ANALYSIS AND DESIGN OF MULTIVIBRATORS

A THESIS

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by

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I wish to express my sincerest appreciation to Associate Professor M. A. Honnell for his interest and valuable assistance in the prosecution of this work.

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DEFINITION OF SYMBOLS USED

A	Voltage amplification of the amplifying devices of the ideal multivibrator.
A_1	Amplifier number 1, or its amplification.
A_2	Amplifier number 2, or its amplification.
C_1	Condenser connected between the output terminal of A_1 and the input terminal of A_2.
C_2	Condenser connected between the output terminal of A_2 and the input terminal of A_1.
C_b	Output capacitance of either vacuum tube.
C_g	Input capacitance of either vacuum tube.
C_m	Coupling condenser connected between the plate terminal of one tube and the grid terminal of the other tube.
E_b	D.C. plate voltage of either tube.
E_c	D.C. grid bias of either tube.
E_e	The change of voltage across R_b when the plate current of its driving tube changes from zero to its maximum value. It consists of two components, E_x and E_s.
E_e'	The change of voltage across R_b when the plate current of its driving tube changes from the quiescent value to zero.
E_{go1}	The critical (negative) grid voltage of VT_1 at which flip-over occurs. Also the critical (negative) input voltage of A_1.
E_{go2}	The critical (negative) grid voltage of VT_2 at which flip-over occurs. Also the critical (negative) input voltage of A_2.
E_s	The step-function component of E_e.
E_x	The exponential-function component of E_e.
e	The base of natural logarithms.
f	frequency.
G_1	The input terminal of A_1.

G_2	The input terminal of A_2.
G_{m1}	The grid-plate transconductance of VT_1.
G_{m2}	The grid-plate transconductance of VT_2.
γ	The ratio of the maximum negative grid voltage to the critical negative grid voltage (for the ideal multivibrator).
γ	The ratio of the equivalent maximum negative grid voltage to the critical negative grid voltage (for the non-ideal multivibrator).
I_b	The D.C. plate current of either vacuum tube.
i_g	The current in R_g.
K_1	Cathode, or ground terminal of A_1.
K_2	Cathode, or ground terminal of A_2.
λ	$= R_g R_e (C_g C_m + C_b C_m + C_b C_g)$
p	d/dt = time derivative operator.
R_1	Resistance of the path over which C_1 discharges.
R_2	Resistance of the path over which C_2 discharges.
R_a	$= R_e(C_b + C_m)/C_m + R_g C_g/C_m$
R_b	The plate load resistance of either tube.
R_e	Equivalent resistance formed by putting R_b in parallel with the plate resistance of its driving tube.
R_g	Resistance connected between the grid terminal of a tube and its cathode terminal.
r_g	Dynamic grid resistance of either tube.
R_i	$= R_g r_g/(R_g + r_g)$
T	Time for one complete cycle of the multivibrator.
T_1	Time during which VT_2 (or A_2) is inactive. (See also equation D1)
T_2	Time during which VT_1 (or A_1) is inactive. (See also equation D4)
t	Time in seconds.

τ_1	Time constant of the circuit over which C_{m1} discharges.
τ_2	Time constant of the circuit over which C_{m2} discharges.
τ_{g1}	Time constant of the circuit over which C_{m1} charges.
τ_{g2}	Time constant of the circuit over which C_{m2} charges.
U	Ideal rectifier.
v_1	The voltage drop across R_{g2}. Also the voltage drop across R_2.
v_2	The voltage drop across R_{g1}. Also the voltage drop across R_1.

ANALYSIS AND DESIGN OF MULTIVIBRATORS

INTRODUCTION

More than a quarter of a century has passed since Abraham and Bloch published a description of a circuit to generate "multiple vibrations". Their device was characterized by two essential features which determined its usefulness: an output voltage wave rich in harmonics, and a willingness to synchronize its fundamental frequency with that of an external voltage. Many early applications made use of these features to provide frequency standards, harmonic generators, and other devices of a similar nature. In more recent years there has grown an increasing demand for signal generators to supply an endless variety of complex wave shapes, in applications varying from television timers to electronic switches. The diversity of these applications has naturally resulted in a diversity of circuits, all of which may be properly called multivibrators. Yet, the basic circuit of Abraham and Bloch is generic to a great many, if not all, of these varied forms.

Much has been written concerning multivibrators, but the literature is inclined to go from one extreme to another, either neglecting too little, or neglecting too much. There is still a need for a treatment comprehensive enough to provide reasonable accuracy, yet simple enough to facilitate rapid engineering calculations.

The following analysis provides a design procedure which the author believes meets this need.

ANALYSIS OF THE IDEAL MULTIVIBRATOR

The circuit diagram in Figure 1 shows the familiar prototype multivibrator. Applying Thevenin's Theorem and the equivalent plate-circuit theorem, the circuit of Figure 2 is obtained. As far as the currents in C_m and R_g are concerned, Figure 2 is equivalent to Figure 1. A clearer understanding of the general aspects of the multivibrator can be gained by idealizing the circuit, as shown in Figure 3. This circuit will be called an "ideal multivibrator". From its behavior we can obtain the fundamental equation for frequency and also determine the basic wave-shapes of the voltages generated. Later it will be shown how these results can be modified so as to apply to actual circuits.

The ideal multivibrator consists of two amplifying devices and associated circuits connected as shown. For the sake of generality, the two devices are assumed to have different coefficients, although they are, of course, of the same nature otherwise. The subscripts 1 and 2 will be used when it is necessary to distinguish the sections, or stages, from each other as indicated in the diagram.

The amplifiers have input terminals K[1] and G, and output terminals K and P. When the input voltage, v, is such as to make G positive, the impedance between G and K is zero; and when G is negative, this impedance is infinite. This property is indicated by the ideal rectifiers, labeled U. When v is between zero and some critical neg-

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[1]A table of symbols is given on page v.

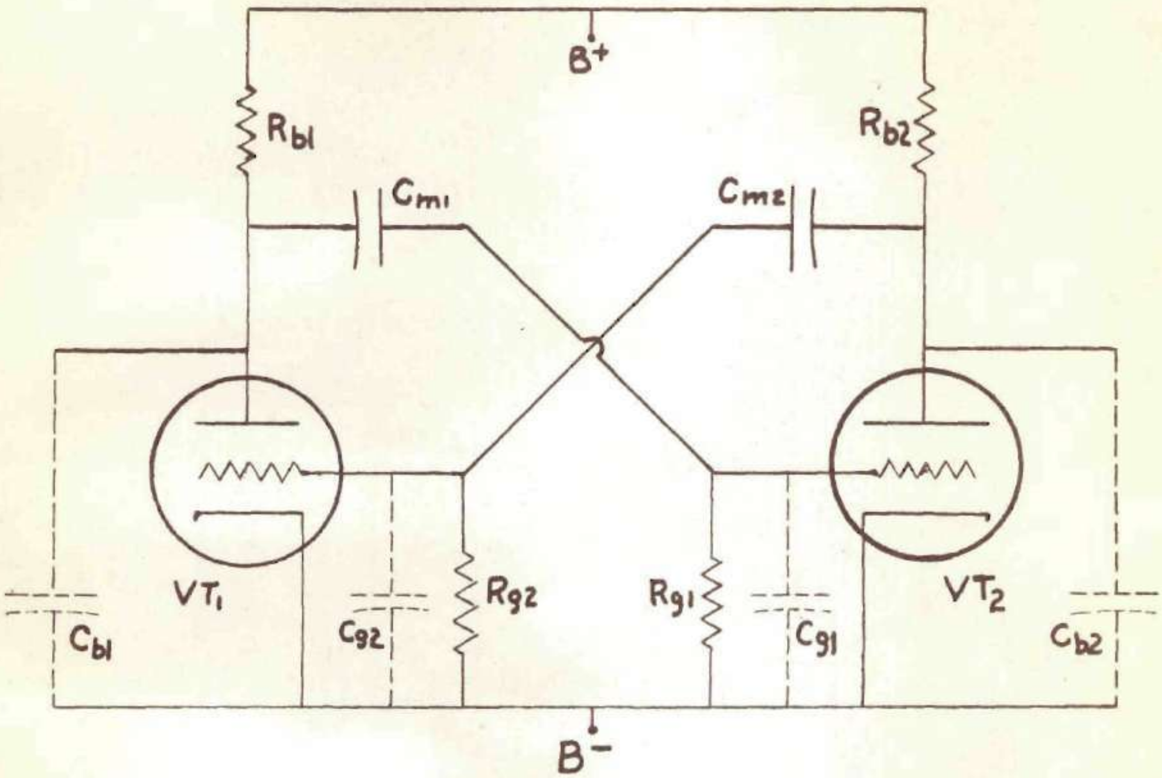

FIG. 1. BASIC MULTIVIBRATOR CIRCUIT

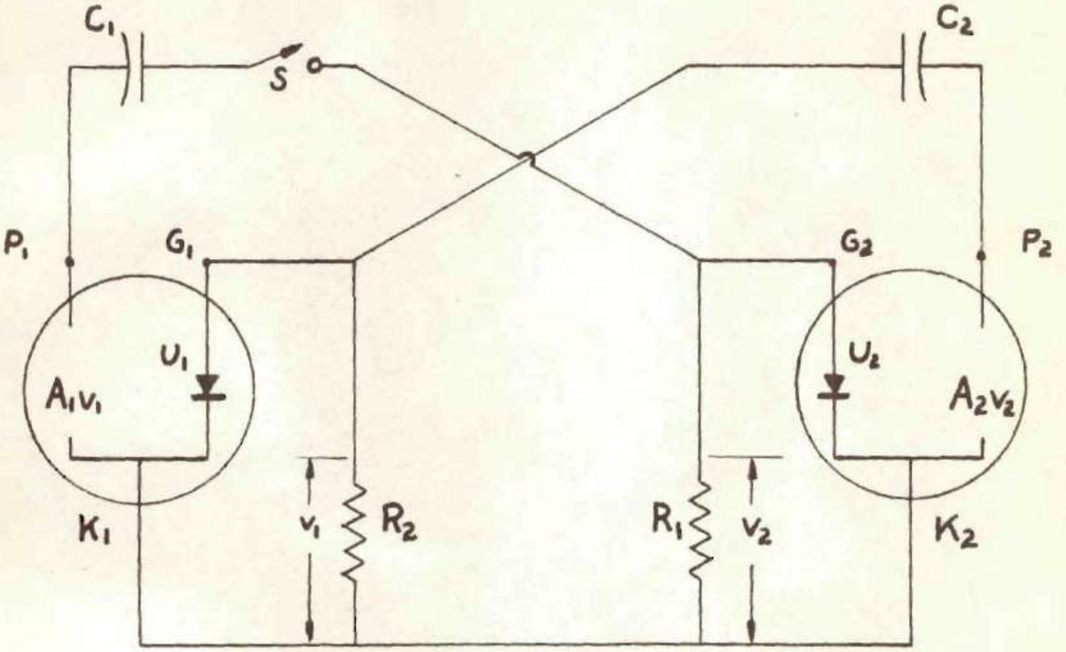

FIG. 3. THE IDEAL MULTIVIBRATOR

ative value, E_{go}, the gain is A. When v is more negative than E_{go}, the gain is zero. (Because of the properties of U, v cannot be positive.) The polarity of the amplified voltage is reversed with respect to the input voltage, so that when G is negative P is positive. Futhermore, the output sections of the amplifiers have zero internal impedance.

Referring to Figure 3, let us suppose that the switch, S, is opened and that the condenser C_1 is charged to a voltage A_1E_{go1}, the terminal towards G_2 being negative. Assume that the charge on C_2 is zero. If S is closed, the voltage A_1E_{go1} instantly appears across R_1; and C_1 begins to discharge. The voltage across R_1 is amplified by A_2; but, since A_2 cannot amplify voltages greater than E_{go2}, the total voltage appearing in its output circuit is A_2E_{go2}. The polarity of this voltage is such as to make G_1 positive and R_2 is, therefore, short-circuited through U_1. Thus C_2 is charged instantly to a voltage A_2E_{go2}.

As long as v_2 is greater than E_{go2}, A_2 cannot amplify and nothing more happens in the circuit of R_2C_2. However, C_1 continues to discharge and the voltage across R_1 decreases exponentially with time so that

$$v_2 = A_1E_{go1}e^{-t/R_1C_1} \tag{1}$$

When $t = T_1$, v_2 will have fallen to the critical value, E_{go2}.

Thus
$$E_{go2} = A_1E_{go1}e^{-T_1/\tau_1} \tag{2}$$

where

T_1 = time required for v_2 to decrease from A_1E_{go1} to E_{go2}. (D1)

$\tau_1 = R_1C_1$ (D2)

Solving equation (2) for T_1 we have

$$T_1 = \tau_1 \ln(A_1 E_{go1}/E_{go2}) \tag{2a}$$

If we let

$$\gamma_1 = A_1 E_{go1}/E_{go2} \tag{D3}$$

Equation (2a) becomes

$$T_1 = \tau_1 \ln \gamma_1 \tag{3}$$

At the end of the interval T_1, A_2 again amplifies, the voltage in its output circuit is reduced to zero[2] (permitting C_2 to discharge), the voltage $A_2 E_{go2}$ appears across R_2, and C_1 is again charged to the voltage $A_1 E_{go1}$. During the interval T_2, C_2 discharges, and we have

$$T_2 = \tau_2 \ln(A_2 E_{go2}/E_{go1}) \tag{4}$$

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[2]At the instant when v_2 becomes less than the critical value, the voltage in the output circuit of A_2 becomes less than $A_2 E_{go2}$. This causes C_2 to begin to discharge, which in turn causes a negative voltage to appear across R_2. The resulting voltage in the output circuit of A_1 tends to make G_2 positive, causing a further decrease in v_2. This is followed by further reduction in the output voltage of A_2, and so on. In actual circuits a finite time is required for this process to be propagated, but in the ideal circuit it is assumed that the events just described occur simultaneously, so that as soon as v_2 becomes less than the critical value, v_2 and $A_2 v_2$ are reduced to zero.

or $$T_2 = \tau_2 \ln \gamma_2 \tag{5}$$

where

T_2 = time required for v_1 to decrease from A_2E_{go2} to E_{go1}. (D4)

$$\tau_2 = R_2C_2$$

$$\gamma_2 = A_2E_{go2}/E_{go1} \tag{D6}$$

The interval T_2 completes one cycle. The total time is

$$T = T_1 + T_2 \tag{6}$$

and $$f = 1/T = 1/(\tau_1 \ln \gamma_1 + \tau_2 \ln \gamma_2) \tag{7}$$

Equation (7) is the basic equation for the frequency of the multivibrator. As far as the ideal circuit is concerned, it involves no approximations and is, therefore, an exact equation for frequency. Figure 4 shows the wave-shapes of the voltages, v_1 and v_2, which will be generated by the ideal multivibrator. These are the wave-shapes commonly associated with multivibrators, shown in ideal form. In fact, expressions similar to equation (7) are well known and have been derived by several writers.

The purpose of the foregoing analysis is two-fold. First, to establish a useful standard by which to evaluate the performance of actual circuits. Second, to establish the meaning of the terms τ and γ. It must not be thought that the use of the term "ideal" means that the ideal multivibrator would be the most desirable of all possible circuits, for it is clear that such a choice would depend on the use to which any particular device would be put. Rather, the circuit is ideal in that it can be analyzed without making any approximations and its performance can be completely determined in terms of the known properties of the circuit.

Although γ_1 is formally defined in equation (D3), its significance is more readily seen from equation (3). With this in mind, we may restate the definition of γ_1 as follows. If the voltage across R_1 follows an exponential law as C_1 discharges, and if it has the value E_{go2} at the end of the interval T_1, then its value at the beginning of the interval was $\gamma_1 A_1 E_{go1}$. τ_1 is, of course, the time constant of the circuit over which C_1 discharges. Similar statements apply to γ_2 and τ_2. The implicationa are obvious. Although actual circuits differ in many respects from the ideal circuit, it is possible (within certain limits) to establish values for the τ's and γ's so that equation (7) can still be used to predict the frequency of oscillations. In other words, the concept of the ideal multivibrator reduces the analysis of actual circuits to the problem of finding the values of τ and γ.

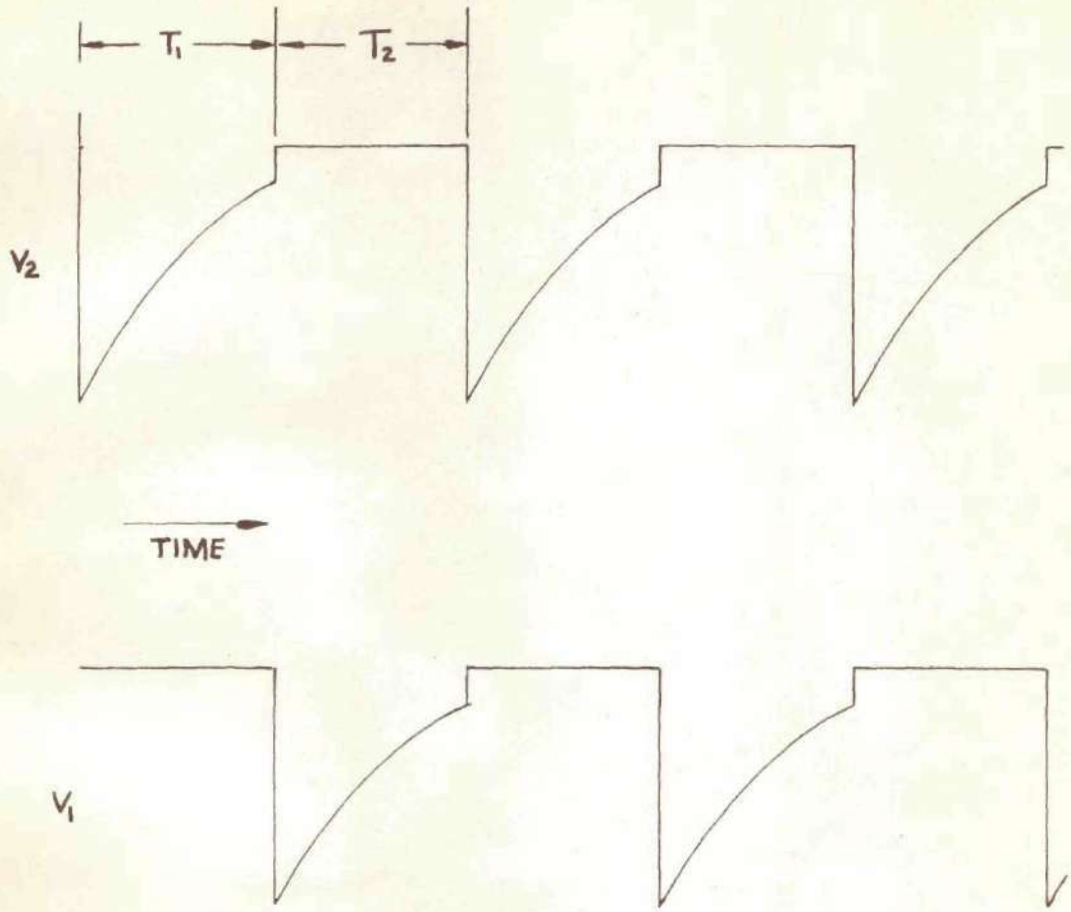

FIG. 4 WAVE-SHAPE OF THE VOLTAGE GENERATED BY THE IDEAL MULTIVIBRATOR

THE NON-IDEAL MULTIVIBRATOR

We now return to the circuit of Figure 1. We know that its general behavior is similar to that of the ideal multivibrator. We also know that every detail of its behavior is different. In the first place, the "flip-over" does not occur instantaneously, but requires a finite time, during which both vacuum tubes follow non-linear operating paths. In the second place, the voltages across the R_g's do not follow exponential laws; and even if they did, the time constants of the circuit would not be given by the simple RC products. In view of these facts, it is apparent that a complete analysis must be comparatively complicated. In order to avoid such complexity, it is necessary to make simplifying assumptions, or approximations. While such approximations may simplify the analysis, they do not simplify the circuit and it is to be expected that they will lead to errors in the results. The first step in the analysis is to decide what approximations are justified and what detail is justified in the light of the desired accuracy.

The circuit is made up of vacuum tubes, resistors, capacitors, etc. It is difficult to measure the properties of such devices to a high order of precision. An accuracy exceeding 1% can hardly be obtained on an engineering basis. Besides, changes of this order of magnitude are likely to occur during normal operation. Accordingly, some means of adjustment must be provided and if a design can be executed to an accuracy of, say, 10% it will be satisfactory for most purposes. It is believed that the treatment which follows meets this specification in predicting frequencies up to a fundamental frequency of at least 50 KC, provided certain requirements are fulfilled in design. Although based on the circuit of Figure 1, assuming approximately equal values for T_1 and T_2, the method employed should prove useful in the analysis and design of many other similar circuits.

The following simplifying assumptions are made:

1. That the "flip-over" time is so short that it has no appreciable influence on the frequency.
2. That the stray inductance of the circuits is so small that it has no appreciable influence on the time constants of the circuit.
3. That the device is free from external disturbances, such as plate supply ripple, stray magnetic or electric fields, etc.
4. That the stray capacitances connected to the input and output terminals of the vacuum tubes may be adequately represented by lumped capacitances.

Referring again to Figure 1, suppose that the grid of VT_1 has just been driven to zero potential, causing a sudden increase in its plate current. During the interval immediately following, the equivalent circuit shown in Figure 5 applies. E_e is the change in voltage across R_{b1}. It is not necessary to specify its exact nature at this time and it may be considered to be merely an equivalent voltage in the circuit. R_e is an equivalent resistance formed by placing R_{b1} and the plate resistance of VT_1 in parallel. We wish to find the equation for

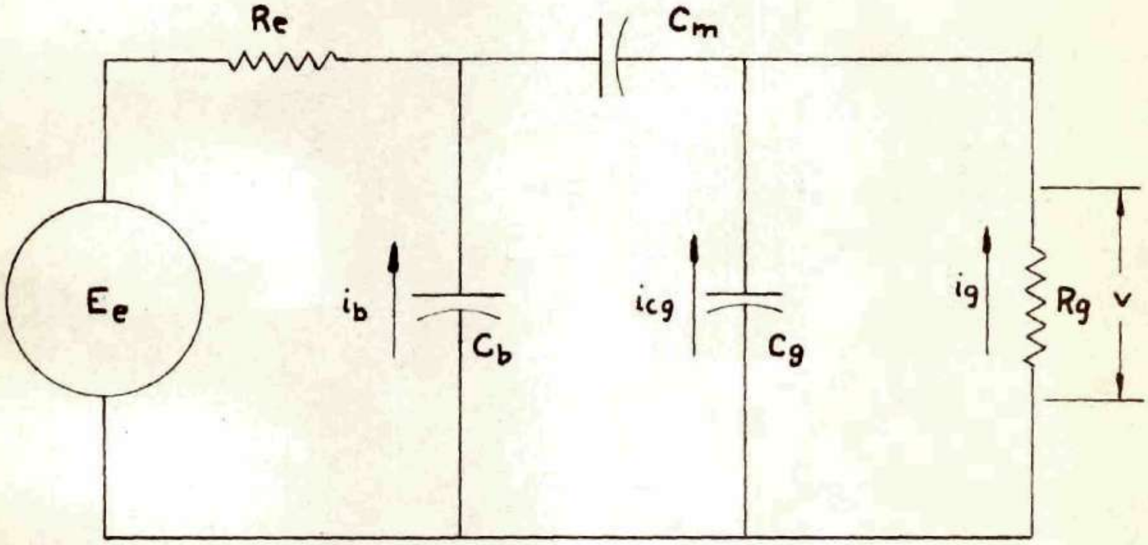

FIG. 5. EQUIVALENT CIRCUIT WHICH HOLDS DURING THE INTERVAL WHEN THE GRID IS NEGATIVE.

the voltage across R_g. Choose current directions as indicated in the figure. Starting with the voltage drop across R_g and summing up the voltage drops in the circuit, we obtain:

$$i_gR_g + (1/pC_m)(i_g + C_gpi_gR_g) + R_e\Big\{i_g + C_gpi_gR_g + pC_b\Big[i_gR_g + (1/pC_m)(i_g + C_gpi_gR_g)\Big]\Big\} = E_e \tag{8}$$

in which

$$p = d/dt = \text{time derivative operator} \tag{D7}$$

$$1/p = \int_0^t ()dt \tag{D8}$$

Rearranging and collecting terms, this gives for the differential equation for i_g:

$$\Big[R_gR_e(C_gC_m + C_bC_m + C_bC_g)p/C_m + R_g(C_m + C_g)/C_m + R_e(C_b + C_m)/C_m + 1/pC_m\Big]i_g = E_e \tag{8a}$$

Let

$$\lambda = R_gR_e(C_gC_m + C_bC_m + C_bC_g)/C_m \tag{D9}$$

and let

$$R_a = R_e(C_b + C_m)/C_m + R_gC_g/C_m \tag{D10}$$

Then equation (8a) becomes:

$$\Big[\lambda p + (R_a + R_g) + 1/pC_m\Big]\, i_g = E_e \tag{9}$$

Equation (9) suggests the equivalent circuit of Figure 6. This circuit will give i_g correctly regardless of the nature of E_e. It shows very clearly the principle effects of the capacitances C_b and C_g, namely: (1) they cause a delay in the increase of voltage across R_g when flip-over occurs; (2) they increase the time constant of the circuit; and (3) they decrease the magnitude of the voltage which appears across R_g.

In order to solve equation (9) for i_g, E_e must be specified. Since flip-over occurs in a very short time, E_e may be assumed to be a step function.[3] This gives:

$$i_g = \left[E_e \Big/ \sqrt{(R_a + R_g)^2/4 - \lambda/C_m}\right] e^{-(t/2\lambda)(R_a + R_g)} \sinh(t/\lambda)\sqrt{(R_a + R_g)^2/4 - \lambda/C_m} \qquad (10)$$

The terms under the radical sign may be written

$$\sqrt{(R_a + R_g)^2/4 - \lambda/C_m} = (1/2)(R_a + R_g)\sqrt{1 - 4\lambda/(R_a + R_g)^2 C_m} \qquad (11)$$

Also, from equation (D9) we find that

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[3] It will be shown presently that this assumption is inadequate. A more accurate definition of E_e is given in connection with equation (17).

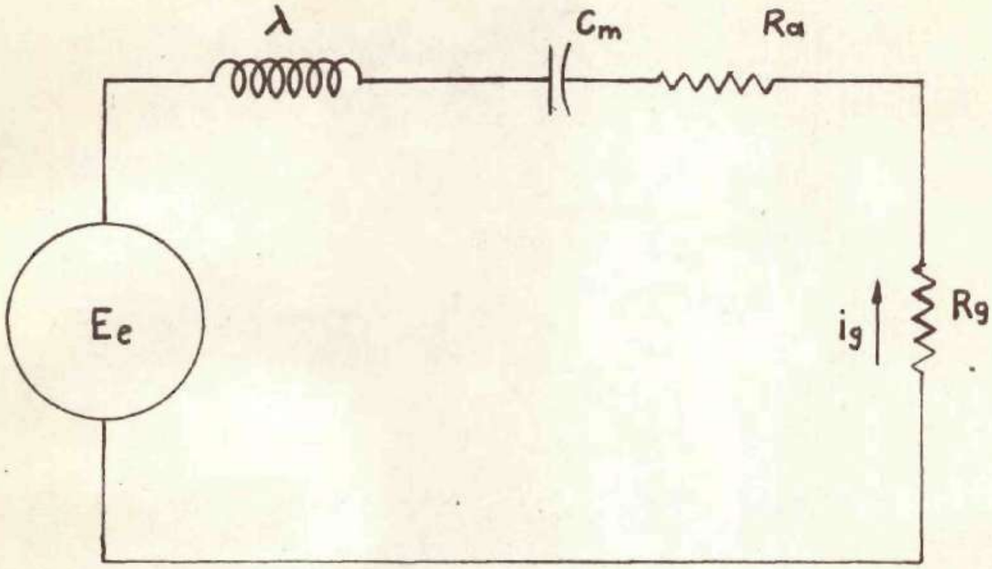


FIG. 6. CIRCUIT WHICH IS EQUIVALENT TO FIG. 5 SO FAR AS i_g IS CONCERNED.

$$4\lambda/(R_a + R_g)^2C_m = 4\left[R_gR_e(C_gC_m + C_bC_m + C_bC_g)/C_m\right]/(R_a + R_g)^2C_m \tag{12}$$

If $C_m >> (C_gC_m + C_bC_m + C_bC_g)$, the numerical value of the left side of equation (12) will be much less than unity and we can make the approximation:

$$(1/\lambda)\sqrt{(R_a + R_g)^2/4 - \lambda/C_m} \approx \left[(R_a + R_g)/(2\lambda)\right]$$

$$\left[1 - 2\lambda/(R_a + R_g)^2C_m\right] \tag{13}$$

Substituting equation (13) in equation (10) gives:

$$i_g = \left\{E_e/(R_a + R_g)\right\}\left\{e^{-t/(R_a + R_g)C_m} - e^{-t\left[(R_a + R_g)/\lambda - 1/(R_a + R_g)C_m\right]}\right\} \tag{14}$$

The second term in the bracket quickly disappears and hence, except for very small values of t,

$$i_g = \left[E_e/(R_a + R_g)\right]e^{-t/(R_a + R_g)C_m} \tag{15}$$

and finally

$$v = i_gR_g = \left[E_eR_g/(R_a + R_g)\right]e^{-t/(R_a + R_g)C_m} \tag{16}$$

It is evident that equations (15) and (16) may be obtained by merely neglecting λ in Figure 6, and this is justified when the value of the left hand side of equation (12) is small compared to unity. Equations (15) and (16) are not valid for very small values of t.

On comparing equation (16) with equation (1) it is seen that for the circuit of Figure 5:

$$\tau = C_m(R_a + R_g) \tag{D11}$$

$$\gamma = (E_e/E_{go})(R_g)/(R_a + R_g) \tag{D12}$$

From a study of equations (16) and (D10) it may be concluded that the most important effect of the capacitances C_b and C_g is equivalent to increasing R_a. This results in an increase in the time constant, τ, but it decreases the gain, γ, and equation (7) shows that these two variations tend to cancel each other as far as frequency is concerned. If γ is small, an increase of C_b or C_g may cause an increase of frequency, whereas if γ is large, the frequency may decrease. As a rule, it is not easy to measure either of these capacitances, and the best way of handling the situation is to keep C_m large in comparison with C_b and C_g.

The grid-to-plate capacitances are usually smaller than C_b and C_g. Although no satisfactory method of taking these into account has been found, they seem to have very little effect on frequency, at least up to 100 KC.

When calculations were made, based on the theory thus far developed, it was found that the operating frequency was invariably higher than the predicted value. The approximations which have been made ought to lead to operating frequencies which are lower than calculated values. In seeking the cause of this discrepancy, a careful study was made of the wave shape of the grid voltage. It was found that this non-exponential curve could be closely approximated by adding together two true exponentials having different time constants. The larger of these time constants was in fair agreement with equation (D11), but the magnitude of the voltage was much less than had been expected. In other words, the value of γ was wrong. This led to a reconsideration of the whole process.

In describing the ideal multivibrator, the input circuits of the amplifying devices were assumed to contain ideal rectifiers which prevented any positive voltage from appearing in their input circuits. In actual circuits employing vacuum tubes this is not the case. When flip-over occurs, the grid of one tube is driven well into the positive region. It remains positive as long as C_m is charging, but it is zero (or even slightly negative) before the end of the half-period is reached. It was at first assumed, therefore, that the "over-shoot" could be disregarded as long as C_m had sufficient time to become completely charged. A more careful analysis shows that this is not permissible. The positive component of v causes a component of plate current which persists for the complete half-period. Furthermore, this component is negative with respect to the current which has already been considered. This results in a reduction of output voltage which is equivalent to reducing the gain. With designs usually employed, this effect is much more important than the effect of the shunt capacitances.

During the interval when the grid of VT_1 is positive, (see Figure 1), the equivalent circuit of Figure 7 applies. Here R_i is the resistance measured across the input terminals of VT_1 when its grid is positive, including the effect of R_g. E_e' is the total change in volt-

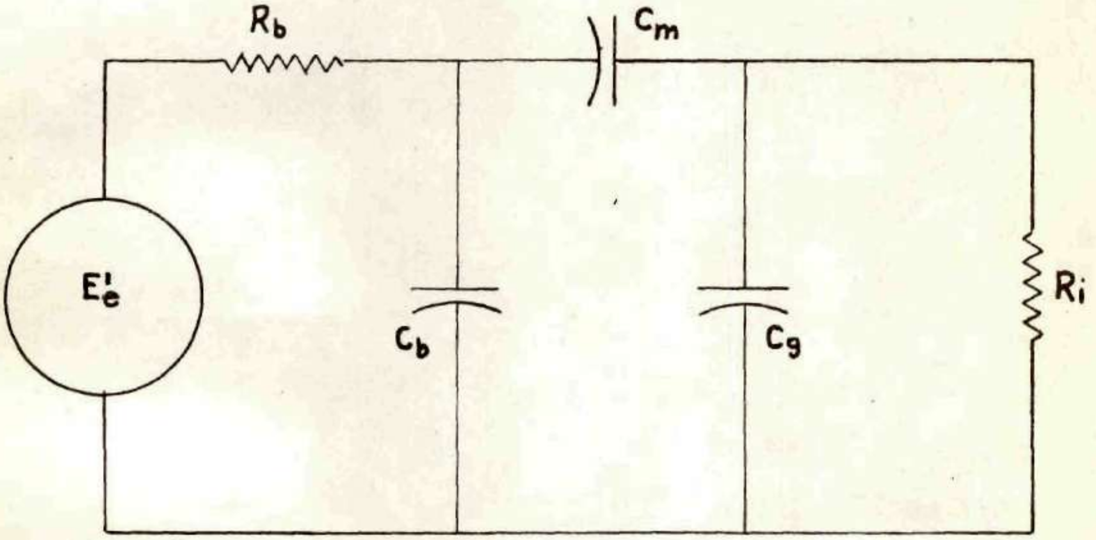


FIG. 7. EQUIVALENT CIRCUIT WHICH APPLIES WHEN THE GRID IS POSITIVE.

age across R_{b2} when the plate current of VT_2 goes to zero. This is, for all practical purposes, a step-function voltage. The grid of VT_1 will be positive as long as current flows in this circuit, and its voltage will follow a nearly-exponential law with a time constant given by:

$$\tau_g = R_b(C_m + C_b) + R_i(C_m + C_g) \tag{D13}$$

The plate circuit of VT_1 will contain a corresponding component of voltage, essentially exponential in wave-shape and having the same time constant as given by equation (D13). Therefore, in equation (8a), E_e is in reality made up of two components, one of which is essentially a step-function and the other essentially an exponential function. If we neglect λ in equation (9), then we can write:

$$\left[(R_a + R_g) + 1/pC_m\right] i_g = E_s + E_x\, e^{-t/\tau_g} \tag{17}$$

Here E_s gives the step-function component and E_x gives the exponential-function component of the voltage E_e. The solution of equation (17) is:

$$i_g = \left[E_s/(R_a + R_g)\right]\left[1 - (E_x/E_s)(\tau_g)/(\tau - \tau_g)\ e^{-t/\tau}\right.$$

$$\left. + \left[E_x/(R_a + R_g)\right]\left[(\tau)/(\tau - \tau_g)\right] e^{-t/\tau_g} \tag{18}$$

If τ is greater than about $2\tau_g$ the second term of equation (18) will be negligible at the time when flip-over occurs, and we have as an approximation:

$$i_g = \left[E_s/(R_a + R_g)\right]\left[1 - (E_x/E_s)(\tau_g)/(\tau - \tau_g)\right] e^{-t/\tau} \tag{19}$$

There are several reasons why it is desirable to keep τ_g small compared to τ . If these should have nearly the same value, equation (19) fails and, of course, equation (18) is not readily solved for t. Besides, if $_g$ is too large, C_m will not have time to completely charge and flip-over will occur while the grid is still positive. The value of E_x for each successive cycle will then be different until equilibrum is established according to equation (18). When operation is desired at very high frequencies (above 100 KC) it may be desirable to forsake the restriction on τ_g and use equation (18), in spite of the ensuing difficulties.

From equation (19):

$$v = i_g R_g = \left[E_s R_g/(R_a + R_g)\right]\left[1 - (E_x/E_s)(\tau_g)/(\tau - \tau_g)\right] e^{-t/\tau} \qquad (20)$$

Then

$$\gamma = \left[(E_s/E_{go})(R_g)(R_a + R_g)\right]\left[1 - (E_x/E_s)(\tau_g)/(\tau - \tau_g)\right] \qquad (D14)$$

When suitable values are used for the two stages, equation (7) gives the frequency.

It should be noted that equation (20) gives v correctly only for values of t which approach T. In other words, a multivibrator having values of γ and τ found from equations (D14) and (D11) will have the same frequency as an ideal multivibrator having these same values. Thus the correlation between the ideal multivibrator and the actual circuit is established.

THE DESIGN PROCEDURE

In order to show how the foregoing analysis may be used to design a multivibrator, calculations will be given for a circuit operating at 50 KC and employing a type 7N7 twin triode tube.

A part of the plate family for one section of this tube is shown in Figure 8, and the grid-plate transfer characteristic corresponding to a plate-supply voltage of 150 is given in Figure 9. The latter curve is plotted on semi-logarithmic coordinates. From it the curve giving G_m as a function of grid voltage is easily obtained in the following manner.

Draw a tangent to the E_c - I_b curve at a point such as E_{c2}, I_{b2} (see Figure 9). On this tangent line locate E_{c1} corresponding to a plate current exactly ten times I_{b2}. The mutual conductance, G_m, corresponding to E_{c2} is then:

$$G_m = 2.302\ I_{b2}/(E_{c2} - E_{c1}) \tag{21}$$

If I_b is in microamperes, G_m is in micromhos.

The design procedure begins with the choice of R_b. The value is not critical, but it is probably better to make it approximately equal to the plate resistance of the tube. For operation at low frequencies considerably larger values may be used. In this case, two resistors having 9600 ohms resistance were available and these were selected. R_g should not be less than twice R_b. Here a relatively large value, 48000 ohms, was chosen so that C_m would be small enough to show the effects of the tube capacitances. A smaller value, say 25000 ohms, would give

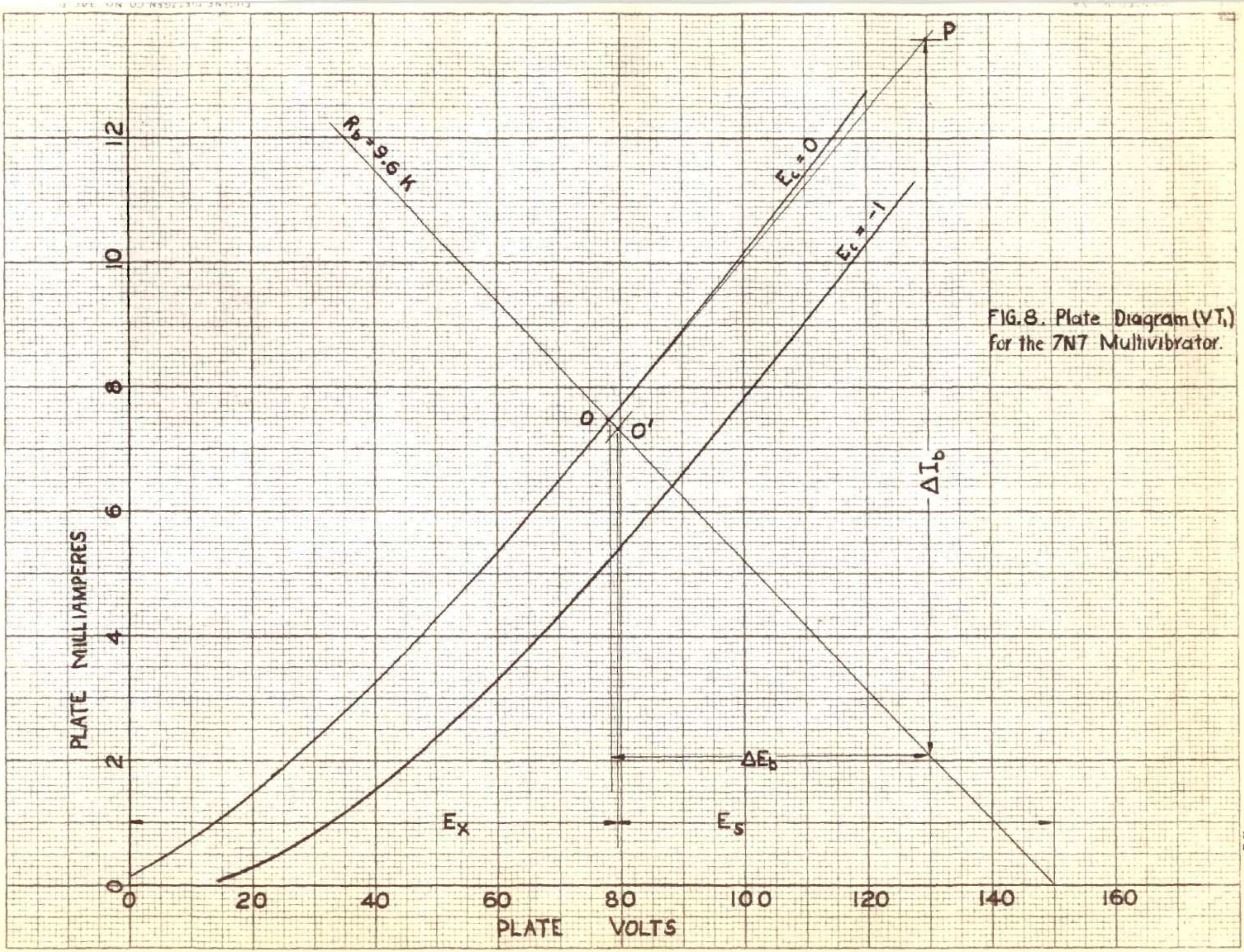


FIG. 8. Plate Diagram (VT_1) for the 7N7 Multivibrator.

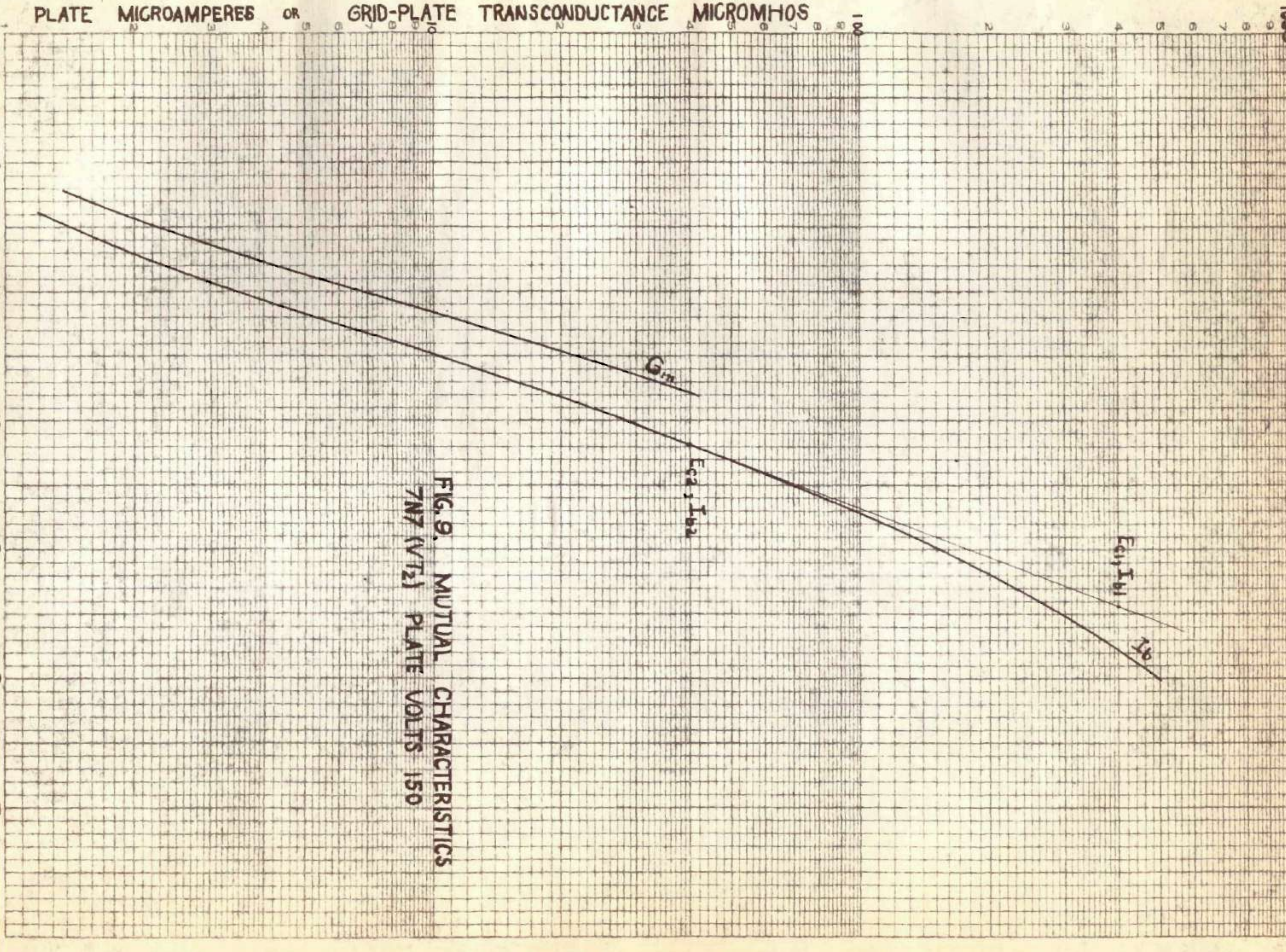


FIG. 9. MUTUAL CHARACTERISTICS
7N7 (VT2) PLATE VOLTS 150

a circuit of lower impedance, and hence one less likely to be disturbed by stray couplings.

When the values of R_b and R_g have been fixed, draw the load line on the plate family. The point O', rather than point O, corresponds to the quiescent value of plate current since the tube developes a small negative grid bias due to its own grid current. The magnitude of this bias depends on both R_g and R_b, and the easiest way to locate the point O' is to measure the plate current with the resistors in place. The dynamic grid resistance must also be measured. This resistance decreases very rapidly as the grid is made more positive and depends to a considerable extent on the plate current. It should be measured with R_b in place and plate voltage applied. The value used is not critical, and may be obtained with a grid voltage of plus one to two volts. For this particular tube, the resistance is 2000 ohms.

R_e may be found directly from the plate diagram of Figure 8. At the point O, draw a tangent line, OP. Then:

$$R_e = \Delta E_b/\Delta I_b = 51.7/11.55 = 4.5 \text{ K ohms}$$

R_i is an equivalent resistance formed by putting R_g in parallel with the dynamic grid resistance.

$$R_i = (2)(48)/(2 + 48) = 1.9 \text{ K ohms}$$

Assume that the foregoing values apply to VT_1. Then flip-over will occur when the G_m of VT_2 becomes large enough to make the net amplification around the circuit equal to unity. From Figure 8 it is found that a change of 1 volt on the grid of VT_1 causes a change of 10 volts across R_b. Taking into account the voltage-dividing effect of R_g, we have for the "normal gain" of VT_1:

$$\text{normal gain of } VT_1 = (10)(48)/52.5 = 9.15$$

When flip-over occurs the plate current of VT_2 will be very small and its plate resistance will be very high. Its amplification is therefore:

$$G_m R_b R_g/(R_b + R_g)$$

The net amplification will equal unity when:

$$(\text{normal gain of } VT_1)(\text{amplification of } VT_2) = 1$$

or when:

$$G_{m2} = (R_b + R_g)/(R_b R_g)(\text{normal gain of } VT_1)$$

Therefore, when the critical grid voltage of VT_2 is reached:

$$G_{m2} = (48 + 9.6)/(48)(9.6)(9.15) = .0137/10^3 \text{ mhos, or } 13.7 \text{ micromhos}$$

From Figure 9 E_{go2} is found to be 10.7 volts. Theoretically E_x is the difference between the plate voltage at O' and the lowest instantaneous plate voltage when the grid is positive. However, the non-linearity of the dynamic grid resistance causes the circuit to behave as if it had a larger time constant than that given by equation (D13), and this can be offset by using a larger value for E_x. A compromise which seems to be satisfactory is to take E_x as being equal to the value of E_b at the point O'. From Figure 8:

$$E_x = 79.6$$

$$E_s = 70.4$$

We now find $\tau_g/(\tau - \tau_g)$. If C_b and C_g are equal, this value is independent of C_m. Thus we have, without much error:

$$\tau_g/(\tau - \tau_g) = (R_b + R_i)/(R_e + R_g - R_b - R_i)$$

$$= (11.5)/(52.5 - 11.5) = 0.28$$

Then from equation (D14), neglecting C_b and C_g:

$$\gamma_1 = (48)(70.4)(1 - 0.28 \times 1.13)/(10.7)(52.5) = 4.1$$

In the same way γ_2 was also found to be 4.1. The two values are not always equal and must be calculated separately if precise results are desired.

Next, find a tentative value of C_m. From equation (3):

$$\tau_1 = T_1/\ln\gamma_1 = 1/2f\ln\gamma_1 = 1/(2 \times 50 \times 10^3 \times 1.41) = 7.1 \times 10^{-6}$$

From equation (D11), neglecting C_b and C_g:

$$C_m = 7.1 \times 10^{-6}/52.5 \times 10^3 = 135 \text{ mmfd.} \quad \text{(tentative)}$$

This value is sufficiently accurate to permit corrections for C_b and C_g. These were estimated to be 7 mmfd. each. Therefore:

$$R_a + R_g = (R_e + R_g)(C_m + C_b)/C_m = (52.5)(135 + 7)/(135) = 55.2 \text{ K ohm.}$$

and the new value of γ_1 is, from equation (D14):

$$\gamma_1 = (4.1)(R_e + R_g)/(R_a + R_g) = (4.1)(52.5)/(55.2) = 3.9$$

Finally, the corrected value of C_m is found to be 133 mmfd.

EXPERIMENTAL VERIFICATION OF THEORY

A multivibrator was set up using a type 7N7 tube with R_b = 9.6 K ohms, R_g = 48 K ohms, and C_m = 133 mmfd. The measured frequency was 49 KC. This is in good agreement with the predicted value of 50 KC.

A series of tests was performed using several different circuit designs, and covering a wide range of frequencies. The results of these tests are given in Table 1. The values of R_b, R_g, and the frequencies calculated by the method described above for C_m = .1 mfd., are indicated at the top of the columns. Below are shown the ratios of the measured frequencies to the calculated frequencies for various values of C_m. In each case the highest measured frequency is given by the numbers in parenthesis. The maximum error is seen to be 10%. This is considered to be good agreement, in view of the wide range of frequencies involved. It is possible that a more rigorous treatment of the E_x voltage would improve the results. However, when it is remembered that variations in the tube characteristics can cause variations in frequency which may be as high as 10%, the additional labor does not seem justified. By keeping R_g large compared to R_b, the effects of overshoot can be minimized. This is evident from a study of Table 1. The circuit in which R_b was 17.7 K and R_g was only 42 K shows larger errors than any of the others. No measurements were made on this circuit for C_m less than .00051 mfd., but it is likely that the error would be larger for smaller values of C_m.

Figure 10 shows the grid-voltage and plate-voltage waves of the multivibrator having R_b = 9.7 K, R_g = 42 K, and C_m = .012 mfd. These

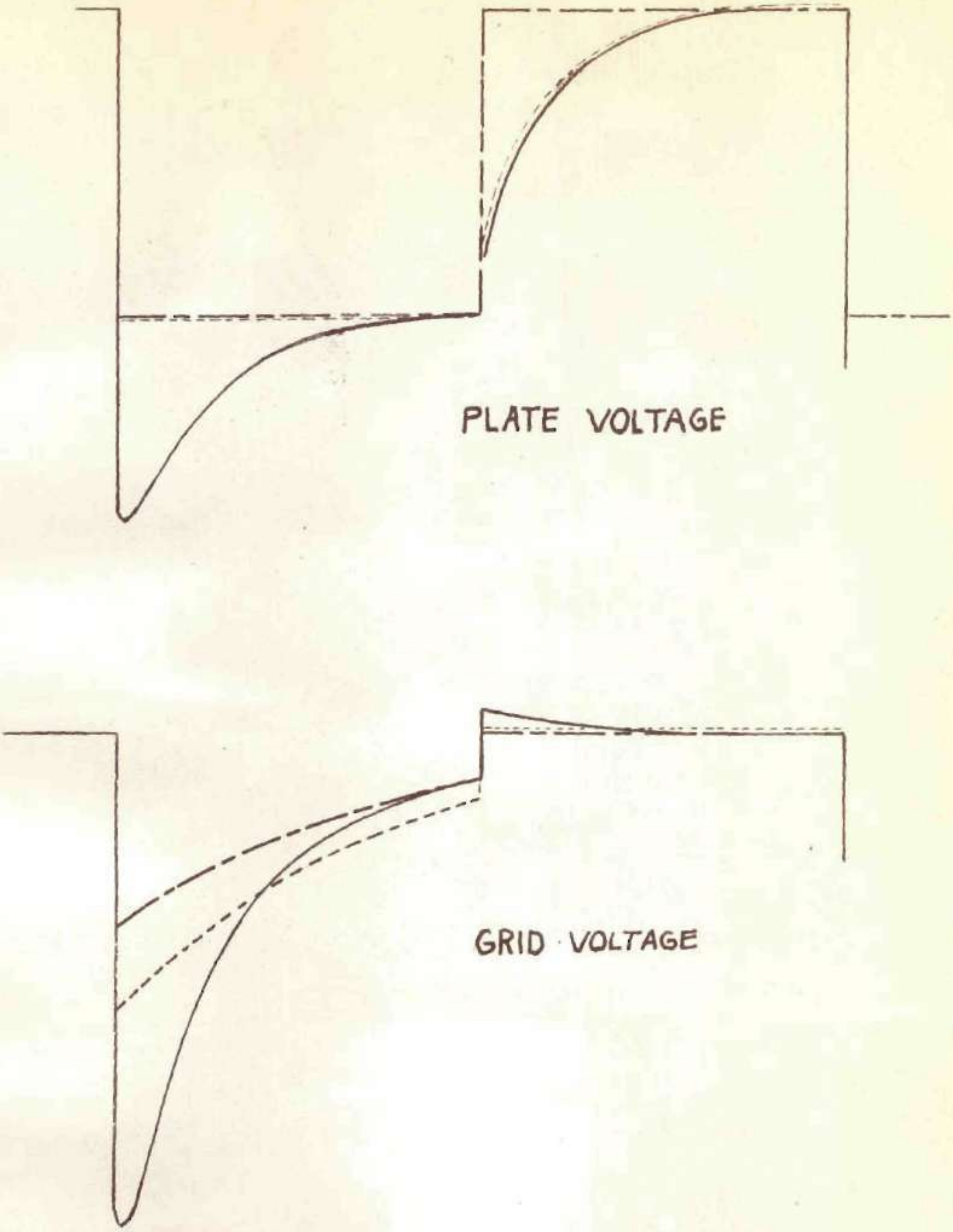
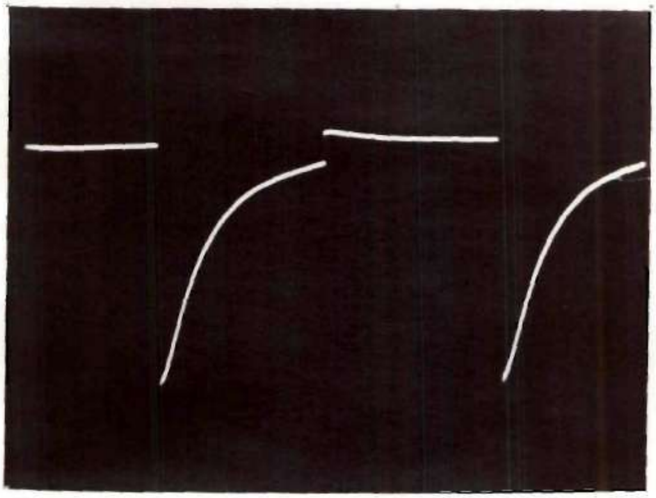


FIGURE 10. VOLTAGES GENERATED BY A MULTIVIBRATOR. THE SOLID LINES SHOW THE ACTUAL VOLTAGES. THE DOTTED LINES SHOW HOW THESE VOLTAGES WOULD APPEAR IF THERE WERE NO OVERSHOOT. THE DASHED LINES ARE FOR AN IDEAL MULTIVIBRATOR WITH THE SAME VALUE OF γ. FREQUENCY = 670 CPS.

Plate voltage



Grid voltage

Fig. 11. Voltages generated by a

multivibrator

curves were copied from the oscillograms in Figure 11. The dotted lines show how these voltages would look if there were no over-shoot. The dashed lines show the voltages of an ideal multivibrator with the same γ.

The effects of the tube capacitances become noticeable for values of C_m in the order of .001 mfd. Besides modifying τ, τ_g, and γ they reduce the "normal gain", and this in turn reduces E_{go}. The change in E_{go} is small and usually can be neglected.

Table 1

	Two type 7A4 tubes			One type 7N7 tube	
R_b	48 K	9.7 K	17.7 K	48 K	9.6 K
R_g	220 K	42 K	42 K	220 K	48 K
Calculated frequency for C_m = .1 mfd.	10.3 cps	79 cps	71 cps	10.3 cps	67 cps
C_m mfd.	measured frequency/calculated frequency				
.107	1.08	1.0	.97	1.05	1.04
.048	1.08	1.0	.97	1.01	1.04
.012	1.08	1.0	---	1.03	1.04
.0053	1.09	.98	.93	1.02	1.02
.00113	1.04	.96	.93	1.05	.99
.00051	1.05	.93	.90 (12.1 KC)	1.00	.97
.000102	---	.92	---	.97	---
.000065	---	.90 (104 KC)	---	---	---
.000053	---	---	---	---	.94 (115 KC)
.000030	1.03 (31 KC)	---	---	1.00	---
.000015	---	---	---	1.00 (57 KC)	---

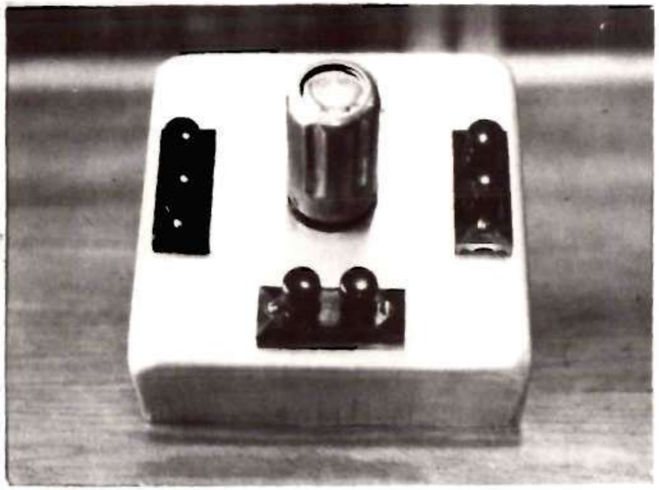
CONCLUSION

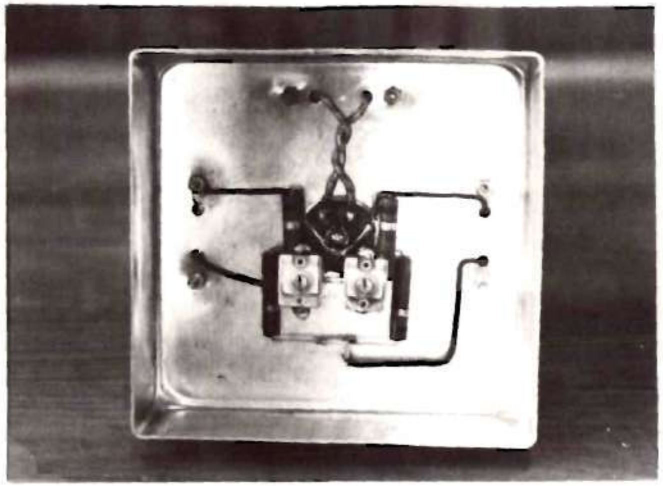
The "ideal multivibrator" has been defined and analyzed. In describing its behavior, the concepts of τ and γ have been introduced. The non-ideal multivibrator has been analyzed and the effects of tube capacitances have been considered. It has been shown that the "overshoot", or positive component of grid voltage, causes a reduction in output voltage which is equivalent to a reduction in the amplification of the tubes. If the grid resistor, R_g, has several times the resistance of the plate resistor, R_b, an approximate method of accounting for the "overshoot" yields reasonably satisfactory results.

Equations have been derived which can be used to predict the operating frequency with an error which does not exceed 10% over a wide range of frequencies.

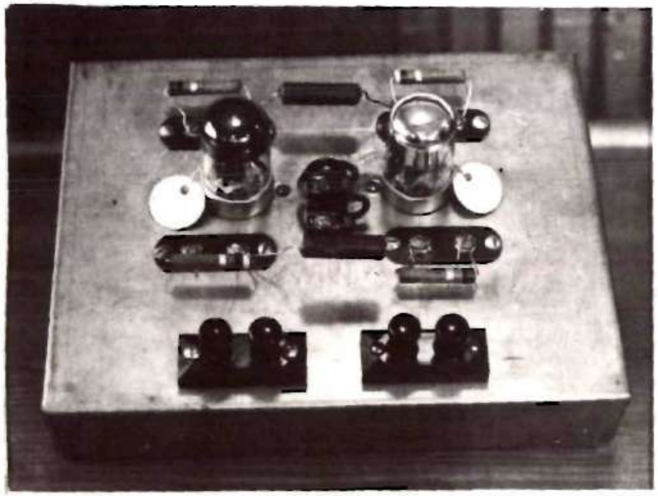
A procedure has been described which can be used to design a multivibrator to operate at a specified frequency.

The writer believes that this analysis is accurate and reliable enough for practical design. Yet there are still some questions which remain unanswered. There is good reason to believe that further investigation along these lines will reduce still more the gap between predicted performance and actual behavior.

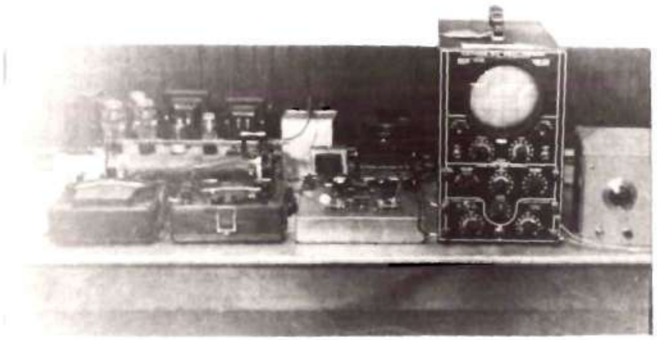




Two views of the 50 KC multivibrator



The chassis used for the tests



Equipment set up for checking wave shapes

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