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Tenancy Default, Excess Demand and the Rental Market

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Abstract:
We develop a model of a competitive rental housing market with an endogenous rate of tenancy default arising from income uncertainty. Potential tenants must choose to engage in a costly search for rental housing, and must commit to a rental agreement before the uncertainty is resolved. We show that there are two possible equilibria in this market : a market-clearing equilibrium and an equilibrium with excess demand. Therefore, individuals might not have access to rental housing because they are unable to afford to look for housing, they are unable to pay their rent, or with excess demand in the market they are simply unable to find a rental unit. We show that government regulations affecting the cost of default to the housing suppliers and the quality of rental units can have different effects on the equilibrium variables of interest – rental rate, quantity demanded and supplied, and access to rental housing – depending on the type of equilibria in the market. A numerical example illustrates these results.

Keywords: Tenancy Default, Excess Demand, Rental Housing Policies

JEL Classification: R21, R31, R38, D41

1. Introduction

Over the past few years public attention has increasingly focused on the issue of 'affordable housing'. Generally housing is considered affordable if the household spends less than 30% of its gross income on housing costs (rent and/or mortgage and associated operating costs, such as, utilities, water, heat, etc.). Concern over the lack of access to affordable housing, in particular for low income households, has been the driving force behind this increased attention. The basis for this concern lies in the following observations. First, lower income households are more likely to rent than higher income households. In 2001, almost two thirds of households in the bottom 20% of the income distribution were renters versus one third of middle-income households and only 10% of households at the top of the income distribution (Canadian Housing Observer, 2006). Further, the likelihood that a rental household spends more than 30% of their gross income on rental housing is significantly higher for rental households at the lower end of the income distribution. In 2001, 70% of low-income rental households spent more than 30% of their gross income on housing costs whereas less than 5% of middle and high income households did. Similar observations have been documented in the United States by Quigley and Raphael (2004). They also show that there has been a steady decline in the proportion of affordable rental housing stock for the median income rental household in the U.S. over the past thirty years. These observations point to both an undersupply of affordable rental housing as well as inequities in rental burdens, with the greatest burden falling on the poorest households.

There are various possible explanations for the lack of affordable rental housing. One explanation is simply that the income of rental households has declined.[1] Other explanations for the undersupply of rental housing rely on some form of government intervention. For example, Quigley and Raphael (2004) argue that reductions in the amount of affordable housing in the 1980s and 1990s in the U.S. were due to increased rental prices that resulted from increased quality of rental housing driven by government building standards. Price regulations and other forms of rent control that restrict rental price could also generate excess demand in the rental housing market. Other government regulations such as land-use or zoning regulations could also inhibit the development of new construction and thereby cause an undersupply of rental housing (Arnott, 1995).

In this paper, we examine another possible explanation for the undersupply of rental housing – tenancy default. For households at the bottom of the income distribution, the lack of access to financial means could be a very binding constraint. As noted in the Canadian Housing Observer (2006): "Due to their limited incomes, low- and moderate-income households face greater challenges in addressing their housing needs and in balancing housing costs against other household expenses." Such households may face a very real risk of defaulting on their rent and being evicted from their homes. The evidence on actual rental housing evictions is somewhat limited as there is no

[1] Quigley and Raphael (2004) argue that this income decline could explain most of the reduction in the stock of affordable rental housing in the US during the 1970s but cannot explain any later decline.

central registry for maintaining such information. Most of the evidence on evictions comes from case-studies and court records. Often the evictions that are documented are those that were the outcome of a forceful eviction in which a court-order was issued. Of course, some tenants might choose to voluntarily leave their homes before being forcefully removed and thus the evidence on evictions might understate the true number of evictions (Hartmann and Robinson, 2003). There are generally three reasons a tenant might be legally evicted; rent arrears, engaging in some form of anti-social behaviour, and renovation or conversion of rental unit to an alternative use. The majority of evictions fall into the first category (CMHC, 2005). For example, in Montreal close to 85% of all complaints filed by landlords cite non-payment of rent as the reason for the complaint (UN-Habitat Urban Indicators, 2005). Further, this study indicates that an eviction ratio of about 3% of the rental stock is standard for most Canadian cities. Such evictions are costly for both the tenant and the landlord, and may also have long term social costs.

We argue that the risk of default for households could explain the lack of access to rental housing even with a perfectly competitive rental market.[2] To show this, we build a model of a perfectly competitive rental housing market in which individuals may not have access to a rental unit for three reasons: First, individuals face some income uncertainty and must commit to a housing decision before this uncertainty is resolved. As such, once their final income is known, some individuals who have found a rental unit will be unable to pay their rent and will be evicted. Second, some individuals may face a prohibitive cost of searching for a rental unit. Third, there may be excess demand in the rental housing market even though the rental market is perfectly competitive. Increasing supply to eliminate the excess demand (or, equivalently increasing the rental price) would increase the average probability of default in the rental housing market. It may therefore be in the interest of each competitive supplier not to increase supply and to keep rental prices below the market-clearing level. As a result, some individuals who choose to look for a rental unit will be unable to find one. Having shown that tenancy default can generate an equilibrium with excess demand, we then examine the impact of government policies on the housing market. As it turns out, changes in government policies (e.g. regulation on the quality of housing or regulation impacting on the cost of default for the housing suppliers) may well exacerbate the problem of the lack of affordable housing. We also explain how the same government policies can have significantly different effects when the economy rests in a an equilibrium with excess demand as opposed to in a standard market clearing equilibrium.

That competitive suppliers may find it in their interest to charge an *efficiency price* different from the market-clearing price was established, among others, by Weiss (1980), Stiglitz and Weiss (1981), and Shapiro and Stiglitz (1984). The paper which is the closest to ours is Stiglitz and Weiss (1981) in which it is shown that banks may prefer to keep interest rates on loans at a low level to avoid attracting only high-risk borrowers. In our paper, a similar "sorting" effect is also present,

[2] Basu and Emerson (2003) also develop a model of excess demand for rental housing. In their model, however, there are monopolistic landlords, no tenancy default and some forms of rent control.

but it works in the opposite direction for firms (landlords): by charging a low rent, firms reduce the probability of default for a given set of tenants, but they also worsen the pool of tenant as lower rents attract relatively poorer tenants. Our analysis also differs from Stiglitz and Weiss by focusing on the rental housing market, which allows us to build a rich model with several housing specific ingredients. Finally, as will be shown, in our analysis the level of excess demand affects demand and supply, a phenomenon which is absent in Stiglitz and Weiss (1981).[3] As we show, the responses of demand and supply to excess demand is important in establishing the impact of policies on key variables of interest.

The remainder of the paper is as follows: The next section outlines our model of the rental housing market. We characterize the types of equilibria arising in this market in Section 3. We then examine the effect of changes in the various parameters on the equilibrium variables of interest in Section 4, and on social welfare in Section 5. In Section 6, we present numerical examples illustrating the two types of equilibria arising in the model as well as some comparative static results. Section 7 briefly concludes.

2. The Model

Consider a world populated with a continuum of individuals extracting utility v from the consumption of housing H and of some composite good c. Utility is given by

$$v(H, c) = u(H) + g(c)$$

with $u' > 0 \geq u''$, $g(c) = c$ for $c \geq c_o$, and $g(c) = -\infty$ for $c < c_o$ where $c_o \geq 0$ is some minimal consumption level of the composite good needed for survival.[4] Utility v is assumed to be bounded below for non-negative levels of housing, that is, $v(H, c \geq c_o) \geq \underline{v} > -\infty,\ \forall\ H \geq 0$. It follows that in choosing their consumption bundle all individuals ensure that $c \geq c_o$. Individuals differ in terms of *ex ante* income y which is assumed to be distributed on $[\underline{y}, \overline{y}]$ with $\underline{y} > 0$ according to the cumulative distribution $G(y)$ with density $G'(y) = g(y) > 0$ for all y. Total population is normalized to unity. Final or *ex post* income is uncertain. *Ex ante* income is affected by an *i.i.d.* shock s drawn from the cumulative distribution $F(s)$ with density $F'(s) = f(s) > 0$ for all s, unit mean $E(s) = 1$, and support $[\underline{s}, \overline{s}]$ with $\underline{s} > 0$. Thus, *post*-shock final income of an individual with *ex ante* income y who has drawn shock s is simply sy, so expected income is $E(sy) = y$. We assume that this income shock is uninsurable.[5]

[3] Note that in Shapiro and Stiglitz (1984), labour supply (effort) depends on excess supply (unemployment). However, Shapiro and Stiglitz (1984) is a moral hazard model while our model and that of Stiglitz and Weiss are adverse selection models.

[4] Assuming separability simplifies the analysis but the same qualitative results should be obtained without separability provided $v_{cH} > 0$.

[5] As we are considering individuals at the bottom of the income distribution, we think it is reasonable

Individuals must decide whether to look for rental housing. Rental housing can only be consumed in a single discrete amount h. Individuals who do not have access to a rental unit either because they did not look for housing, couldn't not find any, or defaulted on their rent receive a level of housing $h_o \geq 0$ which is strictly less than h and is costless.[6] Therefore, $H \in \{h_o, h\}$ with $h > h_o \geq 0$. The actual value of rental housing h represents both quality features, such as location and state of disrepair, and quantity features, such as square footage, of the rental unit. In what follows, we will assume that the government, through regulation, can affect the actual value of h. Indeed, as was often done in the past in Canada or in the U.S., a government can increase the value of h by introducing modifications to the construction or building code.

The price of rental housing or rent is denoted by r and the price of the composite good is unity (that is, c is the numeraire). There is a fixed utility cost of looking for housing denoted by $k > 0$ which could be interpreted as the time it takes to look for housing.[7] If individuals search for a rental unit and are successful in finding one, then they must sign a lease before shock s is realized (i.e. before income uncertainty is resolved). We assume that there is no voluntary default, that is, we assume that the difference between $u(h)$ and $u(h_o)$ is large enough so that $u(h) - k + sy - r > u(h_o) - k + sy$ or $u(h) - r > u(h_o)$.[8] Individuals only default *ex post* because they are forced to. For individuals who find rental housing and are lucky enough to draw a good shock (i.e. those for which $sy \geq r + c_o$), there is no need to adjust H and the final consumption of c is *ex post* income minus spending on housing: $c = sy - r$, so final utility is $v = u(h) - k + sy - r$. However, some individuals draw a bad shock and have to default *ex post* on their rent to ensure that they satisfy $c \geq c_o$. This happens if $sy < r + c_o$. In this case, H is revised to h_o, final consumption is $c = sy$ and final utility is $v = u(h_o) - k + sy$.[9] The worst shock $\underline{s}$ is assumed to

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to assume these individuals have limited access to the financial means to self-insure against adverse income shocks, that is, we assume they have no savings and lack the means to borrow money. Adverse income shocks could reflect loss of employment income due to injury or illness, increases in the price of necessities, or the loss of income from a supporting person or government program. Some of these adverse shocks may be publicly insured but there are often waiting times before benefits are received and it is during these times of no benefit payments that default could occur.

[6] This alternative housing could be living with family/friends, in a shelter or on the street. The utility derived from such housing would obviously depend on the specific alternative.

[7] Incorporating a positive search cost generates an elastic demand for housing. Without this positive search cost, demand for rental housing would be completely inelastic. An alternative way to generate elastic demand would be to allow for a secondary rental housing market with rental units of higher quality/quantity. Having rental units of differing quality, however, would greatly complicate the analysis without changing the possibility of a market equilibrium with excess demand. Another way to generate elastic demand would be to assume that there is some utility cost from defaulting on one's rent. In this case, both demand and supply would be independent of excess demand in the market but an equilibrium with excess demand could still be obtained.

[8] Also note that there is no moral hazard: individuals cannot affect the value of shock s that they draw.

[9] We assume that both *ex ante* income y and *ex post* shock s are private information so re-negotiation

be large enough so that c_o can always be purchased yet not so large as to preclude the possibility that *any* individual may default on their rent.[10]

It is possible that there will be excess demand in the rental market. If there is excess demand, then we assume that rental units are allocated randomly according to a pure Bernouilly mechanism. Let μ denote the probability that an individual will not be allocated a rental unit given that he has opted to look for housing. This probability will be equal to the proportion of total demand for rental housing that is not met in equilibrium, i.e. the ratio of excess demand for rental housing divided by the total demand for rental housing. In other words, μ is determined endogenously in equilibrium. If there is market-clearing then μ will be zero in equilibrium. If there is excess demand in the rental housing market then μ will be positive. Those that look for housing or demand rental housing, but are unsuccessful at obtaining a rental unit, simply end up consuming h_o and receive final utility $u(h_o) - k + sy$.

On the other side of the market, there are M housing suppliers where we normalize M to unity so there is a representative entrepreneur who produces housing units. It is assumed that both *ex ante* income y and *final* income sy are unobservable, so there can only be one price in the market, which is charged to all tenants. The distribution of income and the distribution of shocks is known, so the average probability of default in the rental housing market is also known to the representative supplier. The representative supplier takes both this average probability of default and the rental price as given and chooses the number of units of housing to produce, denoted by n, that maximizes expected profits. The total cost of producing n units of housing of quality/quantity h is $hC(n)$, where $C' > 0$, $C'' \geq 0$. The timing of events is as follows:

1. Taking the rental price as given, individuals decide whether to look for rental housing and the representative housing supplier chooses the number of rental units to supply.
2. Equilibrium in the housing market: Rental price is determined, and the rental housing market may or may not clear.
3. Income shock is realized and individuals who succeeded in finding a place will default if $sy < c_o + r$.

2.1 Demand Side of the Rental Housing Market

An individual with *ex ante* income y must decide whether to look for a rental unit. If he chooses

of the rent *ex post* is not possible, even if that would be beneficial for the owner and the tenant. It is likely that such information is difficult and costly to observe.

[10] The worst shock $\underline{s}$ is such that the poorest *ex post* individual will always be able to afford c_o: $\underline{sy} \geq c_o$ and the individual will the highest *ex ante* income who receives this shock will default on their rent: $\underline{s}\overline{y} < c_o + r$.

to look for a rental unit and finds one, then he will default on his rent if $sy < c_o + r$, or if

$$s < \frac{c_o + r}{y} \equiv \hat{s}(r, y, c_o) \tag{1}$$

where

$$\hat{s}_r(r, y, c_o) = \frac{1}{y} > 0, \quad \hat{s}_y(r, y, c_o) = -\frac{\hat{s}}{y} < 0, \quad \hat{s}_c(r, y, c_o) = \frac{1}{y} > 0, \tag{2}$$

and he will not default if $s \geq \hat{s}(r, y, c_o)$.

An individual's expected utility if he looks for rental housing is given by:

$$\begin{aligned} v_h =& (1-\mu)\left\{\int_{\underline{s}}^{\hat{s}(r,y,c_o)} [u(h_o) - k + sy]dF + \int_{\hat{s}(r,y,c_o)}^{\overline{s}} [u(h) - k + sy - r]dF\right\} \\ &+ \mu\left\{\int_{\underline{s}}^{\overline{s}} [u(h_o) - k + sy]dF\right\} \end{aligned} \tag{3}$$

If he chooses not to look for rental housing, then his expected utility is given by:

$$v_o = \int_{\underline{s}}^{\overline{s}} [u(h_o) + sy]dF = u(h_o) + y \tag{4}$$

Therefore, an individual with a given expected income y will look for rental housing if $v_h - v_o \geq 0$ or if

$$(1-\mu)[1 - F(\hat{s}(r, y, c_o))][u(h) - r - u(h_o)] - k \geq 0 \tag{5}$$

We assume there exists some income level in $[\underline{y}, \overline{y})$ such that the expected utility differential between looking and not looking for rental housing is equal to zero, $v_h - v_o = 0$. Denote this level of income by $\hat{y}(r, \mu, \mathbf{x})$ where we use the summary notation $\mathbf{x} = \{c_o, k, h\}$. Differentiating the expected utility differential with respect to y yields:

$$\frac{\partial(v_h - v_o)}{\partial y} = (1-\mu)[u(h) - r - u(h_o)]f(\hat{s})\frac{\hat{s}}{y} > 0 \tag{6}$$

It follows from (6) that all individuals with $y \geq \hat{y}(r, \mu, \mathbf{x})$ will look for rental housing and all individuals with $y < \hat{y}(r, \mu, \mathbf{x})$ will not search for rental housing. Total demand for rental housing will be given by $D(r, \mu, \mathbf{x}) = 1 - G(\hat{y}(r, \mu, \mathbf{x}))$ and will depend on the endogenous variables $\{r, \mu\}$ and the exogenous parameters $\{c_o, k, h\}$. We can now obtain the following partial derivatives (see Appendix for details):

$$\hat{y}_r > 0, \quad \hat{y}_\mu > 0, \quad \hat{y}_{c_o} > 0, \quad \hat{y}_k > 0, \quad \hat{y}_h < 0 \tag{7}$$

$$D_r < 0, \quad D_\mu < 0, \quad D_{c_o} < 0, \quad D_k < 0, \quad D_h > 0 \tag{8}$$

INSERT Figure 1: Market Demand HERE

As shown in Figure 1, market demand is decreasing in the rental price. The market demand curve will shift in response to changes in the other parameters, c_o, k, and h. In particular, for a given rental rate, individuals are less likely to look for housing, the more income they need for consumption to survive, the more costly it is to search for housing, and the lower the quality/quantity of the rental housing units. In other words, the demand curve will shift down with an increase in c_o, and k, and a decrease in h. Market demand also depends negatively on the probability that an individuals finds rental housing if they search for rental housing, μ. The value of this variable, together with the rental price, is determined in equilibrium but it is instructive to note that for a given rental rate, an increase in μ will shift the demand curve down as shown in Figure 1.

2.2 Supply Side of the Rental Housing Market

To solve the problem of the representative housing supplier, we need to first establish the default rate in the rental market. To do this, we simply have to work out the average probability of default of those individuals who demand rental housing which we denote by π. This average probability of default can be obtained by summing up the probability of default for all individuals who demand rental housing and dividing by the total demand for rental housing. Doing this, we have:

$$\begin{aligned} \pi(r,\mu,\mathbf{x}) &= \frac{1}{[1-G(\hat{y}(r,\mu,\mathbf{x}))]}\int_{\hat{y}(r,\mu,\mathbf{x})}^{\overline{y}}\int_{\underline{s}}^{\hat{s}(r,c_o,y)} dF dG \\ &= \frac{1}{[1-G(\hat{y}(r,\mu,\mathbf{x}))]}\int_{\hat{y}(r,\mu,\mathbf{x})}^{\overline{y}} F\left(\frac{c_o+r}{y}\right) dG \end{aligned} \tag{9}$$

Differentiating (9), we obtain

$$\pi_r = A + B\hat{y}_r, \quad \pi_{c_o} = A + B\hat{y}_{c_o}, \quad \pi_\mu = B\hat{y}_\mu < 0, \quad \pi_k = B\hat{y}_k < 0, \quad \pi_h = B\hat{y}_h > 0 \tag{10}$$

where

$$\begin{aligned} A &= \frac{1}{1-G(\hat{y})}\int_{\hat{y}}^{\overline{y}} f\left(\frac{c_o+r}{y}\right)\frac{1}{y} dG > 0 \\ B &= \frac{g(\hat{y})}{[1-G(\hat{y})]^2}\left[\int_{\hat{y}}^{\overline{y}} F\left(\frac{c_o+r}{y}\right) dG - F\left(\frac{c_o+r}{\hat{y}}\right)(1-G(\hat{y}))\right] < 0. \end{aligned}$$

A change in either r or c_o has both a direct effect and an indirect or selection effect on the average probability of default. These two effects work in opposite directions. The direct effect, denoted by A, is positive. An increase in r or c_o increases the probability of default for a given set of tenants. The indirect or selection effect works through changes in the demand for rental housing or in the set of individuals looking for rental housing, $\hat{y}$. The higher is $\hat{y}$, the higher is the average income of those looking for rental housing and the lower the average probability of default as given by B. An increase in r or c_o reduces the demand for rental housing or increases $\hat{y}$. Any change in μ, k, and h only has an indirect effect on the average probability of default. An increase in μ and k, or a decrease in h reduces the demand for housing or increases $\hat{y}$. Thus, the changes in these parameters have an unambiguous effect on the average probability of default.

The interesting case we consider in this paper is the one in which the direct effect on the average probability of default of an increase in the rental price dominates the selection effect.[11] In this case, the average probability of default will be increasing in both the rental price and the minimum consumption level.[12] We assume the following for the remainder of the paper:

Assumption 1: $\pi_r > 0$ (10)

The representative supplier in the perfectly competitive rental housing market takes the rental price and average probability of default as given. Let $z > 0$ be the cost of having a tenant default on their rent.[13] This could be interpreted as time and/or money cost of evicting tenants, and in what follows, we assume that the government can influence this cost through housing market regulations, for example, by lengthening the procedure for eviction. The supplier's expected profit is given by:

$$nR(r, \mu, \mathbf{m}) - hC(n) \tag{11}$$

where

$$R(r, \mu, \mathbf{m}) = r - \pi(r, \mu, \mathbf{x})(r + z) \tag{12}$$

is the supplier's effective or expected revenue per rental unit and in which we use summary notation $\mathbf{m} = \{c_o, k, h, z\}$. Differentiating (12) and using (10), we obtain the following:[14]

$$\begin{aligned} &R_r(r, \mu, \mathbf{m}) = (1 - \pi) - (r + z)\pi_r, \quad R_\mu(r, \mu, \mathbf{m}) > 0, \\ &R_{c_o}(r, \mu, \mathbf{m}) < 0, \quad R_k(r, \mu, \mathbf{m}) > 0, \quad R_h(r, \mu, \mathbf{m}) < 0, \quad R_z(r, \mu, \mathbf{m}) < 0. \end{aligned} \tag{13}$$

With the possibility of default, the marginal expected unit revenue with respect to the rental rate, R_r, can be positive or negative. A higher rental price increases rental revenue by $1 - \pi$ but it also increases the likelihood that tenants will default on their rent by π_r, in which case the supplier

[11] If the average probability of default was decreasing in the rental rate, then expected revenue per rental unit would always be increasing in the rental rate and there would never be an equilibrium with excess demand. Housing suppliers would always want r to be as large as possible.

[12] The assumption that $\pi_r > 0$ is sufficient to ensure that $\pi_{c_o} > 0$ since $\hat{y}_r > \hat{y}_{c_o}$ as shown in the appendix.

[13] It is possible that z is less than zero so housing suppliers recover some portion of the rent from tenants who default. In this case, we would have to account for where this money is coming from.

[14] The expressions for the remaining partial derivatives are given in the appendix.

does not receive any rental income and has to incur an additional cost of z. The net effect on expected revenue is ambiguous. An increase in the probability of not finding a rental unit, μ, has a positive effect on expected revenue per rental unit since an increase in μ reduces the demand for rental housing and thereby reduces the average probability of default for a given rental price. This effect is illustrated in Figure 2. The remaining parameters also affect expected unit revenue through their effect on the set of individuals demanding rental housing and therefore on the average probability of default. The signs of these partial derivatives are as one would expect.

We assume that there exists a unique rental price, denoted by $\tilde{r}(\mu, \mathbf{m})$, such that the marginal expected revenue per rental unit is exactly equal to zero. That is,

$$R_r(\tilde{r}, \mu, \mathbf{m}) \equiv (1 - \pi(\tilde{r}, \mu, \mathbf{x})) - (\tilde{r} + z)\pi_r(\tilde{r}, \mu, \mathbf{x}) = 0 \tag{14}$$

where it is assumed that for all r

$$R_{rr}(\tilde{r}, \mu, \mathbf{m}) = -2\pi_r - (\tilde{r} + z)\pi_{rr} < 0 \tag{15}$$

so expected unit revenue is strictly concave in the rental price.[15] This implies that for $r < \tilde{r}$, $R_r > 0$, and for all $r > \tilde{r}$, $R_r < 0$, as shown in Figure 2.

***INSERT Figure 2: Expected Revenue Per Rental Unit HERE ***

Any change in the probability of not finding a rental unit, μ, will affect the rental price that maximizes the supplier's expected revenue per rental unit. To see this, differentiate (14) with respect to μ to obtain

$$\tilde{r}_\mu(\mu, \mathbf{m}) = \frac{-R_{r\mu}}{R_{rr}} = \frac{\pi_\mu + (\tilde{r} + z)\pi_{r\mu}}{R_{rr}}. \tag{16}$$

The greater the probability of not finding housing (higher μ), the more costly it is to look for a rental unit and the average income of those looking for a house will be higher. This drives down the probability of default and increases the marginal expected unit revenue. Given $R_{rr} < 0$, expected revenue will be maximized at a higher rental price as given by the first term in the numerator of (16). At the same time, the higher probability of not finding a house affects the marginal probability of default with respect to the rental rate, π_r, as given by the second term in (16). The sign of this effect is not clear and we are unable to sign (16). We therefore make the following assumption as indicated in Figure 2.[16]

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[15] Of course, it is possible that there is a unique, global maximizer of R without having R being strictly concave. The key fact that marginal revenue, R_r, can be both positive and negative and therefore market supply can be increasing and decreasing in the rental rate will still hold.

[16] This assumption is not crucial for our main results. Alternatively, we could assume that $\tilde{r}_\mu(\mu, \mathbf{m}) < 0$.

Assumption 2: $\tilde{r}_\mu(\mu, \mathbf{m}) > 0$

The representative supplier chooses the number of rental units to supply that maximize expected profits given by (11). The first-order condition is

$$R(r, \mu, \mathbf{m}) - hC'(n) = 0 \tag{17}$$

which yields the supplier's (or market) supply $n(r, \mu, \mathbf{m})$.[17]

To determine how market supply is affected by the various parameters, we can totally differentiate the first-order condition (17) to obtain the following partial derivatives which are given in the appendix.

$$\begin{aligned} &n_r(r, \mu, \mathbf{m}) = \frac{R_r}{hC''(n)} \gtrless 0, \quad n_\mu(r, \mu, \mathbf{m}) > 0, \quad n_{c_o}(r, \mu, \mathbf{m}) < 0, \\ &n_k(r, \mu, \mathbf{m}) > 0, \quad n_h(r, \mu, \mathbf{m}) < 0, \quad n_z(r, \mu, \mathbf{m}) < 0. \end{aligned} \tag{18}$$

Whether the supply curve is upward or downward sloping in the rental price will depend on whether the expected unit revenue is increasing or decreasing in the rental rate. Given the assumed properties of the expected unit revenue function, the rental housing supply curve will be upward-sloping for all $r < \tilde{r}$ and backwards-bending for all $r > \tilde{r}$. In other words, the supply curve will be shaped like a backwards 'C' with the maximum supply at $r = \tilde{r}$, as shown in Figure 3.

*****INSERT Figure 3: Market Supply HERE ********

Changes in the (endogenous) probability of not finding a rental unit, μ, and the various other exogenous parameters will shift the market supply of rental housing. Increases in μ or k make it more costly to look for housing so only those with higher incomes will look for rental units. The average probability of default will be lower and the supplier's expected revenue per rental unit will be higher giving the supplier incentive to produce more rental units (for a given r). Therefore, an increase in either μ, or the search cost, k, will shift the supply curve outwards. For illustrative purposes, this is shown for a change in μ in Figure 3. An increase in h will have the opposite effect and will shift market supply inwards. A change in the quality of rental units has a direct negative effect (for a given r) on the supplier's profits. Increases in h increase the cost of producing rental units and reduce the expected revenue per rental unit by inducing lower income individuals to demand rental housing and thereby increasing the average probability of default. A higher cost of default z will shift market supply inwards since it directly reduces expected revenue per rental unit. Likewise, a higher c_o reduces expected unit revenue since it increases the average probability of default and therefore shifts the supply curve inwards for a given r.

[17] The second-order condition is given by $-hC''(n) < 0$, and is satisfied for a maximum.

3. Market Equilibria

In general, there are two possible types of equilibria in this model. An equilibrium with excess demand in which the demand for rental units exceeds the supply of rental units and, a market-clearing equilibrium in which the supply of rental units exactly equals the demand for rental units. We consider each in turn.

We first show that there may exist a rental price compatible with competitive behaviour (i.e. such that firms have no incentive to charge another price) and with excess demand ($\mu > 0$). This rental price will be given by $\tilde{r}(\mu, \mathbf{m})$ for an appropriate equilibrium μ. To save on notation, we suppress $\mathbf{x}$ and $\mathbf{m}$ and write demand as $D(r, \mu)$, supply as $n(r, \mu)$, expected revenue per unit as $R(r, \mu)$, and the expected revenue (per unit) maximizing rental rate as $\tilde{r}(\mu)$. We have the following proposition.

Proposition 1: *If demand $D(r, \mu)$ lies above supply $n(r, \mu)$ when $\mu = 0$, or if, for $\mu = 0$, demand $D(r, \mu)$ crosses supply $n(r, \mu)$ at a rental price greater than $\tilde{r}(0)$, then there is a competitive equilibrium with excess demand.*

Proof: To prove the above Proposition, consider the case in which demand and supply cross at $\check{r} > \tilde{r}(0)$, implying, graphically, that $\check{r}$ is on the decreasing right-hand side portion of the $R(r, 0)$ mountain.[18] It follows that there is excess demand at $\tilde{r}(0)$, i.e. $D(\tilde{r}(0), 0) > n(\tilde{r}(0), 0)$. As discussed above, we focus (without loss of generality) on the case in which $\tilde{r}$ is increasing in μ, i.e. that for which $\tilde{r}_\mu(\mu) > 0$. We are then interested in any $\mu > 0$ such that $\tilde{r}(0) < \tilde{r}(\mu) < \check{r}$. Hence, for any such $\mu > 0$, the expected revenue per housing unit $R(r, \mu)$ is decreasing in r for all $r > \tilde{r}(\mu)$. Now pick some $\bar{r}$ such that $\tilde{r}(\mu) < \bar{r} < \check{r}$ and let $n(\bar{r}, \mu) = \arg\max_n nR(\bar{r}, \mu) - hC(n)$. Then, for some

$$n^o = \frac{R(\bar{r}, \mu)}{R(\tilde{r}(\mu), \mu)} n(\bar{r}, \mu) < n(\bar{r}, \mu),$$

we have

$$n^o R(\tilde{r}(\mu), \mu) - hC(n^o) = n(\bar{r}, \mu) R(\bar{r}, \mu) - hC(n^o) > n(\bar{r}, \mu) R(\bar{r}, \mu) - hC(n(\bar{r}, \mu))$$

where the inequality follows from $n^o < n(\bar{r}, \mu)$. So competitive firms are better off producing n^o rental units at $\tilde{r}(\mu)$ then any number of rental units at $\bar{r} > \tilde{r}(\mu)$.

Of course, if the rental price is $\tilde{r}(\mu)$, then each firm supplies $n(\tilde{r}(\mu), \mu)$=$\arg\max_n nR(\tilde{r}(\mu), \mu)$ $-hC(n)$. Thus, by definition, we have

$$n(\tilde{r}(\mu), \mu) R(\tilde{r}(\mu), \mu) - hC(n(\tilde{r}(\mu), \mu)) \geq n^o R(\tilde{r}(\mu), \mu) - hC(n^o) > n(\bar{r}, \mu) R(\bar{r}, \mu) - hC(n(\bar{r}, \mu))$$

[18] The case in which demand is fully above supply when $\mu = 0$ is straightforward.

Charging a price $\tilde{r}(\mu)$ lower than $\check{r}$ is therefore consistent with competitive firm behaviour because it entails a larger expected unit return and therefore, larger expected profits.

Finally, an equilibrium with excess demand is given by some positive level of μ, denoted by $\tilde{\mu}$ such that:

$$(1-\tilde{\mu})D(\tilde{r}(\tilde{\mu}),\tilde{\mu}) - n(\tilde{r}(\tilde{\mu}),\tilde{\mu}) = 0 \tag{19}$$

where $\tilde{\mu}$ is the equilibrium probability that someone looking for a house does not find one given excess demand in the rental housing market and in which all firms charge price $\tilde{r}(\tilde{\mu}) < \check{r}$. Note that given $\tilde{r}(\mu)$ is increasing in μ, the left-hand side of the equilibrium condition (19) is strictly decreasing in μ for all $\mu > 0$, so that there should be some $\tilde{\mu} \in (0,1)$ such that the above condition (19) is exactly satisfied.[19] This completes the proof.[20] ∎

***INSERT Figure 4a: Excess Demand Equilibrium HERE ***

An excess demand equilibrium is represented in Figure 4a. In equilibrium, the rental rate is $\tilde{r}(\tilde{\mu})$, quantity supplied is $n(\tilde{r}(\tilde{\mu}),\tilde{\mu})$, and quantity demanded is $D(\tilde{r}(\tilde{\mu}),\tilde{\mu})$. In Figure 4a, supply and demand (when $\mu = 0$) cross at $\check{r}$ where supply is downward sloping. While demand and supply clear at $\check{r}$, this rent level is not an equilibrium. This is because any housing supplier (our representative supplier) charging a lower rent $\in [\tilde{r}(0),\check{r}[$ would be able to attract tenants and earn larger profits. Thus, when supply and demand cross above $\tilde{r}(0)$, that is, when the market supply curve is downward-sloping, all suppliers would optimally deviate from $\check{r}$ and in equilibrium, the rental housing market would move to the excess demand equilibrium $(\tilde{r}(\tilde{\mu}),\tilde{\mu})$.

For there to be a market-clearing equilibrium so that $\mu = 0$ and anyone looking for a rental housing unit will find one, the equilibrium rental price r^* defined by

$$D(r^*,0) = n(r^*,0) \tag{20}$$

---

[19] For the case in which $\tilde{r}(\mu)$ is decreasing in μ, a sufficient (non-necessary) condition for excess demand (i.e. the left-hand side of equation (19)) to be decreasing in μ is that the direct effect of an increase of μ on excess demand (i.e. $(1-\mu)D_\mu - n_\mu$) outweighs the indirect effect of the increase in μ through $\tilde{r}$ (i.e. $[(1-\mu)D_r - n_r]\tilde{r}_\mu$).

[20] The above proof is similar in nature to the argument presented in Weiss (1980). Weiss showed that if the expected revenue from hiring a working is an increasing function of the wage, then competitive firms may choose not to lower wages (hire more workers) even when there is excess supply in the labour market. In our model, there could never be an equilibrium with excess supply. With excess supply in the rental housing market only a fraction, say $1-v$, of all rental units will be rented. This reduces the expected revenue per rental unit to $(1-v)R$ and the supply of rental units will be decreasing in v. Both market demand and $\tilde{r}$, on the other hand, will be independent of the vacancy rate, v. Consequently, the representative supplier's expected profits are strictly decreasing in the vacancy rate and therefore the supplier will always want to reduce supply so there is no excess supply in the rental housing market.

must occur where supply is upward-sloping as shown in Figure 4b.[21] In this case, housing suppliers would like to increase the rental price but are unable to attract any tenants at the higher rent. Thus, r^* is a market-clearing equilibrium and we have the following Proposition.

Proposition 2: *If demand $D(r, \mu)$ and supply $n(r, \mu)$ cross when $\mu = 0$ at a point where supply is upward-sloping, then there is a competitive equilibrium in which the demand for rental housing exactly equals the supply for rental housing.*

***INSERT Figure 4b: Market Clearing Equilibrium HERE ***

Define excess demand as the difference between market demand and market supply for a given set of parameters, $E = D - n$. For a stable market-clearing equilibrium, excess demand in equilibrium must be decreasing in the rental price, i.e. $D_r - n_r < 0$ at $r = r^*$ and $\mu = 0$. From (8), market demand is always decreasing in the rental price, and in Proposition 2, we require market supply to be increasing in r at $r = r^*$. Therefore, the market-clearing equilibrium is stable.

4. Comparative Statics: Impact of Government Policies

We now examine how changes in government regulations affecting the cost of default, z, and the quality of housing, h, affect the two different types of equilibria. We begin by considering the more straightforward comparative statics of the market-clearing equilibrium and then turn to the comparative statics of the equilibrium with excess demand. For this section, it is useful to bring back our full notation for demand $D(r, \mu, \mathbf{x})$, supply $n(r, \mu, \mathbf{m})$, expected revenue per unit $R(r, \mu, \mathbf{m})$, and the expected revenue (per unit) maximizing rental rate $\tilde{r}(\mu, \mathbf{m})$.

4.1 Market-Clearing Equilibrium

The market-clearing equilibrium condition (20) yields $r^*(\mathbf{m})$ and the equilibrium quantity demanded (or, equivalently quantity supplied) will be given by $D(r^*(\mathbf{m}), 0, \mathbf{x})$. By totally differentiating these two expressions we obtain Results 1 and 2 (see Appendix for details).

Result 1: *The equilibrium rental price which clears the housing market r^* is increasing in z and h.*

Result 2: *The quantity demanded (or quantity supplied) in a market-clearing equilibrium is decreasing in z.*

These results can also be obtained from considering how changing either the cost of default

[21] If the demand curve crosses both the upward and downward-sloping portions of the supply curve when $\mu = 0$, then the only equilibrium is the market-clearing equilibrium since there will be excess supply at $\tilde{r}(0)$.

or the quality of housing shifts the demand and supply curves.

4.2 Excess Demand Equilibrium

Suppose now the rental housing market is in an equilibrium with excess demand. It should be clear from the proof of Proposition 1 that the equilibrium probability, $\tilde{\mu}$, that someone looking for a rental unit does not find one is a function of the exogenous parameters $\mathbf{m}$. Thus, in what follows, we write $\tilde{\mu}(\mathbf{m})$ and an excess demand equilibrium is the pair $\{\tilde{r}(\tilde{\mu}(\mathbf{m}), \mathbf{m}), \tilde{\mu}(\mathbf{m})\}$.

For an excess demand equilibrium to exist, demand and supply must cross at $\check{r} > \tilde{r}(0, \mathbf{m})$. The equilibrium with excess demand can be obtained from the following two conditions:

$$R_r(\tilde{r}, \tilde{\mu}, \mathbf{m}) = 0,$$

$$(1 - \tilde{\mu})D(\tilde{r}, \tilde{\mu}, \mathbf{x}) - n(\tilde{r}, \tilde{\mu}, \mathbf{m}) = 0,$$

which can be totally differentiated to obtain the following comparative statics result as shown in the Appendix.

Result 3: *The proportion of demand not being filled in any equilibrium with excess demand, denoted by $\tilde{\mu}$, is increasing in both the default cost z and the quality/quantity of housing h.*

To provide more intuition, we illustrate Result 3 graphically for the case of a regulated increase in the supplier's cost of default z.

Effect of a Change Supplier's Cost of Default on Excess Demand Equilibrium

Consider Figures 5a and 5b. In both cases, market demand is initially $D(r, \tilde{\mu}_0)$, market supply is $n(r, \tilde{\mu}_0, z_0)$, and the equilibrium with excess demand consists of a rental price $\tilde{r}(\tilde{\mu}_0, z_0)$, a proportion of demand not being filled $\tilde{\mu}_0$, a quantity demanded $D(\tilde{r}(\tilde{\mu}_0, z_0), \tilde{\mu}_0)$, and a quantity supplied $n(\tilde{r}(\tilde{\mu}_0, z_0), \tilde{\mu}_0, z_0)$, the latter two being consistent with the following expression: $\tilde{\mu}_0 = [D(\tilde{r}(\tilde{\mu}_0, z_0), \tilde{\mu}_0) - n(\tilde{r}(\tilde{\mu}_0, z_0), \tilde{\mu}_0, z_0)]/D(\tilde{r}(\tilde{\mu}_0, z_0), \tilde{\mu}_0)$.[22]

****** INSERT Figures 5a and 5b: Effect on Equilibrium of a Change in z HERE *****

Suppose now that the government changes housing market regulation so that z increases, from z_0 to $z_1 > z_0$. What happens to market demand and supply? Recall, the default cost does not directly affect the demand for housing.[23] Therefore, a change in z has no direct effect on demand

[22] We have suppressed the notation for the remaining parameters for exposition purposes.

[23] Note however that if z was negative and interpreted as a transfer from the tenant to the landlord, then demand would be reduced by a smaller z.

and the demand curve doesn't shift. Supply, on the other hand, is affected negatively by the increase in z. It is now more costly to produce housing. Therefore, the supply curve will shift in to $n(r, \tilde{\mu}_0, z_1)$.

At this new supply curve, the rental rate that maximizes expected unit revenue will be lower and given by $\tilde{r}(\tilde{\mu}_0, z_1)$.[24] At this new rent level, the quantities demanded and supplied are not consistent with $\tilde{\mu}_0$:

$$\tilde{\mu}_0 < \frac{D(\tilde{r}(\tilde{\mu}_0, z_1), \tilde{\mu}_0) - n(\tilde{r}(\tilde{\mu}_0, z_1), \tilde{\mu}_0, z_1)}{D(\tilde{r}(\tilde{\mu}_0, z_1), \tilde{\mu}_0)}$$

It follows that $\tilde{\mu}$ must increase to re-establish equilibrium given Assumption 2 ($\tilde{r}_\mu > 0$).[25] An increase in μ pushes outward the supply curve ($n_\mu > 0$) while the demand curve shifts down ($D_\mu < 0$). The new equilibrium therefore entails a larger level of μ, denoted by $\tilde{\mu}_1$.

In this equilibrium, market demand is $D(r, \tilde{\mu}_1)$, market supply is $n(r, \tilde{\mu}_1, z_1)$, the equilibrium price is $\tilde{r}(\tilde{\mu}_1, z_1)$, the quantity demanded is $\tilde{D}_1 = D(\tilde{r}(\tilde{\mu}_1, z_1), \tilde{\mu}_1)$, and the quantity supplied is $\tilde{n}_1 = n(\tilde{r}(\tilde{\mu}_1, z_1), \tilde{\mu}_1, z_1)$, with the latter two being consistent with the equilibrium level of excess demand: $\tilde{\mu}_1 = (\tilde{D}_1 - \tilde{n}_1)/\tilde{n}_1$. The actual change in the equilibrium values of the rental rate, quantity demanded and quantity supplied will depend on the relative shifts of the demand and supply curves. The curves in Figure 5a are drawn such that an increase in z induces a decrease in the equilibrium rental rate, a decrease in quantity demanded and an increase in quantity supplied. In Figure 5b, the curves are drawn such that an increase in z induces an increase in the equilibrium rental rate, and a reduction in both quantity demanded and quantity supplied.

Although we obtain unambiguous comparative statics on the equilibrium value of the probability of not finding rental housing (Result 3), the effect of changes in policies on the equilibrium value of the rental rate, excess demand, quantity demanded and housing supply are all ambiguous. Figure 5 illustrates just two possible cases that can occur when there is an increase in the cost of default. To fully characterize all the possible cases, we can totally differentiate the equilibrium rental rate $\tilde{r}$, quantity demanded $\tilde{D}$ and quantity supplied $\tilde{n}$ with respect to z.

$$\frac{d\tilde{r}}{dz} = \tilde{r}_\mu \frac{d\tilde{\mu}}{dz} + \tilde{r}_z, \quad \frac{d\tilde{n}}{dz} = n_r \frac{d\tilde{r}}{dz} + n_\mu \frac{d\tilde{\mu}}{dz} + n_z, \quad \frac{d\tilde{D}}{dz} = D_r \frac{d\tilde{r}}{dz} + D_\mu \frac{d\tilde{\mu}}{dz}$$

The last terms in the first two expressions are the direct *negative* effect of a change in z on $\tilde{r}$ and $\tilde{n}$ (since $\tilde{r}_z < 0$ and $\tilde{n}_z < 0$). There is no direct effect of z on demand. A change in z also increases $\tilde{\mu}$ (by Result 3) which has a *positive* indirect effect on both $\tilde{r}$ and $\tilde{n}$ (since $n_\mu > 0$ and $\tilde{r}_\mu > 0$ by assumption) and a *negative* indirect effect on demand (since $D_\mu < 0$). A change in z has an ambiguous effect on these equilibrium values. We can say, however, that if $\tilde{r}$ is increasing in z, then

[24] This follows from totally differentiating (14) to obtain $\tilde{r}_z = \pi_r / R_{rr} < 0$ given the assumption that $R_{rr} < 0$.

[25] Having $\tilde{r}_\mu > 0$ implies the $(D - n)/D$ is decreasing in μ as stated in the proof of Proposition 1.

equilibrium demand must be decreasing in z. Equilibrium supply, however, could be increasing or decreasing. In addition, we note that by definition $\tilde{\mu} = (\tilde{D} - \tilde{n})/\tilde{D}$. Totally differentiating this condition with respect to z and re-writing, we have in equilibrium

$$\frac{d\tilde{D}}{dz}\frac{z}{\tilde{D}} - \frac{d\tilde{n}}{dz}\frac{z}{\tilde{n}} = \frac{d\tilde{\mu}}{dz}\frac{\tilde{D}}{\tilde{n}}z > 0$$

The elasticity of equilibrium quantity demanded with respect to z will be greater than the elasticity of equilibrium quantity supplied with respect to z which rules out the possibility of quantity demanded going down and quantity supplied going up in response to an increase in z. Taken together then, we have the following four possible cases when the default cost z increases:

i.) $\tilde{\mu}\uparrow$, $\tilde{r}\downarrow$, $\tilde{n}\downarrow$, $\tilde{D}\downarrow$ (drawn in Figure 5a)

ii.) $\tilde{\mu}\uparrow$, $\tilde{r}\downarrow$, $\tilde{n}\downarrow$, $\tilde{D}\uparrow$

iii.) $\tilde{\mu}\uparrow$, $\tilde{r}\downarrow$, $\tilde{n}\uparrow$, $\tilde{D}\uparrow$

iv.) $\tilde{\mu}\uparrow$, $\tilde{r}\uparrow$, $\tilde{n}\downarrow$, $\tilde{D}\downarrow$ (drawn in Figure 5b)

It is also worth pointing out that excess demand in equilibrium $E = \tilde{D} - \tilde{n}$ could be increasing or decreasing in z when quantity demanded and quantity supplied move in the same direction. In the case when $\tilde{D}$ goes up and $\tilde{n}$ goes down then excess demand will be larger.

We can conduct a similar graphical exercise for a regulated change in the quality of housing.[26] This exercise is available upon request. Note that to further illustrate these comparative statics, we consider changes in the values of z and h in a numerical example in Section 6. But first we consider social welfare in this economy.

5. Social Welfare

We are interested in determining how changes in policies affects social welfare in the different types of equilibria. The direct utilitarian social welfare function is given by the following expression:

$$W(r, \mu, \mathbf{m}) = U(r, \mu, \mathbf{x}) + \beta\Pi(r, \mu, \mathbf{m}) \tag{21}$$

where

$$\begin{aligned} U(r, \mu, \mathbf{x}) = &\int_{\underline{y}}^{\hat{y}} (u(h_o) + y)dG + \int_{\hat{y}}^{\overline{y}} (1-\mu)\left[F(\hat{s})u(h_o) + (1 - F(\hat{s}))(u(h) - r)\right] dG \\ &+ \int_{\hat{y}}^{\overline{y}} \left[\mu u(h_o) + y - k\right] dG \end{aligned} \tag{22}$$

[26] We can also consider the impact of rent control. By definition, rent control reduces the rental price and increases excess demand under both types of equilibria. It is possible, however, that the supply of rental housing could go up with the imposition of a rental ceiling unlike the standard analysis of rent control. The reason is that tenants may default on their rent in this model. The lower rental rate reduces the aggregate probability of default for a given set of tenants.

is consumer welfare and $\beta > 0$ is the weight on expected maximized profits which are given by

$$\Pi(r, \mu, \mathbf{m}) = n(r, \mu, \mathbf{m})R(r, \mu, \mathbf{m}) - hC(n(r, \mu, \mathbf{m})). \tag{23}$$

How would a change in the rental price and the probability of not finding a house affect social welfare? First note that by the definition of $\hat{y}$ given by (5), the change in consumer welfare with respect to $\hat{y}$ is equal to zero. That is,

$$\frac{\partial U}{\partial \hat{y}} = -\left[(1-\mu)\left(1 - F\left(\frac{c_o + r}{\hat{y}}\right)\right)[u(h) - r - u(h_o)] - k\right] g(\hat{y}) = 0.$$

Consider now the partial effect of a change in the rental price r and the probability of not finding rental housing μ. We have

$$\frac{\partial W(r, \mu, \mathbf{m})}{\partial r} = -(1-\mu)\int_{\hat{y}}^{\overline{y}} [(1 - F(\hat{s})) + f(\hat{s})(1/y)(u(h) - r - u(h_o))]\, dG + \beta \frac{d\Pi}{dr} \tag{24}$$

where by (17) and (23) $d\Pi/dr = nR_r \gtrless 0$ and

$$\frac{\partial W(r, \mu, \mathbf{m})}{\partial \mu} = -\int_{\hat{y}}^{\overline{y}} (1 - F(\hat{s}))\,[u(h) - r - u(h_o)]\, dG + \beta \frac{d\Pi}{d\mu} \tag{25}$$

where by (17) and (23) $d\Pi/d\mu = nR_\mu > 0$.

The first terms in (24) and (25) are both negative for any $\mu \in [0, 1)$. An increase in either the rental price or probability of not finding a rental unit reduces the sum of consumers' (expected) utilities. With excess demand, the rental price is $\tilde{r}$. By definition $R_r = 0$ at $\tilde{r}$ so social welfare is higher the lower the equilibrium rental rate when there is excess demand in the rental market. How social welfare is affected by a change in μ, however, is ambiguous since an increase in μ increases expected maximized profits. With market-clearing equilibrium, $\mu = 0$. The rental price is r^* and by Proposition 2, $R_r > 0$ at r^* so the second term in (24) is positive and the net effect on social welfare of an increase in the equilibrium rental rate is ambiguous.

We can also derive the direct effect of policies on social welfare.

$$\frac{\partial W}{\partial h} = \int_{\hat{y}}^{\underline{y}} (1 - F(\hat{s}))u'(h) + \beta n R_h - C(n) \tag{26}$$

$$\frac{\partial W}{\partial z} = \beta n R_z < 0 \tag{27}$$

In (26), an increase in h has opposite effects on the sum of consumer's (expected) utilities and maximized expected profits (using (13)). An increase in z, however, has an unambiguously negative direct effect on social welfare since z does not affect the consumer's expected utility and has a negative effect on the supplier's expected profits.

Given the above analysis, it is clear that we will be unable to say anything conclusive about the total welfare effects of a change in policies when the economy is in an equilibrium with excess demand. Therefore, we rely on numerical examples to illustrate these comparative statics.

6. Numerical Example

We construct a numerical example to confirm that an excess demand equilibrium can be obtained in a competitive economy and further to illustrate the impact government policies on the rental housing market and on social welfare.

We consider a very simple economy of the type described in previous sections. The following specific functional forms are assumed:

- ◇ The total number of individuals in the economy is one;
- ◇ The utility of housing is given by $u(h) = 2\alpha h$;
- ◇ The distribution of income is uniform on the interval $[\underline{y}, \overline{y}]$;
- ◇ The distribution of shocks is uniform on the interval $[\underline{s}, \overline{s}]$;
- ◇ The cost of producing n housing units of quality h is $hn^2/2$.

Further, the initial parameters are set as follows: $\underline{y} = 2$; $\overline{y} = 4$; $\underline{s} = 0.8$; $\overline{s} = 1.2$; $c_o = 0.5$; $k = 0.5$; $h = 2$; $h_o = 0$; $z = 0.1$. Since individuals may demand only one unit of housing, total supply (denoted S) and demand (denoted D) are numbers lying within the unit interval. In what follows, U denotes expected consumer welfare, Π denotes expected profits, and $W = U + \Pi$ denotes total expected welfare (there is a unit weight β on profits). Also, excess demand is denoted E [with $E = D - S$] and the amount of homelessness is denoted M [with $M = (1-D)+D(\mu+(1-\mu)\pi)$]. To generate the different types of equilibria, we change the value of the demand parameter α. When $\alpha = 2.25$, an excess demand equilibrium obtains, while when $\alpha = 1.25$, the unique equilibrium is of the market-clearing type.

In Table 1, we report the values of several variables in the excess demand equilibrium (EDE) and the values of those variables in the market-clearing allocation (MCA). This allocation is not an equilibrium since $\check{r} > \tilde{r}(0)$ (as shown in Figure 4a) but serves as a useful benchmark as this is the allocation deemed to be an equilibrium if one fails to recognize the possibility of an equilibrium with excess demand. We also report the values of the various variables in the market-clearing equilibrium (MCE).

One of the main message of this paper is that excess demand, i.e. a lack of access to affordable housing, can be an equilibrium phenomenon in the housing market. Table 1 makes clear that such an equilibrium can exist. Indeed, the EDE is characterized by a rate of excess demand (μ) of 8.1%. Of course, there is no excess demand in either the MCA or the MCE. Some other interesting features can be noted.

- ⋄ As predicted by our theoretical model, the equilibrium rental rate is lower in the EDE than in the MCA.
- ⋄ Equilibrium demand and supply (equal to served demand $[(1-\mu)D]$) are larger in the EDE than in the MCA (despite $\mu > 0$ in the EDE).
- ⋄ The probability of default and the amount of homelessness are lower in the EDE than in the MCA.
- ⋄ Profits are larger in the EDE than in the MCA. This, of course, is due to the lower probability of default in the EDE which translates into a larger R (not reported).
- ⋄ Consumer welfare, profits and therefore total welfare are larger in the EDE than in the MCA.

Table 1

Benchmark: MCA, EDE, and MCE

	μ	$r^{\dagger}$	D	S	E	π	M	U	$\mathit{\Pi}$	W
$\alpha = 2.25$										
MCA	0	2.113	0.884	0.884	0	0.155	0.253	7.697	0.781	8.478
EDE	0.081	1.899	0.973	0.894	0.079	0.055	0.155	8.509	0.799	9.309
$\alpha = 1.25$										
MCE	0	1.945	0.922	0.922	0	0.049	0.123	5.217	0.850	6.068

†: For the MCA, the reported r is $\check{r}$, for the EDE, it is $\tilde{r}$, and for the MCE, it is r^*.

Note that we have performed several examples using different values for the parameters and that these results are robust. It follows from those facts that the EDE dominates in virtually all respect the MCA.[27]

In Table 2, we report comparative static results for an economy initially resting in one or the other allocation described in Table 1. Thus, Table 2 indicates the direction of the change in a variable from its original value in Table 1. The first remarkable result that obtains is that an increase in housing quality h, observed in the 1980s and the 1990s when more stringent regulation of the housing construction industry was enacted by governments (Quigley and Raphael, 2004), could have exacerbated the problem of access to affordable housing. Indeed, in our numerical example, an increase in h translates into an increase in homelessness. This is true whether the economy rests in an EDE or a MCE.

However, the second remarkable result that obtains is that the impact of a regulation induced

[27] Note that we cannot make any statements regarding how the EDE compares with the MCE since these equilibria arise under different values for the demand parameter α.

change in z or h is generally different in the EDE as compared to either the MCA of the MCE. For example, whether the change is in z or h, the rental rate moves in opposite directions. The same is true for the impact on demand, supply, the probability of default, consumer welfare and total welfare. It follows from these facts that in choosing housing policies, it is well-advised to understand that their impact in an EDE may substantially differ from that in a MCE. To illustrate this last fact, consider the impact of an increase z achieved, for example, by an increase in the protection of tenants from eviction and often observed in practice (e.g. by imposing a delay before a tenant can be evicted, or by forbidding eviction in the winter). The standard analysis performed by analyzing the market clearing equilibrium suggests that increasing z is very much counter-productive for the tenants as they would then face a higher rent, a higher probability of default and a higher rate of homelessness, and generally, a lower level of welfare. On the contrary, an analysis of this policy in the excess demand equilibrium characterized in this paper points to a number of improvements for the tenants. Indeed, increasing the protection of tenants from eviction lowers the equilibrium rent, probability of default, and amount of homelessness and ultimately increases the level of welfare.

Table 2

Impact of a 10% regulated increase in h and z, relative to the benchmark in Table 1 §

Variable	μ	$r^\dagger$	D	S	E	π	M	U	Π	W
h ($2 \rightarrow 2.2$)										
MCA ($\alpha = 2.25$)	n/a	+	—	—	n/a	+	+	—	—	—
EDE ($\alpha = 2.25$)	+	—	+	—	+	—	+	+	—	+
MCE ($\alpha = 1.25$)	n/a	+	—	—	n/a	+	+	—	—	—
z ($0.1 \rightarrow 0.11$)										
MCA ($\alpha = 2.25$)	n/a	+	—	—	n/a	+	+	—	—	—
EDE ($\alpha = 2.25$)	+	—	+	—	+	—	—	+	—	+
MCE ($\alpha = 1.25$)	n/a	+	—	—	n/a	+	+	—	—	—

§: A "+" indicates an increase in the variable while a "—" indicates a decrease (relative to the initial values in the benchmark).

†: For the MCA, the reported r is $\check{r}$, for the EDE, it is $\tilde{r}$, and for the MCE, it is r^*.

The case of a regulation induced increase in housing quality h is rather similar. Recall that an increase in h translates into a higher utility for a tenant *given he consumes a housing unit*. An increase in h also translates into higher construction costs on the supply side (recall the cost function is $hn^2/2$). Again, in a market clearing equilibrium, an increase in h translates into a general deterioration of the housing market and a lower level of welfare for the tenants. And again, the conclusion is completely reversed if instead, the economy rests in an excess demand equilibrium. In this case, an increase in housing quality leads to an improvement in the market and more generally, to a higher level of welfare for the tenants.

Given the possibility that actual housing markets may be resting in excess demand equilibria, it follows from these observations that standard real world policy prescriptions should be thoroughly re-examined.

7. Concluding Remarks

We have shown that if there is a possibility that tenants default on their rent, then a competitive market equilibrium could exhibit excess demand without any market interventions, i.e., rent control. We demonstrated both algebraically and with numerical examples that whether or not there is excess demand in the rental market has implications for housing policies. There are several housing policies we did not consider, such as the provision of public housing, rental housing insurance, and rent allowances. Some of these policies would require financing. We leave these investigations for future research.

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Appendix

Section 2.1: Comparative Statics of Market Demand

To determine how market demand changes with the various parameters, we can differentiate $v_h - v_o$ given by the expression on the left-hand side of (5) with respect to the various parameters. Doing this, we obtain

$$\begin{aligned}
\frac{\partial(v_h - v_o)}{\partial r} &= -(1-\mu)[u(h) - r - u(h_o)]f(\hat{s})\frac{1}{y} - (1-\mu)[1 - F(\hat{s})] < 0, \\
\frac{\partial(v_h - v_o)}{\partial \mu} &= -[1 - F(\hat{s})][u(h) - r - u(h_o)] < 0, \\
\frac{\partial(v_h - v_o)}{\partial c_o} &= -(1-\mu)[u(h) - r - u(h_o)]f(\hat{s})\frac{1}{y} < 0, \\
\frac{\partial(v_h - v_o)}{\partial k} &= -1 < 0, \quad \frac{\partial(v_h - v_o)}{\partial h} = (1-\mu)[1 - F(\hat{s})]u'(h) > 0.
\end{aligned}$$

Using the above and (6), we obtain the following partial derivatives for $\hat{y}(r, \mu, c_o, k, h)$

$$\begin{aligned}
\hat{y}_r &= \frac{1}{\hat{s}} + \frac{1 - F(\hat{s})}{[u(h) - r - u(h_o)]f(\hat{s})(\hat{s}/y)} > 0, \quad \hat{y}_\mu = \frac{1 - F(\hat{s})}{(1-\mu)f(\hat{s})(\hat{s}/y)} > 0, \quad \hat{y}_{c_o} = \frac{1}{\hat{s}} > 0, \\
\hat{y}_k &= \frac{1}{(1-\mu)[u(h) - r - u(h_o)]f(\hat{s})(\hat{s}/y)} > 0, \quad \hat{y}_h = -\frac{[1 - F(\hat{s})]u'(h)}{[u(h) - r - u(h_o)]f(\hat{s})(\hat{s}/y)} < 0
\end{aligned}$$

which yields (7). Then, from the definition of market demand $D = 1 - G(\hat{y})$ it follows that

$$\begin{aligned}
&D_r = -g(\hat{y})\hat{y}_r < 0, \quad D_\mu = -g(\hat{y})\hat{y}_\mu < 0, \\
&D_{c_o} = -g(\hat{y})\hat{y}_{c_o} < 0, \quad D_k = -g(\hat{y})\hat{y}_k < 0, \quad D_h = -g(\hat{y})\hat{y}_h > 0
\end{aligned}$$

which yields (8).

Section 2.2: Comparative Statics of Market Supply

To determine how market supply changes with the various parameters, we first differentiate the supplier's expected revenue per rental unit given by (12). Doing this, we obtain

$$\begin{aligned}
&R_r = (1-\pi) - (r+z)\pi_r \gtreqless 0, \quad R_\mu = -(r+z)\pi_\mu > 0, \\
&R_{c_o} = -(r+z)\pi_{c_o} < 0, \quad R_k = -(r+z)\pi_k > 0, \\
&R_h = -(r+z)\pi_h < 0, \quad R_z = -\pi < 0
\end{aligned}$$

which yields (13). For later use, we also determine how $\tilde{r}$ changes with the exogenous parameters by totally differentiating (14) to obtain:

$$\tilde{r}_\mu = -\frac{R_{r\mu}}{R_{rr}} \quad \tilde{r}_z = \frac{\pi_r}{R_{rr}} < 0, \quad \tilde{r}_{c_o} = -\frac{R_{rc_o}}{R_{rr}}, \quad \tilde{r}_k = -\frac{R_{rk}}{R_{rr}}, \quad \tilde{r}_h = -\frac{R_{rh}}{R_{rr}}$$

where $R_{rj} = -\pi_j - (r+z)\pi_{rj}$ for $j = \mu, c_o, k, h$ and it is assumed that $R_{rr} < 0$. Totally differentiating (17) to obtain

$$
\begin{aligned}
n_r &= \frac{R_r}{hC''(n)}, \quad n_z = \frac{R_z}{hC''(n)} < 0 \\
n_\mu &= \frac{R_\mu}{hC''(n)} > 0, \quad n_{c_o} = \frac{R_{c_o}}{hC''(n)} < 0 \\
n_k &= \frac{R_k}{hC''(n)} > 0, \quad n_h = \frac{R_h - C'(n)}{hC''(n)} < 0
\end{aligned}
$$

which yields (18).

Section 3: Derivation of Results 1 and 2

Result 1: Totally differentiating the equilibrium condition (20), using (8) and (18) and the stability condition $D_r - n_r < 0$ at $r = r^*$ to obtain

$$
\begin{aligned}
{r_z}^* &= \frac{n_z}{D_r - n_r} > 0, \quad {r_{c_o}}^* = \frac{-D_{c_o} + n_{c_o}}{D_r - n_r}, \\
{r_k}^* &= \frac{-D_k + n_k}{D_r - n_r} < 0, \quad {r_h}^* = \frac{-D_h + n_h}{D_r - n_r} > 0
\end{aligned}
$$

which yields Result 1. ■

Result 2: Totally differentiating $D^* = D(r^*(z, c_o, k, h), 0, c_o, k, h)$ with respect to z, k, c_o, and h to obtain

$$
\begin{aligned}
\frac{dD^*}{dz} &= D_r {r_z}^* < 0 \\
\frac{dD^*}{dc_o} &= D_r {r_{c_o}}^* + D_{c_o} = \frac{-D_{c_o} n_r + n_{c_o} D_r}{D_r - n_r} \\
\frac{dD^*}{dk} &= D_r {r_k}^* + D_k = \frac{-D_k n_r + n_k D_r}{D_r - n_r} \\
\frac{dD^*}{dh} &= D_r {r_h}^* + D_h = \frac{-D_h n_r + n_h D_r}{D_r - n_r}
\end{aligned}
$$

where the expressions for the partials of r^* have been substituted in. Given (8) and (18), we have Result 2. ■

Section 3: Derivation of Result 3

We first state and prove the following Lemma:

Lemma 1: If $R_{r\mu} > 0$, then $R_{rk} > 0$ and $R_{rh} < 0$.

In what follows, we focus, without loss of generality, on the case in which the effect on the average probability of default dominates the effect on the marginal probability of a change in μ, i.e., $R_{r\mu} > 0$ or $\tilde{r}_\mu > 0$. By Lemma 1, we can then sign R_{rk} (and $\tilde{r}_k$) and R_{rh} (and $\tilde{r}_h$). The sign of R_{rc_o}, however, remains ambiguous.

Proof of Lemma 1:

From (14), we have

$$R_{r\mu} = -\pi_\mu - (r+z)\pi_{r\mu}, \quad R_{rk} = -\pi_k - (r+z)\pi_{rk}, \quad R_{rh} = -\pi_h - (r+z)\pi_{rh}$$

Recall, the following

$$\pi_\mu = B\hat{y}_\mu < 0, \quad \pi_k = B\hat{y}_k < 0, \quad \pi_h = B\hat{y}_h > 0$$

where

$$B(c_o, r, \hat{y}) = \frac{g(\hat{y})}{[1-G(\hat{y})]^2}\left[\int_{\hat{y}}^{\overline{y}} F\left(\frac{c_o+r}{y}\right) dG - F\left(\frac{c_o+r}{\hat{y}}\right)(1-G(\hat{y}))\right] < 0$$

and $\hat{y}(r,\mu,c_o,k,h)$ with $\hat{y}_\mu > 0$, $\hat{y}_k > 0$ and $\hat{y}_h < 0$.

Given the order of differentiation is irrelevant for cross-partials, we have that

$$\pi_{r\mu} = \pi_{\mu r} = \frac{dB}{dr}\hat{y}_\mu + B\hat{y}_{\mu r}, \quad \pi_{rk} = \pi_{kr} = \frac{dB}{dr}\hat{y}_k + B\hat{y}_{kr}, \quad \pi_{rh} = \pi_{hr} = \frac{dB}{dr}\hat{y}_h + B\hat{y}_{hr}$$

We can write the expression for $\hat{y}_r$ as follows:

$$\hat{y}_r \equiv Y(h, r, \hat{s}, \hat{y}) = \frac{1}{\hat{s}} + \frac{H(\hat{s})}{\hat{s}}\frac{1}{u(h) - r - u(h_o)}\hat{y}$$

where $H(\hat{s}) = (1-F(s))/f(s)$, $\hat{s} = (c_o+r)/y$ and $\hat{y}(r,\mu,c_o,k,h)$. We do not make any assumptions on the sign of $H'(s)$. Differentiating Y, we obtain

$$\begin{aligned}
Y_h &= -\frac{H(\hat{s})}{\hat{s}}\frac{u'(h)}{[u(h)-r-u(h_o)]^2} < 0 \\
Y_r &= \frac{H(\hat{s})}{\hat{s}}\frac{1}{[u(h)-r-u(h_o)]^2} > 0 \\
Y_{\hat{s}} &= -\frac{1}{\hat{s}} + \frac{H'(\hat{s})}{\hat{s}}\frac{1}{u(h)-r-u(h_o)} - \frac{H(\hat{s})}{\hat{s}^2}\frac{1}{u(h)-r-u(h_o)} \\
Y_{\hat{y}} &= \frac{H(\hat{s})}{\hat{s}}\frac{1}{u(h)-r-u(h_o)} > 0
\end{aligned}$$

We can then determine the following second partial derivatives:

$$\begin{aligned}
\hat{y}_{r\mu} &= \hat{y}_{\mu r} = (Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}})\,\hat{y}_\mu \\
\hat{y}_{rk} &= \hat{y}_{kr} = (Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}})\,\hat{y}_k \\
\hat{y}_{rh} &= \hat{y}_{hr} = (Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}})\,\hat{y}_h + Y_h
\end{aligned}$$

Using the above, we have

$$\begin{aligned}
\pi_\mu + (r+z)\pi_{r\mu} &= B\hat{y}_\mu + (r+z)\left(\frac{dB}{dr}\hat{y}_\mu + B\hat{y}_{\mu r}\right) \\
&= \hat{y}_\mu\left[B + (r+z)\left(\frac{dB}{dr} + B\frac{\hat{y}_{r\mu}}{\hat{y}_\mu}\right)\right]
\end{aligned}$$

$$\pi_k + (r+z)\pi_{rk} = B\hat{y}_k + (r+z)\left(\frac{dB}{dr}\hat{y}_k + B\hat{y}_{kr}\right)$$
$$= \hat{y}_k\left[B + (r+z)\left(\frac{dB}{dr} + B\frac{\hat{y}_{rk}}{\hat{y}_k}\right)\right]$$
$$\pi_h + (r+z)\pi_{rh} = B\hat{y}_h + (r+z)\left(\frac{dB}{dr}\hat{y}_h + B\hat{y}_{hr}\right)$$
$$= \hat{y}_h\left[B + (r+z)\left(\frac{dB}{dr} + B\frac{\hat{y}_{rh}}{\hat{y}_h}\right)\right]$$
$$= \hat{y}_h\left[B + (r+z)\left(\frac{dB}{dr} + B\left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}} + Y_h/\hat{y}_h\right)\right)\right]$$
$$= \hat{y}_h\left[B + (r+z)\left(\frac{dB}{dr} + B\left(Y_{\hat{s}}\hat{s}_y + Y_{\hat{y}}\right)\right)\right] + (r+z)BY_h$$

Since $\hat{y}_{r\mu}/\hat{y}_\mu = \hat{y}_{rk}/\hat{y}_k$, $\hat{y}_\mu > 0$ and $\hat{y}_k > 0$, it follows that $\pi_\mu + (r+z)\pi_{r\mu} < 0$ if and only if $\pi_k + (r+z)\pi_{rk} < 0$. Given $\hat{y}_\mu > 0$ and $\hat{y}_h < 0$, the condition $\pi_\mu + (r+z)\pi_{r\mu} < 0$ implies that $\pi_h + (r+z)\pi_{rh} > 0$. Therefore, we have Lemma 1.

Result 3:

The equilibrium with excess demand can be represented by the following two conditions:

$$R_r(r, \mu, z, c_o, k, h) = 0$$
$$(1-\mu)D(r, \mu, c_o, k, h) - n(r, z, \mu, c_o, k, h) = 0$$

Together, the above conditions yield $\hat{r}$ and $\hat{\mu}$ as functions of (z, c_o, k, h).

Totally differentiating the system of equations yields

$$\begin{bmatrix} R_{rr} & R_{r\mu} \\ (1-\mu)D_r & -D + (1-\mu)D_\mu - n_\mu \end{bmatrix} \begin{bmatrix} d\hat{r} \\ d\hat{\mu} \end{bmatrix} =$$

$$\begin{bmatrix} -R_{rz} & -R_{rc_o} & -R_{rk} & -R_{rh} \\ n_z & -(1-\mu)D_{c_o} + n_{c_o} & -(1-\mu)D_k + n_k & -(1-\mu)D_h + n_h \end{bmatrix} \begin{bmatrix} dz \\ dc_o \\ dk \\ dh \end{bmatrix}$$

Applying Cramer's Rule and using (8) and (18) as well as well as Lemma 1, we obtain the following total partial derivatives

$$\frac{d\hat{\mu}}{dz} = \frac{n_z R_{rr} + R_{rz}(1-\mu)D_r}{Det} > 0$$
$$\frac{d\hat{\mu}}{dk} = \frac{-((1-\mu)D_k - n_k)R_{rr} + R_{rk}(1-\mu)D_r}{Det} < 0$$
$$\frac{d\hat{\mu}}{dh} = \frac{-((1-\mu)D_h - n_h)R_{rr} + R_{rh}(1-\mu)D_r}{Det} > 0$$

where

$$Det = R_{rr}(-D + (1-\mu)D_\mu) - R_{r\mu}(1-\mu)D_r > 0$$

which proves Result 3. Note, in equilibrium $\hat{\mu} = \tilde{\mu}$ as defined in Section 3. ■

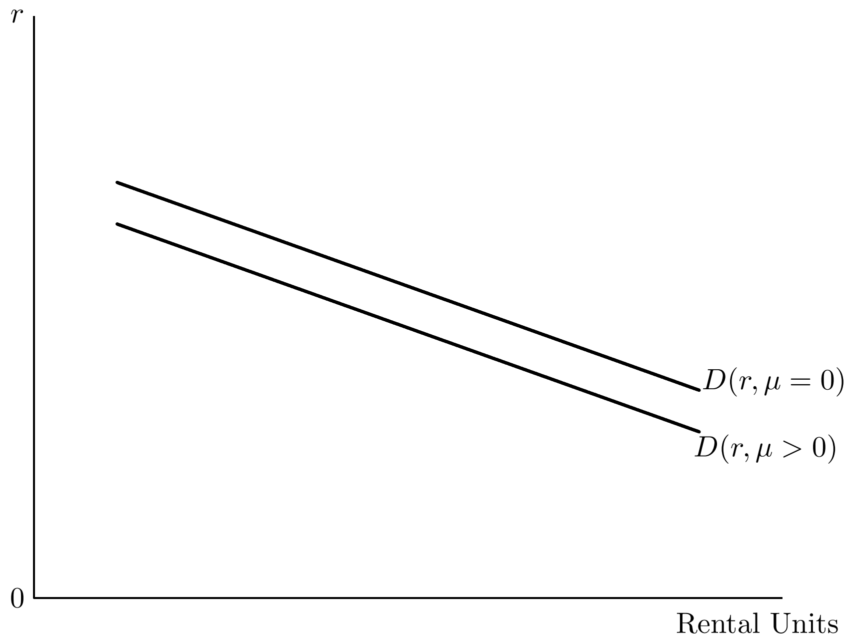

Figure 1: Market Demand for $\mu = 0$ and $\mu > 0$

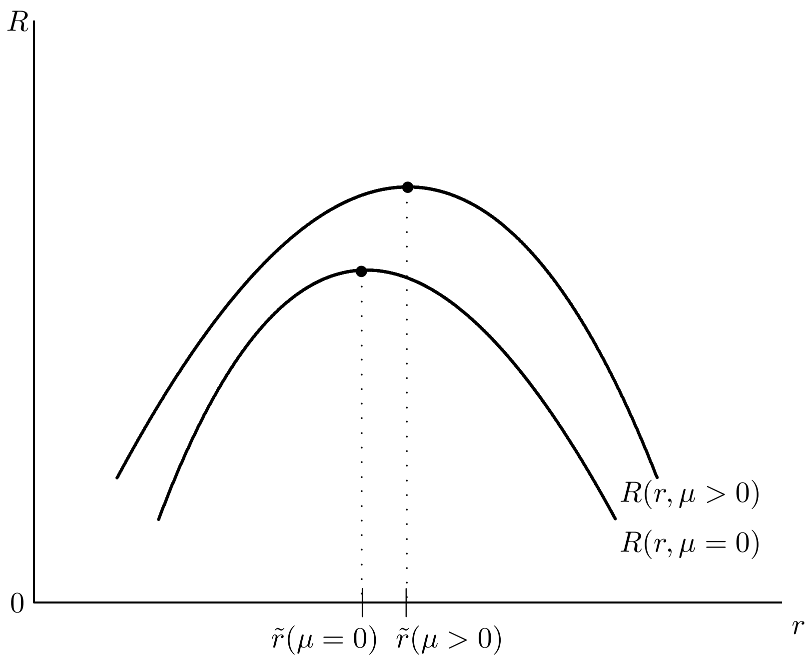

Figure 2: Expected Revenue Per Rental Unit, $\tilde{r}_\mu > 0$

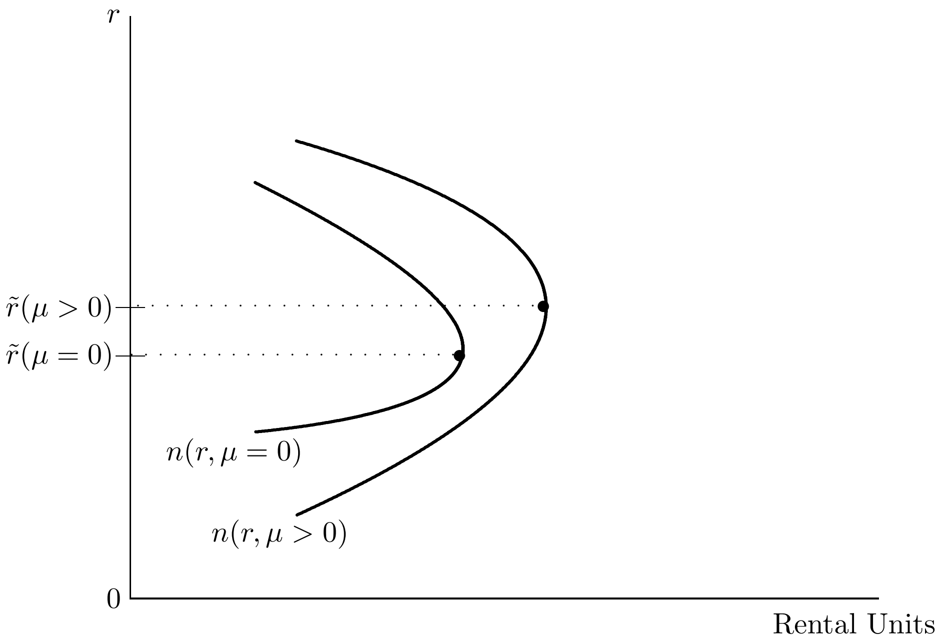

Figure 3: Market Supply for $\mu = 0$ and $\mu > 0$, $\tilde{r}_\mu > 0$

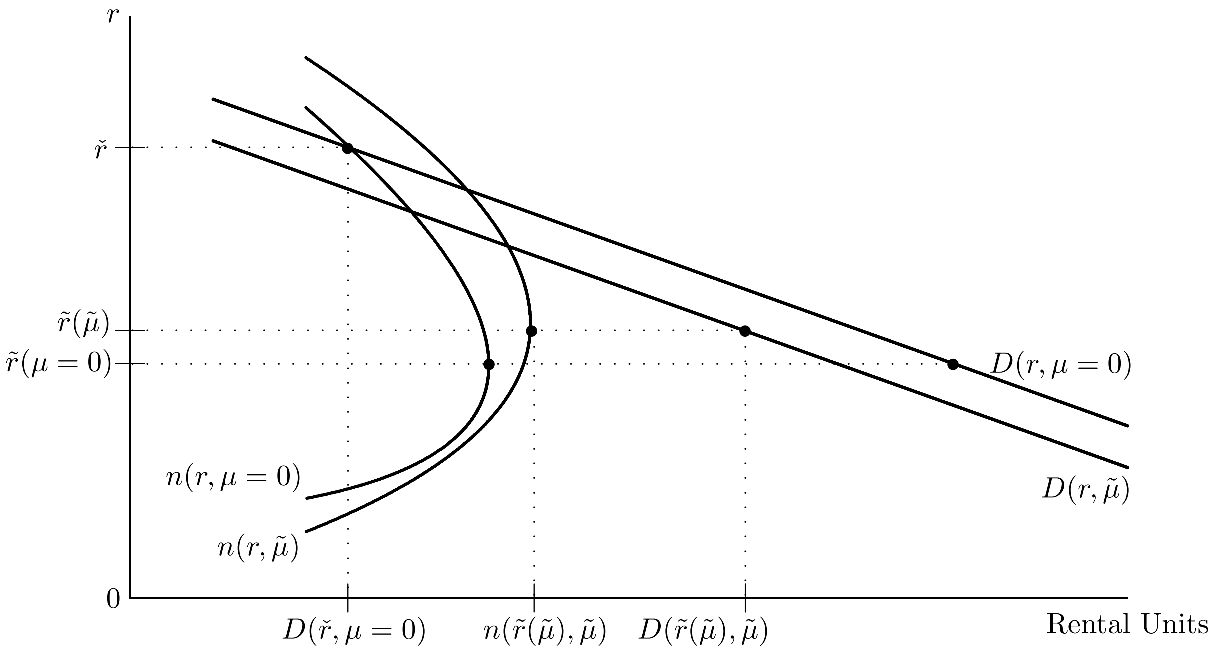

Figure 4a: Excess Demand Equilibrium

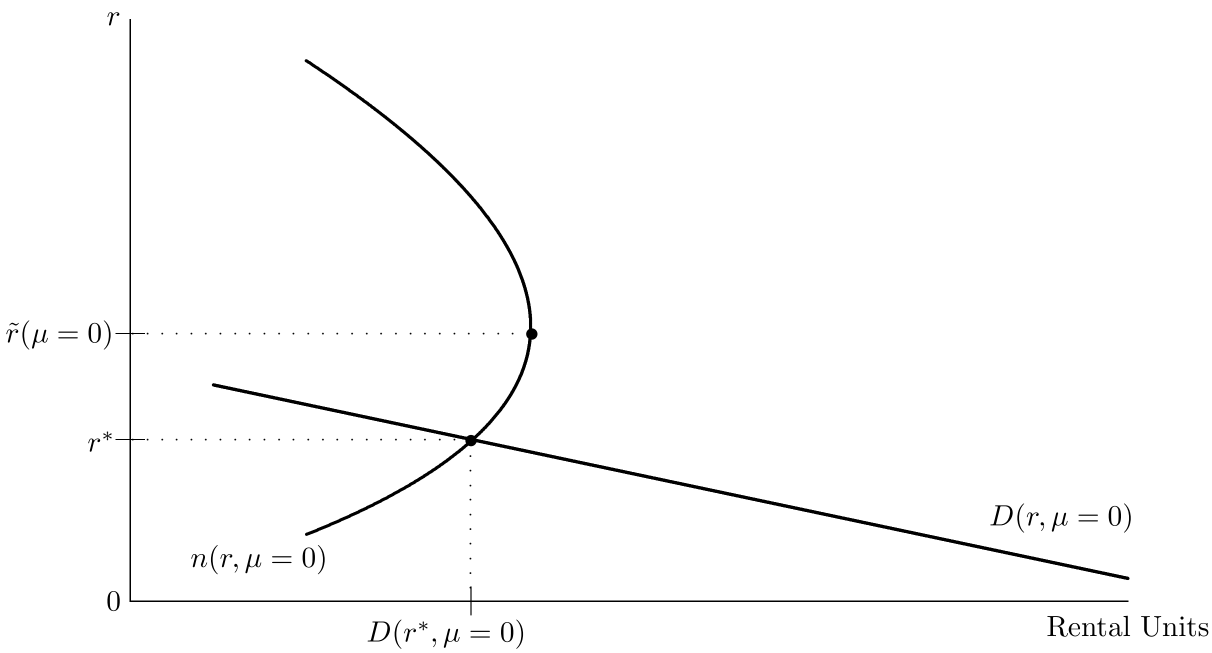

Figure 4b: Market-Clearing Equilibrium

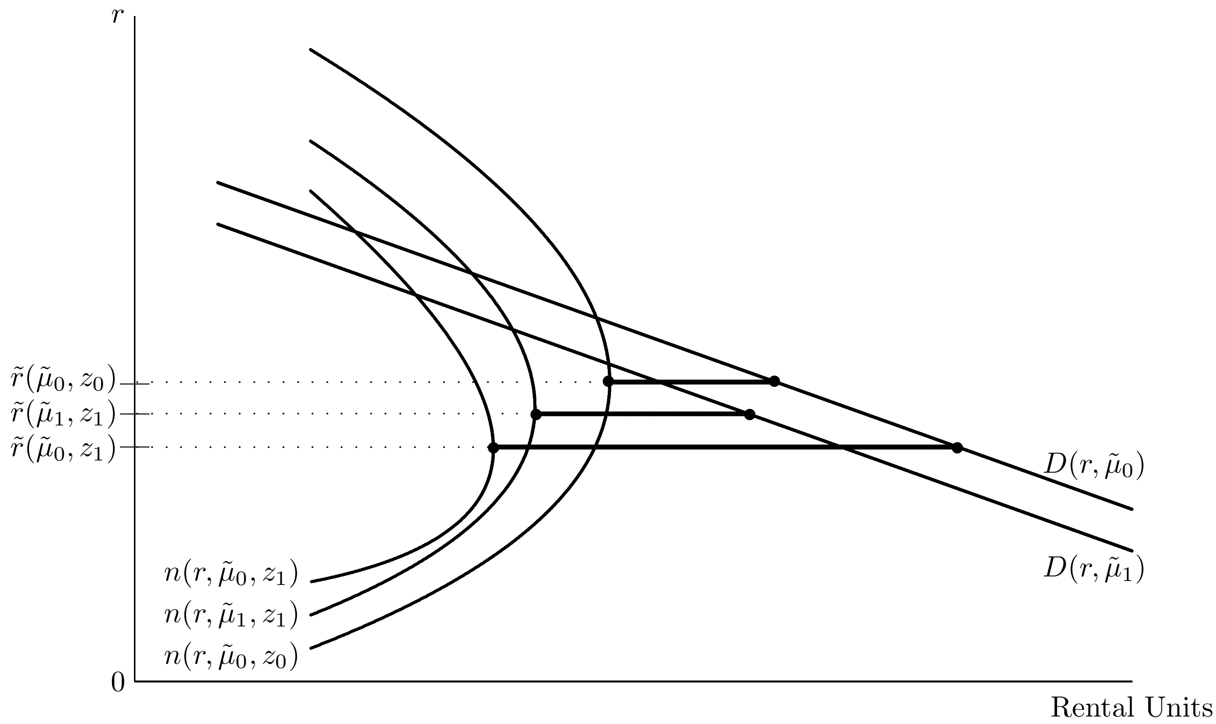

Figure 5a: Comparative Static, Changes in z, Case where $\tilde{r}$ decreases

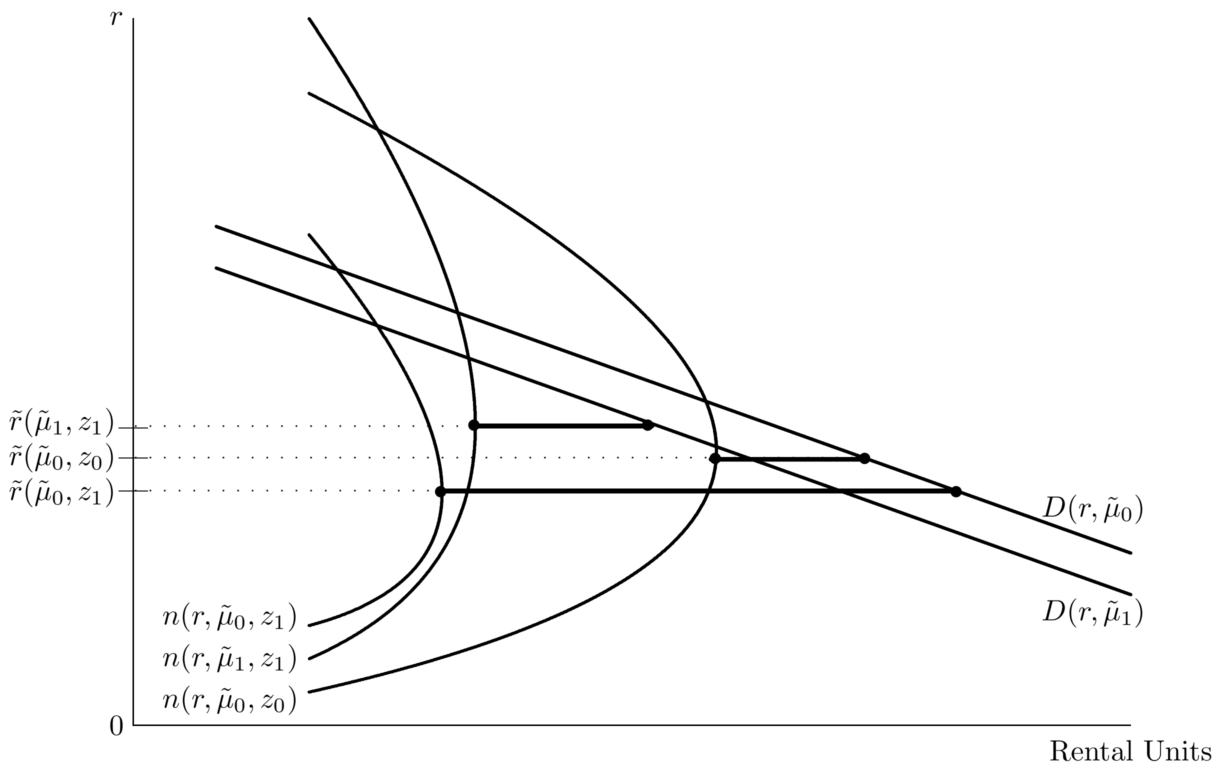

Figure 5b: Comparative Static, Changes in z, Case where $\tilde{r}$ increases