Stability Analysis and Clustering of Electrical Transmission Systems

Sebastian Brato, Dirk Surmann, Sebastian Krey, Jürgen Götze, Uwe Ligges, and Claus Weihs
all at TU Dortmund University, Germany

***Abstract*—A proper understanding and modelling of the behaviour of heavily loaded large-scale electrical transmission systems is essential for a secure and uninterrupted operation. In this paper we present a descriptive analysis especially of low frequency oscillations within an electricity network and methods to assess the stability of the whole system based on an ARMAX model and the ESPRIT algorithm. Further we present two methods to separate the network into local areas, which is necessary for an efficient modelling of a large electrical system. The first method has its foundation in the results of the ARMAX based stability analysis and the second method concentrates on the network topology. In the last part of this paper we present an approach how an modelling of such local areas within an large electrical system based on stochastic differential equation models is possible.**

I. Introduction

The European electrical transmission system is operated increasingly close to its operational limits due to market integration, energy trading and the increased feed-in by renewable energies. For this reason it is necessary to analyse that part of power that permanently oscillates through the electrical transmission system with a low frequency. These so called Low Frequency Oscillations are described and analysed within a smaller electrical system, the `New England Test System`, which guarantees a convenient handling. The analysis results in a new model which describes each node of the transmission system over partly excited mechanical harmonic oscillators. As in a real transmission system, the harmonic oscillators are connected over mechanical components according to the transmission lines of the electrical system. This model, which is based on a system of differential equations, is compared with a well established and much more complex simulation system used at the Institute of Energy Systems, Energy Efficiency and Energy Economics of TU Dortmund University. The current work is to optimise the parameters of the connected mechanical harmonic oscillators (mass, damping, stiffness and excitation) to get the same behaviour for the Low Frequency Oscillations as in the complex simulation. In addition stochastic elements are implemented in the differential equations to analyse their impact on the Low Frequency Oscillations.

To allow the simulation of large transcontinental electrical systems it is necessary to build smaller local models for different areas of the whole system to reduce the amount of transferred data, computational time and communication latencies. Hence it is necessary to structure the whole electrical system into such areas. The available transmission lines constrain the possible structure and it is desirable to have similar dynamic behaviour within an area. In Sec. IV we present a method to use the results of an ARMAX based stability analysis for the construction of clusters within the system. In Sec. V we construct the clusters via Spectral Clustering, a method which uses the static information about the network topology as the main criterion for the clustering process. This results in clusters, which follow the constraints of the available transmission lines. Although these two methods use completely different information from the electrical system, the agreement of the results is large, which we will discuss in Sec. V-A. This makes it possible to get a sensible clustering even if only one of these information is available.

II. Data Generation

The base for our modelling is a simulation of the `New England Test System` (NETS) [1], [3], [4] implemented in the well established energy simulation software `DIgSilent PowerFactory`. The `New England Test System` is a well known reference system for testing and evaluation of methods and algorithms for the control of electrical systems. It consists in total of 39 busbars which connect 10 generators and 18 loads to the network and are interconnected by 37 power lines. An abstracted overview of the system is given in Fig. 1. At all nodes every information about the electrical characteristics is available.

The simulation software `DIgSilent PowerFactory` allows a flexible simulation of complex electrical systems. Beside static load flow calculations also dynamic simulations are possible. To examine the dynamic behaviour of the the electrical system different events are generated. These are changes of the electrical load in parts of the system and failure events. The used events for inducing oscillations into the electrical system are:

- Increase the load at every second node by 5%
- Line outage between node 1 and 2
- Combination of the line outage between node 1 and 2 with a variation of the load at node 4

In our work we use the electro-mechanical simulation mode. In this mode the behaviour of the electrical system is simulated in discrete time steps of at least 10 ms. We use this maximum resolution and record the voltage magnitude and angle at every bus bar in the network. This builds the data base for the

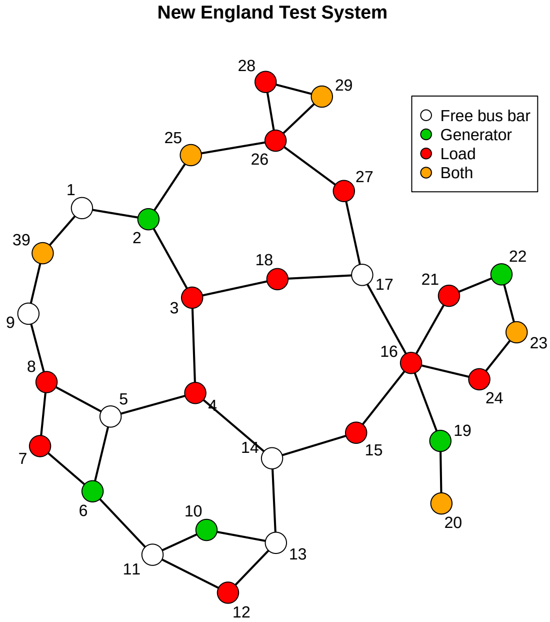

Fig. 1: The New England Test System

descriptive analysis in Sec. III and the stability assessment in Sec. IV.

III. Descriptive Analysis

To generate data sets for the descriptive analysis, different load events were simulated with the established energy simulation software `DIgSilent PowerFactory` in the `New England Test System`. After 20 s of transient effects, all chosen events that excite the electrical transmission system show the characteristics for Low Frequency Oscillations on the analysed voltage angle and magnitude of about 0.17 Hz. In the first stage of the statistical part of project 5[1], the descriptive analysis shows that the different events result in different amplitudes for the nodes of the system. The relative oscillation situation, in contrast, is always the same. This adds up to the assumption that the topology of the transmission system mainly determines which nodes show a higher or lower amplitude of the Low Frequency Oscillation.

The second result of the descriptive analysis is the oscillation behaviour of the nodes and areas of nodes among each other. In a time and spatial based analysis the voltage angle in the `NETS` system shows three regions, in which the nodes oscillate in the similar way. The regions are divided by nodes with a very low amplitude, so that these nodes cannot be count to any region. A part of the whole analysis is shown in Fig. 2. The graphics show the amplitude of the voltage angle proportional to the size of the nodes at the timestamps 33.5 s and 35.5 s. The regions B and C oscillate in the same way and with a 180° phase offset against the region A. Compared to the

[1]see: http://for1511.tu-dortmund.de/cms/de/Projekte/Teilprojekt_5/index.html

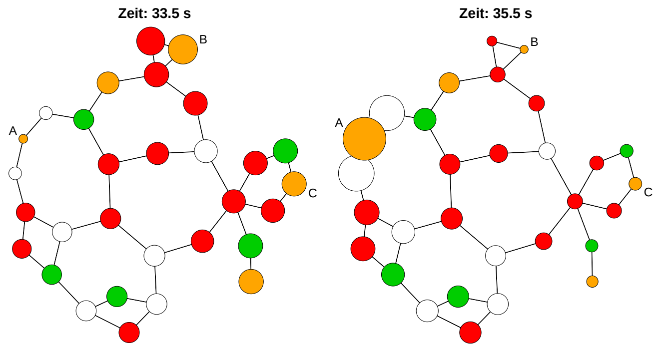

Fig. 2: Visualisation of the voltage angle oscillation in the energy system

voltage angle, the magnitude shows another behaviour, which makes it difficult to outline the four regions. Furthermore the nodes in the system oscillate equally without any phase offsets between the nodes. Only the amplitudes of the nodes are different but much smoother than for the voltage angle. The behaviour over the whole time interval of this scenario is shown in the video at http://www.statistik.tu-dortmund.de/~surmann/NETS_Video/Jede2plus5_Schwingung.avi.

Mainly the antithetic oscillations of the voltage angle makes a direct usage of the first favoured Markov Random Fields model approach useless. Markov Random Fields model neighbouring nodes over correlations, which means that this approach cannot model situations where connected nodes get full information from their neighbours, but show complete different behaviours. Furthermore a model is needed in which it is possible to excite the system close to reality, over a continuous stochastic excitation. Currently a punctual load change provides the excitation of the system, which is not realistic. These issues leads to a model of differential equations, like a mechanical driven harmonic oscillator that is discussed in detail in Sec. VI.

IV. Stability Assessment by System Identification and Clustering

This section deals with the stability estimation of an electrical transmission network. As electrical power systems tend to be operated closer to its limits, the stability analysis is of growing importance. Additionally it is mandatory to get a fast overview when changes happen in the network, so the realtime requirement to stability analysis tools is also very high. The described algorithms are based on the voltage phase angle data derived from the NETS system.

A. ARMAX based stability analysis

The stability of the distribution network is estimated by the measured data from multiple phasor measurement units (PMU) which are placed within the grid. The data from each node is used to process an ARMAX model within each node of the network. For the estimation process of an ARMAX model it is mandatory to have a defined set of input and output data.

With this data an ARMAX model shown in figure 3 can be estimated.

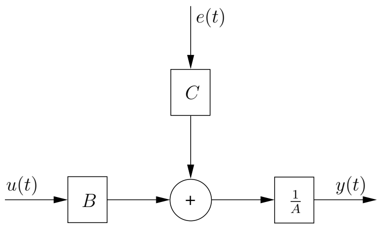

Fig. 3: Signal flow for ARMAX model

The mathematical formulation

$$A(q)y(t) = B(q)u(t - n_k) + C(q)e(t) \tag{1}$$

describes the ARMAX model, where the system polynominal $A(q)$ and $B(q)$ are relevant for the system behaviour. The polynomial $C(q)$ is used to model the error which the signal contains. This error mainly consists of noise which is superimposed to the signal. The three polynomials are computed from a set of output data $y(t)$ and input data $u(t)$ and an estimation for the error $e(t)$. The data used for the modelling process is gained from the node for which the model is currently computed. For this node, the voltage phase angle is treated as output. For the input data, each voltage phase angle of the neighbouring nodes is used. There is no further data of other nodes used in this process such that the network topology can be completely unknown. After the modelling process the eigenvalues of the system polynomial A are estimated. These eigenvalues give an insight on the stability of the system. The eigenvalue which is nearest to the stability border is determined and tracked. This gives a quick overview of the network stbility margin. In order to provide a real-time feedback of the system stability, the ARMAX model is frequently updated and the new eigenvalues are estimated. The whole process of modelling and the determination of eigenvalues is done dynamically for each time-step and uses a data history of about 40s. The sampling of data is done every 10ms, this leads to a very time accurate view of the system and gives a quick response for changes within the network. In Fig. 4 an example is given for the data generated by the simulation of the network. The plot shows the voltage phase angle at a single node while a line outage between nodes 1 and 2 occurs. To analyse a stable and an unstable case the line outage takes place under different loading situations of the network. This is done by varying the load at node 4 between 400MW and 800MW. This power setting is done at $t = 20s$ while the line outage takes place at $t = 250s$. In addition to the excitation by load changes and line outage, the system is subject to noise caused by all of the loadings within the network. As it can be seen in Fig. 4 and 5, for a very high loading of 800MW, the power system reaches an unstable operating condition.

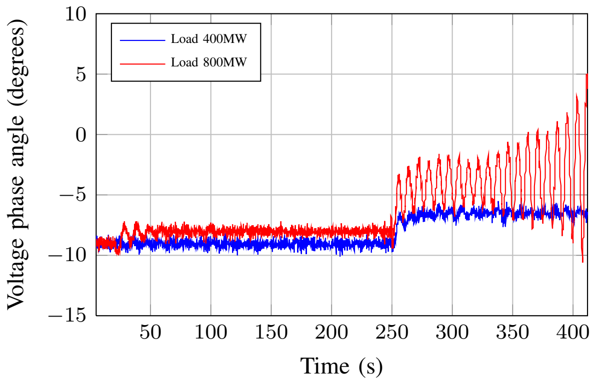

Fig. 4: Voltage phase angle at node 6 over time for different load situations in case of a line outage

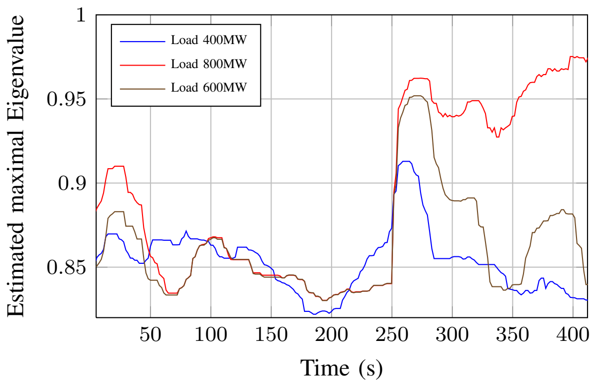

Fig. 5: ARMAX based stability estimation at node 6

B. Esprit

Additionally to the ARMAX based approach, a subspaced based identification technique is used to estimate the frequencies within the power system. This approach is based on the ESPRIT algorithm. It provides a fast and efficient estimation of the spectral components of the measured signal as shown in [5]. While this method is less complex than the ARMAX based stability criterion, it lacks the possibility of generating clusters from the generated data as shown in IV-C. The main advantage of an ESPRIT based stability criterion is the faster computation and the smaller data history used for the calculations. While the data history needed for the ARMAX model is $40s$ the ESPRIT based algorithm suffices with $6s$. This great difference is caused by the large amount of data which is needed to get a reliable representation of the system for an ARMAX model. As there is no modelling of the system for the ESPRIT based algorithm, the dataset required for an estimation of the frequencies and power spectra is therefore much smaller. This leads to a faster reaction on events occuring within the network by using the ESPRIT algorithm.

The ESPRIT algorithm is based on the $m \times m$ correlation matrix $\hat{R}_{yy}$ of the signal $y(t)$, which in this case is the voltage phase angle of the node, where the stability criterion shall be calculated. For this matrix an Eigenvalue decomposition

$\hat{R}_{yy} = \hat{U}\hat{\Lambda}\hat{U}^*$ is computed. Matrix $\hat{U}$ is sorted according to the Eigenvalues in descending order and is separated into noise and signal space $\hat{S}$. The signal space matrix is then decomposed to $\hat{S}_1 = [I_{m-1}\ 0]\hat{(S)}$ and $\hat{S}_2 = [0\ I_{m-1}]\hat{(S)}$ and the Matrix $\hat{\Phi}$ is estimated as $\hat{\Phi} = (\hat{S}_2^* \hat{S}_2)^{-1}\hat{S}_2^*\hat{S}_1$. The Eigenvalues $\hat{\chi}_i$ of $\hat{\Phi}$ yield the frequencies $\hat{\omega}_i = arg(\hat{\chi}_i)$ contained in the original signal $y(t)$. From the frequencies the spectral power is determined by evaluating the following function:

$$f(\omega) = \left[\left| \prod_{k=1}^{n} e^{j\omega} - |\chi_k| e^{j\omega_k} \right|^2 \right]^{-1} \tag{2}$$

As shown in Fig. 7, the ESPRIT based stability criterion is the estimated power of the signal.

$$P(f) = \frac{1}{e^H(f)(\sum\limits_{k=p+1}^{N} v_k v_k^H)e(f)} = \frac{1}{\sum\limits_{k=p+1}^{N} |v_k^H e(f)|^2} \tag{3}$$

For an unstable network setting this estimated power tends to gain great values. Compared to Fig. 5, it can be seen that not only an unstable case can be identified but it is also possible to estimate the severity of the failure by accounting the information of the estimated power value. On the other hand, the ARMAX based approach allows a better stability estimation prior to the failure when the amplitudes of the oscillating signal are still very low.

C. ARMAX based Clustering

As the ARMAX based modelling process is done in each of the network nodes, it is possible to obtain a time series of eigenvalues for each node. When this information is exchanged within the network and gathered at a central place, it is possible to analyse the correlation of these synchronised time series for the whole network. It has been shown that there are certain areas with a very similar behaviour, while other network nodes show a completely different progression. Based on the correlation between these time series data it is possible to cluster the network nodes. As an example Fig. 6 shows the clusterization of the NETS network with an excitation of 800MW at node 4. Additionally the network partitioning is examined in Sec. V. In that section mainly statically known parameters like line impedances are used as basis for the clustering process. Although a different data basis is used, the cluster results of both methods show a very similar behaviour, see Sec.V-A.

D. Further Work

Currently, there are two approaches for improving the stability analysis handled in this section. The first one describes an additional stability indicator based on Eigenvalues. The Eigenvalues estimated from the ARMAX models show the tendency to produce bifurcations. These bifurcation indicate sudden changes in stability for the system and therefore can be used as an additional and reliable stability indicator. To evaluate these bifurcations, it is mandatory to track each Eigenvalue over time. The second approach aims at the clusterization process. To get a better insight into the changes of clusters formed by the network, it can be useful to track the correlation information between the Eigenvalue progression for each node pair. If the correlation between a node pair undergoes a great change, it is an indication for a topological change within the network. This information can then be escalated from the bottom of the network (the nodes) to an information aggregator. As the nodes have the intelligence to see their own changes in cluster membership, there will only be data transmitted if it is required.

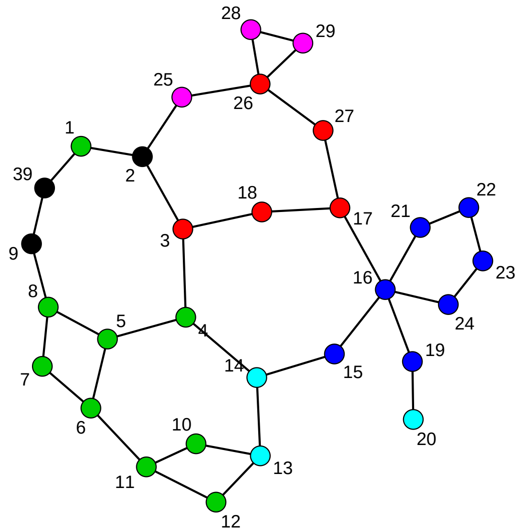

Fig. 6: Network clustering by ARMAX identification

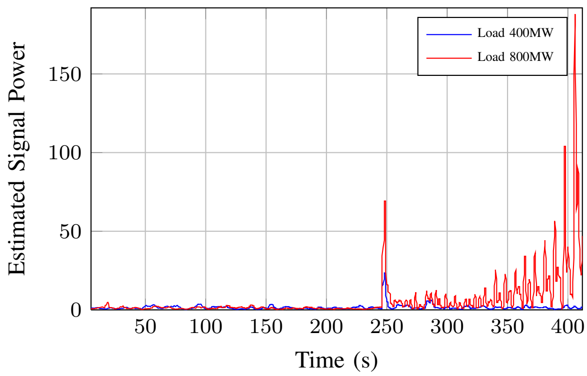

Fig. 7: ESPRIT based stability estimation at node 6

V. Clustering of Network Graph

In highly distributed electrical systems like the transcontinental Continental European system of the European Network of Transmission System Operators for Electricity a centralized stability assessment as described in Sec. IV or modelling of the network behaviour as presented in Sec. VI is usually not feasible. The necessary amount of communication to gather all data and the time for the analysis take often a not acceptable amount of time for the secure control of such a large electrical system. Hence it is necessary to distribute these calculations into local data centers and feed them only with the relevant information for their region. To allow such a distributed analysis, the electrical system has to be divided into regions. In Sec. IV-C we have presented a method to get such a structure out of a stability analysis of the system. For this method only the dynamic behaviour of the nodes is analysed. The network topology is not relevant.

In this section we use the network topology of the electrical system for the clustering. The topology is described by the nodal admittance matrix A. With the total number of nodes n and a_{ij} the admittances of all direct power lines between node i and j, the entries of the admittance matrix are defined by

$$A_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{k=1}^{n} a_{ij} & i = j \end{cases}$$

The admittance can be interpreted as a similarity measure between the nodes in the network graph. If there is no direct connection between two nodes, the corresponding entry in the matrix is zero. The result is a neighborhood matrix, which is singular. In order to make this matrix strict diagonal dominant and in consequence invertible we enlarge all diagonal elements by a factor of 1.1:

$$A'_{ij} = \begin{cases} A_{ij} & i \neq j \\ 1.1 \cdot A_{ij} & i = j \end{cases}$$

We get our final similarity matrix S by inversion of A':

$$S = A'^{-1}$$

Spectral clustering is a method for the clustering of an undirected graph with respect to the neighborhood structure. It is a collective term for clustering methods which are based on the eigenvalue structure of a matrix. Our method uses the algorithm by Ng, Jordan and Weiss [2].

First our similarity Matrix S is normalized using the diagonal matrix D with $D_{ii} = \sum_{j=1}^{n} S_{ij}$:

$$L = D^{-\frac{1}{2}} S D^{-\frac{1}{2}}$$

Then we calculate the eigenvalues and vectors and use the eigenvectors of the K largest eigenvalues in decreasing order, where K is the desired number of clusters, to build the columns of matrix E. The rows of E are standardized by

$$E^s_{ij} = \frac{E_{ij}}{\left(\sum_{j=1}^{K} E_{ij}^2\right)^{\frac{1}{2}}}.$$

In the end the matrix E^s is clustered by K-means. K-means is an iterative optimization method to find the cluster configuration $C = \bigcup_{j=1}^{K} C_j$, where C_j is the set of all network nodes in Cluster j, that minimizes the sum of squared euclidean distances $d_e(x_i, m_j)$ between the nodes x_i and their corresponding cluster center m_j

$$C^o = \arg\min_C \sum_{j=1}^{K} \sum_{m \in C_j} d_2(x_m, m_j)^2,$$

by alternating in every iteration t between the reassignment of all nodes to their new cluster

$$C_j^t = \left\{ x_i | d_2(x_i, m_j^t) \leq d_2(x_i, m_{j'}^t) \forall j' = 1, \ldots, K \right\}$$

and recalculating the new cluster centers

$$m_j^t = \frac{1}{|C_j^t|} \sum_{x_j \in C_j^t} x_j$$

until convergence. The algorithm is initialized by choosing K nodes of the electrical system at random for the initial cluster centers m_j^0.

The drawback of K-means is the risk of not finding the global optimum. Restarting the algorithm for a few times with different initial cluster centers reduces this risk.

An alternative to K-means is the usage of agglomerative hierarchical clustering with average linkage and the Euclidean distance. This method is for large numbers of nodes a lot slower than K-means, but always finds the global optimum. The results of both methods are equivalent if K-means finds the global optimum, hence they can be easily exchanged, whether the higher precision of hierarchical clustering or the higher speed of K-means is more important in the final application.

The final clustering of the NETS into $K = 4$ clusters is shown in Fig.8.

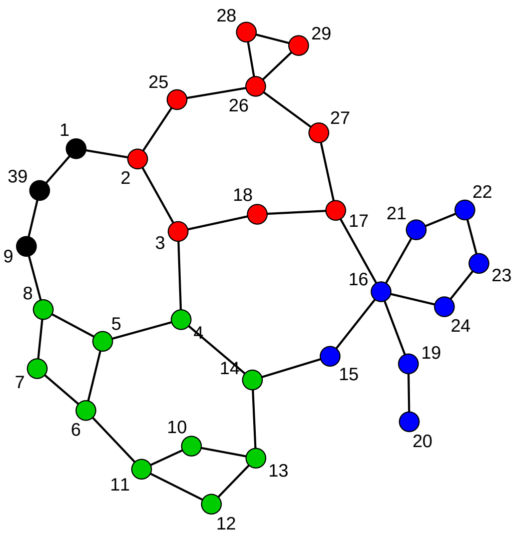

Fig. 8: Spectral Clustering of the New England Test System

One of the important questions for the future work is the definition of a quality criterion for the clustering in order to automatically find the optimal number of clusters. If such a quality criterion has been found, it may be necessary to include further static information into the similarity matrix, for example characteristics of the generators or even the results of a static power flow calculation.

A. Comparison of Clustering results

In Fig. 6 and Fig. 8 the clustering results based on the stability analysis and on the network topology are shown. The clustering based on the stability analysis favours a result with two more clusters ($K = 6$). It splits the red cluster in Fig. 8 into two and uses an additional cluster for the interconnecting nodes between the green and blue cluster. The remaining nodes show a large aggreement on their cluster assignment. Island nodes like the one with number 1 in Fig. 6 are a result of ignoring the network topology in the ARMAX based clustering process. A combination of the network topology and stability based clustering methods could be an interesting goal. As a result we could get a clustering of the electrical system, which is based on the the stability assessment by the ARMAX model and respects the network topology.

VI. Differential Equation Model

A simple example for a Differential Equation Model is illustrated by a mechanical harmonic oscillator, which consists of a mass and a spring. The mass is connected to a fixed point over the spring and oscillates permanently after a punctual displacement, because the spring generates a force proportional to how far it is stretched and acting in the opposite direction to the stretch. A more general harmonic oscillator additionally includes a damper and an external excitation, the so called driven harmonic oscillator. The damper connects the mass to the fixed point, like the spring, and generates a damping force that resists the motion and is proportional to the velocity of the mass. The excitation applies an external force to the mass, which is added to the spring and damping force. This changes the acceleration of the mass over Newtons second law and thereby the impact of the spring and damper. The mechanical driven harmonic oscillator describes a second order differential equation.

A. Identify the Energy System with harmonic oscillators

The driven harmonic oscillator was chosen to describe the differential equations, because it can be identified to the parts of an electromechanical energy system. A mass represents the inertia of a generator or a load, the spring its reactance. Over the damper it is possible to model the resistance, which extracts energy from the system. The excitation, in contrast, adds an external force to the electromechanical system, that can be identified with the turbine. Furthermore it is possible to identify the position of the mass with the magnitude and the velocity with the angle of the voltage. Another reason to choose the mechanical oscillator is the easy way to increase the complexity of the differential equation model without losing the overview. To model a generator with two loads in a row, connect one driven oscillator and two oscillators without an external force at its masses over pairs of springs and dampers. One spring and one damper connects the mass of the driven oscillator to the first non-driven mass, and another pair of a spring and a damper connects the second non-driven mass to the first non-driven mass. In this case, the pairs of springs and dampers between the harmonic oscillators can be identified with the power lines in an electrical transmission system.

B. Stochastic excitation

One requirement for the differential equation model was the potential to model the excitation in a continuous stochastic way. This type of excitation is necessary, because an initial analysis with a fixed sinusoidal driving force at 50 Hz shows, that the differential equation model swings in a steady state condition with a fast disappearing Low Frequency Oscillations. In addition, the analysis shows that a sinusoidal force at 50 Hz is inappropriate to excite the model, because the differential equation model is built to model Low Frequency Oscillations, not the power supply frequency of 50 Hz. This leads to an excitation force $F(t) = 0$, which describes a generator with no differences from the desired frequency. In this case the mass of the harmonic oscillator stands still at its starting point $x = 0$ with its starting velocity $v = 0$, because the modelled generator is in a steady state condition. A continuous force $F(t) = c$, with $c > 0$, models a punctual load change, described in Sec. III. In this case, the mass starts to oscillate, because the modelled generator was disturbed and tries to find the new steady state condition. In this model the external excitation force describes the distance from the desired supply frequency.

To model the excitation in a stochastic way the excitation force F is modelled by an $AR(1)$ process F_t with

$$F_t = c + \alpha \cdot F_{t-1} + \varepsilon_t \tag{4}$$

where c is a constant, $\alpha \in (0, 1)$ is a parameter and $\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$. The process starts at the desired value of $F_0 = 0$. To get an expected value $\mathrm{E}(F_t) = \mu_F = 0$ the constant c must be $c = \mu_F(1 - \alpha) = 0$. The variance $\mathrm{Var}(F_t) = \sigma_F^2$ of the stochastic process is $\sigma_F^2 = \frac{\sigma^2}{1-\alpha^2}$. With this equation the value α can be calculated from a desired variance and vice versa. The AR process rates large difference to the expected value higher, than small differences if $\alpha < 1$, and attract the process to the expected value. The process is weak stationary. This behaviour is close to reality, where a large difference to the desired frequency is worse than small differences.

C. Modelling

To model the voltage angle and magnitude of the simulated `NETS` in `PowerFactory`, a differential equation model with the corresponding characteristics was build. The model consists of 30 harmonic oscillators, one for every node, that are connected over 46 spring-damper pairs to model the power lines. A first analysis of this model, with a manual chosen set of parameters (masses, stiffness and damping constants, excitations) for the harmonic oscillators, shows a Low Frequency Oscillation in the position and velocity of the masses after the transient time interval. As in `PowerFactory` the position and velocity of the masses refer to a referee generator. The observed Low Frequency Oscillation disappears in the following 100 s, because the excitation in this first analysis is not appropriate for this model as described in Sec. VI-B. Currently, a method is developed to estimate the parameters (masses, stiffness and damping constants, excitation amplitudes) of the differential equation model to get a corresponding behaviour of the nodes in the model to the simulation data from `PowerFactory`. The method takes, on the one hand, the information of the existing `NETS` system. For instance, a short power line with a low impedance should be represented by a strong spring, because both forward information directly. On the other hand, the method should work with the simulated data, to ensure an adequate model. After the parameter estimation, an analysis of a simulation study will show whether the stochastic excitation keeps the Low Frequency Oscillation in the model. Furthermore the study will answer the question, if it is possible to excite a Low Frequency Oscillation when switching from a steady state differential equation model with a fixed excitation to a stochastic driving force.

D. Further Work

The future development will show whether a stochastic excitation is sufficient to model the Low Frequency Oscillation in the differential equation model. If this is not the case, it is possible to add stochastic processes to the masses of the loads and after that, to the springs and dampers in the model. In this approach the layout of the stochastic processes is important. If a desired Low Frequency Oscillation requires a stochastic process with a too large variance for the excitation frequency it is not possible to identify the process to real deviations in a power plant. An equivalent argumentation can be obtained to all parameters of the model, because the elements in the model identify parts of the real electromechanical energy system.

An important part of the energy system are the controllers for the frequency of power plants. These controllers adjust the frequency of a power plant that moves away from the desired frequency of 50 Hz. Due to the fact, that every power plant adjusts its frequency by a controller their interactions can cause in an oscillation of power plants generators against each other. This gives a further optimisation potential of the differential equation model by implementing a basic controller to the driven harmonic oscillators that identify the generators in the model. With a controller for the excitation frequency the model gets the ability to adjust its frequency depending on the current frequency of the corresponding moving mass.

From the estimated parameters of the differential equation model, an in-depth analysis of the harmonic oscillators, for example, its eigenfrequencies will be interesting. The analysis can figure out groups of oscillators, that have different parameters, but react similar to external excitations.

VII. Summary

At the beginning of the DFG research unit 1511 the descriptive analysis gives a basic understanding of the simulated data. To decode the dynamic processes in the electrical transmission system was important for the further work. Especially the understanding of the interacting behaviour between the generators helps to find an adequate model approach, the currently favoured differential equation model, which is illustrated by mechanical driven harmonic oscillators.

In the following research a model, with respect to the `New England Test System`, was build, that contains 30 connected driven harmonic oscillators. A first analysis of the differential equation model shows the Low Frequency Oscillations in this model. In the current development a stochastic process in the frequency of the external excitation is studied to analyse its potential to excite the Low Frequency Oscillations. If necessary, it is possible to implement stochastic processes in every part of the harmonic oscillators. A future concept is the implementation of basic controllers, close to controllers in a power plant, that adjusts the excitation frequency if the frequency of the corresponding mass differs from the desired frequency. This can optimise the differential equation model to show antithetic oscillations between generators.

Furthermore, the analysis of the network stability has been discussed. An ARMAX and an ESPRIT based approach has been proposed. In combination these two methods lead to a comprehensive view on the stability level of the observed system. Additionally it is possible to gain clustering information from the implmented ARMAX based modelling methods. These clustering results have been compared to impedance based statical clustering and show a consistency. It can be seen that the cluster centres are the same for both methods. The main differences are experienced to the bordering nodes of a cluster. With this information, it is possible to get a detailed stability estimation of the network which can assist in controlling the network and protection of endangered transmission lines.

References

[1] L.A. Fink, and T.A. Trygar, “Systems Engineering for Power: Emergency Operating State Control,” *Conference Proceedings*, Davos, Switzerland, 1979.

[2] A.Y. Ng, M.I. Jordan, and Y. Weiss, “On Spectral Clustering: Analysis and an Algorithm”, *Neural Information Processing Symposium 2001*, http://www.nips.cc/NIPS2001/papers/psgz/AA35.ps.gz.

[3] M.A. Pai, “Energy Function Analysis for Power System Stability,” McGraw Hill, 1994.

[4] G. Rogers, “Power System Oscillations,” Kluwer Academic Publishers, 2000.

[5] M. Lechtenberg and J. Götze, “Database Assisted Frequency Estimation,” th IEEE International Conference on Computer Science and Information,ICCSIT 2011.