

# Calculation of the Electric Field below Hybrid Overhead Lines

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**Abstract**—The demand for increased transmission capacities in Germany will be covered in part by high voltage direct current (HVDC) lines. In order to reduce the need for new corridors, hybrid systems with AC and DC circuits together on the same tower are planned. Therefore, the characteristics of such arrangements need to be studied. In the first part of this paper, electromagnetic coupling mechanisms between overhead lines are summarized. Next, the method of image charges as a way to calculate the electric field around overhead lines is presented. The method is then used to analyze the electric field on ground level below three different hybrid line configurations.

**Keywords**- HVDC; hybrid line; electric field

## I. HYBRID LINES FOR THE GERMAN “ENERGIEWENDE”

The remaining nuclear power plants in Germany are scheduled to be shut down in a few years according to the agreements of the Energiewende. As they constitute the largest generating units in southern Germany today, a considerable amount of power plant capacity will have to be substituted. Furthermore, continuing expansion of volatile renewable power generation with large regional differences and often far from load centers will lead to increased load flows over large distances.

In order to retain network stability and reliability in spite of these challenges, the transmission capacities of the electrical grid have to be increased. For the first time in Germany, four high voltage direct current (HVDC) lines are among the projected measures [1]. Finding new line corridors is often extremely difficult, therefore the possibility of operating hybrid AC/DC power lines on existing towers is currently investigated by transmission grid operators (TSOs). The TSOs Amprion and TransnetBW plan to put into operation a 340 km hybrid line from Osterath to Philippsburg in 2019, when the Philippsburg nuclear plant will be decommissioned [2].

While the mechanical design of hybrid overhead lines is not very different to that of conventional ones, the close proximity of AC and DC conductors causes electrical coupling effects that have to be studied precisely. Also, it must be ensured that electromagnetic fields around hybrid lines do not exceed the permissible values [3].

In the first part of this paper, coupling mechanisms and their consequences are discussed in a general way. Next, the

method of image charges as an approach to calculate the electrostatic field around overhead lines is explained in detail. Finally, this method is used to calculate the electric field below hybrid overhead lines. Three different tower types and the influence of different placement of the DC poles are considered regarding the maximum field values on ground level. For one configuration, the instantaneous field distribution along an AC cycle is discussed.

## II. COUPLING MECHANISMS BETWEEN OVERHEAD LINE CIRCUITS

In general, three different coupling mechanisms can be distinguished: inductive, (quasi-)ohmic and capacitive coupling.

### A. Inductive Coupling

Magnetic fields resulting from time-dependent current flows in one conductor produce longitudinal voltages in adjacent conductors according to Maxwell's laws. Thus, high transients in one system will induce large overvoltages in systems nearby. In closed circuits, these voltages will also cause currents. On a hybrid line, stationary load flow in the AC conductors results in an alternating current component in the DC system because the DC sources appear as a short circuit to alternating currents.

In addition to an increased voltage drop across a DC reactor, these currents can be harmful especially to any equipment with iron cores. Line commutated HVDC converters will turn the induced fundamental frequency current mainly into a second harmonic and a DC component on the AC side. In the worst case, this offset can lead to core saturation and endanger a safe transformer operation [4].

### B. Ohmic Coupling

High electric field strength on the surface of conductors leads to ionization of surrounding air molecules, the so called corona discharge. While ionized field charges from AC conductors stay close to the wires, field charges originating from DC conductors can move over large distances and thus reach neighboring systems. AC conductors collect the free charges, resulting in a small DC current being injected. The problems from such undesired current components have been discussed in the previous chapter.

Amprion runs a short hybrid line for testing purposes in Datteln, Germany, where they investigated the injection of ion currents extensively. Besides the distance between wires, the current intensity was found to be most influenced by weather conditions. While being in the range of 1 mA/km in fair weather conditions, ten times higher values were observed during heavy rainfall [5].

In addition to ion current injection, also called quasi-ohmic coupling, corona discharges can have a significant influence on the electric field near the hybrid line as well. However, it is a very complex task to take into account all the charge creation, movement and recombination effects and requires tools like the finite element method or finite volume method. In sections III and IV of this paper as well as in many calculations found in the literature, the effect of field charges on the electric field is therefore neglected [6].

### C. Capacitive Coupling

An arrangement of conductors in a dielectric medium results in a network of capacitive couplings between them. Thus, a wire on high potential can induce phase voltages on other wires nearby. The electric field around the line results from the superposition of all the individual conductor's fields. This is of special interest firstly on conductor surfaces in view of corona discharge and secondly on ground level with regard to the line's environmental impact.

The surface voltage stress effects corona discharge, noise emissions and radio interference voltages. In order to prevent heavy corona losses it must not exceed 28.8 kV/cm. In a hybrid line, field strength on the DC conductors will be superimposed by an alternating component, whereas the field stress on AC wires will show a DC offset.

On ground level, regulations regarding electromagnetic field emissions have to be met. In Germany, the Bundesimissionsschutzgesetz (BImSchG) prescribes a limit of 5 kV/m for the electric field strength at 50 Hz. As there is no limit for 0 Hz, the CIGRE recommendation of 25 kV/m can be used for the DC electric field [3], [7].

## III. CALCULATION OF THE ELECTRIC FIELD AROUND OVERHEAD LINES

In this chapter, the method of image charges as an approach to calculate the electrostatic field around overhead lines is explained in detail. The presented procedure is used to calculate the electric field in a certain instant with known line-to-earth voltages and can be repeated for consecutive points in time, e.g. along an AC cycle.

### A. Premises

As already mentioned, the contribution of field charges to the electric field is neglected in this paper. Furthermore, the magnetic and electric fields are considered to be independent from each other, which is usually a valid assumption in electrical power grids as the operating frequencies are rather low. In this case, the electric field  $\mathbf{E}$  is irrotational as shown by (1) and can be expressed in terms of the electrostatic potential  $\varphi$  as in (2).

$$\text{rot } \mathbf{E} = -\partial/\partial t \mathbf{B} = 0 \quad (1)$$

$$\mathbf{E} = -\text{grad } \varphi \quad (2)$$

### B. Conductor Representation

Overhead line wires can be approximated as cylindrical conductors with infinite length as compared to their diameter. In general, the electric field of such objects can be modeled by infinitely long line charges, placed eccentrically inside the conductor. However, taking into account the fact that distances between conductors and to the earth's surface are significantly larger than their diameters, every wire can be described with very good accuracy by a single line charge in its center. The electric field of one such charge configuration in the coordinate origin can be expressed in complex coordinates as follows, where  $Q'$  is the line charge,  $\varepsilon$  the dielectric constant and  $\underline{z}$  the complex vector to the point of observation.

$$\underline{E} = \frac{Q'}{2\pi\varepsilon \underline{z}^*} \quad (3)$$

The potential with the reference point set to infinity yields

$$\varphi = -\frac{Q'}{2\pi\varepsilon} \ln|z|. \quad (4)$$

If more than one conductor is present, the influences of all charges superimpose, so that  $\varphi = \varphi_1 + \varphi_2 + \dots$  and  $\underline{E} = \underline{E}_1 + \underline{E}_2 + \dots$  holds.

Bundled conductors are treated as one wire located at the bundle's center. The radius of this equivalent wire is calculated as follows, where  $n$  is the number,  $r_{\text{wire}}$  the radius and  $a_{1\mu}$  the distance of individual conductors in the bundle.

$$r_{\text{eq}} = \sqrt[n]{r_{\text{wire}} \prod_{\mu=1}^n a_{1\mu}} \quad (5)$$

### C. Method of image charges

In order to calculate the field strength around an overhead line, the location of every conductor in a certain cross section is described by a position vector in the complex plain as shown in Fig. 1 on the next page. The point of origin is chosen to be in the line center on ground level.

To account for the earth's surface as a plain with zero potential, all conductors are mirrored along the real axis and the images are assigned with a charge of the same magnitude but of opposite polarity.

Using (3) and (4) along with the superposition principle, the electrostatic potential and the electric field at any point  $P$  in the cross section is given as follows.

$$\varphi_P = \frac{1}{2\pi\epsilon} \sum_{v=1}^n Q'_v \ln \left| \frac{z_P - z_v^*}{z_P - z_v} \right| \quad (6)$$

$$\underline{E}_P = \frac{1}{2\pi\epsilon} \sum_{v=1}^n Q'_v \left( \frac{1}{z_P^* - z_v^*} - \frac{1}{z_P - z_v} \right) \quad (7)$$

#### D. Matrix Equations to calculate local electric field

In equation (6), the Point  $P$  may as well be located on the surface of one of the conductors. If  $n$  such equations are written down for the potentials of all  $n$  conductors, this gives a system of linear equations.

$$\boldsymbol{\varphi} = \boldsymbol{\alpha} \mathbf{Q}' \quad (8)$$

The matrix  $\boldsymbol{\alpha}$  is called the matrix of potential coefficients. Its elements are:

$$\alpha_{vv} = \frac{1}{2\pi\epsilon} \ln \left| \frac{z_v - z_v^*}{r_v} \right| \quad (9a)$$

$$\alpha_{v\mu} = \frac{1}{2\pi\epsilon} \ln \left| \frac{z_v - z_\mu^*}{z_v - z_\mu} \right| \quad \text{for } v \neq \mu \quad (9b)$$

In order to find the electric field using (7), the charges on the individual conductors are needed. Those can be calculated from the conductor potentials given by their line-to-earth voltages by rearranging (8). The inverse of  $\boldsymbol{\alpha}$  is called matrix of capacity coefficients.

$$\mathbf{Q}' = \boldsymbol{\alpha}^{-1} \boldsymbol{\varphi} = \boldsymbol{\gamma} \boldsymbol{\varphi} \quad (10)$$

#### E. Wire sag

Along the power line, conductor positions in every cross section vary due to wire sag. Assuming a slack wire that cannot transmit shear stress, its curve can be described by a catenary, as is done in a Cartesian coordinate system in (11).

$$y - y_0 = \frac{\sigma}{\delta} \cosh\left(\frac{\delta}{\sigma}(x - x_0)\right). \quad (11)$$

Herein  $\sigma$  is the tensile stress in the wire and  $\delta$  its weight force per cross sectional area and length. If the maximum sag  $f_{\max}$  is known rather than these parameters, either the approximation given by (12) can be used, or (11) has to be iterated numerically [8].

$$\frac{\sigma}{\delta} = \frac{a^2}{8f_{\max}} \quad (12)$$

### IV. DISCUSSION OF RESULTS

Using the described method, the electric field on ground level was analyzed for the three different hybrid tower configurations shown in Fig 2.

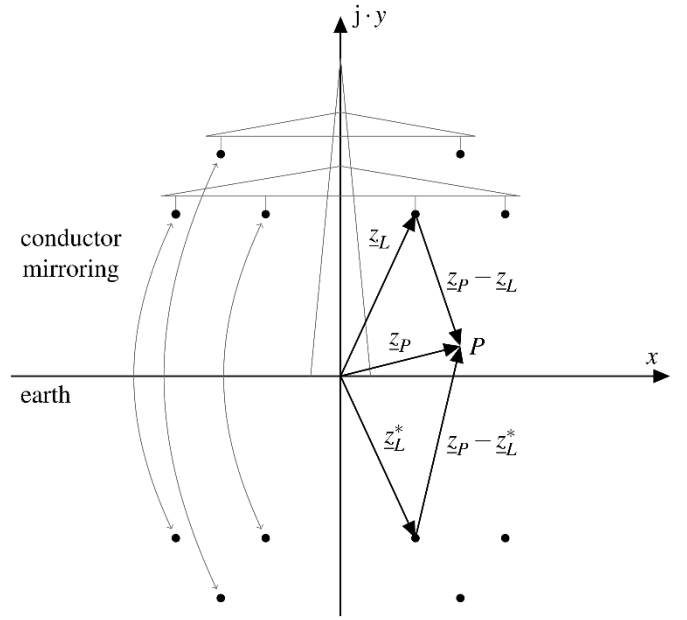


Figure 1. Method of images applied to overhead line conductors.

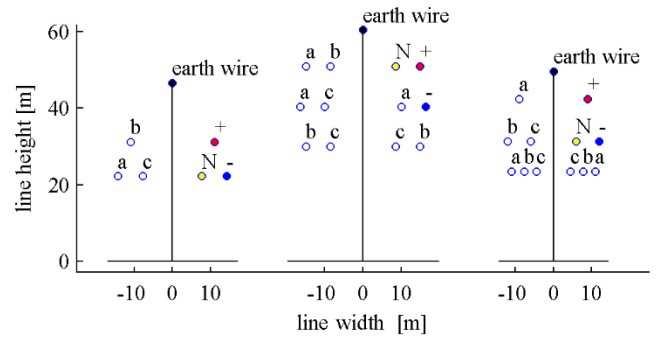


Figure 2. Analyzed tower types A (left), B (middle) and C (right).

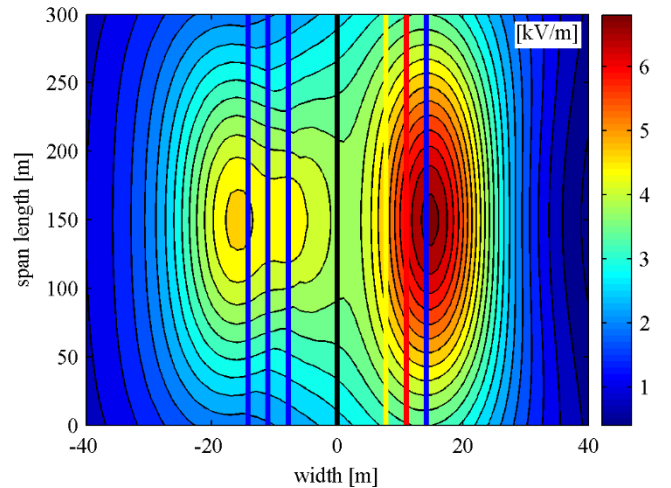


Figure 3. Maximum value of the Electric field at ground level under a 400 kV double circuit hybrid line (configuration A).

Configuration A is a 400 kV double circuit line in delta arrangement. The circuit on one side is replaced by a  $\pm 400$  kV bipolar DC system with plus and minus poles and a neutral conductor. In configuration B one of four 400 kV circuits of a tower in semi-vertical arrangement is replaced. Configuration C is similar to configuration A with two additional 110 kV circuits in horizontal arrangement below. For all configurations, a span length of 300 m and a sag of 8 m are assumed.

#### A. Electric field under a 400 kV double circuit hybrid line

The electric field has been calculated for short time steps along one AC cycle. Fig. 3 shows a contour plot of the maximum occurring field strength on ground level for configuration A.

The DC system clearly dominates with a maximum field of 6.9 kV/m as compared to a maximum of 4.8 kV/m on the AC side. Interestingly, those maximum values are found slightly offset to the actual conductor positions which are indicated by straight lines from top to bottom of the graph. The influence of the line sag can be seen clearly. The highest values in the middle of the span are almost the double of those at the towers, with a maximum of only 3.5 kV/m. These results are very similar to results found in the literature for similar tower configurations, like in [3].

#### B. Comparison between three different tower types

Fig. 4 shows line plots of the maximum fields along three different cross sections at 0 %, 25 % and 50 % of the span length for configuration A, B and C.

It can be seen that the highest electric field occurs for type A, where the DC conductors are closest to the earth with no other conductors in between. In case of configuration B, no distinct predominance of the DC field can be observed, because the DC poles are suspended much higher up and a 400 kV AC system lies between them and the ground, leading to a shielding effect. With regard to configuration C, the DC circuit causes an asymmetrical field pattern, but the overall maximum value of 2.1 kV/m lies markedly below that of configurations A and B. This can be explained firstly by the much lower phase voltage of the horizontally arranged systems closest to the ground. Secondly, these 110 kV circuits exert a strong shielding effect, as removing them results in a maximum field of 3.6 kV/m. The dotted line in the last graph shows the field at 50 % span length without the 110 kV systems.

#### C. Influence of phase positioning

With a fixed tower type, the exact arrangement of plus, minus and neutral poles remains to be determined and has a strong influence on the electric field at ground level. When analyzing absolute values of the maximum electric field over the entire AC cycle, the positions of plus and minus are interchangeable without affecting the result. Thus, three situations depending on the neutral wire's positioning need to be distinguished. For configuration A, they are depicted together with the maximum field at 50 % of the span length in Fig. 5.

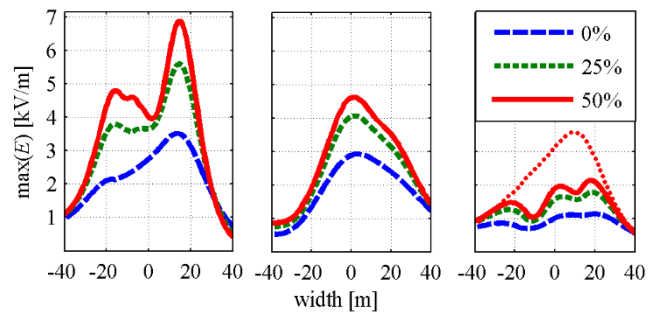


Figure 4. Maximum electric field on ground level for three different tower types A (left), B (middle) and C (right) at 0 %, 25 % and 50 % of the span length. Dotted line: tower type C without the 110 kV systems.

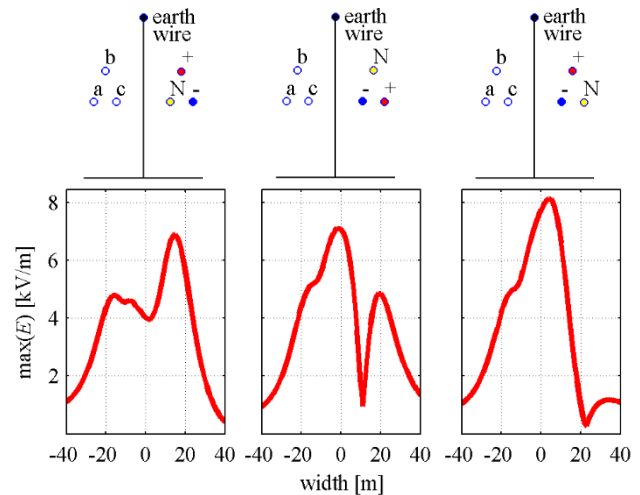


Figure 5. Influence of phase positions on the maximum electric field for configuration A: neutral wire inside (left), on top (middle) and outside (right).

The lowest maximum value occurs when the neutral wire lies closest to the tower center. In this case, the active poles are farthest away from the 400 kV system, preventing strong superposition of the corresponding fields.

When the neutral wire is on top the curves show a sharp drop of the field strength exactly between the horizontally arranged poles. This is due to the fact that the field of these two opposite charges could only have a horizontal component at this point, the earth's surface as an equipotential surface however allows only field lines perpendicular to it.

The highest field value occurs with the neutral wire on the outside, as the described effect of horizontal charges does not occur and the field of the inner DC pole overlaps strongly with the AC field.

#### D. Field strength along one AC period

In this section, the change of the electric field along one AC cycle is analyzed. This is done for configuration A with the most favorable DC conductor placement according to section E, which means that the neutral conductor lies closest to the tower center. The positioning of all phases can be seen in Fig. 5 on the top left.

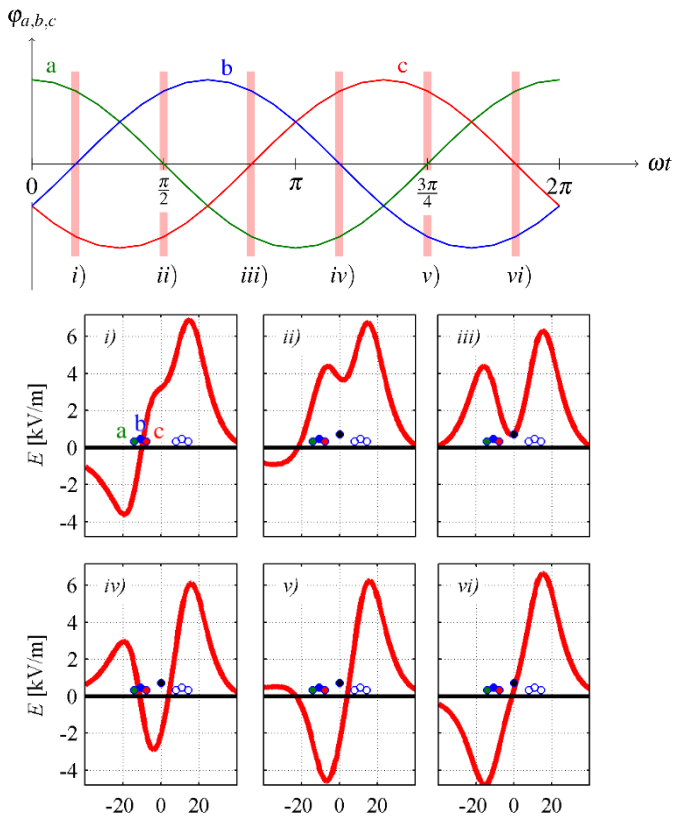


Figure 6. Vertical component of the electric field at 50 % span length for configuration A along one AC cycle.

Fig. 6 shows the AC phase voltages along one AC cycle together with the ground field curves in the middle of the span length corresponding to six different points in time *i) – vi)*. Unlike all previous diagrams, Fig. 6 shows instantaneous values of the field vector's vertical component rather than absolute maximum values. This means that field lines coming from a positive conductor and arriving at the ground result in a negative value, whereas lines starting from the ground result in a positive value.

The points in time *i) – vi)* are chosen in such a way that one AC phase is at potential zero, while the other two have opposite potential. At these times, the ground-level electric field shows maximum amplitudes.

One fact that can be noted is that on the DC side, the negative pole closest to the ground produces a positive field that shows almost no variation during the cycle, which confirms it is a real DC field. On the AC side, a wave-like movement of field amplitudes from the tower center outwards can be observed. This movement is directed inwards, if the AC phase sequence is reversed.

The individual field curves can be explained by considering the instantaneous phase voltages and the position of corresponding conductors as indicated by the colors green, blue and red. Exemplarily at time *i)*, phase *a* is positive and *c* is negative. This leads to a transition from negative values underneath phase *a* to positive values under phase *c*. At time *ii)*,

phase *b* is positive instead of *a*, but its influence on the field at ground level is much weaker because it lies farther away and diagonally offset from the ground. Other instants can be explained in a similar manner.

## V. CONCLUSION

The operation of hybrid lines with AC and DC systems on the same tower is a promising possibility to increase transmission capacities without the need for new corridors. Their electrical characteristics are strongly affected by inductive, (quasi-) ohmic and capacitive coupling mechanisms due to the close vicinity of the systems.

With regard to the electric field on ground level below hybrid lines, the calculations show that DC systems cause significantly higher fields than AC systems with the same nominal voltage. However, the situation depends strongly on the tower configuration. The higher up the DC conductors are suspended, the smaller their dominance of the ground-level field. Systems in between the DC poles and the ground exert a shielding effect. In one of the analyzed configurations, the presence of an underlying AC system results in a reduction of the maximum ground electric field of more than 40 %. The arrangement of plus, minus and neutral phase of the DC system also affects the field distribution. In a double line with delta arrangement for instance, the neutral wire should be placed close to the tower center to avoid strong superposition of the AC and DC fields. Analysis of the hybrid field of a double circuit line along an AC cycle shows that the field on the DC side shows almost no variation in time and can therefore be considered as a pure DC field. This is important with regard to the compliance with emission regulations, as limit values are generally higher for lower frequencies. Taking this into account, none of the analyzed configurations exceeds current limit values for the electric field on ground level.

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