UNIVERSITY OF DORTMUND

REIHE COMPUTATIONAL INTELLIGENCE

COLLABORATIVE RESEARCH CENTER 531

Design and Management of Complex Technical Processes and Systems by means of Computational Intelligence Methods

Using Gene Duplication and Gene Deletion in Evolution Strategies

Karlheinz Schmitt

No. CI-187/04

Technical Report ISSN 1433-3325 November 2004

Secretary of the SFB 531 · University of Dortmund · Dept. of Computer Science/XI
44221 Dortmund · Germany

This work is a product of the Collaborative Research Center 531, "Computational Intelligence," at the University of Dortmund and was printed with financial support of the Deutsche Forschungsgemeinschaft.

Using Gene Duplication and Gene Deletion in Evolution Strategies

Karlheinz Schmitt
Universtität Dortmund
D-44221 Dortmund, Germany
Karlheinz.Schmitt@udo.edu

Abstract- Self-adaptation is a powerfull mechanism in evolution strategies (ES), but it can fail. The reasons for the risk of failure are manyfold. As a consequence premature convergence or ending up in a local optimum in multimodal fitness landscapes can occur. In this article a new approach controlling the process of adapation is proposed. This approach combines the old ideas of gene deletion and gene duplication with the self-adaptation mechanism of the ES. In order to demonstrate the practicability of the new approach several multimodal test functions are used. Methods from statistical design of experiments and regression tree methods are used to improve the performance of a specific heuristic-problem combination.

1 Introduction

In modern synthesis of evolutionary theory, gene duplication emerged as a major force. In particular, redundant gene loci created by gene duplication are permitted to accumulate formerly forbidden mutations and emerge then as additional gene loci with new functions. It is hardly surprising that these major effect is not unconsidered in the design of evolutionary algorithms. The first evolution strategy (ES) using operators like gene duplication and gene deletion can be found in Schwefel [Sch68]. Here the optimization of a nozzle for a two-phase flow leads to surprising good results. A good overview of existing approaches dealing with variable-length representations can be found in [Sch97, Bur98]. Unfortunately, most of the approaches are extremely application-oriented. Perhaps the most important reason for the application oriented design is the restrictive fixed-length, fixed-position representation of the solutions that are used in many search heuristics. Based on this fixed representation, the introduction of duplication and deletion leads to several problems. For instance, the role of positions in a fixed-length solution is destroyed. In order to design genetic operators which are able to generate interpretable solutions, the assignment problem of finding the locus of corresponding genes has to be solved. In most cases those solutions lead to extremely application oriented solutions [Har92, GKD89].

The main focus of this article lies on a more application independent approach. Starting from the idea of introducing gene duplication and gene deletion into ES, additional genetic operators varying the number of used endogenous strategy parameters are introduced to generate a satisfactory self-adaptation in various fitness landscapes. Experiences gained in the last four decades show that self-adaptation is a powerfull mechanism, but it can fail. The reasons for the risk of failure are manyfold. As a consequence premature convergence or ending up in a local optimum in multimodal fitness landscapes can occur. Therefore, various countermeasures should be found in literature [Rec94, Her92, Tri97], but the main countermeasure in nature is gene duplication.

The rest of this article is organized as follows: in Section 2, an overview of the basic principles of the proposed search heuristic is given, followed by a brief description of the implementation details. In Section 3, a stastistical methodology to set up the experiments in an efficient manner is discussed. Experimental results are presented and discussed in Section 4. Finally, Section 5 concludes this article with a summary of the insigths and with directions for further research.

2 Evolution Strategy

This section presents the main aspects of a multimembered evolution strategy (ES), since this was needed for further discussion. For a comprehensive introduction the reader is referred to [BS02, Sch95].

In principle, existing parameters in evolution strategies can be distinguished between exogenous and endogenous parameters. Exogenous parameters like μ (parent population size) or λ (member of descendants) which are kept constant during the optimization run, are a characteristic of most of the modern search heuristics. Endogenous parameters are a pecularity of ES: they are used to control the ability of self-adaptation in ES during the run.

The adaptation of the endogenous parameters - the so called strategy parameters - depends on various adjustments. First of all, the strategy parameters are closely coupled with the object parameters [BS02]. Each individual has its own set of strategy parameters. Like the object parameters, the strategy parameters undergo recombination (together with the object parameters) and mutation and are used to control the mutation of the object parameters. Due to this mechanism, the optimizer can hope - and only hope - that an individual is able to learn the approximately optimal strategy parameters for the specific problem.

The realization of described self-adaptation mechanism above depends further on the kind and the number of strategy parameters to be adapted. In most cases only 1 or N

standard deviations are used. In the sphere function, i.e. only one standard deviation [Sch95] will do the work efficiently, in multimodal fitness landscapes it is favourable to use more than one standard deviation. The question of, how many standard deviations are necessary for a specific algorithm-problem combination or how many are necessary during of evolution, is still open.

Correlated mutations finalize the current self-adaptation mechanism in ES. For a deeper insight of correlated mutations the reader is referred to [Rud92]. The use of correlated mutation introduces $N(N-1)/2$ additional strategy parameters which have to be controlled, too. This may be the reason why correlated mutations are commonly not used. But in many real-world applications where the computational cost of optimization problems is determined mainly by the time-comsuming function evaluations, the computational effort for the optimization task will be relativize.

In order to obtain the best possible self-adaptation for the given problem the specification of the exogenous parameters is required. Table 1 shows the main exogenous parameters used in ES and their common default parameterizations.

Table 1: Exogenous parameters of an ES. Column 1 shows the usual symbols or the parameters. Column 3 holds commomly used values [Bäc96, Kur99].

Symbol	Description	Default Values
μ	number of parent individuals	15
λ	number of offspring individuals	100
$\sigma_i^{(0)}$	initial standard deviation	1.0
n_σ	number of standard deviations	problem-dimension
c_{τ_0}	progress coefficient	1
κ	Maximum age of an individual	$\{1;\infty\}$
β	correlation variablity	0.0873
ρ	Mixing number	2
R_x	Recombination type for the object variables	$r_{(d)}$ local discrete
R_σ	Recombination type for the standard deviations	$r_{(i)}$ local intermediate
R_α	Recombination type for the rotation angles	$r_{(-)}$ no recombination

Some of these values originate from investigations in the sixties [Sch68, Rec71] of the last century about only two artificial test functions (sphere and corridor). Other values - such as the progress coefficient c_{τ_0} - are theoretically very well analyzed [Bey95], but also only for a specific test function. Experimental investigations from [Kur99, Bäc96] have yielded to principle recommendations for the parameter settings i.e. for the type of recombination that must be choosen if the test function is unimodal or multimodal or for the initial standard deviation. But all of them state out that the use of these default values without reflection could be a mistake.

Nevertheless, after a first specification of these parameters, an evolution strategy is performed as follows: The initial parental population of size μ will be generated. A new offspring population is produced then by the rule of the $(\mu/\rho\kappa\lambda)$ - notation. From the parent population of size μ, ρ individuals are randomly choosen as parents for one child. Depending on the specified types of recombination, the recombination of the endogeneous and exogeneous parameters takes place. With respect to the recombination step, the mutation of the strategy parameters is done. The learning parameter τ determines the rate and precision of the self-adaptation of the standard deviations and β determines the adaptation of the rotation angles. After having a new offspring population of size λ, the selection operator is used to select the new parental population for the next iteration. $\kappa = 1$ referres to the well-known comma-selection scheme of an ES, and $\kappa = +\infty$ to the plus-selection scheme.

2.1 Implementation details

As mentioned above, the implementation of the self-adaptation mechanism depends on the kind and the number of strategy parameters. Given an individual $\vec{a} = (\vec{x}, \vec{\sigma}, \vec{\alpha})$, where $\vec{x}$ is the vector of objective variables, $\vec{\sigma}$ holds the set of standard deviations and $\vec{\alpha}$ the rotation angles. Each ES individual may include one up to $N(N+1)/2$ endogenous strategy parameters. For the case $1 < n_\sigma < N$ the standard deviations $\sigma_1, \ldots, \sigma_{N-1}$ are coupled with the corresponding object variables and σ_N is used for the remaining ones. The number of rotation angles n_α depends directly on n_σ [Sch95] or is explicit set to 0.

The new deletion and duplication operator work on the set of standard deviations (n_σ) only. The additional variation operators are defined as follows:

Duplication Operator: With a predefined duplication probalitity ($dup = 0.028$) a duplication may occur if $n_\sigma < N$. The duplicated standard deviation is added then at the end of $\vec{\sigma}$. The rate of gene duplication is taken from an investigation of [ML00]. Within their work an estimation of gene duplication rates of *Drosophila*, and *C. elegans* is given. This is a surprisingly high mutation rate compared to previous estimations that recommended a mutation rate of 0.1% per gene.

Deletion Operator: Vice versa a predefined deletion probability ($del = 0.028$) is used in order to delete the last standard deviation in n_σ if $n_\sigma > 1$ or not. The deletion of the last standard deviation is used because of their direct coupling with the rotation angles.

3 Experimental Environment

3.1 Test Functions

Just as for any other search heuristic, evolution strategies need to be assessed concerning their effectiveness for optimization purposes. To facilitate a reasonably fair comparison of search heuristics a number of artificial test functions are typically used. Well-known test suites of single-criteria parameter optimization problems are those of De Jong [DeJ75], Schwefel [Sch95] and Flaudas et el. [FPA+99]. These test suites serve well as an archive. In most cases only a selection was made taking into account that it is important to cover various topological characteristics of landscapes in order to test the heuristics concerning efficiency and effectiveness. In principle, Whitley et al. [WMRD95] and Bäck and Michalewicz [BM97] propose five basic properties as selection criterias for a fair test suite. The suite should contain unimodal functions in order to test the efficiency, they have to include high-dimensinal, multimodal functions and also constrained problems to simulate typical real-world applications. Due to the possibility of the presence of noise in industrial applications test functions with randomly pertubed objective values have to be included to.

The following test suite is composed disregarding the presence of noise and constraints in real-world applications.

Sphere function (F_1) [DeJ75]: This is an unimodal test function with a minimum at $\vec{x^*} = \vec{0}$, with $f(\vec{x^*}) = 0$. For a test of efficiency, this is the most used fitness function.

$$f(\vec{x}) = \sum_{i=1}^{n} x_i^2. \tag{1}$$

where

$$\text{Start Point: } x_i^0 = 10\, \forall i \in \{1, \ldots, n\}.$$

Double Sum (F_2) [Sch95]:

$$f(\vec{x}) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_i)^2. \tag{2}$$

where

$$\text{Start Point: } x_i^0 = 10\, \forall i \in \{1, \ldots, n\}.$$

Generalized Rastrigin function (F_3) [TZ89]: This is a multimodal function. The constants are give by $A = 10$ and $\omega = 2\pi$. Here the global optima is at $\vec{x^*} = \vec{0}$, with $f(\vec{x^*}) = 0$.

$$f(\vec{x}) = n \cdot A + \sum_{i=1}^{n} x_i^2 - A\cos(\omega x_i). \tag{3}$$

where

$$\text{Start Point: } x_i^0 = 4\, \forall i \in \{1, \ldots, n\}.$$

Generalized Ackley function (F_4) [BRS93]: This general extension of an originally two-dimensional test function is multimodal. Their constants are given by $a = 20$, $b = 0.2$ and $c = 2\pi$.

$$\begin{aligned} f(\vec{x}) &= -a \exp[-b(\tfrac{1}{n}\textstyle\sum_{i=1}^{n} x_i^2)^{1/2}] - \\ &\exp[\tfrac{1}{n}\textstyle\sum_{i=1}^{n} cos(cx_i)] + \\ &a + \exp(1) \cdot \exp(1). \end{aligned} \tag{4}$$

where

$$\text{Start Point: } x_i^0 = 25\, \forall i \in \{1, \ldots, n\}.$$

Fletcher and Powell (F_5) [RF63]: The constants a_{ij}, $b_{ij} = [-100, 100]$ and $\alpha_j \in [-\pi, \pi]$ are randomly choosen and specify the position of the local minima. The minimum is $f(\alpha) = 0$. The matrices A and B are taken from Bäck [Bäc96].

$$f(\vec{x}) = \sum_{i=1}^{n} (A_i - B_i)^2 \tag{5}$$

where

$$\begin{aligned} A_i &= \textstyle\sum_{j=1}^{n} (a_{ij} \sin \alpha_j + b_{ij} \cos \alpha_j) \\ B_i &= \textstyle\sum_{j=1}^{n} (a_{ij} \sin x_j + b_{ij} \cos x_j) \\ \text{Start Point: } x_i^0 &= 2.51818\, \forall i \in \{1, \ldots, n\}. \end{aligned}$$

3.2 Choice of Parameter Settings

As mentioned in Section 2 many of so called default parameterizations are proposed for the multimembered ES. In Table 1 usual default values are listed. But again, each heuristic-problem combination requires its specific parameterization. Therefore, as long as the success or failure of a heuristic depends on (nearly) optimal parameter settings, it is neccessary to look for the optimal set of parameters anew.

Manyfold methods are proposed to tackle this problem. A good overview can be found in [Kle87, BB03]. In this study, tree based methods, fractional factorial designs as well as classical regression analysis are used [BFOS84, Kle87, BBM04] in order to achive good parameter settings and to analyze the obtained results.

Fractional factorial designs are constructed by choosing a certain subset of all the possible 2^k combinations from a full factorial design. The advantage of reducing the computational effort is in opposite to the disadvantage of confounding. From now on it is possible that for several different effects the same algebraic expression is used, so it is impossible to differentiate between these two effects, these effects are called confounded. A good way to handle this problem is the concept of resolution. A 2_R^{k-p} fractional factorial design is of resolution R if no q-factor effect is confounded with another effect that has less than $R - q$ factors [GB78]. For a first screening phase, where only the

Table 2: Fractional factorial design 2^{11-7}_{III}. This design represents the starting design which is used for all test functions using correlated mutations with duplication and deletion probability > 0. If no correlated mutations, or duplication/deletion oparetors are used, the values were set to 0.

	A	**B**	**C**	**D**	**E= ABC**	**F= BCD**	**G= ACD**	**H= ABD**	**I= ABCD**	**J= AB**	**K= AC**
1	−	−	−	−	−	+	+	+	−	+	+
2	+	−	−	−	+	+	−	−	+	−	−
3	−	+	−	−	+	−	+	−	+	−	+
4	+	+	−	−	−	−	−	+	−	+	−
5	−	−	+	−	+	−	−	+	+	+	−
6	+	−	+	−	−	−	+	−	−	−	+
7	−	+	+	−	−	+	−	−	−	−	−
8	+	+	+	−	+	+	+	+	+	+	+
9	−	−	−	+	−	+	+	+	−	+	+
10	+	−	−	+	+	+	−	−	+	−	−
11	−	+	−	+	+	−	+	+	+	−	+
12	+	+	−	+	−	−	−	−	−	+	−
13	−	−	+	+	+	−	−	−	+	+	−
14	+	−	+	+	−	−	+	+	−	−	+
15	−	+	+	+	−	+	−	−	−	−	−
16	+	+	+	+	+	+	+	+	+	+	+

main effects but no interactions are of interest, a resolution three design is sufficient. It ensures only that no main effect is confounded with each other main effect. Table 2 shows a 2^{11-7}_{III} design, where the minus and the plus signs denote to the low and the high levels of the factors. The resulting design matrix for the high and the low levels of the evolution strategy is shown in Table 3.

3.3 Methods of Analysis

Classical methods of statistical analysis such as regression analysis can be extend by tree-based regression methods. A detailed example can be found in [BB03]. Here, it is shown that using regression trees the practitioner is able to screen out important parameter settings. One of their main advantages, besides their simplicity of interpretation, is that they do not require any assumptions about the underlying distribution of the responses.

The construction of a regression tree is a kind of variable selection similar to stepwise selection from classical ones, and rely on three components [BFOS84]:

- a set of questions upon which to base a split,
- splitting rules and goodness-of-split criteria for judging how good a split is and
- the generation of summary statistics for terminal nodes.

In principle, a set of questions of the form

$$\text{Is} \quad X \quad \leq d? \tag{6}$$

Table 3: Corresponding 2^{11-7}_{III} fractional factorial design for the choosen ES parameterization.

μ	λ	$N\sigma$	κ	ρ	dup	del	σ^0_{init}	R_x	R_σ	R_α
10	60	1	1	2	0.001	0.001	0.15	d	d	d
20	60	1	1	20	0.001	0.028	3.0	i	i	i
10	120	1	1	10	0.028	0.001	3.0	i	i	d
20	120	1	1	2	0.028	0.028	0.15	d	d	i
10	60	5	1	10	0.028	0.028	0.15	i	d	i
20	60	5	1	2	0.028	0.001	3.0	d	i	d
10	120	5	1	2	0.001	0.028	3.0	d	i	i
20	120	5	1	20	0.001	0.001	0.15	i	d	d
10	60	1	$+\infty$	2	0.028	0.028	3.0	i	d	d
20	60	1	$+\infty$	20	0.028	0.001	0.15	d	i	i
10	120	1	$+\infty$	10	0.001	0.028	0.15	d	i	d
20	120	1	$+\infty$	2	0.001	0.001	3.0	i	d	i
10	60	5	$+\infty$	10	0.001	0.028	3.0	d	d	i
20	60	5	$+\infty$	2	0.001	0.028	0.15	i	i	d
10	120	5	$+\infty$	2	0.028	0.001	0.15	i	i	i
20	120	5	$+\infty$	20	0.028	0.028	3.0	d	d	d

is given, where X is a variable and d is a constant. The response to such a question is binary (yes/no). Each response partitions the tree into a left and a right node. This recursive procedure will continue, if one node contains enough experimental observations for another split.

4 Experimental Results

The following experiments were performed to investigate the question if the new duplication and deletion operator improve the performance of an evolution strategy when optimizing multimodal fitness functions.

Therefore it is a common practice to compare the performance of the new algorithm including the additional operators with the standard implementation of the original algorithm. In order to ensure a relative fair comparison, both algorithms have to be tuned on the given heuristic-problem combination at first. This tuning step should be done with regression tree methods. In the following a detailed discussion on the 20-dimensional ackley function (F_4) was performed. The other test functions were analyzed in a similar manner.

4.1 A Simple Tuning Step

First experiments, based on the experimental design shown in Table 3, were performed for both types of algorithms. Due to simplification the experiments are devided into two main groups:

- Experiments without correlated mutations, and
- with correlated mutations.

For the first group the β, and the R_α values from Table 3 are set to 0. Each of the 16 parameter settings was repeated

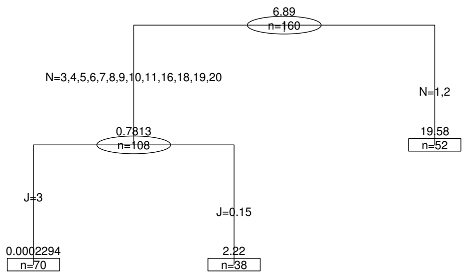


Figure 1: Pruned regression tree for the modified ES optimizing a 20-dimensional Ackley function. The first split partitions the 160 experiments in the root node in two groups of 108 and 52 events. The average fitness value in the left node reads 0.7831 and in the right node 19.58. The first split is performed by the number of used standard deviations (N).

Table 4: Corresponding ANOVA analysis. All observed factors have a significant contribution to the results.

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
J	1	3189.1	3189.1	455.752	$< 2.2e-16$
K	1	229.4	229.4	32.783	$5.843e-08$
N	14	9170.3	655.0	93.609	$< 2.2e-16$
Residuals	143	1000.6	7.0		

ten times, so that 160 observations are available for each algorithm in the group.

Figure 1 shows the pruned tree of the fitness values of function F_4 with correlated mutations using the ES with duplication and deletion operators.

The first split partitions the $N = 160$ observations into groups of 108 (left node) and 52 (right node) observations. The left group contains experimental runs with a great number of standard deviations $N = \{3, \ldots, 20\}$ and an average fitness value of 0.7831, and the right node contains all experimental runs, where the number of used standard deviations remains very small $N = \{1, 2\}$ with an average fitness value of 19.58. Following the tree down to the node with the smallest average fitness value $2.294E - 4$ the regression tree indicates that the initial value of the standard deviation (J) is significant for the success of the evolution runs. The corresponding classical analysis (Table 4) indicates that the initial state (J), the number of used standard deviations (N), and the duplication (K) probability are significant for the obtained results.

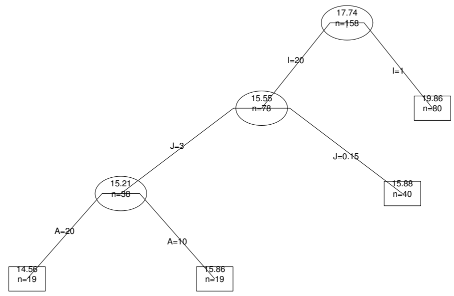


Figure 2: Pruned regression tree for the standard ES optimizing a 20-dimensional Ackley function. The first split partitions the 158 experiments in the root node into two groups of 78 and 80 events. The average fitness value in the left node reads 15.55 and in the right node 19.86. The first split is performed by the number of used standard deviations (I).

The first 160 results indicate that a high value for the initial standard deviation improves the performance. But this is not an unexpected result keeping in mind that the initialization of the first population of every algorithm takes place by choosing a single start point. As a consequent the initial population remains in a relativ small area of the fitness landscape. The extension of this area is defined by the initial standard deviation. The greater the value the greater the covered area. In multimodal fitness landscapes, this type of initialization is not a disdained factor. It may also be the reason, why in many ES the traditional initialization is changed to the initialization that is usually in genetic algorithms.

A relative high number of standard deviations (J) used during the evaluation runs seems to be the most significant effect for the obtained results. Now, it could be conjectured that using the greatest possible number of standard deviations in the experiments ($N_\sigma = N$) is the best choice for this parameter and therefore no additional variation operators are neccessary. Figure 2 shows the pruned regression tree for the standard ES.

Although the amount of the used standard deviations is even significant as in the former case, the heuristic is not able to reach the global attractor area (fitness = 15.21). Only in the initial phase of the optimization run, the self-adaptation process with a great number of standard deviations (left node) is able to steer the population through the multimodal fitness landscape. But a high number of standard deviations does not seem to be the reason in itself for

Table 5: Parameter setting for the discussed heuristic-problem combination without correlated mutations.

Parameter	modified	ES
μ	20	20
λ	120	60
κ	$+\infty$	1
R_x	intermediate	discrete
R_σ	discrete	intermediate
R_α	none	none
ρ	2	2
β	0	0
N_σ	2	20
σ_{init}	3.0	3.0
dup	0.001	0.0
del	0.001	0.0

the good results from Fig. 1. This will be discussed in detail in the next section.

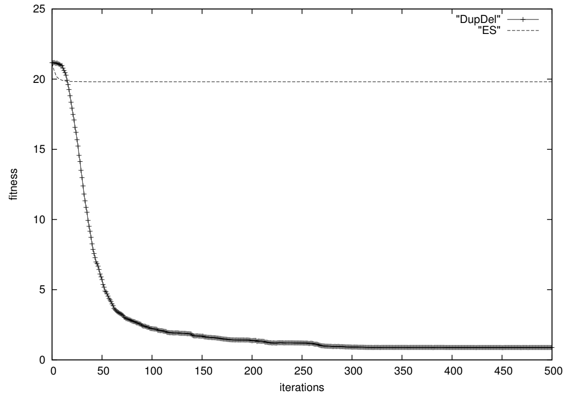
To begin with a fair comparison of both heuristics, regression tree methods as well as classical statistical methods are able to produce first statistically proved hints for nearly optimal parameter settings. For the given heuristic-problem combination the setting read:

4.2 Comparisons

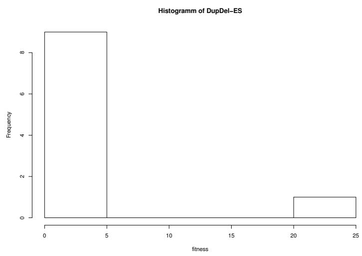
A comparison of the standard ES and the modified ES with gene deletion and duplication operators was performed in this section. Again, each heuristic-problem combination was going through the discussed simple tunig step before the comparison was performed.

Figure 3 shows that – on a 20-dimensional Ackley function - the ES working with duplication and deletion operators (DupDel) outperforms the standard ES (ES) in a significant manner. In Fig. 3(a) the arithmetic mean of ten independent runs without correlated mutations is depicted, respectively. From these ten independent experiments nine runs of (DupDel) are able to reach the global attractor area 3(b). Only one run shows similar results as in the standard ES. In this single run the self-adaptation fails just as in all runs of the standard ES (ES). In the later case, the ES is not able to guide the population through the mulimodal fitness landscape. All populations end up in a local optimum.

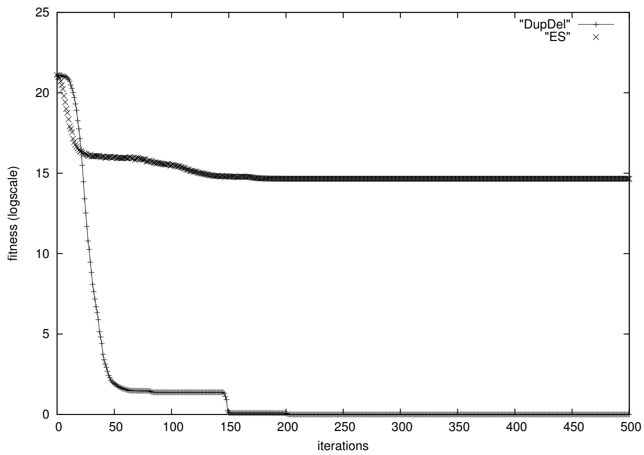
Quite different results can be observed, when using correlated mutations (see Figure 3(c)). The single freak value in the former experiments from the DupDel model can be completely avoided, when using correlated mutations. In addition, a better convergence rate can be observed (150 iterations in contrast to 300). In Figure 4 results for test functions F_2, F_3, and F_5 are presented. In functions F_2 and F_3 similar results as in the Ackley function can be shown. In function F_2 the convergence rate is on multiple regions superior to the standard ES. In case of function F_3 the mod-



(a)



(b)



(c)

Figure 3: Figure (a) shows the arithmetic mean of ten independent runs optimizing a 20-dimensional Ackley function with using correlated mutations. Figure (b) shows the corresponding histogramm plot. Figure (c) shows the arithmetic mean of ten independent runs optimizing a 20-dimensional Ackley function with correlated mutations.

Table 6: Comparison of both models optimizing the 20-dimensional sphere function. The number of performed experiments is set to 10.

Model	median	mean fitness	variance
F_1 ES$_{without}$	$7.357e{-}77$	$1.022e{-}76$	$4.72442e-153$
F_1 DupDel$_{without}$	$3.110e{-}77$	$2.913e{-}77$	$3.753419e{-}155$
F_1 ES$_{with}$	$6.227e{-}77$	$6.287e{-}77$	$9.179366e{-}155$
F_1 DupDel$_{with}$	$2.895e{-}77$	$3.009e{-}77$	$3.664180e{-}155$

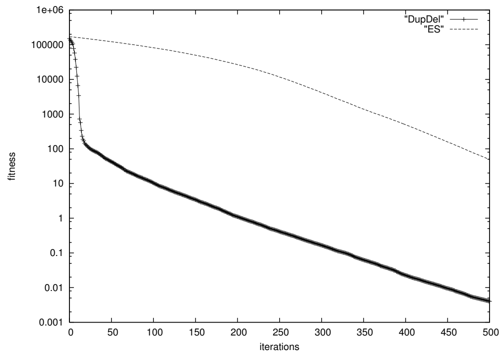
ified model is also able to achive the global attractor area in contrast to the other model. An exception could be found in function F_5. Here, no improvement can be observed.

4.3 Discussion

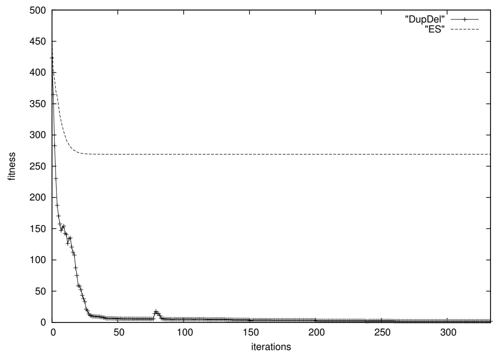
In the last section it was shown that it could be favourable to use additional variation operators mimicing gene deletion and gene duplication from nature in order to solve multimodal problems - but why? Looking at the duplication operator, a duplication take place with a probabilty of $dup = 0.028$ or 0.001. In case of $N_\sigma = 1$ before duplication, the single strategy variable is – depending on the number of iterations – more or less adapted to the local fitness landscape. Now, adding a second strategy variable the first endogenous variable controls the first objective variable only, the second controls all the rest. In the early stage of the optimization this will lead to relative great jumps in the fitness landscape. The more duplication and the greater the initial standard deviations the greater the fluctuations in the fitness values. In multimodal landscapes this effect could be high enough to guide a hole population out of a local optimum. In Figure 5 an interaction plot between the duplication probability ($K = \{0.028, 0.001\}$) and the sizes of the inital standard deviation ($J = \{0.15, 3\}$) is shown. The greater the duplication probability and the greater the initial standard deviation the lower (better) the mean of the fitness (O). Therefore, the conjecture could be confirmed.

On the other side it must be also considered that this effect, which is favourable in the early stage of the evolution run, is counterproductive for the end, when the global attractor area is achieved. In many test functions – here for example F_4, and F_3 – when the global attractor area is achievd, the special case of a sphere function can be found. Therefore, both heuristics were set up on a 20-dimensional sphere function F_1. Table 6 shows the obtained results. An extremely high number of fitness evaluations ($Fit_{eval} = 200000$) was used in order to show the adaptation.

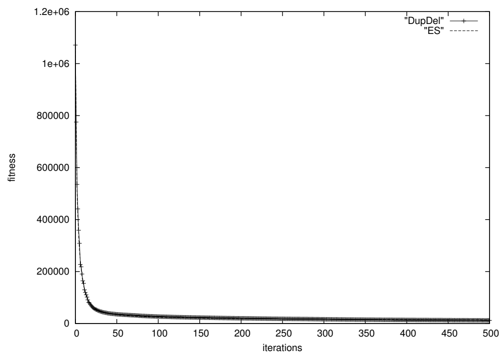
Despite of duplication and deletion, the modified ES is able to adapt the global optima in the same accuracy as the standard ES. The reasons could be found in the progress of the optimization run itself. During the run, the self-



(a)



(b)



(c)

Figure 4: Figure (a) shows the arithmetic mean of ten independent runs optimizing a 20-dimensional Achwefel-1.2 (F_2) function without correlated mutations. Figure (b) shows the obtained results for the generalized Rastrigin function (F_3) with correlated mutations, and finally Figure (c) shows the results of function F_5.

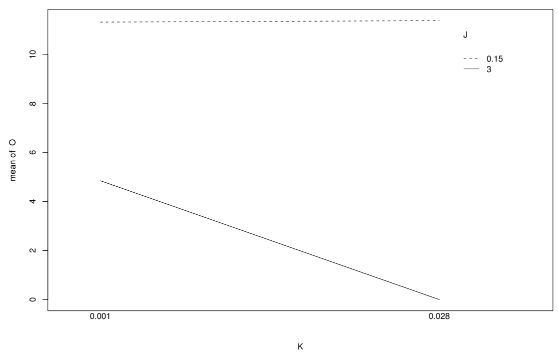


Figure 5: Plot of the means of the fitness values. The label on the x-axis represent different duplication probabilities ($K = \{0.0028, 0.001\}$)

.

adaptation process leads to smaller and smaller standard deviations, so if a duplication or deletion occurs, the noise will become smaller and smaller. Differences between both models seem to have vanished at the end of the evolution run.

5 Summary and Outlook

In this article additional variation operators mimicing gene duplication and gene deletion from nature were developed in order to improve the self-adaptation mechanism of evolution strategies. Self-adaptation is a powerful mechanism. But even in multimodal fitness landscapes it can fail. On four multimodal test functions it was shown that using the new operators the risk of failure of the self-adaptation mechanism can be reduced in a significant manner. This work will be extendend in the following way: To avoid the necessity of correlated mutations, the interaction between gene deletion and gene duplication operators has to be analyzed in detail.

Acknowledgments. Karlheinz Schmitt's research was supported by the DFG as a part of the collaborative research center 'Computational Intelligence' (SFB 531) in Dortmund.

Bibliography

[Bäc96] T. Bäck. *Evolutionary Algorithms in Theory and Practice*. Oxford University Press, New York, 1996.

[BB03] T. Bartz-Beielstein. Experimental analysis of evolution strategies – overview and comprehensive introduction. Interner Bericht des Sonderforschungsbereichs 531 *Computational Intelligence* CI–157/03, Universität Dortmund, November 2003.

[BBM04] T. Bartz-Beielstein and S. Markon. Tuning search algorithms for real-world applications: A regression tree based approach. Interner Bericht des Sonderforschungsbereichs 531 *Computational Intelligence* CI–172/04, Universität Dortmund, März 2004. (im Druck).

[Bey95] H.-G. Beyer. Toward a Theory of Evolution Strategies: On the Benefit of Sex – the $(\mu/\mu, \lambda)$-Theory. *Evolutionary Computation*, 3(1):81–111, 1995.

[BFOS84] L. Breimann, J. Friedman, R. Ohlsen, and C.J. Stone. *Classification and Regression Trees*. Chapman and Hall, New York, 1984.

[BM97] T. Bäck and Z. Michalewicz. Test landscapes. In *Handbook of Evolutionary Computation*, pages B2.7:14 – B2.7.20. Oxford University Press, 1997.

[BRS93] T. Bäck, G. Rudolph, and H.-P. Schwefel. Evolutionary programming and evolution strategies: similarities and differences. In D. B. Fogel and W. Atmar, editors, *Proc. Second Annual Conf. Evolutionary Programming (EP'93)*, pages 11–22, San Diego CA, 1993. Evolutionary Programming Society.

[BS02] H.-G. Beyer and H.-P. Schwefel. Evolution strategies – A comprehensive introduction. *Natural Computing*, 1(1):3–52, 2002.

[Bur98] D. S. Burke. Putting more genetics into genetic algorithms. *Evolutionary Computation*, 6(4):387–410, Winter 1998.

[DeJ75] K.A. DeJong. *An Analysis of the Behavior of a Class of Genetic Adaptive Systems*. PhD thesis, University of Michigan, 1975.

[FPA$^+$99] C.A. Flaudas, P.M. Pardalos, C.S. Adjiman, W.R. Esposito, Z.H. Gümüs, S.T. Harding, J.L. Klepeis, C.A. Meyer, and C. A. Schweiger. *Handbook of Test Problems in Local and Global Optimization*. Kluwer Academic Publishers, 1999.

[GB78] J.S. Hunter G.E.P. Box, W.G. Hunter. *Statistics for experimenters*. Wiley series in probability and mathematical statistics: Applied probability and statistics. Wiley, 1978.

[GKD89] D. E. Goldberg, B. Korb, and K. Deb. Messy genetic algorithms: Motivation, analysis, and first results. *Complex Systems*, 3(5):493–530, October 1989.

[Har92] I. Harvey. The saga cross: The mechanics of recombination for species with variable-length genotypes. In R. Männer & B. Manderick, editor, *Parallel Problem Solving from Nature*, volume 2, pages 269–281, 1992.

[Her92] M. Herdy. Reproductive isolation as strategy parameter in hierarchically organized evolution strategies. In *Parallel Problem Solving from Nature*, pages 207–217, Amsterdam, 1992. Elsevier.

[Kle87] J.P.C. Kleijnen. *Statistical Tools for Simulation Practioners*. Marcel Dekker, New York, 1987.

[Kur99] F. Kursawe. *Grundlegende empirische Untersuchungen der Parameter von Evolutionsstrategien - Metastrategien*. PhD thesis, University of Dortmund, 1999.

[ML00] J.S. Conery M. Lynch. The evolutionary fate and consequences of duplicate genes. *Science*, (290):1151–1155, 2000.

[Rec71] I. Rechenberg. *Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution*. Dissertation, Technical University of Berlin, Berlin, Germany, 1971.

[Rec94] I. Rechenberg. *Evolutionsstrategie '94*, volume 1 of *Werkstatt Bionik und Evolutionstechnik*. Frommann–Holzboog, Stuttgart, 1994.

[RF63] M.J.D. Powell R. Fletcher. A rapidly convergent descent method for minimization. *Comput. J.*, 6:163–168, 1963.

[Rud92] G. Rudolph. On correlated mutations in evolution strategies. In R. Männer and B. Manderick, editors, *Parallel Problem Solving from Nature – Proc. Second Conf. PPSN*, pages 105–114, Free University of Brussels, September 28–30, 1992. Elsevier, Amsterdam.

[Sch68] H. P. Schwefel. Experimentelle Optimierung einer Zweiphasendüse, Teil I. Bericht Nr. 35 zum Projekt MHD–Staustrahlrohr 11.034/68, AEG Forschungsinstitut, Berlin, Oktober 1968.

[Sch95] H. P. Schwefel. *Evolution and Optimum Seeking*. Sixth-Generation Computer Technology. Wiley Interscience, New York, 1995.

[Sch97] M. Schütz. Other operators: Gene duplication and deletion. In Th. Bäck, D. B. Fogel, and Z. Michalewicz, editors, *Handbook of Evolutionary Computation*, pages C3.4:8–15. Oxford University Press, New York, and Institute of Physics Publishing, Bristol, 1997. [Eingeladener Beitrag].

[Tri97] K. Trint. Strukturoptimierung mit geschachtelten Evolutionsstrategien. Master's thesis, TU Berlin, Fachgebiet Bionik und Evolutionstechnik, 1997.

[TZ89] A. Törn and A. Zilinskas. *Global Optimization*. Number 350 in Lecture Notes in Computer Science. Springer, Berlin, 1989.

[WMRD95] D. Whitley, K. Matthias, S. Rana, and J. Dzubera. Building better test functions. In L. Eshelman, editor, *Proc. 6th Int. Conf. on Genetic Algorithms*, pages 239–246, San Francisco, CA, 1995. Morgan Kaufmann.