

Modeling of magnetization reversal processes in magnetic circuits of measuring transformers

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The article describes methods for modeling transient regimes in current and voltage transformers. In most studies measuring transformers are modeled in a stationary mode to determine their metrological characteristics. However, for safe uninterrupted operation of transformers and electrical networks it is necessary to carry out their research in dynamic mode. In particular, the study of the transformers stability to the ferroresonant phenomena occurring during switching of high voltage switches is of practical interest. An algorithm for calculating transient processes in transformers allows to combine field models developed in the software package COMSOL Multiphysics and circuit models developed in the programming environment MATLAB. Special focus is on the approximation of the magnetization curve of transformers. The mathematical and simulation models allow us to investigate transient and steady-state regimes of transformers connected to the external electrical circuits.

Keywords – ferroresonance phenomena, measuring transformers, combine modeling, integration of COMSOL and MATLAB, approximation of the magnetization curve.

I. INTRODUCTION

The widespread introduction of microprocessor relay protection and automation devices impose new requirements to the primary measuring current and voltage transformers. In turn, the mathematical models carried out to study instrument transformers must be able to simulate not only established, but transient modes as well, accurately reproducing the secondary signal in the form of instantaneous values.

At Ivanovo State Power University (department of Electric power systems automatic control) digital current and voltage transformers are under development. The study of the primary converters, i.e. electromagnetic transformers with special properties for the above mentioned transformers was necessary [2].

In particular, the study of instrument transformers with open cores is of interest. In [1] we compare the results of metrological characteristics comparison based on of the field of modeling of design variants of open core voltage transformers. The advantage of the field mathematical modeling is the obtaining of the complete picture of behavior of measuring transformers under study – what parameters and values are studied, for example. However, the interaction of the simulated transformer with external circuits and diagrams is important for

their study in dynamic modes. Also for studying the dynamic properties, including anti-resonant properties of voltage transformers electrical connection between the individual stages in the cascade transformer must be considered. At the same time the field models must be "connected" to the external primary and secondary electrical circuits. Such studies are not possible without creating a circuit model.

II. GROUNDING OF THE APPROACH OF THE COMBINE MODELS

A. Combine modelling

COMSOL Multiphysics and MATLAB software packages are used for creation a combined field-circuit models. Field model created in COMSOL Multiphysics can be "connected" to the circuit by means of built-in SPICE or interact with Simulink and SimPowerSystem applications of MATLAB software package.

Field model is most conveniently constructed on the basis of the equation describing the electromagnetic interaction when written in relation to the magnetic vector potential:

$$\sigma \frac{\partial \vec{A}}{\partial t} + \nabla \times (\mu_0^{-1} \mu^{-1} \nabla \times \vec{A}) = \vec{\delta} \quad , \quad (1)$$

where t - time, μ , μ_0 - relative and absolute permeability, $\vec{\delta}$ - the density vector of external currents. The solution of equation (1) is carried out with a given distribution of the vector current density, which can be easily obtained if the currents in the coils are given (the skin effect and proximity effect due to the small cross section of winding wires are usually neglected).

The complexity of the field and circuit models integration is that current (current density) must be given initially for the calculation of the magnetic field according to the equation (1) while for the circuit calculating of non-linear inductance elements the voltage needs to be given.

Calculation perform using the built-in module COMSOL SPICE with selecting the appropriate settings solver can achieve acceptable accuracy and stability, but often for selected configurations requires a lot of time. Wider opportunities for external electric circuit modeling gives

Simulink + SimPowerSystem in comparison with the module SPICE program COMSOL.

B. Example of the numerical solution for the integration of COMSOL and MATLAB packets

In a first approximation the transformer is represented as an inductance coil. Consider the algorithm by the example of the inductance coil connected to a voltage source. From the theory of electrical circuits it is known:

$$\frac{\partial \psi}{\partial t} = L \frac{\partial i}{\partial t} + i \frac{\partial L}{\partial t} = E - i \cdot R \quad (2)$$

where L – inductance of the coil.

Neglecting the term $i \frac{\partial L}{\partial t}$ pass from derivatives to increments:

$$\frac{\Delta \psi}{\Delta t} = \frac{\psi_k - \psi_{k-1}}{\Delta t} = E - i_{k-1} \cdot R \quad (3)$$

$$L_{k-1} \frac{\Delta i}{\Delta t} = L_{k-1} \cdot \frac{i_k - i_{k-1}}{\Delta t} = E - i_{k-1} \cdot R \quad (4)$$

where k – number of time step.

By expressing i_k in the formula (4), we obtain a formula for the approximate current value at the k -th time step:

$$i_k^{(0)} = i_{k-1} + \frac{E - i_{k-1} \cdot R}{L_k^{(0)}} \cdot \Delta t \quad (5)$$

where $L_k^{(0)}$ – initial approximation of the coil inductance.

Flux linkage on the k -th time step is calculated by the formula:

$$\psi_k = L_k \cdot I_k \quad (6)$$

The residual flux current divided by the time step is calculated by the formula:

$$\frac{\Delta \psi}{\Delta t} = \frac{\psi_k - \psi_{k-1}}{\Delta t} - (E - i_k \cdot R) \quad (7)$$

$$\Delta \psi = \psi_k - \psi_{k-1} - (E - i_k \cdot R) \cdot \Delta t \quad (8)$$

$$\frac{\Delta \psi}{\Delta t} = L_k^{(1)} \cdot \frac{\Delta i}{\Delta t} = L_k^{(1)} \cdot \frac{i_k^{(0)} - i_k^{(1)}}{\Delta t} \quad (9)$$

From the formula (9), we can express the new approximation of the current by the k -th time step:

$$i_k^{(1)} = i_k^{(0)} - \frac{\Delta \psi}{L_k^{(1)}} \quad (10)$$

The algorithm for the current calculation consists of the following steps.

1. Set the initial value of the coil inductance.
2. Calculate the initial value of the current by the formula (5).
3. Calculate the new value of the coil inductance.
4. Calculate the flux linkage by the formula (6).
5. Calculated residual flux current by the formula (8).
6. Calculate the new current value by the formula (10).
7. Calculate the residual strength of current.
8. If the residual strength of the current exceeds a predetermined error, the calculation is repeated from step 3, otherwise the calculations are terminated.

III. RESEARCH ALGORITHM AND CURVE FITTING OF MAGNETIZATION.

For research of the algorithm the toroidal core of rectangular cross section with a tight and evenly wound single-layer winding with n turns was chosen. Core is made of steel 3408 of 0.3 mm thickness.

Accuracy of the algorithm and the subsequent modeling is significantly increased by using the experimentally obtained normal magnetization curve (NMC) of the transformer. Example of passport characteristics of electrical steel and NMC of three transformers having a core of this type of steel differences is shown in Fig. 1. Further, curves were measured for the supply current transformer with the presence and absence of the gap (Fig. 2) and zero-sequence current transformer.

This experimental dependence is necessary to smooth or approximate by the function for later use in the calculation. In [3] it is shown that the monotonicity of $H = F(B)$ affects the convergence of the solver COMSOL Multiphysics and the solving time.

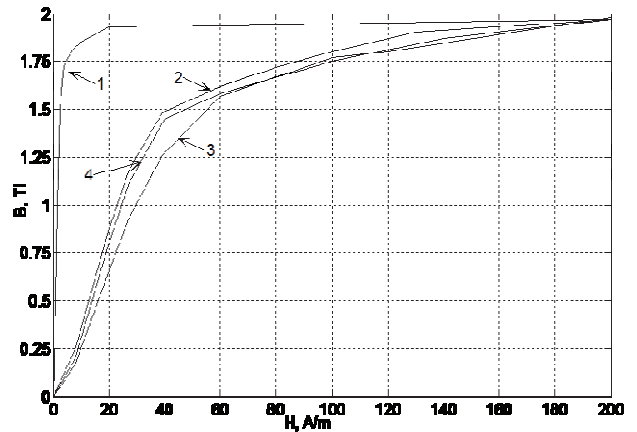


Figure 1. Example characteristic scatter of current transformers TLM-10 (curves 2-4) and steel 3408 magnetic curve (curve 1)

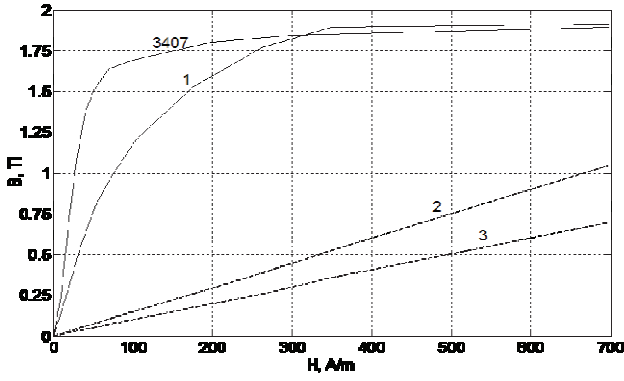


Figure 2. Magnetization curves for steel 3407 and experimental dependence for current transformer: 1 – without gap; 2 for gap width of about 0,15 mm; 3 - for gap width of about 0,3 mm

In [3-6] the following approximation of the magnetization curve are proposed.

- 1) Universal function

$$H = \frac{1}{\mu_0} \cdot \frac{B}{\frac{\mu_i - 1 + C_a (B/B_m)}{1 + C_b (B/B_m) + (B/B_m)^n} + 1} \quad (11)$$

- 2) Hyperbolic sine

$$H(B) = A_1 \cdot \sin h(A_2 \cdot B) \quad (12)$$

- 3) Precise hyperbolic sine

$$H(B) = A_1 \cdot B + A_2 \cdot \sin h(A_3 \cdot B) \quad (13)$$

- 4) Odd polynomial

$$H(B) = \sum_{i=1}^n (A_k \cdot B^{2k-1})$$

$$B(H) = \sum_{i=1}^n (A_k \cdot H^{2k-1}) \quad (14)$$

- 5) Simplified polynomial

$$H(B) = \sum_{i=1}^m (A_k \cdot B^{2k-1}) + A_n \cdot B^{2n-1},$$

$$B(H) = \sum_{i=1}^m (A_k \cdot H^{2k-1}) + A_n \cdot H^{2n-1}, \quad m \ll n \quad (15)$$

- 6) Frohlich formula

$$B(H) = \frac{H}{A_1 + A_2 \cdot H} \quad (16)$$

- 7) Arctangent

$$B(H) = A_1 \cdot \arctg(A_2 \cdot H) \quad (17)$$

- 8) Precise arctangent

$$B(H) = A_1 \cdot \arctg(A_2 \cdot H + A_3) \quad (18)$$

- 9) Adjusted arctangent

$$B(H) = A_1 \cdot \arctg(A_2 \cdot H) + A_3 \cdot H \quad (19)$$

- 10) Advanced arctangent

$$B(H) = A_1 \cdot \arctg(A_2 \cdot H + A_3) + A_4 \cdot H \quad (20)$$

- 11) Exponential dependence

$$B(H) = e^{\frac{H}{A_1 + A_2 \cdot H}} - 1 \quad (21)$$

- 12) Logarithmic dependence

$$B(H) = A_1 \cdot \sqrt{\ln(A_2 \cdot H + 1)} \quad (22)$$

- 13) Hyperbolic tangent

$$B(H) = A_1 \cdot th(A_2 \cdot H) \quad (23)$$

- 14) Adjusted hyperbolic tangent

$$B(H) = A_1 \cdot th(A_2 \cdot H + A_3) \quad (24)$$

- 15) Advanced hyperbolic tangent

$$B(H) = A_1 \cdot th(A_2 \cdot H + A_3) + A_4 \cdot H \quad (25)$$

Table 1 shows the results of the experimental magnetization curve approximation.

Of the considered approximation $H(B)$ types precise hyperbolic sine is the best approximation. It provides a small approximation error and correctly extrapolates dependence $H(B)$. Approximation of hyperbolic sine is an odd function and can be used to calculate the magnetic circuit of a constant and a variable field.

Arctangent approximation is most accurate for the $B(H)$ dependence for the same reasons. Table 2 shows the results of the study according to the convergence of the solver COMSOL Multiphysics on the type of interpolation of the experimental data.

TABLE I. RESULTS OF THE EXPERIMENTAL MAGNETIZATION CURVE APPROXIMATION

Type of approximating function	Root-mean-square deviation
<i>Approximating of H(B) function</i>	
Universal function	17.569
Hyperbolic sine.	12.835
Precise hyperbolic sine	11.033
Odd polynomial 31 degree	6.988
Simplified polynomial 49 degree	10.308
<i>Approximating of B(H) function</i>	
Frohlich formula.	0.01195
Arctangent	0.00833
Precise arctangent.	0.00301
Advanced arctangent	0.00833
Exponential dependence.	0.01162
Logarithmic dependence	0.02393
Hyperbolic tangent	0.00743
Precise hyperbolic tangent	0.00725
Advanced hyperbolic tangent	0.04197

TABLE II. MODELING OF H=F(B) CURVE

Interpolation or approximation type	Solving time, sec
Experimental dependence, cubic spline, partially-cubic interpolation, linear interpolation, nearest-neighbor method	Solver does not converged
Smoothed experimental curve, a cubic spline	148.668
Smoothed experimental dependence, partially cubic interpolation	188.12
Smoothed experimental dependence, linear interpolation	390.656
Smoothed experimental curve, nearest-neighbor method	Solver does not converged
Precise hyperbolic sine	230.943

For the algorithm study the field model of the object was "connected" to the calculation algorithm using the MATLAB programming language. An error in the calculation of current values for the developed algorithm and the algorithm of the SPICE module was calculated relative to the current calculated by the Simulink model with integration (Fig. 3).

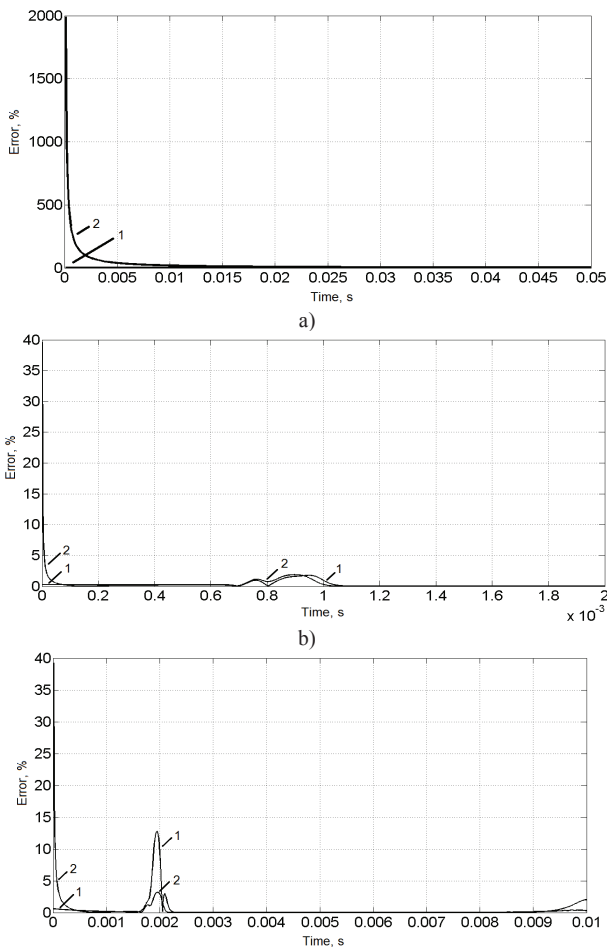


Figure 3. The current: calculation error of a - the DC voltage, b - the DC voltage, the coil with a nonlinear dependence of the magnetic permeability of the material, c - the sinusoidal voltage, with the initial phase of 10°, with a nonlinear magnetic permeability. 1 - calculation of the algorithm with Connecting the Field model COMSOL Multiphysics, 2 - calculation in SPICE module

This algorithm can be used for the calculation of ferroresonance processes study occurring during the switching of high-voltage circuit breakers are in the same circuit with a voltage transformer (Fig. 4).

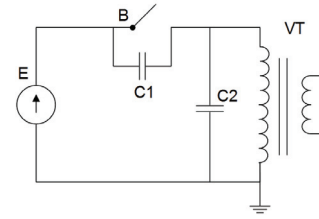


Figure 4. Synthetic scheme for research on Ferroresonance: E – voltage source, B - breaker, VT - voltage transformer

The result of research of transformers NKF-220 and transformer with open magnetic circuits by the proposed algorithm is the regions of dangerous ferroresonance existence (Fig. 5), which can estimate the range of capacities switches and receptacles on the ground, in which there is damage to these transformers. Z-axis in the graphs is the ratio of the amplitude of the current flowing in the windings of transformers at ferroresonance and the amplitude of the nominal current. Plane parallel to XY, determines the maximum current that can flow through the windings of the transformer voltage of this design.

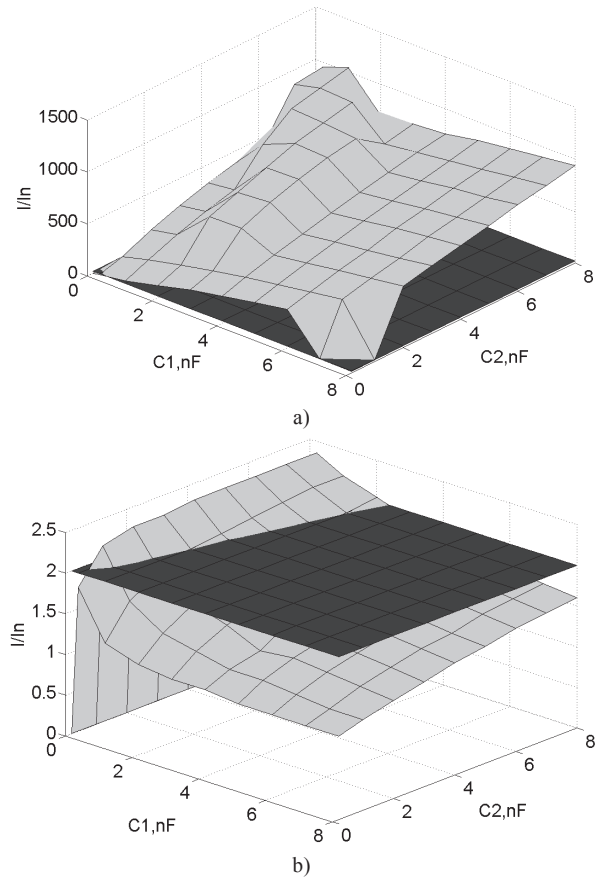


Figure 5. Areas of ferroresonance existence a - NKF-220; b - transformer with open magnetic circuits

IV. MATHEMATICAL MODELS OF CURRENT TRANSFORMERS.
APPLICATION OF DIFFERENT METHODS OF APPROXIMATION

In some cases the usage of combined models (numerical calculation of the current to the field in the COMSOL model and simulation in Simulink) is not justified. For cases such as symmetric magnetic core without a gap the circuit models are used developed with different software [7 – 12]. While solving problems related to the improvement of existing or the development of new relay protection devices, we need not only the primary and secondary current and voltage transducers modeling, but also models of the protection devices, as well as complex systems "electric-power object - relay protection device". For simulation of electric power facilities (EPF) and integrated systems "EPF - relaying device" most widely simulation software package Simulink is used implemented in the environment of a universal system simulation Matlab, which includes a special software package (library) SimPowerSystem. The equations derived from the T-equivalent circuit of the current transformer are used for model basis:

$$\begin{cases} e_2 = W_2 \cdot S \cdot \frac{dB}{dt}; \\ e_2 = i_a \cdot R_0; \\ e_2 = L \cdot \frac{di_2}{dt} + i_2 \cdot R; \\ i_\mu = \frac{H \cdot l}{W_2}; \\ i_1' = i_2 + i_a + i_\mu, \end{cases} \quad (26)$$

where W_2 – the number of turns of the secondary winding, L and R – load inductance and resistance, l - length of the middle line of the transformer, i_a , i_μ – active and reactive components of the magnetizing current of the transformer core.

The equation of dynamic magnetization proposed in [11] was also used:

$$\frac{dB}{dt} = r \cdot \left(1 - \frac{B^2}{B_s^2}\right) \cdot [H(t) - H_{cm}(B)] \quad (27)$$

where $H_{st}(B)$ – static hysteresis loop, r – the eddy current loss and magnetic viscosity.

Implementation of the model in Matlab is shown in Fig. 6.

Several models were developed for current transformers of different purposes and types: zero-sequence current transformer (type TZLM), phase current measuring transformers (such as TLM - 10), the supply current transformers.

The effect of depending $H(B)$ representation was studied. For each of the models the most accurate way to represent the magnetization curve was chosen. Precision of the curve was evaluated by comparison of the secondary variables (voltages and currents) obtained on the model with the experimentally obtained dependencies in a wide range of primary currents

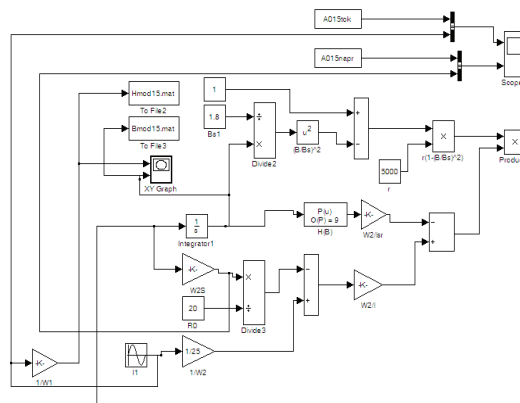


Figure 6. Model of current transformer in Matlab software

(from a few to hundreds of Amperes), and at different values (0,1 to 100 Ohm) of the secondary load. Example of waveforms of the secondary current for a supply current transformer with a different type of approximation NMC, with a load $R = 100$ Ohms is shown in Fig. 7.

To test real transformers in transient modes device RETOM-51 and waveform COMTRADE-damped oscillations in the frequency range from 200 Hz to 500 Hz were used. To obtain transient currents with a higher frequency (up to 5 kHz and above) physical modeling oscillatory discharge capacity was used.

Test of the real samples and models conducted mode small secondary loads (0.1 to 1 ohm). The resulting waveform of the secondary currents of the real sample were compared with the calculated waveforms obtained for models based on the equations of the dynamics of magnetization reversal of magnetic (Fig. 8 and Fig. 9).

These transformer models provide acceptable accuracy (error less than 10%) in a wide range of primary currents (from a few to hundreds of amperes), frequency (from 50 to 10 000 - 20 000 Hz) at different values of the secondary load (from fractions to tens of Ohms for transformer type TZLM and TLM-10 and hundreds of Ohms for the supply transformer) [12].

Such models can be successfully applied to solve problems related to the improvement of existing or the development of new safety devices and can be used in complex models "electric-power object - relay protection device".

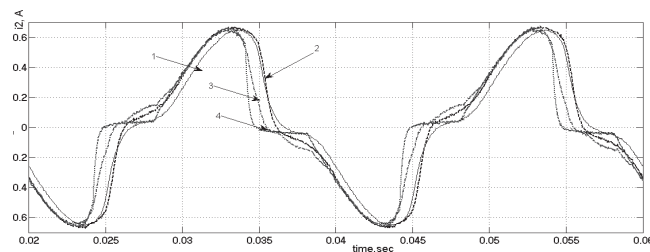


Figure 7. Oscillograms of secondary currents of supply transformer: 1 - experimental curve; 2 - model based on magnetizing curve, approximated by a polynomial n-th power; 3 - model based on magnetizing curve, approximated by Frohlich; 4 - model based on magnetizing curve, approximated by hyperbolic sine.

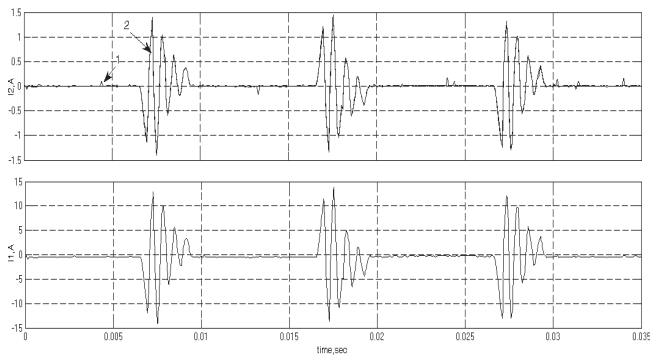


Figure 8. Oscillograms of secondary currents (upper waveform) for different primary signals (lower waveform) for the core of accuracy class 0.5 TLM – 10 and a secondary load 1 Ohm

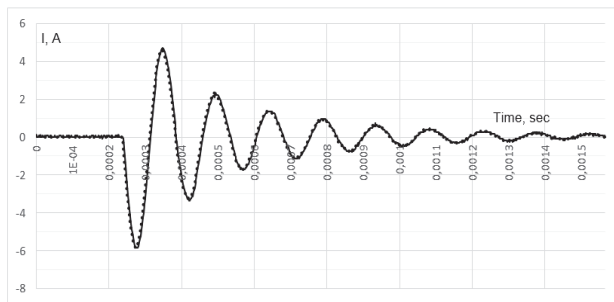


Figure 9. Oscillograms of TZLM secondary currents (experimental curve 1) and model (calculated curve 2) at an oscillation frequency of the transient current $f = 7$ kHz and a secondary load 0.1 Ohm

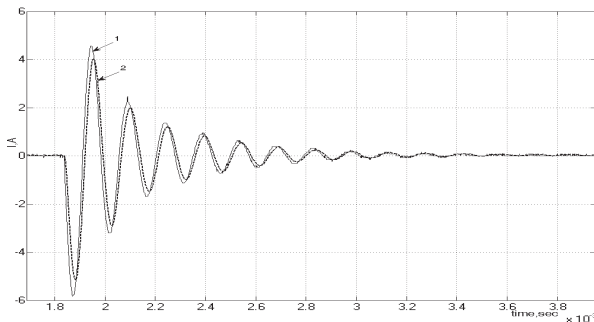


Figure 10. Oscillograms of TZLM secondary currents (experimental curve 1) and model (calculated curve 2) at an oscillation frequency of the transient current $f = 7$ kHz and a secondary load 1 Ohm

V. CONCLUSION

So, the interaction of the simulated transformer with external circuits and diagrams is important for their study in dynamic modes. For studying the dynamic properties, including anti-resonant properties of voltage transformers electrical connection between the individual stages in the cascade transformer must be considered. The field models must be "connected" to the external primary and secondary electrical circuits. Such studies are not possible without creating a combine model. Such a model was described in the

article. For cases such as symmetric magnetic core without a gap the circuit models are used. Such models can be successfully applied to solve problems related to the improvement of existing or the development of new safety devices and can be used in models "electric-power object - relay protection device". These transformer models provide acceptable accuracy in static and dynamic modes.

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