Yu-Ping CHANG, München, Kristina REISS, München, Fou-Lai LIN, Taipei

Mathematical Proof in German and Taiwanese Textbooks: A Perspective on Geometry at the Lower Secondary School

1. Introduction

Results of international evaluation studies (e.g., PISA, TIMSS) on students' mathematical achievement and/or competence show a better performance of students from some East Asian countries like Korea, Singapore, or Taiwan compared to their counterparts in some European countries or the U.S. There are a number of studies discussing these differences. For example, lessons in Germany, Japan, and the U.S. were videotaped and then analyzed with respect to the teaching style. The results suggested important differences in the active involvement of students in the classroom and their consideration of the mathematical content (Klieme & Bos, 2000).

The presentation of mathematical content in the classroom is often reflected by its presentation in textbooks. However, there are hardly any studies which take into consideration how textbooks differ under an international perspective. Therefore, we took this as the guiding idea for an international research project. Our main concern was to compare German and Taiwanese curricular material with respect to their ways of presenting mathematical content. As research provides evidence that basic features of the curriculum, as content, organization, and sequencing, impact students' conception of proof (Chazan, 1993; Harel, 2001; Healy & Hoyles, 2000; Stylianides, 2007), we chose proof in geometry as a topic for the comparison. In the following, we will present first results of this study.

2. Theoretical Background

There are some challenges for students while dealing with mathematical proof and the process of proving. For example, Alibert's study (1988) points out that even university students who learned proof at school still treat the activity of proving as an extraneous task, not as a tool for thinking more deeply about mathematics. Moreover, when proving a statement for which the proof already exists or is intuitively obvious, it often leads students to the perception that proving is a goal oriented activity and not a process of discovery (Harel & Sowder, 1998; Schoenfeld, 1994; Wheeler, 1990).

A German research study identified three general difficulties of students in proof and logical argumentation: 1) lacking *knowledge of facts*; 2) deficient *methodological knowledge* on mathematical proof; 3) difficulties in devel-

oping and implementing a *proof strategy* (Reiss, Hellmich, & Thomas, 2002; Heinze, 2004). Lin and Cheng (2003) conducted a nation-wide investigation on Taiwanese students' development of mathematical argumentation competences. In this research, they found that Taiwanese students could organize their knowledge from elementary school in order to solve difficult and unknown/new questions, but they could not retrieve a simple principle to judge and explain why a property was true.

It is well-known that the process of proving is complex and identifying the statement/proposition is a main obstacle in fulfilling the task. Duval (1998; 2002; 2007) concluded that the meaning of a proposition is determined with respect to three dimensions: 1) A semantical dimension through its content; 2) a knowledge dimension through its epistemic value; 3) a logical dimension through its truth-values. From a cognitive perspective, investigating the relationship between the last two dimensions seems to be especially difficult but essential for understanding students' knowledge of mathematical proof.

3. Research Questions

Within the framework mentioned above, we address these research questions.

- What are the differences between German and Taiwanese curriculum materials (here we focus on textbooks) with respect to mathematical argumentation and proof?
- How is a mathematical statement presented in the curriculum, and particularly in textbooks?

4. Method and First Results

In this research project, we chose the Gymnasium track in the German state ("Bundesland") of Bavaria and the junior high school in Taiwan as representatives for the school systems. In particular, only the Gymnasium track in Bavaria introduces mathematical proof. In Taiwan, there is a single school track at the lower secondary level in which proof is regularly treated. We chose textbooks from grades 7 to 9 for this comparison as they included a sufficient number of topics to be taught in both countries. Six different textbooks, approved by their respective ministries of education (Chang et al., 2011), were selected from Germany and Taiwan, three from each country. We are aware of the fact that textbooks may differ substantially within a country, which is particularly true for German textbooks.

The analysis was based on a developing analytic framework composed of several variables including how to choose the analytic units and how to

discern the complicated content information of each unit, e.g., denotation, calculation, figuralization, decomposition, or mode of argumentation. We concentrated on two topics, namely the sum of interior angles of a triangle and the Pythagorean theorem.

The analysis showed important differences between the textbooks in the two countries. First, both topics were introduced in all textbooks but the methods differed between Germany and Taiwan (Chang et al., 2011). In the German arrangement, the sum of interior angles of a triangle started from introducing different angles, e.g., alternative interior angles or corresponding angles, continued with the axiom of parallels, and concluded with the generalization to the angle sum of a polygon. In the Taiwanese arrangement, the introduction started with figural operations, e.g.. paper folding in order to discover that three angles could be lined to a straight angle and accordingly an angle of 180°. The observation was then regarded as a fact and a mathematical result. The sum of exterior angles was discussed, the presentation finished as well with the generalization to the angle sum of a polygon. Regarding the Pythagorean theorem, German books used the properties of similarity to hypotenuse-leg theorem and leg-leg theorem, and then proved the Pythagorean theorem. The Pythagorean theorem in Taiwanese books is introduced with the help of examples. They show the relationship between side and area of figures in order to sustain the statement $c^2 = a^2 + b^2$ is true for all right-angled triangles. Second, we found some general differences between German and Taiwanese geometry content arrangement in textbooks: 1) German textbooks focused on building geometrical ideas hierarchically with hardly any repetitions in this introductory phase, while Taiwanese textbooks emphasized the transmission of mathematical concepts by elaboration or result-driven illustration; 2) there was no specific room for a proof in German textbooks. In Taiwanese textbooks, a formal (geometric) proof is always presented at the end of a chapter on geometry.

5. Summary and Discussion

Textual forms are diverse among textbooks from different publishers in Germany, however, they are similar among books from different publishers in Taiwan. Besides, from the analysis of introducing the statements of two topics, we found that argumentation as a mode of validation guided the presentation in German textbooks whereas taking conjectures as facts for generalization or application seemed to lead most texts in Taiwanese books.

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