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CAHIER 03-2000

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THE SEEMINGLY UNRELATED DYNAMIC COINTEGRATION REGRESSION MODEL AND TESTING FOR PURCHASING POWER PARITY

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March 2000

Financial assistance from the Mathematics of Information Technology and Complex Systems (MITACS) network is gratefully acknowledged.

RÉSUMÉ

Cet article analyse les modèles de régressions empilées avec régresseurs intégrés et erreurs stationnaires. En ajoutant des retards et des avances des premières différences des régresseurs et en estimant le modèle qui en résulte par moindres carrés quasi-généralisés, nous obtenons un estimateur efficace du vecteur de coïntégration qui a une loi limite normale mixte. Les résultats de simulation suggèrent que ce nouvel estimateur se compare favorablement aux autres déjà proposés dans la littérature. Ce nouvel estimateur est utilisé pour tester la théorie de la parité des pouvoirs d'achat (PPA) parmi les pays du G-7. Le test nous permet de rejeter l'hypothèse nulle de la parité des pouvoirs d'achat pour la plupart des pays.

Mots clés : régressions empilées, estimation efficace, parité des pouvoirs d'achat, coïntégration

ABSTRACT

This paper studies seemingly unrelated linear models with integrated regressors and stationary errors. By adding leads and lags of the first differences of the regressors and estimating this augmented dynamic regression model by feasible generalized least squares using the long-run covariance matrix, we obtain an efficient estimator of the cointegrating vector that has a limiting mixed normal distribution. Simulation results suggest that this new estimator compares favorably with others already proposed in the literature. We apply these new estimators to the testing of purchasing power parity (PPP) among the G-7 countries. The test based on the efficient estimates rejects the PPP hypothesis for most countries.

Key words : seemingly unrelated regressions, efficient estimation, purchasing power parity, cointegration

1 Introduction

Zellner (1962) showed that a feasible generalized least squares (FGLS) estimator is efficient in the seemingly unrelated regression (SUR) model in which regressors are stationary and errors are independent and identical (iid) over time. The efficiency gain is obtained by exploiting cross-sectional correlation information among individual regression equations. Park and Ogaki (1990) demonstrated that this is not true in general if the system of regression equations consists of nonstationary time series regression models that allow for endogenous regressors and serially correlated errors. The FGLS estimator of the integrated SUR model has a nonstandard limit distribution that is skewed and shifted away from the true parameter due to the asymptotic endogeneity of the regressors and the serial correlation of the errors, even though the estimate is consistent (see also Park and Phillips, 1988 and Moon, 1999). This renders inference in these systems difficult.

Two solutions have so far been put forth. First, Park and Ogaki (1991) suggested using the *canonical cointegrating regression* (CCR) estimator first presented in Park (1992). On the other hand, Moon (1999) suggested using *fully-modified* (FM) estimators (*e.g.*, see Phillips and Hansen, 1990, Phillips, 1991, and Phillips, 1995) on these systems. These two sets of estimators have limiting mixed normal distributions, rendering inference straightforward and allowing efficiency comparisons between estimators.

This paper suggests an alternative method for obtaining mixed normal limiting distributions based on results of Saikkonen (1991) and Stock and Watson (1993). The approach (which we call dynamic FGLS) consists of adding leads and lags of the first differences of the regressors and using feasible generalized least squares on this augmented dynamic regression model with the long-run covariance matrix. We prove that with restrictions on the rate at which we add leads and lags as the sample size increases, we obtain estimators with limiting mixed normal distributions which allow for standard inference procedures to be used. Moreover, our estimator is more efficient than equation-by-equation or system-wide ordinary least squares.

We apply these new estimators to the testing of purchasing power parity. There are various forms of the PPP doctrine, but they all share the idea that nominal exchange rates should reflect the behavior of relative price levels. There has been an enormous amount of recent literature devoted to testing this hypothesis. We use our more efficient estimators to develop tests that we believe are more appropriate than those already in the literature.

The outline for the rest of the paper is as follows. The next section introduces our estimators and derives their limiting asymptotic distribution. Section 3 presents results from a simulation experiment comparing estimators of the integrated SUR model. Section 4 presents our empirical methodology and results for testing PPP among industrialized countries over the recent float, while section 5 concludes.

2 Dynamic GLS Estimation of the Integrated Seemingly Unrelated Regression Model

In this section we study a SUR model with integrated regressors. Suppose that there are M individual linear cointegration regression equations,

$$\begin{aligned} y_{m,t} &= \beta_{m,0} + \beta_{m,1}' x_{m,t} + u_{m,t} \\ x_{m,t} &= x_{m,t-1} + v_{m,t}, \end{aligned} \tag{1}$$

where $u_{m,t}$ and $v_{m,t}$ are scalar and L-vector valued stationary processes, respectively, for $m = 1, \cdots, M$, and $t = 1, ..., T$. Assume that $x_{m,0} = O_p(1)$ as $T \to \infty$ for all $m = 1, ..., M$. Let $y_t = (y_{1,t}, \ldots, y_{M,t})'$, $\tilde{x}_t = diag(\tilde{x}_{1,t}, \ldots, \tilde{x}_{M,t})$, $\tilde{x}_{m,t} = (1, x_{m,t}')'$, $x_t = \left(x_{1,t}', ..., x_{M,t}'\right)'$, $u_t = (u_{1,t}, \ldots, u_{M,t})'$, and $v_t = (v_{1,t}', \ldots, v_{M,t}')'$. Then, using vector notation, we rewrite (1) as

$$\begin{aligned} y_t &= \tilde{x}_t' \beta + u_t, \\ x_t &= x_{t-1} + v_t, \end{aligned} \tag{2}$$

where $\beta = \left(\beta_1', ..., \beta_M'\right)'$ and $\beta_m = \left(\beta_{m,0}, \beta_{m,1}'\right)'$.

Define $w_t = \begin{pmatrix} u_t' & v_t' \end{pmatrix}'$. We assume that the partial sum process of w_t converges in distribution to a Brownian Motion,

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} w_t \Rightarrow B(r) \equiv BM(\Omega), \tag{3}$$

where $\Omega = \sum_{h=-\infty}^{\infty} E(w_0 w_h') = \begin{pmatrix} \Omega_{uu} & \Omega_{uv} \\ \Omega_{vu} & \Omega_{vv} \end{pmatrix}$, $B(r) = \left(B_u(r)', B_v(r)'\right)'$, $B_u(r) = \left(B_{1,u}(r), ..., B_{M,u}(r)\right)'$, $B_v(r) = \left(B_{1,v}(r)', ..., B_{M,v}(r)'\right)'$, and the partitions of Ω and $B(r)$ conform to the size of u_t and v_t. The functional central limit theorem assumed in (3) is satisfied under mild regularity conditions on w_t (*e.g.*, see Phillips and Solo, 1992). In this paper we assume that the long-run covariance matrix Ω of w_t is positive definite, which excludes the possibility that there exists a cointegrating relation among the elements of x_t.

The main purpose of this section is to develop an efficient estimation method for the cointegration parameter β in the SUR cointegration model (1) and establish its asymptotic properties. When there is (long-run) correlation between u_t and v_t (as in a simultaneous equation model) and/or serial correlation, it is well known that the limiting distribution of the OLS estimator of model (1) [1] is miscentered and skewed, and this causes difficulties in statistical inference. To overcome the problem, we modify the regression model and make the transformed error asymptotically uncorrelated with the regressors.

To do so, we decompose the stacked vector u_t into two components, the projection onto the sigma field generated by $\{v_t\}_{t=-\infty}^{\infty}$, $\sum_{j=-\infty}^{\infty}\pi_j v_{t-j}$, and a residual, namely

$$u_t = \sum_{j=-\infty}^{\infty}\pi_j v_{t-j} + \xi_t. \tag{4}$$

and in this case, ξ_t is uncorrelated with $\{v_t\}_{t=-\infty}^{\infty}$. Denote the long-run covariance of ξ_t by $\Omega_{uu.v}$. Under mild regularity conditions [2], we know that

$$\Omega_{uu.v} = \Omega_{\mathrm{u}u} - \Omega_{\mathrm{u}v}\Omega_{vv}^{-1}\Omega_{v\,\mathrm{u}}. \tag{5}$$

Now, in view of the decomposition (4) , we can write the model (2) as

$$y_t = \tilde{x}_t'\beta + \sum_{j=-\infty}^{\infty}\pi_j v_{t-j} + \xi_t. \tag{6}$$

In equation (6) we have added an infinite number of regressors $\{v_t\}_{t=-\infty}^{\infty}$, which makes estimating it impossible with a finite number of observations. We propose a feasible version of (6) obtained by truncating the infinite sum and replacing it by $\sum_{j=-K}^{K}\pi_j\Delta x_{t-j}$, where K is assumed to tend to infinity as $T\to\infty$ at an appropriate rate to be specified below. Hence the estimable version of model (6) is

$$\begin{aligned} y_t &= \tilde{x}_t'\beta + \sum_{j=-K}^{K}\pi_j\Delta x_{t-j} + \xi_t^* \\ &= \tilde{x}_t'\beta + \sum_{j=-K}^{K}\left(\Delta x_{t-j}'\otimes I_M\right)vec(\pi_j) + \xi_t^* \\ &= z_t'b + \xi_t^*, \end{aligned} \tag{7}$$

where

$$\xi_t^* = \xi_t + e_t,\ e_t = \sum_{|j|>K}\pi_j v_{t-j},$$

[1] In this case, system OLS estimation is identical to equation-by-equation OLS estimation.

[2] See, for example, Brillinger (1975) and Saikkonen (1991).

$$
\begin{aligned}
z_t &= \begin{pmatrix} \tilde{x}_t \\ \Delta x_{t-K} \otimes I_M \\ \vdots \\ \Delta x_{t+K} \otimes I_M \end{pmatrix}, \ b = \left(\beta', \Pi_K'\right)', \\
\Pi_K &= \left(vec(\pi_{-K})', ..., vec(\pi_K)'\right).
\end{aligned}
$$

Note that the regression model (7) is simply an augmented version of the original SUR model (1) obtained by adding leads and lags of Δx_t.

We may consider estimating the regression model (7) in two ways, either by OLS or by GLS using the long-run correlation in the error ξ_t as is appropriate when the error term has serial correlation. Let

$$
\begin{aligned}
Y_t &= (y_{1,t}, ..., y_{M,t})', \ Y = \left(Y_{K+1}', ..., Y_{T-K}'\right)', \\
\tilde{X} &= (\tilde{x}_{K+1}, ..., \tilde{x}_{T-K})', \\
\Delta_{K,t} &= \left(\Delta x_{t-K}', ..., \Delta x_{t+K}'\right)' \otimes I_M, \ \Delta_K = (\Delta_{K,K+1}, ..., \Delta_{K,T-K})', \\
Z &= \left(\tilde{X}, \Delta_K\right).
\end{aligned}
$$

The system dynamic OLS (hereafter SDOLS) and the dynamic (feasible) GLS (hereafter DGLS) estimators are defined, respectively, as:

$$
\hat{b}_{SDOLS} = \left(Z'Z\right)^{-1} Z'Y, \tag{8}
$$

$$
\hat{b}_{DGLS} = \left(Z'\left(I_{T-2K} \otimes \hat{\Omega}_{uu.v}\right)^{-1} Z\right)^{-1} \left(Z'\left(I_{T-2K} \otimes \hat{\Omega}_{uu.v}\right)^{-1} Y\right) \tag{9}
$$

or in summation notation,

$$
\begin{aligned}
\hat{b}_{SDOLS} &= \left(\sum_{t=K+1}^{T-K} z_t z_t'\right)^{-1} \left(\sum_{t=K+1}^{T-K} z_t y_t\right) \qquad (10) \\
&= b + \left(\sum_{t=K+1}^{T-K} z_t z_t'\right)^{-1} \left(\sum_{t=K+1}^{T-K} z_t \xi_t^*\right), \\
\hat{b}_{DGLS} &= \left(\sum_{t=K+1}^{T-K} z_t \hat{\Omega}_{uu.v}^{-1} z_t'\right)^{-1} \left(\sum_{t=K+1}^{T-K} z_t \hat{\Omega}_{uu.v}^{-1} y_t\right) \qquad (11) \\
&= b + \left(\sum_{t=K+1}^{T-K} z_t \hat{\Omega}_{uu.v}^{-1} z_t'\right)^{-1} \left(\sum_{t=K+1}^{T-K} z_t \hat{\Omega}_{uu.v}^{-1} \xi_t^*\right),
\end{aligned}
$$

where $\hat{\Omega}_{uu.v}$ is a consistent estimate of $\Omega_{uu.v}$. Note that the $DGLS$ estimator $\hat{\beta}_{DGLS}$ is a GLS estimator using the long-run correlation information in the system (7).

An alternative estimator is the dynamic OLS estimator for the cointegrating vector β_m (hereafter IDOLS) in the m^{th} individual regression model of (1). Now define $w_{m,t} = \left(u_{m,t}, v'_{m,t}\right)'$ and $\Omega^{m,n} = \begin{pmatrix} \Omega^{m,n}_{uu} & \Omega^{m,n}_{uv} \\ \Omega^{m,n}_{vu} & \Omega^{m,n}_{vv} \end{pmatrix} = \sum_{h=-\infty}^{\infty} E\left(w_{m,0}w'_{n,h}\right)$. Let

$$\theta_{m,t} = u_{m,t} - \sum_{j=-\infty}^{\infty} \pi_{m,j} v_{m,t-j}, \tag{12}$$

be the linear projection residual of the m^{th} individual equation regression error $u_{m,t}$ on the closed linear space of $\{v_{m,t}\}_{t=-\infty}^{\infty}$ for $m = 1, ..., M$. Denote the long-run variance of $\theta_{m,t}$ by $\Omega^m_{uu.v}$. Under mild regularity conditions,

$$\Omega^m_{uu.v} = \Omega^{m,m}_{uu} - \Omega^{m,m}_{uv} \left(\Omega^{m,m}_{vv}\right)^{-1} \Omega^{m,m}_{vu}. \tag{13}$$

Now we define the following m^{th} individual dynamic regression

$$y_{m,t} = \tilde{x}'_{m,t}\beta_m + \sum_{j=-K}^{K} \pi_{m,j} \Delta x_{m,t-j} + \theta^*_{m,t}, \tag{14}$$

where

$$\begin{aligned} \theta^*_{m,t} &= \theta_{m,t} + \epsilon_{m,t}, \\ \epsilon_{m,t} &= \sum_{|j|>K} \pi_{m,j} v_{m,t-j}. \end{aligned}$$

We denote $\hat{\beta}_m$ the OLS regression estimator for β_m of (14) and $\hat{\beta}_{IDOLS} = \left(\hat{\beta}'_1, ..., \hat{\beta}'_M\right)'$.

Let $\hat{\beta}_{DGLS}$ and $\widehat{\Pi}_{DGLS}$ be sub-vectors of $\hat{b}_{DGLS}$ whose sizes are conformable to those of β and Π. In a similar fashion, define $\hat{\beta}_{SDOLS}$, $\widehat{\Pi}_{SDOLS}$, $\hat{\beta}_{IDOLS}$, and $\widehat{\Pi}_{IDOLS}$. Define

$$\begin{aligned} D_N &= diag\left(\sqrt{N}, NI_L\right), \\ F_N &= diag\left(D_N, ..., D_N\right), \\ G_N &= diag\left(F_N, \sqrt{N} I_{(2K+1)LM^2}\right), \end{aligned}$$

where $N = T - 2K$. We collect the asymptotic distributions of the above estimators in the following proposition.

Proposition Suppose that the functional limit theorem (3) holds and ξ_t in (4) and $\theta_{m,t}$ in (12) have long-run covariance matrices $\Omega_{uu.v}$ in (5) and $\Omega^m_{uu.v}$ in (13) respectively. Further suppose that $\frac{K^3}{T} \to 0$ and $\sqrt{T}\sum_{j>|K|}\|\pi_j\| = o(1)$. Then, as $T \to \infty$,

(a) $F_N\left(\hat{\beta}_{DGLS} - \beta\right) \Rightarrow MN\left(0, \left(\int_0^1 \tilde{B}_v(r)\,\Omega^{-1}_{uu.v}\tilde{B}'_v(r)\,dr\right)^{-1}\right)$,

(b) $F_N\left(\hat{\beta}_{SDOLS} - \beta\right) \Rightarrow MN\left(0, \left(\int_0^1 \tilde{B}_v(r)\,\tilde{B}'_v(r)\,dr\right)^{-1}\left(\int_0^1 \tilde{B}_v(r)\,\Omega_{uu.v}\tilde{B}_v(r)\right)\left(\int_0^1 \tilde{B}_v(r)\,\tilde{B}'_v(r)\,dr\right)^{-1}\right)$,

(c) $F_N\left(\hat{\beta}_{IDOLS} - \beta\right) \Rightarrow MN\left(0, \left(\int_0^1 \tilde{B}_v(r)\,\tilde{B}'_v(r)\,dr\right)^{-1}\left(\int_0^1 \tilde{B}_v(r)\,\tilde{\Omega}_{uu.v}\tilde{B}_v(r)\right)\left(\int_0^1 \tilde{B}_v(r)\,\tilde{B}'_v(r)\,dr\right)^{-1}\right)$,

where $\tilde{\Omega}_{uu.v}$ is the asymptotic long-run covariance of $(\theta_{1,t}, ..., \theta_{M,t})'$.

Moreover, it is easy to show that the limiting variance of $\hat{\beta}_{DGLS}$ is smaller than that of $\hat{\beta}_{SDOLS}$ with probability one because

$$\begin{aligned}&\left(\int_0^1 \tilde{B}_v(r)\,\Omega^{-1}_{uu.v}\tilde{B}'_v(r)\,dr\right)^{-1}\\ < \;&\left(\int_0^1 \tilde{B}_v(r)\,\tilde{B}'_v(r)\,dr\right)^{-1}\left(\int_0^1 \tilde{B}_v(r)\,\Omega_{uu.v}\tilde{B}_v(r)\right)\left(\int_0^1 \tilde{B}_v(r)\,\tilde{B}'_v(r)\,dr\right)^{-1} \qquad (15)\end{aligned}$$

with probability one. Therefore, $\hat{\beta}_{DGLS}$ is asymptotically more efficient than $\hat{\beta}_{SOLS}$.

Moreover, since $\Omega_{uu.v} \leq \tilde{\Omega}_{uu.v}$, we can conclude that $F_N\left(\hat{\beta}_{SDOLS} - \beta\right)$ is asymptotically more efficient than $F_N\left(\hat{\beta}_{IDOLS} - \beta\right)$. Therefore, among the three estimators, we can conclude that the DGLS estimator $\hat{\beta}_{DGLS}$ is the most efficient and the individual dynamic OLS estimator $\hat{\beta}_{IDOLS}$ is the least efficient. This result is also obtained by Park and Ogaki (1991) and Moon (1999) with the CCR method and the FM method, respectively.

In a linear cointegration regression model with no restriction on the cointegrating vectors, it is well known that there is no efficiency gain in GLS estimation over OLS estimation (*e.g.*, Phillips and Park, 1988 and Stock and Watson, 1993). This asymptotic equivalence result is similar in spirit to the classical Grenander and Rosenblatt theorem (1957) for which the polynomial time trend regression is considered with a stationary error.

However, as we verify through (15), in the dynamic augmented model (7) the GLS estimator is asymptotically more efficient than the OLS estimator. As discussed in Park and Ogaki (1991), the asymptotic efficiency gain of GLS in the SUR model comes from overidentification parameter restrictions. To see this, write the SUR model (2) in a multivariate regression form,

$$y_t = BX_t + u_t, \tag{16}$$

where $B = diag\,(\beta_1, ..., \beta_m)$ and $X_t = \left(\tilde{x}'_{1,t}, ..., \tilde{x}'_{M,t}\right)'$. In model (16), overidentifying parameter restrictions are imposed by restricting the off-block diagonal elements of B to be zero, while in the models studied in Phillips and Park (1988) and Stock and Watson (1993), the regression coefficients are exactly identified.

The mixed normality property of the limiting distributions of $\hat{\beta}_{SDOLS}$, $\hat{\beta}_{DGLS}$, and $\hat{\beta}_{IDOLS}$ enables us to use conventional chi-square tests for the null hypothesis of parameter restrictions. Suppose that we are interested in testing a null hypothesis on the parameters,

$$H_0 : \varphi(\beta) = r, \tag{17}$$

where φ is a $(q \times 1)$ vector-valued, continuously differentiable function and the first derivative of φ at the true parameter β ,Π, has full rank q. The Wald test statistic W_T using $\hat{\beta}_{DGLS}$ is then defined as

$$\begin{aligned} W_T \equiv\ & F_T(\varphi(\hat{\beta}_{DGLS}) - r)' \\ & \times \left(\Phi(\hat{\beta}_{DGLS}) \left(\frac{1}{T^2} \sum_{t=1}^{T} F_T^{-1} \tilde{x}_t \hat{\Omega}_{uu.v}^{-1} \tilde{x}'_t F_T^{-1} \right)^{-1} \Phi(\hat{\beta}_{DGLS})' \right)^{-1} \\ & \times F_T(\varphi(\hat{\beta}_{DGLS}) - r), \end{aligned}$$

and it is easy to verify that under the assumptions stated above, as $T \to \infty$, $W_T \Rightarrow \chi^2_q$, a chi-square distribution with q degrees of freedom.

3 Simulation comparison

In this section, we want to compare the relative merit of the various estimators of the integrated SUR model. The data generating process that will be used for this purpose is:

$$\begin{aligned} y_{1t} &= \beta_{10} + \beta_{11} x_{1t} + u_{1t} \\ y_{2t} &= \beta_{20} + \beta_{21} x_{2t} + u_{2t} \end{aligned}$$

where $x_{m,t}$ are correlated random walks:

$$\Delta x_{j,t} = v_{j,t}, \quad j = 1, 2$$

and u_t is correlated with $v_{j,t}$:

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim i.i.d.N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho & \omega & 0 \\ \rho & 1 & 0 & \omega \\ \omega & 0 & 1 & \phi \\ 0 & \omega & \phi & 1 \end{pmatrix} \right).$$

The parameter ρ controls the degree of correlation between the two equations and will therefore affect the efficiency gains of GLS relative to OLS. On the other hand, ω controls the degree of correlation between regressors and disturbances and will therefore affect the behavior of the static OLS and GLS estimators of (2). Finally, ϕ controls the degree of correlation between the regressors of the two equations and will control the relative efficiency of system versus individual equation OLS. We set the covariances (ρ, ω, ϕ) to extreme values, that is $(0,0,0)$, $(0,0.5,0.5)$, $(-0.9,0,0)$, $(0.9,0,0)$, $(0,-0.9,0)$, $(0,0.9,0)$, $(0,0,0.9)$, and $(0,0,-0.9)$.

Without loss of generality, we set $\beta_{m,j} = 1$ for $m,j = 1,2$ and concentrate on the estimation of the coefficients from the first equation. We want to compare $\sqrt{N}\left(\widehat{\beta}_{1,0,e} - 1\right)$ and $N\left(\widehat{\beta}_{1,1,e} - 1\right)$ for $e = IDOLS$, $SDOLS$, $DGLS$, $FM-OLS$ [3], and $FM-GLS$. The sample sizes chosen are $T = 100$ and $T = 500$, and the number of replications is 1000. The number of leads and lags in the dynamic estimators (K) is set at 5 for $T = 100$ and 8 for $T = 500$. We also report the rejection frequency of the hypotheses $\beta_{1,0} = 1$ and $\beta_{1,1} = 1$ for each estimator based on individual t-tests using the asymptotic critical values.

Since the efficiency gains due to GLS-type estimators come from a non-diagonal long-run covariance matrix, it is important to see how each parameter affects it. For this DGP, the long-run covariance matrix $\Omega_{\mathrm{u}u.v}$ is

$$\Omega_{\mathrm{u}u.v} = \begin{pmatrix} 1 - \frac{\omega^2}{1-\phi^2} & \rho + \frac{\phi\omega^2}{1-\phi^2} \\ \rho + \frac{\phi\omega^2}{1-\phi^2} & 1 - \frac{\omega^2}{1-\phi^2} \end{pmatrix}$$

This expression highlights the role played by the three parameters of the DGP.

The results are presented in table 1 for $T = 100$ and table 2 for $T = 500$. The first line for each set of parameters gives the results for $\beta_{1,0}$ and the second line gives the results for $\beta_{1,1}$.

The main features of the results are:

[3] There is an error in $\hat{\beta}_{FM-SOLS}$ defined in Moon (1999). The component $\hat{\pi}_m$ should be

$$\hat{\pi}_m = \begin{pmatrix} 0 \\ \hat{\Delta}_{vu}^{m,m} - \left(\hat{\Delta}_{vv}^{m,\cdot}\right)\left(\left(\hat{\Omega}_{uv}\hat{\Omega}_{vv}^{-1}\right)_{m,\cdot}\right)' \end{pmatrix}.$$

- All estimators are nearly unbiased even with the smaller sample size.
- The convergence of the estimates of the slope coefficient $\left(\hat{\beta}_{1,1}\right)$ is much faster than that of the constant, as expected.
- There are large efficiency gains in using GLS in cases where $|\rho|$ is high. These are cases where the cross-equation correlation is high.
- All estimators suffer from size distortions with $T = 100$. The size of the tests is much improved with $T = 500$. However, FM-OLS and FM-GLS have severe distortions for high $|\rho|$.
- There are noticeable efficiency gains of using system methods relative to individual equation methods when the regressors are endogenous $(\omega \neq 0)$ and correlated $(\phi \neq 0)$.
- The magnitude of ω or ϕ on their own is negligeable. However, they would bias static estimators.

Overall, our estimator performs well in all cases relative to its competitors and has substantial advantages in some cases. This is especially true with the bigger sample size $(T = 500)$.

4 Empirical Application: Testing for PPP

The purchasing power parity (PPP) doctrine states that nominal exchange rates should reflect relative price behavior. Over the years, several versions of it have been proposed. The strongest version is *absolute* PPP which states that the nominal bilateral exchange rate should be equal to the relative price level between the two countries, implying that all goods and services should have the same price in both countries once expressed in a common currency. Due to the different national price indices and the presence of non-traded goods, a more common version is *relative* PPP in which changes in the nominal exchange rate reflect changes in relative price levels (or the inflation differential). Pedroni (1996) makes a further distinction between *weak (*relative) PPP and *strong* (relative) PPP. In the strong form, the coefficient of a regression of the nominal exchange rate on the relative price level is 1, implying that the real exchange rate is stationary. Under weak relative PPP, the coefficient is different from 1, but there exists some value for which the residuals are stationary.

Table 1. Simulation results, $T = 100$

	Mean					Std. error					Size				
	IDOLS	SDOLS	DGLS	FM-OLS	FM-GLS	IDOLS	SDOLS	DGLS	FM-OLS	FM-GLS	IDOLS	SDOLS	DGLS	FM-OLS	FM-GLS
(0,0,0)	1.013	1.012	1.013	1.007	1.009	0.264	0.291	0.295	0.189	0.189	12.2	16.1	17.1	9.4	9.8
	0.998	0.998	0.998	1.000	0.999	0.050	0.053	0.054	0.036	0.036	13.7	16.5	19.3	11.1	13.1
(0,0.5,0.5)	0.999	0.997	0.996	1.000	1.000	0.219	0.220	0.224	0.163	0.164	12.8	15.5	17.4	7.6	7.7
	0.997	0.998	0.998	1.008	1.009	0.041	0.042	0.042	0.032	0.033	12.6	15.5	18.9	8.9	10.6
(-0.9,0,0)	0.989	0.988	0.996	0.995	0.996	0.262	0.273	0.204	0.191	0.148	13.2	15.3	16.2	50.8	45.5
	0.999	1.000	1.000	0.999	0.999	0.047	0.051	0.031	0.035	0.023	12.7	14.6	16.9	50.5	33.1
(0.9,0,0)	0.983	0.985	0.993	0.997	1.000	0.245	0.267	0.196	0.187	0.149	10.4	16.1	16.8	2.5	1.5
	1.002	1.002	1.001	1.002	1.002	0.047	0.051	0.031	0.033	0.022	11.3	14.7	18.6	1.1	0.6
(0,-0.9,0)	1.000	1.001	0.999	0.999	0.998	0.107	0.117	0.120	0.119	0.124	12.3	15.8	17.3	15.2	17.0
	1.000	0.999	0.999	0.983	0.981	0.020	0.022	0.023	0.025	0.025	12.4	15.0	17.5	25.3	28.7
(0,0.9,0)	0.998	0.996	0.996	1.000	0.999	0.112	0.120	0.123	0.133	0.137	13.7	15.4	17.0	15.7	18.0
	1.001	1.001	1.001	1.018	1.020	0.022	0.023	0.024	0.027	0.028	12.4	15.2	19.1	23.5	27.6
(0,0,-0.9)	1.003	1.007	1.006	1.008	1.007	0.240	0.260	0.263	0.178	0.180	10.8	14.2	15.7	8.8	8.8
	0.999	0.999	0.999	1.000	1.000	0.049	0.054	0.054	0.035	0.035	12.9	16.9	17.1	9.6	9.9
(0,0,0.9)	1.005	1.005	1.007	1.003	1.003	0.229	0.258	0.263	0.179	0.180	11.2	14.1	15.1	11.0	11.1
	1.000	1.000	1.000	1.001	1.001	0.048	0.053	0.053	0.034	0.034	11.7	16.2	16.5	10.1	10.3

Tests of absolute PPP are scarce in the literature because of measurement problems. One exception is the paper by Crownover, Pippenger, and Steigerwald (1996) which uses data on price levels used to adjust the salaries of German diplomats over the world. However, there is by now an enormous literature on testing relative PPP. The most common form of these tests consist of testing whether the real exchange rate is stationary as it should be under strong relative PPP. Sample articles include Frankel and Rose (1996), MacDonald (1996), Oh (1996), Wu (1996), Koedjik Schotman, and Dijk (1998), Papell and Theodoridis (1998), Sarno and Taylor (1998), O'Connell (1998), Higgins and Zakrajsek (1999), and Flôres, Jorion, Preumont, and Szafarz (1999). The overall consensus from these studies seem to be that real exchange rates are stationary when using system tests and that PPP holds in the long run, the only exception being the study by O'Connell.

Table 2. Simulation results, $T = 500$

	Mean					Std. error $(\times 10^{-1})$					Size				
	IDOLS	SDOLS	DGLS	FM-OLS	FM-GLS	IDOLS	SDOLS	DGLS	FM-OLS	FM-GLS	IDOLS	SDOLS	DGLS	FM-OLS	FM-GLS
(0,0,0)	1.004	1.004	1.005	1003	1.003	0.849	0.865	0.867	0.782	0.784	6.0	7.0	7.3	6.8	7.2
	1.000	1.000	1.000	1.000	1.000	0.074	0.075	0.075	0.067	0.068	7.5	8.8	8.6	6.3	7.0
(0,0.5,0.5)	0.996	0.996	0.996	0.998	0.998	0.758	0.722	0.721	0.661	0.662	6.8	6.6	6.4	3.7	3.8
	1.000	1.000	1.000	1.001	1.001	0.068	0.065	0.063	0.059	0.058	6.7	7.4	8.3	4.8	4.6
(-0.9,0,0)	0.997	0.997	0.998	0.996	0.998	0.869	0.891	0.629	0.807	0.588	6.6	7.3	6.7	53.7	44.2
	1.000	1.000	1.000	1.000	1.000	0.073	0.076	0.039	0.067	0.035	5.7	7.0	7.0	51.9	24.8
(0.9,0,0)	1.000	1.000	1.000	1.001	1.000	0.845	0.854	0.597	0.796	0.561	7.1	7.0	6.8	1.7	0.6
	1.000	1.000	1.000	1.000	1.000	0.074	0.075	0.038	0.071	0.037	7.2	7.6	6.5	1.5	0.0
(0,-0.9,0)	0.999	0.998	0.998	0.998	0.998	0.372	0.382	0.383	0.357	0.360	6.2	6.9	7.1	6.1	6.2
	1.000	1.000	1.000	0.999	0.999	0.032	0.032	0.033	0.035	0.035	7.5	8.3	9.0	9.9	11.0
(0,0.9,0)	1.000	1.000	1.000	1.000	0.999	0.375	0.385	0.386	0.376	0.380	7.1	8.0	7.8	7.9	8.1
	1.000	1.000	1.000	1.001	1.001	0.032	0.033	0.033	0.033	0.034	6.7	7.3	7.5	8.7	10.3
(0,0,-0.9)	1.002	1.002	1.002	1.003	1.003	0.863	0.881	0.884	0.797	0.799	6.5	7.2	7.3	5.9	6.0
	1.000	1.000	1.000	1.000	1.000	0.069	0.071	0.072	0.064	0.065	6.3	6.1	6.4	5.9	5.9
(0,0,0.9)	1.003	1.003	1.003	1.002	1.002	0.929	0.938	0.944	0.873	0.878	8.1	8.5	8.6	7.2	7.3
	1.000	1.000	1.000	1.000	1.000	0.080	0.080	0.081	0.073	0.073	9.3	9.1	9.0	9.1	9.2

We use the estimator proposed above in testing for strong relative PPP. Because of strong links across markets and the use of a numeraire country in defining real exchange rates, real exchange rates should have high cross-correlation, a fact neglected in many of the above studies and taken into account by our estimator. The papers by O'Connell, HIggins and Zakrajsek, and Flôres, Jorion, Preumont, and Szafarz do control for this heterogeneity but in the short-run only. Even in the long run, we should expect (and we will show) that real exchange rates are cross-correlated. Taking advantage of these long-run correlations in estimation leads to more efficient estimates and more powerful inference.

The data we employ covers the entire recent float for the G-7 countries (1974-1998, 300 observations per country). The data was obtained from IFS and consists of monthly averages of the bilateral exchange rates relative to the US dollar and national consumer price indices.

Table 3 presents the short-run correlation matrix of changes in the real exchange rates of the 6 countries involved and is similar to table 1 in Flores et al. (1999). Three explanations can account for the large correlation, in particular among European countries. The first one is the presence of various nominal exchange rate co-ordination mechanisms among European countries over the period. The second source of correlation, as pointed out by O'Connell (1998) , is the use of a numeraire country (in this case the United States). Finally, the international transmission of shocks among these countries leads to correlation among their real exchange rates.

Table 3. Short-run correlation matrix of real exchange rate changes

Canada	1.000					
Japan	0.059	1.000				
Germany	0.127	0.593	1.000			
France	0.121	0.576	0.919	1.000		
Italy	0.145	0.457	0.756	0.800	1.000	
United Kingdom	0.142	0.445	0.648	0.657	0.650	1.000

Table 4 is identical to the previous table, but it compares the *long-run* correlation of changes in the real exchange rates among the countries involved. There are large off-diagonal entries, especially among European countries, but also between Japan and Europe. Canada is the only country with small correlation with the other countries in the sample. These long-run correlations were neglected by all previous authors and will be used to gain efficiency in testing for PPP.

Table 4. Long-run correlation matrix of real exchange rate changes

Canada	1.000					
Japan	−0.013	1.000				
Germany	−0.028	0.561	1.000			
France	−0.115	0.502	0.926	1.000		
Italy	0.014	0.329	0.784	0.836	1.000	
United Kingdom	−0.061	0.291	0.590	0.693	0.726	1.000

The test that we will use is a test of strong relative PPP. Strong relative PPP implies that β should be unity in the regression:

$$\ln s_t = \alpha + \beta \ln (p_t/p_t^*) + \varepsilon_t \tag{18}$$

where s_t is the nominal bilateral exchange rate defined as the number of local currency units per US dollar, p_t is the local consumer price index, and p_t^* is the US CPI. Absolute PPP would further restrict α to be zero, but we do not impose this or test it as it is obviously violated in our sample.

Our test differs from most other empirical work in that the null hypothesis $\beta = 1$ treats PPP as the null hypothesis. As previously stated, most studies test for PPP by carrying out unit root tests on real exchange rates. The null hypothesis in this case (the unit root) is no PPP. We believe that treating PPP as the maintained hypothesis to be discarded is more natural.

A further advantage of our framework is that, under the null hypothesis of PPP, the linear specification (18) is robust to the choice of a numeraire country. That is, if we choose say Germany as the numeraire country, our null hypothesis of PPP would be a hypothesis on the coefficients of a linear regression of nominal exchange rates on relative price levels. Tests based on autoregressions do not have this property: the specification is sensitive to the choice of the numeraire country as shown by Flôres et al. (1999). That is, if the real exchange rate using the U.S. as numeraire is an AR(1) process, the real exchange rate using Germany as numeraire will not be an AR(1).

Moreover, in the case where equation (18) is not a cointegration regression, that is if ε_t is non-stationary for any value of α and β, our test statistic diverges to infinity (equation (18) becomes a spurious regression, see theorem 1(d) in Phillips (1986)). In this case, our test will correctly conclude that PPP is not supported by the data.

Table 5 presents the results of estimating equation (18) using the same 5 estimators as in the Monte Carlo experiment above. To assess sensitivity to the choice of K, we report results for three values of K close to $T^{\frac{1}{3}}$. It turns out that the results are quite robust to this choice.

First of all, as just mentioned, it is reassuring that no conclusion depends on the choice of K, the number of leads and lags included in the dynamic regressions. With our dynamic GLS, we can reject the null hypothesis for all countries except for Canada and Italy. The non-rejection for these two countries is robust across methods as well. We can only reject Italy with FM-GLS.

There are noticeable differences when passing to system methods (either SDOLS or DGLS). The two system methods allow us to reject the null hypothesis for 4 of the six countries while individual OLS allows us to reject only for France and Japan. The efficiency gains come for highly correlated European countries (Germany and the United Kingdom). Moreover, dynamic GLS provides more precise estimates, suggestive of further efficiency gains though no conclusion is reversed by the choice between OLS or GLS.

As emphasized by other authors, it is important to look at system methods in testing for PPP. This provides efficiency gains and increased power sufficient to reject the hypothesis that purchasing power parity is a reasonable description of long-run exchange rate behavior.

Table 5. Estimates of β

(t-statistics for $H_0 : \beta = 1$ in parentheses, * indicates significance at 5% level)

	$K =$	Canada	Japan	Germany	France	Italy	U.K.
	5	1.422 (1.317)	1.963 * (6.492)	1.031 (0.192)	2.161 * (6.800)	1.108 (1.370)	0.696 (−1.755)
IDOLS	6	1.430 (1.234)	1.983 * (6.148)	1.056 (0.292)	2.178 * (6.523)	1.128 (1.470)	0.691 (−1.544)
	7	1.437 (1.164)	2.014 * (5.992)	1.068 (0.329)	2.187 * (8.289)	1.145 (1.539)	0.678 (−1.417)
	5	1.284 (0.617)	1.585 * (3.136)	1.644 * (3.152)	2.455 * (6.579)	0.968 (−0.365)	0.627 * (−3.142)
SDOLS	6	1.136 (0.306)	1.399 * (2.186)	1.568 * (2.740)	2.656 * (7.000)	0.980 (−0.230)	0.544 * (−4.005)
	7	1.097 (0.208)	1.283 (1.454)	1.446 * (2.109)	2.780 * (6.672)	0.998 (−0.027)	0.472 * (−4.469)
	5	1.319 (1.066)	1.633 * (4.648)	1.434 * (3.115)	2.060 * (7.547)	0.900 (−1.419)	0.680 * (−3.155)
DGLS	6	1.187 (0.765)	1.493 * (3.545)	1.449 * (3.302)	2.273 * (8.817)	0.909 (−1.232)	0.656 * (−3.527)
	7	1.147 (0.632)	1.373 * (2.676)	1.394 * (3.072)	2.457 * (9.119)	0.951 (−0.666)	0.617 * (−4.283)
$FM-OLS$		1.052 (0.132)	1.829 * (7.996)	0.784 * (−1.968)	1.969 * (3.023)	0.953 (−0.784)	0.777 * (−3.382)
$FM-GLS$		1.252 (0.656)	1.930 * (10.424)	0.839 (−1.480)	1.539 (1.785)	0.863 * (−2.262)	0.705 * (−4.610)

5 Conclusion

This paper has proposed new estimators of the SUR model with integrated regressors based on an augmented regression model. We have derived the asymptotic distributions of our estimators and indicated that a feasible generalized least squares estimator of the augmented model using the long-run covariance matrix is the most efficient among them.

Monte Carlo results suggest that our dynamic GLS estimator compares favorably with other estimators and improves noticeably upon them in some situations. Moreover, inference with this estimator has size close to its nominal level. This is not the case of fully-modified estimators which suffer from more severe size distortions.

An application of the methods to testing of purchasing power parity among G-7 countries demonstrates the importance of analyzing this issue in a system framework. With our dynamic system methods, we are able to reject PPP for 4 of the 6 countries in our analysis. This casts doubts on the validity of PPP as a reasonable description of long-run exchange rate behavior. This conclusion is in line with that of O'Connell (1998) but contrary to much of the recent literature. Further work is necessary in assessing the strength and weaknesses of the various methods of testing for PPP.

6 Appendix

Proof of Proposition

The proof uses standard arguments and will be mostly omitted. It suffices to notice that we can write

$$G_N\left(\hat{b}_{DGLS}-b\right)=\begin{pmatrix} F_N\left(\hat{\beta}_{DGLS}-\beta\right) \\ \sqrt{N}\left(\widehat{\Pi}_{DGLS}-\Pi\right)\end{pmatrix}=\begin{pmatrix} A_{11} & A_{12} \\ A_{12}' & A_{22}\end{pmatrix}^{-1}\begin{pmatrix} B_1 \\ B_2\end{pmatrix},$$

where

$$\begin{aligned}
A_{11} &= \sum_{t=K+1}^{T-K} F_N^{-1}\tilde{x}_t\hat{\Omega}_{uu.v}^{-1}\tilde{x}_t' F_N^{-1} \\
A_{12} &= \begin{pmatrix} \frac{1}{\sqrt{N}}\sum_{t=K+1}^{T-K} F_N^{-1}\tilde{x}_t\hat{\Omega}_{uu.v}^{-1}\left(\Delta x_{t-K}'\otimes I_M\right) & \cdots & \frac{1}{\sqrt{N}}\sum_{t=K+1}^{T-K} F_N^{-1}\tilde{x}_t\hat{\Omega}_{uu.v}^{-1}\left(\Delta x_{t+K}'\otimes I_M\right)\end{pmatrix} \\
A_{22} &= \begin{pmatrix} \frac{1}{N}\sum_{t=K+1}^{T-K}\left(\Delta x_{t-K}\Delta x_{t-K}'\otimes\hat{\Omega}_{uu.v}^{-1}\right) & \cdots & \frac{1}{N}\sum_{t=K+1}^{T-K}\left(\Delta x_{t-K}\Delta x_{t+K}'\otimes\hat{\Omega}_{uu.v}^{-1}\right) \\ \vdots & \ddots & \vdots \\ \frac{1}{N}\sum_{t=K+1}^{T-K}\left(\Delta x_{t+K}\Delta x_{t-K}'\otimes\hat{\Omega}_{uu.v}^{-1}\right) & \cdots & \frac{1}{N}\sum_{t=K+1}^{T-K}\left(\Delta x_{t+K}\Delta x_{t+K}'\otimes\hat{\Omega}_{uu.v}^{-1}\right)\end{pmatrix},
\end{aligned}$$

and

$$B_1 = F_N^{-1}\sum_{t=K+1}^{T-K}\tilde{x}_t\hat{\Omega}_{uu.v}^{-1}\xi_t^*$$

$$B_1 = \begin{pmatrix} \frac{1}{\sqrt{N}} \sum_{t=K+1}^{T-K} \left(\Delta x_{t-K} \otimes \hat{\Omega}_{uu.v}^{-1} \xi_t^* \right) \\ \vdots \\ \frac{1}{\sqrt{N}} \sum_{t=K+1}^{T-K} \left(\Delta x_{t+K} \otimes \hat{\Omega}_{uu.v}^{-1} \xi_t^* \right) \end{pmatrix}.$$

Then, with $\frac{K^3}{T} \to 0$ and $\sqrt{T} \sum_{j>|K|} \|\pi_j\| = o(1)$, we can use similar arguments to those in Saikkonen(1991) to show that as $T \to \infty$

$$\begin{aligned} & F_N \left(\hat{\beta}_{DGLS} - \beta \right) \\ \Rightarrow & \left(\int_0^1 \tilde{B}_v(r) \, \Omega_{uu.v}^{-1} \tilde{B}_v'(r) \, dr \right)^{-1} \int_0^1 \tilde{B}_v(r) \, \Omega_{uu.v}^{-1} dB_{uu.v}(r) \\ \equiv & MN \left(0, \left(\int_0^1 \tilde{B}_v(r) \, \Omega_{uu.v}^{-1} \tilde{B}_v'(r) \, dr \right)^{-1} \right), \end{aligned}$$

where $B_{uu.v}(r) = B_u(r) - \Omega_{uv} \Omega_{vv}^{-1} B_v(r)$. The derivation of the other two results is similar. ■

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