma 1 *The multiple-route CAP is NP-hard.*

Proof. We consider the decision problem corresponding to the optimization problem CAP. (CAP Decision Problem) INSTANCE: Given directed graph $G = (N, A)$, capacity on each route, price and demand for each O-D pair and total revenue goal K. QUESTION: Is there an assignment of route capacities to each agent, $\vec{a} = \{\vec{a}_1|\vec{a}_2|...|\vec{a}_{|P|}\}$ where the capacity is used independently by that agent to maximize his own revenue, $Z_p^*(\vec{a}_p) = \sum_{k \in OD_p} p_k y_k^{r*}(\vec{a}_p)$ where $y_k^{r*}(\vec{a}_p)$,$\forall k \in OD_p, r \in RR_k$ is the agent optimal solution, such that $\sum_{p \in P} Z_p^*(\vec{a}_p) \geq K$ and $\sum_{p \in P} \sum_{k \in (S_l^r \cap OD_p)} y_k^{r*}(\vec{a}_p) \leq cap_r, \forall r \in R, l \in L_r$?

Restrict to directed multi-commodity integral flow by allowing instances where each agent has demand for only one O-D pair. In this case, for each capacity allocation assigned to an independent agent p ($\vec{a}_p$), the solution of the AP_p, consisting of the accepted OD pair and the corresponding revenue is known exactly. Thus the decision of finding the set of capacity allocations becomes a question of how to send flow (or commodities) such that revenue achieves a goal and capacities are not violated. (Note that directed multi-commodity integral flow problem is a generalization of directed 2-commodity integral flow problem, which is NP-complete, see Garey and Johnson [29]). ■

The multiple-route CAP problem incorporates agent behavior through the individual AP solutions for each set of capacity allocation. Suppose instead, that one formulated an integer program without the AP solutions, where the decision variables are the capacity limits allocated to each agent and the number of demands accepted; constraints limit the bookings on each leg by ship capacity and each agent's bookings by his capacity allocation. Although the capacity limits are added as decision variables, the solution of this program is equivalent to the CP solution, and it does not model actual agent behavior in a decentralized system. In the decentralized system, a different approach is needed to incorporate agent behavior, ensure feasibility and maximize revenue. If the agent behavior can be explicitly characterized, then it would be possible to solve CAP in one comprehensive model that directly incorporates agent decisions. We show that this is possible for the special case of a single-route.

An alternative allocation scheme to aggregate capacity allocation on each route is *leg-based allocation*, where each agent receives separate capacity limits on each of the legs he can use. In this case, the feasibility of CAP is easily ensured in an integer programming model. In spite of this modelling convenience, a leg-based allocation scheme may not be appealing in practice for large systems. For instance, consider the size of a typical network for a sea cargo carrier operating in Intra-Asia (e.g., [63]). The sales agent in Hong Kong is located on a port that is serviced by 8 different routes. The sales agent would require 52 leg-based capacity limits or 41 O-D pair-based capacity limits compared to 8 aggregate capacity limits out of Hong Kong. Some sea cargo companies prefer the simplicity of assigning aggregate capacity despite the inefficiencies that may result. Moreover, assigning aggregate capacity limits rather than leg-based or O-D pair-based limits may be better as a planning tool since aggregate forecasts of demand are more accurate than forecasted demand for each O-D pair.

The last problem that we model is *CP*, where the central headquarters and the sales agents act as a single unit and jointly determine the best set of demands to accept in order to maximize the overall system revenue. We formulate *CP* as an integer multi-commodity flow problem following the representation in Ahuja et al. [1], where the commodities are the O-D pairs in our setting.

We construct the underlying network $G = (N, A)$, which includes the set of arcs (A_r) and nodes (P_r) corresponding to each route $r \in R$, a source (n_k) and a sink node (t_k) for each O-D pair k. In order to model no multiple loading/unloading of cargo, we replicate the nodes in the routes which are visited more than once. We denote the resulting set of nodes with P'_r. We denote the subgraphs for each O-D pair k with $G^k = (N^k, A^k)$. We summarize the arcs associated with each O-D pair k with O^k and D^k, where O^k is the set of origin arcs, where $O^k = \bigcup_{r \in RR_k}\{(n_k, j) : j \in P'_r, j = op_k\}$ and D^k is the set of destination arcs, where $D^k = \bigcup_{r \in RR_k}\{(j, t_k) : j \in P'_r, j = dp_k\}$.

Then, $G = (N, A)$, where

$N = \{(\bigcup_{r \in R} P'_r) \cup (\bigcup_{k \in OD} n_k) \cup (\bigcup_{k \in OD} t_k)\}$

$A = \{(\bigcup_{r \in R} A_r) \cup (\bigcup_{k \in OD}(n_k, t_k)) \cup (\bigcup_{k \in OD} O^k) \cup (\bigcup_{k \in OD} D^k)\}$.

For each $k \in OD$, $G^k = (N^k, A^k)$, where

$N^k = \{(\bigcup_{r \in R}\{i \in P'_r : i \text{ is visited by O-D pair } k\}) \cup n_k \cup t_k\}$

$A^k = \{(\bigcup_{r \in R}\{(i, j) \in A_r : i, j \in N^k \text{ and } (i, j) \text{ is traversed by O-D pair } k\ \}) \cup (n_k, t_k) \cup O^k \cup D^k\}$.

In Figure 2, we show an example network with two routes, which we use to construct the subgraph for the O-D pair with origin port 1 and destination port 7 in Figure 3. (The subgraph is highlighted with solid arcs.)

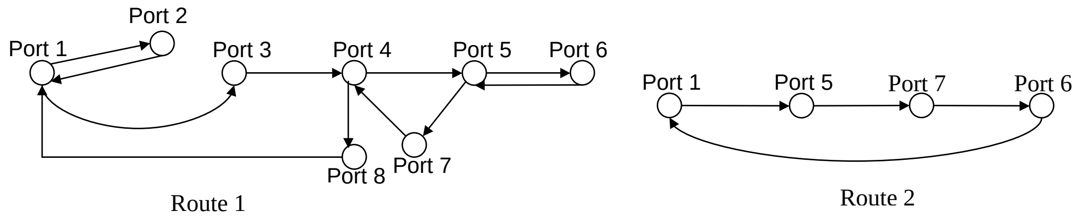


Figure 2: A network with two routes

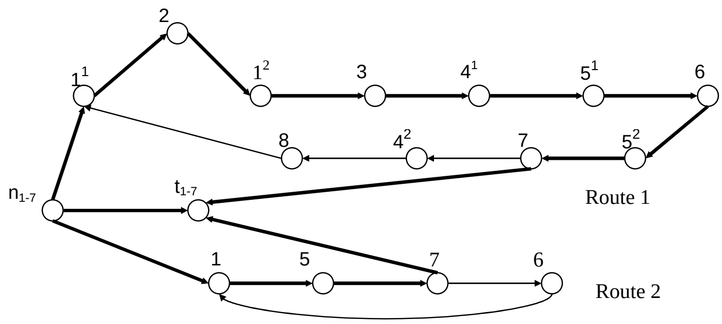


Figure 3: Illustration of the subgraph constructed for O-D pair 1-7 in Figure 2

The problem parameters corresponding to the net supply at node i, (b^i), the per unit cost of flow for commodity k on arc (i,j), (c_{ij}^k), and the upper bound on arc (i,j), (u_{ij}), are as follows:

$$b_i^k = \begin{cases} d_k & \text{if } i = n_k \\ -d_k & \text{if } i = t_k \\ 0 & \text{otherwise} \end{cases} ;\ c_{ij}^k = \begin{cases} -p_k & \text{if } (i,j) \in O^k \\ 0 & \text{otherwise} \end{cases} ;$$

$$u_{ij} = \begin{cases} d_k & \text{if } (i,j) \in (O^k \cup D^k \cup (n_k, t_k)) \\ cap_r & \text{if } (i,j) \in \bigcup_{r \in R} A_r. \end{cases}$$

The decision variables are $x_{ij}^k, \forall k \in OD, \forall (i,j) \in A^k$, denoting the flow of O-D pair k on arc (i,j). We formulate CP as follows:

$$
\begin{aligned}
\textit{(CP)} \quad \min \quad & \sum_{(i,j)\in A}\sum_{k\in OD} c_{ij}^k x_{ij}^k \\
\text{subject to} \quad & \sum_{(i,j)\in A^k} x_{ij}^k - \sum_{(j,i)\in A^k} x_{ji}^k = b_i^k, \ \ \forall k \in OD, \ \forall i \in N^k \\
& \sum_{k\in OD} x_{ij}^k \le u_{ij}, \ \forall (i,j) \in A \\
& 0 \le x_{ij}^k \le d_k, integer, \forall (i,j) \in A, k \in OD
\end{aligned}
$$

The first set of constraints satisfies the flow balance restrictions. The second set of constraints corresponds to the bundle constraints that restrict the total flow of all O-D pairs on each arc with the capacity of the arc. The last set of constraints are individual flow bounds. The objective is to maximize the total revenue from accepted demand.

2.3 Solution Approaches

2.3.1 Special Case: Single Route

In this section, we analyze capacity allocation in a decentralized network with a single-route, and we show several results using the special structure of *CAP*. These results also give us insight into the multiple-route problem and could also apply to other transportation areas, such as the railroad industry.

Similar to the analysis in Section 2.2, we first model the *Single-route Agent Problem (SAP)* to understand the behavior of the agents. Given an agent p, we call the agent's problem SAP_p and show the formulation below.

$$
(SAP_p) \quad \max_{y_k \in \mathbb{Z}^+} \left\{ \sum_{k\in OD_p} p_k y_k \ s.t: \sum_{k\in OD_p} y_k \le a_p, \ y_k \le d_k \ , \ \forall k \in OD_p \right\}.
$$

The objective function is to maximize the revenue of the agent. The first set of constraints ensures that the total number of demands accepted is limited by the agent's capacity allocation. The second set of constraints restricts the number of accepted bookings for each O-D pair by the corresponding demand. SAP_p is equivalent to a continuous knapsack problem with integer data, which is optimized by a greedy algorithm that accepts the O-D pair demands in the order of non-increasing prices. This characterization of the agents' optimal solutions allows us to directly model agent behavior in the single-route case, which we incorporate in a mixed integer linear program for the *Single-route Capacity Allocation Problem (SCAP)*.

To formulate *SCAP*, we order the O-D pairs of each agent from highest to lowest in prices, and we obtain the set $\overline{OD}_p$, $\forall p \in P$. We use an index $l(k)$ to denote the order of O-D pair k in $\overline{OD}_p$, where $l(k) = 1$ denotes the O-D pair k with the highest price, and $l(k) = |OD_p|$ denotes the O-D pair k with the lowest price in OD_p. (Since O-D pair k is exclusively serviced by one agent, $l(k)$ uniquely identifies each O-D pair k.) We use the binary decision variables z_k^p ($p \in P, k \in OD_p$) to incorporate the precedence relations among O-D pairs of each agent. We formulate *SCAP* as follows:

$$
\begin{array}{llll}
(SCAP) & \max & \sum_{k \in OD} p_k y_k & \\
& \text{subject to} & \sum_{k \in S_l} y_k \leq cap & \text{for all } l \in L, \\
& & d_k z_k^p \leq y_k \leq d_k & \text{for all } p \in P, k \in OD_p : l(k) = 1, \\
& & d_k z_k^p \leq y_k \leq d_k z_{k'}^p & \text{for all } p \in P, k, k' \in OD_p : l(k) = l(k') + 1, l(k) > 1, \\
& & y_k \geq 0 & \text{for all } k \in OD, \\
& & z_k^p = \{0, 1\} & \text{for all } p \in P, k \in OD_p.
\end{array}
$$

The first set of constraints ensures that leg capacities are not exceeded. The second set of constraints ensures that the demands accepted for the O-D pairs satisfy each agent's optimal behavior as characterized by the *SAP* solution. We calculate the capacity limits to agent p from the solution of *SCAP* as $a_p = \sum_{k \in OD_p} y_k$, $\forall p \in P$.

Finally, we model the *Single-route Central Problem (SCP)* as an integer program with a constraint matrix that has the circular 1's property in its columns. (See Appendix A.1) for the formulation.) This is best seen when the matrix is formed with rows denoting the legs (in the order that they are traversed according to the sequence of ports) and columns with O-D pairs. Then, the matrix has an entry of 1 when the leg is traversed by the O-D pair in the corresponding column. For a definition of the circular 1's property and results on complexity status of related problems, see Hochbaum and Tucker [38]. Since *SCP* is equivalent to *SCAP* without the precedence constraints, *SCAP* is at least as difficult as

SCP, the complexity status of which is unknown to the best of our knowledge.

2.3.2 Heuristics for Multiple-Route CAP

As mentioned in Section 2.2, the multiple-route CAP is NP-hard. Therefore, we focus on developing efficient heuristics that incorporate agent behavior.

2.3.2.1 Marginal Revenue (MR) Heuristic

In this section, we describe the *Marginal Revenue (MR)* heuristic to solve CAP. Given current allocated capacities, we build a feasible solution by successively assigning fixed increments of capacity to the agent that brings the highest current additional revenue (or highest marginal revenue) without violating system feasibility. The latter is determined by finding the capacity used on each leg considering bookings by all agents. Although designing an allocation mechanism based on marginal revenue is a simple idea, the revenue can be quite close to that of the optimal CAP solution. We summarize the main steps of the *MR* heuristic below and provide a formal description of the algorithm in Appendix A.2.

1. Initialize the capacity limits to all agents at zero.

2. For each agent p on each route $r \in R_p$, temporarily increase the agent's capacity limit on route r by a $stepsize$ and solve AP_p.

3. Find the agent and route pair (p_{max}, r_{max}) that brings the largest increase in the objective function value and satisfies overall system feasibility.

4. Increase the capacity limits to agent p_{max} by $stepsize$.

5. Repeat Steps 2-4 while there exists an agent whose capacity limit can feasibly be increased on a route.

The most time consuming part of the algorithm is finding (p_{max}, r_{max}), since this requires solving multiple APs for each new capacity increment and checking the feasibility of the overall system. An upper bound on the total number of minimum cost flow problems solved is $\max_{p \in P} \left\{ |R_p| \sum_{r \in R_p} (cap_r \cdot nv_p^r) \right\}$. The use of a scalar, nv_p^r, is needed in order to capture the increased capacity resulting from multiple visitations to a port in a particular route. The parameter *stepsize*, which is the size of the capacity increment in the allocation vector, may be increased to reduce runtime, although this can result in a decrease in the solution quality.

2.3.2.2 Priority O-D (POD) Based Allocation Heuristic

The *Priority O-D (POD)* based allocation heuristic is partly motivated by the single route results in Section 2.3.1, where the optimal solution of AP is such that the agents satisfy demands in a greedy fashion according to the ranked prices of the O-D pairs. Thus, for the POD heuristic, we form $\overline{OD}$ by ordering the O-D pairs in the order of decreasing prices. We allocate capacity to the agents according to this priority list, where allocation increments are as large as possible without violating system feasibility. We summarize the main steps of the POD heuristic below and provide a formal description of the algorithm in Appendix A.3.

1. Start with the O-D pair k that has the highest price and find a route $r \in RR_k$ that may accommodate the largest amount of its demand.

2. Temporarily increase the allocation of the agent at the origin port of O-D pair k (op_k) on route r by the demand or the available capacity on that route, whichever is smaller.

3. Solve AP_{op_k}. Make the agent's allocation increment permanent if the new allocation does not violate the system feasibility. Go to Step 5 if O-D pair k is entirely booked.

4. Continue with $\overline{r} \in RR_k$, $\overline{r} \neq r$ and repeat Steps 2 and 3.

5. Continue with the next O-D pair in $\overline{OD}$ until all O-D pairs have been considered or no capacity is remaining.

An alternative we consider to giving priority to the routes with higher allowable space (Step 1) is to solve the linear relaxation of CP and use the dual variables corresponding to the legs of the routes for ranking the routes. Neither of the two approaches to route selection is outperformed by the other in our computational experiments. The runtime of

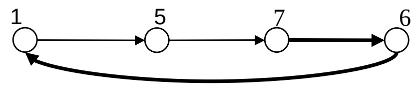


Figure 4: Example illustrating O-D pair replacement

the algorithm is directly proportional to the total number of O-D pairs. For each O-D pair k, at most $|RR_k|$ number of minimum cost flow problems are solved.

While agent behavior is greedy and relatively easy to characterize in the single-route problem, this is not true in the multiple-route case. There may be several types of behaviors where the agent uses capacity differently than as estimated in the *POD* heuristic. In Figure 4, we illustrate a situation, which we call "O-D pair replacement", where the agent may use the (temporarily) allocated capacity in his own self interest and may cause system infeasibility. In this case, "O-D pair replacement" is performed by agent at port 5. Legs $(7, 6)$ and $(6, 1)$ are totally occupied (represented by bold arcs with heavy lines), and the prices of O-D pairs 5-1 and 5-7 are such that $p_{5\text{-}1} > p_{5\text{-}7}$. Note that O-D pair 5-1 needs space on $(7, 6)$ and $(6, 1)$, but O-D pair 5-7 does not. Consequently, the *POD* heuristic does not allocate any capacity to the agent when considering O-D pair 5-1, but may attempt to allocate him capacity for O-D pair 5-7 in order to improve the current solution. However, the agent can use this capacity for O-D pair 5-1 since $p_{5\text{-}1} > p_{5\text{-}7}$. This violates capacity limits on legs $(7, 6)$ and $(6, 1)$. *POD* avoids future O-D replacements by agent 5 by not allocating him more space on this route. In the case of "O-D pair replacement", we can identify the behavior and do not increase the agent's capacity limit so that feasibility is maintained.

Other kinds of agent behaviors may also occur, such as "O-D pair swapping", where the agent may swap routes used for previously considered O-D pairs and book an O-D pair with higher price using the new capacity; thus causing system infeasibility. This behavior is more difficult to characterize since it may result in multiple swaps of O-D pairs among routes. Therefore, we ensure the feasibility of the overall solution by solving *AP* in Step 3 of the heuristic. In this step, if we detect that the optimal *AP* solution causes system infeasibility, we do not assign the capacity increment permanently to the agent.

The *POD* heuristic assigns capacities to the sales agents based on estimated agent behavior according to the priority list of O-D pairs. In contrast, the *MR* heuristic does not make any assumptions about the structure of agent behavior and solves a new *AP* to capture behavior for each potential capacity allocation. The *POD* heuristic allocates as much capacity as possible at each iteration of the algorithm, while the *MR* heuristic allocates fixed capacity increments determined by the *stepsize*. We show in our computational experiments that the *POD* heuristic is much faster than the *MR* heuristic with a small *stepsize*.

2.3.2.3 Benchmark Heuristics

We analyze two simple heuristics, *Equal Allocation (EA)* and *Conservative Allocation (CA)*, which are easy to implement and can be solved very quickly. In general, we do not expect that these heuristics perform well for *CAP*, but they provide another benchmark against which to compare the *MR* and *POD* heuristics. The *EA* heuristic equally distributes ship capacity on a route among the agents that can book demand on the route. The *CA* heuristic accounts for the O-D pair prices by solving a minimum cost flow problem, where the total capacity allocated to all agents on a route is restricted by the ship capacity. In contrast, the sum of the capacity limits in the optimal *CAP* solution may be greater than the ship capacity based on the agents' booking decisions.

The benchmark heuristics guarantee system feasibility independently of how the agents solve their own problems, although this is achieved at the expense of revenues. The *MR* and *POD* heuristics achieve higher revenues by incorporating how agents use their allocated capacities, thus accommodating higher total demand; however, they require more extensive effort to ensure overall feasibility.

2.3.3 Further Analysis of CAP

2.3.3.1 Alternative Agent Incentives

In all the models described so far, we assume that the agents maximize total revenue generated by their O-D pair bookings. This corresponds with the incentives that sales agents currently receive in some sea cargo companies, i.e., a proportion of the total revenue generated by their bookings. The total revenue incentive might be disadvantageous from a system perspective when O-D pairs occupying a high number of legs bring higher revenue. We propose an alternative sales incentive, which we call *revenue per leg (rev/leg)*. Under the rev/leg incentive, an agent receives revenue for an O-D pair based on the route he chooses to transport that demand, where one unit of demand accepted for O-D pair k on a route r improves the agent's revenue by $p_k/|LL_k^r|$. In particular, the formulation of AP_p is modified by adjusting the problem parameters as follows:

$$c_{ij} = \begin{cases} -p_k/|LL_k^r| & \forall (i,j) = (n_k, m_r), \forall k \in OD_p \\ 0 & \text{otherwise.} \end{cases}$$

Consequently, the agent's objective is to maximize the sum of the revenues generated per leg rather than the total revenue, while the central headquarters' objective is to maximize total system revenue under both incentives.

In our computational analysis we show that simply changing the agents' objectives in this way can be effective in moving the decentralized solution towards the centralized solution. One reason for its effectiveness is the following. If an O-D pair traverses over a large number of legs, then it typically shares capacity with a large number of other O-D pairs. The rev/leg incentive essentially penalizes O-D pairs that use a large number of legs, and therefore, a large amount of system capacity.

In some systems it is possible that the price to transport cargo already incorporates the number of legs traversed between its origin and destination ports. In that case, another incentive may be appropriate, such as *revenue/container-mile* if the price is not based on the distance the cargo is transported. Other incentives that may be appropriate in some situations include: i) the total number of demands booked by a sales agent ii) a proportion of the total system revenue. The latter aligns agent's revenue functions with the centralized system, although it may be difficult to implement since the agents may not have full knowledge of demand in other locations.

2.3.3.2 Theoretical Results for CAP

In this section we investigate the performance of the optimal decentralized system under the total revenue and rev/leg incentives. We show through examples that the optimal revenue of the decentralized system may be asymptotically worse than that of the centralized system. This is consistent with theoretical results on the price of anarchy in other kinds of decentralized networks (Perakis [67]). This theoretical result also has practical implications as it indicates that investment in a centralized system may be cost effective. One interesting result is that when the decentralized system performed poorly under one incentive, the other incentive led it to achieve the optimal *CP* solution.

Theorem 1 *As the number of ports goes to infinity, the revenue obtained by the optimal CAP solution with the total revenue incentive may be arbitrarily worse than the revenue obtained by the optimal CP solution.*

To show the theoretical inefficiency of *CAP*, we focus on the special case of the single route, *SCAP*. We construct an example with a network of one simple cycle. That is, port visitations are in the form of $\{1, 2, 3, .., n, 1\}$, where n (the number of ports)≥ 3. We present the formal proof of this theorem and other key results of this chapter in Appendix A.4.

Remark 1 *For the example in Theorem 1, the revenue obtained by the optimal SCAP solution with the rev/leg incentive is the same as the revenue obtained by the optimal SCP solution.*

Remark 1 shows that the rev/leg incentive achieves the best performance for the decentralized system in the example. Unfortunately, this incentive does not always give a good decentralized system outcome. The following is a result on the worst case performance of the rev/leg incentive.

Theorem 2 *As the number of ports goes to infinity, the revenue obtained by the optimal CAP solution with the rev/leg incentive may be arbitrarily worse than the revenue obtained by the optimal CP solution.*

To prove Theorem 2, we construct another example with a similar network structure as in Theorem 1, but with different prices for O-D pairs.

Remark 2 *For the example in Theorem 2, the revenue obtained by the optimal SCAP solution with the total revenue incentive is the same as the revenue obtained by the optimal SCP solution.*

Theorems 1 and 2 indicate that even if the decentralized system operates optimally, its performance may be very poor compared to the optimal centralized system, i.e., the price of anarchy may be high. Remarks 1 and 2 show that one incentive may perform better than another in a particular system, therefore careful choice of incentives is important to improve the decentralized system performance.

2.3.3.3 Upper Bound on Performance of CAP

Since we use heuristics to solve *CAP* for large problem sizes, additional analysis is needed to evaluate the performance of the heuristics. This is particularly important when the heuristics perform badly compared to *CP*; in that case, it is not clear if the gap between the heuristic solution and the *CP* solution is due to the nature of the heuristics or the inefficiency of the decentralized system. In the following, we introduce an upper bound on the performance of *CAP* that can be efficiently computed when it is not practical to solve *CAP* optimally for large problem sizes.

We compute the upper bound by solving the independent *AP*s for all agents where the agents' capacity limits are obtained from the accepted demands in the optimal *CP* solution.

Lemma 2 *The procedure outlined in Steps 1-4 below provides an upper bound on CAP.*

1. *Solve* CP *and obtain the solution vectors of accepted demand out of each port.*

2. *For each agent on each route, sum the accepted O-D pair demand originating from the agent's port in the* CP *solution, and let this be the agent's capacity allocation on that route.*

3. *Solve* AP_p, $\forall p \in P$, *with the agents' capacity limits as found in Step 2.*

4. Sum the objective function values (total revenue) of the AP*s to obtain the upper bound on the optimal* CAP *revenue.*

Given capacity limits from the *CP* solution, the agents can achieve a greater revenue by accepting O-D pairs with higher prices since overall system feasibility is not required. Thus, the total revenue generated by all agents is guaranteed to be at least as high as the objective function value of the *CP* solution. We also know that the *CP* solution is an upper bound on the *CAP* solution; therefore the result holds.

Introducing an upper bound for *CAP* that is weaker than the *CP* solution may seem ineffectual. However, we motivate this strategy by noting that the performance gap between the *CAP* and *CP* solutions may relate to the performance gap between the upper bound and *CP* solutions. For example, we intuitively expect *CAP* to perform badly compared to *CP* when the *CP* solution accepts many short O-D pairs but the agents prefer longer O-D pairs with higher prices. In this case, to ensure feasibility the *CAP* solution assigns relatively small capacity limits to the agents, thus resulting in significantly smaller revenue than the *CP* solution. As a result, the revenue generated in the computation of the upper bound is significantly greater than the *CP* solution, since the agents receive capacity limits from the *CP* solution and they accept longer O-D pairs with higher prices.

Further, when capacity is very large relative to demand, both *CAP* and the upper bound find the *CP* solution. We conjecture that the gap between the upper bound and the *CP* solution is similar to the gap between the *CP* and *CAP* solutions. This is further supported by our numerical experiments. Consequently, the upper bound may give insight on the performance of the heuristics when the optimal *CAP* solution is not known.

2.4 Computational Experiments

The goals in our computational study are twofold. The first is to test the decentralized system performance under different sales incentives in order to understand factors that influence its performance relative to the centralized system. The second is to examine the performance of our heuristics.

2.4.1 Data Generation

The network characteristics in our computational study are intended to be similar to the industry practice of sea cargo logistics companies. These carriers provide service throughout the globe on trade lanes/routes (e.g., Trans-Pacific, Trans-Atlantic, Asia-North America, Asia-Europe, and Intra-Asia.), where each route is further divided into sub-routes. For example, OOCL's Trans-Pacific service area includes 11 routes visiting a total of 29 different ports, and all routes are non-simple cycles with approximately 25% of the ports being revisited in the port sequence [64].

In data generation, we first specify the number of ports and the number of routes. Next, we randomly assign the ports to the routes and determine a port rotation (or sequence) within each route, allowing ports to be visited multiple times. We limit the number of revisitations to 1/3 of the total number of ports.

The price of an O-D pair is a uniform random variable taking values between 0 and 2000, and the demand of an O-D pair is a uniform integer random variable taking values between 0 and $Maxdemand$. In our experiments, we choose the values for $Maxdemand$ as 30, 100 and 500. We calculate the ship capacity on a route as a fraction (*Capacity/demand ratio*) of the total demand for the most highly requested leg on the route. We choose the values of *Capacity/demand ratio* as $0.3, 0.5$, and 0.8, representing low, medium and high capacity levels. In the computational experiments, we generate 30 random instances and average over the iterations. We conduct all computational experiments on a computer with a 2.66 GHz Intel processor and 1 Gb of memory. We implement the formulations for the APs and CP using ILOG's OPT Studio 3.7, and we use the callable component library feature (C++ API) to access ILOG's CPLEX 9.0 solver when solving CAP.

2.4.2 Enumerative Method

The optimal solution of CAP can be found by an enumerative algorithm that checks all possible combinations of capacity limits for agents to find the set with the highest system revenue. However, such an algorithm is impractical for reasonable problem sizes since the number of possible combinations is very large. In an instance with $|P| = 10, |R| =$

$5, |R_p| = 2, cap_r = 1000$, and nv_p^r=1, $\forall p \in P, \forall r \in R$, the total number of combinations is $\prod_{p \in P} (\prod_{r \in R_p} (cap_r \cdot nv_p^r)) = 10^{60}$. Thus, to evaluate the heuristics, we solve the CAP to optimality for small instances. The optimal CAP solution also gives us insight on the performance of the upper bound that we introduced in Section 2.3.3.3, which we calculate for all problems.

Note that for a given vector of allocated capacities, there may exist alternative optimal solutions of APs. In such cases, system feasibility may be violated if the agents use capacity differently than predicted. The solution methods that we propose for CAP are guaranteed to contain a set of optimal AP solutions that constitutes a system feasible solution. In cases where the optimal AP solution can uniquely be found, system feasibility is always ensured. In practice, this can be achieved by the use of an optimization software throughout the company that has the same deterministic solver engine [39]. Perturbing the O-D pair prices such that it is less likely for multiple combinations of O-D pairs to have the same revenue can also be effective in alleviating the existence of alternative optimal solutions.

2.4.3 Experimental Results

In Figure 5, we show the upper bound and the performance of various solution methods for CAP as a percentage of the optimal CP revenue. In Figure 5(a), which is based on small-sized instances, we see that both the optimal and heuristic decentralized system solutions are able to achieve at least 88% of the optimal CP revenue. In the worst case, the heuristics achieve 98% of their respective optimal decentralized system revenues in the CAP solution. The upper bound also has a similar performance to that of the decentralized optimal solution, performing the worst when capacity is tight.

In Figure 5(b), we analyze the performance of heuristics using larger-sized instances. As expected, the MR and POD heuristics perform significantly better than the benchmark heuristics. For example, the former achieve 80% of the centralized revenue under the tightest capacity, while the latter attain no more than 43% in that case. Therefore, it is valuable to improve performance by explicitly solving APs as in the MR and POD heuristics. The MR heuristic with rev/leg incentive has the best performance of the heuristics (within 5%

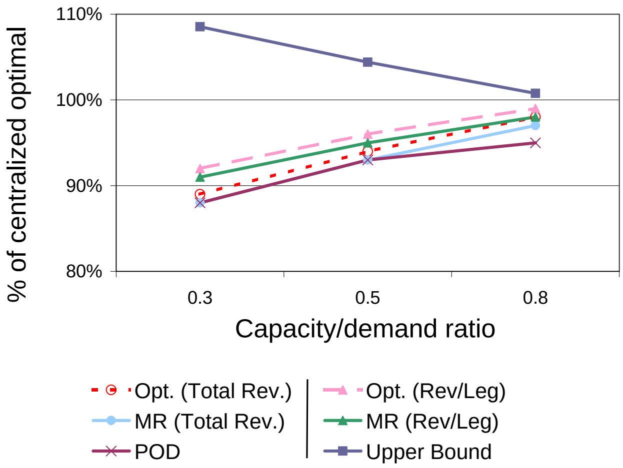
of the optimal *CP* revenue). For the large problems, the upper bound may be used as a proxy for the performance of the decentralized optimal solution; note that the gap between the upper bound and the optimal *CP* revenue as a function of capacity is about the same in both graphs.

One observation that is common to Figures 5(a) and 5(b) is that the performance of all heuristic solutions improves relative to the optimal *CP* solution with an increasing *Capacity/demand ratio*. We can explain this observation as follows. As capacity becomes tight compared to demand, the revenue loss for not accepting the most profitable demands is more important since any loss may have a significant impact.

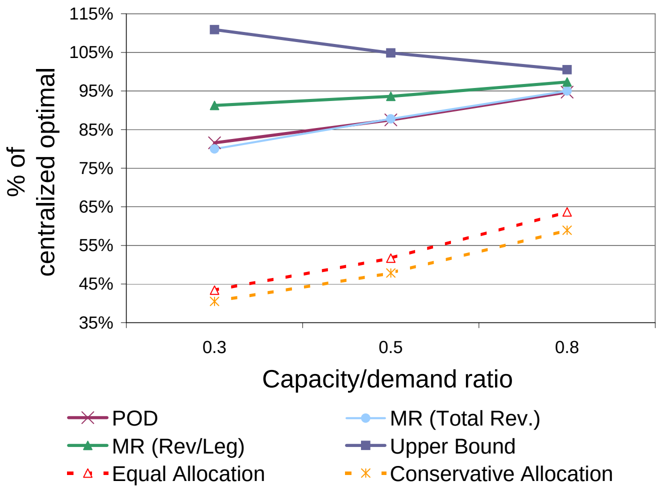
In Figure 6, we show examples on additional networks with different sizes. Note that, the upper bound and the optimal *CAP* solution behave similarly relative to the optimal *CP* solution, which is consistent with our conjecture in Section 2.3.3.3.

Two performance measures that are of interest to the cargo carriers are the service levels of the system and the utilizations of the ships. We calculate the service level for each agent as the ratio of total demand accepted by the agent to the total demand out of the agent's home port. In Figure 7(a), we depict the average service levels across all agents as a function of *Capacity/demand ratio*. One interesting observation is that the *MR* heuristic solution with the rev/leg incentive achieves higher service levels than the optimal *CP* solution. The ship utilization on each route is equal to the average leg occupancy, which is defined as the ratio of total flow on a leg to the route capacity. In Figure 7(b), we show the ship utilizations averaged over all routes. We see that ship utilizations are significantly higher in the optimal *CP* solutions compared to the *MR* heuristics because the *CP* solution is able to use the ship capacities more efficiently without considering agent behavior.

The average runtimes of the algorithms for the decentralized system are given in Table 2 for the instances corresponding to Figure 5. In Table 2(a), which shows the small networks, the runtimes of all algorithms increase as the *Capacity/demand ratio* increases. As we expect, the enumerative algorithm that finds the optimal *CAP* solution increases at a much faster rate than the runtimes of the heuristics. Table 2(b) shows the average runtimes of the heuristics for the larger network. Since *CAP* is solved quarterly or weekly rather than in



(a) #Routes=2, #Ports=6, $Maxdemand$=30



(b) #Routes=5, #Ports=10, $Maxdemand$=500

Figure 5: Decentralized system performance with increasing $Capacity/demand\ ratio$

Table 2: Runtimes of the algorithms in CPU seconds

(a) $\#Routes = 2$, $\#Ports = 6, Maxdemand = 30$

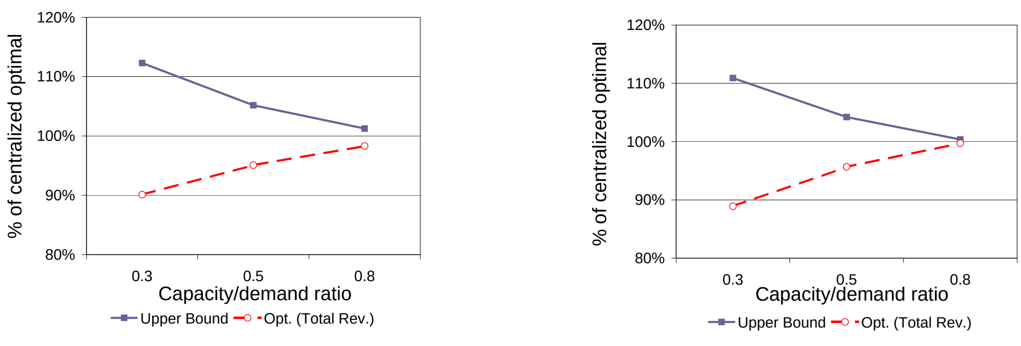
	$Capacity/demand\ ratio$		
	0.3	0.5	0.8
CAP Optimal (total revenue)	7.04	50.59	509.49
CAP Optimal (rev/leg)	7.07	51.92	540.47
MR (total revenue)*	1.23	2.04	3.19
MR (rev/leg)	1.39	2.25	3.28
POD	0.12	0.14	0.17

(b) $\#Routes = 5$, $\#Ports = 10, Maxdemand = 500$

	$Capacity/demand\ ratio$		
	0.3	0.5	0.8
MR (total revenue)	204.18	326.44	489.75
MR (rev/leg)	309.38	434.14	588.87
POD	0.65	0.81	0.95

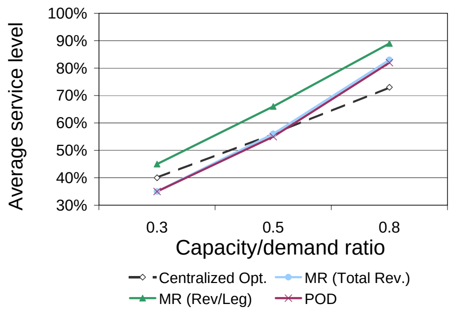
* $stepsize = 1$ for the MR heuristic

real-time, these runtimes are reasonable. Also note that the runtime of the POD heuristic is significantly smaller than the MR heuristic, while its performance in these instances is at most 10% worse than the latter. We may reduce the runtimes of the MR heuristics by choosing larger $stepsize$ values, although note that this can decrease the quality of the solution. In the instances with medium capacity in Table 2(b), the MR heuristic with the total revenue incentive achieves 93% of the centralized optimal revenue with a $stepsize$ of 1, as compared to 87% with a $stepsize$ of 10 and a runtime of 35 CPU seconds.

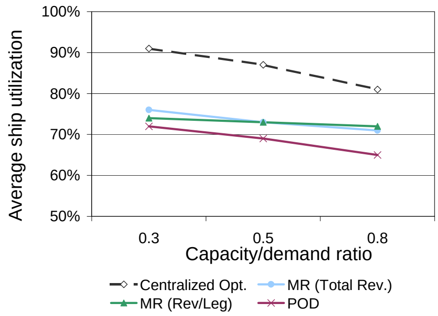


(a) #Routes=1, #Ports=6, $Maxdemand$=30 (b) #Routes=3, #Ports=6, $Maxdemand$=100

Figure 6: Upper bound and optimal CAP solution versus optimal CP solution



(a) #Routes=5, #Ports=10, $Maxdemand$=500



(b) #Routes=5, #Ports=10, $Maxdemand$=500

Figure 7: Average service level and ship utilization with increasing $Capacity/demand$ $ratio$

2.5 Concluding Remarks

In this chapter, we study a Principal-Agent problem on a network motivated by practices in some sea cargo companies. In such companies, the central headquarters, acting as the principal, assigns capacity limits and incentives to sales agents, and the sales agents individually book O-D pair demands originating from their home port. This is an example of a system that operates in a decentralized fashion, and it is sometimes less efficient than a centralized system where all decisions are made by a central organizing body. However, decentralized systems are desirable when centralization requires high investment and operating costs. Our research focuses on managing the decentralized system efficiently by developing optimization tools to determine the capacity limits and incentives for the sales agents.

We formulate a capacity allocation problem for the central headquarters that determines the capacity limits by explicitly incorporating agent behavior with network flow models. For the special case of the single route, we show that the optimal agent behavior can be included in a mixed integer program for the capacity allocation problem. In the general case of the multiple-route, we show that the capacity allocation problem is NP-hard, and we develop several heuristics that integrate agent behavior. We show an asymptotic result that the decentralized system performance can be arbitrarily worse than the centralized system performance. On the other hand, for more typical networks, we use computational experiments to show that the decentralized system performance can be close to the centralized system performance, and its performance improves as capacity increases. We further show that the choice of sales incentive is a factor that can improve the performance of the decentralized system. Our models can be useful in rough-cut capacity planning and for the managers to evaluate “what-if” scenarios.

There are several limitations to our work. First, we do not explicitly model the agents' decisions in choosing sales effort. Therefore, the models are expected to be useful when the effect of incentives on effort levels is limited. Next, we do not include competition among agents or sea cargo companies. We also study a deterministic system as a first step towards studying the more complex real system that is stochastic and dynamic.

Our work develops optimization models to manage a system with individual decision makers. Analyzing decentralized decision-making is important since there are many systems in practice that operate in a decentralized manner, and most optimization models in literature assume a centralized planner. It would be interesting to extend this research by modelling the decentralized booking system in a stochastic setting, where capacity can be re-allocated to the sales agents as demand bookings are realized.

CHAPTER III

SALES PROMOTIONS IN A DECENTRALIZED SUPPLY CHAIN

3.1 Introduction

Much of supply chain management focuses on matching supply and demand through various means such as production flexibility, lead-time management, and channel coordination. In recent years, there has also been a particular focus on manipulating demand through the use of dynamic pricing or promotions to achieve higher profits, e.g., Elmaghraby and Keskinocak [26], Kim et al. [47], and Tang and Tang [76]. This is especially true in industries such as automotive and airlines, where capacity is difficult and costly to adjust. Our goal is to determine under what market conditions (such as demand uncertainty in market potential and price sensitivity) offering customer rebates versus retailer incentives or combined promotions are more profitable or lead to higher sales for the manufacturer. This problem lies at the interface of marketing and operations management.

The main motivation for our research comes from the practices in the auto industry and our discussions with a major auto manufacturer. The production and labor costs of the American auto manufacturers are nearly fixed in part due to union costs, and hence, they focus on increasing revenues, e.g., through the use of promotions. Jakobson [41] states that "Detroit's costs are roughly the same whether a plant is churning out as many cars as it can or standing idle part of the time - so the Big Three produce more cars than their market share justifies, creating gluts that force them to offer large cash incentives to move the excess." Busse et al. [15] mention the auto manufacturers' rigid pricing strategy and state that "Although retail demand for an automobile fluctuates due to changing economic conditions, seasonality, and the stage of the model's life cycle, manufacturers rarely vary published retail and invoice prices of a particular model over the course of the model year." The authors also mention that the auto manufacturers use the "incentive promotions" as an important market strategy tool to respond to fluctuating demand conditions.

Promotions have been frequently used by manufacturers in different industries as a means to increase sales, revenues and market share against competitors by increasing consumers' awareness about their brand, to reduce the inventories of the slow-moving items, or to price discriminate. In the auto industry, different promotional programs such as customer rebate, low percent financing, and employee pricing programs, are offered. In addition to these direct-to-customer promotions, auto manufacturers sometimes offer incentives to the retailers that are generally not publicized to the end customers. Retailer incentives may encourage the dealers to advertise or negotiate with their customers to generate more sales.

As stated by Priddle and Zoia [68], promotions in the auto industry date back to 1912 with rebates offered by Henry Ford on Model Ts. Other developments include the introduction of Chrysler's rebate program (1975) and General Motors' 0% financing program (2001). Automobile manufacturers offered $56 billion in incentives to sell 16.6 million cars and trucks in 2003 (Smith [71]) and the average rebate per model offered by the American auto manufacturers was more than triple that of the Japanese auto manufacturers in 2004 (Henderson [37]). In fact, the American auto manufacturers are known to use frequent and deep customer rebates or cashback, whereas the non-Americans, especially, Japanese, are more inclined to offer incentives to their dealers. Figure 8, based on data sets provided by R. L. Polk & Co. and a major market research firm, shows that the rebate percentages used by the American auto manufacturers are higher and show an increasing trend compared to the Japanese auto manufacturers who offer steadily lower rebates. The promotional choice may depend on the demand characteristics faced by the manufacturers.

It is essential to know which promotion provides higher sales and profits under what kind of market conditions. In this paper, we examine these questions by focusing on two different promotions, namely, the *retailer incentive* and the *customer rebate*. In the former, the manufacturer offers a (lump-sum) incentive to the retailer which can be used in a flexible way by the dealer; in the latter, the manufacturer offers a per-unit customer rebate directly to the end customers. Dealer incentives are offered in different forms by manufacturers, e.g., as lump-sum or per-unit incentive for each vehicle sold. Referring to lump-sum incentives Edmunds.com president Jeremy Anwyl states "here is $100,000 for the dealer to use as he

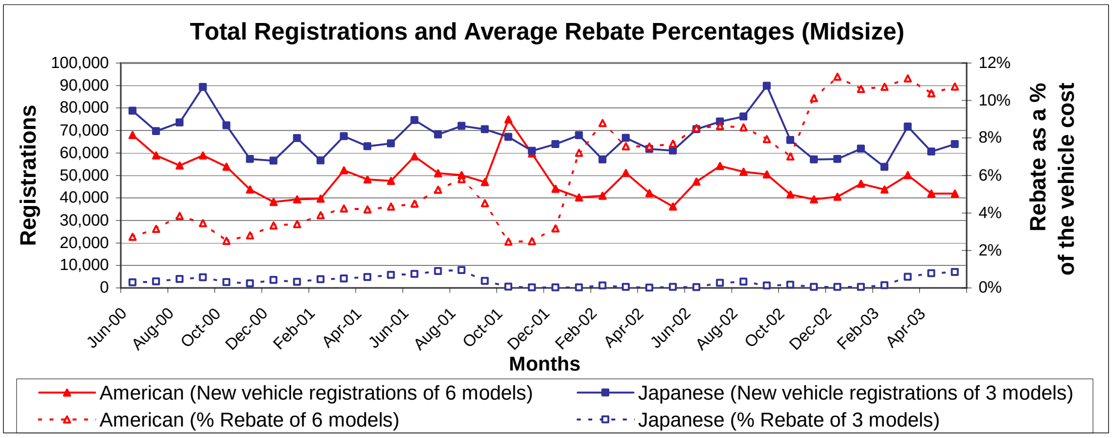


Figure 8: Total registrations and average rebate percentages of midsize models

sees fit" (Anwyl [4]). By contrast, in per-unit incentive scheme, the dealer usually receives a higher payment for each additional vehicle sold beyond a quota set by the manufacturer. The basic tradeoff is that customer rebates are given to every customer, while the retailer controls the use of the retailer incentive and decides how much to give to each customer. While customer rebates are widely publicized, dealer incentives are not. The retailers can also offer their own promotions mostly in the form of customer rebates, but these are generally offered locally and at smaller amounts; thus they are outside the scope of our study.

This chapter is organized as follows. We provide a literature review in Section 3.2, followed by our models, their analysis and comparisons in Section 3.3. We state our conclusions in Section 3.4.

3.2 Literature Review

Several articles on promotions focus on their role as a price discriminating tool, considering single or multiple firms selling directly to the end customers with no intermediary in between. (e.g., see Bester and Petrakis [8], Gerstner and Holthausen [34], Narasimhan [60].) Gerstner and Hess [31] introduce channel issues by analyzing different types of promotions (trade deals and rebates) in a single manufacturer and single retailer setting, where the market has two customer segments with high and low reservation prices. They find that even when all consumers exercise their rebates and price discrimination does not occur, the manufacturers may still find it profitable to offer rebates. Gerstner et al. [33] extend their research by allowing competition between the retailers but only analyze customer rebates.

The retailer pass through rate (the percentage of a trade promotion passed through to consumers) and forward buying (or stockpiling) are two issues stemming from the use of promotions directed towards retailers. Tyagi [81] focuses on temporary price cuts given by the manufacturer to retailers and shows conditions on the market demand function that determines the retailer's pass-through rate. Lal et al. [51] analyze the motivations for manufacturers to offer promotions when the retailers forward buy and do not pass the price discounts to the customers. Kim and Staelin [48] analyze manufacturer allowances (similar to our lump-sum incentives) and retailer pass-through rates in a supply chain with two manufacturers selling through two non-exclusive retailers. In their model, the lump-sum payments are used by the retailers as price discounts for each unit of sales. They find that the manufacturers give retailers side payments (allowances) even though they know the retailers will pocket some portion of it. This finding is mainly a result of the competition among the manufacturers. Side payments that are offered by manufacturers to promote their new product introductions and to convince the retailers to carry their products are called slotting allowances. Lariviere and Padmanabhan [53] analyze slotting allowances to promote new product sales where the manufacturer has private information about demand. The authors show that slotting allowances are only offered if the retailer faces a high opportunity cost of stocking the product. Desai [24] compares slotting allowance and advertising efforts of a manufacturer to promote new product sales, where the retailer faces uncertain demand. The author shows that slotting allowances help the manufacturer more when the market is highly competitive and the retailers have high stocking costs.

In recent years, there have been some extensions within the framework of supply chains analyzing sales promotions as a tool for channel coordination including Gerstner and Hess [32], Krishnan et al. [50], and Taylor [77]. Chen et al. [17] and Aydin and Porteus [6] analyze the effects of rebates on the manufacturer's and retailer's profits in a 2-stage supply chain with stochastic demand. Chen et al. [17] show that rebates always benefit the manufacturer unless all of the buyers redeem their rebates; otherwise they do not necessarily increase the manufacturer's profits. Aydin and Porteus [6] compare per-unit retailer rebate and per-unit customer rebate. The authors conclude that neither the manufacturer nor the retailer always prefers one particular rebate to the other. Sohoni et al. [72] analyze the effects of dealer incentives on sales variability and show that manufacturers may increase profits and decrease sales variability by offering an appropriate stair-step dealer incentive when their dealer is exclusive.

Motivated by practices in the auto industry, our work differs from the cited articles in that the retailer can price discriminate rather than choose a fixed retail price for all customers, which we believe captures the nature of sales by the auto dealers. We analyze retailer incentives that are in the form of lump-sum amounts rather than a wholesale price deduction, motivated by the practices of auto manufacturers who generally keep wholesale prices constant for the model year and offer periodic incentives to the dealers.

Bruce et al. [12, 13] analyze trade promotions (wholesale discounts after a sales quantity target) and cash rebates in the durable goods market, such as automobile, by explicitly incorporating a durability measure for the manufacturer's products to focus on the intertemporal effects of the promotions. In the former research, the authors find in a competitive setting that the manufacturer of the more durable product benefits more from trade promotions. In the latter research, they analyze cash rebates in a single manufacturer and single retailer setting similar to ours. They find that as the durability of the manufacturer's products decreases, the manufacturer finds it more profitable to offer deeper cash rebates. Our research differs from [12, 13] in several aspects. We focus on the effect of demand uncertainty on the manufacturer's choice of promotions, while they focus on the durability of the manufacturers' products. In their analysis, the wholesale price of the manufacturer is contracted simultaneously with the promotions in a deterministic setting, while we allow the wholesale price to be determined under uncertainty. Further, our work focuses on comparing the performances of the customer rebates and retailer incentives in the same model setting where the retailer can price discriminate.

There is a large body of empirical research investigating how promotions work, focusing mostly on non-durables. See Blattberg et al. [10] for a review. Some empirical analysis in durables, such as automobiles includes Busse et al. [15] and Pauwels et al. [66].

In our research, we focus mainly on the manufacturer's wholesale price and promotion decisions and their impact on profits and sales in a decentralized decision framework where the retailer chooses the sales price (for each customer) and the sales quantity. We also contribute to the economics literature by analyzing first-degree (perfect) price discrimination, where different prices may be given to every customer rather than just to segments of customers, which has received little attention by researchers. To the best of our knowledge, Spulber [74] and White and Walker [84] are only two that analyze perfect price discrimination in detail. Spulber [74] analyzes a model where a group of firms that do perfect price discrimination select their output levels simultaneously. The author shows the existence of a unique non-cooperative equilibrium in output levels. In this analysis the firms sell directly to the end customers and no sales promotions are considered. White and Walker [84] analyze the case where there is a variable cost associated with perfect price discrimination. The authors propose a model where perfect price discrimination is selectively practiced only on one portion of the linear demand function, and they show that their model generates higher profits compared to the perfect price discrimination practiced on the entire demand function. Their setting is simplistic, and they analyze only one firm's sales quantity decision, where the firm sells directly to the end customers, while we analyze sales quantity and promotional decisions in two-stage supply chains.

3.3 Models

We consider a two-stage supply chain with a single manufacturer and a single retailer, operating in a market where demand is characterized by a linear inverse demand function, i.e., $P(Q) = a - bQ$, where a is the market potential, b is the price sensitivity and $P(Q)$ is the price when Q units of the product are sold. (Note that $\frac{a}{b}$ is the maximum quantity that may be sold when the price is zero. Since we are interested in changing a while keeping b fixed, we denote a as the market potential for the rest of the paper.) The retailer (or dealer) buys from the manufacturer at a unit wholesale price denoted by w, and has a reservation price, $w + m$, below which he would not be willing to sell. We assume that the retailer can do perfect (first degree) price discrimination. While perfect price discrimination by the retailer can be costly, we do not expect this cost to differ significantly from one customer to another even when the customers differ in terms of their willingness to pay. Therefore, we assume that the cost of price discrimination is constant for each unit of sale. This allows us to absorb the price discrimination cost as part of the retailer's reservation price.

Our setting is applicable to channels where the retailer is a franchisee selling one manufacturer's products exclusively, e.g., the auto industry. Linear demand models have been used by major auto manufacturers in practice, e.g., Biller et al. [9]. In the auto industry, manufacturers usually commit to a constant wholesale price. (Cachon and Lariviere [16] mention that manufacturers hold wholesale prices constant even when capacity is scarce.) Our model mimics the practice, where a retailer's reservation price may depend on the car invoice price (Scott Morton et al. [69], and Zettelmeyer et al. [86]). In our setting, the reservation price of the retailer may be compensated by a combination of the sales price (which should be at least $w+m$ if there is no incentive) and the lump-sum incentive offered by the manufacturer.

The auto dealers have the opportunity to negotiate individually with each customer and collect information about the customer's willingness to pay. Zettelmeyer et al. [85] mention price discrimination as one reason for the retailers to negotiate the prices and state that "Given the high price of a new car, it would not be surprising if the cost of gaining information about a customer's willingness to pay is, in comparison, small enough to make the dealer's effort to assess a customer's valuation and negotiate individual prices more profitable than posting a fixed price." Goldberg [36] empirically finds that dealers can achieve price discrimination through the car model, market-specific properties, and the type of purchase transaction such as first-time purchase and trade-in. Scott Morton et al. [70] add that individual characteristics of car buyers, such as income, education, and search costs are significant factors that affect the dealers' pricing of the cars. Price discrimination through negotiations may also be observed during purchases and sales of houses or other less expensive products through bidding over the internet.

We assume that the manufacturer has ample capacity, and there is no cost to customers associated with rebate redemptions. Since we are interested in comparing the performances of different promotions, we also assume that the administrative cost of offering promotions is zero. We analyze a single selling season; this can be thought of as snapshot in time of a repeated process. Finally, we assume that the manufacturer and the retailer are risk neutral, and both seek to maximize their own (expected) profits.

Table 3 summarizes our notation. Note that the manufacturer cannot make positive profits unless she sells above cost, therefore she chooses $w \geq c$. By the retailer's reservation price, $P(0) \geq w + m$ must hold, which together with $w \geq c$, implies that $a \geq c + m$.

Table 3: Notation for Chapter 3

a^j :	Market potential in demand state $j = l, h$ (l =low, h =high)
b^j :	Price sensitivity of customers in demand state $j = l, h$
Q_i^j :	Retailer's order/sales quantity in demand state $j = l, h$ when promotion type $i \in (o, I, R, R', C)$ is used (o =no promotion, I =retailer incentive, R =customer rebate, R' =customer rebate leading to market expansion, C =combined)
$P(Q)$:	Retail price when Q units are sold
$\Pi_i^{M_j}$:	Profit of the manufacturer under promotion i in demand state j (Π_i^M when demand is deterministic)
$\Pi_i^{D_j}$:	Profit of the retailer under promotion i in demand state j when manufacturer makes her w decision under uncertainty (Π_i^D when demand is deterministic)
Π_i^{SC} :	Profit of the supply chain under promotion i
c :	Production cost of the manufacturer
w_i :	Wholesale price under promotion type i
$w_i + m$:	Reservation price of the retailer under promotion i
K^j :	Lump-sum incentive given to the retailer in demand state j
R^j :	Per unit customer rebate in demand state j

We analyze three demand settings. In Section 3.3.1, we present a deterministic model where there is no uncertainty in the system parameters. (Deterministic demand models have been commonly used in the literature to provide insights and for analytical tractability; see for example, Choi [18] and Corbett and Karmarkar [21].) In Sections 3.3.2 and 3.3.3, we introduce uncertainty ("high" with probability β and "low" with probability $1 - \beta$) to the market potential (a) and price sensitivity (b) parameters of the demand function,

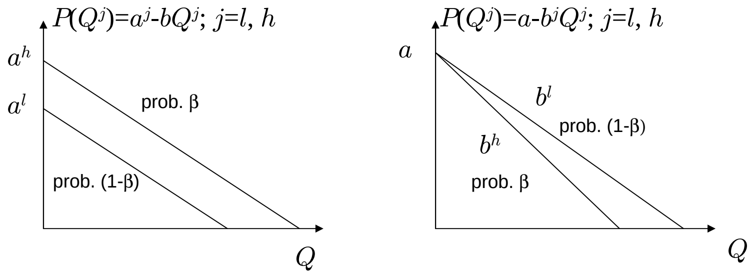


Figure 9: Demand functions for uncertain market settings

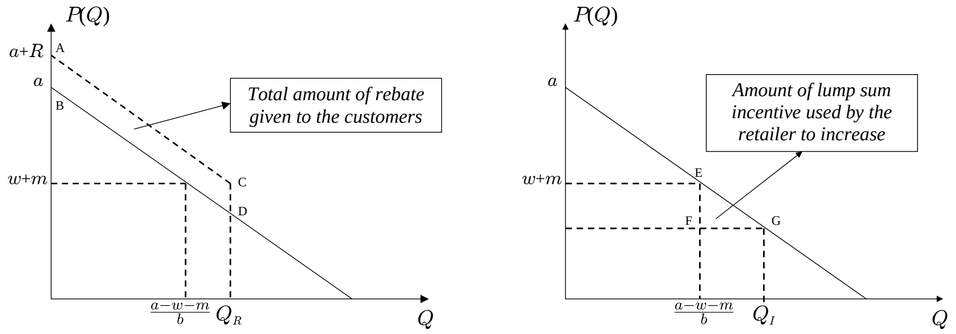
respectively. This type of uncertainty could correspond to scenario planning as is used by many manufacturers in strategic level decisions. In Figure 9, we plot the demand functions corresponding to uncertain market potential and uncertain price sensitivity. Introducing uncertainty has the effect of an up or down shift in the demand function or a change in its slope.

We formulate all models as Stackelberg games, where the manufacturer is the leader and the retailer is the follower. We use backward induction to find the subgame-perfect Nash equilibrium (SPNE). We find the optimal decisions for the manufacturer and the retailer for each model and compare the promotions based on the manufacturer's profit and total sales. We also analyze the profits of the retailer and the supply chain, but our focus is on the manufacturer's choice of promotions. Our goal is to determine situations under which one promotion type is better than another for the manufacturer.

3.3.1 Deterministic Demand Model

In this benchmark setting, demand is deterministic and is common knowledge to both the manufacturer and the retailer. The sequence of decisions starts with the manufacturer determining the wholesale price. Given the manufacturer's decision, the retailer then decides the order/sales quantity.

When the manufacturer offers customer rebate or retailer incentive, Figure 10 describes how we would expect these promotions to affect the retailer's decisions. In the case of no promotion, the retailer has no incentive to sell at a price less than $w + m$, where the corresponding order/sales quantity is $Q_o = \left(\frac{a-w-m}{b}\right)^+$. The manufacturer's purpose in offering promotions is to induce the retailer to order more than Q_o. If she offers customers



(a) Customer rebates

(b) Retailer incentives

Figure 10: The impact of retailer incentives and customer rebates on the retailer's decisions under deterministic demand

a per-unit rebate of R applied to each buyer's price, this has the effect of shifting the demand function up, i.e., $P(Q)-R = a-bQ$ or $P(Q) = (a+R)-bQ$ so that the y-axis is intersected at $a+R$ (Figure 10(a)). The total amount of rebate given to the end customers is the area of $ABCD$. On the other hand, if the manufacturer offers a lump-sum incentive to the retailer, this will have no effect on the shape of the demand function since end customers are not made aware of this incentive, but will make the retailer move downward on the demand function from E down to G as long as he receives the reservation price from the sales of each additional unit (Figure 10(b)). In this case, the triangular area EFG represents the total amount of the lump-sum incentive the retailer is going to use to increase sales, while his reservation price is compensated by this incentive.

Case 0: (No-promotion) The retailer solves the problem in (P1) to decide how much to order/sell to maximize his profit given the wholesale price decision of the manufacturer, and finds that $Q_o = \left(\frac{a-w-m}{b}\right)^+$ where $(x)^+ = \max\{0, x\}$. Note that, for the retailer to satisfy his reservation price, $P(Q) = a - bQ \geq w + m$, i.e., $Q \leq \frac{a-w-m}{b}$.

$$\text{(P1)} \qquad \Pi_o^D = \max_{Q \geq 0} \int_0^Q (a-bQ)dQ - wQ = (a-w)Q - \frac{bQ^2}{2}$$
$$\text{s.t.} \quad Q \leq \frac{a-w-m}{b}$$

The manufacturer's problem is to choose the optimal wholesale price that maximizes her profit, i.e., $\max_{w \geq c}(w-c)Q_o$.

Case 1: (Retailer incentive) The manufacturer offers an incentive (K) to the retailer.

In this case, the retailer chooses his order/sales quantity (Q_I) by solving the problem in (P2), and the manufacturer determines the wholesale price and incentive value simultaneously by solving $\max_{K\geq 0, w\geq c} (w-c)Q_I - K$.

$$\text{(P2)} \qquad \Pi_I^D = \max_{Q\geq 0} \quad (a-w)Q - \frac{bQ^2}{2} + K$$
$$\text{s.t.} \quad \int_{\frac{a-w-m}{b}}^{Q} ((w+m)-(a-bQ))dQ \leq K$$

Note that, given the manufacturer's decisions, (w, K), the retailer has the option of not transferring the incentive to the end customers and choosing $Q_I = \frac{a-w-m}{b}$, where he pockets all of K, and the price of the last unit sold is equal to his reservation price. However, note also that the unconstrained maximizer of the retailer's profit function $\left(Q = \frac{a-w}{b}\right)$ is greater than $\frac{a-w-m}{b}$, i.e., there is potential for the retailer to further increase his profits if he agrees to sell some additional units below his reservation price. On the other hand, to sell additional units beyond $Q = \frac{a-w-m}{b}$ and still make his reservation price $w+m$, the retailer must be compensated by the lump-sum incentive, which is ensured by the constraint. In other words, being the first decision maker in this game, the manufacturer has the advantage of choosing (w, K) to make the retailer voluntarily transfer some part of the incentive to the end customers and increase sales. We also note that while any $K \geq 0$ increases the retailer's profit, it results in higher sales only until $\left(Q = \frac{a-w}{b}\right)$, after which the retailer starts to pocket the incentive. In equilibrium, the manufacturer finds the optimal incentive value such that incentive is completely passed to the end customers and no pocketing occurs. This model captures a dealer behavior observed in practice, where the auto dealers transfer some part of the manufacturers' incentives to the end users. This is investigated by Busse et al. [15] among others. The turn-and-earn system for inventory allocation, and the fact that dealers who sell a high volume may receive a better selection of vehicles, e.g., vehicles with higher profit margin in the future, also motivate this behavior.

Case 2: (Customer rebate) The manufacturer gives the end customers a rebate, R, for each unit purchased. Therefore, the purchasing power of the customers increases by R and the inverse demand function is written as: $P(Q) = a + R - bQ$. The retailer's and the manufacturer's problems are similar to the no-promotion case, where the retailer determines

Table 4: The SPNE for the deterministic demand model with retailer incentive

	$a \leq 2m+c$	$a \geq 2m+c$
w_I	$a-m$	$\frac{a+c}{2}$
Q_I	$\frac{a-m-c}{b}$	$\frac{a-c}{2b}$
K	$\frac{(a-m-c)^2}{2b}$	$\frac{m^2}{2b}$
Π_I^M	$\frac{(a-m-c)^2}{2b}$	$\frac{(a-c)^2-2m^2}{4b}$
Π_I^D	$\frac{(a-m-c)m}{b}$	$\frac{(a-c)^2+4m^2}{8b}$
Π_I^{SC}	$\frac{(a-c)^2-m^2}{2b}$	$\frac{3(a-c)^2}{8b}$

order/sales quantity, and the manufacturer determines the rebate R and the wholesale price w that maximize their own profits.

Theorem 3 *When demand is deterministic, the SPNE corresponding to the cases where the manufacturer offers no promotion, a lump-sum amount of incentive to the retailer, and a per-unit rebate to the end customers are summarized as follows:*

(i) No promotion: $w_o = \frac{a+c-m}{2}$; $Q_o = \frac{a-m-c}{2b}$; $\Pi_o^M = \frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_o^D = \frac{(a-m-c)(a+3m-c)}{8b}$; $\Pi_o^{SC} = \frac{(a-m-c)(3a+m-3c)}{8b}$.

(ii) Retailer incentive: see Table 4.

(iii) Customer rebate: $w_R - R = \frac{a+c-m}{2}$; $Q_R = \frac{a-m-c}{2b}$; $\Pi_R^M = \frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_R^D = \frac{(a-m-c)(a+3m-c)}{8b}$; $\Pi_R^{SC} = \frac{(a-m-c)(3a+m-3c)}{8b}$.

We present the proof of this theorem and other key results of this chapter in Appendix B.1.

In Theorem 3(iii), we find $w_R - R = \frac{a+c-m}{2}$ in equilibrium. Since for any given wholesale price we can find another wholesale price and rebate pair (w, R) resulting in the same profits, this is equivalent to the optimal wholesale price decision for the no-promotion case. Moreover, the manufacturer's sales and profit do not improve with the customer rebate promotion. Bruce et al. [13] find a similar relation between w^* and R^*, and show that the manufacturer does not always find it profitable to give customer rebates, especially when the administrative cost of rebate promotion is high. In our analysis, as a special case of their model, the cost of rebates to the manufacturer is zero. The ineffectiveness of the customer rebates in our setting is mainly a result of the retailer's ability of perfect price discrimination which allows him to capture all the market surplus when the manufacturer offers rebates directly to the end customers.

In our basic model, the effect of the rebates is limited by an additive function of the cash-back amount, i.e., the market potential increases to $a + R$. However, in practice, rebates may cause an additional increase in the market potential (market expansion) since they are often advertised by the manufacturers and/or the retailers, which increase customer awareness. For example, in the auto industry both the dealers and the auto manufacturers use the newspapers and broadcasting for advertising. Since it is costly to advertise these promotions, it is essential that the manufacturer finds the correct balance between the cost of promoting and the revenue from the additional sales. We model this situation with a market expansion factor (α) affecting market potential, and an advertising cost that is a convex increasing function of this expansion factor ($e\alpha^2$). We observe that when rebates lead to market expansion, neither of the promotions is always preferred to another by the manufacturer. We present the complete analysis in Appendix B.2.

Using the results in Theorem 3, we compare the retailer incentive and the customer rebate promotions with respect to two measures that might be of interest to the manufacturer, namely, total profit and quantity sold. The latter is related with the market share, which is a particularly important measure in the auto industry. Although we focus on the manufacturer's promotional decisions, we also compare the retailer's and the supply chain's profits under different promotions to better understand their impact on the entire chain.

Observation 1 *When rebates do not lead to market expansion (i) offering customer rebates is not effective in increasing the quantity sold and does not change the manufacturer's or retailer's profits (ii) the retailer incentive is always better than the customer rebate and no promotion in terms of total sales, the manufacturer's profit, and the supply chain's profit, but not necessarily the retailer's profit.*

Observation 1(i) holds because when rebates do not lead to market expansion, for any (w, R) combination, the manufacturer can choose a wholesale price $w - R \geq 0$ and achieve the same results as in no promotion. Observation 1(ii) is driven by the tradeoff in using the promotion for every buyer (customer rebate) versus only for those who need it (retailer incentive), as well as the retailer's reservation price. Recall that, the retailer is not willing to sell below his reservation price ($w + m$). By using the incentive to cover the difference between the price that the customer pays and his reservation price (only for those customers who cannot afford to pay his reservation price), the retailer is able to generate more sales than the no-promotion case. Although the manufacturer obtains higher profits with higher sales, the retailer's profit does not always increase. For example, if the market potential is already high ($a \geq 3.5m + c$), when the manufacturer offers an incentive, she raises her wholesale price at the same time, which in turn results in higher sales but lower profits for the retailer. (Bruce et al. [13] make a similar observation regarding customer rebate, i.e., the manufacturer offering a cash rebate increases her wholesale price more than the rebate amount. Bruce et al. [12] analyze trade promotions with two manufacturers and two retailers, and find that each retailer makes less profit when both manufacturers offer trade promotions.) A modelling alternative is based on principal-agent framework, where the manufacturer chooses the wholesale price and the retailer incentive such that the retailer's profit with the incentive is at least as high as his profit with no promotion. We present the analysis of this model in Appendix B.3 and show that our qualitative insights are the same as those in Observation 1. This model may have limited applicability in the auto industry where the retailers do not have the flexibility to accept or reject each contract in the case of promotions. Moreover, a dealer generally has a long-standing relationship with the manufacturer, and a dealership does not easily switch from one manufacturer to another.

The retailer's reservation price can be related to his operational costs. However, if we treat m only as a cost incurred for every unit sold, in the retailer incentive case, the entire K is pocketed by the retailer, and therefore, the manufacturer does not offer any incentive. As a result, the customer rebate and retailer incentive cases become identical to the no-promotion case. Assuming the retailer's reservation price is similar to what we observe for customer behavior (willing to buy if price is below reservation price), as also mentioned in several other studies in the automobile industry (Scott Morton et al. [69], and Zettelmeyer et al. [86]), we assume that the reservation price of the retailer does not only correspond to an operational cost per-unit sold, but rather it is the lowest acceptable price for each unit that the retailer is willing to sell.

We analyze the manufacturer's promotional decisions in three other cases: 1) the retailer sets a fixed retail price rather than price discriminating, 2) the retailer incentive is a per-unit payment rather than a lump-sum amount, 3) the retailer has a specific pass-through rate when offered an incentive. We present the analysis of each case in Appendices B.4, B.5, and B.6, respectively. In the first case, we find that the manufacturer is always better off with the retailer incentive than the customer rebate in terms of profit, but not necessarily in terms of sales, which is different than our result for the price discrimination case. This also shows the importance of modelling price discrimination since increased sales with incentives are observed in the auto industry. In the second case, we find that the manufacturer is always better off with a per-unit retailer incentive than a customer rebate (just as we found for lump-sum incentive.) Moreover, when $a \geq 3m + c$, the manufacturer's profit is higher with a per-unit incentive than a lump-sum incentive. Therefore, we expect that our insights with lump-sum incentive would hold even more strongly for other incentive schemes that depend on quantity. Finally, in the third case, we find that our qualitative results in the comparison of promotions do not change, i.e., the manufacturer is better off with a retailer incentive than a customer rebate as long as the pass-through rate is not zero; otherwise, the manufacturer would not offer any incentive.

By Observation 1, we conclude that when the manufacturer and the retailer have the same (and accurate) information about the market conditions and when rebates do not lead to market expansion, the manufacturer always prefers the retailer incentive over customer rebate. If this is the case, then why do the manufacturers offer customer rebates? In practice, rebates are frequently used especially by the American auto manufacturers with the goal of increasing market share. One reason for the use of rebates might be the increased awareness and market potential. In addition, in practice, market demand is most likely stochastic rather than deterministic. In such situations, the timing of the decisions, as well as the information possessed at the time of the decisions become critical for the success

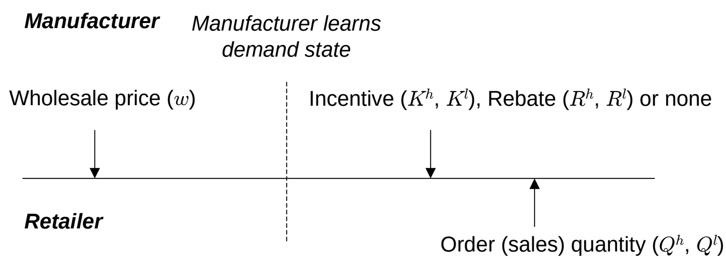


Figure 11: Timeline of decisions (monopoly)

of promotions. In the next section, we show that demand uncertainty may explain the American auto manufacturers' choice of customer rebates over retailer incentives.

3.3.2 Uncertain Market Potential Model

In this section, we introduce uncertainty to the demand through the market potential by considering two demand states, high (h) and low (l), with respective probabilities β and $1-\beta$, $\beta \in [0,1]$. Correspondingly, we represent the inverse demand functions as $P(Q^j) = a^j - bQ^j$, where $j = l, h$. Although this is a simplification of demand uncertainty in reality, it helps us to capture the effects of uncertainty on the decisions of the retailer and the manufacturer. In the auto industry, the dealers are closer to the end market and have more information about the customers, therefore we assume that the retailer knows the demand state. In addition, dealers do not have to report sales directly back to manufacturers, therefore manufacturers may not have timely and accurate demand information. In recent years, it has become possible for the manufacturers to buy demand information although there may be a delay and transactions may be averaged. The manufacturers usually have a forecast based on past sales, or they may estimate whether demand is high or low based on signals from the market, web interest or economic indicators, which may include uncertainties.

In the presence of uncertainty, the manufacturer makes her decisions in two stages. In the first stage, she determines the wholesale price; in the second stage, after the demand state is revealed, she chooses to offer either a retailer incentive, customer rebate or no promotion. As in the deterministic model, the retailer determines the optimal order/sales quantity given the manufacturer's decisions. We illustrate the timeline of decisions in Figure 11.

Similar to Section 3.3.1, we analyze three cases, i.e., the retailer incentive, customer

Table 5: The SPNE for the uncertain market potential model with no promotion

<table>
<tr><th></th><th>$a^h - a^l \leq a^l - m - c$</th><th colspan="2">$a^h - a^l \geq a^l - m - c$</th></tr>
<tr><td></td><td>$\beta \in [0, 1]$</td><td>$\beta \leq \overline{\beta}$</td><td>$\beta \geq \overline{\beta}$</td></tr>
<tr><td>w_o</td><td colspan="2">$\frac{\overline{a}+c-m}{2}$</td><td>$\frac{a^h-m+c}{2}$</td></tr>
<tr><td>Q_o^l</td><td colspan="2">$\frac{(1+\beta)a^l-\beta a^h-m-c}{2b}$</td><td>$0$</td></tr>
<tr><td>Q_o^h</td><td colspan="2">$\frac{(2-\beta)a^h-(1-\beta)a^l-m-c}{2b}$</td><td>$\frac{a^h-m-c}{2b}$</td></tr>
<tr><td>$\Pi_o^{D_l}$</td><td colspan="2">$\frac{(\beta a^h-(1+\beta)al+m+c)(\beta a^h-(1+\beta)a^l-3m+c)}{8b}$</td><td>$0$</td></tr>
<tr><td>$\Pi_o^{D_h}$</td><td colspan="2">$\frac{((\beta-2)a^h+(1-\beta)al+m+c)((\beta-2)a^h+(1-\beta)al-3m+c)}{8b}$</td><td>$\frac{(a^h-m-c)(a^h-c+3m)}{8b}$</td></tr>
<tr><td>Π_o^M</td><td colspan="2">$\frac{(\overline{a}-m-c)^2}{4b}$</td><td>$\frac{\beta(a^h-m-c)^2}{4b}$</td></tr>
</table>

rebate and no promotion, and then compare the results. The solution procedure for finding the SPNE is again backward induction, starting with the retailer's problem, which we solve for both high and low demand states. Next, we solve the manufacturer's problem of determining the optimal promotion values for both high and low states. The final step in the induction is to maximize the manufacturer's expected profit, i.e., $\Pi_i^M = \beta\Pi_i^{M_h} + (1-\beta)\Pi_i^{M_l}$ by choosing the optimal wholesale price, where $\Pi_i^{M_j}$; $j = l, h$ is the manufacturer's profit in demand state j with promotion type i. We denote the expected market potential with $\overline{a} = \beta a^h + (1-\beta)a^l$.

Theorem 4 *When the market potential is uncertain, the SPNE corresponding to the cases where the manufacturer offers no promotion, a lump-sum amount of incentive to the retailer, and a per-unit rebate to the end customers are summarized as follows:*

(i) No promotion: see Table 5, where $\overline{\beta} = \frac{(a^l-m-c)^2}{(a^h-a^l)^2}$.

(ii) Retailer incentive: see Table 6.

(iii) Customer rebate: $w_R - R^j = \frac{a^j+c-m}{2}$; $Q_R^j = \frac{a^j-m-c}{2b}$, $j = l, h$;

$$\Pi_R^M = \frac{\beta(a^h-m-c)^2+(1-\beta)(a^l-m-c)^2}{4b};\ \Pi_R^{D_j} = \frac{(a^j-m-c)(a^j+3m-c)}{8b},\ j = l, h.$$

In the cases of no-promotion and retailer incentive, we obtain a *unique* equilibrium for a given set of system parameters. In both cases, the feasible solutions that are candidates for being the unique equilibrium are of two types: the wholesale price is either driven by the expected market potential, $\overline{a}$, (expectation driven wholesale price or solution) or only by the high market potential, a^h, (high-demand driven wholesale price or solution). In the

Table 6: Dominating feasible solutions for the uncertain market potential demand model with retailer incentive

Feasible Region (F.R.)			Solution
$a^l \leq 2m+c$	$a^h \leq 2m+c$		RI.2, RI.6
	$2m+c \leq a^h \leq 2a^l - c$	$\beta \leq \frac{2m+c-a^l}{a^h-a^l}$	RI.2, RI.3
		$\beta \geq \frac{2m+c-a^l}{a^h-a^l}$	RI.1, RI.3
	$a^h \geq 2a^l - c$	$\beta \leq \frac{2m+c-a^l}{a^h-a^l}$	RI.2, RI.3
		$\frac{2m+c-a^l}{a^h-a^l} \leq \beta \leq \frac{a^l-c}{a^h-a^l}$	RI.1, RI.3
		$\beta \geq \frac{a^l-c}{a^h-a^l}$	RI.3
$a^l \geq 2m+c$	$2m+c \leq a^h \leq 2a^l - 2m - c$		RI.5, RI.1
	$2a^l - 2m - c \leq a^h \leq 2a^l - c$		RI.4, RI.1
	$a^h \geq 2a^l - c$	$\beta \leq \frac{a^l-c}{a^h-a^l}$	RI.4, RI.1
		$\beta \geq \frac{a^l-c}{a^h-a^l}$	RI.4

	RI.1	RI.2	RI.3	RI.4
w^*	$\frac{\overline{a}+c}{2}$	$\overline{a} - m$	$\frac{a^h+c}{2}$	
Q^{l*}	$\frac{-\beta a^h+(1+\beta)a^l-c}{2b}$	$\frac{a^l-m-c}{b}$	0	
Q^{h*}	$\frac{(2-\beta)a^h-(1-\beta)a^l-c}{2b}$	$\frac{a^h-m-c}{b}$	$\frac{a^h-c}{2b}$	
K^{l*}	$\frac{m^2}{2b}$	$\frac{(\overline{a}-m-c)^2}{2b}$	0	
K^{h*}	$\frac{m^2}{2b}$	$\frac{(\overline{a}-m-c)^2}{2b}$	$\frac{m^2}{2b}$	
$\Pi^{D_l^*}$	$\frac{(\beta a^h-(1+\beta)a^l+c)^2+4m^2}{8b}$	$\frac{\beta^2(a^h-a^l)^2}{2b} + \frac{2m(a^l-m-c)}{2b}$	0	
$\Pi^{D_h^*}$	$\frac{(\overline{a}-2a^h+c)^2+4m^2}{8b}$	$\frac{(a^h-a^l)^2(\beta-1)^2}{2b} + \frac{2m(a^h-m-c))}{2b}$	$\frac{(a^h-c)^2+4m^2}{8b}$	
Π^{M^*}	$\frac{(a^l-c)^2-2m^2+\beta^2(a^h-a^l)^2}{4b} + \frac{2\beta(c(a^l-a^h)+a^h a^l-(a^l)^2)}{4b}$	$\frac{(\overline{a}-m-c)^2}{2b}$	$\frac{\beta((a^h-c)^2-2m^2)}{4b}$	
F.R:	$\overline{a} \geq 2m+c$ $\beta \leq \frac{a^l-c}{a^h-a^l}$	$\overline{a} \leq 2m+c$	$a^h \geq 2m+c$ $a^l \leq 2m+c$	$a^h \geq 2a^l - m - c$

	RI.5	RI.6
w^*	$a^l - m$	$a^h - m$
Q^{l*}	0	0
Q^{h*}	$\frac{a^h-a^l+m}{b}$	$\frac{a^h-m-c}{b}$
K^{l*}	0	0
K^{h*}	$\frac{m^2}{2b}$	$\frac{(a^h-m-c)^2}{2b}$
$\Pi^{D_l^*}$	0	0
$\Pi^{D_h^*}$	$\frac{(a^h-a^l+m)^2+m^2}{2b}$	$\frac{m(a^h-m-c)}{b}$
Π^{M^*}	$\frac{\beta(a^l-m-c)(a^h-a^l+m)}{b} - \frac{\beta m^2}{2b}$	$\frac{\beta(a^h-m-c)^2}{2b}$
F.R:	$a^l \geq 2m+c$ $a^h \leq 2a^l - m - c$	$a^h \leq 2m+c$

former, the manufacturer is able to make positive sales whether the realized demand is low or high, whereas in the latter, she cannot generate any sales when the realized demand is low. The manufacturer faces a tradeoff between the loss of profit she incurs in the following two events. In the first event, she keeps the wholesale price "average" (by considering both states) and sees a high demand, which happens with a probability of β. In the second event, she keeps the wholesale price "high" (by only considering the high state) and sees a low demand, which happens with a probability of $1-\beta$. Being a risk neutral decision maker, the manufacturer chooses the event that brings highest expected profit. Intuitively, we expect that the loss incurred in the former event would increase in β. For all expectation driven solutions, we can show that the expected loss of profit in the former event increases in a^h and decreases in a^l. This suggests that the high-demand driven wholesale price brings higher expected profit to the manufacturer when β and $a^h - a^l$ are "high".

Observation 2 *In the case of no-promotion, when the difference between high and low market potentials is relatively "small" (i.e., $a^h - a^l \leq a^l - m - c$), or β is "low" (i.e., $\beta \leq \overline{\beta}$), the equilibrium for the manufacturer is driven by the expectation over the high and low market potentials. Otherwise, the wholesale price depends on the high market potential, but not on the low market potential. Finally, as the gap between a^h and a^l increases, $\overline{\beta}$ decreases. (See Figure 12 for an example.)*

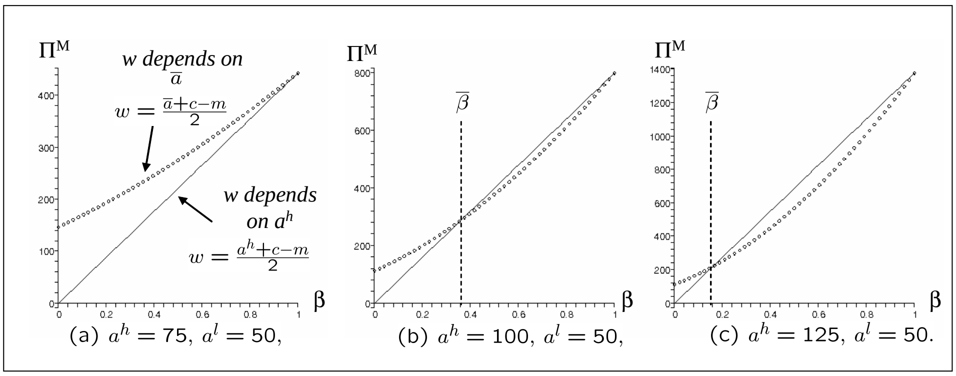


Figure 12: No promotion equilibrium wholesale price with increasing a^h ($m = 5, b = 2, c = 15$)

In Figure 13, we see that the manufacturer's equilibrium behavior in the retailer incentive case is similar to the no-promotion case. In this example, two feasible solutions corresponding to the system parameters are RI.4 (high-demand driven solution) and RI.1 (expectation driven solution) in Table 6. When $a^h - a^l$ is "low", RI.1 provides a higher profit for the manufacturer. As we increase a^h in Figures 13(b) and 13(c), when $\beta \geq \beta^* = \frac{(a^l-c)^2-2m^2}{(a^h-a^l)^2}$, RI.4 results in a higher profit for the manufacturer. Note also that β^* decreases in $(a^h - a^l)$. In Figure 14, we observe a behavior similar to that in Figure 13 when the gap between a^h and a^l is small, but a^l is very close to $m + c$, i.e., the manufacturer's profit is very low in case of low demand.

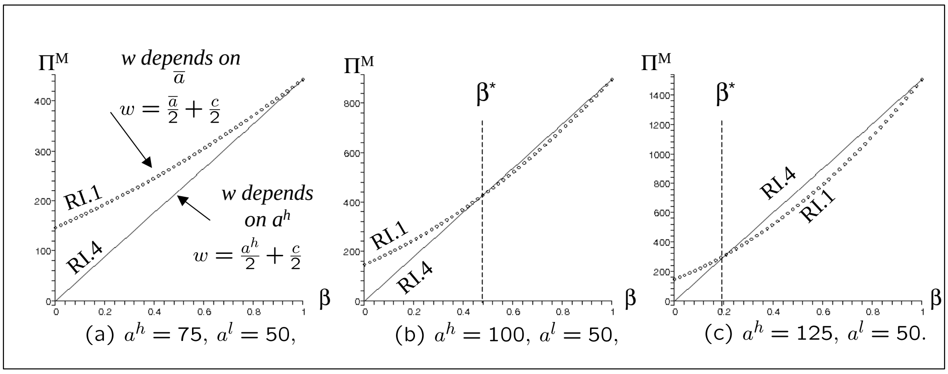


Figure 13: Retailer incentive equilibrium wholesale price with increasing a^h ($m = 5, b = 2, c = 15$)

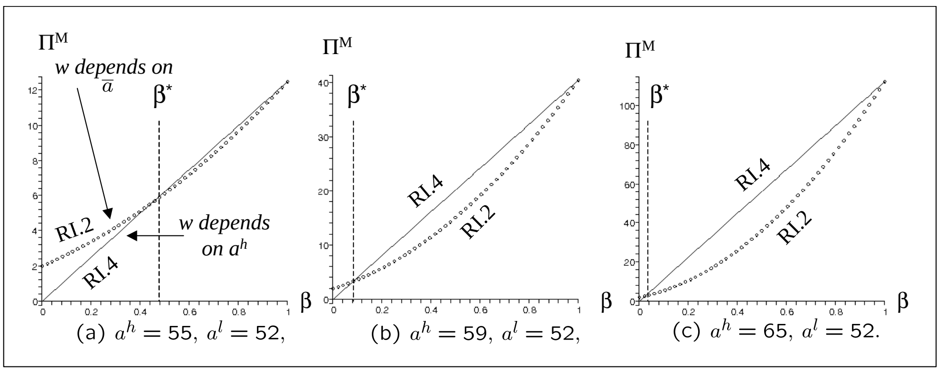


Figure 14: Retailer incentive equilibrium wholesale price with increasing a^h ($m = 20, b = 1, c = 30$)

In Appendix B.7, we extend our analysis with uncertain market potential by adding a “medium” state. While this complicates the analytical tractability considerably, we show from examples that our main insights obtained with two states and three states are similar, i.e., when the probability of a high state is sufficiently high, the equilibrium decision moves from being an expectation-driven solution to a solution only dependent on high demand.

Finally, we analyze the equilibrium in the customer rebate case stated in Theorem 4(iii). Note that, although the manufacturer determines the wholesale price before knowing the demand state, she has the opportunity to adjust it with a rebate according to the demand state. This is obviously an advantage for the manufacturer and as we will see in Observation 3, depending on the system parameters, the manufacturer may be better off offering a customer rebate instead of retailer incentive when there is demand uncertainty.

Next, we compare the promotions under uncertain market potential. From Theorem 4(i)-(iii), we can show that offering no promotion is the least profitable option and generates the lowest sales for the manufacturer. The comparison of the retailer incentive and customer rebate promotions shows that unlike the deterministic demand case in Section 3.3.1, when there is demand uncertainty neither the retailer incentive nor the customer rebate has an absolute dominance over the other. Depending on system parameters, especially related to uncertainty, the manufacturer may find customer rebate or retailer incentive more profitable.

Observation 3 *When the market potential is uncertain and the uncertainty is “high”, i.e., when $(a^h - a^l)$ is “large” and β is in the “middle” of the range, offering a customer rebate may be more profitable than a retailer incentive for the manufacturer. (See Figure 15 for an example.)*

In Figure 15, we continue with the example illustrated in Figure 13. We observe that as $(a^h - a^l)$ increases, the β range (β_1, β_2) where the customer rebate is more profitable than the retailer incentive shifts to the left approaching the origin. We analytically derive this comparative static result for this example in Proposition 1. Note that β^* denotes the threshold value where the equilibrium switches from RI.1 to RI.4. When $\beta_1 \leq \beta \leq \beta^*$, customer rebate gives a higher profit than RI.1, and when $\beta^* \leq \beta \leq \beta_2$, customer rebate

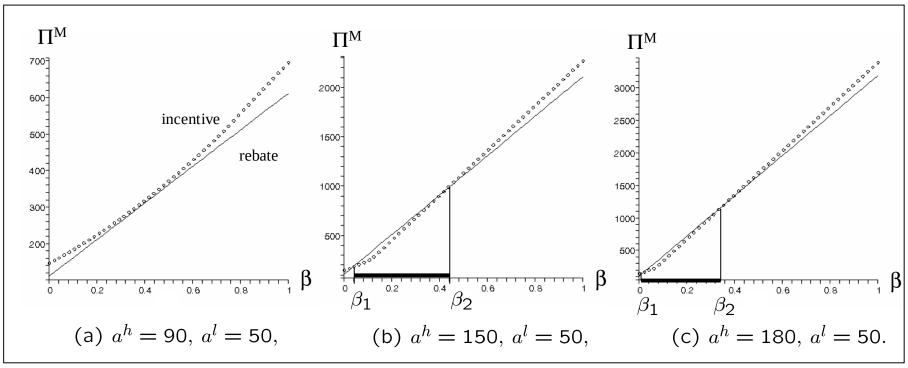


Figure 15: Comparison of retailer incentive and customer rebate with increasing a^h ($m = 5, b = 2, c = 15$)

gives a higher profit than RI.4. Therefore (β_1, β_2) denotes the range where the manufacturer is better off with a customer rebate. We also observe that (β_1, β_2) becomes smaller as $(a^h - a^l)$ increases (Figure 15(c)). This suggests that uncertainty can be one reason for auto manufacturers to offer rebates.

Proposition 1 *When $\beta_1 \leq \beta^* \leq \beta_2$, as a^h increases and a^l decreases ($(a^h - a^l)$ increases), β^*, β_1, and β_2 decrease. ($\beta^* = \frac{(a^l-c)^2-2m^2}{(a^h-a^l)^2}$, $\beta_1 = \frac{a^h-a^l-2m-\sqrt{(a^h-a^l-2m)^2+4m(3m+2c-2a^l)}}{2(a^h-a^l)}$, and $\beta_2 = \frac{(a^l-m-c)^2}{(a^l-c)^2+2m(a^h-a^l-m)}$)*

In Figure 16, we plot the retailer's profits for the example in Figure 15. We see that when the market potential is uncertain and the realized demand state turns out to be "low", the retailer obtains higher profits with a customer rebate. If the realized demand state turns out to be "high", when β is "low", the retailer's profits are higher with a retailer incentive; otherwise he obtains higher profits with a customer rebate. We can explain this observation as follows. When β is "low", the manufacturer's wholesale price decision is driven by expectation when she offers a retailer incentive, and as β decreases w also decreases. After the manufacturer makes the wholesale price decision under uncertainty, if the actual demand state turns out to be "high", the retailer is able to generate higher profits with a retailer incentive than a customer rebate because he is able to order/sell more (with a lower wholesale price), and moreover receives an incentive (K^h) to further increase his order/sales quantity under the "high" demand state.

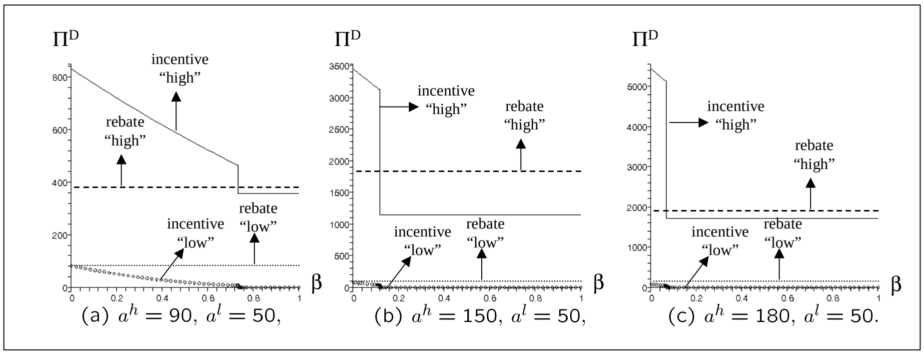


Figure 16: The retailer's profit under retailer incentive and customer rebate with increasing a^h ($m = 5, b = 2, c = 15$)

Table 7: The SPNE for the uncertain price sensitivity model

(a) No Promotion

w_o	$\frac{a+c-m}{2}$
Q_o^j	$\frac{a-m-c}{2b^j}$; $j = l, h$
$\Pi_o^{D_j}$; $j = l, h$	$\frac{(a-m-c)(a+3m-c)}{8b^j}$
Π_o^M	$\frac{(a-m-c)^2}{4}\left(\frac{\beta}{b^h} + \frac{(1-\beta)}{b^l}\right)$

(b) Customer Rebate

$w_R - R^j$	$\frac{a+c-m}{2}$; $j = l, h$
Q_R^j	$\frac{a-m-c}{2b^j}$; $j = l, h$
$\Pi_R^{D_j}$; $j = l, h$	$\frac{(a-m-c)(a+3m-c)}{8b^j}$
Π_R^M	$\frac{(a-m-c)^2}{4}\left(\frac{\beta}{b^h} + \frac{(1-\beta)}{b^l}\right)$

(c) Retailer Incentive

	$a \leq 2m + c$	$a \geq 2m + c$
w_I	$a - m$	$\frac{a+c}{2}$
Q_I^j; $j = l, h$	$\frac{a-m-c}{b^h}$	$\frac{a-c}{2b^h}$
K^j; $j = l, h$	$\frac{(a-m-c)^2}{2b^j}$	$\frac{m^2}{2b^j}$
$\Pi_I^{D_j}$; $j = l, h$	$\frac{(a-m-c)m}{b^j}$	$\frac{(a-c)^2+4m^2}{8b^j}$
Π_I^M	$\frac{(a-m-c)^2}{2}\left(\frac{\beta}{b^h} + \frac{(1-\beta)}{b^l}\right)$	$\frac{(a-c)^2-2m^2}{4}\left(\frac{\beta}{b^h} + \frac{(1-\beta)}{b^l}\right)$

3.3.3 Uncertain Price Sensitivity Model

In this section, we consider uncertainty in price sensitivity, i.e., $P(Q^j) = a - b^j Q^j$; $j = l, h$, where $P(Q^j)$ is the price when Q^j units are sold with price sensitivity b^j in state $j = l, h$. We repeat the analysis in Section 3.3.2, and find the SPNE for the cases of no promotion, retailer incentive, and customer rebate. (See Table 7 for the equilibrium solutions.)

We observe that when there is uncertainty in price sensitivity, offering customer rebate is identical to offering no promotion from the manufacturer's point of view. Moreover, for any market condition, the manufacturer's profit and the quantity sold is higher under retailer incentive compared to customer rebate or no promotion in both demand states. This observation arises because the wholesale price decision is independent of price sensitivity and therefore the manufacturer's decisions are identical to the deterministic demand case.

3.3.4 Combined Promotions: Retailer Incentive and Customer Rebate

In practice, manufacturers sometimes choose to offer a combination of retailer incentive and customer rebate. In this section, we aim to obtain insights on how effective it is to offer both promotions at the same time. We summarize the equilibrium solutions in Table 8.

When demand is deterministic, the manufacturer's profit and total sales when she offers retailer incentive and customer rebate simultaneously are equal to her profit and total sales when she offers retailer incentive alone. In Section 3.3.1 we have seen that the manufacturer's profit remains the same when she offers a customer rebate instead of no promotion, while she increases her profits by offering a retailer incentive. It is indeed expected that combined promotions, being a hybrid of the customer rebate and the retailer incentive promotions, will not do any better than the retailer incentive itself for the deterministic demand case. With the same reasoning, we also expect that combining two promotions will not improve the profits when there is uncertainty in price sensitivity, and confirm by Table 8(b).

However, for the uncertain market potential model, we have observed situations where customer rebate was performing better than the retailer incentive as well as situations where the opposite holds. Clearly, by combining these promotions, the manufacturer is expected to do at least as well as the case when she uses each promotion individually. The question is whether the manufacturer is able to do strictly better when she offers both promotions simultaneously. Our analysis of the equilibrium decisions shows that this is true, i.e., the manufacturer's total profit is higher if she offers two promotions (retailer incentive and customer rebate) at the same time rather than offering any one of them individually. Moreover, the combined promotions generate more sales than the retailer incentive, except for some instances with $a^h \geq a^l \geq 2m + c$ or $a^l \leq 2m + c \leq a^h$. Under all market conditions, combined promotions generate more sales than the customer rebate alone.

Table 8: The SPNE for the combined promotions

(a) Deterministic Demand Model

	$a \leq 2m+c$	$a \geq 2m+c$
$w_C - R$	$a-m$	$\frac{a+c}{2}$
Q_C	$\frac{a-m-c}{b}$	$\frac{a-c}{2b}$
K	$\frac{(a-m-c)^2}{2b}$	$\frac{m^2}{2b}$
Π_C^D	$\frac{(a-m-c)m}{b}$	$\frac{(a-c)^2+4m^2}{8b}$
Π_C^M	$\frac{(a-m-c)^2}{2b}$	$\frac{(a-c)^2-2m^2}{4b}$

(b) Uncertain Price Sensitivity Model

	$a \leq 2m+c$	$a \geq 2m+c$
$w_C - R^j;\ j=l,h$	$a-m$	$\frac{a+c}{2}$
$Q_C^j;\ j=l,h$	$\frac{a-m-c}{b^j}$	$\frac{a-c}{2b^j}$
$K^j;\ j=l,h$	$\frac{(a-m-c)^2}{2b^j}$	$\frac{m^2}{2b^j}$
$\Pi_C^{D_j};\ j=l,h$	$\frac{(a-m-c)m}{b^j}$	$\frac{(a-c)^2+4m^2}{8b^j}$
Π_C^M	$\frac{(a-m-c)^2}{2}\left(\frac{\beta}{b^h}+\frac{(1-\beta)}{b^l}\right)$	$\frac{(a-c)^2-2m^2}{4}\left(\frac{\beta}{b^h}+\frac{(1-\beta)}{b^l}\right)$

(c) Uncertain Market Potential Model

	$a^h \geq a^l \geq 2m+c$	$a^l \leq 2m+c \leq a^h$	$a^l \leq a^h \leq 2m+c$
$w_C - R^l$	$\frac{a^l+c}{2}$	a^l-m	a^l-m
$w_C - R^h$	$\frac{a^h+c}{2}$	$\frac{a^h+c}{2}$	a^h-m
Q_C^l	$\frac{a^l-c}{2b}$	$\frac{a^l-m-c}{b}$	$\frac{a^l-m-c}{b}$
Q_C^h	$\frac{a^h-c}{2b}$	$\frac{a^h-c}{2b}$	$\frac{a^h-m-c}{b}$
K^l	$\frac{m^2}{2b}$	$\frac{(a^l-m-c)^2}{2b}$	$\frac{(a^l-m-c)^2}{2b}$
K^h	$\frac{m^2}{2b}$	$\frac{m^2}{2b}$	$\frac{(a^h-m-c)^2}{2b}$
$\Pi_C^{D_l}$	$\frac{(a^l-c)^2+4m^2}{8b}$	$\frac{m(a^l-m-c)}{b}$	$\frac{(a^l-m-c)m}{b}$
$\Pi_C^{D_h}$	$\frac{(a^h-c)^2+4m^2}{8b}$	$\frac{(a^h-c)^2+4m^2}{8b}$	$\frac{(a^h-m-c)m}{b}$
Π_C^M	$\beta\left(\frac{(a^h-c)^2}{4b}-\frac{m^2}{2b}\right)$ $+(1-\beta)\left(\frac{(a^l-c)^2}{4b}-\frac{m^2}{2b}\right)$	$\beta\left(\frac{(a^h-c)^2}{4b}-\frac{m^2}{2b}\right)$ $+(1-\beta)\frac{(a^l-m-c)^2}{2b}$	$\beta\frac{(a^h-m-c)^2}{2b}$ $+(1-\beta)\frac{(a^l-m-c)^2}{2b}$

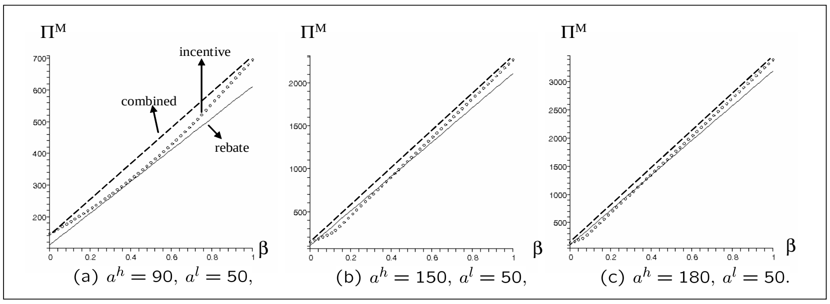


Figure 17: Profit improvement by combined promotions with increasing a^h $(m = 5, b = 2, c = 15)$

Continuing with the example in Figure 15, we show the improvement in the manufacturer's profits when she offers combined promotions as opposed to offering each promotion individually (Figure 17). In this case, the manufacturer is able to increase her profits by as much as 18% by combining two promotions. The improvement in the manufacturer's profits decreases as the gap between the market potentials $(a^h - a^l)$ increases. We can explain this behavior as follows. As $(a^h - a^l)$ increases, the manufacturer is better off with a high-demand driven equilibrium, i.e., behaving as if she is in a deterministic setting with a market potential of a^h. We have seen that when demand is deterministic, the retailer incentive is always better than the customer rebate. Therefore, we expect that the profit increase achieved by combining the two promotions in an effectively deterministic setting is not as high as the profit increase in a setting with uncertainty where both promotions may improve the manufacturer's profits.

3.4 Concluding Remarks

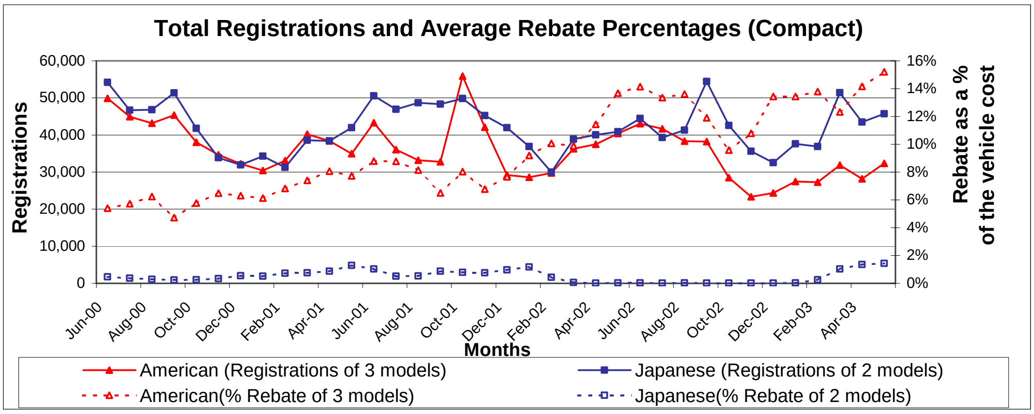
In this chapter, we analyze two different types of promotions, customer rebates and retailer incentives, which are commonly used by manufacturers in the auto industry. We consider several models with different demand characteristics to determine under which market conditions one promotion is more effective than the other in terms of increasing the manufacturer's profits and sales. Our setting is a two stage supply chain with an uncapacitated manufacturer and a retailer who can do perfect price discrimination in a market with price sensitive demand.

We show that when demand is deterministic and rebates do not lead to market expansion, customer rebates are not effective in increasing the manufacturer's sales and profits, and the manufacturer is always better off with a retailer incentive. If the rebates lead to an increase in the market potential possibly through advertising but at an additional cost, the manufacturer may prefer the customer rebate over the retailer incentive when the increase in the market potential is sufficient enough to cover the cost of promoting.

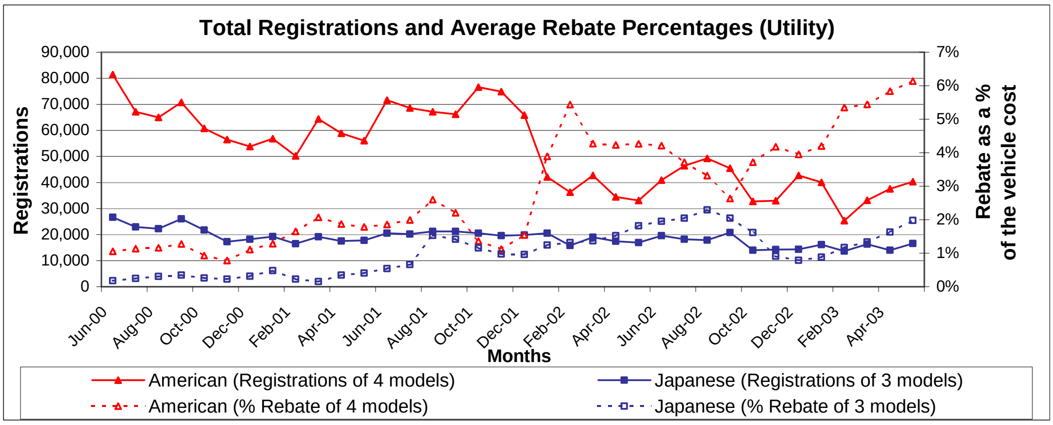
We show that when the market potential is uncertain, neither the retailer incentive nor the customer rebate has an absolute dominance over the other. We observe that when the uncertainty is high, the customer rebate performs better than the retailer incentive. On the other hand, when the uncertainty is on the price sensitivity parameter of the demand function, we obtain identical results to the deterministic setting where the profits and sales generated with a retailer incentive are higher than those of a customer rebate. We also show that offering combined promotions improves the manufacturer's profits and sales only if there is uncertainty in the market potential.

We have seen that uncertainty can be an important factor in determining whether to offer a customer rebate or retailer incentive. We investigate whether our analytical findings are in line with what we observe in practice, in particular, whether different promotions offered by the American auto manufacturers (customer rebates based promotions) and the Japanese auto manufacturers (retailer incentives based promotions) can be explained by the variability of the demand that they observe. Since the actual demand data is not available, we use sales as an approximation of demand and we take the variability of sales to represent uncertainty in demand for this analysis unless described otherwise. We focus on the following two questions: 1) Is there any statistical evidence for the rebates by the American auto manufacturers being higher than those by the Japanese auto manufacturers? 2) Is the sales variability of the American vehicles higher than the Japanese vehicles?

We analyze three car segments: midsize, compact size and utility vehicle. In Figure 18, we plot the rebate percentages for compact size and utility vehicle segments. A quick inspection of the graphs shows that the rebate percentages used by the American auto



(a) Compact size models



(b) Utility vehicle models

Figure 18: Total registrations and average rebate percentages of compact size and utility vehicle models

manufacturers are higher than the Japanese auto manufacturers for all car segments including the midsize segment displayed in Figure 8. We observe that the demand for cars is highly seasonal with the peak seasons in late summer-early fall and late winter-early spring. Table 9 summarizes the basic statistics related with the data sets for the compact, midsize, and utility vehicle segments. Although these values may help to test the hypotheses, one should be careful using them because the registrations and the rebates are time series data with possible autocorrelations and therefore may lack independence. In order to test the variability differences, ideally we need to separate out the effects of uncertainty from seasonality. For these reasons, instead of the actual registrations, we use the normalized data where each data point is normalized with the total registrations in the corresponding month, and we then divide the normalized data points into non-overlapping batches and take the batch means as the data points. (Batch means method is frequently used for the estimation of mean and variances in simulation output analysis; e.g., see Alexopoulos et al. [3].)

Table 9: Summary statistics

	Total number of Registrations				Rebate Percentages			
	Mean		CoV*		Mean %		CoV	
Segment	U.S.	Japan	U.S.	Japan	U.S.	Japan	U.S.	Japan
Midsize	48,574.58	66,942.86	0.18	0.13	6.24	0.33	0.48	0.85
Compact	35,977.94	42,078	0.20	0.16	9.48	0.50	0.33	0.86
Utility	52,459.75	18,729.42	0.29	0.17	2.87	1.02	0.57	0.64

*CoV(Coefficient of variation) $= \frac{\text{standard deviation}}{\text{mean}}$

Our results suggest that the rebates by the American auto manufacturers are statistically significantly higher than the rebates by the Japanese auto manufacturers at the 95% confidence level. We have weaker results for the variability differences; at 90% confidence level, there is evidence that the sales variability of the American auto manufacturers is higher than that of the Japanese auto manufacturers for the midsize and utility vehicle segments, but not for the compact segment. As we mentioned before, we relate the variability in sales with the uncertainty in demand. Therefore, the higher variability in demand by the American auto manufacturers could explain their choice of rebates as an important promotion mechanism as our analytical results suggested that the rebates improve profits and sales most when there is high uncertainty. However, it would be useful to further investigate the effect of rebates on the variability of demand. It might be the case that the American auto manufacturers started using the rebates for some other reason, but then as they offered high rebates in some periods and no rebates in others, they added more variability to the demand that they see. Identifying if rebates cause variability or if rebates are offered because of variability is very difficult and we leave this for future research.

Our work analytically compares different promotions used in the auto industry, which is very important for the manufacturers. However, there are many further questions of interest in this area. For example, it would be useful to add the sales effort decisions of the dealers into the analysis, since effort may play important role in increasing sales. Issues of supply chain coordination in this setting could also be useful as well as the aspects of customer behavior such as strategic buying.

CHAPTER IV

SALES PROMOTIONS IN THE PRESENCE OF COMPETITION

4.1 Introduction

Promotions are widely used by manufacturers and retailers in varying degrees across different industries. In some cases, companies offer promotions to increase sales, advertise new products or to reduce their inventories; in others, promotions are used as strategic tools to react to the competitors' actions. As we have discussed in Chapter 3, the auto industry in the U.S. is one example where promotions play an important role in the marketing strategies of the companies. In this industry, it is commonly observed that promotions offered by one manufacturer lead competitors to start or intensify their own promotions. For example, General Motors introduced zero-percent financing option in September 2001, which was quickly followed by similar promotions by Ford and Chrysler. More recently, in response to increasing gasoline prices in the U.S., Ford started to offer an extra $1,000 cash rebate in addition to rebates of up to $4,000 on certain models. Analysts expect a "new rebate war among U.S. automakers" in 2007 and state that Ford's new rebate program "could cause some manufacturers to increase their incentives programs on select models in response to Ford's actions" [5].

Competition may impact the manufacturers' promotion decisions. In Figure 19, we use data sets from a major market research firm and plot the rebates offered by two American auto manufacturers in the period June 2000-May 2003. The cross-correlation between the two time series data is 0.87, and there is an increasing trend in rebate amounts for both manufacturers. Although the correlation itself is not sufficient to claim that the manufacturers' rebate promotions were closely affected by those of the competitor, there is also significant anecdotal evidence for competitive effects of promotions, e.g., the employee discount program was initiated by General Motors in June 2005, which increased the manufacturer's sales by 41% in that month, and it was matched by Chrysler and Ford the next month.

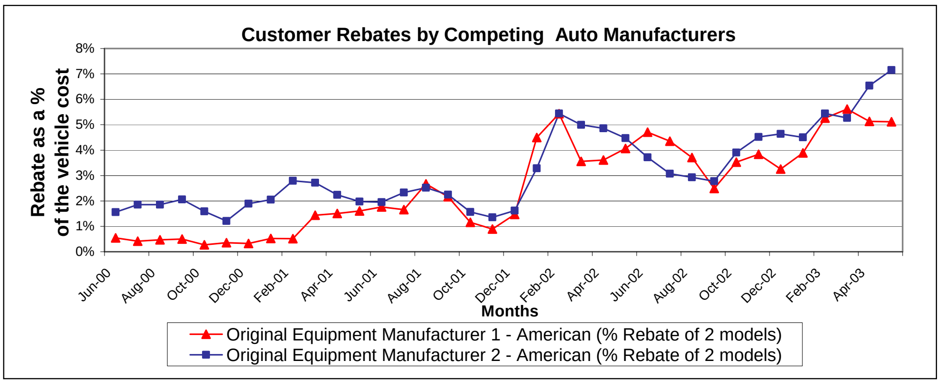


Figure 19: Rebates by two competing American auto manufacturers in the U.S. market

In Chapter 3, we analyzed "customer rebate" and "retailer incentive" promotions in a two-stage supply chain with a single manufacturer and single retailer and determined which promotion would benefit the monopolistic manufacturer under which market conditions. We modelled a customer rebate in the form of a per-unit payment from the manufacturer to the end customer, and a retailer incentive in the form of lump-sum payment from the manufacturer to the dealer; both forms are observed in practice. In this chapter, we extend this research by analyzing a duopoly setting with two competitive manufacturers who sell their products at their exclusive retailers, who are also competitors in the end market. Our goal is to investigate the impact of competition on manufacturers' promotion and pricing decisions, retailers' order/sales quantity decisions, as well as the profits of the firms. One question is whether including competition will change which promotion is better for a manufacturer. We are also interested in using our models to explain the manufacturers' promotion choices observed in practice. We adopt a game theoretical framework to analyze the problems.

This chapter is organized as follows. We review the relevant literature in Section 4.2. In Section 4.3, we explain our assumptions, describe the competitive interactions in the supply chains, and present our models. We summarize our results in Section 4.4.

4.2 Literature Review

Sales promotions have been extensively studied in the marketing and economics literature especially in non-durable goods industries with posted prices. (e.g., see Ailawadi and Neslin [2], Blattberg et al. [10], Neslin and Stone [62] and Wansink and Deshpande [83].) Different than these articles, we focus on industries where the sales price is negotiated. The auto industry is a good example where consumption rates are low, prices are high and can be negotiated with the customers.

Narasimhan [60] analyzes the role of coupons in achieving price discrimination in industries where the manufacturers sell directly to the buyers. In another stream of research, promotions are analyzed in two-stage supply chains where manufacturers sell through retailers. Gerstner and Hess [32], Khrishnan et al. [50], and Taylor [77] focus on the use of promotions in channel coordination, where the latter two articles also consider retailers that exert sales effort to influence demand. Ernst and Powell [27] analyze a setting where the manufacturer can offer an incentive to increase the retailer's service level and to capture additional demand. We do not consider retailers' sales effort or find mechanisms to coordinate the channels. Our focus is on understanding how *competition* affects the manufacturers' promotion decisions, where the manufacturers and their retailers operate in decentralized supply chains similar to those in the auto industry.

Several studies analyze promotions in competitive supply chains, e.g., Lal et al. [52], Narasimhan [61], and Steenkamp et al. [75]. Kim and Staelin [47] study a four-player model where competition exists both at the manufacturers' level and the retailers' level of the supply chain, similar to ours. However, since their model relates to the consumer packaged goods industry, the retailers sell both manufacturers' products at a fixed price, which is different than our setting in the auto industry where the retailers operate exclusively and negotiate prices. The authors only analyze promotions from the manufacturers to the retailers where the promotions are in the form of side payments, corresponding to the lump-sum incentives in our setting; in addition we also analyze rebates to customers. Gerstner et al. [33] study the price discriminating role of a pull-discount promotion (customer rebate) for a monopolist manufacturer where the competition is only among the retailers. The authors analyze a market with two customer segments, one with high and the other with low willingness to pay for one unit of product. Benefiting from the pull-discount is costly for the former, but free for the latter segment. In our analysis, we allow competition both among the manufacturers and the retailers. We assume that customer rebates are redeemed by all buyers without any cost, which is a common practice in the auto industry. Examples of research that analyze competitive supply chains but do not consider promotions include Choi [18], McGuire and Staelin [57], Moorthy [58], and Trivedi [80], where the authors analyze the manufacturers' channel structure decisions. We do not focus on the decisions on channel structure since the auto manufacturers sell their products through exclusive and independent dealers in the U.S. Our focus is on analyzing the promotional decisions of the competing manufacturers and their effects on the entire supply chain. Boyaci and Gallego [11] study a competitive setting similar to ours with two wholesalers supplying two retailers in various scenarios with coordination or decentralization within each supply chain. In their analysis, wholesalers and retailers compete on the basis of customer service where the retailers charge similar prices, while we analyze retailers that price discriminate and compete directly in sales quantity.

In the auto industry, analysis of promotions has attracted some attention from researchers due to the economic significance of the industry and the auto manufacturers' increased use of promotions in the recent years. Empirical studies have investigated diverse issues such as price discrimination by auto dealers (Goldberg [36], Scott Morton et al. [70], and Zettelmeyer et al. [86]), dealers' pass-through rates of promotions to the car buyers (Busse et al. [15] and Crafton and Hoffer [23]), the impact of new product introductions and promotional incentives on firms' profits (Pauwels et al. [66]), and the effects of promotions on customer and dealer behaviors (Keskinocak et al. [46]). Bruce et al. [12, 13] analytically study promotions offered by the durable goods manufacturers using game theoretical models and perform empirical studies with data from the auto industry. Bruce et al. [12] focus on promotions from the manufacturers to the retailers (trade promotions), which are in the form of wholesale price discounts based on the retailers' sales levels. The authors consider a channel structure that is similar to ours where two competing manufacturers sell their products exclusively through two competing retailers, although in their case the dealer sets a fixed price and customer rebates are not analyzed. They incorporate the effects of a secondary market for the goods through a durability measure, and find that the manufacturer with the more durable good benefits more from trade promotions. They also find that in equilibrium, both manufacturers find it optimal to offer trade promotions to their retailers. Bruce et al. [13] analyze cash rebates and the effect of the product's durability on the manufacturer's decisions, but ignores competition. The authors show that lower durability of a manufacturer's products leads to higher cash rebates. In their analysis, the dealer sets a fixed price and dealer incentives are not analyzed.

Our research differs from the cited articles in that the retailers price discriminate. This is mainly motivated by the practices in the auto industry where the final purchase price of a vehicle is negotiated between the dealer and the buyer. As a result, the dealer has the opportunity to learn the customer's willingness to pay and price discriminate accordingly. Spulber [74] and White and Walker [84] analyze models with first-degree (perfect) price discrimination, however neither considers manufacturers' promotion decisions in two-stage supply chains where competition exists both among the manufacturers and the retailers, which is the focus of our study. In Chapter 3, we showed that price discrimination matters even in the monopolistic setting, therefore it is useful to include it in the analysis of competing retailers, since price discrimination and competition are important in the auto industry.

4.3 Models

We analyze promotion and sales decisions in a setting with two competing manufacturers who sell their products through two independent retailers who are also competitors in the end market. We differentiate the supply chains with the subscript $i = 1, 2$, and we assume retailer i sells products of manufacturer i exclusively, which is commonly observed in practice in the form of franchised dealerships of the auto manufacturers. In practice, a dealer may own multiple dealerships or retail outlets although each outlet tends to be operated by a separate management. Auto manufacturers generally keep their wholesale prices constant for the model year and they offer incentives in order to respond to demand changes. Therefore we assume that retailer i purchases from his manufacturer at a wholesale price w_i, which is determined by the manufacturer at the beginning of the model year or selling period.

Below, we summarize further assumptions that are common with those in Chapter 3.

- There is a single selling period.
- Retailers can do perfect price discrimination.
- Retailer i has a reservation price $w_i + m_i$, below which he is not willing to sell.
- Manufacturers have ample capacity.
- There are no administrative and redemption costs associated with the promotions.
- All parameters are common knowledge and all parties seek to maximize their own profits.

Next, we construct the basic demand model that incorporates competition. Since we analyze price discriminating retailers, we use an inverse demand function that relates price to each unit sold. Inverse demand functions are appropriately used to model the firms' profits when they can do perfect price discrimination (Spulber [74] and White and Walker [84]). To model competition, we also need to incorporate the substitution effects between the retailers' products or sales. Most articles in the literature model competition by reflecting the characteristic that a firm's demand (or sales) is downward sloping in the firm's own price, and upward sloping in the competitor's price as a result of substitution (Kim and Staelin [48] and McGuire and Staelin [57]). However, these articles do not consider inverse demand models and use demand functions that relate quantity to the *fixed* selling prices of the firms. We combine characteristics from price discrimination models and competition/substitution models to develop our demand model.

We assume demand is deterministic and has the form shown in Equation 1. The basic structure of the (inverse) demand function is linear, as used in some auto manufacturers (Biller et al. [9]).

$$P_i(Q_i, Q_j) = \begin{cases} a - (b_{io} + b_{ic})Q_i & \text{if } Q_i \leq Q_j \\ a - b_{io}Q_i - b_{ic}Q_j & \text{otherwise} \end{cases} \quad i, j \in \{1, 2\}, i \neq j. \tag{1}$$

$P_i(Q_i, Q_j)$ denotes the price retailer i receives when he sells Q_i units and the competing retailer sells Q_j units. We denote the maximum price with a, which is the market potential when sensitivity is fixed, and we assume that the retailers have identical market potentials. We denote the effect of retailer i's sales on his own price with b_{io} (own price sensitivity), and the effect of the competitor's sales on retailer i's price with b_{ic} (cross price sensitivity). Our demand model incorporates the competitive nature of sales by capturing the following behavior: as the sales/order quantity of one retailer increases, the price that he can receive from each additional unit drops due to his own sales as well as the competitor's sales, both of which are affected by the price sensitivity parameters. (See Appendix C for further discussion on demand model.)

The demand function of retailer 1 is shown in Figure 20 corresponding to the case where he generates higher sales than his competitor. Retailer 1 is competing with retailer 2 on the first segment of the demand function until his sales reach his competitor's sales Q_2, and he is selling additional units beyond Q_2 on the second segment of the demand function. Retailer i is experiencing a faster decrease in the price that the customers are willing to pay on the first segment of his demand function than that on the second segment. We expect that retailer i's own decisions will impact his demand more than his competitor's decisions, therefore we assume $b_{io} \geq b_{ic}$ and $b_{io} \geq b_{jc}$, $i, j \in \{1, 2\}$, $i \neq j$.

Table 10 summarizes our notation. Note that the manufacturers cannot make positive profits unless they sell above their costs, therefore $w_i \geq c_i$, $i = 1, 2$. It follows from the retailers' reservation prices that $P_i(0, 0) = a \geq w_i + m_i$, which together with $w_i \geq c_i$ implies that $a \geq m_i + c_i$, $i = 1, 2$; otherwise either the manufacturer or the dealer would not sell any units. Since the manufacturers do not sell below their costs and the retailers satisfy their reservation prices, $a'_i = a - m_i - c_i$ represents the highest profit retailers can earn, which can be interpreted as the "net" market potential. We can also denote $b'_i = b_{io} + b_{ic}$

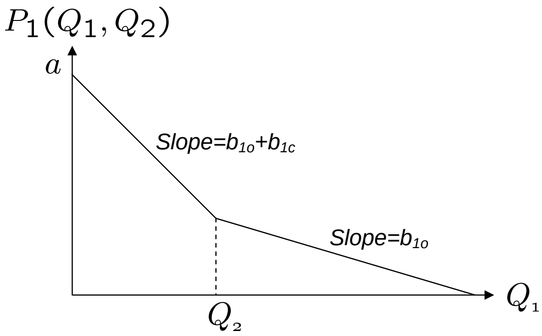


Figure 20: Demand function with competition

as the "total" price sensitivity of manufacturer i since the selling price is affected by both b_{io} and b_{ic} in the region where the retailers sell simultaneously. In the rest of the paper, we denote the manufacturer or the retailer with higher sales quantity as the "market leader".

Table 10: Notation for Chapter 4

a :	Market potential
b_{io} :	Retailer i's price sensitivity of own customers, $i = 1, 2$
b_{ic} :	Retailer i's competitor's price sensitivity of customers affecting retailer i's price
Q_i^k :	Retailer i's order/sales quantity under promotion type $k \in (o, I, R)$ (o =no promotion, I =retailer incentive, R =customer rebate)
$P_i(Q_i, Q_j)$:	Retailer i's price when the retailers sell Q_i, Q_j units respectively ($i, j \in \{1, 2\}$, $i \neq j$)
$\Pi_k^{M_i}$:	Profit of manufacturer i under promotion k
$\Pi_k^{D_i}$:	Profit of retailer i under promotion k
w_i :	Wholesale price of manufacturer i
$w_i + m_i$:	Reservation price of retailer i
c_i :	Production cost of manufacturer i
K_i :	Lump-sum incentive given to retailer i by manufacturer i
R_i :	Per unit customer rebate offered by manufacturer i
δ_i :	The effect of manufacturer i's rebate on the competitor's market potential
A' :	Complement of set A including all elements in the set of real numbers not in A

Figure 21 illustrates the interactions in the supply chains. The manufacturers move first and simultaneously make their wholesale price and promotion decisions. Next, the retailers observe the manufacturers' decisions and simultaneously make their order/sales quantity decisions. Thus, the simultaneous decision games are embedded in a Stackelberg framework

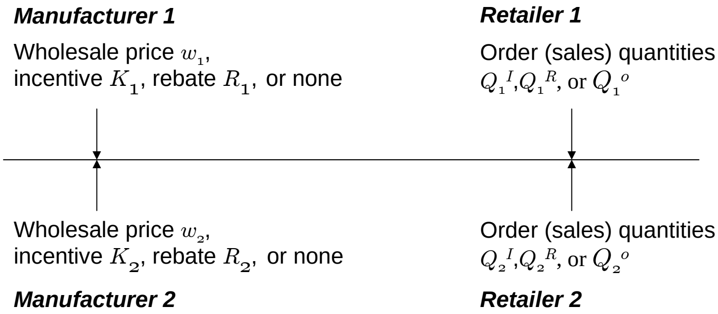


Figure 21: Timeline of decisions (competition)

with the manufacturers acting as the leaders and the retailers as the followers, where the manufacturers consider the retailers' responses while making their own decisions. Unlike the newly introduced promotional programs such as employee discount programs and zero-percent financing where manufacturers' actions may represent leader-follower situations, customer rebates and dealer incentives are more established types of promotions for which it is reasonable to assume neither of the manufacturers acts as the first-mover or has higher market power than the other. We are interested in the SPNE outcomes where none of the firms has an incentive to deviate from their decisions, and we use backward induction to find such equilibria.

We analyze four combinations of promotions. In Section 4.3.1, we present the base model where no promotion is offered by the manufacturers. In Sections 4.3.2 and 4.3.3, we analyze the cases where both manufacturers offer the same type of promotion: retailer incentives K_i and customer rebates R_i, $i = 1, 2$, respectively. Finally, we analyze the case where one manufacturer offers a retailer incentive while the other offers a customer rebate. In each case, we formulate the optimization problems of the manufacturers and the retailers, and we summarize the results from backward induction, which follows the timeline in Figure 21. Given the manufacturers' decisions, we solve the retailers' problems and find the equilibrium as a function of the manufacturers' decisions. In the second step, we solve the manufacturers' problems by embedding the retailers' best responses into the problem formulations. We solve for the SPNE explicitly when possible and provide observations through numerical examples otherwise.

4.3.1 No Promotion

We first consider the case where the manufacturers do not offer any promotions, thus they only make wholesale price decisions and the retailers set their order/sales quantities. The models and results are as follows.

Step 1. The retailers' order/sales quantity decisions: The retailers simultaneously decide how much to order/sell to maximize their own profits given the manufacturers' wholesale price decisions. We show the formulations of the retailers' problems for the case where $Q_1 \leq Q_2$. Symmetric formulations exist when $Q_1 \geq Q_2$.

Retailer 1's problem:

$$\begin{aligned} \Pi_o^{D_1} = \; & \max_{Q_1 \geq 0} \int_0^{Q_1} (a - (b_{1o} + b_{1c})Q_1) dQ_1 - w_1 Q_1 \\ & \text{s.t.} \quad Q_1 \leq \frac{a - w_1 - m_1}{b_{1o+b_{1c}}} \\ & \qquad\;\; Q_1 \leq Q_2 \end{aligned}$$

Retailer 2's problem:

$$\begin{aligned} \Pi_o^{D_2} = \; & \max_{Q_2 \geq 0} \int_0^{Q_1} (a - (b_{2o} + b_{2c})Q_2) dQ_2 + \int_{Q_1}^{Q_2} (a - b_{2o}Q_2 - b_{2c}Q_1) dQ_2 - w_2 Q_2 \\ & \text{s.t.} \quad Q_2 \leq \frac{a - w_2 - m_2}{b_{2o}} - \frac{b_{2c}}{b_{2o}} Q_1 \\ & \qquad\;\; Q_2 \geq Q_1 \end{aligned}$$

Both retailers have the objective of maximizing their profits. The constraints imply no products are sold below the reservation prices of the retailers, where $P_1(Q_1, Q_2) = a - (b_{1o} + b_{1c})Q_1 \geq w_1 + m_1 \Rightarrow Q_1 \leq \frac{a-w_1-m_1}{b_{1o+b_{1c}}}$, and $P_2(Q_2, Q_1) = a - b_{2o}Q_2 - b_{2c}Q_1 \geq w_2 + m_2 \Rightarrow Q_2 \leq \frac{a-w_2-m_2}{b_{2o}} - \frac{b_{2c}}{b_{2o}} Q_1$.

We find the retailers' best responses to the manufacturers' wholesale prices as follows. (Note that, $(x)^+ = \max\{0, x\}$.)

$$Q_1^*(Q_2, w_1, w_2) = \begin{cases} \left(\min\left\{Q_2, \frac{a-w_1-m_1}{b_{1o}+b_{1c}}\right\}\right)^+ = \left(\frac{a-w_1-m_1}{b_{1o}+b_{1c}}\right)^+ & \text{if } Q_2 \geq \frac{a-w_1-m_1}{b_{1o}+b_{1c}} \\ \frac{a-w_1-m_1}{b_{1o}} - Q_2 \frac{b_{1c}}{b_{1o}} & \text{otherwise} \end{cases}$$

$$Q_2^*(Q_1, w_1, w_2) = \begin{cases} \left(\min\left\{Q_1, \frac{a-w_2-m_2}{b_{2o}+b_{2c}}\right\}\right)^+ = \left(\frac{a-w_2-m_2}{b_{2o}+b_{2c}}\right)^+ & \text{if } Q_1 \geq \frac{a-w_2-m_2}{b_{2o}+b_{2c}} \\ \frac{a-w_2-m_2}{b_{2o}} - Q_1 \frac{b_{2c}}{b_{2o}} & \text{otherwise.} \end{cases}$$

Table 11: Retailers' equilibrium under no promotion

	$\frac{a-w_1-m_1}{b_1'} \le \frac{a-w_2-m_2}{b_2'}$	$\frac{a-w_2-m_2}{b_2'} \le \frac{a-w_1-m_1}{b_1'}$
Q_1^*	$\left(\frac{a-w_1-m_1}{b_1'}\right)^+$	$\left(\frac{a-w_1-m_1}{b_{1o}} - \frac{b_{1c}}{b_{1o}}\left(\frac{a-w_2-m_2}{b_2'}\right)^+\right)^+$
Q_2^*	$\left(\frac{a-w_2-m_2}{b_{2o}} - \frac{b_{2c}}{b_{2o}}\left(\frac{a-w_1-m_1}{b_1'}\right)^+\right)^+$	$\left(\frac{a-w_2-m_2}{b_2'}\right)^+$

The retailers' equilibrium in response to the manufacturers' wholesale prices (w_1, w_2) is as shown in Table 11, where $b_i' = b_{io} + b_{ic}$, $i = 1, 2$.

Step 2. The manufacturers' wholesale price decisions: The manufacturers simultaneously choose their wholesale prices to maximize their own profits anticipating the best responses of the retailers.

$$\Pi_o^{M_i} = \max_{w_i \ge c_i} (w_i - c_i) Q_i^*, \quad i = 1, 2$$

Theorem 5 *When there is no promotion offered by the manufacturers, the SPNE is as shown in Table 12, where* $a_i' = a - m_i - c_i$ *and* $b_i' = b_{io} + b_{ic}$*,* $i = 1, 2$*.*

Table 12: The SPNE with no promotion

SPNE.1	$2b_1'a_2' \ge (b_{2c} + 2b_{2o})a_1'$ Feasible Region 1 (F.R.1)
w_1^*	$\frac{a+c_1-m_1}{2}$
w_2^*	$\frac{2b_1'(a-m_2+c_2)-b_{2c}a_1'}{4b_1'}$
$Q_1^{o^*}$	$\frac{a_1'}{2b_1'}$
$Q_2^{o^*}$	$\frac{2b_1'a_2'-b_{2c}a_1'}{4b_{2o}b_1'}$
$\Pi_o^{M_1^*}$	$\frac{(a_1')^2}{4b_1'}$
$\Pi_o^{M_2^*}$	$\frac{(2b_1'a_2'-b_{2c}a_1')^2}{16b_{2o}(b_1')^2}$

SPNE.2	$2b_2'a_1' \ge (b_{1c} + 2b_{1o})a_2'$ (F.R.2)
w_1^*	$\frac{2b_2'(a-m_1+c_1)-b_{1c}a_2'}{4b_2'}$
w_2^*	$\frac{a+c_2-m_2}{2}$
$Q_1^{o^*}$	$\frac{2b_2'a_1'-b_{1c}a_2'}{4b_{1o}b_2'}$
$Q_2^{o^*}$	$\frac{a_2'}{2b_2'}$
$\Pi_o^{M_1^*}$	$\frac{(2b_2'a_1'-b_{1c}a_2')^2}{16b_{1o}(b_2')^2}$
$\Pi_o^{M_2^*}$	$\frac{(a_2')^2}{4b_2'}$

Feasible Region	SPNE
(1) $F.R.1 \cap F.R.2$	SPNE.1, SPNE.2
(2) $F.R.1 \cap F.R.2'$	SPNE.1
(3) $F.R.1' \cap F.R.2$	SPNE.2

Both equilibria can be observed in the region denoted by (1), whereas the supply chain members reach a unique equilibrium in the regions denoted by (2) and (3). (Note that the entire space of feasible parameters is completely partitioned by the preceding feasible regions, where the regions are determined a priori by the system parameters.) From Table 12, when $b_{ic} = 0$, it trivially follows that manufacturer i achieves the monopoly profits of $\Pi^{M_i} = \frac{(a_i')^2}{4b_{io}}$ (Chapter 3), where $a_i' = a - m_i - c_i$, $i = 1, 2$. This may be seen in practice when the manufacturer has a loyal customer base such as for certain luxury vehicles. In Observation 4, we compare the profits of a manufacturer in the monopolistic and competitive settings.

Observation 4 *When $b_{ic} > 0$, $i = 1, 2$, both manufacturers obtain lower profits when there is competition than when they are monopolies. (See Equations 2 and 3 for SPNE.1; the expressions are symmetric for SPNE.2.)*

$$\Pi_1^{SPNE.1} = \frac{(a_1')^2}{4b_1} \leq \frac{(a_1')^2}{4b_{1o}} \tag{2}$$

$$\frac{2b_1 a_2' - b_{2c} a_1'}{2b_1} \leq a_2' \Rightarrow \Pi_2^{SPNE.1} = \left(\frac{2b_1 a_2' - b_{2c} a_1'}{2b_1}\right)^2 \frac{1}{4b_{2o}} \leq \frac{(a_2')^2}{4b_{2o}} \tag{3}$$

In Observation 5, we determine the market leader corresponding to the different SPNE.

Observation 5 *When the supply chain members are in SPNE.1 (SPNE.2), manufacturer 2 (1) is the market leader in terms of sales, which follows by Equations 4 and 5.*

$$Q_2^{o^*}(\text{SPNE.1}) - Q_1^{o^*}(\text{SPNE.1}) = 2b_1' a_2' - (b_{2c} + 2b_{2o})a_1' \geq 0 \tag{4}$$

$$Q_1^{o^*}(\text{SPNE.2}) - Q_2^{o^*}(\text{SPNE.2}) = 2b_2' a_1' - (b_{1c} + 2b_{1o})a_2' \geq 0 \tag{5}$$

In order to predict the outcome of the interactions between the supply chain members, it is desirable to know when a unique equilibrium is achieved. In Observation 6, we identify some conditions where we observe unique equilibria and analyze the sensitivity of the manufacturers' profits to the system parameters.

Observation 6 *When the manufacturers have identical net market potentials, i.e., $a_1' = a_2'$, and manufacturer 2's total price sensitivity is lower than that of manufacturer 1, i.e., $2b_2' \leq b_1' + b_{1o}$, then SPNE.1 is the unique equilibrium, manufacturer 2 is the market leader,* *and* $\Pi_o^{M_1^*} \leq \Pi_o^{M_2^*}$. *Manufacturer 2's profit changes at a faster rate in response to changes in* b_{2o}, m_2, *and* c_2 *compared to the changes in* b_{2c}, m_1, *and* c_1, *respectively, which implies that the market leader's profit is more sensitive to own parameters than those of the competitors. All results are symmetric for SPNE.2, i.e., if* $2b_1' \leq b_2' + b_{2o}$, *then SPNE.2 is the unique equilibrium, manufacturer 1 is the market leader, and* $\Pi_o^{M_2^*} \leq \Pi_o^{M_1^*}$.

Table 13 shows the comparative statics for the manufacturers' profits when SPNE.1 is the unique equilibrium.

Table 13: The impact of parameters on the profits of the manufacturers. $\swarrow$ implies decrease and $\nearrow$ implies increase with increasing values of the parameters.

	b_{1o}	b_{1c}	b_{2o}	b_{2c}	m_1	m_2	c_1	c_2
$\Pi_o^{M_1^*}$	$\swarrow$	$\swarrow$	-	-	$\swarrow$	-	$\swarrow$	-
$\Pi_o^{M_2^*}$	$\nearrow$	$\nearrow$	$\swarrow$	$\swarrow$	$\nearrow$	$\swarrow$	$\nearrow$	$\swarrow$

In the U.S., the sales of the American auto manufacturers show a declining trend while the non-American auto manufacturers have increased their market shares in the recent years [59]. We also find from our data sets that the new vehicle registrations of the compact and midsize vehicles by the American auto manufacturers totaled 4.5M and 6M, respectively, versus 5.2M and 7.4M by the Japanese auto manufacturers. Although this can be due to several reasons, one empirical work shows that the American manufacturers face higher price sensitivity in their demands for the compact and midsize car segments than the Japanese auto manufacturers (Keskinocak et al. [46]). These findings are consistent with Observation 6, which suggests that the manufacturer with lower total price sensitivity is the market leader.

In Examples 1 and 2, we show that the manufacturers' profits are complex functions of all system parameters making it difficult to characterize general relationships that hold in all cases. In Example 1, we show that the market leader (in sales quantity) can make less profit than the competing manufacturer. This situation is more likely to happen when the competitor's production cost and the reservation price of his retailer is lower than those of the market leader.

Example 1 *When* $a = 150, b_{1o} = 3, b_{1c} = 1, b_{2o} = 1.5, b_{2c} = 1, c_1 = 2, c_2 = 10, m_1 = 5, m_2 = 35$*, SPNE.1 is the unique equilibrium with the following decisions:* $w_1^* = 73.60, w_2^* = 53.56, Q_1^{o^*} = 17.88, Q_2^{o^*} = 29.04$*, where* $\Pi_o^{M_1^*} = 1278.06, \Pi_o^{M_2^*} = 1265.13$.

In Example 2, we show that lower production cost and retailer's reservation price do not always guarantee higher profits for a manufacturer.

Example 2 *When* $a = 50, b_{1o} = 6, b_{1c} = 1.5, b_{2o} = 2, b_{2c} = 1, c_1 = 15, c_2 = 20, m_1 = 10, m_2 = 15$*, SPNE.1 is the unique equilibrium with the following decisions:* $w_1^* = 27.50, w_2^* = 26.67, Q_1^{o^*} = 1.67, Q_2^{o^*} = 3.33$*, where* $\Pi_o^{M_1^*} = 20.83, \Pi_o^{M_2^*} = 22.22$.

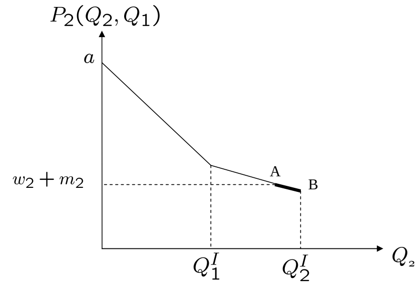
Although manufacturer 1's production cost is lower, she sells less units than manufacturer 2 partly due to higher own and cross price sensitivities, and she ends up with lower profit.

4.3.2 Retailer Incentive

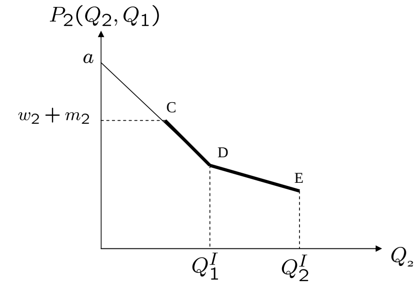
In this section, we look at the case where manufacturer i offers a lump-sum incentive K_i to retailer i, $i = 1, 2$. The incentives allow the retailers to sell to customers they would not have been able to reach otherwise. The retailers want to satisfy their reservation prices $w_i + m_i$ from each unit sold, therefore there is no reason they should use any incentive for buyers who are willing to pay at least $w_i + m_i$. For other customers, the retailer can use the incentive to compensate for the difference between the price that the customer pays and the retailer's reservation price. Although the auto manufacturers do not have direct control over the use of dealer incentives and the dealers determine whether to use the incentives during sales or not, empirical studies show that dealers indeed pass-through some part of the incentives to the end customers (Busse et al. [15]). Other reasons for the retailers' voluntary participation in transferring the incentives to customers include the turn-and-earn system for inventory allocation and advantages in receiving a better selection of vehicles in the future, which is facilitated by a high volume of sales by the dealers.

Similar to the analysis in Section 4.3.1, we identify two cases while formulating the retailers' problems, i.e., $Q_1^I \leq Q_2^I$ and $Q_1^I \geq Q_2^I$. However, we need to analyze additional cases to determine the total amount of incentives used by the retailers.

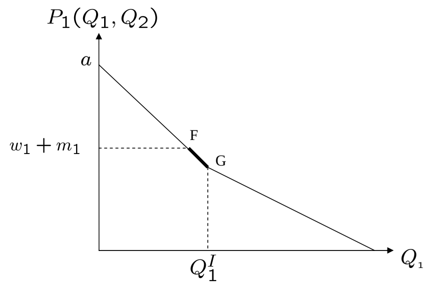
In Figure 22, we analyze the case $Q_1^I \leq Q_2^I$ where retailer 2 is market leader. Note that retailer 2 uses the incentive only for those sales with prices below $w_2 + m_2$. In Figure 22(a), he uses the incentive on the second segment of his demand function where he is generating additional sales beyond his competitor's sales, and uses the incentive for all units between the points denoted by A-B. By contrast, in Figure 22(b), the retailer uses the incentive on both segments of his demand function for all units between the points denoted by C-E. Each of these situations implies a different constraint on the reservation price requirement for retailer 2. On the other hand, since $Q_1^I \leq Q_2^I$, retailer 1 is selling entirely on the first segment of his demand function where both retailers are capturing market share. Therefore, he uses the incentive only on this segment, which is illustrated in Figure 22(c) between the points denoted by F-G.



(a) Retailer 2 uses K_2 on the second segment of the demand function



(b) Retailer 2 uses K_2 on both segments of the demand function



(c) Retailer 1 uses K_1 on the first segment of the demand function

Figure 22: Retailers' demand functions when both manufacturers give incentives and retailer 2 is the market leader.

Below, we summarize all possible cases to consider, where the reservation price requirement leads to different optimization problems for the retailers depending on the manufacturers' wholesale price decisions: (Note that Case 1.a and Case 1.b are illustrated in Figures 22(a) and 22(b), respectively.)

Case 1) $Q_1^I \leq Q_2^I$	**Case 2)** $Q_1^I \geq Q_2^I$
Case 1.a)$\exists\, Q_2 \geq Q_1^I : P_2(Q_2, Q_1^I) \leq w_2{+}m_2$	**Case 2.a)**$\exists\, Q_1 \geq Q_2^I : P_1(Q_1, Q_2^I) \leq w_1{+}m_1$
Case 1.b)$\exists\, Q_2 \leq Q_1^I : P_2(Q_2, Q_1^I) \leq w_2{+}m_2$	**Case 2.b)**$\exists\, Q_1 \leq Q_2^I : P_1(Q_1, Q_2^I) \leq w_1{+}m_1$

We provide the formulations of the retailers' problems for Case 1.a and Case 1.b; the analysis of the symmetrical cases is similar.

Case 1.a) Retailer 1's problem

$$\begin{aligned}
\max_{Q_1^I \geq 0} \quad & \int_0^{Q_1^I} (a - (b_{1o} + b_{1c})Q_1^I)dQ_1^I - w_1 Q_1^I + K_1 \\
\text{s.t.} \quad & \int_{\frac{a-w_1-m_1}{b_{1o}+b_{1c}}}^{Q_1^I} (w_1 + m_1 - (a - (b_{1o} + b_{1c})Q_1^I))dQ_1^I \leq K_1 \\
& Q_1^I \leq Q_2^I
\end{aligned}$$

Retailer 2's problem

$$\begin{aligned}
\max_{Q_2^I \geq 0} \quad & \int_0^{Q_1^I} (a - (b_{2o} + b_{2c})Q_2^I)dQ_2^I + \int_{Q_1^I}^{Q_2^I} (a - b_{2o}Q_2^I - b_{2c}Q_1^I)dQ_2^I - w_2 Q_2^I + K_2 \\
\text{s.t.} \quad & \int_{\frac{a-w_2-m_2}{b_{2o}} - \frac{b_{2c}}{b_{2o}}Q_1^I}^{Q_2^I} (w_2 + m_2 - (a - b_{2c}Q_1^I - b_{2o}Q_2^I))dQ_2^I \leq K_2 \\
& Q_2^I \geq Q_1^I
\end{aligned}$$

In Case 1.b, the formulation of retailer 1's problem and the objective function of retailer 2 remain the same as above. However, the constraints for retailer 2's problem are modified to reflect the retailer's reservation price requirement as follows.

Case 1.b) Retailer 2's constraints

$$\int_{\frac{a-w_2-m_2}{b_{2o}+b_{2c}}}^{Q_1^I} (w_2{+}m_2{-}(a{-}(b_{2o}{+}b_{2c})Q_2^I))dQ_2^I {+} \int_{Q_1^I}^{Q_2^I} (w_2{+}m_2{-}(a{-}b_{2c}Q_1^I{-}b_{2c}Q_2^I))dQ_2^I {\leq}\, K_2$$

$$Q_2^I \geq Q_1^I$$

We state the best responses of the retailers to manufacturers' wholesale price and incentive decisions for Case 1.a and Case 1.b in Equations 6-9.

$$\textbf{Case 1.a)} \qquad Q_1^{I^*} = \left(\min\left\{Q_2^I, \tfrac{a-w_1}{b_1'}, \tfrac{a-w_1-m_1+\sqrt{2K_1b_1'}}{b_1'}\right\}\right)^+ \tag{6}$$

$$Q_2^{I^*} = \left(\max\left\{Q_1^I, \min\left\{\tfrac{a-w_2}{b_{2o}} - \tfrac{b_{2c}}{b_{2o}}Q_1^I, \tfrac{a-w_2-m_2-b_{2c}Q_1^I+\sqrt{2b_{2o}K_2}}{b_{2o}}\right\}\right\}\right)^+ \tag{7}$$

$$\textbf{Case 1.b)} \qquad Q_1^{I^*} = \left(\min\left\{Q_2^I, \tfrac{a-w_1}{b_1'}, \tfrac{a-w_1-m_1+\sqrt{2K_1b_1'}}{b_1'}\right\}\right)^+ \tag{8}$$

$$Q_2^{I^*} = \left(\max\left\{Q_1^I, \min\left\{\tfrac{a-w_2}{b_{2o}} - \tfrac{b_{2c}}{b_{2o}}Q_1^I, \tfrac{a-w_2-m_2}{b_{2o}} - \tfrac{b_{2c}}{b_{2o}}Q_1^I + \tfrac{\sqrt{(b_2')^2(2b_{2o}K_2+(Q_1^I)^2b_{2c}b_2'+2b_{2c}(w_2+m_2-a)Q_1^I)+(b_{2c}(w_2+m_2-a)^2b_2')}}{b_{2o}b_2'}\right\}\right\}\right)^+ \tag{9}$$

Given the best responses of the retailers, manufacturer i's problem is as follows:

$$\Pi_I^{M_i} = \max_{w_i \geq c_i, K_i \geq 0} (w_i - c_i)Q_i^{I^*} - K_i$$

In our analysis, we model competition at both stages of the supply chain: the retailers observe the manufacturers' decisions and compete in sales; the manufacturers determine their wholesale prices and promotion amounts by predicting the retailers' equilibrium, and they compete in sales and profits. Competitive interactions at both stages complicate the analytical characterization of the equilibria considerably. This is also mentioned by Boyaci and Gallego [11], where the authors study a setting similar to ours in terms of supply chain structure and the timeline of decisions. Although the authors derive analytical expressions for the retailers' equilibrium, they use numerical schemes to compute the wholesalers' Nash equilibrium decisions. In our analysis, we were able to analytically derive the SPNE for the base model in the no-promotion case (Section 4.3.1), however, we resort to computational methods to find the equilibria (if any exist) in the case of retailer incentives.

Fudenberg and Tirole [28] discuss one iterative method where players take turns to make their decisions, and each player makes a decision that is a best response to the opponent's decision one iteration before. Unfortunately, there is no guarantee that this process will converge; however, if it converges to a stable point, then it results in a Nash equilibrium. In Table 14, we provide an algorithm that implements an iterative process that is similar to the one discussed in Fudenberg and Tirole [28]. We iteratively search for the manufacturers'

Table 14: Iterative procedure

Manufacturers' equilibrium
1) Initialize the decisions of the manufacturers: $w_i = c_i$, $K_i = 0$, $i = 1, 2$.
2) Without loss of generality, start with manufacturer $i = 2$. Given the current decisions of manufacturer $3 - i$, find the best response of manufacturer i by iterating over all (w_i, K_i) pairs and using the retailers' equilibrium from steps 4-6. The best response of manufacturer i is the (w_i, K_i) pair that brings the highest profit. Update the values of (w_i, K_i) and repeat step 2 for manufacturer $i = 1$.
3) If the successive values of manufacturers' profits do not differ more than a tolerance level ϵ, stop.
Retailers' equilibrium
4) Initialize the decisions of the retailers: $Q_j = 0$; $j = 1, 2$
5) Without loss of generality, start with retailer $j = 2$. Given the current decisions of retailer $3 - j$ and the current decisions of the manufacturers (w_1, K_1, w_2, K_2), calculate the best response of retailer j by using Equations 6-9. Update the value of Q_j and repeat step 5 for retailer $j = 1$.
6) If the successive values of retailers' profits (or quantities) do not differ more than a tolerance level ϵ, stop.

wholesale price and retailer incentive decisions, where we use the best responses of the retailers in Equations 6-9 to find the retailers' equilibrium.

Although the algorithm outlined in Table 14 does not guarantee convergence, in our computational tests, we have always been able to achieve convergence. However, we have also observed that the algorithm can converge to multiple equilibria depending on which manufacturer takes the first turn in Step 2. We also note that multiple equilibria for the retailers' interactions are possible, although in our computational tests we have always been able to achieve a unique stable point for the retailers' equilibrium in response to manufacturers' decisions.

The iterative procedure helps us observe the manufacturers' competitive actions in response to each other. In Table 15, we give an example where one manufacturer's higher incentive triggers the other manufacturer to increase her own retailer incentive. In this example, both manufacturers find it optimal to offer incentives in equilibrium, which suggests that competition can be one reason for the manufacturers to offer incentives to their retailers.

Table 15: Best responses of the manufacturers towards equilibrium ($a = 75$, $b_{1o} = 3$, $b_{1c} = 2$, $b_{2o} = 2$, $b_{2c} = 1$, $c_1 = 10$, $c_2 = 10$, $m_1 = 5$, $m_2 = 5$).

Iteration	w_1	K_1	w_2	K_2
0	15	0	-	-
1	15	0	41*	22*
3	45*	10*	41	22
4	45*	10*	42*	25*

It is interesting to investigate whether the incentives always increase the retailer's profits under competition. In Chapter 3, we showed in a monopolistic setting that the retailer earns higher profit with the incentive only if the market potential is sufficiently low ($a \leq 3.5m+c$), while the manufacturer is always better off with the incentives than with no promotion. In Example 3, we analyze the impact of the market potential on the manufacturers' and the retailers' profits under competition, and observe a similar result to that in the monopolistic setting.

Example 3 *Table 16 compares the manufacturers' and retailers' profits and sales in the no-promotion equilibrium versus the retailer incentive equilibrium, where the market potential takes "low" and "high" values. The manufacturers obtain higher sales and profits in both cases when they offer incentives, whereas the retailers can achieve higher profits with incentive only when a is "low". Note also that the manufacturers' wholesale prices are higher with retailer incentives.*

Table 16: Comparison of equilibria when no promotion is offered versus when both manufacturers offer retailer incentives, NP: No promotion, RI: Retailer incentive ($b_{1o} = 3$, $b_{1c} = 2$, $b_{2o} = 2.5$, $b_{2c} = 1$, $c_1 = 12$, $c_2 = 10$, $m_1 = 6$, $m_2 = 8$).

		w_1	w_2	K_1	K_2	Q_1	Q_2	Π^{D_1}	Π^{D_2}	Π^{M_1}	Π^{M_2}
	NP	33.0	28.9	-	-	4.2	7.6	69.3	140.7	88.2	142.9
$a = 60$	RI	36	32.7	3.6	12.8	4.8	9.0	61.2	126.0	111.6	191.4
	NP	15.5	13.2	-	-	0.7	1.3	5.4	12.3	2.5	4.0
$a = 25$	RI	18.5	16.9	3.6	8.5	1.3	2.3	7.9	18.24	4.9	7.3

In a competitive setting, Bruce et al. [12] study a different kind of dealer promotion that is in the form of wholesale price-cuts based on the retailer's sales. Similar to Example 3, they also find that while the manufacturers are better off with trade promotions the retailers are worse off when both manufacturers offer them.

Next question we analyze is how the retailer incentive affects the manufacturers' sales and profits when the incentive is offered by only one of the manufacturers. To answer this question, we consider a scenario in Example 4 that mimics some characteristics in the U.S. auto market. The American auto manufacturers are known to have higher production costs ([78]), and it has also been found that they have higher price sensitivity in demand than the non-American auto manufacturers (Keskinocak et al. [46]). In our example, we assume manufacturer 1 has higher production cost and total price sensitivity compared to manufacturer 2, and we fix the manufacturers' wholesale prices at arbitrary values assuming manufacturer 1 charges a higher wholesale price than manufacturer 2 as a result of higher production cost. We compare three cases: (i) neither of the manufacturers offers any promotion (Table 17(a)), (ii) only manufacturer 1 offers an incentive (Table 17(b)), (iii) only manufacturer 2 offers an incentive (Table 17(c)). In Tables 17(b) and 17(c), we fix one of the manufacturer's incentive to zero and find the optimal value of the incentive offered by the other manufacturer using the iterative procedure in Table 14.

Example 4 *In Table 17(a), manufacturer 2 is the market leader and obtains higher profits than manufacturer 1. In Table 17(b), the market leader does not offer any incentive but his competitor does, then the market leader loses sales and profits while the competitor generates additional sales and profits. In Table 17(c), the market leader is the only manufacturer offering an incentive. The market leader further increases his sales and profits, although the competitor's sales and profits are not affected. The effects of incentives on the retailers' profits are similar to those on the manufacturers' profits across Tables 17(a)-17(c).*

In Table 17(c), the market leader offers an incentive and encourages her retailer to sell additional units to the customers who cannot afford the retailer's reservation price. As a result, the market leader expands her market. Manufacturer 1 does not provide any incentive for her retailer to sell at prices lower than his reservation price; thus her

Table 17: Effect of retailer incentives ($w_1 = 30$, $w_2 = 20$, $a = 75$, $b_{1o} = 3$, $b_{1c} = 2$, $b_{2o} = 2$, $b_{2c} = 1$, $c_1 = 20$, $c_2 = 10$, $m_1 = 10$, $m_2 = 10$).

(a) No promotion

Q_1	Q_2	Π^{D_1}	Π^{D_2}	Π^{M_1}	Π^{M_2}
7	19	192.5	575.5	70	190

(b) $K_2 = 0$

K_1^*	Q_1	Q_2	Π^{D_1}	Π^{D_2}	Π^{M_1}	Π^{M_2}
10	9	18	212.5	544.5	80	180

(c) $K_1 = 0$

K_2^*	Q_1	Q_2	Π^{D_1}	Π^{D_2}	Π^{M_1}	Π^{M_2}
25	7	24	192.5	625.5	70	215

sales are restricted by the reservation price requirement and insensitive to his competitor's retailer incentive. Further, since retailer 1 makes his sales entirely on the first segment of his demand function, the incentive given to retailer 2 does not worsen his situation. Preceding observations suggest that the manufacturers with characteristics similar to those of the American auto manufacturers can offer incentives to their dealers to improve their profits and to compensate for their high wholesale prices and price sensitivities when their competitor does not offer incentives.

4.3.3 Customer Rebate

In this section, we analyze the case where both manufacturers offer customer rebates. Rebates increase the purchasing power of the customers, thus increase the market potential, or maximum price dealers may receive. However, as found in Keskinocak et al. [46], rebates offered by one manufacturer may reduce the competitor's sales and profits in a linear fashion. We consider both effects by adjusting the demand function as shown in Equation 10.

$$P_i(Q_i, Q_j, R_i, R_j) = \begin{cases} (a+R_i-\delta_j R_j)-(b_{io}+b_{ic})Q_i & \text{if } Q_i \le Q_j \\ (a+R_i-\delta_j R_j)-b_{io}Q_i-b_{ic}Q_j & \text{otherwise.} \end{cases} \quad i,j \in \{1,2\}, i \neq j. \tag{10}$$

While one manufacturer's own rebates increase, her competitor's rebates decrease the customers' willingness to pay, where $0 < \delta_i \le 1$ can be interpreted as the degree of substitution between the two manufacturers' products. The decisions of the manufacturers are w and R. The retailers' equilibrium in response to the manufacturers' decisions is similar to that in the no-promotion equilibrium (Table 11), with $a - w_i - m_i$ modified as

$a + R_i - \delta_j R_j - w_i - m_i$, $i, j \in \{1, 2\}, i \neq j$.

We analytically characterize the SPNE in Proposition 2.

Proposition 2 *When both manufacturers offer customer rebates, there exists a continuum of equilibria with* $w_i - R_i = c_i$, $w_j = \frac{a-m_i-c_i}{\delta_j} + \frac{a-\delta_i R_i - m_j + c_j}{2}$, $R_j = \frac{a-m_i-c_i}{\delta_j}$, $i, j \in \{1, 2\}, i \neq j$. *When* $w_i = c_i$, $R_i = 0$, *there exists an equilibrium solution where manufacturer i obtains zero profit while manufacturer j obtains monopoly profit, where* $w_j = \frac{a-m_i-c_i}{\delta_j} + \frac{a-m_j+c_j}{2}$, $R_j = \frac{a-m_i-c_i}{\delta_j}$.

See Appendix C for the proof. In the monopolistic setting the customer rebates are not effective in increasing the manufacturer's profits and sales when demand is deterministic (Chapter 3). Proposition 2 shows that customer rebates can be highly effective when there is competition.

In Table 18, we give examples of equilibria where one of the manufacturers offers high rebates and generates monopoly profit while driving the competitor out of business. Note that, in equilibrium any combination of w_i, R_i such that $w_i - R_i = c_i$ brings zero profit to manufacturer i driving him out of business; however, the competing manufacturer's (market leader) sales and profits are sensitive to the individual values of w_i and R_i. For example, any $R_i > 0$ can hurt the market leader's sales and profits as we demonstrate in Figure 23, where $w_1 - R_1 = c_1$ results in $\Pi^{M_1} = 0$ on all curves, but Π^{M_2} decreases as R_1 increases.

Table 18: Equilibrium decisions and profits with customer rebates ($a = 60$, $b_{1o} = 3$, $b_{1c} = 2$, $b_{2o} = 2.5$, $b_{2c} = 1$, $c_1 = 12$, $c_2 = 10$, $m_1 = 6$, $m_2 = 8$, $\delta_1 = 0.5$, $\delta_2 = 0.5$).

		w_i	R_i	Q_i	Π^{D_i}	Π^{M_i}
SPNE 1	Supply chain $i = 1$	117	84	7	115.5	147
	Supply chain $i = 2$	10	0	0	0	0
SPNE 2	Supply chain $i = 1$	12	0	0	0	0
	Supply chain $i = 2$	115	84	8.4	155.4	176.4

Although one of the manufacturers can theoretically do very well by offering high customer rebates, in practice it is more likely that the rebate amounts are limited by an upper bound, e.g., the production cost. While we can identify the equilibrium decisions explicitly

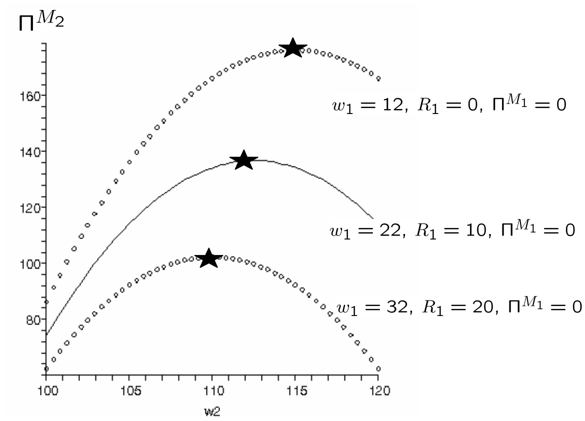


Figure 23: Continuum of equilibria with customer rebates

when there are no restrictions on the rebate values, when there are restrictions, the SPNE is not equal to those characterized in Proposition 2. In the latter case, we use an iterative procedure similar to the one outlined in Table 14 to find a stable point where none of the firms would want to unilaterally deviate from.

We continue with the example in Table 18, but we restrict the rebate values by the production costs for both manufacturers and show an equilibrium outcome in Table 19. In this case, although manufacturer 1 is not driven completely out of the business under rebates, she obtains lower sales and profits than those in the equilibrium of the no-promotion case. The profits and sales of the market leader (manufacturer 2) are higher with the customer rebate than no promotion.

Table 19: Equilibrium with customer rebates where $R_1 \leq c_1$ and $R_2 \leq c_2$, NP: No promotion, CR: Customer rebate ($a = 60$, $b_{1o} = 3$, $b_{1c} = 2$, $b_{2o} = 2.5$, $b_{2c} = 1$, $c_1 = 12$, $c_2 = 10$, $m_1 = 6$, $m_2 = 8$, $\delta_1 = 0.5$, $\delta_2 = 0.5$).

	w_1	w_2	R_1	R_2	Q_1	Q_2	Π^{M_1}	Π^{M_2}
NP	33.0	28.9	-	-	4.2	7.6	88.2	142.9
CR	30.5	39.1	0	10	3.7	7.7	68.5	146.6

4.3.4 Retailer Incentive and Customer Rebate (Hybrid)

In practice, we can observe situations where manufacturers offer different types of promotions. For example, as we also mentioned in Chapter 3, the American auto manufacturers are well-known for their frequent and deep customer rebates, whereas the non-American, especially Japanese, auto manufacturers seldom offer those and instead may give incentives to their dealers. Manufacturers may have different characteristics; the American auto manufacturers have higher production costs partly due to labor unions, and they sell to customers who are more price sensitive to their products than those of the Japanese auto manufacturers (Keskinocak et al. [46]).

One question is which promotion should the manufacturers choose in order to receive high profits and sales. To answer this question, we analyze a scenario where we characterize manufacturer 1 with low production cost and total price sensitivity and manufacturer 2 with high production cost and total price sensitivity; note the correspondence between the former (latter) and the Japanese (American) auto manufacturers. Either manufacturer may choose to offer a retailer incentive or a customer rebate, where the rebates are restricted by the production costs of the manufacturers.

Table 20: Comparison of equilibria in different cases; manufacturer 1 has lower production cost and total price sensitivity than manufacturer 2 ($a = 75$, $b_{1o} = 2$, $b_{1c} = 1$, $b_{2o} = 3$, $b_{2c} = 2$, $c_1 = 10$, $c_2 = 20$, $m_1 = 10$, $m_2 = 10$, $\delta_1 = 0.5$, $\delta_2 = 0.5$).

	w_1	w_2	K_1	K_2	R_1	R_2	Q_1	Q_2	Π^{M_1}	Π^{M_2}
Case 0 No promotion	35.3	42.5	-	-	-	-	12.6	4.5	318.8	101.3
Case 1 Retailer incentive	39.8	47.5	25	10	-	-	14.9	5.5	417.5	141.2
Case 2 Customer rebate	45.5	40	-	-	10	0	12.8	4	325	80
Case 3 Hybrid (K_1, R_2)	40.2	42.4	25	-	-	0	15.1	4.5	432.2	101.3
Case 4 Hybrid (R_1, K_2)	45	45	-	10	10	-	12.5	5	312.3	115

Table 20 shows the equilibrium decisions and the resulting sales and profits of the supply chain members for the four combinations of promotions we have analyzed. In Figure 24, we show the manufacturers' profits. Manufacturer 1 consistently makes higher profits than manufacturer 2 in all cases and obtains the highest profit when she offers a retailer incentive while her competitor offers a customer rebate (Case 3). Manufacturer 1's high profitability and market share are due to the advantages of low production cost and total price sensitivity in demand. Manufacturer 2 is also better off with a retailer incentive but obtains the highest profit in Case 2 when both manufacturers offer retailer incentives. Note that manufacturer 2 offers a retailer incentive in Cases 2 and 4, but obtains lower profit in the latter where his competitor offers a customer rebate. This suggests that a customer rebate by one manufacturer can have a significant impact on the competitor's profits while a retailer incentive has more impact within a supply chain than on the competitor. In other words, the manufacturers are more vulnerable to their competitor's customer rebates than their retailer incentives.

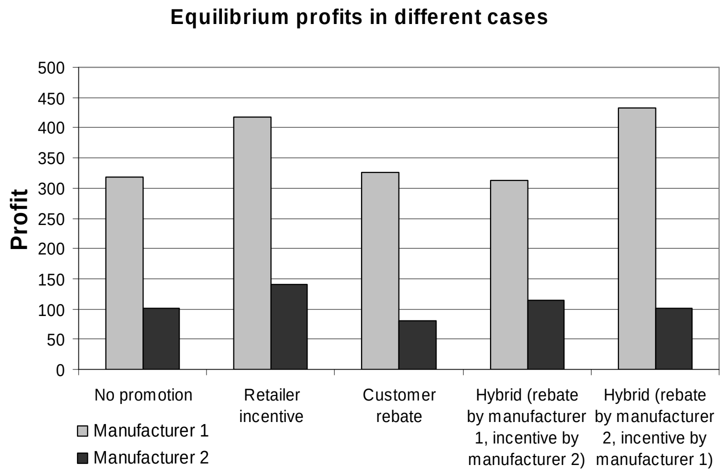


Figure 24: Manufacturers' profits under various equilibria where manufacturer 1 has lower production cost and total price sensitivity than manufacturer 2

4.4 Conclusions

In this chapter, we analyze sales promotions in a competitive environment by considering a duopoly setting. Our analysis extends the research in Chapter 3 where we studied sales promotions in a monopolistic setting. We are motivated by the practices in the U.S. auto industry where sales promotions play an important role in marketing and revenue management strategies of auto manufacturers. Customer rebates and retailer incentives are two types of promotions commonly used by auto manufacturers to increase their sales and profits when market demand is not strong. Customer rebates are per-unit payments from the manufacturer to the end customers, which are used as instant discounts for all purchases. On the other hand, retailer incentives are offered to dealers who have flexibility on whether to transfer these payments to the purchase prices of the customers. One important characteristic of our work is that we analyze a price discriminating retailer, which captures the nature of sales based on negotiations between the dealers and the car buyers.

We analyze a competitive setting with two manufacturers who sell their products through their exclusive but independent retailers. We model a single selling period where the retailers make the order/sales quantity decisions upon observing the manufacturers' wholesale price and promotion decisions. We analyze several models with different promotion decisions by the manufacturers. To understand competition under price discrimination, we analyze a benchmark model with no promotion, which we also use to quantify the effect of the promotions on the manufacturers' and the retailers' profits and sales. We use backward induction and explicitly derive the expressions for the SPNE decisions. We find that under certain market conditions two equilibria are possible, which determine the manufacturer (or the retailer) which is the market leader in sales. We find that low production cost and price sensitivity provide advantage to a manufacturer to achieve higher market share and profits. This is also observed in practice where the American auto manufacturers with higher costs and price sensitivities than the Japanese auto manufacturers obtain lower sales in compact and midsize vehicle segments. We also find that manufacturers obtain lower profits when there is competition than when they are monopolies. In another model, we analyze the case where both manufacturers offer incentives to their retailers. We design an algorithm that numerically converges to the equilibrium decisions in our computational tests, although no convergence guarantee is provided. We observe that retailer incentives increase the manufacturers' sales and profits, but they may reduce the retailers' profits in some cases. We also analyze the case where both manufacturers offer customer rebates. We find that customer rebates can be effectively used by a manufacturer to drive the competitor's profits to zero while achieving high sales and profits. This implies that competition may be one reason why customer rebates are frequently offered in the auto industry. In the last model that we study, we look at the case where one of the manufacturers offers a customer rebate and the other a retailer incentive. We observe in numerical examples that the impact of the customer rebates is more pronounced on the competitor's profits and sales than the retailer incentives. Our results suggest that the manufacturers with high cost and price sensitivities, e.g., the American auto manufacturers, can improve their profits by offering incentives to their retailers.

Our analysis has some limitations due to deterministic demand model and single-period selling season. However, these simplifications enable us to focus on competition, which is an important factor for the manufacturers' decisions. We can extend our work in a multiple-period model to investigate the effects of promotions on retailers' sales over time periods with different promotion offers. One extension area is to analyze other kinds of retailer incentive promotions. For example, it is interesting to analyze cases where the competing retailers receive incentives in different forms, such as per-unit versus lump-sum incentive.

CHAPTER V

CONCLUSIONS

In this thesis, we studied models to analyze and control decentralized systems focusing on issues in demand and revenue management. Decentralized systems operate according to the actions and interactions of the system members who are equipped with localized decision-making responsibilities, whereas centralized systems operate in an integrated way with a single decision maker making system-wide decisions. There are several reasons for companies or systems to decentralize their operations. For example, implementation of centralization may be uneconomical for large systems since it may require high investment to store and process data. In organizations, decentralization facilitates better accountability for actions by independent departments since the responsibilities for certain decisions are clearly identified. The companies can use this accountability to motivate the members by rewarding good performance, which eventually adds to the overall system efficiency. Distributing the decision-making power to individual members may lead to fast decision-making in real time which might be desirable for highly dynamic environments. To realize such advantages, it is essential to address problems and issues arising in decentralized operations. One important problem observed in some cases is the misalignment of members' objectives in the system, which can deteriorate the system performance unless effective mechanisms are developed to account for possible conflicts. Our focus is on developing models to optimize such mechanisms to improve the performance of decentralized systems.

In the first part of the thesis (Chapter 2), we considered a decentralized booking system in the sea cargo industry. The system is composed of a central headquarters and sales agents; the former assigns capacity limits and incentives to the sales agents, and the latter independently handles the accept/demand decisions for the incoming demand requests and chooses the route to transport the accepted cargo. Each sales agent contributes to the revenue of the overall system and receives revenue based on the sales incentive assigned by the central headquarters. We focused on the perspective of the central headquarters, who is concerned with utilizing the available resources most efficiently while maximizing the total revenue generated in the overall system. We modelled the central headquarters' capacity allocation problem and analyzed several sales incentives to improve the decentralized system performance. We developed heuristic methods to solve the firm's capacity allocation problem, which we proved to be NP-hard. When designing heuristic methods, we characterized agent behaviors depending on the sales incentives determined by the central headquarters. We conducted extensive computational experiments to test our heuristics for systems with various sizes and characteristics, and we observed that the heuristics performed reasonably well and fast. We proved a worst-case result for an extreme instance where the number of locations goes to infinity, which shows that the decentralized system performance can be arbitrarily worse than that of the centralized system. However, in our computational experiments with practical instances, we observed that the decentralized system performance can be quite close to that of the centralized system, and a revenue per leg sales incentive improves the decentralized system performance.

Our research in Chapter 2 is limited by its deterministic nature and can be extended by introducing stochasticity to the booking process. In practice, uncertainty is observed in the demand requests as well as in other operational issues relevant to the sea cargo practices, for example, demand requests with unknown destinations. In stochastic settings, models would be developed to facilitate dynamic space exchanges between the sales agents to make the capacity available to the agents with high demand and revenue expectations. It may be possible to incorporate strategic gaming by agents, or information mechanisms that may further improve the decentralized system performance. Future research also includes the analysis of other decentralized systems that could benefit from the design of optimization models to explicitly capture agent behavior and mechanism design.

In the second part of the thesis (Chapters 3 and 4), we analyzed sales promotions by manufacturers in decentralized supply chains. In such environments, due to decentralization and non-direct sales of products through retailers, the manufacturers have the option of whether to direct their promotions to the end customers or to the retailers. Promotions in the former case are generally defined as customer rebates and in the latter case as retailer incentives. The auto industry in the U.S. is one example where both types of promotions are offered by the manufacturers. Specific characteristics of the auto market such as an oversupply of vehicles, largely fixed production costs of manufacturers, and intense competition, contribute to the extensive use of promotions as an important marketing strategy by the companies. To obtain high market shares and profits, the companies need to know which type of promotions benefit them most under which type of market conditions, as well as how their profits and sales are affected by the competitors' promotions. We studied these problems in two separate works. In the first part, we analyzed a setting with a single manufacturer and single retailer, and we focused on the effects of promotions on the manufacturers' sales and profits. We analyzed several cases where the manufacturer's promotional schemes varied from no promotion to simultaneous use of customer rebate and retailer incentive. We analyzed a base model with deterministic demand and found that the manufacturer is always better off with a retailer incentive than a customer rebate. In the more general model with uncertain demand, we found that either of the promotions can be more profitable and generate higher sales for the manufacturer than the other depending on the degree of uncertainty. One observation is that the customer rebate performs better than the retailer incentive when uncertainty in a market potential parameter of demand function is high. In the second part, we extended the analysis of promotions by analyzing a setting with two competing manufacturers and two exclusive retailers that are also competitors in the end market. Although our models realistically capture the interactions in practice by incorporating competition at two stages of the supply chains, this complicates the analytics considerably. Therefore, we resort to computational schemes to find the equilibria and gain insights through numerical examples. One of our analytical finding is that unlike the monopolistic setting, the customer rebates can be highly effective even leading monopoly profits for one manufacturer and zero profit for the competitor.

There are several directions that our research in sales promotions can be extended. First, the models in Chapters 3 and 4 are single-period models, which are useful to obtain insights while focusing on price discriminating retailers and competition. One extension would be to analyze the manufacturers' promotional decisions in a multi-period model, where the secondary market of vehicles is incorporated into the analysis along with price discrimination. In such a model, it would be interesting to investigate the effects of promotions on the secondary market and the profits of the manufacturers from new vehicles. Within a multi-period framework, another extension can be to analyze the customers' strategic behaviors reacting to the manufacturers' promotions over multiple periods. It would be useful to analyze retailer incentives with other forms, such as incentives dependent on sales levels. Future research also includes analysis of promotions with a demand model that is sensitive to the retailers' effort levels.

APPENDIX A

APPENDIX FOR CHAPTER 2

A.1 SCP Formulation

We use the decision variable y_k to denote the number of containers accepted on O-D pair $k \in OD$. *SCP* is formulated as follows:

$$
\begin{aligned}
\textit{(SCP)} \quad \max \quad & \sum_{k \in OD} p_k y_k \\
\text{subject to} \quad & \sum_{k \in S_l} y_k \leq cap, \forall l \in L \\
& y_k \leq d_k, integer, \ \forall k \in OD
\end{aligned}
$$

A.2 MR Heuristic

We formally describe the algorithm in Table 21. In implementation, we store the optimal AP solutions of all agents, based on their most recent allocated capacities in a *solutions array.*

Table 21: Marginal revenue (MR) heuristic

1. Initialize the allocation vectors at zero ($a_p^r = 0$, $\forall p \in P$, $\forall r \in R_p$).
2. For all $p \in P$, $r \in R_p$ do
   solve $AP_p(\vec{a}_p + stepsize \cdot e_p^r)$. Record the solutions in *solutions array*[1].
3. Do begin
   $(p_{max}, r_{max}) = argmax_{p \in P, r \in R_p} \{Z_p^*(\vec{a}_p + stepsize \cdot e_p^r) - Z_p^*(\vec{a}_p) :$
   $\sum_{q \in P \setminus \{p\}} \sum_{\{k \in OD_q \cap S_l^r\}} y_k^{r'*}(\vec{a}_q) + \sum_{\{k \in OD_p \cap S_l^r\}} y_k^{r'*}(\vec{a}_p + stepsize \cdot e_p^r) \leq cap_{r'}, \forall r' \in R, \, l \in L_{r'}\}$
   $change = Z_{p_{max}}^*(\vec{a}_{p_{max}} + stepsize \cdot e_{p_{max}}^{r_{max}}) - Z_{p_{max}}^*(\vec{a}_{p_{max}})$
   if $change > 0$ then begin
     $\vec{a}_{p_{max}} = \vec{a}_{p_{max}} + stepsize \cdot e_{p_{max}}^r$
     for all $r \in R_{p_{max}}$ do begin
       if $(a_{p_{max}}^r + stepsize \leq cap_r \cdot nv_{p_{max}}^{r_{max}})$ then
         solve $AP_{p_{max}}(\vec{a}_{p_{max}} + stepsize \cdot e_{p_{max}}^{r_{max}})$. Update *solutions array* for p_{max}.
     end for
   end if
   end while ($change > 0$)

A.3 POD Heuristic

In order to formally describe the algorithm in Table 22, we introduce the following notation: r_l denotes the booked capacity on leg l, $\overline{d}_k$ denotes the remaining demand for O-D pair k, w_k^r denotes the available space on route r that may be devoted to booking of O-D pair k. As defined in Table 1, $\overline{OD}$ denotes the ordered set of all O-D pairs with non-increasing prices, where the first O-D pair has the highest price. For easy representation, we consider the routes with no particular order in the description of the algorithm. See discussion on the POD heuristic in Section 2.3.2.2 for the choice of the order of routes.

Table 22: Priority OD (POD) based allocation heuristic

1. Initialization: $a_p^r = 0, \forall p \in P, \forall r \in R_p$; $r_l = 0, \forall l \in L$; $\overline{d}_k = d_k, \forall k \in OD$; $w_k^r = cap_r, \forall r \in R, \forall k \in RR_k$
2. Starting with the first O-D pair in $\overline{OD}$, for all $k \in \overline{OD}$ do begin
   for all $r \in RR_k$ do begin
     if $(\overline{d}_k > 0)$ then begin
       $w_k^r = \min_{l \in LL_k^r} (cap_r - r_l)$
       solve $AP_{op_k}(\vec{a}_{op_k} + min(w_k^r, \overline{d}_k) \cdot e^r_{op_k})$
       if $\Big(\sum_{\{m \in OD_{op_k} \cap S_l^r\}} y_m^{r*}(\vec{a}_{op_k} + min(w_k^r, \overline{d}_k) \cdot e^r_{op_k}) +$
         $\sum_{q \in P \setminus \{op_k\}} \sum_{\{m \in OD_q \cap S_l^r\}} y_m^{r*}(\vec{a}_q) \le cap_r, \forall r \in R, l \in L_r \Big)$ then begin
         $\vec{a}_{op_k} = \vec{a}_{op_k} + min(w_k^r, \overline{d}_k) \cdot e^r_{op_k}$
         $\overline{d}_k = \overline{d}_k - min(w_k^r, \overline{d}_k)$
         $r_l = r_l + \sum_{\{m \in OD_{op_k} \cap S_l^r\}} y_m^{r*}(\vec{a}_{op_k}), \forall r \in R, l \in L_r$
       end if
     end if
   end for
   end for.

A.4 Proofs of Theorems and Remarks

For each O-D pair k, let ij be an alternative index where $i = op_k$ and $j = dp_k$. Parallel to Table 1, $\overline{OD}_i$ is the ordered set of O-D pairs of the agent at port i, representing the priority list of the agent under the total revenue incentive. Let $\overline{OD}'_i$ represent the priority list under the rev/leg incentive. We let $F^*_{SCAP}(n)$ and $\overline{F}^*_{SCAP}(n)$ denote the optimal revenue of the $SCAP$ solution with the total revenue and rev/leg incentive, respectively. We denote the optimal revenue of the SCP solution by $F^*_{SCP}(n)$.

Proof. (Theorem 1)

In the example with travel sequence $\{1,2,3,..,n,1\}$, let $cap = 1$ and $d_{ij} = 1\ \forall i,j : 1 \leq i < j \leq n$ and $\forall i \neq 1, j = 1$. Assume no demand exists for the other O-D pairs. We let $0 < \gamma < \frac{p}{n}, 0 < \epsilon < \frac{p}{n}$, and we define the parameters as follows:

$p_{1n} = p$; $p_{n1} = p - \epsilon$;

$p_{i(i+1)} = p - \epsilon$, $\forall i = 1,..,(n-1)$;

$p_{ij} = p - \epsilon + \gamma(|LL_{ij}| - 1)$, $\forall (i,j)$ s.t: $|LL_{ij}| > 1, (i,j) \neq \{(1,n),(n,1)\}$.

In this example, we show that $lim_{n\to\infty} \frac{F^*_{SCAP}(n)}{F^*_{SCP}(n)} = \frac{2p-\epsilon}{n(p-\epsilon)} = 0$. First, note that: $\overline{OD}_i = \{OD_{i1}, OD_{in}, OD_{i(n-1)}, .., OD_{i(i+1)}\}$, $\forall i : 1 < i \leq (n-1)$. Since capacity is 1, each agent may at most receive an allocation of 1, which he will use to accept the highest priority O-D pair of the agent, i.e., OD_{1n} for agent 1, OD_{n1} for agent n, and OD_{i1} for the other agents. However, since the highest priority O-D pairs of all agents except 1 and n share at least one leg, these agents cannot be assigned a positive allocation at the same time in a feasible solution. Therefore, the set of feasible solutions of $SCAP$ with the total revenue incentive is:

1. $a_1 = 1$, $a_n = 1$, $a_i = 0, \forall i : 1 < i \leq (n-1)$. In this case, $F_{SCAP}(n) = 2p - \epsilon$ with accepted demand $y_{1n} = y_{n1} = 1$ and $y_{ij} = 0$ for all other O-D pairs.

2. For each agent $i : 2 \leq i \leq (n-1)$, a feasible solution of the form $a_i = 1$ with solution $y_{i1} = 1$ and $F_{SCAP}(n) = p_{i1}$. The solution $y_{21} = 1$ and $y_{ij} = 0$, for all other O-D pairs with objective value $F_{SCAP}(n) = p - \epsilon + (n-2)\gamma$ has the highest revenue in this set of solutions.

Since $\gamma < \frac{p}{n} < \frac{p}{(n-2)} \Rightarrow 2p - \epsilon > p - \epsilon + (n-2)\gamma$, the optimal solution value is $F^*_{SCAP}(n) = 2p - \epsilon$ corresponding to the solution in 1 above.

Next we show that $F^*_{SCP}(n) = n(p - \epsilon)$, which corresponds to the optimal solution $y^*_{i(i+1)} = 1, \forall i : 1 \leq i < n$ and $y^*_{n1} = 1$. For this instance, any accepted demand from $SCAP$ that is an O-D pair with more than one leg may be replaced by the 1-leg O-D pairs occupying the same set of legs but with higher total revenue. This is possible in the centralized system because there is no need to incorporate agent behavior. In a solution, if OD_{ij} demand is accepted, we can improve this solution by rejecting OD_{ij} demand and instead accepting the $|LL_{ij}|$ 1-leg OD demands since $p-\epsilon+(|LL_{ij}|-1)\gamma < |LL_{ij}|(p-\epsilon)$ for any $(i,j) : |LL_{ij}| > 1, (i,j) \neq (1,n)$. This is also true for $(1,n)$ since $(2p-\epsilon) < n(p-\epsilon)$. Since any solution that accepts an OD_{ij} with $|LL_{ij}| > 1$ may be improved by such replacements, $F^*_{SCP}(n) = n(p-\epsilon)$. Therefore, we have that $\frac{F^*_{SCAP}(n)}{F^*_{SCP}(n)} = \frac{2p-\epsilon}{n(p-\epsilon)}$, and the limit of this worst case ratio approaches 0 as the number of ports grows. ■

Proof. (Remark 1)

In the example used for the proof of Theorem 1, $\overline{OD}'_i = \{OD_{i(i+1)}, OD_{i(i+2)}, .., OD_{in}\}$, since $\frac{p_{ij}}{|LL_{ij}|}, \forall i : 1 < i \leq (n-1)$ is decreasing. Since shorter O-D pairs have higher priorities, the optimal centralized solution that accepts all of the 1-leg O-D pairs is feasible for the optimal decentralized problem and is the best that is possible for the decentralized problem. So, we can conclude that under the rev/leg incentive, in this problem $\overline{F}^*_{SCAP}(n) = F^*_{SCP}(n)$.

■

Proof. (Theorem 2)

We use the same network as in the proof of Theorem 1. We let $\epsilon \leq \frac{p}{n}$, and we modify the price parameters as follows:

$p_{1i} = (i-1)p - (i-2)\epsilon$, $\forall i = 2, .., n$;

$p_{n1} = p$;

$p_{ij} = (j-i)(\frac{p}{n}) - (|LL_{ij}|-1)\epsilon$, $\forall (ij) : |LL_{ij}| \geq 1$ and $j > i \neq 1$;

$p_{i1} = (n-(i-1))(\frac{p}{n}) - (n-i)\epsilon$, $\forall i = 2, .., (n-1)$.

In this example,, we show that $lim_{n\to\infty} \frac{\overline{F}^*_{SCAP}(n)}{F^*_{SCP}(n)} = \frac{(\frac{3n-2}{n})p}{np-(n-2)\epsilon} = 0$. Note that the priority lists of the agents for the rev/leg incentive are the same as those in the proof of Theorem 1 since $\frac{p_{ij}}{|LL_{ij}|}$ is decreasing. If we give an allocation of 1 to each agent, each agent will satisfy the demand for the O-D pair with the shortest travel.

With a similar argument as used for Remark 1, we may conclude that the optimal $SCAP$ solution is $y^*_{i(i+1)} = 1, \forall i : 0 \leq i \leq (n-1)$ and $y^*_{n1} = 1$, and has objective value $\overline{F}^*_{SCAP}(n) = p_{12} + \sum_{i=2}^{n-1} p_{i(i+1)} + p_{n1} = (\frac{3n-2}{n})p$.

We next show that SCP has the optimal solution $y^*_{1n} = y^*_{n1} = 1, y^*_{ij} = 0$ for all other

O-D pairs, with objective value $F^*_{SCP}(n) = p_{1n} + p_{n1} = np - (n-2)\epsilon$. This result follows from Remarks 3 and 4.

Remark 3 *In the optimal solution of SCP for this example,* $y_{n1} = 1 \Rightarrow y_{1n} = 1$.

Proof. Assume first that OD_{n1} is accepted, then we have to find the best way of using capacity from 1 to n. In total, there are 2^{n-2} combinations, but due to the structure of the price parameters we can consider a limited set. We first show that the best use of ship capacity between all ports (i,j):$1 < i < (n-1)$, $i < j \leq n$ is to accept all of the 1-leg O-D pairs between i and j. We prove this by induction. There is only one choice of demand or path from port $(n-1)$ to n. Since there is only one unit of capacity, the choice of which demand to accept is equivalent to finding the longest path where distance is determined by the price of the O-D pairs. First assume that the best capacity use from $(n-i) \geq 2$ to n is to accept the i 1-leg O-D pairs. i.e., $\{OD_{(n-i)(n-i+1)}, OD_{(n-i+1)(n-i+2)}, .., OD_{(n-1),n}\}$. We need to show that the best path from $n-i-1$ to n is to use the $(i+1)$ 1-leg O-D pairs, i.e., $\{OD_{(n-i-1)(n-i)}, OD_{(n-i)(n-i+1)}, .., OD_{(n-1)n}\}$. If we choose to go from $n-i-1$ to $n-i+j, \forall j : 0 \leq j \leq i$, then by the induction hypothesis, the best path to n from $n-i+j$ is to choose the $(i-j)$ 1-leg O-D pairs between those two ports. The path from $n-i-1$ directly to $n-i+j$ and from $n-i+j$ in the best way to n has a total revenue of $(j+1)\frac{p}{n} - j\epsilon + (i-j))\frac{p}{n} = (i+1)\frac{p}{n} - j\epsilon$, which is maximized at $j = 0$. This means that the best choice from $n-i-1$ to n is to use all of the 1-leg O-D pairs between the two ports.

Now we find the best path from port 1 to port n. We consider a path that visits j. We have shown that the best choice from j to n is to use all of the 1-leg O-D pairs between the ports j and n. Then, the value of this path is $(j-1)p - (j-2)\epsilon + (n-j)\frac{p}{n}$, which is maximized at $j = n$. Therefore, $y_{n1} = 1 \Rightarrow y_{1n} = 1$. ■

Remark 4 *Any solution of SCP with* $y_{n1} = 0$ *can be improved by modifying the solution so that* $y_{n1} = 1$.

Proof. If $y_{n1} = 0$, then exactly one of OD_{j1}, $j = 2, .., n-1$ must have been accepted. Otherwise we can accept OD_{n1} without violating feasibility. Consider the path from j

directly to 1, which has a value of $(n-j+1)\frac{p}{n} - (n-j)\epsilon$. We can modify this path such that it is composed of two partial paths: the first from j directly to n, and the second from n to 1. The modified path has a value of $(n-j)\frac{p}{n}+p$, which is greater than $(n-j+1)\frac{p}{n}-(n-j)\epsilon$; therefore the SCP solution improves. ■

Based on Remarks 3 and 4, we have that $lim_{n\to\infty}\frac{\overline{F}^*_{SCAP}(n)}{F^*_{SCP}(n)} = \frac{(\frac{3n-2}{n})p}{np-(n-2)\epsilon} = 0$. ■

Proof. (Remark 2) The result can be proved with a similar argument to that used for Remark 1. ■

APPENDIX B

APPENDIX FOR CHAPTER 3

B.1 Proofs of key results

Proof of Theorem 3.

No Promotion : The results follow from the backward induction steps presented below.

Step 1. The retailer's order quantity decision: Given w, find Q that maximizes the retailer's profit by solving problem (P1) in Section 3.3.1. Π_o^R is concave in Q ($\frac{\partial^2 \Pi_o^R(Q)}{\partial Q^2} = -b < 0$). From first order conditions (FOC) we get $Q = \frac{a-w}{b}$. Considering the upper bound on Q, we find the retailer's best response to w as $Q_o = max\left\{0, min\left\{\frac{a-w}{b}, \frac{a-w-m}{b}\right\}\right\} = \left(\frac{a-w-m}{b}\right)^+$.

Step 2. The manufacturer's wholesale price decision: Given the best response of the retailer to w, find w that maximizes the manufacturer's profit by solving the following problem.

$$\Pi_o^M = \max_{w \geq c} \quad (w-c)\left(\frac{a-w-m}{b}\right)^+$$

When $\frac{a-w-m}{b} \geq 0$, the objective function is concave in w ($\frac{\partial^2 \Pi_o^M}{\partial w^2} = \frac{-2}{b} < 0$). It follows from FOC and the lower bound on w that $w^* = max\left\{c, min\left\{\frac{a+c-m}{2}, a-m\right\}\right\} = \frac{a+c-m}{2}$. Therefore, $w_o = \frac{a+c-m}{2}$; $Q_o = \frac{a-m-c}{2b}$; $\Pi_o^M = \frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_o^D = \frac{(a-m-c)(a+3m-c)}{8b}$; $\Pi_o^{SC} = \frac{(a-m-c)(3a+m-3c)}{8b}$.

Retailer Incentive: The results follow from the backward induction steps presented below.

Step 1. The retailer's order quantity decision: Given w and K, find Q that maximizes the retailer's profit by solving the problem (P2) in Section 3.3.1. Π_I^R is concave in Q ($\frac{\partial^2 \Pi^R}{\partial Q^2} = -b$). From FOC we get $Q = \frac{a-w}{b}$, and by considering the boundary condition on Q enforced with the constraint, we find the retailer's best response as $Q_I = \left(min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2Kb}}{b}\right\}\right)^+$.

Step 2. The manufacturer's wholesale price and retailer incentive decision:

Given the retailer's best response, find w and K that maximizes the manufacturer's profit:

$$\Pi_I^M = \max_{K\geq 0, w\geq c} \quad (w-c)\left(min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2Kb}}{b}\right\}\right)^+ - K$$

In order to solve the manufacturer's problem, we proceed in two steps; first, we characterize the optimal value for the retailer incentive, K^*, for a given w, and next, we find the optimal wholesale price, by embedding K^* in the manufacturer's objective function and maximizing it over w. In the first step, we obtain the expression in (11) for K^*.

$$K^* = \begin{cases} min\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\} & \text{if } K \leq \frac{m^2}{2b} \text{ and } \sqrt{2Kb} \geq w-(a-m) \text{ where } w \leq a \\ 0 & \text{if } K \leq \frac{m^2}{2b} \text{ and } \sqrt{2Kb} \leq w-(a-m) \text{ where } w \geq a-m \\ \frac{m^2}{2b} & \text{if } K \geq \frac{m^2}{2b} \text{ where } w \leq a. \end{cases} \tag{11}$$

Note that K^* is identified under different cases which lead to different Q decisions. Characterizing K^* for a given w, in the second step, we find the optimal wholesale price, w_I, by embedding K^* in the manufacturer's objective function and maximizing it over w in the subregions defined by the branching in Figure 25. (FS stands for "feasible solution".)

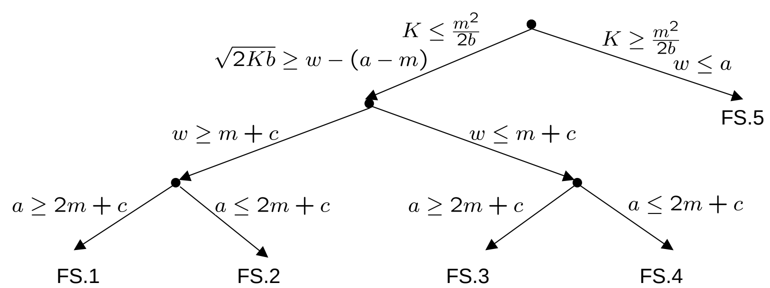


Figure 25: Decomposition of the feasible region for determining w in the deterministic demand model with retailer incentive

To solve the manufacturer's problem, we identify four cases and summarize the retailer's response in each case as follows:

Case 1) $K \leq \frac{m^2}{2b}$; $\sqrt{2Kb} \geq w-(a-m)$; $\Rightarrow Q^* = \frac{a-w-m+\sqrt{2Kb}}{b}$

Case 2) $K \geq \frac{m^2}{2b}$; $w \leq a \Rightarrow Q^* = \frac{a-w}{b}$

Case 3) $K \geq \frac{m^2}{2b}$; $w \geq a \Rightarrow Q^* = 0$

Case 4) $K \leq \frac{m^2}{2b}$; $\sqrt{2Kb} \leq w-(a-m)$; $\Rightarrow Q^* = 0$

The manufacturer has the feasible solution of setting $w = c$ and $K = 0$, and receiving zero profit. Therefore, we can omit Cases 3 and 4, since they can not do any better than resulting in a profit of zero for the manufacturer.

Case 1) $K \leq \frac{m^2}{2b}$; $\sqrt{2Kb} \geq w-(a-m)$; $\Rightarrow Q^* = \frac{a-w-m+\sqrt{2Kb}}{b}$. First we solve for K:

$$
\begin{aligned}
\text{(P3)} \quad \max_{K} \quad & (w-c)(\frac{a-w-m+\sqrt{2Kb}}{b}) - K \\
\text{s.t.} \quad & 0 \leq K \leq \frac{m^2}{2b} \\
& \sqrt{2Kb} \geq w-(a-m)
\end{aligned}
$$

The objective function is concave in K, ($\frac{\partial^2 \Pi^M}{\partial K^2} = \frac{-(w-c)\sqrt{2}}{4\sqrt{b}\sqrt{K^3}} \leq 0$). From FOC, we obtain the point $K = \frac{(w-c)^2}{2b}$. We have two cases to consider: $w \geq a-m$ and $w \leq a-m$. In the first case, $K^* = \max\left\{\frac{(w-(a-m))^2}{2b}, \min\left\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\right\}\right\}$. Note that, the feasibility condition for $K \leq \frac{m^2}{2b}$ and $\sqrt{2Kb} \geq w-(a-m)$ to hold is $w \leq a$. This condition and our assumption $a \geq c+m$ imply that $\frac{(w-(a-m))^2}{2b} \leq \frac{(w-c)^2}{2b}$ and $\frac{(w-(a-m))^2}{2b} \leq \frac{m^2}{2b}$. Hence, $K^* = \min\left\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\right\}$. When $w \leq a-m$, the second constraint becomes inactive and we have the same expression for K^*, i.e., $K^* = \min\left\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\right\}$. Alternatively, by using the assumption $a \geq c+m$, we can show that $K = \frac{(w-c)^2}{2b}$ satisfies the second constraint, and thus we can conclude that $K^* = min\{\frac{(w-c)^2}{2b}, \frac{m^2}{2b}\}$.

Next, we solve for optimal w. We need to consider the following cases: $w \leq c+m$ and $w \geq c+m$, where $K^* = \frac{(w-c)^2}{2b}$ and $K^* = \frac{m^2}{2b}$ respectively.

Case 1.a) $w \geq c+m$

$$
\begin{aligned}
\max_{w} \quad & (w-c)(\frac{a-w}{b}) - \frac{m^2}{2b} \\
\text{s.t.} \quad & m+c \leq w \leq a
\end{aligned}
$$

$w^* = max\{m+c, min\{a, \frac{a+c}{2}\}\} = max\{m+c, \frac{a+c}{2}\}$

Case 1.a.1) (FS.1) $a \geq 2m+c \Rightarrow w^* = max\{m+c, \frac{a+c}{2}\} = \frac{a+c}{2}$; $Q^* = \frac{a-c}{2b}$; $K^* = \frac{m^2}{2b}$; $\Pi^M = \frac{(a-c)^2-2m^2}{4b}$.

Case 1.a.2) (FS.2) $a \leq 2m+c \Rightarrow w^* = max\{m+c, \frac{a+c}{2}\} = m+c$; $Q^* = \frac{a-m-c}{b}$; $K^* = \frac{m^2}{2b}$; $\Pi^M = \frac{m(2a-3m-2c)}{2b}$.

Case 1.b) $w \leq c + m$

$$\max_{w} \quad (w-c)(\frac{a-m-c}{b}) - \frac{(w-c)^2}{2b}$$
$$\text{s.t.} \quad c \leq w \leq m+c$$

$w^* = max\{c, min\{m+c, a-m\}\} = min\{m+c, a-m\}$

Case 1.b.1) (FS.3) $a \geq 2m + c \Rightarrow w^* = min\{m+c, a-m\} = m+c$; $Q^* = \frac{a-m-c}{b}$; $K^* = \frac{m^2}{2b}$; $\Pi^M = \frac{m(2a-3m-2c)}{2b}$.

Case 1.b.2) (FS.4) $a \leq 2m + c \Rightarrow w^* = min\{m+c, a-m\} = a-m$; $Q^* = \frac{a-m-c}{b}$; $K^* = \frac{(a-m-c)^2}{2b}$; $\Pi^M = \frac{(a-m-c)^2}{2b}$.

Case 2) $K \geq \frac{m^2}{2b}$; $w \leq a \Rightarrow Q^* = \frac{a-w}{b}$. First, we solve for K:

$$\max_{K} \quad (w-c)(\frac{a-w}{b}) - K$$
$$\text{s.t.} \quad K \geq \frac{m^2}{2b}$$

$K^* = \frac{m^2}{2b}$. Next, we solve for w:

$$\max_{w} \quad (w-c)(\frac{a-w}{b}) - \frac{m^2}{2b}$$
$$\text{s.t.} \quad c \leq w \leq a$$

$w^* = max\{c, min\{a, \frac{a+c}{2}\}\} = \frac{a+c}{2}$; $Q^* = \frac{a-c}{2b}$; $K^* = \frac{m^2}{2b}$; $\Pi^M = \frac{(a-c)^2-2m^2}{4b}$. We denote this feasible solution by **FS.5.**

We present all feasible solutions in Table 23. The SPNE in Table 4 (Section 3.3.1) is the solution that is feasible in a region and has the highest profit. (In the last row of Table 23, the SPNE that dominates the feasible solution in the respective column is shown under the title DS, standing for "dominating solution".)

Customer Rebate: The retailer's best response is $Q_R^* = \left(\frac{a+R-w-m}{b}\right)^+$, and the manufacturer's problem is as follows:

$$\max_{w \geq c+R, R \geq 0} \quad (w-c-R)\left(\frac{a+R-w-m}{b}\right)^+$$

We solve the manufacturer's problem in a two-step procedure, where in the first step we characterize the optimal rebate value given a wholesale price, and in the second step we substitute the optimal rebate value back into the manufacturer's profit function to solve for

Table 23: All feasible solutions for the deterministic demand model with retailer incentive

	FS.1	FS.5	FS.2	FS.3	FS.4
F.R:	$a \geq 2m + c$	$a \geq m + c$	$a \leq 2m + c$	$a \geq 2m + c$	$a \leq 2m + c$
w^*	$\frac{a+c}{2}$		$m + c$		$a - m$
Q^*	$\frac{a-c}{2b}$		$\frac{a-m-c}{b}$		$\frac{a-m-c}{b}$
K^*	$\frac{m^2}{2b}$		$\frac{m^2}{2b}$		$\frac{(a-m-c)^2}{2b}$
Π^M	$\frac{(a-c)^2-2m^2}{4b}$		$\frac{m(2a-2c-3m)}{2b}$		$\frac{(a-m-c)^2}{2b}$
DS:	-		FS.4	FS.1	-

the optimal wholesale price. We find $R = w - \frac{a+c-m}{2}$. After back substitution, the objective function becomes independent of w, and therefore any w, R satisfying $w_R - R = \frac{a+c-m}{2}$ is optimal.

Proof of Observation 1 Observation 1(i) is trivial since $Q_o = Q_R$ and $\Pi_o^M = \Pi_R^M$. Observation 1(ii) follows from Table 24.

Table 24: Comparison of retailer incentive and customer rebate for the deterministic demand model

	$a \leq 2m + c$	$a \geq 2m + c$
$Q_I - Q_R$	$\frac{(a-m-c)}{2b} \geq 0$	$\frac{m}{2b} \geq 0$
$\Pi_I^M - \Pi_R^M$	$\frac{(a-m-c)^2}{4b} \geq 0$	$\frac{m(2a-2c-3m)}{4b} \geq 0$
$\Pi_I^D - \Pi_R^D$	$\frac{(a-m-c)(5m+c-a)}{8b} \geq 0$	$\frac{m(7m+2c-2a)}{8b} \geq 0$ for $a \leq \frac{7}{2}m + c$
$\Pi_I^{SC} - \Pi_R^{SC}$	$\frac{(a-m-c)(a+3m-c)}{8b} \geq 0$	$\frac{m(2a+m-2c)}{8b} \geq 0$

Proof of Theorem 4(i).

No Promotion

Retailer's best response in "high" and "low" state is: $Q^{j*} = \left(\frac{a^j - w - m}{b}\right)^+; \quad j = l, h.$

Case 1) $w \leq a^l - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^l = \frac{a^l - w - m}{b}$

Case 2) $a^l - m \leq w \leq a^h - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^l = 0$

Case 3) $w \geq a^h - m \Rightarrow Q^h = 0$; $Q^l = 0$

Note that we can omit Case 3 since the manufacturer has the feasible solution of setting $w = c$ and receive zero profit. We analyze Cases 1 and 2 and find the manufacturer's optimal wholesale price in each case.

Case 1) $w \leq a^l - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^l = \frac{a^l - w - m}{b}$

$$\begin{aligned} \max \quad & \beta(w-c)Q^h + (1-\beta)(w-c)Q^m \\ \text{s.t} \quad & w \leq a^l - m \end{aligned}$$

$w^* = min\{a^l - m, \frac{\beta a^h + (1-\beta)a^l - m + c}{2}\}$

Case 1.a) (NP.1) $\beta \leq \frac{a^l - m - c}{a^h - a^l} \Rightarrow w^* = \frac{\beta a^h + (1-\beta)a^l - m + c}{2}$; $Q^h = \frac{(2-\beta)a^h - (1-\beta)a^l - m - c}{2b}$; $Q^l = \frac{(1+\beta)a^l - \beta a^h - m - c}{2b}$; $\Pi^M = \frac{(\beta a^h + (1-\beta)a^l - m - c)^2}{4b}$

Case 1.b) (NP.2) $\beta \geq \frac{a^l - m - c}{a^h - a^l} \Rightarrow w^* = a^l - m$; $Q^h = \frac{a^h - a^l}{b}$; $Q^l = \frac{a^h - a^l}{b}$; $\Pi^M = \frac{\beta(a^l - m - c)(a^h - a^l)}{b}$

Case 2) $a^l - m \leq w \leq a^h - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^l = 0$

$$\begin{aligned} \max \quad & \beta(w-c)Q^h \\ \text{s.t} \quad & a^l - m \leq w \leq a^h - m \end{aligned}$$

$w^* = max\{a^l - m, min\{a^h - m, \frac{a^h - m + c}{2}\}\}$

Case 2.a) (NP.3) $a^h - a^l \leq a^l - m - c \Rightarrow w^* = a^l - m$; $Q^h = \frac{a^h - a^l}{b}$; $\Pi^M = \frac{\beta(a^l - m - c)(a^h - a^l)}{b}$

Case 2.b) (NP.4) $a^h - a^l \geq a^l - m - c \Rightarrow w^* = \frac{a^h - m + c}{2}$; $Q^h = \frac{a^h - m - c}{2b}$; $\Pi^M = \frac{\beta(a^h - m - c)^2}{4b}$

We summarize the feasible solutions obtained in different regions in Table 25.

Table 25: All feasible solutions for the uncertain market potential model with no promotion

	NP.1	NP.2	NP.3	NP.4
w^*	$\frac{\bar{a} - m + c}{2}$	$a^l - m$		$\frac{a^h - m + c}{2}$
Q^{l*}	$\frac{(1+\beta)a^l - \beta a^h - m - c}{2b}$	0		0
Q^{h*}	$\frac{(2-\beta)a^h - (1-\beta)a^l - m - c}{2b}$	$\frac{a^h - a^l}{b}$		$\frac{a^h - m - c}{2b}$
Π^M	$\frac{(\bar{a} - m - c)^2}{4b}$	$\frac{\beta(a^l - m - c)(a^h - a^l)}{b}$		$\frac{\beta(a^h - m - c)^2}{4b}$
F.R:	$\beta \leq \frac{(a^l - m - c)}{(a^h - a^l)}$	$\beta \geq \frac{(a^l - m - c)}{(a^h - a^l)}$	$2a^l \geq a^h + m + c$	$2a^l \leq a^h + m + c$

Next, we show that the SPNE in Table 5 (Section 3.3.2) is the dominating solution in the region it is feasible.

- When $a^l - m - c \geq a^h - a^l$, it follows that $\beta \leq \frac{a^l-m-c}{a^h-a^l}$, and feasible solutions are NP.3 and NP.1. The manufacturer chooses NP.1 since $\Pi^M(NP.1) - \Pi^M(NP.3) = \frac{(\beta a^h-(1+\beta)a^l+m+c)^2}{4b} \geq 0$.

- When $a^l - m - c \leq a^h - a^l$, either $\beta \leq \frac{a^l-m-c}{a^h-a^l}$ or $\beta \geq \frac{a^l-m-c}{a^h-a^l}$ may hold. NP.4 is feasible in this region independent of the value of β. In addition to this, NP.1 is feasible when $\beta \leq \frac{a^l-m-c}{a^h-a^l}$ and NP.2 is feasible when $\beta \geq \frac{a^l-m-c}{a^h-a^l}$. Note the following:

  $\Pi^M(NP.4) - \Pi^M(NP.2) = \frac{\beta(a^h-2a^l+m+c)^2}{4b} \geq 0.$

  $\Pi^M(NP.1) - \Pi^M(NP.4) = \frac{(1-\beta)(\beta(a^h-a^l)^2-(a^l-m-c)^2)}{4b}$

  In the second equality, when $\beta \leq \frac{(a^l-m-c)^2}{(a^h-a^l)^2}$ $\Pi^M(NP.1) \geq \Pi^M(NP.4)$; otherwise $\Pi^M(NP.4) \geq \Pi^M(NP.1)$.

Proof of Proposition 1

Note that, a necessary condition for β_1 to exist is $(a^h-a^l-2m)^2+4m(3m+2c-2a^l) \geq 0$.

$\frac{\partial \beta^*}{\partial a^h} = \frac{2(a^h-a^l)(2m^2-(a^l-c)^2)}{(a^h-a^l)^4} \leq 0$, since $a^l - c \geq 2m \Rightarrow (a^l-c)^2 \geq 4m^2 \geq 2m^2$.

$\frac{\partial \beta^*}{\partial a^l} = \frac{2((a^l-c)(a^h-c)-2m^2)}{(a^h-a^l)^3} \geq 0$, since $a^h - c \geq a^l - c \geq 2m \Rightarrow (a^h-c)(a^l-c) \geq 2m^2$.

$\frac{\partial \beta_1}{\partial a^h} = \frac{(a^h-a^l-2m-A)(a^l-a^h-A)}{2A(a^h-a^l)^2} \leq 0$, where $A = \sqrt{(a^h-a^l-2m)^2+4m(3m+2c-2a^l)}$.

The inequality follows since $a^l \geq 2m+c \Rightarrow 3m+2c-2a^l \leq 0$, $(a^h-a^l-2m-A) \geq 0$, and $a^l - a^h - A \leq 0$.

$\frac{\partial \beta_1}{\partial a^l} = \frac{m(3a^h+a^l-8m-4c-A)}{A(a^h-a^l)^2} \geq \frac{m(3a^h+a^l-8m-4c-a^h+a^l+2m)}{A(a^h-a^l)^2} = \frac{m(2a^h+2a^l-6m-4c)}{A(a^h-a^l)^2} \geq 0$, where $A = \sqrt{(a^h-a^l-2m)^2+4m(3m+2c-2a^l)}$. The inequalities follow since $a^l \geq 2m+c \Rightarrow 2a^l \geq 3m+2c$, and $A \leq (a^h-a^l-2m)$.

$\frac{\partial \beta_2}{\partial a^h} = \frac{-2m(a^l-m-c)^2}{((a^l-c)^2+2m(a^h-a^l-m))^2} \leq 0.$

$\frac{\partial \beta_2}{\partial a^l} = \frac{2m(a^l-m-c)(2a^h-3m-2c)}{((a^l-c)^2+2m(a^h-a^l-m))^2} \geq 0$, since $a^h \geq 2m+c \Rightarrow 2a^h \geq 3m+2c$.

B.2 Analysis of the model with market expansion

We model this situation with an increased market potential $a^D = (1+\alpha)a$, $\alpha \geq 0$, when the manufacturer offers a rebate R. In this case, the retailer's best response to a given R, w, and α is $Q_{R'} = \left(\frac{(1+\alpha)a+R-w-m}{b}\right)^+$. The manufacturer's problem is to find the optimal values for R and α to maximize profit as follows:

$$\max_{R\geq 0,\alpha\geq 0,w\geq c+R} \quad (w-c-R)\left(\frac{(1+\alpha)a+R-w-m}{b}\right)^{+} - e\alpha^2$$

When the manufacturer offers a customer rebate and the rebate leads to market expansion, the SPNE is shown in Table 26.

Table 26: Equilibrium decisions when customer rebates expand market potential

	$a^2 \geq 4be$ or $be \leq a^2 \leq 2be$	$a^2 \leq be$ or $2be \leq a^2 < 4be$
$w_{R'} - R$	$a-m$	$\frac{a^2c+2be(m-c-a)}{a^2-4be}$
α	$\frac{(a-m-c)a}{2be}$	$\frac{(a-m-c)a}{4be-a^2}$
$Q_{R'}$	$\frac{(a-m-c)a^2}{2b^2e}$	$\frac{(a-m-c)2e}{4be-a^2}$
$\Pi^M_{R'}$	$\frac{(a-m-c)^2a^2}{4b^2e}$	$\frac{(a-m-c)^2e}{4be-a^2}$
$\Pi^D_{R'}$	$\frac{a^2(a-m-c)(a^2(a-m-c)+4bem)}{8b^3e^2}$	$\frac{2e(a-m-c)(be(a+3m-c)-a^2m)}{(4be-a^2)^2}$
$\Pi^{SC}_{R'}$	$\frac{a^2(a-m-c)(a^2(a-m-c)+2be(a+m-c))}{8b^3e^2}$	$\frac{e(a-m-c)(-a^2(a+m-c)+2be(a+m-3c))}{(4be-a^2)^2}$

In Table 27, we compare the manufacturer's profits and sales with the retailer incentive and the customer rebate in two feasible regions and see that the manufacturer can be better off with alternate promotions depending on the system parameters.

Table 27: Comparison of retailer incentive and customer rebate for the deterministic demand model when rebates lead to market expansion

Condition		$\Pi^M_I - \Pi^M_{R'}$	$Q_I - Q_{R'}$
$a \leq 2m+c$	$eb \leq a^2 \leq 2be$	$\frac{(2be-a^2)(a-m-c)^2}{4b^2e} \geq 0$	$\frac{(2be-a^2)(a-m-c)}{2b^2e} \geq 0$
	$2be \leq a^2 \leq 4be$	$\frac{(2be-a^2)(a-m-c)^2}{2b(4be-a^2)} \leq 0$	$\frac{(2be-a^2)(m+c-a)}{b(a^2-4be)} \leq 0$

B.3 Analysis of the model where $\Pi^D_I \geq \Pi^D_o$ is enforced in the equilibrium

In the equilibrium where there is no promotion offered by the manufacturer, the retailer receives a profit equal to $\Pi^D_o = \frac{(a-m-c)(a+3m-c)}{8b}$. We analyze the case where the manufacturer offers an incentive K to the retailer while making sure that the retailer receives a profit at least as high as Π^D_o in equilibrium. From the analysis in Appendix B.1, the retailer's best response is as follows: $Q^* = \left(\min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2Kb}}{b}\right\}\right)^+$

The Manufacturer's Problem:

Case 1) $K \leq \frac{m^2}{2b}; \sqrt{2Kb} \geq w-(a-m) \Rightarrow Q = \frac{a-w-m+\sqrt{2Kb}}{b}$

$$
\begin{aligned}
\text{(P4)} \quad \max_{w,K} \quad & (w-c)\left(\frac{a-w-m+\sqrt{2Kb}}{b}\right) - K \\
\text{s.t.} \quad & \frac{(a-w)^2 - m^2 + 2m\sqrt{2Kb}}{2b} \geq \frac{(a-m-c)(a+3m-c)}{8b} \\
& \sqrt{2Kb} \geq w-(a-m) \\
& K \leq \frac{m^2}{2b}
\end{aligned}
$$

Note that the first constraint is to ensure that $\Pi_I^D \geq \Pi_o^D$, i.e., $\int_0^{\frac{a-w-m+\sqrt{2Kb}}{b}}(a-bQ)dQ - w\left(\frac{a-w-m+\sqrt{2Kb}}{b}\right) + K \geq \frac{(a-m-c)(a+3m-c)}{8b}$.

Note the correspondence with the problem denoted by (P4) and the problem denoted by (P3) in the proof of Theorem 3 (Appendix B.1). We can show that the equilibrium denoted by FS.1 that is found in the proof of Theorem 3 satisfies the additional constraint in (P4) when $4m+2c \leq 2a \leq 7m+2c$. When $a \leq 2m+c$, the equilibrium denoted by FS.4 satisfies the additional constraint, therefore, they are also optimal for (P4) for the specified regions. (These solutions are the equilibrium solutions in Table 4.)

Case 2) $K \geq \frac{m^2}{2b}; w \leq a \Rightarrow Q = \frac{a-w}{b}$

$$
\begin{aligned}
\max_{w,K} \quad & (w-c)\left(\frac{a-w}{b}\right) - K \\
\text{s.t.} \quad & \frac{(a-w)^2}{2b} + K \geq \frac{(a-m-c)(a+3m-c)}{8b} \\
& K \geq \frac{m^2}{2b} \\
& c \leq w \leq a
\end{aligned}
$$

Note that the first constraint is to ensure that $\Pi_I^D \geq \Pi_o^D$, i.e., $\int_0^{\frac{a-w}{b}}(a-bQ)dQ - w\frac{(a-w)}{b} + K \geq \frac{(a-m-c)(a+3m-c)}{8b}$. We follow a two step procedure to find w^* and K^*, where we characterize K^*, and embed this into the manufacturer's objective function to find w^*.

$K^* = max\left\{\frac{m^2}{2b}, \frac{(a-m-c)(a+3m-c)}{8b} - \frac{(a-w)^2}{2b}\right\}$

Case 2.a) $\frac{m^2}{2b} \geq \frac{(a-m-c)(a+3m-c)}{8b} - \frac{(a-w)^2}{2b} \Rightarrow K = \frac{m^2}{2b}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{b}\right) - \frac{m^2}{2b} \\ \text{s.t.} \quad & 4w^2 - 8aw \geq -3a^2 - 7m^2 + 2ma - 2ca - 2mc + c^2 \\ & c \leq w \leq a \end{aligned}$$

$$w^* = \begin{cases} \frac{(a+c)^2}{2} & \text{if } 2a \leq 7m + 2c \\ a - \frac{1}{2}\sqrt{(a-c)^2 + m(2a-2c-7m)} & \text{if } 2a \geq 7m + 2c. \end{cases} \tag{12}$$

We find two feasible solutions which can be the equilibrium:

1. $2a \leq 7m + 2c \Rightarrow w^* = \frac{(a+c)^2}{2}$; $K^* = \frac{m^2}{2b}$; $Q^* = \frac{a-c}{2b}$; $\Pi^M = \frac{(a-c)^2-2m^2}{4b}$; $\Pi^D = \frac{(a-c)^2+4m^2}{8b}$.

2. $2a \leq 7m + 2c \Rightarrow w^* = a - \frac{1}{2}\sqrt{(a-c)^2 + m(2a-2c-7m)}$; $K^* = \frac{m^2}{2b}$; $Q^* = \frac{\sqrt{(a-c)^2+m(2a-2c-7m)}}{2b}$; $\Pi^M = \frac{(a-c)(2\sqrt{(a-c)^2+m(2a-2c-7m)}-(a-c))+m(-2a+2c+5m)}{4b}$; $\Pi^D = \frac{(a-m-c)(a+3m-c)}{8b}$.

Case 2.b) $\frac{m^2}{2b} \leq \frac{(a-m-c)(a+3m-c)}{8b} - \frac{(a-w)^2}{2b} \Rightarrow K = \frac{(a-m-c)(a+3m-c)}{8b} - \frac{(a-w)^2}{2b}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{b}\right) - \frac{(a-m-c)(a+3m-c)}{8b} + \frac{(a-w)^2}{2b} \\ \text{s.t.} \quad & 4w^2 - 8aw \leq -3a^2 - 7m^2 + 2ma - 2ca - 2mc + c^2 \\ & c \leq w \leq a \end{aligned}$$

We find one feasible solution which can be the equilibrium:
$w^* = a - \frac{1}{2}\sqrt{(a-c)^2 + m(2a-2c-7m)}$; $K^* = \frac{m^2}{2b}$; $Q^* = \frac{\sqrt{(a-c)^2+m(2a-2c-7m)}}{2b}$; $\Pi^M = \frac{(a-c)(2\sqrt{(a-c)^2+m(2a-2c-7m)}-(a-c))+m(-2a+2c+5m)}{4b}$; $\Pi^D = \frac{(a-m-c)(a+3m-c)}{8b}$.

In Table 28, we summarize the equilibrium solutions for the retailer incentive case when the manufacturer ensures that the retailer receives a profit with the incentive that is at least as high as that of the no-promotion equilibrium.

We compare the retailer incentive equilibria with the no-promotion equilibrium stated in Theorem 3(i). We show that when $2a \geq 7m+2c$ sales and manufacturer's profit increase, while the retailer's profit remain the same when the manufacturer offers an incentive.

First, we show that $Q_I \geq Q_o$.

Table 28: The SPNE for the deterministic demand model with retailer incentive when the manufacturer ensures the profit in the no-promotion equilibrium for the retailer

	$a \leq 2m + c$	$4m + 2c \leq 2a \leq 7m + 2c$	$2a \geq 7m + 2c$
w_I	$a - m$	$\frac{a+c}{2}$	$a - \frac{1}{2}\sqrt{(a-c)^2 + m(2a - 2c - 7m)}$
Q_I	$\frac{a-m-c}{b}$	$\frac{a-c}{2b}$	$\frac{\sqrt{(a-c)^2+m(2a-2c-7m)}}{2b}$
K	$\frac{(a-m-c)^2}{2b}$	$\frac{m^2}{2b}$	$\frac{m^2}{2b}$
Π_I^M	$\frac{(a-m-c)^2}{2b}$	$\frac{(a-c)^2-2m^2}{4b}$	$\frac{(a-c)(2\sqrt{(a-c)^2+m(2a-2c-7m)}-(a-c))+m(-2a+2c+5m)}{4b}$
Π_I^D	$\frac{(a-m-c)m}{b}$	$\frac{(a-c)^2+4m^2}{8b}$	$\frac{(a-m-c)(a+3m-c)}{8b}$
Π_I^{SC}	$\frac{(a-c)^2-m^2}{2b}$	$\frac{3(a-c)^2}{8b}$	$\frac{(a-c)(4\sqrt{(a-c)^2+m(2a-2c-7m)}-(a-c))+m(-2a+2c+7m)}{8b}$

$$Q_I = \frac{\sqrt{(a-c)^2+m(2a-2c-7m)}}{2b} \leq \frac{a-m-c}{2b} = Q_o$$
$$\Leftrightarrow \sqrt{((a-c)-m))^2} \leq \sqrt{(a-c)^2 + m(2a-2c-7m)}$$
$$\Leftrightarrow ((a-c)-m)^2 \leq (a-c)^2 + m(2a-2c-7m) \Leftrightarrow m(m-2a+2c) \leq m(2a-2c-7m).$$

The result follows since $2a \geq 7m + 2c \Rightarrow -4a + 4c + 8m \leq 0$.

Next, we show that $\Pi_I^M \geq \Pi_o^M$

$$\Pi_I^M = \frac{(a-c)(2\sqrt{(a-c)^2+m(2a-2c-7m)}-(a-c))+m(-2a+2c+5m)}{4b}$$
$$\Pi_o^M = \frac{(a-m-c)^2}{4b}$$
$$\Pi_I^M - \Pi_o^M = \frac{-(a-c)^2+2m^2+(a-c)\sqrt{(a-c)^2+m(2a-2c-7m)}}{2b}.$$

Note that, $(a-c)\sqrt{(a-c)^2 + m(2a-2c-7m)} \geq (a-c)^2$, since $2a \geq 7m+2c$. Therefore $\Pi_I^M - \Pi_o^M \geq 0$.

B.4 Analysis of fixed retail price

The analysis with perfect price discrimination has the demand function of the form $P(Q) = a - bQ$. We model the alternative case where the retailer sets a fixed price r instead of charging a different price for each unit that is equal to the customer's willingness to pay. For a fair comparison of these models, we derive the following correspondence between the demand functions: $r = P(Q) = a - bQ \Rightarrow Q = \frac{a-r}{b}$. Then, the retailer's decision becomes to find the optimal retail price instead. We make the same assumptions as in the base model such as the retailer's reservation price requirement, and analyze the no-promotion, retailer incentive, and customer rebate cases with deterministic demand.

<u>**No Promotion**</u>

The Retailer's Problem:

$$\begin{aligned} \max_{r} \quad & (r-w)\left(\frac{a-r}{b}\right) \\ \text{s.t.} \quad & r \geq w+m \end{aligned}$$

$r^* = \max\left\{w+m, \frac{a+w}{2}\right\}$

Case 1) $w \geq a-2m \Rightarrow r^* = w+m.$

The Manufacturer's Problem:

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w-m}{b}\right) \\ \text{s.t.} \quad & a-m \geq w \geq a-2m \\ & w \geq c \end{aligned}$$

$w^* = max\left\{a-2m, min\left\{\frac{a-m+c}{2}, a-m\right\}\right\}$

Case 1.a) $a \leq 3m+c$

$w^* = \frac{a-m+c}{2}$; $r^* = \frac{a+m+c}{2}$; $\Pi^M = \frac{(a-m-c)^2}{4b}$

Case 1.b) $a \geq 3m+c$

$w^* = a-2m$; $r^* = a-m$; $\Pi^M = \frac{m(a-2m-c)}{b}$

Case 2) $w \leq a-2m \Rightarrow r^* = \frac{a+w}{2}.$

The Manufacturer's Problem:

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{2b}\right) \\ \text{s.t.} \quad & c \leq w \leq a-2m \end{aligned}$$

$w^* = min\left\{\frac{a+c}{2}, a-2m\right\}$

Case 2.a) $a \geq 4m+c$

$w^* = \frac{a+c}{2}$; $r^* = \frac{3a+c}{4}$; $\Pi^M = \frac{(a-c)^2}{8b}$

Case 2.b) $a-2m \leq a \leq 4m+c$

$w^* = a-2m$; $r^* = a-m$; $\Pi^M = \frac{m(a-2m-c)}{b}$

The SPNE for the no-promotion case is summarized in Table 29. The SPNE for the no-promotion case in perfect price discrimination setting is found in Section 3.3.1 as follows:

$w_o = \frac{a+c-m}{2}$; $Q_o = \frac{a-m-c}{2b}$; $\Pi_o^M = \frac{1}{b}\left(\frac{a-m-c}{2}\right)^2$; $\Pi_o^D = \frac{(a-m-c)(a+3m-c)}{8b}$;
$\Pi_o^{SC} = \frac{(a-m-c)(3a+m-3c)}{8b}$.

We can show that both the retailer and the manufacturer are strictly better off in terms of profits and sales when the retailer has the ability to perfectly price discriminate versus when he sets a fixed retail price, except when $c+m \leq a \leq c+3m$, where the sales and the manufacturer's profit are identical in both models.

Table 29: The SPNE for the deterministic demand model with no promotion where the retailer sets a fixed retail price

	$c+m \leq a \leq c+3m$	$c+3m \leq a \leq c+4m$	$a \geq c+4m$
w_o	$\frac{a+c-m}{2}$	$a-2m$	$\frac{a+c}{2}$
r_o	$\frac{a+c+m}{2}$	$a-m$	$\frac{3a+c}{4}$
Q_o	$\frac{a-m-c}{2b}$	$\frac{m}{b}$	$\frac{a-c}{4b}$
Π_o^M	$\frac{(a-m-c)^2}{4b}$	$\frac{m(a-2m-c)}{b}$	$\frac{(a-c)^2}{8b}$
Π_o^D	$\frac{m(a-m-c)}{2b}$	$\frac{m^2}{b}$	$\frac{(a-c)^2}{16b}$
Π_o^{SC}	$\frac{(a-m-c)(a+m-c)}{4b}$	$\frac{m(a-m-c)}{b}$	$\frac{3(a-c)^2}{16b}$

Retailer Incentive

The Retailer's Problem:

$$\begin{aligned} \max_{r} \quad & (r-w)\left(\frac{a-r}{b}\right)+K \\ \text{s.t.} \quad & (w+m-r)\left(\frac{a-r}{b}\right) \leq K \end{aligned}$$

Case 1) $w \leq a-2m \Rightarrow r^* = \frac{a+w}{2}$

The Manufacturer's Problem:

$$\begin{aligned} \max_{w,K} \quad & (w-c)\left(\frac{a-w}{2b}\right)-K \\ \text{s.t.} \quad & c \leq w \leq a-2m \end{aligned}$$

$w^* = min\{\frac{a+c}{2}, a-2m\}$; $K^* = 0$

Case 1.a) $a \geq 4m+c$

$w^* = \frac{a+c}{2}$; $r^* = \frac{3a+c}{4}$; $K^* = 0$; $\Pi^M = \frac{(a-c)^2}{8b}$; $\Pi^D = \frac{(a-c)^2}{16b}$

Case 1.b) $a-2m \leq a \leq 4m+c$

$w^* = a-2m$; $r^* = a-m$; $K^* = 0$; $\Pi^M = \frac{m(a-2m-c)}{b}$; $\Pi^D = \frac{m^2}{b}$

Case 2) $w \geq a-2m \Rightarrow r^* = \left(min\left\{\frac{a+w}{2}, \frac{a+w+m-\sqrt{(a-w-m)^2+4Kb}}{2}\right\}\right)^+$

Case 2.a) $\frac{a+w}{2} \geq \frac{a+w+m-\sqrt{(a-w-m)^2+4Kb}}{2} \geq 0$

$$\begin{aligned} \max_{K} \quad & (w-c)\left(\frac{a-w-m+\sqrt{4Kb+(a-w-m)^2}}{2b}\right) - K \\ \text{s.t.} \quad & \frac{a(w+m)}{b} \geq K \geq \frac{m^2-(a-w-m)^2}{4b} \end{aligned}$$

$K^* = max\left\{\frac{m^2-(a-w-m)^2}{4b}, min\left\{\frac{(a-m-c)(2w+m-a-c)}{4b}, \frac{a(w+m)}{b}\right\}\right\}$
$= max\left\{\frac{m^2-(a-w-m)^2}{4b}, \frac{(a-m-c)(2w+m-a-c)}{4b}\right\}$

Case 2.a.1) $\frac{m^2-(a-w-m)^2}{4b} \geq \frac{(a-m-c)(2w+m-a-c)}{4b} \Rightarrow (w-c)^2 \leq m^2$

$K^* = \frac{m^2-(a-w-m)^2}{4b}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w}{2b}\right) - \frac{m^2-(a-w-m)^2}{4b} \\ \text{s.t.} \quad & a-2m \leq w \\ & (w-c)^2 \leq m^2 \\ & w \geq c \end{aligned}$$

$w^* = max\{a-2m, m+c\}$

Case 2.a.1.a $a \geq 3m+c$

$w^* = a-2m$; $K^* = 0$; $\Pi^M = \frac{a(a-2m-c)}{b}$

Case 2.a.1.b $2m+c \leq a \leq 3m+c$

$w^* = m+c$; $K^* = \frac{(3m-a+c)(a-m-c)}{4b}$; $\Pi^M = \frac{(a-m-c)^2}{4b}$

Case 2.a.2) $\frac{m^2-(a-w-m)^2}{4b} \leq \frac{(a-m-c)(2w+m-a-c)}{4b} \Rightarrow (w-c)^2 \geq m^2$

$K^* = \frac{(a-m-c)(2w+m-a-c)}{4b}$

$$\begin{aligned} \max_{w} \quad & (w-c)\left(\frac{a-w-m+\sqrt{4K^*b+(a-w-m)^2}}{2b}\right) - K^* \\ \text{s.t.} \quad & a-2m \leq w \\ & (w-c)^2 \geq m^2 \\ & w \geq c \end{aligned}$$

Objective function becomes independent of w when $K^* = \frac{(a-m-c)(2w+m-a-c)}{4b}$. Any (w, K) pair that satisfies the following relations is optimal with profit $\Pi^M = \frac{(a-m-c)^2}{4b}$:
$K^* = \frac{(a-m-c)(2w+m-a-c)}{4b}$; $w \geq a-2m$; $(w-c)^2 \geq m^2$; $w \geq \frac{a+c-m}{2}$.

Case 2.b) $0 \leq \frac{a+w}{2} \leq \frac{a+w+m-\sqrt{(a-w-m)^2+4Kb}}{2}$

$$\begin{aligned} \max_K \quad & (w-c)\left(\frac{a-w}{2b}\right) - K \\ \text{s.t.} \quad & K \leq \frac{m^2-(a-w-m)^2}{4b} \end{aligned}$$

Trivially, $K^* = 0$.

$$\begin{aligned} \max_w \quad & (w-c)\left(\frac{a-w}{2b}\right) \\ & a - 2m \leq w \leq a \\ & c \leq w \end{aligned}$$

$w^* = max\{\frac{a+c}{2}, a-2m\}$

Case 2.b.1) $a \leq 4m + c$

$w^* = \frac{a+c}{2}$; $r^* = \frac{3a+c}{4}$; $K^* = 0$; $\Pi^M = \frac{(a-c)^2}{8b}$

Case 2.b.2) $a \geq 4m + c$

$w^* = a - 2m$; $r^* = a - m$; $K^* = 0$; $\Pi^M = \frac{m(a-2m-c)}{b}$

Summary of the SPNE:

- When $a \leq c + 3m$, SPNE is the solution obtained in Case 2.b.1 ($w_I = \frac{a+c}{2}$; $K = 0$; $r_I = \frac{3a+c}{4}$; $Q_I = \frac{a-c}{4b}$; $\Pi_I^M = \frac{(a-c)^2}{8b}$; $\Pi_I^D = \frac{(a-c)^2}{16b}$).
- When $c + 3m \leq a \leq c + 4m$, solution obtained in Case 2.a.2 or Case 2.b.1 can be the unique equilibrium depending on the parameters, where $\Pi_I^M = max\left\{\frac{(a-c)^2}{8b}, \frac{(a-m-c)^2}{4b}\right\}$
- When $a \geq c+4m$, the solution obtained in Case 2.a.2 is the unique equilibrium, where $\Pi_I^M = \frac{(a-m-c)^2}{4b}$

For each region we can show that $\Pi_I^M \geq \Pi_o^M$ when the retailer uses fixed price. This is identical to our result in the case of perfect price discrimination, where offering a retailer incentive increases the manufacturer's profits and sales. However, when the retailer sets a fixed retail price, offering a retailer incentive does not always increase sales. (This result is trivial to prove in cases that are not listed below.)

- When $a \leq c + 2m$, $\Pi_I^M = \frac{(a-c)^2}{8b} \geq \frac{(a-m-c)^2}{4b} = \Pi_o^M$ follows since $a - c \leq 2m$ and $a - c \leq 2a - 2c - m$. $Q_o = \frac{a-m-c}{2b} \geq Q_I = \frac{a-c}{4b}$.

- When $c+2m \le a \le c+3m$, $\frac{(a-c)^2}{8b} - \frac{(a-m-c)^2}{4b}$ is decreasing in a, and takes its smallest value when $a = c+3m$. We show that $\frac{(a-c)^2}{8b} - \frac{(a-m-c)^2}{4b} \ge 0$ when $a = c+3m$.

- When $a \ge c+4m$, $\frac{(a-m-c)^2}{4b} - \frac{(a-c)^2}{8b}$ is increasing in a, and takes its smallest value when $a = c+4m$. We show that $\frac{(a-m-c)^2}{4b} - \frac{(a-c)^2}{8b} \ge 0$ when $a = c+4m$.

B.5 Analysis of the deterministic demand model with per-unit retailer incentive

Let k denote the per-unit payment given by the manufacturer to the retailer for each unit of sale. We start the backward induction steps by solving the retailer's problem:

$$\begin{aligned}\Pi_I^D = \quad & \max_{Q\ge 0} \quad \int_0^Q (a-bQ)dQ - (w-k)Q \\ & \text{s.t.} \quad \int_{\frac{a-w-m}{b}}^Q ((w+m)-(a-bQ))dQ \le kQ \\ & \qquad\quad Q \ge \tfrac{a-w-m}{b}\end{aligned}$$

$Q^* = \left(\min\left\{\frac{a-w+k}{b}, \frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b}\right\}\right)^+.$

In the next step, we solve the manufacturer's problem in a two step procedure similar to our analysis for the case of lump-sum incentive.

Case 1) $0 \le \frac{a-w+k}{b} \le \frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b} \Rightarrow m^2 \le k^2 + 2k(a-w-m)$

$$\begin{aligned}\max_{k} \quad & (w-c-k)\left(\frac{a-w+k}{b}\right) \\ \text{s.t.} \quad & k \ge (w-a)^+ \\ & k^2 + 2k(a-w-m) - m^2 \ge 0\end{aligned}$$

$k^* = \max\left\{w - \frac{a+c}{2}, -a+w+m+\sqrt{(a-w-m)^2+m^2}\right\}$

Case 1.a) $a \ge 2m+c$; $\frac{(a-2m-c)^2}{4} \ge (a-w-m)^2 + m^2 \Rightarrow k^* = w - \frac{a+c}{2}$

$$\begin{aligned}\max_{w} \quad & \frac{(a-c)^2}{4b} \\ \text{s.t.} \quad & c + k^* \le w \le a - m \\ & \frac{(a-2m-c)^2}{4} \ge (a-w-m)^2 + m^2 \\ & (k^*)^2 + 2k^*(a-w-m) - m^2 \ge 0\end{aligned}$$

Note that when $a \geq 4m + c$, any w that satisfies $a - m - \frac{1}{2}\sqrt{(a-c)(a-c-4m)} \leq w \leq a - m$ is a feasible solution with $\Pi_I^M = \frac{(a-c)^2}{4b}$.

Case 1.b) $a \geq 2m + c$; $\frac{(a-2m-c)^2}{4} \leq (a-w-m)^2 + m^2 \Rightarrow k^* = -a + w + m + \sqrt{(a-w-m)^2 + m^2}$

Case 1.c) $a \leq 2m + c \Rightarrow k^* = -a + w + m + \sqrt{(a-w-m)^2 + m^2}$

In Case 1.b and Case 1.c, $c + k^* \leq w \Leftrightarrow \sqrt{(a-w-m)^2 + m^2} \leq a - m - c$. Note also that $c \leq w \leq a - m \Rightarrow \sqrt{(a-m-c)^2 + m^2} \leq a - m - c$, which is a contradiction. So there are no feasible solutions obtained in these regions.

Case 2) $\frac{a-w+k}{b} \geq \frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b} \geq 0 \Rightarrow m^2 \geq k^2 + 2k(a-w-m)$

$$\begin{aligned} \max_{k} \quad & (w-c-k)\left(\frac{a-w-m+k+\sqrt{k^2+2k(a-w-m)}}{b}\right) \\ \text{s.t.} \quad & k \leq w - c \\ & k^2 + 2k(a-w-m) - m^2 \leq 0 \end{aligned}$$

$k^* = \min\left\{\frac{(w-c)^2}{2(a-m-c)}, -a + w + m + \sqrt{(a-w-m)^2 + m^2}\right\}$

Case 2.a) $\frac{(w-c)^2}{2(a-m-c)} \leq -a + w + m + \sqrt{(a-w-m)^2 + m^2}$

$$\begin{aligned} \max_{w} \quad & (w-c-k^*)\left(\frac{a-w-m+k^*+\sqrt{(k^*)^2+2k^*(a-w-m)}}{b}\right) \\ \text{s.t.} \quad & c + k^* \leq w \leq a - m \\ & (k^*)^2 + 2k^*(a-w-m) - m^2 \leq 0 \\ & \frac{(w-c)^2}{2(a-m-c)} \leq -a + w + m + \sqrt{(a-w-m)^2 + m^2} \end{aligned}$$

where $k^* = \frac{(w-c)^2}{2(a-m-c)}$.

The objective function can be simplified to: $(w-c-k^*)\left(\frac{(a-w-m)+k^*+\frac{(w-c)}{2(a-m-c)}|w-2a+2m+c|}{b}\right)$. Note that $w - 2a + 2m + c \geq 0$ conflicts with $w \leq a - m$. Therefore, $w - 2a + 2m + c \leq 0$.

$$\begin{aligned} \max_{w} \quad & \frac{(w-c)(-w+2a-2m-c)}{2b} \\ \text{s.t.} \quad & w \leq a - m \\ & \frac{(w-c)^4}{(2a-2c-2m)^2} + \frac{2(w-c)^2(a-w-m)}{(2a-2m-2c)} - m^2 \leq 0 \\ & \frac{(w-c)^2}{2(a-m-c)} \geq -a + w + m + \sqrt{(a-w-m)^2 + m^2} \end{aligned}$$

$$w^* = \begin{cases} a-m & \text{if } a \le 3m+c \\ a-m-\sqrt{(a-m-c)(a+m-c)} & \text{if } a \ge 3m+c \end{cases} \tag{13}$$

Case 2.a.1) $a \le 3m+c \Rightarrow w^* = a-m$; $k^* = \frac{a-m-c}{2}$; $Q^* = \frac{a-m-c}{b}$; $\Pi_I^M = \frac{(a-m-c)^2}{2b}$

Case 2.a.2) $a \ge 3m+c \Rightarrow w^* = a-m-\sqrt{(a-m-c)(a+m-c)}$; $k^* = a-c-\sqrt{(a-m-c)(a+m-c)}$; $\Pi_I^M = \frac{(a-m-c)^2-(a-m-c)(a+m-c)}{2b}$

Case 2.b) $\frac{(w-c)^2}{2(a-m-c)} \ge -a+w+m+\sqrt{(a-w-m)^2+m^2}$

$$\begin{aligned} \max_k \quad & (w-c-k^*)\left(\frac{a-w-m+k^*+\sqrt{(k^*)^2+2k^*(a-w-m)}}{b}\right) \\ \text{s.t.} \quad & c+k^* \le w \le a-m \\ & \frac{(w-c)^2}{2(a-m-c)} \le -a+w+m+\sqrt{(a-w-m)^2+m^2} \end{aligned}$$

where $k^* = -a+w+m+\sqrt{(a-w-m)^2+m^2}$.

Note that $c+k^* \le w \Leftrightarrow \sqrt{(a-w-m)^2+m^2} \le a-m-c$. Note also that $c \le w \le a-m \Rightarrow \sqrt{(a-m-c)^2+m^2} \le a-m-c$, which is a contradiction. So there is no feasible solution obtained in this region.

The SPNE solutions are summarized as follows:

- When $a \le 3m+c$, SPNE is the solution obtained in Case 2.a.1, where $w^* = a-m$; $k^* = \frac{a-m-c}{2}$; $Q^* = \frac{a-m-c}{b}$; $\Pi_I^M = \frac{(a-m-c)^2}{2b}$.
- When $a \ge 3m+c$, SPNE is the solution obtained in Case 1.a, where $a-m-\frac{1}{2}\sqrt{(a-c)(a-c-4m)} \le w^* \le a-m$; $\Pi_I^M = \frac{(a-c)^2}{4b}$.

We can show that in each region, $\Pi_I^M \ge \Pi_o^M$. Therefore, the manufacturer is always better off with a per-unit retailer incentive. (Same result holds for a lump-sum incentive.) Moreover, when $a \ge 3m+c$, the manufacturer's profit is higher with a per-unit incentive than a lump-sum incentive.

B.6 *Analysis of the retailer incentive case in the deterministic demand model with pass-through rate* $0 < \rho \leq 1$

The Retailer's Problem:

$$\begin{aligned} \Pi^R = \quad & \max_{Q \geq 0} \quad \int_0^Q (a - bQ)dQ - wQ \\ & \text{s.t.} \quad \int_{\frac{a-w-m}{b}}^{Q} ((w+m) - (a - bQ))dQ \leq \rho K \\ & \qquad\quad Q \geq \frac{a - w - m}{b} \end{aligned}$$

$Q^* = \left(min\left\{\frac{a-w}{b}, \frac{a-w-m+\sqrt{2\rho Kb}}{b}\right\}\right)^+$.

Case 1) $K \leq \frac{m^2}{2b\rho}$; $\sqrt{2\rho Kb} \geq w - (a - m)$; $\Rightarrow Q^* = \frac{a-w-m+\sqrt{2\rho Kb}}{b}$. First we solve for K:

$$\begin{aligned} \max_{K} \quad & (w - c)(\frac{a - w - m + \sqrt{2\rho Kb}}{b}) - K \\ \text{s.t.} \quad & 0 \leq K \leq \frac{m^2}{2b\rho} \\ & \sqrt{2\rho Kb} \geq w - (a - m) \end{aligned}$$

$K^* = min\{\frac{\rho(w-c)^2}{2b}, \frac{m^2}{2b\rho}\}$. Next, we solve for w. We need to consider the following cases: $w \leq c + \frac{m}{\rho}$ and $w \geq c + \frac{m}{\rho}$, where $K^* = \frac{\rho(w-c)^2}{2b}$ and $K^* = \frac{m^2}{2b\rho}$, respectively.

Case 1.a) $w \geq c + \frac{m}{\rho}$

$$\begin{aligned} \max_{w} \quad & (w - c)(\frac{a - w}{b}) - \frac{m^2}{2b\rho} \\ \text{s.t.} \quad & \frac{m}{\rho} + c \leq w \leq a \end{aligned}$$

$w^* = max\{\frac{m}{\rho} + c, min\{a, \frac{a+c}{2}\}\}$

Case 1.a.1) $\rho a \geq 2m + \rho c \Rightarrow w^* = \frac{a+c}{2}$; $Q^* = \frac{a-c}{2b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(a-c)^2}{4b} - \frac{m^2}{2b\rho}$.

Case 1.a.2) $\rho a \leq 2m + \rho c \Rightarrow w^* = \frac{m}{\rho} + c$; $Q^* = \frac{a-\frac{m}{\rho}-c}{b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(m+\rho c)(\rho a-m-\rho c)}{\rho^2 b} - \frac{m^2}{2b\rho}$.

Case 1.b) $w \leq c + \frac{m}{\rho}$

$$\begin{aligned} \max_{w} \quad & (w - c)(\frac{a - (1-\rho)w - m - \rho c}{b}) - \frac{\rho(w - c)^2}{2b} \\ \text{s.t.} \quad & c \leq w \leq c + \frac{m}{\rho} \end{aligned}$$

$w^* = max\{c, min\{c + \frac{m}{\rho}, \frac{a-m-(\rho-1)c}{2-\rho}\}\}$

Case 1.b.1) $\rho a \geq 2m + \rho c \Rightarrow w^* = \frac{m}{\rho} + c$; $Q^* = \frac{a-\frac{m}{\rho}-c}{b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(m+\rho c)(\rho a-m-\rho c)}{\rho^2 b} - \frac{m^2}{2b\rho}$.

Table 30: The SPNE for the deterministic demand model with retailer incentive when the incentive pass-through rate is $0 < \rho \leq 1$

	$\rho a \leq 2m + \rho c$	$\rho a \geq 2m + \rho c$
w_I	$\frac{a-m-(\rho-1)c}{2-\rho}$	$\frac{a+c}{2}$
Q_I	$\frac{a-m-c}{(2-\rho)b}$	$\frac{a-c}{2b}$
K	$\frac{\rho(a-m-c)^2}{(\rho-2)^2 2b}$	$\frac{m^2}{2b\rho}$
Π_I^M	$\frac{(a-m-c)^2}{2b(2-\rho)}$	$\frac{(a-c)^2}{4b} - \frac{m^2}{2b\rho}$
Π_I^D	$\frac{(a-m-c)((1-\rho)(a-c)-m(\rho-3))}{b}$	$\frac{\rho(a-c)^2+4m^2}{8\rho b}$

Case 1.b.2) $\rho a \leq 2m + \rho c \Rightarrow w^* = \frac{a-m-(\rho-1)c}{2-\rho}$; $Q^* = \frac{a-m-c}{(2-\rho)b}$; $K^* = \frac{\rho(a-m-c)^2}{(\rho-2)^2 2b}$; $\Pi^M = \frac{(a-m-c)^2}{2b(2-\rho)}$.

Case 2) $K \leq \frac{m^2}{2b\rho}$; $w \leq a \Rightarrow Q^* = \frac{a-w}{b}$. We obtain the following solution:

$w^* = \frac{a+c}{2}$; $Q^* = \frac{a-c}{2b}$; $K^* = \frac{m^2}{2b\rho}$; $\Pi^M = \frac{(a-c)^2}{4b} - \frac{m^2}{2b\rho}$.

We summarize the SPNE in Table 30.

Note that, $\Pi_R^M = \frac{(a-m-c)^2}{4b}$. We can show that the manufacturer is better off with the retailer incentive than the customer rebate where the equilibrium for the latter is stated in Theorem 3(iii).

$\Pi_I^M - \Pi_R^M = \frac{m(-2\rho c+2a\rho-m\rho-2m)}{4b\rho} \geq 0$, when $\rho a \geq 2m + \rho c$, since $\rho a \geq 2m + \rho c \Rightarrow 2a\rho \geq 4m + 2\rho c \geq (2+\rho)m + 2\rho c$.

$\Pi_I^M - \Pi_R^M \geq 0$, when $\rho a \leq 2m + \rho c$, since $0 < \rho \leq 1$.

B.7 Analysis of the no-promotion case, where demand can be in "high", "medium", and "low" states with probabilities β_h, β_m, and $1 - \beta_h - \beta_m$, respectively.

No Promotion

Retailer's best response is as follows: $Q^{j*}(w) = \left(\frac{a^j-w-m}{b}\right)^+$; $j = l, m, h$.

Case 1) $a^m - m \leq w \leq a^h - m \Rightarrow Q^h = \frac{a^h-w-m}{b}$; $Q^m = 0$; $Q^l = 0$

Case 2) $a^l - m \leq w \leq a^m - m \Rightarrow Q^h = \frac{a^h-w-m}{b}$; $Q^m = \frac{a^m-w-m}{b}$; $Q^l = 0$

Case 3) $w \leq a^l - m \Rightarrow Q^h = \frac{a^h-w-m}{b}$; $Q^m = \frac{a^m-w-m}{b}$; $Q^l = \frac{a^l-w-m}{b}$

Case 4) $w \geq a^h - m \Rightarrow Q^h = 0$; $Q^m = 0$; $Q^l = 0$

Note that we can omit Case 4 since the manufacturer has the feasible solution of setting

$w = c$ and receive zero profit. We analyze the other cases and find the manufacturer's optimal wholesale price in each case.

Case 1) $a^m - m \leq w \leq a^h - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^m = 0$; $Q^l = 0$

$$\begin{aligned} \max \quad & \beta_h (w - c) Q^h \\ \text{s.t} \quad & a^m - m \leq w \leq a^h - m \end{aligned}$$

$w^* = max\left\{a^m - m, min\left\{a^h - m, \frac{a^h - m + c}{2}\right\}\right\}$

Case 1.a) (FS.1) $2a^m \leq a^h + m + c \Rightarrow w^* = \frac{a^h - m + c}{2}$; $Q^h = \frac{a^h - m - c}{2b}$; $Q^m = Q^l = 0$; $\Pi^M = \frac{\beta_h (a^h - m - c)^2}{4b}$

Case 1.b) (FS.2) $2a^m \geq a^h + m + c \Rightarrow w^* = a^m - m$; $Q^h = \frac{a^h - a^m}{b}$; $Q^m = Q^l = 0$; $\Pi^M = \frac{\beta_h (a^m - m - c)(a^h - a^m)}{b}$

Case 2) $a^l - m \leq w \leq a^m - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^m = \frac{a^m - w - m}{b}$; $Q^l = 0$

$$\begin{aligned} \max \quad & \beta_h (w - c) Q^h + \beta_m (w - c) Q^m \\ \text{s.t} \quad & a^l - m \leq w \leq a^m - m \end{aligned}$$

$w^* = max\left\{a^l - m, min\left\{a^m - m, \frac{\beta_h (a^h - m + c) + \beta_m (a^m - m + c)}{2(\beta_h + \beta_m)}\right\}\right\}$

Case 2.a) (FS.3) $\frac{\beta_h}{\beta_h + \beta_m} \geq \frac{a^m - m - c}{a^h - a^m} \Rightarrow w^* = a^m - m$; $Q^h = \frac{a^h - a^m}{b}$; $Q^m = Q^l = 0$; $\Pi^M = \frac{\beta (a^m - m - c)(a^h - a^m)}{b}$

Case 2.b) $\frac{\beta_h}{\beta_h + \beta_m} \geq \frac{a^m - m - c}{a^h - a^m}$

Case 2.b.1) (FS.4) $2(\beta_h + \beta_m) a^l \geq \beta_h a^h + \beta_m a^m + (\beta_h + \beta_m)(m + c) \Rightarrow w^* = a^l - m$; $Q^h = \frac{a^h - a^l}{b}$; $Q^m = \frac{a^m - a^l}{b}$; $Q^l = 0$; $\Pi^M = \frac{(\beta_h (a^h - a^l) + \beta_m (a^m - a^l))(a^l - m - c)}{b}$

Case 2.b.2) (FS.5) $2(\beta_h + \beta_m) a^l \leq \beta_h a^h + \beta_m a^m + (\beta_h + \beta_m)(m + c) \Rightarrow$
$w^* = \frac{\beta_h (a^h - m + c) + \beta_m (a^m - m + c)}{2(\beta_h + \beta_m)}$; $Q^h = \frac{(\beta_h + 2\beta_m) a^h - \beta_m a^m - (\beta_h + \beta_m)(c + m)}{2(\beta_h + \beta_m) b}$; $Q^l = 0$;
$Q^m = \frac{(2\beta_h + \beta_m) a^m - \beta_h a^h - (\beta_h + \beta_m)(c + m)}{2(\beta_h + \beta_m) b}$; $\Pi^M = \frac{(\beta_h a^h + \beta_m a^m - (\beta_h + \beta_m)(m + c))^2}{4(\beta_h + \beta_m) b}$

Case 3) $w \leq a^l - m \Rightarrow Q^h = \frac{a^h - w - m}{b}$; $Q^m = \frac{a^m - w - m}{b}$; $Q^l = \frac{a^l - w - m}{b}$

$$\begin{aligned} \max \quad & \beta_h (w - c) Q^h + \beta_m (w - c) Q^m + (1 - \beta_h - \beta_m) Q^l \\ \text{s.t} \quad & w \leq a^l - m \end{aligned}$$

$w^* = min\left\{a^l - m, \frac{\beta_h a^h + \beta_m a^m + (1 - \beta_h - \beta_m) a^l - m + c)}{2}\right\}$

Table 31: The SPNE with no promotion when demand can be in "high", "medium", and "low" states with probabilities β_h, β_m, and $1-\beta_h-\beta_m$, respectively.

<table>
<tr><th colspan="4">Feasible Region (F.R.)</th><th>Solution</th></tr>
<tr><td rowspan="3">$2a^m \geq a^h+m+c$</td><td rowspan="3"></td><td>$2(\beta_h+\beta_m)a^l \geq \beta_h a^h + \beta_m a^m + (\beta_h+\beta_m)(m+c)$</td><td></td><td>FS.6</td></tr>
<tr><td rowspan="2">$2(\beta_h+\beta_m)a^l \leq \beta_h a^h + \beta_m a^m + (\beta_h+\beta_m)(m+c)$</td><td>$(1+\beta_h+\beta_m)a^l \geq \beta_h a^h + \beta_m a^m + m + c$</td><td>FS.5, FS.6</td></tr>
<tr><td>$(1+\beta_h+\beta_m)a^l \leq \beta_h a^h + \beta_m a^m + m + c$</td><td>FS.5</td></tr>
<tr><td rowspan="5">$2a^m \leq a^h+m+c$</td><td rowspan="3">$\frac{\beta_h}{\beta_h+\beta_m} \leq \frac{a^m-m-c}{a^h-a^m}$</td><td>$2(\beta_h+\beta_m)a^l \geq \beta_h a^h + \beta_m a^m + (\beta_h+\beta_m)(m+c)$</td><td></td><td>FS.1, FS.6</td></tr>
<tr><td rowspan="2">$2(\beta_h+\beta_m)a^l \leq \beta_h a^h + \beta_m a^m + (\beta_h+\beta_m)(m+c)$</td><td>$(1+\beta_h+\beta_m)a^l \geq \beta_h a^h + \beta_m a^m + m + c$</td><td>FS.1, FS.5, FS.6</td></tr>
<tr><td>$(1+\beta_h+\beta_m)a^l \leq \beta_h a^h + \beta_m a^m + m + c$</td><td>FS.1, FS.5</td></tr>
<tr><td rowspan="2">$\frac{\beta_h}{\beta_h+\beta_m} \geq \frac{a^m-m-c}{a^h-a^m}$</td><td rowspan="2"></td><td>$(1+\beta_h+\beta_m)a^l \geq \beta_h a^h + \beta_m a^m + m + c$</td><td>FS.1, FS.6</td></tr>
<tr><td>$(1+\beta_h+\beta_m)a^l \leq \beta_h a^h + \beta_m a^m + m + c$</td><td>FS.1, FS.7</td></tr>
</table>

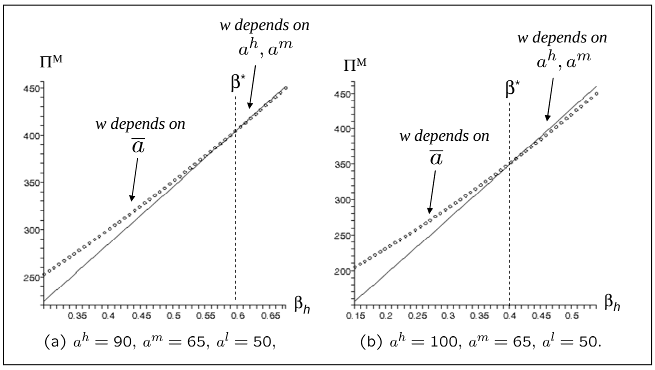
Case 3.a) (FS.6) $(1+\beta_h+\beta_m) \geq \beta_h a^h+\beta_m a^m+m+c \Rightarrow w^* = \frac{\beta_h a^h+\beta_m a^m+(1-\beta_h-\beta_m)a^l-m+c}{2}$; $Q^h = \frac{(2-\beta_h)a^h-\beta_m a^m-(1-\beta_h-\beta_m)a^l-m-c}{2b}$; $Q^m = \frac{-\beta_h a^h+(2-\beta_m)a^m+(1-\beta_h-\beta_m)a^l-m-c}{2b}$; $Q^l = \frac{-\beta_h a^h-\beta_m a^m+(1+\beta_h+\beta_m)a^l-m-c}{2b}$; $\Pi^M = \frac{(\beta_h a^h+\beta_m a^m+(1-\beta_h-\beta_m)a^l-m-c)^2}{4b}$

Case 3.b) (FS.7) $(1+\beta_h+\beta_m) \leq \beta_h a^h+\beta_m a^m+m+c \Rightarrow w^* = a^l-m$; $Q^h = \frac{a^h-a^l}{b}$; $Q^m = \frac{a^m-a^l}{b}$; $Q^l = 0$; $\Pi^M = \frac{(\beta_h(a^h-a^l)+\beta_m(a^m-a^l))(a^l-m-c)}{b}$

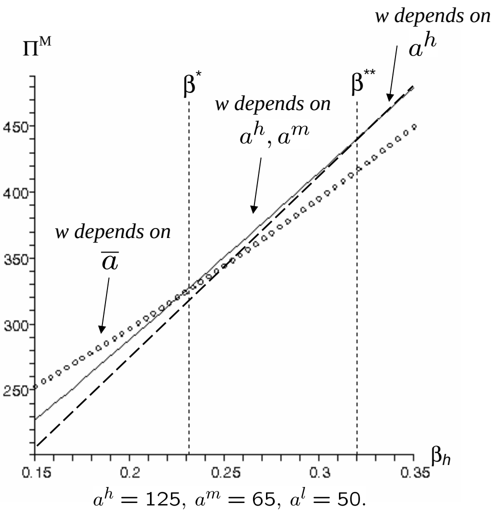
We summarize the feasible solutions in Table 31. We give some examples to understand the equilibrium behavior.

Example 5 *We focus on the region where* $2a^m \geq a^h+m+c$, $2(\beta_h+\beta_m)a^l \leq \beta_h a^h + \beta_m a^m + (\beta_h+\beta_m)(m+c)$, *and* $(1+\beta_h+\beta_m)a^l \geq \beta_h a^h+\beta_m a^m+m+c$. *In this region two feasible solutions are FS.5 (driven only by the high and medium states) and FS.6 (driven by all states).*

In Figure 26(a), we fix β_m *at a feasible value in the specified region, and plot* Π^M *with respect to* β_h *within the limits that are enforced by the feasible region. We see that when* $\beta_h \leq \beta^*$, *FS.6 is the equilibrium solution, whereas when* $\beta_h \geq \beta^*$ *FS.5 is the equilibrium. This observation is similar to Observation 2, where for* β *values that are greater than a threshold value, the manufacturer is better off by the solution which is driven only by the* *high demand state. In other words, the wholesale price does not depend on the low market potential.*



(a) With increasing a^h



(b) No-promotion equilibrium wholesale price

Figure 26: No-promotion equilibrium wholesale price ($m = 5, b = 2, c = 15$)

Example 6 *We focus on the region where* $2a^m \leq a^h + m + c$, $\frac{\beta_h}{\beta_h+\beta_m} \leq \frac{a^m-m-c}{a^h-a^m}$, $2(\beta_h + \beta_m)a^l \leq \beta_h a^h + \beta_m a^m + (\beta_h + \beta_m)(m + c)$, *and* $(1 + \beta_h + \beta_m)a^l \geq \beta_h a^h + \beta_m a^m + m + c$. *In this region three feasible solutions are FS.1 (driven only by the high state), FS.5 (driven only by the high and medium states), and FS.6 (driven by all states).*

In Figure 26(b), we see that as β_h *switches from the range* $\beta_h \leq \beta^*$ *to the range* $\beta^* \leq$

$\beta_h \leq \beta^{**}$, and to the range $\beta_h \geq \beta^*$, the equilibrium switches from FS.6 to FS.5, and to FS.1, respectively. The equilibrium behavior is similar to Observation 2, where as β gets sufficiently large, the lower value if the market potential drops from the manufacturer's wholesale price decision.

APPENDIX C

APPENDIX FOR CHAPTER 4

Demand Model with Competition: Consider the following demand models for two competing firms:

$$Q_1 = \alpha - \beta_1 p_1 + \beta_2 p_2 \tag{14}$$

$$Q_2 = \alpha - \gamma_2 p_2 + \gamma_1 p_1 \tag{15}$$

Note that Q_1 and Q_2 denote sales quantities of firms 1 and 2, p_1 and p_2 are the prices set by firms 1 and 2, β_1 and β_2 are own and cross price sensitivities of firm 1, γ_2 and γ_1 are own and cross price sensitivities of firm 2, and α is the market size of each firm. Demand models in Equations 14 and 15 are commonly used in operations management literature (Kim and Staelin [48] and McGuire and Staelin [57]), where the sales quantities are expressed as functions of the prices, while our demand model in Equation 1 has the inverse representation relating the prices to each sales quantity. Although we cannot completely derive our demand model from Equations 14 and 15, we can provide some intuitive explanation on the correspondence between the two models.

We can assume that $\beta_1 \geq \beta_2 \geq 0$ and $\gamma_2 \geq \gamma_1 \geq 0$, which is similar to our assumptions $b_{1o} \geq b_{1c}$ and $b_{2o} \geq b_{2c}$ (Section 4.3). We further assume that $\beta_1\gamma_2 - \beta_2\gamma_1 > 0$. We consider the first segment of our demand model, i.e., $P_1(Q_1, Q_2) = a - (b_{1o} + b_{1c})Q_1$ corresponding to the case where $Q_1 \leq Q_2$.

When $Q_1 \leq Q_2$ we use Equations 14 and 15 and derive the following expressions:

$$Q_1 \leq Q_2 \Rightarrow \exists\, \epsilon \geq 0 \; s.t: Q_1 + \epsilon = Q_2$$

$$Q_1 + \epsilon = Q_2 \Rightarrow -\beta_1 p_1 + \beta_2 p_2 + \epsilon = -\gamma_2 p_2 + \gamma_1 p_1 \Rightarrow (\beta_2 + \gamma_2)p_2 + \epsilon = (\beta_1 + \gamma_1)p_1 \tag{16}$$

$$Q_1 = \alpha - \beta_1 p_1 + \beta_2 p_2 \Rightarrow p_2 = \frac{Q_1 - \alpha + \beta_1 p_1}{\beta_2} \tag{17}$$

Substituting Equation 17 into 16, we obtain the following:

$$\frac{(\gamma_2\beta_1 - \beta_2\gamma_1)}{\beta_2} p_1 = -\frac{(\beta_2 + \gamma_2)}{\beta_2} Q_1 + \frac{(\beta_2 + \gamma_2)}{\beta_2}\alpha - \epsilon$$

$$\Rightarrow p_1 = \left(\frac{\beta_2 + \gamma_2}{(\gamma_2\beta_1 - \beta_2\gamma_1)}\alpha - \epsilon'\right) - \left(\frac{\beta_2}{(\gamma_2\beta_1 - \beta_2\gamma_1)} + \frac{\gamma_2}{(\gamma_2\beta_1 - \beta_2\gamma_1)}\right) Q_1 \tag{18}$$

In Equation 18, dependence of p_1 on Q_1 is similar to that in the expression $P_1(Q_1, Q_2) = a - (b_{1o} + b_{1c})Q_1$, where price is decreasing in sales quantity since $\frac{\beta_2}{(\gamma_2\beta_1 - \beta_2\gamma_1)} \geq 0$ and $\frac{\gamma_2}{(\gamma_2\beta_1 - \beta_2\gamma_1)} \geq 0$. Given $P_i(Q_i, Q_j) = a - (b_{io} + b_{ic})Q_i$ when $Q_i \leq Q_j$, we can derive the expression in the second part of the demand function ($Q_i \geq Q_j$) in Equation 1 as follows:

$$a - (b_{io} + b_{ic})Q_j - b_{io}(Q_i - Q_j) = a - b_{io}Q_i - (b_{io} + b_{ic} - b_{io})Q_j = a - b_{io}Q_i - b_{ic}Q_j.$$

Proof of Proposition 2

Proof. We show that the decisions of the manufacturers ($w_1 - R_1 = c_1$ and $w_2 = \frac{a-m_1-c_1}{\delta_2} + \frac{a-\delta_1 R_1 - m_2 + c_2}{2}$; $R_2 = \frac{a-m_1-c_1}{\delta_2}$) are best responses to each other. The best response of manufacturer 1 when $w_2 = \frac{a-m_1-c_1}{\delta_2} + \frac{a-\delta_1 R_1 - m_2 + c_2}{2}$ and $R_2 = \frac{a-m_1-c_1}{\delta_2}$ is $w_1 - R_1 = c_1$, which follows from the following inequalities:

$$a + R_1 - \delta_2 R_2 \geq P_1(Q_1, Q_2) \geq w_1 + m_1 \Rightarrow a - m_1 - \delta_2 R_2 \geq w_1 - R_1 \Rightarrow c_1 \geq w_1 - R_1.$$

We also know that $w_1 - R_1 \geq c_1$ in equilibrium, therefore $w_1 - R_1 = c_1$.

To find the best response of manufacturer 2 when $w_1 - R_1 = c_1$, we consider the cases with $Q_1 \leq Q_2$ and $Q_2 \leq Q_1$. In each case, we formulate the problem of manufacturer 2, and we find an upper bound on his profit. We show that the decisions $w_2 = \frac{a-m_1-c_1}{\delta_2} + \frac{a-\delta_1 R_1 - m_2 + c_2}{2}$ and $R_2 = \frac{a-m_1-c_1}{\delta_2}$ are feasible and result in a profit equal to the upper bound.

Case 1) $0 \leq \frac{a-\delta_2 R_2 - c_1 - m_1}{b_{1o}+b_{1c}} \leq \frac{a+R_2-\delta_1 R_1 - w_2 - m_2}{b_{2o}+b_{2c}}$

$Q_1 \leq Q_2$; $Q_1 = \frac{a-\delta_2 R_2 - c_1 - m_1}{b_{1o}+b_{1c}}$; $Q_2 = \frac{a+R_2-\delta_1 R_1 - w_2 - m_2}{b_{2o}} - \frac{b_{2c}}{b_{2o}}\left(\frac{a-\delta_2 R_2 - c_1 - m_1}{b_{1o}+b_{1c}}\right)$

Manufacturer 2's problem:

$$\begin{aligned}
\max_{w_2, R_2} \quad & (w_2 - c_2 - R_2)\left(\frac{a + R_2 - \delta_1 R_1 - w_2 - m_2}{b_{2o}} - \frac{b_{2c}}{b_{2o}}\left(\frac{a - \delta_2 R_2 - m_1 - c_1}{b_{1o} + b_{1c}}\right)\right) \\
\text{s.t.} \quad & c_2 \leq w_2 - R_2 \leq a - \delta_1 R_1 - m_2 \\
& w_2 - R_2\left(1 + \delta_2 \tfrac{b_{2o}+b_{2c}}{b_{1o}+b_{1c}}\right) \leq \left(\tfrac{b_{2o}+b_{2c}}{b_{1o}+b_{1c}}\right)(c_1 - a + m_1) + (a - \delta_1 R_1 - m_2) \\
& R_2 \leq \tfrac{a-m_1-c_1}{\delta_2}
\end{aligned}$$

Note that an upper bound on the objective function value is $(w_2 - c_2 - R_2)\left(\frac{a+R_2-\delta_1 R_1 - w_2 - m_2}{b_{2o}}\right)$. Consider the following unconstrained problem: $\max_{w_2, R_2}(w_2 - c_2 - R_2)\left(\frac{a+R_2-\delta_1 R_1 - w_2 - m_2}{b_{2o}}\right)$. We can show that the optimal solution of this problem is $w_2 - R_2 = \frac{a-\delta_1 R_1 - m_2 + c_2}{2}$, and the objective function value is $\frac{(a-\delta_1 R_1 - m_2 - c_2)^2}{4b_{2o}}$. Note also that the objective function value of manufacturer 2's problem evaluated at $w_2 = \frac{a-m_1-c_1}{\delta_2} + \frac{a-\delta_1 R_1 - m_2 + c_2}{2}$ and $R_2 = \frac{a-m_1-c_1}{\delta_2}$

is equal to $\frac{(a-\delta_1 R_1-m_2-c_2)^2}{4b_{2o}}$. Next, we show that these w_2 and R_2 decisions satisfy the constraints in the manufacturer 2's problem. All constraints evaluated at the preceding values of w_2 and R_2 reduce to the inequality $R_1 \leq \frac{a-m_2-c_2}{\delta_1}$, which holds by the feasibility requirements in the formulation of the manufacturer 2's problem.

Case 2) $\frac{a-\delta_2 R_2-c_1-m_1}{b_{1o}+b_{1c}} \geq \frac{a+R_2-\delta_1 R_1-w_2-m_2}{b_{2o}+b_{2c}} \geq 0$

$Q_1 \geq Q_2$; $Q_1 = \frac{a-\delta_2 R_2-c_1-m_1}{b_{1o}} - \frac{b_{1c}}{b_{1o}}\left(\frac{a+R_2-\delta_1 R_1-w_2-m_2}{b_{2o}+b_{2c}}\right)$; $Q_2 = \frac{a+R_2-\delta_1 R_1-w_2-m_2}{b_{2o}+b_{2c}}$

Manufacturer 2's problem:

$$\begin{aligned}
\max_{w_2,R_2} \quad & (w_2-c_2-R_2)\left(\frac{a+R_2-\delta_1 R_1-w_2-m_2}{b_{2o}+b_{2c}}\right) \\
\text{s.t.} \quad & c_2 \leq w_2-R_2 \leq a-\delta_1 R_1-m_2 \\
& w_2-R_2\left(1+\delta_2\frac{b_{2o}+b_{2c}}{b_{1o}+b_{1c}}\right) \geq \left(\frac{b_{2o}+b_{2c}}{b_{1o}+b_{1c}}\right)(c_1-a+m_1)+(a-\delta_1 R_1-m_2) \\
& R_2 \leq \frac{a-m_1-c_1}{\delta_2}
\end{aligned}$$

Consider the following unconstrained problem: $\max_{w_2,R_2}(w_2-c_2-R_2)\left(\frac{a+R_2-\delta_1 R_1-w_2-m_2}{b_{2o}+b_{2c}}\right)$. We can show that the optimal solution of this problem is $w_2-R_2 = \frac{a-\delta_1 R_1-m_2+c_2}{2}$, and the objective function value is $\frac{(a-\delta_1 R_1-m_2-c_2)^2}{4(b_{2o}+b_{2c})}$. Note that the feasible decisions $w_2 = \frac{a-m_1-c_1}{\delta_2} + \frac{a-\delta_1 R_1-m_2+c_2}{2}$ and $R_2 = \frac{a-m_1-c_1}{\delta_2}$ with an objective function value of $\frac{(a-\delta_1 R_1-m_2-c_2)^2}{4b_{2o}}$ dominate any decision in Case 2, and consequently we can eliminate this case. ■

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