SLIP VELOCITIES OF PARTICLES
FLOWING VERTICALLY IN AN AIR STREAM

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LIST OF SYMBOLS

Latin Letters

A	Pipe cross-sectional area, ft.^{2}
C_D	Drag coefficient
D	Pipe diameter, ft.
d_s	Particle diameter, ft.
f	Fanning friction factor
F	force, lbs. force
L	Pipe length, ft.
P	Pressure, lbs. force/ft.^{2}
Q	Volume rate of flow, ft.^{3}/sec.
R	Specific loading
Re	Air Reynolds number
Re_p	Particle Reynolds number
u	Velocity, ft./sec.
W	Weight rate of flow, lbs. mass/ft.^{2}-sec.

Greek Letters

Δ	Finite difference
μ	Dynamic viscosity of air, lbs. mass/hr.-ft.
ρ	Density, lbs. mass/ft.^{3}

Subscripts

a	Air
aa	Acceleration, air
ap	Acceleration, particle
ds	Dispersed solids
fa	Friction, air
ff	Free-fall
fp	Friction, particle
ft	Friction, total
g	Gas
p	Particle
r	Relative
s	Solids
sa	Static, air
sp	Static, particle
t	Total

Superscript

*	Equilibrium condition

SUMMARY

The importance of pneumatic transport systems has increased greatly during recent years, especially since the application of fluidized catalyst processes has become widespread. Pneumatic conveying systems have also been found useful in transporting hazardous solid substances and heavy concentrations of granular materials.

Early investigations were concerned with the development of empirical relationships for estimating the energy requirements of the pneumatic process. More recent studies have been concerned with the evaluation of empirical constants in terms of particle and system properties.

The present investigation was concerned with evaluating slip velocities of a typical solid material being transported through a vertical riser in an upward flowing air stream. The primary objective was to determine the relationship between air velocity and slip velocity for this material. Secondary objectives were to determine correlations between the slip velocity and other design parameters, such as particle Reynolds numbers, free-fall velocity, and drag coefficients.

Pressure-drop measurements were used as a source of information for obtaining slip velocities. All experiments were conducted using a vertical Lucite pipe two inches in diameter.

Pressure drops were measured at irregular intervals along a total length of twelve feet.

The solid material used was a silica-alumina catalyst 65×10^{-4} centimeters in diameter. Solids loadings ranged from 11.5 to 30.2 pounds per square foot per second; air velocities ranged from 30 to 80 feet per second for each solids loading.

Slip velocities were calculated from relationships developed in previous studies at the Georgia Institute of Technology; other parameters were developed from information available in the literature.

The slip velocity was found to be a linear function of the air velocity and to be independent of the solids loading. Figures present the graphical correlations between particle Reynolds number and drag coefficient, between u_a/u_{ff} and drag coefficient, and between u_a/u_{ff} and $u_{ff}/(u_a - u_s)$.

The correlations developed in this investigation permit the estimation of particle Reynolds numbers, drag coefficients, and slip velocities from the properties of the solid material and the air velocity. Similar studies with a variety of materials and pipe sizes should show like correlations and permit the development of general relationships suitable for accurate design calculations for pneumatic conveying systems.

CHAPTER I

INTRODUCTION

Literature review.--Interest in the pneumatic conveying of solids has increased steadily during the past thirty years. The recent development of fluidized catalyst systems has led to more intensive studies of the dynamics of the transported particles.

Gasterstadt (1), studying the pneumatic transportation of grain, defined a dimensionless factor α as the ratio of the pressure drop of the solid suspension to the pressure drop for air alone at the same velocity. He concluded that a linear relationship existed between α and the specific loading R, defined as the weight ratio of solid flow to air flow. Segler (2) confirmed the results of Gasterstadt, but others, e.g., Farbar (3), have found that no such simple relationship exists.

Cramp (4) presented a detailed analysis of the force terms to be considered in the estimation of pressure drops in pneumatic conveying systems. Jennings (5) and Chatley (6) gave theoretical treatments of the evaluation of forces discussed by Cramp, and their disagreement with respect to estimating particle velocity is evident. Wood and Bailey (7) analyzed in detail the momentum transfer between the conveying air and the solids, using an injector system. Davis (8) presented an analysis of the minimum fluid velocity necessary to raise a particle of fixed size and keep it in suspension in both clear and saturated streams.

Korn (9) has suggested three separate classifications of solid flow, for which the criterion is the amount of contact between the particles and the pipe walls.

The importance of terminal particle velocity in pneumatic conveying studies was mentioned by Gasterstadt and was discussed by Wagon (10). Vogt and White (11) used the dimensionless pressure-drop ratio α suggested by Gasterstadt, and correlated their data by the equation

$$\alpha - 1 = A\left[\frac{D}{d}\right]^2\left[\frac{W_g}{W_s} \cdot \frac{R}{Re}\right]^K \qquad (1)$$

where D is the pipe diameter, d the particle diameter, W_g and W_s the densities of the gas and of the solids, respectively, R the weight ratio of the solids flow per unit time to the air flow per unit time, and Re the air Reynolds number. The constants A and K are functions of the dimensionless group

$$\left[\frac{(\rho_s - \rho_g)\rho_g g_c d^3}{3\mu^2}\right]^{0.5}$$

which is the product of the Reynolds number and the square root of the drag for a spherical particle under free-settling conditions. No velocity term is involved, and the effect of particle shape is ignored.

Hariu and Molstad (12) studied the transport of closely sized silica-alumina catalysts in vertical pipes 0.267 and 0.532 inch in diameter. Particle sizes ranged from 0.0043 to 0.0198 inch. These investigators emphasized the importance of a knowledge of particle velocity in the correlation of data on vertical transport, and they calculated the particle velocity through measurements of the "disperse-density" of the solids. The continuity equation they applied to the solids flow is

$$\frac{G_s}{A} = \frac{u_s}{\rho_{ds}} \qquad (2)$$

where G_s is the mass flow of the solids, A the cross-sectional area of the pipe, u_s the particle velocity, and ρ_{ds} the weight of solids dispersed per unit volume. The particle velocity was easily calculated once the mass rate and the disperse-density were known.

Hariu and Molstad (12) also observed that the equilibrium velocity of the solids was independent of the loading, that the pressure drop due to the acceleration of the solids was a significant portion of the total pressure drop, and that the average particle velocity for the material used was about one-half the gas velocity. Good correlation was achieved by considering the total pressure drop as a sum of the pressure drop due to the carrier gas plus a pressure drop due to entrained solid particles.

Belden and Kassel (13) also studied pneumatic conveying in vertical tubes. They presented data for the transport of spherical catalysts approximately 0.04 and 0.08 inch in diameter in transfer lines 0.473 and 1.023 inches in diameter. The correlation developed in this work expresses the total pressure drop as a function of, first, a static term based upon actual particle density in the transfer line and, second, a friction term which involves the particle mass velocity but which is independent of particle diameter and density. The greater part of the data presented can be correlated by the equation

$$f(Re)^{0.2} = 0.049 + 0.22 \frac{G_g G_s}{(G_g + G_s)^2} \qquad (3)$$

where f is the Fanning friction factor, Re the Reynolds number, G_g the mass velocity of the carrier gas, and G_s the mass velocity of the solids. According to these authors the correlation proposed by Vogt and White (11) involves an incorrect dependence upon the ratio of tube diameter to particle diameter. Korn (9) confirmed the findings of Belden and Kassel that the pressure drop is nearly independent of this ratio. It should be pointed out, however, that acceleration losses were not determined, and that the measured pressure drops were corrected for acceleration on a speculative basis, which may account for the fact that anomalous negative friction factors appear in a few of the experiments.

Farbar (3) investigated the flow characteristics of a silica-alumina catalyst mixture with particle sizes ranging from 10 to 220 microns. The glass conveying tube was 17 mm. inside diameter, and the air velocity was varied from 50 to 150 feet per second. Several types of nozzles for feeding the solids were investigated, and qualitative observations have been presented on the flow in the solids feed line, the mixing nozzle, the horizontal and vertical test sections, and bends and on the behavior of a cyclone separator. No measurement of particle velocity was attempted. The data obtained in this study were not included, but plots of the specific pressure drop versus specific loading for both the horizontal and vertical sections were offered.

Lapple (14) discussed the contributions of Gasterstadt (1) and DallaValle (15), reviewed the various forces to be considered in the pneumatic design problem, and suggested the following equation for calculating the pressure drop due to the friction of both the air and the solids:

$$P_f = \frac{4fLG_a^{\ 2}}{2g_cD} (1 + R)\left[\frac{1}{\rho_a} + \frac{R}{\rho_s}\right] \qquad (4)$$

where f is the Fanning friction factor, L the length of the pipe in feet, G_a is the air mass velocity in pounds per second per square foot, D is the pipe diameter in feet, R the specific loading, and ρ_a and ρ_s the densities of the air and of the solid, respectively. Friction factors are determined from plots of f versus Re, using Re as defined by the equation

$$Re = \frac{DG}{\mu} (1 + R) \qquad (5)$$

where μ is the viscosity of the air. The pressure drops predicted from equation (4) were somewhat lower than those encountered in experimental work.

The study of Khudyakov and Chukhanov (16) was concerned with the movement of sand particles 70, 200, and 845 microns in average size in a stream of gas. Pipes 14, 20, and 32 mm. in diameter were used. No data on pressure drops were presented. The work of Khudyakov and Chukhanov is only applicable to the early phase of the acceleration period, as their equation predicts an ultimate particle velocity equal to the gas velocity. This prediction is contradicted by experimental evidence.

Uspenskii (17) stated that all energy losses in the pneumatic conveying of granular solids are functions of the particle velocity. His calculations of particle velocity were accomplished in the same manner as discussed by Hariu and Molstad (12). Particles of 0.82, 0.105, and 0.142 mm. in average diameter were transported in a tube 41 mm. in diameter, and pressure drops were measured. The friction coefficient was assumed to be the same in the accelerating region as in the region of uniform velocity. The data of Hinkle (18) did not support this assumption.

Albright et al. (19) studied the flow of dense coal-air mixtures with specific loadings up to 200 pounds of coal per pound of air. The coal was sized so that 90 per cent of the particles would pass through 200 mesh; the mixture was conveyed through tubing 3/8, 5/16, and 1/2 inch in diameter. No particle velocity data were obtained. None of the methods of correlation proposed thus far applied to their investigation, although the authors felt that a modification of the Vogt and White correlation might be useful.

Zenz (20) obtained data on pressure drops for the flow of three samples of essentially uniform particles 0.231, 0.0366, and 0.066 inch in diameter, and of one material with a mean diameter of 0.0066 inch and a five-fold variation in particle size. All experiments were carried out in a 1.75-inch inside diameter Lucite tube. Particle velocity was not measured. Correlation for the vertical-tube data was offered in the form of a plot of specific pressure drop versus fluid velocity divided by choking velocity. Choking velocity was defined as the fluid velocity at which the particles began to choke up and travel in distinct slugs. The pressure-drop data reported by Zenz were high, owing to the short accelerating section provided. Although the graphical correlations offered were limited in general application, the dependence of pressure drop upon choking velocity was apparent.

Culgan (21) examined the horizontal conveying of materials of approximately unit specific gravity, average particle size ranging from 0.03 to 0.33 inch, in a three-inch pipe. Only a few measurements of particle velocity were made. Correlation of data was achieved through the use of an empirical correlating factor, and the pressure drop per unit length of pipe was expressed by an equation of the form

$$f(Re_m) = \frac{2g_c D h_m}{u_a^2 L}\left[\frac{\rho_m}{\rho_s}\right]^{0.25} \qquad (6)$$

where h_m is the head loss in feet of air, ρ_m the density of the mixture of air and solids, ρ_s the density of the solid, and Re_m the Reynolds number based on the air velocity and mixture density; the other terms are defined as in the present study. The mixture density, ρ_m, was calculated by the equation

$$\rho_m = \frac{W_s}{Q_a} + \rho_a \tag{7}$$

where W_s is the feed rate of the solids in pounds per minute, Q_a the volumetric air rate in c.f.m., and ρ_a the density of the conveying air.

As Hinkle (18) has pointed out, the mixture density, ρ_m, in pounds per cubic foot is equal to the pounds of solids dispersed per cubic foot, ρ_{ds}, plus the pounds of air per cubic foot, ρ_a, and, therefore, Culgan has essentially defined the disperse-density as

$$\rho_{ds} = \frac{W_s}{Q_a}. \tag{8}$$

Continuity equations for the solids and for the air may be expressed as

$$W_s = Au_s\rho_{ds} \tag{9}$$

and

$$W_a = Au_a\rho_a \tag{10}$$

where A is the cross-sectional area of the pipe. Dividing equation (9) by equation (10) and solving for ρ_{ds} leads to

$$\rho_{ds} = \frac{W_s \rho_a}{W_a} \cdot \frac{u_a}{u_s}. \tag{11}$$

Substituting Q_a for its equivalent W_a/ρ_a reduces equation (11) to

$$\rho_{ds} = \frac{W_s}{Q_a} \cdot \frac{u_a}{u_s}. \tag{12}$$

A comparison of equation (8) with equation (12) shows that Culgan's calculated values of the disperse-density are low by the amount of the prevailing slip factor, u_a/u_s.

The correlating factor introduced by Culgan evidently compensated to some extent for his neglecting the slip factor, since equation (6) was found by Hinkle (18) to express his data within ten per cent.

Hinkle (18) studied the horizontal flow of particles having a size range of 0.014 to 0.33 inch in diameter in two- and three-inch glass pipes. Particle velocities were measured by high-speed photographic techniques. The solids friction effects were treated by a method analogous to the Fanning equation for fluids. Hinkle's studies showed that the slope of the line resulting from a plot of specific pressure drop versus loading was a function of particle velocity, air velocity, air friction factor, and solids friction factor. For the case of vertical conveying, Hinkle developed a method of presenting friction pressure-drop data which satisfied data already existing in the literature. His equation was

$$\frac{\Delta P_{ft}/\Delta L}{\Delta P_{fa}/\Delta L} = 1 + \frac{f_p u_p{}^* R}{f_a u_a} + \frac{2 g_c D R}{f_a u_a u_p{}^*} \qquad (13)$$

where the symbols used are defined the same as in this study.

Objectives of investigation.--The literature review shows that the major contributions to the field of pneumatic conveying indicate progress, although some confusion and disagreement remain. While numerous empirical correlations have been developed, very little attention has been given to the study of particle velocities and slip velocities and their correlation with design parameters.

In this study particle velocities and slip velocities were calculated from pressure-drop measurements. The primary objective was to determine the relationship between the air velocity, slip velocity, and solids loading. Other objectives were to determine what correlations might exist between the slip velocity and the particle Reynolds number, the drag coefficient, and the particle free-fall velocity.

CHAPTER II

THEORETICAL DISCUSSION

Pressure drops in vertical pneumatic conveying.--The total measured pressure drop in a vertical riser can be regarded as being composed of several separate terms, as shown in the following equation:

$$\Delta P_t = \Delta P_{aa} + \Delta P_{sa} + \Delta P_{fa} + \Delta P_{ap} + \Delta P_{sp} + \Delta P_{fp}. \qquad (14)$$

The first component, ΔP_{aa}, is the pressure drop occurring in the acceleration of the air to its final velocity. The second term, ΔP_{sa}, represents the pressure drop due to the static head of air in the riser. The third term, ΔP_{fa}, is the pressure drop resulting from the friction of the air in the riser. The fourth, fifth, and sixth terms are similar terms for the solid particles carried along in the air stream.

If consideration is limited to the section of the riser after the acceleration of air and solids is completed, equation (14) becomes

$$\Delta P_{ft} = \Delta P_{fa} + \Delta P_{sp} + \Delta P_{fp} + \Delta P_{sa}. \qquad (15)$$

The term ΔP_{sa} is negligible in comparison to the other terms; if this term is neglected, equation (15) can be rearranged to obtain a relationship involving the specific pressure drop. This is

$$\frac{\Delta P_{ft}/\Delta L}{\Delta P_{fa}/\Delta L} = 1 + \frac{\Delta P_{fp}/\Delta L}{\Delta P_{fa}/\Delta L} + \frac{\Delta P_{sp}/\Delta L}{\Delta P_{fa}/\Delta L}. \tag{16}$$

Using this relationship, Hinkle developed equation (13) from these considerations:

1. The air-friction pressure drop can be found from the Fanning equation,

$$\frac{\Delta P_{fa}}{\Delta L} = \frac{f_a u_a^2}{2 g_c D} (\rho_a). \tag{17}$$

2. The solids friction effects can be expressed by a relationship analogous to the Fanning equation, as Hinkle found through correlation of his data with those of previous observers, i.e.,

$$\frac{\Delta P_{fp}}{\Delta L} = \frac{f_p u_p^2}{2 g_c D} (\rho_{ds}). \tag{18}$$

3. The pressure drop in the carrier gas due to supporting the solids may be regarded as a solids static head of density ρ_{ds},

$$\frac{\Delta P}{\Delta L} = \rho_{ds} = \frac{G_p}{u_p}. \tag{19}$$

He then substituted equations (17), (18), and (19) into (16) and arrived at

$$\frac{\Delta P_{ft}/\Delta L}{\Delta P_{fa}/\Delta L} = 1 + \frac{f_p u_p{}^* R}{f_a u_a} + \frac{2g_c DR}{f_a u_a u_p{}^*}. \tag{13}$$

All of the elements in equation (13) can be determined directly from experimental measurements except for u_p and f_p. Knowledge of either u_p or f_p would permit the direct calculation of the other variable from equation (13). For example, if f_p were known, u_p could be calculated by letting $\frac{\Delta P_{ft}/\Delta L}{\Delta P_{fa}/\Delta L} = F$ and solving equation (13) with the quadratic formula:

$$u_p = \frac{f_a u_a(F - 1) \pm \sqrt{\left[f_a u_a(F - 1)\right]^2 - 8g_c f_p DR^2}}{2f_p R}. \tag{20}$$

The density of the dispersed solids can then be determined from

$$\rho_{ds} = \frac{G_p}{u_s}. \tag{21}$$

<u>Calculation of drag coefficient</u>.--The relationship between the drag coefficient, C_D, and the particle Reynolds number Re_p is given in many sources (22, 23) where C_D and Re_p are defined by the following expressions for a single particle:

$$F_r' = \frac{u_s^2 C_D A_s \rho_a}{2g_c} \tag{22}$$

and

$$Re_p = \frac{d_s \rho_a u_a}{\mu_a}. \tag{23}$$

Equations (22) and (23) were developed for free-falling bodies, i.e., at zero gas velocity. In the case of pneumatic conveying, however, the velocity terms appearing in these equations are actually the difference between the gas velocity, u_a, and the solids velocity, u_s, as Hinkle (18) has pointed out. This difference is the "slip" velocity. Equations (22) and (23) rewritten in terms of the slip velocity are

$$F_r' = \frac{(u_a - u_s)^2 C_D A_s \rho_a}{2g_c} \tag{24}$$

and

$$Re_p = \frac{d_s \rho_a (u_a - u_s)}{\mu_a}. \tag{25}$$

Equation (24) written in terms of the particle diameter is

$$F_r' = \frac{C_D \pi d_s^2 \rho_a (u_a - u_s)^2}{8g_c}. \tag{26}$$

The mass of a single particle is $\pi d_s^3 \rho_s/6$ and the total mass for a section ΔL of the column is $\rho_{ds} \pi D^2 \Delta L/4$. Hence, the number of particles is

$$N = \frac{6\rho_{ds}D^2\Delta L}{4d_s{}^3\rho_s}. \tag{27}$$

The total force acting on the particles in the section is then $F_r{}' \times N$ or

$$F_r = \frac{C_D 3\ \rho_a \rho_{ds}(u_a - u_s)^2 D^2 \Delta L}{16 g_c d_s \rho_s}. \tag{28}$$

However, the pressure drop in the section ΔL is given by

$$\Delta P = \frac{F_r}{A} = \frac{4F_r}{\pi D^2}. \tag{29}$$

Therefore,

$$\frac{\Delta P}{\Delta L} = \frac{3C_D \rho_a \rho_{ds}(u_a - u_s)^2}{4 g_c d_s \rho_s}, \tag{30}$$

which can be solved explicitly for the drag coefficient as

$$C_D = \frac{4 g_c d_s \rho_s}{3\rho_a \rho_{ds}(u_a - u_s)^2} \cdot \frac{\Delta P}{\Delta L}. \tag{31}$$

DallaValle (15) has summarized the criteria for particle Reynolds numbers. He states that if a fluid is in turbulent motion, the motion of a particle injected into it will be turbulent, regardless of the particle Reynolds number. In the present study the air Reynolds number was in the turbulent region in all experiments.

Calculation of free-fall velocity.--If a single body falling under the action of gravity is considered, the body will attain a constant terminal velocity in free fall, u_{ff}, when the resisting upward drag force, F_r', is equal to the net gravitational accelerating force, F_g'. If the mass of the particle is m, the gravitational pull is

$$F_g' = mg_c \frac{\rho_s - \rho_a}{\rho_s}. \tag{32}$$

The drag force is given by equation (22). When $F_r' = F_g'$, $u_s = u_{ff}$. Equating equations (32) and (22) and solving for u_{ff} gives

$$u_{ff} = \sqrt{\frac{2mg_c(\rho_s - \rho_a)}{C_D \rho_s \rho_a A_s}}. \tag{33}$$

For spherical particles $m = d_s^3 \rho_s \pi/6$, and $A_s = d_s^2 \pi/4$. Then

$$u_{ff} = \sqrt{\frac{4g_c d_s(\rho_s - \rho_a)}{3\rho_a C_D}}. \tag{34}$$

Isaac Newton derived the following expression for the terminal velocity of a free-falling body under conditions of turbulent flow and negligible viscous forces:

$$u_{ff} = \sqrt{\frac{\pi g_c d_s(\rho_s - \rho_a)}{6K_n\rho_a}} \tag{35}$$

where K_n is a constant indeterminate from theoretical study. As Lapple (14) points out, equation (35) is identical with equation (34) for a constant value of C_D when $K_n = \pi C_D/8$. Experimental data have indicated that this is a good approximation for K_n in the turbulent range of particle Reynolds numbers, where C_D is substantially constant.

CHAPTER III

EXPERIMENTAL METHODS

Apparatus.--The essential features of the apparatus used for the vertical pneumatic conveying studies are presented in Figure 1. The solid material to be conveyed entered the screw conveyor hopper through an opening in the Lucite reservoir immediately below the cyclone separator. Enough solid material was added to the hopper-reservoir system to keep the Lucite reservoir nearly half full of solids during the experimental runs, about 250 pounds of solids. This level was maintained to prevent any of the air under pressure in the system from escaping through the screw conveyor and hopper. At the bottom of the hopper was a screw conveyor operated by a variable-speed motor. From this conveyor the solids fell into a horizontally flowing stream of air from the blower.

Air was supplied by a rotary positive Roots-Connersville blower (Type AF-59) with a rated capacity of 200 cubic feet per minute at a discharge pressure of 3 psig. The volume of air passed through the test section was controlled by using variously sized sheaves on the blower motor and by installing a gate valve on the discharge line to exhaust some of the air to the atmosphere.

The air-solids mixture flowed horizontally for about two feet from where the solids were introduced before entering the

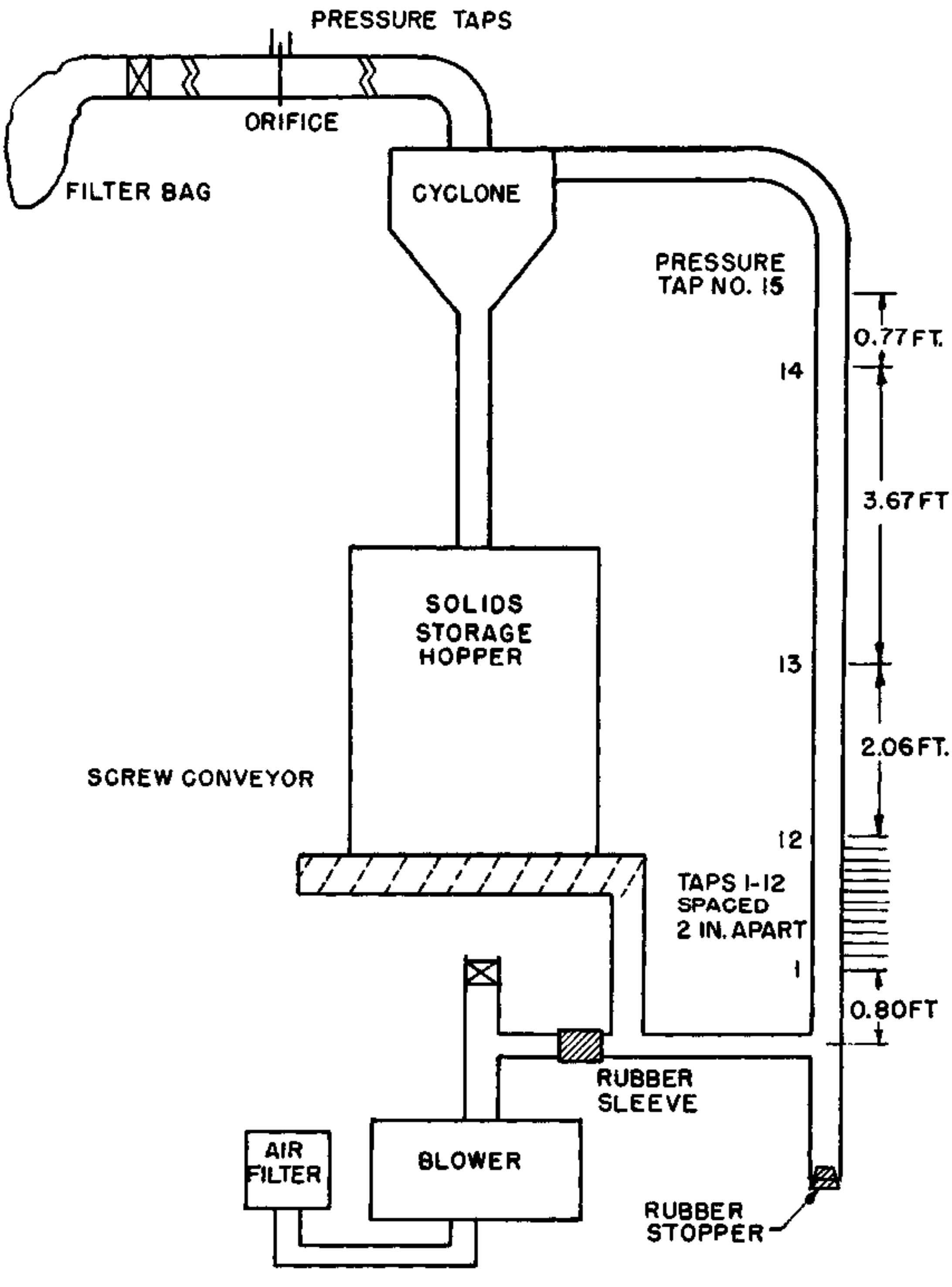

Figure 1. Schematic Diagram of Apparatus

vertical test section. After passing through the test section, the mixture entered a cyclone separator from which the solids returned to the hopper and the air passed through an orifice to measure its velocity. From the orifice the air went through a gate valve and out into the atmosphere through a filter bag.

The test section was made of two-inch Lucite tubing with an inside diameter of 1.970 inches. The sections were carefully aligned and joined together in the manner shown in Figure 2. Pressure taps were made of 0.125-inch inside diameter Lucite tubes cemented into the test section.

The flow rate of air through the pipe was measured with an orifice 2.174 inches in diameter located in the standard three-inch line between the cyclone exhaust and the gate valve. The orifice installation used was the same as that used by Hinkle (18) in previous work of this type. The orifice was installed according to the instructions of Stearns et al. (24); hence, the orifice coefficients used in subsequent calculations were evaluated by the methods outlined by Stearns. For purposes of calibration, Hinkle had installed a pitot tube in his system, and the measurements made with this pitot tube gave excellent agreement with those obtained with the orifice. Hinkle's plot of air velocity calculated from orifice measurements versus air velocity calculated from pitot-tube measurements is given as Figure 3.

An indirect check of the accuracy of orifice measurements was obtained through pressure-drop measurements for the air alone. The experimental results for the conveying air are presented in

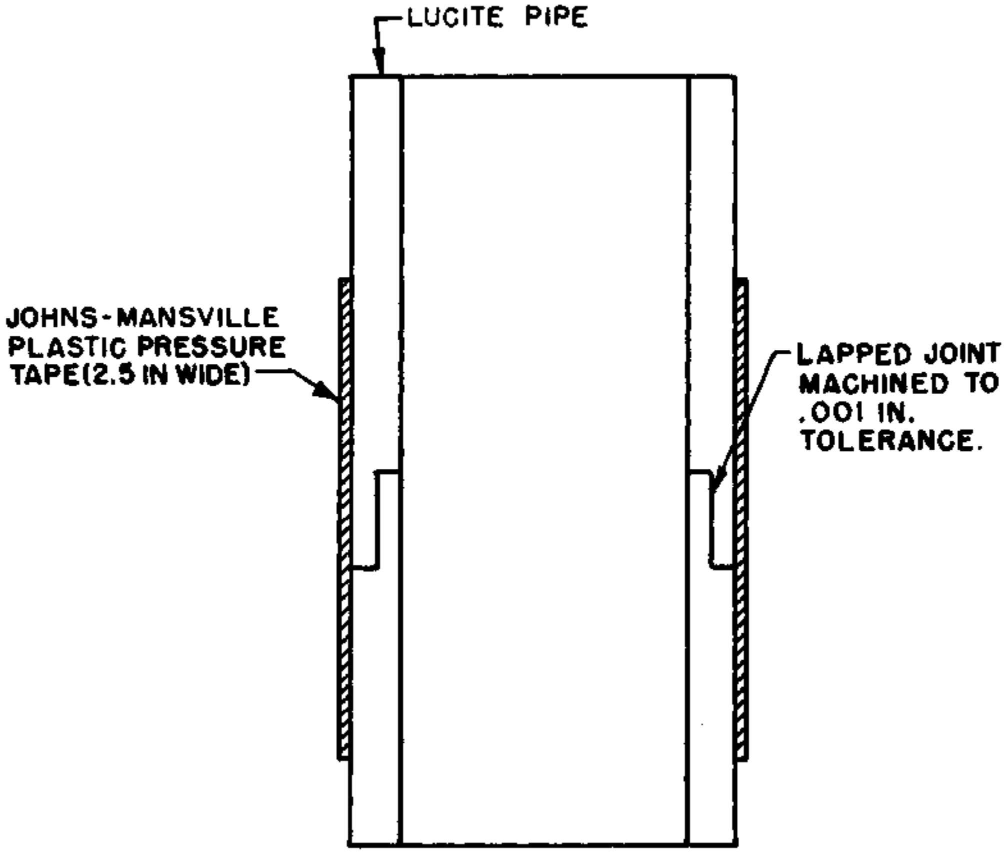

Figure 2. Typical Lucite Pipe Joint

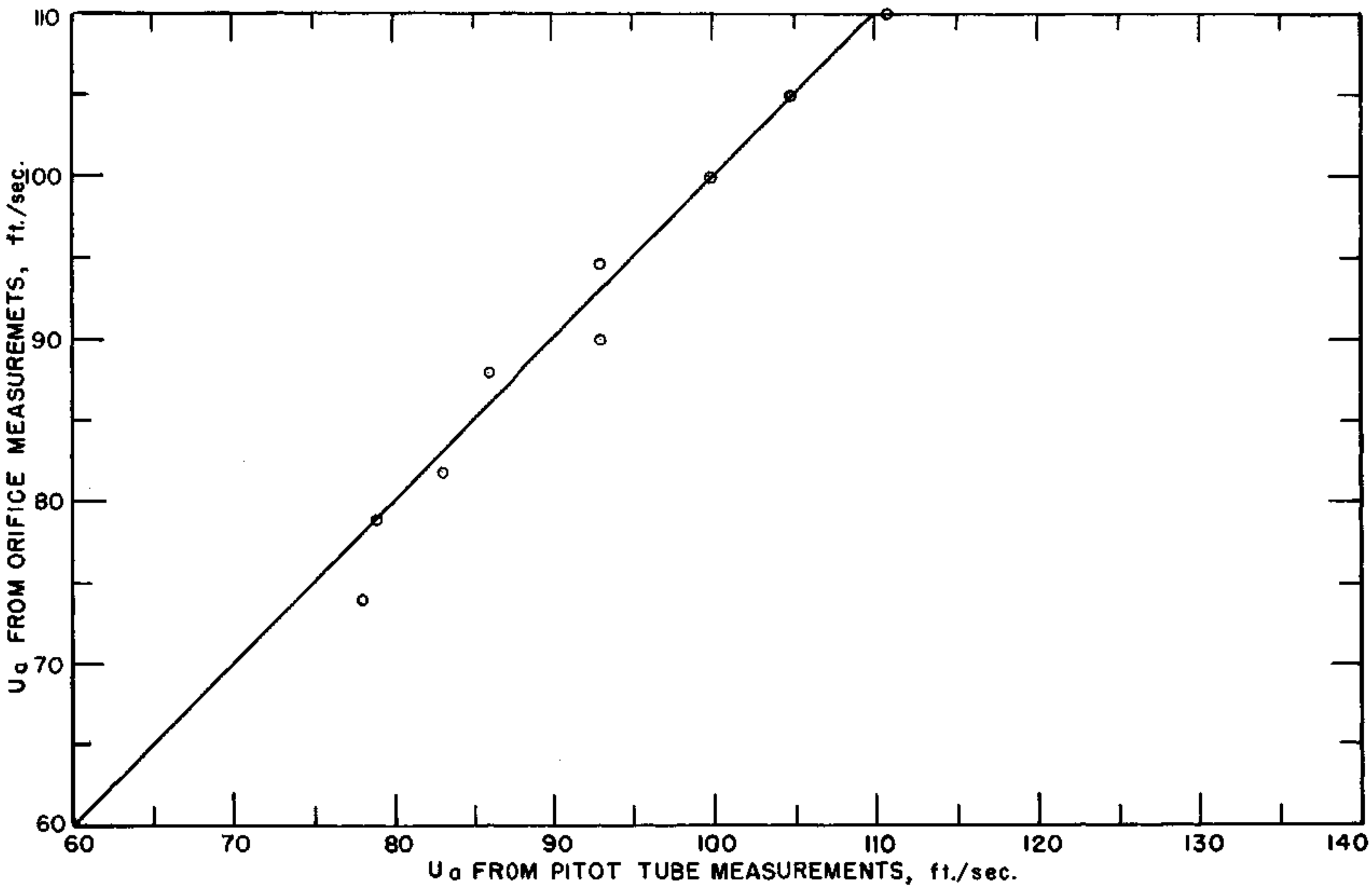

Figure 3. Comparison of Air Velocities Measured with the Orifice Meter and Pitot Tube (after Hinkle, Figure 27)

Table 1 in the Appendix. The pressure drop of air in the pipe was calculated by the Fanning equation using a roughness factor suggested for smooth glass tubing and friction factors corresponding to the air Reynolds number. The agreement between calculated and experimentally determined pressure drops for the two-inch pipe is given in Figure 4. Static pressures and differential pressures across the orifice were measured by manometers manufactured by the Uehling Company. A Uehling differential manometer was used to measure the differential pressures across the orifice in cases where the pressure drop was less than three inches of water. Indicating fluid was supplied with the differential manometer. For differential pressures across the orifice greater than three inches of water a U-tube manometer containing water with a small amount of dye added was used. The static pressure ahead of the orifice was measured with a Uehling pressure gage calibrated in inches of mercury.

The static pressures in the test section were read from a U-tube manometer containing mercury as the indicating fluid. The lines from the pressure taps on the test section were all run into a manifold from which one line led to the manometer. The tap connections were opened or closed individually by means of screw clamps.

The screw conveyor was calibrated for the material used by disconnecting the blower and test-section lines from the hopper and conveyor; the material discharged by the conveyor in a unit time at various settings of the variable-speed motor was then weighed. This was done a minimum of five times at each conveyor speed. The results of this calibration are presented in Figure 5.

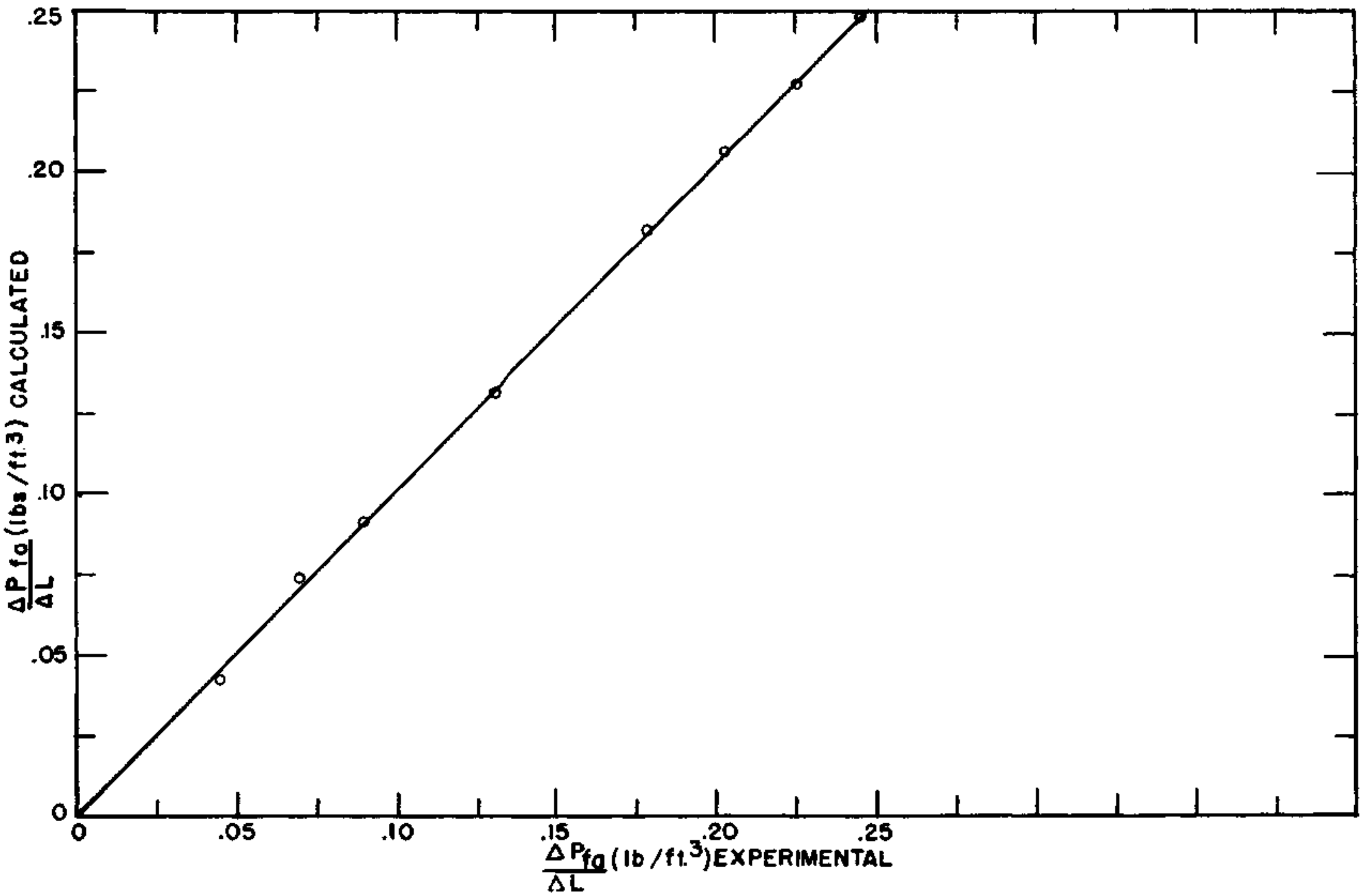

Figure 4. Comparison of Experimental and Calculated Pressure Drops for the Conveying Air

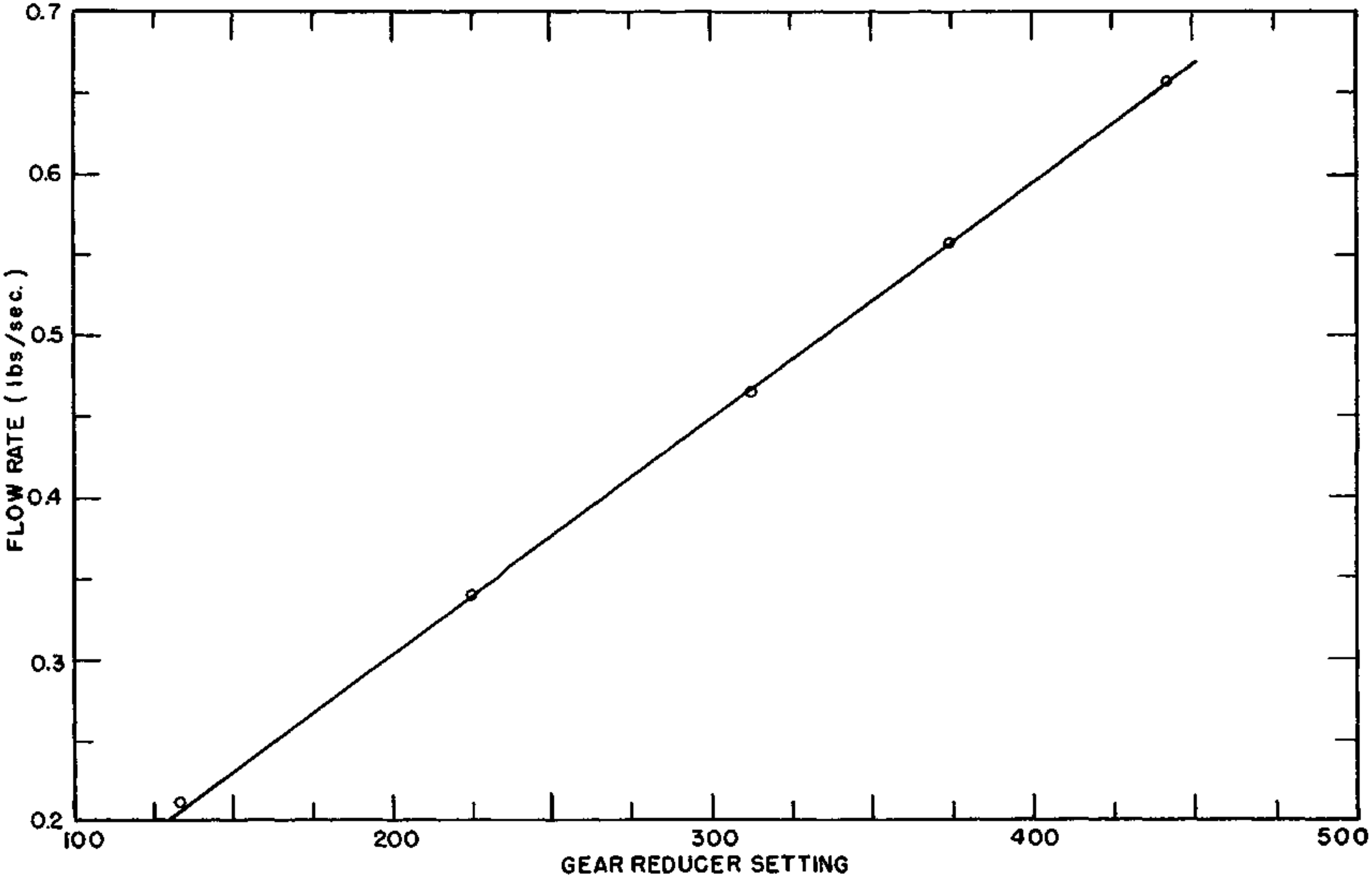

Figure 5. Calibration of the Screw Conveyor

Much static electricity was noted during preliminary runs. This was eliminated by wrapping strips of aluminum foil around the pipe at convenient places and grounding the strips to utility pipes.

Material.--The solid material used in this study was M-S(C-2) silica-alumina fluid-cracking catalyst provided through the courtesy of the Davisson Chemical Company of Baltimore, Maryland. This material had the following properties:

Particle diameter (weighted mean) = 65 x 10^{-4} cm.
Bulk density = 71 lbs./cu. ft.
Absolute density = 111 lbs./cu. ft.
Void fraction = 0.341
Pore space = 0.117

Procedure.--Preliminary runs (S1L-S10L) were confined to learning the operating characteristics of the equipment. It was found that the system reached equilibrium almost immediately, requiring from 30 to 60 seconds of operation to reach conditions at which reproducible pressure-drop measurements could be made in the test section.

During experimental runs the system was operated for five minutes during each run before any measurements were made, then a series of three readings was taken with the same solids loading and air velocity. This was done at four or five air velocities at one solids loading, depending upon the air velocity at which "choking" commenced and the pressure readings became erratic. The solids loading was then changed and pressure-drop readings were again made at four or five air velocities.

The range of solids loadings and air velocities used was determined by the capacities of the conveyor and blower, respectively. The conveyor delivered from 11.52 to 30.22 pounds per square foot per second of the solid material and air velocities of up to 78.2 feet per second were possible with the blower.

Four solids loadings were used in the study, each at four or five air velocities, and three runs were made at each combination of solid loading and air velocity. Since three measurements were made during each run, this gave a total of nine determinations for each air-solid mixture.

To start the apparatus, the blower was turned on; then the solids conveyor was started; then the by-pass valve on the blower and the gate valve below the orifice were regulated as necessary to obtain the desired operating air velocity. After the five-minute stabilizing period, pressure-drop readings were begun. The system was checked for leaks before and after each run.

In order to prevent the loss of any solids to the outside air, a filter bag had been attached to the air exhaust line; however, the cyclone separator was extremely efficient in separating the mixture. After operation of the equipment for about three hours during the preliminary runs at the highest solids loading, only about one-half pound of solids was found in the filter bag. Therefore, the minute amounts of solids passing out through the orifice were assumed to be negligible losses.

CHAPTER IV

DISCUSSION OF RESULTS

The experimental pressure drops and equilibrium conditions at each loading are summarized in Table 2 in the Appendix. The experimental pressure-drop data were plotted as a function of the distance above the entrance to the vertical test column for each set of experimental conditions. Typical plots are shown in Figure 6. The slope of the straight-line portion of each curve was measured, and the value so obtained is given as $\Delta P_{ft}/L$ at equilibrium in Table 2 for each set of conditions. The data presented in Figure 6 and Table 2 show that $\Delta P_{ft}/L$ becomes constant in the upper half of the test section, indicating that acceleration of the solid material is completed in the lower part of the column, in most cases within four feet of the point of entrance of the solids into the test section.

Equation (13) expresses the pressure-drop relationships in a vertical column after the solids acceleration is completed. This equation, developed and tested by Hinkle with data already presented in the literature, is the basic equation used to correlate the pressure-drop data obtained in this study.

The particle velocities corresponding to the experimental data were calculated using equation (13) as solved for u_p by the

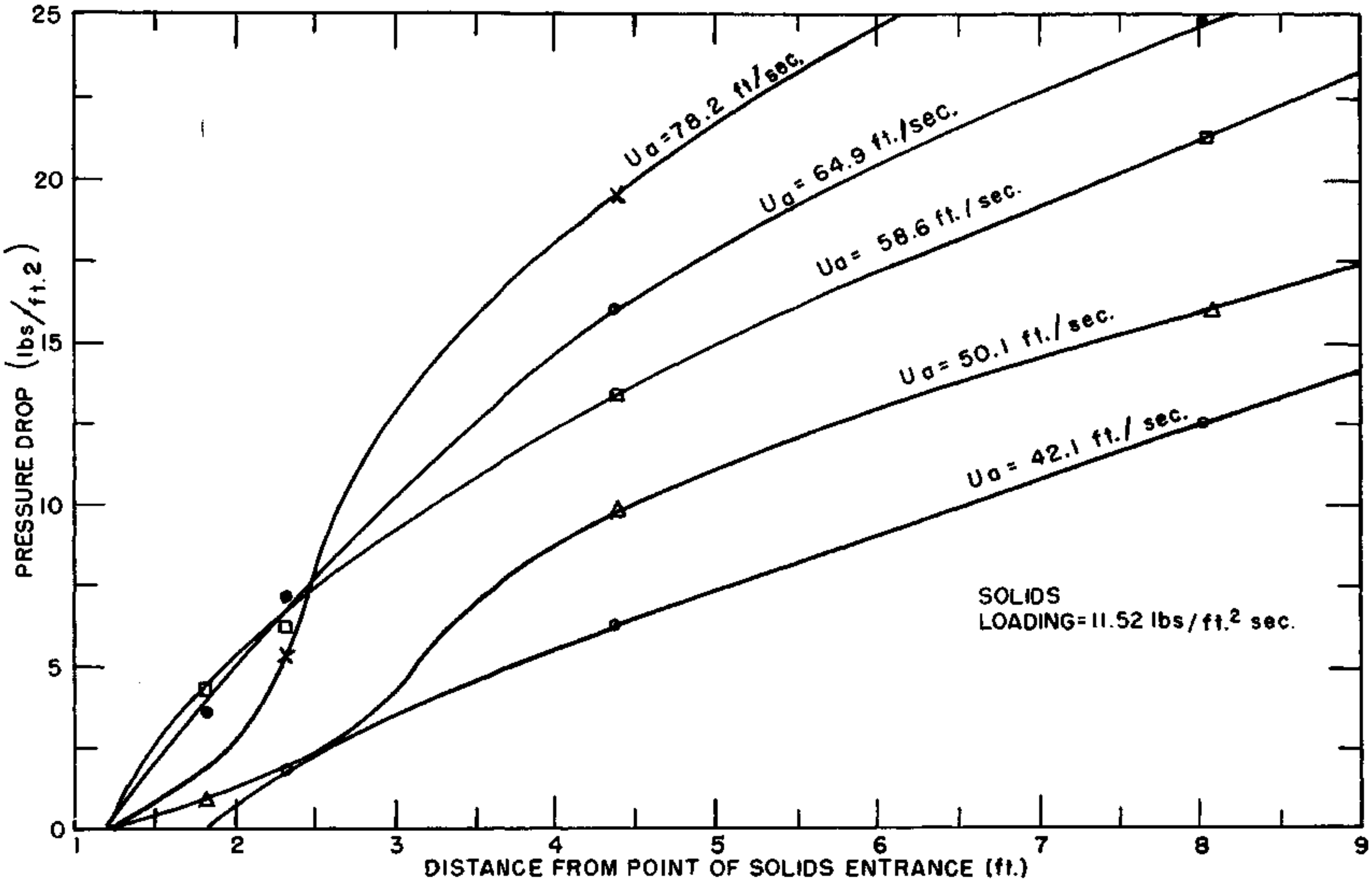

Figure 6. Typical Pressure Drops of Air-Solids Mixtures in the Vertical Test Section

quadratic formula

$$u_p = \frac{f_a u_a(F - 1) + \sqrt{\left[f_a u_a(F - 1)\right]^2 - 8g_c f_p DR^2}}{2f_p R}. \qquad (20)$$

In addition to the experimental data, some estimate of f_p is required in order to use this relationship. Hariu and Molstad recommend a value of $f_p = 0.001$ for the materials used in their study. Hinkle calculated f_p's ranging from 0.003 to 0.02 from his data. In view of the variations obtained by other investigators, it was decided to use several different values for f_p and to see what effect these differences had on the derived quantities. Calculations were made using f_p's of 0.01, 0.005, 0.001, and f_a. The results of these calculations are presented in Table 3 in the Appendix.

The data in Table 3 show that the slip velocity and other derived parameters have only a very slight dependence upon the magnitude of f_p. In Figure 7 the slip velocities corresponding to $f_p = f_a$ and $f_p = 0.001$ are plotted as a function of the corresponding air velocities. For air velocities above 50 feet per second comparatively large deviations exist. Velocities at $f_p = 0.01$ and $f_p = 0.005$ show similar relationships. The same straight line fits all four sets of values equally well. The slip velocity is apparently a linear function of the air velocity and is independent of the solids loading, as has been found by previous investigators. This relationship can be expressed as

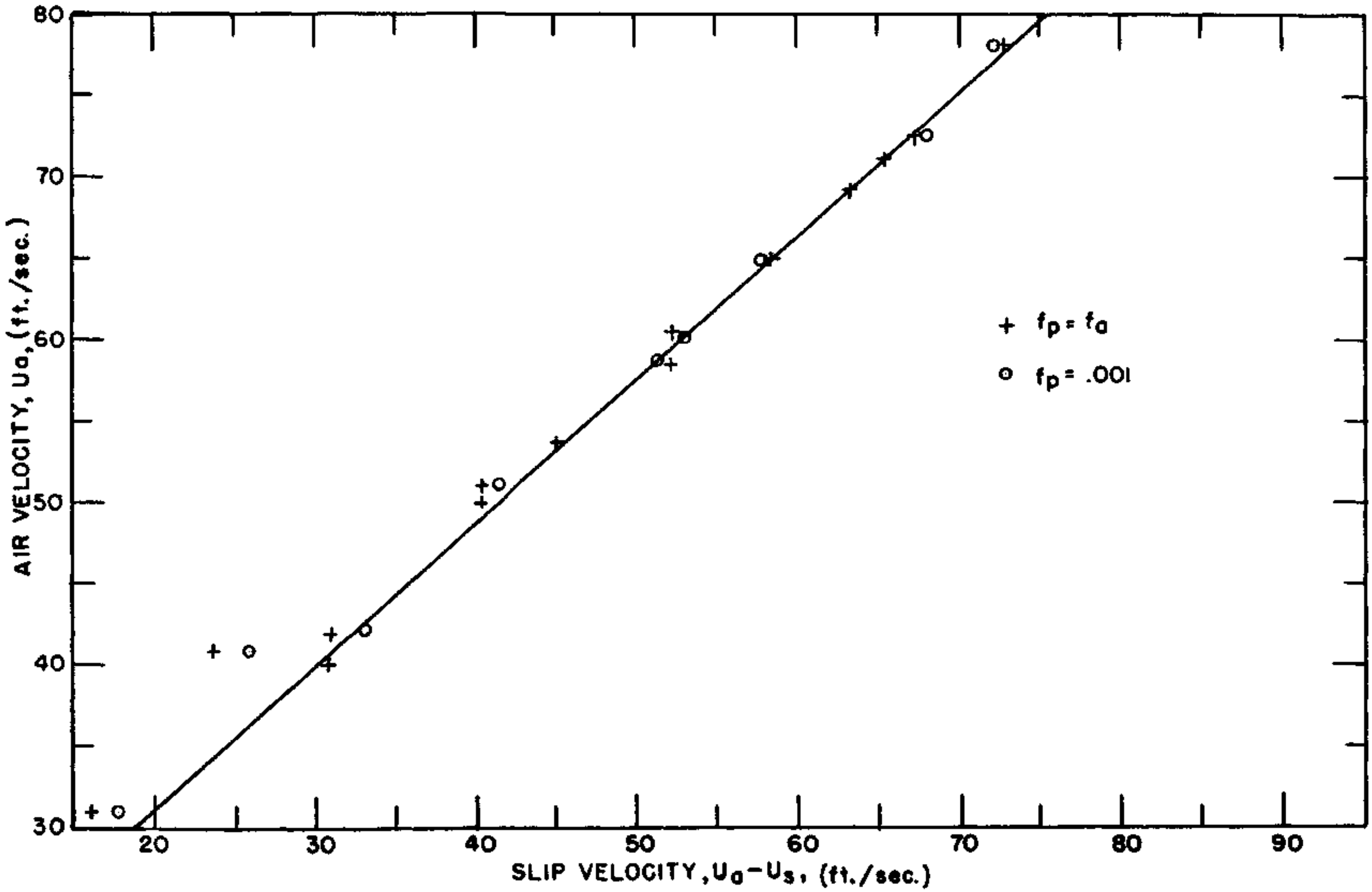

Figure 7. Slip Velocity as a Function of Air Velocity

$$u_a = K(u_a - u_s) + A. \tag{36}$$

For the material and riser used in this study, K = 0.88 and A = 13.3. The value of K is probably a function of the riser diameter, and A is probably a function of the material being transported. It is interesting that in this case A was found to have a value slightly more than twice the free-fall velocity of the solid material (u_{ff} = 5.65 ft./sec.). The significance of these two constants is, of course, conjecture; further experimentation with different riser sizes and different materials would be necessary to establish the exact relationship.

For values of the air velocity less than approximately 40 feet per second, wide deviations from the straight-line relationship to slip velocity are apparent, indicating that the choking velocity is being approached. The choking velocity is usually regarded as that velocity at which the solids begin travelling in slugs, rather than in a smooth, continuous stream. This value would be characterized in experimental data by erratic pressure-drop measurements and corresponding deviations in the values derived from them.

Some of the slip-velocity data obtained by Hariu and Molstad are plotted in Figure 8. Unfortunately, the air velocities used in their study are less than the minimum velocity obtainable without choking in the present study. Since different materials and different riser sizes were used in each study, the effects of the particle size and density and the riser diameter and material cannot be determined by comparing the data.

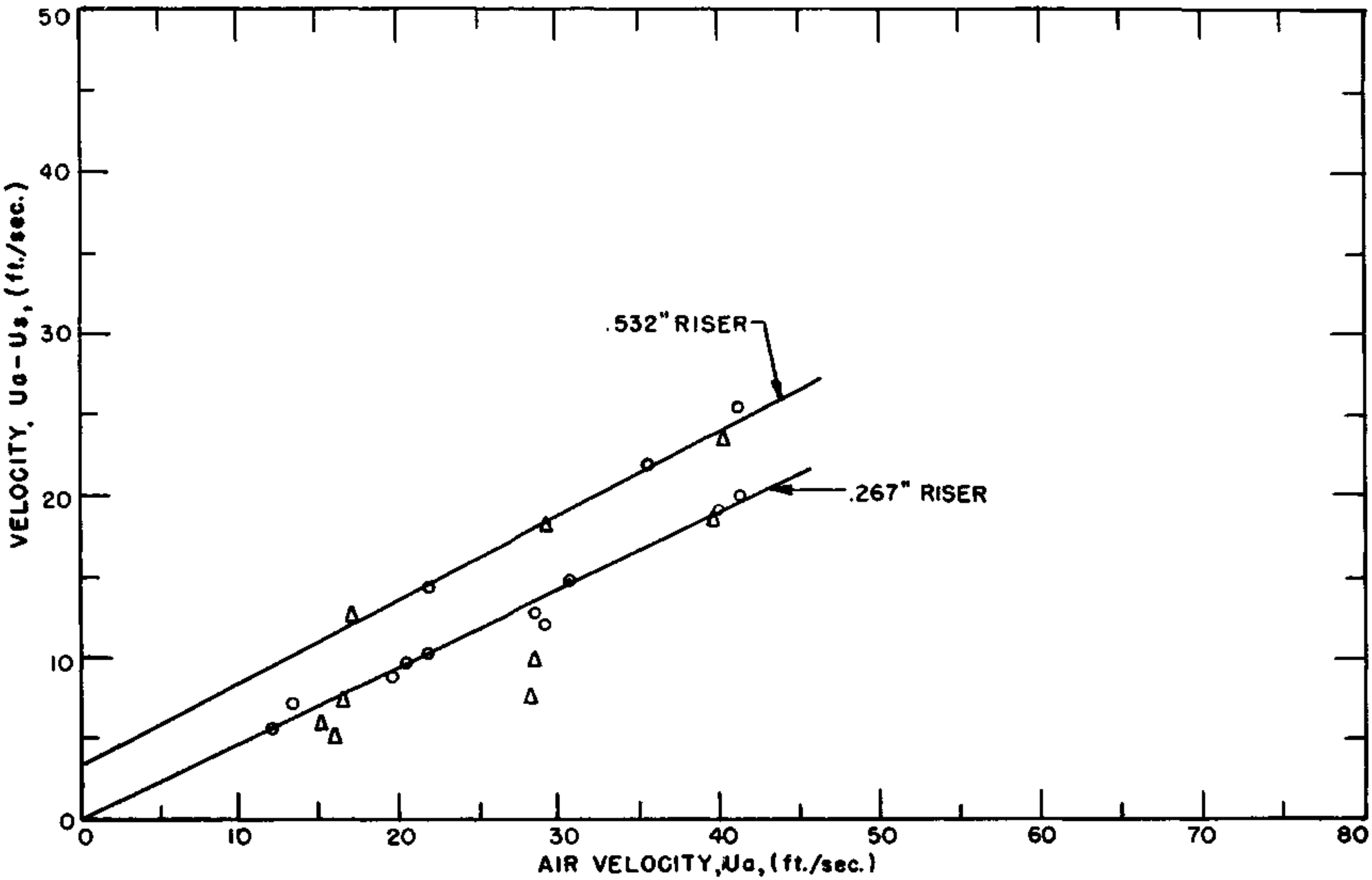

Figure 8. Slip Velocities Obtained by Hariu and Molstad

The drag coefficient, C_D, can be calculated from equation (31) for each set of experimental conditions. The particle Reynolds number, Re_p, can be calculated similarly from equation (25). The relationships express the drag coefficient and particle Reynolds number as a function of the slip velocity rather than of the velocity of the particles themselves. Plotting C_D as a function of Re_p on log-log paper, as in Figure 9, gives a straight line with very slight experimental deviations. This relationship between C_D and Re_p demonstrates the utility of using the slip velocity as a primary consideration in design calculations.

Other graphical correlations useful in design calculations can also be demonstrated. The drag coefficient plotted as a function of u_a/u_{ff} for the solid particles gives a smooth curve on log-log paper (Figure 10); the graphing of u_a/u_{ff} as a function of $u_{ff}/(u_a - u_s)$ gives a smooth curve on semi-log paper (Figure 11). These relationships illustrate direct methods of obtaining drag coefficients, slip velocities, and particle Reynolds numbers when the properties of the solid material are known. All of these figures show that the magnitude of the solids loading has very little, if any, effect on the correlations shown.

These correlations were observed on only one material and using one riser diameter in the experimental phase. It seems reasonable to assume, however, that similar correlations exist for other materials and different riser diameters. If similar studies were made with other materials and under other experimental conditions, it should be possible to develop general correlations which

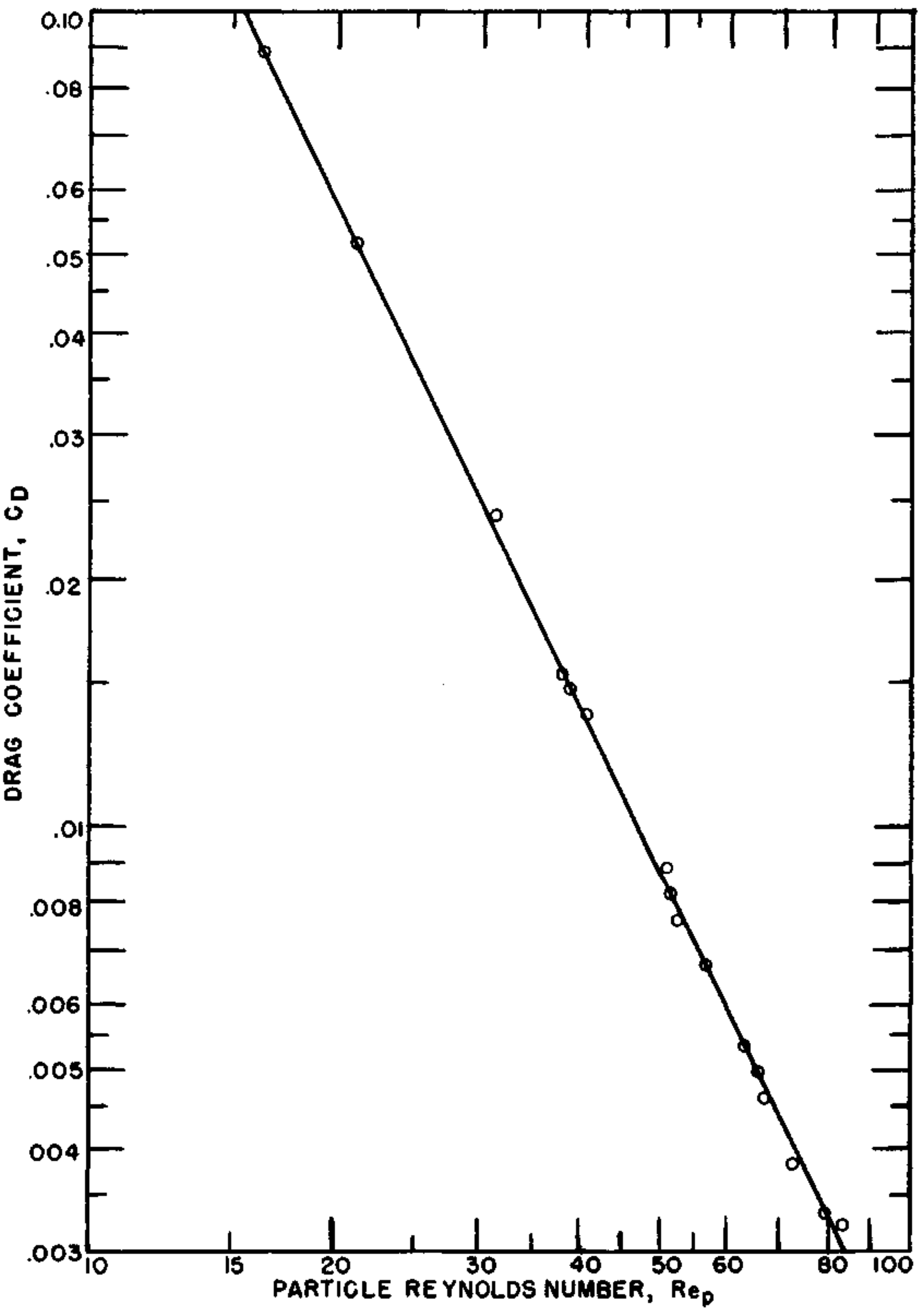

Figure 9. Correlation of Drag Coefficients with Particle Reynolds Numbers

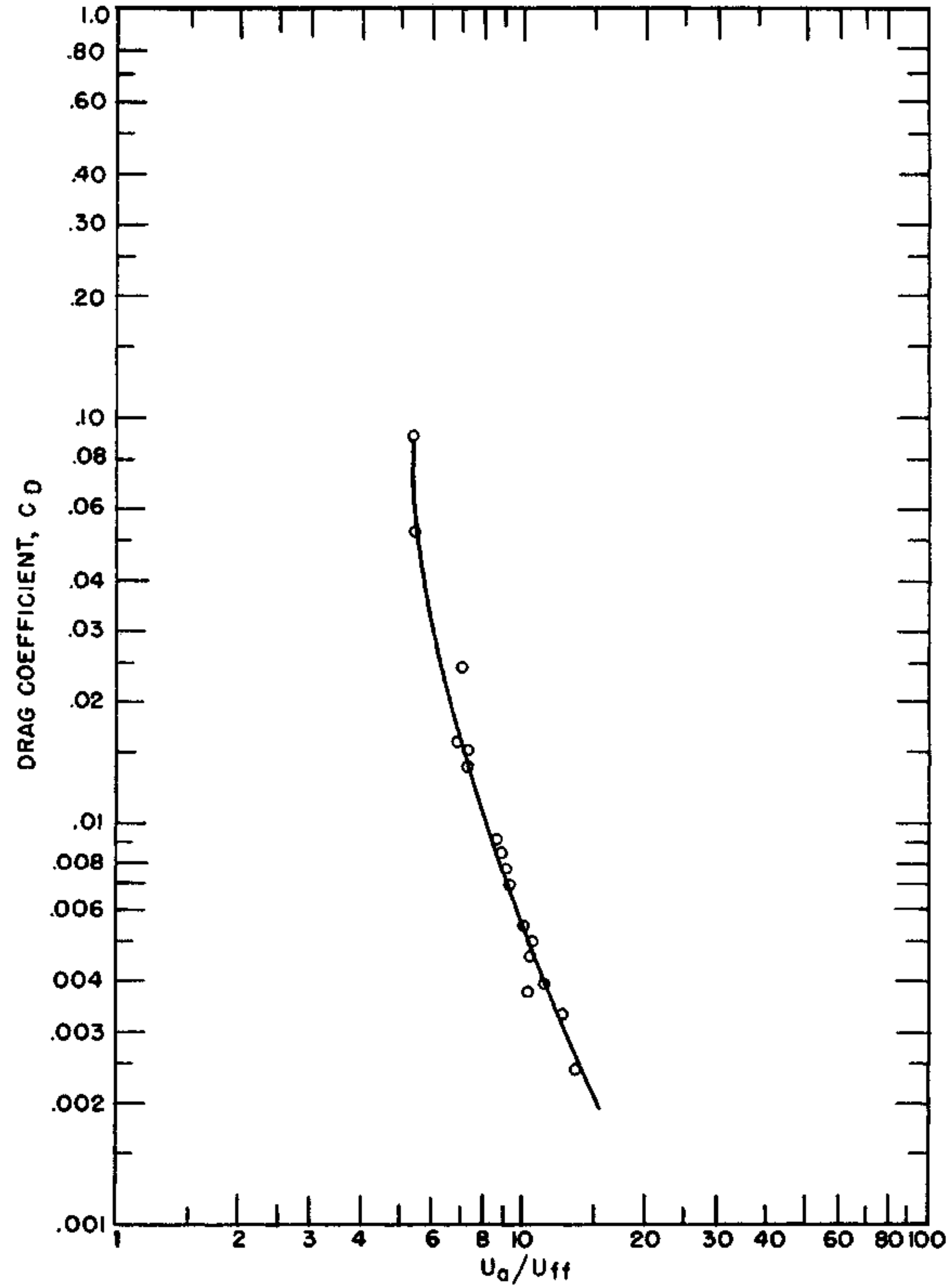

Figure 10. Relation between Drag Coefficient and u_a/u_{ff} for Vertical Transport of Solids

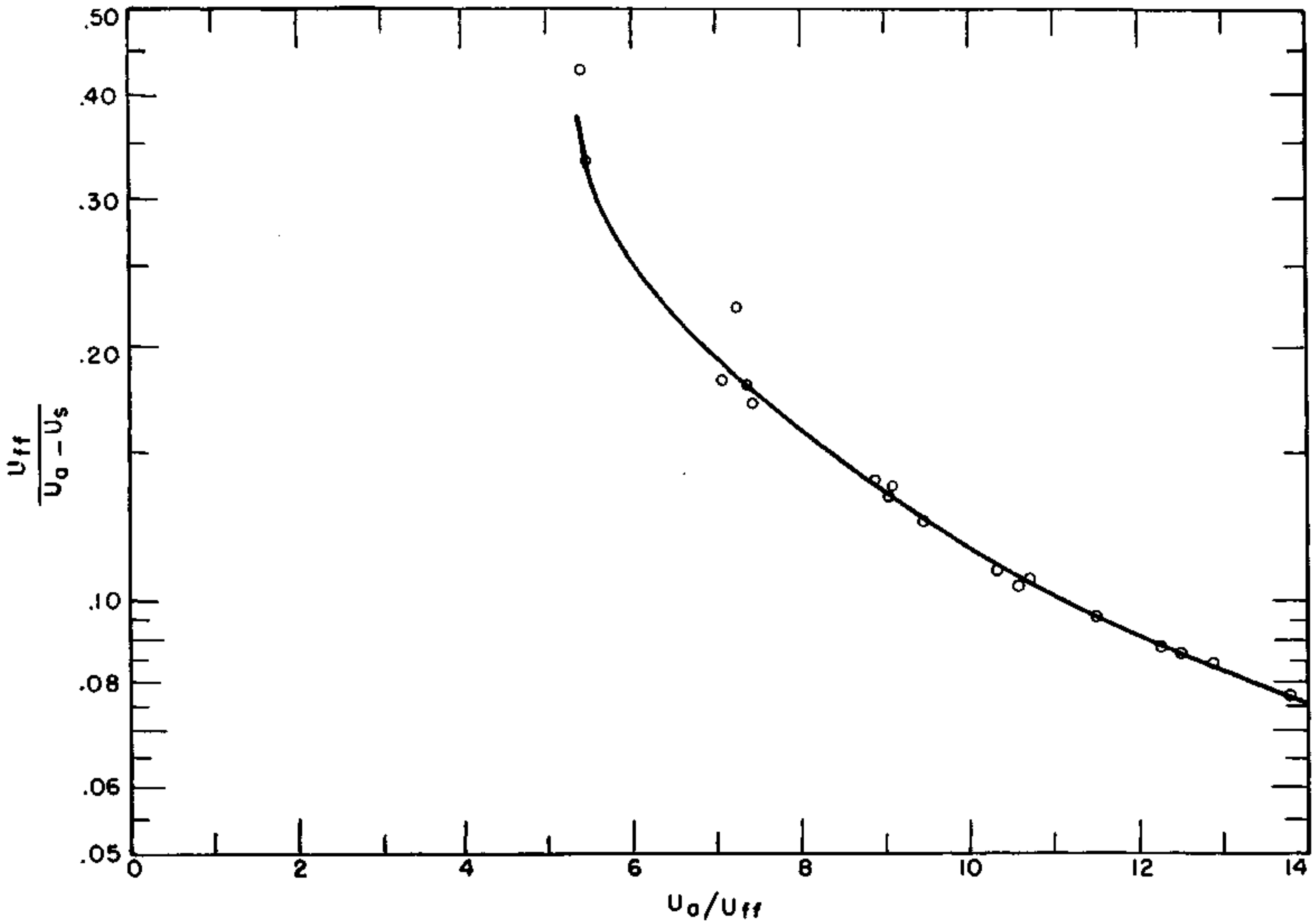

Figure 11. Relation among u_{ff}, u_a, and u_s for Vertical Transport of Solids

would permit the design of pneumatic transport systems from the physical properties of the solid material and the feasible air velocities.

CHAPTER V

CONCLUSIONS

Analysis of the data presented in this study has led to these conclusions:

1. The slip velocity, $u_a - u_s$, has an arithmetic relationship to the air velocity and is independent of the solids loading.

2. An assumed value for f_p equal to that of f_a is a practical approximation for design purposes.

3. The slip velocity, drag coefficient, and particle Reynolds number can be calculated from a knowledge of the properties of the solid material alone.

These conclusions are based upon the data obtained with the M-S(C-2) silica-alumina catalyst and the two-inch riser. They appear valid for the general case of vertical transportation of solids, but similar studies with other solid materials at different air velocities and with different riser sizes must be made before the general relationship is established.

APPENDIX

Table 1. Pressure-Drop Data for the Conveying Air

Run Number	Orifice Reading (in. H_2O)	Orifice Static Pressure (in. H_2O)	$\frac{\Delta P_{fa}}{\Delta L}$ (lb./ft.^{3})	u_a (ft./sec.)
A1, 9, 19	0.25	1.25	0.044	32
A2, 10, 20	0.48	2.30	0.070	42
A3, 11, 21	0.70	3.22	0.090	50
A4, 12, 22	1.20	5.10	0.132	60
A5, 13, 23	1.50	6.15	0.180	.72
A6, 14, 24	1.75	6.70	0.205	78
A7, 15, 25	2.05	7.50	0.227	84
A8, 16, 26	2.40	8.60	0.247	91

Each value is the average of three runs.

The temperature during all the runs was 32° C.; the barometric pressure was 740 mm. of mercury.

Table 2. Experimental Data

Run No.	Solids Loading (lbs./ft.^{2}-sec.)	Air Velocity (ft./sec.)	Pressure Drop* (lbs./ft.^{2}) between Taps 5-9	Taps 9-12	Taps 12-13	Taps 13-14	Taps 14-15	$\Delta P_{ft}/L$ at Equilibrium (lbs./ft.^{3})
S29L	11.5	42.1	-5.66	1.41	3.54	4.95	1.05	1.35
S28L	11.5	50.1	0.71	0.71	6.37	4.95	1.00	1.35
S27L	11.5	58.6	3.54	1.41	5.66	6.37	1.32	1.74
S26L	11.5	64.9	2.83	2.83	7.07	7.07	1.48	1.93
S25L	11.5	78.2	1.41	2.83	11.32	8.49	1.77	2.32
S24L	15.3	30.5	--	--	2.83	3.54	0.75	0.97
S23L	15.3	41.7	--	-1.41	4.95	5.66	1.20	1.54
S22L	15.3	51.1	-2.12	1.41	6.37	6.37	1.35	1.74
S21L	15.3	60.0	2.83	3.54	5.66	14.85	1.87	2.43
S20L	15.3	72.5	1.41	9.90	8.49	11.32	2.38	3.09
S19L	21.9	40.9	-9.19	0.71	7.07	5.66	1.20	1.54
S18L	21.9	51.1	-4.95	4.24	6.37	8.49	1.82	2.32
S17L	21.9	60.5	3.54	3.54	9.90	10.61	2.20	2.89
S16L	21.9	71.0	3.54	3.54	7.07	14.85	3.11	4.05
S15L	30.2	31.0	--	0.00	9.19	8.49	1.80	2.32
S32L	30.2	40.0	-9.19	0.71	8.49	12.73	2.72	3.47
S14L	30.2	42.1	--	--	9.19	7.78	1.45	1.85
S31L	30.2	50.1	-11.32	5.66	8.49	11.32	2.13	2.81
S13L	30.2	53.5	3.54	1.41	7.78	14.15	2.76	3.86
S12L	30.2	60.5	4.24	3.54	7.07	14.85	3.15	4.05
S30L	30.2	69.3	4.24	8.49	19.80	19.10	4.02	5.21
S11L	30.2	69.3	-8.49	3.54	10.61	11.32	2.15	2.81

*An average of the nine values obtained during three runs.

The temperature during all the runs varied between 32° and 35° C.; the barometric pressure varied between 738 and 742 mm. of mercury.

Table 3. Calculated Data

Run Number	Solids Velocity u_s (ft./sec.)	Slip Velocity $u_a - u_s$ (ft./sec.)	Drag Coefficient C_D	u_a/u_{ff}	Re_p	$\frac{u_{ff}}{u_a - u_s}$	f_a
For $f_p = 0.01$:							
S29L	9.76	32.2	0.0147	7.45	40.0	0.175	--
S28L	10.1	40.0	0.0096	8.87	49.7	0.141	--
S27L	7.54	51.0	0.0057	10.4	63.3	0.111	--
S26L	6.71	58.2	0.0043	11.5	72.2	0.097	--
S25L	5.67	72.5	0.0028	13.8	90.0	0.078	--
S24L	---	--	--	5.40	--	---	--
S23L	27.2	14.5	0.174	7.38	18.0	0.389	--
S22L	10.3	40.8	0.0093	9.04	50.7	0.138	--
S21L	6.93	53.1	0.0052	10.6	65.9	0.106	--
S20L	5.38	67.1	0.0032	12.8	83.3	0.084	--
S19L	20.9	20.0	0.0493	7.24	24.8	0.282	--
S18L	11.0	40.1	0.0098	9.04	49.7	0.140	--
S17L	8.48	52.0	0.0056	10.7	64.6	0.108	--
S16L	5.86	65.1	0.0034	12.6	80.8	0.087	--
S15L	16.8	14.2	0.088	5.49	17.7	0.398	--
S32L	9.70	30.3	0.017	7.08	37.6	0.186	--
S13L	8.58	44.9	0.0074	9.47	55.8	0.126	--
S12L	8.32	52.2	0.0056	10.7	64.8	0.108	--
S30L	6.12	63.2	0.0036	12.3	78.4	0.089	--

Continued

Table 3 Continued. Calculated Data

Run Number	Solids Velocity u_s (ft./sec.)	Slip Velocity $u_a - u_s$ (ft./sec.)	Drag Coefficient C_D	u_a/u_{ff}	Re_p	$\frac{u_{ff}}{u_a - u_s}$	f_a
For f_p = 0.005:							
S29L	9.37	32.7	0.014	7.45	40.6	0.173	--
S28L	9.56	40.5	0.0086	8.87	50.3	0.139	--
S27L	7.35	51.3	0.0055	10.4	63.6	0.110	--
S26L	6.50	58.4	0.0042	11.5	72.5	0.097	--
S25L	5.42	72.8	0.0026	13.8	90.3	0.078	--
S24L	19.8	10.7	0.148	5.40	13.3	0.528	--
S23L	10.9	30.8	0.0155	7.38	38.3	0.183	--
S22L	9.76	41.3	0.0086	9.04	51.3	0.137	--
S21L	6.79	53.2	0.0051	10.6	66.0	0.106	--
S20L	5.40	67.1	0.0032	12.8	83.3	0.084	--
S19L	16.7	24.2	0.0270	7.24	30.0	0.233	--
S18L	10.4	40.7	0.0089	9.04	50.6	0.139	--
S17L	8.14	52.4	0.0053	10.7	65.0	0.108	--
S16L	5.64	65.4	0.0033	12.6	81.1	0.086	--
S15L	14.5	16.5	0.057	5.49	20.5	0.342	--
S32L	9.27	30.7	0.016	7.08	38.1	0.184	--
S13L	8.32	45.2	0.0071	9.47	56.1	0.125	--
S12L	7.96	52.5	0.0053	10.7	65.2	0.107	--
S30L	6.12	63.2	0.0036	12.3	78.4	0.089	--

Continued

Table 3 Continued. Calculated Data

Run Number	Solids Velocity u_s (ft./sec.)	Slip Velocity $u_a - u_s$ (ft./sec.)	Drag Coefficient C_D	u_a/u_{ff}	Re_p	$\frac{u_{ff}}{u_a - u_s}$	f_a
For f_p = 0.001:							
S29L	9.23	32.9	0.133	7.45	40.8	0.172	--
S28L	9.12	41.0	0.0084	8.87	50.9	0.138	--
S27L	7.17	51.4	0.0053	10.4	63.8	0.110	--
S26L	6.50	58.4	0.0042	11.5	72.5	0.097	--
S25L	5.42	72.8	0.0026	13.8	90.3	0.078	--
S24L	19.8	13.2	0.0859	5.40	16.3	0.430	--
S23L	10.9	30.8	0.0155	7.38	38.3	0.183	--
S22L	9.69	41.4	0.0086	9.04	51.4	0.136	--
S21L	6.68	53.3	0.0050	10.6	66.2	0.106	--
S20L	5.16	67.3	0.0030	12.8	83.6	0.084	--
S19L	14.9	26.0	0.0207	7.24	32.3	0.217	--
S18L	9.95	41.2	0.0084	9.04	51.1	0.137	--
S17L	7.98	52.5	0.0051	10.7	65.2	0.108	--
S16L	4.70	66.3	0.0027	12.6	82.3	0.085	--
S15L	13.3	17.7	0.045	5.49	21.9	0.320	--
S32L	4.30	30.7	0.0053	7.08	44.3	0.158	--
S13L	8.32	45.2	0.0071	9.47	56.1	0.125	--
S12L	7.96	52.5	0.0053	10.7	65.2	0.108	--
S30L	5.88	63.4	0.0034	12.3	78.7	0.089	--

Concluded

Table 3 Concluded. Calculated Data

Run Number	Solids Velocity u_s (ft./sec.)	Slip Velocity $u_a - u_s$ (ft./sec.)	Drag Coefficient C_D	u_a/u_{ff}	Re_p	$\frac{u_{ff}}{u_a - u_s}$	f_a
For $f_p = f_a$:							
S29L	9.44	32.7	0.0138	7.45	40.5	0.173	0.0057
S28L	9.61	40.5	0.0090	8.87	50.3	0.140	0.0054
S27L	6.82	51.8	0.0050	10.4	64.3	0.109	0.0052
S26L	6.50	58.4	0.0035	11.5	72.5	0.097	0.0050
S25L	5.53	72.7	0.0027	13.8	90.2	0.078	0.0049
S24L	21.0	9.52	0.197	5.40	11.8	0.593	0.0062
S23L	11.1	30.6	0.0160	7.38	38.0	0.184	0.0058
S22L	9.75	41.4	0.0086	9.04	51.3	0.137	0.0054
S21L	6.83	53.2	0.0051	10.6	66.0	0.106	0.0052
S20L	5.40	67.1	0.0032	12.8	83.3	0.084	0.0050
S19L	17.2	23.7	0.0290	7.24	29.4	0.238	0.0058
S18L	10.5	40.6	0.0090	9.04	50.4	0.139	0.0054
S17L	8.25	52.3	0.0053	10.7	64.8	0.108	0.0052
S16L	5.64	65.4	0.0033	12.6	81.1	0.086	0.0050
S15L	14.94	16.1	0.0560	5.49	19.9	0.352	0.0061
S32L	9.40	30.6	0.0159	7.08	38.0	0.185	0.0058
S13L	8.21	45.3	0.0070	9.47	56.2	0.125	0.0053
S12L	7.94	52.6	0.0052	10.7	65.2	0.107	0.0051
S30L	6.12	63.2	0.0036	12.3	78.4	0.089	0.0050

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