A GENERAL APPROACH TO THE PLANNING OF A TRANSMISSION NETWORK

A THESIS

Presented to

The Faculty of the Division of Graduate Studies

By

Athanasios Panayotis Meliopoulos

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

in the School of Electrical Engineering

Georgia Institute of Technology

July, 1976

A GENERAL APPROACH TO THE PLANNING

OF A TRANSMISSION NETWORK

Approved:

Atif S. Debs, Chairman

Roger P. Webb

Edward W. Kamen

Leo J. Rindt

Date approved by Chairman August 2, 1976

Dedicated to

my parents,

Panayotis and Victoria

ACKNOWLEDGMENTS

The author wishes to gratefully thank his thesis advisor, Professor Atif S. Debs, who provided valuable counsel during the course of completion of this research.

The author also wishes to express deep thanks to Professor Roger P. Webb. His guidance, understanding, encouragement, and continued assistance throughout the author's graduate program at Georgia Tech were without equal.

Mr. Leo J. Rindt and Mr. John T. Day of the Westinghouse Electric Corporation deserve the author's special thanks for their contribution to the work of this thesis with a series of meetings to the purpose of exchanging ideas. Mr. Leo Rindt is further acknowledged for his assistance as a member of the reading committee.

The author is also grateful to Professor Edward W. Kamen of the EE Department, Marvin B. Sledd of the Math Department, John J. Jarvis of the ISyE Department, and H. Neal Nunnally of the EE Department for their careful reading of the thesis draft and their valuable comments.

Last, but not least, Mrs. Kathy Massett deserves the author's thanks for her patience and effort to successfully complete the typing of this thesis.

This research has been supported by the department of Advanced Systems Technology of the Westinghouse Electric Corporation. This

ACKNOWLEDGMENTS (CONCLUDED)

support is gratefully acknowledged.

Special permission was received from the Division of Graduate Studies to deviate from the manual requirements for figure captions in order that figures earlier developed could be used. This permission is gratefully acknowledged.

TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Concluded)

SUMMARY

This thesis addresses the problem of optimal expansion planning of an electric power transmission network over a finite planning interval. A procedure has been developed which yields an expansion policy, optimal with respect to a flexible economic criterion, and which incorporates realistic network constraints.

The problem is formulated as a discrete time deterministic optimal control problem.

A control at a time t is defined as the transmission facilities to be put in service at this time. To be determined is an optimal sequence of controls which will provide an admissible transmission system at any time during the planning period. An admissible transmission system is defined in terms of operational constraints of network security and reliability. These constraints require the solution to the problem of power flow on the network. The flow of power is modeled with the Kirchhoff's network laws, simplified at some degree.

The objective is the minimization of the economic cost of the expansion policy. This cost consists of investment cost, cost of energy and power losses, and financial charges. The definition of the economic cost is very general. The terminal value of the system at the end of the planning period is automatically accounted for.

The controls are assumed to be discrete because of standardization of transmission facilities. There is a tremendous number of controls

which can be applied for the expansion of a transmission system. However, because of the existing network coherency (Chapter III) in power transmission systems and by using upper bounds created by the optimizing algorithm, the number of admissible discrete controls is limited to a manageable one. An algorithm, consisting of a detection scheme, a feasibility condition, and an optimality condition, generates the admissible controls.

The optimization problem is solved by a non Linear Branch and Bound method. It is developed from an enumerative algorithm. Historically, enumerative algorithms have not been considered for this problem because of its size. However, the transmission network planning problem with finite planning period is a bounded problem. The non Linear Branch and Bound method, with the aid of the algorithm which generates the admissible controls, is able to compute the bounds at the beginning of the algorithm. Enumeration of the trajectories is then limited by these bounds.

The non Linear Branch and Bound has low storage requirements. In-core solution can be obtained for even large transmission systems. Cost escalation and construction lead times are handled without extra complications.

The general transmission planning problem can be solved by Dynamic Programming too. However, application of Dynamic Programming to the problem of this thesis encounters huge practical difficulties. A tremendous amount of data is required to be stored and retrieved during the computations. The important cases of cost escalation and lead time of the construction of transmission lines tend to increase the dimensionality of the problem. These practical difficulties are discussed in Chapter V. This chapter is basically independent from the rest of this thesis and may be skipped without loss of continuity.

It has been mentioned that the flow of power is modeled with the Kirchhoff's network laws, simplified at some degree. This simplification is not necessary. The exact Kirchhoff's laws can be used for the power flow model. Accuracy is increased at the expense of efficiency. Chapter VI is denoted in a discussion of the computational requirements with the exact power flow model. Again, this chapter is independent from the rest of this thesis and may be skipped without loss of continuity.

The planning procedure of this thesis has been implemented and tested. Two test systems have been used. A detailed evaluation of the performance of the algorithm is given in Chapter VII. The conclusions of this evaluation are: (a) The storage requirements of the algorithm are low. As a matter of fact, in-core solutions can be achieved for even large networks. (b) The present planning algorithm yields the global optimum with high level of confidence. (c) The execution time of the algorithm is reasonable.

The method of this thesis is very flexible. Operational controls in power systems can be included in the formulation of the transmission planning problem. The impact of just one operational control, the corrective rescheduling of the generator outputs, on the transmission planning problem has been investigated. It is concluded that corrective rescheduling yields considerable savings.

In summary, this thesis reports the successful application of an enumerative optimization process to a huge discrete optimization problem, the electric power transmission network planning over a finite period of time.

CHAPTER I

INTRODUCTION

General

The subject of this dissertation is the long range planning of a power transmission network. The specific problem considered is to determine the most economical expansion policy of an electric power transmission network over a finite planning period. The problem is formulated as a discrete time deterministic optimal control one. The solution of this problem is achieved by the non Linear Branch and Bound method. This optimization algorithm is developed from an enumerative procedure by taking advantage of specific properties of the problem.

The problem of planning a transmission network is a huge computational problem. The reasons are:

(a) The decisions the planner has to make are discrete because of standardization of transmission equipment.

(b) There is an enormous number of discrete alternative decisions for expanding a transmission network.

(c) The constraints to be satisfied by the transmission network are numerous and non linear.

Because of the size of the problem enumerative approaches have not even been considered for its solution. Enumerative approaches, however, possess great advantages:

(a) They provide the optimal solution for any class of problems.

(b) Non-linearities in the equations are easily handled.

(c) They provide flexibility in the mathematical modeling of the problem.

(d) The implementation of an enumerative algorithm is relatively simple.

Because of the forementioned advantages, an enumerative approach is attractive for problems which assume discrete solutions and which are conceptually complex. The transmission planning problem falls in this class of problems. In general, it can be formulated as an optimal control problem. The controls or alternative decisions to expand a transmission network are discrete and numerous. This thesis reports that information from the optimization method and constraints which have to be satisfied by the controls can be used in order to prove that the majority of the discrete controls are not qualified to be in the optimal trajectory. The controls which can not be disqualified are limited in number. These controls should enter the optimizing algorithm. An enumerative approach is practically feasible in this case. The non Linear Branch and Bound is developed from an enumerative approach. It takes advantage of the specific properties of the problem in order to disqualify the majority of the discrete controls. This function is analyzed in Chapters III and IV.

In the following sections the general objectives and requirements of a planning study for a transmission network will be stated in loose terms. The existing methods for the solution of the problem will be presented. Their capabilities and shortcomings of the most representative methods will be discussed. Conclusions will be drawn which lead from these methods to the method of this thesis.

Objectives and Requirements

In the 20th century the use of electricity has been spread in almost every human endeavor. This is so because it is easy and simple to convert electric energy in any other form of energy. Today the economic life of a community depends heavily on the availability of electric energy. Large power systems generate and distribute electric energy to the users.

Figure I.1 shows the basic structure of a power system. Generation plants convert energy of some type (thermal, hydro, nuclear) into electric energy. The electric energy is transmitted through network type systems to the consumers of electric energy.

Vertically, the power system is divided roughly into four layers:

(a) Distribution level

(b) Subtransmission level

(c) Transmission level

(d) Tie line system (which connects a number of power systems into a power pool).

Horizontally, each layer divides into a large number of systems which are isolated electrically (and usually geographically) from their neighboring systems of the same level, and they are electrically connected with each other only through the systems of higher vertical layers. The purpose of connecting the individual power systems by tie lines is to pool their facilities with the aim of mutual economy and for assisting each other during emergencies.

The demand for electric energy is increasing at a rate of seven to ten per cent annually. This trend is likely to continue for many years to come. Power systems will have to increase their generation and network capacity in order to meet the demand. In view of the extremely high investment costs of the power systems, it is imperative to thoroughly analyze the way of increasing the capacity of the power system in order to make maximum use of the available resources. The power system planner is facing a challenging and complex problem. It is, however, a worthwhile problem because even small improvements in the planning practices will mean large savings.

This thesis addresses itself to the problem of planning a transmission network.

The transmission network consists of transmission lines which carry the electric energy from the plants, where it is generated, to places close to consumption centers. There the electric energy is supplied to the subtransmission system. The transmission network

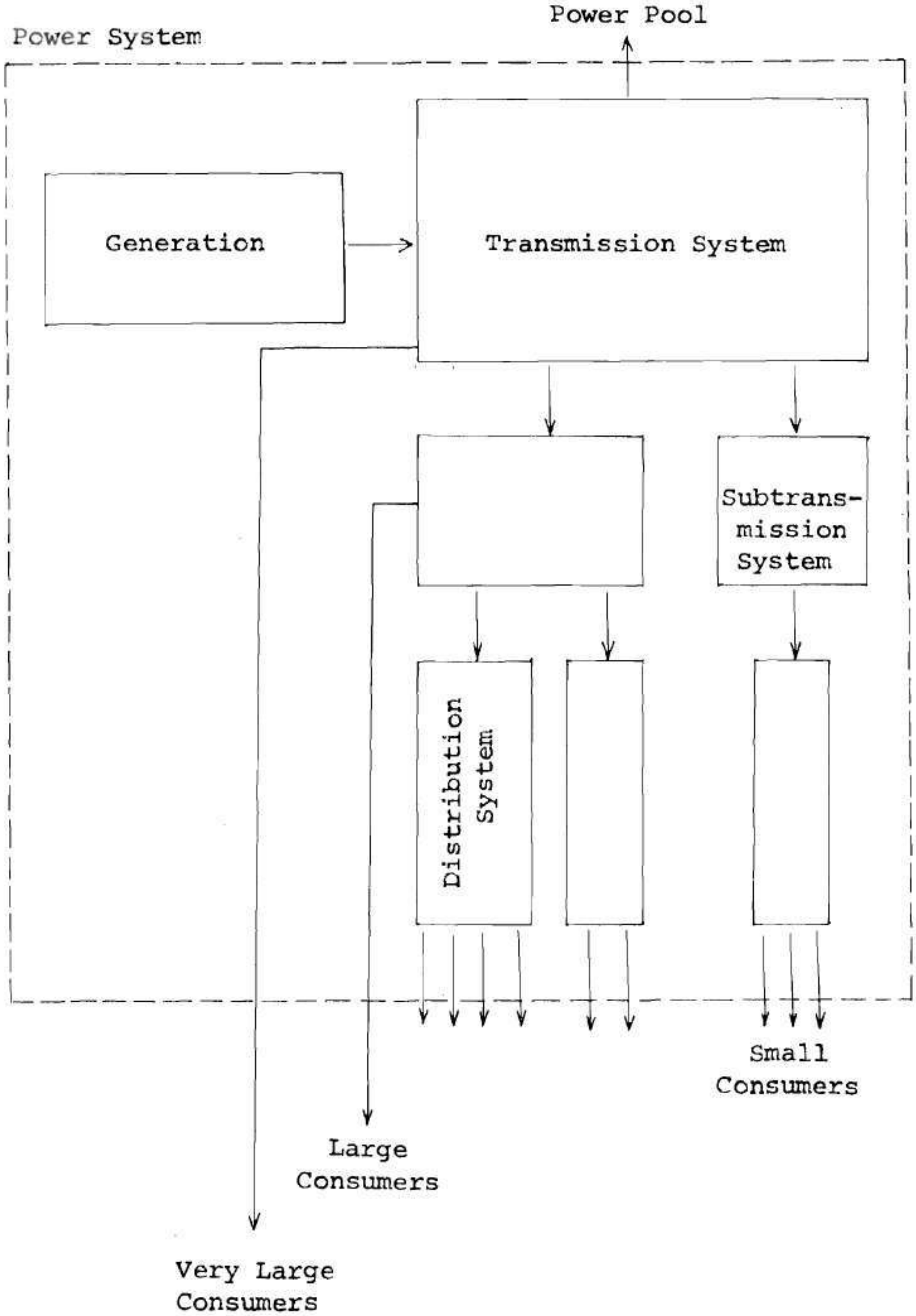


Figure I.1. Block Diagram Representation of a Power System.

carries bulk amounts of power at usually extra high voltages and above.

The transmission network should meet certain requirements which are discussed below.

Reliability

The system should be able to provide the customers with electric energy continuously. Any interruptions will cause inconvenience to the customers and will curtail revenues for the power company since there will be customers willing to buy electric energy but cannot do so. Furthermore, in the case of an interruption, restoration of service is always costly. Since the transmission network carries bulk amounts of energy, an unreliable network may cause frequent interruptions of service to a large number of consumers, a highly unfavorable performance to both the electric utility and the customers.

Security

A transmission network is subject to random events such as lightning, short circuits, accidents, etc. which may cause the loss of transmission lines. If such an event did happen and the system lost some of its facilities, would the remaining system be able to operate safely? It should be understood that the operation of the power system is a dynamic phenomenon and any sudden disturbance will cause oscillations. If the transmission network is not well designed, the oscillations might drive the system out of stability and possibly to a complete or partial blackout. This has actually happened in many systems.

Efficiency

The transmission of power should be done with minimum losses on the transmission network. Energy lost is money lost. Furthermore, a lossy transmission network calls for more installed generating capacity and therefore higher investment costs. It is, therefore, desirable to transmit power in an efficient way.

The demand for electric energy increases with time and in order to meet the reliability, security, and efficiency requirements, more capacity should be added to the system. Transmission capacity can be added only in discrete quantities. The characteristics of the lines to be added to the system have to be chosen among several standard types (e.g. 230 kV, 345 kV, 500 kV, etc.). This presents mathematical difficulties since we are dealing with variables which take discrete values.

Finally, the economics is of major concern. We are facing the problem of achieving the maximum result with minimum use of resources. Every available resource (labor, land, capital, etc.) can be translated into dollar figures. It is then appropriate to talk about cost. The major objective in long range planning is to minimize the cost over a long period. The investment level of individual decisions is not of primary interest. However, in the actual implementation of a timed series of decisions, the investment level might be a burden because of budgetary limitations.

Because of the extremely high investment costs of the transmission networks, it is imperative to have procedures for adding the right kind of equipment at the right time in the right location to achieve the desired level of quality of service at lowest cost over a long period. It is believed that the use of high speed electronic computers in the field of system planning should be directed towards optimization rather than mechanization of planning procedures.

In this work, the problem of planning a transmission network is formulated as an optimization one. The objective is to minimize a flexible economic criterion subject to security, reliability, and discrete circuit additions constraints.

State of the Art

The problem of choosing an optimal transmission network expansion plan is an extremely complex problem that has not yet been satisfactorily solved. It is difficult to quantify the costs and constraints of a transmission network. Since the early days of digital computers, however, attempts have been made to solve the problem. The first attempts amounted to a mechanized procedure: A performance standard is established and whenever the system does not meet the standards new constructions are decided upon until the system satisfies the criterion. Definition of performance standards is controversial.

In view of the extremely high investment costs of the transmission networks, it is believed that the use of high speed electronic computers in the field of system planning should be directed towards optimization rather than mechanization of planning procedures. It is common to express the power flow laws, reliability, security, and quality of service, as constraints. The objective is the minimization of the cost of the system.

The approaches for the solution of the problem can be classified into two categories: static and dynamic. The static transmission planning problem seeks to design an optimal network which will accommodate the needs of a certain system at a target year without considering the time of construction of the network reinforcements. The dynamic approach seeks to determine an optimal sequence (in time) of network reinforcements which will prove the system sufficient to accommodate the dynamically growing needs of the system at every time.

The static approach tends to exaggerate the economic impact of the economy of scale on the system. By economy of scale we mean the fact that the acquisition cost per unit capacity of an installation decreases as the capacity of the installation increases.

With respect to the power flow laws, the methods can be divided into two categories:

1. Those which use a transportation model.
2. Those which use Kirchoff's laws to determine the power flow.

Transportation models fail to reproduce the actual flow of power on the network and therefore will be excluded from this discussion.

Available Methods

The combined costs method [10] formulates the problem as a linear program through use of simulation techniques. The solution is obtained by iterating between simulation and the linear programming problem.

The merit of the method is based on the fact that a similar model can be developed for the generation planning problem and the two problems can be concurrently solved. It is, however, impractical to incorporate reliability constraints or the effects of controls on the expansion of the system.

Another method [5] defines as economic cost the following performance index:

$$PI = \sum_{\ell=1}^{M} \left(\frac{W_\ell}{2n} \left(\frac{P_\ell}{P_\ell^d} \right)^{2n} + K_\ell y_\ell \right)$$

where:

M - number of rights of way in the network

W_ℓ - weight factor

P_ℓ - actual power flow through the lines on the right of way ℓ

P_ℓ^d - power transmission capability of the lines on right of way ℓ

K_ℓ - cost of one unit of susceptance on the right of way ℓ

y_ℓ - equivalent susceptance of the lines on the right of way ℓ

n - an externally defined integer number

which is to be minimized subject to DC-power flow constraints

$$P_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} y_{ij}(\theta_i - \theta_j), \; i = 1,2, \; . \; . \; . \; ,N$$

where:

N - number of nodes in the network

θ_i - phase angle of the voltage at node i and limits on the number of lines on each right of way.

$$y_\ell^{min} \le y_\ell \le y_\ell^{max}, \ \ell = 1,2, \ . \ . \ . \ ,M$$

The gradient ∇(PI) with respect to admittance y on the rights of way indicates the most effective rights of way in minimizing the performance index. Based on this indication, combinations of discrete line additions are considered and the optimal will be that combination which yields the smallest value of the performance index.

The overall approach is static and therefore unable to evaluate the economic impact of the economy of scale on the system. Another drawback of the method is the fact that the state of the system is evaluated in one single number which might prove the method highly deficient for certain situations.

Another method [13] linearizes the DC load flow equations around the operating point in order to define the problem as a linear program.

Minimize:

$$K = \sum_m K_m \Delta y_m$$

subject to:

$$-\overline{\psi}_k \le \psi_k + \sum_{m=1}^{M} \frac{\partial \psi_k}{\partial y_m} \Delta y_m \le \overline{\psi}_k, \ k = 1,2, \ . \ . \ . \ ,M$$

$$\Delta y_m \ge q_m, \ m = 1,2, \ . \ . \ . \ ,M$$

where:

K_m - is the cost of one unit of capacity on the right of way m

Δy_m - is the decision variable = transmission capacity on the right of way m

$\bar{\psi}_k$ - is the absolute maximum permissible phase angle difference on the right of way k

The above model is the point of departure for the method in reference 14.

For each right of way the optimal cost versus capacity curve is calculated subject to discrete line additions and space constraints. The result is a staircase function for each right of way.

The decision variables are X_{ij}:

$$X_{ij} = \begin{cases} 1 & \text{if line additions equal to the } j^{th} \text{ step of the } i^{th} \text{ right of way is made} \\ 0 & \text{otherwise} \end{cases}$$

Linearizing around the operating point one can define the problem to be:

Minimize:

$$Z = \sum_{i=1}^{M} \sum_{j=1}^{n(i)} C_{ij} X_{ij}$$

subject to:

$$\sum_{j=1}^{n(i)} X_{ij} \leq 1 \qquad i = 1,2, \ldots ,M$$

$$\sum_{i=1}^{M} \sum_{j=1}^{n(i)} A_{ijk} X_{ij} \geq b_k \qquad k = 1,2, \ldots ,p$$

where:

M - number of rights of way considered

$n(i)$ - number of discrete steps in the cost-capacity curve of the i^{th} right of way

p - number of overloads

C_{ij} - cost associated with the j^{th} step in the cost capacity curve of the i^{th} right of way

A_{ijk} - $(\partial\psi_k/\partial y_{ij})\Delta y_{ij}$

Δy_{ij} - admittance associated with the j^{th} step of the i^{th} right of way

b_k - the amount of the k^{th} overload

A branch and bound algorithm is employed for the solution of the above problem.

Each right of way is replaced with a number of decision variables therefore increasing the dimension of the problem. But the major drawbacks of this method and any other method which linearizes around the operating point, are as follows:

1. There are numerous operating points of the

system (contingencies, generation schedules, load levels, etc.) which are of interest to the planner. Linearization around these points will increase the number of constraints tremendously, and more important:

2. The derivatives of line flows (or phase angle difference) relative to admittance increments change drastically with even one line removed or added to the system. Therefore, these derivatives cannot be used quantitatively in the decision-making process.

Another method [15] uses discrete dynamic programming and a mathematical stochastic model of the alternative expansion plans to arrive at an answer which is optimal within a certain level of confidence.

A strategy $S_j = (a_1, a_2, \ldots, a_H)$ is an ordered set of numbers which completely defines an expansion plan through the years 1,2,3, . . . ,H. The idea is to confine the optimization algorithm to a subset of all possible strategies, called a neighborhood. A neighborhood is generated randomly, the optimization is carried out and then another neighborhood is selected and the procedure is repeated. The process is stopped when a heuristic criterion is met. Specifically, the objective function is:

$$\pi_t(a_1, a_2, \ldots, a_t) = - \left\{ \begin{array}{l} \text{total accumulated present worth} \\ \text{cost for given alternatives at} \\ \text{stages 1 through t} \end{array} \right\}$$

$C(a_t)$ = present worth cost of an alternative at stage t

$$P_t(a_1, a_2, \ldots, a_t) = \begin{cases} 1 & \text{if the plan satisfies the performance criteria} \\ \infty & \text{otherwise} \end{cases}$$

At each stage t the forward dynamic programming is used:

$$\pi_t(a_1, \ldots, a_{t-1}, a_t) = -C(a_t)$$

$$+ \max[\pi_{t-1}(a_1, \ldots, a_{t-1}) P_t(a_1, \ldots, a_{t-1}, a_t)]$$

$$a_{t-1} \in AN_{t-1}$$

$$\text{for all } a_t \in AN_t$$

where:

AN_t - the set of alternatives at stage t.

The imperfections of the method are:

1. No theory is provided for the construction of the neighborhoods.
2. It fails to recognize that some rights of way are ineffective in reinforcing the network.

3. The definition of the state of the system at a stage t does not admit the problem to an effective application of dynamic programming.

In applying dynamic programming, the formulation of the problem is very important. In reference 16, a judicious definition of the state of the system is introduced and the optimization is achieved by dynamic programming. The state of the system coincides with given standard designs of the transmission network. The method is exact if the designs of the optimal strategy are assumed to be included in the given standard designs. Therefore, further work is required in order to fill in this gap.

Conclusions

The available methods are suboptimal due to either considerable approximations of the model or omission of important problem constraints. These approximations or omissions jeopardize the validity of the results. Furthermore, none of the methods considers the possibility of alleviating contingencies by on-line control action instead of construction. With the ever increasing applications of on-line corrective controls in power systems, it is imperative to evaluate the impact of such practices in the area of planning.

It is apparent that a general formulation of the transmission network planning problem is needed. This formulation should be free of controversial approximations. Furthermore, it should be flexible in order to incorporate new practices in the area of power system operation such as the corrective controls. A formulation which meets the stated requirements is presented in the next chapter.

CHAPTER II

THE GENERAL TRANSMISSION PLANNING PROBLEM

Formulation

The general transmission planning problem considered in this work can be formulated as a discrete-time deterministic optimal control problem. The statement of this problem is as follows:

(i) A system described by the linear difference equation

$$x(k+1) = x(k) + u(k) \tag{I}$$

where:

x = state matrix, LxM dimensioned

M = number of rights of way

L = number of discrete circuit types

u = Control matrix, LxM dimensioned

k = Index of stage variable

Note: The entry a_{ij} of either matrix x or u equals the number of circuits type i existing on the right of way j.

(ii) A variational performance criterion

$$J = \sum_{k=o}^{N-1} \frac{1}{(1+r)^k} \{ \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+r)^{\lambda-k}} + \ell_2[x(k+1),k+1] \} \tag{II}$$

where:

$\ell_1(u(k))$ = investment cost plus interest of control u(k)

$\ell_2(x(k+1),k+1)$ = operational cost

r = interest rate (per stage)

(iii) Constraints

$$u \in U(x(k), k+1) \qquad \text{(III)}$$

$$x \in X(x(k-1), u(k-1), k) \qquad \text{(IV)}$$

where:

$U(x(k),k+1)$ = set of admissible controls at state x, stage k.

$X(x(k-1),u(k-1),k)$ = set of admissible states at stage k.

Notes:

1. The determination of the set of admissible controls at state x, stage k+1, $U(x(k),k+1)$, is a difficult problem by itself and it is presented in another chapter under the name "Automatic Generation of Alternatives."

2. The set of admissible states at stage k, $X(x(k-1),$ $u(k-1,k)$, can be generated in a straightforward manner from the set of admissible controls at state x, stage k-1, by using equation (I).

(iv) An initial state

$$x(0) = c \qquad \text{(V)}$$

Find:

The control sequence $u(0), u(1), \ldots, u(N-1)$ such that J in equation (II) is minimized, subject to the system equation (I) the constraint equations (III) and (IV) and the initial condition (V).

The defined problem with the relations I through V is the general statement of the long range transmission network planning. Because of its generality, many important aspects of the problem are hidden. For example, the actual constraints which determine the admissibility of a control are not explicitly spelled out. Therefore, it is expedient to undertake a thorough explanation of the presented formulae. The following sections are devoted to this task.

The State of the System

The system matrix is defined as follows: The entry x_{ij} of the matrix $x(k)$ equals the number of circuits type i ($1 \le i \le L$) existing on the right of way j ($1 \le j \le M$) during stage k.

The number of discrete circuit types for a typical transmission network is very small. The Georgia Power Company transmission network consists of three discrete types of circuits: (1) 115 kV, (2) 230 kV, and (3) 500 kV. For this network $L = 3$.

The system matrix provides information about the transmission facilities existing in the system and their topology. The expression "Base case configuration of the network" denotes the same information.

The Controls

The control matrix u is dimensionally identical to the state matrix x and it is defined in a similar way: The entry u_{ij} of the matrix $u(k)$ equals the number of circuits type i ($1 \le i \le L$) to be constructed on the right of way j ($1 \le j \le M$) during stage k ($0 \le k \le N-1$).

In practice the number of circuits under construction at a particular stage is very small. This means that the matrix u is highly sparse.

The controls (decisions to reinforce the transmission network) are assumed to be applied at discrete time intervals, i.e. at the beginning of a stage. This is quite desirable indeed in long-range planning of a transmission network because the aim of the decision maker is to make in advance relatively large investments to compensate for long range demand trends. Optimal real time adjustments are not the objective of long range planning. On the other hand, electric power demand exhibits daily, weekly, monthly, seasonal, and annual peaks. In most instances, the annual peak is considerably higher than the other peaks and most importantly it occurs in the same period of the year for example, July-August. If reinforcements of the network are necessary, they should be implemented and ready to operate before this period of the year. Therefore, it is realistic to assume that the length of a stage equals one year. Furthermore, the control $u(k)$ is assumed to denote the transmission facilities which are ready to operate at the beginning of year $k+1$ (stage $k+1$). If the transmission facilities described by the control matrix $u(k)$ require τ time to be constructed (τ = construction lead time) then the decision for implementing the control $u(k)$ should be taken at time

$$t = k+1-\tau \tag{1}$$

Of course if $t<0$, then the control $u(k)$ is inadmissible since it is not conceivable to make a decision prior to the present time. Equation (1) allows for fractional construction lead times.

The above discussion of the construction lead times is a simplification because the control u(k) may involve the construction of different transmission facilities with different construction lead times. The purpose of the discussion was to make clear that the present formulation allows for construction lead times. However, the objective of this long-range planning is not to analyze the decisions in real-time but rather to determine when more transmission facilities are needed, where should they be located and what should they be.

Performance Criterion

The performance of the system can be measured with the following variational performance criterion:

$$J = \sum_{k=o}^{N-1} \frac{1}{(1+r)^k} \{ \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+r)^{\lambda-k}} + \ell_2(x(k+1),k+1) \} \qquad (II)$$

In practice the above expression represents the "economic cost" of expanding and operating the transmission network throughout the planning period. It is necessary to point out that the "economic cost" can not be universally defined. The term "economic cost" means different things to different companies and it is rather dependent on the economic environment in which the activities of a particular company are placed.

In long-range planning the level of investment itself is not of primary interest but rather the overall cost of the system in a relatively remote future time. The problem becomes complex because of the economy of scale resulting from relatively large investments. Furthermore, an investment made now might have an economic impact on the system for a period longer than the planning period. It is imperative, therefore, to define a performance criterion which automatically satisfies all these requirements.

The performance is defined to be the sum of the investment costs and the operational cost both of them converted into present value.

Investment Cost. As it has been mentioned an investment made during the planning period might have economic effects on the system beyond the end of the planning period. This is always the case. A transmission line has an expected economic life of over 40 years while we are interested in planning periods 10 to 30 years.

To avoid problems of this nature, we make the following assumption: Suppose there is an infinite source of capital. We can borrow money from this source at any desired amount but in return we have to pay back the capital plus interest at an annual rate r. The first payment is due the year of the energization of the equipment and the rest of them one per annum for as many years as the expected economic life of the equipment is. All payments are equal.

The described assumption yields the following calculations. Let the implementation of a decision call for investing A_i at time x_i, $i = 1,2, \ldots ,n$. Further, energization of the equipment takes place in year k. Assume N_E is the expected economic life of the equipment. Then payments of level C are due at years ℓ, $\ell = k,k+1, \ldots ,k+N_E-1$. Of course the present worth value of the investment and the payments should be equal.

$$P.W.V. = \sum_{i=1}^{n} \frac{A_i}{(1+r)^{x_i}} = \sum_{\ell=k}^{k+N_E-1} \frac{C}{(1+r)^{\ell}}$$

and

$$C = \frac{\sum_{i=1}^{n} \frac{A_i}{(1+r)^{x_i}}}{\sum_{\ell=k}^{k+N_E-1} \frac{1}{(1+r)^{\ell}}} \qquad (2)$$

Note that x_i need not be an integer.

In our case it is pertinent to consider the decisions consistent from the following unit: Construct in year k a transmission line of type j on the right of way m. Then the amount C can be dependent on the indices k, j, and m.

$$C = C(k,j,m) \qquad (3)$$

The above model of the costs is very general and it can account for escalation of cost. This is so because of the index k: the same type of transmission line, j, on the same right of way, m, costs different if constructed at different times.

Note that investments are not limited in occuring in intervals of integer number of years. Only payments have to be made in intervals of integer number of years.

The above cost model is very flexible to incorporate trends and policies of particular companies. For example, if the retrieval of the invested amount of money is desired in a short period of time, then a shorter expected life in the computation of C(k,j,m) will reflect this policy. Or, if the money market is tight, then a higher interest rate will be appropriate. In any case, the computation of C(k,j,m) is a task to be defined by the administration of the particular company.

Assuming the values C(k,j,m) are given, the function $\ell_1(u(k))$ is of the following simple form.

$$\ell_1(u(k)) = \sum_{m=1}^{M} \sum_{j=1}^{L} C(k,j,m) \cdot u_{jm} \tag{4}$$

The above cost model automatically accounts for salvage values of the equipment at the end of the planning period. The proof follows.

Assume that energization of a transmission line, type j, on the right of way m, occurred in year k. Further assume that N is the number of years in the planning period. The cost of this transmission line over the planning period is:

$$\text{Cost} = \sum_{\ell=k}^{N} \frac{C(k,j,m)}{(1+r)^{\ell}} \tag{5}$$

On the other hand, the salvage value of this line at the end of the planning period is

$$\text{Salvage value} = \sum_{\ell=N+1}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell-N-1}} \tag{6}$$

Now the cost can be rewritten

$$\text{Cost} = \sum_{\ell=k}^{N} \frac{C(k,j,m)}{(1+r)^{\ell}} = \sum_{\ell=k}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell}} - \sum_{\ell=N+1}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell}}$$

$$= \sum_{\ell=k}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell}} - \sum_{\ell=N+1}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell-N-1}} \cdot \frac{1}{(1+r)^{N+1}}$$

It is easy to recognize the terms:

$$\sum_{\ell=N+L}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell-N-1}}$$

is the salvage value at the end of the planning period.

$$\sum_{\ell=k}^{k+N_E-1} \frac{C(k,j,m)}{(1+r)^{\ell}}$$

is the total investment cost referred at the start of the planning period.

Therefore,

Cost = Present worth value of total investment minus salvage value at the end of the planning period.

Operational Costs. Operational costs mainly stem from losses on the transmission network.

Let us discuss the general nature of the losses on the transmission network. At every instant t, the energy balance equation holds

$$P_{Gen} = P_{Load} + P_{Loss} \tag{7}$$

The load P_{Load} is an exogenous variable. The losses P_{Loss}, however, are functionally dependent on the network and the generation schedule. At every instant t, they have to be satisfied. In other words, in an interval Δt, the losses P_{Loss} call for the following: (1) An amount of energy equal $P_{Loss} \cdot \Delta t$ has to be generated. (2) In the interval t, t + Δt, we need to have generation excess with respect to the demand P_{Load} equal to P_{Loss}. This fact should be considered irrespectively from reserve requirements.

The above considerations make clear that operational cost can be split into two categories: (1) cost of energy losses (heat dissipation on the circuits), and (2) cost of installed generating capacity to compensate losses on the network.

These costs are directly associated with the transmission network.

Cost of Energy Losses. Energy losses in year k can be computed from the following integral

$$\begin{matrix}\text{Energy losses} \\ \text{in stage k}\end{matrix} = \int_{t=(k-1)T}^{t=kT} \left(\sum_{\ell=1}^{M} r_\ell(k,t) I_\ell^2(k,t)dt \right. \tag{8}$$

where $r_\ell(k,t)$ is the equivalent resistance of the circuits on the right of way ℓ, and $I_{\ell}(k,t)$ is the total current flowing through the circuits on the right of way ℓ.

In reality $r_{\ell}(k,t)$ is not constant throughout the stage k due mainly to outages and switching practices. Other reasons are construction of new circuits. Considering, however, our assumption that energization of new equipment takes place at the end or the beginning of a stage, we conclude that $r_{\ell}(k,t)$ is not affected by construction of new circuits within a stage. Also, outages last for a very short time and switching is applied only in special cases. Therefore, the equivalent resistance $r_{\ell}(k,t)$ of the circuits on the right of way ℓ is constant during stage k except for a small fraction of the duration of the stage. Therefore, we can write:

$$r_{\ell}(k,t) = r_{\ell}(k) \tag{9}$$

The above equation can be stated in another way: For purposes of computing the energy losses on a transmission network, the network configuration can be considered invariant throughout the duration of a stage.

The total current, $I_{\ell}(k,t)$, through the circuits on the right of way ℓ, is in reality a random process, which is in a functional relationship with three other random processes: (a) network configuration, $x(k)$; (b) power demand, $P_L(k,t)$; and (c) generation schedule, $P_G(k,t)$.

The network configuration can be considered constant during a stage for the same reasons the equivalent resistance $r_{\ell}(k,t)$ is considered constant during a stage. We can write

$$I_\ell(k,t) = f_\ell(x(k),P_L(k,t),P_G(k,t)) \tag{10}$$

In theory, by knowing the statistics of the vector random processes $P_L(k,t)$ and $P_G(k,t)$ and the functional f_ℓ, it is possible to determine the statistics of $I_\ell(k,t)$.

However, our approach is deterministic because of the fact that the main random process $P_L(k,t)$ can be predicted with great accuracy for a period of few future years. Therefore,

$$I_\ell(k,t) = f_\ell(x(k),\overline{P}_L(k,t),\overline{P}_G(k,t)) \tag{11}$$

where an upper bar means expected value.

The integral (8) is then computed by simulating the operation of the system throughout the stage k and using the functional relationship (11).

It is, however, expedient to make use of the coefficients of losses defined as follows.

$$\varepsilon_\ell(k) = \frac{\int_{t=(k-1)T}^{kT} r_\ell(k) \cdot I_\ell^2(k,t)dt}{(I_\ell^{peak}(k))^2 \cdot T} \tag{12}$$

where $I_\ell^{peak}(k)$ is the current on the circuits of the right of way ℓ

during peak hour in year k.

The coefficients of losses are rather insensitive to small variations of the network configuration. This fact can be proven very important from the computational point of view.

In terms of the coefficients of losses, the total losses on the network during stage k are:

$$\begin{matrix}\text{Energy losses}\\ \text{(stage k)}\end{matrix} = Tp(k) \sum_{\ell=1}^{M} \varepsilon_{\ell}(k) \cdot r_{\ell}(k) \cdot (I_{\ell}^{peak}(k))^2 \tag{13}$$

where p(k) is the price of one unit of energy during stage k. The price p(k) of one unit of energy is considered constant throughout the duration of the stage, but it may differ from stage to stage due to escalation of fuel costs.

<u>Cost of Installed Generating Capacity to Compensate Losses on the Network</u>. This component of the cost stems from the fact that when there are losses on the system, the generating plants have not only to produce the energy losses but also to have adequate generating capacity in order not to curtail any revenue creating load. To clarify this point, recall the equation

$$P_{Gen} = P_{Load} + P_{Loss} \tag{7}$$

We can rewrite this equation in the following form

$$P_{GA} = P_{Load} + P_{Loss} + R \tag{14}$$

where P_{GA} = total generating capacity of the system, and R = reserve generating capacity.

Of course the reliability [28] of the system is a function of R:

$$\text{Reliability} = f(R) = f(P_{GA} - P_{Load} - P_{Loss}) \tag{15}$$

In this approach P_{GA} and P_{Load} are considered to be deterministically known. In any event, they are independent of the state of the transmission network. On the contrary, the variable P_{Loss} is dependent on the state of the transmission network. Therefore

$$\text{Reliability} = f(R) = f'(P_{Loss}) = f''(x(k))$$

It is trivial to assess [29] that the function f(R) is monotonically increasing and therefore the function f' is monotonically decreasing. In other words, a lossy system is less reliable than a less lossy system with the same generating capacity and topology.

The question at hand is what is the cost of losing reliability because of the losses. From equation (14) it is obvious that in order to maintain a specified generation reserve R over the demand P_{Load}, we need extra generating capacity of P_{Loss} MW. Therefore, it is expedient to consider as cost the annual investment cost plus interest of installing generating capacity equal to P_{Loss}. This is rather a simplification since one can install generating capacity only in big chunks but it is rather acceptable as a reliability penalty.

Let g be the annual investment cost plus interest to install one unit of generating capacity. It is computed in exactly the same way as C(k,j,m) and it is considered constant for simplicity. Then the cost of installed generating capacity to compensate losses on the network is

$$g\ P_{Loss} = g \cdot \sum_{\ell=1}^{M} r_{\ell}(k)(I_{\ell}^{peak}(k))^{2} \tag{16}$$

The above formula is based on the realistic assumption that the maximum losses on the system occur during peak load condition. In summary, the operational cost of the transmission network at stage k is

$$\ell_{2}(x(k),k) = p(k)T\sum_{\ell=1}^{M} \varepsilon_{\ell}(k)r_{\ell}(k)(I_{\ell}^{peak}(k))^{2} + g\sum_{\ell=1}^{M} r_{\ell}(k)(I_{\ell}^{peak}(k))^{2} \tag{17}$$

Constraints

The constraints are expressed with the relationship (III) and (IV) which are cited again

$$u \in U(x(k),k+1) \tag{III}$$

$$x \in X(x(k-1),u(k-1),k) \tag{IV}$$

where $U(x(k),k+1)$ is the set of admissible controls at state x, stage k+1, and $X(x(k-1),u(k-1),k)$ is the set of admissible states at stage k. It is appropriate in this point to clarify the following:

A. The set of admissible controls, $U(x(k),k+1)$, at state $x(k)$, stage k+1 is conditional, the condition being that at stage k the state of the system is $x(k)$. This condition has a tremendous impact on the size of the set $U(x(k),k+1)$. This problem is investigated in Chapters III and V.

B. The set of admissible states, $X(x(k-1),u(k-1),k)$ at stage k, is conditional too. The condition being that the state of the system at stage k-1 is $x(k-1)$. Same comments, as in A, can be applied about the size of the set X.

C. We need to define admissibility of a state only, since the equation of motion (I) is invertible, i.e. given the states of the system in stages k, and k+1, the control $u(k)$ is completely defined. Admissibility of a control $u(k)$ is then defined in terms of admissibility of a state: The control $u(k)$ is admissible if and only if the state of the system $x(k+1) = x(k) + u(k)$ is admissible. The state $x(k)$ is assumed to be admissible.

D. Given the set of admissible controls $U(x(k),k+1)$, the set $X(x(k),u(k),k+1)$ is uniquely determined.

From the above discussion, it is obvious that we are confronted with two problems.

1. We need a rational definition of an admissible state $x(k)$, in stage k.
2. We need to determine the set of admissible controls $U(x(k),k+1)$.

Problem (2) is very important and it is thoroughly analyzed in Chapter III.

Regarding problem (1), it should be noted that a lot of confusion exists in what is an acceptable transmission network. This is rather justified because the operation of a transmission network is very complex. The fact that a transmission network is subject to random events such as faults on equipment, or abrupt change of generating output/or load create transient phenomena which might lead to transmission line outages. Furthermore, operating practices and assisting media have evolved and are still evolving through research and today operating and controlling a transmission network is a rather sophisticated and complex task.

In the following, operational considerations will be taken into account in order to define admissibility in a rational way. In particular, two different definitions of an admissible state will be given. Both definitions are rather rational because they consider events with appreciable probability only according to the recommendations of the Federal Power Commission. The first definition does not take into account the possibility of alleviating contingencies by on-line control action while the second one does.

It should be clear that the formulation of the problem is not dependent on the definition of an admissible state. In any case the operating department of the particular company can specify what is acceptable and what is not.

Before the admissibility of a state is defined, it is expedient to discuss the power flow model for the transmission network.

Power Flow Model. The flow of power on a transmission network is an electromagnetic phenomenon which is commonly described by the Kirchhoff's laws. These laws lead to the so-called AC-load flow equations which are non-linear and their solution requires an iterative scheme. In a planning study a simplified model is desirable because of the computational size of the overall problem. This is the so-called DC-load flow model which is derived in reference 13.

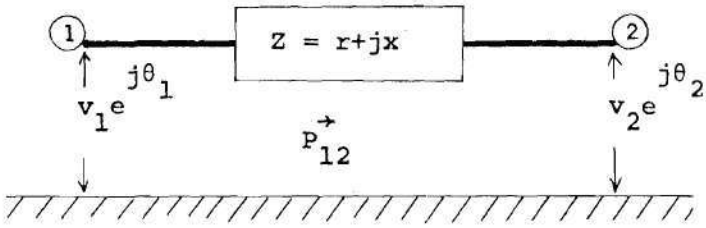
The simplified model is a reasonable approximation to the complete AC-load flow equations based on the following assumptions:

1. The voltages are assumed to be constant at any node due to the action of perfect regulators at each node.
2. The reactive part of a circuit's impedance is much higher than the active part.
3. The voltage phase angle difference across a circuit is relatively small.

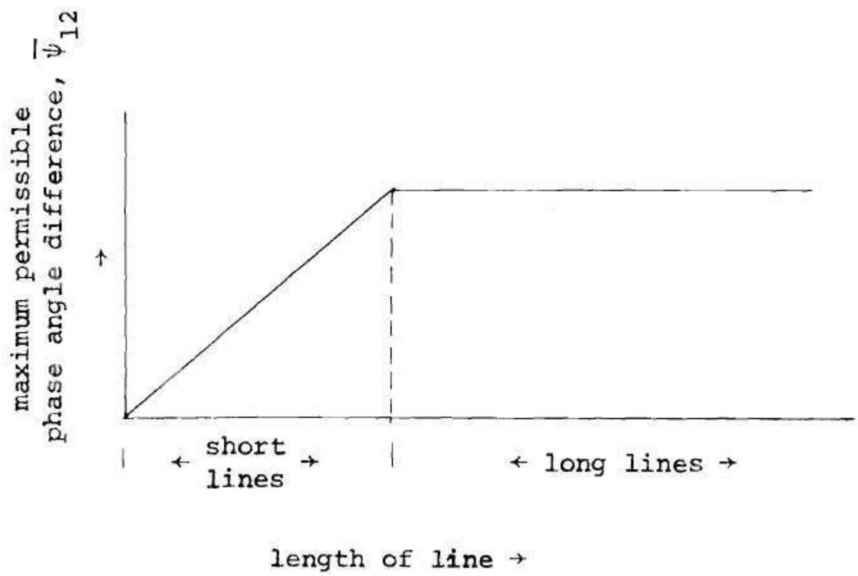
The above assumptions are very close to reality for most transmission systems.

The DC model of a transmission line is illustrated in Figure II.1. The flow of power in a circuit with impedance z and voltages $V_1^{ej\theta_1}$, $V_2^{ej\theta_2}$ at the terminals is given by

$$P_{12} = Y_{12}(\theta_1 - \theta_2) \tag{18}$$



$$P_{12} = \frac{x}{r^2+x^2}(\theta_1-\theta_2)$$



$$|\theta_1-\theta_2| \le \bar{\Psi}_{12}$$

Figure II.1. DC Model of an Electric Power Transmission Line.

where:

$$y_{12} = \frac{x}{r^2+x^2} \tag{19}$$

The quantity y_{12} is called the transmission "capacity" of the circuit 1,2. If similar equations are written for all circuits in the network, we obtain, in matrix form,

$$Y(k)\theta(k) = P_b(k) \tag{20}$$

where:

Y is the matrix of the transmission "capacities" y_{ij}

θ is the vector of voltage phase angles, and

P_b is the vector of generation minus load at each node.

More details for the DC model can be found in the references [13] and [21].

All developments in this thesis have been based on the DC power flow model. However, it will be shown that the exact AC power flow model can be used (Chapter VI) without major modifications. The penalty for using the accurate AC-model will be longer execution time.

<u>Loading Capabilities of Transmission Lines</u>. The loading capability of a transmission line is determined by either thermal limitations of the conductor materials or stability considerations. The limits as they are dictated by the above two reasons, should be calculated and the minimum will be the loading capability of the line.

In a practical system thermal limitations determine loading capability of short lines and stability determines loading limits for long lines. The borders between short and long lines depend on the system's layout.

Short Lines. For each conductor material there is a temperature limit above which the material loses its mechanical strength. By adopting a safety factor, the safe temperature limit is readily determined. The current carrying capability of the conductor is defined as the maximum current through the conductor which will not cause the temperature of the conductor to raise above the limit. The current carrying capability can be translated into power carrying capability. Finally, the power carrying capability can be translated into maximum permissible phase angle difference across the line.

Long Lines. For long lines, the stability of the system is the main factor for determining loading capability. Therefore, a stability study should determine the maximum permissible load on a long line. But this would be computationally infeasible for planning studies.

An approximate stability constraint [13] is defined as follows:

$$|\psi_\ell| \le \overline{\psi}_\ell$$

where:

ψ_ℓ is the actual phase angle difference across the transmission line ℓ, and

$\overline{\psi}_\ell$ is the maximum permissible phase angle difference across the transmission line ℓ.

In summary, the loading limit of a transmission line can be expressed with the maximum permissible phase angle difference across the line.

$$|\theta_i - \theta_j| \leq \overline{\psi}_{ij} \tag{21}$$

The maximum permissible phase angle difference $\overline{\psi}_{ij}$ across the line i,j is a function of the length of the line as it is illustrated in Figure II.1.

The Power Injections. The loading level of the transmission lines in a network depends on the power injections at the nodes of the network. It is expected that the power injections play an important role in the planning of transmission networks. Therefore, a discussion on this subject is pertinent.

Each node of a transmission network can be classified into three categories: (1) nodes connected to a generating plant, (2) nodes connected to a load or to the subtransmission system, and (3) nodes connected to a generating plant and a load or the subtransmission system.

In any case, if P_{Gi} is the output of the generation plant [$P_{Gi}=0$ if the node is in category 2] and P_{Li} is the load or the power injected to the subtransmission system, then the power injected to the node i is

$$P_i = P_{Gi} - P_{Li}$$

The power injections P_i, i=1, . . . ,n constitute the vector of injections P. If P_G is the vector of generated power at the nodes of the network, and P_L is the vector of the loads at the nodes of the network, then

$$P = P_G - P_L \tag{22}$$

P_L is basically a random vector. Econometric or forecasting models [35], [36] can predict the statistics of the vector P_L. The level of confidence in these models is high for short periods of time in the future. Furthermore, most of these models yield the expected value of the vector P_L and its standard deviation.

The standard deviation is an increasing function of time. The important fact is that for several years it is very, very small. This means that the load vector P_L can be predicted with high level of confidence. This is one reason to formulate the problem of planning a transmission network as a deterministic optimization one. In this thesis the load vector P_L is assumed to be an exogenous deterministic variable equal to the expected value of the random process P_L.

The vector P_G is also random but in a slightly different way. Forced outages of generating units may occur any time. It is common practice to determine the vector P_G by economically dispatching the electric power demand among the available generating units. Therefore, if the available units are known, the vector P_G can be accurately determined for a given load level. If forced outages with appreciable probability are only to be considered, then for a given load level there will be as many discrete generation schedules P_G as the forced outages are.

$$P_G^{(\nu)}, \quad \nu = 1,2, \ . \ . \ . \ ,\mu$$

where:

$P_G^{(\nu)}$ is the generation schedule as determined by the economic dispatcher for the given load level and the ν^{th} forced outage.

μ is the number of forced outages whose probability to occur is not negligible.

The generation schedule, when all units are available, is denoted by $P_G^{(0)}$.

The vector of power injections P for a given load level will be

$$P^{(\nu)} = P_G^{(\nu)} - P_L, \quad \nu = 0,1,2, \ . \ . \ . \ ,\mu$$

The conclusion is that the vector of power injections which is a random process can be substituted by a small set of vector values for planning purposes. It is then necessary to determine the set of vectors which will put maximum stress on the transmission network over a given period of time. The time period should be one year because it has been assumed that additions of new facilities can occur only in the beginning or ending of a stage (year). For this purpose, it is assumed that the maximum stress on the transmission network will occur during the peak hour demand for the year under consideration. Then, the vectors of power injections will be

$$P^{(\nu)}(k) = P_G^{(\nu)}(k) - P_L(k), \quad \nu = 0,1, \ . \ . \ . \ ,\mu \tag{23}$$

where $P_L(k)$ is the vector of electric power demand during peak hour at stage k, and μ is the number of unit outages with appreciable probability to occur during peak hour.

In conclusion, the determination of the vector(s) of power injections to be considered for planning purposes takes engineering judgement and experience with the particular system. The formulation of the overall problem is very flexible in accepting any defined vector of power injections.

In the definitions of an admissible state only one vector of power injections is considered, namely $P^{(0)}(k)$.

The Generation Schedule. In this section a simplified procedure will be presented which determines the generation schedule given the load level and the available generating units.

We consider a time interval during which loads remain constant. We assume we know the set of the available thermal generating units, and the values of the real power outputs of the hydroelectric and nuclear units if any.

The lossless case of economic dispatch with quadratic cost functions is considered. Furthermore, since in a planning study it is not known a priori which units are on-line, it is necessary to couple the economic dispatch problem with the unit commitment problem. The statement of the combined problem is:

Minimize

$$z = \sum_j f_j(P_j) \tag{24}$$

subject to:

$$f_j(P_j) = a_j + b_j P_j + c_j P_j^2 \tag{25}$$

$$P_j^{min} \leq P_j \leq P_j^{max} \text{ or } P_j = 0 \tag{26}$$

$$\sum_j P_j = P_L \tag{27}$$

$$\alpha_1 c_1 + \alpha_2 c_2 \geq \beta \tag{28}$$

where $j = 1,2, \ldots, n_g$ are the available generation plants; P_j is the actual real power output of plant j; a_j, b_j, c_j are constants; and P_L is the total real power demand (a scalar).

Inequality (28) is a simplified constraint for the spinning reserve requirements and it is derived from the following observations:

1. A generation plant which is on-line can respond "immediately" at a demand within certain limits. The limits depend on the

output of the unit. In an emergency, for example, if a unit shuts down, the running units will be able to provide power

$$C_1 = \sum_i r_i(P_i) \tag{29}$$

where i is a running unit, and $r_i(P_i)$ is the response limit which depends on the output P_i. If the spinning reserve C_1 is adequate to accommodate the load, even if any unit shuts down, we shall say we have a "secure global spinning reserve."

2. Fast start units (gas turbines, hydro) can be brought on-line in a short notice (10 to 15 minutes). Therefore, if there are enough fast start units, it is possible to synchronize them in a short time to compensate generation deficiency (loss of a unit, unexpected load increase). If P_j^{max} is the capacity of the j^{th} fast start unit, then $C_2 = \sum_j P_j^{max}$ is the spinning reserve capacity of fast start units.

3. Conventional thermal units have long lead times to start, synchronize and carry load. This lead time can be reduced by maintaining the boiler in a banked state. Units in this condition are designated as hot reserve. Hot reserve can carry load in a notice of one hour approximately. Because of this long delay time, hot reserve is not considered in this simplified model of spinning reserve.

4. The risk level of finding the system short in generation is a function of the time delay of the spinning reserve capacity. [25-28] It can be approximated as a linear combination of the spinning reserve capacities C_1 and C_2.

$$\text{Risk level} = \alpha_1' C_1 + \alpha_2' C_2$$

where $\alpha_1' < \alpha_2'$ since the spinning reserve capacity C_1 has shorter delay time than C_2. Then, if we are given a tolerable risk, the spinning reserve requirement will be

$$\alpha_1' C_1 + \alpha_2' C_2 \leq \text{Specified Tolerable Risk (S.T.R.)}$$

Let $\alpha_1' = 1-\alpha_1$, and $\alpha_2' = 1-\alpha_2$. The constraint on the risk level will become

$$\alpha_1 C_1 + \alpha_2 C_2 \geq \beta \tag{28}$$

where

$$\alpha_1 > \alpha_2$$

$$\beta = C_1 + C_2 - \text{(S.T.R.)} = \text{a constant}$$

Inequality (28) is an approximate spinning reserve constraint.

The problem defined by (24), (25), (26), (27), and (28) can provide the expected generation schedule given the available generating plants and the load level. This problem, however, is a mixed optimization one and an exact solution will be tedious. A suboptimal method has been developed for the solution of the above problem. It is presented in Appendix C.

Security/Reliability Constraints. To check a given state $x(k)$ of the system at stage k, with regard to reliability of operation, a series of outage tests must be conducted and compared against some reliability criterion. For each of these tests, a certain combination of lines is temporarily removed from the system, and the phase angles are computed using the DC-load flow equations. The removed lines are restored before the procedure steps to the next test.

In this thesis only single outages are considered in which case the reliability criterion can be stated as: if one line, possibly the highest capacity line, is subjected to an outage, no other lines shall be overloaded resulting in their loss at any time of the year, including peak periods.

Therefore, to check a configuration, P outage tests must be conducted, where P is the number of lines in the configuration under consideration. In most instances, however, it is only necessary to conduct M single outages where M is the number of rights of way with circuits. Specifically, for each outage test the highest capacity line on the right of way m is removed and the DC-load flow equations are solved.

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P^{(\nu)}(k) \qquad (30)$$

The rest of the lines are checked for overloads

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq \bar{\bar{\psi}}_\ell(x(k),m) \qquad (31)$$

$$\ell = 1,2, \ldots ,M$$

The removed line is restored before the procedure steps to the next test. The procedure stops when:

1. At least one of the inequalities (31) is violated. In this case the configuration $x(k)$ is classified as inadmissible at stage k.
2. When all rights of way have been considered. In this case the state $x(k)$ is classified as secure and reliable at stage k.

In summary, the security/reliability constraints can be expressed as follows:

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P^{(\nu)}(k) \tag{30}$$

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq \overline{\psi}_\ell(x(k),m) \tag{31}$$

$$\nu = 0,1,2, \ldots ,\mu$$

$$m = 1,2, \ldots ,M$$

$$\ell = 1,2, \ldots ,M$$

where $Y^{(m)}(x(k))$ is the "capacity" matrix of the configuration $x(k)$ when the highest capacity line from the right of way m is removed; $P^{(\nu)}(k)$ is the vector of peak injections during stage k, unit outage ν; $\overline{\psi}_\ell(x(k),m)$ is the maximum permissible phase angle difference across the circuits on the right of way ℓ for the configuration $x(k)$ with the stated outage; and $\theta^{(m)}$ is the vector of the voltage phases for the above condition.

Definition of an Admissible State I. A state $x(k)$ of the transmission network is said to be admissible if and only if it satisfies the following set of relationships.

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P^{(0)}(k) = P_G^{(0)}(k) - P_L(k) \tag{32}$$

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq \overline{\psi}_\ell(x(k),m) \tag{31}$$

where:

$$m = 0,1,2, \ldots ,M$$

$$\ell = 1,2, \ldots ,M$$

$P_G^{(0)}(k)$ is determined by a generation scheduling algorithm for peak load conditions at stage k and all units available; $P_L(k)$ is the vector of the peak load at stage k; $Y^{(m)}(x(k))$ is the "capacity" matrix of the system when the highest capacity line from the right of way m is removed, when m=0, no line is removed; and $\overline{\psi}_\ell(x(k),m)$ is the maximum permissible phase angle difference across the right of way ℓ when the highest capacity line from the right of way m is removed.

Operational Controls

It has been mentioned that operating practices and assisting media have evolved and are still evolving. Power companies install control centers which are capable to predict vulnerable situations and take corrective action. Control centers improve the security and reliability of a given power system. In this case the security/reliability constraints [31] and [32] are very strict and will lead to a very conservative expansion plan for the transmission network. It is apparent that the operations performed by control centers will have an impact on planning practices.

Operational controls related to transmission networks can be of the following type:

(a) Changes in scheduled power output of some of the power plants.

(b) Changes in the scheduled exchange of power with the neighboring systems.

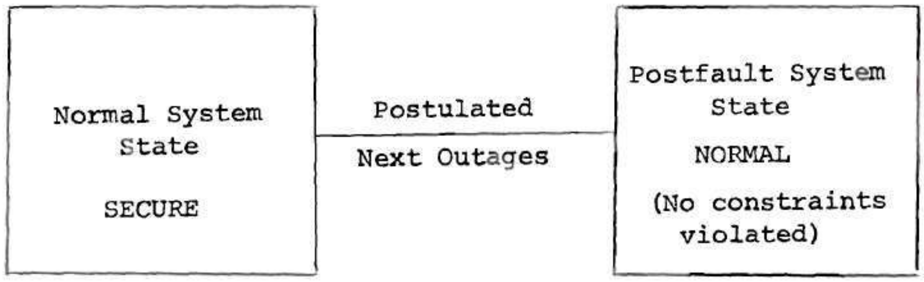
(c) Prearranged curtailment of some interruptible loads.

(d) Changes in the network configuration (switching).

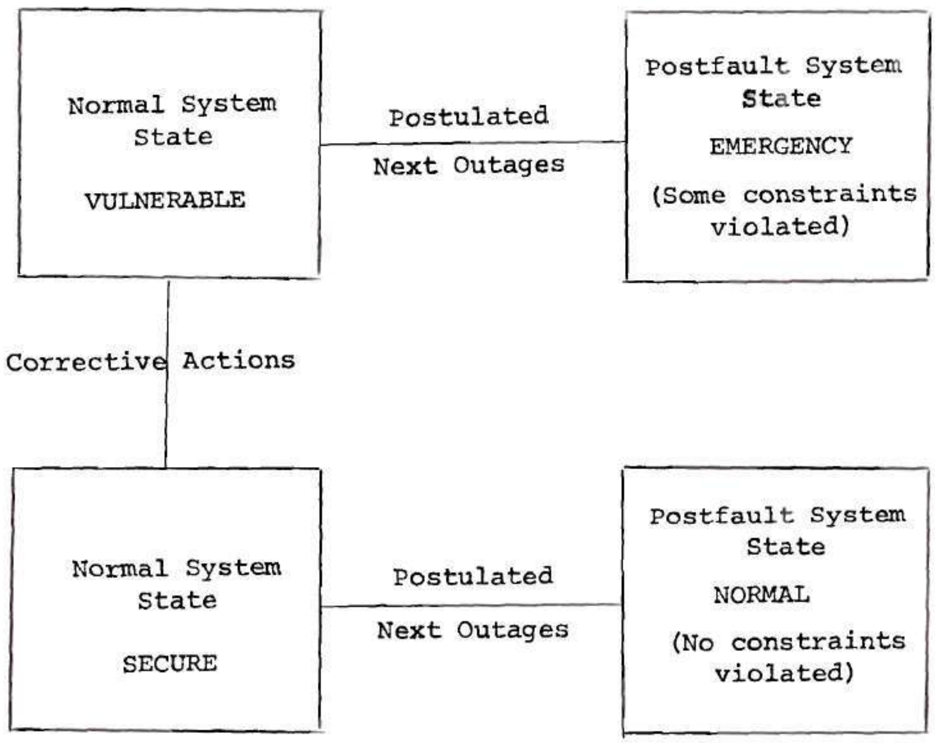
(e) Changes in control logic and protection philosophy.

In the following discussion only the first type of operational control is considered. In the literature it is referred to as corrective rescheduling or security dispatch. The basic idea is depicted in Figure II.2.

A state is secure if the postfault state of the system is normal in the sense that no constraints are violated. Otherwise, the state of



CASE 1: No Corrective Action Necessary.



CASE 2: Corrective Actions Required to Bring a Normal but Vulnerable System into a Secure Operating State.

Figure II.2. Application of Corrective Controls in Power Systems.

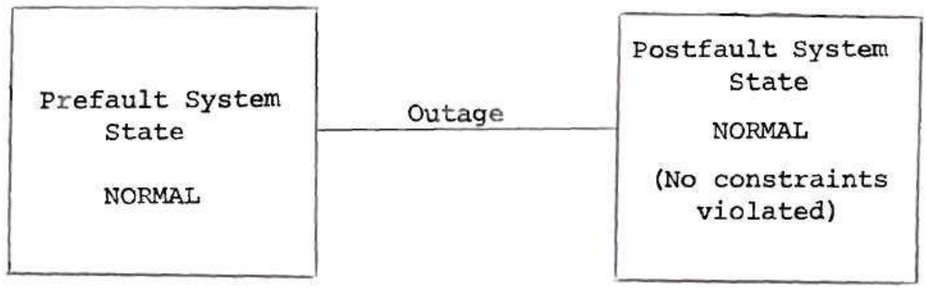
the system is vulnerable and corrective action is required to make this state secure.

From the operational point of view there exist a state of operation and there are several outages which have a probability to occur in the next hour or so. Assuming that an outage did occur and that the postfault state of the system does not satisfy the constraints, the question is: Can a new schedule of the generation be found with the least deviation from the present one and such that the state of the system will be normal under any of the above outages.

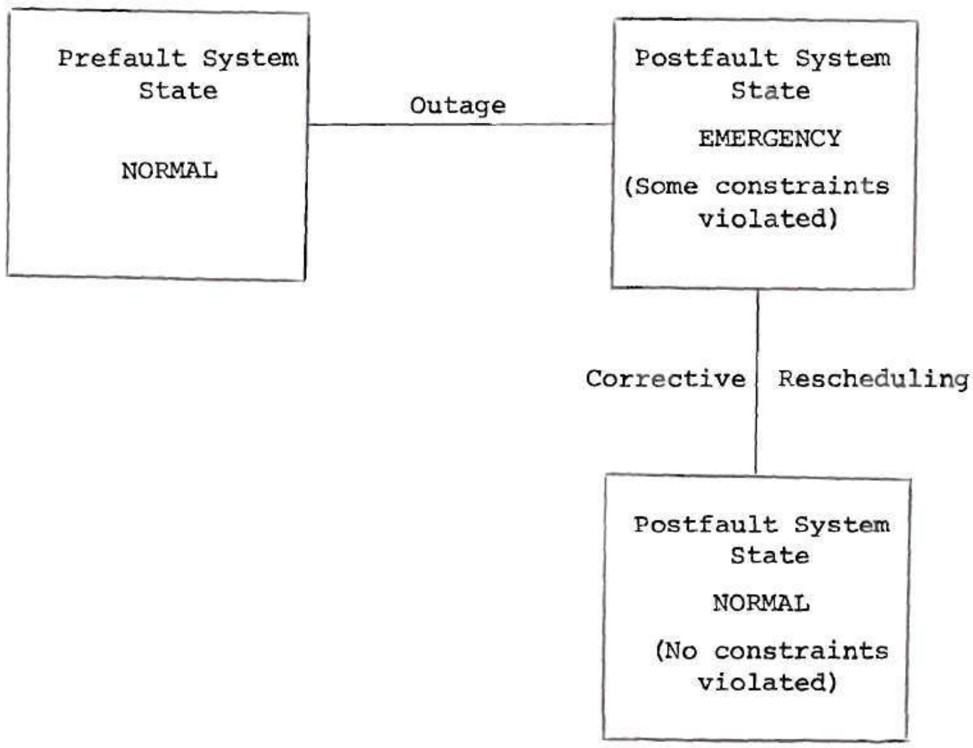
For planning purposes the problem can be simplified. The question is whether the system will be able to operate at a normal state (no constraint violations) under any postulated outages and any load level including peak level. Since a transmission line can withstand a small overload for a short period of time (the thermal time lag of a transmission line is about 15 minutes), the philosophy of approach may differ to the effect that instead of taking preventive action, it is possible to let the emergency state occur first and then take action. This is justifiable since the permissible limits for line currents are greater just after a trip than for a steady state. On the other hand, to reach a new normal steady state, 15 minutes are at our disposal, and we may change the generator outputs during this interval. The application of corrective rescheduling for planning purposes is depicted in Figure II.3.

"Soft" and "Hard" Constraints

It has been mentioned that the permissible limits for line currents are greater just after a trip than for a steady state. For



CASE 1: No Corrective Rescheduling is Necessary.



CASE 2: Corrective Rescheduling is Necessary to Bring the System From the Emergency State into a Normal One.

Figure II.3. Corrective Rescheduling Philosophy for Planning Purposes.

the DC-model we can state that the maximum permissible phase angle difference across a line is greater just after a trip than for a steady state. Therefore, there exist two discrete constraints: one for steady state which is called "soft" constraint, and one for states 10-15 minutes after a trip which we shall call "hard" constraints. The period 10-15 minutes corresponds to the thermal time lag of transmission lines.

$$|\psi| \leq \overline{\psi} \qquad \text{steady state} \qquad (33)$$

$$|\psi| \leq x_h\overline{\psi} \qquad \text{after a trip} \qquad (34)$$

where x_h has a value greater than one. There is an upper bound on the value of x_h which is determined by the settings of the protective devices.

Corrective Rescheduling

The application of corrective rescheduling in the planning algorithm is depicted in Figure II.3.

Specifically, the system is considered operating in a normal state (no "soft" constraints violated) and with the base case configuration. The vector of power injections is assumed known. The load is constant. The discussion will be confined to line outages only but a generalization to include generating unit outages will be obvious.

The prefault state of the system satisfies the following relations

$$Y^{(0)}(x(k)) \cdot \theta^{(0)} = P = P_G - P_L$$

$$|\psi_\ell| = |\theta_i - \theta_j| \leq \overline{\psi}_\ell(x(k),0), \ \ell = 1,2, \ . \ . \ . \ ,M$$

Now assume the highest capacity line of the right of way m is removed. The DC-load flow equations will be

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P_G - P_L$$

Consider the constraints

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq \overline{\psi}_\ell(x(k),m), \ \ell = 1,2, \ . \ . \ . \ ,M \qquad (31)$$

and

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq x_h\overline{\psi}_\ell(x(k),m), \ \ell = 1,2, \ . \ . \ . \ ,M \qquad (35)$$

If some of the constraints (35) are violated, we shall say the state x(k) is not admissible. If constraints (31) are satisfied, the state x(k) is secure during outage m. If some of the constraints (31) are violated while the constraints (35) are satisfied, the system is in an emergency state. In this case, a short period of time is at our disposal to reschedule the generation in such a way that the new steady state is normal.

The statement of the corrective rescheduling is: Given the power flow equations during an outage m

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P_G - P_L \tag{32}$$

the "soft" constraints,

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq \overline{\psi}_\ell(x(k),m), \ \ell = 1,2, \ . \ . \ . \ ,M \tag{31}$$

with some of them violated while the "hard" constraints,

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq x_h \overline{\psi}_\ell(x(k),m) \tag{35}$$

are satisfied. The limits of the real power outputs P_G of the generation plants

$$P_G^{min} \leq P_G \leq P_G^{max} \tag{26}$$

determine a feasible change of the generation schedule ΔP_G such that the new steady state, described by

$$Y^{(m)}(x(k))\theta'^{(m)} = P_G + \Delta P_G - P_L \tag{36}$$

satisfies the "soft" constraints

$$|\psi_\ell'^{(m)}| = |\theta_i'^{(m)} - \theta_j'^{(m)}| \leq \overline{\psi}_\ell(x(k),m), \ \ell = 1,2, \ . \ . \ . \ ,M \tag{37}$$

The objective will be to have minimal cost deviation.

The vector ΔP_{Gi} should satisfy the equation

$$\sum_i \Delta P_{Gi} = 0 \tag{38}$$

since the load remains constant.

The stated problem can be simplified by linearizing around the operating point. This is justified since the changes ΔP_{Gi} are very small.

The phase angle difference across the right of way ℓ, ψ_ℓ, is a function of the vector P_G

$$\psi_\ell^{(m)} = \phi_\ell^{(m)}(P_G)$$

For small deviations ΔP_G, we obtain

$$\psi_\ell'^{(m)} = \psi_\ell^{(m)} + \left(\frac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_G}\right)^T \Delta P_G + \text{higher order terms}$$

where

$$\frac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_G} = \begin{bmatrix} \dfrac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_{G1}} \\ \\ \dfrac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_{G2}} \\ \cdot \\ \cdot \end{bmatrix}$$

By neglecting the higher order terms and assuming quadratic cost functions of the generating units, the problem can be stated as follows:

Minimize the cost deviation

$$\Delta C = B^T \cdot \Delta P_G + (\Delta P_G)^T C \, \Delta P_G \tag{39}$$

subject to

$$\left| \psi_\ell^{(m)} + \left(\frac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_G}\right)^T \Delta P_G \right| \leq \overline{\psi}_\ell(x(k),m), \quad \ell = 1,2, \ldots ,M \tag{40}$$

$$\sum_i \Delta P_{Gi} = 0 \tag{38}$$

$$\Delta P_{min} \leq \Delta P_G \leq \Delta P_{max} \tag{41}$$

where B is a constant vector with $B_i = b_i + 2c_i P_{Gi}$ and C is a constant diagonal matrix with $C_{ii} = c_i$, and b_i, c_i, are the coefficients of the quadratic cost function of plant i.

The above problem can be reduced to a standard quadratic programming problem with linear constraints. However, a quadratic solution of a large problem as the above will be time consuming for a planning algorithm. For this reason, a fast but suboptimal algorithm has been developed. It is presented in Appendix A.

Once a solution to the corrective rescheduling problem has been found, the removed line is restored and the procedure steps to the next outage.

<u>Definition of an Admissible State II</u>. A state $x(k)$, at stage k, is said to be admissible if:

(a) it satisfies the "soft" constraints with the base case configuration

$$Y^{(0)}(x(k))\theta^{(0)} = P^{(0)}(k)$$

$$|\psi_\ell^{(0)}| = |\theta_i^{(0)} - \theta_j^{(0)}| \leq \overline{\psi}_\ell(x(k),0)$$

(b) it satisfies the "hard" constraints during an outage m, $m = 1,2, \ldots, M$

$$Y^{(m)}(x(k))\theta^{(m)} = P^{(0)}(k)$$

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq x_h\overline{\psi}_\ell(x(k),m)$$

(c) For the outage m, $m = 1,2, \ldots, M$, it is possible, if necessary, to find a change ΔP to the generation schedule which is feasible and which will make the system to satisfy the "soft" constraints.

$$Y^{(m)}(x(k))\theta^{(m)} = P^{(0)}(k) + \Delta P$$

$$|\psi_\ell^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq \overline{\psi}_\ell(x(k),m), \quad m = 1,2, \ldots, M$$

The External System

In almost every case the transmission network under study is interconnected with neighboring system for the purpose of assisting each other. The interconnections influence the flow of power in the system under consideration. It is, therefore, necessary to have an equivalent representation of the external systems which will accurately reproduce the power flows in the internal system.

Many steady state equivalencing techniques have been developed. An excellent review is presented in reference 37. In the same paper the equivalent model of the external systems is obtained with an optimization technique whereby the best equivalent representation is generated given the available information about the external systems. This is mostly desirable for planning purposes.

Let us assume that at stage k the equivalent model of the external system is known as well as the state x(k) of the system under study. The external system is taken into account if the equivalent model is used in the construction of the system's matrix. This is depicted in Figure II.4.

Inclusion of the equivalent representation of the external system in a planning study gives realistic results while keeping the size of the system under study small.

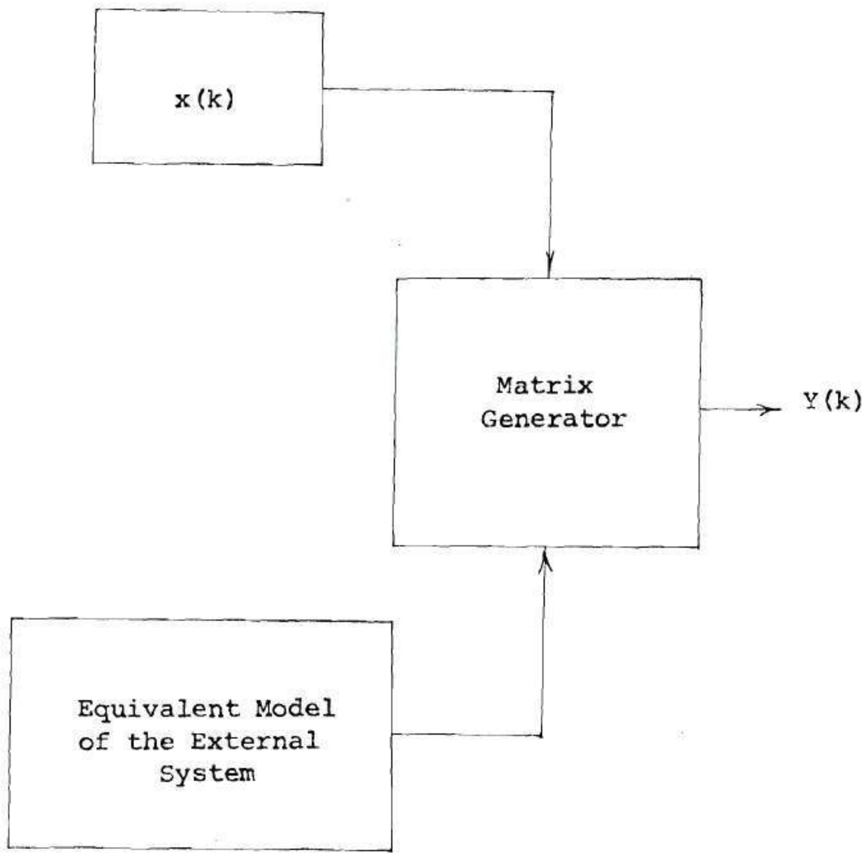


Figure II.4. Inclusion of the Equivalent Model of the External System.

CHAPTER III

THE AUTOMATIC GENERATION OF ALTERNATIVES

General

In this Chapter the problem of defining the set of admissible controls $U(x(k),k+1)$ is considered. The state of the system $x(k)$ at stage k is assumed to be known. The set $U(x(k),k+1)$ is then defined as the set of controls $u(k)$ which will yield an admissible system state $x(k+1)$ at stage k+1.

A control $u(k)$, otherwise referred to as an alternative, is a discrete combination of facilities (transmission lines) which will be in service for the stage k+1.

Given the discrete types of transmission lines and the available locations or rights of way for construction, the conceivable controls $U(k)$ at stage k can be obtained by considering all the possible combinations. This, however, leads to an enormous number of discrete controls. This thesis reports that most of these controls are either inadmissible or they are not qualified to be in the optimal trajectory. This Chapter substantiates the above statement and provides techniques for the automatic generation of alternatives.

The set of all possible controls $S_a(k)$, which can be applied at stage k, state $x(k)$ is defined as the set of all possible ways of expanding exactly M* rights of way with transmission lines chosen from L* discrete types. M* is the set of rights of way which are available for construction. L* is the set of discrete types of transmission lines which may be used for the expansion of the system. Because of standardization in the design of transmission lines and transformers, the number L* is very small. The usual case is L*=2. This corresponds to the case where the transmission network is expanded with a certain type of line, for example 230 kV lines, and at the same time an overlay of lines operating at higher voltage, (i.e. 500 kV) is to be started.

The number of all possible controls is very large. Suppose K* is the maximum number of circuits allowed in an alternative and further assume that only one circuit is allowed on a right of way. Then the number of controls in the set $U(k)$ is:

$$na = \sum_{i=o}^{K^*} \binom{M^*}{i} (L^*)^i \tag{42}$$

This number is a large number by itself. On the other hand, the number of possible trajectories is much larger. Assuming the same number of controls, na, for each state and stage, and the same number of available rights of way, then the number of trajectories will be:

$$nt = (na)^N = \left(\sum_{i=o}^{K^*} \binom{M^*}{i} (L^*)^i \right)^N \tag{43}$$

where N is the number of stages in the planning period.

The numbers na and nt are very large for even small networks. Therefore, the problem of planning a transmission network appears to be computationally infeasible. However, the research of this thesis has revealed the following facts: (1) If the construction of the na alternatives at a given state x(k) and stage k+1 is done concurrently with the optimization procedure (in a general planning algorithm), then it is possible to restrict the number na in a computationally manageable number. This is so because the majority of the possible alternatives na fails to satisfy optimality conditions which may be generated by the optimizing algorithm; and (2) The problem of planning the expansion of a transmission network can be viewed as capacity expansion in order to alleviate circuits which become loaded over their capacity or close to it as demand increases. It has been observed that alleviation of the overloads can be achieved, in an economic way, by circuit additions to a limited number of rights of way. We shall call these rights of way effective for network reinforcement. The number of effective rights of way for network reinforcement represents a small percentage of the total number. In this way a reduction of the size of the problem is achieved.

Optimality conditions are discussed in Chapter IV. In this Chapter the detection of effective rights of way for network reinforcement is investigated. Two different detection schemes are presented. And finally, the construction of the controls is discussed.

Network Coherency

The flow of power on a network is a dynamic phenomenon which is governed by Kirchhoff's laws. Under given constant power injections at the nodes of the network, the power flow on a given circuit is a function of the impedances of the existing circuits. In DC-model terminology, the power flow can be equivalently represented by phase angle difference across the circuit

$$\psi_\ell = \phi_\ell(y_1, y_2, \ . \ . \ . \ , y_M) \tag{44}$$

where y_i is the "capacity" of the circuits on the right of way i.

From the planning point of view, the following question is very important: If the capacity y_i on the right of way i is increased, what will happen to the power flow on the right of way ℓ, or equivalently to the phase angle difference ψ_ℓ. If the variation of y_i is very small, then we can assume

$$\Delta\psi_\ell \simeq \frac{\partial\phi_\ell}{\partial y_i} \Delta y_i$$

The derivative

$$\frac{\partial\phi_\ell}{\partial y_i}$$

indicates the direction of change of the power flow on right of way ℓ when the capacity on the right of way i is changed.

Let us consider the vector

$$\frac{\partial\phi_\ell}{\partial y} = [\frac{\partial\phi_\ell}{\partial y_i}] \quad i = 1,2, \ . \ . \ . \ ,M$$

It has been observed that only few components of the vector $\frac{\partial\phi_\ell}{\partial y}$ have relative high value and the rest of them have value orders of magnitude less. The components with high relative value define a set of rights of way. Changes in the transmission "capacity" of these rights of way have considerable effect on the power flow on the circuits of the right of way ℓ. The power flow on the right of way ℓ is insensitive to changes of the transmission capacity of the rest of the rights of way. Therefore, there is a kind of coherency in the network.

We shall call a right of way i coherent to the power flow on the right of way ℓ if the following relation is satisfied:

$$\left| \frac{\dfrac{\partial \psi_\ell}{\partial y_i}}{\dfrac{\partial \psi_\ell}{\partial y_\ell}} \right| \geq x_{coh}$$

where x_{coh} is a defined threshold value for coherency.

Therefore, coherency is defined in terms of a threshold x_{coh}. Values in the neighborhood of 0.10 are very reasonable. In this case, the number of rights of way which are "coherent" to the power flow on a particular circuit is very small compared to the total number of rights of way. This observation is of great practical value.

Unfortunately, this coherency is dependent on the power injections at the nodes of the network. In Appendix B, an expression for the derivatives $\frac{\partial \phi_\ell}{\partial y_i}$ is given:

$$\frac{\partial \phi_\ell}{\partial y_i} = -A_{\ell i} \psi_i \tag{45}$$

where $A_{\ell i} = e_\ell^T Y^{-1} e_i$ is dependent only on the system's parameters, and ψ_i is the phase angle difference across the right of way i.

The problem of planning the expansion of a transmission network can be viewed as capacity expansion in order to alleviate circuits which become loaded over their capacity or close to it as demand increases. For such a circuit there is practically a small number of coherent rights of way on which construction of new circuits may alleviate the undesirable condition. Construction of new circuits in other areas of the system will affect the undesirable condition very little, practically none. Therefore, the existing coherency in a transmission network provides a basis for size reduction of the planning problem. Since the coherency depends on both network topology (and parameter values) and power flow on the network and since both may vary widely in a period of several years, it is then imperative to consider a certain coherency pattern to be valid only for a short period of time, for example, one year.

Sensitivity Analysis

The expansion of the transmission network can be viewed as a sequence of network reinforcements throughout the planning period. At a given time (stage) in the future, several circuits will be overloaded or very close to being overloaded. These circuits can be obtained by solving the power flow equations for the conditions prevailing at that future time and under all postulated outages.

Given the circuits which need reinforcement, the problem is to find the rights of way on which construction of new circuits may alleviate the undesirable loading of the circuits.

From the previous section it follows that candidates are all rights of way which are coherent to the above circuits. It is therefore necessary to formulate a procedure for detecting coherency. Furthermore, since cost of new circuits is a decisive factor, it is imperative to include cost considerations in the detection of the rights of way which are effective for reinforcing the overloaded circuits.

Two detection schemes are presented in this chapter. Each one involves the computation of the sensitivity coefficients

$$\frac{\partial \phi_\ell}{\partial y_i}\ , \ i = 1, \ . \ . \ . \ , M$$

for every right of way ℓ on which a circuit is overloaded or near overloaded. This task is referred to as sensitivity analysis. In Appendix B the technicalities of the computations are presented.

The detection schemes to be presented are simple and represent the conclusion of long experimentation.

Single Outage Analysis

Given a state of the transmission network $x(k)$ at stage k, and a set of conditions (power injections at the nodes of the network) for the next stage k+1, it is desirable to detect all circuits which may be critically loaded during the next stage and for all single contingency conditions. This information is very useful. The expansion of the transmission network is then directed towards reinforcing these critically loaded circuits. It should be understood that in a transmission network there is plenty of transmission capacity, so to speak, which cannot be fully employed because of the dynamic nature of power flow (Kirchoff's laws) and because the operator of the system has limited control in channeling the flow of power.

We shall say a circuit is critically loaded if the relation

$$\left|\frac{P}{\overline{P}}\right| \geq x_{over}$$

is satisfied for at least one single contingency condition or the base case conditions where P is the actual power flowing through the circuit, $\overline{P}$ is the maximum permissible power to flow through this circuit and X_{over} is an externally defined parameter.

Using the above definition of a critically loaded circuit and the DC power flow model, the detection of the critically loaded circuits requires the solution of the following relations:

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P^{(0)}(k+1) \tag{46}$$

$$|\psi_{\ell}^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq x_{over}\overline{\psi}_{\ell}(x(k),m) \tag{47}$$

$$m = 0,1,2, \ldots ,M$$

$$\ell = 1,2, \ldots ,M$$

The symbols have been defined in Chapter II. A circuit, ℓ, is critically loaded if inequality (47) is violated for this circuit for at least one value of m. Furthermore, an outage m which causes a circuit to be overloaded is called a critical outage.

The set of rights of way with critically loaded circuits is denoted by S_u and the set of critical outages is denoted by S_c. The solution of the relations (46) and (47), which is referred to as single outage analysis, yields both sets S_u and S_c.

Detection Scheme I

The statement of this detection scheme is: A network configuration, x(k) the power injections at the nodes of the network at stage k+1, and the set of rights of way with critically loaded circuits S_u, are given. Find the rights of way on which construction of new circuits may eliminate the critical loading of the circuits S_u in an economic way.

Let S_E denote the set of rights of way to be detected. Then the detection scheme I involves the following steps:

1. $S_E = \phi$.
2. Consider one circuit at a time from the set S_u. Let it be circuit ℓ.
3. Compute the effectiveness ratio vector [E.R.V.].

$$\text{E.R.V.} = \left[\frac{\dfrac{\partial \phi_\ell}{\partial y_i} \cdot x_\ell d_\ell^2}{\dfrac{\partial \phi_\ell}{\partial y_\ell} \cdot x_i d_i^2} \right] \quad i = 1,2, \ldots ,M$$

where:

x_ℓ is the relative acquisition cost of one mile long transmission line on the right of way ℓ

$\frac{\partial \phi_\ell}{\partial y_i}$ is the sensitivity coefficient computed with the base case network configuration

d_ℓ is the length of the right of way ℓ.

4. The rights of way i, which satisfy the inequality

$$\frac{\frac{\partial \phi_\ell}{\partial y_i} x_\ell d_\ell^2}{\frac{\partial \phi_\ell}{\partial y_\ell} x_i d_i^2} \geq x_{cut}$$

where:

x_{cut} is a threshold parameter

Form a set, let it be S_E'.

5. $S_E = S_E \cup S_E'$

6. If all circuits in the set S_u have been considered the detection scheme has been completed. Otherwise return to step (2).

This detection scheme is very simple and fast from the computational point of view. Sensitivity coefficients are computed for the base case network configuration. The threshold x_{cut} is defined externally. A value of 0.2 to 0.4 is adequate. The detection is very reliable because each critically loaded circuit is processed separately.

Detection Scheme II

The statement of this detection scheme is: A network configuration, x(k), the power injections at the nodes of the network at stage k+1, the set of rights of way with critically loaded circuits S_u, and the set of critical outages S_c are given. Find the rights of way on which construction of new circuits may eliminate the critical loading of the circuits S_u in an economic way and for all single contingency conditions.

Again let S_E denote the set of rights of way to be detected. Then the detection scheme II involves the following steps:

1. $S_E = \phi$.
2. Consider one circuit at a time from the set S_u. Let it be circuit ℓ.
3. Compute the effectiveness ratio vectors (E.R.V.), one for each critical outage from the set S_c

$$(E.R.V.)^j = \left[\frac{\dfrac{\partial \phi_\ell^{(j)}}{\partial y_i} \cdot x_\ell d_\ell^2}{\dfrac{\partial \phi_\ell^{(j)}}{\partial y_\ell} \cdot x_i d_i^2} \right]_{i = 1,2, \ldots ,M}$$

and $j \epsilon S_c$. The variables have been defined in detection scheme I.

4. The rights of way i, which satisfy the inequality

$$\left[\frac{\dfrac{\partial\phi_\ell^{(j)}}{\partial y_i}\cdot x_\ell d_\ell^2}{\dfrac{\partial\phi_\ell^{(j)}}{\partial y_\ell}\cdot x_i d_i^2}\right] \geq x_{cut},\ j\epsilon S_c$$

form a set, let it be S_E'.

5. $S_E = S_E \cup S_E'$.

6. If all circuits in the set S_u have been considered, the detection scheme has been completed. Otherwise, return to step (2).

This detection scheme is more complicated than the first one. For each circuit in the set, S_u, the effectiveness ratio vectors for a series of network configurations are computed. This is justified because the sensitivity coefficients $\frac{\partial\phi_\ell}{\partial y_i}$ change drastically for two configurations differing by only one line. This detection scheme requires more computations but it is reliable and a higher value of the threshold parameter x_{cut} is sufficient.

Availability of Right of Way

If the availability of the rights of way is constraint, then a further operation on the set S_E is necessary. If S_A is the set of the available for construction rights of way, then

$$S_E = S_E \cap S_A.$$

Construction of Alternatives

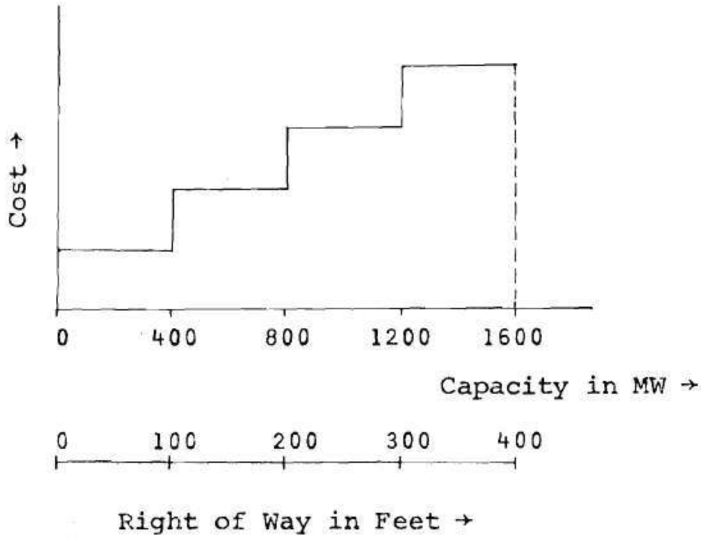
Given the set of effective rights of way, S_E, it is easy to construct the alternatives (controls) u(k). To this purpose, engineering judgement, reasonability, and general policies of the particular company should be considered.

A general discussion of the problem is presented in this section.

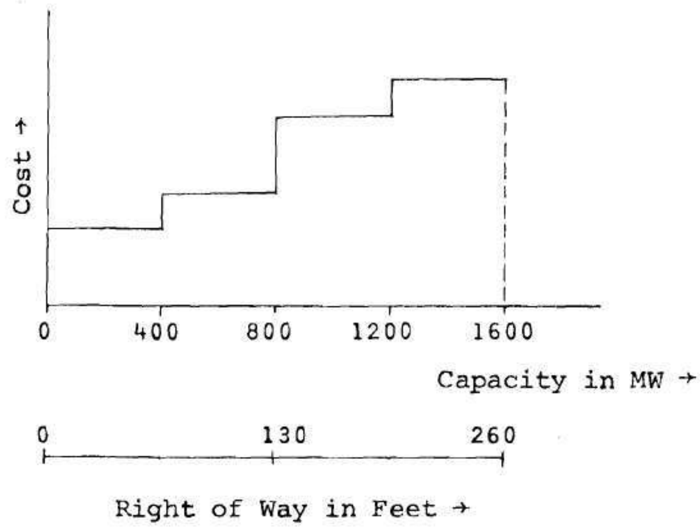
The problem of constructing the controls u(k), given the set of rights of way S_E and L* types of transmission lines to be used for the expansion of the system, is basically a combinatorics problem. This problem can be partitioned into two subproblems.

(1) Given L* types of transmission lines, find all the possible combinations of these facilities which can be constructed on a given right of way. This is a fairly simple problem. However, it may happen two discrete combinations to be identical from the operational point of view but different in cost. In this case, the combination with the higher cost should not be included in the optimization algorithm. This statement is obvious. It is now imperative to develop a systematic way to exclude these combinations. A primitive cost-capacity curve for each line type in a particular right of way is constructed. Figure III.1 shows two primitive cost-capacity curves. The curves are truncated when the available space of the right of way is exceeded.

Let the primitive cost-capacity and right of way curves be



(a) Primitive Cost-Capacity Curve of a 400 MW Single-Circuit Line Requiring 100 ft. Right of Way.



(b) Primitive Cost-Capacity Curve of a 800 MW Double-Circuit Tower Requiring 130 ft. Right of Way.

Figure III.1. Primitive Cost-Capacity Curves denoted by the following:

$$\text{Cost:} \quad C_i(x_i),\ x_i = 0, d, 2d, \ldots, M_i d$$

$$\text{Right of Way:} \quad r_i(x_i),\ x_i = 0, d, 2d, \ldots, M_i d$$

where d is a capacity increment common to all line types, and $M_i d$ is the maximum capacity of the i^{th} type line which can be constructed within the limits of the available right of way.

The optimal cost-capacity curve is derived from an operation similar to convolution of these primitive curves

$$C(x) = C_1(x_1) * C_2(x_2) * \ldots * C_m(x_m) \tag{48}$$

where m is the number of primitive curves and C(x) is the optimal cost-capacity curve for the right of way under consideration. The convolution type operation denoted by the operator * is defined as follows:

$$C_{ij}(x_{ij}) = C_i(x_i) * C_j(x_j)$$

$$= \text{Min}\{L_{ij}(x_{ij}-v, v)\,[C_i(x_{ij}-v) + C_j(v)]\}$$

(except zero)

$$v = 0, d, 2d, \ldots, M_i d$$

and

$$L_{ij}(x_i,x_j) = \begin{cases} 0 & \text{if } r_i(x_i) + r_j(x_j) \geq r_T \\ 1 & \text{otherwise} \end{cases}$$

and r_T is the total available space in this right of way.

Equation (48) yields a staircase function. Each step corresponds to a certain combination of transmission lines. This combination is optimal in the sense that there is not another combination of transmission lines which has the same transmission capacity and which costs less.

(2) Assuming that the optimal cost capacity curves for each right of way are known, it is simple to construct the alternatives. If η_ℓ is the number of steps in the optimal cost-capacity curve of the ℓ^{th} right of way, then the total number of alternatives is

$$\eta a = \prod_{\ell \in S_E} (\eta_\ell + 1) \tag{49}$$

since there are η_ℓ+1 ways to expand the right of way ℓ (the possibility of no addition of transmission capacity to the right of way ℓ has been considered).

The described partition of the problem reduces the complexity of the overall problem without impairing the generality of the approach.

The mechanics of determining the set of controls $U(x(k),k+1)$ at stage k, stage $x(k)$ should be obvious. A control matrix $u(k)$ belonging to this set is defined as follows:

$$u_{i\ell} = 0 \qquad \text{if } \ell \notin S_E$$
$$\text{any } i$$

$$\underset{i=1,...,L}{u_{i\ell}} = S_{i\ell} \text{ if } \ell \in S_E \tag{50}$$

where:

$S_{i\ell}$ is the number of transmission lines of type i in the combination of lines corresponding to the selected step of the optimal cost versus capacity curve of the right of way ℓ.

The set of controls $U(x(k),k+1)$ has na (Equation 49) elements (control matrices $u(k)$).

Optimality and Feasibility Conditions

A procedure of determining the controls (alternatives) which can be applied at a state $x(k)$ of the system at stage k has been presented. The number of these controls, na, is moderately large yet lower than the total number of controls. On the other hand, since we are interested in the long range planning of the system, we would like to know the possible ways of expanding the system throughout the planning period. If $na(k,i)$ are the controls applicable at stage k, state $i(x_i(k))$, then the possible number of expansions of the network throughout the planning period is

$$nt = \sum_{k=0}^{N-1} \sum_{i} na(k,i) \tag{51}$$

This number can be extremely large and therefore the problem appears to be computationally infeasible.

This conclusion is inevitable if the construction of alternatives is considered independent from the optimization algorithm. However, if the generation of controls is performed concurrently with the optimization, it is possible to tremendously limit the number of alternatives by using information from the optimizing algorithm. Of course this procedure will not impair the optimality of the results.

Consider the alternatives na(k,j) at stage k, state $x_j(k)$. Every alternative, in order to enter the optimization algorithm, should satisfy the feasibility and optimality conditions.

Feasibility Condition

An alternative u(k) from the set $U(x_j(k),k+1)$ will enter the optimizing algorithm if the state x(k+1),

$$x(k+1) = x_j(k) + u(k) \tag{I}$$

is admissible for stage k+1. The state x(k+1) should be checked with the constraints which define the state admissibility (I or II, Chapter II). Because the majority of the constraints are ineffective, it is expedient to retain only those constraints which are effective or close to being effective. The control u(k) meets the feasibility condition if the state x(k+1) satisfies the set of effective or close to being effective constraints.

Optimality Condition

An alternative u(k) from the set $U(x_j(k),k+1)$ satisfies the optimality condition if and only if

$$C(u(k)) \le \bar{J} - J_p \tag{52}$$

The inequality (52) is derived in Chapter IV. The optimality condition is simple from the computational point of view and therefore should be applied first.

The number of controls u(k) which meet the feasibility and optimality conditions is relatively small for even large networks. These controls, which form a set $S^*_{ju}(k)$, will enter the optimizing algorithm. Obviously,

$$S^*_{ju}(k) \subset U(x_j(k),k+1)$$

In summary, the automatic generation of controls algorithm generates a large number of controls (alternatives). However, information obtained from the optimizing algorithm and a feasibility condition can be used in order to prove that the majority of these controls do not belong to the optimal trajectory.

Discussion

The detection schemes actually reduce the size of the overall problem. The number of the effective rights of way is actually a small percentage of the total number of rights of way. Therefore, the detection schemes define a subproblem of the problem. Then the optimization method will yield the optimal solution to the subproblem. It can be argued that this optimal solution might not be the global optimum to the problem. This, however, is only a theoretical argument. Practically, the solution of the subproblem is the global optimum if the values of the parameters x_{over} and x_{cut} are appropriately selected.

It should be obvious that the degree of the problem size reduction is controllable through the parameters x_{over} and x_{cut}. If

$$x_{over} = 0$$

or

$$x_{cut} = -\infty$$

then no reduction of the problem size is performed. The solution of the overall planning problem with $x_{over} = 0$ or $x_{cut} = -\infty$ will yield the global optimum. This is possible for small networks. It has been observed, however, that relatively high values of the parameters x_{over} and x_{cut} still yield the known global optimum. For the test system A (see Chapter VII), the known global optimum was obtained with $x_{over} = .98$ and $x_{cut} = .80$. This experimental result signifies the merit of the automatic generation of alternatives.

In Chapter VII the performance of the automatic generation of controls algorithm is evaluated. The criteria are:

1. Computational effort.
2. Number of controls in the set $S^*_{ju}(k)$.

The criteria are self-explanatory. In Chapter VII quantitative measures of the performance of the algorithm with respect to the above two criteria are defined. These measures are computed for two test systems and for various values of the parameters x_{over}, x_{cut}, and the stage variable k.

Conclusion

The number of alternatives that can be selected at a particular state of the transmission network, x(k), and stage k is enormous. This fact renders the planning problem an insurmountable computational burden.

The dynamic nature of the power flow on the network practically restricts the number of rights of way which are effective in reinforcing the network. Two detection schemes, based on sensitivity analysis, have been developed. They detect all rights of way which are effective in reinforcing the network at a given time. Two parameters x_{over} and x_{cut} control the output of the detection schemes. A reduction of the problem size is immediately achieved.

The construction of the set of alternatives or controls U(x(k), k+1) is based on combinatorics and problem partitioning. The most important finding is the fact that when the construction of the set U(x(k),k+1) is done concurrently with the optimizing algorithm, it is possible to disqualify the majority of the generated controls without impairing the optimality of the results. This is achieved with optimality and feasibility conditions.

CHAPTER IV

THE NON LINEAR BRANCH AND BOUND

General

The non Linear Branch and Bound is an enumerative approach to our problem. It takes advantage of the fact that the problem of transmission planning is bounded and therefore the set of the controls is finite.

Enumeration of the discrete controls yields to a tree-like structure as in Figure IV.1. This figure illustrates a four stage planning period. There are ten discrete trajectories. It is obvious that in order to obtain the optimal trajectory, it is sufficient to:

(a) Determine the set of admissible trajectories.

(b) Compute the performance criterion J for each admissible trajectory. The trajectory with the smallest J is optimal.

A trajectory is admissible if and only if it yields an admissible state at any stage.

We call $v_j(k)$ the predecessor of $v_i(k+1)$, which in turn is called a successor of its predecessor. Note that a vertex has a unique predecessor but generally more than one successor.

Separation

The successors of a vertex $v_j(k)$ determine a finite set $S_j^*(k)$ of

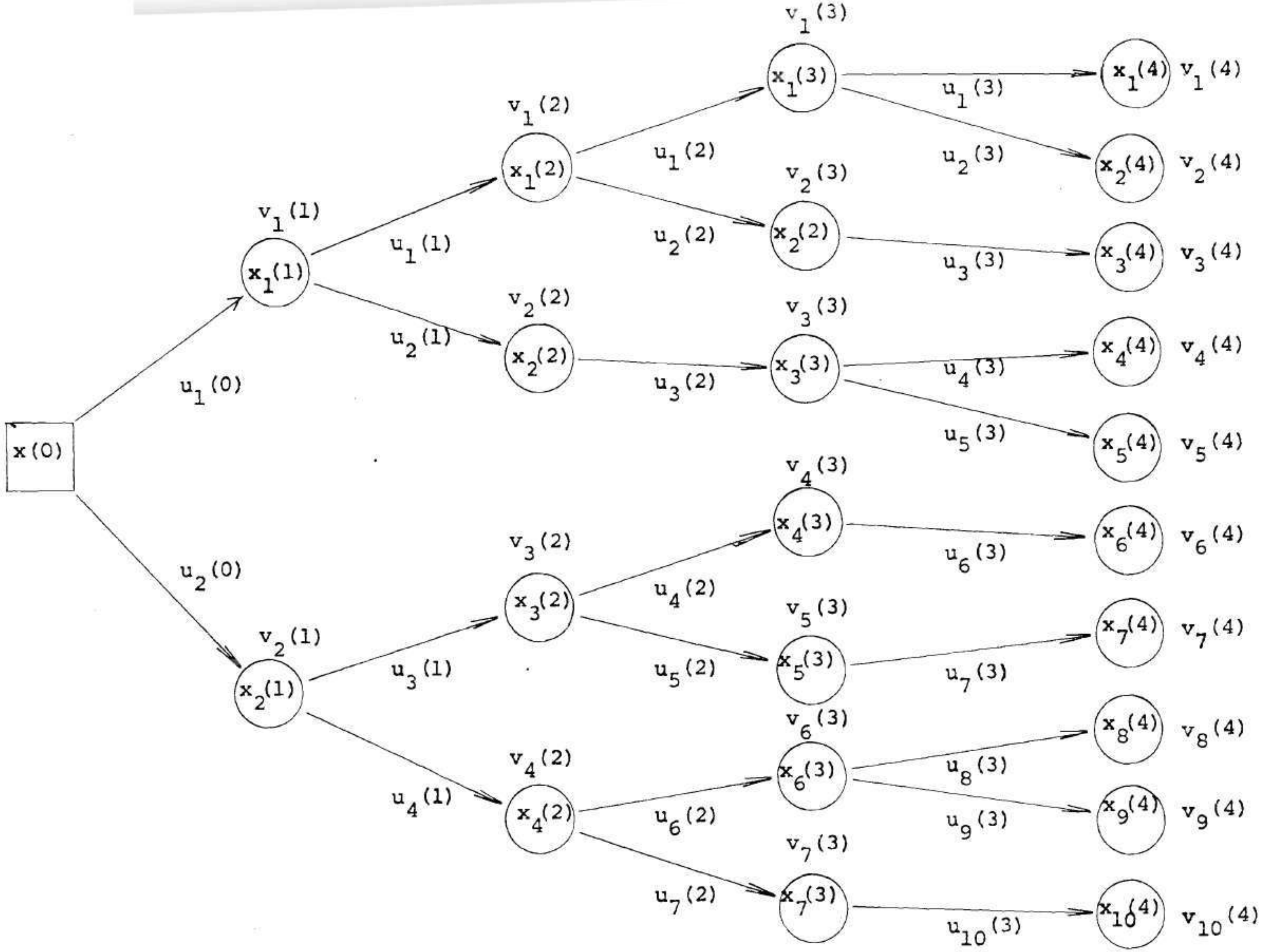


Figure IV.1. Enumeration Tree.

subsets of $S_j(k)$, where $S_j(k)$ is the set of trajectories emanating from vertex $v_j(k)$. Obviously,

$$\bigcup_{t \in S_j^*(k)} t = S_j(k)$$

The set $S_j^*(k)$ is called a separation of $S_j(k)$.

It is important to note that $S_j^*(k)$, for the transmission network planning problem, is a partition of $S_j(k)$ and generally a very small set compared to $S_j(k)$.

<u>Branching</u>

A vertex that is not fathomed and whose corresponding constraint set has not been separated is called a live vertex. Branching means choosing a live vertex to consider next for fathoming or separation. There are many possible rules for branching. Here branching is performed to one of the successor vertices of the vertex under consideration. If the current vertex $v_j(k)$ is fathomed, one simply <u>backtracks</u> along the trajectory until a vertex having at least one live successor is encountered. One of these successor vertices is chosen for branching. If there are no more live vertices, the enumeration has been completed.

Branch and Bound is an optimization technique that uses the basic tree enumeration described previously. It involves calculating upper and lower bounds on the objective function in order to accelerate the fathoming process and thereby curtail the enumeration.

The transmission network planning has been formulated as a minimization problem which can be solved with the non Linear Branch and

Bound method. The efficiency of the method depends on the effectiveness of the fathoming process which ultimately depends on the nature of the problem. This thesis reports that the fathoming process is very effective for the transmission network problem.

On the other hand, the concept of separation yields good storage requirements since it is only necessary to retain one separation per stage.

In the following we will describe the tasks of separation, branching, and bounding to our problem.

Description of the Non Linear Branch and Bound Method

This section presents a systematic description of the concepts and the essential tasks performed by the non Linear Branch and Bound.

The Concept of a Vertex

It is easy to describe the concept of a vertex with the aid of Figure IV.1. This figure represents an enumeration tree for a four-stage problem. Each circle represents a vertex. Each vertex is associated with a state of the system.

Consider vertex $v_1(2)$ which is associated with the state $x_1(2)$. The vertex determines:

(a) A unique trajectory which brings the system from the initial state to the state which is associated with the vertex.

(b) A set of trajectories, namely all trajectories which include the vertex under consideration.

For example, the vertex which is associated with the state $x_1(2)$ uniquely determines the trajectory $\{u_1(0),\ u_1(1)\}$ which brings the system from the state $x(0)$ to $x_1(2)$ and also represents the set of the trajectories

$$\{u_1(0),\ u_1(1),\ u_1(2),\ u_1(3)\}$$

$$\{u_1(0),\ u_1(1),\ u_1(2),\ u_2(3)\}$$

$$\{u_1(0),\ u_1(1),\ u_2(2),\ u_3(3)\}$$

A vertex is said to be fathomed if and only if it can be proven that further exploration is not profitable.

A vertex may have one or more successor vertices but only one predecessor vertex.

It is obvious that if a vertex is fathomed it is not necessary to evaluate the successor vertices. These vertices are said to be implicitly enumerated.

A vertex uniquely determines the state of the system. This statement is irreversible since one state may be associated with more than one vertex.

Two trajectories with the same number of steps are said to be complimentary if and only if terminate at the same state. For example, the trajectories

$$\{u_1(0),\ u_1(1),\ u_2(2)\}$$

$$\{u_2(0),\ u_4(1),\ u_6(2)\}$$

are complimentary if and only if

$$x_2(3) \equiv x_6(3)$$

From the above discussion, it is obvious that the number of vertices in one stage is always greater or equal to the number of discrete states at that stage.

Before explaining the mechanics of fathoming, it is necessary to present the lower and upper bounds.

Upper Bound

The upper bound is defined to be the performance criterion J of the best-up-to-date trajectory t^*:

$$\bar{J} = J(t^*) \tag{53}$$

If no best-up-to-date trajectory is available, the upper bound is infinity

$$\bar{J} = \infty \tag{54}$$

Note that this definition of the upper bound is independent of the vertex at which the enumeration might be. It also remains unchanged unless a better trajectory has been found in which case the upper bound will be updated.

Lower Bound

Suppose that the enumeration is at vertex $v_j(k)$ in the tree. Recall that vertex $v_j(k)$ represents a set of trajectories, namely those which include vertex $v_j(k)$. The lower bound is defined to be the lower bound of the function J (performance criterion) for the above mentioned set of trajectories. That is, the lower bound $\underline{J}$ satisfies

$$\underline{J} \leq J(t(v_j(k))) \tag{55}$$

where $t(v_j(k))$ is a trajectory which includes vertex $v_j(k)$.

There are many ways to determine a lower bound $\underline{J}$ which satisfy relation (55). A desirable feature of these ways will be simplicity and speed in computing it. Two very simple ways are described below.

(a) Recall that the vertex $v_j(k)$ uniquely determines the trajectory which brings the system from the initial state to the state of vertex $v_j(k)$, namely $t_p = \{u(0), u(1), \ldots, u(k-1)\}$. It is obvious that a lower bound of the performance criterion of the trajectories $t(v_j(k))$ will be

$$\underline{J} = \sum_{m=0}^{k-1} \frac{1}{(1+i)^m} \cdot \{ \sum_{\lambda=m}^{N-1} \frac{\ell_1(u(m))}{(1+i)^{\lambda-m}} + \ell_2(x(m+1), m+1) \} \tag{56}$$

It is trivial to show that

$$\underline{J} \leq J(t(v_j(k)))$$

(b) In practical situations it is expedient to fully understand the nature of the performance criterion and to take advantage of observed behavioral patterns. For example, it has been observed that in a practical transmission system the total energy losses during a year is a monotonically increasing function of the year variable. This observation can be introduced as an assumption which will be used for the definition of a better lower bound. Assumption: The operational cost, $\ell_2(x(k),k)$, is a monotonically increasing function of the stage variable k. Then the lower bound can be defined as:

$$\underline{J} = \sum_{m=0}^{k-1} \frac{1}{(1+i)^m} \{ \sum_{\lambda=m}^{N-1} \frac{\ell_1(u(m))}{(1+i)^{\lambda-m}} + \ell_2((m+1),m+1) \} +$$

$$+ \ell_2(x(k),k) \cdot \sum_{m=k+1}^{N} \frac{1}{(1+i)^{m-1}} \qquad (57)$$

Given the stated assumption, it is easy to prove

$$\underline{J} \leq J(t(v_j(k)))$$

It should be emphasized that every vertex $v_j(k)$ has a lower bound $\underline{J}$. It is therefore expedient to write

$$\underline{J} = \underline{J}(v_j(k))$$

in order to explicitly denote the dependence of the lower bound on the vertex.

Separation

Consider vertex $v_j(k)$. Let us recall that this vertex represents a set of trajectories $S_j(k)$. Every trajectory in the Set $S_j(k)$ contains vertex $v_j(k)$. Consider vertex $v_\lambda(k+1)$ and assume that $v_\lambda(k+1)$ is a successor of $v_j(k)$. Vertex $v_\lambda(k+1)$ represents a set, $S_\lambda(k+1)$, of trajectories which is a subset of $S_j(k)$:

$$S_\lambda(k+1) \subset S_j(k) \tag{58}$$

To prove this, it is only necessary to observe that every trajectory of the set $S_\lambda(k+1)$ includes vertex $v_j(k)$ and therefore belongs to the set $S_j(k)$ too.

The set $S_j^*(k)$ of the sets $S_\lambda(k+1)$ is called a separation of $S_j(k)$. There is a one to one correspondence between the elements of the set $S_j^*(k)$ and the controls $u(k)$ which can be applied at the vertex $v_j(k)$ because a successor vertex can be uniquely defined by a control. Therefore, the set $S_j^*(k)$ is finite (Chapter III). This observation can be used as the basis to prove the finiteness of the overall problem (next section). On the other hand, the fact that there is a one to one correspondence between the set of controls $u(k)$, $S_j^*(k)$, which can be applied at a vertex $v_j(k)$ and the elements of the set $S_j^*(k)$ establishes an equivalence between these two sets. Therefore, the two sets, $S_{ju}^*(k)$ and $S_j^*(k)$, can be used interchangeably. The above observations will be used in the discussion of optimality.

Finally, the set $S_j^*(k)$ represents a partition of $S_j(k)$. This is so because a trajectory t cannot belong to two different elements of the set $S_j^*(k)$.

Finiteness

It is always useful to guarantee that the non Linear Branch and Bound algorithm will terminate after a finite number of steps. To prove this it is only necessary to prove that the number of possible trajectories is finite:

Since the set $S_j^*(k)$ contains a finite number of elements, it is possible to find a number H such that:

$$\text{number of elements in } S_j^*(k) \leq H$$

$$\text{for any } j,k$$

By inspection of the enumeration tree:

$$\text{number of trajectories} \leq H^N$$

where N is the number of stages.

Therefore, the enumeration is bounded and the algorithm will terminate after a finite number of steps.

Fathoming

A vertex $v_j(k)$ is fathomed if:

(a) $\overline{J} \leq \underline{J}(v_j(k))$ (59)

(b) $S_j^*(k) = \phi, \quad k \neq N$ (60)

(a) The fathoming here is performed by bounds. If relationship (59) holds, it is easy to prove that further exploration from vertex $v_j(k)$ is not profitable in the sense that no one trajectory which includes vertex $v_j(k)$ can yield a performance criterion J less than $\overline{J}$.

Consider the set of trajectories $S_j(k)$. If $t \epsilon S_j(k)$, then t includes vertex $v_j(k)$. By definition of the lower bound

$$J_{t \epsilon S_j(k)}(t) \geq \underline{J}(v_j(k))$$

$$J_{t \epsilon S_j(k)}(t) \geq \overline{J} \text{ by using (59)}$$

Therefore, every trajectory $t \epsilon S_j(k)$ yields performance criterion higher than $\overline{J}$ and vertex $v_j(k)$ should be abandoned.

(b) Equation (60) states that if no successor vertices can be found for the vertex $v_j(k)$, then vertex $v_j(k)$ is fathomed.

Let us recall that the number of elements in the set $S_j^*(k)$ equals the number of elements in the set $S_{ju}^*(k)$. Then

$$S_j^*(k) = \phi \leftrightarrow S_{ju}^*(k) = \phi.$$

Therefore, a vertex $v_j(k)$ is fathomed if no admissible control can be found to be applied at vertex $v_j(k)$.

Branching

A vertex that is not fathomed and whose corresponding set of trajectories has not been separated is called a live vertex. Branching is the task of choosing a live vertex to consider next for fathoming or separation.

Consider vertex $v_j(k)$, which has not been fathomed and separated. In this case we know the set $S^*_{ju}(k)$ of controls $u(k)$ which can be applied to the state of vertex $v_j(k)$. Every control $u(k) \epsilon S^*_{ju}(k)$ defines a successor vertex $v_\lambda(k+1)$ of $v_j(k)$. This vertex is, by definition, a live vertex. It is now obvious that branching can be easily performed by choosing one control, $u(k)$, from the set $S^*_{ju}(k)$.

Branching from a vertex, $v_j(k)$, is possible as long as at least one live successor vertex exists.

The Non Linear Branch and Bound Algorithm

The described tasks can be organized into an algorithm which will be equivalent to a complete enumeration of all trajectories. Figure IV.2 presents the flowchart of the basic algorithm.

Block A represents the task of separation. Separation is performed by determining the set of admissible controls at the vertex under consideration. Therefore, block A is the automatic generation of alternatives (controls). This is fully justifiable since the equivalence between the sets $S^*_{ju}(k)$ and $S^*_j(k)$ has been established.

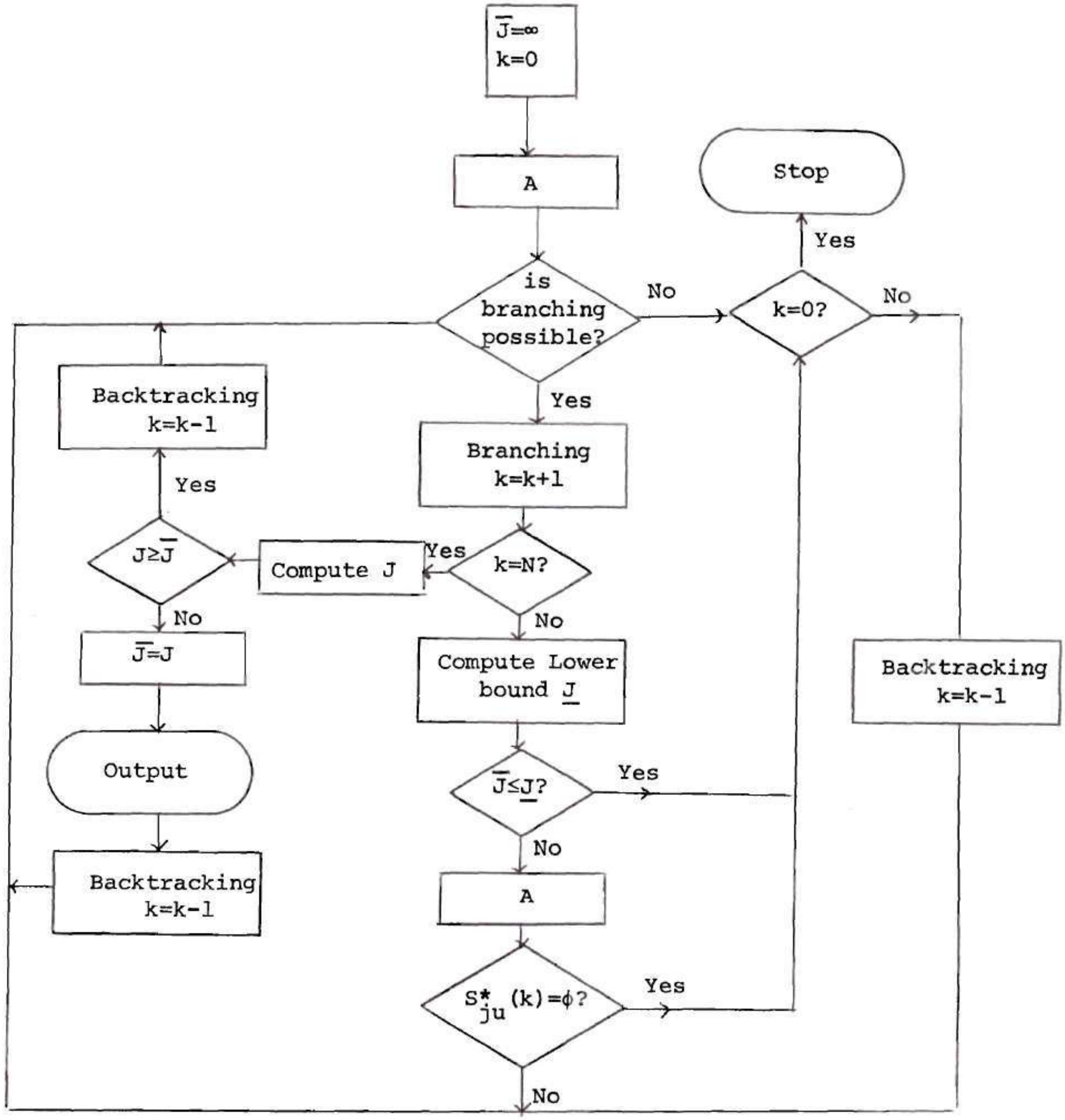


Figure IV.2. The Basic Non Linear Branch and Bound Algorithm.

Efficiency

In a problem as large as the one considered in this thesis, the question of efficiency is of great importance. Loosely speaking, an optimization algorithm is efficient if with a small amount of computations it can yield the optimum. An evaluation of the flow diagram of Figure IV.2 reveals that the bulk computations are performed in block A. The number of times the algorithm goes through block A is an indication of how efficient the algorithm is. On the other hand, this number depends on the size of the problem that is the total number of vertices in the complete enumeration tree of the problem. A reasonable normalized index of efficiency can be defined as:

$$\text{E.I.(Efficiency Index)} = \frac{\text{number of times block A was called}}{\text{total number of vertices}} \tag{61}$$

The above defined efficiency index depends on several factors which will be discussed in the following. Numerical evaluation of this dependence will be given in Chapter VII.

Ordering of Alternatives

Consider vertex $v_j(k)$ which cannot be fathomed by bounds. Therefore, next task (from the flowchart) is to call block A which will determine the admissible controls at that particular vertex. Furthermore, for each control, $u(k)$, the following quantity will be computed:

$$C(u(k)) = \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+i)^{\lambda}} + \frac{\ell_2(x(k+1),k+1)}{(1+i)^{k}}$$

It is expedient to compute C(u(k)) because the performance criterion can be readily computed as a sum of C(u(k))

$$J = \sum_{k=0}^{N-1} C(u(k))$$

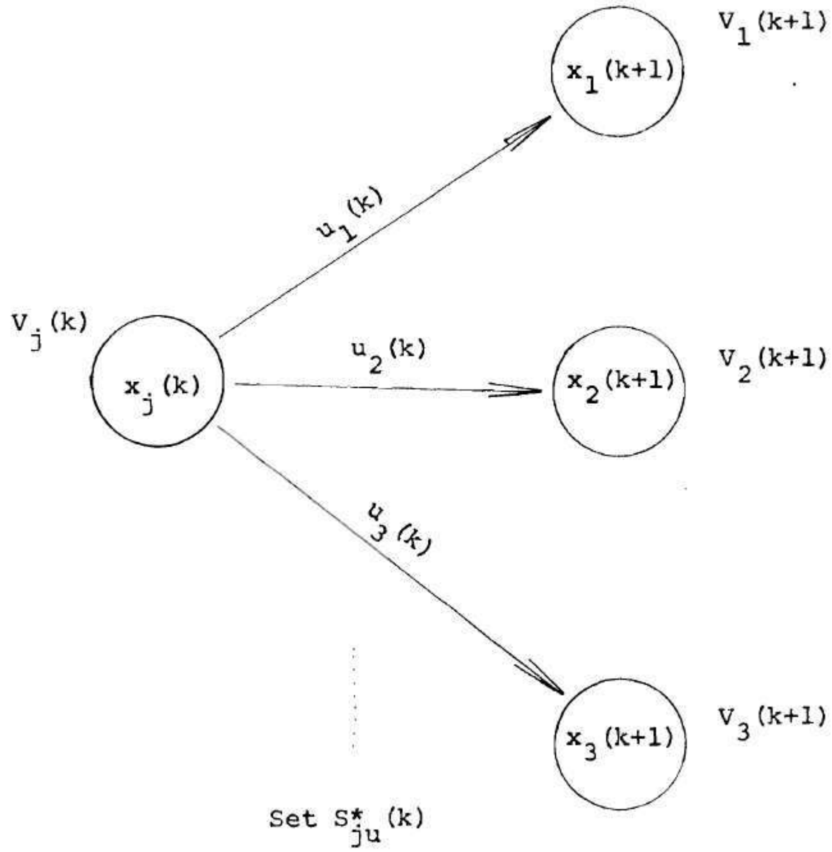
Furthermore, C(u(k)) can be used to order the controls u(k). Since the problem has been formulated as a minimization one, the first control will be the one with minimum C(u(k)), as in Figure IV.3.

Then during the first branching from vertex $v_j(k)$, vertex $v_1(k+1)$ will be selected, during the second branching from $v_j(k)$, vertex $v_2(k+1)$ will be selected, and so on. With a little bit of imagination one can expect that an ordering of the alternatives like the one described will yield a "better" upper bound $\overline{J}$ of the performance criterion in the sense that the value $\overline{J}$ is closer to the optimal value J^*. This fact has been actually observed. Now by inspecting equation (59), it is obvious that fathoming by bounds has a better chance if the upper bound $\overline{J}$ has a low value. Therefore, a better upper bound will speed up the fathoming and therefore will improve the efficiency of the algorithm.

Optimality Condition

A very attractive feature of the method is the following: Once a feasible trajectory has been found, it is possible to generate constraints which will suppress the computational burden.

In this section a constraint for the cost of the individual controls will be generated. This constraint will be used in the automatic



$$C(u_1(k)) \leq C(u_2(k)) \leq \ldots\ldots$$

Figure IV.3. Optimal Ordering of Controls.

generation of admissible controls in order to limit the number of generated controls while retaining optimality. As such, this constraint is rather an Optimality Condition.

The derivation of the optimality condition assumes that a feasible trajectory $t^1 = \{u^1(0), u^1(1), \ldots, u^1(N-1)\}$ has been found and therefore its performance criterion $\bar{J}$ is known, $\bar{J} = J(t^1)$. Recall that $\bar{J}$ is an upper bound of the performance criterion since we are interested in trajectories which yield a return J less than $\bar{J}$ (minimization).

Let us consider a vertex, $v_j(k)$, with the associated state $x_j(k)$. The partial trajectory $t_p = (u(0), u(1), \ldots, u(k-1))$ which brings the system from the initial state $x(0) = C$ to the state $x_j(k)$, is uniquely determined by the vertex. Therefore, the return function can be computed for the stages $1, 2, \ldots, k$ and it is:

$$J_p = \sum_{m=0}^{k-1} \frac{1}{(1+i)^m} \left\{ \sum_{\lambda=m}^{N-1} \frac{\ell_1(u(m))}{(1+i)^{\lambda-m}} + \ell_2(x(m+1), m+1) \right\}$$

It is obvious that any trajectory which includes the vertex $v_j(k)$, under consideration, i.e.

$$t_{t \in S_j(k)} = \{u(0), u(1), \ldots, u(k-1), *, *, \ldots\}$$

will have a return function

$$J = J_p + \varepsilon$$

where ε is a non-negative quantity, unknown at the present time. This statement is obvious by inspecting the definition of the performance criterion.

However, we are interested in the controls u(k) that can be applied at stage k. It is expedient, therefore, to analyze the non-negative quantity ε as follows:

$$\varepsilon = \frac{1}{(1+i)^k} \left\{ \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+i)^{\lambda-k}} + \ell_2(x(k+1),k+1) \right\} + \varepsilon'$$

where ε' is another non-negative quantity, unknown at the present time.

Recall that given a vertex ($v_j(k)$ in this case) and an admissible control u(k), a successor vertex is uniquely determined, say $v_i(k+1)$. Consider now the set of trajectories which include the vertex $v_i(k+1)$:

$$t = \{u(0),u(1), \ldots ,u(k-1),u(k),*,*,*, \ldots\}$$

$$t \varepsilon S_i(k+1)$$

The performance criterion of any of these trajectories is:

$$J = J_p + \frac{1}{(1+i)^k} \left\{ \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+i)^{\lambda-k}} + \ell_2(x(k+1),k+1) \right\} + \varepsilon'$$

It is obvious now that the set $S_i(k+1)$ may include a trajectory "better" than t^1 if and only if

$$J_p + \frac{1}{(1+i)^k}\left\{\sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+i)^{\lambda-k}} + \ell_2(x(k+1),k+1)\right\} + \varepsilon' < \bar{J} \tag{62}$$

A trajectory t is "better" than t^1 if and only if it yields a return function J less than $\bar{J}$ (minimization problem). This definition eliminates the necessity to prove condition (62).

Condition (62) can be rewritten:

$$\frac{1}{(1+i)^k}\left\{\sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+i)^{\lambda-k}} + \ell_2(x(k+1),k+1)\right\} < \bar{J} - J_p - \varepsilon' \le \bar{J} - J_p$$

The last inequality follows from the fact that ε' is a non-negative quantity

$$\frac{1}{(1+i)^k}\left\{\sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+i)^{\lambda-k}} + \ell_2(x(k+1),k+1)\right\} < \bar{J} - J_p \tag{63}$$

Inequality (63) is an optimality condition for all controls u(k) which can be applied at vertex $v_j(k)$. It is important to stress the fact that the optimality condition (63) depends on the vertex since J_p is generally different at different vertices.

The optimality condition (63) can be transformed into more useable forms. For example, it can be transformed into maximum permissible number of circuit miles in the control u(k).

The above derived optimality condition, very simple in principle, has a tremendous impact on the efficiency of the overall algorithm. It can be used directly in the automatic generation of controls algorithm to eliminate a large number of controls which do not belong in the optimal trajectory. Numerical evaluation of the impact of the optimality condition on the efficiency of the algorithm is given in Chapter VII.

Starting Upper Bound

At the beginning of the algorithm it is not known what the values of the performance criterion might be. On the other hand, it is desirable to have an upper bound on the value of the performance criterion J in order to speed up the fathoming procedure. If such an upper bound is not known, it is necessary to assume that J is unbounded until a bound has been found. In other words:

$$\bar{J} = \infty \tag{54}$$

where $\bar{J}$ denotes an upper bound for the criterion J.

It should be noted that the algorithm can work with the starting upper bound of equation (54). However, the overall algorithm speeds up if we relax equations (III) and (IV) until we find a "better" upper bound than the one in equation (54). For the minimization problem an upper bound $\bar{J}_2$ is "better" than $\bar{J}_1$ if and only if

$$\bar{J}_2 < \bar{J}_1$$

Relaxation of the relations (III) and (IV) is meant in the sense that only controls (states) which belong to a subset of $\cup(x(k),k+1)$ $(X(x(k-1),u(k-1),k))$ are considered. The reason for doing so is that the objective here is to find a feasible trajectory as fast as possible. The performance criterion computed for this trajectory, which will be finite, shall be an upper bound for the overall problem, better than the one of the equation (54).

Figure IV.4 depicts the algorithm which by relaxing constraints (III) and (IV) searches for a feasible trajectory and then computes the performance criterion of this trajectory. This value can be used as the starting upper bound on J in the main algorithm. The key idea in Figure IV.4 is to force the automatic generation of alternative algorithm to generate only one admissible control if possible (in the worst case only few admissible controls). The number of generated admissible controls can be controlled with the parameters x_{over} and x_{cut} [Chapter III]. However, since strict values of the parameters x_{over} and x_{cut} might yield no admissible control, an iterative loop, in which the values of these parameters are successively lowered, is necessary to guarantee the detection of at least one admissible control. The rest of the flowchart is self-explanatory.

The above idea can be viewed as follows: The optimization algorithm is divided into two phases.

<u>First phase</u>: Determine, as fast as possible, a feasible trajectory and compute its performance criterion which is to be used as the starting upper bound for phase two.

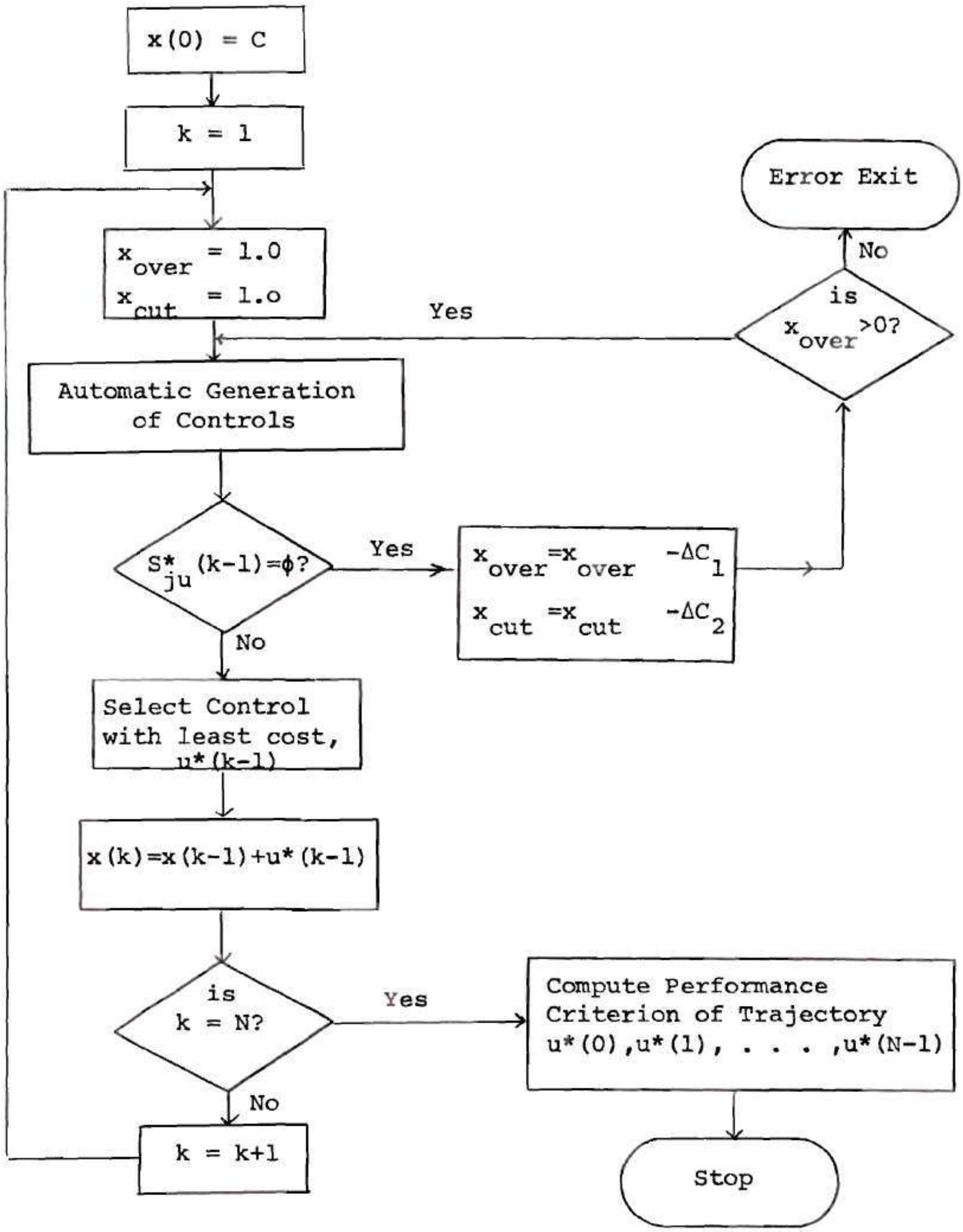


Figure IV.4. Search for a Finite Upper Bound on the Performance Criterion J.

Second phase: Let the upper bound of J be the one found in the first phase and proceed to the solution of the problem with the non Linear Branch and Bound.

The use of starting upper bound increases the efficiency of the non Linear Branch and Bound algorithm. Quantitative evaluation is given in Chapter VII.

In summary, the efficiency increasing modifications of the algorithm can be incorporated in the basic algorithm. The result is illustrated in the flowchart of Figure IV.5.

Computational Aspects and Storage Requirements

A very attractive feature of the method is the low storage requirements. It basically requires the storage of N-states x(k), k=0, 1, . . . ,N-1 and N sets of controls $S^*_{ju}(k)$, k=0,1, . . . ,N-1 and the data. In this way, in core solutions can be achieved.

Now let us consider the state x(k) and the corresponding set $S^*_{ju}(k)$, as in Figure IV.3.

Let us assume that the DC-load flow matrix of the state x(k) is known and has been triangulated. To evaluate a successor state $x_i(k+1)$, a series of load flows is required. Normally, we should form the DC-load flow matrix for the state $x_i(k+1)$ then triangulate it and proceed to the load flow analysis. However, because the control matrix $u_i(k)$ is highly sparse, a faster approach can be used. Load flow analysis of the state $x_i(k+1)$ can be performed with the triangulated DC-load flow matrix of the state x(k) and the control $u_i(k)$ by using the well known matrix inversion lemma. In so doing, the triangulation of the DC-load

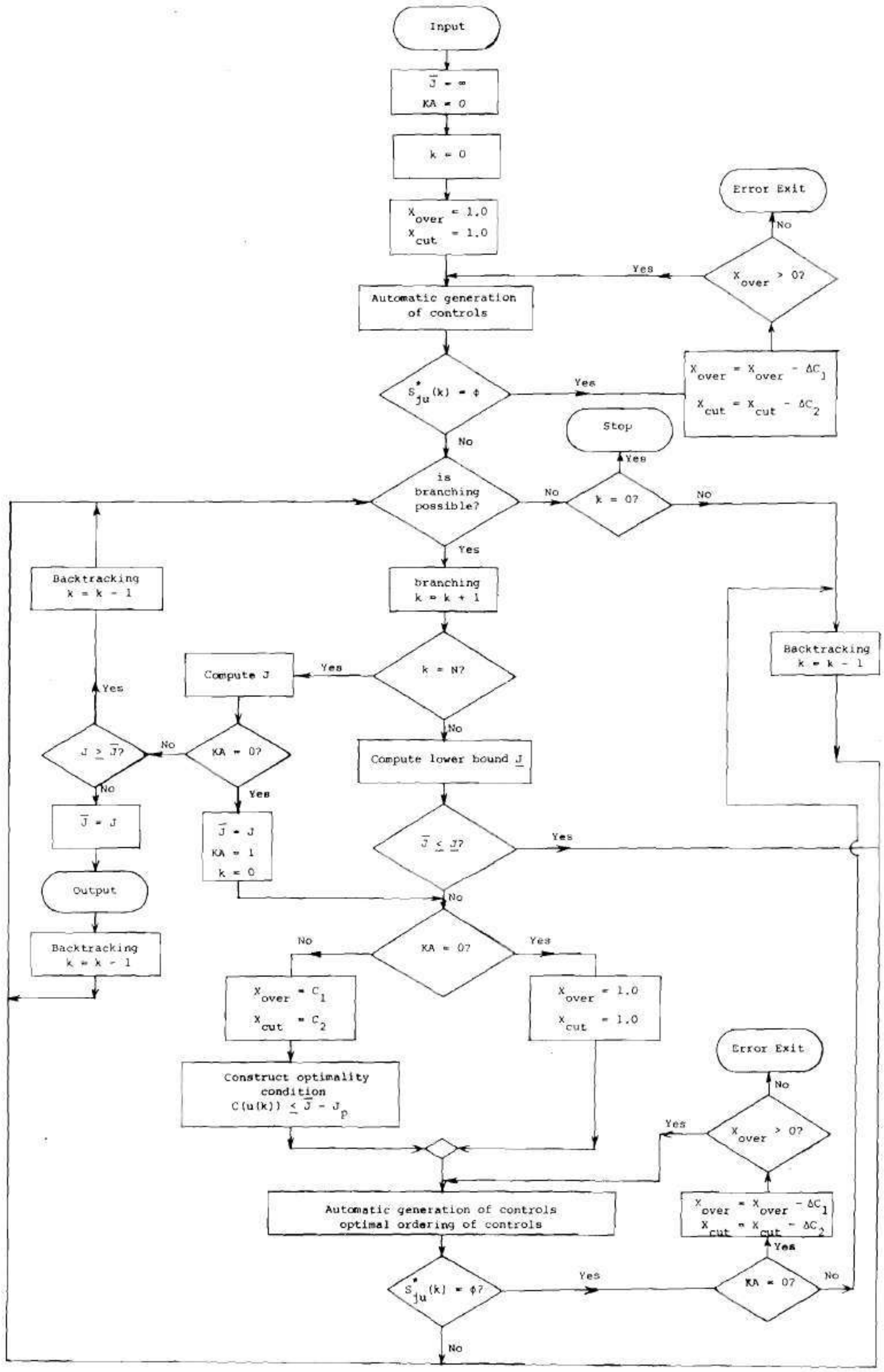


Figure IV.5. The Planning Algorithm

flow matrix of state $x_i(k+1)$ is avoided.

On the other hand, the algorithm is receptive to problem reduction ideas: For the state $x(k+1)$, stage $k+1$, it can be readily determined which security-reliability constraints are effective or near effective. These constraints shall be called the working constraints. The working constraints represent a small percentage of the total number of constraints. Now the states $x_1(k+1), x_2(k+1), \ldots$ can be checked for admissibility with the small set of the working constraints. One should not conclude that in doing so there is the possibility that an inadmissible state might be taken as admissible. This is so because a state which satisfies the working constraints and whose corresponding vertex cannot be fathomed, is checked to determine if the set of security-reliability constraints and the set of working constraints are equivalent for that particular state and stage. If yes, no action is taken. If not, the admissibility of the state has to be rechecked.

A problem associated with computational and storage requirements is present in the non Linear Branch and Bound algorithm. To illustrate this problem let us assume that at stage k and state $x(k)$ (vertex $v_j(k)$) the automatic generation of controls has been called to generate the set $S^*_{ju}(k)$. Furthermore, assume that the controls $u(k)$ of $S^*_{ju}(k)$ have been accounted for and therefore vertex $v_j(k)$ has been abandoned as well as the set $S^*_{ju}(k)$. Now assume that later in the algorithm the vertex $v_i(k)$ is considered. Assume that the trajectories from the initial state to $v_j(k)$ and $v_i(k)$ are complimentary. Therefore the associated state with vertex $v_i(k)$ is $x(k)$ identical to the one with vertex $v_j(k)$. If the separation at vertex $v_i(k)$ is needed, the automatic generation of controls algorithm has to be executed to determine the set $S^*_{ju}(k)$, since this set is not available at the present time. It is obvious that a repetition of the same exact calculations has occurred. It is possible to avoid this repetition but at a price: instead of disposing the sets $S^*_{ju}(k)$ according to the basic algorithm, they can be stored and recalled appropriately. The price to be paid is the increased storage requirements.

Optimality

An enumerative approach to an optimization problem always provides the global optimum for any class of problems assuming that the enumeration is complete. Therefore the question of optimality is actually a question of completeness of the enumerative scheme. By inspecting the enumeration tree of Figure IV.1, it is obvious that the enumeration is complete if and only if the set $S^*_{ju}(k)$ is complete in the sense that all admissible controls have been included.

Recall the way the set $S^*_{ju}(k)$ is constructed. Let $S_a(k)$ be the set of all possible controls at some vertex $v_j(k)$--the set $S_a(k)$ may or may not depend on vertex $v_j(k)$. The automatic generation of controls yields a set of controls $S_b(k)$[†] which is a subset of $S_a(k)$

$$S_b(k) \subset S_a(k) \tag{64}$$

The controls in the set $S_a(k) - S_b(k)$ have been left out of the

---

[†] The set $S_b(k)$ is identical to the set $U(x(k),k+1)$ of Chapter III.

optimization process. Each control in the set $S_b(k)$ is checked with the feasibility and optimality conditions. The controls which satisfy both feasibility and optimality conditions constitute the set $S^*_{ju}(k)$.

The question now is: Does the disposal of the controls u(k), $u(k) \epsilon (S_a(k) - S_b(k))$ affect the optimality of the solution? To answer this question, a discussion is presented in this section and numerical results in Chapter VII.

The formation of the set $S_b(k)$ [Chapter III] is not random but rather sophisticated and experimentally successful selection of controls which have a chance of being admissible and close to the optimal. Therefore, the set $(S_a(k) - S_b(k))$ consists of controls which have very low chance of satisfying both the admissibility and optimality conditions. Of course, if it can be assured that the set $S_a(k) - S_b(k)$ does not possess a control u(k) which can satisfy both the feasibility and optimality conditions, then the overall method will be globally optimum. Another way to guarantee global optimality is to make

$$S_b(k) \equiv S_a(k) \tag{65}$$

However, it is not practical to adopt the above suggestion in order to guarantee global optimality. The reasons are:

(a) The number of controls in the set $S_a(k) - S_b(k)$, which satisfy both feasibility and optimality conditions, is very small. A normalized measure of the above quantity is defined as follows:

$$p_s = \frac{\text{Number of Controls in the Set } (S_a(k)-S_b(k)) \text{ which Satisfy Feasibility and Optimality Conditions}}{\text{Total Number of Controls in the Set } S_a(k)} \quad (66)$$

Obviously, if $p_s = 0$, the planning method of this thesis is globally optimal.

It is possible to compute p_s for small networks. This has been done and the results are illustrated in Chapter VII, Figure VII.11.

2. It has been experimentally observed that there is a minimum size of the set $S_b(k)$ which yields the global optimum to a certain problem. Further increase of the size of the set $S_b(k)$ is not profitable. Since the size of the set $S_b(k)$ depends on the values of the parameters x_{over} and x_{cut}, it will be expedient to talk about the values of x_{over} and x_{cut} instead of the size of the set $S_b(k)$.

Post-Optimality Analysis

Following the attainment of the optimal solution to the transmission planning problem it is always desirable to study the effect of discrete changes in the various parameters of the problem on the current optimal solution. One way to accomplish this is to solve the problem anew. This, however, may be computationally inefficient. If one makes use of the properties of the non Linear Branch and Bound method, the additional computational effort to determine the optimal for new values of the variables is considerably reduced. This is the objective of post-optimality analyses.

Post-optimality analyses can be of different kinds depending on the changes considered:

1. Changes in the parameters of the performance criterion.
2. Changes in the constraints, i.e. changes in the definition of an admissible network.

1. Post-optimality analysis, when changes in the parameter values of the performance criterion occur, is easily performed.

The optimal trajectory t* specifies a value of the performance criterion J*. This value is basically a function (functional to be precise) of various parameters such as cost of one unit of energy, interest rate, annual investment cost plus interest of a transmission line type A constructed on the right of way m at stage k, etc. For simplicity, let us write

$$J^* = J^*(p_1, p_2, \ldots, p_\nu)$$

where $p_1, p_2, \ldots$ are the various parameters.

Let us assume a change Δp_i in the parameter p_i.

$$p_i' = p_i + \Delta p_i$$

Further, assume that solution to the problem has been attained for the new parameter value p_i' and that the new optimal trajectory is t'* (may or may not be identical to t*) and the new value of the performance criterion is J'*. On the other hand, evaluation of the performance criterion along the trajectory t'* and parameter value p_i (old value) yields

$$J'^* = J^* + \Delta J^*(\Delta p_i)$$

where $\Delta J^* \geq 0$. The change in the performance criterion is a functional of the change in the parameter value.

In the real world, each parameter takes values in a range with some probability distribution. The procedure just described yields a range of values for $\Delta J^*(p_i/i = 1,2, \ldots ,\nu)$. This mapping of the statistics of the parameters into the statistics of ΔJ^* is straightforward but computationally huge task. However, if it is assumed that the statistics of ΔJ^* are known, then given a probability level p we can find a number $\overline{\Delta J}$ such that

$$\Delta J^* \leq \overline{\Delta J} \text{ with probability } p.$$

It follows that given:

(a) The ranges of the parameter values p_i, $i = 1,2, \ldots ,\nu$ and their probability distribution.

(b) The solution of the optimization problem [trajectory t*, performance criterion J*] with parameter values equal to their expected values.

Then the optimization problem solved again with fixed upper bound at the level of

$$\bar{J} = J^* + \Delta J = x_c J^*, \; x_c = 1 + \frac{\overline{\Delta J}}{J^*}$$

will yield a set of trajectories S_t, namely

$$t \epsilon S_t \leftrightarrow J(t) \leq J^* + \overline{\Delta J}$$

which is not the null set since $t^* \epsilon S_t$. With probability p, the set S_t contains the optimal trajectory for any combination of parameter values.

If the value x_c were known from the beginning, we can attain the set S_t with only one solution of the non Linear Branch and Bound algorithm: in the flowchart of Figure IV.5, it is sufficient to replace the upper bound $\bar{J} = J$ with $\bar{J} = x_c J$. It is obvious that the so defined upper bound $\bar{J} = x_c J$ will always be greater or equal to $x_c J^*$.

$$\bar{J} = x_c J \geq x_c J^*$$

Therefore, the set of trajectories, S_t' attained in the above way will possess all trajectories t, satisfying

$$J(t) \le x_c J^*$$

and possibly some other trajectories. Therefore,

$$S_t \subset S'_t$$

Given the set S_t (or S'_t), post-optimality analysis is easily performed.

In practice it is possible to obtain a good estimate for the value of x_c. In most cases the parameters p_i, $i=1,2, \ldots, \nu$, which enter in the computation of the performance criterion, have a very narrow distribution. In this case, experience with the non Linear Branch and Bound for the transmission planning problem shows that the variable $\overline{\Delta J}/J^*$ has a very narrow distribution too. Therefore, a good estimate for the variable x_c will be a value few percents over unity.

To summarize post-optimality analysis, the following tasks are involved:

(a) If J^* is known:

1. Estimate a value for x_c.
2. Solve the non Linear Branch and Bound problem again with $\overline{J} = x_c J^* =$ constant. The set S_t will be attained.
3. Perform post-optimality computations for the set S_t only.

(b) If J^* is not known:

1. Estimate a value for x_c.

2. Solve the non Linear Branch and Bound problem with the relationship $\overline{J} = J$ substituted by $\overline{J} = x_c J$. The set S_t' will be attained.

3. Perform post-optimality computations for the set S_t' only.

2. When changes in the constraints are considered--for example, a control center is installed sometime during the planning period and therefore, the definition of an admissible network should change--post-optimality analysis cannot be performed as easily as in 1. The reason is that when there are changes in the constraints, the value of the performance criterion may change drastically. Therefore, it is recommended that in this case the problem should be solved anew.

CHAPTER V

A COMPARISON BETWEEN THE NON LINEAR BRANCH AND BOUND AND DYNAMIC PROGRAMMING

General

The transmission planning problem which was formulated in Chapter II can be solved by Dynamic Programming too--at least in theory. There are, however, many practical difficulties which will be investigated in this chapter.

Dynamic Programming was originally developed by Richard Bellman. It is a powerful approach for solving multistage optimization problems. It has been applied extensively in many fields such as inventory theory, allocation problems, control theory, chemical engineering design, production scheduling, capital budgeting, and others. The approach has many advantages, some of which follow.

(a) The problem formulation can be very general. Nonlinearities in the equations can easily be handled.

(b) Variables can be discrete.

(c) Constraints can be applied to both decision and state variables (constraints usually reduce the computational burden instead of increasing it as opposed to many other optimization methods).

(d) Questions of the uniqueness of the solution are avoided. As long as the problem is feasible, the direct procedure guarantees that the optimum (within the rights of the model) will be found.

(e) The optimal solution is obtained in a feedback form, i.e. optimal decisions are obtained for each admissible state of the system at each instant of time.

The above features of Dynamic Programming make it suitable for solving multistage decision processes. Long range transmission network planning is a typical multistage decision process. Unfortunately, this problem is dimensionally large and application of Dynamic Programming is very difficult for the following reasons:

1. Due to the large number of stages that have to be considered in the optimization process, a large amount of high-speed storage is required during the computations.
2. The important problems of escalation of costs and construction lead time tend to increase the dimensionality of the problem.

In the following, we will analyze the above two restrictive reasons in applying Dynamic Programming in long range transmission network planning.

Storage Requirements

The transmission network planning problem can be considered as a sequential process of discrete control actions. This problem can be formulated to be solved by Dynamic Programming. The efficiency of the Dynamic Program depends strongly on the formulation. From this point of view the definition of the state of the system is very crucial. The most judicious definition for application of Dynamic Programming to the transmission planning problem has been the one in reference 16. This definition of the state of the system coincides with the one we have presented in Chapter II. Then the problem can be formulated as follows.

(i) A system described by the equation of motion

$$x(k+1) = x(k) + u(k) \tag{I}$$

where:

x = Base case state matrix, LxM dimensioned

M = number of rights of way

L = number of discrete circuit types

u = transition matrix, LxM dimensioned

k = index of stage variable

(ii) A variational performance criterion

$$J = \sum_{k=0}^{N-1} \frac{1}{(1+r)^k} \{ \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+r)^{\lambda-k}} + \ell_2[x(k+1),k+1] \} \tag{II}$$

where:

J = total present worth cost

ℓ_1 = annual investment cost plus interest of the transition

ℓ_2 = operational cost

r = annual interest rate

(iii) Constraints

$$u \epsilon U(k) \tag{III}$$

$$x \epsilon X(k) \tag{IV}$$

where:

$X(k)$ = set of admissible states at stage k

$U(k)$ = set of feasible transitions

(iv) An initial state

$$x(0) = C \tag{V}$$

Find:

The state sequence $x(1), x(2), \ldots, x(N)$ such that J in equation II is minimized, subject to the equation of motion I, the constraints III and IV, and the initial condition V.

The basic difference between the above formulation and the one presented in Chapter II is that here the decision variables are the discrete states of the system while in Chapter II the decision variables are the discrete controls.

The two formulations are equivalent since the sequence of controls (or transitions for the present formulation) $\{u(0), u(1), \ldots, u(N-1)\}$ can be obtained from the sequence of states $\{x(1), \ldots, x(N)\}$ with the aid of the equation of motion I.

The actual system equations and inequalities which determine if a state $x(k)$ is admissible remain the same as in Chapter II. Therefore, the term admissible state will refer to a state which satisfies the definition of admissibility I or II (Chapter II).

Dynamic Programming analyzes the multistage decision process into a series of single stage optimization problems. One of these single stage optimization problems can be stated as follows: Given a set of states $X(k)$ at stage k and a set of states $X(k+1)$ at stage $k+1$, find for each state of the set $X(k+1)$ the optimal transition.

It is obvious that solution of the above single stage optimization will require that the sets $X(k)$ and $X(k+1)$ be a priori known. And taking these arguments one step further for a N-stage problem, the set X should be given, which is defined as follows:

$$X = X(1) \cup X(2) \cup \ldots \cup X(N) \tag{67}$$

The size of the set X is very crucial because each element of the set X requires a large number of variables to be stored. And if the set X is large, then the storage requirements may be unbearable.

The number of possible discrete states for a transmission network is mighty large. For comparison purposes, however, it will be beneficial to ask the following question: How many discrete states should be included in the set X in order to establish equivalence between the planning problem solved by the method described in this thesis and by Dynamic Programming. The answer to the above problem is achieved by the following procedure. The N-stage transmission planning problem is solved by the method of this thesis where the fathoming procedure is relaxed. During execution the vertices are stored in N sets, one for each stage. This procedure will yield the sets V(1),V(2), . . . ,V(N) where V(k) is the set of vertices generated in stage k. It is obvious that the set V(k) can yield the set X(k) by consolidating the states associated with the set of vertices V(k). And finally, the set X is obtained from (67).

The above procedure has been applied to a small network, namely the test system A (Chapter VII). The test system A is a 5-node, 7-branch system. For even this small network the set X was large. Table V.1 shows the results.

Similar results for large networks are almost impossible to obtain because of the large number of discrete states in the set X. Then the storage requirements are tremendous and application of Dynamic Programming to the transmission network planning will require an enormous amount of fast storage devices. On the other hand, the non Linear Branch and Bound method alleviates the storage problem since it is only necessary to store, for each stage, a small set of controls which corresponds to a subset of X (see Separation, Chapter IV).

In reference [16] the state of the system is defined with only one state variable. This variable is an identification one which identifies one network configuration from another. The storage problem is still

Table V.1.* Size of the Set X for

Test System A (Five-Bus, Seven-Branch)

Number of Stages in the Planning Period	Number of Discrete States in the Set X
3	177
4	432

* Parameter values used for the automatic generation of controls

$x_{over} = 0.95$ $\qquad$ $x_{cut} = 0.20$

Assumptions:

1. At most one circuit per right of way and stage
2. Only one type of circuit is used for expansion

there since each network configuration, which is identified by one variable, requires storage space which depends on the size of the network.

In summary, the application of Dynamic Programming to the transmission planning problem encounters the burden of excessive storage requirements. It is difficult to handle this problem with present computers. On the other hand, the non Linear Branch and Bound method has moderate storage requirements (Chapter IV). As a matter of fact, in core solutions can be obtained for even large networks (Chapter VII).

Construction Lead Time

In planning a transmission network, a decision to invest has to be made before the investment is actually needed. This is because some lead time is required to implement this decision. In this case the transition from one state to another is constrained. For example, if the transition from state x_i to the state x_j is decided in year k and a lead time of λ years is required, then

$$x(t) = x_i, \qquad t = k, k+1, \ldots, k+\lambda-1$$

$$x(k+\lambda) = x_j \tag{68}$$

In this example it has been assumed that the construction lead time is always an integer number of stages and that once a specific transition has been decided, no other transition can be decided until the current transition has been terminated.

Regarding this problem, the following observations are important:

1. If each admissible state of the system is identified by one variable only, the equation of motion describes transitions from one state to another. On the other hand, it is imperative to consider as decision variables the admissible states of the system for a judicious formulation of the problem for application of Dynamic Programming. This fact has its impact which is described in the following observations.
2. As long as the lead time for a transition is greater than one stage, there will be a set of constraints similar to (68).
3. If the controls are considered as the decision variables of the problem, then the problem of lead time has automatically become independent of the planning problem. This is so because a sequence of controls can always be analyzed into a sequence of decisions. In this case the only constraint will be that a decision can not be made prior to the beginning of the planning period if it has not been made before. Therefore, the formulation presented in this thesis (Chapters II, III, and IV) automatically solves the lead time problem because it decouples

it from the general planning problem.

4. Constraints similar to (68) tend to increase the dimensionality of the problem.

The above observations make it clear that in applying Dynamic Programming for the transmission planning problem extra developments are necessary if lead time is to be considered. In reference 16 two methods are proposed. One method increases the state space by artificial states which correspond to the constraints (68) and then the solution is achieved with a usual Dynamic Program. The other method does not increase the state space, but requires that, at each stage k, the optimal expected returns at several posterior stages be known. Actually, those returns at stages $k+1, k+2, \ldots, k+\lambda+1$ where λ is the largest lead time corresponding to a decision allowed in stage k.

In conclusion, the inclusion of construction lead times in a Dynamic Program for the transmission network planning increases the dimensionality of the problem. On the other hand, the non Linear Branch and Bound method decouples the problem of lead times from the planning problem and therefore automatically solves the planning problem with construction lead times.

Escalation of Costs

In every planning task the escalation of cost is a very important factor. It has been the case and will always be that inflationary trends and changes in the economic environment in general bring about changes in the cost of the same resource. In a planning study it is desirable to evaluate the impact of the escalation of cost on the overall cost of expanding the system over a given period of time.

In the general formulation of the problem, the various costs will be a function of the stage variable if escalation of cost is considered. The non Linear Branch and Bound can handle the problem automatically because it is basically an enumerative optimization algorithm. On the other hand, the usual Dynamic Program can not solve this problem unless certain modifications are made. As a matter of fact, it is easy to make a counterexample where the usual Dynamic Program may skip the known optimal trajectory if escalation of cost is present.

As in the case of construction lead time, it is possible to modify the problem in order to account for escalation of cost. Here we propose two methods:

1. The state variables of the system are increased by another vector, the vector of equipment age. Then two discrete states of the system can differ by as little as the age of one equipment. In this case the transition cost from one state to another will incorporate any foreseeable escalation of cost. A usual Dynamic Program can solve the above problem.
   It should be emphasized, however, that the number of states increases drastically because of the introduction of the additional state vector.
2. For each state, at a stage k, a set of values

> of the performance criterion should be stored, corresponding to all possible trajectories which take the system from the initial state to the present state. Each state will enter the next stage computations with a set of values of the performance criterion.

Both proposed methods increase the dimensionality of the problem and may impose heavy storage requirements.

It should be emphasized that the above two proposed methods lead in a natural way to the concept of a vertex, which has been introduced in Chapter IV. In a planning study a state of the system, defined by the existing equipment, is not a uniquely defined economic entity. The trajectory which brings the system from the initial state to the state under consideration may differentiate the economic cost of the system (performance criterion). The reason, of course, is the escalation of cost. Therefore, it is expedient in a planning study to consider a state of the system, not alone, but with the trajectory which creates this state from the initial state. This is the concept of a vertex (Chapter IV) from another point of view.

In summary, the cost escalation case of the planning problem can be solved by Dynamic Programming. This, however, leads to further increase of the dimensionality of the problem.

Conclusion

Dynamic Programming is a powerful technique with unlimited theoretical possibilities. However, application of Dynamic Programming to the planning of a transmission network encounters huge practical limitations. For even small networks, an enormous number of states has to be accessible for computations at each stage. This number increases drastically with the number of stages in the planning period. On the other hand, researchers who have applied dynamic programming to network planning consider the problem of defining the admissible states of the system to be separated from the optimization problem. This thesis has shown that these two tasks should be coupled because very useful information for the determination of the admissible states can be obtained from the optimization method. Dynamic Programming is susceptible to application of this finding.

Two important cases of the planning problem, the construction lead time and the escalation of cost cases can be handled by Dynamic Programming at least in theory. Modification of the definition of the state of the system will embed the general transmission planning problem into a problem solvable by Dynamic Programming. The dimensionality of the problem is, however, further increased and the practicality of the application of Dynamic Programming for this problem is questioned.

CHAPTER VI

FORMULATION AND COMPUTATIONAL ASPECTS WITH EXACT POWER FLOW MODEL

Introduction

The formulation of the transmission planning problem, which was introduced in Chapter II with the equations I, II, III, IV, and V, is very general. The power flow model of the transmission network does not explicitly enter these equations. However, for purposes of computing the performance criterion (Equation II) and the set of admissible controls (Relation III), it is necessary to compute the actual flow of power on the network. Kirchhoff's network laws describe the power flow. Solution of the power flow equations will give the answer to the above problem. These equations are, however, non linear for networks operating with alternating current. Historically, the exact solution is referred to as AC load flow. Many algorithms have been proposed for the solution of the AC load flow problem. Reference 21 presents a concise review of all these methods. All methods involve an iterative scheme, and therefore require considerable computing effort. For this reason, in Chapter II the AC load flow has been replaced with the DC load flow. The DC load flow is obtained from the AC load flow if certain approximations are introduced in the power flow equations. The name comes from the resemblance of these approximate equations to the Kirchhoff's equations for a network operating with direct current. DC load flow solution is obtained from the simultaneous solution of a set of linear equations. No iterative scheme is required. For the above reason, the approximate DC load flow is preferred for planning purposes.

In recent years, another method has been developed for the solution of the AC load flow problem. This method is known as the fast decoupled load flow (FDLF) and is published in reference 20. The important feature of this method is that, implementation wise, does not differ from the DC load flow. From the computational point of view, it is a few times slower than a DC load flow. The planning method of this thesis can be implemented with the fast decoupled load flow at minimum effort.

In this chapter, the transmission planning problem is formulated with the exact AC load flow. The emphasis is put on the computational aspects.

Formulation

The general transmission planning problem is again formulated with the relations I, II, III, IV, and V, which are cited again.

(i) A system described by the linear difference equation

$$x(k+1) = x(k) + u(k) \tag{I}$$

(ii) A variational performance criterion

$$J = \sum_{k=0}^{N-1} \frac{1}{(1+r)^k} \{ \sum_{\lambda=k}^{N-1} \frac{\ell_1(u(k))}{(1+r)^{\lambda-k}} + \ell_2[x(k+1),k+1] \} \tag{II}$$

(iii) Constraints

$$u \epsilon U(x(k),k+1) \tag{III}$$

$$x \epsilon X(x(k-1),u(k-1),k) \tag{IV}$$

(iv) An initial state

$$x(0) = C \tag{V}$$

Find the control sequence u(0),u(1), . . . ,u(N-1) such that J in equation II is minimized, subject to the system equation (I) the constraint equations (III) and (IV), and the initial condition (V). The variables have been defined in Chapter II.

In the above formulation, the following tasks require the solution of the power flow problem:

1. Computation of the operational cost $\ell_2(x(k),k)$.
2. Determination of the set U(X(k),k+1).
3. Determination of the set X(x(k),u(k), k+1).

Since the set X(x(k),u(k),k+1) is uniquely determined from the set U(x(k),k+1) and the equation of motion I, it is only necessary to discuss tasks 1 and 2.

In the following sections, the fast decoupled load flow will be presented, its similarities to the DC load flow will be explained and then the details of using the fast decoupled load flow in the above tasks (1 and 2) will be discussed.

The Fast Decoupled Load Flow

In recent years a highly efficient method for the solution of the AC load flow has been proposed [20]. It is an exact method for the solution of the power flow equations for a power transmission network. It is an attractive method because of the following reasons:

1. It minimizes storage requirements.
2. Its implementation is rather simple.
3. The speed of convergence is slightly slower than the Newton-Raphson method but the overall solution speed is higher.

The fast decoupled load flow can be best described with the well known Newton method which is an iterative algorithm for solving a set of simultaneous non linear equations in an equal number of unknowns.

$$F(x) = 0$$

where x is the vector of independent variables. In this case, the equations are the power flow equations which are derived from Kirchhoff's network equations. At a given iteration point, each function $f_i(x)$ is approximated by its tangent hyperplane. This linearized problem is constructed as the Jacobian-matrix equation.

$$F(x) = -J \cdot \Delta x \tag{69}$$

which is then solved for the correction Δx. The square Jacobian matrix J is defined by

$$J_{ik} = \frac{\partial f_i}{\partial x_k}$$

and represents the slopes of the tangent hyperplanes.

The nodes of a power network are classified into three classes for load flow purposes:

(a) PQ nodes where the externally injected real and reactive power are known.

(b) PV nodes where the externally injected real power and the magnitude of the voltage are known.

(c) Slack nodes (one in the system) where the voltage phasor is specified (magnitude and phase angle).

Let n_b be the number of nodes in the network and n_g the number of PV nodes. Then for the derivation of the fast decoupled load flow, the vector X is defined as follows:

$$X = \begin{bmatrix} \theta \\ V \end{bmatrix}$$

where:

θ is the vector of node voltage phase angles, dimension n_b-1, and

V is the vector of node voltage magnitude, dimension n_b-n_g.

The equations F(x) are

$$P(\theta,V) = 0, \qquad n_b-1 \text{ equations}$$

$$Q(\theta,V) = 0, \qquad n_b-n_g \text{ equations}$$

where $P_i(\theta,V) = 0$ is the total real power injected at node i, one for each node except the slack node, and $Q_i(\theta,V) = 0$ is the total reactive power injected at node i, one for each PQ node.

Then the Jacobian matrix equation can be written as

$$\begin{bmatrix} P(\theta,V) \\ Q(\theta,V) \end{bmatrix} = - \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$$

where:

$$H = \frac{\partial P(\theta,V)}{\partial\theta}$$

$$N = \frac{\partial P(\theta,V)}{\partial V}$$

$$J = \frac{\partial Q(\theta,V)}{\partial\theta}$$

$$L = \frac{\partial Q(\theta,V)}{\partial V}$$

The elements of the submatrices N and J have a small relative value and therefore represent a weak coupling between the vectors P and V on one hand and Q and θ on the other. These submatrices may be neglected yielding the two independent matrix equations:

$$P(\theta,V) = -H\Delta\theta$$

$$Q(\theta,V) = -L\Delta V$$

A series of approximations to the matrices H and L, which are justified by the physical properties of the power systems, lead to the transformation of the above equations into the following:

$$\frac{P(\theta,V)}{V} = B'\Delta\theta \qquad (70)$$

$$\frac{Q(\theta,V)}{V} = B''\Delta V \qquad (71)$$

where:

$$B'_{ik} = -\frac{1}{X_{ik}} \qquad (i \neq k)$$

$$B'_{ii} = -\sum_{k \in S_i} B'_{ik}, \qquad (S_i = \text{set of nodes connected to node } i)$$

$$B''_{ik} = B'_{ik}$$

X_{ik} = equivalent reactance of the circuits between the nodes i and k.

There are several logarithmic possibilities for the solution of the problem. The most efficient one involves successive solutions of (70) and (71). At the end of each solution [of (70) or (71)], the corresponding variable vector is updated (θ or V) and the maximum absolute mismatch

$$\left(\left|\frac{P(\theta,V)}{V}\right| \text{ or } \left|\frac{Q(\theta,V)}{V}\right|\right)$$

is computed. If the mismatch is less than a specified value, the algorithm is terminated. Figure VI.1 shows the flow diagram of the iterative scheme.

Both matrix equations (70) and (71) contain a constant matrix, namely, B' and B". The solution of either equation is obtained by first computing the symbolic inverse of the matrix (Gaussian elimination, table of factors) and then by forward and back substitution on the driving vector

$$\left[\frac{P(\theta,V)}{V}\right] \text{ or } \left[\frac{Q(\theta,V)}{V}\right]$$

The convergence of the method is good. Typical real cases yield

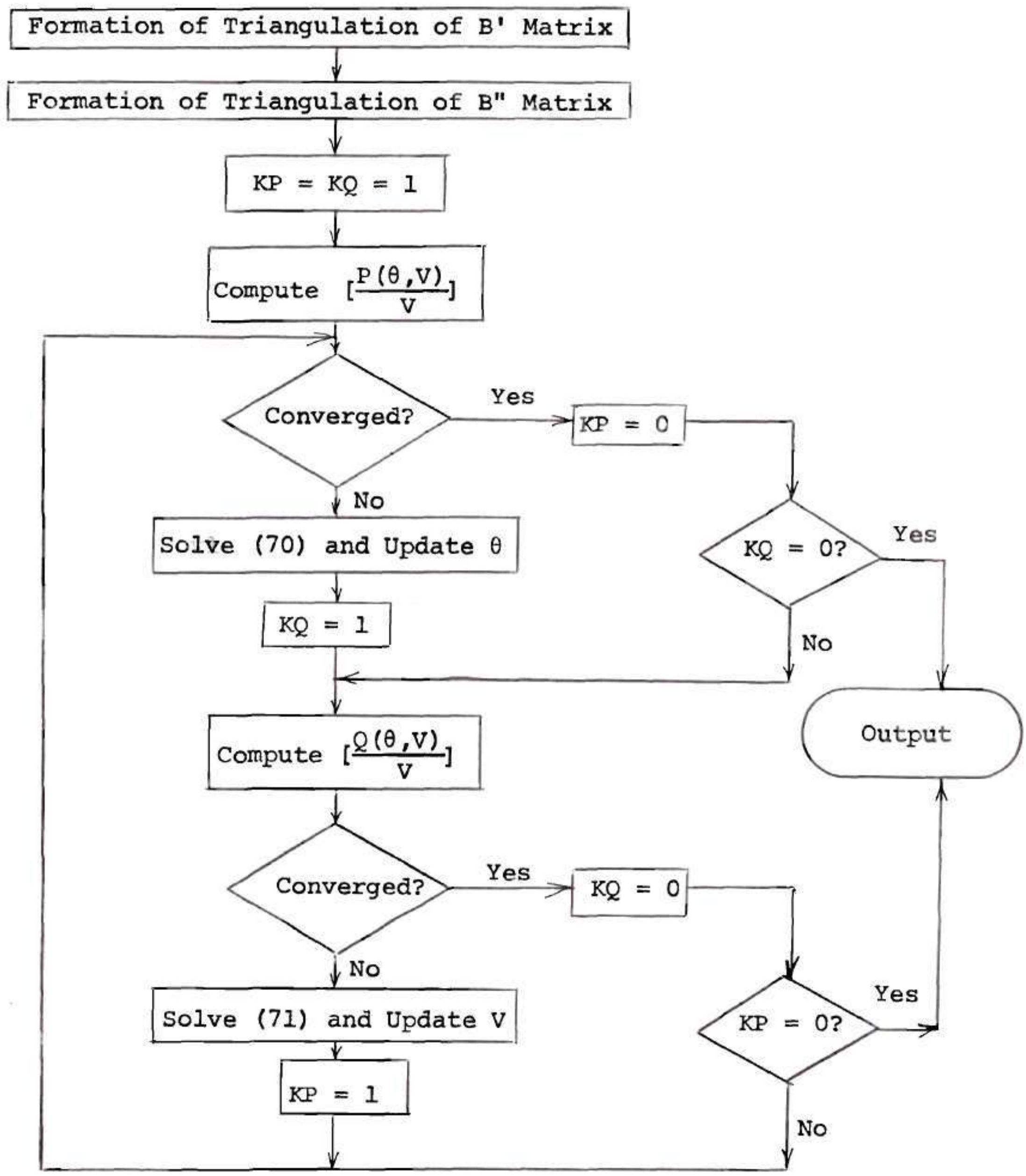


Figure VI.1. Flow Diagram of the Fast Decoupled Load Flow.

accuracy in the order of 0.1 MVA in three iterations starting from a "flat" start $[[\theta] = [0], [V] = [1]]$.

Computationally, the solution of the equation (70) is equivalent to the DC load flow. For comparison purposes, Table VI.1 presents a list of the tasks involved by the DC load flow and the fast decoupled load flow. The relative execution time for a 118 node system is given (data have been taken from reference 20). Assuming K=3, the execution of the fast decoupled load flow is 2.605 times longer than that of the DC load flow.

Reactive Power Sources

At the PV buses there are sources of reactive power. Their output is automatically adjusted in such a way that the voltage magnitudes remain constant. This control, however, is possible as long as the capabilities of the reactive power sources are not violated. In other words, the following constraints exist for PV buses

$$Q_i^{min} \le Q_i \le Q_i^{max} \tag{72}$$

where:

Q_i - actual injected reactive power at bus i

i - index for PV buses

Q_i^{min}, Q_i^{max} - minimum and maximum capability of the reactive power source at bus i

If in the final solution the above inequalities are violated at some bus(es), then the voltage magnitude can not remain constant at that

Table VI.1. A Comparison Between the DC Load Flow and the Fast Decoupled Load Flow.

Relative Execution Time* of the Tasks Involved in the Above Two Load Flow Methods (Data from Reference 20). K is the Number of Iterations.

	DC Load Flow	Fast Decoupled Load Flow
Formation and Triangulation of B'	.848	.848
Formation and Triangulation of B"	----	.212
Calculation of [P/V] and Convergence Test	----	.181K
Solution of (70) and θ Update	.152	.152K
Calculation of [Q/V] and Convergence Test	----	.152K
Solution of (71) and V Update	----	.03K

* For the IEEE test system (118 nodes).

bus (at least at the specified level). Then this bus has to be converted into a PQ bus.

From the planning point of view, it is necessary as the transmission capacity of the system expands, to expand the reactive power sources too. This problem is referred to as optimal VAR planning. The objective of the optimal VAR planning problem is to determine the location and the amount of controllable VAR sources in order that a given transmission network maintain adequate voltage levels and assist in optimal operation under normal and emergency conditions.

Several solutions have been proposed for the optimal VAR planning. However, since the cost of reactive power sources is few orders of magnitude smaller than the cost of the transmission facilities, it will be unwise to couple the VAR planning problem with the transmission planning problem. For this reason, we propose the following two approaches:

1. Assume that at each PV node there is a controllable reactive power source of unlimited capacity. Then the voltage magnitude at this node will be constant and at the specified level.

2. The performance criterion J (Equation II) is augmented with the following penalty function which basically represents the cost of controllable reactive power sources at the PV nodes.

$$J_{VAR} = \sum_i f_i\{\max_m[\max(y_{1i}S(y_{1i}),\ y_{2i}S(y_{2i}))]\}$$

where:

$$y_{1i} = Q_i^{(m)}(\theta^{(m)}, V^{(m)}) - Q_i^{max}$$

$$y_{2i} = Q_i^{min} - Q_i^{(m)}(\theta^{(m)}, V^{(m)})$$

$Q_i^{(m)}(\theta^{(m)}, V^{(m)})$ = reactive power necessary to maintain the voltage of node i at the desired level during outage m

Q_i^{min}, Q_i^{max} = minimum and maximum capability of the reactive power source at node i

$$S(y) = \begin{cases} 1 \text{ if } y>0 \\ 0 \text{ otherwise} \end{cases}$$

i = index of PV nodes

m = 0,1,2, . . . ,M index for single outages (Chapter II)

$f_i(x) = C_i X$

C_i = the annual investment cost plus interest for one unit of installed reactive power source at node i

The former approach is straightforward. The latter requires single contingency analysis. This requirement, however, does not create any deviation from the planning algorithm of the Chapters II, III, and IV since single contingency analysis is performed anyway in order to establish the admissibility of a state.

Computation of the Operational Cost $\ell_2(x(k),k)$

Computation of the operational cost $\ell_2(x(k),k)$ involves the solution of the power flow problem at different load levels. The load duration curve during the stage is assumed to be known. R points are selected on the load duration curve. Each point specifies the demand on the system. For this case, the outputs of the generation plants are known except that of the slack node. The load flow problem is solved and the losses are computed. This task is repeated for every selected point on the load duration curve. The total losses are computed as a weighted sum of the losses found in the above R solutions. The weights are defined as the duration of each load level during the stage.

Therefore computation of the operational cost requires the solution of R power flow problems. For comparison purposes, a listing of the computations required by the DC load flow and the fast decoupled load flow is given in Table VI.2.

Determination of the Set of Admissible Controls with the Fast Decoupled Load Flow

In Chapter III, the determination of the set $U(x(k),k+1)$ has been described. We cite again the basic tasks involved:

1. A single outage analysis which determines the set of the critically loaded circuits and critical outages.
2. A detection scheme which determines the set of rights of way which are effective for network reinforcement.
3. Construction of the controls based on the set of effective rights of way.
4. The generated controls are checked with the optimality and feasibility conditions. Those which meet the above conditions form the set

Table VI.2. Computational Requirements of the Operational Cost With the DC Load Flow and the Fast Decoupled Load Flow.

	DC Load Flow	Fast Decoupled Load Flow
Formation and Triangulation of B'	1	1
Formation and Triangulation of B"	----	1
Calculation of [P/V] and Convergence Test	----	R•K
Solution of (70)	R	RK
Calculation of [Q/V] and Convergence Test	----	RK
Solution of (71)	----	RK
Computation of Losses	R	R

$U(x(k),k+1)$.

The accuracy of the DC model is adequate for the detection scheme. Task number 3 does not involve a power flow. Therefore, only the single outage analysis and feasibility condition need to be formulated with the exact load flow.

Single Outage Analysis and Feasibility Condition. The computations which are involved in the single outage analysis and the feasibility condition are similar since the feasibility condition is basically a truncated single outage analysis. For this reason, only the implementation of the feasibility condition, with the exact power flow, will be discussed. The feasibility condition, in AC load flow notation, is written as follows:

$$P^{(m)}(\theta^{(m)},V^{(m)}) = 0 \tag{73}$$

$$Q^{(m)}(\theta^{(m)},V^{(m)}) = 0 \tag{74}$$

$$|P_\ell^{(m)}(\theta^{(m)},V^{(m)}) + jQ_\ell^{(m)}(\theta^{(m)},V^{(m)})| \leq \overline{S}_\ell(x(k),m) \tag{75}$$

$$m \in S_c$$

$$\ell \in S_u$$

where:

S_c - set of critical outages

S_u - set of critically loaded circuits

Equations (73) and (74) are the power flow equations during outage m,

$P_{\ell}^{(m)}(\theta^{(m)},V^{(m)})+jQ_{\ell}^{(m)}(\theta^{(m)},V^{(m)})$ is the power flowing on the circuits of the right of way ℓ, during outage m, and $\overline{S}_{\ell}(x(k),m)$ is the maximum permissible power to flow on the circuits of the right of way ℓ, during outage m.

The computational problem at hand appears as follows: Given the state x(k) of the system at stage k, and a control u(k), the state of the system at stage k+1 is then uniquely defined by the equation I

$$x(k+1) = x(k) + u(k)$$

Does the state x(k+1) satisfy the relations (73), (74), and (75) at stage k+1?

There are several algorithmic possibilities for this problem. Because the control u(k) involves only few circuit additions, we will base our discussion on the following algorithm.

1. Form and triangulate matrices B' and B" of the system x(k).
2. Use the well known matrix inversion lemma in the iterative scheme in order to obtain the solution $\theta^{(m)},V^{(m)}$ of the load flow for the system x(k+1) at stage k+1 and during outage m.
3. Check inequalities (75).

Table VI.3 lists the computations required for the feasibility condition by the DC load flow and the fast decoupled load flow. L_u is

Table VI.3. Computational Requirements of the Feasibility Condition With the DC Load Flow and the Fast Decoupled Load Flow.

	DC Load Flow	Fast Decoupled Load Flow
Formation and Triangulation of B'	1	1
Formation and Triangulation of B"	----	1
Computation of [P/V] and Convergence Test	----	M'K
Solution of (70)	$(L_u+2)M'$	$(L_u+1)M'K$
Computation of [Q/V] and Convergence Test	----	M'K
Solution of (71)	----	$(L_u+2)M'K$
Small Matrix Inversion	M'	M'K

NOTES: M' is the number of critical outages

K is the number of iterations

L_u is the number of circuits in the control u(k) the number of discrete circuits in the control u(k). The table has been prepared in accordance with the above algorithm. This algorithm is similar to the one used in a planning program implemented with the DC load flow. It is efficient for the feasibility condition with the DC load flow but inefficient when the fast decoupled load flow is used. In this case, optimization of the algorithm is imperative. For this reason, Table VI.3 overestimates the augmentation of the computational requirements with the fast decoupled load flow.

Conclusion

The general transmission planning method which has been presented in Chapters II, III, and IV can be formulated with an exact load flow model instead of the approximate DC model. The most important implementation details have been described and the additional computational requirements have been listed. The increase of the computational effort depends on required accuracy. An estimate of the relative computational effort increase for a given system and specified accuracy can be obtained from Tables VI.1, VI.2, and VI.3.

CHAPTER VII

PERFORMANCE EVALUATION

General

The described planning method of this thesis has been implemented and tested. The objective of the testing was to demonstrate the specific properties of the problem of power transmission network planning which lead to the present planning algorithm. From this point of view, the automatic generation of controls is very important. Detailed evaluation of this algorithm will be given.

The developed computer program can accommodate a network as large as 100 nodes, and 200 branches. The planning period can be as long as ten stages. In-core solution of this program requires approximately 74 K of core memory. For longer planning periods, the storage requirements increase by 3.7 K per stage.

Two test systems have been used for the evaluation of the planning algorithm. These systems and the associated data are presented in the next section.

The Test Systems

Figures VII.1 and VII.2 present the two test systems. Test system A has been taken from reference 15. It is a 5-bus, 7-branch system. The system is a model of certain parts of the Bonneville Power Administration transmission network. It is an "overall" system model. It consists of the main transmission arteries (230 kV and above) terminating at nodes that represent a number of geographically adjacent substations. Table VII.1 describes the state of the system at the beginning of the planning period and Table VII.2 the net power injections at year one. The net power injections are assumed to grow at a 5% annual rate. Finally, Table VII.3 gives the characteristics of the transmission lines.

Test System B is the Georgia Power Company transmission system. Transmission lines operating at 230 kV and above have been retained. Figure VII.2 illustrates the system. Table VII.4 describes the state of the system at the beginning of the planning period, while Table VII.5 lists the bus data for year one. It is assumed that the load at each bus increases at a 8.5% annual rate. Furthermore, it is assumed that in the third year of the planning period the generating capacity of the plant HATCH is increased by 800 MW. Similarly, during year five, the generating capacity of plant VOGTLE is increased at 850 MW. Finally, Table VII.6 lists the characteristics of the transmission lines.

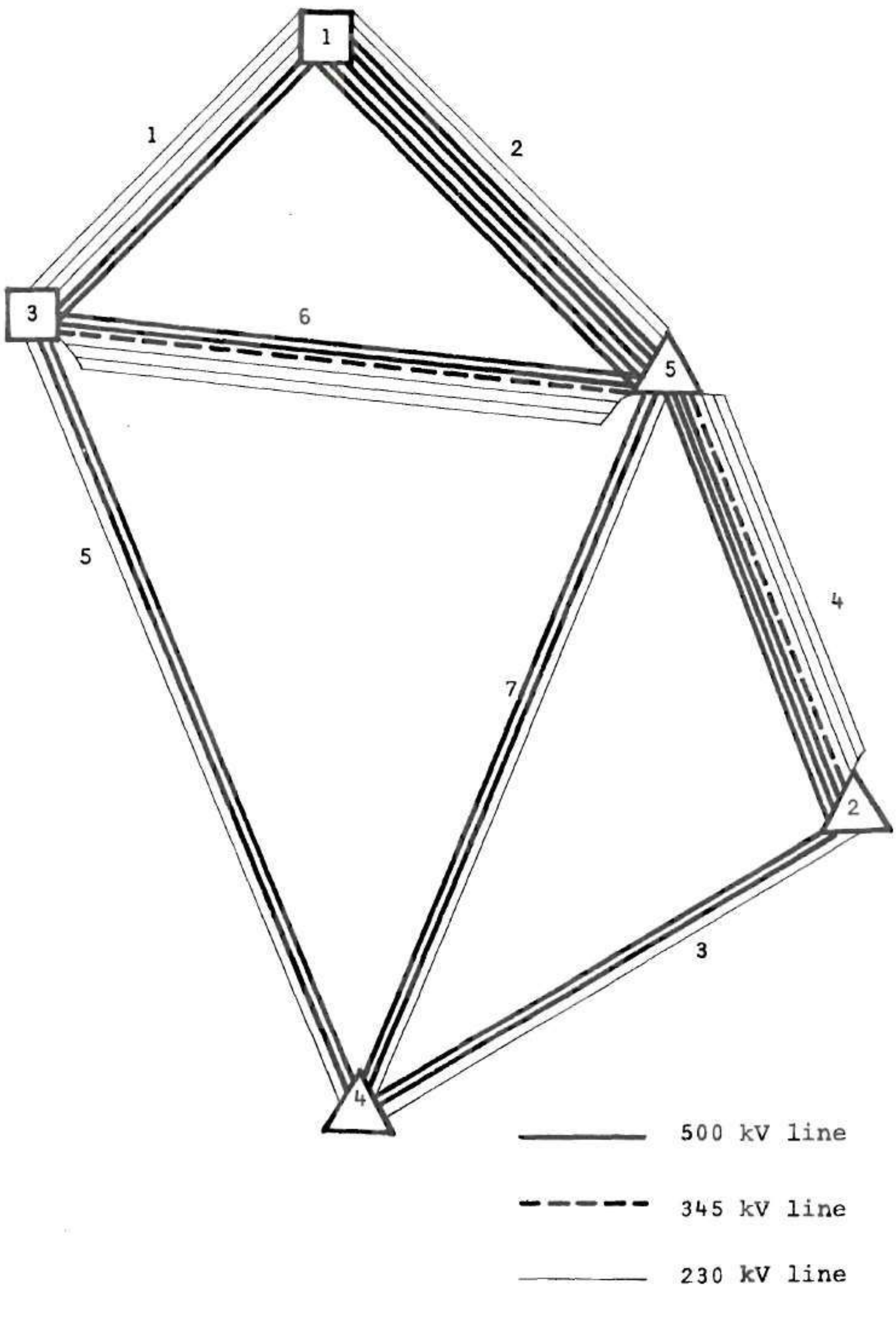


Figure VII.1. Test System A. Network Graph.

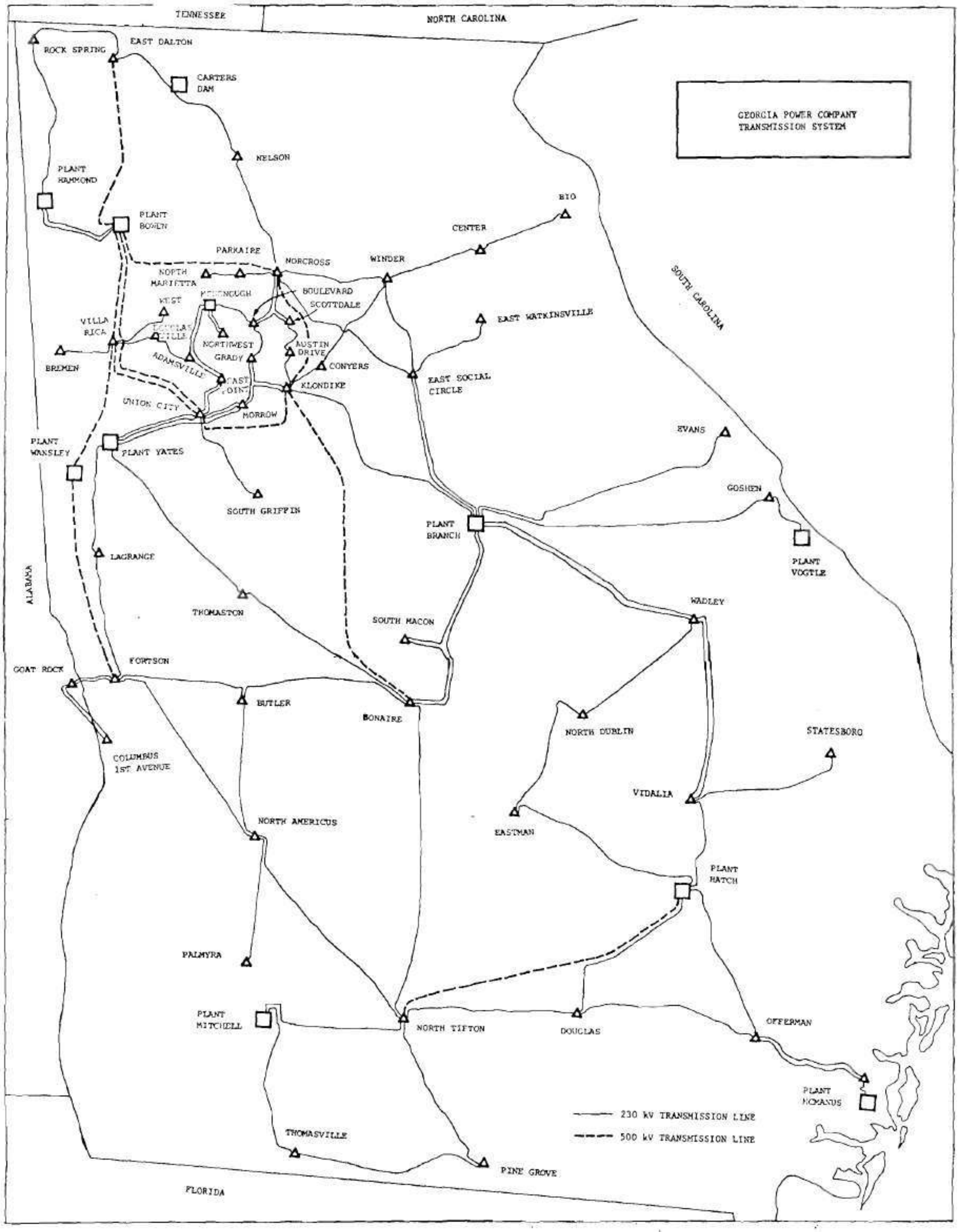


Figure VII.2. Test System B. Network Graph.

Table VII.1. Test System A. State of the System at Year Zero.

Branch	Sending End Node	Receiving End Node	Length (miles)	Type of Line	Quantity
1	1	2	40.0	1	2
				3	3
2	1	5	85.0	1	5
				3	2
5	4	3	200.0	1	2
				3	1
6	3	5	90.0	1	2
				2	1
				3	3
3	2	4	90.0	1	2
				3	1
7	4	5	170.0	1	2
				3	1
4	2	5	100.0	1	3
				2	1
				3	3

Table VII.2. Test System A. Net Power Injections at Year One

Node	Net Injection (MW)
2	-5000.0
3	4000.0
4	-1000.0
5	-5000.0

Table VII.3. Test System A. Properties of Transmission Lines Studied.

Type	Operating Voltage (KV)	Impedance (Ω/mile)	Maximum Permissible Real Power Flow (MW)
1	500.0	.0571+j.571	1200.0
2	345.0	.0803+j.803	440.0
3	230.0	.081+j.81	180.0

Table VII.4. Test System B. List of Existing Transmission Lines at Year Zero.

Sending End Bus	Receiving End Bus	Type of Line	Length (Miles)
PLANT BOWEN	PLANT HAMMOND	230 kV	30.30
PLANT BOWEN	PLANT HAMMOND	230 kV	30.30
PLANT BOWEN	VILLA RICA	500 kV	28.44
PLANT BOWEN	NORCROSS	500 kV	45.65
PLANT BOWEN	EAST DALTON	500 kV	48.50
PLANT BOWEN	UNION CITY	500 kV	57.73
PLANT BOWEN	NELSON	----	43.00
ADAMSVILLE	EAST POINT	230 kV	6.55
ADAMSVILLE	PLANT MCDONOUGH	230 kV	5.87
ADAMSVILLE	DOUGLASVILLE	230 kV	17.33
EAST POINT	PLANT MCDONOUGH	230 kV	12.42
EAST POINT	VILLA RICA	----	48.00
EAST POINT	UNION CITY	230 kV	11.07
EAST POINT	UNION CITY	230 kV	11.07
PLANT MCDONOUGH	BOULEVARD	230 kV	10.45
PLANT MCDONOUGH	NORTHWEST	230 kV	4.64
PLANT MCDONOUGH	NORTHWEST	230 kV	4.64
DOUGLASVILLE	VILLA RICA	230 kV	10.75
AUSTIN DRIVE	KLONDIKE	230 kV	12.57
AUSTIN DRIVE	SCOTTDALE	230 kV	11.58
KLONDIKE	BONAIRE	500 kV	89.26
KLONDIKE	PLANT BRANCH	230 kV	64.41
KLONDIKE	NORCROSS	500 kV	27.38
KLONDIKE	CONYERS	230 kV	6.01
KLONDIKE	MORROW	230 kV	17.01
KLONDIKE	UNION CITY	500 kV	36.24
KLONDIKE	GRADY	230 kV	16.46
SCOTTDALE	BOULEVARD	230 kV	10.46

Sending End Bus	Receiving End Bus	Type of Line	Length (Miles)
SCOTTDALE	NORCROSS	230 kV	13.15
BIO	CENTER	230 kV	31.23
CENTER	WINDER	230 kV	22.30
BONAIRE	BUTLER	230 kV	43.53
BONAIRE	NORTH TIFTON	230 kV	75.98
BONARIE	SOUTH MACON	230 kV	34.65
BONAIRE	THOMASTON	230 kV	50.46
BONAIRE	PLANT BRANCH	230 kV	55.16
BONAIRE	EASTMAN	----	45.00
BUTLER	THOMASTON	----	25.00
BUTLER	FORTSON	230 kV	36.24
BUTLER	NORTH AMERICUS	230 kV	33.16
NORTH TIFTON	NORTH AMERICUS	230 kV	59.88
NORTH TIFTON	DOUGLAS	230 kV	41.45
NORTH TIFTON	PLANT HATCH	500 kV	82.96
NORTH TIFTON	PINE GROVE	230 kV	46.77
NORTH TIFTON	PLANT MITCHELL	230 kV	35.43
SOUTH MACON	PLANT BRANCH	230 kV	42.96
THOMASTON	PLANT YATES	230 kV	54.25
THOMASTON	SOUTH GRIFFIN	----	28.00
BOULEVARD	NORCROSS	230 kV	13.24
BOULEVARD	GRADY	230 kV	4.24
PLANT HAMMOND	ROCK SPRINGS	230 kV	46.33
PLANT YATES	MORROW	230 kV	35.15
PLANT YATES	UNION CITY	230 kV	23.42
PLANT YATES	UNION CITY	230 kV	23.42
PLANT YATES	LAGRANGE	230 kV	37.58
VILLA RICA	WEST MARIETTA	230 kV	20.85
VILLA RICA	UNION CITY	500 kV	30.16
VILLA RICA	PLANT WANSLEY	500 kV	26.50

Sending End Bus	Receiving End Bus	Type of Line	Length (Miles)
VILLA RICA	BREMEN	230 kV	22.00
PLANT BRANCH	EAST SOCIAL CIRCLE	230 kV	40.42
PLANT BRANCH	EAST SOCIAL CIRCLE	230 kV	40.42
PLANT BRANCH	EVANS	230 kV	72.73
PLANT BRANCH	GOSHEN	230 kV	78.46
PLANT BRANCH	WADLEY	230 kV	57.27
PLANT BRANCH	WADLEY	230 kV	57.27
WEST MARIETTA	NORTH MARIETTA	----	15.00
EAST SOCIAL CIRCLE	NORCROSS	230 kV	37.52
EAST SOCIAL CIRCLE	WINDER	230 kV	24.26
EAST SOCIAL CIRCLE	EAST WATKINSVILLE	230 kV	22.67
GOSHEN	PLANT VOGTLE	230 kV	19.80
GOSHEN	WADLEY	----	45.00
NORCROSS	WINDER	230 kV	28.35
NORCROSS	NELSON	230 kV	34.97
NORCROSS	PARKAIRE	230 kV	10.88
WADLEY	NORTH DUBLIN	230 kV	37.94
WADLEY	PLANT VOGTLE	----	46.00
WADLEY	VIDALIA	230 kV	46.16
WADLEY	VIDALIA	230 kV	46.16
SOUTH GRIFFIN	UNION CITY	230 kV	32.67
FORTSON	GOAT ROCK	230 kV	12.20
FORTSON	GOAT ROCK	230 kV	12.20
FORTSON	NORTH AMERICUS	230 kV	55.13
FORTSON	PLANT WANSLEY	500 kV	66.50
FORTSON	LAGRANGE	230 kV	36.22
GOAT ROCK	COLUMBUS 1ST AVENUE	230 kV	1.41
GOAT ROCK	COLUMBUS 1ST AVENUE	230 kV	1.41
NORTH AMERICUS	EASTMAN	----	70.00
NORTH AMERICUS	PALMYRA	230 kV	33.33
WINDER	CONYERS	230 kV	27.57
MORROW	UNION CITY	230 kV	11.72

Sending End Bus	Receiving End Bus	Type of Line	Length (Miles)
MORROW	UNION CITY	230 kV	11.72
MORROW	GRADY	230 kV	9.56
DOUGLAS	OFFERMAN	230 kV	46.92
DOUGLAS	PLANT HATCH	230 kV	46.22
OFFERMAN	PLANT HATCH	230 kV	38.42
OFFERMAN	PLANT MCMANUS	230 kV	38.80
OFFERMAN	PLANT MCMANUS	230 kV	38.80
PLANT HATCH	EASTMAN	230 kV	57.23
PLANT HATCH	VIDALIA	230 kV	22.99
EAST DALTON	ROCK SPRINGS	230 kV	27.50
EAST DALTON	CARTERS DAM	230 kV	22.35
NELSON	CARTERS DAM	230 kV	25.68
PINE GROVE	THOMASVILLE	230 kV	50.50
NORTH MARIETTA	PARKAIRE	230 kV	8.45
EASTMAN	NORTH DUBLIN	230 kV	32.57
VIDALIA	STATESBORO	230 kV	42.50
PLANT MITCHELL	THOMASVILLE	230 kV	43.69
PLANT MITCHELL	PALMYRA	----	17.00

Table VII.5. Test System B. List of Nodes, Load and Generating Capabilities in the First Year of the Planning Period.

Bus Type	Bus	Load (MW)	Gen. Type	PGMIN (MW)	PGMAX (MW)	BGEN	CGEN
1	PLANT BOWEN	0.0	STEAM	200	1400	1.25	.00536
0	ADAMSVILLE	18.2					
0	EAST POINT	336.4					
1	PLANT MCDONOUGH	0.0	STEAM	100	774	1.4	.0065
0	DOUGLASVILLE	130.3					
0	AUSTIN DRIVE	56.5					
0	KLONDIKE	2.7					
0	SCOTTDALE	186.3					
0	BIO	118.9					
0	CENTER	166.1					
0	BONAIRE	294.7					
0	BUTLER	21.6					
0	NORTH TIFTON	52.2					
1	SOUTH MACON	236.4	STEAM	50	190	1.5	.009
0	THOMASTON	120.4					
0	BOULEVARD	318.9					
1	PLANT HAMMOND	436.0	STEAM	200	800	1.4	.006
1	PLANT YATES	0.0	STEAM	200	1250	1.26	.0055

Bus Type	Bus	Load (MW)	Gen. Type	PGMIN (MW)	PGMAX (MW)	BGEN	CGEN
0	VILLA RICA	0.0					
1	PLANT BRANCH	0.0	STEAM	200	1540	1.23	.005
0	WEST MARIETTA	145.2					
0	EAST SOCIAL CIRCLE	93.5					
0	EVANS	143.3					
0	GOSHEN	166.4					
0	NORCROSS	432.7					
0	WADLEY	127.1					
0	SOUTH GRIFFIN	116.9					
0	FORTSON	81.0					
1	GOAT ROCK	100.0	HYDRO	50	180	----	----
0	NORTH AMERICUS	114.5					
0	WINDER	138.6					
0	COLUMBUS 1ST AVENUE	131.7					
0	CONYERS	91.5					
0	MORROW	272.9					
0	DOUGLAS	71.8					
0	OFFERMAN	82.4					
1	PLANT HATCH	18.3	NUCLEAR	200	800	----	----
0	EAST DALTON	145.3					
0	NELSON	80.8					

Bus Type	Bus	Load (MW)	Gen. Type	PGMIN (MW)	PGMAX (MW)	BGEN	CGEN
0	PINE GROVE	85.8					
0	UNION CITY	16.7					
0	NORTH MARIETTA	90.3					
0	EAST WATKINSVILLE	75.1					
0	EASTMAN	12.7					
0	NORTH DUBLIN	57.8					
1	PLANT VOGTLE	0.0	COMBUSTION	50	351	1.99	.0099
0	GRADY	278.5					
0	ROCK SPRINGS	75.6					
0	VIDALIA	142.1					
0	NORTHWEST	315.0					
1	PLANT MCMANUS	102.9	STEAM	50	271	1.8	.0075
1	PLANT MITCHELL	0.0	STEAM	50	288	1.75	.0075
0	THOMASVILLE	16.7					
0	PALMYRA	65.5					
0	PARKAIRE	167.5					
1	PLANT WANSLEY	0.0	STEAM	100	884	1.34	.0056
0	BREMEN	31.0					
0	LAGRANGE	114.1					
0	STATESBORO	78.4					
1	CARTERS DAM	0.0	HYDRO	50	250	----	----

Table VII.6. Test System B. Properties of the 230 kV and 500 kV Transmission Lines Used by the Georgia Power Company.

Type	Operating Voltage	Resistance Per Mile	Reactance Per Mile	Maximum Permissible Power Flow
1	500 kV	.00001292*	.00021141*	14.00*
2	230 kV	.000152*	.001432*	5.00*

* Per unit values

Base System

Power: 100 MVA

Voltage: The operating voltage

The Automatic Generation of Alternatives

The automatic generation of alternatives algorithm generates the set of controls which will enter the optimizing algorithm. The successful application of the non Linear Branch and Bound depends on the following two specific properties of the automatic generation of controls algorithm:

1. The algorithm requires minimal computational effort
2. The number of generated controls which are qualified to enter the optimization algorithm is small.

For the purpose of demonstrating the above properties of the algorithm, it is expedient to analyze the automatic generation of alternatives algorithm into subtasks and evaluate each subtask separately. Figure VII.3 illustrates the various subtasks and their relation to the rest of the planning algorithm. The results of the evaluation are presented in the next sections.

For the test cases and regarding the construction of controls, given the effective rights of way for network reinforcement, the following assumptions have been used:

1. At most, one circuit per right of way and per year (stage) is allowed in a control.
2. A control (alternative) may be constructed with circuits of only one type.

Assumption 1 is a generally acceptable constraint for real systems.

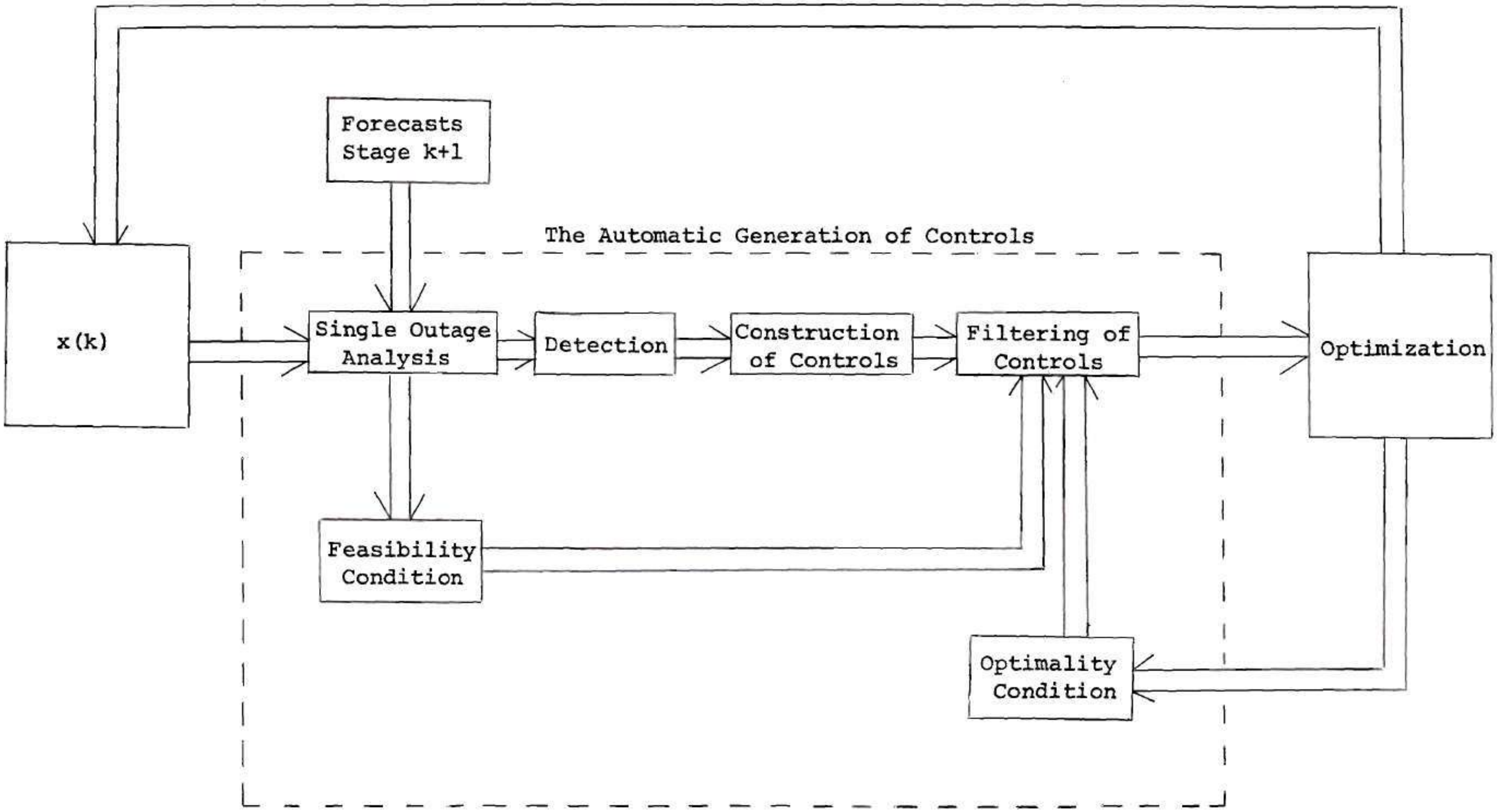


Figure VII.3. Block Diagram of the Automatic Generation of Controls Algorithm.

Assumption 2 is a simplification which has been taken for practical reasons.

Critically Loaded Circuits and Critical Outages

The first task in the automatic generation of alternatives algorithm is the determination of the critically loaded circuits and critical outages. A single outage analysis is employed to this purpose. Specifically, given the state of the system at stage k, x(k), and the power injections on the system at stage k+1, the following relationship, are checked (Chapter III)

$$Y^{(m)}(x(k)) \cdot \theta^{(m)} = P^{(0)}(k+1) \quad (46)$$

$$|\psi_{\ell}^{(m)}| = |\theta_i^{(m)} - \theta_j^{(m)}| \leq x_{over}\overline{\psi}_{\ell}(x(k),m) \quad (47)$$

$$m = 0,1, \ldots, M$$

$$\ell = 1,2, \ldots, M$$

where x_{over} is a parameter with value less than one.

The result of the above analysis will be the set S_u of rights of way with critically loaded circuits and the set S_c of critical outages. Normalized measures of these results are defined as follows:

$$p_u = \frac{\text{Number of Rights of Way with Critically Loaded Circuits}}{\text{Total Number of Rights of Way}} \quad (73)$$

$$p_c = \frac{\text{Number of Critical Outages}}{\text{Total Number of Rights of Way}} \quad (74)$$

The dependence of the quantities p_u and p_c on the parameter x_{over} and the stage variable is illustrated in Figures VII.4, VII.5, VII.6, and VII.7 for the two test systems. It is obvious that only a small percentage of the existing circuits may reach a loading level close to the permissible loading level. Of course this loading level may never be reached if the corresponding outage, which causes this loading, did not occur during the stage. On the other hand, only a small percentage of the outages are critical.

The above two experimental facts have the following impact in planning algorithms:

1. The stage to stage expansion of the transmission network is directed towards reinforcement of only a few circuits.
2. The majority of the constraints which define an admissible state [see definition of state admissibility, Chapter II] are ineffective. Only a small percentage of these are effective or close to being effective.

Later in the algorithm the controls are checked for admissibility. The admissibility of a control u(k) is checked with the feasibility condition (Chapter III). The feasibility condition consists of those constraints which correspond to a critically loaded circuit or to a critical outage. We shall call these constraints the <u>working constraints</u>. Let p_w be defined as follows:

$$p_w = \frac{\text{Number of Working Constraints}}{\text{Total Number of Constraints}} \qquad (75)$$

Figure VII.8 illustrates the quantity p_w as a function of x_{over} and the stage variable k for the test system B. The value of the quantity p_w signifies the relative computational effort in order to determine the feasibility of a state (or alternatively, the feasibility of a control). The extremely low values of p_w demonstrate the efficiency of the automatic generation of controls algorithm.

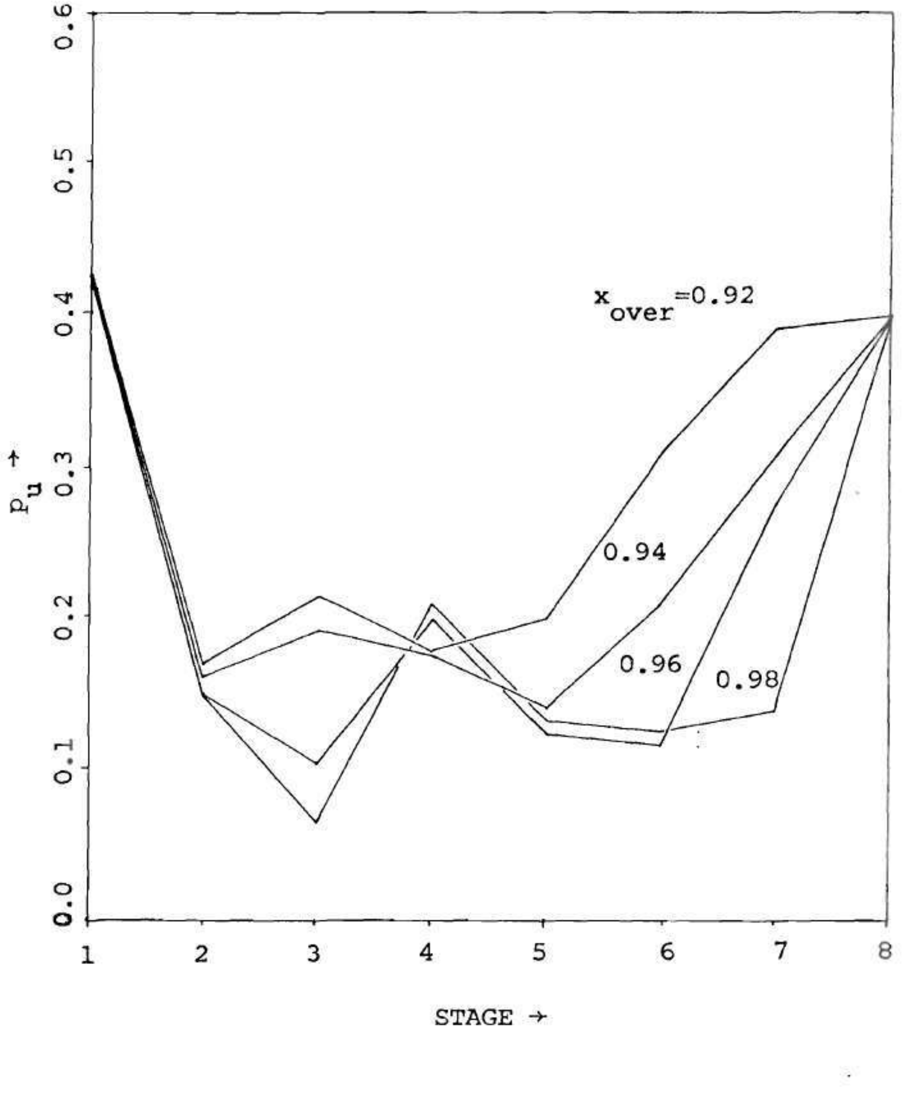


Figure VII.4. Test System A. Portion of Rights of Way With a Critically Loaded Circuit as a Function of the Parameter x_{over} and Stage Variable.

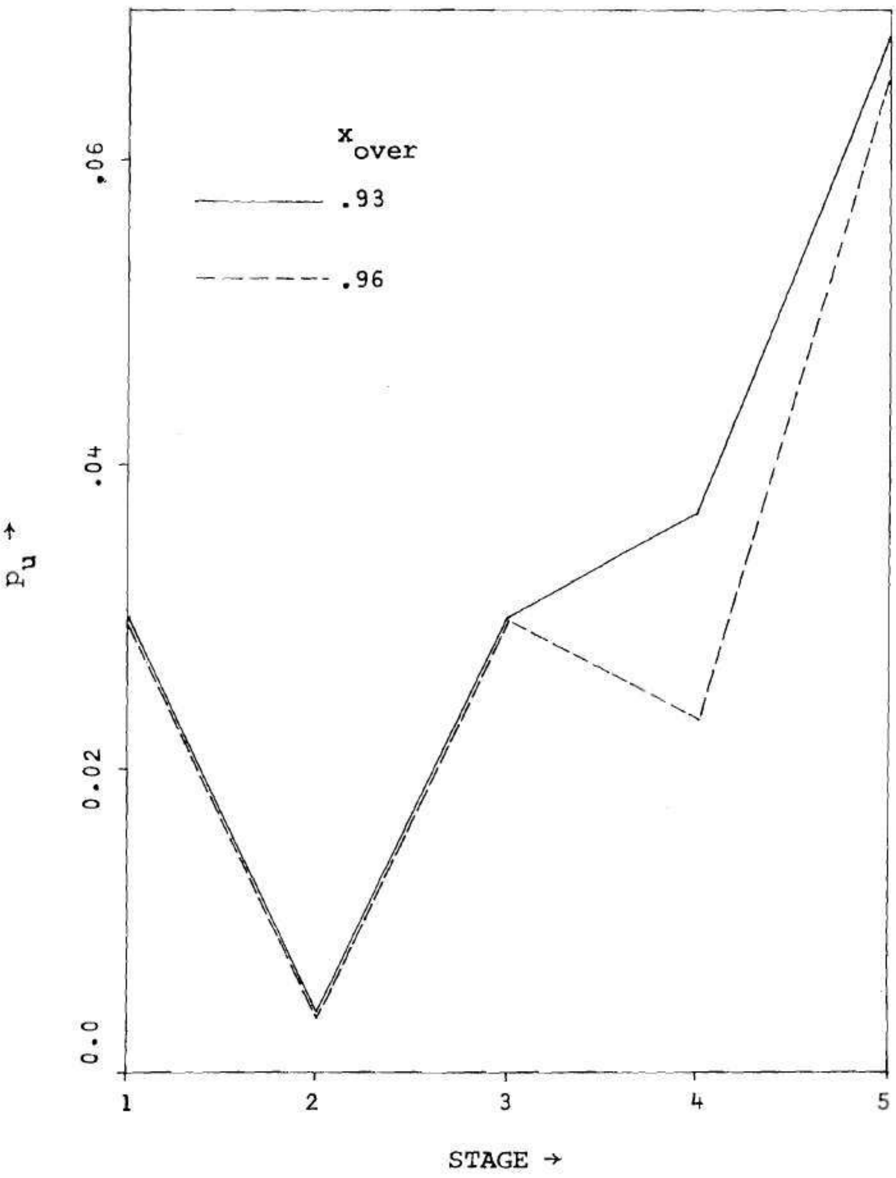


Figure VII.5. Test System B. Portion of Rights of Way With a Critically Loaded Circuit as a Function of the Parameter x_{over} and Stage Variable.

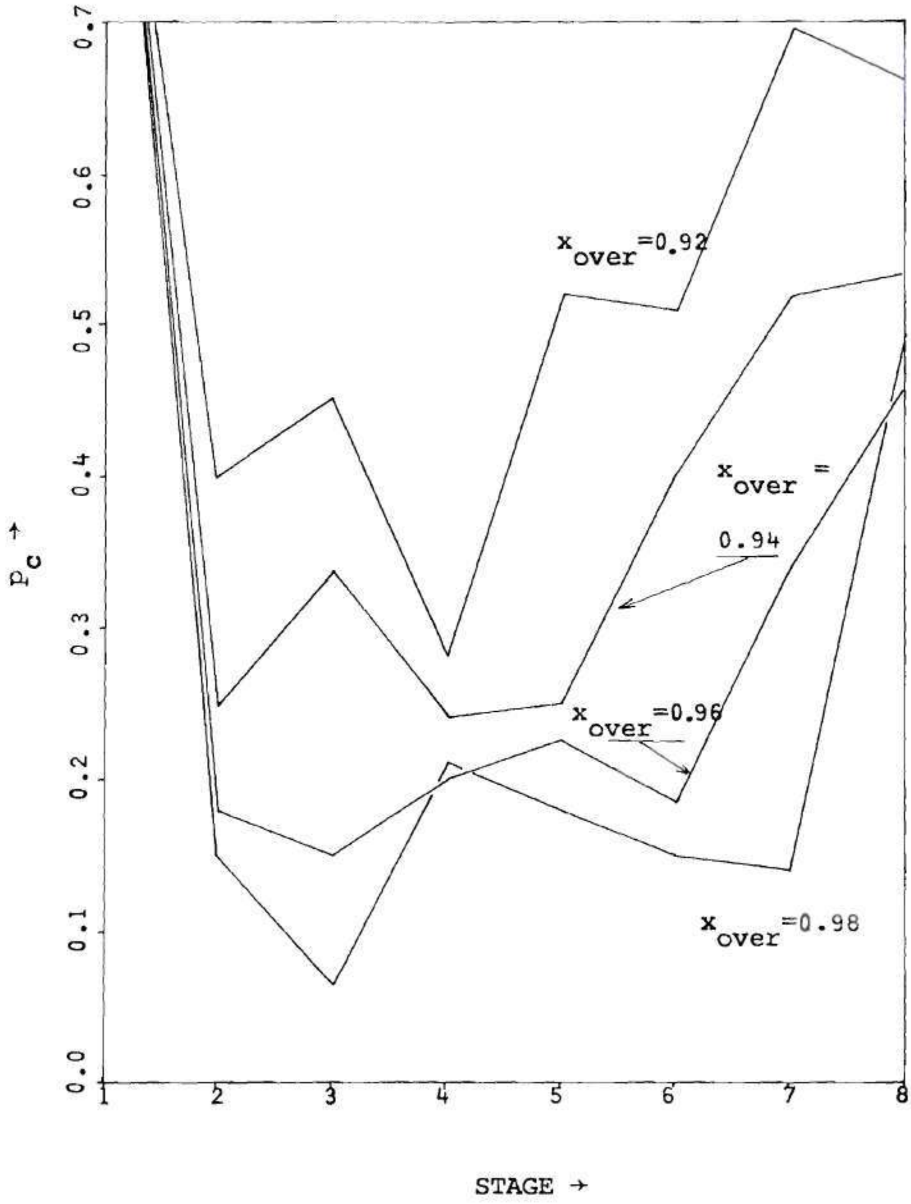


Figure VII.6. Test System A. Portion of Circuit Outages Which Are Critical as a Function of the Parameter x_{over} and the Stage Variable.

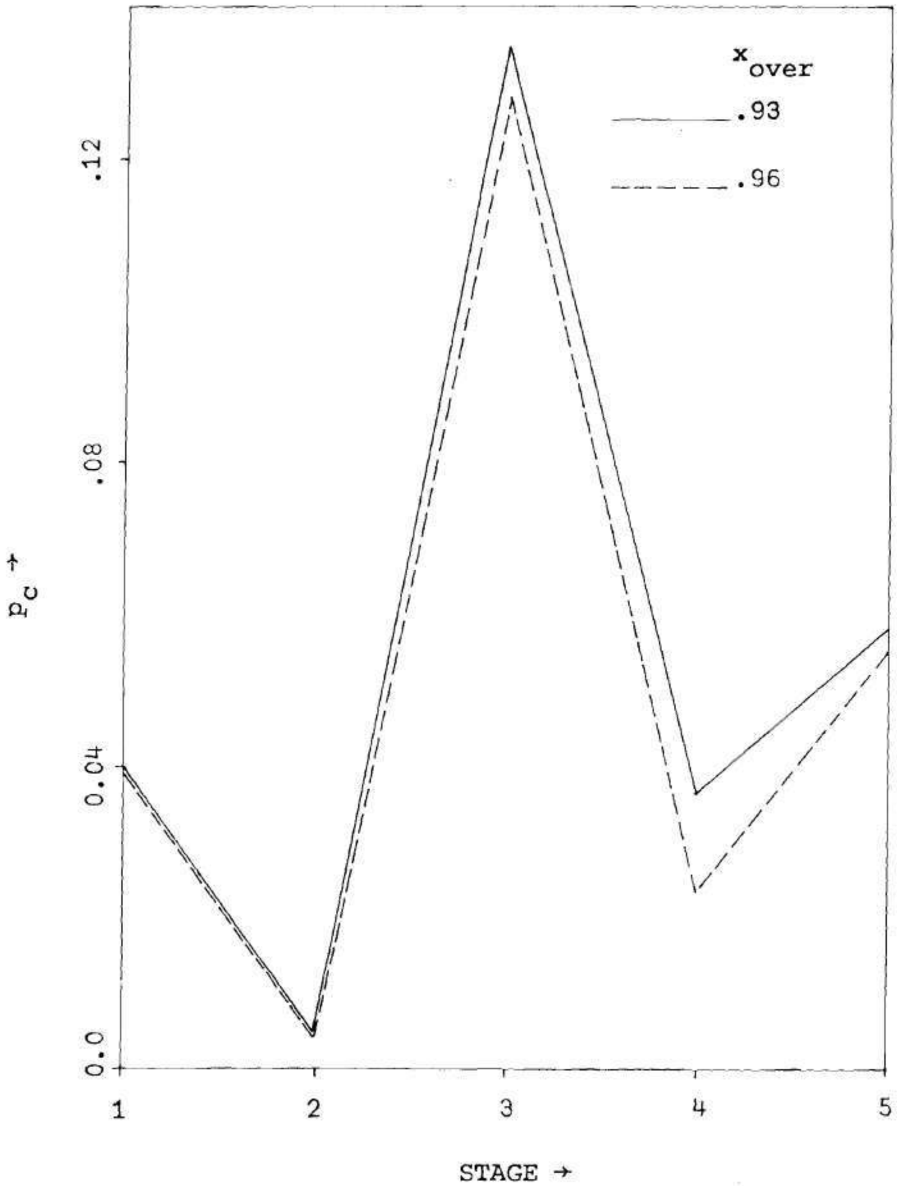


Figure VII.7. Test System B. Portion of Circuit Outages Which Are Critical as a Function of the Parameter x_{over} and the Stage Variable.

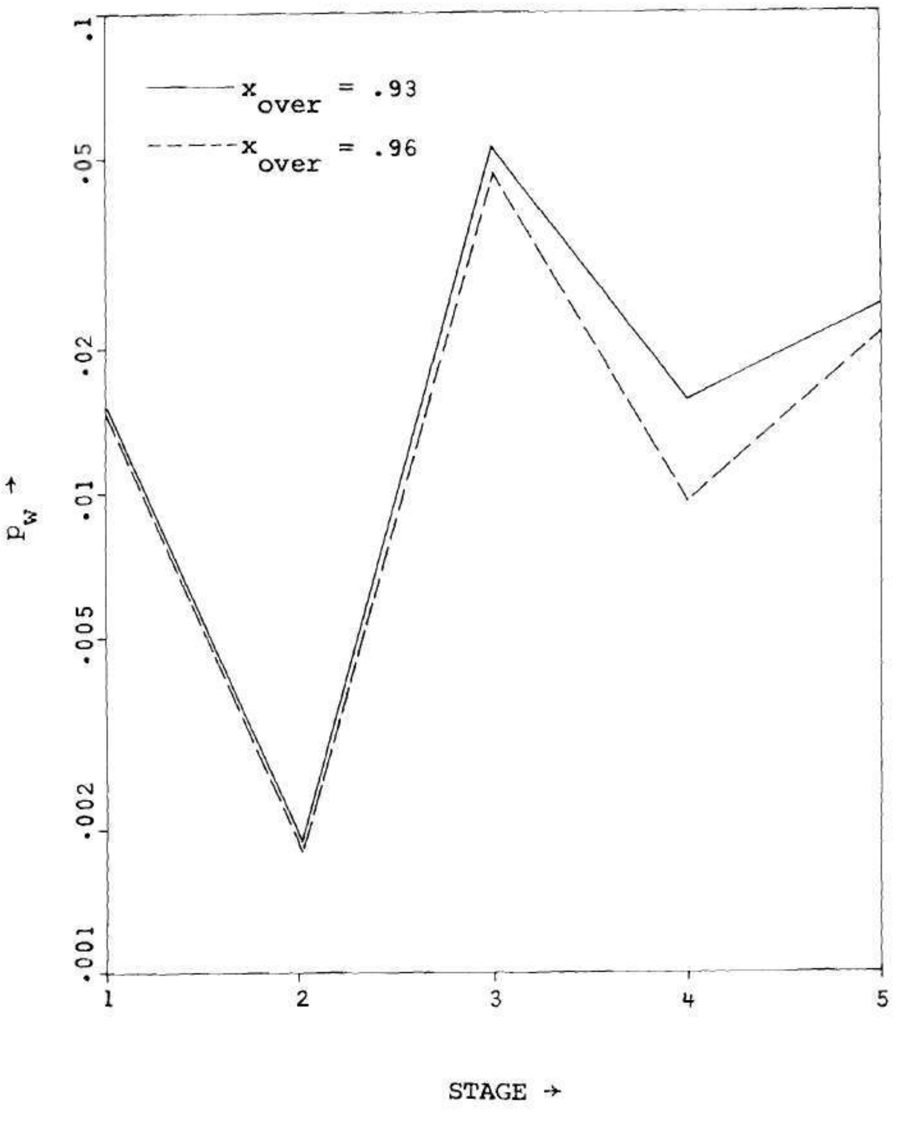


Figure VII.8. Test System B. Portion of Constraints Which Are Included in the Feasibility Condition as a Function of the Parameter x_{over} and the Stage Variable.

Detection of the Effective Rights of Way for Network Reinforcement

The next task in the automatic generation of alternatives is the determination of those rights of way which are effective for network reinforcement. The detection scheme (I or II) is employed. A normalized measure of the outcome is defined as follows:

$$p_e = \frac{\text{Number of Effective Rights of Way for Network Reinforcement}}{\text{Total Number of Rights of Way}} \qquad (76)$$

The dependence of this quantity on the parameter x_{over} and x_{cut} and the stage variable is illustrated in Figures VII.9 and VII.10. Only a small portion of the total number of rights of way is effective for reinforcing the critically loaded circuits. This fact demonstrates the network coherency as has been defined in Chapter III. By comparing Figures VII.9 and VII.10, it should be concluded that coherency is more profound in realistic transmission networks. Or, to put it in another way, test system A forms one coherent region. Detection scheme II yields higher values of p_e, as is expected. For networks similar to test system A (one coherent region), detection scheme I is adequate in the sense that it yields the same set of effective rights of way as detection scheme II. For realistic networks, however, this is not true. The reason is that for realistic networks the sensitivity coefficients $\left(\frac{\partial \phi_\ell}{\partial y_i}\right)_{prefault}$ and $\left(\frac{\partial \phi_\ell}{\partial y_i}\right)_{postfault}$ may considerably differ. Furthermore, it has been observed that for the test system B the use of the detection scheme II yields the optimum with even higher values of the parameter x_{cut} [x_{cut} = 0.90].

Detection scheme II can be used with high values of x_{cut} while detection scheme I should be used with low values of x_{cut}. It is recommended that the use of detection scheme II should be preferred for realistic networks.

In Chapter IV the question of optimality of the overall planning procedure of this thesis was analyzed. It was shown that the overall planning procedure is globally optimal if the automatic generation of controls algorithm generates all possible controls. The set of all possible controls for a stage k is denoted by $S_a(k)$. The automatic generation of controls algorithm, however, generates a smaller set of controls, namely $S_b(k)$. The size of the set $S_b(k)$ depends on the parameter x_{over} and x_{cut}. It is obvious that the set of controls $(S_a(k) - S_b(k))$ has been left out of the optimizing algorithm. The question is if this truncation of controls jeopardizes the optimality of the overall planning algorithm. For the test system A, the global optimum is known. For this system, the planning algorithm of this thesis yielded the known global optimum for considerably high values of the parameters x_{over} and x_{cut} [$x_{over} = 0.98$, $x_{cut} = 0.80$]. This fact demonstrates the perfection of the detection scheme and the automatic generation of controls algorithm in general. A quantitative indicator of the merit of the detection scheme has been defined in Chapter IV. It is cited again:

$$p_s = \frac{N_s}{N_a} \tag{66}$$

where:

N_a - number of controls in the set $S_a(k)$.

N_s - number of controls in the set $S_a(k) - S_b(k)$ which satisfy the feasibility and optimality conditions.

Figure VII.11 illustrates the dependence of the quantity p_s on the parameters x_{over} and x_{cut}, and the stage variable k. It can be concluded that only a small number of controls are left out of the optimizing algorithm because of the detection scheme. This number is

$$n_c(k) = p_s(k) \cdot N_a$$

Then, on the average, the projected number of trajectories which will be left out because of the detection scheme is:

$$n_t = \prod_k n_c(k)$$

On the other hand, the total number of trajectories is

$$n_T = (N_a)^N$$

With the assumption that each trajectory t has equal probability of being the optimal, the quantity

$$\overline{p}_s = 1 - \frac{n_t}{n_T} = 1 - \prod_k p_s(k)$$

expresses the level of confidence that the planning algorithm will yield the global optimum. This is a hypothetical interpretation of the results. A rigorous interpretation will require a complete enumeration of the trajectories.

Filtering of the Controls

Once the effective rights of way for network reinforcement have been determined, the construction of the controls is performed. These controls are checked with the feasibility and optimality conditions. The controls which satisfy the above conditions will enter the optimizing algorithm. The effectiveness of the above two conditions is reflected in the following normalized quantity.

$$p_a = \frac{\text{Number of Controls Which Satisfy Feasibility and Optimality Conditions}}{\text{Number of Generated Controls}} \qquad (77)$$

Figure VII.12 illustrates the dependence of the quantity p_a on the parameter x_{over} and x_{cut}, and the stage variable for the test system A. Figure VII.13 shows the same variation of the quantity p_a for the test system B and the number of generated alternatives. Even if this number is quite large, the number of alternatives which will enter the optimizing algorithm is very small. This fact justifies the use of an enumerative optimization algorithm for the transmission planning problem.

The extremely low values of the quantity p_a at stages close to the end of the planning period, are mainly caused by the optimality condition.

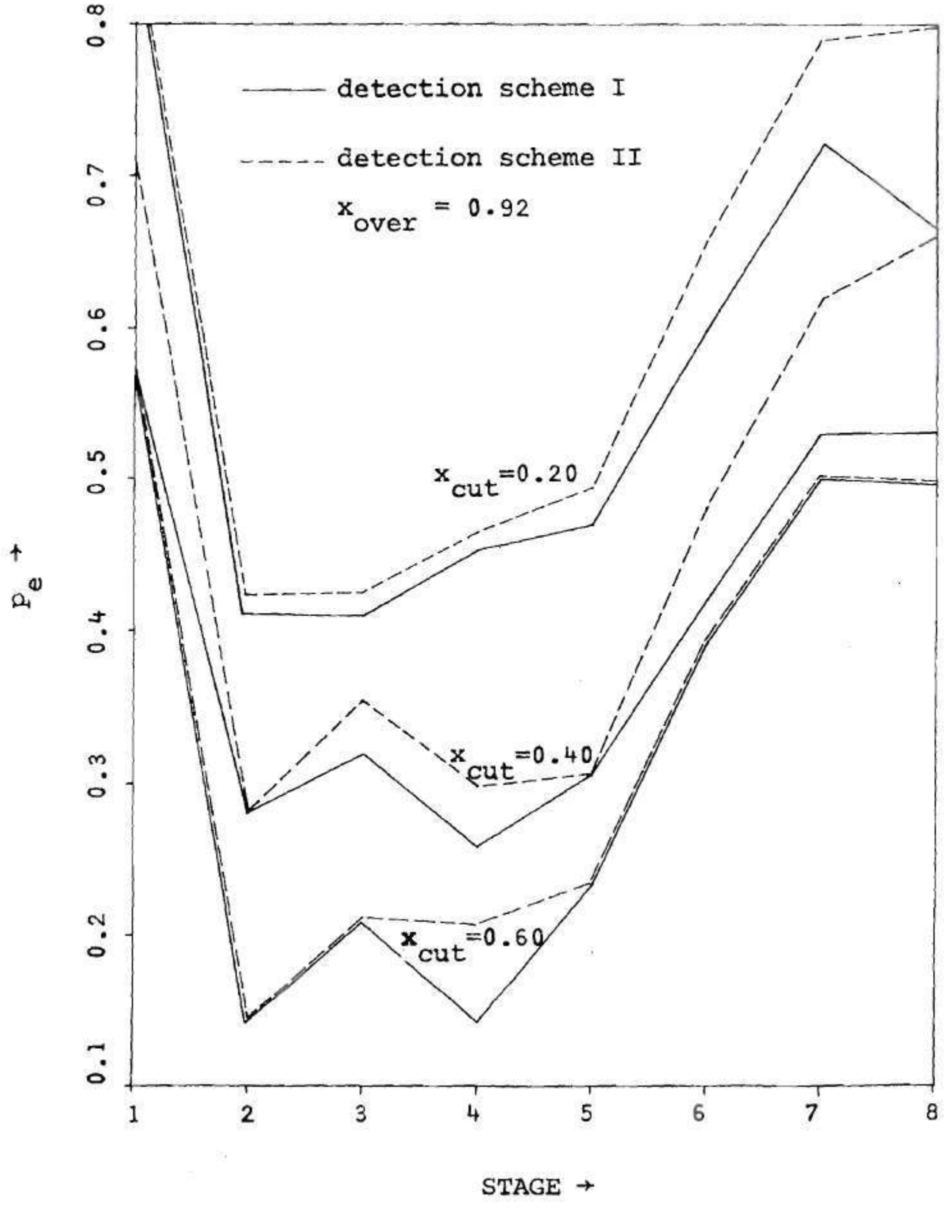


Figure VII.9. Test System A. Portion of Rights of Way Which Have Been Detected as Effective For Network Reinforcement.

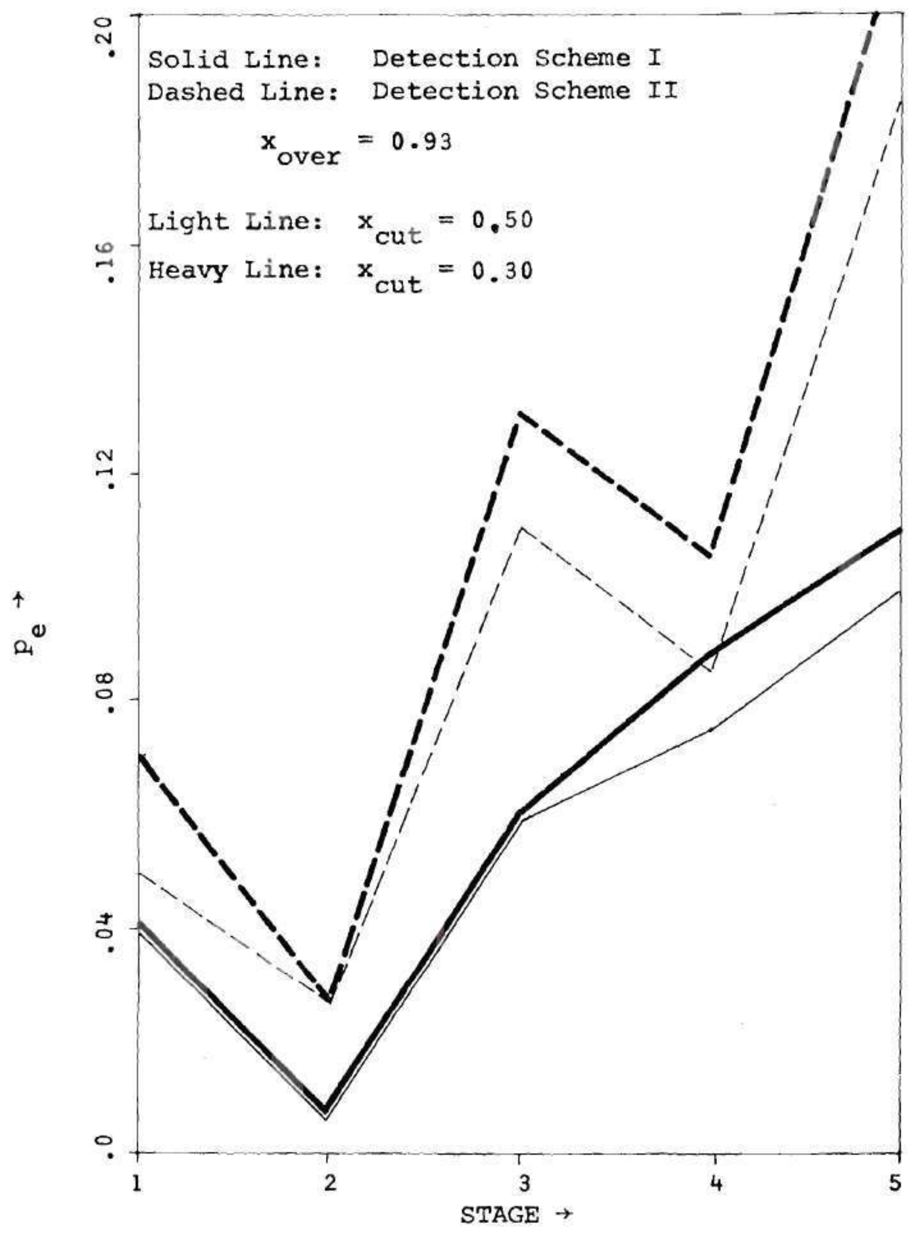


Figure VII.10. Test System B. Portion of Rights of Way Which Have Been Detected as Effective For Network Reinforcement.

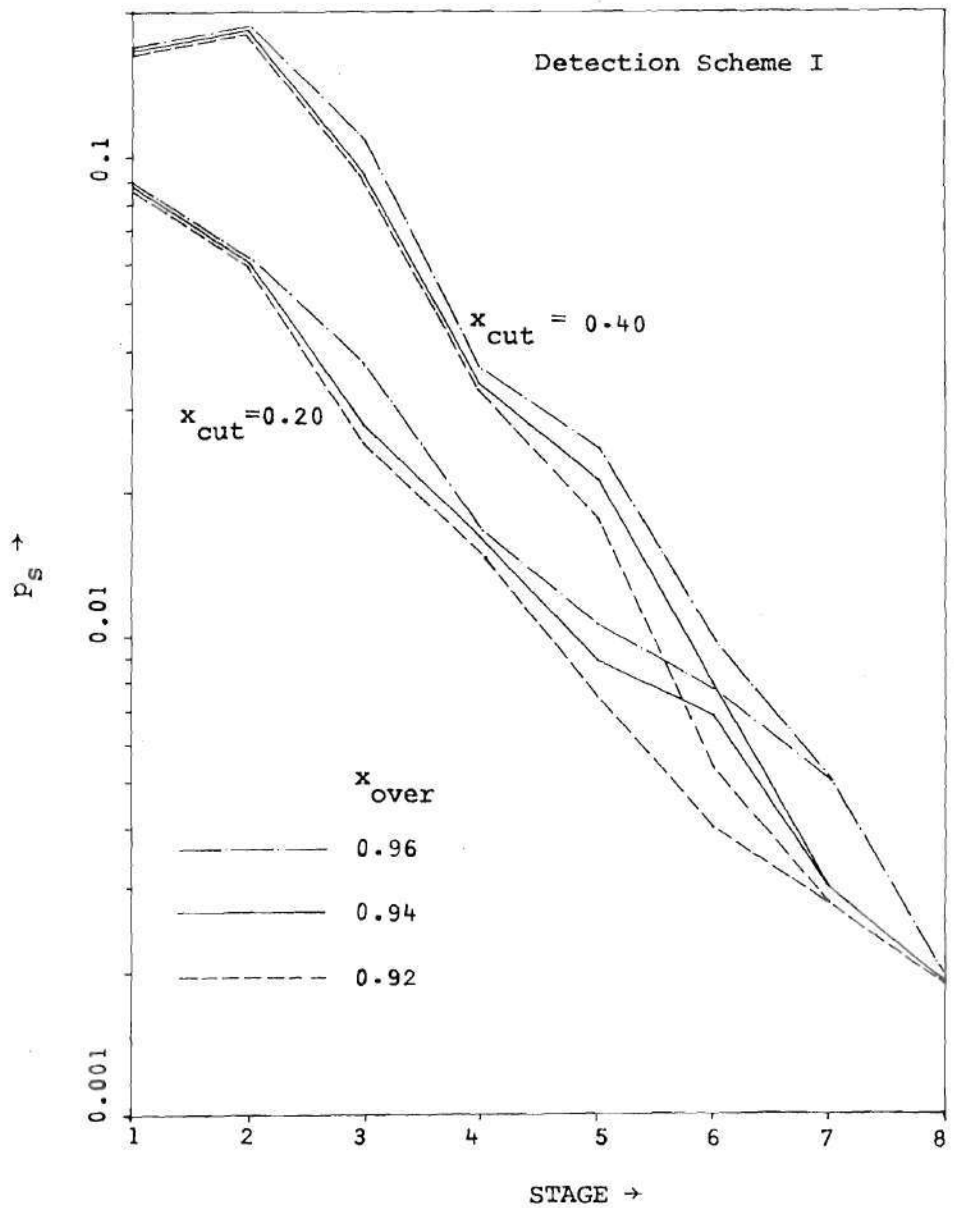


Figure VII.11. Test System A. Dependence of the Quantity p_s on the Parameter x_{over} and x_{cut}, and the Stage Variable.

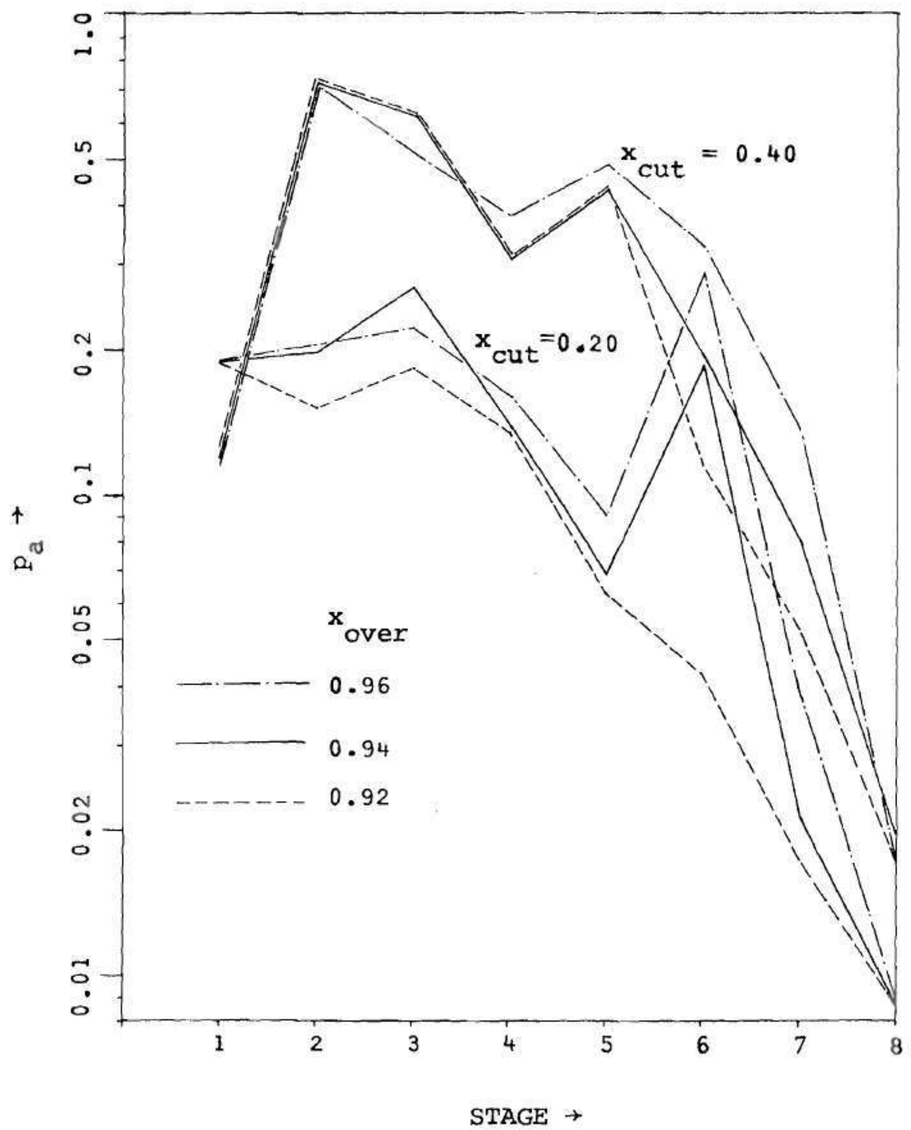


Figure VII.12. Test System A. Portion of the Generated Controls Which Satisfy Feasibility and Optimality Conditions as a Function of the Parameters x_{over} and x_{cut} and the Stage Variable

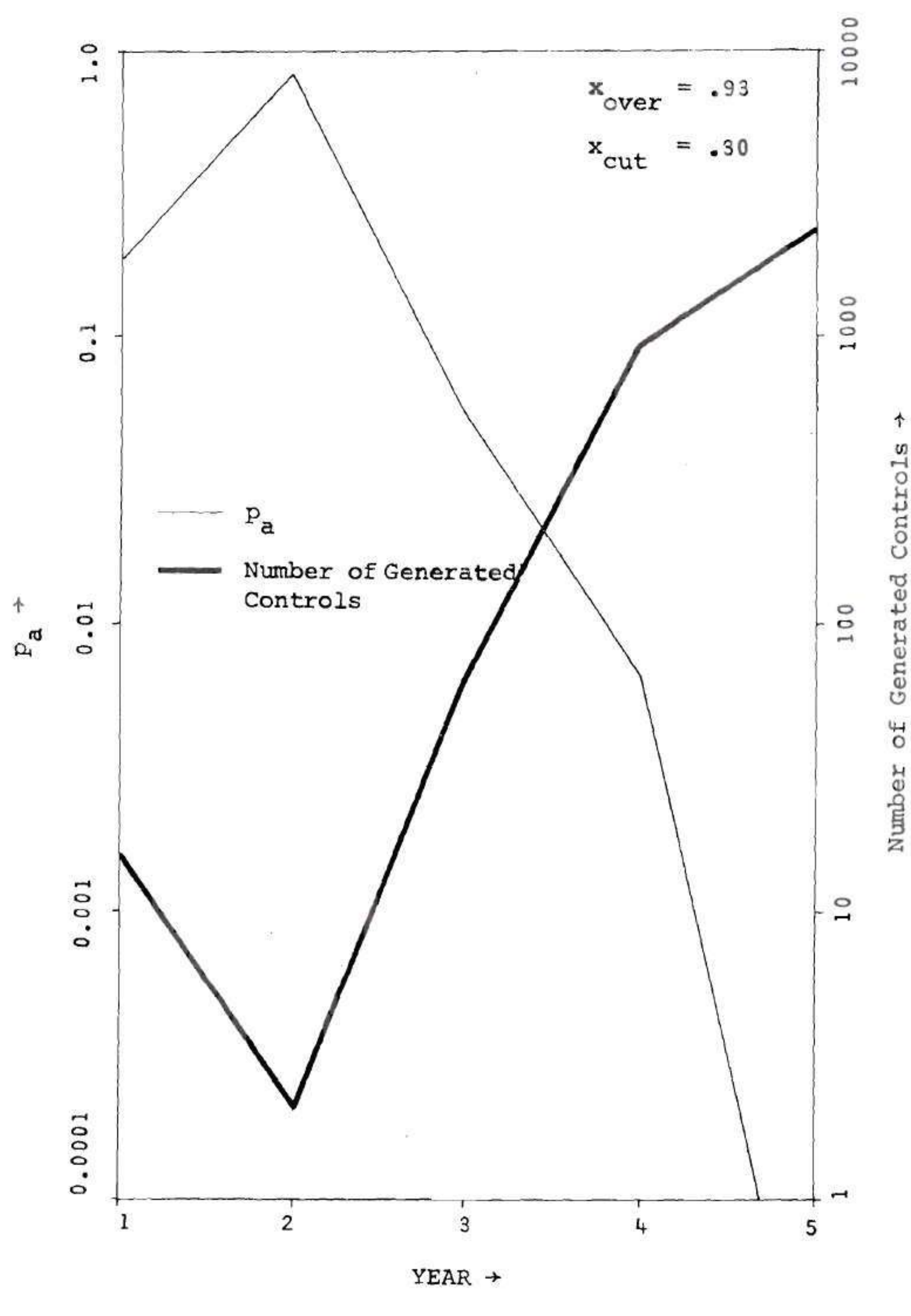


Figure VII.13. Test System B. Portion of Generated Controls Which Satisfy Feasibility and Optimality Conditions and Number of Generated Controls.

The Non Linear Branch and Bound

Efficiency is the main criterion for the evaluation of the non Linear Branch and Bound method. In Chapter IV an efficiency index (E.I.) has been defined for this optimization algorithm. The definition is cited again:

$$E.I. = \frac{\text{Number of Times the Automatic Generation of Controls Was Called}}{\text{Total Number of Vertices}} \qquad (61)$$

It is, however, difficult to compute the efficiency index because the total number of vertices is never known unless a complete enumeration of the vertices is made. A complete enumeration is practically impossible. For this reason the total execution time of the planning algorithm is used as an efficiency index.

The effect of the optimality condition, the starting upper bound, and the optimal ordering of alternatives on the total execution time, is tabulated in Table VII.7 for the test system A and for several values of the parameters x_{over} and x_{cut}. The dependence of the execution time on the optimality condition is very profound. The use of starting upper bound considerably decreases the total execution time. And finally, the optimal ordering of controls has a small influence on the total execution time.

Similar results for the test system B require a tremendous amount of computer time. For this reason such results are not available.

The performance of the non Linear Branch and Bound method with respect to the number of stages in the planning period and the parameter

Table VII.7. Test System A. The Effect of (a) Optimal Ordering of Controls, (b) Optimality Condition, and (c) Starting Upper Bound on the Total Execution Time for Various Values of the Parameters x_{cut} and x_{over}.

x_{cut}	x_{over}	Optimal Ordering of Controls	Optimality Condition	Starting Upper Bound	Execution Time (Sec)*
0.2	0.94	YES	YES	YES	7.261
0.2	0.94	YES	YES	NO	11.633
0.2	0.94	YES	NO	YES	41.283
0.2	0.94	YES	NO	NO	42.014
0.2	0.94	NO	YES	YES	8.021
0.2	0.94	NO	YES	NO	11.803
0.2	0.94	NO	NO	YES	34.129
0.2	0.94	NO	NO	NO	39.907
0.2	0.96	YES	YES	YES	5.37
0.2	0.96	YES	YES	NO	7.328
0.2	0.96	YES	NO	YES	18.106
0.2	0.96	YES	NO	NO	17.696
0.2	0.96	NO	YES	YES	6.089
0.2	0.96	NO	YES	NO	7.406
0.2	0.96	NO	NO	YES	16.775
0.2	0.96	NO	NO	NO	18.614
0.4	0.94	YES	YES	YES	4.417
0.4	0.94	YES	YES	NO	5.003
0.4	0.94	YES	NO	YES	9.19
0.4	0.94	YES	NO	NO	9.39
0.4	0.94	NO	YES	YES	3.921
0.4	0.94	NO	YES	NO	4.51
0.4	0.94	NO	NO	YES	10.792
0.4	0.94	NO	NO	NO	9.745
0.4	0.96	YES	YES	YES	2.684
0.4	0.96	YES	YES	NO	2.777
0.4	0.96	YES	NO	YES	4.999
0.4	0.96	YES	NO	NO	3.755
0.4	0.96	NO	YES	YES	2.573
0.4	0.96	NO	YES	NO	2.294
0.4	0.96	NO	NO	YES	4.161
0.4	0.96	NO	NO	NO	4.577

*CYBER 74

x_c is illustrated in Figures VII.16 and VII.17. The size of the set S_t is very sensitive to the value of the parameter x_c and the number of years in the planning period.

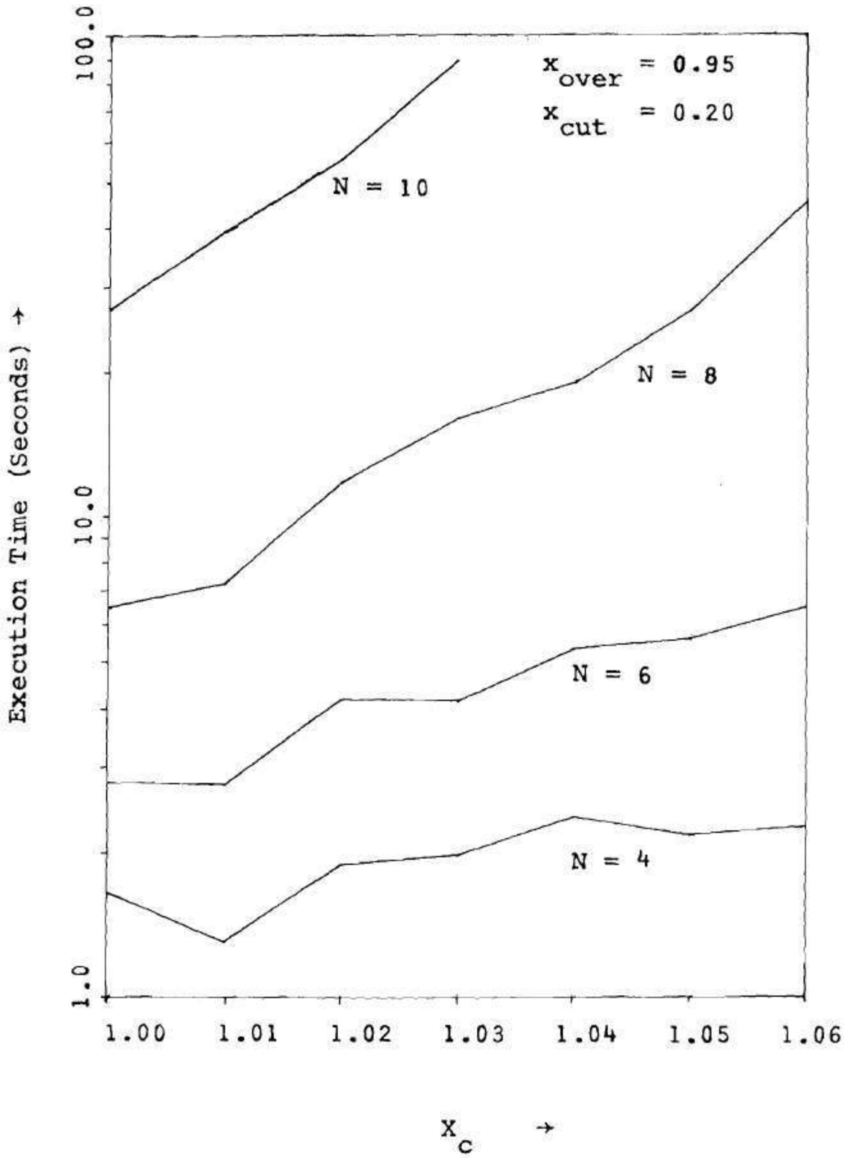


Figure VII.14. Test System A. Execution Time in Seconds as a Function of the Parameter X_c and the Number of Stages in the Planning Period.

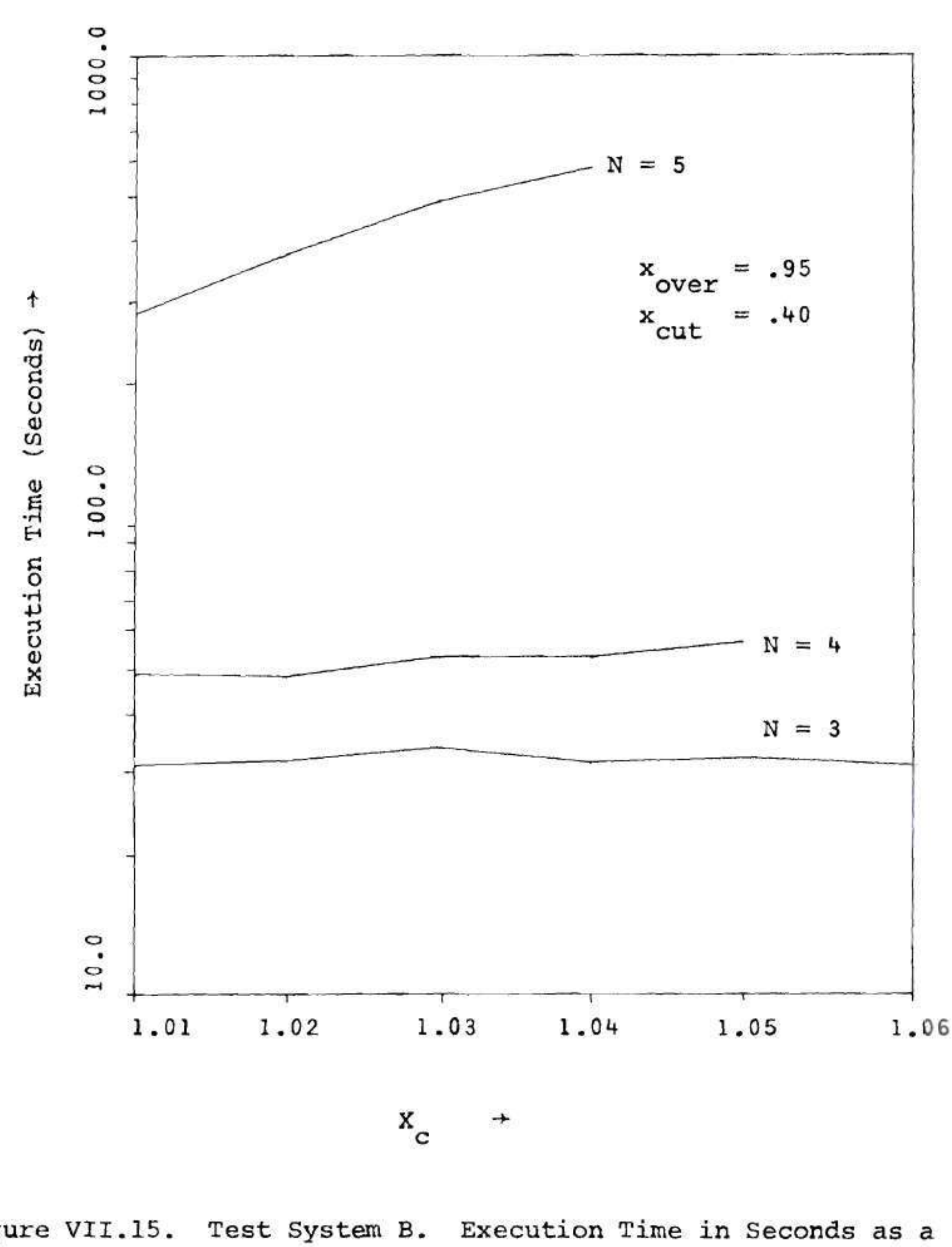


Figure VII.15. Test System B. Execution Time in Seconds as a Function of the Parameter X_c and the Number of Stages in the Planning Period.

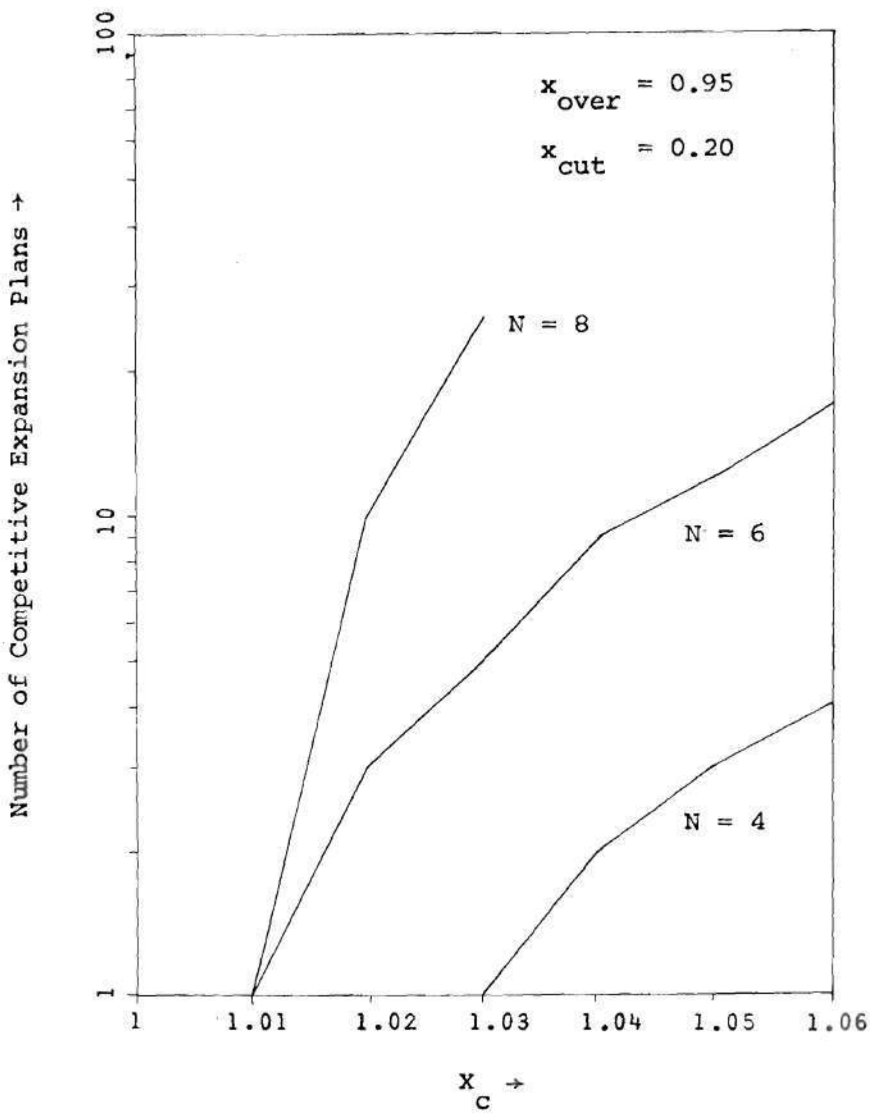


Figure VII.16. Test System A. Number of Competitive Expansion Plans as a Function of the Parameter X_c and the Number of Stages in the Planning Period.

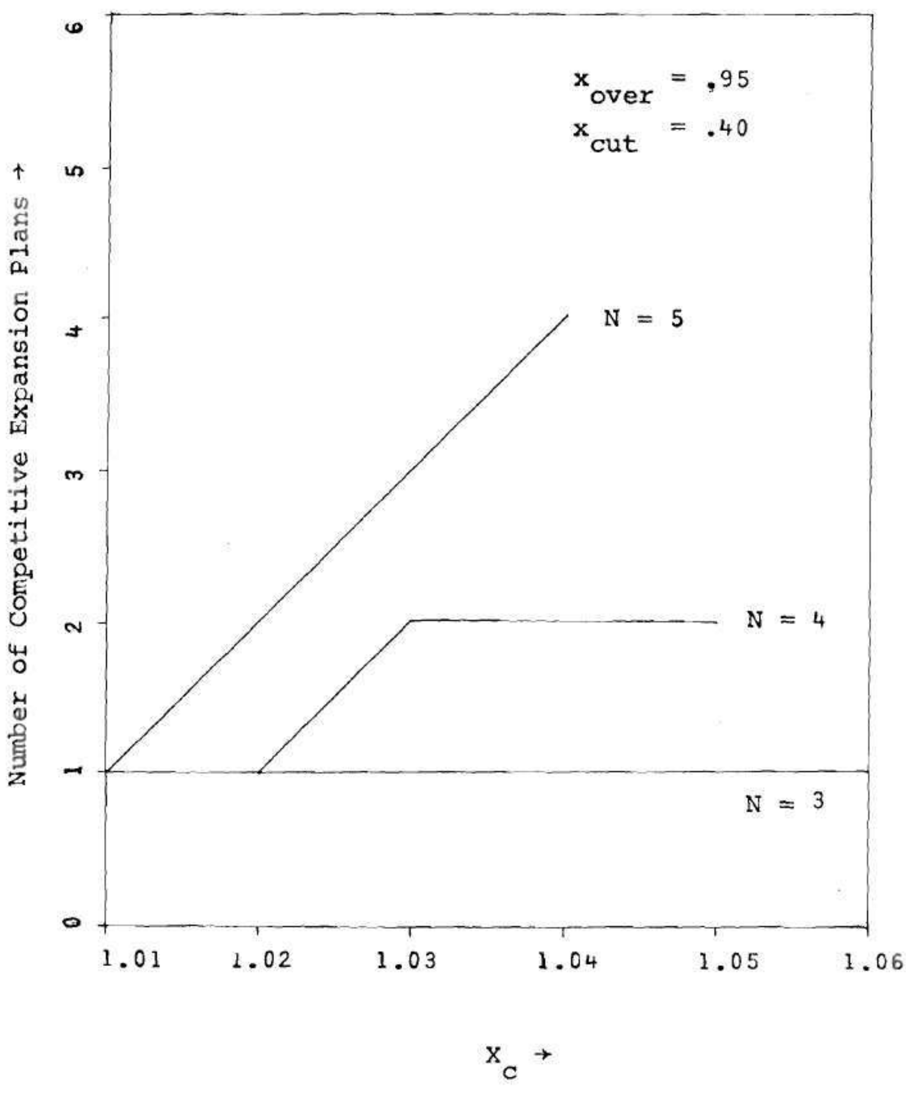


Figure VII.17. Test System B. Number of Competitive Expansion Plans as a Function of the Parameter X_c and the Number of Stages in the Planning Period.

Stage by Stage Optimization Versus Long Range Optimization

The planning algorithm of this thesis can yield the stage by stage optimum to a transmission planning problem as well as the N-stage optimal trajectory. Table VII.8 tabulates the results which have been obtained with test system A. There is a profound difference between the stage by stage optimum and the N-stage optimum. This difference mainly stems from the economy of scale.

The Impact of Operational Controls to the Planning of Transmission Networks

In Chapter II, the admissibility of a state was defined in two alternative ways. The first one does not recognize operational controls while the second one takes into account one form of operational control, the corrective rescheduling of the generator outputs (see Chapter II).

Test system A has been used for an evaluation of the impact of corrective rescheduling to the planning of transmission networks. It has been assumed that the generator outputs at nodes 1 and 3 can change by as much as five per cent if necessary. Then the problem of planning the expansion of the system was solved twice. Once using the definition of state admissibility I and then using the definition of state admissibility II. The procedure was repeated for various lengths of the planning period. The results are tabulated in Table VII.9. It is obvious that the practice of corrective rescheduling reduces the cost of expanding a transmission network by a considerable amount.

Conclusions

The various specific properties of the transmission planning

Table VII.8. Test System A. Comparison Between the Stage by Stage Optimal Trajectory and the N-Stage Optimal Trajectory

Number of Years in the Planning Period	Performance Criterion of Optimal Trajectory* (in $1000)		
	Stage by Stage	N-Stage	Per Cent Change
5	222303.51	191877.78	15.85
6	275389.06	236434.10	16.47
7	341249.25	278620.94	22.47

*Two types of circuits (345 kV and 500 kV transmission lines) have been used for the expansion of the transmission network.

Table VII.9. Test System A. Impact of Corrective Rescheduling Practices to the Planning of Transmission Networks.

Number of Years in the Planning Period	Performance Criterion of Optimal Trajectory (in $1000) I*	II**	Per Cent Change
5	191877.78	186761.97	2.739
6	236434.10	227352.03	3.994
7	278620.94	269305.95	3.458

* Definition of State Admissibility I (Chapter II) has been used.

** Definition of State Admissibility II (Chapter II) has been used.

problem have been demonstrated. These properties influenced the development of the planning algorithm of this thesis. Specifically, the existence of the coherency in realistic transmission networks which was claimed in Chapter III becomes obvious from the results of the detection schemes. The automatic generation of controls algorithm generates a fairly large number of controls. The number of controls can be controlled by the parameters x_{over} and x_{cut}. The size of the optimization problem to be solved is therefore **determined** by the parameters x_{over} and x_{cut}.

From the computational point of view, the present planning algorithm is efficient. This is so because the majority of the generated controls do not meet feasibility and optimality conditions and therefore the number of controls which will enter the optimizing algorithm is small. This fact justifies the use of an enumerative optimization method for the problem of transmission planning.

For the above reasons, the non Linear Branch and Bound method is very efficient. Reasonable execution times are obtained for even larger systems. A set of competitive expansion plans may be obtained for post optimality analysis at the expense of longer execution time.

Finally, the impact of operational controls on the planning of transmission networks is important. Significant reduction of the cost of expanding transmission networks over a long range is achieved with the application of a particular operational control, the corrective rescheduling of the generators outputs.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This thesis has presented the successful application of an enumerative approach to a large discrete optimization problem. The problem is the planning of an electric power transmission network over a long period of time. In general, this problem can be formulated as a discrete time deterministic optimal control problem. The controls or alternative ways to expand a transmission network are discrete because of standardization of transmission facilities. The optimal trajectory is obtained by a non Linear Branch and Bound method. This method has been developed from an enumerative procedure.

In general, the controls are numerous. The solution space (space of trajectories) contains an insurmountable number of elements. For this reason, the transmission planning problem is a huge computational one. The computational burden is alleviated with the following actions.

1. A simple optimality condition is generated by the optimizing algorithm. This condition is very effective in disqualifying the majority of the discrete controls with minimal computations. A control is disqualified if it can be proven that it does not belong to the optimal trajectory.

2. The size of the problem is reduced by taking advantage of specific properties of the transmission planning problem. In particular,

this problem exhibits the following properties:

(a) Network coherency (Chapter III).

(b) The transmission planning problem can be viewed as capacity expansion in order to reinforce circuits which become overloaded or close to being overloaded as demand increases.

The above properties provide the basis for the automatic generation of alternatives algorithm which, given the state of the network at some stage, generates a subset of the set of all possible controls for expanding the network for the next stage. In this way a subproblem is defined. The optimizing algorithm will yield the optimal trajectory to the subproblem. It has been shown, however, that the defined subproblem is equivalent to the complete problem with very high probability. In this sense the automatic generation of controls is successful.

The success of the non Linear Branch and Bound method stems from the fact that the bounds of the return function are computed at the beginning of the algorithm. Enumeration of the feasible trajectories is then limited between these bounds. The process continues with always better estimation of the bounds until the optimal trajectory is isolated.

The overall planning procedure has the following advantages: (a) The storage requirements are low. As a matter of fact, in-core solutions can be obtained for even large networks, (b) It yields the global optimum with high level of confidence, and (c) The execution time is reasonable.

Another advantage of the method is the fact that it is very flexible in accepting any mathematical model of the transmission network. Therefore, the accuracy of the results is controllable. To demonstrate this flexibility two different power flow models for the transmission network have been used as well as two different definitions of state admissibility. The power flow model determines the accuracy of the computations while the definition of state admissibility reflects the operational practices of the particular company.

Recommendations

In general, a planning procedure can be divided into three phases:

1. Principal planning phase
2. Advanced planning phase
3. Project planning phase

The objective of the first phase is to isolate a number of solutions to the planning problem which are feasible and which are economical. The advanced planning phase involves detailed evaluation of the solutions produced in the previous phase. And finally, human decisions will carry out the project planning phase.

The contribution of this thesis is directed to the principal planning phase. This phase can be formulated as an optimization problem. Because of the complexity of operation of power systems, it is recommended that the mathematical model of the system should reflect the operational practices of the system under consideration. Furthermore, it is the belief of the writer that, in this planning phase, a less accurate mathematical model of the system can be used, given that the optimization problem will yield all the solutions which lie in a specified neighborhood of the optimal. In this case, the errors introduced by a simplified mathematical model can be detected and corrected in the advanced planning phase.

A controversial issue in a planning study is the length of the planning period. From another point of view, the same issue can be stated as: for how long in the future a present decision will have a sound economic impact on the system. The issue is complex and the answer depends on the system, the rate of demand increase, and the economic environment. Furthermore, the following facts increase the complexity of the problem: (a) there is uncertainty in the load forecast, (b) there is uncertainty in the future economic environment, and (c) research and development introduces innovations.

The planning method of this thesis can solve the following related problem. Determine the minimum value of N (number of stages in the planning period) which does not affect the first stage decision. To solve this problem the solution to the transmission planning problem for different values of N should be obtained and then the minimum value of N which does not affect the first year decision can be obtained by inspection. This process yields values of N in the neighborhood of three to five years. Based on this evidence, we recommend that a planning period in the neighborhood of ten years will be sufficient in most cases.

The uncertainty in the electric power demand and the cost has been neglected in the work of this thesis. Inclusion of this uncertainty in an enumerative optimization process is straightforward. However, in view of the fact that forecasting methods have advanced to the degree of predicting the electric power demand for several years in the future and with small deviations, it is recommended that the uncertainty should be taken into account in the advanced planning phase.

APPENDIX A

In Chapter II the corrective rescheduling problem has been formulated as follows:

$$\text{Minimize } \Delta C = B^T P_G + (\Delta P_G)^T C \Delta P_G, \text{ subject to} \tag{39}$$

$$\left| \psi_\ell^{(m)} + \left(\frac{\partial \phi_\ell^{(m)} (P_G)}{\partial P_G} \right)^T \Delta P_G \right| \leq \overline{\psi}_\ell (x(k), m) \qquad \ell = 1, 2, \ldots, M \tag{40}$$

$$\sum_i \Delta P_{Gi} = 0 \qquad i = 1, \ldots, n_g \tag{38}$$

$$\Delta P_{min} \leq \Delta P_G \leq \Delta P_{max} \tag{41}$$

The solution to the above problem, if it exists, can be found with a standard quadratic program. For planning purposes, however, it will be impractical from the computational and storage point of view. On the other hand, it has been observed that in most cases only one "soft" constraint is violated or few of them in the worst cases. The majority of the inequalities (40) are ineffective and a tremendous reduction of the problem is achieved if the ineffective constraints are neglected. This logic leads to the following suboptimal algorithm.

1. Let $S_{c.r.}$ represent the subset of the constraints (40) which are effective.

2. Let p_i be the participation factor of the generation plant

i. These factors are determined externally according to the generating margins and incremental fuel cost.

3. Define

$$s_i = \sum_{\ell \in S_{c.r.}} \frac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_{Gi}}, \qquad i = 1,2,3, \ldots, n_g$$

4. Set

$$\Delta P_{Gi} = \begin{cases} s_i p_i f^+ \alpha & \text{if } s_i > 0 \\ s_i p_i \alpha & \text{otherwise} \end{cases}$$

$$i = 1,2, \ldots, n_g$$

p_i is the participation factor

f^+, α non-negative constant to be determined.

5. Compute the constant f^+ from equation (38). If no value for f^+ can be defined, the algorithm terminates. If yes, proceed to the next step.

6. The constraints

$$\left|\psi_\ell^{(m)} + \left(\frac{\partial \phi_\ell^{(m)}(P_G)}{\partial P_G}\right)^T \Delta P_G\right| \leq \overline{\psi}_\ell(x(k),m), \quad \ell \in S_{c.r.}$$

$$\Delta P_{min} \leq \Delta P_G \leq \Delta P_{max}$$

contain only one unknown, the constant α. If there is not a value of α such that it satisfies the above constraints, the algorithm terminates. If there is one or more values of α satisfying all the constraints, then let α^* be their minimum.

7. The solution is

$$P_{Gi} = \begin{cases} s_i p_i f^+ \alpha^* & \text{if } s_i > 0 \\ s_i p_i \alpha^* & \text{otherwise} \end{cases}$$

The described algorithm is simple and very fast. The objective, minimization of the incremental cost ΔC, can be taken into account in the participation factors p_i:

$$p_i = \begin{cases} \dfrac{1}{\dfrac{\partial \Delta C}{\partial P_{Gi}}} & \text{if generating plant i is participating in the corrective rescheduling} \\ 0 & \text{otherwise} \end{cases}$$

Since in the search of solution, only the effective constraints were considered, it is possible that the new generation schedule may force other "soft" constraints to be violated. It is, therefore, necessary to check the solution. The load flow problem is solved with the new vector of power injections and the "soft" constraints are checked. If they are satisfied, the corrective rescheduling was successful.

The computation of the derivatives

$$\frac{\partial \phi_{\ell}^{(m)}}{\partial P_{Gi}}$$

is rather straightforward. According to the DC-model, the power flow, when the highest capacity line from the right of way m is removed, is described by

$$Y^{(m)}(x(k))\theta^{(m)} = P_G - P_L$$

Differentiating both sides, we obtain

$$Y^{(m)}(x(k))d\theta^{(m)} = dP_G$$

since $Y^{(m)}(x(k))$ and P_L are constant. Then,

$$d\theta^{(m)} = [Y^{(m)}(x(k))]^{-1}dP_G$$

Assume

$$dP_G = Z \cdot dP_{Gi}$$

where Z is a vector defined as follows:

$$Z_j = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{otherwise} \end{cases}$$

Then

$$d\theta^{(m)} = [Y^{(m)}(x(k))]^{-1}Z \cdot dP_{Gi}$$

Since

$$\phi_\ell^{(m)} = e_\ell^T \theta^{(m)}$$

it follows

$$d\theta_\ell^{(m)} = e_\ell^T d\theta^{(m)} = e_\ell^T [Y^{(m)}(x(k))]^{-1}Z \cdot dP_{Gi}$$

and therefore

$$\frac{\partial \phi_\ell^{(m)}}{\partial P_{Gi}} = e^T [Y^{(m)}(x(k))]^{-1}Z$$

APPENDIX B

In this appendix an expression for the derivative

$$\frac{\partial \phi_\ell}{\partial y_i}$$

is derived. A reciprocity type relation will be proved and used in the computation of the effectiveness ratio vector.

The DC load flow equations are in matrix notation

$$Y\theta = P \tag{B1}$$

The power injections vector P is assumed to be constant. Differentiating both sides of the equation (B1), we obtain

$$Y \cdot d\theta + dY \cdot \theta = 0$$

or

$$d\theta = -Y^{-1} \cdot dy \cdot \theta$$

If the "capacity" of the i^{th} right-of-way has only changed by dy_i, then

$$dY = e_i e_i^T \cdot dy_i$$

where

$$e_i = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ -1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \\ -k \\ \\ \\ \\ \\ \\ -n \\ \\ \\ \\ \\ \\ \end{matrix}$$

if the right of way i terminates at buses k and n.

Then

$$d\theta = -Y^{-1} e_i e_i^T \cdot \theta \cdot dy_i \tag{B2}$$

Since

$$\phi_\ell = \psi_\ell = e_\ell^T \theta$$

$$\rightarrow \qquad d\phi_\ell = e_\ell^T d\theta \tag{B3}$$

Substituting (B2) into (B3), we obtain

$$d\phi_{\ell} = -e_{\ell}^{T}Y^{-1}e_{i}e_{i}^{T}\theta \cdot dy_{i}$$

$$\rightarrow \qquad \frac{\partial\phi_{\ell}}{\partial y_{i}} = -e^{T}Y^{-1}e_{i}e_{i}^{T}\theta$$

Since $\psi_{i} = e_{i}^{T}\theta$, it follows

$$\frac{\partial\phi_{\ell}}{\partial y_{i}} = -e_{\ell}Y^{-1}e_{i}\psi_{i}$$

The quantity $A_{\ell i} = e_{\ell}Y^{-1}e_{i}$ is dependent only on the system's parameters. ψ_{i} is the phase angle difference across the right of way i, which is a function of the power injections at the nodes of the network. Therefore, the derivative

$$\frac{\partial\phi_{\ell}}{\partial y_{i}}$$

can be factored as follows:

$$\frac{\partial\phi_{\ell}}{\partial y_{i}} = -A_{\ell i} \cdot \psi_{i}$$

If matrix Y is symmetric,* then

---

*Matrix Y is symmetric for almost all power networks.

$$A_{\ell i} = A_{i\ell}$$

Since

$$-A_{\ell i} = \frac{1}{\psi_i} \frac{\partial \phi_\ell}{\partial y_i}$$

and

$$-A_{i\ell} = \frac{1}{\psi_\ell} \frac{\partial \phi_i}{\partial y_\ell}$$

It follows

$$\frac{1}{\psi_i} \cdot \frac{\partial \phi_\ell}{\partial y_i} = \frac{1}{\psi_\ell} \cdot \frac{\partial \phi_i}{\partial y_\ell} \tag{B4}$$

The above relationship may be recognized as the reciprocity theorem in the networks. It can be used to speed up the computations of the effectiveness ratio vector which is defined in Chapter III. The definition is cited below.

$$\text{E.R.V.} = \left[\frac{\frac{\partial \phi_\ell}{\partial y_i} x_\ell d_\ell^2}{\frac{\partial \phi_\ell}{\partial y_\ell} x_i d_i^2} \right]$$

$$i = 1, 2, \ldots, M$$

From (B4), we obtain

$$\frac{\partial \phi_\ell}{\partial y_i} = \frac{\psi_i}{\psi_\ell} \cdot \frac{\partial \phi_i}{\partial y_\ell}$$

Then

$$\text{E.R.V.} = \left[\frac{\dfrac{\psi_i}{\psi_\ell} \; \dfrac{\partial \phi_i}{\partial y_\ell} \, x_\ell d_\ell^2}{\dfrac{\partial \phi_\ell}{\partial y_\ell} \, x_i d_i^2} \right]$$

$$i = 1, 2, \ldots, M$$

The above vector can be computed with only one forward and back substitution.

APPENDIX C

In Chapter II, the generation scheduling problem was defined as a mixed optimization problem. The statement of this problem is:

$$\text{Minimize } Z = \sum_j f_j(P_j), \text{ subject to} \tag{24}$$

$$f_j(P_j) = a_j + b_j P_j + c_j P_j^2 \tag{25}$$

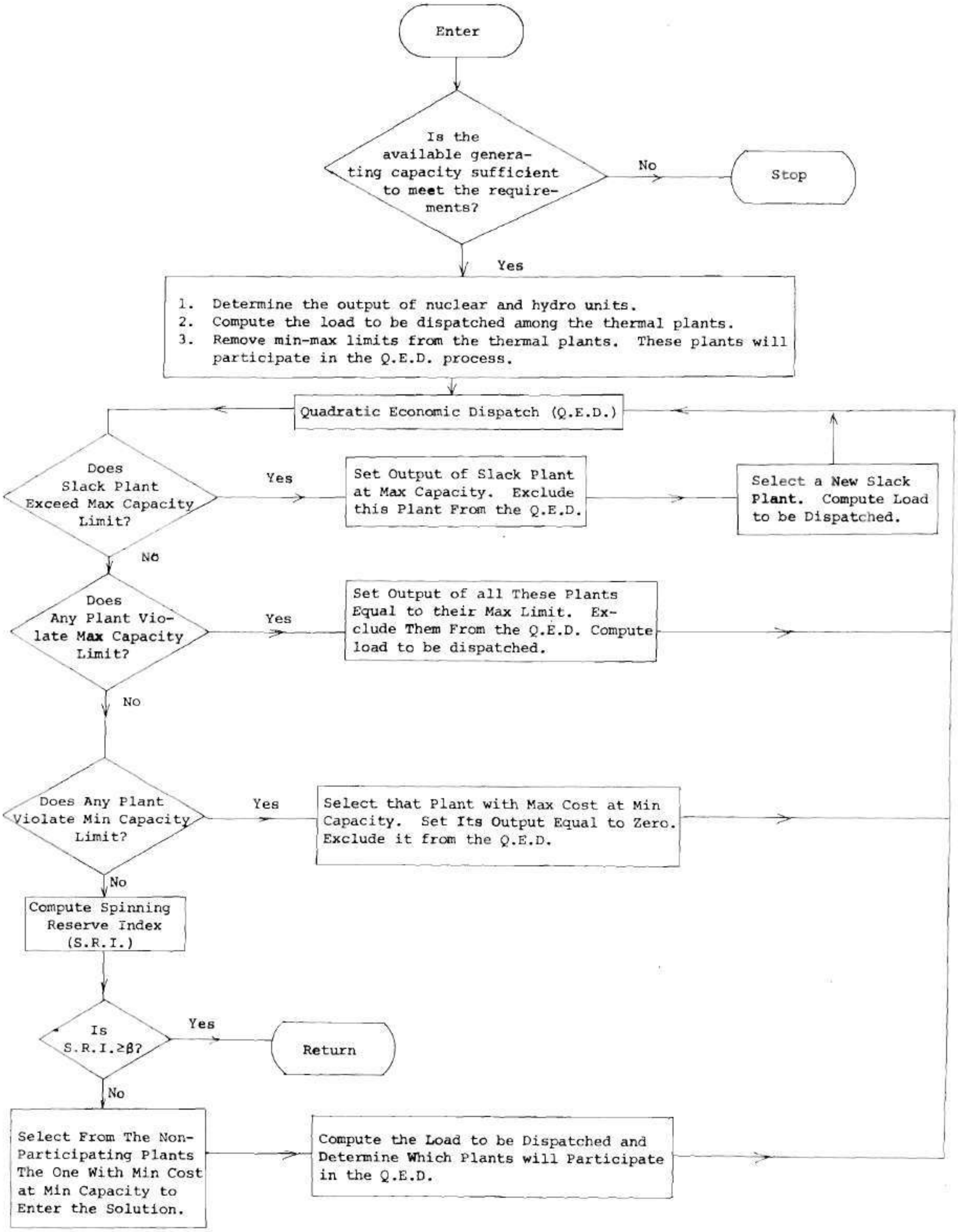
$$P_j^{min} \le P_j \le P_j^{max} \text{ or } P_j = 0 \tag{26}$$

$$\sum_j P_j = P_L \tag{27}$$

$$a_1 C_1 + a_2 C_2 \ge \beta \tag{28}$$

The variables have been defined in Chapter II.

An optimization procedure has been developed which is suboptimal with respect to the spinning reserve constraint (Inequality 28). This procedure is depicted in the following flowchart.



Generation Scheduling

BIBLIOGRAPHY

1. A. M. Sasson and H. M. Merril, "Some Applications of Optimization Techniques to Power System Problems," Proceedings of the IEEE, Vol. 62, No. 7, pp. 959-972, July 1974.

2. C. A. DeSalvo and H. L. Smith, "Automatic Transmission Planning With AC Load Flow and Incremental Transmission Loss Evaluation," IEEE Trans. Power App. Syst., Vol. PAS, pp. 156-163, February 1965.

3. R. Billinton and M. P. Bhavaraju, "Transmission Planning Using A Reliability Criterion, Part I: A Reliability Criterion," IEEE Trans. Power App. Syst., Vol. PAS-89, No. 1, pp. 28-34, January 1970.

4. M. P. Bhavaraju and R. Billinton, "Transmission Planning Using A Reliability Criterion, Part II: Transmission Planning," IEEE Trans. Power App. Syst., Vol. PAS-90, No. 1, pp. 70-78, Jan./Feb. 1971.

5. W. R. Puntel, N. D. Reppen, R. J. Ringlee, J. E. Platts, W. A. Ryan, and P. J. Sullivan, "An Automated Method for Long-Range Planning of Transmission Networks," in Proc. 8th PICA Conf. (Minneapolis, Minn., 1973), pp. 38-46.

6. S. W. Director, and A. Rohrer, "The Generalized Adjoint Network and Network Sensitivities," IEEE Trans. on Circuit Theory, Vol. CT-16, No. 3, August 1969.

7. L. L. Garver, "Transmission Network Estimation Using Linear Programming," IEEE Trans. Power App. Syst., Vol. PAS-89, pp. 1688-1697, Sept./Oct. 1970.

8. J. E. Platts, R. M. Sigley, and L. L. Garver, "A Method for Horizon-Year Transmission Planning," Conference Paper presented at IEEE Power Engineering Society Winter Meeting, 1972.

9. V. G. Knight, R. R. Booth, S. A. Mallard, and D. M. Lewis, "Computers in Power System Planning," Proceedings of the IEEE, Vol. 62, No. 7, pp. 872-883, July 1974.

10. F. Beglari, and M. A. Laughton, "The Combined Costs Method for Optimal Economic Planning of an Electrical Power System," Paper presented at the IEEE PES Summer Meeting and Energy Resources Conf., Anaheim, Calif., July 14-19, 1974.

11. N. Adams, F. Beglari, M. A. Laughton, and G. Mitra, "Mathematical Programming Systems in Electrical Power Generation, Transmission and Distribution Planning," in Proc. 4th PSCC (Grenoble, France, 1972) Paper 1.1/13.

12. R. N. Adams, and M. A. Laughton, "A Dynamic Programming/Network Flow Procedure for Distribution System Planning," in Proc. 8th PICA Conf. (Minneapolis, Minn., 1973), pp. 348-354.

13. J. C. Kaltenbach, J. Peschon, and E. H. Gehrig, "A Mathematical Optimization Technique for the Expansion of Electric Power Transmission Systems," IEEE Trans. Power App. Syst., Vol. PAS-89, No. 1, pp. 113-119, Jan. 1970.

14. S. T. Y. Lee, K. L. Hicks, and E. Hnyilicza, "Transmission Expansion by Branch-and-Bound Integer Programming with Optimal Cost-Capacity Curves," Paper presented at the IEEE PES Winter Meeting, New York, Jan./Feb. 1974.

15. Y. P. Dusonchet, and A. H. El Abiad, "Transmission Planning Using Discrete Dynamic Optimizing," IEEE Trans. Power App. Syst., Vol. PAS-92, pp. 1358-1371, July/Aug. 1973.

16. P. H. Henault, R. B. Eastvedt, J. Peschon, and L. P. Hajdn, "Power System Long Term Planning in the Presence of Uncertainty," IEEE Trans. Power App. Syst., Vol. PAS-89, No. 1, pp. 156-164, Jan. 1970.

17. IEEE Committee Report, "Present Practices in the Economic Operation of Power Systems," Paper 71 TP 97-PWR, presented at the IEEE Winter Power Meeting, New York, Jan./Feb. 1971.

18. W. F. Tinney, and W. S. Meyer, "Solution of Large Sparse Systems by Ordered Triangular Factorization," IEEE Trans. Automatic Control, Vol. AC-18, No. 14, August 1973.

19. W. F. Tinney, and J. W. Walker, "Direct Solutions of Sparse Network Equations by Optimality Ordered Triangular Factorization," Proceedings of the IEEE, Vol. 55, No. 11, November 1967.

20. B. Stott, O. Alsac, "Fast Decoupled Load Flow," IEEE Trans. on Power App. Syst., Vol. PAS-93, No. 3, pp. 859-869, May/June 1974.

21. B. Stott, "Review of Load-Flow Calculation Methods," Invited paper, Proceedings of the IEEE, Vol. 62, No. 7, July 1974.

22. A. S. Debs, and A. R. Benson, "Security Assessment of Power Systems," Final Report to Energy Research and Development Administration prepared under contract to Georgia Institute of Technology, July 1975.

23. J. C. Kaltenbach, L. P. Hajdn, "Optimal Corrective Rescheduling for Power System Security," IEEE Trans. on Power App. Syst., Vol. PAS-90, No. 2, March/April 1971.

24. B. F. Wollenberg, W. O. Stadlin, "A Real Time Optimizer For Security Dispatch," IEEE Transactions, PAS-93, Sept./Oct. 1974, pp. 1640-1649.

25. R. Billinton, A. V. Jain, "Unit Derating Levels in Spinning Reserve Studies," IEEE Winter Power Meeting, New York, Feb. 1971.

26. R. Billinton, A. V. Jain, "The Effect of Rapid Start and Hot Reserve Units in Spinning Reserve STudies," Paper presented at the IEEE Summer Meeting and International Symposium on High Power Testing, Portland, Oregon, July 1971.

27. R. Billinton and A. V. Jain, "Interconnected System Spinning Reserve Requirements," Paper presented at the IEEE Summer Meeting and International Symposium on High Power Testing, Portland, Oregon, July 1971.

28. V. M. Cook, C. D. Galloway, M. J. Steinberg, A. J. Wood, "Determination of Reserve Requirements of Two Interconnected Systems," IEEE Trans. on Power App. Syst., April 1963, pp. pp. 18-33.

29. R. Billinton, Power System Reliability Evaluation, Gordon and Breach, Science Publishers, New York, 1970.

30. J. L. Carpentier, "Differential Injections Method, A General Method For Secure and Optimal Load Flows," 1973 PICA Conf. Proceedings.

31. J. L. Carpentier, "Total Injections Method, A Method For the Solution of the Unit Commitment Problem Including Secure and Optimal Load Flow," 1973 PICA Conf. Proceedings.

32. R. S. Garfinkel and G. L. Nemhauser, Integer Programming, John Wiley and Sons.

33. Cooper and Steinberg, Methods of Optimization, W. B. Saunders Company.

34. H. A. Taha, Operations Research, An Introduction, McMillan Publishing Company.

35. K. N. Stanton and P. C. Gupta, "Forecasting Annual or Seasonal Peak Demand in Electric Utility Systems," IEEE Trans. on Power App. and Syst., Vol. PAS-89, No. 5, May/June 1970.

36. P. C. Gupta, "Model Identification in Peak Power Demand Forecasting," Paper THP4.5, Proceedings of Conference on Decision and Control, November 1974.

37. G. Contaxis and A. S. Debs, "Network Equivalents for On-Line Power System Security Assessment," Paper presented at the IEEE Southeastern Conference, Clemson, SC, April 1976.

38. IEEE Committee Report, "Bibliography on Power Capacitors 1967-1970," Paper T72 210-8, IEEE Winter Meeting, New York, Jan./Feb. 1972.

39. E. Zolezzi Ch., F. Vervloet S., "Automatic Allocation of Static Capacitors in Power Systems Using a Linearized Network Model," in Proc. 8th PICA Conf. (Minneapolis, Minn., 1973) pp. 333-338.

40. A. M. Pretelt, "Automatic Allocation of Network Capacitors," Paper 70 TP 47-PWR presented at the IEEE Winter Power Meeting, New York, 1970.

41. J. B. Young, "Optimal Static Capacitor Allocation by Discrete Programming: Development of Theory," Paper 70 TP 68-PWR presented at the IEEE Winter Power Meeting, New York, 1970.

42. R. M. Maliszewski, L. L. Garver, and A. J. Wood, "Linear Programming as an Aid in Planning Kilovar Requirements," IEEE Trans. Power App. Syst., Vol. PAS-87, pp. 1963-1968, December 1968.

43. S. S. Sachdeva, R. Billinton, "Optimum Network VAR Planning by Nonlinear Programming," Paper T 73 111-2, Presented at the IEEE PES Winter Meeting, New York, Jan./Feb. 1973.

VITA

Athanasios Panayotis Meliopoulos (nickname Sakis) was born in Katerini, Greece, on March 19, 1949. Mr. Meliopoulos graduated with the Diploma in Electrical and Mechanical Engineering from the National Technical University in Athens, Greece, in 1972. As an undergraduate, he was honored with several awards. He graduated with a M.S. in Electrical Engineering from the Georgia Institute of Technology in 1974.

The School of Electrical Engineering at the Georgia Institute of Technology granted Mr. Meliopoulos a Doctor of Philosophy degree in 1976. He was appointed to positions of graduate research and teaching assistantships during his graduate studies.

Mr. Meliopoulos is a member of the Society of Professional Engineers of Greece and the IEEE.