

# **THE EFFECT OF SOLUTE DISSOLUTION KINETICS ON CLOUD DROPLET FORMATION**

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# **THE EFFECT OF SOLUTE DISSOLUTION KINETICS ON CLOUD DROPLET FORMATION**

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# TABLE OF CONTENTS

Acknowledgments	
List of Tables	ii
List of Figures	iii
List of Symbols or Abbreviations	iv
Summary	vi
Chapter 1. Introduction	1
Chapter 2. Dissolution kinetics model	4
2.1. Equation Formulation	4
2.2. Integration procedure	7
2.3. Simulations Considered	8
2.4. Simulation Results	9
Chapter 3. Introducing Solute Dissolution Kinetics into Köhler Theory	17
3.1. Parameterization of the numerical simulations	17
3.2. Introduction to Köhler Theory	20
3.3. Implications of solute dissolution kinetics for cloud droplet formation.	21
Chapter 4. Summary	26
References	27

## LIST OF TABLES

Table 1: Aqueous diffusivity of organic compounds found in atmospheric aerosols.	5
Table 2: Parameters varied in numerical simulations.	9

## LIST OF FIGURES

- Figure 1: Illustration of the problem geometry and the solute concentration profile.  $R_s$  represents the location where the concentration gradient becomes effectively zero. 10
- Figure 2: Transient radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ , and  $s = 1\%$ . 11
- Figure 3: Steady-state radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $s = 1\%$ , and for  $D_{Aw}$  between  $1 \times 10^{-9}$  and  $1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ . 12
- Figure 4: Steady-state radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ , and for  $s$  between 0.1% and 1%. 13
- Figure 5: Steady-state droplet surface  $C_A$  normalized with  $C_{eq}$ , as a function of core diameter and ambient supersaturation. Simulations are shown for (a)  $D_p = 1\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ , and, (b)  $D_p = 5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ . 14
- Figure 6: Parameterized vs. Simulation Surface Solute Concentration Values. Simulations are shown for which  $C^* / C^{eq} \leq 0.95$ . 19
- Figure 7: Köhler curves modified to include the effect of dissolution kinetics. The dry aerosol is 75 nm in diameter and composed of a partially soluble substance with  $1750 \text{ kg m}^{-3}$  density,  $0.132 \text{ kg mol}^{-1}$  molar mass, van't Hoff factor of 2 and a solubility of  $10^{-2} \text{ kg kg}^{-1}$ . Ambient supersaturation is assumed to be 1%. Calculations are presented as (a)  $S_{eq}$  vs. wet diameter for a range of  $D_{Aw}$ , and, (b) change in  $S_{eq}$ , relative to the instantaneous dissolution. 22
- Figure 8: Same as Figure 7, but for a 100 nm dry particle diameter 25

## LIST OF SYMBOLS AND ABBREVIATIONS

$C_A(r,t)$  -- denotes the concentration of  $A$  [moles/m<sup>3</sup>] at time  $t$  [sec] distance  $r$  [m] from the core droplet center

$C^{eq}$  -- solubility in water

$C^*$  -- is the steady-state concentration of  $A$  at the droplet surface.

CCN – Cloud Condensation Nuclei

$D_{A,w}$  -- the diffusivity of  $A$  in the droplet aqueous

$D_p$  -- the droplet diameter [m], diameter corresponding radial distance  $R_p$

$\frac{dD_p}{dt}$  -- droplet growth rate

$\Delta D_p$  -- change is droplet diameter from droplet surface to previous droplet diameter

$dr$  -- infinitesimal thickness, , adjacent to the droplet surface (which later on we will refer to as “surface layer”)

$D_s$  -- diameter corresponding radial distance  $R_s$

$D_v'$  -- mass transfer coefficient of water vapor from the gas to the droplet.

G – growth constant parameter defined,

$\Delta H_v$  -- the latent heat of vaporization for liquid water,

$M_w$  -- the molecular weight,

$n_A$  -- moles of solute A

$\frac{dn_A}{dt}$  -- moles of  $A$  in the added condensed water vapor volume

$n_s$  -- the moles of solute dissolved in the droplet,

$k_a'$  -- the thermal conductivity of air

$p^\circ(T)$  -- the vapor pressure at the ambient temperature  $T$ ,

$\rho_w$  -- density of water

$r$  -- radial distance from core droplet center

$R$  -- the universal gas constant

$R_c$ , -- radial distance at the core surface,

$R_s$  -- the distance where the dilution effect of water condensation on the droplet surface (dilution) balances diffusional supply of solute from the core

$R_p$  --radial distance at droplet surface

$s = S - 1$  is the supersaturation ratio

$S$  -- ambient water vapor saturation ratio

$S_c$  -- critical supersaturations

$S_{eq}$  -- equilibrium saturation ratio of the growing droplet,

$\sigma_w$  -- Surface tension of water,

$t$  -- time in seconds

$V$  , droplet Volume

$\frac{dV}{dt}$  , change volume of the droplet from water vapor condensation,

$\nu$  -- Van't Hoff factor,

$\omega = \frac{D_p}{D_c}$  -- ratio of particle diameter to core diameter

$\delta = 1 - \frac{D_c}{D_s}$  -- Parameterized variable.



## **SUMMARY**

This study focuses on the importance of solute dissolution kinetics for cloud droplet formation. To comprehensively account for the kinetics, a numerical model of the process was developed. Simulations of cloud droplet growth were performed for solute diffusivity, droplet growth rates, dry particle and droplet diameters relevant for ambient conditions. Simulations suggest that high ambient supersaturations and a decrease in solute diffusivity are major contributors to significant decreases in effective solute surface concentrations. The numerical simulations were incorporated into Köhler theory to assess the impact of dissolution kinetics on the droplet equilibrium vapor pressure. For CCN composed of partially soluble material, a significant increase was found in the equilibrium supersaturation of CCN.

## CHAPTER 1: INTRODUCTION

Organics constitute a major component of natural and anthropogenic particulate matter (Sloane, 1991; Wolff, 1991; Chow, 1994; Seinfeld and Pandis, 1997), yet much remains to be grasped about their effects on cloud droplet formation. Although not completely understood, carbonaceous aerosol can readily form cloud droplets (Novakov and Penner, 1993) and potentially have a strong effect on cloud formation and the hydrological cycle (Shulman et al., 1996; Facchini et al., 1999; IPCC, 2001; Feingold and Chuang, 2002; Nenes et al., 2002; Lance et al., 2004)

The theory used to describe the formation of cloud droplets from precursor aerosol, or, “cloud condensation nuclei” (CCN), was first developed by Köhler (Köhler, 1936) and has successfully been applied to CCN composed of deliquescent inorganic salt aerosols (ammonium sulfate and sodium chloride) and low molecular weight organic species that exhibit hygroscopic behavior (e.g, adipic acid and glutaric acid) (Cruz and Pandis, 1997; Raymond and Pandis, 2002). Unfortunately, the theory is less successful in describing the behavior of less hygroscopic compounds, such as those found in ambient aerosol (Cruz and Pandis, 1997; Raymond and Pandis, 2002). Analysis of ambient CCN measurements (Chuang P. Y., 2003; VanReken, 2003) typically show large deviations between predicted and measured CCN concentration under polluted conditions, which are often attributed to the complex interaction of organics with water. Organics, depending on their solubility, can contribute solute (Shulman et al., 1996; Laaksonen et al., 1998). Hydrophobic compounds with multiple functional groups may act as strong surfactants and considerably depress surface tension (Shulman et al., 1996; Facchini et al., 1999; Nenes et al., 2002); compressed surfactant layers may act as “film-forming compounds”,

and may influence droplet growth kinetics enough to affect droplet number and spectral dispersion (Blanchard, 1964; Gill et al., 1983; Feingold and Chuang, 2002; Nenes et al., 2002; Chuang P. Y., 2003). Polymerization reactions, thought to occur in regions of secondary organic aerosol formation (Limbeck et al., 2003), may also have a considerable impact on the CCN properties of carbonaceous aerosol (VanReken et al., 2005). A thorough review of organics and their interactions with water vapor can be found in Kanakidou et al., 2005.

A common assumption for partially soluble compounds is that the solute instantaneously dissolves and distributes uniformly throughout the drop (Laaksonen et al., 1998; Raymond and Pandis, 2002; Shantz et al., 2003). Compared to electrolytes, the majority of organic compounds are not very soluble in water, do not deliquesce, have a higher molar mass and thus diffuse more slowly in aqueous solutions. The implication for a growing droplet is that mass transfer of the dissolving organics may not be fast enough to assure uniform distribution of solute through the droplet volume; this may decrease the solute concentration at the droplet surface and increase the droplet equilibrium vapor pressure. If sufficient, the latter may delay or even hinder droplet formation. Thus, assuming instantaneous dissolution and distribution of solute throughout the droplet volume may overestimate the effect of slightly soluble compounds on CCN activation. This kinetic limitation mechanism is fundamentally different from the growth delay identified by Shantz et al. (2003), which arises from differences in the equilibrium curves between inorganic and organic CCN.

This study focuses on exploring the effects of solute dissolution kinetics on cloud droplet formation. A numerical model is developed to simulate the dissolution of solute

from a solid core located at the center of the droplet and its diffusion throughout the aqueous phase of the growing drop. Conditions are determined for which a significant decrease in surface solute concentration is expected. The numerical simulations are parameterized and introduced into Köhler theory for a thorough analysis of dissolution kinetics on CCN behavior.

## CHAPTER 2: DISSOLUTION KINETICS MODEL

### 2.1 Equation Formulation

The numerical model is based on the conservation of mass for the dissolving substance in a spherically symmetric droplet. The solute originates from a spherical solid core located at the center of the droplet; after dissolution, we assume that the solute mass transport occurs via molecular diffusion from the core to the droplet surface. Convective transport within the droplet phase is neglected, because i) the solution is assumed to be dilute enough so that Stefan convection is negligible, and ii) the low terminal velocity for droplets smaller than 20  $\mu\text{m}$  in diameter (i.e., most CCN during their activation phase) yields negligible shear forces on their surface (Seinfeld and Pandis, 1997) hence negligible re-circulations within the droplet volume. Assuming that the solid core is composed of a slightly soluble substance A, the transport of dissolved solute from the core into the droplet aqueous phase can be described by,

$$\frac{\partial C_A(r,t)}{\partial t} = D_{Aw} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C_A(r,t)}{\partial r} \right) \right] \quad (1)$$

where  $C_A(r,t)$  denotes the concentration of A at time  $t$  and distance  $r$  from the core droplet center, and  $D_{Aw}$  is the diffusivity of A in water.  $D_{Aw}$  depends on temperature and solute molecular size (Table 1).

**Table 1:** Aqueous diffusivity of organic compounds found in atmospheric aerosols.

Compound	Formula	$D_{A,w}$ (298 K) $\text{m}^2 \text{s}^{-1}$	$D_{A,w}$ (273 K) $\text{m}^2 \text{s}^{-1}$
Sodium Chloride*	NaCl	$1.26 \times 10^{-9}$	
Methanol $^\diamond$	CH <sub>4</sub> O	$1.94 \times 10^{-9}$	$1.02 \times 10^{-9}$
Caffeine $^\diamond$	C <sub>8</sub> H <sub>10</sub> N <sub>4</sub> O <sub>2</sub>	$6.43 \times 10^{-10}$	$3.38 \times 10^{-10}$
Adipic acid $^\diamond$	C <sub>6</sub> H <sub>10</sub> O <sub>4</sub>	$8.19 \times 10^{-10}$	$4.31 \times 10^{-10}$
Succinic acid $^\diamond$	C <sub>4</sub> H <sub>6</sub> O <sub>4</sub>	$1.00 \times 10^{-9}$	$5.27 \times 10^{-10}$
Suberic $^\diamond$	C <sub>8</sub> H <sub>14</sub> O <sub>4</sub>	$7.04 \times 10^{-10}$	$3.70 \times 10^{-10}$
Cholesterol $^\diamond$	C <sub>27</sub> H <sub>46</sub> O	$3.73 \times 10^{-10}$	$1.96 \times 10^{-10}$
C <sub>50</sub> (e.g, humic-like substances) $^\diamond$	C <sub>50</sub> H <sub>104</sub>	$2.47 \times 10^{-10}$	$1.30 \times 10^{-10}$

\* obtained from (Perry et al., 1997).  $^\diamond$  obtained from (Hines and Maddox, 1985).

The concentration of  $A$  throughout the droplet volume is initially equal to its solubility in water,  $C^{eq}$ ,

$$C_A(0, r) = C^{eq} \quad (2)$$

In terms of spatial boundary conditions, we assume that the solution is saturated with  $A$  at the core-solution interface,

$$C_A(t > 0, R_c) = C^{eq} \quad (3)$$

The boundary condition at the droplet surface is somewhat more complex, as solute diffusion and water condensation affect the surface concentration of  $A$ .  $C_A(t, R_p)$  at the droplet surface can be rewritten in terms of the rate of change of freshly condensed water

volume,  $\frac{dV}{dt}$  and the molar flux  $\frac{dn_A}{dt}$  of  $A$  into  $V$ ,

$$\frac{dC_A(t, R_p)}{dt} = \frac{d}{dt} \left[ \frac{n_A}{V} \right] = \frac{-n_A}{V^2} \left( \frac{dV}{dt} \right) + \frac{1}{V} \left( \frac{dn_A}{dt} \right) = \frac{-C_A}{V} \left( \frac{dV}{dt} \right) + \frac{1}{V} \left( \frac{dn_A}{dt} \right) \quad (4)$$

The freshly condensed water (hereon referred to as the “surface layer”) is assumed to occupy a layer of infinitesimal thickness,  $dr$ , adjacent to the droplet surface.

$\frac{dV}{dt}$  is equal to the rate of change of droplet volume and can be expressed in terms of the

droplet diameter,  $D_p$ , as

$$\frac{dV}{dt} = \frac{\pi D_p^2}{2} \cdot \frac{dD_p}{dt} \quad (5)$$

$\frac{dn_A}{dt}$  is equal to the flux of solute from the bulk of the droplet,

$$\frac{dn_A}{dt} = -D_{Aw} \frac{\partial C_A(t, R_p - dr)}{\partial r} 4\pi(R_p - dr)^2 \approx -D_{Aw} \frac{\partial C_A(t, R_p)}{\partial r} \pi D_p^2 \quad (6)$$

In equation (6), we assumed that the surface layer has negligible thickness compared to the droplet radius, so that  $R_p - dr \approx R_p$ . Introducing Equations (5) and (6) into (4)

yields,

$$\frac{\partial C_A(t, R_p)}{\partial t} = \frac{-3}{D_p} \left[ C_A \frac{dD_p}{dt} + 4D_{Aw} \frac{\partial C_A(t, R_p)}{\partial D_p} \right] \quad (7)$$

Equation (7) expresses the rate of change of  $C_A$  at the growing droplet surface and is an exact boundary condition for equation (1).  $\frac{dD_p}{dt}$  can be written in terms of the

ambient conditions (Seinfeld and Pandis, 1997) as,

$$\frac{dD_p}{dt} = \frac{1}{D_p} \frac{(S - S_{eq})}{G} \quad (8)$$

where  $S = \frac{p^w}{p^\circ(T)}$  is the ambient saturation ratio,  $p^w$  is the water vapor partial pressure,  $p^\circ(T)$  is the water saturation vapor pressure at the ambient temperature  $T$  and  $S_{eq}$  is the equilibrium saturation ratio of the growing droplet.  $G$  is a kinetic growth parameter defined as  $\frac{\rho_w RT}{4p^\circ(T)D'_v M_w} + \frac{\Delta H_v \rho_w}{4k_a' T} \left( \frac{\Delta H_v M_w}{TR} - 1 \right)$  where  $M_w$  and  $\rho_w$  are the molecular weight and density of water, respectively,  $R$  is the universal gas constant,  $\Delta H_v$  is the latent heat of vaporization for liquid water,  $k_a'$  is the thermal conductivity of air, and  $D'_v$  is mass transfer coefficient of water vapor from the gas to the droplet. Since  $S_{eq} \approx 1$  and if we neglect the size-dependence of  $G$ , Equation (8) is inversely proportional to  $D_p$  and simplifies to:

$$\frac{dD_p}{dt} \approx \frac{1}{G} \frac{S-1}{D_p} = \frac{1}{G} \frac{s}{D_p} \quad (9)$$

where  $s = S - 1$  is the ambient water vapor supersaturation. Substitution of Equation (9) into Equation (7) gives,

$$\frac{\partial C_A(t, R_p)}{\partial t} = \frac{-3}{D_p} \left[ \frac{1}{G} \frac{s C_A(t, R_p)}{D_p} + 4D_{Aw} \frac{\partial C_A(t, R_p)}{\partial D_p} \right] \quad (10)$$

Equation (10) is the droplet surface boundary condition used to integrate Equation (1).

## 2.2 Integration procedure

Equation (1), with the initial and boundary conditions expressed by Equations (2), (3) and (10) is numerically integrated using finite differences. The solution procedure



entails applying the initial condition (Equation 2) to all grid points, and then integrating the equations over time using central-differencing in the spatial direction and backward differencing in time. The boundary condition at  $r = R_c$  (Equation 3) is directly applied, while the boundary condition at  $r = R_p$  (Equation 10) is applied after approximating the derivatives with finite differences:

$$C_A(t + \Delta t, R_p) = C_A(t, R_p) - \frac{3\Delta t}{D_p} \left[ \frac{1}{G} \frac{sC_A(t, R_p)}{D_p} + 4D_{Aw} \frac{C_A(t, R_p) - C_A(t, R_p - \Delta r)}{\Delta r} \right] \quad (11)$$

where the “new” time is “ $t+\Delta t$ ”, the “old” time is “ $t$ ”, and  $\Delta t$ ,  $\Delta r$  are the time step and grid spacing used respectively.  $5 \times 10^4$  grid points between  $r = R_c$  and  $r = R_p$  are used; the equations are integrated over time until steady-state is achieved.

### 2.3 Simulations Considered

Examination of Equation (1) and its boundary conditions (Equations 2, 3 and 10) suggests that four parameters influence the extent of surface solute concentration depression from dissolution kinetics:  $D_{Aw}$ ,  $s$ ,  $D_p$  and  $D_c$ . We explore the dependence of  $C_A$  to these parameters by performing a series of sensitivity simulations. To avoid any effects from transients and initial conditions, we focus on the steady-state profiles of  $C_A(t, r)$ ; thus, to facilitate the numerical integration, we keep  $R_c$  and  $R_p$  constant during the integration.

Overall, a total of 500 simulations were completed.  $D_{Aw}$  was varied between  $1 \times 10^{-10}$  to  $1 \times 10^{-9}$  m s<sup>-2</sup>, which encompasses the diversity of water-soluble organic compounds found in ambient aerosol (Table 2). Ambient supersaturations were chosen to represent globally important cloud types; we consider radiative fog ( $s = 0.01\%$ ),

stratiform ( $s = 0.25\%$ ), cumuliform ( $s = 0.5\%$ ) and convective clouds ( $s = 1\%$ ) (Pruppacher and Klett, 1997; Seinfeld and Pandis, 1997; Nenes et al., 2001). Five different droplet diameters were considered (0.1, 0.5, 1.0, 5.0 and 10  $\mu\text{m}$ ) to represent the transition from deliquesced aerosol to activated cloud droplets (Seinfeld and Pandis, 1997). The slightly soluble core size was also varied, expressed as a fraction of the wet diameter (0.5%, 1%, 5%, 10% and 20% of  $D_p$ ).

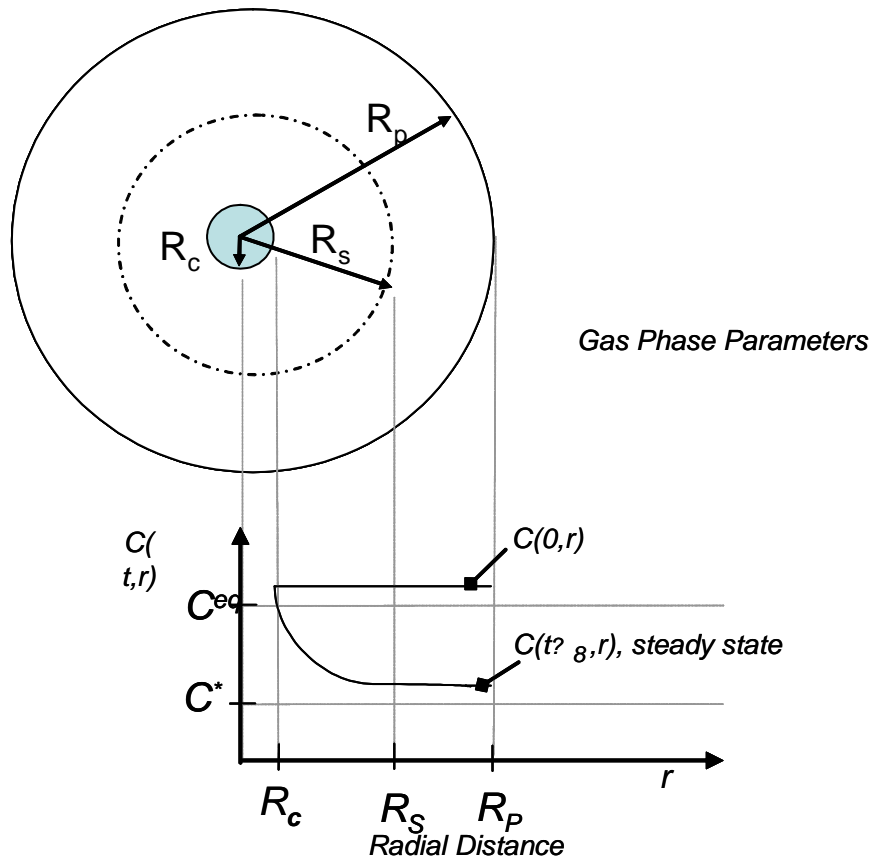
**Table 2:** Parameters varied in numerical simulations.

$D_{A,w}$ ( $\text{m}^2 \text{s}^{-1}$ )	$s$ (%) (Cloud type)	Drop Diameter (m)	$\frac{D_c}{D_p}$
$1 \times 10^{-10}$	0	$0.1 \times 10^{-6}$	0.005
$2.5 \times 10^{-10}$	0.01 (fog)	$0.5 \times 10^{-6}$	0.01
$5 \times 10^{-10}$	0.25 (stratiform)	$1.0 \times 10^{-6}$	0.05
$1 \times 10^{-9}$	0.5 (cumulus)	$5.0 \times 10^{-6}$	0.1
	1 (cumulus)	$10.0 \times 10^{-6}$	0.2

## 2.4 Simulation Results

Figure 1 illustrates the characteristics of all numerical simulations. Initially,  $C_A$  is uniform throughout the droplet volume, being equal to  $C^{eq}$ . As water begins to condense,  $C_A$  decreases at the surface layer, progressively diluting much of the droplet volume. This can be seen in Figure 2, which presents transient radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ , and  $s = 1\%$ . The simulations

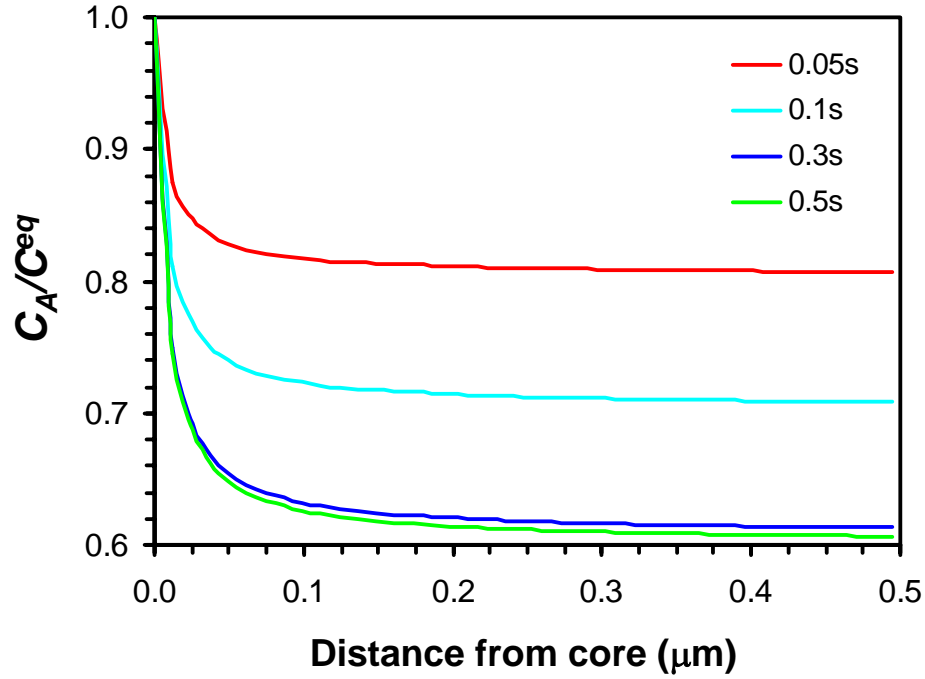
indicate that throughout most of the simulation,  $C_A$  drops steeply with distance from the core until a characteristic distance,  $R_s$  (Figure 1), after which the gradient of  $C_A$  becomes effectively zero (at steady state).



**Figure 1: Illustration of the problem geometry and the solute concentration profile.  $R_s$  represents the location where the concentration gradient becomes effectively zero.**

Figure 2 suggests that  $R_s$  may not significantly change location, and that the steady-state solution can be achieved in a fraction of a second, i.e. within the timescale of cloud droplet activation. It is therefore sufficient to focus on the steady-state concentration

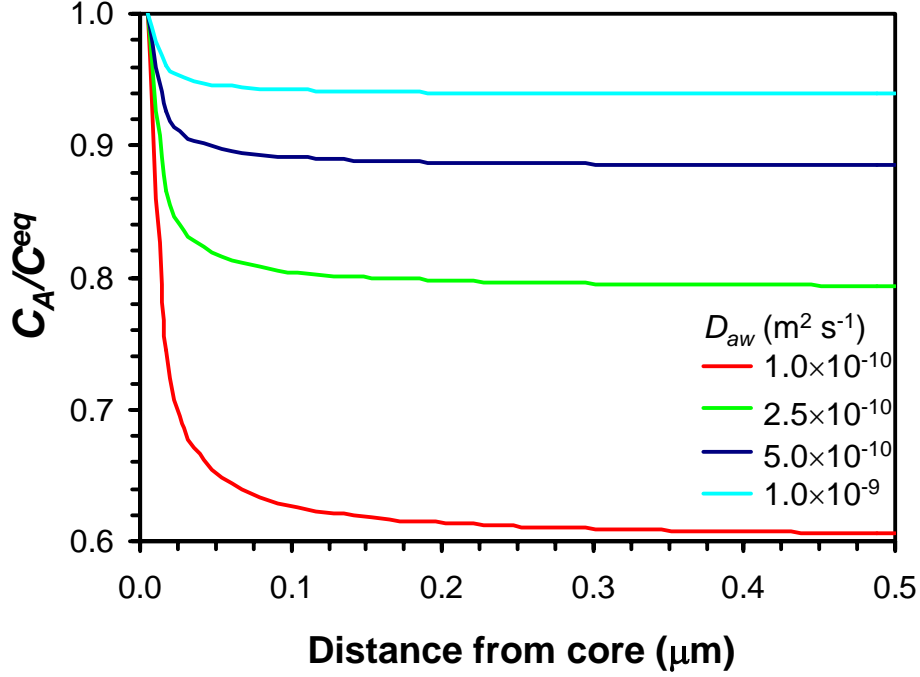
profile, with an emphasis on the steady-state  $C_A(t, R_p)$  (hereon referred to as  $C^*$ ) because it determines the droplet equilibrium vapor pressure. Therefore, it is important to explore the dependence of the  $C^*$  on all four parameters varied in Table 1.



**Figure 2: Transient radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $D_{Aw} = 1\times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ , and  $s = 1\%$ .**

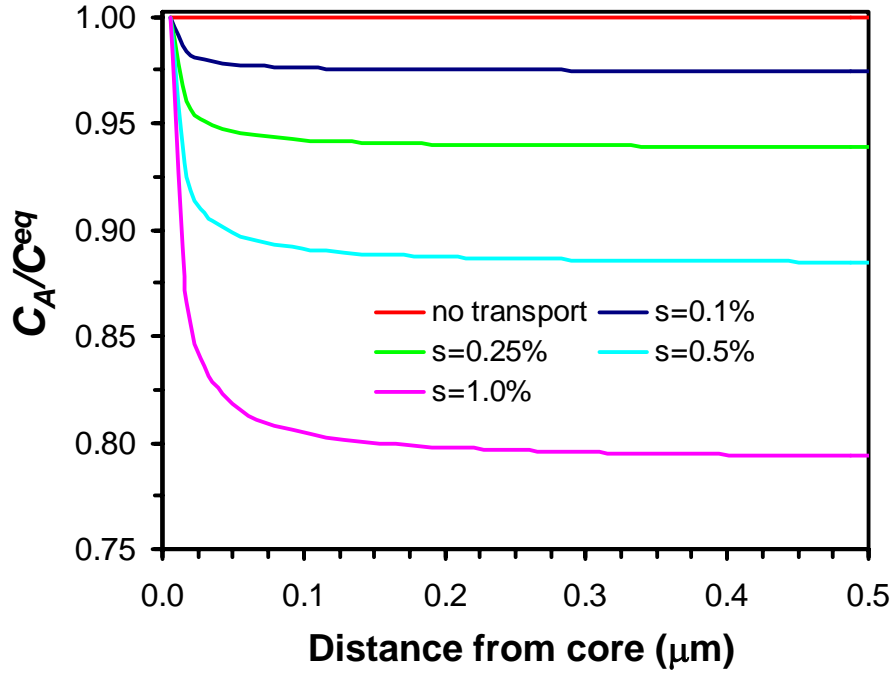
Figure 3 presents steady-state radial profiles of  $C^*/C_{eq}$  for  $D_{Aw}$  ranging between  $1\times 10^{-9}$  and  $1\times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$  and  $s = 1\%$ . As expected, the most pronounced decrease in  $C^*$  occurs for the lowest molecular diffusivity ( $\sim 10^{-10} \text{ m}^2 \text{ s}^{-1}$ ), i.e., when mass transfer is the slowest. The simulations suggest

that the concentration depression is minor ( $\sim 5\%$ ) for rapid mass transfer ( $D_{Aw} = 10^{-9} \text{ m}^2 \text{ s}^{-1}$ ) but becomes significant ( $>10\%$ ) for  $D_{Aw} < 5 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ . The % depression in  $C^*$  seems to be inversely proportional to  $D_{Aw}$ .



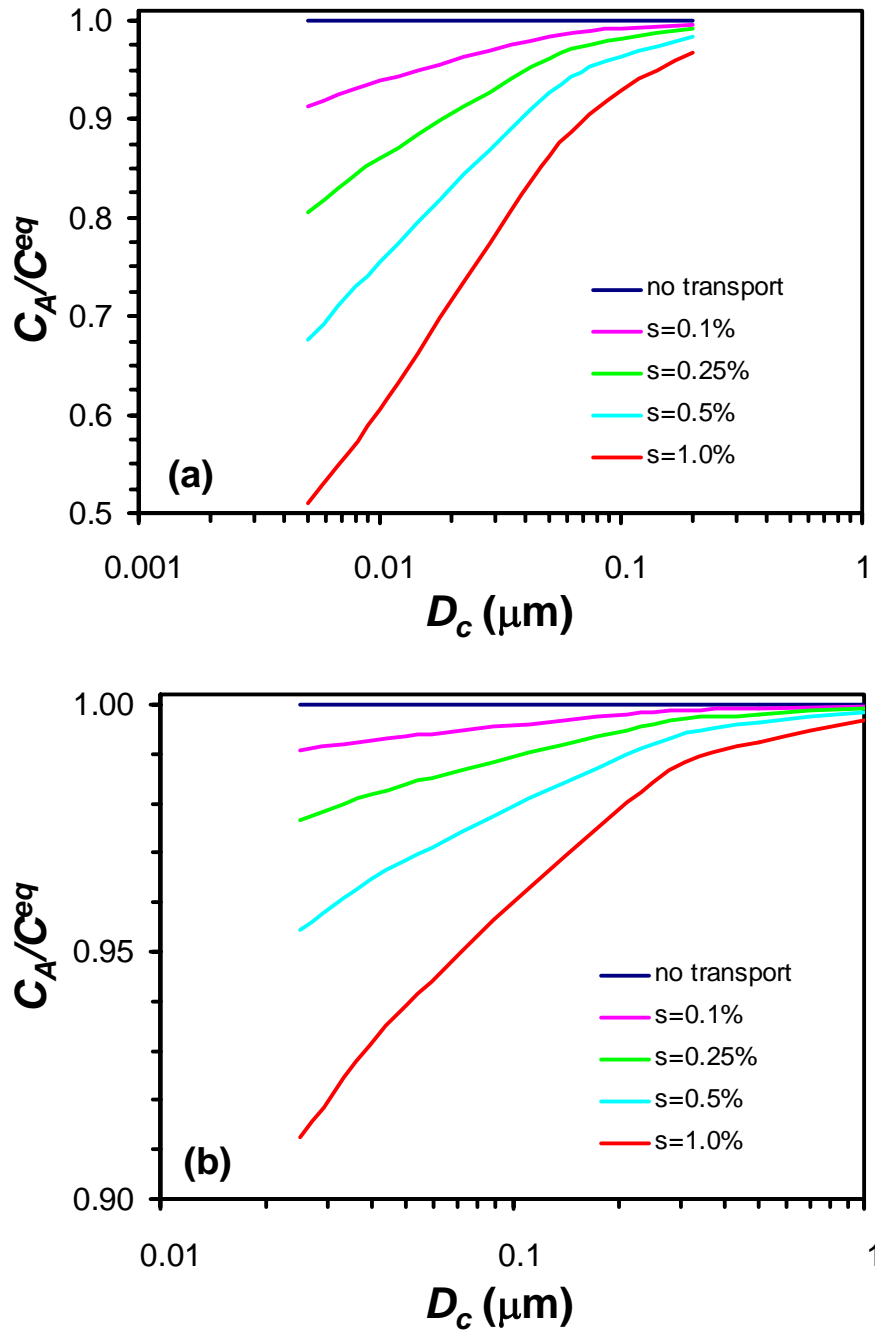
**Figure 3: Steady-state radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $s = 1\%$ , and for  $D_{Aw}$  between  $1 \times 10^{-9}$  and  $1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ .**

In addition to  $D_{Aw}$ ,  $C^*$  depends on the droplet growth rate (i.e., the ambient supersaturation). This is depicted in Figure 4, which presents steady-state radial profiles of  $C^*/C^{eq}$  for  $s$  ranging between 0.1% and 1%. Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ . The largest drop in  $C^*$  is seen for high  $s$ , as this is the condition for which dilution of A from water condensation at the surface layer is strongest. Compared to  $D_{Aw}$ , an order of magnitude change in  $s$  leads to a smaller (but still significant) decrease in  $C^*$ .



**Figure 4: Steady-state radial profiles of  $C_A$ . Simulations are shown for  $D_p = 1\mu\text{m}$ ,  $D_c = 0.5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ , and for  $s$  between 0.1% and 1%.**

Finally,  $C^*$  depends on the droplet and core diameters. This is depicted in Figure 5, which shows  $C^*/C^{eq}$  as a function of core diameter and ambient supersaturation. Simulations are shown for (a)  $D_p = 1\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ , and, (b)  $D_p = 5\mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ . It is clear that decreasing the core size and increasing the droplet size yield a larger depression in  $C^*$ , because mass transport becomes less efficient in supplying solute at the droplet surface. Simulations suggest that  $C^*$  can be fifty percent lower than  $C^{eq}$ .



**Figure 5: Steady-state droplet surface  $C_A$  normalized with  $C_{eq}$ , as a function of core diameter and ambient supersaturation. Simulations are shown for (a)  $D_p = 1 \mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-10} \text{ m}^2 \text{ s}^{-1}$ , and, (b)  $D_p = 5 \mu\text{m}$ ,  $D_{Aw} = 1 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ .**

The trends seen for  $C^*$  in the simulations can be rationalized through the following analysis. Applying the steady state requirement on the droplet surface boundary condition (Equation 10),

$$0 = -\frac{3}{D_p} \left[ C_A(t, R_p) \left( \frac{dD_p}{dt} \right) + 4D_{Aw} \left( \frac{dC_A(t, R_p)}{dD_p} \right) \right] \quad (12)$$

$\left( \frac{dC_A(t, R_p)}{dD_p} \right)$  refers to the gradient of  $C_A$  at the droplet surface, and can be expressed as

$$\left( \frac{dC_A(t, R_p)}{dD_p} \right) \approx \frac{C^* - C^{eq}}{\Delta D_p}, \text{ where } \Delta D_p \text{ is an appropriate spatial scale (more will be}$$

discussed in section 3.1). Substitution of the above into Equation (8) and solving for  $C^*$  yields,

$$C^* = \frac{C^{eq}}{\frac{\Delta D_p}{4 \cdot D_{Aw}} \left( \frac{dD_p}{dt} \right) + 1} \quad (13)$$

Substituting Equation (9) into (13) gives,

$$C^* = \frac{C^{eq}}{\frac{s}{4 \cdot D_{Aw} G} \left( \frac{\Delta D_p}{D_p} \right) + 1} \quad (14)$$

Equation 14 confirms the numerical simulation trends; when  $D_{Aw}$  is large,  $C^* \approx C^{eq}$  and when it decreases,  $C^* < C^{eq}$ . Similarly, smaller droplet sizes or large ambient supersaturations will lead to large growth rates, and will decrease  $C^*$  because of excessive dilution of the surface layer. However, when the ambient supersaturation is very small or the droplet diameter is very large,  $\frac{dD_p}{dt}$  is negligible and  $C^* \approx C^{eq}$ .  $D_c$  also affects  $C^*$ ;



smaller sizes imply that the solute needs to diffuse over a larger distance, hence  $\Delta D_p$  increases and  $C^*$  decreases.

### 3. INTRODUCING DISSOLUTION KINETICS INTO KÖHLER THEORY

#### *3.1. Parameterization of the numerical simulations*

It is desirable to introduce appropriate modifications to Köhler theory when assessing the effects of solute mass transfer kinetics on cloud droplet formation. This could be accomplished if a relationship is established between  $C^*$  and the dissolution kinetics parameters  $D_{Aw}$ ,  $s$ ,  $D_p$  and  $D_c$ . The steady-state droplet boundary condition (Equation 14) is ideally suited for this purpose; the free parameter  $\Delta D_p$  however must first be defined. In deriving Equation (14), we assumed that  $\left( \frac{dC_A(t, R_p)}{dD_p} \right) \approx \frac{C^* - C^{eq}}{\Delta D_p}$ . Based

on Figure 1, most of the variation of  $C_A$  takes place in a region close to the core; therefore the flux of A at the surface of the droplet can, at steady state, can be expressed as,

$$\left. \frac{dC_A(t, R_p)}{dD_p} \right|_{R_p} = \left. \frac{dC_A(t, R_p)}{dD_p} \right|_{R_s} \left( \frac{R_s^2}{R_p^2} \right) \approx \frac{C^* - C^{eq}}{R_s - R_c} \left( \frac{R_s^2}{R_p^2} \right) \quad (15)$$

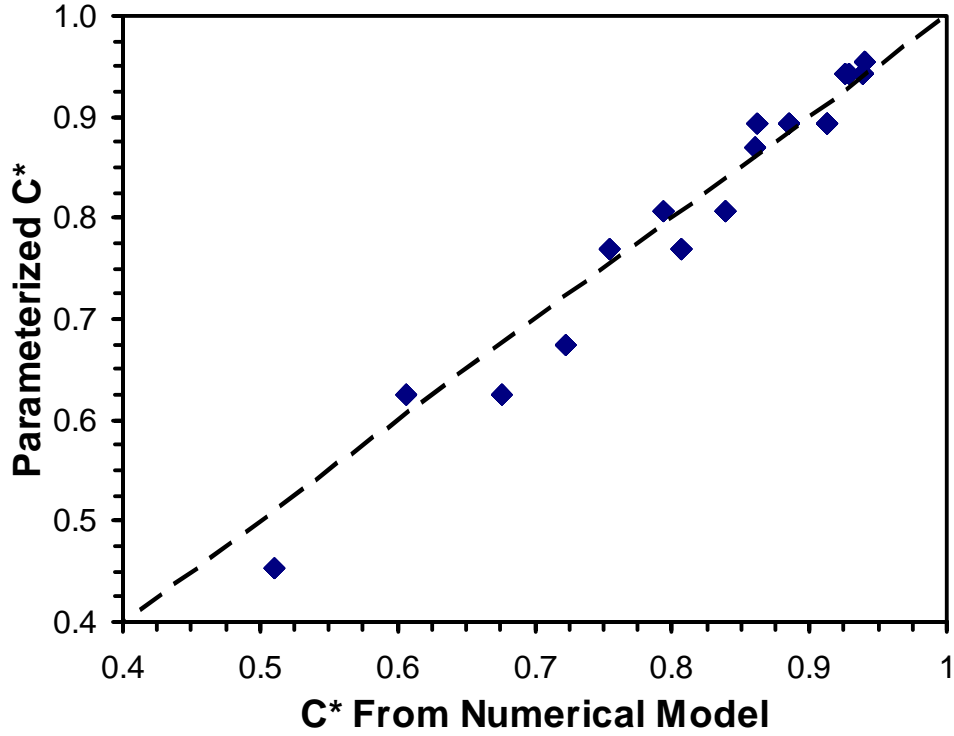
where  $R_s$  is the distance where the concentration gradient becomes effectively zero

(Figure 1). Defining  $\omega = \frac{D_p}{D_c}$  and  $\delta = 1 - \frac{D_c}{D_s}$  (where  $D_s$  and  $D_p$  are diameters

corresponding to  $R_s$  and  $R_p$ , respectively), Equation (15) becomes,

$$C^* = \frac{C^{eq}}{1 + \delta(1 - \delta) \frac{\omega s}{4GD_{Aw}}} \quad (16)$$

Equation (16) describes the solute surface concentration at the droplet surface in terms of solubility, ambient supersaturation, solute diffusivity, droplet and core size, and the non-dimensional coefficient  $\delta$  which is related to the mass transfer kinetics. In agreement with the numerical simulations, Equation (16) indicates that a decrease in solute diffusivity and an increase ambient supersaturation both yield a decrease in the solute surface concentration. A small core size (i.e., increasing  $\omega$ ) implies that the solute has to diffuse a large distance;  $C^*$  will decrease because water vapor condensation is more efficient in diluting the droplet surface layer. We use the numerical simulations to constrain  $\delta$  so that Equation (16) reproduces the steady-state  $C^*$  derived from the simulations. Least squares minimization yields  $\delta = 0.933$  with which the numerical simulations are reproduced to within 10% (Figure 6).



**Figure 6: Parameterized vs. Simulation Surface Solute Concentration Values. Simulations are shown for which  $C^* / C^{eq} \leq 0.95$ .**

An issue arises for small core diameters; as the core dissolves and approaches zero,  $\omega \rightarrow \infty$ ; under such conditions, Equation (16) predicts that  $C^* \rightarrow 0$ . This is of course does not happen in reality but arises because Equation (16) was derived assuming steady-state for  $C_A(t,r)$ . When  $D_c \rightarrow 0$ , the steady state timescale becomes exceedingly long, and Equation (16) does not become applicable. This limitation is easily overcome by considering the droplet growth dynamics. Initially, droplet growth is at a rate that steady state assumptions can be made. The core begins to dissolve, shrink and eventually reaches a size in where Equation (16) is not applicable. At this instance, we will assume that  $C^*$  does not change anymore; thus a lower-limit constraint is applied on  $C^*$ . We

consider that a timescale relevant for cloud droplet formation is  $\sim 1$  sec; under this constraint, numerical simulations suggest that  $C^* \geq 0.5C^{eq}$ . Therefore, the surface concentration of solute when mass transfer kinetics are considered is parameterized as,

$$C^* = \max \left\{ 0.5C^{eq}, \frac{C^{eq}}{1 + 0.0625 \frac{\omega S}{4GD_{AB}}} \right\} \quad (17)$$

### 3.2. Introduction to Köhler Theory

Solute mass transfer kinetics can be included in Köhler theory by appropriate modifications to existing theory. Assuming instantaneous dissolution for slightly solubles, the equilibrium supersaturation,  $s_{eq}$ , of a CCN composed of soluble (deliquescent) and a slightly soluble compounds can be expressed as (Shulman et al., 1996),

$$s_{eq} = \frac{4M_w \sigma_w}{RT \rho_w D_p} - \frac{6n_s M_w \nu_s}{\pi \rho_w D_p^3} - \frac{6n_{ss} M_w \nu_{ss}}{\pi \rho_w D_p^3} \quad (18)$$

where  $n_s, n_{ss}$  represents the moles of solute dissolved in the droplet from the soluble and slightly soluble species, respectively;  $\nu_s, \nu_{ss}$  are the corresponding effective van't Hoff factors.

If the solubility of the partially soluble compound,  $C^{eq}$ , is expressed in moles  $m^{-3}$ , then in the presence of a soluble core (i.e., when there is not enough water in the CCN to completely dissolve the core) Equation (18) becomes,

$$s_{eq} = \frac{4M_w \sigma_w}{RT \rho_w D_p} - \frac{6n_s M_w \nu_s}{\pi \rho_w D_p^3} - \frac{M_w}{\rho_w} C^{eq} \nu_{ss} \quad (19)$$

After the core has completely dissolved, Equation (18) is used with  $n_{ss}$  equal to the total moles of slightly soluble species available in the CCN.

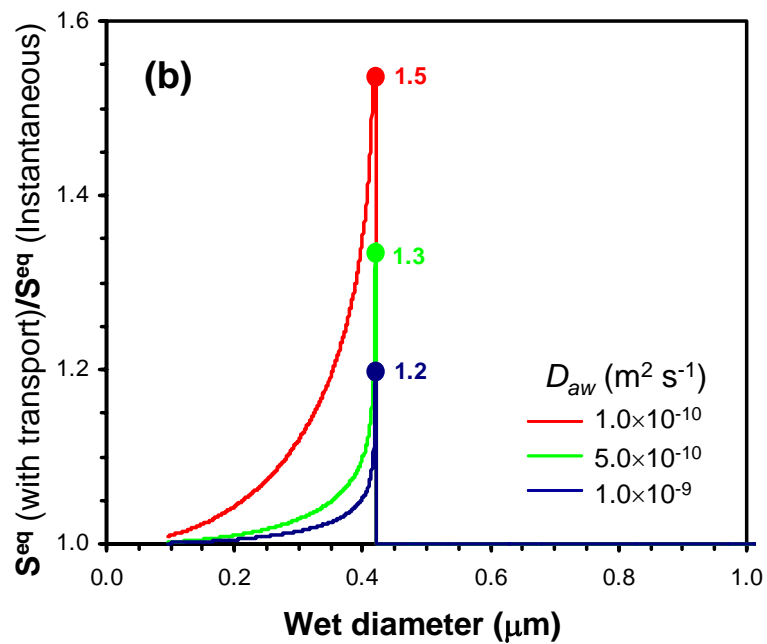
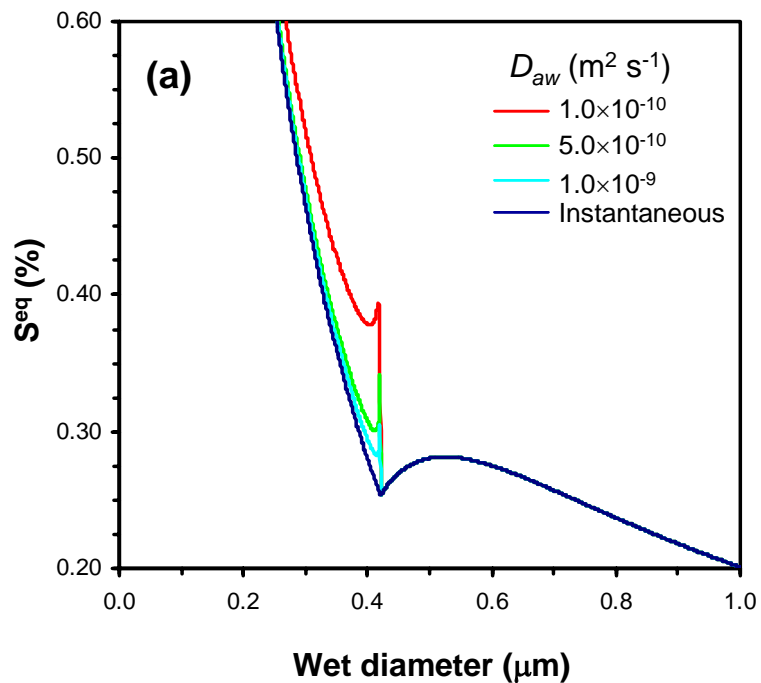
Solute transport kinetics can be introduced in Equation (19) by replacing  $C^{eq}$  with  $C^*$  (Equation 17),

$$s_{eq} = \frac{4M_w\sigma_w}{RT\rho_w D_p} - \frac{6n_s M_w v_s}{\pi\rho_w D_p^3} - \frac{M_w}{\rho_w} v_{ss} \max \left\{ 0.5C^{eq}, \frac{C^{eq}}{1 + 0.0625 \frac{\omega s}{4GD_{AB}}} \right\} \quad (20)$$

We assume that after the core has completely dissolved, Equation (18) is used with  $n_{ss}$  equal to the total moles of slightly soluble species available in the CCN. This implies that mass transfer kinetics are not important after the core has dissolved; in reality there is a relaxation time associated with solute transport from the bulk of the droplet to the surface layer, which for simplicity we have neglected.

### ***3.3. Implications of solute transport kinetics for cloud droplet formation***

For solute dissolution kinetics to have an important influence on droplet formation: (a) a partially soluble core needs to be present during the CCN activation, and, (b) the amount of partially soluble solute provided is a significant fraction of the total solute. The latter requirement implies that solute dissolution kinetics may not influence droplet formation if the CCN contains substantial amounts of inorganic electrolytes (e.g.,  $(\text{NH}_4)_2\text{SO}_4$ ). In the subsequent analysis, we consider CCN composed of only partially soluble substances so that the effects of dissolution kinetics on cloud droplet formation are maximized. In the atmosphere, such particles could correspond to aged secondary organic aerosol (VanReken et al., 2005).



**Figure 7: Köhler curves modified to include the effect of dissolution kinetics. The dry aerosol is 75 nm in diameter and composed of a partially soluble substance with  $1750 \text{ kg m}^{-3}$  density,  $0.132 \text{ kg mol}^{-1}$  molar mass, van't Hoff factor of 2 and a solubility of  $10^{-2} \text{ kg kg}^{-1}$ . Ambient supersaturation is assumed to be 1% . Calculations are presented as (a)  $S_{eq}$  vs. wet diameter for a range of  $D_{Aw}$ , and, (b) change in  $S_{eq}$ , relative to the instantaneous dissolution.**

Figure 7 displays Köhler curves with and without the effect of solute dissolution kinetics. In these calculations, the dry CCN is 75 nm in diameter and composed of a partially soluble substance with  $1750 \text{ kg m}^{-3}$  density,  $0.132 \text{ kg mol}^{-1}$  molar mass, van't Hoff factor of 2 and a solubility of  $10^{-2} \text{ kg kg}^{-1}$ . Ambient supersaturation is assumed to be 1%. Assuming that dissolution and mass transfer of the solute is instantaneous (blue curves) yields “typical” Köhler curves (Figure 7a). At small wet diameters ( $< 0.4 \text{ }\mu\text{m}$ ), the amount of liquid water is insufficient to completely dissolve the core, hence the concentration of solute is constant throughout the droplet volume until complete dissolution (here at  $\sim 0.4 \text{ }\mu\text{m}$ ). For larger wet diameters, the droplet dilutes as it grows and develops the characteristic “Köhler” maximum in  $s_{eq}$ . When dissolution kinetics are considered, the decrease in  $C^*$  (Equation 17) shifts  $s_{eq}$  to higher levels at small wet diameters; this effect is rather small for large solute diffusivity ( $\sim 10^{-9} \text{ m}^2 \text{ s}^{-1}$ ) but becomes significant at modest to low diffusivity ( $\sim 5 \times 10^{-10}$  to  $10^{-10} \text{ m}^2 \text{ s}^{-1}$ ). This effect is more clearly seen if  $s_{eq}$  is normalized to  $s_{eq}$  from instantaneous dissolution (Figure 7b); the region where the ratio is larger than unity indicates an increase in equilibrium vapor pressure from the effect of dissolution kinetics. The effect of dissolution kinetics maximizes close to the point of complete dissolution of the core because the diffusion length scale (i.e.,  $\omega$ ) becomes maximum. In Figure 7b, the increase in  $s_{eq}$  is at most 20% for large diffusivity ( $\sim 10^{-9} \text{ m}^2 \text{ s}^{-1}$ ), which is rather small. However, when the diffusivity is decrease,  $s_{eq}$  becomes significantly higher ( $\sim 50\%$ ), exceeding the local maximum at  $\sim 0.5 \text{ }\mu\text{m}$ . The effects of dissolution kinetics become even stronger if a larger CCN is considered (not shown).



In addition to the diffusion coefficient and solubility, the droplet growth rate (i.e., ambient supersaturation) also affects  $s^{eq}$ . This is shown in Figure 8, which displays Köhler curves with the effect of solute dissolution kinetics for various ambient supersaturations. The CCN characteristics are the same as in Figure 7, and  $D_{Aw} = 10^{-10}$  m  $s^{-2}$ . Calculations show that the Köhler curves exhibit rather important sensitivity to  $s$  (Figure 8a). As the ambient supersaturation drops and droplet growth slows down, dilution of the droplet surface layer from water condensation is less efficient, hence  $C^*$  approaches  $C^{eq}$  and  $s^{eq}$  converges to the “classical” Köhler (i.e., instantaneous transfer) curve. This effect is more clearly seen if  $s_{eq}$  is normalized to  $s_{eq}$  from instantaneous dissolution (Figure 8b); as in Figure 8b, dissolution kinetics maximizes close to the point of complete dissolution of the core.

Both Figures 7 and 8 imply that in the presence of solute dissolution kinetics, CCN could have a “dynamical” equilibrium saturation ratio that is considerably different from that obtained using only thermodynamic arguments. Although this may not be sufficient to completely inhibit CCN from becoming droplets, the modified droplet growth kinetics may have a strong influence on the water vapor availability during cloud droplet nucleation and feedback into cloud droplet number. For this, our newly developed theory needs to be incorporated into a numerical cloud parcel model (e.g., Nenes *et al.*, 2001); this will be the subject of a future study.

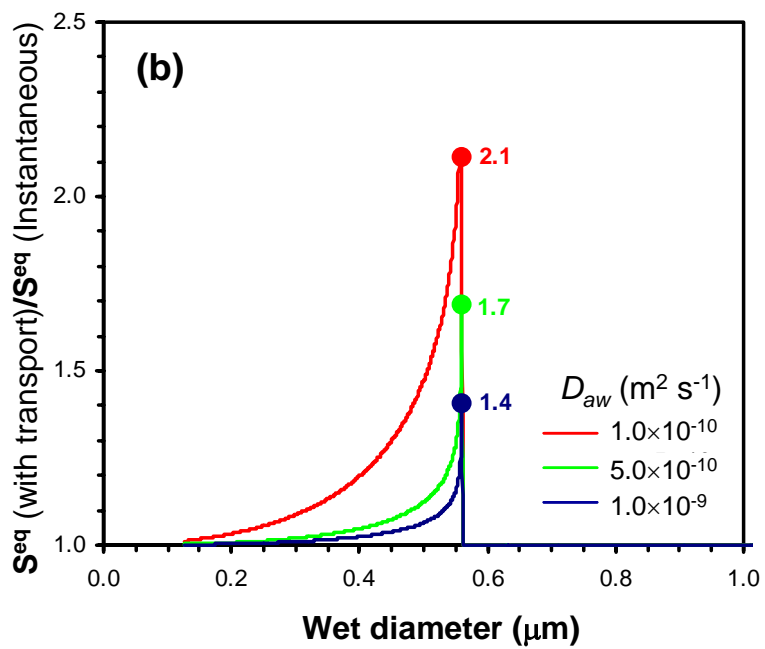
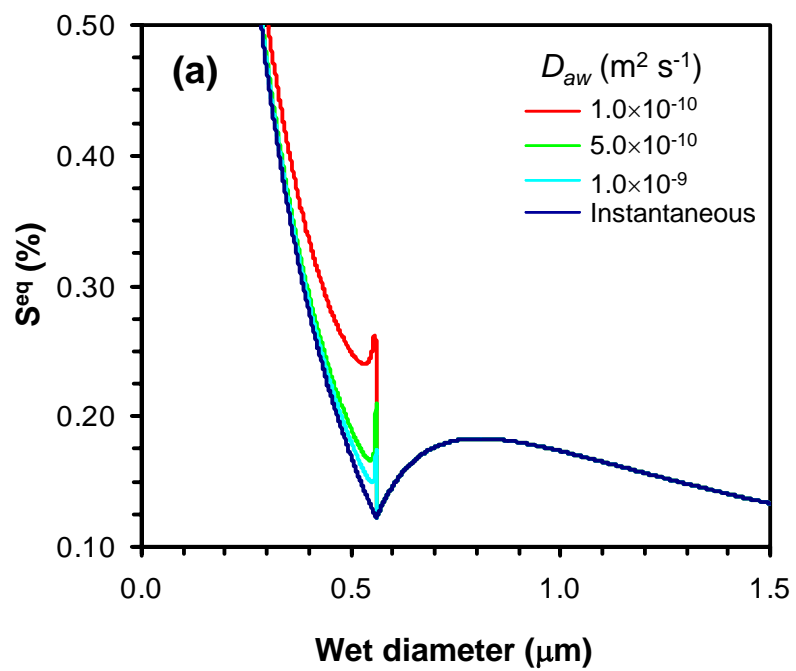


Figure 8: Same as Figure 7, but for a 100 nm dry particle diameter

## CHAPTER 4: SUMMARY

Partially soluble substances, although able to affect the ability of cloud condensation nuclei (CCN) to form droplets, need a finite time to dissolve and diffuse to the droplet surface before they can affect droplet growth. This study focuses on the importance of dissolution kinetics for cloud droplet formation. To comprehensively account for the kinetics, a numerical model of the process was developed. Simulations of cloud droplet growth were performed for solute diffusivity, droplet growth rates, dry particle and droplet diameters relevant for ambient conditions. Simulations suggest that high ambient supersaturations and a decrease in solute diffusivity are major contributors to significant decreases in effective solute surface concentrations. Our simulations suggest that the effect of dissolution kinetics can lead up to a 50% decrease in surface solute concentration.

The steady-state numerical simulations were then parameterized and incorporated into Köhler theory to assess the impact of dissolution kinetics on the droplet equilibrium vapor pressure. For CCN composed of partially soluble material, a significant increase was found in the equilibrium supersaturation of CCN. Although this may not be sufficient to completely inhibit CCN from becoming droplets, the modified droplet growth kinetics may have a strong influence on the water vapor availability during cloud droplet nucleation and have a strong feedback on cloud droplet number. For this, our newly developed theory needs to be incorporated into a numerical cloud parcel model (e.g., Nenes *et al.*, 2001); this will be the subject of a future study.

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