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How Migration can Contribute to Achieving a Stationary Population[1]

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Abstract

Methods from mathematics of finance and demography are presented in order to investigate the influence of migration on the long-term population development. Methods from mathematics of finance do not take the age structure of a population into consideration and can therefore only be used as an approximation. The less the age structures in question deviate from those of stable populations, the more exact the approximation will be. In the empirical section quantitative measures for population policy are described and analyzed using the population of Germany and of the world as examples. The long-term goal of quantitative population policy is zero growth. Whereas in less developed countries, this goal can be achieved for the most part only by a reduction of fertility, it is possible in more developed countries with below-replacement fertility to achieve stationarity by means of immigration. Under the assumptions made here, Germany would have to take in between 350.000 and 500.000 immigrants each year for the population to remain at the present level. Immigration has demographic consequences for the age structure and the composition of the population which will be described at the end.

Keywords: Population Projection, Demography, Leslie Model, Mathematics of Finance, Population Growth

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[1] This paper is a translation of Pflaumer, Peter (1993): Stationäre Bevölkerungen: Anwendungsmöglichkeiten finanzmathematischer und demographischer Methoden zur Analyse des Einflusses von Wanderungen auf die langfristige Bevölkerungsentwicklung, Diskussionsbeiträge: Serie II, Sonderforschungsbereich 178 "Internationalisierung der Wirtschaft", Universität Konstanz No. 207, 1993.

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SUMMARY

Methods from mathematics of finance and demography are presented in order to investigate the influence of migration on the long-term population development. Methods from mathematics of finance do not take the age structure of a population into consideration and can therefore only be used as an approximation. The less the age structures in question deviate from those of stable populations, the more exact the approximation will be. In the empirical section quantitative measures for population policy are described and analyzed using the population of Germany and of the world as examples. The long-term goal of quantitative population policy is zero growth. Whereas in less developed countries, this goal can be achieved for the most part only by a reduction of fertility, it is possible in more developed countries with below-replacement fertility to achieve stationarity by means of immigration. Under the assumptions made here, Germany would have to take in between 350.000 and 500.000 immigrants each year for the population to remain at the present level. Immigration has demographic consequences for the age structure and the composition of the population which will be described at the end.

A. INTRODUCTION

In the past two decades, the birth rate has greatly decreased in all of the more developed countries. The net reproduction rate in Germany, for example, has fallen to about 0,7. Assuming this reproductive behavior continues and no immigration takes place, a rapid population decrease would occur after the turn of the next century. In the long term, a negative growth of approximately 1,3 would have to be reckoned with. On the other hand, the population growth continues unabated in most less developed countries. In the case of unchanged fertility, these number of people in these countries would rise to over 55 billion in the year 2100 (cf. Lutz and Prinz, 1991). Even with a decrease in fertility, the population in the less developed countries will in all probability exceed the ten billion limit. The situation of the world population can be described in the following way: "population growth with an overproportionally large percentage of young people in less developed countries and a population decrease with an overproportionally large percentage of old people in more developed countries." The consequences are predictable. The population pressure in less developed countries and the bad economic situation due to an insufficient availability of employment lead to increasing migration movements to the economically better situated more developed countries. The population surplus on the one side and the population decrease on the other side cause migration waves from the poorer South in the direction of the wealthier North.

The problem of a quantitative population policy arises, which should be understood as the total of all goal-oriented measures concerning the numerical development of a population and its age structure. In the long term, the goal of population policy is stationarity. A stationary population is a population that (on the average) neither grows nor shrinks.

Population policy measures can try to influence either fertility or migration. If one assumes the below-replacement level fertility in the more developed countries to be a permanent phenomenon, then only constant immigration can prevent a population decrease in the long term. If the goal of population policies is stationarity, then the level of yearly net immigration must be determined. A constant immigration has long-term consequences for the age structure of the total population and for the composition of the foreign and native population groups. These consequences should be known to the population politicians. In the less developed countries, the measures for reaching a stationary population should aim above all to reduce births, since population growth cannot be slowed down by emigration in a realistic dimension alone.

First, simple methods from mathematics of finance will be presented in order to analyze the influence of migration on the long-term population development. These methods do not take the age structure of a population into consideration. They can therefore only be used as a first approximation. More exact results can be obtained from the demographical analysis of populations divided up by age, which follows the analysis from mathematics of finance. The less the age structure of a population deviates from that of a stable population, however, the better the approximation will be. Nevertheless, methods from mathematics of finance do have the advantage that the basic proofs that appear in connection with migration and population developments are elementary to show and are easy to understand. In the empirical section, quantitative measures of population policy will be described and analyzed using the population of Germany and of the world as examples.

B. METHODS

1. Methods from Mathematics of Finance

The model for the population development is as follows:

$$P_t = P_0 \cdot q^t \ ,$$

whereby

P_t = population size at time t

q = growth factor.

In the case of an increasing population $q>1$, and in the case of a shrinking population $0<q<1$.The following relation exists between the continuous growth rate r and the growth factor q:

$$q = e^r.$$

If $x>0$ denotes the (constant) number of immigrants at the end of a period, one then obtains

$$\begin{aligned} P_1 &= P_0 q + x \\ P_2 &= P_1 q + x = P_0 q^2 + qx + x \\ &\vdots \\ P_t &= P_{t-1} q + x = P_0 q^t + xq^{t-1} + \ldots + xq + x \end{aligned}$$

and

$$P_t = P_0 q^t + x\tfrac{q^t - 1}{q-1}, \text{ if } q \neq 1$$

and

$$P_t = P_0 + t \cdot x, \text{ if } q = 1.$$

Here it is assumed that the immigrants have a fertility and mortality that is equally as high as the population of the immigration country. This assumption is certainly unrealistic. However, nothing about the results fundamentally changes if one assumes at least a gradual adjustment. A gradual adjustment to the domestic fertility rates can be seen with all immigrant groups. In the empirical section of this paper, the effects on the population level in the case of a delayed fertility adjustment will be studied.

The growth rate can be calculated as

$$\frac{\frac{dP_t}{dt}}{P_t} = r_t = \frac{\left(P_0 + \frac{x}{q-1}\right) \ln q q^t}{P_0 q^t + x \frac{q^t - 1}{q-1}}.$$

If $q>1$, then $r_t>0$. The limit is then:

$$\lim_{t \to \infty} r_t = \ln q = r.$$

In the long term, a constant amount of immigration does not influence the growth of the population in the immigration country.

If $q<1$, i.e. the population of the immigration country decreases, then the growth rate depends first on the level of immigration. The growth rate is

a) $r_t > 0$, if $x/P_0 > 1 - q$ or $x > (1-q)P_0$

b) $r_t = 0$, if $x/P_0 = 1 - q$ or $x = (1-q)P_0$

c) $r_t < 0$, if $x/P_0 < 1 - q$ or $x < (1-q)P_0$.

The limit for $q<1$ is.

$$\lim_{t\to\infty} r_t = 0.$$

The population converges to a stationary level

$$\lim_{t\to\infty} P_t = P_s = \frac{x}{1-q}.$$

In a stablely shrinking population, a constant number of immigrants each year leads in the end to a stationary population if the immigrants take on the fertility behavior of the immigration country immediately or after a certain time. The level of the stationary population depends only on the level of immigration and the domestic fertility and mortality situation, which are reflected in q (cf. Cerone, 1987; Espenshade/ Bouvier/Arthur, 1982; Mitra, 1990).

If ℓ denotes the survival rate of the population, then the number of immigrants born in a foreign country is

$$x + x\ell + x\ell^2 + x\ell^3 + \ldots = \frac{x}{1-\ell}, \text{ whereby } \ell < q < 1.$$

The percentage of foreigners in the stationary population

$$\pi = \frac{\frac{x}{1-\ell}}{\frac{x}{1-q}} = \frac{1-q}{1-\ell}$$

is independent of the level of immigration x.

In the case of an increasing population ($q>1$),

$$2P_0 = P_0 q^d + x\tfrac{q^d-1}{q-1}$$

leads to the doubling time

$$d = \frac{\ln\left(\frac{2(q-1)+\frac{x}{P_0}}{(q-1)+\frac{x}{P_0}}\right)}{\ln q}.$$

If $x=0$, then the equation for the doubling time is simplified to:

$$d = \frac{\ln 2}{\ln q}.$$

In the case of a shrinking population in the immigration country (q<1), $x > 2(1-q)P_0$ or $\frac{x}{P_0} > 2(1-q)$ must hold in order for immigration to cause a doubling of the initial population.

The total population consists of the original (old) and the (new) population of immigrants and their descendents:

$$P_t = P_0 q^t + x\tfrac{q^t-1}{q-1} = P_A + P_N.$$

The old population P_A dies out in the long term (q<1), whereas the new population P_N gradually increases to a stationary level. Time t*, at which both populations are equally large, is of particular interest. If one sets $P_A = P_N$, one obtains

$$t^* = -\frac{\ln\left(1+\frac{P_0}{x}(1-q)\right)}{\ln q}.$$

If $\frac{x}{P_0} = (1-q)$ or $r_t = 0$, then

$$t^* = -\frac{\ln 2}{\ln q}.$$

If, for example, a population with a negative growth rate of 1% compensates for the population decrease through immigration at the level of $x = 0{,}01\,P_0$, then the new population will be as large as the old one after 70 years.

In the case of an increasing population (q>1), the growth rate of the old population is

$$r_A = \ln q$$

and the growth rate of the new population is

$$r_{tB} = \ln q\left(\tfrac{q^t}{q^t-1}\right).$$

In the long term it coverges toward ln q, since

$$\lim_{t\to\infty} r_{tB} = \ln q.$$

The growth rate of the total population is

$$r_t = \frac{\left(P_0 + \frac{x}{q-1}\right)\ln q q^t}{P_0 q^t + x\frac{q^t-1}{q-1}}$$

$$= \ln q\left(\frac{P_0 q^t}{P_t}\right) + \ln q\left(\frac{q^t}{q^t-1}\right)\frac{x\frac{q^t-1}{q-1}}{P_t},$$

where $\left(\frac{P_0 q^t}{P_t}\right) \mathrel{\hat{=}} \alpha_t$: portion of the old population and $\frac{x\frac{q^t-1}{q-1}}{P_t} \mathrel{\hat{=}} 1-\alpha_t$: portion of the new population.

The growth rate of the total population is thus a weighted arithmetic mean of the growth rates of the sub-populations:

$$r_t = r_A \alpha_t + r_{tN}(1-\alpha_t)$$

with

$$\lim_{t\to\infty} r_t = \ln q \alpha_t + \ln q(1-\alpha_t) = \ln q .$$

The proportion of the old population γ depends on the level of the (constant) immigration and on the domestic growth rate. It is namely:

$$\gamma = \frac{P_0 q^t}{P_0 q^t + x\frac{q^t-1}{q-1}}$$

$$\lim_{t\to\infty} \gamma = \frac{P_0(q-1)}{P_0(q-1)+x} = \frac{1}{1+\frac{x}{P_0(q-1)}} ;$$

$$\frac{x}{P_0(q-1)} = \frac{\text{Immigration}}{\text{Natural Growth}} .$$

The stable equivalent of an increasing population with a constant level of immigration is defined as follows:

$$R_0 = \frac{P_t}{q^t} = P_0 + \frac{x}{q^t}\frac{q^t-1}{q-1}, \text{ if } q>1.$$

It holds for big t:

$$R_0 \approx P_0 + \frac{x}{q-1} .$$

It follows that:

$$R_0 q^t = P_0 q^t + x\frac{q^t-1}{q-1} \approx \left(P_0 + \tfrac{x}{q-1}\right)q^t .$$

A permanent immigration at the level of x functions like one-time immigration at the level of $x/_{q-1}$ at time 0.

If $x>0$ is the (constant) number of emigrants per period, then one obtains

$$P_t = P_0 q^t - x\frac{q^t-1}{q-1}$$

as the population size of an increasing population ($q>1$) at time t.

The growth rate

$$r_t = \frac{\left(P_0 - \frac{x}{q-1}\right)\ln q q^t}{P_t}$$

is

a) $r_t > 0$, if $x/P_0 < (q-1)$ or $x < (q-1)P_0$

b) $r_t = 0$, if $x/P_0 = (q-1)$ or $x = (q-1)P_0$

c) $r_t < 0$, if $x/P_0 > (q-1)$ or $x > (q-1)P_0$.

The limit is (case a):

$$\lim_{t\to\infty} r_t = \ln q \ .$$

In case c) a constant growth rate means negative population levels. It is obvious that in the long term, the emigration cannot be larger than the natural growth.

If $x = (q-1)P_0$, then the stablely increasing population is transformed into a stationary population:

$$P_t = P_0 q^t - (q-1)P_0 \tfrac{q^t-1}{q-1} = P_0 \ .$$

In the long term, a constant level of emigration has no influence on the growth rate of an increasing population if $x < (q-1)P_0$. Rather, a constant level of emigration functions like a one-time reduction of the population at time 0, since

$$P_0 q^t - x\tfrac{q^t-1}{q-1} \approx \left(P_0 - \tfrac{x}{q-1}\right)q^t \ .$$

If one divides the world population into the population of the more developed countries with $q_I < 1$ and the population of the less developed countries with $q_E > 1$, then the population permanently decreases in the more developed countries, whereas it continually increases in the less developed countries.

Without migration between the two blocks, the world population would explode while the population of the more developed countries decreases at the same time. The question arises, how high the migration movements from the less developed countries to the more developed countries would have to be for the world population to reach a stationary level (cf. also Bouvier/Espenshade, 1989).

The growth rate of the population in the less developed countries is zero when

$$x = (q_E - 1)P_o^E \ .$$

This amount of immigration leads in the case of the more developed countries to a stationary population

$$P_S^I = \frac{x}{1-q_I} = -\frac{1-q_E}{1-q_I} P_0^E .$$

The stationary level of the world population is

$$P_S = P_0^E + P_S^I = \frac{q_E - q_I}{1-q_I} \cdot P_0 .$$

Example:

$$P_0^E = 4\text{Billion}$$
$$q_E = 1,018$$
$$q_I = 0,99$$
$$P_S^I = -\tfrac{-0,018}{0,01} \cdot 4\text{Billion} = 7,2\text{Billion}$$
$$P_S = 11,2\text{Billion}$$

The example makes clear that migration cannot be a suitable means of effectively slowing down population growth in the world. With the present mortality, a stationary world population can only be obtained by a decrease in fertility. Even if the more developed countries would agree to take in such a high level of immigration, it is doubtful whether stationarity would result, since the assumption of an immediate or gradual decrease in fertility would no longer hold true in the case of such massive migration.

2. Discrete-Time Demographic Models

The projection model can be formally described as follows, whereby only the female portion of the population is considered:

$$n_1 = Ln_0 + w$$
$$n_2 = Ln_1 + w = L^2 n_0 + Lw + Iw$$
$$\vdots$$
$$n_t = L^t n_0 + L^{t-1}w + \ldots + Lw + Iw$$

or

$$n_t = L^t n_0 + (I-L)^{-1}(I-L^t)w .$$

The vector n_t represents the number of women in the individual age groups at time t. L is the projection or Leslie matrix, which contains fertility rates in the first row and survival rates in the subdiagonals. w is the vector of the constant net immigration. I is the unity matrix. Details of the calculation of the elements of the projection matrix can be taken, for example, from Keyfitz (1977) or Pflaumer (1988).

The similarity between the demographic model divided up by age and the model from mathematics of finance which is not divided up by age is obvious.

To investigate the theoretical features of the projection process, for reasons of clarity the study will be restricted to a Leslie matrix L which only comprises the fertile age groups. In the case of groups with a age span of 15 years (0-15, 15-30, 30-45), L has the following appearance:

$$L = \begin{pmatrix} b_1 & b_2 & b_3 \\ s_1 & 0 & 0 \\ 0 & s_2 & 0 \end{pmatrix}$$

b_i : fertility rates $\quad (b_i > 0)$
s_i : survival rates $\quad (0 < s_i \leq 1)$

The population can be projected in 15-year intervals with this matrix.

It is easy to show that

$$\det L = s_1 \cdot s_2 \cdot b_3$$

$$L^{-1} = \begin{pmatrix} 0 & \frac{1}{s_1} & 0 \\ 0 & 0 & \frac{1}{s_2} \\ \frac{1}{b_3} & -\frac{b_1}{s_1 b_3} & -\frac{b_2}{s_2 b_3} \end{pmatrix}$$

$$\det(I - L) = 1 - b_1 - s_1 \cdot b_2 - s_1 \cdot s_2 \cdot b_3$$
$$= 1 - R_0$$
$$\text{with } R_0 = b_1 + s_1 \cdot b_2 + s_1 \cdot s_2 \cdot b_3 \ .$$

R_0 is the net reproduction rate, which indicates the average number of daughters that a woman will bear in the course of her life considering mortality.

One obtains the characteristic value λ_i of L by solving the following characteristic polynomial:

$$\lambda^3 - b_1\lambda^2 - s_1 \cdot b_2\lambda - s_1 \cdot s_2 \cdot b_3 = 0 \ .$$

Since L is an irreducible, primitive matrix, there exists a positive characteristic value that is absolutely bigger than all other characteristic values (cf. Huppert, 1990).
In the long term, the positive characteristic value λ_1 is the growth factor of the population in each projection step. The characteristic vector λ_1 belonging to δ_1 contains only positive elements and reflects the age structure of the stable population. It is

$$\delta_1 = \begin{pmatrix} 1 \\ s_1 \cdot \lambda_1^{-1} \\ s_1 \cdot s_2 \cdot \lambda_1^{-2} \end{pmatrix}.$$

The characteristic value λ_1 cannot be specified in general. However, the following relationship exists between the characteristic value λ_1 and the net reproduction rate R_0 (cf. for example Keyfitz, 1977):

$$\lambda_1 > 0 \leftrightarrow R_0 > 1$$
$$\lambda_1 = 0 \leftrightarrow R_0 = 1$$
$$\lambda_1 < 0 \leftrightarrow R_0 < 1.$$

The stable growth rate is calculated as

$$r = \frac{\ln \lambda_1}{T}$$

(T = projection step or length of an age group).

The stable growth rate can be approximately determined by the relation

$$r \approx \frac{\ln R_0}{m}$$

whereby m is the average interval between generations (cf. Keyfitz, 1977).

To calculate the powers of L, it is useful to diagonalize the Leslie matrix, i.e.

$$L = QDQ^{-1},$$

whereby D contains the characteristic values on the main diagonals and Q contains the corresponding characteristic vectors of L.

The powers can now be easily calculated. It namely holds that

$$L^t = QD^tQ^{-1},$$

in the case that Q^{-1} exists.

An example will make the relations clear:

Let the following projection matrix, which reflects the present situation in Germany, be given:

$$L = \begin{pmatrix} 0{,}2 & 0{,}45 & 0{,}1 \\ 0{,}995 & 0 & 0 \\ 0 & 0{,}99 & 0 \end{pmatrix}.$$

The net reproduction rate is R_0 = 0.7463. Let the vector of the female population in age groups of 15 years be:

$$n_0 = \begin{pmatrix} 5456,8 \\ 6889,2 \\ 6373,7 \end{pmatrix}.$$

The characteristic values of L are obtained by solving the following characteristic polynomial:

$$\lambda^3 - 0,2\lambda^2 - 0,4475\lambda - 0,098505 = 0 .$$

The characteristic values in matrix D are:

$$D = \begin{pmatrix} 0,857 & 0 & 0 \\ 0 & -0,328 - 0,084i & 0 \\ 0 & 0 & -0,328 + 0,084i \end{pmatrix}.$$

Matrix Q is as follows:

$$Q = \begin{pmatrix} 1 & 1 & 1 \\ 1,161 & -2,847 + 0,729i & -2,847 - 0,729i \\ 1,341 & 7,535 - 4,13i & 7,535 + 4,13i \end{pmatrix}.$$

The projection model not taking migration into consideration is

$$n_t = L^t n_0 = QD^tQ^{-1}n_0$$
$$= 0,857^t \cdot 5589,67 \begin{pmatrix} 1 \\ 1,161 \\ 1,341 \end{pmatrix} + 0,339^t \begin{pmatrix} -132,9\cos 2,89t - 29,3\sin 2,89t \\ 399,6\cos 2,89t - 13,5\sin 2,89t \\ 328,11\sin 2,89t - 1122\cos 2,89t \end{pmatrix}$$

for the example using the trigonometric form of the complex numbers.
The entire population P_t develops according to

$$P_t = 0,857^t \cdot 19575 + (285,3\sin 2,89t - 855,3\cos 2,89) \cdot 0,339^t$$

The population follows a geometric trend with decreasing oscillations.

In the long term, the population development can be represented by a geometric trend model (cf. the model from mathematics of finance), i.e.

$$P_t = 19575 \cdot 0,857^t$$

and

$$P_t = P_0 \lambda_1^t ,$$

whereby P_0 is the stable equivalent.

The characteristic vector of λ_1

$$\delta_1 = \begin{pmatrix} 1 \\ 1{,}161 \\ 1{,}341 \end{pmatrix}$$

of the matrix L determines the age structure of stable population.

The population decreases every 15 years by the factor 0,857. The stable growth rate is accordingly

$$r = \frac{\ln 0{,}857}{15} = \text{-}0{,}010\,.$$

Using the net reproduction rate, one obtains

$$r \approx \frac{\ln 0{,}7463}{27{,}5} = \text{-}\,0{,}011\,.$$

as the stable growth rate.

Taking the immigration vector

$$w = \begin{pmatrix} 750 \\ 1200 \\ 495 \end{pmatrix}$$

into consideration, the projection model results as

$$\begin{aligned} n_t &= QD^tQ^{-1}n_0 + (I - QDQ^{-1})^{-1}(I - QD^tQ^{-1})w \\ &= 0{,}857^t \cdot 5589{,}67 \begin{pmatrix} 1 \\ 1{,}161 \\ 1{,}341 \end{pmatrix} + 0{,}339^t \begin{pmatrix} -132{,}9\cos 2{,}89t - 29{,}3\sin 2{,}89t \\ 399{,}6\cos 2{,}89t - 13{,}5\sin 2{,}89t \\ 328{,}11\sin 2{,}89t - 1122\cos 2{,}89t \end{pmatrix} \\ &+ \begin{pmatrix} 5747{,}11 \\ 6918{,}37 \\ 7344{,}20 \end{pmatrix} - 0{,}857^t \begin{pmatrix} 5808{,}72 \\ 6745{,}66 \\ 7794 \end{pmatrix} + 0{,}339^t \begin{pmatrix} 61{,}62\cos 2{,}89t - 3{,}65\sin 2{,}89t \\ 54{,}35\sin 2{,}89t - 172{,}7\cos 2{,}89t \\ 49{,}78\cos 2{,}89t - 280{,}16\sin 2{,}89t \end{pmatrix} \end{aligned}$$

and

$$P_t = -773{,}3 \cdot 0{,}857^t + (55{,}89\sin 2{,}89t - 516{,}60\cos 2{,}89t) \cdot 0{,}339^t + 20009{,}7\,.$$

In the long term, the old population shrinks and a new population is built up:

$$\lim_{t\to\infty} n_t = \begin{pmatrix} 5747{,}11 \\ 6918{,}37 \\ 7344{,}20 \end{pmatrix}.$$

If δ_1 is the characteristic vector of λ_1, then one obtains

$$\lim_{t\to\infty} n_t = (I-L)^{-1}\,\delta_1 = \frac{1}{1-\lambda_1}\delta_1 = \frac{1}{1-0{,}857}\delta_1$$
$$= 6{,}993\delta_1$$

Thus, if the age structure of the immigrants has the structure of the stablely shrinking population without migration, then the age structures of the stationary and the stablely shrinking populations are the same. The stationary population is approximately seven times larger than the sum of the immigrants in 15 years.

As a rule, the age structure of the immigrants is young, and thus the age structure of the stationary population is generally younger than the age structure of the stable population which results if no migration takes place.

The stationary population can be approximatively calculated with

$$P_S = \frac{x_{15}}{1-0{,}857} \quad \text{(15-year intervals)}$$

or

$$P_S = \frac{x_1}{1-0{,}857^{1/15}} = \frac{x_1}{1-0{,}99}\left(=\frac{x_1}{1-q}\right) \quad \text{(1-year interval)},$$

whereby x_{15} represents the immigration during 15 years and x_1 the immigration during one year.

The approximation is, however, only good when the age structure of the immigrants does not deviate too much from the age structure of the stablely shrinking population without migration. This assumption is not fulfilled in practice.

If $\lambda_1<1$, it namely holds that

$$\lim_{t\to\infty} n_t = (I-L)^{-1}w = (I+L+L^2+L^3+\ldots)w$$

with

$$(I-L)^{-1} = \begin{pmatrix} \frac{1}{1-R_0} & \frac{b_2+s_2\cdot b_3}{1-R_0} & \frac{b_3}{1-R_0} \\ \frac{s_1}{1-R_0} & \frac{1-b_1}{1-R_0} & \frac{s_1\cdot b_3}{1-R_0} \\ \frac{s_1\cdot s_2}{1-R_0} & \frac{s_2(1-b_1)}{1-R_0} & \frac{1-b_1-s_1\cdot b_2}{1-R_0} \end{pmatrix}.$$

The age structure of the stationary population is, as the following summary shows, a weighted arithmetic mean of the columns of $(I-L)^{-1}$, whereby the weights are the immigrants in the individual age groups. The older the age structure of the immigrants is, the older the age structure of the stationary population and the smaller the size of the stationary population will be, since the reproductive value of older immigrants is lower than that of the younger ones.

Summary: Structure of a stationary population

The immigration vectors

$$\text{a)}\quad \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad \text{b)}\quad \begin{pmatrix}0\\1\\0\end{pmatrix}, \qquad \text{c)}\quad \begin{pmatrix}0\\0\\1\end{pmatrix}$$

yield the following stationary populations:

$$\text{a)}\quad \begin{pmatrix} 1+R_0+R_0^2+R_0^3+\ldots \\ s_1(1+R_0+R_0^2+R_0^3+\ldots \\ s_1 s_2(1+R_0+R_0^2+R_0^3+\ldots \end{pmatrix} = \begin{pmatrix} 3,941 \\ 3,921 \\ 3,882 \end{pmatrix}$$

stationary population level: 11,744; relative age structure: $\begin{pmatrix} 0,336 \\ 0,334 \\ 0,330 \end{pmatrix}$

$$\text{b)}\quad \begin{pmatrix} (b_2+s_2 b_3)(1+R_0+R_0^2+R_0^3+\ldots \\ (1-b_1)(1+R_0+R_0^2+R_0^3+\ldots \\ s_2(1-b_1)(1+R_0+R_0^2+R_0^3+\ldots \end{pmatrix} = \begin{pmatrix} 2,164 \\ 3,153 \\ 3,121 \end{pmatrix}$$

stationary population level: 8,438; relative age structure: $\begin{pmatrix} 0,256 \\ 0,374 \\ 0,370 \end{pmatrix}$

$$\text{c)}\quad \begin{pmatrix} b_3(1+R_0+R_0^2+R_0^3+\ldots \\ s_1 \cdot b_3(1+R_0+R_0^2+R_0^3+\ldots \\ (1-b_1-s_1 b_2)(1+R_0+R_0^2+R_0^3+\ldots \end{pmatrix} = \begin{pmatrix} 0,394 \\ 0,392 \\ 1,388 \end{pmatrix}$$

stationary population level: 2,174; relative age structure: $\begin{pmatrix} 0,181 \\ 0,180 \\ 0,638 \end{pmatrix}$

Stationary population of the initial example:

$$750\begin{pmatrix} 3,941 \\ 3,921 \\ 3,882 \end{pmatrix} + 1200\begin{pmatrix} 2,164 \\ 3,153 \\ 3,121 \end{pmatrix} + 495\begin{pmatrix} 0,394 \\ 0,392 \\ 1,388 \end{pmatrix} = \begin{pmatrix} 5747,11 \\ 6918,37 \\ 7344,20 \end{pmatrix}$$

stationary population level: 20,009.68; relative age structure: $\begin{pmatrix} 0,287 \\ 0,346 \\ 0,367 \end{pmatrix}$

If λ_i is a characteristic value of L, then $1-\lambda_i$ is a characteristic value of I-L and $1/_{1-\lambda_i}$ a characteristic value of $(I-L)^{-1}$. Therefore, it holds for the characteristic vector δ_i that

$$(I-L)^{-1}\delta_i = \frac{1}{1-\lambda_i}\delta_i .$$

The composition of the immigrants born in a foreign country is given by

$$\begin{aligned} m_1 &= Mm_0 + w \\ m_2 &= Mm_1 + w = M^2 m_0 + Mw + w \\ &\vdots \\ m_t &= M^t m_0 + (I-M)^{-1}(I-M^t)w \end{aligned}$$

whereby

$$M = \begin{pmatrix} 0 & 0 & 0 \\ s_1 & 0 & 0 \\ 0 & s_2 & 0 \end{pmatrix}$$

is the matrix of the survival rates and

$$m_0 = w .$$

The following stationary age structure of the foreigners results for the present example:

$$\lim_{t\to\infty} m_t = (I-M)^{-1} w = \begin{pmatrix} 1 & 0 & 0 \\ s_1 & 1 & 0 \\ s_1 \cdot s_2 & s_2 & 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0,995 & 0 & 0 \\ 0,98505 & 0,99 & 1 \end{pmatrix}\begin{pmatrix} 750 \\ 1200 \\ 495 \end{pmatrix} = \begin{pmatrix} 750 \\ 1946,25 \\ 2421,78 \end{pmatrix}.$$

The following relationship exists between the absolute and the relative age structure of the immigrants:

$$w = kw^*$$

with

$$w^* = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 = (1-\alpha_1-\alpha_2) \end{pmatrix} \quad \alpha_i \geq 0 , \quad \sum \alpha_i = 1.$$

k indicates the number of immigrants per period.

The proportion of foreigners = $\frac{\text{Foreign born immigrants}}{\text{Total population}}$ in the stationary population is calculated as

$$\pi = [e'(I-M)^{-1} k \cdot w^*] [e'(I-L)^{-1} kw^*]^{-1}$$
$$= [e'(I-M)^{-1} w^*] [e'(I-L)^{-1} w^*]^{-1} = 0{,}256$$
$$\text{with} \quad e' = (1,1,...,1)$$

and is independent of the number of immigrants in each period (cf. Mitra, 1987; Feichtinger/ Steinmann, 1992).

The percentage of foreigners is especially easy to calculate when all entries occur in the first age group, i.e. $\alpha_1=1$ is:

$$\pi = \frac{1+s_1+s_1 \cdot s_2}{\frac{1}{1-R_0}(1+s_1+s_1 \cdot s_2)} = 1 - R_0 \ ,$$

whereby $R_0<1$.

The asymptotic behavior of the population is independent of the initial population and is only determined by the Leslie matrix, i.e. by the fertility and mortality rates. The age structure of the projected population asymptotically approaches a stable age structure. This is the content of the theorem of strong ergodicity in demography.

The projection model, however, only leads to a stable age distribution if L is an irreducible, primitive matrix. L is a n×n-matrix and is primitive if $L^k>0$ for all $k \geq n^2-2n+2$ (cf. Huppert, 1990, p. 372). L is then primitive when there are two consecutive fertile age groups (cf. for example Pollard, 1973).

The following Leslie matrix is a reducible, but non-primitive matrix:

$$L = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}.$$

The characteristic values of L are

$$\lambda_1 = 1$$
$$\lambda_2 = -\tfrac{1}{2} + \tfrac{\sqrt{3}i}{2}$$
$$\lambda_3 = -\tfrac{1}{2} - \tfrac{\sqrt{3}i}{2}$$

The amount of the positive characteristic value λ_1 is not larger than the other two characteristic values.

Taking the population vector

$$n_0 = \begin{pmatrix} 50 \\ 30 \\ 20 \end{pmatrix}$$

into consideration, the projection model can in the end be represented as follows, using the trigonometric form of the complex numbers after diagonalization:

$$P_t = 5{,}7735 \sin 2{,}09439t + 100 \ .$$

The population fluctuates with constant oscillations around 100.
The development of the population divided up according to age is shown in the following table.

t	0	1	2	3	...
0-15	50	40	30	50	
15-30	30	50	40	30	
30-45	20	15	25	20	
	100	105	95	100	

The projection matrix

$$L = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{pmatrix}$$

with the characteristic values

$$\lambda_1 = 0,7937$$
$$\lambda_2 = -0,3969 - 0,6874i$$
$$\lambda_3 = -0,3969 + 0,6874i$$

leads to the following population trajectory when taking the above population vector into consideration:

$$P_t = 100,779 \cdot 0,7937^t + (6,841 \sin 2,0947t - 0,7791 \cos 2,0947t) \cdot 0,7937t\ .$$

Here we have a sinking population with decreasing cycles.

t	0	1	2	3	...
0-15	50	20	15	25	
15-30	30	50	20	15	
30-45	20	15	25	10	
	100	85	60	50	

The age structure is also subject to a cyclical development.

In the last case, a constant yearly immigration would lead to a stationary population, since the cycles decrease.

At the end of this section, the number of emigrants necessary in a given age structure for a growing population to become a stationary one will be determined. The projection matrix

$$L = \begin{pmatrix} 0,6 & 1,1 & 0,12 \\ 0,9 & 0 & 0 \\ 0 & 0,95 & 0 \end{pmatrix}$$

with the characteristic values

$$\lambda_1 = 1,37454$$
$$\lambda_2 = -0,661746$$
$$\lambda_3 = -0,112796$$

is typical for the fertility and mortality rates in less developed countries. The stable growth rate is approximately 2.1% per year.

Assuming the population vector

$$n_0 = \begin{pmatrix} 450 \\ 340 \\ 210 \end{pmatrix}$$

the population development can be indicated by

$$n_t = 1,374^t \begin{pmatrix} 474,70 \\ 310,81 \\ 214,82 \end{pmatrix} + 0,662^t \begin{pmatrix} 25,36 \\ -34,49 \\ 49,52 \end{pmatrix} + 0,113^t \begin{pmatrix} -0,67 \\ 5,31 \\ -44,70 \end{pmatrix}$$

Taking the emigration vector

$$w = -k \begin{pmatrix} 23 \\ 46 \\ 31 \end{pmatrix} \text{ mit } k \geq 0$$

into consideration, the long-term population development can be approximated by

$$n_t \approx 1,374^t \begin{pmatrix} 474,70 - 108,75k \\ 310,81 - 71,21k \\ 214,82 - 49,21k \end{pmatrix} + k \begin{pmatrix} 119,208 \\ 61,2878 \\ 27,2235 \end{pmatrix}$$

The long-term population development depends on the factor k, which regulates the amount of emigration. If $k = 4{,}365$, then a stationary population results:

$$n_S = 4,365 \begin{pmatrix} 119,208 \\ 61,2878 \\ 27,2235 \end{pmatrix} \approx \begin{pmatrix} 520 \\ 268 \\ 119 \end{pmatrix}.$$

If $k > 4{,}365$, then the amount of emigration is larger than the natural population growth. The population of the emigration country decreases. In reality, such a situation is, however, not possible in the long term. If $k < 4{,}365$, the population of the emigration country continues to increase in the long term with the same growth rate.

A constant level of permanent emigration has a considerable influence on the age structure of the emigration country, as the following table makes clear:

Age structures

	initial population	emigration	stable population (no emigration)	sationary population (emigration)
0-15	0,45	0,23	0,475	0,573
15-30	0,34	0,46	0,310	0,296
30-45	0,21	0,31	0,215	0,131
	1	1	1	1

If the age structure of the emigrants is identical to that of the stable population, then the age structure of the resulting stationary population converges to that of the stable population. The stationary population is 1000. The number of emigrants each period is $1,374 \cdot 1000 - 1000 = 374$ in the above example.

C. EMPIRICAL RESULTS

1. A Stationary Population in Germany

Since the mid 1960's the net reproduction rate in Germany has continuously decreased. It currently fluctuates around 0,7. Without immigration, the population would decline in the long term by 30% each generation; this would mean a decrease of about 1,3% yearly. The deviation of the actual growth rate from the stable one can be explained on the one hand by age structure effects and on the other by a surplus of immigration. The cohorts with high birth rates from the 1960's are entering their reproductive phase and thereby contribute to a reduction of the population decline. In the long term, however, a population decrease in the case of unchanged fertility can only be avoided by an increase in immigration. Since the mid 1950's Germany has been a de facto immigration country. The immigration of Italian guestworkers began at the end of 1955. They were followed by guestworkers from Spain, Greece, Portugal, Yugoslavia and Turkey. An all-time high of almost one million immigrants was recorded in the year 1970. It was only with the recruitment stop of 1973 that the number of new immigrants considerably decreased. Whereas the immigration from the traditional guest worker countries has been sinking since the beginning of the 1980's, the immigration of foreigners seeking asylum, especially from Eastern Europe and Asia, has considerably increased. Because of the political upheavals in Eastern Europe, the number of ethnic German immigrants from those countries has likewise experienced a considerable increase. At the end of 1990, the percentage of foreigners was approximately 8% of an entire population of 80 million in Germany.

In the following, we will investigate the question of how many immigrants Germany needs each year in order to reach a stationary population of roughly 40 million women. Since the age structures of the domestic population and of the immigrants deviate considerably from the stable age structure, this question cannot be answered with the common equations presented in the previous section. Therefore, population projections will be carried out with the Leslie model.

The starting point of the projection is the female resident population of Germany in the year 1990, which has been divided up according to age and sex (see Fig. 1). The projection is made in stages of five years. A net reproduction rate of 0,7 is assumed for the domestic population. The age structure of the immigrants (see Fig. 1) corresponds to the typical age structure of the guest workers in the 1980's (cf. Pflaumer, 1992). At first it will be assumed that the immigrants and the domestic population have the same mortality and fertility rates.

The results of the projection indicate that around 253,000 (female) immigrants are necessary each year in order for a (female) stationary population of 40 million to be attained. The number of necessary immigrants is far less than that which would result with equations from mathematics of finance. The difference can be explained by the young age structure of the immigrants. Despite this young age structure, the stationary population nevertheless shows the typical urn shape of a shrinking population (cf. Fig. 1). Although this structure is younger than the stable age structure which would result without immigration, it is older than the age structure of the life table population. Several important characteristics of the age structures of different populations can be taken from Table 1, whereby the measures are defined as follows:

$$\text{Old Age Dependency Ratio (OADR)} = \frac{\text{People over 60 years old}}{\text{People between 20 and 60 years old}}$$

$$\text{Youth Dependency Ratio (YDR)} = \frac{\text{People under 20 years old}}{\text{People between 20 and 60 years old}}$$

Total Dependency Ratio (TDR) = OADR +YDR.

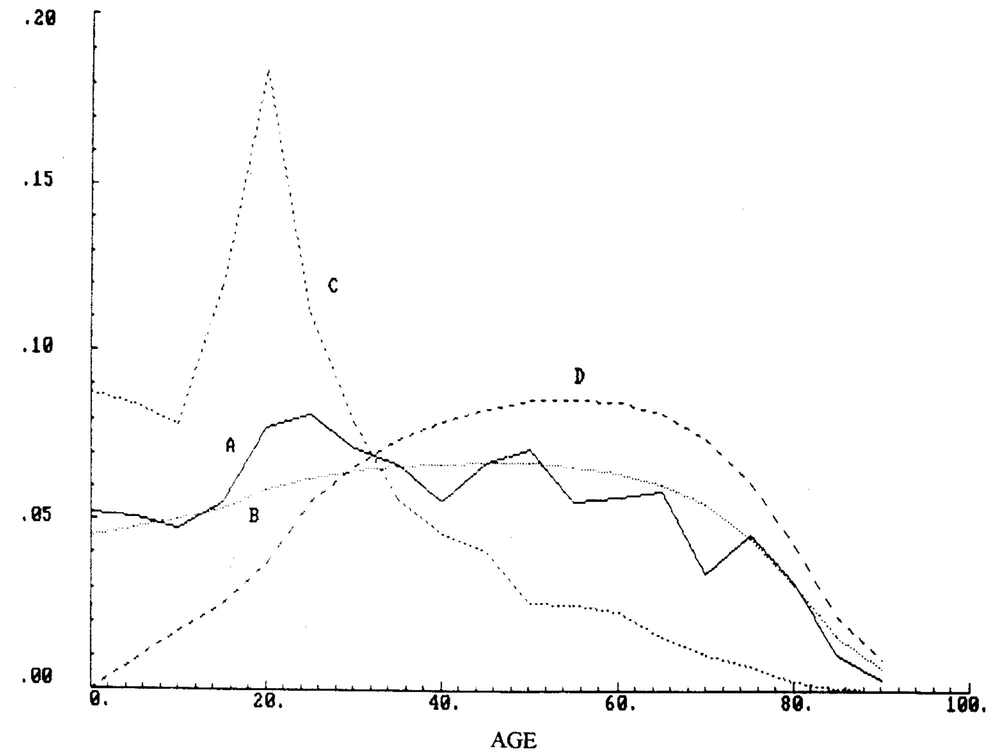

Fig. 1: Age Structures: Resident Population 1990 (A), Stationary Population (B), Annual Immigration (C), Foreigners in the Stationary Population (D)

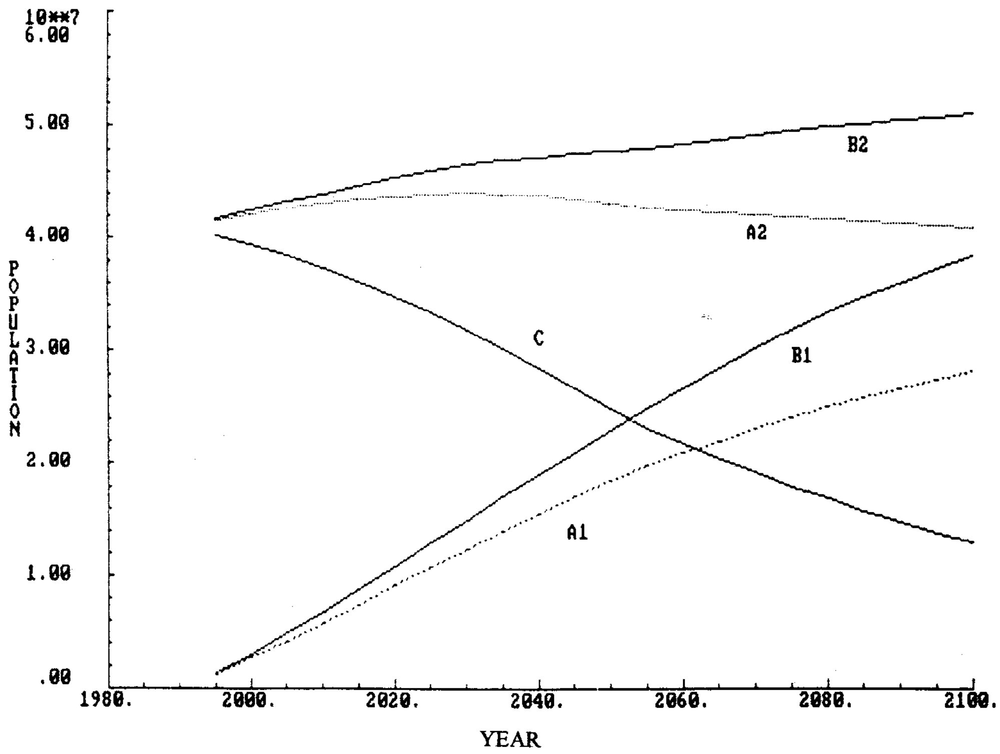

Fig. 2: Development of the Total Population (A2, B2), New Population (A1, B1), Old Population (C) under Different Fertility Assumptions

Table 1: Age-Structure Characteristics

	Resident Population 1990	Immigrants	Stationary Population	Stable Population without migration
Old age dependency ratio	0,444	0,108	0,535	0,743
Youth dependency ratio	0,374	0,649	0,375	0,344
Total dependency ratio	0,818	0,757	0,909	1,087

In Fig. 2 the development of the original (old) population and the immigrant population and their children (new population) can be seen. As early as the year 2060, the two populations will be equal. The number of immigrants who were born in a foreign country amounts to roughly 13,76 million; their age structure is presented in Fig. 1. If these immigrants are compared to the stationary population of 40 million, the resulting proportion of foreigners is 34,3%. The proportion of foreigners is presented for the individual age classes of the stationary population in Fig. 3.

Fig. 4 shows the temporal development of the old age and youth dependency ratios. The old age quotient will sharply increase starting in 2020, since the children of the baby boom of the 1960's will be reaching retirement age. The old age dependency ratio is an indicator of the financial burden on the old age social security system. The old age dependency ratio rises despite immigration with a young age structure. In the long term this ratio is independent of the number of immigrants, since a stationary population results. In the short and middle term, however, the development of the old age dependency ratio does depend on the number of immigrants each year (cf. Pflaumer, 1991). The higher this is, the lower the old age dependency ratio turns out to be. In this context, the static-comparative analyzes, which study the influence of immigration on a population with a fertility under replacement level, can be criticized (cf. e.g. Espenshade et al., 1982; Mitra, 1990; Cerone, 1987). The initial state will be compared with the stationary end state, without considering the temporal development which occurs between these two states.

In the following population projection, the no doubt more realistic assumption is made that domestic and foreign fertilities differ. It is taken into consideration that the immigrants take on the behavior of the new country only gradually, generally after one to two generations. The higher initial fertility (net reproduction rate = 1,645) should fit the domestic fertility after 30 years, whereby a geometric trend is assumed.

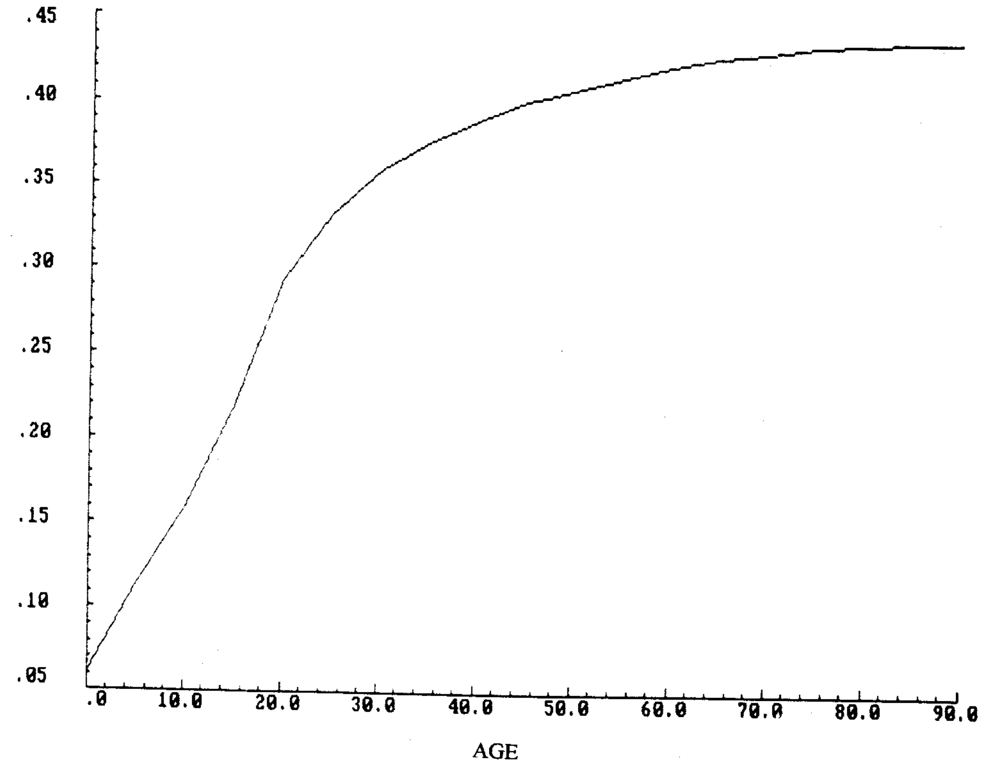

Fig. 3: Age-Structured Proportions of Foreigners in the Stationary Population

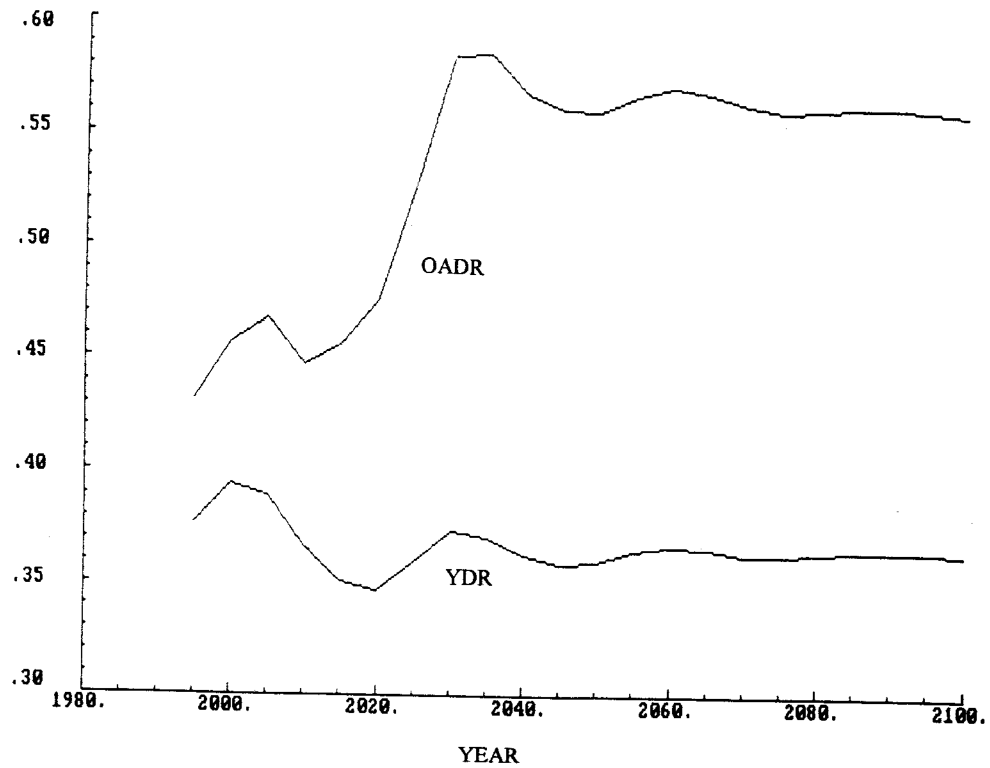

Fig. 4: Old Age Dependency Ratio (OADR) and Youth Dependency Ratio (YDR)

The development of the old and new population groups can be taken from Fig. 2. Because of the higher fertility, the new population group increases faster than in the previous case. The stationary population rises to around 60 million. In the case of a stationary target population of 40 million, only slightly more than 175,000 immigrants would be necessary each year.

2. World Population Scenarios

The world population has crossed the five billion mark. Its growth is roughly 1,7% yearly. Whereas in the more developed countries, the population will in all probability decrease or stagnate in the long run, it will grow further in less developed countries. It is indisputable that stationarity can basically only be achieved in the less developed countries by reducing the net reproduction rate to one. In the following, the extent to which immigration movements from less developed countries to more developed countries can contribute as a supporting measure to the goal of stationarity will be analyzed. For this purpose, the global population will be divided into the population of the more developed countries (1990: 0,62 billion women) and the population of the less developed countries (1990: 1,98 billion women). Let the net reproduction rate in the more developed countries be uniformly 0,7 (the fertility and mortality rates of the Federal Republic of Germany in 1987 are assumed for the more developed countries; cf. Pflaumer, 1988) and in less developed countries 1,645 (the fertility and mortality rates of Mauritius in 1970 are assumed for the less developed countries; these rates adequately reflect the currently valid conditions in less developed countries and correspond to a stable growth rate of 1,8% per year; cf. Keyfitz/Flieger, 1990). The population in each group is separately projected. The above model does not presume to make realistic projections of the world population, since the degree of aggregation is much too large. Rather, it is basically the influence of migration on the long-term population growth that is to be analyzed.

Fig. 5 shows four scenarios for the development of the female population in less developed countries, whereby constant mortality conditions and the abscence of migration are assumed.

Scenario I: The net reproduction rate of 1,645 of the initial population does not change. The female population increases to roughly 16 billion at the end of the next century. This would mean a total population of ca. 32 billion people in the less developed countries. However, because of the high degree of aggregation, the present model underestimates the population development when constant fertility conditions are assumed. Many countries in Africa have a net reproduction rate which is more than 2. Using a model which divides the world into six regions, Lutz and Prinz (1991) project a global population of over 56 billion people in the year 2100 under the assumption of constant fertility.

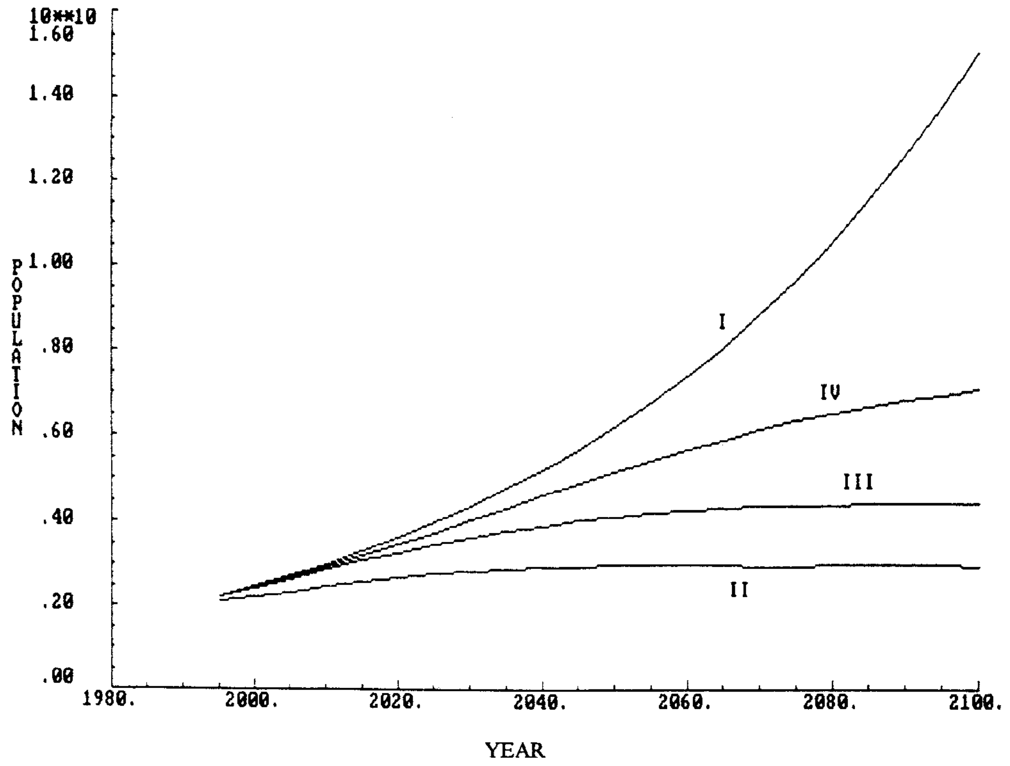

Fig. 5: Scenarios of the Development of the Female Population in LDC

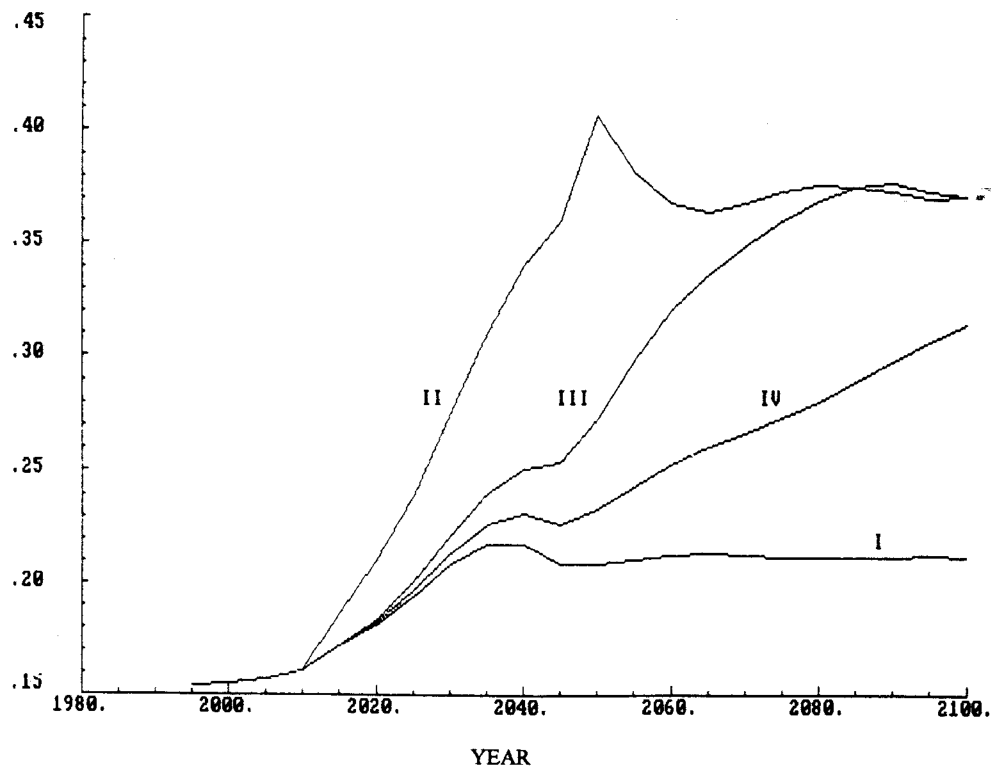

Fig. 6: Scenarios of the Development of the Old Age Dependency Ratio in LDC

Scenario II: Fertility immediately sinks to replacement level. Due to age structure effects, the population in the less developed countries does, however, temporarily increase to 3 billion, but then reaches the stationary level of 2,99 billion. Taking into consideration the total population in the more developed countries and the male population in the less developed countries, the total stationary world population would be around 7 billion. Lutz and Prinz (1991) arrive at similar results with their more realistic model.

Scenario III: Rapid decline in fertility to replacement level after 40 years. The stationary female population in the less developed countries is 4,41 billion (total world population: 9,8 billion).

Scenario IV: Slow decrease in fertility down to replacement level after 100 years. The stationary female population in the less developed countries amounts to 7,46 billion. (Total global population: 16 billion.)

Since scenario I and scenario II are very unrealistic, the global population will probably lie between 10 and 16 billion people at the end of the next century. Comparable results have been published by other authors as well (cf. e.g. Lutz/Prinz, 1991 or United Nations, 1992). This confirms that it is not so much the degree of aggregation of the models, but the assumptions concerning mortality and fertility that are the determining factors for the results.

Fig. 6 shows the development of the age dependency ratio in the different scenarios. With a stationary population, the age dependency ratio would ceteris paribus increase to approximately 0,37 in these countries. Because of the higher mortality, the age dependency ratio of the stationary population is smaller in less developed countries than in the more developed ones (cf. Fig. 4). Since the youth dependency ratio sinks with scenarios II to IV, the total dependency ratio decreases from 1,08 to 0,9. In the case of the continuously growing population of scenario I, the total dependency ratio stabilizes at 1,1.

Taking into consideration the results from mathematics of finance presented in Section B, the number of yearly female emigrants from less developed countries would have to be roughly 36 million in order for the population there not to increase further. If one assumes the age structure of the immigrants in Fig. 1 (age structure C) for the emigrants, then the number of emigrants necessary to attain stationarity sinks to 32.417 million per year. If the number of emigrants is lower, then the population in the less developed countries increases in the long term by 1,8% yearly if fertility remains constant (stable growing population; cf. Fig. 8). The effects of the emigration on the age structure in the less developed countries can be seen in Fig. 7. Such large numbers of immigrants are, however, not conceivable for the more developed countries. Assuming a net reproduction rate of 0,7 in more developed countries, which is

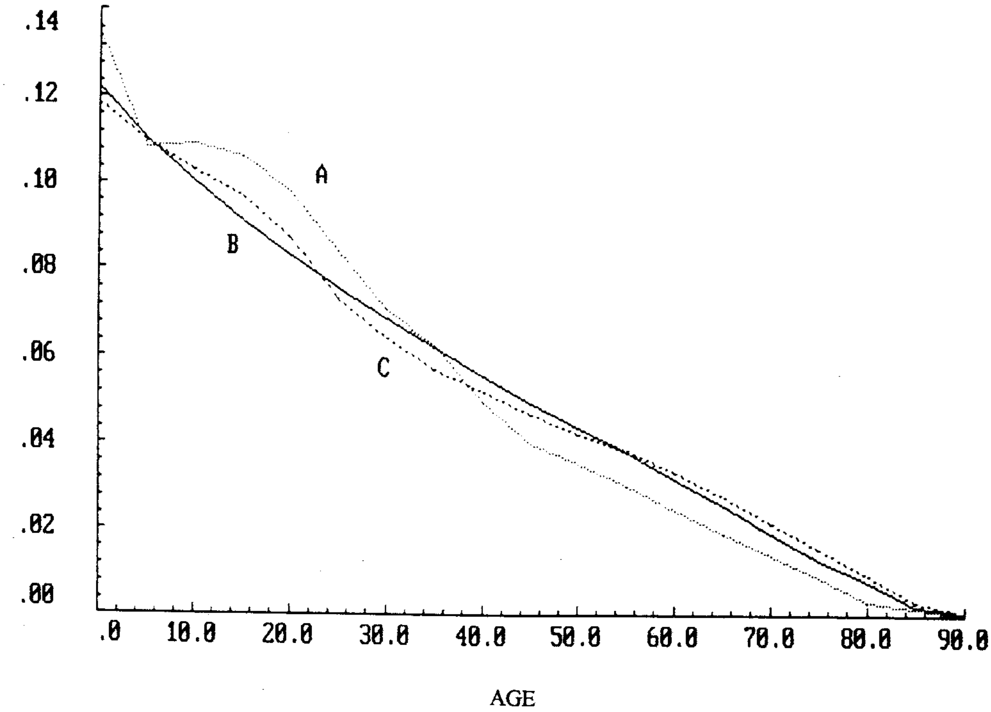

Fig. 7: Age Structures: Population in LDC 1990 (A), Stable Population (B), Stationary Population (C)

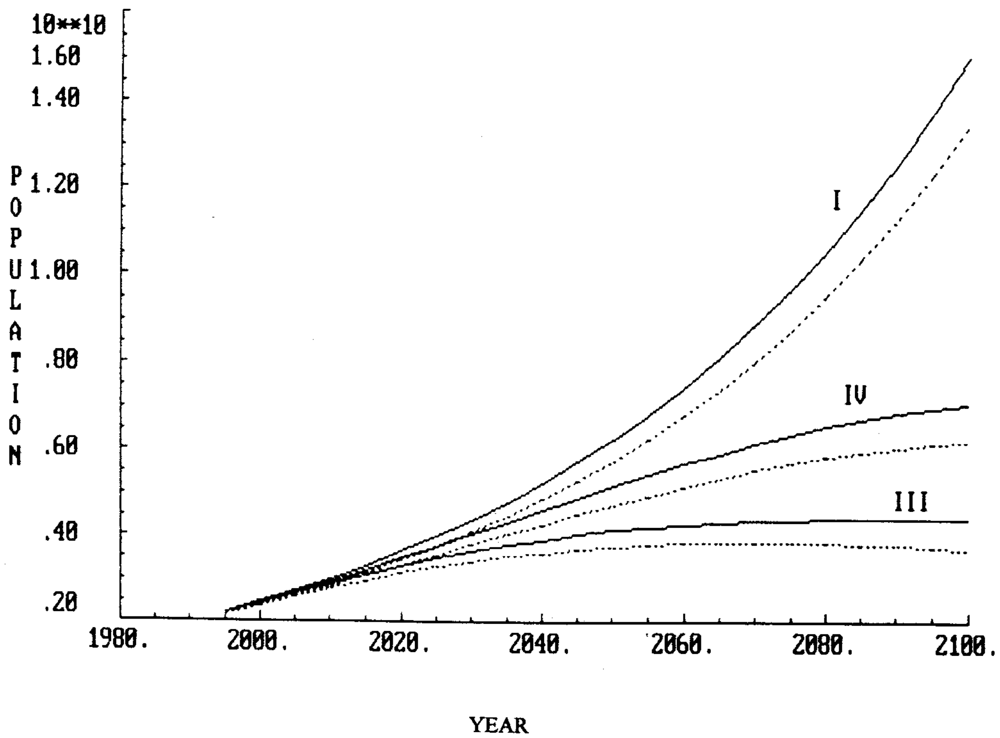

Fig. 8: Scenarios of the Development of the Female Population in LDC with (- - -) and without (—) migration

completely unrealistic with such large migration waves, a stationary (female) population of 4,8 billion would result.

Emigration in more realistic dimensions has on the other hand hardly any influence on the population development in the less developed countries, as Fig. 8 and Table 2 show. It is assumed that the number of female emigrants from less developed countries amounts to 4 million. This amount of immigration leads to a stationary population of 0,613 billion women in the more developed countries. The level of the stationary population roughly corresponds to that of the actual population in the year 1990.

Table 2: World Population Scenarios

	Scenario I		Scenario II		Scenario III	
	without migration	with migration	without migration	with migration	without migration	with migration
female population in less developed countries	15,06	13,47	4,41	3,73	7,08	6,20
female population in more developed countries	0,24	0,68	0,24	0,68	0,24	0,68
total	15,30	14,15	4,65	4,41	7,32	6,88

D. CONCLUSIONS

In the long term, the goal of quantitative population policies must be zero growth. This goal leads to a stationary population. If one does not consider the demographic variable mortality, then the only instruments that come into consideration for this goal are those that influence fertility and mortality. Whereas the population in the less developed countries will increase further, it will decrease in the more developed countries if immigration does not take place. It has been made clear that in less developed countries the goal of stationarity can basically only be attained through a decrease in fertility. The faster the decrease in fertility is, the smaller the stationary population will be. In the more developed countries stationarity can be attained by an increase in fertility or through immigration. The calculations have shown that under the assumptions made here, Germany has to taken in between 350,000 and 500,000 immigrants each year - depending on the age structure and fertility of the immigrants - in order for the population to stay at 80 million.

Immigration policies nevertheless have different demographic consequences for the age structure and the composition of the population than pro-natalist ones do. Despite the young age structure of the immigrants, the stationary population will show the typical urn shape of a stable shrinking population. The question whether immigration can replace children can be answered as follows: In the case of a net reproduction rate which is smaller than one, immigration prevents a population from dying out; in this respect it is a substitute for children. Nevertheless, it does not prevent the aging of the population; in this respect it is not a substitute for children. The age structure can be effectively rejuvenated only by an increase in fertility. If stationarity is obtained in a population by immigration, then one must deal with a high proportion of foreigners. In Germany, the percentage of foreigners will rise to 30% because of the current fertility situation. After two or three generations, the number of immigrants and their descendents will already have surpassed that of the indigenous population. Whether the national community can be maintained in the face of such large numbers of immigrants, or whether the population will disintegrate into a multitude of autonomous competing rival groups, as the French orientalist Maxime Rodinson (cf. Chimelli, 1989) fears, depends on the ability of the immigrants to integrate, as well as on qualitive population measures, whose aspects are not examined in this purely quantitatively oriented study.

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