ALL-TOLERANCE MULTIPARAMETER SENSITIVITY AND MULTIVARIABLE CONTINUOUSLY EQUIVALENT NETWORKS

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SUMMARY

There are two main purposes for conducting the sensitivity study. One is for the comparative evaluation of different circuits realizing the same network function. The other is for designing better circuits from the sensitivity viewpoint. To accomplish these, the most important step is to find a criterion that gives correct sensitivity information of the network performance. The objective of this research is to develop a new deterministic and statistical multiparameter sensitivity measure, and to develop a new formulation and algorithm of continuously equivalent networks which are used for sensitivity optimization of a given network function.

The contribution of this research includes:

1. Pointing out the shortcomings of the existing sensitivity measures.
2. Proposing a new deterministic and statistical sensitivity measure.
3. Developing a new multivariable continuously equivalent network formulation.
4. Applying the new sensitivity measure to evaluate different circuits realizing the same network function and comparing the results with those obtained from traditional sensitivity measures.
5. Using the newly proposed multiparameter sensitivity measure and multivariable continuously equivalent network theory to

obtain an optimal network for a prescribed network function. The optimal network obtained by this method shows that the network performance has a far less variation from the nominal point for finite element tolerances than those obtained by the traditional methods.

Several examples have been worked out based on the new measure and the new formalism. These examples include both active and passive networks. In the example of the optimization of a passive network, the new measure for the best network designed on the basis of the first-order sensitivity is 2.473. This measure has been reduced to 0.044 when the new technique is applied. This represents an improvement by a factor of approximately 56.

CHAPTER I

INTRODUCTION

Circuit sensitivity problem was first studied by Bode [1]. His classical definition of network sensitivity is closely related to the usual definition of the differential sensitivity

$$S_x^F = \frac{x}{F} \cdot \frac{dF}{dx} = \frac{d(\ell n\ F)}{d(\ell n\ x)}$$

where F is the network function and x is the value of an element.

Multiparameter Sensitivity Measures

Since Bode, a number of different definitions of multiparameter sensitivity of a circuit have been proposed. Those definitions can be classified into two categories:

1. Small-Change (or First-Order) Multiparameter Sensitivity

So far, five different definitions of multiparameter sensitivities have been proposed. Mikulski [2] proposed the multiparameter sensitivity measure to be

$$M = \sum_i \frac{\partial(\ell nF)}{\partial(\ell nx_i)}\, d(\ell nx_i) = \sum_i S_{x_i}^F\, d(\ell nx_i) \tag{1.1}$$

where x_i is the value of the ith element of the network.

Kuo and Goldstein [3] defined the multiparameter sensitivity as

$$M = \left|\sum_i S_{x_i}^F\right| \tag{1.2}$$

Shoeffler [4] used another multiparameter sensitivity measure

$$M = \sum_i |S^F_{x_i}|^2 \tag{1.3}$$

in optimizing his continuously equivalent networks.

Assuming the variations of the elements of the network to be random variables with zero mean and known statistics, Rosenblum and Ghausi [5] defined the multiparameter sensitivity measure in a frequency range as

$$M = E\left[\int_{\omega_1}^{\omega_2} \left|\frac{[\nabla_x F]^t}{F}\,\Delta x\right|^2 d\omega\right] \tag{1.4}$$

where E denotes the expected value, $\nabla_x F = [\partial F/\partial x_1 \ldots \partial F/\partial x_k]^t$ (k is the number of elements in the network), ω_1 to ω_2 is the network operating frequency range, and Δx is the element variation vector. The measure defined in (1.4) is not very useful because it is difficult to carry out the calculation, therefore it was modified to read

$$M = \int_{\omega_1}^{\omega_2} \left[\frac{\nabla_a F}{F}\right]^{*t} C_1{}^t P C_1 \left[\frac{\nabla_a F}{F}\right] d\omega + \int_{\omega_1}^{\omega_2} 2\ \mathrm{Re}\left(\left[\frac{\nabla_a F}{F}\right]^{*t} C_1{}^t P C_2 \left[\frac{\nabla_a F}{F}\right]\right) d\omega$$

$$+ \int_{\omega_1}^{\omega_2} \left[\frac{\nabla_b F}{F}\right]^{*t} C_2^t P C_2 \left[\frac{\nabla_b F}{F}\right] d\omega$$

where

$$P = E[\Delta x \cdot \Delta x^t]$$

$$\nabla_a F = [\partial F/\partial a_{n-1}, \ldots, \partial F/\partial a_0]^t$$

$$\nabla_b F = [\partial F/\partial b_{n-1}, \ldots, \partial F/\partial b_0]^t$$

$$F = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}{s^m + a_{m-1} s^{m-1} + \ldots + a_1 s + a_0}$$

$$C_1 = \begin{bmatrix} x_1 \frac{\partial a_{m-1}}{\partial x_1} & \cdots & x_1 \frac{\partial a_0}{\partial x_1} \\ \vdots & & \\ x_k \frac{\partial a_{m-1}}{\partial x_k} & \cdots & x_k \frac{\partial a_0}{\partial x_k} \end{bmatrix} \quad C_2 = \begin{bmatrix} x_1 \frac{\partial b_n}{\partial x_1} & \cdots & x_1 \frac{\partial b_0}{\partial x_1} \\ \vdots & & \\ x_k \frac{\partial b_n}{\partial x_k} & \cdots & x_k \frac{\partial b_0}{\partial x_k} \end{bmatrix}$$

Shenoi [6] used Hilberman's [7] biquadratic approach to sensitivity in forming a multiparameter sensitivity measure as

$$M = \min_{\substack{x_i \\ i=1,2,\ldots,N_f}} \left[\max_{\omega_1 < \omega < \omega_2} \left(\left| \mu_{\Delta\gamma}(j\omega) + g\sigma_{\Delta\gamma}(j\omega) \right| \right) \right] \qquad (1.5)$$

where N_f is the number of degrees of freedom in choosing the element values of a network, g is chosen according to the percentage yield specified, and

$$\mu_{\Delta\gamma} = (\mu_R + \mu_C) \sum_i \left[- S_{n_{1i}}^{\gamma_{Ni}} - 2\, S_{n_{0i}}^{\gamma_{Ni}} + 2\, S_{d_{1i}}^{\gamma_{Di}} + 2\, S_{d_{0i}}^{\gamma_{Di}} \right] + \mu_{A_o} \Sigma A_o$$

$$\sigma_{\Delta\gamma}^{2} = \sum_{i,j,k}\left[S_{n_k}^{Ni} S_{x_j}^{n_k} - S_{d_k}^{Di} S_{x_j}^{d_k}\right]\sigma_j^{2}$$

$$\gamma_{Ni}(\omega) = \ell n\left|- n_{2i}\omega^2 + jn_{1i}\omega + n_{0i}\right|$$

$$\gamma_{Di}(\omega) = \ell n\left|- d_{2i}\omega^2 + jd_{1i}\omega + d_{0i}\right|$$

$$\gamma(\omega) \equiv \ell n\left|F(j\omega)\right| = \sum_i\left(\gamma_{Ni}(\omega) - \gamma_{Di}(\omega)\right)$$

$$F(s) = \prod_{i=1}^{n} \frac{Ni(s)}{Di(s)} = \prod_{i=1}^{n} \frac{n_{2i}s^2 + n_{1i}s + n_{0i}}{d_{2i}s^2 + d_{1i}s + d_{0i}}$$

$$S_{n_k}^{\gamma_{Ni}} = n_k \frac{\partial\gamma_{Ni}}{\partial n_k}$$

$$S_{x_j}^{n_k} = \frac{x_j}{n_k}\frac{\partial n_k}{\partial x_j}$$

$$S_{d_k}^{\gamma_{Di}} = d_k \frac{\partial\gamma_{Di}}{\partial d_k}$$

$$S_{x_j}^{d_k} = \frac{x_j}{d_k}\frac{\partial d_k}{\partial x_j}$$

σ_R: the standard deviation of resistors

σ_C: the standard deviation of capacitors

μ_R: the mean value of the resistors

μ_C: the mean value of the capacitors

μ_{A_o}: the mean value of the operational amplifier gain

A_o: the gain of the operational amplifier.

The measures defined in (1.1) through (1.5) all depend only on the first derivative or the linear term of the Taylor's series expansion of a network function about the nominal point. These measures are only good and meaningful if the higher-order terms are negligible.

2. Large-Change Multiparameter Sensitivity

Hakimi and Cruz [8] defined the multiparameter sensitivity measure at a single frequency ω_k as

$$M = \frac{\max \Delta F[j\omega_k]}{F(j\omega_k)[\delta_1 \cdot \delta_2 \ldots \delta_n]} \qquad 0 \leq |\varepsilon_i| \leq \delta_i \tag{1.6}$$

where δ_i is the tolerance on element x_i and ε_i is the actual per-unit deviation, $\Delta x_i / x_i$. Kelly [9] extended this single-frequency definition to a range of frequency

$$M = \int_{\omega_1}^{\omega_2} \lambda(\omega) \left| F_{max}(\omega) - F_{min}(\omega) \right| d\omega \tag{1.7}$$

where $F_{max}(\omega)$ and $F_{min}(\omega)$ are the maximum and minimum values, respectively, that the network function $F(\omega)$ can attain for element values contained in the element constraint set R.

$$R = [x_i (1-\delta_i) x_{iN} \leq x_i \leq (1+\delta_i) x_{iN}]$$

where x_{iN} is the nominal value of element x_i and δ_i is the tolerance of element x_i. $\lambda(\omega)$ is a weighting factor.

Among these definitions, the M's defined in (1.3) and (1.4) are statistical and the remainder are deterministic.

Shortcomings of the Existing Multiparameter Sensitivity Measures

In general, first-order sensitivity measures only give information on the characteristics of a network function in the vicinity of its nominal point. These quantities may yield quite useful results in some applications when element variations or tolerances are small. However, for fairly large element variations such as those that may occur in integrated and hybrid circuits, the first-order sensitivity is not only inadequate to describe the behavior but also misleading in many cases.

For instance, the Hamilton-Sedra circuit [11] realizing the voltage transfer function of a band-pass filter

$$T(s) = \frac{V_o}{V_{in}} = \frac{1.5s}{s^2 + \frac{1}{Q}s + 1}$$

is shown in Figure 1. For Q = 10, ω = 1 radian per second and with the following nominal element values:

$C_1 = C_{10} = 1.00001$ farads

$C_2 = C_{20} = 1.0001$ farads

$C_3 = C_{30} = 2.00002$ farads

$R_4 = R_{40} = 0.649$ ohm

$R_5 = R_{50} = 2.167$ ohms

$R_6 = R_{60} = 1.0$ ohm

$R_7 = R_{70} = 1.0$ ohm

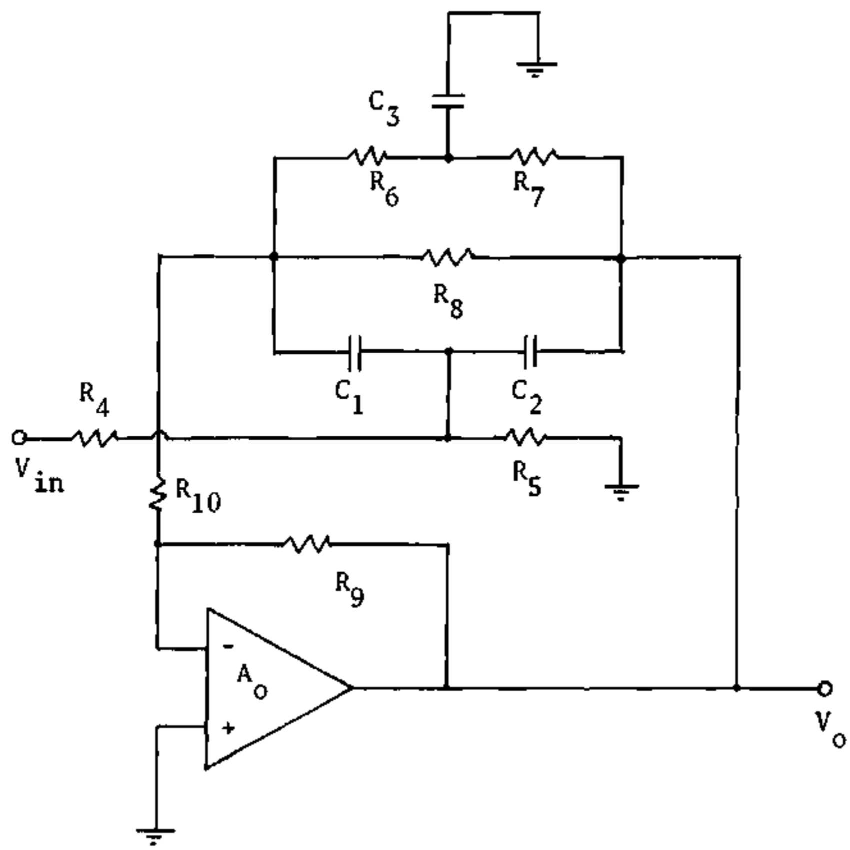


Figure 1. Hamilton-Sedra Circuit

$$R_8 = R_{80} = 10{,}000 \quad \text{ohms}$$
$$R_9 = R_{90} = 39.12 \quad \text{ohms}$$
$$R_{10} = R_{100} = 1.0 \quad \text{ohm}$$

the variations of the magnitude of the normalized transfer function versus circuit elements C_1 and C_2 in the neighborhood of nominal point are depicted in Figure 2. When all the elements are at their nominal values the transfer function at ω_o is denoted by T_o. $T(C_2)$ represents the variation of the transfer function when C_2 is varied and all other elements are kept at their nominal values. Similarly $T(C_1)$ represents the variation of the transfer function when C_1 is varied and all other elements are kept at their nominal values. The solid line in Figure 2 represents the variation of the magnitude of the normalized transfer function $T(C_1)$, while the dotted line represents that of $T(C_2)$. Since at nominal point b the slope of the dotted curve $T(C_2)$ is smaller than that of the solid curve $T(C_1)$, the first-order sensitivity with respect to C_2 is less than that with respect to C_1. Nevertheless, when the values of the elements C_1 and C_2 are considerably less than their nominal values ($C_1/C_{10} = C_2/C_{20} = 1.0$), the degradation in performance of the transfer function from the nominal with respect to C_2 is worse. For instance, when the element values C_1 and C_2 are changed from their nominal values point c to point d, the variation of the network function from the nominal point caused by the change C_1 is ef and the variation of the network function from the nominal point caused by the change C_2 is eg. Obviously, eg is larger than ef. This contradicts the conclusion one might infer from the first-order sensitivity

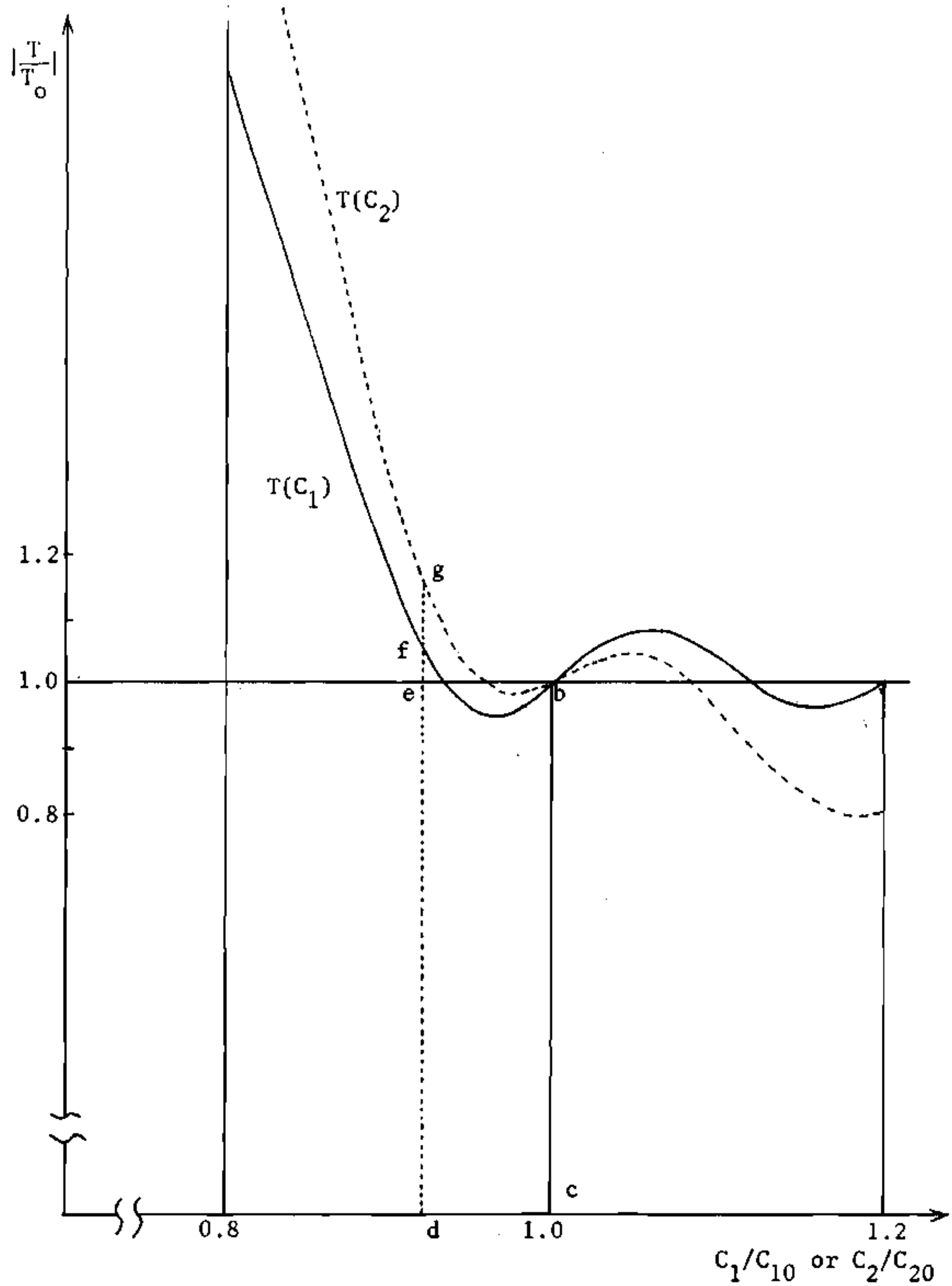


Figure 2. Transfer Function Plot vs. C_1 and C_2 in Figure 1

measures.

Another example of the situation in which the first-order sensitivity can give misleading information is the circuit of Figure 3. Assume the nominal element values are $R_o = 0.01$ ohm, $L_o = 1.0$ henry, and $C_o = 1.0$ farad, the variation of the magnitude of the driving-point admittance at $\omega = 1$ radian per second, when R and L are kept at their nominal values and C is considered to vary, is depicted in Figure 4. The first-order sensitivity of the driving-point admittance magnitude with respect to C at the nominal point is zero, $\left.\frac{\partial |Y(j1)|}{\partial C}\right|_{C=C_o} = 0$. According to the first-order sensitivity, this circuit is extraordinarily insensitive to the change in C. However, a 1 percent increase in C results in a 90 percent decrease in $|Y(j1)|$. Even a 0.2 percent increase in C results in a 29.1 percent decrease in $|Y(j1)|$.

From these examples, it is seen that the first-order sensitivity can frequently give inaccurate information on network behavior. In other words, <u>the first-order sensitivity is not always reliable as an index of the sensitivity of a network</u>.

The large-change sensitivity measures cited earlier give information at two extreme points for each element value change rather than at the nominal point. This usually gives a better description of the network behavior within the element tolerance limit. However, in general, <u>the behavior of a network function is dependent upon all points in the element tolerance limit</u>. For example, the two assumed network performance curves shown in Figure 5 behave differently except that they have the same magnitude at the nominal point b and the end points of the tolerance limit (a and c). Even though the network performance on the

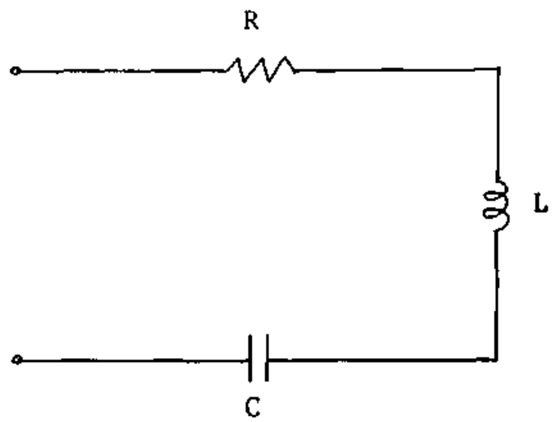


Figure 3. An RLC Circuit

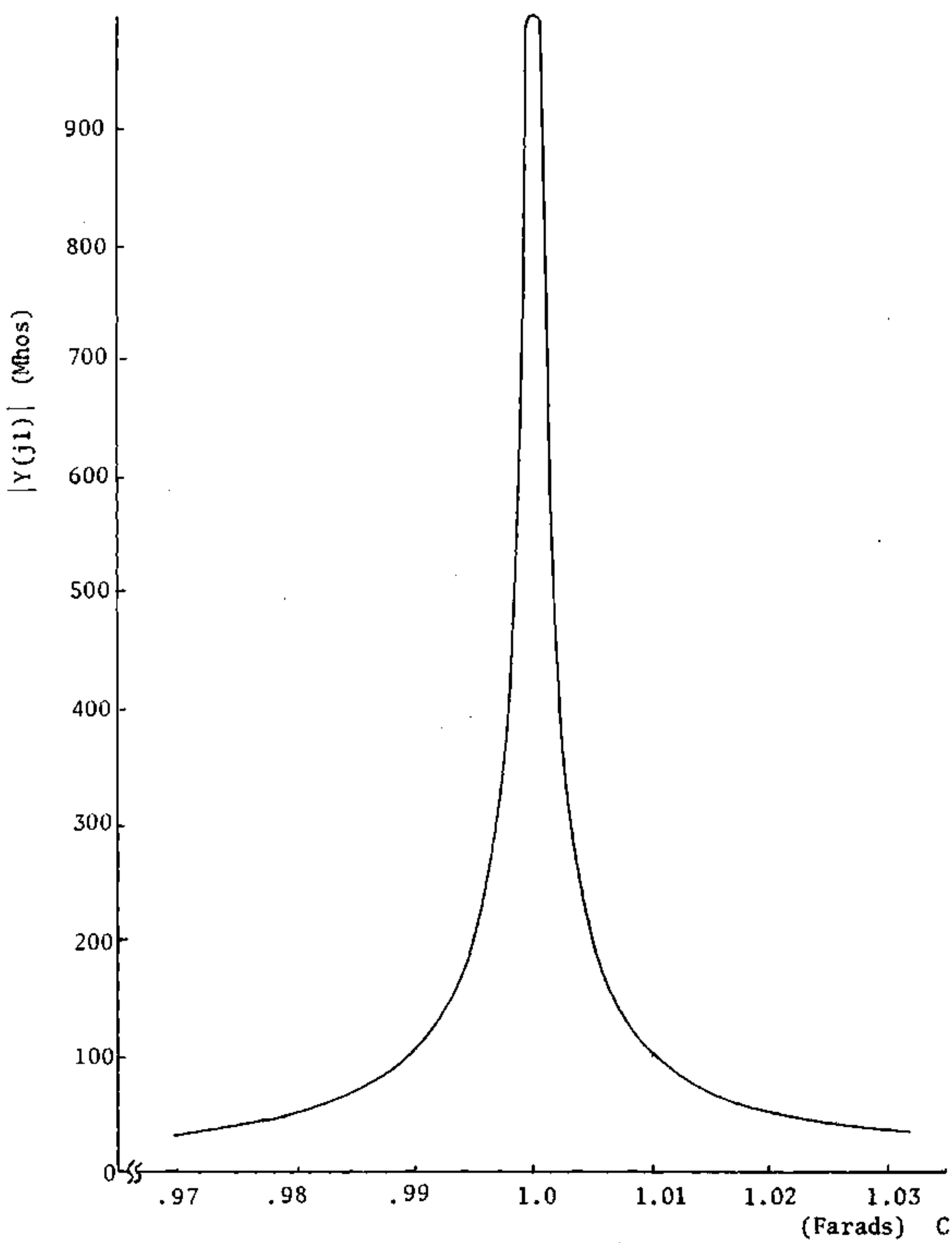


Figure 4. Plot of $|Y(j1)|$ vs. C in Figure 3

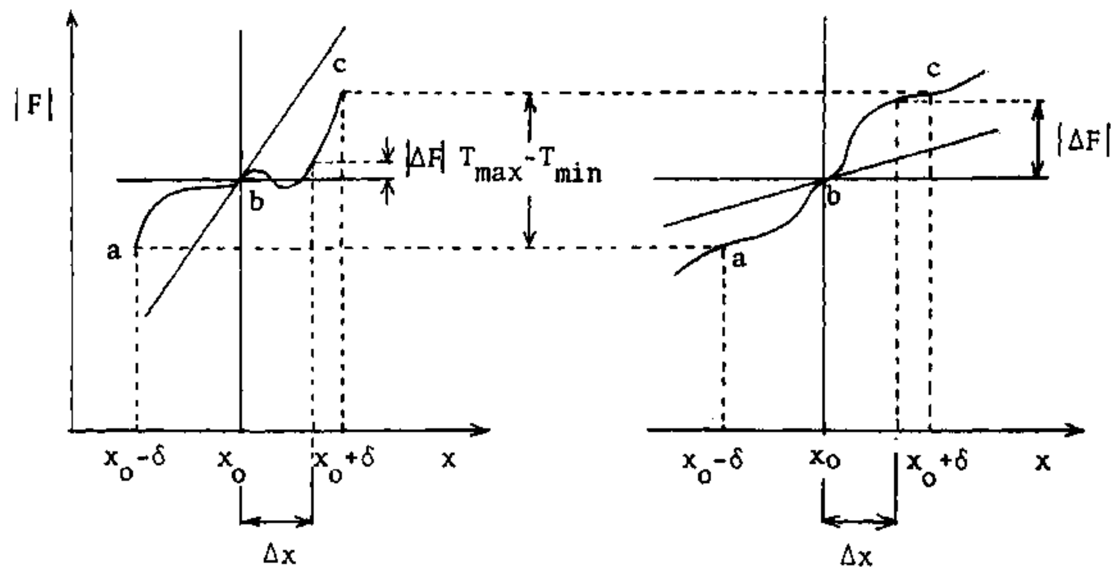


Figure 5. Two Assumed Different Network Performance Curves

right behaves worse than the one on the left because for more values of Δx it has a higher value of $|\Delta F|$, the definitions of large-change sensitivity given in (1.6) and (1.7) would both indicate that the two networks have the same sensitivity measure and, presumably, performs equally well. The misleading information that the first-order sensitivities give is also shown clearly in Figure 5 where the slopes at the nominal point would indicate that the network on the right would perform better than the one on the left. This is exactly the opposite of the true picture.

Continuously Equivalent Networks

Once the criterion of sensitivity has been established, a network designer can use the sensitivity to evaluate network performances based on that criterion. Methods can then be developed to optimize the design for a given network function. There has been some work done in this area [4, 12-20]. Optimization is usually done by minimizing the sensitivity measure with respect to the designed (or nominal) element values either for a fixed network configuration or for a series of continuously equivalent networks [4]. The current theory of continuously equivalent network is derived from the work of Howitt [21] which is based on the first-order sensitivity measure.

The Howitt theory says: Given an n+1 terminal n-port network N_o described by the admittance matrix Y_o, consider an n-port network N_1 whose admittance matrix Y_1 is such that

$$Y_1 = \mathcal{A}\, Y_o\, A$$

with

$$\mathcal{A} = \underbrace{\begin{bmatrix} \underbrace{1_r}_{r} & \mathcal{A}_{12} \\ 0 & \mathcal{A}_{22} \end{bmatrix}}_{n} \begin{matrix} \}r \\ \end{matrix} \Bigg\} n \qquad\qquad A = \begin{bmatrix} \underbrace{1_r}_{r} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{matrix} \}r \\ \end{matrix} \Bigg\} n$$

where 1_r is a unit matrix of $r \times r$, and $\mathcal{A}$ and A are nonsingular. Then network N_1 is equivalent to network N_o as far as the first r ports are concerned. If $\mathcal{A} = A^t$, the congruence transformation is called the Howitt transformation.

Schoeffler chose $\mathcal{A}^t = A = 1 + B\Delta x$ where $B = \begin{bmatrix} 0 \\ \cdots \\ b \end{bmatrix} \}r$, 1 is a unit matrix and proved that any solution to the differential equation

$$\frac{dY(x)}{dx} = B^t Y(x) + Y(x)B \ , \ Y(0) = Y_o \qquad (1.7)$$

(where Y_o is the $n \times n$ admittance matrix of the original network) represents a network which is r-equivalent to the original network (i.e., they are equivalent at the r ports formed by nodes $1,2,\ldots,r$ and the reference node, $0 < r < n$). The network component vector satisfies the equations

$$\frac{dG}{dx} = MG$$

$$\frac{dC}{dx} = MC$$

$$\frac{d\Gamma}{dx} = M\Gamma$$

where

$$G^t = [g_1, g_2, \ldots, g_n]^t$$

$$C^t = [C_1, C_2, \ldots, C_n]^t$$

$$\Gamma^t = [\Gamma_1, \Gamma_2, \ldots, \Gamma_n]^t$$

$g_1, g_2, \ldots, g_n$ are the conductances; C_1, C_2, C_n are the capacitances; $\Gamma_1, \Gamma_2, \ldots, \Gamma_n$ are the inverse inductances of the network components. The elements of the matrix M are linear sum of the elements of matrix B.

Cheetham [22] also provided an alternative technique to solve the same problem. Instead of choosing A = 1 + Bx, Cheetham chose A = 1 + Bx to perform the Howitt transformation and yielded the equation

$$Y(x) = Y_o + (B^t Y_o + Y_o B)x + B^t Y_o B x^2 \tag{1.8}$$

the transformation represented by this equation may be expressed for each component vector as

$$E(x) = M_1(x) E_o$$

where E_o is the appropriate component vector of the original network and the m × m matrix $[m = \frac{1}{2} n(n+1)]$ M(x) is in the form of

$$M_1(x) = 1 + Px + Dx^2 \tag{1.9}$$

where P and D are real m × m matrix whose elements are linearly dependent on those of matrix B. The calculation of the quadratic equation (1.9) is simpler than solving the differential equation (1.7). Nevertheless, his sensitivity optimization was still limited to the traditional first-order sensitivity shown in equation (1.3).

In the time domain, Leon and Yokomoto [23] used the Howitt theory on the state equation of a network, and thereby obtained new equivalent networks. The method is described briefly below. Assume that a given network N_o is represented by the state equations

$$M_o\dot{\underline{x}} = - N_o\underline{x} + \underline{b}u \tag{1.10}$$

$$w = \underline{d}^t\underline{x} \tag{1.11}$$

where

$$M_o = \begin{bmatrix} C_o & 0_1 \\ 0_2 & L_o \end{bmatrix} \tag{1.12}$$

$$N_o = \begin{bmatrix} G_o & \beta \\ S & R \end{bmatrix} \tag{1.13}$$

a new network N_1 represented by another state equations

$$M_1\dot{\underline{x}} = -N_1\underline{x} + \underline{b}u$$

$$w = \underline{d}^t\underline{x}$$

such that

$$M_1 = A^tM_oA \qquad A^tb = b$$

$$N_1 = A^tN_oA \qquad d^tA = d^t$$

is r equivalent to N_o.

The matrices C_o and G_o are $r \times r$ matrices whose elements are linear sums of the network capacitances c_i and conductances g_i respectively with coefficients ± 1; L and R are $(n-r) \times (n-r)$ matrices whose elements are linear sums of the network inductances ℓ_i and resistances r_i respectively; β and S^t are $r \times (n-r)$ matrices whose elements are ± 1, -1, or zero; and 0_1 and 0_2 are $r \times (n-r)$ null matrices. The elements g_i in G are co-tree conductances and the elements r_i in R are tree resistances.

Because of the restrictions on state equations in describing a network, this state-equation time-domain method has more limitations in generating equivalent networks than the preceding two methods. The optimization criterion Leon and Yokomoto used was also the traditional first-order sensitivity measure shown in equation (1.3).

Since networks generated by the Howitt theory are known to be incomplete, continuously equivalent networks derived from it are also incomplete [24]. In Chapter IV a new formulation and algorithm of continuously equivalent network will be presented.

Outline for the Thesis

This research includes the following five facets and will be presented in that order.

1. To point out the shortcomings of the existing sensitivity measures which are either limited in their validity or giving inaccurate or misleading information of the behavior of the network in the element tolerance space. This item has already been discussed in Chapter I.

2. To propose a new deterministic and statistical sensitivity measure which will overcome the shortcomings of the existing sensitivity

measures. This will be discussed in Chapter II. The newly proposed sensitivity measure will be called "All-tolerance Multiparameter Sensitivity."

3. To develop a new formulation and algorithm for continuously equivalent networks in Chapter III. Continuously equivalent networks obtained by this new scheme will be called "Multivariable Continuously Equivalent Networks."

4. A comparative evaluation of different circuits realizing the same network function will be discussed in Chapter IV. The evaluation includes: (i) Applying the all-tolerance multiparameter sensitivity measure to compare more meaningfully which of the circuits realizing the same network function is less sensitive. (ii) Comparing a series of equivalent networks by the newly proposed sensitivity measure with those obtained by the traditional sensitivity measure.

5. To use the newly proposed multiparameter sensitivity measure and multivariable continuously equivalent network theory to obtain an optimal design for a prescribed network function. This will be discussed in Chapter V.

Finally, the conclusions derived from the research and recommendations for further work are presented in Chapter VI.

CHAPTER II

THE ALL-TOLERANCE MULTIPARAMETER SENSITIVITY MEASURE

For the comparative evaluation of different circuits realizing the same network function and for the optimization scheme for realizing better networks to be meaningful, a sensitivity measure that gives accurate information about the behavior of the network function throughout the entire region of element values within their tolerance limits must first be formulated. As demonstrated in Chapter I, none of the existing sensitivity measures serves this purpose well.

In this chapter, a new deterministic and statistical sensitivity measure that will be more meaningful than the existing sensitivity measures will be proposed. The proposed deterministic multiparameter sensitivity measure is used primarily to evaluate and design networks in small production volumes. The statistical one is more applicable to large-scale productions of units such as integrated or hybrid circuits. The proposed statistical multiparameter sensitivity measure will include the consideration of tolerance, the type of the probability density function (PDF), and the mean change of the element values. Since the new deterministic and statistical sensitivity measure is applicable to both the small-change and the large-change sensitivities, the new sensitivity measure will be called the "all-tolerance multiparameter sensitivity."

New Deterministic and Statistical Multiparameter Sensitivity Measure

The newly proposed statistical all-tolerance multiparameter sensitivity is:

$$I = \frac{1}{||x_o||^n} \int_{\omega_1}^{\omega_2} \lambda(\omega) \int_{x_o-\Delta x}^{x_o+\Delta x} |F(x,\omega) - F(x_o,\omega)|^k \, P_x^{\ell} \, d\delta d\omega \qquad (2.1)$$

where $x = (x_1, x_2, \ldots, x_n)^t$: the element value vector of the network (or the random variables of the network)

$x_o = (x_{1o}, x_{2o}, \ldots, x_{no})^t$: the nominal element vector of the network

$\Delta x = (\Delta x_1, \Delta x_2, \ldots, \Delta x_n)^t$: the tolerance vector of the elements

$||x_o|| = \sqrt{x_{1o}^2 + x_{2o}^2 + \ldots + x_{no}^2}$: the norm of x_o

P_x = joint probability density function of all elements $x_1, x_2, \ldots, x_n$

$d\delta = dx_1, dx_2, \ldots, dx_n$

ω_2, ω_1 = the operating frequency range of the network ($\omega_2 > \omega_1$)

λ = weighting factor

k = a positive number

ℓ = a positive number

When $\ell = 0$, equation (2.1) becomes the deterministic all-tolerance multiparameter sensitivity measure, i.e.,

$$I = \frac{1}{||x_o||^n} \int_{\omega_1}^{\omega_2} \lambda(\omega) \int_{x_o-\Delta x}^{x_o+\Delta x} |F(x,\omega) - F(x_o,\omega)|^k \, d\delta d\omega \qquad (2.2)$$

Simplified n One-Dimensional Integrals

Equation (2.1) and (2.2) are (n+1)-dimensional integrals. The computation time required to evaluate such an integral may be excessive. In order to make the calculation of the sensitivity measure practical, it is proposed that each of the integrals be replaced by the sum of a number of n one-dimensional integrals.

For the statistical sensitivity measure of equation (2.1) the correlated case and the uncorrelated case have to be considered separately.

(a) Correlated Case

For the correlated case, equation (2.1) is replaced by

$$I = \sum_{i}^{m} \sum_{j}^{n} \frac{1}{x_{j_o}} \int_{x_{j_o}-\Delta x_j}^{x_{j_o}+\Delta x_j} \lambda(\omega_i) \left| F(x_j,\omega_i) - F(x_{j_o},\omega_i) \right|^k P_x^{\ell} \, dx_j \qquad (2.3)$$

where m is the number of frequency points being calculated; n is the number of network elements; and P_x is the joint probability density function of element parameters $x_1, x_2, \ldots, x_n$.

It is noted that the integral with respect to frequency ω within the operating frequency range ω_1 to ω_2 is replaced by the summation of aggregated values that approximate the n one-dimensional points. The number m is chosen arbitrarily. When m is sufficiently large, the results from integration and summation will become very close.

(b) Uncorrelated Case

For the uncorrelated case, equation (2.1) is replaced by

$$I = \sum_{i}^{m} \sum_{j}^{n} \frac{1}{x_{j_o}} \int_{x_{j_o}-\Delta x_j}^{x_{j_o}+\Delta x_j} \lambda(\omega_i) \left| F(x_j,\omega_i) - F(x_{j_o},\omega_i) \right|^k P_{x_j}^{\ell} \, dx_j \qquad (2.4)$$

where P_{x_j} is the probability density function of the element parameter x_j.

Similarly, the deterministic all-tolerance multiparameter sensitivity measure of (2.2) is replaced by the following summation of n one-dimensional integrals

$$I = \sum_{i}^{m} \sum_{j}^{n} \frac{1}{x_{j_o}} \int_{x_{j_o}-\Delta x_j}^{x_{j_o}+\Delta x_j} \lambda(\omega_i) \left| F(x_j,\omega_i) - F(x_{j_o},\omega_i) \right|^k dx_j \qquad (2.5)$$

In general, the integral evaluation needs to be carried out through the computer by using numerical methods. Various numerical integration methods such as the trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, or Simpson's 3- and 5-point approximations are available in the form of computer subroutines. The number of points at which the integrand is to be evaluated depends on the degree of accuracy required. Usually a maximum permissible error between successive evaluations of the integral is set in advance. The type of error comparison between successive values of the integral can be relative or absolute. For the purpose of evaluating the new measure, I, it is recommended that the relative type error comparison be used since the value of the new measure, I, usually has a different range for different problems. The absolute type error comparison can only be used when the range of I is already known in advance, which is usually not the case.

For a certain maximum permissible error between successive calculated values of the integral, the reduction of the number of points at which the integrand is to be evaluated can be quite substantial when the n-dimensional integral is reduced to n one-dimensional integral. For instance, assume that 100 points are required for a relative maximum error of 10^{-2} in one-dimension. Further assume that n is 6. Then the total number of points that the integrand must be calculated for n-dimensional integral will be $(100)^n = 100^6$ or 10^{12}. However, the number of points that the integrand needs to be calculated for n one-dimensional integral is $n \times 100 = 6 \times 100$ or 600, which is dramatically smaller than 10^{12}.

Even though the n one-dimensional integrals give much less information on the network behavior than the (n+1)-dimensional integral, it still gives much more information than any of the sensitivity measures mentioned in Chapter I. This is to say that the quantity I defined in any of the equations (2.3), (2.4), and (2.5) still gives considerably more informative than the existing sensitivity measures.

Figure 5 is redrawn in Figure 6 in conjunction with the probability density functions of different kinds of element parameters. From this figure it is seen that not only the type of the PDF but also the mean change (μ) of the element parameters play an important role on the network performance. For instance, when $\mu = 0$, the network function F_1 has a better performance when the probability

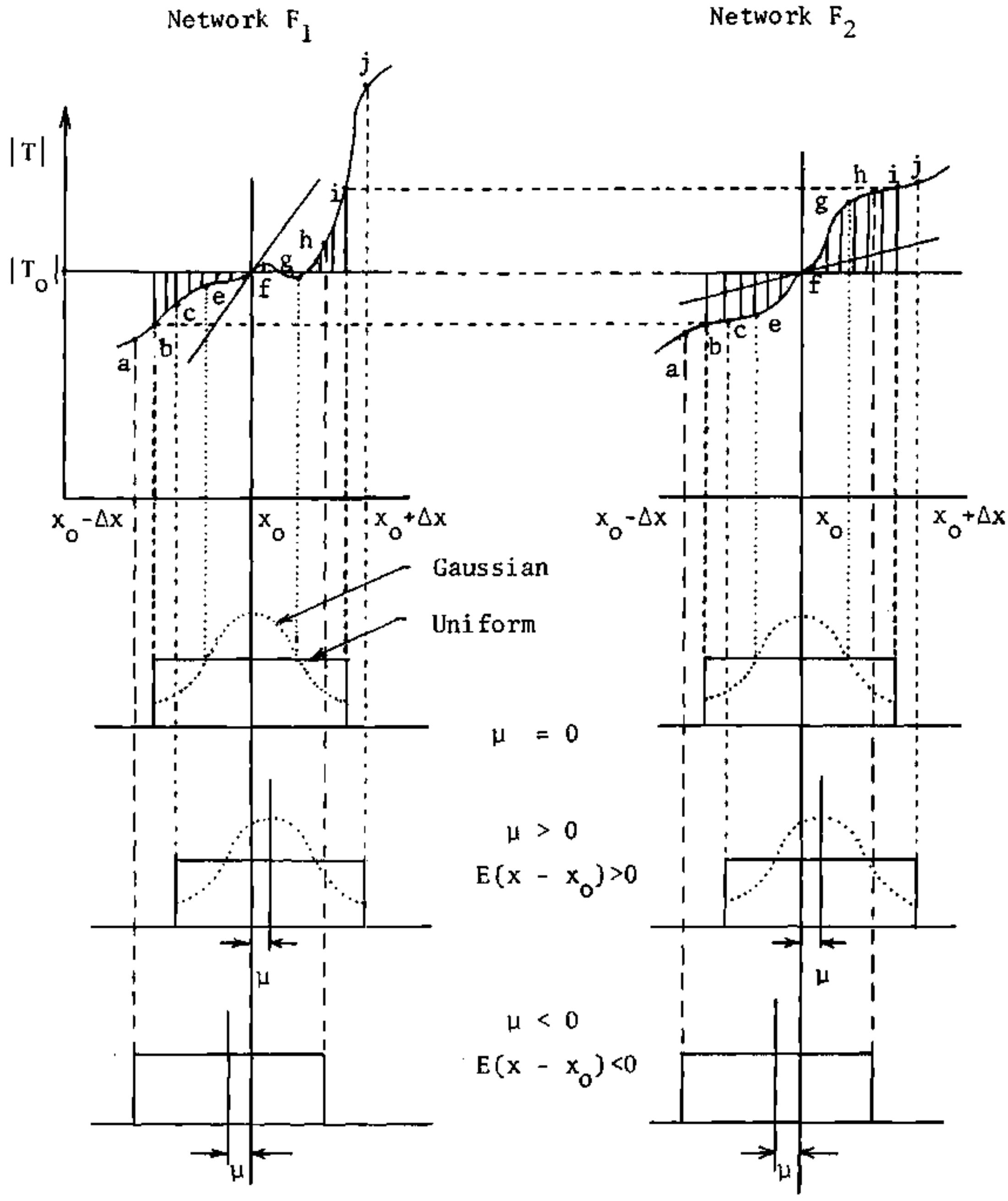


Figure 6. Network Function and PDF

density function of the element parameter is Gaussian than when the PDF of the element parameter is uniform. This is because of the fact that the area between that part of the performance curve labeled efg and the $|T| = |T_o|$ is weighted more heavily when the PDF is Gaussian than when it is uniform; while the opposite is true for those parts labeled bce and ghi. For the same reasoning, network F_2 also performs better when the PDF of the network element parameter is Gaussian than when the network element parameter is uniform.

If the PDF of the network element parameter is uniform, network F_2 in Figure 6 will have a worse performance when μ is positive ($\mu > 0$) than when μ is negative ($\mu < 0$). The difference in the measure is dependent on the relative areas of the deviation that are weighted in one PDF but not in the other. When $\mu > 0$, the area in question is between that part of the performance curve labeled hij and $|T| = |T_o|$; while when $\mu < 0$, the area is between abc and $|T| = |T_o|$. The former is larger than the latter. The same conclusion cannot be reached for network F_1 by an inspection of Figure 6 since the relative areas of the network performance curve is no longer obvious. The new measure I needs to be calculated in order to determine what kind of μ, positive or negative, will give a better performance when the PDF of the network element parameter is uniform.

When $\ell = 0$ both equations (2.3) and (2.4) become equation (2.5). That is to say that the deterministic sensitivity measure is a special case of the statistical sensitivity measure. When $k = 1$, $m = 1$, $n = 1$ and both $F(x_j, \omega_i)$ and $F(x_{j_o}, \omega_i)$ are real or are in the same phase the value of I in equation (2.5) represents the shaded area shown in

Figure 6. For practical purposes it is suggested that $k = 2$, $\ell = 1$ be used for the statistical cases of equations (2.3) and (2.4), and that $k = 2$ be used for the deterministic case of equation (2.5). The value $k = 1$ is not recommended especially when the sensitivity measure is being minimized, since this value of k will cause trouble in evaluating the values of the partial derivatives of I with respect to the element parameters. In the numerical integration algorithm, the denominator of the integrand of the partial derivative of I with respect to any element parameters becomes zero at the nominal point. This, in turn, will make that integrand infinitely large and keep the computer from proceeding further without errors.

A new formulation and algorithm of continuously equivalent networks--the multivariable continuously equivalent networks--will be presented in Chapter III. The application of the new all-tolerance multiparameter sensitivity measure will be discussed in Chapter IV along with a series of continuously equivalent networks. In Chapter V, the new all-tolerance multiparameter sensitivity measure and the multivariable continuously equivalent networks will be applied to obtain an optimal network for a prescribed network function.

CHAPTER III

THE MULTIVARIABLE CONTINUOUSLY EQUIVALENT NETWORKS

Continuously equivalent networks are a series of networks whose network functions are identical to that of a given network but whose element values are varied from one network to the next by an incremental amount. Noting that the removal of an element between two nodes is the same as replacing it by an admittance of value zero and that adding an element between the nodes is equivalent to replacing a zero value admittance element by one with finite conductance, the topology or the configuration of the equivalent network can often be changed by this process.

The purpose of generating a series of continuously equivalent networks is to establish a series of networks from which an optimal network can be found. In particular, the new techniques are more suitable for thin-film and integrated circuits since the engineering of these circuits has changed some of the criteria by which networks are evaluated. For instance, the new technique places less emphasis on the number of network elements but requires designs which are fairly insensitive to changes in the element value. Under this circumstance, the continuously equivalent networks will serve as a good tool for finding an optimal network if (i) the networks generated by the theory and algorithm are complete, and (ii) the optimality criterion or the sensitivity measures used in the process of finding the optimal network is effective. In Chapter I, it is mentioned that none of the existing theories and algorithms can generate continuously equivalent networks that are complete. In Chapter II, a sensitivity measure that serves the purpose of optimization has been proposed. In this chapter, a new formulation and algorithm for continuously equivalent networks will be developed. This new formulation and algorithm will have the following outstanding features: (a) the new scheme can grow both new meshes and new nodes with all types of new elements (R, L, and C) and, thereby, change the topology of the network; (b) the continuously equivalent networks produced by the new scheme are complete under certain conditions.

Instead of dealing with the continuously equivalent networks with one variable, the new scheme will use more than one variable. Therefore, the continuously equivalent networks obtained by the new scheme shall be called multivariable continuously equivalent networks. Two methods of changing the topology of a given network will be presented. One is the element growing method between any pair of existing nodes; the other is the node and element growing method.

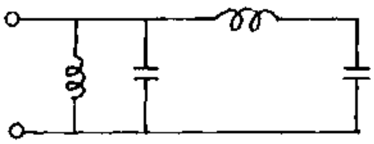
Element Growing Method Between a Pair of Existing Nodes

Given a network, one can assume that there is a component of each type (R, L, and C) between every pair of nodes. For those components which are not needed in the new equivalent network, the admittance values are set equal to zero. As an example, when this principle is applied to Schoeffler's original network, one can get the topology of the equivalent networks Schoeffler and Cheetham got or the topology of the equivalent network Leon and Yokomoto got immediately. The process is demonstrated in Figure 7.

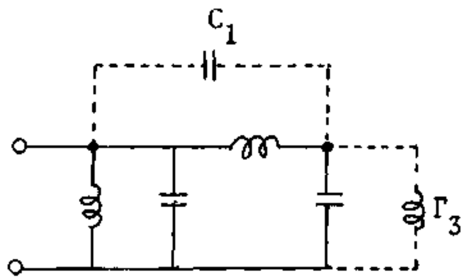
In this group of examples, network elements are limited to the lossless type (Γ and C). Schoeffler's original network is shown in Figure 7(a). In Figure 7(b) a component of each type is assumed to exist between every pair of nodes. The newly grown elements C_1 and Γ_3 are shown as dotted branches. If one decides to include both newly grown elements C_1 and Γ_3 in the new equivalent network, the topology of the new equivalent network is exactly the same as the one obtained by Schoeffler and Cheetham which is shown in Figure 7(c). On the other hand, if one decides to include only one of the newly grown elements, say C_1, in the new equivalent network, the topology of the new equivalent network will be exactly the same as the one obtained by Leon and Yokomoto which is shown in Figure 7(d).

Once the new topology of continuously equivalent networks is obtained, a new network function can be formed in terms of the element parameters. Assuming the number of elements of the new network is n, the number of independent coefficients in the given network function is N_c. By equating each of the N_c coefficients, expressed in terms of element parameters in the newly formed network function to the coefficients of the given network function there remain $n - N_c$ unknown parameters. These $n - N_c$ parameters are then used as independent variables. The number $n - N_c$ is usually greater than one. This is why the equivalent networks generated by this scheme are called multivariable continuously equivalent networks.

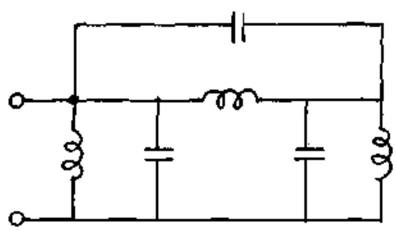
The following example illustrates the proposed procedure.



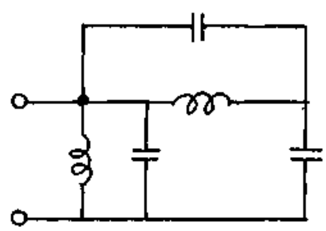
(a) Schoeffler's original network



(b) Network with newly grown elements



(c) Schoeffler's and Cheetham's equivalent network



(d) Leon-Yokomoto's equivalent network

Figure 7. Formulation of New Networks

Example

Given a network function

$$Z(s) = \frac{5s^3 + 12s}{6s^4 + 22s^2 + 18} = \frac{\frac{5}{6}s^3 + 2s}{s^4 + \frac{11}{3}s^2 + 3} = \frac{b_3 s^3 + b_1 s}{s^4 + a_2 s^2 + a_0} \tag{3.1}$$

which is realized in Figure 8 with element values shown below

$$\Gamma_1 = 0.02 \text{ henry}^{-1}$$

$$\Gamma_2 = 1.50 \text{ henry}^{-1}$$

$$C_2 = 1.20 \text{ farads}$$

$$C_3 = 0.00833 \text{ farads}$$

Following the element growing method stated above, a new equivalent network having the same Z(s) as in Figure 8 is obtained in Figure 9. The network function with its coefficients expressed in terms of element parameters is

$$Z(s) = \frac{\frac{x_1+x_3}{x_1(x_2+x_3)+x_2x_3}s^3 + \frac{x_4+x_6}{x_1(x_2+x_3)+x_2x_3}s}{s^4 + \frac{x_4(x_2+x_3)+x_5(x_1+x_3)+x_6(x_1+x_2)}{x_1(x_2+x_3)+x_2x_3}s^2 + \frac{x_4(x_5+x_6)+x_5x_6}{x_1(x_2+x_3)+x_2x_3}}$$

$$= \frac{B_3(x)s^3 + B_1(x)s}{s^4 + A_2(x)s^2 + A_0} \tag{3.2}$$

Setting equation (3.2) equal to (3.1), one gets

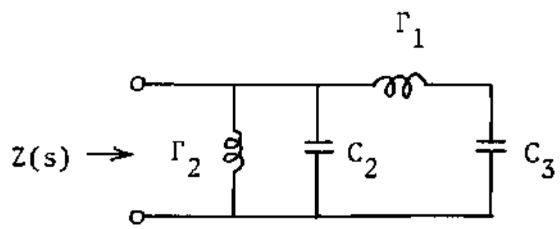


Figure 8. Circuits Realizing Equation (3.1)

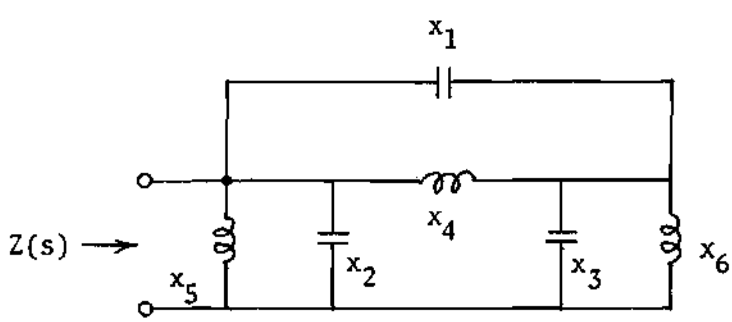


Figure 9. A New Equivalent Network

$$B_3(x) = \frac{x_1 + x_3}{x_1(x_2+x_3) + x_2x_3} = \frac{5}{6} = b_3 \tag{3.3}$$

$$B_1(x) = \frac{x_4 + x_5}{x_1(x_2+x_3) + x_2x_3} = 2 = b_1 \tag{3.4}$$

$$A_2(x) = \frac{x_4(x_2+x_3) + x_5(x_1+x_3) + x_6(x_1+x_2)}{x_1(x_2+x_3) + x_2x_3} = \frac{11}{3} = a_2 \tag{3.5}$$

$$A_0(x) = \frac{x_4(x_5+x_6) + x_5x_6}{x_1(x_2+x_3) + x_2x_3} = 3 = a_0 \tag{3.6}$$

In equation (3.1) the number of independent coefficients is 4, (a_0,a_2, b_1 and b_3), i.e., $N_c = 4$. In Figure 9 the number of elements in the new network is 6, i.e., $n = 6$. Solving equations (3.3), (3.4), (3.5), and (3.6), there remain $n - N_c = 6 - 4 = 2$ unknown parameters. These two unknown parameters are then used as independent variables. The new networks of two independent variables are multivariable continuously equivalent networks compared to Schoeffler's and Cheetham's equivalent networks of a single variable.

After the new equivalent network with newly grown elements are determined, the general procedure for getting the values of the elements of the new network for the general case is outlined below.

Given a network function

$$F(s) = \frac{b_p s^p + b_{p-1}s^{p-1} + \dots + b_1 s + b_0}{s^q + a_{q-1}s^{q-1} + \dots + a_1 s + a_0} \tag{3.7}$$

where $p + q + 1 = N_c$ is the number of independent coefficients, a new network function with coefficients as functions of the element parameters $x_1, x_2, \ldots, x_n$ is formed from the new equivalent network. Using the notation $x = (x_1, x_2, \ldots, x_n)$ the new network function becomes

$$F(s) = \frac{B_p(x)s^p + B_{p-1}(x)s^{p-1} + \ldots + B_1(x)s + B_0(x)}{s^q + A_{q-1}(x)\ s^{q-1} + \ldots + A_1(x)s + A_0(x)} \quad (3.8)$$

Setting equation (3.8) equal to equation (3.7), N_c equations are obtained as follows.

$$\begin{array}{ll} B_p(x) = b_p & A_{q-1}(x) = a_{q-1} \\ B_{p-1}(x) = b_{p-1} & A_{q-2}(x) = a_{q-2} \\ \vdots & \vdots \\ B_1(x) = b_1 & A_1(x) = a_1 \\ B_0(x) = b_0 & A_0(x) = a_0 \end{array} \quad (3.9)$$

With N_c element values constrained by the N_c equations in (3.9), the remaining $n - N_c$ element values are free to vary. That is to say, the values of $n - N_c$ element parameters can be chosen arbitrary as one desires. Once the values of $n - N_c$ element parameters are chosen, the remaining values of N_c parameters can be obtained by solving the N_c simultaneous equations in equation (3.9).

Node and Element Growing Method

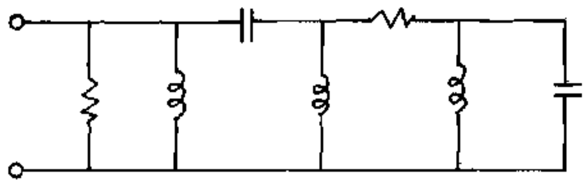
In addition to the fact that elements can be added between existing nodes, new nodes and new elements can be grown out of a given network. This is illustrated in Figure 10. The original network is shown in Figure 10(a). The new network with newly grown nodes and elements is shown in Figure 10(b) where N_1, N_2, and N_3 are newly grown nodes and Γ_1, g_1, S_1, ℓ_1, and ℓ_2 are newly grown elements. It is noted when the values of Γ_1, g_1, S_1, ℓ_1, and ℓ_2 are zero, the topology of the new equivalent network in Figure 10(b) is the same as the topology of the original network in Figure 10(a). It is also noted that newly grown elements are not in the same unit. The unit for Γ_1 is henry^{-1}, g_1 mho, S_1 farad^{-1}, and ℓ_1 & ℓ_2 henry. The rule to determine the unit of a newly grown element in an equivalent network is that when the value of a newly grown element is zero, the topology of the new equivalent network should be the same as the original network.

Once the new network is grown, the algorithm of getting the values of the element parameters is basically the same as the element growing method in Section 3.1 except some minor modifications.

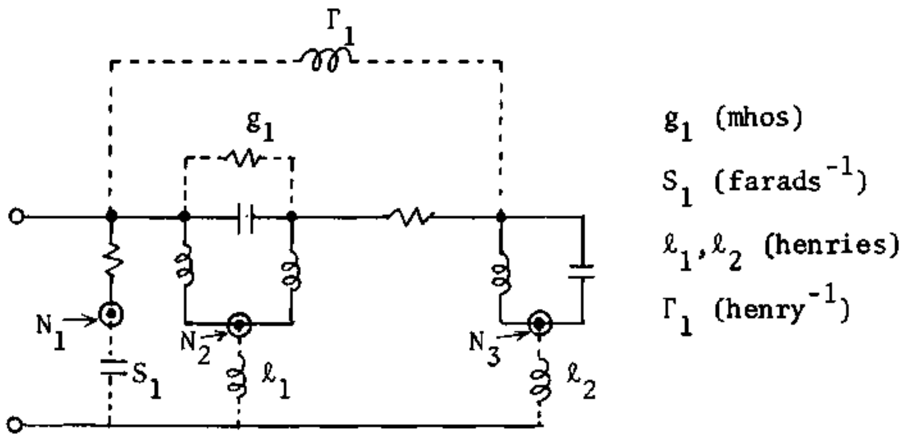
When nodes are grown, the network function of the newly grown equivalent network will not have the same degree as the original given network function. Therefore, function in equation (3.7) is modified to equation (3.10) below.

$$F(s) = \frac{B_{p+k}(x)s^{p+k}+\ldots+B_{p+1}s^{p+1}+B_p(x)s^p+B_{p-1}s^{p-1}+\ldots+B_1(x)s+B_0(x)}{A_{q+k}(x)s^{q+k}+\ldots+A_q(x)s^q+A^{q-1}(x)s^{q-1}+\ldots+A_1(x)s+A_0(x)} \tag{3.10}$$

Equation (3.9) is correspondingly modified to (3.11) below.



(a) Original network



(b) New equivalent network with newly grown elements (g_1, S_1, ℓ_1, ℓ_2, and Γ_1) and nodes (N_1, N_2, and N_3)

Figure 10. Node and Element Growing Scheme

$$
\begin{array}{ll}
B_{p+k}(x) = 0 & A_{q+k}(x) = 0 \\
\vdots & \vdots \\
B_{p+1}(x) = 0 & A_{q+1}(x) = 0 \\
B_p(x) = b_p & A_q(x) = 1 \\
B_{p-1}(x) = b_{p-1} & A_{q-1}(x) = a_{q-1} \\
\vdots & \vdots \\
B_1(x) = b_1 & A_1(x) = a_1 \\
B_0(x) = b_0 & A_0(x) = a_0
\end{array}
\tag{3.11}
$$

Now the number of constraint equations becomes $N_c = p + q + 1 + 2k$. The rest of the algorithm is the same as the element growing method in the previous section.

Completeness of the Proposed Multivariable Continuously Equivalent Networks

Given a network function F(s) in equation (3.7) which is repeated below.

$$F(s) = \frac{b_p s^p + b_{p-1} s^{p-1} + \ldots + b_1 s + b_0}{s^q + a_{q-1} s^{q-1} + \ldots + a_1 s + a_1} \tag{3.7}$$

If all possible values of the elements in a network with certain topology satisfy the given network function cannot be obtained by the algorithm of a continuously equivalent network, then the continuously equivalent networks generated by this algorithm are incomplete. In Chapter I it is mentioned that the existing theories and algorithms of continuously equivalent networks are derived from the Howitt theory. The networks generated by the Howitt theory are known to be incomplete. Therefore the continuously equivalent networks generated by the existing algorithms are incomplete [24].

For the proposed multivariable continuously equivalent networks the constrained equations (3.9) of obtaining the element values of the new network are tied to the given network function directly as shown below. All element values that satisfy the given network function

$$\begin{array}{ll} B_p(x) = b_p & A_{q-1}(x) = a_{q-1} \\ B_{p-1}(x) = b_{p-1} & A_{q-2}(x) = a_{q-2} \\ \vdots & \vdots \\ B_1(x) = b_1 & A_1(x) = a_1 \\ B_0(x) = b_0 & A_0(x) = a_0 \end{array} \tag{3.9}$$

can be obtained from equation (3.9) if all the roots of equation (3.9) can be found. If the roots of equation (3.9) can be obtained in closed form, there is no doubt that all the roots can be found. Then the continuously equivalent networks obtained are complete. In other words, there are no networks which satisfy equation (3.7) that cannot be obtained from equation (3.9) as long as all the roots of equation (3.9) can be obtained. Hence the multivariable continuously equivalent networks obtained by the proposed algorithm are complete if equation (3.9) can be solved in closed form or if all the roots of equation (3.9) can be found. On the other hand, the multivariable continuously equivalent networks would not be complete if the equations in (3.9) are nonlinear and cannot be solved explicitly.

In Chapter IV, the new all-tolerance multiparameter sensitivity measure will be applied to evaluate different networks realizing the same network function and the results will be compared with those obtained from the traditional sensitivity measures. The convergence problem of carrying out the numerical integration in finding the value of the new measure will also be discussed in Chapter IV. The variation of the new measure, I, versus the variation of network elements will be surveyed first in Chapter V. Then the new measure will be applied to obtain an optimal network along with the multivariable continuously equivalent networks.

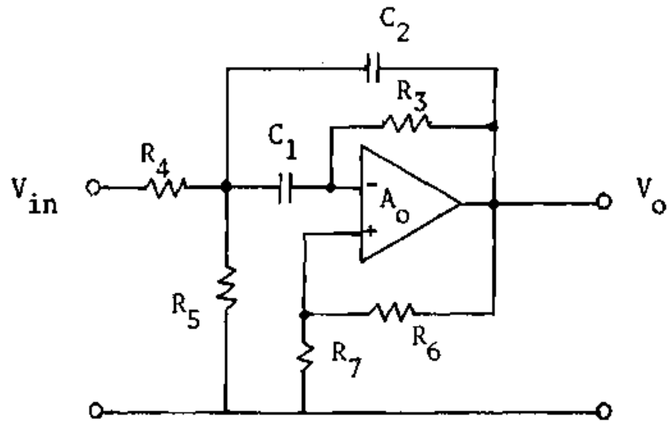
CHAPTER IV

COMPARATIVE EVALUATION OF DIFFERENT CIRCUITS

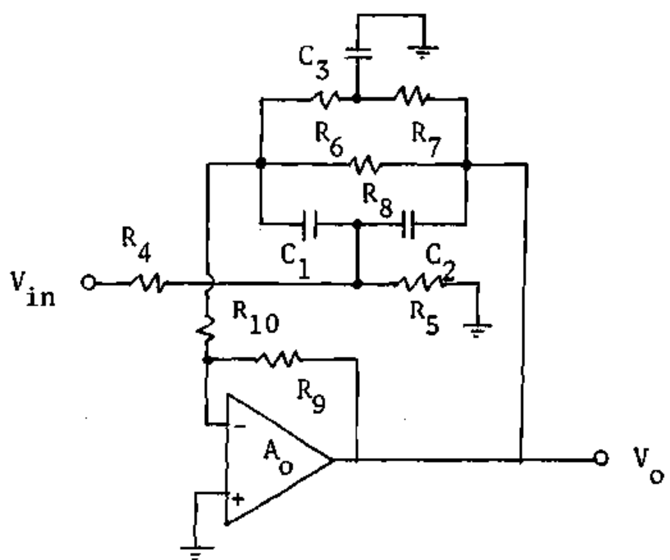
For a given network function, there exist many different circuits that realize that same network function. To determine which amongst all of them gives the best performance is important especially from the point of view of mass production. For instance, there are nearly 100 active-RC circuit configurations that have been proposed during the last ten years for the realization of low-pass, band-pass, all-pass response, etc. Without a good criterion one will have a great deal of difficulty in choosing the right one to use. In this chapter, the proposed new sensitivity measure will be applied to compare more meaningfully which of the circuits realizing the same network function is less sensitive. The new sensitivity measure will also be used to evaluate a series of continuously equivalent networks and the results will be compared with those obtained by the traditional sensitivity measure.

Applying the All-Tolerance Multiparameter Sensitivity Measure to Compare Circuits Realizing the Same Network Function

Two biquadratic band-pass circuits that are practical and known to have low sensitivity performance will be chosen for the purpose of comparison. They are the Deliyannis-Friend circuit [25,26] and the Hamilton-Sedra circuit [11] shown in Figure 11. Since the sensitivity measure of a band-pass circuit generally decreases as the number of



(a) Deliyannis-Friend Circuit



(b) Hamilton-Sedra Circuit

Figure 11. Two Band-pass Circuits, D-F and H-S Circuits

operational amplifiers increases, the comparison of the circuits should be made only between circuits that have the same number of operational amplifiers. The Deliyannis-Friend circuit and the Hamilton-Sedra circuit both use a negative beedback around the single operational amplifier. The open-loop gain of the operational amplifier is chosen to be 90.4 dB, or 33,110, at 1 kHz for a typical operational amplifier provided with some internal compensation. This frequency is normalized to a radian frequency of unity and the nominal band-pass transfer function is given by

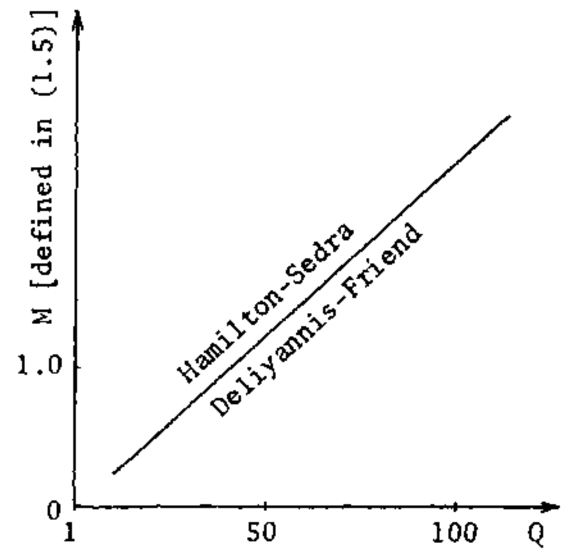
$$T(s) = \frac{\mp 1.5s}{s^2 + (\frac{1}{Q})s + 1} \tag{4.1}$$

By using the sensitivity measure M defined in equation (1.5) Shenoi was unable to tell the difference in network performance between the two circuits. This is shown in Figure 12(a). The values of the elements of the two circuits for different values of Q for Figure 12(a) are shown in Tables 1 and 2.

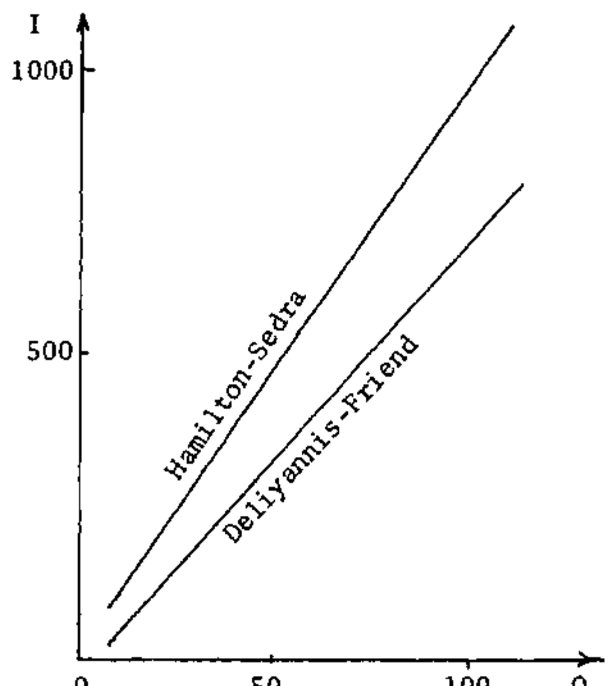
The proposed new sensitivity measures with $k = 1$, $\ell = 1$ have been calculated for the various values of Q using the following equations for the Deliyannis-Friend circuit by letting $x_1 = C_1$, $x_2 = C_2$, $x_3 = R_3$, $x_4 = R_4$, $x_5 = R_5$, $x_6 = R_6$, $x_7 = R_7$, and $x_8 = A_o$.

$$I = \sum_{i=1}^{5} \sum_{j=1}^{8} \frac{1}{x_j} \int_{-0.1x_j}^{+0.1x_j} |T_j(x_j+y,\omega_i) - T_o(x_j,\omega_i)|^k \, P_{x_j}^{\ell} \, dy$$

$$= \sum_{i=1}^{5} \sum_{j=1}^{8} \frac{1}{x_j} \int_{-0.1x_j}^{+0.1x_j} \{[Re\{T_j\}-Re\{T_o\}]^2 + [Im\{T_j\}-Im\{T_o\}]^2\}^{\frac{1}{2}} \, P_{x_j}^{\ell} \, dy \tag{4.2}$$



(a) Shenoi's measure for H-S and D-F circuits



(b) New measure, I, for H-S and D-F circuits

Figure 12. Two Different Measures for H-S and D-F Circuits

Table 1. Element Values for Deliyannis-Friend Circuit

Q	10	25	50	80
C_1	1.0	1.0	1.0	1.0
C_2	0.666	0.666	0.666	0.666
R_3	24.578	56.854	94.091	119.818
R_4	1.0	1.0	1.0	1.0
R_5	0.064	0.027	0.016	0.012
R_6	1.0	1.0	1.0	1.0
R_7	.0001	.0001	.0001	.0001
A_o	33,110	33,110	33,110	33,110

ohms, farads

Table 2. Element Values for Hamilton-Sedra Circuit

Q	10	25	50	80
C_1	1.0	1.0	1.0	1.0
C_2	1.0	1.0	1.0	1.0
C_3	2.0	2.0	2.0	2.0
R_4	0.649	0.659	0.663	0.664
R_5	2.167	2.062	2.030	2.019
R_6	1.0	1.0	1.0	1.0
R_7	1.0	1.0	1.0	1.0
R'_8	.996	.990	.980	.968
R'_9	38.967	98.800	198.205	316.976
R'_{10}	.0004	.0010	.020	.032
A_o	33,110	33,110	33,110	33,110

ohms, farads

where

$$\mathrm{Re}\{T_o\} = \frac{B_o F_o \omega}{(D_o - \omega)^2 + (B_o \omega)^2} \tag{4.3}$$

$$\mathrm{Im}\{T_o\} = \frac{F_o \omega (D_o - \omega^2)}{(D_o - \omega^2)^2 + (B_o \omega)^2} \tag{4.4}$$

$$B_o = \frac{x_4 x_5 x_8 [x_1 (x_6 + x_7) + x_2 x_6] + x_1 x_3 (x_4 + x_5)(x_6 - x_7 x_8)}{(1 + x_8) x_1 x_2 x_3 x_4 x_5 x_6} \tag{4.6}$$

$$D_o = \frac{x_4 + x_5}{x_1 x_2 x_3 x_4 x_5}$$

$$F_o = \frac{x_8 (x_6 + x_7)}{(x_8 + 1) x_2 x_4 x_6} \tag{4.6}$$

$$\mathrm{Re}\{T_j\} = \frac{B_j F_j \omega}{(D_j - \omega^2)^2 + (B_j \omega)^2} \tag{4.7}$$

$$\mathrm{Im}\{T_j\} = \frac{F_j \omega (D_j - \omega^2)}{(D_j - \omega^2)^2 + (B_j \omega)^2} \tag{4.8}$$

$$B_1 = \frac{x_4 x_5 x_8 [(x_1 + y)(x_6 + x_7) + x_3 x_6] + (x_1 + y) x_3 (x_4 + x_5)(x_6 - x_7 x_8)}{(1 + x_8)(x_1 + y) x_2 x_3 x_4 x_5 x_6} \tag{4.9}$$

$$D_1 = \frac{x_4 + x_5}{(x_1 + y) x_2 x_3 x_4 x_5} \tag{4.10}$$

$$F_1 = \frac{x_8(x_6+x_7)}{(x_8+1)x_2x_4x_6} \tag{4.11}$$

$$B_2 = \frac{x_4x_5x_8[x_1(x_6+x_7)+(x_2+y)x_6] + x_1x_3(x_4+x_5)(x_6-x_7x_8)}{(1+x_8)x_1(x_2+y)x_3x_4x_5x_6} \tag{4.12}$$

$$D_2 = \frac{x_4 + x_5}{x_1(x_2+y)x_3x_4x_5} \tag{4.13}$$

$$F_2 = \frac{x_8(x_6+x_7)}{(x_8+1)(x_2+y)x_4x_6} \tag{4.14}$$

$$\vdots$$

$$B_8 = \frac{x_4x_5(x_8+y)[x_1(x_6+x_7)+x_2x_6]+x_1x_3(x_4+x_5)(x_6-x_7(x_8+y))}{(1+x_8+y)x_1x_2x_3x_4x_5x_6} \tag{4.15}$$

$$D_8 = \frac{x_4 + x_5}{x_1x_2x_3x_4x_5} \tag{4.16}$$

$$F_8 = \frac{(x_8+y)(x_6+x_7)}{(x_8+y+1)x_2x_4x_6} \tag{4.17}$$

$$Q = \frac{1}{B_o}$$

$$\omega_1 = \omega_0 = 1.0 \quad \text{radian /sec} \tag{4.18}$$

$$\omega_2 = \omega_0 - \frac{1}{2Q} \quad \text{radians/sec} \tag{4.19}$$

$$\omega_3 = \omega_0 + \frac{1}{2Q} \quad \text{radians/sec} \tag{4.20}$$

$$\omega_4 = \omega_0 - \frac{1}{Q} \quad \text{radians/sec} \tag{4.21}$$

$$\omega_5 = \omega_0 + \frac{1}{Q} \quad \text{radians/sec} \tag{4.22}$$

The evaluation of the integral (4.2) was carried out by using the trapezoidal-rule numerical integration. The calculated new measures for different values of Q for the Deliyannis-Friend circuit are plotted in Figure 12(b).

Similarly, the new sensitivity measures for the Hamilton-Sedra circuit was evaluated in the same manner from (4.2) except that $j = 11$; $x_1 = C_1$, $x_2 = C_2$, $x_3 = C_3$, $x_4 = R_4$, $x_5 = R_5$, $x_6 = R_6$, $x_7 = R_7$, $x_8 = R_8$, $x_9 = R_9$, $x_{10} = R_{10}$; $x_{11} = A_o$ and

$$B_o = \frac{P_{10} + P_{40} + P_{50} + P_{80}}{P_{20} + x_2} \tag{4.23}$$

$$D_o = \frac{P_{30} + P_{60} + P_{70} + P_{90}}{P_{20} + x_2} \tag{4.24}$$

$$F_o = \frac{1}{x_4(P_{20} + x_2)} \tag{4.25}$$

where

$$P_{10} = \frac{[(1+x_{11})x_{10}+x_9][(x_4+x_5)x_6x_8 + (x_6+x_8)x_4x_5]}{x_{11}x_4x_5x_6x_8x_9} \tag{4.26}$$

$$P_{20} = \frac{[(1+x_{11})x_{10} + x_9]x_2}{x_{11}x_9} \tag{4.27}$$

$$P_{30} = \frac{[(1+x_{11})x_{10}+x_9](x_6+x_8)(x_4+x_5)}{x_{11}x_4x_5x_6x_8x_9x_1} \tag{4.28}$$

$$P_{40} = \frac{[(1+x_{11})x_{10}+x_9](x_6+x_8)x_2}{x_{11}x_6x_8x_9x_1} \tag{4.29}$$

$$P_{50} = \frac{x_1 + x_2}{x_1x_8} \tag{4.30}$$

$$P_{60} = \frac{x_4 + x_8}{x_1x_4x_5x_8} \tag{4.31}$$

$$P_{70} = \frac{x_9x_{11}-x_9-(1+x_{11})x_{10}}{x_1x_6x_7x_9x_{11}} \tag{4.32}$$

$$P_{80} = \frac{(x_1+x_2)(1+x_{11})}{x_1x_9x_{11}} \tag{4.33}$$

$$P_{90} = \frac{(1+x_{11})(x_4+x_5)}{x_1x_4x_5x_9x_{11}} \tag{4.34}$$

B_j, D_j, and F_j can be obtained from B_o, D_o, and F_o by replacing every x_j with $x_j + y$ for j = 1 through 11. For example, when j = 1 one gets

$$B_1 = \frac{P_{11} + P_{41} + P_{51} + P_{81}}{P_{21} + x_2} \tag{4.35}$$

$$D_1 = \frac{P_{31} + P_{61} + P_{71} + P_{91}}{P_{21} + x_2} \tag{4.36}$$

$$F_1 = \frac{1}{x_4(P_{21} + x_2)} \tag{4.37}$$

where

$$P_{11} = P_{10} \tag{4.38}$$

$$P_{21} = P_{20} \tag{4.39}$$

$$P_{31} = \frac{[(1+x_{11})x_{10}+x_9](x_6+x_8)(x_4+x_5)}{x_{11}x_4x_5x_6x_8x_9(x_1+y)} \tag{4.40}$$

$$P_{41} = \frac{[(1+x_{11})x_{10}+x_9](x_6+x_8)x_2}{x_{11}x_6x_8x_9(x_1+y)} \tag{4.41}$$

$$P_{51} = \frac{(x_1 + y) + x_2}{(x_1 + y)x_8} \tag{4.42}$$

$$P_{61} = \frac{x_4 + x_8}{(x_1+y)x_4x_5x_8} \tag{4.43}$$

$$P_{71} = \frac{x_7x_{11} - x_9 - (1+x_{11})x_{10}}{(x_1+y)x_6x_7x_9x_{11}} \tag{4.44}$$

$$P_{81} = \frac{(x_1+y+x_2)(1+x_{11})}{(x_1+y)x_9x_{11}} \tag{4.45}$$

$$P_{91} = \frac{(1+x_{11})(x_4+x_5)}{(x_1+y)x_4x_5x_9x_{11}} \tag{4.46}$$

The new sensitivity measures, I, for different values of Q for the Hamilton-Sedra circuit are also plotted in Figure 12(b). The values of the elements of the two circuits, Deliyannis-Friend and Hamilton-Sedra, used in calculating the new measure are the same as those used by Shenoi in Tables 1 and 2. Some values of the measure I for Figure 12(b) are given in Table 3 where $\Delta x_j/x_j = 0.1$; or, equivalently, a tolerance of 10 percent. The probability density function of the circuit elements were assumed to be uniformly distributed. The value of $\lambda(\omega)$ used in five different frequencies was unity. Thus, we may conclude that the Deliyannis-Friend circuit has a smaller 10 percent tolerance sensitivity than the Hamilton-Sedra circuit.

Table 3. New Measure, I, for Two Different Circuits

$$I = \sum_{i}^{m} \sum_{j}^{n} \frac{1}{x_{j_o}} \int_{x_{j_o}-\Delta x_j}^{x_{j_o}+\Delta x_j} \lambda(\omega_i) \left| F(x_j,\omega_i) - F(x_{j_o},\omega_i) \right|^k P_{x_j}^{\ell} \, d_{x_j}$$

Q	Deliyannis-Friend Circuit	Hamilton-Sedra Circuit
10	24.253	85.435
25	145.041	222.225
50	322.584	446.520
75	491.142	669.147
100	662.015	890.246
125	835.236	1115.437

Comparing the Measures of Continuously Equivalent Networks

According to Schoeffler, networks $\eta(0)$, $\eta(1)$, $\eta(2)$, and $\eta(3)$ in Figure 13 perform increasingly well in that sequence from the standpoint of traditional first-order sensitivity measures. Though Leed-Ugron [13] and Leon-Yokomoto did not agree with Schoeffler in numerical values, they agreed that the sensitivities are decreasing according to the sequence $\eta(1)$, $\eta(2)$, $\eta(3)$. Leon-Yokomoto and Cheetham claimed that their equivalent networks $\eta(4)$ and $\eta(5)$ of Figure 14 are even better than Schoefflers. This is evidenced by the measures tabulated in Table 4.

Table 4. First-Order Sensitivity Measures for Six Equivalent Networks

*	Schoeffler	Leon-Yokomoto	Cheetham	Leeds-Ugron	
$\eta(0)$	23.4	23.48	23.48	23.03	Original Network
$\eta(1)$	17.9	26.46	-	33.26	Schoeffler's Continuously Equivalent Networks
$\eta(2)$	11.9	20.17	-	20.16	
$\eta(3)$	5.52	19.14	-	19.14	
$\eta(4)$	-	18.35	-	-	Leon-Yokomoto's Equivalent Networks
$\eta(5)$	-	-	14.69	-	Cheetham's Equivalent Network

*Sensitivity measure calculated by.

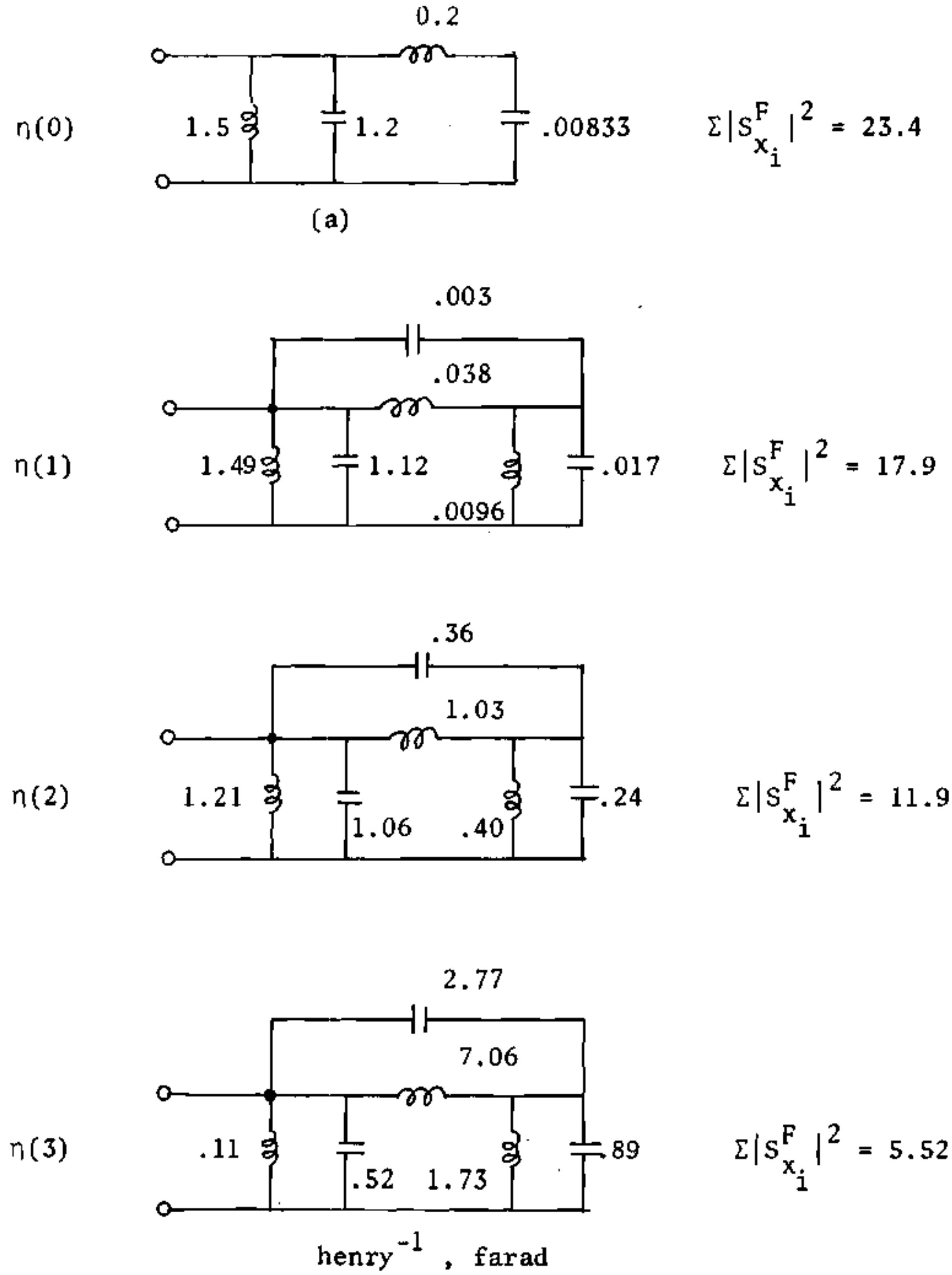
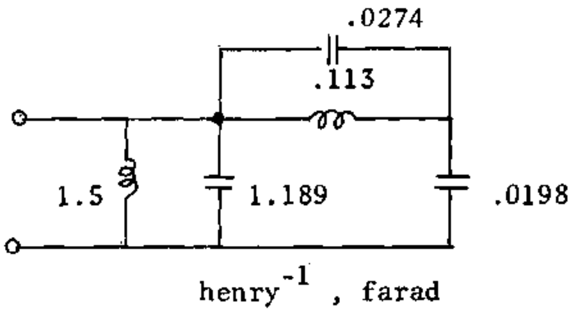


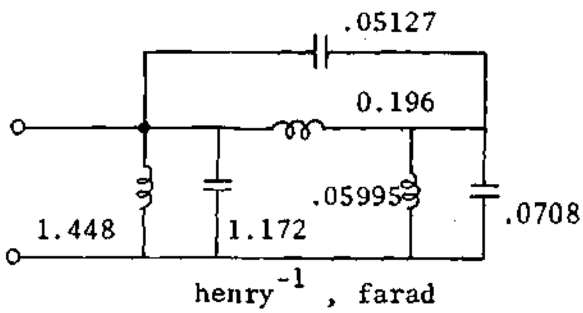
Figure 13. A Series of Continuously Equivalent Networks

η(4)



$\Sigma|S^F_{x_i}|^2 = 18.35$

η(5)



$\Sigma|S^F_{x_i}|^2 = 14.69$

Figure 14. Two Continuously Equivalent Networks

The traditional sensitivity measures according to Schoeffler, Leon-Yokomoto, and Cheetham are plotted versus the network sequence $\eta(0)$, $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$ in Figure 15 and Figure 16. The newly proposed measures for the same six networks; $\eta(0)$, $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$; for various element tolerances are calculated from the following equations at $\omega = 1.5$ radians per second.

$$I = \sum_{i=1}^{6} \frac{1}{x_i} \int_{-Ax_i}^{Ax_i} |F(x_1,\ldots x_i+y,\ldots x_6) - F(x_1,\ldots x_i,\ldots x_6)|^2 dy$$

$$= \sum_{i=1}^{6} \frac{1}{x_i} \int_{-Ax_i}^{Ax_i} |F_i - F_o|^2 dy$$

$$= \sum_{i=1}^{6} \frac{1}{x_i} \int_{-Ax_i}^{Ax_i} \{[\mathrm{Re}\{F_i\} - \mathrm{Re}\{F_o\}]^2 + [\mathrm{Im}\{F_i\} - \mathrm{Im}\{F_o\}]^2\} dy \quad (4.47)$$

where

$$\mathrm{Re}\{F_o\} = 0 \quad (4.48)$$

$$\mathrm{Re}\{F_i\} = 0 \quad (4.49)$$

$$\mathrm{Im}\{F_o\} = \frac{A_o\omega^4 - B_o\omega^2 + C_o}{\omega(D_o - F_o\omega^2)} \quad (4.50)$$

$$\mathrm{Im}\{F_i\} = \frac{A_i\omega^4 - B_i\omega^2 + C_i}{\omega(C_i - F_i\omega^2)} \quad (4.51)$$

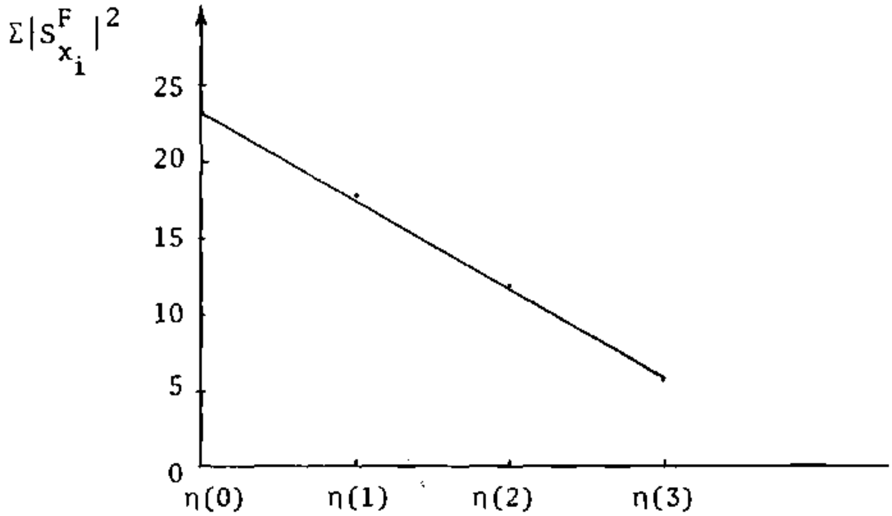


Figure 15. Sensitivity Measures Given by Schoeffler

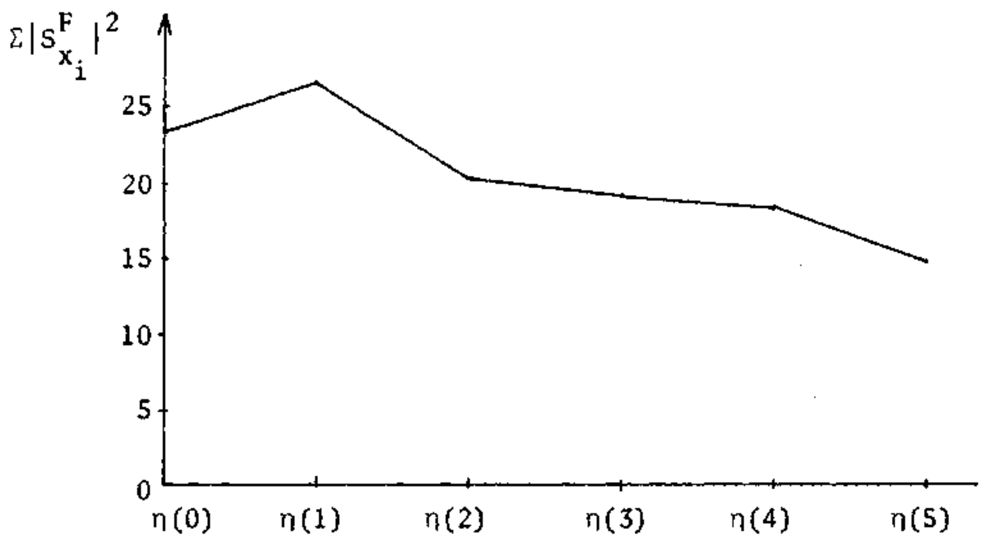


Figure 16. Sensitivity Measure Given by Leon-Yokomoto and Cheetham

$$A_o = x_1(x_2 + x_3) + x_2x_3 \tag{4.52}$$

$$B_o = x_4(x_2 + x_3) + x_5(x_1 + x_3) + x_6(x_1 + x_2) \tag{4.53}$$

$$C_o = x_4(x_5 + x_6) \tag{4.54}$$

$$D_o = x_4 + x_6 \tag{4.55}$$

$$F_o = x_1 + x_3 \tag{4.56}$$

A_i, B_i, C_i, D_i, and F_i for i = 1 to i = 6 can be obtained from A_o, B_o, C_o, D_o, and F_o by replacing each x_i with $x_i + y$. For instance,

$$A_1 = (x_1 + y)(x_2 + x_3) + x_2x_3 \tag{4.57}$$

$$B_1 = x_4(x_2 + x_3) + x_5(x_1 + y + x_3) + x_6(x_1 + y + x_2) \tag{4.58}$$

$$C_1 = x_4(x_5 + x_6) \tag{4.59}$$

$$D_1 = x_4 + x_6 \tag{4.60}$$

$$F_1 = x_1 + y + x_3 \tag{4.61}$$

The elements x_1, x_2, x_3, x_4, x_5, and x_6 corresponding to the six networks $\eta(0)$, $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$ are shown in Figure 17 and their values are tabulated in Table 5. The number A in equation (4.47) is the tolerance.

The calculated new measures for various element tolerances are tabulated in Table 6. These measures are plotted in Figures 18 through 21. For element tolerance of 5 percent ($\Delta x/x_o = 0.05$, or A = 0.05), it is seen that, according to the measure I, the five

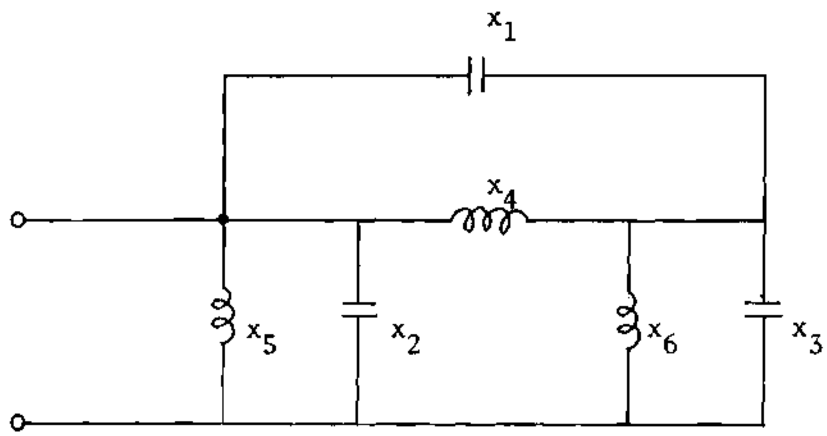


Figure 17. Elements Assignment for Evaluating the New Measure, I

Table 5. Element Values for Six Equivalent Networks

Networks \ Elements	x_1	x_2	x_3	x_4	x_5	x_6
$\eta(0)$	0	1.2	0.0833	.02	1.5	0
$\eta(1)$	.003	1.12	.017	.038	1.49	.0096
$\eta(2)$	.36	1.06	.24	1.03	1.21	.40
$\eta(3)$	2.77	.52	.89	7.06	.11	1.73
$\eta(4)$	.0274	1.189	.0198	.113	1.50	0
$\eta(5)$	.05127	1.172	.0708	0.196	1.448	.05995

Table 6. New Measure, I, for Six Equivalent Networks

Networks \ Element Tolerance $\Delta x/x_o$	0.05	0.10	0.15	0.20
$\eta(0)$	.00712	1.172	1.196	12.258
$\eta(1)$	.00697	1.992	2.031	2.075
$\eta(2)$	.00267	0.874	18.627	30.409
$\eta(3)$	.00181	12.804	14.865	18.007
$\eta(4)$	.00448	1.586	3.115	5.197
$\eta(5)$	.00171	2.473	7.591	7.795

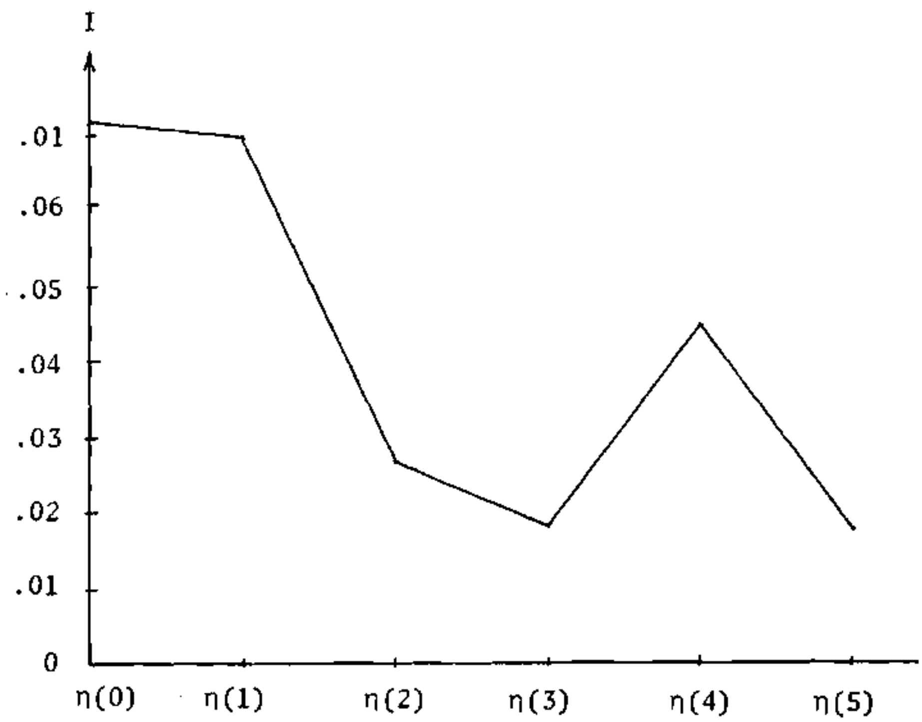


Figure 18. New Measure, I, for $\Delta x/x_o = 0.05$

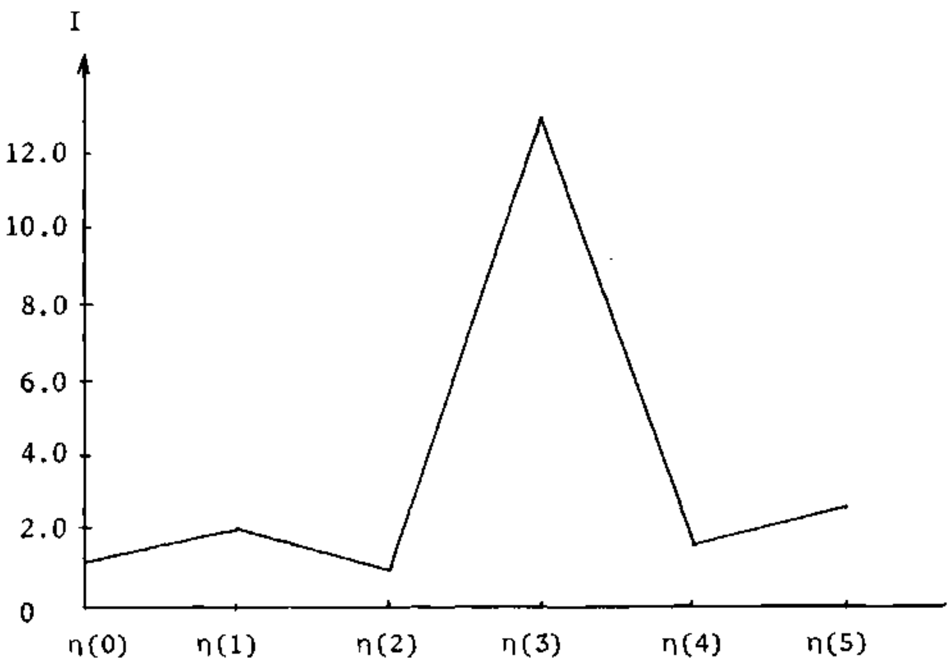


Figure 19. New Measure, I, for $\Delta x/x_o = 0.10$

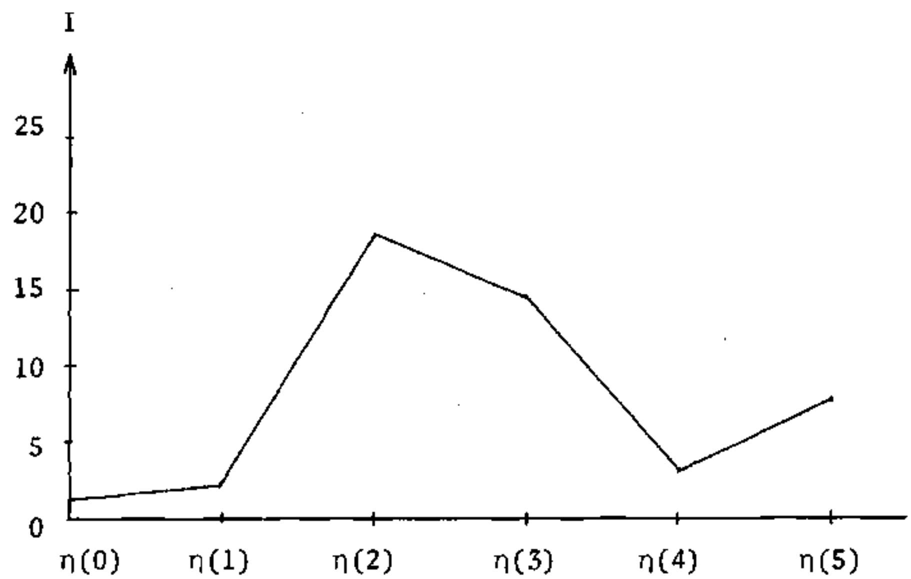


Figure 20. New Measure, I, for $\Delta x/x_o = 0.15$

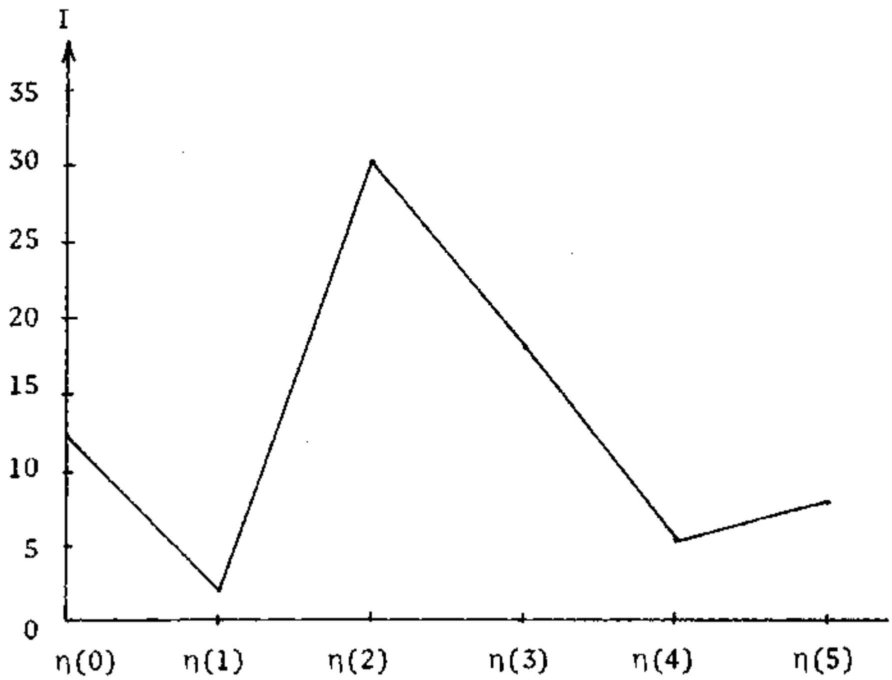


Figure 21. New Measure, I, for $\Delta x/x_o = 0.20$

networks, $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$ do not perform increasingly well in that order. From Figure 18 to Figure 21, it is found that the performance of a network depends heavily on the tolerances of the network elements.

From these facts it can be concluded that the property that the sensitivity measure decreases as the number of elements increases in continuously equivalent networks claimed by Leeds and Ugron [13] is not necessarily true when the element tolerances are finite.

Convergence of Numerical Integration of the New Measure

The evaluation of the integral of the new measure in (4.47) was carried out by the trapezoidal rule numerical integration method. In earlier runs for a tolerance of 10 percent, several difficulties were encountered. The maximum permissible relative error between successive evaluations of the integral was set to 10^{-4} and the maximum permissible number of evaluations of the integral to be computed before non-convergence status was declared was set to 50. In several runs, the computer estimated run time of one minute for the Univac 1108 computer was exceeded. The computer run time was extended until the maximum permissible number of evaluations of the integral was reached. Nevertheless, the relative error between successive evaluations of the integral was still larger than 10^{-3}. Other numerical integration methods such as Simpson's 1/3 rule and Simpson's 3/8 rule were also tried, but the results were equally unsatisfactory. Then the interval of integration was divided into ten sections between $x/x_o = 0.9$ and $x/x_o = 1.1$ as shown in Figure 22; and the numerical integration was

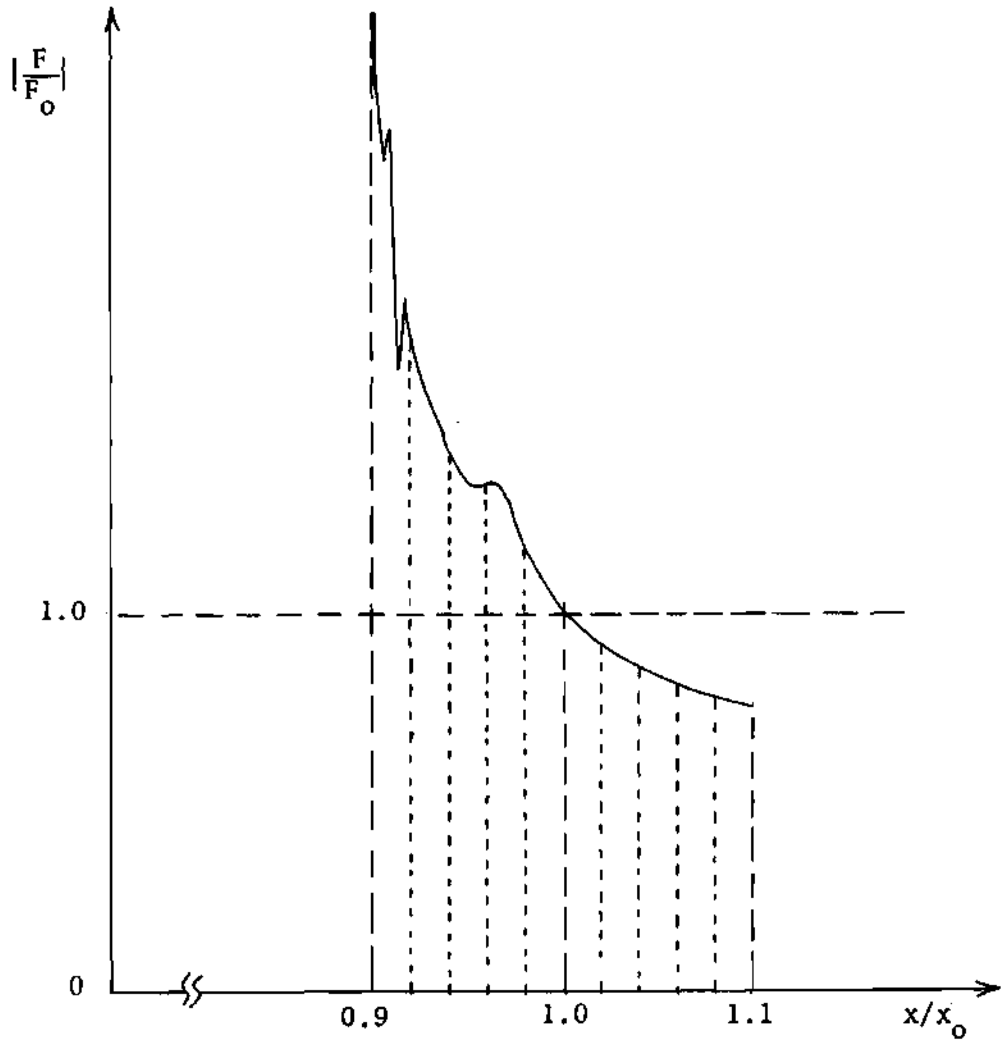


Figure 22. Divisions of Integration Interval

carried out independently for each section. It was found that the numerical integration converged in most of the sections. The integrand in those sections for which the computation did not converge was calculated and inspected for a fairly fine interval of 0.001 x_o. The tabulated values revealed that there was an infinity in that section as shown in Figure 23. The infinity was then handled by setting it equal to an arbitrary large number, say 10^4. The division of the whole interval between the lower and higher limits of the tolerance into many sections is a good way to save the computer time and detect any troublesome regions. For those parts of a performance curve that are relatively smooth it is not necessary to have a large number of functions evaluated as those parts that are not smooth, before the error between successive evaluation of the integral reaches the preset small error figure. Only in these sections in which the curve is not smooth did the algorithm require very small steps of the independent variable at which the function is to be evaluated. This scheme reduces the total computer time greatly.

The role of an infinity within the tolerance limit in the network optimization will be discussed in Chapter V.

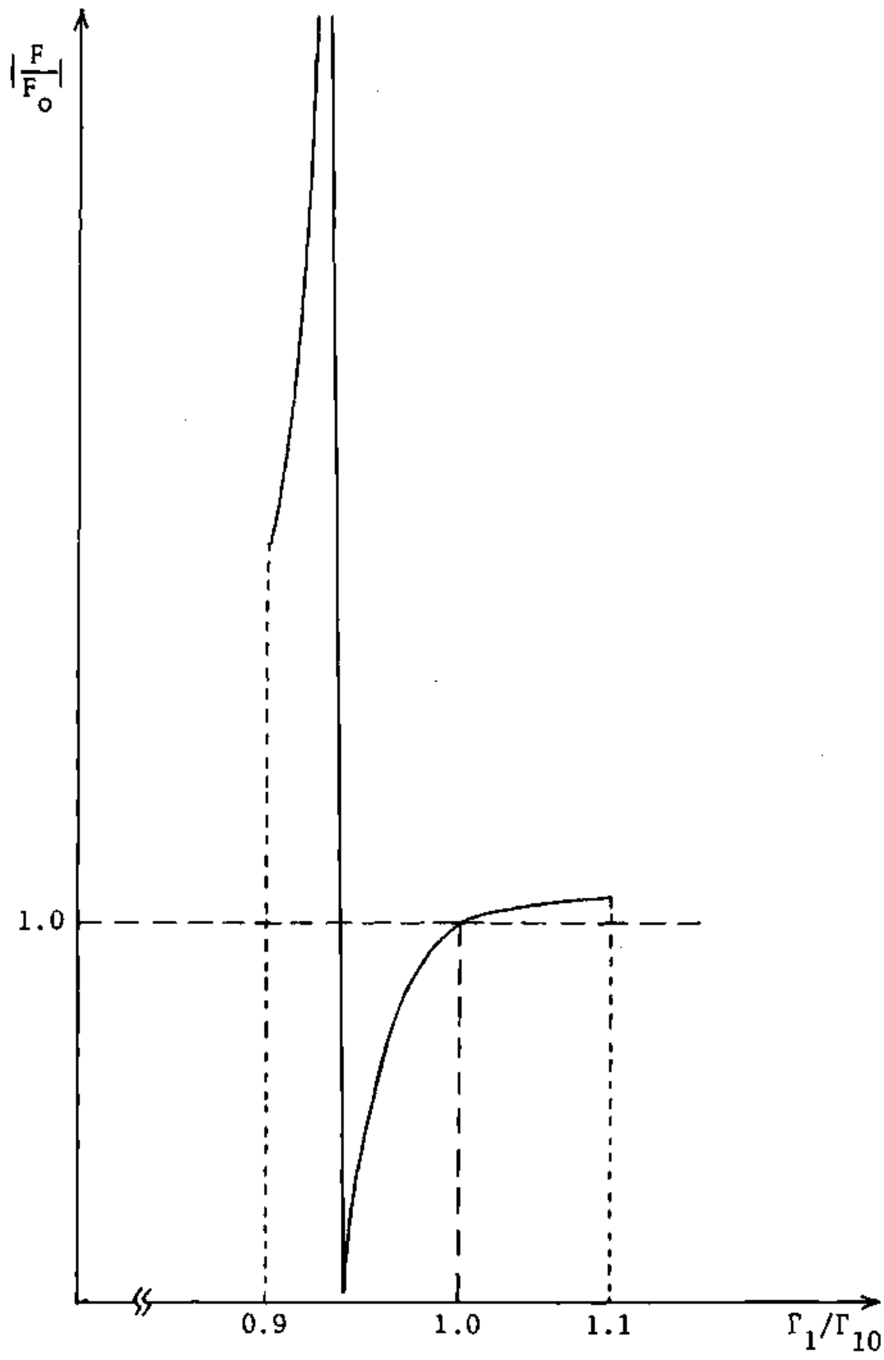


Figure 23. Infinity on $\eta(0)$

CHAPTER V

OPTIMIZATION OF NETWORK DESIGN BASED ON THE NEW MEASURE AND MULTIVARIABLE CONTINUOUSLY EQUIVALENT NETWORKS

One of the two purposes of sensitivity study--the comparative evaluation of circuits realizing the same network function--was discussed in the preceding chapter. The other purpose of sensitivity study--the optimal network design--will be discussed in this chapter. The variation of the new all-tolerance multiparameter sensitivity measure with respect to circuit elements of a circuit will be studied first. Then methods of obtaining optimal networks will be discussed. Finally, an example of getting an optimal network by minimizing the new sensitivity measure along with multivariable continuously equivalent networks will be presented.

Variation of the New Measure versus Circuit Element Values

In many circuit design problems, particularly in active networks, there exists one or more parameters that can be chosen arbitrarily insofar as the realization of a given network function is concerned. The existence of these parameters indicates that there are several degrees of freedom that can be utilized to good advantage. It offers an opportunity to effect a good design under certain figures of merit or criterion. An example of such a utilization is the Horowitz decomposition in active network synthesis [27].

In this research, when such opportunities are present, the possibility of minimizing the new all-tolerance multiparameter sensitivity will be investigated. Given a certain network function and after a certain circuit configuration has been chosen, the question becomes: How does the new sensitivity measure, I, vary with respect to the arbitrary parameter(s)? Does it have a minimum? If so, where and what is the minimum? How can the parameter(s) be chosen to render the minimum sensitivity measure? The following example illustrates a typical situation in which these questions are answered.

Example 1

A well-known practical active RC circuit is the Sallen and Key filter section shown in Figure 24. This filter circuit [28] has been discussed by many [see, for example, 5, 29]. This circuit is chosen as an example here for the purpose of demonstrating the variation of the new measure of the filter versus the variation of an arbitrary element value.

Assume that the circuit in Figure 24 is to realize the following voltage transfer function

$$T(s) = \frac{10}{s^2 + \frac{1}{Q} s + 1} \tag{5.1}$$

It is desired to find the variation of the new sensitivity measure versus the variation of some of the circuit elements when $Q = 2$, $R_1 = 1$ ohm, and element tolerance is 10 percent.

In order to find the new measure, I, the first step is to write the voltage transfer function with its coefficients represented in

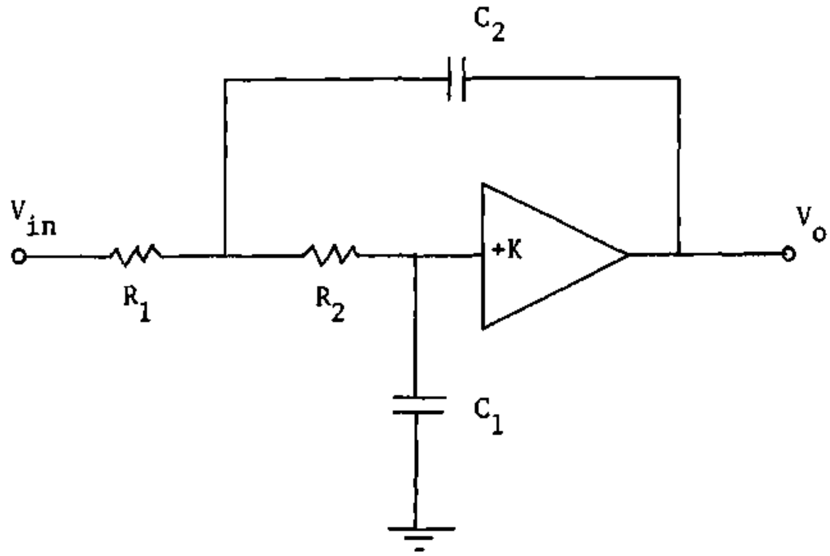


Figure 24. Sallen and Key Active Filter

terms of element parameters. It is easily shown that

$$T(s) = \frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2 + \dfrac{R_1C_1 + R_1C_2 + R_2C_2 - KR_1C_1}{R_1R_2C_1C_2} s + \dfrac{1}{R_1R_2C_1C_2}} \tag{5.2}$$

Comparing (5.1) and (5.2), three constraint equations are obtained.

$$\frac{K}{R_1R_2C_1C_2} = 10 \tag{5.3}$$

$$\frac{1}{R_1R_2C_1C_2} = 1 \tag{5.4}$$

$$\frac{R_1C_1 + R_1C_2 + R_2C_2 - KR_1C_1}{R_1R_2C_1C_2} = \frac{1}{Q} \tag{5.5}$$

Now we suppose that the value of K is fixed to be 10. Since R_1 is fixed to be 1 ohm; for a given value of R_2, the values of C_1 and C_2 that realize (5.1) can be found from (5.4) and (5.5).

$$C_2 = \frac{\dfrac{1}{Q} \pm \sqrt{\dfrac{1}{Q^2} - 4\,\dfrac{(R_1+K_2)(1-K)}{R_2}}}{2(R_1+R_2)} \tag{5.6}$$

$$C_1 = \frac{1}{C_2R_1K_2} \tag{5.7}$$

That is, when the nominal value of R_2 is varied in a certain range, one will be able to see how the sensitivity measure, I, of this circuit varies.

Since the circuit in Figure 24 is a low-pass filter the new measure, I, is calculated at the following different frequency points

$\omega_1 = 0$ radian per second

$\omega_2 = 0.2$ radian per second

$\omega_3 = 0.4$ radian per second

$\omega_4 = 0.6$ radian per second

$\omega_5 = 0.8$ radian per second

$\omega_6 = 1.0$ radian per second

Let $x_1 = K$, $x_2 = R_1$, $x_3 = R_2$, $x_4 = C_1$, and $x_5 = C_2$, the new measure is calculated by

$$I = \sum_{i=1}^{6} \sum_{j=1}^{5} \int_{-0.1x_j}^{+0.1x_j} |T_j(x_j + y, \omega_i) - T_o(x_j, \omega_i)|^2 dy$$

$$= \sum_{i=1}^{6} \sum_{j=1}^{5} \int_{-0.1x_j}^{+0.1x_j} \{[\mathrm{Re}\{T_j\} - \mathrm{Re}\{T_o\}]^2 + [\mathrm{Im}\{T_j\} - \mathrm{Im}\{T_o\}]^2\} dy$$

where

$$\mathrm{Re}\{T_o\} = \frac{H_o(D_o - \omega_i^2)}{(D_o - \omega_i^2)^2 + (B_o\omega_i)^2} \tag{5.8}$$

$$\mathrm{Im}\{T_o\} = \frac{-H_o B_o \omega_i}{(D_o - \omega_i)^2 + (B_o\omega_i)^2} \tag{5.9}$$

$$\mathrm{Re}\{T_j\} = \frac{H_j(D_j - \omega_i^2)}{(D_j - \omega_i^2)^2 + (B_j\omega_i)^2} \tag{5.10}$$

$$\operatorname{Im}\{T_j\} = \frac{-H_j B_j \omega_i}{(D_j - \omega_i^2)^2 + (B_j \omega_i)^2} \tag{5.11}$$

$$B_o = \frac{x_2x_4 + x_2x_5 + x_3x_5 - x_1x_2x_4}{x_2x_3x_4x_5} \tag{5.12}$$

$$D_o = \frac{1}{x_2x_3x_4x_5} \tag{5.13}$$

$$H_o = \frac{x_1}{x_2x_3x_4x_5} \tag{5.14}$$

B_j, D_j, and H_j are obtained from B_o, D_o, and H_o by replacing x_j with $x_j + y$. For instance, B_2, D_2, and H_2 are obtained from B_o, D_o, and H_o by replacing x_2 with $x_2 + y$

$$B_2 = \frac{(x_2+y)x_4 + (x_2+y)x_5 + x_3x_5 - x_1(x_2+y)x_4}{(x_2+y)x_3x_4x_5} \tag{5.15}$$

$$D_2 = \frac{1}{(x_2+y)x_3x_4x_5} \tag{5.16}$$

$$H_2 = \frac{x_1}{(x_2+y)x_3x_4x_5} \tag{5.17}$$

Some of the calculated values of C_1, C_2, and the new measure, I, versus R_2 are tabulated in Table 7. The data in Table 7 are plotted in Figure 25 where the ordinate is I and the abscissa is R_2 in ohms. This is a rough sketch of the variation of the new measure, I, versus the variation of R_2. It is seen from the figure that there is a

Table 7. Variation of I versus a Wide Range of R_2 for SK #1 Circuit

$A = 10$, $K = 100$, $R_1 = 1$			
R_2	C_1	C_2	I*
10^{-4}	33.3072	300.235	5.44503
10^{-3}	10.5185	95.0710	8.11952
10^{-2}	3.32230	30.0997	15.3207
10^{-1}	1.07811	9.27547	28.6952
1	.444444	2.25000	5.44550
10	.322927	.309668	.17567
10^{2}	.308368	.032429	4.78951
10^{3}	.306877	.003259	4.98391
10^{4}	.306744	.000652	4.99738
ohms , farads			

*Element Tolerance = 10%

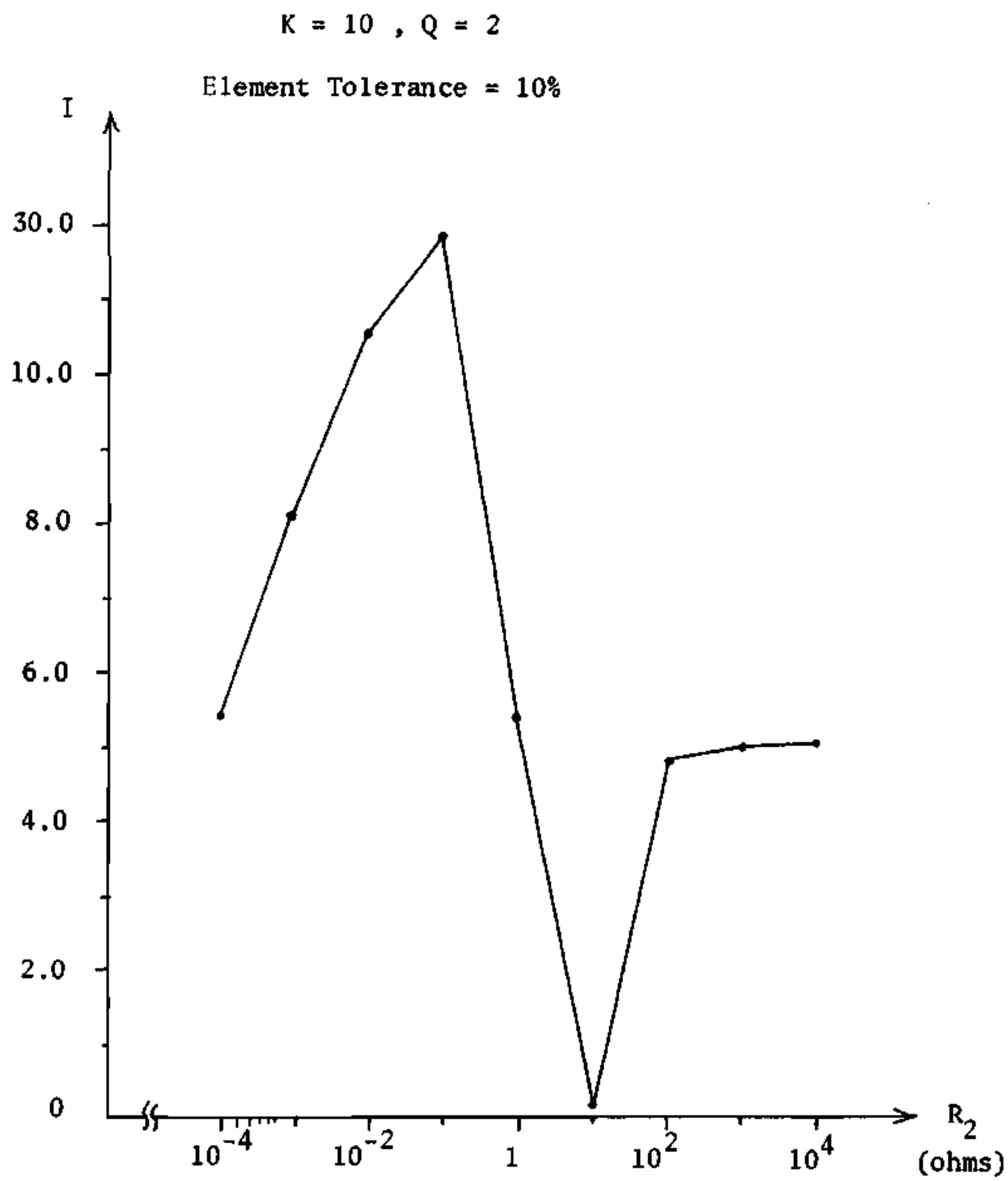


Figure 25. Rough Variation of I versus R_2 for SK #1 Circuit

minimum of I in the neighborhood of R_2 = 10 ohms. Another set of data in the neighborhood of R_2 = 10 ohms are taken and are tabulated in Table 8. The data in Table 8 are plotted in Figure 26. It is seen that the variation of I, for 10-percent element tolerance, versus R_2 is a smooth one. When K = 10, Q = 2 and R_1 = 1 ohm, the new measure, I, exhibits a minimum in the neighborhood of R_2 = 9.9 ohms.

Methods of Obtaining Optimal Design of a Network

(a) Single Independent Variable

In obtaining an optimal design of a network from the sensitivity viewpoint, we minimize the sensitivity measure. The preceding example shows that for K = 10, Q = 2 and R_1 = 1 ohm, I has a minimum when R_2 = 9.9 ohms, C_1 = 0.3231 farad, and C_2 = 0.3126 farad. In other words, if K, Q, and R_1 are fixed at K = 10, Q = 2 and R_1 = 1 ohm, the optimal design of the network is obtained by varying R_2 from a small value to a large value until a minimum measure I is found. The values of C_1 and C_2 for the optimal design are obtained from the two constraint equations (5.4) and (5.5). That is to say: <u>If there is only one independent variable, all one needs to do in finding the minimum measure I for an optimal design is to vary the independent variable within a range such that all the corresponding element values from the constraint equations are in the acceptable ranges</u>. When a minimum I is found, the corresponding value of the independent variable is the optimal design value for that element. The optimal values of other elements are then obtained from the constraint equations.

Table 8. Variation of I in the Neighborhood of R_2 = 10 Ω for SK #1 Circuit

Q = 2 , K = 10 , R_1 = 1 , Tolerance = 10%			
R_2	C_1	C_2	I
9.1	.324490	.338655	1.85142
9.2	.324302	.335168	1.02285
9.3	.324117	.331753	.638396
9.4	.323937	.328407	.425308
9.5	.323760	.325127	.299862
9.6	.323586	.321913	.226094
9.7	.323417	.318762	.185582
9.8	.323250	.315671	.167705
9.9	.323087	.312641	.165848
10.0	.322927	.309668	.175668
10.2	.322616	.303888	.219212
10.4	.322317	.298321	.283071
10.6	.322029	.292954	.358785
10.8	.321752	.287777	.441306
11.0	.321484	.282780	.527489
12.0	.320277	.260192	.961903
13.0	.319252	.240948	1.39542
14.0	.318371	.224356	1.69638
15.0	.317606	2.09904	1.99005

ohms, farads

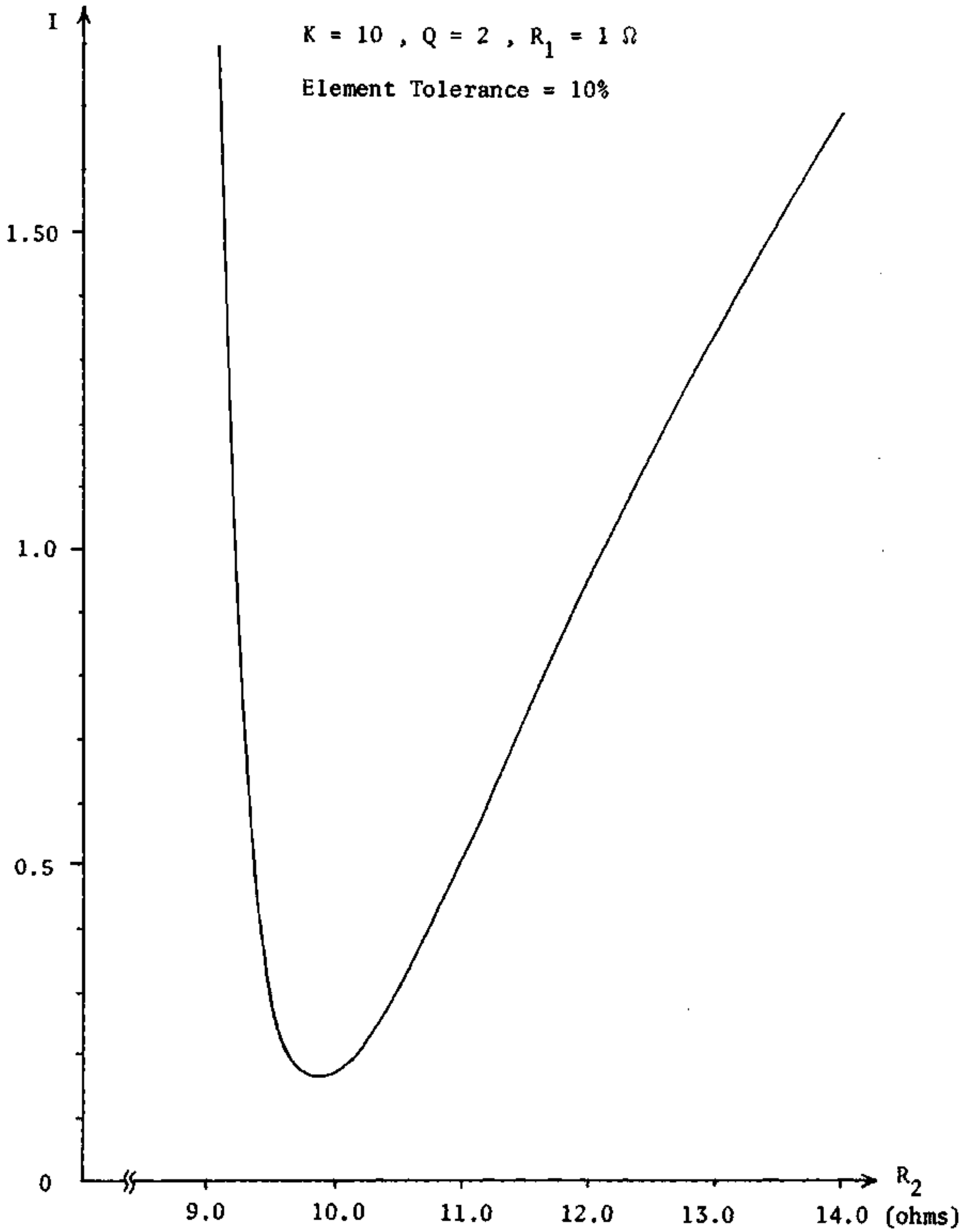


Figure 26. Variation of I versus R_2 for SK #1 Circuit

Example 2

We shall now extend the same design and example in Example 1 to (a) find the optimal designs of the circuit in Figure 24 to realize (5.1) for Q = 2, 4, 6, 8, and 10 when the element tolerance is 10 percent, and (b) find how the sensitivity measure, I, of the optimal circuit in Figure 24 varies with Q in (5.1) when the element tolerance is 10 percent.

Repeating the same design carried out in Example 1 for Q = 4, 6, 8, and 10, the final results are tabulated in Table 9. It is seen that when Q varies from 2 to 10, the new measure, I, varies from 0.16498 to 0.87698. This is also depicted in Figure 27. It is seen that as Q increases, measure I increases almost proportionally.

Table 9. Optimal Designs for Various Values of Q for SK #1 Circuit

Q	K	R_1	R_2	C_1	f_1	I
2	10	1.0	9.86	.323152	.313846	.164983
4	10	1.0	9.33	.337128	.317924	.413432
6	10	1.0	9.23	.341789	.316986	.619712
8	10	1.0	9.19	.344125	.316205	.768519
10	10	1.0	9.17	.345527	.315609	.876976

Element Tolerance = 10% , ohms, farads

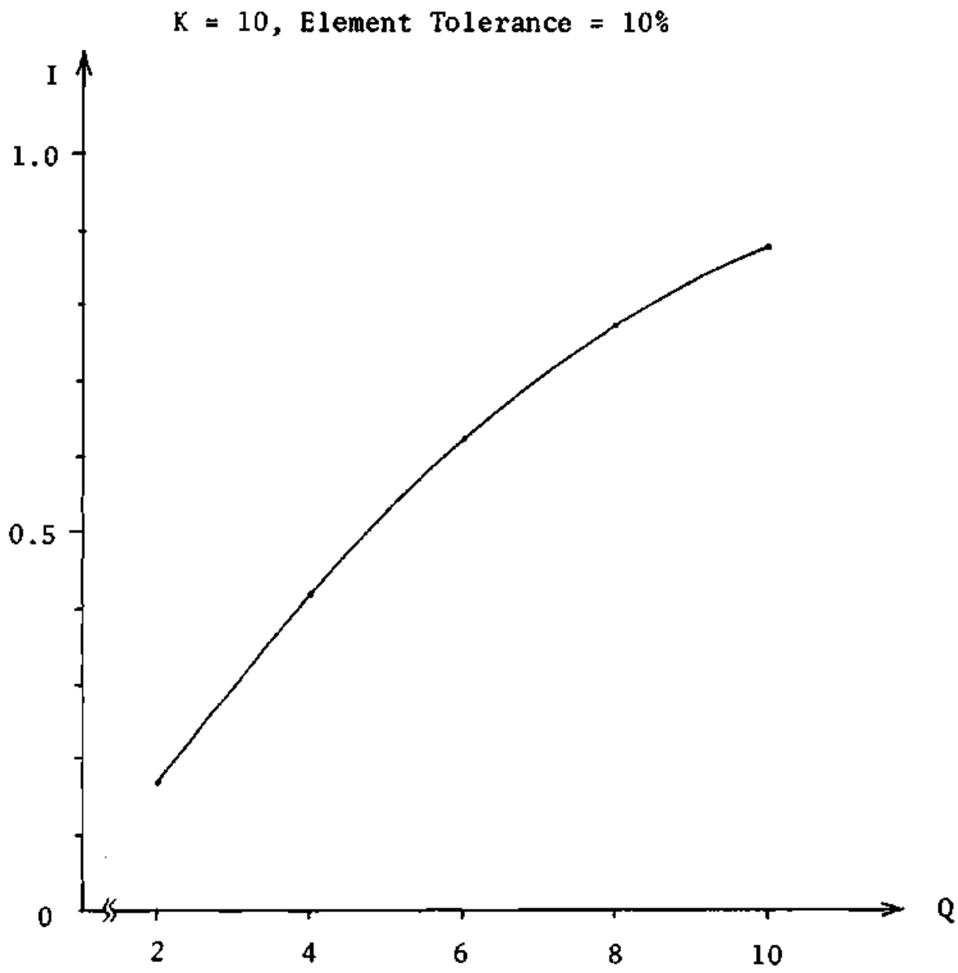


Figure 27. I versus Q for Optimal Design of SK #1 Circuit

(b) Multiple Independent Variables

For a network that has more than one available degree of freedom in realizing a network function, the method of getting a minimum measure for the optimal design of the network is much more complicated. The minimum measure can no longer be obtained readily by simply varying the independent variables in the applicable ranges. It is especially difficult to find the minimum measure when there are a large number of independent variables. Under this circumstance, some kind of minimization scheme needs to be employed. There are several schemes available such as the steepest-descent method, the conjugate-gradient method, and the Fletcher-Powell method [30]. The Fletcher-Powell algorithm which is known to be one of the best algorithms available for function minimization and has been used extensively in many applications will be chosen to serve this purpose.

There are always some constraints on the values of the network elements. For instance, all the network element values are constrained to be nonnegative. The network elements are also subjected to the constraint equations such that the network satisfies the given network function. Because of these reasons, the minimization of the new measure for any network belongs to constrained function minimization. The Fiaco and McCormick method enables us to transform a constrained minimization problem into an unconstrained one.

Transformation of a Constrained Problem Into an Unconstrained Problem

The first step of the minimization of the proposed measure, I, with respect to $(x_1,x_2,\ldots,x_j,\ldots,x_n)$, where $j = n - N_c$ is the number of independent variables subject to constraints $x_m \geq 0$, $m = 1,2,3,\ldots,n$, and n is the number of network elements, is to transform the constrained minimization problem into a sequence of unconstrained problems in such a way that the difficulties associated with the motion along the constrained boundary are avoided. The particular transformation proposed by Carroll [31] and proved by McCormick [32, 33] will be used in this section. The transformation is accomplished by the use of the function

$$P(x,r) = I(x) + r \sum_{i=1}^{m} \frac{w_i}{g_i(x)} \tag{5.18}$$

where $I(x)$ is the objective function or the new measure, $g_i(x)$ is the ith constraint function, w_i is a positive weighting function, and r is a member of a monotone decreasing sequence. For any fixed r, the unconstrained minimization technique of Fletcher-Powell can be used on equation (5.18). In using the technique one needs to find the partial derivatives of the function $P(x,r)$ with respect to each of the $n - N_c$ independent variables. Once the minimum point x_{min} is found, the element values, $x_1,x_2,\ldots,x_j$ are the optimal design values of the network. The remaining N_c element values of the optimal design are determined by the N_c equations in (3.9).

Partial Derivatives of the New Measure with Respect to the Independent Network Elements

In order to use the Fletcher-Powell minimization technique to find the optimal network element values, it is necessary to find the partial derivative of the cost function $P(x,r)$ of equation (5.1) with respect to each of the independent network elements. In other words, one needs to find the partial derivative of the new measure, $I(x)$, and the equation $1/g_i(x)$ with respect to each of the independent network elements. Since the constraint equations $g_i(x)$, $i = 1,\ldots,m$, usually are not complicated, their partial derivatives will not be discussed here. A general form of partial derivative of any network function F with respect to any independent network element at a certain frequency is presented in the following. Assume that the probability density function of the element parameter x_j is uniform, then the all-tolerance multiparameter sensitivity measure, I, in equation (2.5) can be written as

$$I = \sum_{i=1}^{n} \frac{1}{x_i} \int_{-A_i x_i}^{A_i x_i} |F(x_1,\ldots,x_i+y,\ldots,x_n) - F(x_1,\ldots,x_i,\ldots,x_n)|^2 dy \tag{5.19}$$

where A_i is the tolerance of the ith element parameter. Further assume

$$F_i = F(x_1,\ldots,x_i+y,\ldots,x_n)$$

$$F_o = F(x_1,\ldots,x_i,\ldots,x_n)$$

equation (5.19) can be written as

$$I = \sum_{i=1}^{n} \frac{1}{x_i} \int_{-A_i x_i}^{A_i x_i} |F_i - F_o|^2 dy$$

$$= \sum_{i=1}^{n} \frac{1}{x_i} \int_{-A_i x_i}^{A_i x_i} |[\mathrm{Re}\{F_i\} - \mathrm{Re}\{F_o\}] + j[\mathrm{Im}\{F_i\} - \mathrm{Im}\{F_o\}]|^2 dy$$

$$= \sum_{i=1}^{n} \frac{1}{x_i} \int_{-A_i x_i}^{A_i x_i} \{[\mathrm{Re}\{F_i\} - \mathrm{Re}\{F_o\}]^2 + [\mathrm{Im}\{F_i\} - \mathrm{Im}\{F_o\}]^2\} dy$$

$$= \sum_{i=1}^{n} \frac{1}{x_i} \int_{-A_i x_i}^{A_i x_i} f_i(y,x_i)\, dy \tag{5.20}$$

where

$$f_i(y,x_i) = [\mathrm{Re}\{F_i\} - \mathrm{Re}\{F_o\}]^2 + [\mathrm{Im}\{F_i\} - \mathrm{Im}\{F_o\}]^2 \tag{5.21}$$

The partial derivative of I with respect to x_j, the nominal value of an independent element parameter, is

$$\frac{\partial I}{\partial x_j} = \frac{1}{\partial x_j} \left[\sum_{i=1}^{n} \frac{1}{x_i} \int_{-A_i x_i}^{A_i x_i} f_i(y,x_i)\, dy \right]$$

$$= \sum_{i=1}^{n} \left[\frac{\partial}{\partial x_j} \left(\frac{1}{x_i}\right) \int_{-A_i x_i}^{A_i x_i} f_i(y,x_i)\, dy + \frac{1}{x_i} \frac{1}{\partial x_j} \int_{-A_i x_i}^{A_i x_i} f_i(y,x_i)\, dy \right]$$

$$= \sum_{i=1}^{n} - \frac{1}{x_i^2} \frac{\partial x_i}{\partial x_j} \int_{-A_i x_i}^{A_i x_i} f_i(y,x_i)\,dy$$

$$+ \sum_{i=1}^{n} \frac{1}{x_i} \left(\int_{-A_i x_i}^{A_i x_i} \frac{\partial}{\partial x_j} [f_i(y,x_i)]\,dy + A_i \frac{\partial x_i}{\partial x_j} [f_i(A_i x_i, x_i) + f_i(-A_i x_i, x_i)] \right)$$

(5.22)

Substituting equation (5.4) into equation (5.5), $\partial I/\partial x_j$ becomes

$$\frac{\partial I}{\partial x_j} = \sum_{i=1}^{n} (G_1 + G_2 + G_3) \tag{5.23}$$

where

$$G_1 = - \frac{1}{x_i^2} \frac{\partial x_i}{\partial x_j} \int_{-A_i x_i}^{A_i x_i} \left([\mathrm{Re}\{F_i\} - \mathrm{Re}\{F_o\}]^2 + [\mathrm{Im}\{F_i\} - \mathrm{Im}\{F_o\}]^2 \right) dy \tag{5.24}$$

$$G_2 = \frac{2}{x_i} \left\{ \int_{-A_i x_i}^{A_i x_i} \left([\mathrm{Re}\{F_i\} - \mathrm{Re}\{F_o\}] \cdot \left[\frac{\partial \mathrm{Re}\{F_i\}}{\partial x_j} - \frac{\partial \mathrm{Re}\{F_o\}}{\partial x_j}\right] \right. \right.$$

$$\left. \left. + [\mathrm{Im}\{F_i\} - \mathrm{Im}\{F_o\}] \cdot \left[\frac{\partial \mathrm{Im}\{F_i\}}{\partial x_j} - \frac{\partial \mathrm{Im}\{F_o\}}{\partial x_j}\right] \right) dy \right\} \tag{5.25}$$

$$G_3 = \frac{A_i}{x_i}\frac{\partial x_i}{\partial x_j}\Big([\mathrm{Re}\{F(x_1,\ldots(1+A_i)x_i,\ldots x_n)\} - \mathrm{Re}\{F(x_1,\ldots x_i,\ldots x_{no})\}]^2$$

$$+ [\mathrm{Im}\{F(x_1,\ldots(1+A_i)x_i,\ldots x_n)\} - \mathrm{Re}\{F(x_1,\ldots x_i,\ldots x_{no})\}]$$

$$+ [\mathrm{Re}\{F(x_1,\ldots(1-A_i)x_i,\ldots x_n)\} - \mathrm{Re}\{F(x_1,\ldots x_i,\ldots x_{no})\}]^2$$

$$+ [\mathrm{Im}\{F(x_1,\ldots(1-A_i)x_i,\ldots x_n)\} - \mathrm{Im}\{F(x_1,\ldots x_i,\ldots x_{no})\}]^2\Big) \tag{5.26}$$

Once the partial derivative $\partial I/\partial x_j$ in equation (5.23) is obtained, it is easy to find all of the partial derivatives of I with respect to every independent element parameter of a network. Once all partial derivatives are obtained, it is just a routine matter to use the Fletcher-Powell minimization technique to find an optimal design of a network.

Starting Point and Optimal Step Size

The Fletcher-Powell algorithm is used to find the unconstrained local minimum of equation (5.18). The success of finding the local minimum depends heavily on the proper choice of the starting point of the independent network elements and on the choice of a scalar step size. If the scalar step size chosen is too large the next values chosen for the independent network elements may be beyond the boundaries of the constraints. On the other hand, if the step size chosen is too small, the number of iterations may be excessive.

One way of finding a starting point is to tabulate some of the values of the cost function by varying the values of the independent network elements in a random way. The point that has a smallest cost function amongst the tabulated points is then chosen as a starting point. When the number of the independent network elements are small, a rough, systematic tabulation of the cost function versus the network elements can even be employed in locating a good starting point.

In order to prevent the values of the independent network elements from moving over the constraint boundaries, two additional steps are incorporated in the Fletcher-Powell algorithm. The first step is to choose a scalar step size that produces only a small change in the function, roughly of the order of 1 percent. Then the value of the scalar step size is changed in a doubling fashion (1,2,4,8,...) as long as the function is decreasing up to and including the first time the function increases. The second step is to check on every iteration if the new values of the independent variables obtained satisfy the constraints. If they satisfy the constraints, the increasing of the scalar step size in doubling fashion is continued. If they do not satisfy the constraint, the last iteration is repeated by dividing the scalar size by 1.5. This process is repeated until all the independent variables fall within the constraint boundaries.

Example 3

Given a network function

$$Z(s) = \frac{5s^3 + 12s}{6s^4 + 22s^2 + 18} = \frac{\frac{5}{6}s^3 + 2s}{s^4 + \frac{11}{3}s^2 + 3} \tag{5.27}$$

which is realized by the circuit in Figure 28. An optimal design of circuit realizing equation (5.27) will be obtained at ω = 1.5 radians per second by using the multivariable continuously equivalent network theory and the new all-tolerance multiparameter sensitivity measure. The result will be compared with those equivalent circuits obtained by Schoeffler, Leon-Yokomoto, and Cheetham.

By the element growing method of multivariable continuously equivalent network theory, an equivalent circuit having the same Z(s) as in Figure 28 is obtained in Figure 29. The network function with its coefficients expressed in terms of element parameters is

$$Z(s) = \frac{\dfrac{x_1+x_3}{x_1(x_2+x_3)+x_2x_3}\, s^3 + \dfrac{x_4+x_6}{x_1(x_2+x_3)+x_2x_3}\, s}{s^4 + \dfrac{x_4(x_2+x_3) + x_5(x_1+x_3) + x_6(x_1+x_2)}{x_1(x_2+x_3)+x_2x_3}\, s^2 + \dfrac{x_4(x_5+x_6)+x_5x_6}{x_1(x_2+x_3)+x_2x_3}} \tag{5.28}$$

The constrained equations are obtained from equations (5.27) and (5.28) as follows

$$\frac{x_1+x_3}{x_1(x_2+x_3)+x_2x_3} = \frac{5}{6} \tag{5.29}$$

$$\frac{x_4+x_6}{x_1(x_2+x_3)+x_2x_3} = 2 \tag{5.30}$$

$$\frac{x_4(x_2+x_3)+x_5(x_1+x_3)+x_6(x_1+x_2)}{x_1(x_2+x_3)+x_2x_3} = \frac{11}{3} \tag{5.31}$$

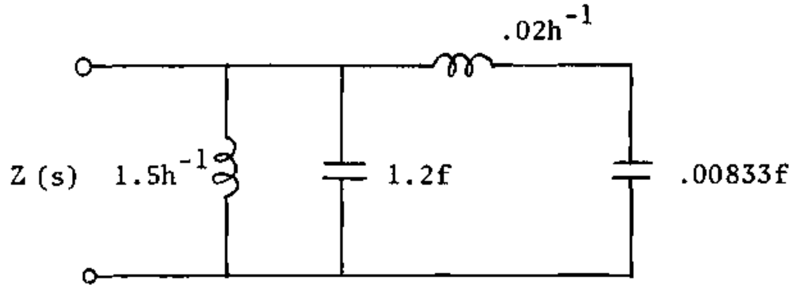


Figure 28. Circuit Realizes (5.27)

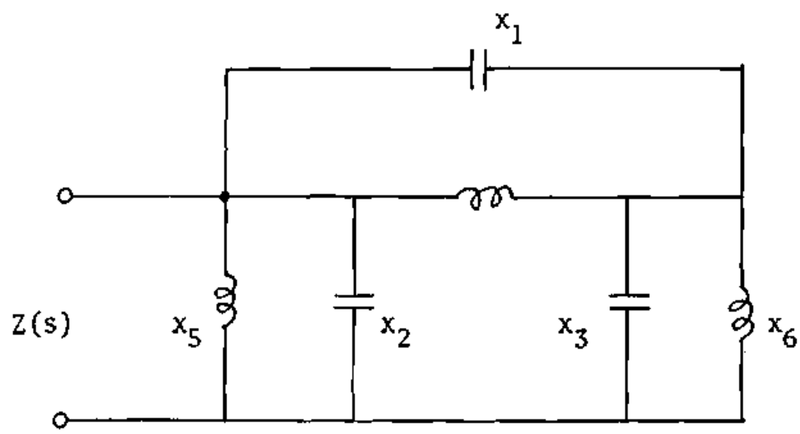


Figure 29. Multivariable Continuously Equivalent Network for Optimization

$$\frac{x_4(x_5+x_6)}{x_1(x_2+x_3)+x_2x_3} = 3 \tag{5.32}$$

Now, there are four equations, (5.29), (5.30), (5.31), and (5.32), and six variables, x_1, x_2, x_3, x_4, x_5, and x_6. In other words $N_c = 4$, $n = 6$, and the number of independent variables is $n - N_c = 6 - 4 = 2$.

Assume x_1 and x_3 are the two element parameters chosen as independent variables, then the remaining four element parameters, x_2, x_4, x_5, and x_6, can be obtained in terms of x_1 and x_3 by solving the four equations above. The results are

$$x_2 = (x_1 + x_3) - \frac{5}{6}(x_1 + x_3)/(\frac{5}{6}(x_1 + x_3)) \tag{5.33}$$

$$x_6 = \frac{g_b \pm \sqrt{g_b^2 + 4g_ag_c}}{2g_a} \tag{5.34}$$

where
$$g_a = \frac{x_1 + x_3}{2a} \tag{5.35}$$

$$a = x_1(x_2 + x_3) + x_2x_3 \tag{5.36}$$

$$g_b = 2x_3 \tag{5.37}$$

$$g_c = \frac{11}{3}a - 2a(x_2 + x_3) - \frac{3}{2}(x_1 + x_3) \tag{5.38}$$

$$x_4 = 2a - x_6 \tag{5.39}$$

$$x_5 = \frac{3a - 2ax_6 + x_6^2}{2a} \tag{5.40}$$

The partial derivatives $\partial I/\partial x_1$ and $\partial I/\partial x_3$ are obtained from equation (5.23). The tolerance is assumed to be 10 percent, i.e., A = 0.1. The constraints $g_i(x)$ for minimizing equation (5.18) in order to get the optimal values of the element parameters are

$$g_1(x) = x_1 \geq 0 \tag{5.41}$$

$$g_2(x) = x_3 \geq 0 \tag{5.42}$$

$$g_3(x) = x_2 \geq 0 \tag{5.43}$$

$$g_4(x) = g_b^2 + 4g_a g_c \geq 0 \tag{5.44}$$

$$g_5(x) = x_6 \geq 0 \tag{5.45}$$

$$g_6(x) = x_4 \geq 0 \tag{5.46}$$

$$g_7(x) = x_5 \geq 0 \tag{5.47}$$

Since there are only two independent variables, x_1 and x_3, the initial point was obtained by a preliminary tabulation of the cost function versus x_1 and x_3. The initial values of x_1 and x_3 were chosen as

$$x_1 = 0.50 \quad \text{farad}$$

$$x_3 = 0.50 \quad \text{farad}$$

The values of w_i for i = 1,2,3,...,7 in equation (5.18) were chosen to be unity. The value of r in (5.18) was chosen to be 0.1. For each successive iteration r was divided by 4.0. By using the Fletcher-Powell function minimization algorithm with information provided above, an optimal design of network was obtained. The values of the elements of the optimal network are

$$x_1 = 0.512 \quad \text{farad}$$

$$x_2 = 0.9408 \quad \text{farad}$$

$$x_3 = 0.525 \quad \text{farad}$$

$$x_4 = 1.452 \quad \text{henry}^{-1}$$

$$x_5 = 0.895 \quad \text{henry}^{-1}$$

$$x_6 = 1.037 \quad \text{henry}^{-1}$$

The optimal design of the network, $\eta(6)$, that realizes equation (5.27) is shown in Figure 30. The all-tolerance multiparameter sensitivity measure for this optimal network is I = 0.044. The final value of the weighted penalty function in (5.18)

$$r \sum_{i=1}^{7} \frac{W_i}{g_i(x)}$$

is 10^{-7}. This is sufficient to indicate the value of the cost function $P(x,r)$ in equation (5.18) finally converges to the minimum value of $I(x)$ in (5.18).

Comparison of the Optimal Network in Example 3 with Those Obtained by Schoeffler, Leon-Yokomoto, and Cheetham

The original network, $\eta(0)$, realizing equation (5.27) is shown in Figure 31(a). The remaining networks $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and

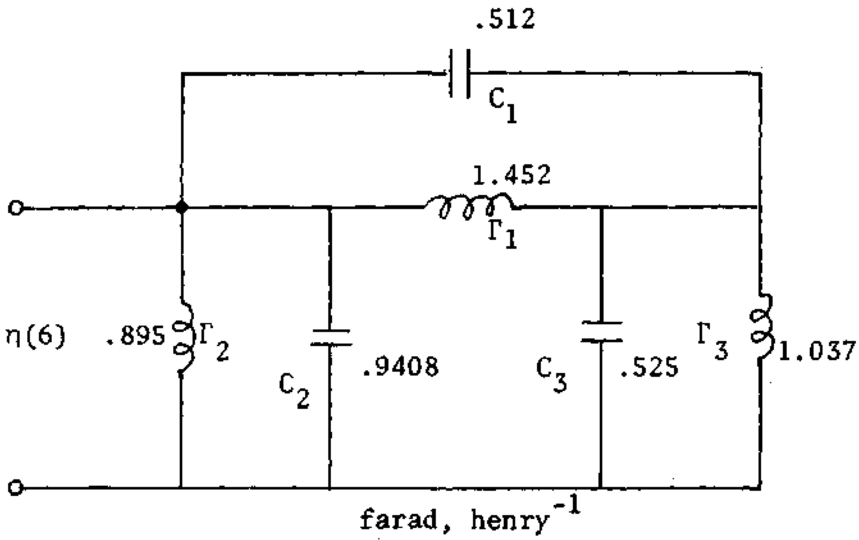
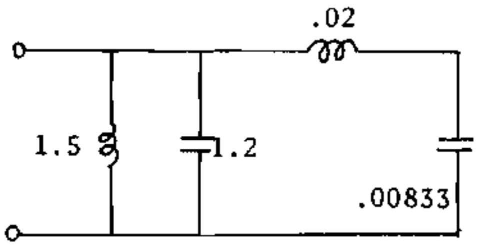
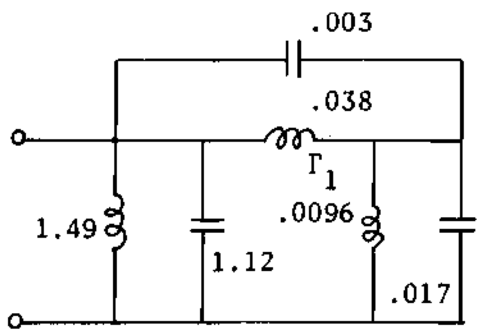


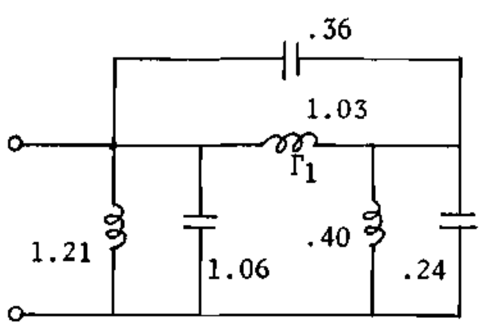
Figure 30. Network Realizing the Impedance of (5.27) with Minimum I



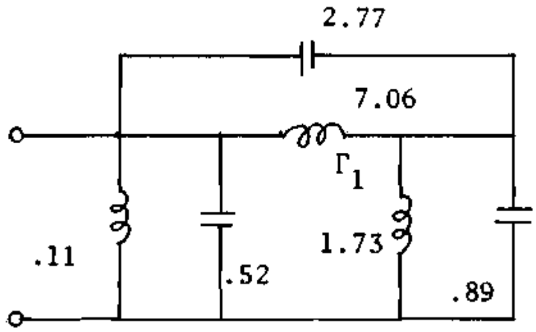
(a) η(0), Original



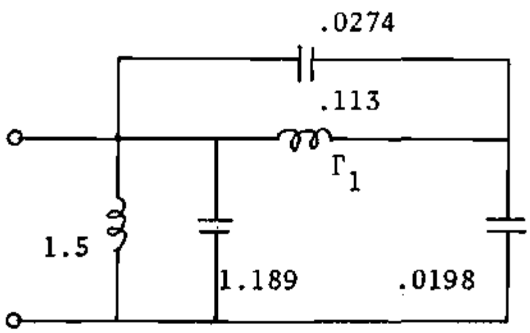
(b) η(1), Schoeffler



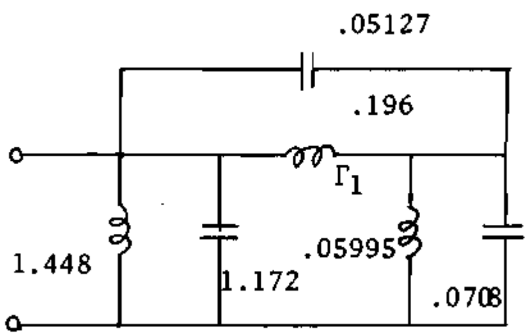
(c) η(2), Schoeffler



(d) η(3), Schoeffler



(e) η(4), Leon-Yokomoto



(f) η(5), Cheetham

henry^{-1}, farads

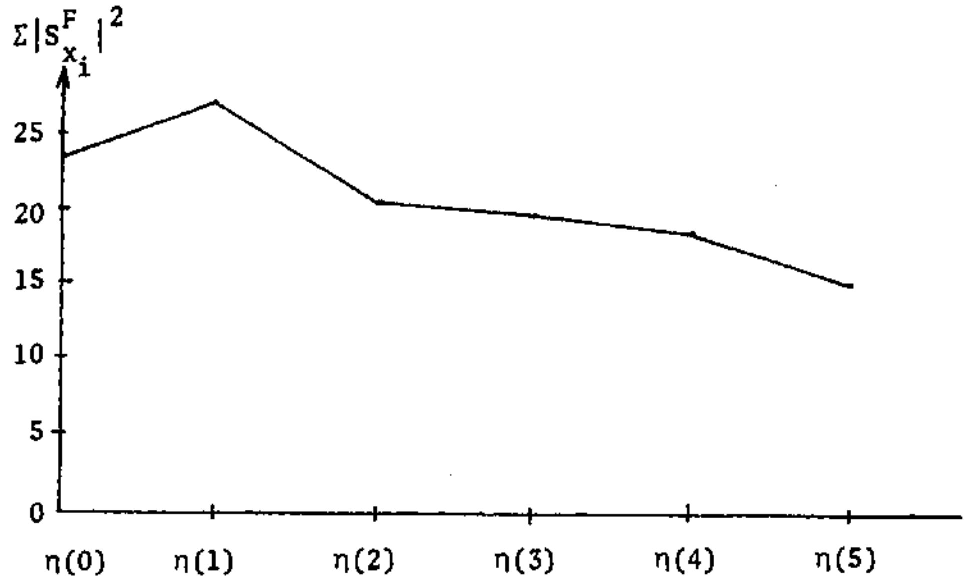
Figure 31. A Series of Continuously Equivalent Networks

$\eta(5)$ in Figure 31 were claimed to perform increasingly well in the sequence according to the traditional first-order sensitivity measure. This is shown in Figure 32(a). However, according to the all-tolerance multiparameter sensitivity measure, I, networks $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$ do not perform increasingly well in that sequence as shown in Figure 32(b). The optimal network $\eta(6)$ in Example 3 has the least measure as shown in Figure 32(b), i.e., I = 0.044. (For values of I of other networks, see Table 6.)

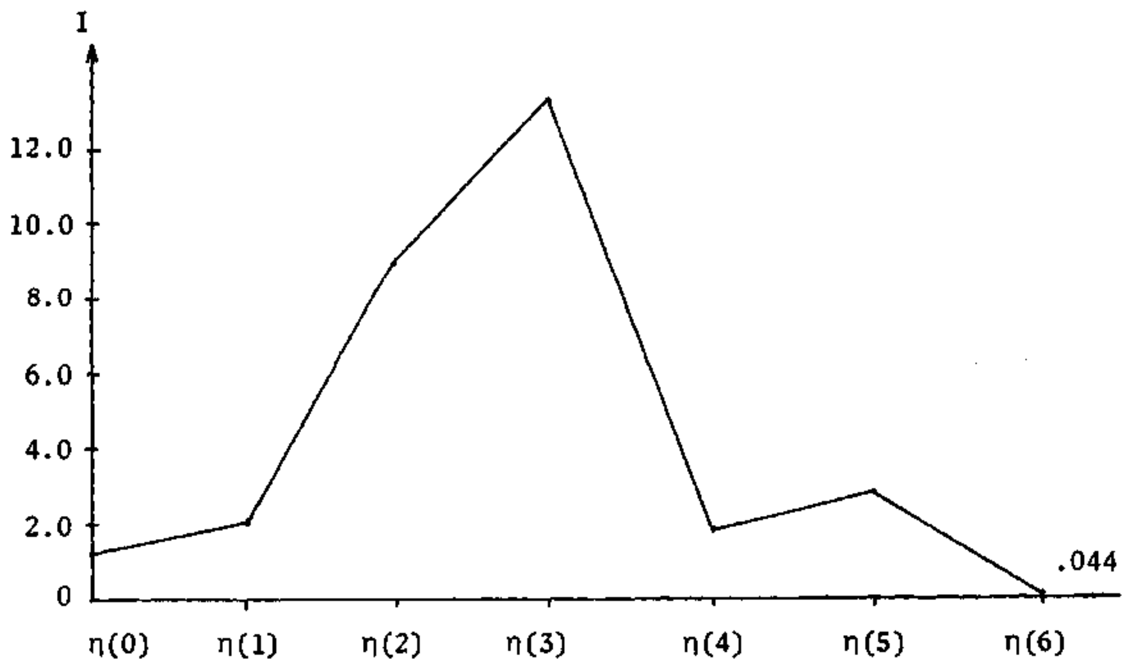
How much does the optimal network $\eta(6)$ outperform the networks in Figure 31 obtained by Schoeffler, Leon-Yokomoto, and Cheetham can also be seen in Figure 33 where the performance of different networks versus the element Γ_1 are depicted in the vicinity of nominal point. For a tolerance of 10 percent, $\Delta\Gamma_1/\Gamma_1 = 0.1$, networks $\eta(0)$, $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$ all show large variations of network performance. All of them have an infinity and almost a null within the tolerance limit. On the other hand, the performance of the optimal network $\eta(6)$ is shown in Figure 34, which shows neither an infinity nor a null in the tolerance limit. This is the reason why the optimal network $\eta(6)$ outperforms $\eta(0)$, $\eta(1)$, $\eta(2)$, $\eta(3)$, $\eta(4)$, and $\eta(5)$ as far as the element parameter Γ_1 is concerned.

It is also of interest to show how the performance of a network function varies as other element values are changed from their nominal values. These variations are shown in Figure 35. Again, neither an infinity nor a null exists.

What this example demonstrates is that the result obtained by the optimization scheme based on the new all-tolerance multivariable



(a) First-order sensitivity measure



(b) New measure for $\Delta x/x = 0.10$

Figure 32. Two Different Sensitivity Measures

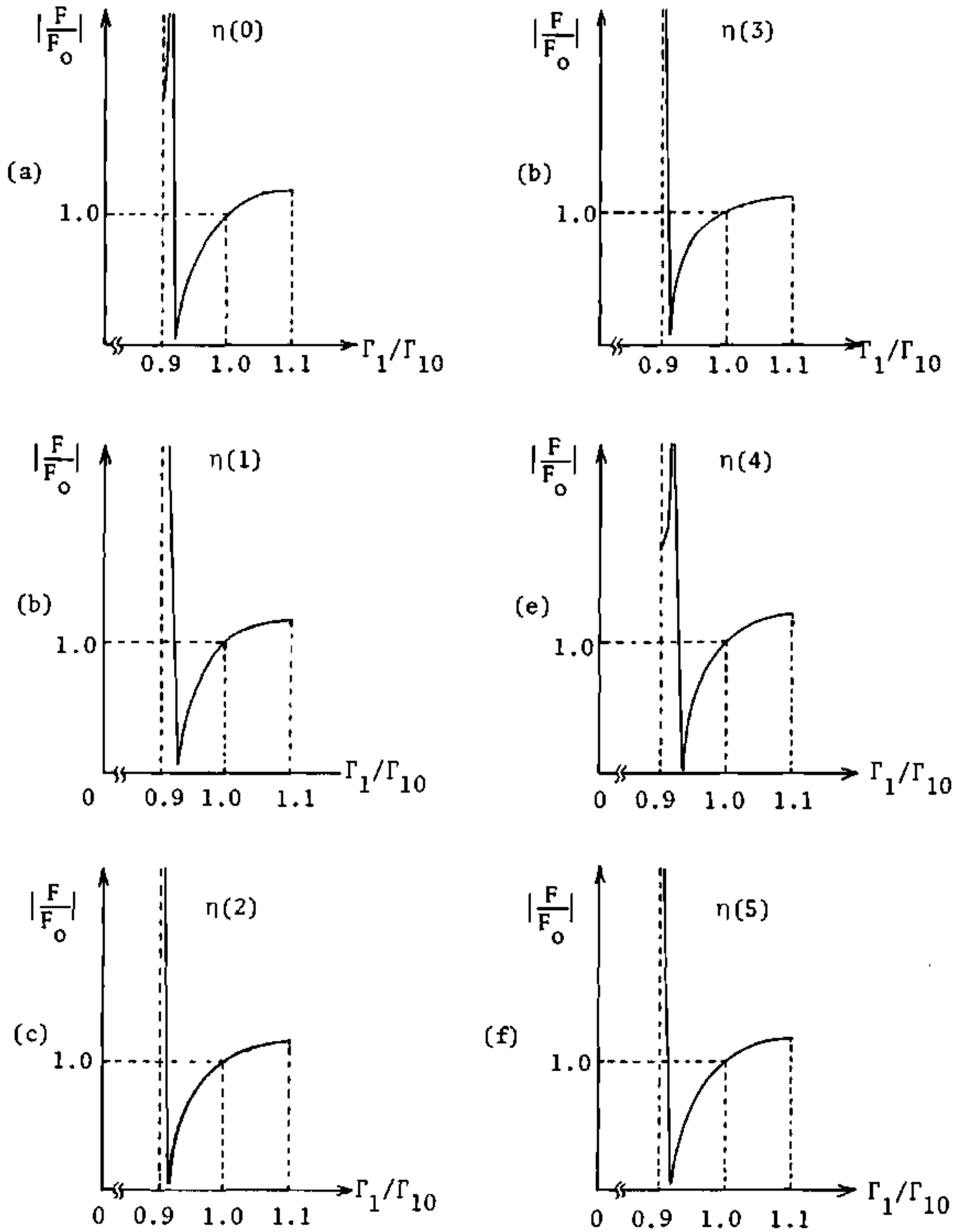


Figure 33. Different Network Performances versus element Γ_1 in Figure 31

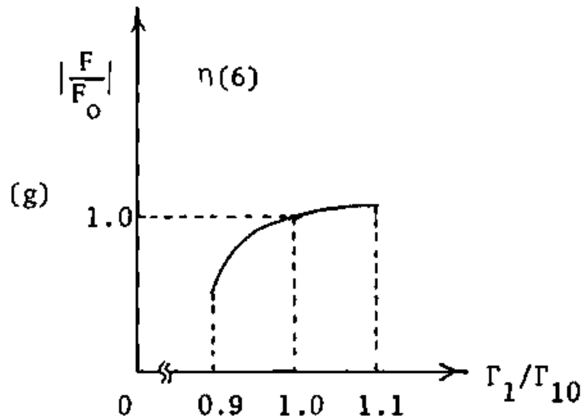


Figure 34. Network Performance of η(6) versus Element Γ_1

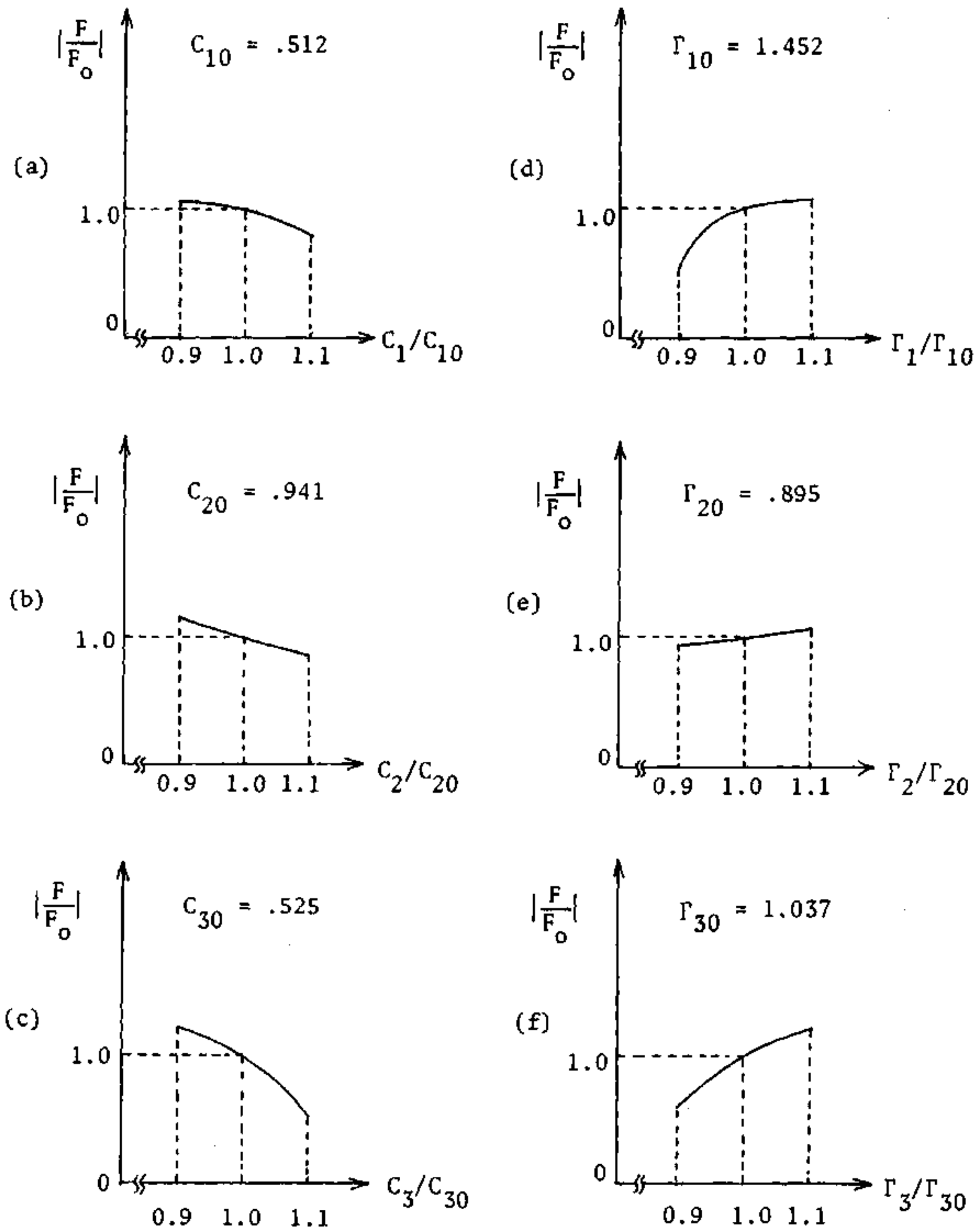


Figure 35. Network Performance of $\eta(6)$ versus All of Its Elements

sensitivity measure is far better than that based on the traditional first-order sensitivity measure. If a large variation of the network performance exists within the tolerance limit, it will manifest itself through the value of I.

A Comment on Sensitivity Minimization

The traditional minimum-sensitivity design is to reduce the slope of the network function with respect to the element values at the nominal point [4, 7, 8, 16, 19, 20, 23]. This approach may actually introduce a worse performance than the original one when the tolerances of the network elements are finite. For instance, in the process of optimization, Schoeffler obtained a better sensitivity of network function with respect to element C_1 for network $\eta(3)$ of Figure 31(d) compared to network $\eta(2)$ of Figure 31(c). That is true at nominal point and is shown in Figure 36. At the nominal point ($|Z/Z_o| = 1.0$, $C_1/C_{10} = 1.0$) the slope of the dotted line is less than the slope of the solid line. However, by reducing the slope of the network function at the nominal point one can see from this figure, an infinity is introduced within the element tolerance limit. Hence, some networks optimized by the traditional sensitivity measure may be completely unacceptable even though their element values fall within the tolerance limit.

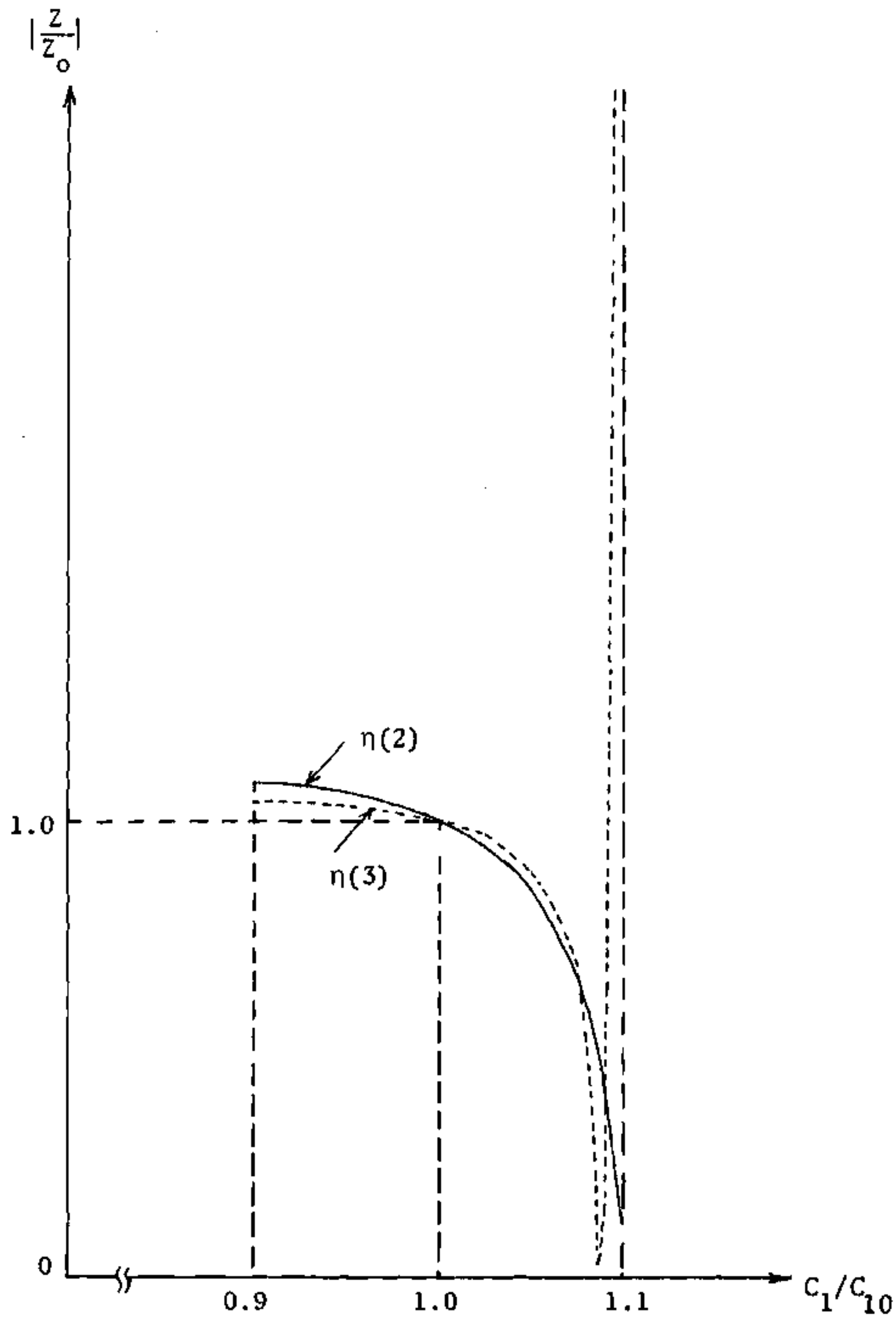


Figure 36. Effect of Optimization at the Nominal Point

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The shortcomings of the existing sensitivity measures were pointed out and a new all-tolerance multiparameter sensitivity measure that overcomes these shortcomings was presented. Examples in which the traditional first-order sensitivity measures gave inaccurate and misleading information were given. Since one of the purposes of sensitivity study is to make comparative evaluation of different circuits realizing the same network function, the new sensitivity measure was applied to evaluate different circuits realizing the same network function so that a better circuit can be chosen for practical application.

A new formulation and algorithm for continuously equivalent network called "multivariable continuously equivalent networks" was presented. The continuously equivalent networks generated by the existing single-variable continuously equivalent network theory are incomplete. However, the continuously equivalent networks generated by the new multivariable continuously equivalent network algorithm are complete as long as the constraint equations can be solved in closed form.

In addition to applying the all-tolerance multiparameter sensitivity measure to compare more meaningfully which of the circuits realizing the same network function is less sensitive, the new sensitivity measure was used to evaluate a series of continuously equivalent networks and the results were compared with those obtained by the traditional sensitivity measure. The results show that tolerances play an important role in network sensitivity.

It was also pointed out that the optimization scheme based on the traditional first-order sensitivity measure could give a worse result. Since the other purpose of sensitivity study is to design a better network from the sensitivity viewpoint, the new sensitivity measure and multivariable continuously equivalent network theory were incorporated into a scheme to obtain an optimal network design. It was found that the optimal network obtained by minimizing the new sensitivity measure outperformed the "optimized network" obtained by using the traditional first-order sensitivity measure. This is because, in effect, the new sensitivity measure takes into account the actual variation of the network performance when the element values are changed by finite amounts.

Fiacco and McCormick method was employed to transform the constrained optimization problem into an unconstrained optimization problem. Then the Fletcher-Powell minimization technique was used to find the optimal network. The success of the optimization depends heavily on the proper choice of the starting points and the weighting factor for the step size. It is recommended that further work can be done in this area so that a more workable and efficient technique can be found.

The completeness of the multivariable continuously equivalent networks depends on whether or not all roots of the constraint equations can be found. All roots can be found if the solutions are in the closed form. A numerical method can be used if closed form solutions are not obtainable. Methods for finding all roots of a set of nonlinear equations by the numerical method is apparently an area that is yet to be investigated.

The all-tolerance multiparameter sensitivity measure is recommended to be used to evaluate those filter designs published in the last ten years so that we can determine which ones have low large-change sensitivities and presumably more suitable for practical applications.

Applying the multivariable continuously equivalent networks along with the new measure to automated network design is another open area that further work can be pursued.

BIBLIOGRAPHY

1. Bode, H. W., "Network Analysis and Feedback Amplifier Design," Princeton, N.J.: Van Nostrand, 1945.

2. Mikulski, J. J., "A Correlation Between Classical and Pole-Zero Sensitivity," Technical Note No. 5, Contract No. AF 49 (638)-63, Electrical Engineering Research Labs., University of Illinois.

3. Goldstein, A. J., and Kuo, F. F., "Multiparameter Sensitivity," IRE Trans. on Circuit Theory, Vol. CT-8, pp. 177-178, June, 1961.

4. Schoeffler, J. D., "The Synthesis of Minimum Sensitivity Networks," IEEE Trans. Circuit Theory, Vol. CT-11, pp. 271-276, June, 1964.

5. Rosenblum, A. L., and Ghausi, M. S., "Multiparameter Sensitivity in Active RC Networks," IEEE Trans. on Circuit Theory, Vol. CT-18, pp. 592-599, November, 1971.

6. Shenoi, B. A., "On a New Statistical Variability Criterion for Optimal Design and Comparative Evaluation of RC-Active Filters," Proc. 1973 International Symposium on Circuit Theory.

7. Hilberman, Dan, "A Biquadratic Approach to the Sensitivity and Statistical Variability of Filters," IEEE Trans. on Circuit Theory, Vol. CT-20, pp. 382-390, July, 1973.

8. Hakimi, S. L., and Cruz, J. B., Jr., "Measure of Sensitivity for Linear Systems with Large Multiple Parameter Variation," IRE Wescon Convention Record, Part 2, pp. 109-115, August, 1960.

9. Kelly, Gerry L., "Computer-Aided Worst Case Sensitivity Analysis of Electrical Networks Over a Frequency Interval," IEEE Trans. on Circuit Theory, Vol. CT-19, pp. 91-93, January, 1972.

10. Butler, Edward M., "Realistic Design Using Large-Change Sensitivities and Performance Contours," IEEE Trans. on Circuit Theory, Vol. CT-18, pp. 58-66, January, 1971.

11. Hamilton, T. A., and Sedra, A. S., "A Single-Amplifier Biquad Active Filter," IEEE Trans. on Circuit Theory, Vol. CT-19, pp. 398-403, July, 1972.

12. Schoeffler, J. D., "Continuously Equivalent Networks and Their Applications," IEEE Trans. on Communications and Electronics, Vol. 83, pp. 763-767, November, 1964.

13. Leeds, J. V., and Ugron, G. I., "Simplified Multi-parameter Sensitivity Calculation and Continuously Equivalent Networks," IEEE Trans. on Circuit Theory, Vol. CT-14, pp. 188-191, June, 1967.

14. Schmidt, G., and Kasper, R., "On Minimum Sensitivity Networks," IEEE Trans. on Circuit Theory, Vol. CT-14, pp. 438-440, December, 1967.

15. Blostein, M. L., "Generation of Minimum Sensitivity Network," IEEE Trans. on Circuit Theory, Vol. CT-14, pp. 87-88, March, 1967.

16. Calahan, D. A., "Computer Generation of Equivalent Networks," IEEE International Convention Record, Pt. 1, pp. 330-333, 1964.

17. Shirakawa, I., Temma, T., and Ozaki, H., "Synthesis of Minimum Sensitivity Networks Through Some Classes of Equivalent Transformations," IEEE Trans. on Circuit Theory, Vol. CT-17, pp. 2-7, February, 1970.

18. Kuh, E. S., and Lau, C. G., "Sensitivity Invariants of Continuously Equivalent Networks," IEEE Trans. on Circuit Theory, Vol. CT-15, pp. 175-177, September, 1968.

19. Director, S. W., and Rohrer, R. A., "The Generalized Adjoint Network and Network Sensitivities," IEEE Trans. on Circuit Theory, Vol. CT-16, pp. 318-323, August, 1969.

20. Director, S. W., and Rohrer, R. A., "Automated Network Design--The Frequency-Domain Case," IEEE Trans. on Circuit Theory, Vol. CT-16, pp. 330-337, August, 1968.

21. Howitt, N., "Group Theory and the Electric Circuit," Physics Review, Vol. 37, pp. 1583-1595, June 15, 1931.

22. Cheetham, B. M. G., "A New Theory of Continuously Equivalent Networks," IEEE Trans. on Circuits and Systems, Vol. CAS-21, pp. 7-20, January, 1974.

23. Leon, B. J., and Yokomoto, C. F., "Generation of a Class of Equivalent Networks and Its Sensitivities," IEEE Trans. on Circuit Theory, Vol. CT-19, pp. 2-8, January, 1972.

24. Newcomb, R. W., "The Noncompleteness of Continuously Equivalent Networks," IEEE Trans. on Circuit Theory, Vol. CT-13, pp. 207-208, June, 1966.

25. Deliyannis, T., "High-Q Factor Circuit with Reduced Sensitivity," Electronics Letters, Vol. 4, pp. 577-579, December 26, 1963.

26. Friend, J. J., "A Single Operational Amplifier Biquadratic Filter Section," Digest of Papers, International Symposium on Circuit Theory, pp. 179-180, 1970.

27. Su, K. L., Active Network Synthesis, McGraw-Hill, Inc., 1965, pp. 207-221.

28. Sallen, R. P., and Key, E. L., "A Practical Method of Designing RC Active Filters," IRE Trans. Circuit Theory, Vol. CT-2, pp. 74-85, March, 1955.

29. Geffe, P. R., "Toward High Stability in Active Filters," IEEE Spectrum, Vol. 7, pp. 63-66, May, 1970.

30. Fletcher, R., and Powell, M. D., "A Rapid Convergent Descent Method for Minimization," The Computer Journal, Vol. 6, pp. 163-168, 1963.

31. Caroll, C. W., "The Created Response Surface Technique for Optimizing Nonlinear Restrained Systems," Operations Research, Vol. 9, pp. 169-184, 1961.

32. Fiacco, A. V., and McCormick, G. P., "Programming Under Nonlinear Constraints by Unconstrained Minimization: A Prime-Dual Method," Research Analysis Corporation, RAC-TP-96, September, 1963.

33. Fiacco, A. V., and McCormick, G. P., "Computational Algorithm for the Sequential Unconstrained Minimization Technique for Nonlinear Programming," Management Science, Vol. 10, pp. 604-617, July, 1964.

VITA

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