New physics in the lepton sector

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Contents

Introduction

> The best that most of us can hope to achieve in physics is simply to misunderstand at a deeper level.
>
> Wolfgang Pauli

The Standard Model, elaborated over the last decades, is one of the most accurate theories of nature ever built. It passed successfully a number of tests until the recent discovery of a particle with all the expected properties of the Higgs boson, which closes a chapter of particle physics. In spite of this, several issues remain, that point towards new physics beyond the Standard Model. For instance, we know little about the nature of the largest part of the energy content of the Universe, apart from the fact that it cannot be made of known particles. The reason why the Higgs boson mass is so small seems rather arbitrary. The origin of matter itself is a nontrivial question of both particle physics and cosmology. However, no clear indication on the nature of the new physics that could solve these issues has been uncovered so far.

The lepton sector may provide this missing piece of information. On one hand, in spite of their ubiquity, neutrinos and their properties are still little known. The origin of their mass and the question of their nature, Dirac or Majorana, could give the first hint about new physics. Great effort is being deployed to test and measure the properties of neutrinos, which gives encouraging prospects. On the other hand, flavour violation in processes involving charged lepton could be a powerful tool to look at deviations from the Standard Model with a great sensitivity.

The first part of this thesis is dedicated to an introduction to these topics. In chapter 1, we give a quick review of the Standard Model, both historical and technical, and at the same time introduce notations. We also discuss broadly the main challenges of physics beyond the Standard Model. Chapter 2 focuses more precisely on the lepton sector. In particular, we introduce the problem of neutrino masses and mention one of the most promising solutions, the seesaw mechanism. We also discuss the topic of flavour violation in the sector of charged lepton, and its potential links with new physics.

Chapter 3 approaches the problem of the matter-antimatter asymmetry of the Universe: the existence of the matter structures in our universe requires an asymmetry between baryons and antibaryons. Baryogenesis through leptogenesis is an elegant solution to this problem, that connects this issue with that of neutrino masses through the seesaw mechanism, and provides therefore an interesting link between high energy and low energy physics. After introducing the main ideas of leptogenesis, we discuss the case of leptogenesis with a scalar triplet. We present a flavour-covariant formalism to treat Boltzmann equations in the regime where lepton flavours are undistinguishable, and show that flavour effects may have a significant impact. We apply this formalism to a particular model, in which the lepton asymmetry is directly related to neutrino parameters.

In the last part of this thesis, we are interested in some supersymmetric models. Supersymmetry may be advocated for several different reasons. It is the most general space-time symmetry that can be built, it provides a solution to the hierarchy problem, and it leads to the unification of the gauge coupling constants with a troubling precision. In chapter 4, after introducing supersymmetry, we discuss a model in which the fermionic partner of a pseudo-Goldstone boson plays the role of a sterile neutrino, that could explain experimental anomalies in the neutrino sector. Finally, in chapter 5, we study supersymmetry breaking and its mediation. If supersymmetry is indeed realized in nature, it is necessarily broken, because otherwise it would have been discovered long ago. Gauge mediated supersymmetry breaking is an elegant and predictive option but is in tension with the measured value of the Higgs boson mass. Extended gauge mediation can overcome this issue and lead to an interesting phenomenology. In particular, we study how the seesaw mechanism can fit in models of extended gauge mediation and lead to predictions for flavour-changing observables.

The most technical points, such as the derivation of flavoured Boltzmann equations through the closed time-path formalism will be displayed in the appendix.

Chapter 1

A review of the Standard Model

Qui serait assez insensé pour mourir sans avoir fait au moins le tour de sa prison ?

L'Œuvre au noir
Marguerite Yourcenar

More than eighty years separate the formulation of the Dirac equation, which gave the first relativistic and quantum description of the electron and lead to the theorization of the positron, from the discovery of the Higgs boson at the LHC, that was the achievement of two decades of extensive searches. In between, our understanding of particle physics has improved step by step, thanks to contributions of several generations of physicists, to form the picture that we know today, the Standard Model. This justifies the words of Sheldon Glashow, for whom the Standard Model of particle physics is "a tapestry made by many artisans working together".

1.1 Historical overview

The early steps

In 1928, Paul Dirac achieved his goal of including special relativity in a quantum mechanical equation describing the electron. The result was the Dirac equation [1]

$$i\gamma_\mu \partial^\mu \psi - m\psi = 0 \ . \tag{1.1.1}$$

The solutions to this equation have four degrees of freedom, twice as many as expected for a spin-1/2 particle like the electron. In particular, Dirac was puzzled by the possibility of negative-energy solutions, which he thought should not be simply ignored. At first, he assumed that there was a "sea of negative energy states", which had to be completely filled in order to prevent an electron from jumping from a positive energy to a negative energy state. Then, he raised the possibility of the proton being a hole in the sea of negative-energy electrons [2], but this was not consistent with the proton being about 2000 times as heavy as the electron. Finally, in 1931, Dirac predicted the existence of a new particle, the positron, having the same mass as the electron but with an opposite electric charge [3]. He also postulated that a positron and an electron colliding would annihilate. After first experimental evidence observed by Dmitri Skobeltsyn and Chung-Yao Chao, the positron was formally discovered in 1932 by Carl D. Anderson [4]. This was the first proof of the existence of antiparticles.

At the same time, the spectrum of the electron and the nucleus emitted in beta decay had been firmly established to be continuous [5], which was inconsistent with this phenomenon being a two-body decay of the form

$$ {}^{A}_{Z}X \to {}_{Z+1}^{\;\;A}X + e^- \ , \tag{1.1.2}$$

for which the center-of-mass energy of the final state particles should be uniquely determined. Either the conservation of energy was violated in beta decay, or this phenomenon was not fully understood. Another issue was the conservation of spin, which is not satisfied by eq. (1.1.2). This lead Wolfgang Pauli to postulate the existence of an invisible particle emitted along with the electron and the final nucleus in beta decay, which should have spin 1/2. Such a particle restored the conservation of both spin and energy. He exposed this idea in a letter to Lise Meitner, Hans Geiger and other "radioactive people", and named this hypothetical fermion "neutron". In 1932, when the neutron, as we know it today, was discovered by Chadwick, Pauli's particle was renamed "neutrino". Despite Pauli's pessimism regarding the detection of the neutrino, it was finally observed in 1956 by Reines and Cowan [6].

In 1933, Enrico Fermi included the neutrino in the first theory aiming to describe beta decay. In Fermi's theory, the beta decay is the result of a four-fermion contact interaction, of the form

$$\mathcal{L}_{\mathrm{Fermi}} = \frac{G_F}{\sqrt{2}} \left(\bar{p}\, \gamma_\mu\, n\right) \left(\bar{e}\, \gamma^\mu\, \nu\right) \ , \tag{1.1.3}$$

where p and n are respectively the proton and the neutron, and $G_F = 1{,}116637\ \mathrm{GeV}^2$ is known as the Fermi constant. Fermi's paper was refused by *Nature* for being too speculative, so that it was initially only issued in Italian and German [7, 8]. In addition to beta decay, this theory also predicted scatterings such as

$$p + e \to n + \nu \ , \tag{1.1.4}$$

and gave remarkably precise results. However, the cross section of this process growing like the center-of-mass energy squared raised doubt about the validity of Fermi's theory up to arbitrary energies.

In 1935, Hideki Yukawa built a theory of strong interactions, in which the cohesion of protons and neutrons in atomic nuclei is ensured by the exchange of scalar particles, the pions [9]. With this explanation for strong interactions, a full picture of particle physics seemed at hand. In the late 40's and the early 50's, the pions were discovered, confirming the prediction of Yukawa. Charged pions were discovered first in 1947 [10, 11], and the neutral pion three years later [12].

However, the unexpected discovery of the muon in 1937 [13, 14] raised new questions. This particle, with the same electric charge as the electron but 200 times heavier, was first mistaken with one of Yukawa's pions, until it was established that it did not interact strongly. This new state, that was just a heavy copy of the electron, and whose existence was not required in our understanding of nature, seemed incongruous. It is only in the 1960's, with the electroweak unification, that the muon found naturally its place, along with the electron and the associated neutrinos, in the lepton sector.

Quantum electrodynamics

The first completed piece in the formulation of what was going to be the Standard Model of particle physics was Quantum Electrodynamics (QED). In the 1920's, Dirac had been the first to formulate a theory describing the interaction between radiation and matter. He had also introduced annihilation and creation operators to deal with particles. Progress was carried out in the following year, so that the formulation of a complete theory describing electrons and photons seemed only a matter of time. A major obstacle was however reported, first by Robert Oppenheimer in 1930 [15], then by Felix Bloch and Arnold Nordsieck in 1937 [16] and by Victor Weisskopf in 1939 [17]. They showed that the computations performed were reliable only at first order in perturbation theory, while at higher order, infinities appeared. What is now a well-known feature of quantum field theories was thought to make no physical sense at the time. This raised concern about a possible incompatibility of special relativity and quantum mechanics.

A way out began to emerge in 1947, when Hans Bethe tried to compute the Lamb shift of a hydrogen atom [18]. He was the first to apply the idea of renormalization, suggested by Hans Kramers, to solve the problem. With his method, he obtained a remarkable agreement with the measured shift of the levels of a hydrogen atom. His computation was non-relativistic, but this new way of absorbing infinities was later extended to a relativistic framework. The modern formulation of Quantum Electrodynamics was elaborated in the late 1940's by Sin-Itiro Tomonaga [19], Julian Schwinger [20, 21], Freeman Dyson [22, 23] and Richard Feynman [24–26]. At the same time, Feynman introduced a new powerful tool to deal with perturbation theory, the Feynman diagrams.

The resulting theory is renown for its precision: QED gave extremely accurate predictions of physical quantities. It also provided a blueprint for the elaboration of every subsequent quantum field theory.

The electroweak unification

Some thirty years after Fermi's first theory of weak interactions, a big step further was accomplished in the 1960's. In 1961, Glashow proposed the first model of electroweak unification, based on the gauge group $SU(2) \times U(1)$ [27]. In 1964, Abdus Salam and John Clive Ward [28] used the same symmetry group to build a model describing electrons and muons. Unfortunately, gauge theories seemed to require massless vector bosons, whereas the weak interaction is so short-ranged that it has to be carried by very massive particles. The mechanism of spontaneous symmetry breaking, elaborated by Robert Brout, François Englert [29], Peter Higgs [30], Gerald Guralnik, Carl Richard Hagen, and Tom Kibble [31] brought the solution to this puzzle by explaining how gauge bosons can become massive. In 1967, Salam [32] and Weinberg [33] had independently the idea of incorporating this mechanism in Glashow's model of electroweak unification. Finally, Gerard 't Hooft demonstrated in 1971 the renormalizability of a spontaneously broken gauge theory with operators of mass dimension four or less, under the condition that the theory is anomaly-free [34]. This was the last step in proving the theoretical consistency of this model.

The resulting theory of electroweak interactions contains a massless gauge boson which is the photon, and three massive ones. Two of them, W^+ and W^-, are electrically charged and allow to reformulate Fermi's theory in terms of the exchange of a massive particle. The Fermi constant is then understood as

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2} \ , \tag{1.1.5}$$

g being the true fundamental coupling of the theory and M_W the mass of the charged gauge bosons. The third one, Z^0, is neutral and gives rise to a new interaction that was not predicted by Fermi. The observation of neutral currents, followed by the formal discovery of these gauge bosons in 1983 [35–38] confirmed further the validity of the electroweak theory. Until recently, only one piece was missing to this theory, the Higgs boson predicted by the mechanism of spontaneous symmetry breaking.

The quark sector

Before the discovery of pions, the only known hadrons were the proton and the neutron. In the 1950's, thanks to the invention of bubble chambers, an increasing number of hadrons were observed. They were first classified by charge and isospin, then also by strangeness when the first "strange" particles were detected. This classification lead to an intriguing pattern, the eightfold way, highlighted by Murray Gell-Mann [39] and Yuval Ne'eman [40]. They both discovered that the mesons and the spin-1/2 baryons could be arranged into octets while the spin-3/2 baryons could almost fit into a decuplet to which one particle was missing. This lead Gell-Mann and Ne'eman to predict the existence of a new spin-3/2 resonance. This new state, Ω^-, was discovered three years later and completed this picture.

In 1964, Gell-Mann [41] and George Zweig [42, 43] independently found that hadrons could be thought of as made of smaller fermions with 3 flavours (up, down and strange), related by a $SU(3)$ flavour symmetry, which would explain this pattern. Gell-Mann coined the term "quark", found in James Joyce's novel *Finnegans Wake*, to label these particles, which he himself considered at first only as mathematical entities introduced for convenience, not as physical particles, in the sense that free quarks could not be observed.

This raised a new problem, because some of the spin-3/2 baryons had to be composed of three quarks of the same flavour with parallel spins, which should be forbidden by Pauli's exclusion principle. In 1965, considering the Ω^- hyperon composed of three strange quarks, Nikolay Bogolyubov, Boris Struminsky and Albert Tavkhelidze pointed out the necessity for an additional degree of freedom to solve this issue [44]. At the same time, considering a similar problem for the resonance Δ^{++}, Moo-Young Han with Yoichiro Nambu [45] and Oscar W. Greenberg [46] independently proposed the existence of a new $SU(3)$ gauge degree of freedom, called "color". Consequently, three kinds of quarks should interact with a color octet of gauge bosons called gluons. Quarks and gluons were collectively labelled as partons.

Because partons are confined inside hadrons, it was doubtful that strong interaction could be described by a quantum field theory like QED. Instead, some physicists began working on a different formulation, the S-matrix theory. However, James Bjorken pointed out a characteristic feature of pointlike partons that could be revealed by deep inelastic scatterings of electrons and muons on hadrons [47], and which was indeed discovered in 1969 at SLAC [48, 49], entitling the quarks and gluons to the status of physical particles. Consequently, the S-matrix theory was abandoned and a quantum field theory of strong interaction, Quantum Chromodynamics (QCD), was developed. In 1973, Gross and Wilczek [50] and Politzer [51] discovered a fundamental property of Quantum Chromodynamics, asymptotic freedom, that makes it weakly coupled at large momentum, and allows to perform perturbative calculations. This discovery was a decisive argument in favour of the quantum field theory formulation.

At this point, only the up, down and strange quarks were known. However, a fourth quark had been first predicted in 1964 by Bjorken and Glashow [52], and, in 1970, Glashow, Iliopoulos and Maiani [53] proposed a mechanism to explain the suppression of flavour-changing neutral currents in the quark sector, that required the existence of this additional quark. The fourth quark, known as the charm, was discovered in 1974 [54, 55]. In 1973, Makoto Kobayashi and Toshihide Maskawa proposed a third generation of quarks because the observation of CP violation in kaon decay was inconsistent with only two generations [56]. The discovery of the bottom quark at Fermilab in 1977 [57] confirmed their intuition, and finally, the top quark was discovered in 1995 [58, 59], completing the third fermion generation.

The end of the quest?

In 2012, the collaborations Atlas [60] and CMS [61] from the LHC announced the discovery of a new particle that presents the expected properties of the Higgs boson, with a mass

$$m_h = 125.6 \pm 0.3 \text{ GeV} . \tag{1.1.6}$$

Moreover, until now, no significant deviation from what is predicted by the Standard Model has been observed. This particle was the last missing piece of the theory, and its discovery puts an end to the construction of the Standard Model. However, some remaining interrogations, which will be reviewed in section 1.3, motivate the search for physics beyond the Standard Model.

1.2 The framework

1.2.1 The gauge theory

When studying the Standard Model, it is crucial to separate the right- and left-handed components of the fermions, defined according to

$$\psi_R = \frac{1}{2}(1+\gamma_5)\,\psi = P_R\,\psi \ , \qquad \psi_L = \frac{1}{2}(1-\gamma_5)\,\psi = P_L\,\psi \ , \tag{1.2.1}$$

and the Dirac field is given in terms of its chiral components by $\psi = \psi_L + \psi_R$. The discrete symmetries C, P and T act on fermion fields in the following way. Charge conjugation C exchanges particles and antiparticles of the same chirality, for instance

$$\mathcal{C}\,\psi_L\,\mathcal{C}^{-1} = \psi_L^c \ , \tag{1.2.2}$$

whereas parity flips the chirality of a field,

$$\mathcal{P}\,\psi_L\,\mathcal{P}^{-1} = \psi_R \ , \tag{1.2.3}$$

and T reverses time. C, P and T can be broken, but the symmetry CPT is always preserved. Since the Standard Model is a chiral theory with right- and left-handed fields transforming under different representations of the gauge group, the most relevant symmetry between particles and antiparticles is CP, which exchanges for instance left-handed particles transforming under a given representation and right-handed antiparticles transforming under the conjugate representation,

$$(\mathcal{CP})\,\psi_L\,(\mathcal{CP}^{-1}) = \psi_R^c = (\psi_L)^c \ . \tag{1.2.4}$$

Note that for a gauge theory based on $SU(2)$, a representation and its conjugate are always identical, therefore CP-conjugate fermions transform under the same representation. A CP transformation can be represented by mean of the matrix C expressed in terms of Dirac matrices by $C = i\gamma^0\gamma^2$,

$$\psi_R^c = C\bar{\psi}_L^T \ , \qquad \psi_L^c = C\bar{\psi}_R^T \ . \tag{1.2.5}$$

The Standard Model is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. $SU(3)_c$ describes the strong interactions, whereas $SU(2)_L \times U(1)_Y$ provides the framework for the electroweak interaction. We will label the $SU(3)$, $SU(2)$ and $U(1)$ field strengths as $G^a_{\mu\nu}$, $W^a_{\mu\nu}$ and $B_{\mu\nu}$ respectively. The corresponding couplings are referred to as g_s, g and g' respectively.

Fermions fall into irreducible representations of this group, and are divided in two types. Quarks are $SU(3)$ triplets and therefore sensitive to strong interactions, whereas leptons are $SU(3)$ singlets and interact only through electroweak interactions.

Another division occurs, based on chirality. Left-handed fermions are gathered in $SU(2)$ doublets, whereas right-handed fermions are singlets with respect to this group. As announced previously, things are reversed when considering antiparticles, because right-handed antiparticles belong to $SU(2)$ doublets, like for instance

$$\ell^c = \begin{pmatrix} e_R^c \\ \nu_R^c \end{pmatrix} \ , \tag{1.2.6}$$

while left-handed antiparticles are $SU(2)$ singlets. Each fermion generation contains a doublet of left-handed quarks including an up-type quark and a down-type quark, a doublet of left-handed leptons including a charged lepton a neutrino, right-handed up- and down-type quarks and a right-handed charged lepton. There is no right-handed neutrino since it would be a gauge singlet without any interaction, and therefore its presence is unnecessary at least as long as we neglect neutrino masses. This field

	Field	$SU(3)\times SU(2)_L\times U(1)_Y$	B	L
Quarks	$Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	$(3,\ 2,\ +1/6)$	$1/3$	0
	u_{Ri}	$(3,\ 1,\ +2/3)$	$1/3$	0
	d_{Ri}	$(3,\ 1,\ -1/3)$	$1/3$	0
Leptons	$\ell_\alpha = \begin{pmatrix} \nu_{L\alpha} \\ e_{L\alpha} \end{pmatrix}$	$(1,\ 2,\ -1/2)$	0	1
	$e_{R\alpha}$	$(1,\ 1,\ -1)$	0	1

Table 1.1: Fermionic content of the Standard Model.

content is summarized in table 1.1.

Before the spontaneous breaking of the electroweak symmetry, the gauge-invariant Lagrangian for the fermions and gauge fields is

$$\mathcal{L}_{\text{fermions}} = \sum_{i=1,2,3} i\left(\bar{Q}_i \not{D} Q_i + \bar{u}_{Ri}\not{D} u_i + \bar{d}_{Ri}\not{D} d_{Ri}\right) + \sum_{\alpha=e,\mu,\tau} i\left(\bar{\ell}_\alpha \not{D}\ell_\alpha + \bar{e}_{R\alpha}\not{D} e_{R\alpha}\right) \ , \tag{1.2.7}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} \ , \tag{1.2.8}$$

It does not contain any mass term, because gauge boson masses would break gauge invariance, while for fermions, mass terms have the form

$$-\mathcal{L} = m\bar{\psi}\psi = m\bar{\psi}_R\psi_L + m\bar{\psi}_L\psi_R \ . \tag{1.2.9}$$

Since right- and left-handed fermions belong to different representations of $SU(2)$, such a mass term would also violate $SU(2)$ gauge invariance. In the absence of fermion mass terms, the Lagrangian possesses an additional global symmetry $SU(3)^5$, which corresponds to the transformations

$$f_i \to (V_f)_{ij} f_j, \qquad f = Q, u_R, d_R, \ell, e_R \ . \tag{1.2.10}$$

However, to be a viable theory of nature, the Standard Model must account for the masses of the fermions and the gauge bosons of the weak interaction.

1.2.2 The Higgs mechanism and its consequences

This discrepancy is solved by the spontaneous breaking of $SU(2)_{EW}\times U(1)_Y$ to the $U(1)_{EM}$ subgroup describing electromagnetism. This spontaneous breaking is triggered by the Higgs field, which is an $SU(2)_{EW}$ doublet with hypercharge $Y = +1/2$,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \ . \tag{1.2.11}$$

The Higgs field participates in the Lagrangian through the following terms

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) - V(H) \tag{1.2.12}$$

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{i,j}\left(y^u_{ij}\bar{Q}_i\tilde{H}\, u_{Rj} + y^d_{ij}\bar{Q}_i H\, d_{Rj}\right) - \sum_{\alpha,\beta} y^e_{\alpha\beta}\bar{\ell}_\alpha H\, e_{R\beta} + \text{h.c.} \ , \tag{1.2.13}$$

where we defined $\tilde{H} = i\sigma_2 H^*$. Up to this point, gauge invariance is preserved by every term appearing in the Lagrangian.

The scalar potential responsible for the spontaneous breaking of $SU(2)_L \times U(1)_Y$ is

$$V(H) = \mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 \ . \tag{1.2.14}$$

λ should be positive, otherwise the potential is unbounded from below, which means that there is no stable vacuum. When μ^2 is positive, there is a single minimum in $H = 0$, and the gauge symmetry is unbroken. When μ^2 is negative, the minima of the potential are defined by

$$H^\dagger H = \frac{-\mu^2}{2\lambda} \ . \tag{1.2.15}$$

There is an infinite number of degenerated vacua, defined as minima of the scalar potential. Therefore, the Higgs fields acquires a vacuum expectation value (v.e.v.), which we can choose to be

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \ , \qquad v = \langle H^0 \rangle = \sqrt{\frac{-\mu^2}{2\lambda}} \ . \tag{1.2.16}$$

Because H is charged under $SU(2) \times U(1)$, this vacuum expectation value breaks spontaneously the electroweak symmetry and the kinetic term of the Higgs field gives rise to the gauge boson masses. Using the explicit expression of the covariant derivative

$$D^\mu H = \left(\partial^\mu - igW_A^\mu \tau_A - i\frac{g'}{2}B^\mu\right) H \tag{1.2.17}$$

and replacing H with its vacuum expectation value in the kinetic term of eq. (1.2.12) yields

$$\delta\mathcal{L}_{\text{gauge}} = \frac{v^2}{4}\left[g^2\left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right) + \left(-gW_\mu^3 + g' B_\mu\right)^2\right] \ . \tag{1.2.18}$$

The physical states W^+ and W^- correspond to the combinations

$$W_\mu^\pm = \frac{W_\mu^1 \pm iW_\mu^2}{\sqrt{2}} \ . \tag{1.2.19}$$

Consequently, W^+ and W^- are conjugate fields with the squared mass

$$M_W^2 = \frac{g^2 v^2}{2} \ . \tag{1.2.20}$$

The two electrically neutral gauge bosons W^3 and B mix through the squared mass matrix

$$\begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g^2 v^2/4 & -gg' v^2/4 \\ -gg' v^2/4 & g'^2 v^2/4 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \ . \tag{1.2.21}$$

Diagonalizing this matrix, we get the massive gauge boson Z^0 and its mass,

$$Z^0_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \ , \tag{1.2.22}$$

$$M_Z^2 = \frac{g^2 + g'^2}{2} v^2 \ , \tag{1.2.23}$$

where we introduced the Weinberg angle, defined by

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \ . \tag{1.2.24}$$

The masses of the W and Z^0 boson are therefore related by $M_W = M_Z \cos\theta_W$. The fourth gauge boson, which we identify as the photon, corresponds to the unbroken subgroup of electromagnetism $U(1)_{EM}$. It remains massless, and is given in terms of the original fields by

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \ . \tag{1.2.25}$$

The charged bosons W^+ and W^- couple to left-handed fermions with a coupling constant g, whereas the coupling of the photon to a field with weak isospin T_3 and hypercharge Y is $(T_3 + Y) \times gg'/\sqrt{g^2 + g'^2}$, so we can identify the electromagnetic coupling constant e and the electric charge Q of the field as respectively

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad Q = T_3 + Y \ . \tag{1.2.26}$$

Finally, the coupling of the neutral boson Z^0 to a field with weak isospin T_3 and hypercharge Y is

$$g\cos\theta_W T_3 - g'\sin\theta_W Y = \frac{g}{\cos\theta_W}\left(T_3 - \sin^2\theta_W Q\right) \ . \tag{1.2.27}$$

Electroweak interactions can be described as interactions of the W and Z^0 bosons and the photon with charged and neutral currents

$$\mathcal{L}_{\text{weak}} = (W_\mu \, j^\mu_{\text{CC}} + \text{h.c.}) + Z_\mu \, j^\mu_{\text{NC}} + A_\mu \, j^\mu_{\text{EM}} \tag{1.2.28}$$

where the charged current involves only left-handed fields, so that interactions involving the $W^\pm$ bosons violate maximally parity,

$$j^\mu_{\text{CC}} = -\frac{g}{\sqrt{2}}\left\{\sum_i \bar{u}_{Li}\gamma^\mu d_{Li} + \sum_\alpha \bar{\nu}_{L\alpha}\gamma^\mu e_{L\alpha}\right\} \ , \tag{1.2.29}$$

whereas the neutral current coupling to the Z^0 boson involves both left- and right-handed fields, but still violates parity. The neutral current is given by[1]

$$\begin{aligned} j^\mu_{\text{NC}} = \frac{g}{\cos\theta}\Bigg\{&\sum_i \left(\bar{u}_i \frac{1}{2}\gamma^\mu \left(c_V^u - c_A^u\gamma_5\right) u_i + \bar{d}_i \frac{1}{2}\gamma^\mu \left(c_V^d - c_A^d\gamma_5\right) d_i\right) \\ &+ \sum_\alpha \left(\bar{e}_\alpha \frac{1}{2}\gamma^\mu \left(c_V^e - c_A^e\gamma_5\right) e_\alpha + \bar{\nu}_{L\alpha}\gamma^\mu \nu_{L\alpha}\right)\Bigg\} \ . \end{aligned} \tag{1.2.30}$$

Field	c_V	c_A
u_i	$1/2 - 4/3\sin\theta_W^2$	$1/2$
d_i	$-1/2 + 2/3\sin\theta_W^2$	$-1/2$
e_α	$-1/2 + 2\sin\theta_W^2$	$-1/2$

Table 1.2: Axial and vector couplings of the Standard Model fermions.

The c_V and c_A coefficients are respectively the vector and axial couplings of the fermions which are specified in table 1.2. The electromagnetic interaction preserves parity because left- and right-handed components have the same electric charge, and the associated current is

$$j^\mu_{\rm EM} = e\left\{\sum_i \left(\frac{2}{3}\bar{u}_i\gamma^\mu u_i - \frac{1}{3}\bar{d}_i\gamma^\mu d_i\right) - \sum_\alpha \bar{e}_\alpha\gamma^\mu e_\alpha\right\} \ . \tag{1.2.31}$$

The Yukawa couplings give rise to the fermion masses: after the electroweak symmetry breaking, replacing H with its vacuum expectation value in the Yukawa couplings of eq. (1.2.13), we get the fermion mass terms

$$\mathcal{L}_{\rm mass} = -\sum_{i,j=1,2,3}\left(m^u_{ij}\bar{u}_{Li}u_{Rj} + m^d_{ij}\bar{d}_{Li}d_{Rj}\right) - \sum_{\alpha,\beta=e,\mu,\tau} y^e_{\alpha\beta}\bar{e}_{L\alpha}e_{R\beta} + \text{h.c.} \ , \tag{1.2.32}$$

with $m^f_{ij} = y^f_{ij}\, v$. If we want the charged currents of eq. (1.2.29) to keep a diagonal form in flavour space, it is possible to diagonalize simultaneously the mass matrices of the charged leptons and of one type of quark, for instance the up-type one. To achieve this we perform the following rotations,

$$f \to V_f f \ , \tag{1.2.33}$$

where the V_f are unitary matrices. Alternatively, we can choose to diagonalize simultaneously the three mass matrices. Then, in a basis where the three matrices m^u, m^d and m^e are diagonal, the charged current involving the quarks takes the form

$$j^\mu_{{\rm CC},\,q} = -\frac{g}{\sqrt{2}}\sum_i \bar{u}_{Li}\left(V_u^\dagger V_d\right)_{ij}\gamma^\mu d_{Lj} \ . \tag{1.2.34}$$

$V_u^\dagger V_d \ = V_{CKM}$ is known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix [56]. It governs the transitions between quarks of different generations, like for instance the bottom quark decay

$$b \to c + W^- \ , \tag{1.2.35}$$

where the W^- subsequently decays, for instance into an antineutrino and a charged lepton. As long as neutrinos are supposed massless, there is no flavour-changing process in the lepton sector. Things will change when we introduce neutrino masses, which

[1] Here, unless specified otherwise, we refer to Dirac spinors including both left- and right-handed components, for instance $u_i = u_{Li} + u_{Ri}$.

will involve the equivalent of the CKM matrix for the lepton sector, the Pontecorvo-Maki-Nakagawa-Sakato (PMNS) matrix.

The global symmetry $SU(3)^5$ defined by eq. (1.2.10) is broken by the fermion mass terms. What remains is an accidental symmetry $U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$. $U(1)_B$ corresponds to baryon number, and the $U(1)_{L_\alpha}$ to the flavoured lepton numbers. Quarks have a baryon number $B = 1/3$ ($B = -1/3$ for antiquark), while the leptons of flavour α have a lepton number $L_\beta = \delta_{\alpha\beta}$ ($L_\beta = -\delta_{\alpha\beta}$ for antileptons of flavour α). Neglecting neutrino masses, the Standard Model Lagrangian is invariant under the transformations

$$(Q_i,\, u_{Ri},\, d_{Ri}) \to e^{i\theta_B/3}\,(Q_i,\, u_i,\, d_i)\ , \tag{1.2.36}$$

$$(\ell_\alpha,\, e_{R\alpha}) \to e^{i\theta_{L_\alpha}}\,(\ell_\alpha,\, e_{R\alpha})\ . \tag{1.2.37}$$

After the introduction of neutrino masses, the three lepton numbers are not conserved anymore.

1.3 Unanswered questions

The Standard Model proved to be an incredibly accurate description of nature. However, a few issues remain, which are not resolved in this framework. Some of these issues, such as Dark Matter and Dark Energy, are based on observation, and are therefore compelling evidence that the Standard Model cannot be the ultimate theory of Nature. Some other, like the hierarchy problem, are rather unsatisfying features which do not make the theory inconsistent but suggest to look for a bigger picture.

The invisible energy content of the Universe

In 1933, Fritz Zwicky was the first to notice a discrepancy between the motion of galaxies in the Coma cluster and the visible amount of matter [62]. He suggested the existence of Dark Matter to solve this inconsistency. Later, Vera Rubin observed a similar effect when looking at the motion of stars in the Andromeda galaxy [63]. Her measurement was the first conclusive evidence of the existence of Dark Matter. Further evidence was provided by the Cosmic Microwave background. Indeed, anisotropies in the CMB are due to acoustic oscillations of the primordial plasma, and the shape of their spectrum can be related to the amount of dark matter [64]. This new, invisible kind of matter should represent around 80% of the total amount of matter in the Universe, but despite being the most widely accepted explanation, its nature remains a mystery. Several hypotheses have already been excluded and today, the only certainty is that Dark Matter is not made of Standard Model particles. Among the possibilities which are still being explored, Dark Matter could be made of a new, heavy species of sterile neutrinos, or of Weakly Interacting Massive Particles (WIMPS) [65], which could for instance arise in supersymmetry.

Another puzzle concerning the energy content of the Universe was raised at the end of the 90's, when it was found that the expansion of the Universe is accelerating instead of decelerating [66], as could be reasonably expected because of gravity attracting the galaxies to one another. This discovery resurrected the cosmological constant introduced in 1917 by Einstein, who at the time wanted to model a static universe [67]. However, the cosmological constant is a rather ad hoc solution. Instead, some theories suggest that this acceleration is due to a new kind of field which behaves like a fluid with negative pressure, called Dark Energy, which counterbalances the effect of gravity.

Ordinary matter represents only about 5% of the energy content of the universe, whereas Dark Matter represents around 25%, and Dark Energy nearly 70%. In the end, more than 95% of the energy content of the Universe is not described by the Standard Model.

The Hierarchy Problem

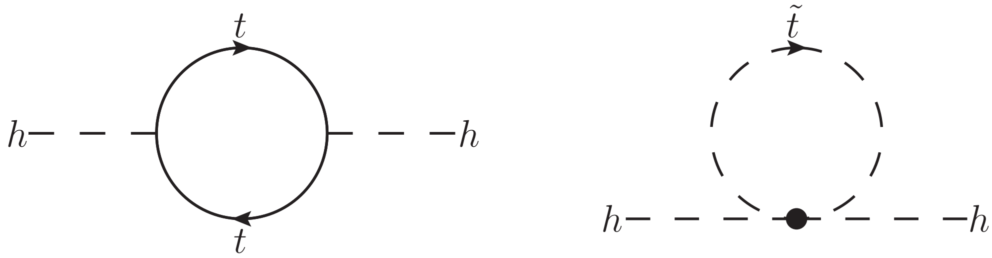


Figure 1.1: Quantum corrections to the Higgs mass coming from a top quark loop (left), and from a stop loop in supersymmetry (right).

Another issue, which is essentially theoretical, is the hierarchy problem [68–72]. A scalar field like the Higgs boson receives corrections to its squared mass coming from diagrams like those displayed in fig. 1.1. Using cutoff regularization to compute these corrections, they grow like Λ^2, Λ being the cutoff of the theory. For instance in the Standard Model, the main correction comes from the coupling with the top quark, and is of the order

$$\delta m_h^2 \sim -\frac{y_t^2}{8\pi^2}\Lambda^2 \ . \tag{1.3.1}$$

This can be a problem if the cutoff is very large. For instance, if it is the Planck scale $M_P = 2.4 \times 10^{18}$ GeV, the squared mass of the Higgs boson receives huge corrections (the abovementioned term is of the order of $10^{35}\,\text{GeV}^2$). In this case, the smallness of the physical mass $m_h \simeq 126\,\text{GeV}$ has to involve a miraculous cancellation between the bare mass parameter appearing in the Lagrangian and the quantum corrections. Even if one dismissed the physical meaning of cutoff regularization and used dimensional regularization instead to get rid of the leading correction above, new physics at a heavy scale would bring back the problem. For instance, any new scalar with a mass M that couples to the Higgs boson with a coupling $-\lambda(H^\dagger H)(S^\dagger S)$ generates a correction to the Higgs squared mass given by

$$\delta m_h^2 \sim \frac{\lambda}{16\pi^2} M^2 \log\frac{M^2}{\mu^2} \ , \tag{1.3.2}$$

where μ is the renormalization scale. The leading correction goes now like M^2 and a fine tuning is again needed to recover the physical Higgs mass. To summarize, the Higgs boson mass is very sensitive to the highest scale in the theory. As a consequence, the hierarchy problem appears as soon as there is new physics beyond the Standard Model, unless this new physics takes a very peculiar form.

There are essentially two type of solutions to the Hierarchy Problem. One solution is to assume that the Higgs boson is not a fundamental scalar, but rather a composite particle, just like the pion in QCD. Hence, the cutoff is lowered to the scale Λ at which the fundamental components of the Higgs boson appear as free particles, which is somehow the equivalent of the confinement scale Λ_{QCD} in Quantum Chromodynamics.

Since a fermion loop gives a negative correction to the Higgs squared mass, and a scalar loop a positive one, another possibility is to counterbalance the negative contributions of the fermions with positive contributions coming from scalars. This is possible if a new symmetry ensures that each fermion is associated to a boson that couples to the Higgs field in the same way, and vice versa. This idea is realized by supersymmetry, in which particles fit in supermultiplets, each one containing the same number of bosonic and fermionic degrees of freedom, which ensures the cancellation of all the divergent contributions to the Higgs boson mass. Supersymmetry will be introduced in 4.1.

The strong CP problem

Another fine-tuning issue is the so-called Strong CP problem. It stems from the fact that in principle, nothing forbids the following CP-violating term in the QCD Lagrangian,

$$\mathcal{L}_{\cancel{CP}} = \frac{g_s^2 \Theta_{QCD}}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \ . \tag{1.3.3}$$

This term is a total derivative, but its contribution does not vanish because of instanton field configurations which are characteristic of non-abelian Yang-Mills theories. If at least one species of quark q was massless, it would be possible to render this CP violation unphysical, by mean of the axial rotation

$$q \to e^{i\Theta\gamma_5} q \ . \tag{1.3.4}$$

The associated chiral current is defined as

$$j_5^\mu = \bar{q}\gamma_5\gamma^\mu q \ . \tag{1.3.5}$$

Because of the chiral anomaly, this rotation modifies the Lagrangian by

$$\delta\mathcal{L} = \Theta\partial_\mu j_5^\mu = -\frac{g_s^2\Theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \ , \tag{1.3.6}$$

which makes it possible to bring the CP violation to zero without any physical consequence, by simply choosing $\Theta = \Theta_{QCD}/2$ in eq. (1.3.4). However, all quarks are massive, and the chiral transformation (1.3.4) for a quark of mass m gives an additional shift to the Lagrangian

$$\delta\mathcal{L} = \partial_\mu j_5^\mu = -\frac{g_s^2\Theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} - 2i\Theta m \bar{q}\gamma_5 q \ , \tag{1.3.7}$$

so that it is only possible to transfer the CP violation from the gauge Lagrangian to the quark mass matrix or conversely. Thus, CP should be violated by strong interactions, but it is not the case. More precisely, the experimental bound on Θ_{QCD} is [73]

$$|\Theta_{QCD}| \lesssim 10^{-10} \ . \tag{1.3.8}$$

The Standard Model does not give any reason why this angle is so small. Again, this does not make the theory inconsistent, but is rather an inelegant feature.

A solution to this problem was proposed in 1977 by Roberto Peccei and Helen Quinn [74, 75]. The idea is to replace the constant angle Θ_{QCD} with a dynamical field. This field has a potential which is minimal for $\Theta_{QCD} = 0$, which means that the vacuum of the theory preserves CP. It requires the introduction of a new scalar particle called the axion. Axions can also play the role of Dark Matter candidate, giving an additional motivation to their search.

Nonzero neutrino masses

Among the remaining issues, the problem of neutrino masses comes from the fact that the Standard Model does not contain right-handed neutrinos, since for a long time they were not required to explain the phenomenology. This implies that neutrinos are massless, but the phenomenon of neutrino oscillation proves the opposite. To this day, neutrino masses are the only evidence for new physics that can be tested in lab experiments. This issue and possible solutions will be discussed in section 2.1.

The matter-antimatter asymmetry

Finally, the fact that our Universe seems to contain only matter and no antimatter is a nontrivial issue. If the discrete symmetry CP, that relates particles and antiparticles, is exact, a universe with an equal amount of matter and antimatter would not contain any structure such as stars and galaxies. This is because every particle would annihilate with its antiparticle, leaving a much smaller amount of matter than what is observed, and an equal amount of antimatter. At some point in the early universe, there must have been an imbalance between particles and antiparticles, but the question remains whether this asymmetry is just the result of the initial conditions or of a dynamical physical process. Thus, the visible energy content of the universe is actually also an issue. This will be addressed in more detail in chapter 3.

Chapter 2

The lepton sector

2.1 Neutrino masses

At this stage, one thing distinguishes neutrinos from the other fermions. Indeed, the Standard Model does not contain right-handed neutrinos, therefore neutrinos should be massless. However, the experimental observation of neutrino oscillations proves that they have tiny masses. The existence of neutrino masses raises a new question: are neutrinos Dirac particles, like the other fermions of the Standard Model, or Majorana particles, with a mass term mixing neutrinos and antineutrinos?

2.1.1 Neutrino oscillations and their interpretation

First predicted by Bruno Pontecorvo in 1957 [76], neutrino oscillations have been observed since then by several experiments looking for disappearance of neutrinos of a given flavour, or for the appearance of a new flavour in a neutrino beam.

Principle

The explanation for oscillations rely on neutrinos being massive particles [77]. Neutrinos are produced in flavour eigenstates ν_e, ν_μ or ν_τ by weak interactions such as the muon decay $\mu \to e + \nu_e + \bar{\nu}_\mu$. If these flavour eigenstates do not coincide with the mass eigenstates ν_1, ν_2 and ν_3, the propagation will induce oscillations.

To introduce the phenomenon of oscillations, it is convenient to study a simplified case where there are only two flavours, ν_e and ν_μ, related by a rotation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} , \tag{2.1.1}$$

Let us consider a neutrino of flavour e produced at a time $t = 0$

$$|\nu_e(0)\rangle = \cos\theta\, |\nu_1(0)\rangle + \sin\theta\, |\nu_2(0)\rangle \tag{2.1.2}$$

For simplicity, we consider plane waves with momentum p. A correct description should involve wave packets, but plane waves allow to grasp the main idea while being much simpler to study. The states ν_1 and ν_2 are energy eigenstates, therefore their evolution over time is given by

$$|\nu_i(t)\rangle = e^{-iE_i t} |\nu_i(0)\rangle \ , \tag{2.1.3}$$

with $E_i = \sqrt{p^2 + m_i^2} \simeq p + m_i^2/(2p)$. At a distance $L = ct$ from the source, the probability of transition to the state ν_μ is

$$\begin{aligned} P_{e\mu}(L) &= |\langle \nu_\mu | \nu_e(t) \rangle|^2 \\ &= \left| (-\sin\theta \langle \nu_1| + \cos\theta \langle \nu_2|) (\cos\theta e^{-iE_1 t} |\nu_1\rangle + \sin\theta e^{-iE_2 t} |\nu_2\rangle) \right|^2 \\ &= \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \\ &= \sin^2 2\theta \sin^2 \left(\frac{\pi L}{L_0} \right) \ , \end{aligned} \tag{2.1.4}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ is the squared mass difference, $E = p$ is the beam energy, and the characteristic oscillation scale L_0 is

$$L_0 = \frac{4\pi E}{\Delta m_{21}^2} \ . \tag{2.1.5}$$

Thus, the observation of neutrino oscillations allows to determine the squared mass splitting Δm_{ij}^2. Very far from the source ($L \gg L_0$), the transition probability can be averaged,

$$P_{e\mu}(L) \to \frac{1}{2} \sin^2 2\theta \ . \tag{2.1.6}$$

This result generalizes to any number n of flavours related by a unitary mixing matrix U,

$$|\nu_\alpha\rangle = U_{\alpha i}^* |\nu_i\rangle \ . \tag{2.1.7}$$

Note that the relation between the operators is $\nu_\alpha = U_{\alpha i}\, \nu_i$, but $|\nu_\alpha\rangle$ is created by the creation operator $\bar{\nu}_\alpha = U_{\alpha i}^*\, \bar{\nu}_i$. The probability of transition from flavour α to flavour β is

$$P_{\alpha\beta}(L) = \left| \sum_{i=1}^n U_{\alpha i} U_{\beta i}^* e^{-iE_i t} \right|^2 = \sum_{i,j=1}^n U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} e^{-i\Delta m_{ij}^2 L/(2E)} \ . \tag{2.1.8}$$

Parametrization

Let us start with general considerations. The mass terms of n Dirac fermions $\psi_\alpha = \psi_{L\alpha} + \psi_{R\alpha}$ appear in the Lagrangian as

$$\mathcal{L}_{\text{Dirac}} = -\bar{\psi}_{R\alpha} M_{\alpha\beta} \psi_{L\beta} + \text{h.c.} \ . \tag{2.1.9}$$

In the case of Majorana fermions, i.e. satisfying the Majorana condition $\psi_i = C\bar{\psi}_i^T$ [78], where C is the charge conjugation matrix, this turns into

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\psi_{L\alpha}^T C M_{\alpha\beta}\psi_{L\beta} + \text{h.c.} \ . \tag{2.1.10}$$

We bring the mass matrix to a diagonal form by mean of a unitary transformation

$$\psi_\alpha = U_{\alpha i}\psi_i \ , \tag{2.1.11}$$

so that the mass terms can be rewritten as

$$\mathcal{L}_{\text{Dirac}} = -\bar{\psi}_{R\alpha}\ U_{\alpha i} M_i U_{\beta i}^*\ \psi_{L\beta} + \text{h.c.} \ , \tag{2.1.12}$$

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\psi_{L\alpha}^T C\ U_{\alpha i}^* M_i U_{\beta i}^*\ \psi_{L\beta} + \text{h.c.} \ . \tag{2.1.13}$$

A $n \times n$ unitary matrix has generally n^2 parameters, including $n(n-1)/2$ mixing angles and $n(n+1)/2$ phases. However, in the case of Dirac fermions, $2n-1$ phases can be eliminated in redefinitions of the fields,

$$\begin{cases} \psi_{L\alpha} \to e^{i\phi_{L\alpha}}\psi_{L\alpha} \\ \psi_{R\alpha} \to e^{i\phi_{R\alpha}}\psi_{R\alpha} \end{cases} . \tag{2.1.14}$$

There are a priori $2n$ such possible rephasings, but we can see from eq. (2.1.12) that choosing the same value for all the phases $\phi_{L\alpha}$ and $\phi_{R\alpha}$ has no effect, therefore only $2n-1$ phases can be eliminated. In the case of Majorana fermions, there are only n possible rephasings, one for each $\psi_{L\alpha}$, so n phases can be eliminated and there remains $n(n-1)/2$ physical phases.

Thus, if neutrinos are Dirac fermions. the PMNS matrix should include three mixing angles and one phase. The standard parametrization is the following,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} , \tag{2.1.15}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ is the CP-violating phase, referred to as the Dirac phase to distinguish it from those arising when neutrinos are Majorana fermions.

When neutrinos are Majorana fermions, two new physical phases are present, which can be parametrized by multiplying the previous matrix by a matrix of phases, for instance

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \times \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\sigma} \end{pmatrix} . \tag{2.1.16}$$

Experimental measurements

The mixing angles of the PMNS matrix[1], as well as the squared mass differences, have been measured by a wide range of experiments looking for appearance or disappearance of flavours in neutrino beams coming from the Sun, the atmosphere or nuclear reactors. A remaining uncertainty is the ordering of neutrino masses. The squared mass splitting Δm^2_{21} is always positive, whereas Δm^2_{31} can be positive or negative. A positive value for Δm^2_{31} gives the mass ordering $m_{\nu 1} < m_{\nu 2} < m_{\nu 3}$, also called Normal Hierarchy (NH), whereas a negative value gives $m_{\nu 3} < m_{\nu 1} < m_{\nu 2}$, referred to as the Inverted Hierarchy (IH). The solar and atmospheric mass $m_{\nu\text{sun}}$ and $m_{\nu\text{atm}}$ are often defined as

$$m_{\nu\text{sun}} = \sqrt{\Delta m^2_{21}} \ , \qquad m_{\nu\text{atm}} = \sqrt{|\Delta m^2_{31}|} \ . \tag{2.1.17}$$

Let us review briefly some of the key experiments that measured the oscillation parameters of neutrinos.

(i) The first evidence of neutrino oscillations was found in 1998 by Super-Kamiokande [79]. The Super-Kamiokande experiment, looking for up-going atmospheric muon neutrinos, found missing events in $\nu_\mu \to \nu_\mu$. The survival probability in this configuration can be approximated by

$$P_{\nu_\mu \to \nu_\mu} \simeq \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{32} L}{4E} \right) \ . \tag{2.1.18}$$

MINOS [80–84] also looked for the disappearance of muon neutrinos, but in a beam produced by an accelerator. These two kinds of experiments are complementary, since atmospheric neutrino experiments give the most precise measurement of the mixing angle θ_{23}, whereas accelerator neutrino experiments allow to measure the splitting Δm^2_{32} with a greater precision.

(ii) The CHOOZ experiment [85], looking at a beam of antineutrinos produced by a nuclear reactor, did not detect disappearance in $\bar{\nu}_e \to \bar{\nu}_e$. For this experiment, the probability of survival can be approximated by

$$P_{\nu_e \to \nu_e} \simeq 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m^2_{31} L}{4E} \right) \ , \tag{2.1.19}$$

and therefore CHOOZ was able to set an upper bound on θ_{13}. The angle θ_{13}, which until recently was thought to be very small, was finally measured by the experiments Daya Bay [86] and Double Chooz [87]. The measured value differs significantly enough from zero,

$$\sin^2 2\theta_{13} \sim 0.1 \ . \tag{2.1.20}$$

(iii) For solar neutrino experiments, both vacuum and matter oscillations play a role. Given that $L \gg 4E/\Delta m^2_{21}$, the survival probability can be approximated by

[1] More precisely, the parameters measured are the $\sin^2 2\theta_{ij}$.

Parameter	Experimental value
Δm^2_{21}	$(7.59 \pm 0.185) \times 10^{-5}$ eV^2
Δm^2_{31}	$(2.47 \pm 0.07) \times 10^{-3}$ eV^2 for Normal Hierarchy (NH)
Δm^2_{32}	$\left(-2.43 \begin{array}{c} +0.042 \\ -0.065 \end{array}\right) \times 10^{-3}$ eV^2 for Inverted Hierarchy (IH)
$\sin^2\theta_{12}$	0.30 ± 0.013
$\sin^2\theta_{23}$	$\left(0.41 \begin{array}{c} +0.037 \\ -0.025 \end{array}\right)$ for the first octant $0.59 \quad \pm 0.022$ for the second octant
$\sin^2\theta_{13}$	0.023 ± 0.0023
δ	$\left(300 \begin{array}{c} +66 \\ -138 \end{array}\right)^\circ$

Table 2.1: Physical parameters responsible for neutrino oscillations. Values are taken from ref. [95].

the following formula as long as the interaction of neutrinos with matter can be neglected,

$$P_{\nu_e \to \nu_e} \simeq 1 - \frac{1}{2} \sin^2 2\theta_{12} \ . \tag{2.1.21}$$

In the opposite case (relevant for SNO [88] and Super-Kamiokande [89]), the MSW effect describing oscillations of neutrinos in matter [90, 91] should be taken into account, which gives

$$P^{\text{matter}}_{\nu_e \to \nu_e} \simeq \sin^2\theta_{12} \ . \tag{2.1.22}$$

KamLAND [92–94] studied disappearance of electron antineutrinos produced in surrounding nuclear reactors

$$P_{\bar{\nu}_e \to \bar{\nu}_e} \simeq c^2_{13} \left(1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{21} L}{4E}\right) \tag{2.1.23}$$

Together, these experiments gave measurements of the mixing angle θ_{12} and the square mass splitting Δm^2_{21}.

The measured values of the parameters are summarized in table 2.1. The Dirac phase δ is still unknown, as is the absolute scale of neutrino masses, defined for instance by the mass of the lightest neutrino, which cannot be determined by oscillation experiments. Cosmology, on the other hand, gives an upper bound on the sum of neutrino masses.

We should signal that one experimental observation could allow us to discriminate between Dirac and Majorana neutrinos without any ambiguity. Some unstable nuclei experience double beta decay. If neutrinos are Dirac fermions, double beta decay is just a superposition of two simultaneous ordinary beta decays [96], i.e.

$$^A_Z N \to \, ^{\;\;A}_{Z+2} N + 2e^- + 2\bar{\nu}_e \ . \tag{2.1.24}$$

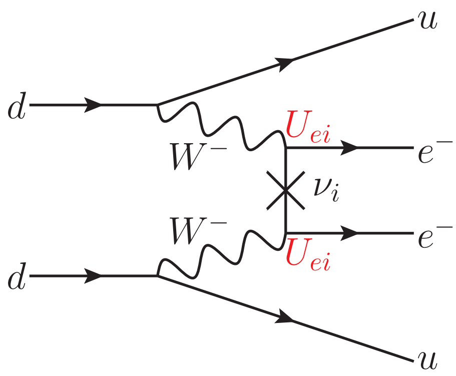


Figure 2.1: Neutrinoless double beta decay. The cross denotes a Majorana mass insertion.

If neutrinos are Majorana fermions, there is another possibility which is neutrinoless double beta decay [97],

$$ {}^{A}_{Z}N \to {}^{\;\;\;A}_{Z+2}N + 2e^- \ . \tag{2.1.25}$$

The principle of this phenomenon is shown in fig. 2.1. Neutrinoless double beta decay clearly violates lepton number by two units and is a characteristic signature of Majorana neutrinos. More interestingly, its rate is related to the entry $m_{\nu ee}$ of the neutrino mass matrix, also called in this context $m_{\beta\beta}$: the inverse half-life of the isotope is proportional to $|m_{\nu ee}|^2$,

$$\left(T^{0\nu}_{1/2}\right)^{-1} \propto \Gamma \propto |m_{\nu ee}|^2 \ . \tag{2.1.26}$$

The observation of neutrinoless double beta decay would therefore be an unambiguous proof that neutrinos are Majorana fermions, and would give us access to an entry of the mass matrix which is related in some sense to the absolute neutrino mass scale (although not in a simple way). The current best bound on the half-life, given by the KamLAND-Zen experiment [98], is

$$T^{0\nu}_{1/2} > 3.4 \times 10^{25} \text{ years} \ , \tag{2.1.27}$$

which in turns gives the following upper bound on the matrix element

$$|m_{\nu ee}| < (120 - 250) \,\text{meV} \ . \tag{2.1.28}$$

Unfortunately, the opposite is not true. The non-observation of neutrinoless double beta decay does not necessarily mean that neutrinos are Dirac fermions, they could be instead Majorana fermions but with a vanishingly small matrix element $|m_{\nu ee}|$. In fact, no experiment could tell us for sure that neutrinos are Dirac particles.

Anomalies

Some neutrino experiments reported results that are in tension with theoretical predictions. These anomalies could point towards a more complex neutrino sector, including more than the three neutrinos of Standard Model. Indeed, one of the possible explanations is the existence of additional species of neutrinos. If this is the case, these new species cannot interact through weak interactions, because otherwise the Z boson would have new decay channels of the kind $Z^0 \to \nu\bar{\nu}$, and its width would be larger than what is measured. Such light particles that do not interact weakly are referred to as sterile neutrinos. Here is a short review of these anomalies.

(i) The LSND experiment looked for $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions [99], while MiniBooNE looked for both $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ [100–103]. The results of LSND indicate an excess of events that cannot be satisfactorily explained. The antineutrino data from MiniBooNE is compatible with LSND, but the neutrino data do not exhibit the same excess. If $\bar{\nu}_e$ appearance was due to a new oscillation, the new squared mass splitting and mixing angle should satisfy $\Delta m^2 \lesssim 1$ eV^2 and $\sin^2 2\theta \sim 10^{-2}$ [104].

(ii) The GALLEX and SAGE experiments use gallium to detect solar neutrinos, thanks to the following reaction,

$$\nu_e +^{71}\mathrm{Ga} \to^{71}\mathrm{Ge} + e^- \ . \tag{2.1.29}$$

These experiments were calibrated with radioactive sources. The results obtained with these sources and the theoretical predictions are in disagreement: the number of events is smaller than expected. For instance, the two GALLEX experiments found the following ratio [105],

$$r = 0.882 \pm 0.078 \ . \tag{2.1.30}$$

This is known as the "Gallium anomaly" [106, 107]. This disappearance of $\bar{\nu}_e$ could be explained by a new oscillation, with a squared mass splitting $\Delta m^2 \gtrsim 1$ eV^2. For instance, ref [107] gives a best-fit point at $\Delta m^2 = 2.24$ eV^2.

(iii) After a recalculation of the $\bar{\nu}_e$ flux from reactors [108–110], it appears that all short baseline experiments detecting reactor neutrinos seem to have measured less events than expected. This is known as the "reactor anomaly". Again, the disappearance of $\bar{\nu}_e$ could be explained by a new oscillation, with a squared mass splitting Δm^2 around 1 eV^2. However, it was recently noticed that the shape of the spectra measured at Daya Bay [86], RENO [111] and Double CHOOZ [87] are not consistent with predictions [112]. This could be unrelated to the reactor anomaly, but it could also mean that both inconsistencies are due to little-known effects coming from reactor physics. Thus, new experiments would be needed to conclude on the origin of the anomaly.

The interpretation of the LSND/MiniBooNE results in terms of sterile neutrinos is in tension with other experimental data [104]. On the other hand, light sterile neutrinos could still be an explanation for the Gallium and reactor anomalies. Such particles would also play a role in cosmology, because if they were thermalized in the early universe, they would contribute to the effective number of neutrinos N_{eff} and to the sum of neutrino masses, which are constrained by the Planck experiment [113],

$$N_{\text{eff}} = 3.15 \pm 0.23 \ , \tag{2.1.31}$$

$$\sum m_\nu < 0.23 \ \text{ eV} \ . \tag{2.1.32}$$

Such results are consistent with the existence of three neutrino family, and the explanation of neutrino anomalies in terms of sterile neutrinos could be in tension with the ΛCDM model of cosmology.

2.1.2 The theoretical puzzle

Until now, we have been reviewing the phenomenology of neutrino oscillations without paying attention to the problems raised by nonzero neutrino masses. Indeed, neutrino masses lead to two questions: how are they generated, and why are they so tiny in comparison with the other fermions? Another issue, which is related to the previous ones, is to know whether neutrinos are Dirac or Majorana particles.

Dirac or Majorana?

At first sight, it seems that it is possible to answer the first question without bringing significant modifications to the Standard Model. Indeed, a first guess would be to introduce precisely what is needed to write Dirac mass terms, i.e. three right-handed neutrinos

$$\mathcal{L}_{\text{Dirac}} = -m^\nu_{\alpha\beta}\, \bar{\nu}_{R\alpha}\nu_{L\beta} + \text{h.c.} \ . \tag{2.1.33}$$

This mass term can be generated after the electroweak symmetry breaking just like those of the other fermions. More fundamentally, it derives from the $SU(2)$-invariant Yukawa coupling

$$\mathcal{L}_{\text{Yukawa}} = -y^\nu_{\alpha\beta}\, \bar{\nu}_{R\alpha} H^T i\sigma_2 \ell_\beta + \text{h.c.} \ , \tag{2.1.34}$$

The neutrino mass matrix is related to the Yukawa coupling and the Higgs v.e.v. by

$$m^\nu_{\alpha\beta} = y^\nu_{\alpha\beta} v \ . \tag{2.1.35}$$

This solution appears clearly as the most simple choice, since it reproduces exactly the same pattern as for the other fermions. Right-handed neutrinos being Standard Model singlets, they are sterile and therefore their introduction does bring any new phenomenology. However, this solution does not answer our second question. The smallness of neutrino masses just follows from the smallness of the Yukawa coupling $y^\nu_{\alpha\beta}$, that has to be at least five order of magnitude smaller than for instance the electron Yukawa y_e, without further explanation. Moreover, since right-handed neutrinos are Standard Model singlets, nothing forbids in principle the following Majorana mass term,

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} m_{R\alpha\beta}\, \nu_{R\alpha}^T C \nu_{R\beta} + \text{h.c.} \ , \tag{2.1.36}$$

where C is the charge conjugation matrix and right-handed neutrinos satisfy the Majorana condition $\nu_R = C\bar{\nu}_R^T$. Contrary to the usual fermion mass terms in the Standard Model, this one is in no way related to the electroweak symmetry breaking, which means that it could be arbitrarily large. In particular, it could be much larger than the electroweak symmetry breaking scale. Another feature of this mass term is that it necessarily breaks lepton number: if we want the Yukawa couplings of neutrinos to conserve lepton number, we should give a lepton number $L = +1$ to right-handed neutrinos, but then the Majorana mass term has a lepton number $L = +2$, breaking lepton number by two units. The seesaw mechanism of type I, imagined by Peter Minkowski [114], exploits precisely this idea.

Alternatively, left-handed neutrinos being the only electrically neutral fermions in the Standard Model, they could also be Majorana particles, satisfying the condition $\nu_L = C\bar{\nu}_L^T$. Their low-energy mass term would be

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} m_{\nu\alpha\beta}\, \nu_{L\alpha}^T C \nu_{L\beta} + \text{h.c.}\ . \tag{2.1.37}$$

Again, this mass term clearly violates the global lepton number, but this is not a problem since lepton number is only an accidental symmetry of the Standard Model. However, since left-handed neutrinos belong to $SU(2)$ doublets, this mass term is not gauge-invariant, and therefore can only be valid in the low energy effective theory, where the electroweak symmetry is broken. Trying to make it invariant at higher energies, where the electroweak symmetry is restored, is a problem similar to explaining for instance how the electron mass can be consistent with the fact that the left- and right-handed electron belong to different representations. For the electron, this was solved by the Higgs mechanism and the introduction of Yukawa couplings. However, the solution is not so simple here, since it is impossible to write a Yukawa coupling able to generate the Majorana mass of eq. (2.1.37). It turns out that the lowest-dimensional operator involving only Standard Model fields that can give rise to this mass term is the Weinberg operator [115],

$$\mathcal{L}_{\text{Weinberg}} = \frac{1}{2} \frac{\kappa_{\alpha\beta}}{\Lambda} (\ell_\alpha^T i\sigma_2 H) C (H^T i\sigma_2 \ell_\beta)\ . \tag{2.1.38}$$

This operator has dimension 5 and is suppressed by the scale Λ at which it should be generated by new physics. This would explain why neutrino masses are so small, namely because they scale like v^2/Λ, with $v \ll \Lambda$. On the other hand, the origin of this operator still requires an explanation.

Finally, focusing for the moment on the low energy theory, the most general mass term for neutrinos, involving both the three Standard Model left-handed neutrinos and n right-handed neutrinos, is

$$\mathcal{L}_{\text{mix}} = -m_{i\beta}^D\, \bar{\nu}_{Ri} \nu_{L\beta} - \frac{1}{2} m_{\alpha\beta}^L\, \nu_{L\alpha}^T C \nu_{L\beta} - \frac{1}{2} m_{ij}^R\, \nu_{Ri}^T C \nu_{Rj} + \text{h.c.}\ , \tag{2.1.39}$$

where the subscripts α and β run over the three flavours e, μ and τ, and the subscripts i and j run over right-handed neutrinos. We can rewrite this as a single $(3+n)\times(3+n)$ Majorana mass matrix

$$\mathcal{L}_{\text{mix}} = -\frac{1}{2} m_{AB}\, {\nu'_A}^T C \nu'_B + \text{h.c.}\ , \tag{2.1.40}$$

where the indices A and B run from 1 to $3+n$ and

$$\nu'_A = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \bar{\nu}_1^T \\ \vdots \\ \bar{\nu}_n^T \end{pmatrix} \;, \quad m_{AB} = \begin{pmatrix} m^L & \left(m^D\right)^T \\ m^D & m^R \end{pmatrix} \;. \tag{2.1.41}$$

Let us review some limiting cases.

(i) If $m^L = m^R = 0$, we recover our first option, with purely Dirac neutrinos.

(ii) If $m^D = 0$, the Standard Model neutrinos are Majorana particles. In this case, the presence of right-handed neutrinos, if they exist, is physically irrelevant because they do not interact at all with the Standard Model, not even through their mass term. If the Dirac mass term is small but non-vanishing, mass eigenstates are mixtures of active and sterile neutrinos.

(iii) Finally, $m^L = 0$ and $m^D \ll m^R$ leads to the seesaw mechanism of type I.

The seesaw mechanism of type I

In this scenario, the smallness of neutrino masses is related to the magnitude of the Majorana mass of right-handed neutrinos, without requiring a significant suppression of the Yukawa coupling. From now on, we label the heavy Majorana neutrinos N_i to distinguish them from the Standard Model neutrinos. There could be in principle an arbitrary number of these particles, but, as we will see, at least two of them are needed to match the observations on neutrino oscillations. Let us call n the number of Majorana neutrinos. Assuming that their mass matrix is diagonal, i.e. $M = \mathrm{diag}(M_1,...,M_n)$ (it is always possible to perform a change of basis to recover this situation), the seesaw Lagrangian reads

$$\mathcal{L}_{\text{seesaw I}} = -y^\nu_{i\alpha}\,\bar{N}_i \ell^T_\alpha i\sigma_2 H - \frac{1}{2} M_i\, N_i^T C N_i + \text{h.c.}\,, \tag{2.1.42}$$

which, after the electroweak symmetry breaking, becomes

$$\mathcal{L}_{\text{seesaw I}} = -m^D_{i\alpha}\,\bar{N}_i \nu_\alpha - \frac{1}{2} M_i\, N_i^T C N_i + \text{h.c.}\,, \tag{2.1.43}$$

where $m^D_{i\beta} = y^\nu_{i\beta}\, v$. We recover the Lagrangian of eq. (2.1.39) in the situation $m^L = 0$. Again, it is possible to rewrite this as a single $(3+n)\times(3+n)$ Majorana mass

$$\mathcal{L}_{\text{seesaw I}} = -\frac{1}{2} m_{AB}\, {\nu'_A}^T C \nu'_B + \text{h.c.}\,, \tag{2.1.44}$$

where

$$\nu'_A = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \bar{N}_1^T \\ . \\ . \\ . \\ \bar{N}_n^T \end{pmatrix} , \quad m_{AB} = \begin{pmatrix} 0 & \left(m^D\right)^T \\ m^D & M \end{pmatrix} . \tag{2.1.45}$$

Bringing this matrix to a block diagonal form, it turns out that there are n heavy mass eigenstates, which are approximately the heavy neutrinos N_i with masses M_i, and three light states corresponding to the three Standard Model neutrinos, with a 3×3 Majorana mass matrix

$$m_\nu \simeq \left(m^D\right)^T M^{-1} m^D . \tag{2.1.46}$$

Neutrino oscillation experiments prove that at least two of the Standard Model neutrinos are massive, which implies that the mass matrix m_ν should have at least rank 2, for which at least two heavy neutrinos are required. In the most commonly studied case, there are exactly three heavy neutrinos: this is the minimal content needed to give nonzero masses to the three Standard Model neutrinos, and moreover, in some Grand Unified Theories, there is a right-handed neutrino included in each fermion generation.

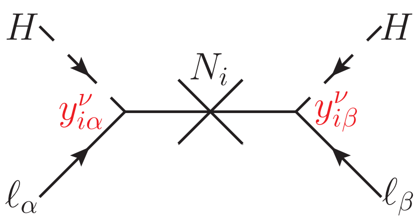


Figure 2.2: Diagram giving rise to the Weinberg operator in the type I seesaw. The cross denotes a chirality flip due to a mass insertion.

Another way to look at the seesaw mechanism is to study the effective theory obtained by integrating out the heavy right-handed neutrinos. First, let us consider the diagram of fig. 2.2. At low energy, after integrating out the right-handed neutrinos, this generates precisely the Weinberg operator

$$\mathcal{L}_{\text{Weinberg}} = \frac{1}{2} \sum_i \frac{y^\nu_{i\alpha} y^\nu_{i\beta}}{M_i} (\ell^T_\alpha i\sigma_2 H) C (H^T i\sigma_2 \ell_\beta) . \tag{2.1.47}$$

This allows us to recover the result obtained by diagonalizing the $(3+n) \times (3+n)$ mass matrix

$$m_{\nu\alpha\beta} = \sum_i \frac{y^\nu_{i\alpha} y^\nu_{i\beta}}{M_i} v^2 . \tag{2.1.48}$$

If we want the coupling $y^\nu_{i\alpha}$ to be of order 1, and for hierarchical neutrinos ($m_{\nu 1} \ll m_{\nu 2} \sim m_{\rm sun}$, $m_{\nu 3} \sim m_{\rm atm}$), the right-handed neutrinos should have masses of the order of 10^{15} GeV, which is not very far from the GUT scale. This can be an additional motivation for implementing this scenario in a grand unified framework. Of course, this is not strictly required and, even with lighter right-handed neutrinos, we can recover the right order of magnitude without extremely small values for y^ν.

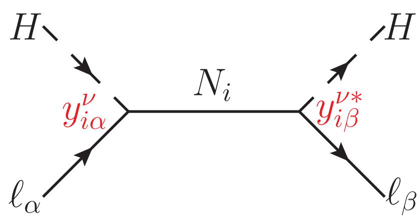


Figure 2.3: Diagram giving rise to the dimension 6 operator in the type I seesaw.

In addition to the Weinberg operator, the type I seesaw gives rise to a unique dimension 6 operator, which comes from the diagram displayed in fig. 2.3 [116],

$$\mathcal{L}_{d=6} = \sum_i \frac{(y^\nu_{i\alpha})^* \, y^\nu_{i\beta}}{M_i^2} (\bar{\ell}_\alpha i\sigma_2 H^*) \, \partial\!\!\!/ (H^T i\sigma_2 \ell_\beta) \ , \tag{2.1.49}$$

which, after the electroweak symmetry breaking, is nothing but a correction to the kinetic term of the neutrinos, which becomes

$$\mathcal{L}_{\nu,\,\rm kin} = i\bar{\nu}_{L\alpha} \left(\delta_{\alpha\beta} + \epsilon^N_{\alpha\beta}\right) \partial\!\!\!/ \, \nu_{L\beta} \ , \tag{2.1.50}$$

$$\epsilon^N_{\alpha\beta} = \frac{v^2}{2} \sum_i \frac{(y^\nu_{i\alpha})^* \, y^\nu_{i\beta}}{M_i^2} \ . \tag{2.1.51}$$

To normalize canonically the kinetic term of neutrinos, we have to do the following rescaling (given at the order M^{-2}),

$$\nu_{L\alpha} \longrightarrow \nu'_{L\alpha} \simeq \left(\delta_{\alpha\beta} + \frac{1}{2}\epsilon^N_{\alpha\beta}\right) \nu_{L\beta} \tag{2.1.52}$$

Because of this, the charged current involving leptons becomes

$$j^\mu_{{\rm CC},\,\ell} = \sum_{\alpha,\beta} \bar{\nu}_{L\alpha} \left(\delta_{\alpha\beta} - \frac{1}{2}\epsilon^N_{\alpha\beta}\right) \gamma^\mu e_{L\beta} = \sum_{\alpha,\,i} \bar{\nu}_{Li} U^*_{\alpha i} \left(\delta_{\alpha\beta} - \frac{1}{2}\epsilon^N_{\alpha\beta}\right) \gamma^\mu e_{L\beta} \ , \tag{2.1.53}$$

which means that the PMNS matrix is replaced with a non-unitary mixing matrix,

$$N = \left(1 - \frac{1}{2}\epsilon^N\right) U \ . \tag{2.1.54}$$

Because of this non-unitarity ($NN^\dagger = 1 - \epsilon^N$, $N^\dagger N = 1 - U^\dagger \epsilon^N U$), there are now flavour-changing neutral currents in the neutrino sector,

$$j^\mu_{{\rm NC},\,\nu} = \sum_\alpha \bar{\nu}_{Li} \left(N^\dagger N\right)_{ij} \gamma^\mu \, \nu_{Lj} \ . \tag{2.1.55}$$

However, these effects are generally very suppressed because they arise only at the order $\mathcal{O}(1/M^2)$, therefore they lead to observable effects only for low-scale seesaw, with right-handed neutrinos not too far from the TeV scale. Such a low scale for right-handed neutrinos brings back the issue of explaining the smallness of neutrino masses.

Type II seesaw

Pursuing the idea of Majorana neutrinos, another possible strategy is to look for models able to generate the Weinberg operator. This is indeed the lowest-dimensional operator that generates a coupling of neutrinos to the vacuum expectation value of the Higgs field, and therefore the simplest way to give a Majorana mass to neutrinos. In addition to the type I seesaw, two extensions of the Standard Model are able to generate this operator at tree-level with a simple additional content. The type II seesaw [117–120] relies on the introduction of a scalar electroweak triplet Δ with hypercharge $Y = 1$. Unless mentioned otherwise, we use the following parametrization for Δ,

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} . \tag{2.1.56}$$

Besides its kinetic term, it gives the following contribution to the Lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{seesaw II}} = &-\frac{1}{2}\left(f_{\alpha\beta}\ell_\alpha^T C i\sigma_2 \Delta \ell_\beta + \mu H^T i\sigma_2 \Delta^\dagger H + \text{h.c.}\right) \\ &- M_\Delta^2 \,\text{tr}(\Delta^\dagger\Delta) - \delta V\left(\Delta, H\right) \ , \end{aligned} \tag{2.1.57}$$

where we separated the terms which are directly involved in the generation of neutrino masses from the others, which are gathered in the additional scalar potential δV. δV can in principle contain the following couplings,

$$\begin{aligned} \delta V\left(\Delta, H\right) = &\frac{\lambda_2}{2}\left[\text{tr}(\Delta^\dagger\Delta)\right]^2 + \lambda_3 H^\dagger H \,\text{tr}(\Delta^\dagger\Delta) \\ &+ \sum_{i=1}^{3}\left\{\frac{\lambda_4}{2}\left(\vec{\Delta}^\dagger T_i \vec{\Delta}\right)^2 + \lambda_5\left(\vec{\Delta}^\dagger T_i \vec{\Delta}\right)\left(H^\dagger \sigma_i H\right)\right\} \ , \end{aligned} \tag{2.1.58}$$

where the T_i are the dimension-3 representations of the $SU(3)$ generators, and we used the notation

$$\vec{\Delta} = \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} (\Delta^{++} + \Delta^0)/\sqrt{2} \\ i(\Delta^{++} - \Delta^0)/\sqrt{2} \\ \Delta^+ \end{pmatrix} . \tag{2.1.59}$$

In this framework, the Weinberg operator comes from the diagram of fig. 2.4. It can be expressed in terms of the parameters as

$$\mathcal{L}_{\text{Weinberg}} = \frac{1}{4}\frac{\mu f_{\alpha\beta}}{M_\Delta^2}(\ell_\alpha^T i\sigma_2 H) C (H^T i\sigma_2 \ell_\beta) \ , \tag{2.1.60}$$

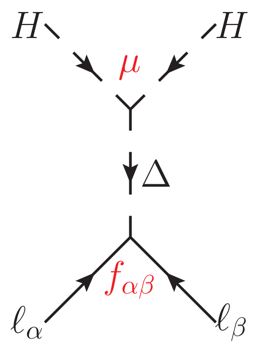


Figure 2.4: Diagram giving rise to the Weinberg operator in the type II seesaw.

giving for the neutrino mass matrix (after reabsorbing a minus sign with the phases)

$$m_{\nu\alpha\beta} = \frac{1}{2}\mu f_{\alpha\beta}\frac{v^2}{M_\Delta^2} \ . \tag{2.1.61}$$

The simple proportionality relation between the neutrino mass matrix and the coupling matrix f is a distinctive feature of this model, unlike in the type I seesaw where the knowledge of the neutrino mass matrix would not give much information about the Yukawa coupling.

Integrating out the triplet also gives rise to an effective dimension-4 operator that generates a correction to the quartic Higgs coupling,

$$\delta\mathcal{L}_{d=4} = \frac{|\mu|^2}{4M_\Delta^2}(H^\dagger H)^2 \ , \tag{2.1.62}$$

as well as three dimension-6 operators [121, 122], summarized in

$$\mathcal{L}_{d=6} = \frac{1}{4}\frac{f_{\alpha\beta}f^*_{\gamma\delta}}{M_\Delta^2}\left(\ell_\alpha^T C i\sigma_2\vec{\sigma}\ell_\beta\right).\left(\bar{\ell}_\delta\vec{\sigma}i\sigma_2 C\bar{\ell}_\gamma^T\right) - 2(\lambda_3+\lambda_5)\frac{|\mu|^2}{M_\Delta^4}(H^\dagger H)^3 \\ + \frac{|\mu|^2}{4M_\Delta^2}\left(H^\dagger\vec{\sigma}\tilde{H}\right)\left(\overleftarrow{D}_\mu \overrightarrow{D}^\mu\right)\left(\tilde{H}^\dagger\vec{\sigma}H\right) \ , \tag{2.1.63}$$

with $\tilde{H} = i\sigma_2 H^*$. Contrary to the type I seesaw, there is no deviation from unitarity in the lepton mixing matrix, but the first term in eq. (2.1.63) contributes to lepton flavour violation in the charged sector.

Type III seesaw

The third way to generate the Weinberg operator at tree-level is to add fermionic triplets Σ_i with hypercharge $Y = 0$, which are self-conjugate [123]. The type III seesaw, as it is known, is by many aspects very similar to the type I. The Lagrangian is

$$\mathcal{L}_{\text{seesaw III}} = -y^\Sigma_{i\alpha}\,\bar{\vec{\Sigma}}_i.(H\vec{\sigma}\ell_\alpha) - \frac{1}{2}M_i\,(\vec{\Sigma}_i^T C).\vec{\Sigma}_i + \text{h.c.} \ , \tag{2.1.64}$$

with $\vec{\Sigma}_i$ defined as

$$\vec{\Sigma}_i = \begin{pmatrix} (\Sigma_i^- + \Sigma_i^+)/\sqrt{2} \\ i(\Sigma_i^+ - \Sigma_i^0)/\sqrt{2} \\ \Sigma_i^0 \end{pmatrix} \ . \tag{2.1.65}$$

Like for the type I seesaw, at least two fermionic triplets are needed to explain the phenomenology of neutrino oscillations. The Weinberg operator is generated in a way

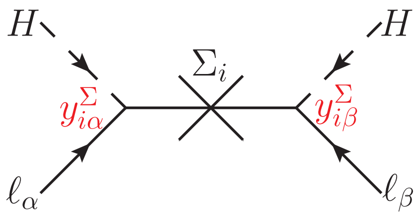


Figure 2.5: Diagram giving rise to the Weinberg operator in the type III seesaw.

which is very similar to the type I seesaw (see fig. 2.5), as is its expression in terms of the parameters,

$$\mathcal{L}_{\text{Weinberg}} = \frac{1}{2} \sum_i \frac{y^{\Sigma}_{i\alpha} y^{\Sigma}_{i\beta}}{M_i} (\ell^T_\alpha i\sigma_2 H) C (H^T i\sigma_2 \ell_\beta) \ , \tag{2.1.66}$$

while differences appear only in higher order corrections.

Like the type I seesaw, the type III gives rise to a unique dimension 6 operator, which is again very similar to the one of eq. (2.1.49) [122],

$$\mathcal{L}_{d=6} = \sum_i \frac{(y^{\Sigma}_{i\alpha})^* \, y^{\Sigma}_{i\beta}}{M_i^2} (\bar{\ell}_\alpha i\sigma_2 H^*) \, \not{D} (H^T i\sigma_2 \ell_\beta) \ . \tag{2.1.67}$$

The only difference is that this time, it involves a covariant derivative, which, after the electroweak symmetry breaking, gives corrections to the kinetic terms of both neutrinos and charged leptons,

$$\mathcal{L}_{\ell,\,\text{kin}} = i\bar{\nu}_{L\alpha} \left(\delta_{\alpha\beta} + \epsilon^{\Sigma}_{\alpha\beta}\right) \not{\partial} \, \nu_{L\beta} + i\bar{e}_{L\alpha} \left(\delta_{\alpha\beta} + 2\epsilon^{\Sigma}_{\alpha\beta}\right) \not{\partial} \, e_{L\beta} \ , \tag{2.1.68}$$

as well as to their couplings to the $SU(2)_L$ bosons,

$$\begin{aligned} \mathcal{L}_{\ell W} = \sum_{\alpha,\beta} \Bigg\{ & \frac{g}{\sqrt{2}} \left[\bar{\nu}_{L\alpha} \left(\delta_{\alpha\beta} + 2\epsilon^{\Sigma}_{\alpha\beta}\right) \not{W}^+ e_{L\beta} + \text{h.c.}\right] \\ & - \frac{g}{2} \bar{e}_{L\alpha} \left(\delta_{\alpha\beta} + 4\epsilon^{\Sigma}_{\alpha\beta}\right) \not{W}^3 e_{L\beta} \Bigg\} + \sum_\alpha \frac{g}{2} \bar{\nu}_{L\alpha} \not{W}^3 \nu_{L\alpha} \ . \end{aligned} \tag{2.1.69}$$

Now, to normalize canonically the kinetic terms of left-handed leptons, we have to do the following rescalings,

$$\nu_{L\alpha} \longrightarrow \nu'_{L\alpha} \simeq \left(\delta_{\alpha\beta} + \frac{1}{2}\epsilon^{\Sigma}_{\alpha\beta}\right) \nu_{L\beta} \ , \tag{2.1.70}$$

$$e_{L\alpha} \longrightarrow e'_{L\alpha} \simeq \left(\delta_{\alpha\beta} + \epsilon^{\Sigma}_{\alpha\beta}\right) e_{L\beta} \tag{2.1.71}$$

The main consequences of this are again the non-unitarity of the lepton mixing matrix, which is now given in terms of the PMNS matrix by

$$N = \left(1 + \frac{1}{2}\epsilon^{\Sigma}\right) U \ , \tag{2.1.72}$$

and the apparition of flavour-changing neutral currents for both neutrinos and charged leptons,

$$j^{\mu}_{\mathrm{NC},\,\ell} = \sum_{i,j} \bar{\nu}_{Li} \left[\left(N^{\dagger}N\right)^{-1}\right]_{ij} \gamma^{\mu}\, \nu_{Lj} + \sum_{\alpha,\beta} \bar{e}_{L\alpha} \left[\left(NN^{\dagger}\right)^{2}\right]_{\alpha\beta} \gamma^{\mu}\, e_{L\beta} \ . \tag{2.1.73}$$

Again, these effects are generally too small to be observable, unless the fermionic triplet are relatively light.

2.2 Charged lepton flavour violation

Because of nonzero neutrino masses and neutrino oscillations, the family lepton numbers are actually violated. However, this violation is undetectable in the Standard Model minimally extended to accommodate neutrino masses. If they were observed, flavour-violating processes involving charged leptons would therefore offer an unmistakable proof of the existence of new physics.

2.2.1 The Glashow-Iliopoulos-Maiani mechanism

The Glashow-Iliopoulos-Maiani (GIM) mechanism [53] was first introduced in 1970 to explain the absence of flavour-changing neutral currents in the quark sector. At the time, only the up, down and strange quarks had been discovered. With this particle content, the charged current involving quarks reads

$$j^{\mu}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \bar{u}_L \gamma^{\mu} \left(\cos\theta_C d_L + \sin\theta_C s_L\right) \ , \tag{2.2.1}$$

where θ_C is the Cabibbo angle. Problems arise when trying to include these three quarks in the gauge theory for the electroweak interaction based on $SU(2) \times U(1)$. Gathering left-handed quarks in an $SU(2)$ multiplet $q_L^T = (u_L^T, d_L^T, s_L^T)$, the charged current can be rewritten as[2]

$$j^{\mu}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} \bar{q}_L \gamma^{\mu} T^{+} q_L \ , \tag{2.2.2}$$

with

$$T^{+} = \begin{pmatrix} 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ . \tag{2.2.3}$$

[2] We follow here the original reasoning as summarized in ref. [124].

T^+ can be interpreted as a raising operator for $SU(2)$, while the corresponding lowering operator is $T^- = (T^+)^\dagger$. These can also be expressed in terms of the $SU(2)$ generators as

$$T^+ = T_1 + iT_2, \qquad T^- = T_1 - iT_2 \ . \tag{2.2.4}$$

Consequently, the third generator is given by

$$T_3 = \frac{i}{2}\left[T_1, T_2\right] = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\cos^2\theta & -\cos\theta\sin\theta \\ 0 & -\cos\theta\sin\theta & -\sin^2\theta \end{pmatrix} \ . \tag{2.2.5}$$

The Z^0 boson couples to a neutral current which is a combination of the (flavour-conserving) electromagnetic current and the current associated to T_3,

$$J_3^\mu = -\frac{g}{\sqrt{2}}\bar{q}_L\gamma^\mu T_3 q_L \ , \tag{2.2.6}$$

and therefore its interactions should violate flavour at tree-level, in contradiction with the experimental data. The solution was to introduce a fourth quark, the charm, that couples to the orthogonal combination $-\sin\theta d_L + \cos\theta s_L$. On can enlarge the previous reasoning to include the fourth quark, with $q_L^T = (u_L^T, c_L^T\, d_L^T, s_L^T)$ and

$$T^+ = \begin{pmatrix} 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \ . \tag{2.2.7}$$

This leads to

$$T_3 = \begin{pmatrix} 1_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & -1_{2\times 2} \end{pmatrix} \ , \tag{2.2.8}$$

so that there is no flavour-changing neutral current anymore.

The GIM mechanism can also be applied to the lepton sector. First, we consider the minimal extension of the Standard Model with Dirac neutrinos. The leptonic part of the charged current of eq. (1.2.29) can be written as

$$j^\mu_{\mathrm{CC},\,\ell} = \sum_\alpha \bar{\nu}_{L\alpha}\gamma^\mu e_{L\alpha} = \sum_{\alpha,\,i} \bar{\nu}_{Li} U^*_{\alpha i}\gamma^\mu e_{L\alpha} \ , \tag{2.2.9}$$

which means that the family lepton numbers are violated. The neutral current involving neutrinos remains diagonal

$$j^\mu_{\mathrm{NC},\,\nu} = \sum_{i,\,j,\,\alpha} \bar{\nu}_{Li} U^*_{\alpha i} U_{\alpha j}\gamma^\mu \nu_{Lj} = \sum_i \bar{\nu}_{Li}\gamma^\mu \nu_{Li} \ , \tag{2.2.10}$$

because the unitarity of the PMNS matrix implies that $U^*_{\alpha i}U_{\alpha j} = \delta_{ij}$, and so there is no flavour-changing neutral current. This means that charged lepton flavour violation can happen at higher order. Indeed, nothing forbids processes such as $\mu^\pm \to e^\pm + \gamma$ or

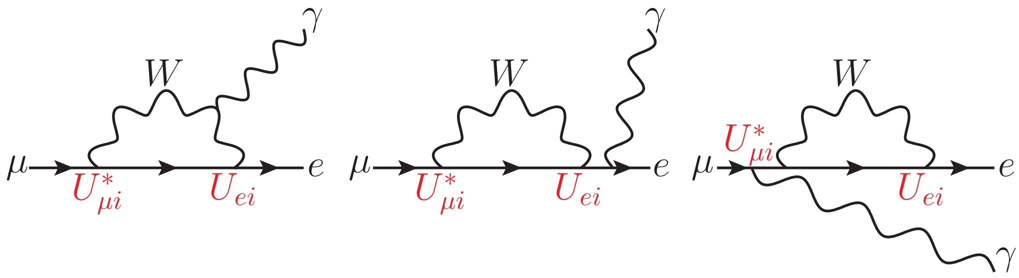


Figure 2.6: One-loop contributions to the radiative decay $\mu \to e\gamma$ in the Standard Model with Dirac neutrinos.

$\mu^\pm \to e^\pm + e^+ + e^-$. For instance, the former comes from the diagrams displayed in fig. 2.6. However, the rate and branching ratio for this process are given by [125, 126]

$$\Gamma(\mu \to e\gamma) = \frac{G_F^2\, m_\mu^5}{192\pi^3}\left[\frac{3}{32}\alpha_{EM}\left|\sum_i U^*_{\mu i} U_{ei}\frac{m_{\nu_i}^2}{M_W^2}\right|^2\right] , \tag{2.2.11}$$

$$\mathcal{B}r(\mu \to e\gamma) = \frac{3}{32}\alpha_{EM}\left|\sum_i U^*_{\mu i} U_{ei}\frac{m_{\nu_i}^2}{M_W^2}\right|^2 . \tag{2.2.12}$$

Both are extremely suppressed due to the smallness of neutrino masses. More precisely, the branching ratio for the muon decaying into an electron and a photon is of the order of 10^{-54}, and therefore far beyond the experimental reach, which makes this kind of process very interesting in the quest for new physics. Indeed, its observation would be a very clear indication of new physics.

This is true in general for flavour-violating processes involving charged leptons. In the Standard Model plus massive Dirac neutrinos, since lepton flavour is only broken by neutrino masses which are very small, these processes are effectively unobservable. Some of the flavour-changing processes involving charged leptons (namely the radiative decays and three-body decays) as well as the experimental bounds on their branching ratios are listed in table 2.2.

2.2.2 New physics and charged lepton flavour violation

The seesaw mechanism

The heavy fields involved in type II and type III seesaw, which are charged, can contribute to charged lepton flavour violation, under the condition that their mass scale is low enough. On the other hand, type I seesaw does not bring any significant contribution to these processes.

The type II seesaw mechanism gives a tree-level contribution to the 3-body decays $\tau^- \to e^-_\alpha e^-_\beta e^+_\gamma$ and $\mu \to e^- e^- e^+$, while the radiative decays such as $\mu^- \to e\gamma$ arise only from loop-diagrams involving a charged scalar Δ^- or Δ^{--}. These new contributions to lepton-flavour violating processes, displayed in fig. 2.7, are in general suppressed

Process	Bound on $\mathcal{B}r$ at 90% CL [73]
$\tau \to \mu\gamma$	4.4×10^{-8}
$\tau \to e\gamma$	3.3×10^{-8}
$\mu \to e\gamma$	5.7×10^{-13}
$\tau^- \to \mu^-\mu^-\mu^+$	2.1×10^{-8}
$\tau^- \to e^-\mu^-\mu^+$	2.7×10^{-8}
$\tau^- \to \mu^-\mu^-e^+$	1.7×10^{-8}
$\tau^- \to e^-e^-\mu^+$	1.5×10^{-8}
$\tau^- \to \mu^-e^-e^+$	1.8×10^{-8}
$\tau^- \to e^-e^-e^+$	2.7×10^{-8}
$\mu \to e^-e^-e^+$	1.0×10^{-12}

Table 2.2: Some flavour-changing processes involving charged leptons and the current bounds on their branching ratios.

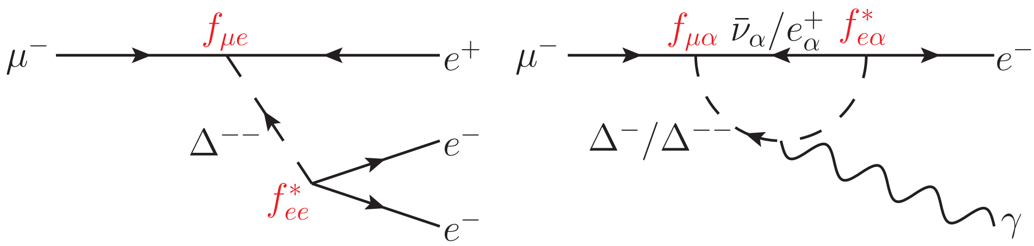


Figure 2.7: Diagrams contributing to $\mu^- \to e^-e^+e^+$ (left) and $\mu^- \to e^-\gamma$ (right) in the type II seesaw framework.

because of the very high scale of the triplet mass M_Δ, but can be sizeable for low-scale seesaw.

We consider for instance the 3-body decay $\mu^- \to e^-e^-e^+$, which has one of the most stringent bounds. Integrating out the triplet generates the effective four-fermion operator,

$$\mathcal{L}_{4F} = \frac{1}{4}\frac{f_{\alpha\beta}f^*_{\gamma\delta}}{M^2_\Delta}\left(\ell^T_\alpha C\vec{\sigma}\ell_\beta\right).\left(\bar{\ell}_\delta\vec{\sigma}C\bar{\ell}^T_\gamma\right) \quad , \tag{2.2.13}$$

which gives a direct contribution to this process. The corresponding decay width can be expressed in terms of the coupling and the triplet mass,

$$\Gamma(\mu^- \to e^-e^-e^+) = \frac{m^5_\mu}{3072\,\pi^5}\frac{|f_{e\mu}|^2\,|f_{ee}|^2}{M^4_\Delta} \quad . \tag{2.2.14}$$

Using $\Gamma_\mu \simeq \Gamma(\mu^- \to e^-\nu_\mu\bar{\nu}_e)$, this gives for the corresponding branching ratio [122]

$$\mathcal{B}r(\mu^- \to e^-e^-e^+) \simeq \frac{\Gamma(\mu^- \to e^-e^-e^+)}{\Gamma(\mu^- \to e^-\nu_\mu\bar{\nu}_e)} = \frac{1}{16M^4_\Delta G^2_F}\,|f_{e\mu}|^2\,|f_{ee}|^2 \quad . \tag{2.2.15}$$

If the couplings $f_{\alpha\beta}$ are of order unity, the constraint on this branching ratio gives a lower bound on the triplet mass,

$$M_\Delta \gtrsim 150 \text{ TeV} . \tag{2.2.16}$$

Of course, this bound gets relaxed if the couplings are smaller. Notice that, in particular, the expression of the neutrino mass matrix (2.1.61) gives a constraint on the factor $f_{\alpha\beta} \times \mu/M_\Delta^2$, which has to be small enough to reproduce the right order of magnitude for $m_{\nu\alpha\beta}$. If the triplet is as light as 150 TeV and the couplings $f_{\alpha\beta}$ are of order unity, the effective coupling μ/M_Δ should be extremely suppressed, namely $\mu/M_\Delta \lesssim 10^{-9}$.

The radiative decay $\mu \to e\gamma$ involves a loop with a triplet and is therefore suppressed by an additional factor. The branching ratio for this process is [122]

$$\mathcal{B}r(\mu \to e\gamma) = \frac{\alpha}{48\pi} \times \frac{25}{256} \times \frac{1}{M_\Delta^4 G_F^2} \left| \sum f_{\mu\alpha} f_{e\alpha}^* \right|^2 . \tag{2.2.17}$$

Considering again couplings of order unity, this process gives a less stringent bound on the triplet mass because of the loop suppression,

$$M_\Delta \gtrsim 20 \,\text{TeV} . \tag{2.2.18}$$

More precisely, the two previous processes allow to derive bounds on $|f_{\mu e}||f_{ee}|/M_\Delta$ or $\left|\sum f_{\mu\alpha} f_{e\alpha}^*\right| / M_\Delta$. Considering other contributions of the triplet to the processes listed in table 2.2, similar constraints can be derived for the other entries of the coupling matrix f. This was done in ref. [122]. For the high-scale seesaw models that we will consider in chapter 3, with $M_\Delta \gtrsim 10^9$ GeV, those bounds are never reached.

The type III seesaw also gives contribution to lepton flavour-violating processes. It generates flavour-changing neutral currents, as can be seen in eq. (2.1.73). As a consequence, the decay of the Z^0 boson into charged leptons can violate flavour [122],

$$\Gamma(Z^0 \to e_\alpha^- e_\beta^+) = \frac{G_F M_Z^3}{12\sqrt{2}\pi} \left[\left(N N^\dagger N N^\dagger \right)_{\alpha\beta} \right]^2 . \tag{2.2.19}$$

For the same reason, 3-body decays such as $\mu^- \to e^- e^- e^+$ arise at tree-level from the exchange of a Z^0. Finally, the radiative decays still involve a loop but get new contributions from the flavour-violating couplings to the W bosons of eq. (2.1.69).

Supersymmetry

In supersymmetric models, the main source of lepton flavour violation is the slepton mixing. Indeed, the breaking of supersymmetry can induce squared mass matrices for the superpartners of the left-handed lepton doublets, labelled $\tilde{\ell}_\alpha = \left(\tilde{\nu}_\alpha ,\, \tilde{e}_{L\alpha} \right)$, and those of the right-handed leptons, labelled $\tilde{e}_{R\alpha}$, as well as trilinear scalar couplings between sleptons and the down-type Higgs, parametrized as follows,

$$\mathcal{L}_{\cancel{\text{SuSy}}} \supset -(m_{\tilde{\ell}}^2)_{\alpha\beta} \tilde{\ell}_\alpha \tilde{\ell}_\beta^\dagger - (m_{\tilde{e}}^2)_{\alpha\beta} \tilde{e}_{R\alpha} \tilde{e}_{R\beta}^\dagger - A_{\alpha\beta}^e \tilde{\ell}_\alpha \tilde{e}_{R\beta}^c H_d . \tag{2.2.20}$$

The last term generates a contribution to the mixing between right- and left-handed charged sleptons proportional to the v.e.v. of the down-type Higgs v_d,

$$(m_{LR}^2)_{\alpha\beta} = A_{\alpha\beta}^e v_d \tag{2.2.21}$$

These squared mass matrices are not necessarily diagonal in the basis of charged lepton mass eigenstates, and can therefore contribute to the processes listed previously (2.2). However, one can generally use the following parametrization for $\alpha \neq \beta$,

$$\delta^{LL}_{\alpha\beta} = \frac{(m^2_{\tilde{\ell}})_{\alpha\beta}}{\bar{m}^2_{\tilde{\ell}}} \ , \qquad \delta^{RR}_{\alpha\beta} = \frac{(m^2_{\tilde{e}})_{\alpha\beta}}{\bar{m}^2_{\tilde{e}}} \ , \qquad \delta^{LR}_{\alpha\beta} = \frac{A^e_{\alpha\beta} v_d}{\bar{m}_{\tilde{\ell}} \bar{m}_{\tilde{e}}} \ . \tag{2.2.22}$$

where the $\bar{m}^2_a$ are the average slepton masses, and the $\delta^{XY}_{\alpha\beta}$ are small parameters.

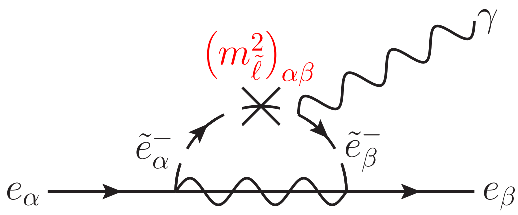


Figure 2.8: One of the contributions of (left-handed) sleptons to $\mu \to e\gamma$, with the exchange of a photino $\tilde{\gamma}$ or a zino $\tilde{Z}^0$. Right-handed sleptons give a similar contribution, with the replacement $m^2_{\tilde{\ell}} \leftrightarrow m^2_{\tilde{e}}$.

Radiative decays typically get contributions from diagrams such as the one of fig. 2.8. There are other similar diagrams, that can be obtained from this one by moving the photon vertex like in fig. 2.6. It is also possible to have sneutrinos instead of charged sleptons $\tilde{e}_L$ in the loop. The exact expression for the branching ratio $\mathcal{B}r(\mu \to e\gamma)$ and other similar processes is rather complicated, but a good understanding can be obtained from the so-called mass insertion approximation [127, 128]: using the parameters defined above, the main contribution to the decay $\mu \to e\gamma$ gives [129–133]

$$\mathcal{B}r(\mu \to e\gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|\delta^{LL}_{e\mu}|^2}{m^4_{\tilde{\ell}}} \tan^2\beta \ . \tag{2.2.23}$$

where $\tan\beta = v_u/v_d$ is the ratio of the v.e.v.'s of the two Higgs doublets. Using the bounds given in table 2.2, one can therefore constrain models of supersymmetry breaking.

Chapter 3

Leptogenesis

Au début, il n'y avait rien. Enfin, ni plus ni moins de rien qu'ailleurs.

Proverbe Shadok

3.1 From baryogenesis to leptogenesis

The apparent symmetry between particles and antiparticles introduces a new puzzle, which is the very existence of matter itself. If the discrete symmetry CP, that relates particles and antiparticles, is exact, a universe containing initially an equal amount of matter and antimatter would not contain any structure such as stars and galaxies. This is because every particle would annihilate with its antiparticle, leaving essentially radiation.

Thus, the simple existence of matter structures implies that the densities of particles and antiparticles are not equal today. This imbalance is characterized by the Baryon Asymmetry of the Universe, which is usually measured by the ratio of the density of baryon number over the density of photons. Two independent estimations using CMB data and Big Bang Nucleosynthesis agree on a value of the baryon-to-photon ratio around 6×10^{-10}. The most recent value was given by the Planck experiment [113],

$$\frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.10 \pm 0.06) \times 10^{-10} \qquad (68\% \text{ C.L.}) \,, \tag{3.1.1}$$

where n_b and $n_{\bar{b}}$ are respectively the densities of baryons and antibaryons. We could in principle define accordingly a Lepton Asymmetry of the Universe, but because neutrinos and antineutrinos are very difficult to detect, this quantity could not be measured.

To explain the origin of the Baryon Asymmetry, two possibilities come to mind. The first one is to assume that this asymmetry was already part of the initial conditions of the universe. This explanation is very simple but, besides being not fully satisfactory, it is completely unverifiable. Moreover, if the universe underwent a phase of inflation [134–136], for which there are strong motivations, it probably erased any pre-existing asymmetry. Indeed, during this phase, the scale factor was multiplied by a huge number, typically e^{50}, diluting dramatically any species of particles and therefore any asymmetry.

The second possibility is that the Baryon Asymmetry was generated dynamically after inflation, during a phase called baryogenesis. This possibility was first explored in 1967 by Andreï Sakharov, who derived the three necessary conditions for this scenario [137].

1. The first condition is, for an obvious reason, the violation of baryon number: the initial state of baryogenesis is fully symmetric, with $n_b = n_{\bar{b}}$, so it has necessarily a vanishing baryon number, unlike the final state.

2. The second condition is the violation of C and CP. If C (resp. CP) was an exact symmetry, the rate of any process involving particles labelled a_1, ..., a_n would be equal to the rate of the equivalent process involving the corresponding C-conjugate states (resp. CP-conjugate states), that carry opposite charges. Even in presence of baryon number-violating processes, baryons would be created and destroyed just as fast as antibaryons, thus no excess would ever be generated.

3. There must be a departure from thermal equilibrium because otherwise, CPT imposes that processes increasing and decreasing baryon number compensate each other. More formally, in equilibrium at the temperature T, the thermally averaged baryon number satisfies

$$\begin{aligned}\langle B\rangle_T &= \frac{\mathrm{tr}\left[Be^{-\beta\mathcal{H}}\right]}{\mathrm{tr}\left[e^{-\beta\mathcal{H}}\right]} \\ &= \frac{\mathrm{tr}\left[(\mathcal{CPT})^{-1}(\mathcal{CPT})Be^{-\beta\mathcal{H}}\right]}{\mathrm{tr}\left[e^{-\beta\mathcal{H}}\right]} \\ &= \frac{\mathrm{tr}\left[(\mathcal{CPT})Be^{-\beta\mathcal{H}}(\mathcal{CPT})^{-1}\right]}{\mathrm{tr}\left[e^{-\beta\mathcal{H}}\right]} \ .\end{aligned} \tag{3.1.2}$$

Since CPT is preserved, we should have $[\mathcal{H},\, (\mathcal{CPT})^{-1}] = 0$, thus the previous expression becomes

$$\langle B\rangle_T = \frac{\mathrm{tr}\left[(\mathcal{CPT})B(\mathcal{CPT})^{-1}e^{-\beta\mathcal{H}}\right]}{\mathrm{tr}\left[e^{-\beta\mathcal{H}}\right]} \ , \tag{3.1.3}$$

but, since CPT exchanges particles and antiparticles, $\mathcal{CPT})B(\mathcal{CPT})^{-1} = -B$, an so

$$\langle B\rangle_T = \frac{\mathrm{tr}\left[-Be^{-\beta\mathcal{H}}\right]}{\mathrm{tr}\left[e^{-\beta\mathcal{H}}\right]} = -\langle B\rangle_T \ , \tag{3.1.4}$$

which is necessarily vanishing. Thus, the emergence of a non-vanishing baryon number can only happen out of equilibrium.

3.1.1 Baryon number nonconservation in the Standard Model

At first, investigations were carried to check if Sakharov's conditions can be satisfied within the Standard Model. At least the second condition is satisfied in the Standard Model because CP is violated, as can be seen for instance in kaon mixing [138]. Baryon and lepton number are seemingly conserved, but they are only accidental symmetries. This conservation holds actually only at the perturbative level, while non-perturbatively, baryon and lepton number are broken by the electroweak sphalerons [139], in a way which is related to the anomalies of the associated currents.

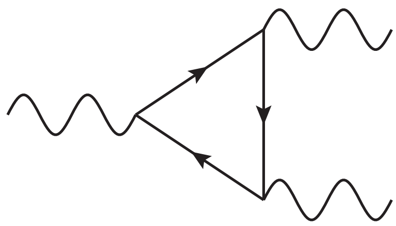


Figure 3.1: Triangle diagram contributing to the anomaly, with leptons or quarks circulating in the loop.

The B and L currents are defined respectively as

$$j_B^\mu = \frac{1}{3} \sum_{i=1,2,3} \left(\bar{Q}_i \gamma^\mu Q_i + \bar{u}_i \gamma^\mu u_i + \bar{d}_i \gamma^\mu d_i \right) \ , \tag{3.1.5}$$

$$j_L^\mu = \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_\alpha \gamma^\mu \ell_\alpha + \bar{e}_\alpha \gamma^\mu e_\alpha \right) \ . \tag{3.1.6}$$

They are classically conserved, but at the quantum level, because of the triangle anomalies shown in fig. 3.1, their divergence is given in terms of the field operators by

$$\partial^\mu j_\mu^B = \partial^\mu j_\mu^L = \frac{N_f}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma} \left(-g^2 W^a_{\mu\nu} W^a_{\rho\sigma} + g'^2 B^{\mu\nu} B^{\rho\sigma} \right) \ , \tag{3.1.7}$$

where N_f is the number of fermion generations. Therefore baryon and lepton number are not conserved, but instead

$$\frac{d}{dt}(B-L) = 0 \ , \tag{3.1.8}$$

$$\frac{dB}{dt} = \frac{dL}{dt} = \int d^3x\, \frac{N_f}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma} \left(-g^2 W^a_{\mu\nu} W^a_{\rho\sigma} + g'^2 B^{\mu\nu} B^{\rho\sigma} \right) \ , \tag{3.1.9}$$

The terms inside the parenthesis are total derivatives, but the integral does not vanish. Indeed, for a non-abelian gauge theory, there are several different vacua corresponding to different topological configurations of the gauge fields. These configurations are indexed by an integer n_{CS}, the Cherns-Simons number. For a transition between two vacua [140],

$$\begin{aligned} \Delta B = \Delta L &= N_f \int_{t_i}^{t_f} dt \int d^3x\, \frac{1}{32\pi^2}\, \epsilon^{\mu\nu\rho\sigma} \left(-g^2 W^a_{\mu\nu} W^a_{\rho\sigma} \right) \\ &= N_f\, \Delta n_{CS} \ . \end{aligned} \tag{3.1.10}$$

Since $N_f = 3$ in the Standard Model, each transition changes baryon and lepton number by a multiple of three units, and more precisely satisfies $\Delta B/3 = \Delta L_e = \Delta L_\mu = \Delta L_\tau$. This process can be formally represented by an effective operator

$$\mathcal{O}_{\text{eff}} \propto \prod_{i=1,2,3}^{\alpha=e,\mu,\tau} Q_i Q_i Q_i \ell_\alpha \ . \tag{3.1.11}$$

At zero temperature, the rate of these processes is exponentially suppressed, namely

$$\Gamma_{\text{sphal}} \propto e^{-4\pi/\alpha_{EW}} = \mathcal{O}\left(10^{-165}\right) \ , \tag{3.1.12}$$

and the probability of quantum tunneling is way too small to be of any relevance. However, at nonzero temperature, because of thermal fluctuations, the energy barrier can be crossed classically. More precisely, in a thermal bath at temperature T, the space-time density of reaction is [141, 142]

$$\gamma_{\text{sphal}} \simeq 26\alpha_{EW}^5 T^4 \ . \tag{3.1.13}$$

Sphalerons come in thermodynamic equilibrium between the temperatures $T \simeq 10^2$ GeV and $T \simeq 10^{12}$ GeV.

Sphalerons have two important implications for baryogenesis, as was shown by Kuzmin, Rubakov and Shaposhnikov [143]. First, if $B - L$ is conserved and zero, any preexisting baryon asymmetry would have been erased by sphalerons when they were in equilibrium. On the other hand, sphalerons could be themselves responsible for the Baryon Asymmetry of the Universe, as it is the case in electroweak baryogenesis: starting with a symmetric universe ($B = L = 0$), CP-violating sphalerons would generate equal baryon and lepton numbers, giving $B = L \neq 0$ today. Of course, because of Sakharov's third condition, this could only happen at the electroweak phase transition, when sphalerons go out of equilibrium. Moreover, in order to satisfy this condition, the phase transition should be strongly first order.

However, it is unclear whether the amount of CP violation in the CKM matrix is large enough to fully satisfy Sakharov's second condition and anyway, the out-of-equilibrium condition is not satisfied in the Standard Model. More precisely, with the field content of the Standard Model, this condition would be fulfilled only if the Higgs boson was lighter than about 45 GeV [144, 145], far from the 126 GeV of the particle discovered at LHC.

Thus, baryogenesis requires new physics. The main options are the following.

(i) A possibility is to try to rehabilitate electroweak baryogenesis by introducing new fields in order to bring the sphalerons out of equilibrium at the electroweak phase transition [146–148]. This can happen for instance in the MSSM if one of the stops is light enough (although the constraints on this scenario are becoming stringent). Another possibility is the 2-Higgs-Doublet Model (2HDM).

(ii) In a class of models known as GUT baryogenesis [149–157], a baryon asymmetry is generated at high temperature by the out-of-equilibrium decay of heavy bosons. However, these models faced difficulties, such as the constraints on proton decay. Moreover, in the simplest models, only $B + L$ is violated, not $B - L$, therefore the asymmetry is erased by electroweak sphalerons.

(iii) Affleck-Dine baryogenesis [158, 159] takes place in a supersymmetric GUT. In this scenario, scalar fields, which carry a non-vanishing baryon or lepton number (e.g. squarks or sleptons), develop a vacuum expectation value in the early universe, which breaks $B-L$. Later, the decay of the scalar fields generate a nonzero $B-L$ stored in quarks and leptons.

(iv) In a large variety of models, it is assumed that the dark matter density is also due to an asymmetry between dark matter particles and antiparticles. It is then natural to think that both the so-called asymmetric dark matter and the baryon asymmetry are generated by a single mechanism, which can be related to other models for generating an asymmetry [160–162]. In the end, the asymmetries in the baryon sector and in the dark sector are comparable or even equal, which explains simply why the energy densities of dark matter and ordinary matter are of the same order of magnitude.

(v) In leptogenesis, a scenario imagined first by Masatake Fukugita and Tsutomu Yanagida [163], the asymmetry is generated in the lepton sector before being converted to a baryon asymmetry by the electroweak sphalerons. We are going to investigate this last possibility in this chapter.

3.1.2 Neutrino masses and leptogenesis

In this class of scenarios, new physics at a high scale breaks $B-L$ in the lepton sector, generating a lepton asymmetry. After this new physics comes to a freeze-out, one is left with a nonzero value of $B-L$, which will be conserved by any subsequent physical process. Then, fast electroweak sphalerons transfer a part of this asymmetry to the baryon sector. The equilibrium condition has been derived in ref. [164] and is defined by

$$C_{\text{sphal}} = \frac{8N_f + 4(N_H+2)}{22N_f + 13(N_H+2)} \ , \tag{3.1.14}$$

N_f being the number of generations and N_H the number of Higgs doublets. The reason why it is interesting to proceed through a violation of lepton number only in a first place is because lepton number violation is at the root of the seesaw mechanism. Thus, leptogenesis provides answers to two apparently unrelated issues, neutrino masses and the Baryon Asymmetry of the Universe.

The general idea is simple: the seesaw mechanism requires the existence of a heavy particle with couplings to lighter fields, which is therefore unstable. Moreover, one of these couplings violates lepton number, so that the decay of the heavy particle is able to generate a lepton asymmetry. Sakharov's conditions are still required, but since baryon number is automatically broken by sphalerons, the violation of baryon number is replaced with a violation of lepton number.

3.1.3 The Standard scenario: leptogenesis with right-handed neutrinos

The "standard" leptogenesis scenario [163, 165] involves three heavy right-handed neutrinos, and relies on the Lagrangian of eq. (2.1.42). The Standard Model neutrinos have a mass matrix which is given by the seesaw formula of eq. (2.1.48).

A minimal realization

The right-handed neutrinos are assumed to be hierarchical, that is M_1 is much smaller than M_2 and M_3, so that N_3 and N_2 decay first, usually when the temperature of the Universe is around $T \sim M_3$ and $T \sim M_2$ respectively, and when N_1 decays around $T \sim M_1$, the two other species have already disappeared.

An interesting feature of right-handed neutrinos is that they are gauge singlets. Contrary to other particles which are maintained close to thermodynamic equilibrium by fast gauge interactions, their density is usually far from its equilibrium value, which allows to satisfy Sakharov's third condition. Besides, if there is no other interaction that creates them have to be created and brought to their equilibrium density by inverse decays like $\ell H \to N$ and $\ell^c H^c \to N$. Because of this, and because of the hierarchy between right-handed neutrinos, any asymmetry generated in the decay of N_3 and N_2 would be erased to create N_1. Thus, it is possible to focus on the decay of the lightest right-handed neutrino N_1 only, starting from a time at which the two other species have already decayed and the lepton asymmetry is vanishing.

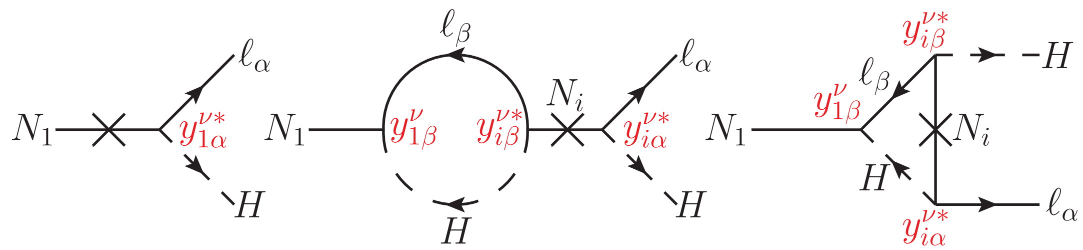


Figure 3.2: Diagrams involved in the violation of CP in the decay of N_1 at one loop. At lowest order, the CP asymmetry comes from the interference of the tree-level diagram with one of the two other.

The decay of N_1 violates CP at the order of one loop, because of interferences between the diagrams displayed in fig. 3.2. The CP asymmetry is defined as

$$\epsilon_{N_1} = \frac{\Gamma(N_1 \to \ell H) - \Gamma(N_1 \to \ell^c H^c)}{\Gamma_{N_1}} \; . \tag{3.1.15}$$

The computation gives

$$\epsilon_{N_1} = \frac{1}{8\pi} \sum_{i=2,3} \frac{\Im\left[(y^\nu y^{\nu\dagger})^2_{1i}\right]}{(y^\nu y^{\nu\dagger})_{11}} \left[F^S\left(\frac{M_i}{M_1}\right) + F^V\left(\frac{M_i}{M_1}\right)\right] \; , \tag{3.1.16}$$

where F^S and F^V are kinematic functions coming from the self-energy correction and the vertex correction respectively. For very hierarchical right-handed neutrinos with $M_1 \ll M_2 \ll M_3$, this can be simplified,

$$\epsilon_{N_1} \simeq -\frac{3}{16\pi} \sum_{i=2,3} \frac{\Im\left[(y^\nu y^{\nu\dagger})^2_{1i}\right]}{(y^\nu y^{\nu\dagger})_{11}} \frac{M_1}{M_i} \; . \tag{3.1.17}$$

The two first Sakharov conditions are satisfied, so that the success of leptogenesis depends on the dynamics of the creation of the lepton asymmetry in the early Universe. More precisely, we need to solve Boltzmann equations describing the evolution of the density of right-handed neutrinos Y_{N_1} and the lepton asymmetry $\Delta_\ell = Y_\ell - Y_{\ell^c}$. We do not take care for the moment of spectator processes redistributing the asymmetry in right-handed leptons and baryons, they will be mentioned in section 3.2.3. The evolution is described by the following system of Boltzmann equations [165],

$$sHz\frac{dY_{N_1}}{dz} = -\left(\frac{Y_{N_1}}{Y_{N_1}^{\rm eq}} - 1\right)\gamma_D^{N_1} - S \ , \tag{3.1.18}$$

$$sHz\frac{d\Delta_\ell}{dz} = \epsilon^{N_1}\left(\frac{Y_{N_1}}{Y_{N_1}^{\rm eq}} - 1\right)\gamma_D^{N_1} - W \ , \tag{3.1.19}$$

where S represents scatterings consuming right-handed neutrinos, like $N_1\ell \to Q_3 t_R^c$, and W is the washout of the lepton asymmetry due to inverse decays $\ell H \to N_1$, and scatterings like for instance $\ell H \to \ell^c H^c$.

This system relies on the so-called single flavour approximation, that is the computation is performed as if there existed only one lepton flavour. At very high temperature, this treatment is justified. Indeed, in the Standard Model, the three lepton generations are distinguished only by their Yukawa couplings, as can be seen from eqs. (1.2.7) and (1.2.13). When the temperature of the Universe is above 10^{12} GeV, every interaction mediated by the lepton Yukawa couplings is out of equilibrium, which means that the three lepton doublets are effectively undistinguishable. In the present model, because of the hierarchy between the right-handed neutrinos, one can integrate N_2 and N_3 out of the Lagrangian, and rewrite the couplings involving N_1,

$$-y_{1\alpha}^\nu\,\bar{N}_1\ell_\alpha^T i\sigma_2 H + \text{h.c.} \to -y_0^\nu\,\bar{N}_1 H.\ell_0 + \text{h.c.} \ , \tag{3.1.20}$$

where we defined

$$\ell_0 = \sum_\alpha \frac{y_{1\alpha}^\nu \ell_\alpha}{y_0^\nu} \ , \qquad y_0^\nu = \sqrt{\sum_\alpha |y_{1\alpha}^\nu|^2} \ . \tag{3.1.21}$$

It appears clearly that, at tree-level at least, N_1 decays only to the final state $\ell_0 H$ (and its CP-conjugate). This ensures that, at the order we are considering, no asymmetry is generated in the two lepton combinations $\ell_1^\perp$ and $\ell_2^\perp$ orthogonal to ℓ_0[1]. Thus, it is enough to solve only one Boltzmann equation describing the asymmetry stored in the flavour ℓ_0. When the temperature decreases, interactions mediated by the lepton Yukawa couplings become faster and reach equilibrium, which means that we should distinguish the asymmetries stored in the various flavours and therefore write several Boltzmann equations to describe the evolution of the lepton asymmetries Δ_{ℓ_α}.

This dynamics generates an asymmetry which is stored in lepton doublets, then transferred to the baryon sector by electroweak sphalerons. In the simplified case where leptogenesis is over when sphalerons proceed, the final baryon asymmetry is

$$\frac{n_B}{n_\gamma} \simeq -\frac{12}{37} \times 7.04 \times \Delta_\ell^f \ , \tag{3.1.22}$$

[1] To be exact, this is only true as long as one can neglect the scatterings mediated by N_2 and N_3

where Δ_ℓ^f is the lepton asymmetry left by the decay of N_1. The factor $12/37$ is the conversion factor due to sphalerons, and the factor 7.04 takes care of the fact that the quantity on the left-hand side is normalized over the density of photon, whereas the quantity on the left is normalized over the entropy density.

A difficulty when studying this scenario is that the Yukawa coupling matrix y^ν cannot be expressed in a simple way in terms of the neutrino mass matrix. Moreover, the combinations of Yukawa couplings that appear in the expression of the neutrino mass matrix (2.1.48) and the CP asymmetry (3.1.16) are different. Thus, there is in general a lot of arbitrariness in the parametrization, even though in some specific scenarios [166–171], it is possible to relate the CP asymmetry to lepton flavour observables or to parameters of the PMNS matrix. In the general case, for hierarchical right-handed neutrinos, it is possible to estimate the minimal value of the mass of N_1 that is needed for a successful leptogenesis. Using the Casas-Ibarra parametrization [172],

$$y^\nu_{i\alpha} = \frac{1}{v}\left(D_M^{1/2} R D_{m_\nu}^{1/2} U^\dagger\right)_{i\alpha} \,, \tag{3.1.23}$$

where $D_M = \mathrm{diag}\,(M_1,\, M_2,\, M_3)$, $D_{m_\nu} = \mathrm{diag}\,(m_{\nu 1},\, m_{\nu 2},\, m_{\nu 3})$ and R is an orthogonal matrix $RR^T = R^T R = 1$, the CP asymmetry can be rewritten as

$$\epsilon_{N_1} = -\frac{3}{16\pi}\frac{M_1}{v^2}\frac{\sum_i \Im\left(R_{1i}^2\right) m_{\nu i}^2}{\sum_k |R_{1k}|^2 m_{\nu k}} \,. \tag{3.1.24}$$

This, in turn, gives the Davidson-Ibarra bound [173].

$$|\epsilon_{N_1}| < \epsilon^{\mathrm{DI}} = \frac{3}{16\pi}\frac{M_1}{v^2}\frac{m_{\nu\mathrm{atm}}^2}{m_{\nu 1} + m_{\nu 3}} \,. \tag{3.1.25}$$

Since the maximal lepton asymmetry that can be generated in the decay of N_1 is $\Delta_\ell^{\max} \sim \epsilon_{N_1} Y_{N_1}^{\mathrm{eq0}}$, where $Y_{N_1}^{\mathrm{eq0}}$ is the equilibrium density of N_1 before its decay, this imposes that

$$\frac{12}{37} \times 7.04 \times \epsilon^{\mathrm{DI}} \times Y_{N_1}^{\mathrm{eq0}} \gtrsim \frac{n_B}{n_\gamma} \,. \tag{3.1.26}$$

In order for this condition to be satisfied, N_1 should have a mass greater than 10^{-9} GeV approximately. When the right-handed neutrinos are not hierarchical, this bound can be relaxed [174]. In particular when N_1 and N_2 have very close masses, it is possible to realize resonant leptogenesis [175, 176] which can give the correct baryon asymmetry at a much lower scale. Alternative possibilities to lower the scale of leptogenesis were investigated in ref. [177].

Beyond the single-flavour approximation

Let us now explore the flavour issue [178–182] in more details. Regarding flavour, there are several temperature regimes, depending of Yukawa-mediated scatterings being in equilibrium or not, as explained in section 3.1.3. When these scatterings are much slower than the expansion of the Universe, they are out of equilibrium and can be neglected.

When the temperature of the Universe is above 10^{12} GeV, every interaction mediated by the lepton Yukawa couplings is out of equilibrium, which means that the three lepton doublets are effectively undistinguishable. As we explained previously, the problem in this temperature range is generally studied in the single-flavour approximation. However, there are several situations in which it does not hold. This is the case for instance close to the transition, when the tau Yukawa-mediated scatterings begin to be fast but not enough to completely destroy the coherence of the combination ℓ_0. It is also the case when more than one right-handed neutrino is taken into account, and in particular in resonant leptogenesis: unless the couplings of the two neutrinos N_1 and N_2 are aligned, it is not possible to define a single linear combination ℓ_0 that couples to both.

The consistent way to describe quantum undistinguishable particles in a general way is the density matrix, which is simply a generalization of the ordinary particle asymmetry given in terms of the field operator by

$$\Delta n_f = n_f - n_{f^c} = \langle : f^\dagger f : \rangle \ . \tag{3.1.27}$$

This applies to leptogenesis in the following way. If it is not possible to perform the same trick as in eq. (3.1.20), the lepton asymmetry should be described by a 3×3 density matrix in flavour space

$$(\Delta n_\ell)_{\alpha\beta} = \langle : \ell_\alpha^\dagger \ell_\beta : \rangle \ , \quad \alpha, \beta \in \{e, \mu, \tau\} \ . \tag{3.1.28}$$

The diagonal elements have the same physical interpretation as before, i.e.

$$(\Delta n_\ell)_{\alpha\alpha} = n_{\ell_\alpha} - n_{\ell_\alpha^c} \ , \tag{3.1.29}$$

whereas the off-diagonal elements have no classical equivalent: they represent purely quantum correlations between the various flavours. Since we work with densities normalized over the entropy density, we define

$$(\Delta_\ell)_{\alpha\beta} = \frac{1}{s} \langle : \ell_\alpha^\dagger \ell_\beta : \rangle \ . \tag{3.1.30}$$

Under a unitary rotation in flavour space $\ell_\alpha \to \ell'_i = V^*_{i\alpha} \ell_\alpha$, this object transforms covariantly as

$$(\Delta_\ell)_{\alpha\beta} \to (\Delta'_\ell)_{ij} = \left(V \Delta_\ell V^\dagger \right)_{ij} \ . \tag{3.1.31}$$

In particular, the trace of the matrix, which describes the total asymmetry stored in the lepton doublets, is unaffected by this change of basis. In fact, the single flavour approximation could be rephrased in the density matrix formalism: the basis $(\ell_0,\, \ell_1^\perp, \ell_2^\perp)$ is the one in which, because of the structure of the interactions, the entries $(\Delta_\ell)_{ij}$, for $i,\, j \neq 0$, remain zero. The off-diagonal entries $(\Delta_\ell)_{0i}$, $(\Delta_\ell)_{i0}$ are actually nonzero but do not participate in the equation for $(\Delta_\ell)_{00}$, therefore it is sufficient to solve the latter to obtain the total lepton asymmetry. In another basis, this would no longer be true, but the final result of the computation performed in the density matrix formalism would be the same. In fact, picking the basis $(\ell_0,\, \ell_1^\perp, \ell_2^\perp)$ is just a convenient choice to carry out the computation.

The derivation of the flavour-covariant transport equation for the density matrix is not as straightforward as that of usual Boltzmann equations. In the context of leptogenesis with right-handed neutrinos, it was performed in ref. [180] in a semi-classical approach, and a quantum treatment of the density matrix formalism was introduced in refs. [183, 184]. This formalism was also applied to resonant leptogenesis, where it can also be used to describe the oscillation of quasi-degenerate right-handed neutrinos [185, 186]. In section 3.2.3, we will apply this formalism to scalar triplet leptogenesis.

As the temperature drops, Yukawa-mediated interactions become faster. Below 10^{12} GeV, the interactions mediated by the tau Yukawa coupling become fast enough to reach thermodynamic equilibrium, which allows to distinguish the tau doublets from the two others. In this case, a general description should involve a 2×2 density matrix and a separate tau asymmetry Δ_{ℓ_τ}.

Finally, when the temperature reaches 10^9 GeV, interactions mediated by the muon Yukawa coupling reach equilibrium as well. From this moment, the three lepton flavour become distinguishable, even though the electron Yukawa-mediated interactions do not reach equilibrium until the temperature drops below 10^5 GeV.

Towards an exhaustive treatment

In addition to lepton flavour, an exhaustive study of thermal leptogenesis would require to take into account several effects, which have generally been studied apart from one another in the context of leptogenesis with right-handed neutrinos.

(i) The fact that leptogenesis takes place at a temperature $T \sim M_1$ means that quantities computed in zero-temperature quantum field theory should in principle be corrected by including finite-temperature effects [187]. They are for instance corrections to the couplings, to the propagator of massless fields and to the CP asymmetry. Altogether, these effects can affect the result by $\mathcal{O}(1)$ corrections. To a good approximation, corrections to the couplings are given by zero-temperature renormalization at the scale $\Lambda \sim 2\pi T$, which we took into account in our study of leptogenesis with a scalar triplet.

(ii) Several Standard Model processes can affect the creation of the baryon asymmetry. They include scatterings mediated by the Yukawa couplings of quarks and leptons, electroweak sphalerons which redistribute the asymmetry between leptons and quarks and QCD sphalerons. They are generally referred to as spectator processes, because they do not directly modify $B - L$. Nevertheless, they intervene indirectly in the dynamics and can modify the results substantially. These processes were investigated in refs. [188–190]. We incorporated them in leptogenesis with a scalar triplet by the mean of chemical equilibriums.

(iii) A new approach, based on Kadanoff-Baym equations in the closed time-path formalism, allows to derive a quantum formulation of leptogenesis. This features novelties such as non-markovian memory effects [191–194] and finite densities effects [195, 196]. It also allows to derive flavour-covariant equations for the density matrix in a natural way [183–186]. However a fully quantum treatment of leptogenesis with right-handed neutrinos is still work in progress [197, 198].

3.2 Leptogenesis with a scalar triplet: a general approach

The type II seesaw is one possible alternative to generate neutrino masses. Like the type I seesaw, it allows to implement leptogenesis [199–204]. Indeed, the decay of the scalar triplet violates lepton number by two units, which matches at least the second Sakharov's condition. In this section, we study a general scenario of leptogenesis with a scalar triplet, before turning to a more specific model in the next one.

3.2.1 The setup

The present scenario is described by the Lagrangian of eq. (2.1.57). The scalar triplet Δ has two decay channels: $\Delta \to \ell^c \ell^c$ and $\Delta \to HH$. To simplify notations, we define the effective couplings $\lambda_\ell = \sqrt{\mathrm{tr}(f f^\dagger)}$ and $\lambda_H = |\mu|/M_\Delta$. The tree-level width and branching ratios of Δ are given by

$$\Gamma_\Delta = \frac{M_\Delta}{32\pi}\left(\lambda_\ell^2 + \lambda_H^2\right) , \tag{3.2.1}$$

$$B_\ell = \frac{\lambda_\ell^2}{\lambda_\ell^2 + \lambda_H^2} , \tag{3.2.2}$$

$$B_H = \frac{\lambda_H^2}{\lambda_\ell^2 + \lambda_H^2} . \tag{3.2.3}$$

Unfortunately, CP is not violated by these decays, or at least not at a sufficiently low order [205, 199]. Solving this problem requires the introduction of new states and interactions, which typically generate new contributions to neutrino masses. For instance, the presence of right-handed neutrinos in addition to the scalar triplet may be motivated by a left-right symmetry [120] or by a SO(10) grand unified theory [117, 118]. Thus, the simple proportionality relation between the neutrino mass matrix and the coupling matrix f, highlighted in 2.1.2, breaks down. Nevertheless, it is still a viable option. In order to keep this study as general as possible, we will simply assume that the additional states, whatever they are, are much heavier than the scalar triplet, and that their main contribution at the triplet mass scale is summarized by an effective operator, which is nothing else than the Weinberg operator (2.1.38) [203],

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\frac{\kappa_{\alpha\beta}}{\Lambda}(\ell_\alpha^T i\sigma_2 H)C(H^T i\sigma_2 \ell_\beta) . \tag{3.2.4}$$

Like before, we define the effective coupling $\lambda_\kappa = \sqrt{\mathrm{tr}(\kappa\kappa^\dagger)}$. As a consequence of this new term, the neutrino mass matrix is the sum of two independent contributions,

$$m_{\nu\alpha\beta} = \frac{1}{2}\mu f_{\alpha\beta}\frac{v^2}{M_\Delta^2} + \frac{1}{2}\kappa_{\alpha\beta}\frac{v^2}{\Lambda} . \tag{3.2.5}$$

Before going on, it should also be noticed that another effective operator is possible,

$$\mathcal{L}_{\text{add}} = -\frac{1}{4}\frac{\eta_{\alpha\beta\gamma\delta}}{\Lambda^2}\left(\ell_\alpha^T C i\sigma_2\vec{\sigma}\ell_\beta\right).\left(\bar{\ell}_\gamma\vec{\sigma} i\sigma_2 C\bar{\ell}_\delta^T\right) . \tag{3.2.6}$$

It appears at least if the additional heavy state is another scalar triplet (see eq. (2.1.63) for instance). In this case, labelling M'_Δ and $f'_{\alpha\beta}$ the mass and the coupling to leptons of this heavier scalar, the new coupling reads explicitly

$$\frac{\eta_{\alpha\beta\gamma\delta}}{\Lambda^2} = \frac{f'_{\alpha\beta} f'^*_{\gamma\delta}}{M'^2_\Delta} \ . \tag{3.2.7}$$

We will disregard this contribution because it is suppressed by an additional power of the high scale, even though it could still be relevant for particular configurations of the parameters[2]. If the additional heavy state is a heavy neutrino or a fermion triplet, this contribution does not exist at this order so it can safely be disregarded.

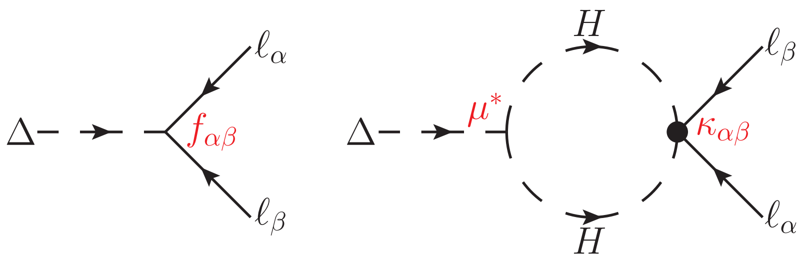


Figure 3.3: Diagrams responsible for the violation of CP in the decay of Δ at one loop.

Let us consider now the decay of Δ. At the 1-loop order, this decay violates CP because of the interference between the two processes displayed in fig. 3.3. The overall CP asymmetry is defined as

$$\epsilon^\Delta = 2\frac{\Gamma(\Delta^c \to \ell\ell) - \Gamma(\Delta \to \ell^c\ell^c)}{\Gamma_{\Delta^c} + \Gamma_\Delta} \ , \tag{3.2.8}$$

with a factor 2 accounting for the fact that each decay produces two leptons. Computing this quantity, one obtains [203]

$$\epsilon^\Delta = \frac{1}{8\pi\Lambda}\frac{\Im\left[\mu^*\mathrm{tr}(f^\dagger\kappa)\right]}{\lambda_\ell^2 + \lambda_H^2} = \frac{M_\Delta}{4\pi v^2}\sqrt{B_\ell B_H}\frac{\Im\left[\mathrm{tr}(m_\Delta^\dagger m_\kappa)\right]}{\bar{m}_\Delta} \ , \tag{3.2.9}$$

where m_Δ and m_κ are the two contributions to the neutrino mass matrix of eq. (3.2.5),

$$(m_\Delta)_{\alpha\beta} = \frac{1}{2}\mu f_{\alpha\beta}\frac{v^2}{M_\Delta^2} \ , \qquad (m_\kappa)_{\alpha\beta} = \frac{1}{2}\kappa_{\alpha\beta}\frac{v^2}{\Lambda} \ . \tag{3.2.10}$$

and $\bar{m}_\Delta = \sqrt{\mathrm{tr}(m_\Delta^\dagger m_\Delta)}$.

[2]For instance, if the coupling of the heavier scalar to the Higgs is very suppressed while its coupling to leptons is sizeable, the dimension-6 operator will not be negligible in comparison with the dimension-5 one.

3.2.2 A simplified model

At this stage, it is instructive to write down a minimal set of Boltzmann equations, neglecting all the subtleties such as flavour effects or spectator processes. In other words, we study a toy model which contains scalar triplets, Higgs doublets and a single species of leptons. Leaving aside for now the effect of sphalerons, these fields communicate with the rest of the universe only through perturbative electroweak interactions.

First, like in leptogenesis with right-handed neutrinos, there is an equation describing the evolution of the total density of the triplets $\Sigma_\Delta = Y_\Delta + Y_{\Delta^c}$ [203],

$$sHz\frac{d\Sigma_\Delta}{dz} = -\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} - 1\right)\gamma_D^\Delta - 2\left[\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}}\right)^2 - 1\right]\gamma_A^\Delta \ . \tag{3.2.11}$$

The novelty with respect to the scenario with right-handed neutrinos is that the triplets are sensitive to electroweak interactions, which has an important consequence. Indeed, their density is maintained closed to its equilibrium value by fast gauge interactions, which may seem contradictory with Sakharov's third condition. These gauge interactions are described by the second term in the right-hand side of eq. (3.2.11) with

$$\gamma_A^\Delta = \gamma(\Delta\Delta^c \to WW,\, BB,\, HH^c, f f^c) \ . \tag{3.2.12}$$

In particular, this term describes the annihilation of the triplets, which is in competition with their decay.

Then, since the triplet is not a self-conjugate state, it also stores an asymmetry $\Delta_\Delta = Y_\Delta - Y_{\Delta^c}$. Thus, there should be a priori three Boltzmann equations, describing the asymmetries stored in the leptons doublet Δ_ℓ, the Higgs doublet Δ_H and the triplets Δ_Δ. However, these equations are not independent because of the conservation law

$$2\Delta_\Delta - \Delta_\ell + \Delta_H = 0 \ . \tag{3.2.13}$$

Using this relation, we can express one of the asymmetries as a function of the others, for instance

$$\Delta_H = C_H^k \Delta_k \ , \tag{3.2.14}$$

with

$$C_H^k = \begin{pmatrix} 1 & -2 \end{pmatrix}, \quad \Delta_k = \begin{pmatrix} \Delta_\ell \\ \Delta_\Delta \end{pmatrix} \ . \tag{3.2.15}$$

Then, the two equations for Δ_ℓ and Δ_Δ are given by [203],

$$\begin{aligned} sHz\frac{d\Delta_\ell}{dz} &= \epsilon^\Delta\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} - 1\right)\gamma_D - 2B_\ell\left(\frac{\Delta_\ell}{Y_\ell^{\rm eq}} + \frac{\Delta_\Delta}{\Sigma_\Delta^{\rm eq}}\right)\gamma_D^\Delta \\ &\quad - 2\left(\frac{\Delta_\ell}{Y_\ell^{\rm eq}} + \frac{C_H^k\Delta_k}{Y_H^{\rm eq}}\right)\left(2\gamma_{H^cH^c}^{\ell\ell} + \gamma_{\ell^cH^c}^{\ell H}\right) \ , \end{aligned} \tag{3.2.16}$$

$$sHz\frac{d\Delta_\Delta}{dz} = \left(-\frac{\Delta_\Delta}{\Sigma_\Delta^{\rm eq}} - B_\ell\frac{\Delta_\ell}{Y_\ell^{\rm eq}} + B_H\frac{C_H^k\Delta_k}{Y_H^{\rm eq}}\right)\gamma_D^\Delta \ . \tag{3.2.17}$$

Let us review the main effects. Annihilations, which produce a symmetric final state, are generally in thermodynamic equilibrium and compete with the decays, which create the lepton asymmetry. If the triplets annihilate much faster than they decay, no asymmetry can be generated. Thus, this model requires the decay rate γ_D to be at least comparable to γ_A. Again, this seems to be in contradiction with Sakharov's third condition because these fast reactions will quickly reach thermodynamic equilibrium. However, one of the decay channels of the triplet can still be out of equilibrium if the corresponding branching ratio is small enough, allowing to produce a sizeable asymmetry.

To illustrate how this can work, we study an extreme case, in which

$$\gamma_D^\Delta \gg \gamma_A^\Delta \ , \tag{3.2.18}$$

whereas B_ℓ is small enough to ensure that the decay channel $\Delta \to \ell^c \ell^c$ is out of equilibrium, which implies $B_\ell \ll B_H \simeq 1$. Because of eq. (3.2.18), the Boltzmann equation for the triplet density can be approximated by.

$$sHz\frac{d\Sigma_\Delta}{dz} = -\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta \ . \tag{3.2.19}$$

Then, assuming that the scattering rates are small, which is generally the case if the couplings are not too large, and because B_ℓ is tiny, the Boltzmann equation for the lepton asymmetry can be approximated by

$$sHz\frac{d\Delta_\ell}{dz} \simeq \epsilon^\Delta \left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta \ . \tag{3.2.20}$$

Considering eqs. (3.2.19) and (3.2.20), the solution is simply

$$\Delta_\ell(z) \simeq -\epsilon^\Delta \left(\Sigma_\Delta(z) - \Sigma_\Delta^0\right) \ , \tag{3.2.21}$$

where Σ_Δ^0 is the triplet abundance before any asymmetry is generated. Since Δ is sensitive to gauge interactions, it is initially produced thermally, so that Σ_Δ^0 is simply given by thermodynamics,

$$\Sigma_\Delta^0 \simeq \frac{45\zeta(3)}{\pi^2 g_*} \ , \tag{3.2.22}$$

where g_* is the effective number of relativistic degrees of freedom in thermal equilibrium. Finally, at the end of leptogenesis the triplets have disappeared, which gives

$$\Delta_\ell \simeq \epsilon^\Delta \Sigma_\Delta^0 \ . \tag{3.2.23}$$

The efficiency η is often defined as

$$\eta = \frac{\Delta_\ell}{\epsilon^\Delta \Sigma_\Delta^0} \ . \tag{3.2.24}$$

In the case we are considering, $\eta \simeq 1$, which is its maximal value given that triplets are initially in thermal equilibrium. The resulting baryon asymmetry is

$$\frac{n_B}{n_\gamma} \simeq \frac{12}{37} \times 7.04 \times \epsilon^{\Delta} \Sigma^0_\Delta \, . \tag{3.2.25}$$

A maximal efficiency $\eta \sim 1$ can similarly be obtained if $B_H \ll B_\ell$. Of course, this is oversimplified. In particular, as can be seen from eq. (3.2.9), if one of the branching ratios is negligible, the CP asymmetry becomes very small. In this minimal scenario, successful leptogenesis requires an interplay between a large asymmetry, favoured by $B_\ell \sim B_H$, and a sufficient efficiency. As we will see, the situation improves when flavour effects are included, because the increase of the number of decay channels makes it easier to find one which is out of equilibrium, without the requirement $B_\ell \ll B_H$.

3.2.3 Introduction of flavour and spectator processes

Taking into account flavour effects and spectator processes – by spectator processes we mean processes that do not directly affect $B - L$ – requires us to split the problem in several temperature ranges and keep only the relevant processes in each one of them, in order to simplify a problem otherwise tremendously difficult. For instance, we consider spectator processes, which come from Standard Model physics, to be either negligible or very fast, which is generally a good approximation except at transitions between two temperature ranges. When they are very fast, their effect can be encoded in relations between chemical potentials, which can be translated in relations between the asymmetries. Since the asymmetry stored in lepton doublets is affected by both Yukawa-mediated interactions of the type $\ell_\alpha + t_R \to e_\alpha + Q_3$ and electroweak sphalerons, it is more convenient to study the evolution of the $B - L$ density, which is unaffected by every Standard Model process. Then, since the washout terms due to new physics are still functions of the lepton doublet asymmetry Δ_ℓ, we need to express the latter as a function of the asymmetry stored in $B - L$, or more precisely as a function of the asymmetries stored in the charges $B/3 - L_\alpha$, just as we expressed Δ_H as a function of Δ_ℓ and Δ_Δ previously.

The impact of Flavour

The relevant temperature range regarding flavour have been discussed in section 3.1.3. Flavour effects in scalar triplet leptogenesis have been first investigated in refs. [206, 207]. Those papers focused mainly on the low-temperature regimes where the tau Yukawa at least is in equilibrium. In particular, ref. [207] provided the correct flavour structure of Boltzmann equations for temperatures lower than 10^9 GeV where all lepton flavours can be treated as distinguishable states. However, both references disregarded flavour effects at higher temperatures by making the assumption that, like in leptogenesis with right-handed neutrinos, the problem could be reduced to one single flavour in the high-temperature regime ($T > 10^{12}$ GeV), and two flavours in the intermediate regime ($10^{12}\,\text{GeV} > T > 10^9$ GeV) where the tau Yukawa is in equilibrium. This approach gives a qualitative understanding of leptogenesis with a scalar triplet and is a good approximation in some regions of parameter space, but it is not completely correct because, as mentioned in 3.1.3 in the case of resonant leptogenesis, it is not possible to consistently define a single flavour that couples to the heavy state. Let us discuss in more details the flavour structure in the three temperature ranges.

(i) Above 10^{12} GeV, the three lepton flavours are undistinguishable, but, because of the structure of the coupling $\Delta\ell\ell$ which is not linear in the lepton doublets, it is in general not possible to find a basis $(\ell_0,\, \ell_1^{\perp}, \ell_2^{\perp})$ such that the triplet decays only to ℓ_0, like it was done in 3.1.3 for the right-handed neutrino. This means that the single flavour approximation is never accurate in this scenario, even for temperatures larger than 10^{12} GeV. Instead, since the decay of the triplet generates states which are mixtures of the three undistinguishable flavours, the lepton asymmetry should be described by a 3×3 density matrix $(\Delta_\ell)_{\alpha\beta}$.

In this high-temperature regime, it is convenient to define the $B - L$ asymmetry as a 3×3 density matrix as well as follows,

$$\Delta_{\alpha\beta} = \frac{\delta_{\alpha\beta}}{9} \sum_{i=1,2,3} (\Delta_{Q_i} + \Delta_{u_i} + \Delta_{d_i}) - (\Delta_\ell)_{\alpha\beta} \ , \quad \alpha, \beta \in \{e, \mu, \tau\} \ , \tag{3.2.26}$$

where Δ_{Q_i}, Δ_{u_i} and Δ_{d_i} are the asymmetries stored in the quarks. The motivations for this choice are the following: the diagonal elements of this matrix describe the asymmetries stored in the charges $B/3 - L_\alpha$, while the trace gives the total density of $B - L$. Moreover, this is also flavour-covariant: under a flavour rotation, it transforms as in eq. (3.1.31). The reason why we do not include the right-handed lepton asymmetry in this definition is because, in this regime, this asymmetry is always zero since no interaction able to create it is fast enough.

(ii) Between 10^{12} and 10^9 GeV, fast interactions mediated by the tau Yukawa come in equilibrium. These interactions will typically project the states created in the decay of Δ on the tau direction or on the $e - \mu$ space. As a consequence, the off-diagonal elements $(\Delta_\ell)_{\alpha\tau}$ and $(\Delta_\ell)_{\tau\alpha}$ decay very quickly. Thus, the asymmetry stored in the tau doublet can be described separately while the asymmetry stored in the $e - \mu$ space is still described by a 2×2 density matrix. Another consequence is that, because of interactions such as $\ell_\tau + t_R \to \tau_R + Q_3$, a part of the asymmetry stored in the tau doublets will be transferred to the right-handed tau.

In this regime, we split the $B - L$ asymmetry into a 2×2 density matrix and a separate asymmetry for $B/3 - L_\tau$,

$$\Delta_{\alpha\beta} = \frac{\delta_{\alpha\beta}}{9} \sum_{i=1,2,3} (\Delta_{Q_i} + \Delta_{u_i} + \Delta_{d_i}) - (\Delta_\ell)_{\alpha\beta} \ , \quad \alpha,\, \beta \in \{e,\, \mu\} \ , \tag{3.2.27}$$

$$\Delta_\tau = \frac{1}{9} \sum_{i=1,2,3} (\Delta_{Q_i} + \Delta_{u_i} + \Delta_{d_i}) - \Delta_{\ell_\tau} - \Delta_{\tau_R} \ . \tag{3.2.28}$$

(iii) Below 10^9 GeV, the situation is simpler to apprehend, because we do not have to deal with undistinguishable states as before, therefore we recover a situation which is easier to understand intuitively, with three lepton doublet asymmetries Δ_{ℓ_e}, Δ_{ℓ_μ} and Δ_{ℓ_τ}, and the asymmetries in the $B/3 - L_\alpha$ charges are given by

$$\Delta_\alpha = \frac{1}{9} \sum_{i=1,2,3} (\Delta_{Q_i} + \Delta_{u_i} + \Delta_{d_i}) - \Delta_{\ell_\alpha} - \Delta_{e_\alpha} \ . \tag{3.2.29}$$

Note that in this regime, we could still in principle work with a 3×3 density matrix, but it would be diagonal. In this regime, our approach and those of refs. [206, 207] agree.

In the present work, for a given triplet mass, we fix the flavour structure once and for all based on which Yukawa-mediated scatterings are in equilibrium at $T = M_\Delta$. Concretely, this means that we work with a 3×3 density matrix for $M_\Delta > 10^{12}$ GeV, a 2×2 density matrix and a separate asymmetry for the tau for $10^9 < M_\Delta < 10^{12}$ GeV, and three number asymmetries for $M_\Delta < 10^9$ GeV. A fully rigorous approach would be to include explicitly Yukawa-mediated scatterings in Boltzmann equations. In particular, in the context of leptogenesis with right-handed neutrinos, it was pointed out in ref. [208] that, even when these scatterings are in equilibrium, they do not necessarily affect the flavour structure of Boltzmann equation as long as they are slower than inverse decays $\ell H \to N_1$. Ref. [207] adapted this discussion to leptogenesis with a scalar triplets. To summarize, the flavour structure is in reality determined by whichever process is faster. The difficulty comes from the fact that this changes over time: for instance, inverse decays can be faster around $T = M_\Delta$, but at later times, Yukawa-mediated scatterings always dominate. In 3.2.4, we will see how our simplifying approximation affects the results.

Spectator processes

We also have to include the various spectator processes. As already said, in each temperature range, the fast reactions in equilibrium impose relations between the chemical potentials and therefore between the asymmetries. These relations take the form of a linear system of equations. In the end, every asymmetry can be expressed as a function of the $B - L$ asymmetry and the triplet asymmetry. Then, to write Boltzmann equations, we will only need the explicit expression of the Δ_ℓ's and Δ_H, which we will write formally as $\Delta_a = C_a^k \Delta_k$.

The relevant processes for the computation of chemical potentials in the context of scalar triplet leptogenesis were exhaustively discussed in ref. [207] in a non-flavour-covariant framework (see previous discussion). We adapted the expressions presented in this work to our flavour-covariant formalism: in our case, the coefficients C_a^k are defined by the following relations.

(i) In the high-temperature range $T > 10^{12}$ GeV, where both the lepton asymmetry and the asymmetry in $B - L$ are 3×3 matrices,

$$\begin{aligned} (\Delta_\ell)_{\alpha\beta} &= C_{\alpha\beta}^{\gamma\delta} \Delta_{\gamma\delta} + C_{\alpha\beta}^{\Delta} \Delta_\Delta \ , \\ \Delta_H &= C_H^{\gamma\delta} \Delta_{\gamma\delta} + C_H^{\Delta} \Delta_\Delta \ . \end{aligned} \tag{3.2.30}$$

(ii) In the intermediate range 10^9 GeV $< T < 10^{12}$ GeV, we define

$$\begin{aligned} (\Delta_\ell)_{\alpha\beta} &= C_{\alpha\beta}^{\gamma\delta} \Delta_{\gamma\delta} + C_{\alpha\beta}^{\tau} \Delta_\tau + C_{\alpha\beta}^{\Delta} \Delta_\Delta \ , \quad (\alpha, \beta, \gamma, \delta) \in \{e, \mu\} \\ \Delta_{\ell_\tau} &= C_\tau^{\gamma\delta} \Delta_{\gamma\delta} + C_\tau^{\tau} \Delta_\tau + C_\tau^{\Delta} \Delta_\Delta \ , \\ \Delta_H &= C_H^{\gamma\delta} \Delta_{\gamma\delta} + C_H^{\tau} \Delta_\tau + C_H^{\Delta} \Delta_\Delta \ . \end{aligned} \tag{3.2.31}$$

(iii) Finally, in the low temperature range 10^9 GeV $> T$, this reduces to

$$\begin{aligned} \Delta_{\ell_\alpha} &= C_\alpha^\gamma \Delta_\gamma + C_\alpha^\Delta \Delta_\Delta \ , \\ \Delta_H &= C_H^\gamma \Delta_\gamma + C_H^\Delta \Delta_\Delta \ . \end{aligned} \tag{3.2.32}$$

Let us review quickly the various effects which are relevant for the computation of these coefficients. A reaction $a_1 + ... + a_n \to b_1 + ... + b_n$ in equilibrium enforces the equality on the chemical potentials of the a_i's and $b_i's$

$$\mu_{a_1} + ... + \mu_{a_n} - \mu_{b_1} - ... - \mu_{b_n} = 0 \ . \tag{3.2.33}$$

Using the relation between the chemical potential and the asymmetry

$$n_f - n_{\bar{f}} = \frac{g_f T^3}{6} \mu_f \quad \text{for a fermion} \ , \tag{3.2.34}$$

$$n_b - n_{\bar{b}} = \frac{g_b T^3}{6} \mu_b \quad \text{for a boson} \ , \tag{3.2.35}$$

where g_f and g_b are the numbers of degrees of freedom of each field, we can translate eq. (3.2.33) in a condition on the asymmetries. First of all, the conservation of the hypercharge is always valid and implies

$$\sum_{i=1,2,3} (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i}) \ - \sum_{\alpha=e,\mu,\tau} (\mu_{\ell_\alpha} + \mu_{e_\alpha}) + 2\mu_H + 6\mu_\Delta \ = \ 0 \ . \tag{3.2.36}$$

Then, the QCD and electroweak sphalerons, which come in equilibrium below 10^{13} and 10^{12} GeV respectively, enforce

$$\sum_{i=1,2,3} (2\mu_{Q_i} - \mu_{u_i} - \mu_{d_i}) = 0 \quad \text{(QCD)} \ , \tag{3.2.37}$$

$$\sum_{i=1,2,3} 3\mu_{Q_i} + \sum_{\alpha=e,\mu,\tau} \mu_{\ell_\alpha} = 0 \quad \text{(EW)} \ . \tag{3.2.38}$$

The various Yukawa-mediated scatterings translate into

$$\mu_{Q_i} - \mu_{u_i} + \mu_H = 0 \quad \text{(up-type quark)} \ , \tag{3.2.39}$$

$$\mu_{Q_i} - \mu_{d_i} - \mu_H = 0 \quad \text{(down-type quark)} \ , \tag{3.2.40}$$

$$\mu_{\ell_\alpha} - \mu_{e_\alpha} - \mu_H = 0 \quad \text{(charged lepton)} \ . \tag{3.2.41}$$

These reactions and the range in which they are relevant are summarized in table 3.1. We can use all the previous constraints to compute the C coefficients.

Boltzmann equations

The Boltzmann equation for the triplet density Σ_Δ is unaffected by these new effects, therefore eq. (3.2.11) remains valid in any temperature range. The $B - L$ density is affected only by interactions contained in the Lagrangian of eq. (2.1.57), so that deriving the Boltzmann equation for the matrix $\Delta_{\alpha\beta}$ or for Δ_α is not more difficult than writing the Boltzmann equation for the lepton asymmetry when spectator processes

Temperature range (GeV)	SM reactions in equilibrium
$T > 10^{15}$	None
$[10^{13}, 10^{15}]$	y_t
$[10^{12}, 10^{13}]$	+ QCD sphalerons
$[10^{9}, 10^{12}]$	$+ y_b + y_\tau + y_c$ + EW sphalerons
$[10^{5}, 10^{9}]$	$+ y_\mu + y_s$
$T < 10^{5}$	$+ y_u + y_d + y_e$

Table 3.1: SM reactions in equilibrium depending on the temperature. y_a stands for scatterings mediated by the Yukawa coupling of particle a.

are neglected. The only subtlety is that only the part of the $B - L$ density which is stored in the lepton doublets is affected by the washout, so we should write the right-hand side like before in terms of Δ_ℓ and then make the substitution $\Delta_\ell \to C_\ell^k \Delta_k$. For temperatures larger than 10^{12} GeV, the Boltzmann equation for the $B-L$ density and the triplet asymmetry is given by

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = -\epsilon^\Delta_{\alpha\beta}\left(\frac{\Sigma_\Delta}{\Sigma^{\rm eq}_\Delta} - 1\right)\gamma^\Delta_D + W^{ID}_{\alpha\beta} + W^{\ell H}_{\alpha\beta} + w^{4\ell}_{\alpha\beta} + w^{\ell\Delta}_{\alpha\beta}\,. \tag{3.2.42}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -\frac{1}{2}\,\mathrm{tr}\left[W^{ID}\right] - B_H\left(\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} - \frac{C^k_H\Delta_k}{Y^{\rm eq}_H}\right)\gamma^\Delta_D\,. \tag{3.2.43}$$

The terms $W^{ID}_{\alpha\beta}$ and $W^{\ell H}_{\alpha\beta}$ are washout terms respectively due to inverse decays $\ell\ell \to \Delta^c$ and lepton-Higgs scatterings $\ell\ell \to H^cH^c$ and $\ell H \to \ell^c H^c$. These terms where already present in our simplified model eq. (3.2.16). The two last terms on the right-hand side of eq. (3.2.42), labelled with a lower case w, are new, because they describe pure flavour effects such as $\ell_\alpha\ell_\beta \to \ell_\gamma\ell_\delta$, which do not modify the total lepton number, and therefore satisfy $\mathrm{tr}[w] = 0$. The source and washout terms come with opposite sign with respect to eq. (3.2.16), because we consider now the evolution of the $B-L$ density.

The derivation of flavoured source and washout terms for the density matrix, using the Closed Time-Path formalism (CTP), is described in Appendix A.4. For temperatures larger than 10^{12} GeV, the 3×3 matrix of CP asymmetry is

$$\epsilon^\Delta_{\alpha\beta} = \frac{1}{8\pi i}\frac{M_\Delta}{v^2}\sqrt{B_\ell B_H}\,\frac{(m_\kappa m_\Delta^\dagger - m_\Delta m_\kappa^\dagger)_{\alpha\beta}}{\bar{m}_\Delta}\,. \tag{3.2.44}$$

For the clarity of notations, we write the washout terms as functions of Δ_ℓ and Δ_H, but it should be remembered that, in order to give a closed form to Boltzmann equations, these quantities are to be expressed as $C^k_\ell\Delta_k$ and $C^k_H\Delta_k$. The washout term due to inverse decays is given in terms of Δ_ℓ by

$$W^{ID}_{\alpha\beta} = \frac{2B_\ell}{\lambda_\ell^2}\left[(ff^\dagger)_{\alpha\beta}\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} + \frac{1}{4Y^{\rm eq}_\ell}\left(2f\Delta_\ell^T f^\dagger + ff^\dagger\Delta_\ell + \Delta_\ell ff^\dagger\right)_{\alpha\beta}\right]\gamma^\Delta_D\,. \tag{3.2.45}$$

The term $W^{\ell H}_{\alpha\beta}$, describing scatterings between lepton doublets and Higgs doublets, is given by the somewhat cumbersome expression

$$\begin{aligned}W^{\ell H}_{\alpha\beta} \;=\; & \frac{2}{\lambda_\ell^2}\left[(ff^\dagger)_{\alpha\beta}\frac{\Delta_H}{Y_H^{\rm eq}}+\frac{1}{4Y_\ell^{\rm eq}}\left(2f\Delta_\ell^T f^\dagger + ff^\dagger\Delta_\ell + \Delta_\ell f f^\dagger\right)_{\alpha\beta}\right]\gamma^{\Delta}_{\ell H}\\
+&\frac{2}{\Re\left[\mathrm{tr}(f\kappa^\dagger)\right]}\left[(f\kappa^\dagger)_{\alpha\beta}\frac{\Delta_H}{Y_H^{\rm eq}}+\frac{1}{4Y_\ell^{\rm eq}}\left(2f\Delta_\ell^T \kappa^\dagger + f\kappa^\dagger\Delta_\ell + \Delta_\ell f \kappa^\dagger\right)_{\alpha\beta}\right]\gamma^{\mathcal{I}}_{\ell H}\\
+&\frac{2}{\Re\left[\mathrm{tr}(f\kappa^\dagger)\right]}\left[(\kappa f^\dagger)_{\alpha\beta}\frac{\Delta_H}{Y_H^{\rm eq}}+\frac{1}{4Y_\ell^{\rm eq}}\left(2\kappa\Delta_\ell^T f^\dagger + \kappa f^\dagger\Delta_\ell + \Delta_\ell \kappa f^\dagger\right)_{\alpha\beta}\right]\gamma^{\mathcal{I}}_{\ell H}\\
&+\frac{2}{\lambda_\kappa^2}\left[(\kappa\kappa^\dagger)_{\alpha\beta}\frac{\Delta_H}{Y_H^{\rm eq}}+\frac{1}{4Y_\ell^{\rm eq}}\left(2\kappa\Delta_\ell^T \kappa^\dagger + \kappa\kappa^\dagger\Delta_\ell + \Delta_\ell \kappa\kappa^\dagger\right)_{\alpha\beta}\right]\gamma^{\kappa}_{\ell H}\ ,\end{aligned}\tag{3.2.46}$$

where $\gamma^{\Delta}_{\ell H}$, $\gamma^{\kappa}_{\ell H}$ and $\gamma^{\mathcal{I}}_{\ell H}$ are respectively the contributions of the triplet, the effective operator of eq. (3.2.4) and the interference term to $\ell\ell \to H^c H^c$ and $\ell H \to \ell^c H^c$. More precisely we define

$$\gamma_{\ell H} = 2\gamma'^{\ell\ell}_{H^cH^c} + \gamma^{\ell H}_{\ell^c H^c}\ ,\tag{3.2.47}$$

where the prime means that we subtracted the contribution of an on-shell intermediate state, as it is explained in appendix A.2, and split $\gamma_{\ell H}$ into these three contributions

$$\gamma_{\ell H} = \gamma^{\Delta}_{\ell H} + 2\gamma^{\mathcal{I}}_{\ell H} + \gamma^{\kappa}_{\ell H}\ .\tag{3.2.48}$$

A way to check the consistency of these two first washout terms is to make sure that when taking the single flavour approximation, we recover the washout terms of eq. (3.2.16). The two purely flavoured washout terms are given by

$$w^{4\ell}_{\alpha\beta} \;=\; \frac{2}{\lambda_\ell^4}\left[\frac{\lambda_\ell^2}{4Y_\ell^{\rm eq}}\left(2f\Delta_\ell^T f^\dagger + ff^\dagger\Delta_\ell+\Delta_\ell ff^\dagger\right)_{\alpha\beta} - \frac{\mathrm{tr}(\Delta_\ell ff^\dagger)}{Y_\ell^{\rm eq}}(ff^\dagger)_{\alpha\beta}\right]\gamma_{4\ell}\,,\tag{3.2.49}$$

$$w^{\ell\Delta}_{\alpha\beta} \;=\; \frac{1}{\mathrm{tr}(ff^\dagger ff^\dagger)}\left[\frac{1}{2Y_\ell^{\rm eq}}\left(ff^\dagger ff^\dagger\Delta_\ell - 2ff^\dagger\Delta_\ell ff^\dagger + \Delta_\ell ff^\dagger ff^\dagger\right)_{\alpha\beta}\right]\gamma_{\ell\Delta}\,,\tag{3.2.50}$$

where we defined

$$\gamma_{4\ell} = 2\gamma'^{\ell\ell}_{\ell\ell} + \gamma^{\ell\ell^c}_{\ell\ell^c}\ ,\tag{3.2.51}$$

$$\gamma_{\ell\Delta} = \gamma^{\ell\Delta}_{\ell\Delta} + \gamma'^{\ell\Delta^c}_{\ell\Delta^c} + \gamma^{\ell\ell^c}_{\Delta\Delta^c}\ ,\tag{3.2.52}$$

the rates being summed over every flavour. Again, we can check that these two purely flavoured washout processes vanish in the limit of a single flavour.

Another important check is that the equation obtained is covariant in flavour space, as was explained in the introduction of 3.2.3. Under a unitary rotation $\ell_\alpha \to \ell'_i = V^*_{i\alpha}\ell_\alpha$, the density matrix and the couplings transform according to

$$\begin{aligned}\Delta_{\alpha\beta}\to\Delta'_{ij} &= (V\Delta V^\dagger)_{ij}\ , &\qquad (\Delta_\ell)_{\alpha\beta}\to(\Delta'_\ell)_{ij} &= (V\Delta_\ell V^\dagger)_{ij}\ ,\\ f_{\alpha\beta}\to f'_{ij} &= (VfV^T)_{ij}\ , & \kappa_{\alpha\beta}\to\kappa'_{ij} &= (V\kappa V^T)_{ij}\ .\end{aligned}\tag{3.2.53}$$

which ensures that all the source and washout term are flavour covariant, namely that they transform like the density matrix[3],

$$S_{\alpha\beta} \to S'_{ij} = \left(VSV^\dagger\right)_{ij} \ , \qquad W_{\alpha\beta} \to W'_{ij} = \left(VWV^\dagger\right)_{ij} \ . \tag{3.2.54}$$

When the temperature drops below 10^{12} GeV, Boltzmann equations for the 2×2 density matrix and the τ asymmetry can easily be deduced from the previous case by performing the replacements $\Delta_{\alpha\tau} \to \delta_{\alpha\tau}\Delta_\tau$ and $(\Delta_\ell)_{\alpha\tau} \to \delta_{\alpha\tau}\Delta_{\ell_\tau}$. Formally, the system to solve becomes

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = -\epsilon^\Delta_{\alpha\beta}\left(\frac{\Sigma_\Delta}{\Sigma^{\rm eq}_\Delta}-1\right)\gamma^\Delta_D + W^{ID}_{\alpha\beta} + W^{\ell H}_{\alpha\beta} + w^{4\ell}_{\alpha\beta} + w^{\ell\Delta}_{\alpha\beta} \ , \quad \alpha,\beta = e,\mu \ , \tag{3.2.55}$$

$$sHz\frac{d\Delta_\tau}{dz} = -\epsilon_{\tau\tau}\left(\frac{\Sigma_\Delta}{\Sigma^{\rm eq}_\Delta}-1\right)\gamma^\Delta_D + W^{ID}_\tau + W^{\ell H}_\tau + w^{4\ell}_\tau + w^{\ell\Delta}_\tau \ , \tag{3.2.56}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -\frac{1}{2}\left(W^{ID}_{ee} + W^{ID}_{\mu\mu} + W^{ID}_\tau\right) - B_H\left(\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} - \frac{C^k_H\Delta_k}{Y^{\rm eq}_H}\right)\gamma^\Delta_D \ , \tag{3.2.57}$$

with, for instance,

$$W^{ID}_{\alpha\beta} = \frac{2B_\ell}{\lambda^2_\ell}\left[(ff^\dagger)_{\alpha\beta}\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} + \frac{1}{4Y^{\rm eq}_\ell}\left(2f(\Delta_\ell)^T_{|e,\mu}f^\dagger + ff^\dagger(\Delta_\ell)_{|e,\mu} + (\Delta_\ell)_{|e,\mu}ff^\dagger\right)_{\alpha\beta} + \frac{1}{2Y^{\rm eq}_\ell}\, f_{\alpha\tau}f^*_{\beta\tau}\Delta_{\ell_\tau}\right]\gamma^\Delta_D \ , \tag{3.2.58}$$

where we used the notation $(\Delta_\ell)_{|e,\mu}$ to make clear that this is the 2×2 matrix describing the $e-\mu$ subspace, and

$$W^{ID}_\tau = \frac{2B_\ell}{\lambda^2_\ell}\left[(ff^\dagger)_{\tau\tau}\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} + \frac{1}{2Y^{\rm eq}_\ell}\left(\sum_{\alpha,\beta=e,\mu} f_{\alpha\tau}f^*_{\beta\tau}(\Delta_\ell)_{\beta\alpha} + \left((ff^\dagger)_{\tau\tau} + |f_{\tau\tau}|^2\right)\Delta_{\ell_\tau}\right)\right]\gamma^\Delta_D \ . \tag{3.2.59}$$

Finally, as the temperature drops below 10^9 GeV, Yukawa-mediated scatterings bring all the off-diagonal elements to zero. Again the flavour structure Boltzmann equations can be deduced from the high-temperature case by setting $(\Delta_\ell)_{\alpha\beta} \to \delta_{\alpha\beta}\Delta_{\ell_\alpha}$. Formally, the system describing the asymmetries now includes three equations for the

[3] This can be checked explicitly for the various terms, for instance

$$f\Delta^T_\ell f^\dagger \to f'\Delta'_\ell f'^\dagger = \left(VfV^T\right)\left(V^*\Delta_\ell V^T\right)\left(V^*fV^\dagger\right) = V\left(f\Delta_\ell f\right)V^\dagger \ .$$

Δ_α's with $\alpha = e,\, \mu,\, \tau$ and one for Δ_Δ,

$$sHz\frac{d\Delta_\alpha}{dz} = -\epsilon_{\alpha\alpha}\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} - 1\right)\gamma_D^\Delta + W_\alpha^{ID} + W_\alpha^{\ell H} + w_\alpha^{4\ell} + w_\alpha^{\ell\Delta}\ , \tag{3.2.60}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -\frac{1}{2}\sum_{\alpha=e,\mu,\tau} W_\alpha^{ID} - B_H\left(\frac{\Delta_\Delta}{\Sigma_\Delta^{\rm eq}} - \frac{C_H^k\Delta_k}{Y_H^{\rm eq}}\right)\gamma_D^\Delta\ . \tag{3.2.61}$$

A qualitative word can be said before proceeding to the numerical computations. When discussing about the simplified model in 3.2.2, we showed that the CP asymmetry is efficiently converted into a lepton asymmetry when the washout is weak. This remark remains true once flavour effects and spectator processes are included in the computation: in particular, in the cases studied in our simplified model, with $\lambda_\ell \ll \lambda_H$ or $\lambda_H \gg \lambda_\ell$, the efficiency will also be large. For instance, we focus on the case $\lambda_\ell \ll \lambda_H$ in the high-temperature regime $T > 10^{12}$ GeV. The washout of the asymmetry stored in lepton doublets, which is essentially governed by λ_ℓ, can be neglected in first approximation (this holds as long as λ_κ/Λ is not too large). Then, the equation for the $B-L$ asymmetry can be approximated by

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = -\epsilon_{\alpha\beta}^\Delta\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} - 1\right)\gamma_D^\Delta\ . \tag{3.2.62}$$

Taking the trace, we recover the unflavoured equation

$$sHz\frac{d\Delta_{B-L}}{dz} = -\epsilon^\Delta\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} - 1\right)\gamma_D^\Delta\ , \tag{3.2.63}$$

and therefore the result is the same. The same kind of reasoning applies if $\lambda_H \ll \lambda_\ell$, leading to a large efficiency in both the flavoured and the unflavoured computation. However, for us, such situations are not the most interesting ones, because then, flavour effects are of little relevance. The novelty here is that there can also be situations in which, even though the total washout is strong, some flavours are weakly washed out, so that the $B-L$ asymmetry can grow anyway. Therefore, flavour is more likely to be relevant for situations with $B_H \sim B_\ell$, for which the unflavoured computation would give a small efficiency. We expect flavour effects to play a particularly important role when the asymmetry is stored mainly in flavours that are little washed out, because such cases are overlooked by the unflavoured computation.

3.2.4 Numerical approach

Typical solutions of Boltzmann equations are displayed in fig. 3.4. These computations were performed in the high-temperature regime, using the 3×3 density matrix formalism. However, the qualitative behaviour of the solutions displayed here is not a characteristic feature of this flavour regime. For the parameters chosen here, the CP asymmetry is pretty large, namely $\epsilon^\Delta = -3.30\times 10^{-4}$.

The difference between the two plots of fig. 3.4 is the initial condition. For the left plot, we chose for the initial triplet density the equilibrium value $\Sigma_\Delta^{\rm eq}$ defined in eq. (3.2.22), which seems a good choice for a gauge multiplet. In this case, the triplet

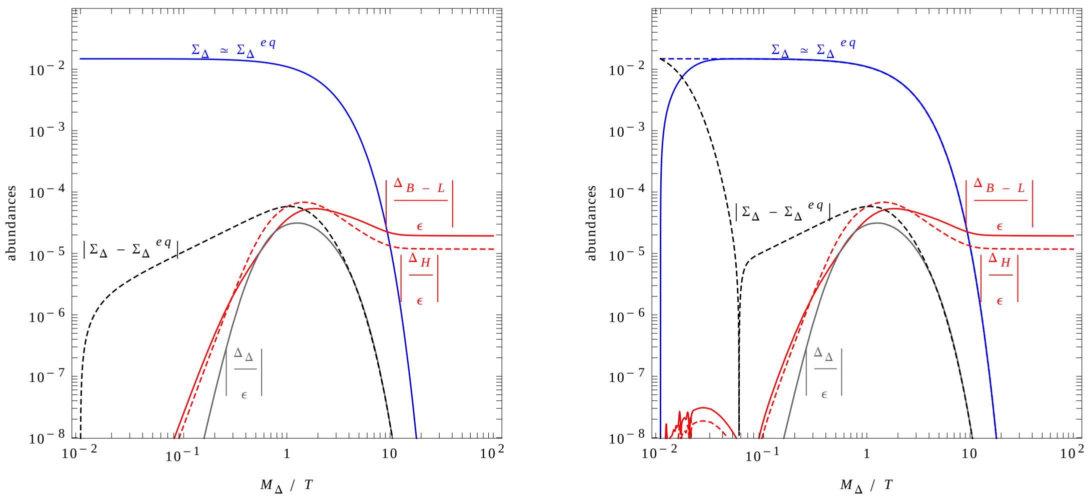


Figure 3.4: Comoving number density of triplets and antitriplets Σ_Δ and asymmetries Δ_Δ, Δ_H and $\Delta_{B-L} = \mathrm{tr}(\Delta_{\alpha\beta})$ as a function of $z = M_\Delta/T$, for $M_\Delta = 5 \times 10^{12}$ GeV, $\lambda_H = 0.1$ and $m_\Delta = i\, m_\nu$. The initial triplet density at $z = 10^{-2}$ is chosen to be the equilibrium density $\Sigma_\Delta^{\rm eq}$ (left) or zero (right). Also shown is the departure of Σ_Δ from its equilibrium value. Asymmetries are plotted in units of the total CP asymmetry in triplet decays ϵ^Δ.

density remains always very close to equilibrium, but nevertheless a sizeable baryon asymmetry can grow, leading to a baryon-to-photon ratio $n_B/n_\gamma = 1.46 \times 10^{-8}$. When we assume instead a bit arbitrarily a vanishing initial abundance for the triplets (right plot), it appears they are anyway quickly generated by gauge interactions and reach soon their equilibrium density. After this first phase, we recover the same behavior as in the previous case, and the final baryon-to-photon ratio is barely modified, $n_B/n_\gamma = 1.45 \times 10^{-8}$. This justifies retrospectively the choice of an initial equilibrium density.

In what follows, we will always perform computations starting from $z_{\rm min} = 10^{-2}$, with triplets initially in equilibrium $\Sigma_\Delta(z_{\rm min}) = \Sigma_\Delta^{\rm eq}$, knowing that this choice has little influence on the final result, besides being better motivated. We will now present more general results, obtained by exploring parameter space. The results summarized here were presented in details in ref. [209]

Computations

In what follows we will compare our results, obtained by solving numerically the Boltzmann equations derived in section 3.2.3 with the relevant constraints coming from chemical equilibriums, which for convenience we will refer to as the "full computation" – even though we cannot claim that it is exhaustive since, for instance, it is still performed in the Boltzmann approximation – to those obtained when performing the following approximations.

(i) In order to identify the effect of spectator processes alone, we perform the approximation of including flavour but neglecting completely spectator processes.

More precisely, we solve the Boltzmann equation for the 3×3 density matrix (3.2.42) and we use the relations

$$(\Delta_\ell)_{\alpha\beta} = -\Delta_{\alpha\beta} \ , \tag{3.2.64}$$
$$\Delta_H = -\operatorname{tr}[\Delta] - 2\Delta_\Delta \ , \tag{3.2.65}$$

that apply when no spectator process is in equilibrium, to express Δ_ℓ and Δ_H whatever the temperature range. In other words, we use in every range a Boltzmann equation that, strictly speaking, is valid only above 10^{15} GeV. We will refer to this computation as FC (Flavour-Covariant)

(ii) Conversely, in order to identify the effect of flavour alone, and more precisely in the high temperature regimes where the density matrix formalism should apply, we perform a computation that does not use this formalism, but includes this time spectator processes through the relations coming from the relevant chemical equilibriums. More precisely, for $T > 10^{12}$ GeV, we consider one single flavour and use relations of the form

$$\Delta_{\ell_0} = C_0^0 \Delta_0 + C_0^\Delta \Delta_\Delta \ , \tag{3.2.66}$$
$$\Delta_H = C_H^0 \Delta_0 + C_H^\Delta \Delta_\Delta \ . \tag{3.2.67}$$

where $\Delta_0 = \Delta_{B-L}$ is the total $B-L$ asymmetry. For 10^9 GeV $< T < 10^{12}$ GeV, we consider two flavours (τ and a combination of e and μ that we call ℓ_0), and the C coefficients are defined as

$$\Delta_{\ell_0} = C_0^0 \Delta_0 + C_0^\tau \Delta_\tau + C_0^\Delta \Delta_\Delta \ , \tag{3.2.68}$$
$$\Delta_{\ell_\tau} = C_\tau^0 \Delta_0 + C_\tau^\tau \Delta_\tau + C_\tau^\Delta \Delta_\Delta \ , \tag{3.2.69}$$
$$\Delta_H = C_H^0 \Delta_0 + C_H^\tau \Delta_\tau + C_H^\Delta \Delta_\Delta \ . \tag{3.2.70}$$

We will refer to this computation as 1F+SP (1 Flavour + Spectator Processes) or 2F+SP (2 Flavours + Spectator Processes) depending on the temperature range.

(iii) Finally, we will compare the result with that obtained by solving the simplified model described by eqs. (3.2.11), (3.2.16) and (3.2.17), that does not include flavour nor spectator processes. We will refer to this approach as 1F.

The comparison between the full computation, involving flavour effects and spectator processes, and this latter approximation shows the relevance of flavour even at high temperatures, which is a new feature compared to the leptogenesis scenario presented in 3.1.3.

Parametrization

The main difficuly in the numerical approach is related to the large number of free parameters, just like in the model presented in 3.1.3. Indeed, we need to choose numerically the entries of the two 3×3 coupling matrices, as well as the mass of the triplet M_Δ and its coupling to the Higgs μ (or equivalently the effective coupling λ_H). It is convenient to fix first M_Δ, λ_H and the unknown parameters of the neutrino mass matrix m_ν, which are the Dirac phase δ, the two Majorana phases ρ and σ, the hierarchy, and the mass of the lightest neutrino $m_{\nu 1}$ (NH) or $m_{\nu 3}$ (IH), and finally to parametrize the two contributions to the neutrino mass matrix. Actually, eq. (3.2.5) simplifies this task a little bit, because once the contribution of the triplet m_Δ is chosen, m_κ is fully determined.

The main effect of the running of the neutrino mass matrix at the scale M_Δ is to multiply the eigenvalues by a common factor $r(M_\Delta)$[4] which varies between 1.2 and 1.4 as the triplet mass goes from 10^8 to 10^{14} GeV. We will take this into account simply by rescaling the squared mass splittings Δm^2_{ij} by a factor r^2. To fix things once and for all in this sector, we consider a normal hierarchy with a lightest neutrino mass $m_{\nu 1} = 10^{-3}$ eV at the scale M_Δ, and vanishing phases $\delta = \rho = \sigma = 0$.

The contribution m_Δ is a 3×3 complex matrix, so in full generality it contains three masses, three mixing angles, and six phases. As explained in section 2.1.1, the phases of the lepton fields have already been fixed in order to obtain the structure of the PMNS matrix given in eq. (2.1.16). Thus, we do not have the freedom to eliminate any phase from m_Δ through rephasings of the lepton fields, so that its six phases are physical. We parametrize m_Δ as

$$m_\Delta = U^* \times V^* \times \mathrm{diag}\,(m_{\Delta 1},\, m_{\Delta 2},\, m_{\Delta 3}) \times V^\dagger \times U^\dagger \ , \tag{3.2.71}$$

where U is the PMNS matrix. This parametrization makes explicit the rotation between the basis of neutrino mass eigenstates and the basis in which m_Δ is diagonal, which is encoded in the unitary matrix V. We adopt for V a parametrization similar to that of the PMNS matrix, but with three additional phases, i.e.

$$V = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_3} \end{pmatrix} \times R(\phi_{23},\, \phi_{13},\, \phi_{12},\, \gamma) \times \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix} \ , \tag{3.2.72}$$

where R is defined as

$$R(\phi_{23},\, \phi_{13},\, \phi_{12},\, \gamma) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\gamma} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\gamma} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\gamma} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\gamma} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\gamma} & c_{13}c_{23} \end{pmatrix} \ ,$$
$$c_{ij} = \cos\phi_{ij} \ , \qquad s_{ij} = \sin\phi_{ij} \ . \tag{3.2.73}$$

The choice $\phi_{12} = \phi_{13} = \phi_{23} = 0$ gives $R_{kl} = \delta_{kl}$, which corresponds to the particular situation where the matrix m_Δ is diagonal in the basis of neutrino mass eigenstates.

The matrix m_κ is then simply defined by

$$m_\kappa = m_\nu - m_\Delta \ . \tag{3.2.74}$$

[4]In principle, r also depends on the mass of the lightest neutrino, but this dependence is weak and can be neglected in most cases. An exception to this occurs for quasi-degenerate neutrinos, which we will not consider here.

Thanks to this relation, we can conveniently rewrite the total CP asymmetry of eq. (3.2.9) by noticing that

$$\frac{\Im\left[\mathrm{tr}(m_\Delta^\dagger m_\kappa)\right]}{\bar{m}_\Delta} = \frac{\Im\left[\mathrm{tr}(m_\Delta^\dagger m_\nu)\right]}{\bar{m}_\Delta} \ . \tag{3.2.75}$$

Since $\mathrm{tr}(m_\Delta^\dagger m_\nu)$ is a scalar product, according to Cauchy-Schwarz inequality, the total CP asymmetry is maximal for the choice $m_\Delta \propto i \times m_\nu$, which implies $\beta_1 = \beta_2 = \beta_3 = \pi/4$, $\alpha_1 = \alpha_2 = \gamma = 0$ and $\phi_{12} = \phi_{13} = \phi_{23} = 0$. In this case, the other contribution m_κ is far from being negligible, namely $m_\kappa = \sqrt{2}e^{-i\pi/4}m_\nu$, and the CP asymmetry is simply

$$\epsilon^\Delta = \frac{M_\Delta \bar{m}_\nu}{4\pi v^2}\sqrt{B_\ell B_H} \ , \tag{3.2.76}$$

with $\bar{m}_\nu = \sqrt{\mathrm{tr}(m_\nu^\dagger m_\nu)}$. This is the upper bound for given values of M_Δ and λ_H. However, since we want to study leptogenesis in a flavoured context, the choice that makes the overall CP asymmetry maximal is not necessarily the one that gives the largest baryon asymmetry in the end, because of the interplay between the various decay channels of the triplet. More generally, inserting the expression of eq. (3.2.71) in eq. (3.2.44), the flavoured CP asymmetry is

$$\epsilon^\Delta_{\alpha\beta} = \frac{1}{8\pi i}\frac{M_\Delta}{v^2}\frac{\sqrt{B_\ell B_H}}{\bar{m}_\Delta}\sum_{i,j,k} m_{\nu i} m_{\Delta k}\left[U^*_{\alpha i}U_{\beta j}V_{ik}V_{jk} - U^*_{\alpha j}U_{\beta i}V^*_{ik}V^*_{jk}\right] \ . \tag{3.2.77}$$

In order to focus on the case where the contribution m_Δ is of the same order of magnitude as m_ν (so that m_κ is also of the same order of magnitude or smaller), we will always take $\bar{m}_\Delta = \bar{m}_\nu$. Of course, it would be difficult to cover the whole parameter space, but let us now consider a few interesting and simple ansätze.

Comparison of the various computations

The case of alignment is the most simple one. Indeed, when $\phi_{12} = \phi_{13} = \phi_{23} = 0$, V reduces to a diagonal matrix of phases. Moreover, in this case it is possible to absorb the α_k's into redefinitions of the β_k's, so that V is simply

$$V = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}) \ . \tag{3.2.78}$$

Then the explicit expression of the flavoured CP asymmetry is

$$\epsilon^\Delta_{\alpha\beta} = \frac{M_\Delta}{4\pi v^2}\frac{\sqrt{B_\ell B_H}}{\bar{m}_\Delta}\sum_i m_{\nu i} m_{\Delta i} U^*_{\alpha i}U_{\beta i}\sin 2\beta_i \ , \tag{3.2.79}$$

and, taking the trace, the overall asymmetry is

$$\epsilon^\Delta = \frac{M_\Delta}{4\pi v^2}\frac{\sqrt{B_\ell B_H}}{\bar{m}_\Delta}\sum_i m_{\nu i} m_{\Delta i}\sin 2\beta_i \ . \tag{3.2.80}$$

It is worth noticing that it is possible to be in a situation where the entries of the matrix ϵ^{Δ} are nonzero, but the trace vanishes because of a compensation between the $\sin\beta_i$. In this case, leptogenesis could be successful due to flavour effects, even though the overall CP asymmetry is zero. In the case where the two matrices m_Δ and m_ν are aligned, we considered the two following ansätze.

(i) In the special case already mentioned $m_\Delta = i\, m_\nu$, the overall CP asymmetry is maximal, and the elements of the CP asymmetry matrix are

$$\epsilon^{\Delta}_{\alpha\beta} = \frac{M_\Delta}{4\pi v^2} \frac{\sqrt{B_\ell B_H}}{\bar{m}_\nu} \sum_i {m_\nu}_i^2\, U^*_{\alpha i} U_{\beta i} \ . \tag{3.2.81}$$

In this case, we do not expect flavour effects to be spectacular. For instance, in the high-temperature regime $T > 10^{12}$ GeV, because of flavour covariance, the computation can be performed in the basis (ℓ_1, ℓ_2, ℓ_3) corresponding to neutrino mass eigenstates, in which the CP asymmetry becomes simpler,

$$\epsilon^{\Delta}_{ij} = \frac{M_\Delta}{4\pi v^2} \frac{\sqrt{B_\ell B_H}}{\bar{m}_\nu} {m_\nu}_i^2 \delta_{ij} \ . \tag{3.2.82}$$

Then, it is easy to see that, because $\bar{m}_\nu \simeq m_{\nu 3}$, the CP asymmetry matrix is dominated by its (3, 3) entry,

$$\epsilon^{\Delta}_{ij} \sim \frac{M_\Delta}{4\pi v^2} \sqrt{B_\ell B_H} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{m}_\nu \end{pmatrix} , \tag{3.2.83}$$

so that the asymmetry will be essentially produced in ℓ_3, but since the washout is governed by the couplings f and κ that have the same hierarchy,

$$f_{ij} = i\lambda_\ell\, \delta_{ij}\, \frac{m_{\nu i}}{\bar{m}_\nu} \sim i\lambda_\ell \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{3.2.84}$$

$$\kappa_{ij} = e^{-i\pi/4} \lambda_\kappa\, \delta_{ij}\, \frac{m_{\nu i}}{\bar{m}_\nu} \sim e^{-i\pi/4} \lambda_\kappa \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{3.2.85}$$

it will also principally affect the flavour ℓ_3. In first approximation, it seems that is is enough to consider only this flavour to get the dominant effects. Of course, the discussion above is a bit oversimplified, since the hierarchy between neutrino masses is not huge, and an asymmetry could also develop in the flavours ℓ_1 and ℓ_2, but nevertheless, flavour effects will not change the order of magnitude of the result.

(ii) We also studied the case where $\beta_1 = \beta_2 = \beta_3 = \pi/4$, which optimizes the asymmetry, but the eigenvalues of the two matrices m_ν and m_Δ are different. We will use the parametrization

$$m_{\Delta 1} = \sqrt{1 - x^2} y\, \bar{m}_\nu \ , \quad m_{\Delta 2} = x\, y\, \bar{m}_\nu \ , \quad m_{\Delta 3} = \sqrt{1 - y^2} \bar{m}_\nu \ , \tag{3.2.86}$$

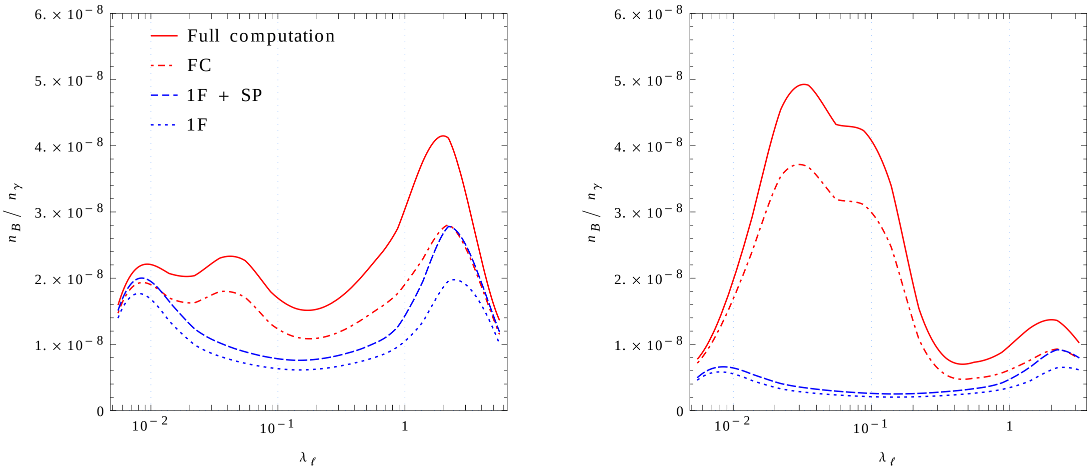


Figure 3.5: Comparison of the final baryon asymmetries obtained via different computations, for a triplet mass $M_\Delta = 5 \times 10^{12}$ GeV, as a function of λ_ℓ, for $m_\Delta = i\, m_\nu$ (left) and $m_\Delta = m_\Delta^{xy}(x = 0.05,\ y = 0.95)$ (right). The equality $B_\ell = B_H$ occurs for $\lambda_\ell \simeq 0.148$. The legend uses the notations defined at the beginning of section 3.2.4.

and study the evolution of the baryon asymmetry when x and y vary. We will refer to this ansatz as m_Δ^{xy}. Note that the previous case can be recovered through the particular choice

$$x = \frac{m_{\nu 2}}{\sqrt{m_{\nu 1}^2 + m_{\nu 2}^2}} \ , \qquad y = \frac{\sqrt{m_{\nu 1}^2 + m_{\nu 2}^2}}{\bar{m}_\nu} \ . \tag{3.2.87}$$

More generally, this ansatz, by relaxing the condition that gives the maximal overall CP asymmetry, allows to look for a possible enhancement of the efficiency due to flavour effects, while giving a large overall CP asymmetry.
This ansatz gives significant flavour effects when λ_ℓ and λ_H are not too different. In the opposite case, we recover a situation where the unflavoured computation gives already a large efficiency, and the introduction of flavour does not modify this result. For $\lambda_\ell \sim \lambda_H$, it appears that the choice $x = 0.95$, $y = 0.05$ is the one that maximizes the final baryon asymmetry, even though it is quite far from the situation $m_\Delta = i m_\nu$ that maximizes the total CP asymmetry.

In figs. 3.5 and 3.6, we compared the results of the various computations for these two ansätze. This confirms that flavour effects are not very large when $m_\Delta = i m_\nu$. At best, they merely enhance the result by a factor 2 with respect to the unflavoured computations. The ansatz m_ν^{xy}, on the other hand, shows how relevant flavour effects can be, with a final baryon asymmetry up to 10 times larger than in the unflavoured approximations.

In the high-temperature regime (fig. 3.5), it appears that the flavour-covariant computation that does not include spectator processes is a decent approximation to the full computation. In comparison, the one-flavour approximations neglect the most

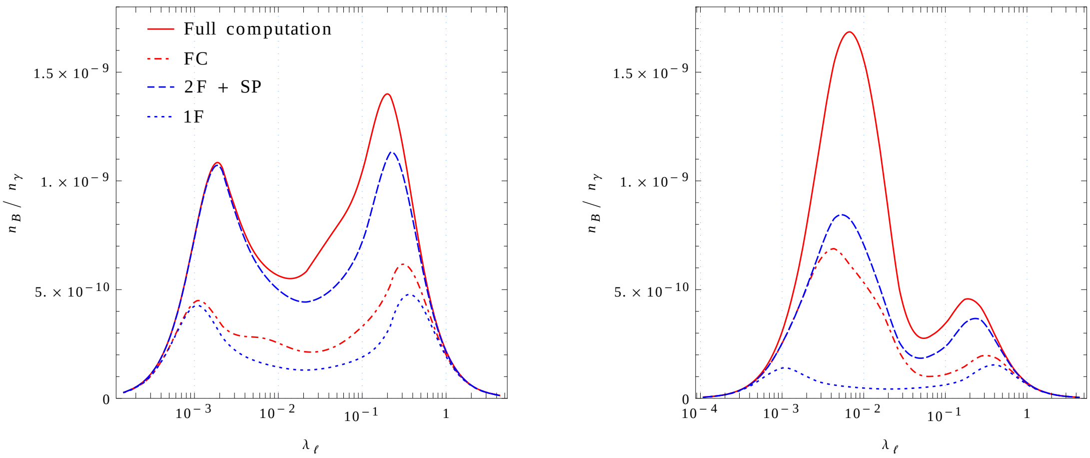


Figure 3.6: Comparison of the final baryon asymmetries obtained via different computations, for a triplet mass $M_\Delta = 10^{11}$ GeV, as a function of λ_ℓ, for $m_\Delta = i\,m_\nu$ (left) and $m_\Delta = m_\Delta^{xy}(x = 0.05,\ y = 0.95)$ (right). The equality $B_\ell = B_H$ occurs for $\lambda_\ell \simeq 2.08 \times 10^{-2}$.

important effect in this regime. When the tau Yukawa coupling is in equilibrium, (fig. 3.6), the two-flavour computation that includes spectator processes is a much better approximation. The reason for this is that, in this regime, one of the main effects of the spectator processes is precisely to destroy the 3×3 flavour covariance. However, there exists situations in which the two-flavour computation is not even a good approximation in the regime where the tau Yukawa is in equilibrium, as we are going to see in the next part.

The comparison of the two ansätze also shows that the choice $m_\Delta = im_\nu$, that gives the maximal overall CP asymmetry does not necessarily maximize the final baryon asymmetry, once flavour effects are properly included.

Discussion on the temperature ranges (continued)

As explained in 3.2.3, in the present work, we choose the flavour structure depending on the triplet mass only, even though leptogenesis could take place in two successive regimes. This approximation may affect the result when the triplet mass is not too far from the temperature at which Yukawa-mediated scatterings reach equilibrium: indeed, if such is the case, the choice of a flavour structure rather than another may seem a bit arbitrary.

In order to evaluate the error made with our approximation, we take $M_\Delta = 5 \times 10^{11}$ GeV, which means that the tau Yukawa-mediated scatterings reach equilibrium during the leptogenesis era. Three computations can then be performed.

- One that includes explicitly Yukawa-mediated scatterings in Boltzmann equations. Explicitly, we add the following washout term [180],

$$W_{\alpha\beta}^{\text{Yukawa}} = -\left[\frac{(\Delta_\ell)_{\alpha\tau}\delta_{\alpha\tau} + (\Delta_\ell)_{\tau\beta}\delta_{\beta\tau}}{2Y_\ell^{\text{eq}}} - \frac{(\Delta_\ell)_{\tau\tau}\delta_{\alpha\tau}\delta_{\beta\tau}}{Y_\ell^{\text{eq}}}\right]\gamma_\tau \ . \tag{3.2.88}$$

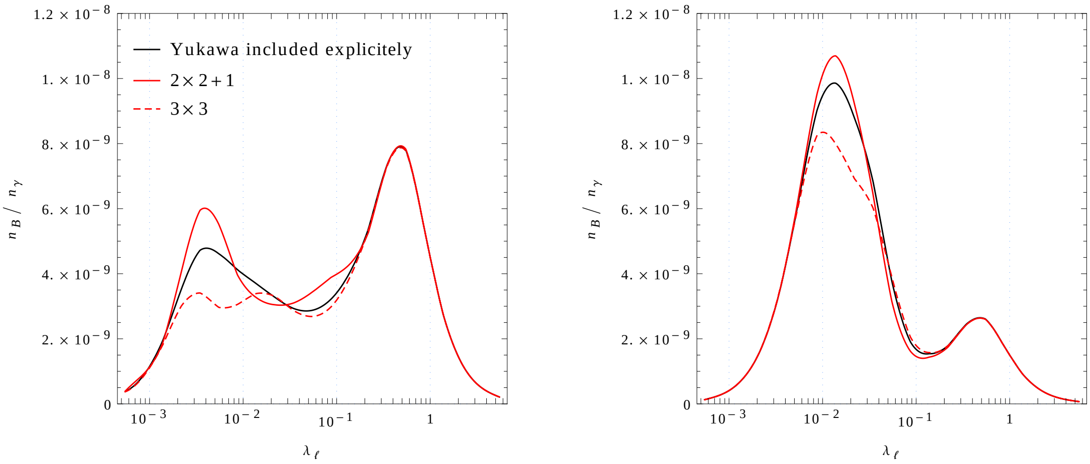


Figure 3.7: Comparison of the final baryon asymmetries obtained via different computations, for a triplet mass $M_\Delta = 5 \times 10^{11}$ GeV, as a function of λ_ℓ, for $m_\Delta = i\, m_\nu$ (left) and $m_\Delta = m_\Delta^{xy}(x = 0.05,\, y = 0.95)$ (right).

- Since the tau Yukawa is in equilibrium at $T = M_\Delta$ (which is the criterium used throughout this work), one can choose to perform the computation with a 2×2 density matrix and a separate tau asymmetry. This amounts to assume that the decay of the off-diagonal elements $\Delta_{\alpha\tau}$ and $\Delta_{\tau\alpha}$ is fast enough. This is the approximation made in the rest of this work. We label this computation "$2 \times 2 + 1$".

- Since inverse decays are in general faster than Yukawa-mediated scatterings at $T = M_\Delta$, one could also choose to perform the computation with a 3×3 density matrix. In this case, one neglects completely the decay of the off-diagonal elements $\Delta_{\alpha\tau}$ and $\Delta_{\tau\alpha}$. We label this approximation "3×3".

On fig. 3.7, we compare the results of these various computations. As could be expected, the result of the computation that includes explicitly Yukawa-mediated scatterings is comprised between the results of the two approximations, which both give errors within a few 10%. Sometimes the agreement is much better, in particular for large and small values of λ_ℓ: this is no surprise because, as already discussed, flavour effects are not very important in such situations, so that the results depend little on the flavour structure. For $m_\Delta = i\, m_\nu$, the approximation "$2 \times 2 + 1$" is slightly better (with an error of at most 25%, against 45% for the approximation "3×3"), while for $m_\Delta = m_\Delta^{xy}(x = 0.05,\, y = 0.95)$, it is in good agreement with the computation that includes explicitly Yukawa-mediated scatterings.

To conclude, the approximation that we use here is not completely accurate when the triplet mass is close to the scale at which Yukawa-mediated scatterings reach equilibrium. This is due to the fact that these scatterings are not really fast enough to destroy completely off-diagonal elements of the density matrix. However, the error remains within 25%, and decreases quickly when the triplet mass is further from the transition scale.

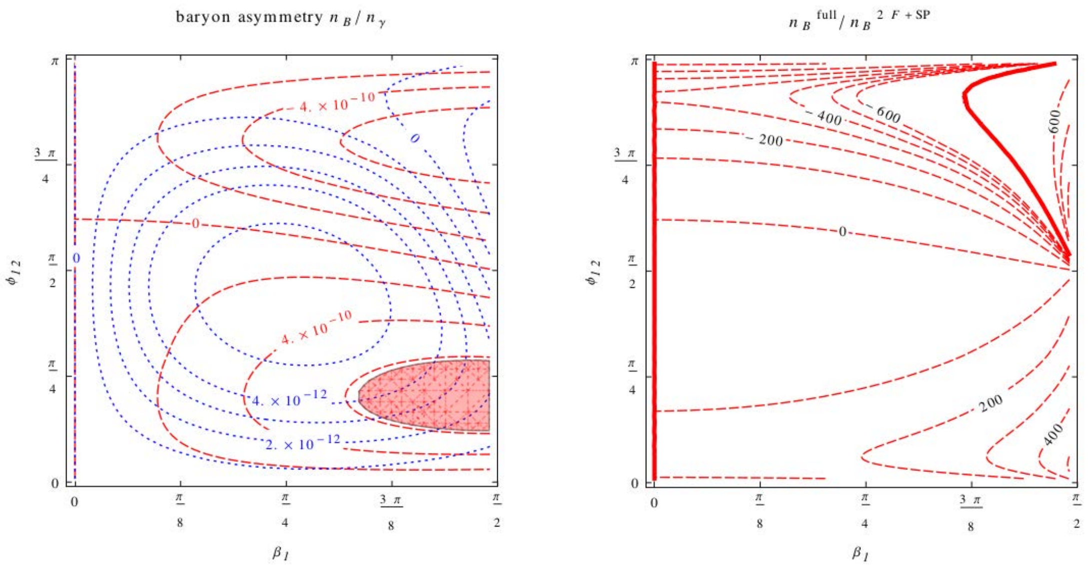


Figure 3.8: Left: isocurves of the final baryon asymmetry as a function of β_1 and ϕ_{12} for the ansatz $m_\Delta = m_\Delta^{\beta\phi}$, $M_\Delta = 5 \times 10^{11}$ GeV and $\lambda_H = 0.1$. Red dashed lines: result of the full computation. Blue dotted lines: result of the 2-flavour computation with spectator processes (2F+SP). The red region indicates values larger than the observed baryon-to-photon ratio 3.1.1. Right: ratio of the results of the two computations $n_B^{\text{full}}/n_B^{2\text{F+SP}}$. The thick red lines signal a vanishing baryon asymmetry from the 2F+SP computation.

Maximal flavour effects

It is also interesting to study the case in which the two matrices m_Δ and m_ν are misaligned, that is when one of the ϕ angles at least is nonzero. We focus on the situation where only the angle ϕ_{12} is nonzero, because it leads to interesting flavour effects. We set the α phases to zero, and studied the case where $\beta_2 = \beta_3 = 0$, whereas β_1 and ϕ_{12} vary. In this case, the rotation matrix is

$$V = \begin{pmatrix} c_{12}e^{i\beta_1} & s_{12}e^{i\beta_1} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.2.89}$$

This ansatz highlights the role of flavour: considering again the high-temperature regime which makes the qualitative discussion simpler, it appears that, in the basis (ℓ_1, ℓ_2, ℓ_3) corresponding to neutrino mass eigenstates, the asymmetry will be essentially generated in the two flavours ℓ_1 and ℓ_2, whereas the washout will essentially affect ℓ_3. Thus, this ansatz gives a non-maximal CP asymmetry but allows for a large enhancement of the final baryon asymmetry due to flavour effects. In the intermediate regime, this discussion is a bit more complicated. However, as we are going to see, it appears from the numerical computation that the final baryon asymmetry is hugely enhanced by flavour effects, and that the two-flavour computation is not a good approximation.

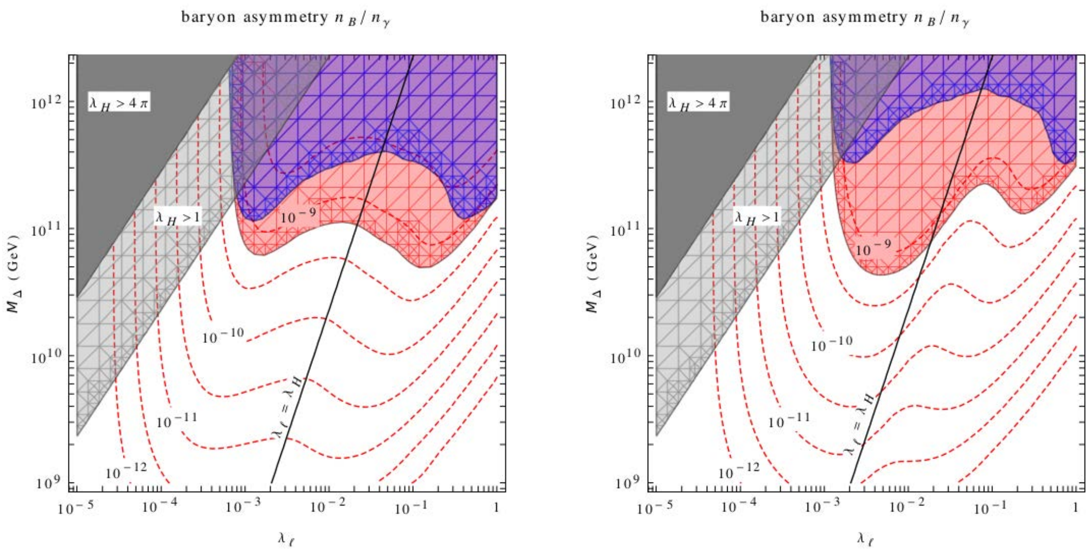


Figure 3.9: Isocurves of the final baryon asymmetry as a function of λ_ℓ and M_Δ, for $m_\Delta = i\, m_\nu$ (left) and $m_\Delta = m_\Delta^{xy}(x = 0.05,\ y = 0.95)$. The black line indicates the equality $\lambda_\ell = \lambda_H$, whereas the region shaded in gray correspond to large (light gray) or non-perturbative (dark gray) values of λ_H. The red and blue regions indicate respectively where the full computation and the approximation 1F reproduce at least the observed value of the baryon-to-photon ratio 3.1.1.

Fig. 3.8 shows the results obtained when performing the full computation and the 2-flavour approximation in the plane $\beta_1 - \phi_{12}$ for the ansatz $m_\Delta^{\beta\phi}$ defined above. The triplet mass is chosen to be $M_\Delta = 5 \times 10^{11}$ GeV, so that leptogenesis essentially takes place in the regime where the tau Yukawa is in equilibrium. This time, taking flavour effects into account in the $e - \mu$ subspace gives a spectacular enhancement of the baryon asymmetry (namely up to a factor $10^2 - 10^3$ compared to the 2-flavour computation), In particular the observed value of the baryon-to-photon ratio, defined in eq. (3.1.1), can be reached, which seems impossible when looking at the result of the 2-flavour computation alone. These results can be understood from the fact that this ansatz reveals a rich flavour structure in the $e - \mu$ subspace, that is overlooked by the two-flavour computation.

Bound on the triplet mass

Finally, in order to study of the order of magnitude of the triplet mass needed to reproduce the baryon-over-photon ratio, we computed the baryon asymmetry generated as a function of M_Δ and λ_ℓ, for $m_\Delta = i\, m_\nu$ and $m_\Delta = m_\Delta^{xy}$ $(x = 0.05,\ y = 0.95)$, which are the two ansätze which optimize the final result, if not the flavour effects. On fig. 3.9, we compare the results obtained to that of the single flavour approximation without spectator processes (1F), that was originally presented in [203]. The lowest bound on the triplet mass is a little bit relaxed by the inclusion of flavour effects and spectator processes. For $m_\Delta = i\, m_\nu$, the lowest bound on the triplet mass are

$$M_\Delta^{\rm min} = 1.3 \times 10^{11}\,\text{GeV} \quad \text{(approximation 1F)}\ , \tag{3.2.90}$$
$$M_\Delta^{\rm min} = 5.0 \times 10^{10}\,\text{GeV} \quad \text{(full computation)}\ . \tag{3.2.91}$$

For the other ansatz the improvement is somewhat clearer, with a lowest bound that decreases by a factor 8 approximately.

$$M_\Delta^{\rm min} = 3.3 \times 10^{11}\,\text{GeV} \quad \text{(approximation 1F)}\ , \tag{3.2.92}$$
$$M_\Delta^{\rm min} = 4.3 \times 10^{10}\,\text{GeV} \quad \text{(full computation)}\ . \tag{3.2.93}$$

These results can be compared to those shown in fig. 3.6. For $m_\Delta = i\, m_\nu$, we saw on fig. 3.6 that, for a given triplet mass, the proper inclusion of flavour effects and spectator processes increases the maximal baryon asymmetry by a factor 3. The same effects allows to lower the bound on the triplet mass by a factor 2.6. On the other hand, for the other ansatz, fig. 3.6 shows that for a given triplet mass the final baryon asymmetry can be enhanced by a factor 10, which in turns allows to achieve successful leptogenesis with a significantly lighter triplet. Consistently with fig. 3.6, one can see that, in the latter case, the most favorable situation is when λ_ℓ is slightly smaller than λ_H, because this gives both a nearly maximal CP asymmetry and a good efficiency due to a weak washout in some of the leptonic decay channels.

Conclusion

To summarize, the inclusion of flavour effects and spectator processes in the computation of the baryon asymmetry increases the parameter space available for successful leptogenesis. However, the lower bound on the triplet mass does not decrease significantly. This bound can be relaxed when the mass of the heavier state responsible for the contribution m_κ to neutrino masses is lowered. In this situation, the nature of this heavier state should be specified, and the computation would become more model-dependent. In particular, it is possible to perform resonant leptogenesis with two scalar triplets that are close in mass, which could relax considerably the bound on the triplet mass [210].

3.3 A more predictive scenario

As we observed, implementing leptogenesis in the type II seesaw framework requires to give up the simple relation between the neutrino mass matrix and the coupling matrix f. Thus, leptogenesis in this framework is not more predictive than in the general type I case. However, a way out can be found in more elaborated frameworks. We consider here a scenario motivated by a Grand Unified Theory based on the gauge group $SO(10)$ [211]. Theories based on $SO(10)$ usually allow the implementation of the type I seesaw mechanism, but this scenario contains all the ingredients for type II seesaw and scalar triplet leptogenesis.

3.3.1 The setup

The principle is the following. Instead of gathering all the Standard Model fermions of a generation in a single multiplet, they are distributed in two different representations, along with new heavy states. More precisely, the left-handed fermions of the Standard Model fit in dimension 16 representations labelled 16_i and dimension 10 representations labelled 10_i.

As in the "standard" $SO(10)$ theory, the representation 16_i contains the quark doublet Q_i, the up-type antiquark singlet u_i^c and the antielectron singlet e_i^c, along with an antineutrino singlet ν_i^c. On the other hand, instead of the lepton doublet ℓ_i and the down-type antiquark singlet d_i^c, this representation contains two heavy fields labelled $\mathcal{L}_i$ and $\bar{\mathcal{D}}_i$. $\mathcal{L}_i$ has the same quantum number as ℓ_i with respect to the Standard Model, so we call it a heavy lepton, while $\bar{\mathcal{D}}_i$ has the same quantum number as d_i^c with respect to the Standard Model so we call it a heavy antiquark.

The lepton doublet ℓ_i and the antiquark singlet d_i^c fit in the 10_i representation, along with two other heavy fields labelled $\bar{\mathcal{L}}_i$ and $\mathcal{D}_i$. The former is a left-handed electroweak doublet with hypercharge $+1/2$, which looks like an antilepton doublet, except for its chirality: antilepton doublets of the Standard Model are right-handed whereas this one is left-handed. After $SO(10)$ is broken, $\mathcal{L}_i$ and $\bar{\mathcal{L}}_i$ form a vectorlike pair. Consequently, we give a lepton number $L = -1$ to $\bar{\mathcal{L}}_i$. Similarly, $\mathcal{D}_i$ is a left-handed heavy quark singlet with hypercharge $+1/3$, that forms a vectorlike pair with $\bar{\mathcal{D}}_i$.

This choice of lepton number for the new states $\mathcal{L}$ and $\bar{\mathcal{L}}$ is justified by the existence of the following coupling in the $SO(10)$ theory,

$$\mathcal{L} \supset -y_{ij}^u 16_i 16_j 10_H + \text{h.c.} \,, \tag{3.3.1}$$

which gives rise to both the up-type quark Yukawa coupling and to the following operator,

$$-y_{ij}^u \bar{e}_i \tilde{H} \, \mathcal{L}_j + \text{h.c.} \,. \tag{3.3.2}$$

With the choice made here, this interaction preserves $B - L$. Because of this coupling, the main decay channel of heavy leptons is $\mathcal{L}_j \to e_i H$, so that in leptogenesis, any asymmetry stored in the heavy leptons will be transferred to the Standard Model leptons before being processed by electroweak sphalerons (see ref. [211] for the supersymmetric version of this discussion).

In addition to that, the scalar sector of the theory is extended to include a self-conjugate dimension 54 representation. This representation contains, among other things, an electroweak triplet Δ with hypercharge $+1$ and its conjugate, a self-conjugate electroweak triplet T (with hypercharge zero) and a self-conjugate Standard Model singlet S. This field content is summarized in table 3.2. From now on, we restrict ourselves to the fields which are involved in leptogenesis. They include the heavy lepton doublets $\mathcal{L}$ and $\bar{\mathcal{L}}$, the three scalars Δ, S and T, in addition to Standard Model fields.

We use the same representation as in eq. (2.1.56) for Δ, and we define similarly

$$T = \begin{pmatrix} T^0/\sqrt{2} & T^+ \\ T^- & -T^0/\sqrt{2} \end{pmatrix} \tag{3.3.3}$$

Representation	Field	$SU(3)\times SU(2)_L\times U(1)_Y$	B	L
16_i	Q_i	$(3,\,2,\,+1/6)$	$+1/3$	0
	u_i^c	$(3,\,1,\,-2/3)$	$-1/3$	0
	e_i^c	$(1,\,1,\,+1)$	0	-1
	$\mathcal{L}_i$	$(1,\,2,\,-1/2)$	0	$+1$
	$\bar{\mathcal{D}}_i$	$(3,\,1,\,+1/3)$	$-1/3$	0
10_i	ℓ_i	$(1,\,2,\,-1/2)$	0	$+1$
	d_i^c	$(3,\,1,\,+1/3)$	$-1/3$	0
	$\bar{\mathcal{L}}_i$	$(1,\,2,\,+1/2)$	0	-1
	$\mathcal{D}_i$	$(3,\,1,\,-1/3)$	$+1/3$	0
54	Δ	$(1,\,3,\,+1)$	0	0
	Δ^c	$(1,\,3,\,-1)$	0	0
	T	$(1,\,3,\,0)$	0	0
	S	$(1,\,1,\,0)$	0	0

Table 3.2: Field content of the model.

The Lagrangian contains the following interaction terms,

$$\mathcal{L} = -M_\Delta^2 \,\mathrm{tr}\left(\Delta^\dagger\Delta\right) - \frac{1}{2}M_T^2\,\mathrm{tr}\left(T^2\right) - \frac{1}{2}M_S^2 S^2 - \sum_{\alpha=1,2,3}\left(M_\alpha \bar{\mathcal{L}}_\alpha \mathcal{L}_\alpha + \text{h.c.}\right)$$
$$-\left\{\frac{1}{2}\left(f_{\alpha\beta}\ell_\alpha^T C i\sigma_2 \Delta \ell_\beta + \mu H^T i\sigma_2 \Delta^\dagger H\right) + f_{\alpha\beta}\,\bar{\mathcal{L}}_\alpha^T C\left(c_S S + c_T T\right)\ell_\beta + \text{h.c.}\right\} . \quad (3.3.4)$$

$c_S = \sqrt{3/10}$ and $c_T = 1$ are nothing but Clebsch-Gordan coefficients determined by the underlying theory. The neutrino mass matrix is given by eq. (2.1.61) with no additional contribution.

Like before, the scalar triplet Δ can decay to two antileptons and two Higgs doublets. If it is kinematically allowed, it can also decay to two heavy antileptons $\bar{\mathcal{L}}$, with the following partial width,

$$\Gamma(\Delta \to \bar{\mathcal{L}}\,\bar{\mathcal{L}}) = \frac{1}{32\pi}M_\Delta \sum_{\alpha,\beta}\left|f_{\alpha\beta}\right|^2 F\left(\frac{M_\alpha}{M_\Delta}, \frac{M_\beta}{M_\Delta}\right) . \quad (3.3.5)$$

The kinematic function F is given by

$$F(x,y) = \theta\left(1-x-y\right)\left(1-x^2-y^2\right)\sqrt{1-(x+y)^2}\sqrt{1-(x-y)^2}\ , \quad (3.3.6)$$

where θ is the Heaviside function. Defining λ_ℓ and λ_H like before and

$$\lambda_{\bar{\mathcal{L}}} = \sqrt{\sum_{\alpha,\beta}\left|f_{\alpha\beta}\right|^2 F\left(\frac{M_\alpha}{M_\Delta}, \frac{M_\beta}{M_\Delta}\right)}\ , \quad (3.3.7)$$

the total width of Δ and its branching ratios are now given by

$$\Gamma_\Delta = \frac{1}{32\pi^2} M_\Delta \left(\lambda_\ell^2 + \lambda_H^2 + \lambda_{\bar{\mathcal{L}}}^2\right) , \tag{3.3.8}$$

$$B_\ell = \frac{\lambda_\ell^2}{\lambda_\ell^2 + \lambda_H^2 + \lambda_{\bar{\mathcal{L}}}^2} , \tag{3.3.9}$$

$$B_H = \frac{\lambda_H^2}{\lambda_\ell^2 + \lambda_H^2 + \lambda_{\bar{\mathcal{L}}}^2} , \tag{3.3.10}$$

$$B_{\bar{\mathcal{L}}} = \frac{\lambda_{\bar{\mathcal{L}}}^2}{\lambda_\ell^2 + \lambda_H^2 + \lambda_{\bar{\mathcal{L}}}^2} . \tag{3.3.11}$$

The two new scalar S and T decay into a Standard Model lepton and a heavy antilepton $\bar{\mathcal{L}}$, or the CP-conjugate state. These decays preserve the total lepton number as defined in table 3.2, and the corresponding widths are

$$\Gamma_S = \frac{1}{4\pi} c_S^2 M_S \lambda_S^2 , \tag{3.3.12}$$

$$\Gamma_T = \frac{1}{8\pi} c_T^2 M_T \lambda_T^2 , \tag{3.3.13}$$

$$\lambda_\Phi = \sqrt{\sum_\alpha (f f^\dagger)_{\alpha\alpha} \left(1 - \frac{M_\alpha^2}{M_\Phi^2}\right) \theta\left(M_\Phi - M_\alpha\right)} , \quad \Phi = S, T . \tag{3.3.14}$$

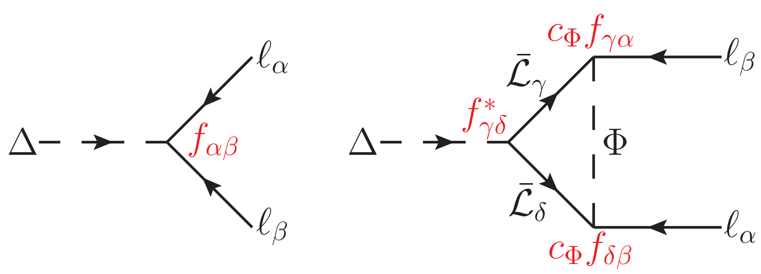


Figure 3.10: Diagrams contributing to the violation of CP in the decay of Δ at one loop. Φ denotes one of the two scalars S or T.

Only Δ has couplings that violate the total lepton number, therefore it appears as the best candidate for leptogenesis. Let us compute the CP asymmetry in its decay into leptons, which is defined like in eq. (3.2.8). The diagrams involved are displayed in fig. 3.10. Summing over flavour, we get

$$\epsilon^\Delta = \frac{B_L}{4\pi\lambda_\ell^2} \sum_{\rho,\sigma} \Im\left[f_{\rho\sigma}(f^\dagger f f^\dagger)_{\rho\sigma}\right] \left[c_S^2 G\left(\frac{M_S}{M_\Delta}, \frac{M_\rho}{M_\Delta}, \frac{M_\sigma}{M_\Delta}\right) + \frac{c_T^2}{2} G\left(\frac{M_T}{M_\Delta}, \frac{M_\rho}{M_\Delta}, \frac{M_\sigma}{M_\Delta}\right)\right] , \tag{3.3.15}$$

whith

$$\begin{aligned} G(x,y,z) &= \theta\left(1-y-z\right)\left[\sqrt{\lambda(1,y^2,z^2)}\right. \\ &\qquad \left. - x^2 \log\left(\frac{1+2x^2-y^2-z^2+\sqrt{\lambda(1,y^2,z^2)}}{1+2x^2-y^2-z^2-\sqrt{\lambda(1,y^2,z^2)}}\right)\right] , \qquad (3.3.16) \\ \lambda(a,\, b,\, c) &= (a-b-c)^2 - 4bc \ . \qquad (3.3.17) \end{aligned}$$

In other words, only the pairs $(\bar{\mathcal{L}}_\rho,\, \bar{\mathcal{L}}_\sigma)$ such that $M_\rho + M_\sigma < M_\Delta$ contribute to the CP asymmetry. Thus, we recover the fact that the violation of CP requires the existence of an on-shell intermediate state. This implies that at least one species of heavy lepton must be light enough. Moreover, since

$$\lim_{y,\,z\to 0} G(x,y,z) = 1 - x^2 \log\left(\frac{1+x^2}{x^2}\right) = \tilde{G}(x) \ , \qquad (3.3.18)$$

if all the heavy leptons have masses much smaller than M_Δ, the kinematic factors in eq. (3.3.15) can be taken out of the sum and the asymmetry is proportional to $\Im\left[\mathrm{tr}\left(ff^\dagger ff^\dagger\right)\right]$, which is zero. For these reasons, we will consider the limiting case where the lightest species $\bar{\mathcal{L}}_1$ is much lighter than Δ, while the remaining two, $\bar{\mathcal{L}}_2$ and $\bar{\mathcal{L}}_3$, are so heavy that they decouple from the theory. This choice simplifies the expression of the asymmetry, which becomes

$$\epsilon^\Delta = \frac{B_L}{4\pi\lambda_\ell^2} \Im\left[f_{11}\left(f^\dagger f f^\dagger\right)_{11}\right]\left[c_S^2 \tilde{G}\left(\frac{M_S}{M_\Delta}\right) + \frac{c_T^2}{2}\tilde{G}\left(\frac{M_T}{M_\Delta}\right)\right] . \qquad (3.3.19)$$

In this scenario, the decay of Δ to two Higgs doublets does not violate CP, but its decay to two heavy leptons does

$$2\frac{\Gamma(\Delta^c \to \bar{\mathcal{L}}^c\bar{\mathcal{L}}^c) - \Gamma(\Delta \to \bar{\mathcal{L}}\bar{\mathcal{L}})}{\Gamma_{\Delta^c} + \Gamma_\Delta} = -2\frac{\Gamma(\Delta^c \to \ell\ell) - \Gamma(\Delta \to \ell^c\ell^c)}{\Gamma_{\Delta^c} + \Gamma_\Delta} \ . \qquad (3.3.20)$$

In order for ϵ^Δ to be sizeable, S and T should not be too heavy compared to Δ, since the function $\tilde{G}$ vanishes in this limit. We can assume that the masses of all the scalars are comparable, which would be consistent with the $SO(10)$ symmetry. To summarize we have the following mass hierarchy,

$$M_1 \ll M_\Delta, M_S, M_T \ll M_2, M_3 \ , \qquad (3.3.21)$$

which simplifies a lot the previous expressions. Now, the effective couplings reduce to

$$\lambda_{\bar{\mathcal{L}}} = |f_{11}| \ , \qquad (3.3.22)$$

$$\lambda_S = \lambda_T = \sqrt{(ff^\dagger)_{11}} \ . \qquad (3.3.23)$$

As mentioned previously, the decay of S or T does not violate the total lepton number, however it does violate lepton flavour, and this should be taken into account

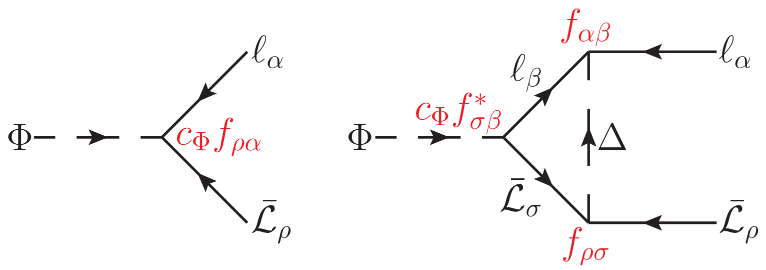


Figure 3.11: Diagrams contributing to the violation of CP in the decay of $\Phi = S$ or T at one loop.

in our scenario. In particular, this decay also violates CP, and we define the associated asymmetry as

$$\epsilon^\Phi = \frac{\Gamma(\Phi \to \ell\,\bar{\mathcal{L}}) - \Gamma(\Phi \to \ell^c\,\bar{\mathcal{L}}^c)}{\Gamma_\Phi} \;, \quad \Phi = S, T \;. \tag{3.3.24}$$

This asymmetry arises at one loop as a consequence of the interference between the two diagrams of fig. 3.11 , which look very similar to those of fig. 3.10. Summing again over flavour, we get, in the general case,

$$\epsilon^\Phi = -\frac{d_\Phi}{16\pi\lambda_\Phi^2} \Im\left[f_{\rho\sigma}(f^\dagger f f^\dagger)_{\rho\sigma}\right] G'\left(\frac{M_\Delta}{M_\Phi}, \frac{M_\rho}{M_\Phi}, \frac{M_\sigma}{M_\Phi}\right) \;, \tag{3.3.25}$$

where $d_S = 3$ and $d_T = 1$, and this time the kinematic factor is

$$G'(x,y,z) = \theta(1-y-z)\left\{x^2(1-z^2) - \frac{1}{1-y^2}\log\left[1 + x^2(1-y^2)(1-z^2)\right]\right\} \;, \tag{3.3.26}$$

but because of eq. (3.3.21), this reduces to

$$\epsilon^\Phi = -\frac{d_\Phi}{16\pi\lambda_\Phi^2} \Im\left[f_{11}(f^\dagger f f^\dagger)_{11}\right] \tilde{G}\left(\frac{M_\Delta}{M_\Phi}\right) \;. \tag{3.3.27}$$

A nice feature of this model is that, since there is no other contribution to the neutrino mass matrix, the coupling matrix f can be expressed in terms of neutrino mass parameters up to a constant,

$$f_{\alpha\beta} = \frac{\lambda_\ell}{\bar{m}_\nu} m_{\nu\alpha\beta} \;, \qquad \bar{m}_\nu = \sqrt{\mathrm{tr}(m_\nu^\dagger m_\nu)} \;, \tag{3.3.28}$$

which allows us to express the CP asymmetries as functions of these parameters thanks to the following relation,

$$\begin{aligned}\Im\left[f_{11}(f^\dagger f f^\dagger)_{11}\right] &= \frac{\lambda_\ell^4}{\bar{m}_\nu^4} \Im\left[m_{\nu 11}(m_\nu^\dagger m_\nu m_\nu^\dagger)_{11}\right] \\ &= \frac{\lambda_\ell^4}{\bar{m}_\nu^4} \sum_{i,j} m_{\nu i} m_{\nu j}^3 \Im\left[U_{ei}^{*2} U_{ej}^2\right] \;. \end{aligned} \tag{3.3.29}$$

Using the parametrization $U_{e.} = \begin{pmatrix} c_{12}c_{13}e^{i\rho} & s_{12}c_{23} & s_{13}e^{i\sigma}\end{pmatrix}$, where we absorbed δ in the redefinition of σ with respect to eq. (2.1.16), this gives

$$\Im\Big[f_{11}(f^\dagger f f^\dagger)_{11}\Big] = \frac{\lambda_\ell^4}{\bar{m}_\nu^4}\Big(-m_{\nu 1}m_{\nu 2}\Delta m_{21}^2 c_{12}^2 c_{13}^4 s_{12}^2 \sin 2\rho \\ +m_{\nu 1}m_{\nu 3}\Delta m_{31}^2 c_{12}^2 c_{13}^2 s_{13}^2 \sin 2(\sigma-\rho) + m_{\nu 2}m_{\nu 3}\Delta m_{32}^2 c_{13}^2 s_{12}^2 s_{13}^2 \sin 2\sigma\Big) \ . \tag{3.3.30}$$

3.3.2 Boltzmann equations

Because of the increase in the number of particles involved in leptogenesis, the system of Boltzmann equations to solve becomes far more complex than before. In addition to the equation for the density Σ_Δ which remains unchanged, there are Boltzmann equations describing the evolution of the densities of S and T. The equation for Y_T is very similar to the equation for Σ_Δ, whereas the equation for Y_S is simpler because S is a gauge singlet and therefore does not annihilate. The following equations, which are not affected by considerations such as flavour or spectator processes, describe the evolution of the scalar densities,

$$sHz\frac{d\Sigma_\Delta}{dz} = -\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}}-1\right)\gamma_D^\Delta - 2\left[\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}}\right)^2-1\right]\gamma_A^\Delta\,, \tag{3.3.31}$$

$$sHz\frac{dY_T}{dz} = -\left(\frac{Y_T}{Y_T^{\rm eq}}-1\right)\gamma_D^T - 2\left[\left(\frac{Y_T}{Y_T^{\rm eq}}\right)^2-1\right]\gamma_A^T\,, \tag{3.3.32}$$

$$sHz\frac{dY_S}{dz} = -\left(\frac{Y_S}{Y_S^{\rm eq}}-1\right)\gamma_D^S\,. \tag{3.3.33}$$

A large number of reactions may affect directly the asymmetries. Scatterings involving lepton doublets can be divided in two categories: reactions that change the overall asymmetry Δ_ℓ stored in the Standard Model lepton doublets, and reactions that only transfer the asymmetry from a flavour to another.

The first category includes scalar-mediated scatterings between Standard Model leptons and Higgs doublets, Standard Model leptons and heavy leptons, and fermion-mediated scatterings such as $\ell\Delta \to \bar{\mathcal{L}}\Phi$. The second category comprises four-lepton scatterings such as $\ell\ell \to \ell\ell$ mediated by the exchange of a heavy scalar, and fermion-mediated scatterings like $\ell\Delta \to \ell\Delta$. All these reactions are gathered in table 3.3. In addition to that, scatterings between heavy leptons and Higgs like $\bar{\mathcal{L}}\bar{\mathcal{L}} \to HH$ are also possible, and modify the asymmetry stored in the heavy leptons

Our new heavy leptons are vector-like, i.e. left- and right-handed components belong to the same Standard Model representation. As a consequence, they do not participate in the triangle anomaly, and are not affected by the electroweak sphalerons. More generally, the chemical equilibriums coming from Standard Model reactions are not modified by the new fields, so the reactions relevant for each temperature range are still those displayed in table 3.1, and eqs. (3.2.37) to (3.2.41) are sill valid. The only novelty with respect to the previous scenario is that heavy leptons should be included

	Label	Reaction	Intermediate State (channel)
Scatterings changing the total Δ_ℓ	$W^{\ell H}$	$\ell\ell \longleftrightarrow H^c H^c$	$\Delta\ (s^*)$
		$\ell H \longleftrightarrow \ell^c H^c$	$\Delta\ (t)$
	$W^{\ell\mathcal{L}}$	$\ell\ell \longleftrightarrow \bar{\mathcal{L}}^c\bar{\mathcal{L}}^c$	$\Delta\ (s^*),\ S,\ T\ (t)$
		$\ell\bar{\mathcal{L}} \longleftrightarrow \ell^c\bar{\mathcal{L}}^c$	$\Delta\ (t),\ S,\ T\ (s^*,\ u)$
	$W^{\ell\mathcal{L}\Phi\Delta}$	$\ell\Delta \longleftrightarrow \bar{\mathcal{L}}\Phi$	$\ell\ (s),\ \bar{\mathcal{L}}\ (u^*)$
		$\ell\Phi \longleftrightarrow \bar{\mathcal{L}}\Delta^c$	$\ell\ (u^*),\ \bar{\mathcal{L}}\ (s)$
		$\ell\bar{\mathcal{L}}^c \longleftrightarrow \Phi\Delta^c$	$\ell\ (t),\ \bar{\mathcal{L}}\ (u)$
Purely flavoured washout	$w^{4\ell}$	$\ell\ell \longleftrightarrow \ell\ell$	$\Delta\ (s^*)$
		$\ell\ell^c \longleftrightarrow \ell\ell^c$	$\Delta\ (t)$
	$w^{\ell\mathcal{L}}$	$\ell\bar{\mathcal{L}} \longleftrightarrow \ell\bar{\mathcal{L}}$	$S,\ T\ (s^*)$
		$\ell\bar{\mathcal{L}}^c \longleftrightarrow \ell\bar{\mathcal{L}}^c$	$S,\ T\ (u)$
	$w^{\ell\Delta}$	$\ell\Delta \longleftrightarrow \ell\Delta$	$\ell\ (s)$
		$\ell\Delta^c \longleftrightarrow \ell\Delta^c$	$\ell\ (u^*)$
		$\ell\ell^c \longleftrightarrow \Delta\Delta^c$	$\ell\ (t)$
	$w^{\ell\Phi}$	$\ell\Phi \longleftrightarrow \ell\Phi$	$\mathcal{L}\ (s,\ u^*)$
		$\ell\ell^c \longleftrightarrow \Phi\Phi$	$\bar{\mathcal{L}}\ (t)$

Table 3.3: Scatterings contributing to the washout of the asymmetry stored in lepton doublets. Stars indicate the possibility for the intermediate state to be on-shell, in which case the procedure described in appendix A.2 should apply.

in the hypercharge conservation relation. If we assume that no asymmetry is generated in $\bar{\mathcal{L}}_2$ and $\bar{\mathcal{L}}_3$, which seems reasonable, eq. (3.2.36) is slightly modified,

$$\sum_{i=1,2,3} (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i}) \ + 2\mu_{\bar{\mathcal{L}}_1} - \sum_{\alpha=e,\mu,\tau} (\mu_{\ell_\alpha} + \mu_{e_\alpha}) + 2\mu_H + 6\mu_\Delta \ = \ 0 \ , \quad (3.3.34)$$

where we used the fact that, because of the mass term $M_1\bar{\mathcal{L}}_1\mathcal{L}_1$, the chemical potentials of the two fields should satisfy $\mu_{\mathcal{L}_1} = -\mu_{\bar{\mathcal{L}}_1}$.

Let us focus on the high temperature regime, with $T > 10^{12}$ GeV. The asymmetries Δ_ℓ and Δ_H should now be expressed as functions of the $B - L$ asymmetry (in which we do not include the contribution of the heavy leptons), the triplet asymmetry and the asymmetry stored in $\bar{\mathcal{L}}_1$,

$$(\Delta_\ell)_{\alpha\beta} = C^{\gamma\delta}_{\alpha\beta}\Delta_{\gamma\delta} + C^{\Delta}_{\alpha\beta}\Delta_\Delta + C^{\bar{\mathcal{L}}_1}_{\alpha\beta}\Delta_{\bar{\mathcal{L}}_1} \ , \quad (3.3.35)$$

$$\Delta_H = C^{\gamma\delta}_{H}\Delta_{\gamma\delta} + C^{\Delta}_{H}\Delta_\Delta + C^{\bar{\mathcal{L}}_1}_{H}\Delta_{\bar{\mathcal{L}}_1} \ . \quad (3.3.36)$$

Once the chemical equilibriums are known, the system of Boltzmann equations for the asymmetries including all the relevant processes can be formally written as

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = -S_{\alpha\beta} + W^{ID}_{\alpha\beta} + W^{\ell H}_{\alpha\beta} + W^{\ell\bar{\mathcal{L}}}_{\alpha\beta} + W^{\ell\bar{\mathcal{L}}\Phi\Delta}_{\alpha\beta} + w^{\ell\Delta}_{\alpha\beta} + w^{4\ell}_{\alpha\beta} + w^{\ell\bar{\mathcal{L}}}_{\alpha\beta} + w^{\ell\Phi}_{\alpha\beta} \,. \quad (3.3.37)$$

$$sHz\frac{d\Delta_{\bar{\mathcal{L}}_1}}{dz} = \mathrm{tr}\,[S] - W^{ID}_{\bar{\mathcal{L}}} - W^{\bar{\mathcal{L}}H} - \mathrm{tr}\left[W^{\ell\bar{\mathcal{L}}}\right] + \mathrm{tr}\left[W^{\ell\bar{\mathcal{L}}\Phi\Delta}\right] \,. \quad (3.3.38)$$

$$sHz\frac{d\Delta_{\Delta}}{dz} = -\frac{1}{2}\,\mathrm{tr}\left[W^{ID}\right] + \frac{1}{2}W^{ID}_{\bar{\mathcal{L}}} + \mathrm{tr}\left[W^{\ell\bar{\mathcal{L}}\Phi\Delta}\right] - B_H\left(\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} - \frac{C^k_H\Delta_k}{Y^{\rm eq}_H}\right)\gamma^{\Delta}_D \,. \quad (3.3.39)$$

As in the previous section, lower case w's label purely flavoured processes satisfying $\mathrm{tr}[w]=0$. Again, the W^{ID}'s represent the washout due to inverse decays $\ell\ell\to\Delta^c$ and $\ell\bar{\mathcal{L}}\to S,T$. $W^{\bar{\mathcal{L}}H}$ is the washout term due to the scatterings $\bar{\mathcal{L}}\bar{\mathcal{L}}\to HH$ and $\bar{\mathcal{L}}H^c\to\bar{\mathcal{L}}^cH$, and the scatterings contributing to the other washout terms can be read in table 3.3.

This system is pretty cumbersome, but in many cases, the couplings are not too large and therefore most scattering terms are negligible in comparison with the washout coming from inverse decays. If such is the case, in a good approximation, the system can be simplified as follows,

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = -S_{\alpha\beta} + W^{ID}_{\alpha\beta} \,. \quad (3.3.40)$$

$$sHz\frac{d\Delta_{\bar{\mathcal{L}}_1}}{dz} = \mathrm{tr}\,[S] - W^{ID}_{\bar{\mathcal{L}}} \,. \quad (3.3.41)$$

$$sHz\frac{d\Delta_{\Delta}}{dz} = -\frac{1}{2}\,\mathrm{tr}\left[W^{ID}\right] + \frac{1}{2}W^{ID}_{\bar{\mathcal{L}}} - B_H\left(\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} - \frac{C^k_H\Delta_k}{Y^{\rm eq}_H}\right)\gamma^{\Delta}_D \,. \quad (3.3.42)$$

The source term gets contributions from the CP-violating decays of the three scalars Δ, S and T,

$$S_{\alpha\beta} = \epsilon^{\Delta}_{\alpha\beta}\left(\frac{\Sigma_\Delta}{\Sigma^{\rm eq}_\Delta} - 1\right)\gamma^{\Delta}_D + \epsilon^{S}_{\alpha\beta}\left(\frac{Y_S}{Y^{\rm eq}_S} - 1\right)\gamma^{S}_D + \epsilon^{T}_{\alpha\beta}\left(\frac{Y_T}{Y^{\rm eq}_T} - 1\right)\gamma^{T}_D \,, \quad (3.3.43)$$

where the expression of the flavoured CP asymmetries are obtained from eqs. (3.3.19) and (3.3.27) as follows,

$$\epsilon^{\Delta,S,T}_{\alpha\beta} = \frac{[f_{11}f_{\alpha\gamma}f^*_{1\gamma}f^*_{1\beta} - f^*_{11}f^*_{\beta\gamma}f_{1\gamma}f_{1\alpha}]}{2\,i\,\Im\,[f_{11}(f^\dagger f f^\dagger)_{11}]}\epsilon^{\Delta,S,T} \,. \quad (3.3.44)$$

The flavour-covariant washout term due to inverse decays $\ell\ell\to\Delta^c$ and $\ell\bar{\mathcal{L}}\to S,T$ is

$$W^{ID}_{\alpha\beta} = \frac{2B_L}{\lambda^2_\ell}\left[(ff^\dagger)_{\alpha\beta}\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} + \frac{1}{4Y^{\rm eq}_\ell}(2f\Delta^T_\ell f^\dagger + ff^\dagger\Delta_\ell + \Delta_\ell ff^\dagger)_{\alpha\beta}\right]\gamma^{\Delta}_D + \frac{1}{2\lambda^2_\Phi}\left[\frac{1}{2Y^{\rm eq}_\ell}\left(f_{1\alpha}(f^\dagger\Delta_\ell)_{1\beta} + (f\Delta^T_\ell)_{1\alpha}f^*_{1\beta}\right) + \frac{\Delta_{\bar{\mathcal{L}}_1}}{Y^{\rm eq}_{\mathcal{L}}}f_{1\alpha}f^*_{1\beta}\right]\left(\gamma^S_D + \gamma^T_D\right) \,, \quad (3.3.45)$$

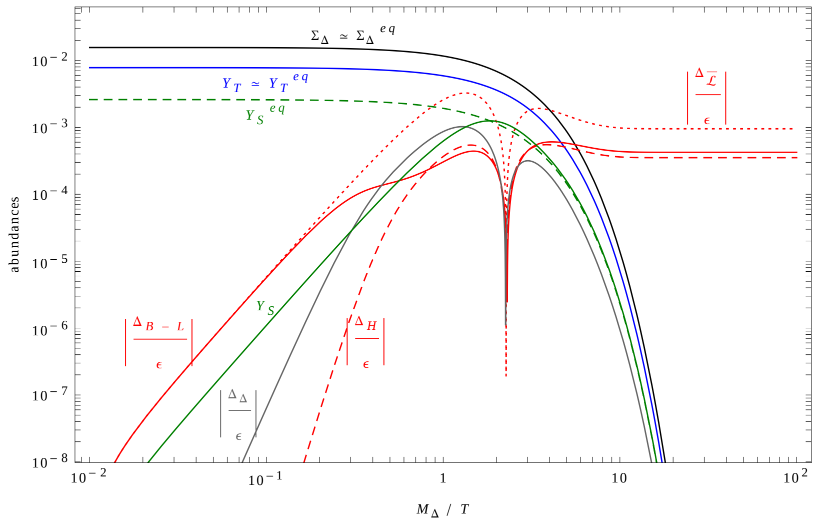


Figure 3.12: Comoving number densities of scalars Σ_Δ, Y_T and Y_S and asymmetries Δ_Δ, Δ_H, $\Delta_{\bar{\mathcal{L}}_1}$ and $\Delta_{B-L} = \mathrm{tr}(\Delta_{\alpha\beta})$ as a function of $z = M_\Delta/T$, for $M_\Delta = 10^{13}$ GeV and $\lambda_H = 0.2$. The asymmetries are displayed in units of $\epsilon = \epsilon^\Delta + \epsilon^S + \epsilon^T$.

where the factor $1/2$ in front of the second term is due to the fact that both S and T have two decay channels (without taking care of the different flavours), $S, T \to \ell\,\bar{\mathcal{L}}$ and $S, T \to \ell^c\,\bar{\mathcal{L}}^c$, and each channel has a branching ratio $1/2$. For heavy leptons, the equivalent term is

$$W^{ID}_{\bar{\mathcal{L}}} = 2B_{\bar{\mathcal{L}}}\left(\frac{\Delta_{\bar{\mathcal{L}}_1}}{Y_{\bar{\mathcal{L}}^{\rm eq}}} - \frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta}\right)\gamma^\Delta_D + \frac{1}{2\lambda^2_\Phi}\left[\frac{1}{Y^{\rm eq}_\ell}(f^\dagger \Delta_\ell f)_{11} + \frac{\Delta_{\bar{\mathcal{L}}_1}}{Y^{\rm eq}_{\mathcal{L}}}\lambda^2_\Phi\right]\left(\gamma^S_D + \gamma^T_D\right) . \tag{3.3.46}$$

3.3.3 Numerical approach

As in 3.2.4, we show first a typical example of the behaviour of the solutions, before turning to a more systematic approach. Fig. 3.12 shows the evolution of the various densities and asymmetries for a common scalar mass $M_\Delta = M_S = M_T = 10^{13}$ GeV, and $\lambda_H = 0.2$. Motivated by the discussion of 3.2.4, we chose for the two $SU(2)$ triplets Δ and T an initial density in equilibrium. On the other hand, we chose a vanishing initial abundance for the scalar singlet S, which is justified by the fact that it is not produced by any fast interaction. The singlets are therefore produced by inverse decays of Standard Model leptons and heavy leptons, before decaying. This explains a new feature of the evolution of the asymmetries, which change sign around $z = 2$, a little after the singlets have reached their equilibrium density and start to decay.

Parameter space

This scenario possesses much fewer free parameters than the general case studied in section 3.2. Indeed, with the choice of eq. (3.3.21), once the masses of the scalars and the effective coupling λ_H are fixed, everything can be expressed in terms of neutrino parameters thanks to the seesaw relation of eq. (2.1.57). The neutrino masses and squared mass differences are known and summarized in table 2.1. The remaining free parameters are low energy parameters: the yet undetermined Majorana phases ρ and σ, the hierarchy and the mass of the lightest neutrino $m_{\nu 1}$ (NH) or $m_{\nu 3}$ (IH). In numerical computations, in order to avoid numerical instabilities, we set $M_\Delta = M_S = M_T$.

In what follows, we choose values of ρ and σ that give a sizeable total CP asymmetry. For Normal Hierarchy, we choose $\rho = 0$ and $\sigma = -\pi/4$. This is especially favourable in the case of very hierarchical neutrinos with $m_{\nu 1} \ll m_{\nu 3}$. Explicitly, the numerical factor of eq. (3.3.30) becomes

$$\Im\left[f_{11}(f^\dagger f f^\dagger)_{11}\right] = -\frac{\lambda_\ell^4}{\bar{m}_\nu^4}\left(m_{\nu 1}m_{\nu 3}\Delta m_{31}^2 c_{12}^2 c_{13}^2 s_{13}^2 + m_{\nu 2}m_{\nu 3}\Delta m_{32}^2 c_{13}^2 s_{12}^2 s_{13}^2\right) . \tag{3.3.47}$$

For Inverted Hierarchy, we set $\rho = \sigma = \pi/4$, which gives for this factor

$$\Im\left[f_{11}(f^\dagger f f^\dagger)_{11}\right] = \frac{\lambda_\ell^4}{\bar{m}_\nu^4}\left(-m_{\nu 1}m_{\nu 2}\Delta m_{21}^2 c_{12}^2 c_{13}^4 s_{12}^2 + m_{\nu 2}m_{\nu 3}\Delta m_{32}^2 c_{13}^2 s_{12}^2 s_{13}^2\right) , \tag{3.3.48}$$

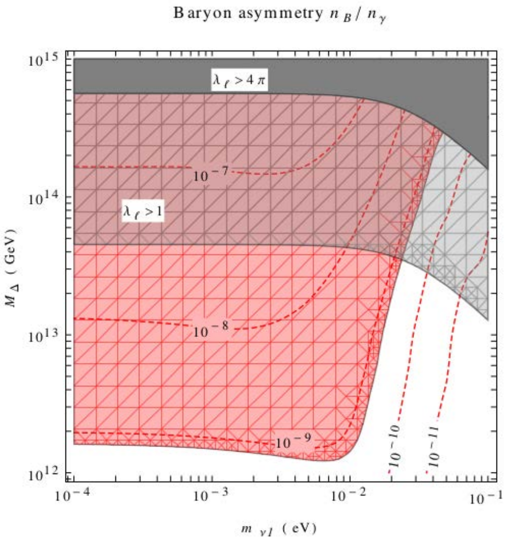
with Δm_{32}^2 negative. Note that the sign of the final baryon asymmetry obtained is irrelevant, because, as can be seen from eq. (3.3.30), it can be switched by changing the sign of ρ and σ.

Because of the simple relation between the neutrino mass matrix and the coupling matrix $f_{\alpha\beta}$ involved in leptogenesis, this scenario allows to study the dependence of the final baryon asymmetry on low-energy parameters, some of which are in principle measurable. However, a point should be mentioned here. Like in the previous case 3.2, we took the effect of the renormalization group into account. The relevant parameters for leptogenesis, for instance the masses appearing in eqs. (3.3.47) and (3.3.48), are those renormalized at the scale M_Δ, but from a phenomenological point of view it is more interesting to see how things depend on parameters that would be measured at low energy. Thus, for the computations, we fixed the mass of the lightest neutrino at the weak scale, then computed the running up to M_Δ and performed computations using the renormalized parameters. In the following figures, the neutrino masses appearing are always the low-scale ones.

A few comments can be made before proceeding to numerical computations. Like in the previous scenario, an efficient conversion of the CP asymmetries requires at least one of the decay channels to be out of equilibrium. When flavour effects are not taken into account, one always has $\Gamma(\Delta \to \bar{\mathcal{L}}_1\bar{\mathcal{L}}_1) < \Gamma(\Delta \to \ell^c\ell^c)$, therefore the channel $\Gamma(\Delta \to \bar{\mathcal{L}}_1\bar{\mathcal{L}}_1)$ has to be out of equilibrium, as was noticed in ref. [211]. This in turns implies that the coupling $|f_{11}|$ must be small enough. This discussion is not drastically modified by the inclusion of flavour, even though there are more decay channels. Indeed, the washout rates are controlled by the entries of the coupling matrix $f_{\alpha\beta}$, which are all roughly of the same order of magnitude since they are proportional to the $m_{\nu\alpha\beta}$. Thus, the condition that $|f_{11}|$ (and therefore $|m_{\nu ee}|$) must be small enough is still valid. For the same reason, flavour effects are less important in this scenario than in the previous one. Indeed, the various channels are always washed out with a comparable strength, unlike in the previous case where it was possible to choose couplings such that the washout is much weaker in one channel, as was discussed in 3.2.4.

Normal hierarchy

We focus first on normal hierarchy. Fig. 3.13 displays the baryon asymmetry generated



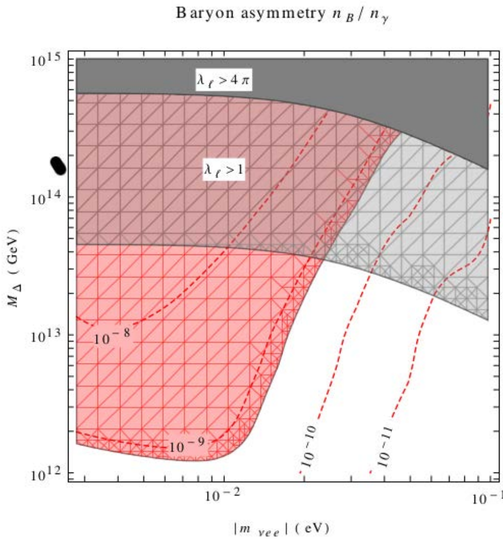


Figure 3.13: Baryon asymmetry produced as a function of $m_{\nu 1}$ and M_Δ (left) and $|m_{\nu ee}|$ and M_Δ (right) for normal hierarchy, with $\lambda_H = 0.2$. The region colored in red is where the baryon asymmetry is greater than the BAU. The region colored in gray is where the coupling λ_ℓ is large (light gray) or nonperturbative (dark gray).

as a function of the scalar mass M_Δ on one hand, and the lightest neutrino mass $m_{\nu 1}$ (left) or the so-called effective Majorana mass $|m_{\nu ee}|$ (right) on the other hand. The dependence on the effective Majorana mass is interesting because this parameter could be measured experimentally if neutrinoless double beta decay was observed, as it appears from eq. (2.1.26). Interestingly, the success of leptogenesis in this scenario imposes an upper bound on the parameters $m_{\nu 1}$ and $|m_{\nu ee}|$: the absolute upper bound is of the order of 0.05 eV for both. However, if we want to accommodate lower values for the triplet mass, they should rather satisfy

$$m_{\nu 1},\, |m_{\nu ee}| \lesssim 0.01\,\mathrm{eV}\ . \tag{3.3.49}$$

This means that this scenario is favoured by hierarchical neutrinos. It also imposes a more stringent bound on the triplet mass than the previous scenario, namely

$$M_\Delta^{\min} = 1.2 \times 10^{12}\,\mathrm{GeV}\ . \tag{3.3.50}$$

We also indicated on this plot the region where the effective coupling λ_ℓ is large or nonperturbative. Note that, strictly speaking, the condition of perturbativity should be $|f_{\alpha\beta}| < 4\pi$ for all flavour indices α and β, which would enlarge a little bit the region of parameter space available, but in practice it is not a too bad approximation to define perturbativity from the condition $\lambda_\ell < 4\pi$.

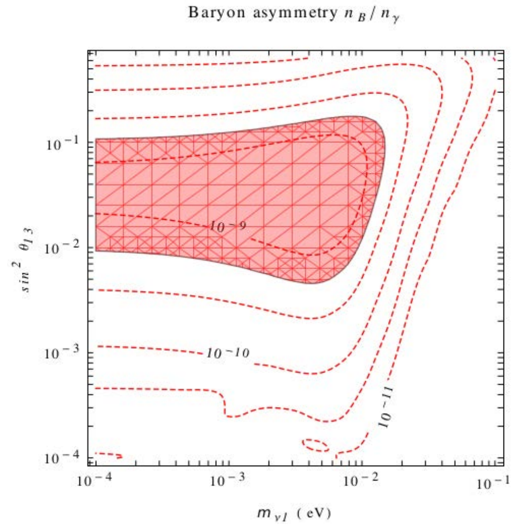


Figure 3.14: Baryon asymmetry produced as a function of $m_{\nu1}$ and $\sin^2\theta_{13}$, for normal hierarchy, with a triplet mass $M_\Delta = 2 \times 10^{12}$ GeV and $\lambda_H = 0.2$. The region colored in red is where the baryon asymmetry is greater than the BAU.

Another interesting feature is the variation of the baryon asymmetry as a function of the mixing angle θ_{13}, which was measured in 2012 [86]. Fig. 3.14 shows how the final baryon asymmetry depends on $m_{\nu1}$ and θ_{13}. It turns out that the measured value $\sin^2\theta_{13} = 0.023$ is rather favourable to this scenario. A value closer to zero would indeed increase the lower bound on the triplet mass, and therefore shrink the region of parameter space available for successful leptogenesis. The reason for this is simply that the CP asymmetry is proportional to s_{13}^2, as can be seen from eq. (3.3.47). Larger values of θ_{13}, already ruled out by the CHOOZ experiment [85], would also be unfavourable.

Inverted hierarchy

We turn now to the inverted hierarchy. The isocurves of the final baryon asymmetry as a function of M_Δ and $m_{\nu3}$ or $|m_{\nu ee}|$ are displayed in fig. 3.15. This case shows important differences with respect to normal hierarchy. It appears that, even though the observed value of the baryon-to-photon ratio can be reached, inverted hierarchy is much less favourable to this scenario, because it leaves only a reduced portion of parameter space available. In particular, it seems that the success of leptogenesis requires a triplet heavier than 5×10^{13} GeV. However, in this region where the couplings are large, the validity of our approximation of neglecting scatterings is more doubtful, which means that, in reality, the baryon asymmetry is probably a bit smaller than what is shown here due to a stronger washout.

Another way to see this shrinking of the available parameter space is to compute

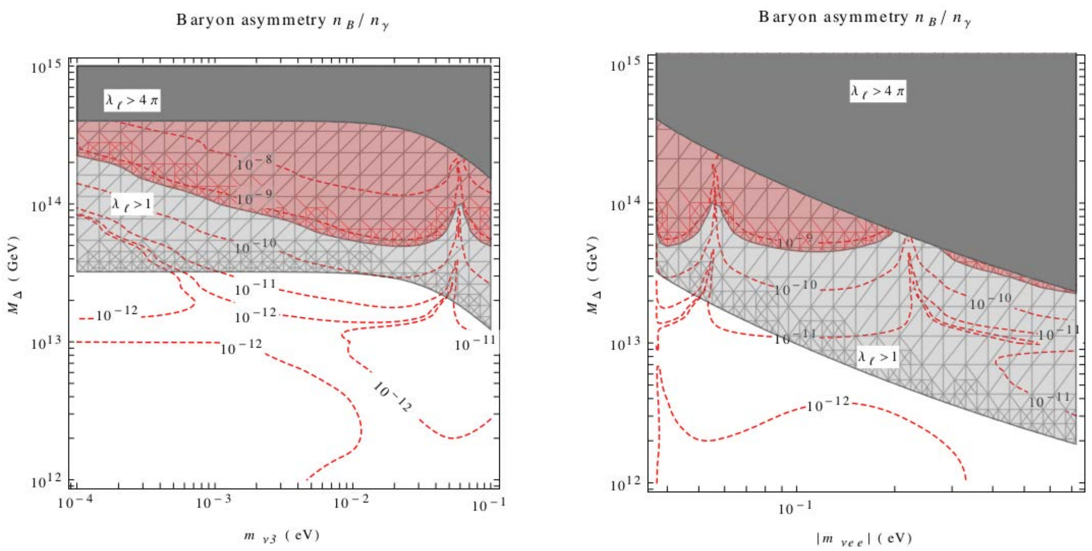


Figure 3.15: Baryon asymmetry produced as a function of $m_{\nu 3}$ and M_Δ (left) and $|m_{\nu ee}|$ and M_Δ (right) for inverted hierarchy, with $\lambda_H = 0.2$. The region colored in red is where the baryon asymmetry is grater than the BAU. The region colored in gray is where the coupling λ_ℓ is large (light gray) or nonperturbative (dark gray).

the baryon asymmetry as a function of λ_ℓ and λ_H, for a fixed $m_{\nu 3} = 10^{-3}$ eV. The results are displayed in fig. 3.16. The observed value of the baryon-to-photon ratio is never reached in this plane, which means that larger values of the effective couplings λ_ℓ and λ_H are required, leading to the same problem as previously.

The reason why this scenario does not work well in inverted hierarchy is related to the previous discussion on the conditions for a large efficiency. As was mentioned, a large efficiency is obtained when the coupling $|f_{11}|$, which is proportional to $|m_{\nu ee}|$, is small enough, but in inverted hierarchy, this quantity is bounded from below. As a consequence, the washout is always strong and the conversion of the CP asymmetry is inefficient.

Summary

This model shows how leptogenesis with a scalar triplet can be consistent with a predictive type II seesaw scenario, in which the neutrino mass matrix is proportional to a single coupling matrix. This has several consequences for the neutrino mass parameters. For instance, the Majorana phases ρ and σ should provide a sufficient CP violation. This was not the case in the model studied in 3.2, where the CP asymmetry did not depend on the phases of the neutrino mass matrix, but only on the relative phases between the two contributions m_Δ and m_κ.

This scenario is favoured by hierarchical neutrinos with a small effective Majorana mass $|m_{\nu ee}| < 3 - 4 \times 10^{-2}$ eV, which is still well below the upper bound given by experiments looking for neutrinoless double beta decay [98]. The corresponding upper bound on the lightest neutrino mass is $m_{\nu 1} < 3 - 4 \times 10^{-2}$ eV. Quasi-degenerate neutrinos would prevent successful leptogenesis through this mechanism, whereas inverted

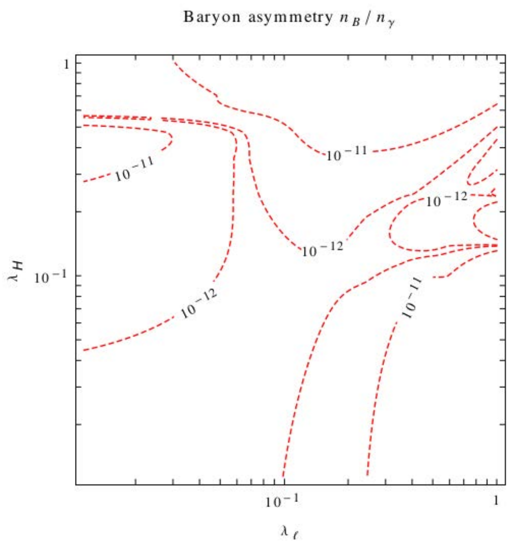


Figure 3.16: Baryon asymmetry produced as a function of λ_ℓ and λ_H for inverted hierarchy and $m_{\nu 3} = 10^{-3}$ eV.

hierarchy would strongly diminish the probability of its realization. In both cases, Sakharov's third condition is not satisfied because all the washout rates are large. It is not possible to overcome this by adjusting the couplings, because they are already constrained by the neutrino mass matrix.

Since the CP asymmetry grows with $\sin^2\theta_{13}$, a small value of θ_{13} could also have disqualified this model, but it appears that the measured value is close to optimizing the production of the baryon asymmetry.

This scenario requires the scalar triplet to be heavier than 10^{12} GeV, in agreement with the estimation of ref. [211]. This is well above the bounds derived in 3.2 for the general model of scalar triplet leptogenesis. This means that leptogenesis always takes place in a regime where the lepton Yukawa are out of equilibrium. Consequently, the computation has to be performed in the density matrix formalism to take flavour effects into account, but their impact is much less important than in the general scenario studied in 3.2: here, the couplings involved are to a large extent determined by the neutrino mass matrix, and there is little freedom left to play with the couplings as was done previously in order to maximize flavour effects.

Chapter 4

Sterile neutrinos as pseudo-Goldstone fermions

> I have done a terrible thing, I have postulated a particle that cannot be detected.
>
> Wolfgang Pauli

4.1 Introduction to Supersymmetry

Supersymmetry was first imagined in 1966 by Hironari Miyazawa as a symmetry between baryons and mesons [212]. Premises were developed in the following years [213–216], and the first supersymmetric four-dimensional quantum field theory was then written in 1973 by Julius Wess and Bruno Zumino [217].

In 1967, Coleman and Mandula had discovered a no-go theorem stating that the symmetry group of any quantum field theory has to be the direct product of the Poincaré group and the internal symmetry group [218]. This seemed to rule out any attempt to extend the Poincaré algebra. There was however a loophole, due to the fact that they only considered bosonic internal symmetries, that can change the spin of particles by integer values only. Allowing for the possibility of fermionic symmetries, that can exchange bosons and fermions, Haag, Lopuszanski and Sohnius discovered that supersymmetry is the only way to extend non-trivially the Poincaré group [219]. This validated a posteriori the work of Wess and Zumino.

Supersymmetric extensions of the Standard Model were built in the late seventies [220–223], leading to what is now known as the Minimal Supersymmetric Standard Model (MSSM). A few years later, it was discovered that supersymmetry not only provides a solution to the hierarchy problem [224–228], as mentioned in 1.3, but also ensures the unification of gauge coupling constants to a very good precision [229].

4.1.1 Superspace and superfields

Here, as in the previous chapters of this thesis, we use the metric signature $(+,-,-,-)$.

Basic properties

The generators of supersymmetry are operators that change the spin of particles by half-integer values. They must therefore have a spin 1/2, and transform as spinors under the Poincaré group. They are labelled Q^I_α, $\bar{Q}^J_{\dot{\alpha}}$, where α, $\dot{\alpha} = 1, 2$ are spinor indices and $I, J = 1,..., N$, N being the number of supersymmetries. The 2×2 antisymmetric tensor is used to raise, lower or contract spinor indices. From now on, $N = 1$ and we drop the indices I and J. The generators satisfy the following anticommutation relations [219],

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \ , \tag{4.1.1}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \ , \tag{4.1.2}$$

where $\sigma^\mu = (1, \sigma^i)$ and we also define $\bar{\sigma}^\mu = (1, -\sigma^i)$. They also satisfy the following commutation relations with generators of the Poincaré algebra,

$$[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot{\alpha}}] = 0 \ , \tag{4.1.3}$$

$$[M_{\mu\nu}, Q_\alpha] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta \ , \tag{4.1.4}$$

$$[M_{\mu\nu}, \bar{Q}^{\dot{\alpha}}] = \frac{1}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{Q}^{\dot{\beta}} \ , \tag{4.1.5}$$

with

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\right) \ , \qquad \bar{\sigma}^{\mu\nu} = \frac{i}{2}\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right) \ . \tag{4.1.6}$$

These (anti-)commutation relations, together with those of the Poincaré algebra, define the Super-Poincaré algebra.

A convenient representation of supersymmetry is given by superspace. Superspace is defined by the introduction of fermionic coordinates in addition to the usual space-time coordinates. These coordinates, labelled as θ^α, $\bar{\theta}_{\dot{\alpha}}$, have mass dimension $-1/2$ and are anticommuting. We use the following shorthand notations,

$$\theta^2 = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta, \qquad \bar{\theta}^2 = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}}, \qquad \theta^4 = \theta^2\bar{\theta}^2, \tag{4.1.7}$$

A remarkable feature of fermionic coordinates is that integration and derivation with respect to them give the same results,

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \int d\theta_\alpha\, \theta^\beta = \delta^\beta_\alpha \ . \tag{4.1.8}$$

Moreover, since the θ, $\bar{\theta}$ coordinates anticommute, any product of more than two θ or two $\bar{\theta}$ is necessarily zero. As a consequence, any function of superspace coordinates $\mathcal{F}(x, \theta, \bar{\theta})$, or superfield, can be expanded in a series that terminates at order θ^4. The most general superfield can therefore be parametrized as follows,

$$\begin{aligned}\mathcal{F}(x, \theta, \bar{\theta}) = a(x) &+ \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta^2 b(x) + \bar{\theta}^2 c(x) + \bar{\theta}\bar{\sigma}^\mu\theta v_\mu(x) \\ &+ \bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}\bar{\eta}(x) + \theta^4 d(x) \ ,\end{aligned} \tag{4.1.9}$$

where a, b, c and d are scalars, ψ, χ, λ and η are left-handed Weyl spinors and v_μ is a four-vector, so that the numbers of fermionic and bosonic degrees of freedom are equal. This field can be fundamental, in which case it represents elementary particles related by supersymmetry, or composite.

An explicit representation of supersymmetry generators is then given by

$$Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu \ , \tag{4.1.10}$$

$$\bar{Q}^{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + (\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu \ , \tag{4.1.11}$$

and an infinitesimal supersymmetry transformation acting on a superfield can be written as

$$\delta_\epsilon\mathcal{F}(x,\theta,\bar{\theta}) = -i\left(\epsilon Q + \bar{\epsilon}\bar{Q}\right)\mathcal{F}(x,\theta,\bar{\theta}) \ , \tag{4.1.12}$$

with a spinorial transformation parameter ϵ_α. This transformation is nothing but a translation in superspace

$$(1+\delta_\epsilon)\mathcal{F}(x^\mu,\theta,\bar{\theta}) = \mathcal{F}(x^\mu - i\epsilon\sigma^\mu\bar{\theta} - i\bar{\epsilon}\bar{\sigma}^\mu\theta, \theta+\epsilon, \bar{\theta}+\bar{\epsilon}) \ , \tag{4.1.13}$$

Looking more precisely at the transformations of the various components, we can see that this does indeed transform fermionic and bosonic degrees of freedom into one another. For instance, the scalar a and the spinor ψ transform according to

$$\delta_\epsilon a(x) = \epsilon\psi(x) + \bar{\epsilon}\bar{\chi}(x) \ , \tag{4.1.14}$$

$$\delta_\epsilon\psi_\alpha(x) = 2\epsilon_\alpha b(x) - (\sigma^\mu\bar{\epsilon})_\alpha\left(v_\mu(x) - i\partial_\mu a(x)\right) \ . \tag{4.1.15}$$

Chiral superfields

Chiral covariant derivatives are defined as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu \ , \tag{4.1.16}$$

$$\bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu \ . \tag{4.1.17}$$

They are consistent with supersymmetry in the sense that they anticommute with supersymmetry generators, so that

$$D_\alpha\left(\delta_\epsilon\mathcal{F}(x,\theta,\bar{\theta})\right) = \delta_\epsilon\left(D_\alpha\mathcal{F}(x,\theta,\bar{\theta})\right) \ . \tag{4.1.18}$$

These covariant derivatives also have the following anticommutation relations,

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{4.1.19}$$

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \ , \tag{4.1.20}$$

which are identical to those of the supersymmetry generators Q_α and $\bar{Q}_{\dot{\alpha}}$.

A chiral superfield Φ is defined by the condition $\bar{D}^{\dot{\alpha}}\Phi = 0$. Changing variables for $(y, \theta, \bar{\theta})$, with $y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}$, the covariant derivative $\bar{D}_{\dot{\alpha}}$ takes a very simple form

$$\bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \; , \tag{4.1.21}$$

This makes it obvious that if $\Phi(y, \theta, \bar{\theta})$ is a chiral superfield, it should not depend on $\bar{\theta}$. Thus, its general form is

$$\Phi(y,\theta,\bar{\theta}) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \; , \tag{4.1.22}$$

where ϕ is a complex scalar and ψ a left-handed Weyl fermion, whereas F is an auxiliary field that does not propagate. If this chiral superfield is fundamental, it has mass dimension 1 in order to be consistent with the dimensionality of fundamental scalars and fermions. The expression of Φ in the usual system of coordinates $(x, \theta, \bar{\theta})$ can be recovered from this,

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) = \phi(x) &+ i\theta\sigma^\mu\bar{\theta}\,\partial_\mu\phi(x) - \frac{1}{4}\theta^4\partial_\mu\partial^\mu\phi(x) + \sqrt{2}\,\theta\psi(x) \\ &+ \frac{i}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta^2 F(x) \; ,\end{aligned} \tag{4.1.23}$$

The conjugate superfield $\Phi^\dagger$ is antichiral and satisfies the condition $D_\alpha\Phi^\dagger = 0$. Any holomorphic function of chiral superfields is itself a chiral superfield.

Since the matter fields of the Standard Model are chiral fermions, in supersymmetric extensions of the Standard Model, they should be embedded in chiral superfields. More precisely the left-handed quarks and lepton doublets and the left-handed antiquark and antilepton singlets belong to chiral superfields, while their CP-conjugate belong to antichiral superfields.

Vector superfields

A vector (or real) superfield V is defined by the condition $V^\dagger = V$, which gives

$$\begin{aligned}V(x,\theta,\bar{\theta}) = a(x) &+ i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + i\theta^2 b(x) - i\bar{\theta}^2 b^\dagger(x) + \bar{\theta}\bar{\sigma}^\mu\theta A_\mu(x) \\ &- i\bar{\theta}^2\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right) + i\,\theta^2\bar{\theta}\left(\bar{\lambda}(x) - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right) \\ &+ \theta^4\left(\frac{1}{2}D(x) - \frac{1}{4}\partial_\mu\partial^\mu a(x)\right) \; .\end{aligned} \tag{4.1.24}$$

Vector superfields are used to represent gauge interactions, in which case A_μ is the associated gauge boson. A fundamental vector superfield is therefore dimensionless. The Weyl fermion λ is called a gaugino and D is an auxiliary field.

For an abelian gauge theory, the supersymmetric version of a gauge transformation takes the form

$$V(x,\theta,\bar{\theta}) \longrightarrow V(x,\theta,\bar{\theta}) + i\left(\Lambda^\dagger(x,\theta,\bar{\theta}) - \Lambda(x,\theta,\bar{\theta})\right) \; , \tag{4.1.25}$$

where Λ is a chiral superfield. This transformation preserves the condition $V = V^\dagger$, and it allows to get rid of the scalars a, b and the fermion χ, whereas the gaugino λ and the auxiliary field D remain unaffected. For the vector field, the gauge transformation takes its usual form,

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \left(\phi(x) + \phi^\dagger(x)\right) \ . \tag{4.1.26}$$

The gauge in which a, b and χ are absent is the Wess-Zumino gauge, in which the vector superfield takes the following simple form,

$$V_{WZ}(x,\theta,\bar\theta) = \bar\theta\bar\sigma^\mu\theta A_\mu(x) - i\bar\theta^2\theta\lambda(x) + i\,\theta^2\bar\theta\bar\lambda(x) + \frac{1}{2}\theta^4 D(x) \ . \tag{4.1.27}$$

Restricting oneself to the Wess-Zumino gauge, one can still perform an ordinary gauge transformation on A_μ alone by choosing for Λ a chiral superfield that reduces to its scalar component.

This gauge transformation can be generalized to the non-abelian case. Using the shorthand notations $\mathbf{V} = 2gV_A T_A$ and $\mathbf{\Lambda} = 2g\Lambda_A T_A$, where the T_A are the generators of the gauge group, a general gauge transformation is given by

$$e^V \to e^{i\mathbf{\Lambda}^\dagger} e^{\mathbf{V}} e^{-i\mathbf{\Lambda}} \ , \tag{4.1.28}$$

where g is the coupling constant. Using the Baker-Campbell-Hausdorff formula to expand the exponential, one can check that, at first order in (Φ, $\Phi^\dagger$), this reduces to eq. (4.1.25) in the case of an abelian gauge theory. In the general case, the gauge boson and gaugino transform as

$$A^\mu_A(x) \longrightarrow A^\mu_A(x) + \partial_\mu \left(\phi_A(x) + \phi^\dagger_A(x)\right) + gf_{ABC}A^\mu_B(x)\left(\phi_C(x) + \phi^\dagger_C(x)\right) \ , \tag{4.1.29}$$

$$\lambda_A(x) \longrightarrow \lambda_A(x) + gf_{ABC}\lambda_B(x)\left(\phi_C(x) + \phi^\dagger_C(x)\right) \ , \tag{4.1.30}$$

f_{ABC} being the structure constant defined by $[T_A, T_B] = if_{ABC}T_C$.

4.1.2 Supersymmetric theories

Supersymmetric Lagrangians

In a supersymmetric theory, the action should be invariant under supersymmetry transformations. Since the θ^4 component of a general superfield transforms as a total derivative, the integral of any superfield over superspace (which selects the highest degree component) meets this requirement. Moreover, since the action must be real, this superfield has to be real as well. The action can therefore get a contribution of the following form,

$$\mathcal{S} \supset \int d^4x \int d^2\theta d^2\bar\theta\, V(x,\,\theta,\,\bar\theta) \ , \tag{4.1.31}$$

where V is a vector superfield. Equivalently, the Lagrangian receives a contribution of the form

$$\mathcal{L} \supset [V]_D = \int d^2\theta d^2\bar\theta\, V(x,\,\theta,\,\bar\theta) = \frac{1}{2}D - \frac{1}{4}\partial_\mu\partial^\mu a \ , \tag{4.1.32}$$

also called a D-term contribution. Another possibility is to consider the integral over ordinary space-time coordinates and over $d^2\theta$ only (resp. $d^2\bar{\theta}$) of a chiral (resp. antichiral) superfield. Indeed, under supersymmetry, this transforms as

$$\int d^2\theta \delta_\epsilon \Phi = \frac{1}{2\sqrt{2}} \bar{\theta}^2 \epsilon \partial_\mu \partial^\mu \psi + i\bar{\theta}\bar{\sigma}^\mu \partial_\mu \left[i\sigma^\nu \bar{\epsilon} \partial_\nu \phi + \epsilon F \right] - i\sqrt{2} \partial_\mu \psi \sigma^\mu \bar{\epsilon} \ , \tag{4.1.33}$$

which is a total derivative, ensuring that the action is invariant. This gives a so-called F-term contribution to the Lagrangian, which should also contain its hermitian conjugate to make sure that the action is real,

$$\mathcal{L} \supset [\Phi]_F + \text{h.c.} = \int d^2\theta \, \Phi(x, \theta, \bar{\theta}) + \int d^2\bar{\theta} \, \Phi^\dagger(x, \theta, \bar{\theta}) = F + F^\dagger + \text{ total derivatives} \ . \tag{4.1.34}$$

One can equivalently define the F-term contribution as

$$\mathcal{L} \supset [\Phi]_F + \text{h.c.} = \int d^2\theta d^2\bar{\theta} \left[\delta^{(2)}(\bar{\theta}) \Phi(x, \theta, \bar{\theta}) + \delta^{(2)}(\theta) \Phi^\dagger(x, \theta, \bar{\theta}) \right] \ . \tag{4.1.35}$$

Kähler potential

A way to construct a vector superfield with fundamental chiral superfields only is to take the product $\Phi^\dagger\Phi$. The associated D-term contribution takes the explicit form

$$\left[\Phi^\dagger \Phi \right]_D = F^\dagger F + \partial_\mu \phi^\dagger \partial^\mu \phi - i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi \ . \tag{4.1.36}$$

This contains the kinetic term of the component fields. More generally, kinetic terms can be described by the Kähler potential, which is a function of chiral superfields with mass dimension 2. In a renormalizable supersymmetric theory, it takes the form

$$K = Z_{ij} \Phi_i^\dagger \Phi_j \ , \tag{4.1.37}$$

where $Z_{ij} = Z_{ji}^*$ is the wavefunction renormalization, which can be absorbed through a rescaling of the fields to recover the canonical form,

$$K = \Phi_i^\dagger \Phi_i \ . \tag{4.1.38}$$

The Kähler potential could in principle contain holomorphic contributions of the form

$$K_{\text{holomorphic}} = \mathcal{H}_{ij} \Phi_i \Phi_j \ , \tag{4.1.39}$$

but, when integrated over $d^2\theta d^2\bar{\theta}$, what remains of this term is a total derivative, at least as long as supersymmetry is unbroken. Therefore, it does not give any physical contribution in a truly supersymmetric theory. However, such a term can play a role when considering supersymmetry-breaking effects. In nonrenormalizable theories, the Kähler potential can also contain higher order terms.

In a gauge theory, there should also be terms accounting for gauge interactions of chiral superfields. This is done as in non-supersymmetric theories by requiring the invariance of the Lagrangian under

$$\Phi_{ia} \to \left(e^{2ig\Lambda_A t_A} \right)_a^{\ b} \Phi_{ib} \ , \qquad \Phi_{ia}^\dagger \to \Phi_{ib}^\dagger \left(e^{-2ig\Lambda_A^\dagger t_A} \right)^b_{\ a} \tag{4.1.40}$$

where Λ_A is a chiral superfield, the t_A are the generators of the gauge group in the representation containing Φ_i, a and b are gauge indices and g is the gauge coupling. The kinetic terms have to be generalized as follows,

$$\left[\Phi^\dagger_{ia}\left(e^{2gt_AV_A}\right)^{ab}\Phi_{ib}\right]_D \ , \tag{4.1.41}$$

where the vector superfields transform as in eq. (4.1.28). In the Wess-Zumino gauge, every term of the gauge superfield contains at least one power of θ and $\bar{\theta}$, so the expansion of the exponential terminates at second order in V. Thus, the contribution to the Lagrangian is,

$$\begin{aligned}\left[\Phi^\dagger_{ia}\left(e^{2gt_AV_A}\right)^{ab}\Phi_{ib}\right]_D &= F^\dagger_{ia}F^{ia} + D_\mu\phi^\dagger_{ia}D^\mu\phi_{ia} - i\bar{\psi}_{ia}\bar{\sigma}^\mu D_\mu\psi_{ia} \\ &+ i\sqrt{2}g\phi^\dagger_{ia}t^{ab}_A\psi_{ib}\lambda_A - i\sqrt{2}g\bar{\lambda}_A\bar{\psi}_{ia}t^{ab}_A\phi_{ib} + g\phi^\dagger_{ia}t^{ab}_A\phi_{ib}D_A \ .\end{aligned} \tag{4.1.42}$$

The first line contains the ordinary kinetic terms, while the second one contains the supersymmetric counterpart of gauge interactions.

Kinetic terms of gauge superfields

A gauge theory obviously requires kinetic terms for the gauge fields. For an abelian gauge theory, one defines

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \ . \tag{4.1.43}$$

$\mathcal{W}_\alpha$ is a chiral superfield, because, as can be seen from the anticommutation relation of eq. (4.1.20), the product of more than two (anti)chiral derivatives is zero, so that $\bar{D}_{\dot{\beta}}\mathcal{W}_\alpha = 0$. In the Wess-Zumino gauge, $\mathcal{W}_\alpha$ reads explicitly

$$\mathcal{W}_\alpha = -i\lambda_\alpha + \theta_\alpha D + \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2(\sigma^\mu\partial_\mu\bar{\lambda})_\alpha \ . \tag{4.1.44}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the (abelian) field strength tensor. Since $\mathcal{W}^\alpha$ is a chiral superfield, $\mathcal{W}^\alpha\mathcal{W}_\alpha$ is also chiral and one obtains the corresponding Lagrangian density by taking the F-term. For an abelian gauge theory, the kinetic terms for the components of the gauge superfield are therefore

$$\frac{1}{4}[\mathcal{W}^\alpha\mathcal{W}_\alpha]_F + \text{h.c.} = \frac{1}{2}D^2 - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ . \tag{4.1.45}$$

For a general gauge theory, one defines instead $\mathcal{W}_\alpha$ in the following way,

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2(e^{-\mathbf{V}}D_\alpha e^{\mathbf{V}}) \ , \qquad \bar{\mathcal{W}}_{\dot{\alpha}} = \frac{1}{4}D^2(e^{-\mathbf{V}}\bar{D}_{\dot{\alpha}} e^{\mathbf{V}}) \ , \tag{4.1.46}$$

where we use again $\mathbf{V} = 2gT_AV_A$. $\mathcal{W}_\alpha$ has a simpler form in the Wess-Zumino gauge, where one can use the Baker-Hausdorff-Campbell formula together with the fact that the product of more than two $\mathbf{V}$'s vanishes to obtain

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha\mathbf{V} + \frac{1}{8}\bar{D}^2[\mathbf{V}, D_\alpha\mathbf{V}] \ . \tag{4.1.47}$$

One can see that, in the special case of an abelian gauge theory, the right-hand side of eq. (4.1.47) would reduce to its first term, which is in agreement with eq. (4.1.43) up to a factor of $2g$. In the case of a non-abelian theory, the second term is needed to replace the ordinary derivatives of eq. (4.1.45) with covariant derivatives. Defining now the field strength tensor as $F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + g f_{ABC} A^B_\mu A^C_\nu$, the F-term contribution to the Lagrangian is the following,

$$\mathrm{tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha]_F = 2g^2 D_A D_A - 4ig^2 \lambda_A \sigma^\mu D_\mu \bar{\lambda}_A - g^2 F^A_{\mu\nu} F^{\mu\nu}_A + \frac{ig^2}{2}\epsilon_{\mu\nu\rho\sigma} F^{\mu\nu}_A F^{\rho\sigma}_A \; . \tag{4.1.48}$$

The last term is a total derivative, but it can nevertheless play a physical role in a non-abelian gauge theory where it accounts for CP violation, as already mentioned in 1.3. As a consequence, in full generality, the gauge kinetic term of a non-abelian theory is

$$\left(\frac{1}{8g^2} - i\frac{\Theta}{64\pi^2}\right)\mathrm{tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \mathrm{h.c.} = \frac{1}{2} D_A D_A - i\lambda_A \sigma^\mu D_\mu \bar{\lambda}_A - \frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A + \frac{g^2\Theta}{64\pi^2}\epsilon_{\mu\nu\rho\sigma} F^{\mu\nu}_A F^{\rho\sigma}_A \; . \tag{4.1.49}$$

Θ is the so-called vacuum angle, that measures the amount of CP violation. Together with eq. (4.1.42), this allows to derive the equation of motion for the non-dynamical auxiliary terms D_A,

$$\frac{\partial \mathcal{L}}{\partial D_A} = 0 \Rightarrow D_A = -g\phi^\dagger_{ia} t^{ab}_A \phi_{ib} \; . \tag{4.1.50}$$

Superpotential

An F-term contribution to the Lagrangian can be built by considering an holomorphic function of chiral superfields, which is therefore itself a chiral superfield, the superpotential. In a renormalizable theory, the interactions in the superpotential must have dimension 3 or less, so that the most general form that it can take is

$$W = L_i \Phi_i + \frac{1}{2} M_{ij} \Phi_i \Phi_j + \frac{1}{6}\lambda_{ijk} \Phi_i \Phi_j \Phi_k \; . \tag{4.1.51}$$

The first term, which we will drop from now on, is allowed only if Φ_i is a gauge singlet. Expanding W in powers of θ and integrating over fermionic coordinates, one finds the following contribution to the Lagrangian,

$$[W]_F = W_i F_i - \frac{1}{2} W_{ij} \psi_i \psi_j \; , \tag{4.1.52}$$

with

$$W_i = \frac{\partial W}{\partial \Phi_i}_{|\Phi_i = \phi_i}, \qquad W_{ij} = \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}_{|\Phi_i = \phi_i, \Phi_j = \phi_j} \; . \tag{4.1.53}$$

Together with the D-term contribution coming from the Kähler potential, this allows to derive the equation of motion for the auxiliary field F_i, which is simply

$$\frac{\partial \mathcal{L}}{\partial F_i} = 0 = F_i^\dagger + W_i \Rightarrow F_i = -W_i^\dagger \ , \tag{4.1.54}$$

so that the auxiliary fields can be eliminated. Finally, the contributions of the superpotential to the Lagrangian are

$$[W]_F + \text{h.c.} = -\frac{1}{2} M_{ij}\psi_i\psi_j - \frac{1}{2}\lambda_{ijk}\phi_i\psi_j\psi_k + \text{h.c.} - \mathcal{V}(\phi_i) \ . \tag{4.1.55}$$

The scalar potential $\mathcal{V}$ is obtained after replacing the auxiliary fields as indicated in eq. (4.1.54)

$$\begin{aligned}\mathcal{V}(\phi_i) = F_i^\dagger F_i = M_{im}^* M_{mj}\phi_i^\dagger\phi_j + \frac{1}{2}M_{im}\lambda_{mjk}^*\phi_i\phi_j^\dagger\phi_k^\dagger + \frac{1}{2}M_{im}^*\lambda_{mjk}\phi_i^\dagger\phi_j\phi_k \\ + \lambda_{ijm}\lambda_{mkl}^*\phi_i\phi_j\phi_k^\dagger\phi_l^\dagger \ .\end{aligned} \tag{4.1.56}$$

In a gauge theory, the scalar potential also receives contributions obtained after replacing the auxiliary field D_A in eq. (4.1.42) with its expression (4.1.50), so that the full scalar potential is

$$\begin{aligned}\mathcal{V}(\phi_i) = F_i^\dagger F_i + \frac{1}{2}D_A D_A = M_{im}^* M_{mj}\phi_i^\dagger\phi_j + \frac{1}{2}M_{im}\lambda_{mjk}^*\phi_i\phi_j^\dagger\phi_k^\dagger + \frac{1}{2}M_{im}^*\lambda_{mjk}\phi_i^\dagger\phi_j\phi_k \\ + \lambda_{ijm}\lambda_{mkl}^*\phi_i\phi_j\phi_k^\dagger\phi_l^\dagger + \sum_A \frac{g^2}{2}\left(\sum_i \phi_{ia}^\dagger t_A^{ab}\phi_{ib}\right)^2 \ ,\end{aligned} \tag{4.1.57}$$

where, except for the last term, we omitted gauge indices to simplify the notations.

A nonrenormalization theorem [230, 231] states that the parameters of the superpotential are free from renormalization. As a consequence, divergences in loop diagrams only come from wavefunction renormalization and are at most logarithmic. This property is important, because it ensures that supersymmetry provides a solution to the hierarchy problem by cancelling quadratic divergences. More precisely, keeping the Kähler potential under a canonical form as in eq. (4.2.6) requires to rescale the fields according to

$$\Phi_i \Rightarrow V_{ij}\Phi_j \ , \tag{4.1.58}$$

with $Z = V^\dagger V$. In turn, this implies a rescaling of the superpotential parameters. As a consequence, the renormalization group equations take the following form,

$$\frac{dV_{ij}}{dt} = -\gamma_{ik}V_{kj} \tag{4.1.59}$$

$$\frac{dM_{ij}}{dt} = \gamma_{im}M_{mj} + \gamma_{jm}M_{im} \tag{4.1.60}$$

$$\frac{d\lambda_{ijk}}{dt} = \gamma_{im}\lambda_{mjk} + \gamma_{jm}\lambda_{imk} + \gamma_{km}\lambda_{ijm} \ , \tag{4.1.61}$$

where γ is the matrix of anomalous dimensions.

Soft terms

As can be seen from eqs. (4.1.55) and (4.1.56), the squared mass matrix of the scalars and the mass matrix of the chiral fermions can be diagonalized in the same basis. In this basis, it is clear that the fermion and scalar of a given superfield have the same mass. However, if such was the case in our universe, superpartners of Standard Model fermions should have been discovered long ago. Supersymmetry must therefore be broken to explain why the superpartners are (until now) out of experimental reach.

A solution would be to break spontaneously supersymmetry, as will be explained in chapter 5. Alternatively, since it is not an easy task, one can simply use an effective parametrization involving interactions that break explicitly supersymmetry. However, these interactions should preserve an important feature of supersymmetry, which is the cancellation of quadratic divergences in corrections to the Higgs squared mass. If this condition is satisfied, supersymmetry breaking is said to be soft.

The list of possible soft couplings was established in ref. [232]. In particular, soft couplings must have a strictly positive mass dimension. Soft terms include non-holomorphic and holomorphic squared masses for the scalars, trilinear couplings between the scalars and Majorana masses for gauginos,

$$\mathcal{L}_{\text{soft}} \supset -(m^2)_{ij}\phi_i^\dagger\phi_j - \left(\frac{1}{2}B_{ij}\phi_i\phi_j + \frac{1}{6}A_{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}M_A\lambda_A\lambda_A + \text{h.c.}\right) . \tag{4.1.62}$$

One could also introduce mass terms for the chiral fermions $m'_{ij}\psi_i\psi_j$, but this could be eliminated through a redefinition of the supersymmetric mass M_{ij} in eq. (4.1.51) and of the first term of eq. (4.1.62).

If the theory contains a chiral superfield Φ_a in the adjoint representation of a non-abelian gauge group, soft terms can also include a mixing between the corresponding chiral fermions and the gauginos, which are then referred to as Dirac gauginos [233],

$$\mathcal{L}_{\text{soft}} \supset -M_a\lambda_a\psi_a + \text{h.c.} . \tag{4.1.63}$$

Finally, another possibility is the non-holomorphic trilinear coupling,

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2}C_{ijk}\phi_i^\dagger\phi_j\phi_k + \text{h.c.} . \tag{4.1.64}$$

This term is soft as long as the theory does not contain any chiral superfield that is a gauge singlet. In the opposite case, it can lead to quadratically divergent tadpoles for the singlets. However, this term is usually not generated by spontaneous symmetry breaking, and we will not consider it.

4.1.3 The MSSM

To conclude this quick introduction to supersymmetry, the minimal supersymmetric extension of the Standard Model (MSSM) can be built by embedding the fermions of the Standard Model in chiral superfields and the gauge vectors in vector superfields [220–223]. Two Higgs doublets with opposite hypercharges, H_u ($Y = +1/2$) and H_d ($Y = -1/2$), which belong to chiral superfields, are needed for two different reasons. First, any chiral fermion charged under $SU(2) \times U(1)$ contributes to the electroweak gauge anomalies. In the Standard Model, the contributions of the various fermions cancel each other, but this cancellation is spoiled if one introduces just one new chiral fermion, as it is the case if there is a single Higgs superfield. Thus, one should instead introduce a vectorlike pair of new fermions, which should belong to two chiral superfield with opposite hypercharges. Second, since the superpotential is holomorphic, it cannot contain $H^\dagger$, so one cannot simply supersymmetrize the Lagrangian of eq. (1.2.13) by promoting every field to a chiral superfield. Instead, one has to introduce an additional Higgs doublet H_d with the same quantum numbers as $H^\dagger$ to write all the Yukawa couplings.

From now on, we use the same notation for chiral superfields as for their SM components, while squarks, sleptons and gauginos are labelled with a tilde. Similarly, we use the notation H_u and H_d both for the Higgs superfields and for their scalar components, while their fermionic counterparts, the higgsinos, are labelled with a tilde. The most general renormalizable and holomorphic superpotential that respects the gauge symmetry is

$$\begin{aligned} W = {} & y^u_{ij} H_u Q_i u^c_j + y^d_{ij} H_d Q_i d^c_j + y^e_{\alpha\beta} H_d \ell_\alpha e^c_\beta + \mu H_u H_d \\ & + \frac{1}{2}\lambda_{\alpha\beta\gamma} \ell_\alpha \ell_\beta e^c_\gamma + \lambda'_{\alpha ij} \ell_\alpha Q_i d^c_j + \frac{1}{2}\lambda''_{ijk} u^c_i d^c_j d^c_k + \mu_\alpha H_u \ell_\alpha \ . \end{aligned} \tag{4.1.65}$$

The terms in the first line give rise to the Yukawa couplings as well as to their supersymmetric counterparts, and to the Higgs squared mass parameter. The terms in the second line violate either lepton number or baryon number and give rise to severely constrained processes such as proton decay. Thus, either these couplings are extremely small in order to evade the experimental bounds, or they are forbidden by a symmetry. The second option is often preferred, with a discrete $\mathbb{Z}_2$ symmetry. One can choose for instance matter parity [234], defined as

$$P_m = (-1)^{3(B-L)} \ . \tag{4.1.66}$$

Since all the terms in the second line of eq. (4.1.65) have $P_m = -1$, they are forbidden. Imposing matter parity is equivalent to imposing the conservation of $B-L$, but remarkably, it forbids all the perturbative interactions that violate baryon or lepton number alone (nevertheless, they are still violated nonperturbatively by the electroweak sphalerons). Another possibility is R-parity [222], which can be defined as

$$P_R = (-1)^{3(B-L)+2s} \ , \tag{4.1.67}$$

where s is the spin of the particle. R-parity also forbids the couplings in the second line of eq. (4.1.65), but has an additional interesting feature. It turns out that the Standard Model fermions and gauge bosons as well as the Higgs scalars have a charge $P_R = +1$, whereas all their supersymmetric partners have $P_R = -1$. The lightest particle with $P_R = -1$ is automatically stable, because its decay cannot simultaneously respect R-parity and be kinematically allowed, so it could be a Dark Matter candidate. A last possibility is to forbid only baryon or lepton number-violating couplings. In particular, if only baryon number violation is forbidden, as will be the case in sec. 4.2, the lepton number violation gives rise to Majorana masses for neutrinos [235, 236]. This solution to the problem of neutrino masses is distinct from those mentioned in 2.1 and peculiar to supersymmetric theories.

In the MSSM, supersymmetry breaking is simply parametrized by the following soft terms (assuming matter parity or R-parity),

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &-(m^2_{\tilde{Q}})_{ij}\tilde{Q}^\dagger_i\tilde{Q}_j - (m^2_{\tilde{u}})_{ij}\tilde{u}^{c\dagger}_i\tilde{u}^c_j - (m^2_{\tilde{d}})_{ij}\tilde{d}^{c\dagger}_i\tilde{d}^c_j - (m^2_{\tilde{\ell}})_{\alpha\beta}\tilde{\ell}^\dagger_\alpha\tilde{\ell}_\beta \\ &- (m^2_{\tilde{e}})_{\alpha\beta}\tilde{e}^{c\dagger}_\alpha\tilde{e}^c_\beta - m^2_{H_d}H^\dagger_d H_d - m^2_{H_u}H^\dagger_u H_u \\ &- M_1\tilde{B}\tilde{B} - M_2\tilde{W}_A\tilde{W}_A - M_3\tilde{G}_A\tilde{G}_A \\ &- \left(A^u_{ij}H_u\tilde{Q}_i\tilde{u}^c_j + A^d_{ij}H_d\tilde{Q}_i\tilde{d}^c_j + A^e_{\alpha\beta}H_u\tilde{\ell}_\alpha\tilde{e}^c_\beta + B_\mu H_u H_d + \text{h.c.}\right) .\end{aligned} \tag{4.1.68}$$

The terms on the two first lines are the non-holomorphic squared masses of the scalars, while the third line contains Majorana masses for the gauginos associated to the $U(1)$, $SU(2)$ and $SU(3)$ gauge group respectively. The only B-term allowed by the symmetries is the holomorphic squared mass $B_\mu H_u H_d$, that plays an important role in the electroweak symmetry breaking.

This parametrization of supersymmetry breaking leaves us with 105 new free parameters with respect to the Standard Model. However, as explained in 2.2.2, the squared mass matrices of the squarks and sleptons as well as the A-terms generate flavour-changing neutral currents, which are strongly constrained by experiments. In order to satisfy these constraints, these soft terms should have a very peculiar flavour structure. For instance, it is sometimes assumed that the squared mass matrices are proportional to the identity matrix in flavour space, while the A-terms are proportional to the Yukawa couplings, for instance

$$A^u_{ij} = A^u_0 y^u_{ij} . \tag{4.1.69}$$

Of course, these assumptions are a bit arbitrary, and should be justified by explicit models of supersymmetry breaking. Such models will be investigated in the next chapter.

4.2 Pseudo-Goldstone fermion Lagrangian

As was said previously, a supersymmetric framework with broken R-parity can give rise to nonzero neutrino masses. We consider here such a scenario, in which the masses of neutrinos arise from their mixing with the fermionic partners of neutral bosons. As was mentioned in 2.1.1, some anomalies in neutrino experiments may be explained by the existence of a sterile neutrino with a mass around 1 eV. However, as we said in 2.1.2, a fermionic singlet could have an arbitrarily large Majorana mass, so that such a small value should be justified. There are a few possibilities for this.

(i) For instance, in the singular seesaw mechanism [237, 238], there are three right-handed neutrinos but their mass matrix has only rank 2 because of some symmetry, so that there is an additional light mass eigenstate.

(ii) In theories with more than four space-time dimensions, charged fields live on a 4-dimension brane, but since right-handed neutrinos are gauge singlets, they can live in the bulk. They can then be decomposed in an infinite tower of Kaluza-Klein modes. Standard Model neutrinos get a Dirac mass mostly with zero modes, while sterile neutrinos are mostly made of higher modes [239–241], with masses proportional to $1/R \sim 10^{-3}\,\text{eV}$, R being the compactification radius of the extra dimension. In their minimal realization, such models are now excluded because they are in disagreement with the energy spectrum of solar neutrinos measured by Super-Kamiokande [89].

(iii) In the context of superstring theories, the existence of neutral fields (moduli), which are massless at the perturbative level, is expected. The fermionic components of these fields get a small mass and mixing with neutrino from nonperturbative effects in supergravity and play the role of sterile neutrinos [242, 243].

(iv) Here, similarly to what was proposed in refs. [244–246], the lightness of the new state is justified by the fact that it is the supersymmetric partner of a pseudo-Goldstone boson.

The motivations for these models were initially the following. Before the Super-Kamiokande results, solar neutrinos oscillations could be explained by active-sterile oscillations, which was ruled out since then. Active-sterile oscillations were also thought to be the best explanation for the LSND anomaly. Moreover, light sterile neutrinos were also seen as candidates for hot dark matter, which is now excluded. On the other hand, recent papers on the topic focus on phenomenological aspect: assuming the existence of light sterile neutrinos, they study the experimental constraints on these neutrinos or try to fit the anomalies (see for instance refs. [247, 248, 104]). Here, we study a model that aims at explaining the origin of the sterile neutrino and the active-sterile mixing in the light of recent experimental results.

In this scenario, a global $U(1)$ symmetry spontaneously broken at a high scale f gives rise to a Goldstone chiral superfield below f,

$$A = \frac{s + ia}{\sqrt{2}} + \sqrt{2}\theta\chi + \theta^2 F \ . \tag{4.2.1}$$

The components of the superfield $\sigma = f \exp(A/f)$ are massless if both the global symmetry and supersymmetry are unbroken. After the breaking of the global $U(1)$, the theory features a residual shift symmetry $A \to A + i\alpha f$, or more precisely $a \to a + \sqrt{2}\alpha f$. a being a Goldstone boson, its interactions can only be derivative ones, which means that A cannot appear in the superpotential, but only in the Kähler potential.

In our scenario, the symmetry $U(1)$ is actually explicitly broken, which gives a small nonzero mass to all components of the pseudo-Goldstone superfield A. After supersymmetry breaking, the masses of the fermionic and scalar components are different. In particular we will assume $m_\chi \sim 1\,\text{eV} \ll m_s, m_a$. Ref. [249] explains in detail how the fermion acquires a mass of the same order as the gravitino mass in a context where A is the axion superfield invoked to solve the strong CP problem: in supergravity, the following effective operator is generated

$$\mathcal{L} \supset \int d^4\theta \lambda \frac{X + X^\dagger}{M_P} (A + A^\dagger)^2 \ , \tag{4.2.2}$$

where X is the field that breaks supersymmetry (see chapter 5 for more details), with in particular $\langle X\rangle = M_X + \theta^2 F_X$. This gives the following mass to the pseudo-Goldstone fermion,

$$m_\chi = \frac{2\lambda F_X}{M_P} \ , \tag{4.2.3}$$

to be compared to the gravitino mass,

$$m_{3/2} = \frac{F_X}{\sqrt{3}M_P} \ . \tag{4.2.4}$$

4.2.1 The framework

We assume that R-parity is broken, which means that nothing distinguishes the down-type Higgs superfield from the lepton superfields, so they can mix. Accordingly, we use the shorthand notation $\ell_a \equiv (H_d,\ \ell_\alpha)$ $(a = 0,1,2,3,\ \alpha = e,\mu,\tau)$, with $\ell_0 = H_d$. We will also write generically $\nu_a = (H_d^0, \nu_\alpha)$ and $e_{La} = (H_d^-, e_{L\alpha})$. However, R-parity-violating parameters should remain small in order to obtain a consistent neutrino phenomenology. On the other hand, we assume that baryon number is conserved. From these principles, the most general superpotential that we can write is

$$W = \mu_a H_u.\ell_a + \frac{1}{2} y^e_{ab\gamma} \ell_a \ell_b e^c_\gamma + y^d_{aij} \ell_a Q_i d^c_j + y^u_{ij} H_u Q_i u^c_j \ . \tag{4.2.5}$$

This contains a generalized μ-term (with $\mu_0 \gg \mu_1,\ \mu_2,\ \mu_3$) and the ordinary Yukawa couplings, as well as the lepton-number violating interaction terms $\ell_\alpha \ell_\beta e^c_\gamma$ and $\ell_\alpha Q_i d^c_j$, mentioned previously. Up to the order $1/f$, the most general general R-parity violating Kähler potential for the Higgs and lepton doublets and the Goldstone superfield reads:

$$\begin{aligned} K = {} & A^\dagger A + H_u^\dagger H_u + \ell_a^\dagger \ell_a + C_u H_u^\dagger H_u \frac{A + A^\dagger}{f} \\ & + C_{ab} \ell_a^\dagger \ell_b \frac{A + A^\dagger}{f} + \left(C_{ua} H_u.\ell_a \frac{A + A^\dagger}{f} + \text{h.c.} \right) + \mathcal{O}\left(\frac{1}{f^2}\right) \ . \end{aligned} \tag{4.2.6}$$

Supersymmetry breaking is parametrized by soft terms, which include in particular

$$\begin{aligned} -\mathcal{L}_{\text{soft}} \supset {} & m^2_{ab} \tilde{\ell}_a^\dagger \tilde{\ell}_b + m_u^2 H_u^\dagger H_u + \frac{1}{2} m_a^2 a^2 + \frac{1}{2} m_s^2 s^2 + \Big(B_a H_u \tilde{\ell}_a + \frac{1}{2} A^e_{ab\gamma} \tilde{\ell}_a \tilde{\ell}_b \tilde{e}^c_\gamma \\ & + A^d_{aij} \tilde{\ell}_a \tilde{Q}_i \tilde{d}^c_j + A^u_{aij} \tilde{\ell}_a \tilde{Q}_i \tilde{u}^c_j + \text{h.c.} \Big) \ . \end{aligned} \tag{4.2.7}$$

As a consequence of the mixing between the leptons and the down-type Higgs, the sneutrinos can get a vacuum expectation value after the electroweak symmetry breaking. In all generality, the minimization of the scalar potential gives

$$\langle H_u^0 \rangle = v_u \ , \qquad \langle H_d^0 \rangle = v_d \ , \qquad \langle \tilde{\nu}_\alpha \rangle = v_\alpha \ , \qquad \langle s \rangle = v_s \ . \tag{4.2.8}$$

Here, v_s should be much smaller than the scale f of the $U(1)$ symmetry breaking, so that, generically, v/f will always be a small parameter.

Then, we redefine fields in such a way that, once the scalar s gets a v.e.v., the charged fields that appear in eq. (4.2.6) have canonical kinetic terms, whereas the neutral fields ν_α, H_u^0 and A mix. To achieve this we have to perform a rescaling of the fields, that brings the Kähler potential for the Higgs and lepton superfields to the following form,

$$K = A^\dagger A + d_u H_u^\dagger H_u + d_{ab}\ell_a^\dagger \ell_b + C_u H_u^\dagger H_u \frac{A+A^\dagger}{f} + C_{ab}\ell_a^\dagger \ell_b \frac{A+A^\dagger}{f} + \left(C_{ua} H_u.\ell_a \frac{A+A^\dagger}{f} + \text{h.c.}\right) + \mathcal{O}\left(\frac{1}{f^2}\right) , \tag{4.2.9}$$

where the coefficients d_u and d_{ab} satisfy

$$d_u + 2\frac{v_s}{f} C_u = 1 , \qquad d_{ab} + 2\frac{v_s}{f} C_{ab} = \delta_{ab} . \tag{4.2.10}$$

After shifting the neutral fields A, H_u^0 and ν_a by the v.e.v. of their scalar component, i.e. $\Phi^0 \to v_\phi + \hat{\Phi}^0$, the Kähler potential will become

$$K = \hat{A}^\dagger \hat{A} + \hat{H}_u^\dagger \hat{H}_u + \hat{\ell}_a^\dagger \hat{\ell}_a + \left(z_{uA}\hat{H}_u^{0\dagger}\hat{A} + z_{aA}\hat{\nu}_a^\dagger \hat{A} + \text{h.c.}\right) + C_u \hat{H}_u^\dagger \hat{H}_u \frac{\hat{A}+\hat{A}^\dagger}{f} + C_{ab}\hat{\ell}_a^\dagger \hat{\ell}_b \frac{\hat{A}+\hat{A}^\dagger}{f} + \left(C_{ua}\hat{H}_u.\hat{\ell}_a \frac{\hat{A}+\hat{A}^\dagger}{f} + \text{h.c.}\right) , \tag{4.2.11}$$

where we dropped the constants and the terms that give total derivatives after integrating on $d^4\theta$. The mixing terms are

$$z_{uA} = \frac{C_u v_u - C_{ua} v_a}{f} , \qquad z_{aA} = \frac{C_{ab} v_b - C_{ua} v_u}{f} . \tag{4.2.12}$$

The next step is to rescale neutral fields only in order to give them canonical kinetic terms once the fields are shifted by their vacuum expectation value. This can be done by the rescaling $\Phi^0 \to V\Phi^0$, with $Z = V^2$. In the neutral sector, Z is defined by

$$\hat{\Phi}^0 Z \hat{\Phi}^0 = (\hat{H}_u^{0\dagger}, \hat{\nu}_a^\dagger, \hat{A}^\dagger) \begin{pmatrix} 1 & 0_{1\times 4} & z_{uA} \\ 0_{1\times 4} & 1_{4\times 4} & z_{aA} \\ z_{uA}^* & z_{bA}^* & 1 \end{pmatrix} \begin{pmatrix} \hat{H}_u^0 \\ \hat{\nu}_b \\ \hat{A} \end{pmatrix} . \tag{4.2.13}$$

so that, at order 1 in v/f,

$$V = \begin{pmatrix} 1 & 0_{1\times 4} & z_{uA}/2 \\ 0_{1\times 4} & 1_{4\times 4} & z_{aA}/2 \\ z_{uA}^*/2 & z_{bA}^*/2 & 1 \end{pmatrix} . \tag{4.2.14}$$

In this new basis, the relevant part of the Kähler potential becomes

$$K = \hat{A}^\dagger \hat{A} + \hat{H}_u^\dagger \hat{H}_u + \hat{\ell}_a^\dagger \hat{\ell}_a + C_u \hat{H}_u^\dagger \hat{H}_u \frac{\hat{A}+\hat{A}^\dagger}{f} + C_{ab}\hat{\ell}_a^\dagger \hat{\ell}_b \frac{\hat{A}+\hat{A}^\dagger}{f} + \left(C_{ua}\hat{H}_u.\hat{\ell}_a \frac{\hat{A}+\hat{A}^\dagger}{f} + \text{h.c.}\right) + \mathcal{O}\left(\frac{1}{f^2}\right) . \tag{4.2.15}$$

The vacuum expectation values of neutral fields are modified by this redefinition, but at this order, it does not affect the d and z coefficients defined in eq. (4.2.10) and (4.2.12). The superpotential is also modified. We write it here in terms of the unhatted fields (i.e. before performing the shift $\Phi^0 \to v_\phi + \hat{\Phi}^0$) because it makes its expression simpler. It becomes

$$W \to W + \frac{\mu_a}{2}\left(z_{uA}A\nu_a + z_{aA}AH_u^0\right) - \frac{y^e_{ab\gamma}}{2}z_{aA}Ae_{Lb}e^c_{R\gamma} - \frac{y^d_{aij}}{2}z_{aA}AQ_i d^c_j - \frac{y^u_{ij}}{2}z_{uA}AQ_i u^c_j \ . \tag{4.2.16}$$

Finally, the last step consists in a unitary transformation on the $SU(2)$ doublets ℓ_a such that, in the new basis, $v_a = v_d\delta_{a0}$, and $y^e_{\alpha b\gamma}(v_b - z_{bA}v_s/2) = m_\alpha\delta_{\alpha\beta}$ (as we will see later, this last condition ensures that the mass matrix of charged leptons is diagonal). This does not affect the structure of the Kähler potential and the superpotential, so we can assume that the various coefficients μ_a, C_u, C_{ab} and C_{ua} are defined in this basis.

4.2.2 Chargino and neutralino mass matrices

In the MSSM, because of the electroweak symmetry breaking, the higgsinos mix with the gauginos of the $SU(2)\times U(1)$ gauge group. The mass eigenstates arising from the mixing of charged fields $\tilde{H}_u^+$, $\tilde{H}_d^-$, $\tilde{W}^\pm$ are referred to as charginos, while those coming from the mixing of neutral fields $\tilde{H}_u^0$, $\tilde{H}_d^0$, $\tilde{W}^3$ and $\tilde{B}$ (or equivalently $\tilde{Z}^0$ and $\tilde{\gamma}$) are called neutralinos. In the present scenario, since R-parity is broken in the lepton sector, charged leptons and neutrinos mix with charginos and neutralinos respectively. In addition to that, the pseudo-Goldstone fermion χ also mixes with neutralinos. The mass matrices of the fermions receive three different contributions at tree level. One-loop corrections may also contribute to the final expression of the neutrino mass matrix [235, 250–261]. However, we assume here that they are negligible, as it is the case if R-parity-violating parameters are small enough. For instance, ref. [262] gives the following estimation for the contribution to the neutrino mass matrix from the $y^e_{\alpha\beta\gamma}$, assuming that all slepton masses are of the same order of magnitude $\tilde{m}$ and that the mixing between left- and right-handed sleptons is roughly given by $(m^2_{LR})_{\alpha\beta} \simeq \tilde{m}y^e_{\alpha\beta}v_d$,

$$\delta m_{\nu_{\alpha\beta}}^{\text{1-loop}} \simeq \frac{1}{8\pi^2}\left[y^e_{\alpha\tau\tau}y^e_{\beta\tau\tau}\frac{m_\tau^2}{\tilde{m}} + \left(y^e_{\alpha\mu\tau}y^e_{\beta\tau\mu} + y^e_{\alpha\tau\mu}y^e_{\beta\mu\tau}\right)\frac{m_\tau m_\mu}{\tilde{m}} + y^e_{\alpha\mu\mu}y^e_{\beta\mu\mu}\frac{m_\mu^2}{\tilde{m}}\right] \ . \tag{4.2.17}$$

Assuming $\tilde{m} \sim 1$ TeV, we find an upper bound on the $y^e_{\alpha\beta\gamma}$ $(\alpha,\beta,\gamma = e,\mu,\tau)$ of approximately 5×10^{-5} in order to keep their contribution to the neutrino mass matrix at one loop smaller than 10^{-4} eV. Fortunately, this constraint does not apply to couplings that respect R-parity, such as $y^e_{0\tau\tau} \cong y_\tau$.

Superpotential and soft terms

Some of the mass terms can be read directly from the Lagrangian of the model. For the chiral fermions, there is a contribution coming from the superpotential, which can be deduced from eq. (4.1.52),

$$M_{ij}^W = \langle W_{ij} \rangle \ . \tag{4.2.18}$$

In particular, the generalized μ-term mixes the up-type higgsinos with the leptons and down-type higgsinos, whereas the Yukawa couplings give the ordinary lepton masses as well as a mixing between right-handed leptons and the down-type higgsino,

$$M^W_{\tilde{H}_u^+ e_{La}} = \mu_a \ , \qquad M^W_{\tilde{H}_u^0 \nu_a} = -\mu_a \ , \qquad M^W_{e_{La} e^c_{R\gamma}} = \sum_b y^e_{ab\gamma} \left(v_b - \frac{z_{bA}}{2} v_s \right) \ . \tag{4.2.19}$$

Additionally, because of the redefinition $\Phi^0 \to V\Phi^0$, the neutrinos and neutral higgsinos also mix with the pseudo-Goldstone fermion χ,

$$M^W_{\tilde{H}_u^0 \chi} = \sum_a \frac{z_{aA}}{2} \mu_a \ , \qquad M^W_{\nu_a \chi} = \frac{z_{uA}}{2} \mu_a \ . \tag{4.2.20}$$

The masses of the gauginos come from the soft supersymmetry-breaking Lagrangian (4.2.7). In the basis of the physical gauge bosons $W^\pm, Z^0, \gamma$, the charged gauginos $\tilde{W}^\pm$ have simply a mass M_2, while the mass matrix of the photino and zino is given, in the basis $(\tilde{\gamma}, \tilde{Z}^0)$, by

$$M = \begin{pmatrix} c_W^2 M_1 + s_W^2 M_2 & c_W s_W (M_2 - M_1) \\ c_W s_W (M_2 - M_1) & s_W^2 M_1 + c_W^2 M_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{pmatrix} \ . \tag{4.2.21}$$

Contribution of the Kähler potential

There is a contribution to the fermion masses coming from the Kähler potential, due to the fact that it contains higher-order nonrenormalizable terms. Indeed, performing the integral of K over $d^4\theta$, one gets terms of the following form,

$$\left[\Phi_l^\dagger \Phi_i \Phi_j \right]_D \supset -F_k^\dagger \psi_i \psi_j \ . \tag{4.2.22}$$

Here, the Kähler potential has the following structure,

$$K = \Phi_i^\dagger \Phi_i + \frac{1}{2} C_{\bar{k}ij} \Phi_k^\dagger \Phi_i \Phi_i \ . \tag{4.2.23}$$

Thus, if the F-terms have non-vanishing v.e.v.'s, fermion masses will receive the following contribution,

$$M_{ij}^K = C_{\bar{k}ij} \langle F_k^\dagger \rangle \ . \tag{4.2.24}$$

Using the equations of motion to integrate out the F-terms and taking the v.e.v. (here, since the Kähler potential contains nonrenormalizable operators, the equation of motion for the F-term differs from eq. (4.1.54), but this does not affect the v.e.v.), we get

$$\langle F_k^\dagger \rangle = \langle W_k \rangle \ , \tag{4.2.25}$$

so that, finally, the contribution to the fermion mass matrix reads

$$M_{ij}^K = -C_{\bar{k}ij} \langle W_k \rangle \ . \tag{4.2.26}$$

This term is nonzero only if Φ_k is a neutral field, because otherwise the v.e.v. would break the electric charge. The various coefficients $C_{\bar{k}ij}$ can be read from eq. (4.2.11),

$$C_{\bar{u}uA} = \frac{C_u}{f} \ , \qquad C_{\bar{a}bA} = \frac{C_{ab}}{f} \ , \qquad C_{\bar{A}ua} = \frac{C_{ua}}{f} \ . \tag{4.2.27}$$

In the end, at order 1 in v/f, this gives the following contributions to the mass matrix of neutralinos.

$$M_{\tilde{H}_u^0 \chi}^K = \frac{C_u}{f} \sum_b \mu_b v_b \ , \tag{4.2.28}$$

$$M_{\tilde{\nu}_a \chi}^K = \sum_b \frac{C_{ba}}{f} \mu_b v_u \ . \tag{4.2.29}$$

Other contributions are supressed by an additional power of v/f, so that we do not take them into account.

Contribution of the gauge interactions

Finally, because the scalars get vacuum expectation values, the matter fermions and the gauginos mix. This contribution to the mass matrix reads

$$M_{jA}^{\text{gauge}} = -i\sqrt{2} g \langle \phi_i^\dagger \rangle t_A \left(\delta_{ij} + C_{\bar{i}jk} \langle \phi_k \rangle \right) \ . \tag{4.2.30}$$

Here, things are complicated by the fact that, after the redefinition of the fields, gauge interactions of H_u, ℓ_a and A are modified. In particular, the new A has gauge interactions inherited from its mixing with lepton and Higgs superfields. To see this, let us go back a few steps. Before the shift $\Phi^0 \to v_\phi + \hat{\Phi}^0$ and the rescaling $\Phi^0 \to V\Phi^0$, the relevant gauge interaction terms read

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = {} & i\sqrt{2} H_u^\dagger \left(g\tilde{W}_A \tau_A + \frac{g'}{2} \tilde{B} \right) \left[\left(d_u + C_u \frac{\sqrt{2}s}{f} \right) \tilde{H}_u + C_u H_u \frac{\chi}{f} \right] \\ & + i\sqrt{2} \tilde{\ell}_a^\dagger \left(g\tilde{W}_A \tau_A - \frac{g'}{2} \tilde{B} \right) \left[\left(d_{ab} + C_{ab} \frac{\sqrt{2}s}{f} \right) \ell_b + C_{ab} \ell_b \frac{\chi}{f} \right] \ . \end{aligned} \tag{4.2.31}$$

After the rescaling $\Phi^0 \to V\Phi^0$, this lagrangian is modified as follows,

$$\begin{aligned} \mathcal{L}_{\text{gauge}} \to \mathcal{L}_{\text{gauge}} & - i \frac{z_{uA}^*}{2} (s - ia) \left[\frac{g}{\sqrt{2}} \tilde{W}^- \tilde{H}_u^+ + \frac{1}{2} \left(-g\tilde{W}_3 + g'\tilde{B} \right) \tilde{H}_u^0 \right] \\ & + \frac{i}{\sqrt{2}} H_u^{0\dagger} \left(-g\tilde{W}_3 + g'\tilde{B} \right) \frac{z_{uA}}{2} \chi \\ & - i \frac{z_{aA}^*}{2} (s - ia) \left[\frac{g}{\sqrt{2}} \tilde{W}^+ \tilde{\ell}_a^- + \frac{1}{2} \left(-g\tilde{W}_3 + g'\tilde{B} \right) \nu_a \right] \\ & - \frac{i}{\sqrt{2}} \tilde{\nu}_a^\dagger \left(-g\tilde{W}_3 + g'\tilde{B} \right) \frac{z_{aA}}{2} \chi \ . \end{aligned} \tag{4.2.32}$$

Finally, after shifting the fields by their v.e.v., at order 1 in v/f, the following mass terms are generated for charginos,

$$M^{\text{gauge}}_{\tilde{H}_u^+\tilde{W}^-} = -ig\left(v_u - \frac{z^*_{uA}}{2}v_s\right)\ , \qquad M^{\text{gauge}}_{e_{La}\tilde{W}^+} = -ig\left(v_a - \frac{z^*_{aA}}{2}v_s\right)\ . \tag{4.2.33}$$

while for neutralinos,

$$M^{\text{gauge}}_{\tilde{H}_u^0\tilde{Z}} = i\frac{g}{\sqrt{2}c_W}\left(v_u - \frac{z^*_{uA}}{2}v_s\right)\ , \qquad M^{\text{gauge}}_{\nu_a\tilde{Z}} = -i\frac{g}{\sqrt{2}c_W}\left(v_a - \frac{z^*_{aA}}{2}v_s\right)\ ,$$

$$M^{\text{gauge}}_{\chi\tilde{Z}} = -i\frac{g}{2\sqrt{2}c_W}\left[\sum_{a,b} C_{ab}\frac{v_a v_b}{f} - C_u\frac{v_u^2}{f}\right]\ . \tag{4.2.34}$$

Summary

In the previous part, we dropped additional contributions to the $\nu_a - \chi$ and $H_u^0 - \chi$ mixings proportional to m_χ, because m_χ is already a small parameter, and these new terms will feature an additional power of v/f so they can safely be neglected.

Putting this together, we can write the mass matrices of the fermions. Writing the mass term for charginos as

$$\left(-i\tilde{W}^-,\, e_{La}\right) M_C \begin{pmatrix} -i\tilde{W}^+ \\ \tilde{H}_u^+ \\ e^c_{R\gamma} \end{pmatrix}\ , \tag{4.2.35}$$

the mass matrix reads

$$M_C = \begin{pmatrix} M_2 & g\left(v_u - \frac{z^*_{uA}}{2}v_s\right) & 0_{1\times 3} \\ g\left(v_a - \frac{z^*_{aA}}{2}v_s\right) & \mu_a & y^e_{ab\gamma}\left(v_b - \frac{z_{bA}}{2}v_s\right) \end{pmatrix}\ . \tag{4.2.36}$$

When writing the neutralino mass matrix, we can now distinguish the neutrinos from the down-type higgsino. We will use the following notations,

$$M_u = -iM^{\text{gauge}}_{\tilde{H}_u^0\tilde{Z}}\ , \qquad M_d = iM^{\text{gauge}}_{\tilde{H}_d^0\tilde{Z}}\ , \tag{4.2.37}$$

$$m_{Z\alpha} = iM^{\text{gauge}}_{\nu_\alpha\tilde{Z}}\ , \qquad m_{Z\chi} = iM^{\text{gauge}}_{\chi\tilde{Z}}\ , \tag{4.2.38}$$

$$m_{\alpha\chi} = M^W_{\nu_\alpha\chi} + M^K_{\nu_\alpha\chi}\ , \quad m_{d\chi} = M^W_{\tilde{H}_d^0\chi} + M^K_{\tilde{H}_d^0\chi}\ , \quad m_{u\chi} = M^W_{\tilde{H}_u^0\chi} + M^K_{\tilde{H}_u^0\chi}\ . \tag{4.2.39}$$

The neutralino mass matrix is defined by

$$\psi^{0T}M_N\psi^0 = \left(-i\tilde{\gamma},\, -i\tilde{Z},\, \tilde{H}_u^0,\, \tilde{H}_d^0,\, \nu_\alpha,\, \chi\right)\begin{pmatrix} M_{4\times 4} & \mu_{4\times 4} \\ \mu^T_{4\times 4} & m_{4\times 4} \end{pmatrix}\begin{pmatrix} -i\tilde{\gamma} \\ -i\tilde{Z} \\ \tilde{H}_u^0 \\ \tilde{H}_d^0 \\ \nu_\beta \\ \chi \end{pmatrix}\ . \tag{4.2.40}$$

Remarkably, this turns out to have a seesaw structure, because there is a large hierarchy between the upper-left block and the other ones. The 4×4 block describing the heavy fields $\tilde{\gamma}$, $\tilde{Z}^0$, $\tilde{H}_u^0$ and $\tilde{H}_d^0$ is given by

$$M_{4\times 4} = \begin{pmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{12} & M_{22} & -M_u & M_d \\ 0 & -M_u & 0 & -\mu_0 \\ 0 & M_d & -\mu_0 & 0 \end{pmatrix} , \tag{4.2.41}$$

The coefficients of this block are functions of parameters related to supersymmetry breaking (M_1 and M_2) or to the electroweak symmetry breaking (μ_0, v_u and v_d), so they should lie roughly between 10^2 and 10^3 GeV. The determinant of M is given by

$$\det M = 2\mu_0 M_{11} M_u M_d - \mu_0^2 (M_{11} M_{22} - M_{12}^2) . \tag{4.2.42}$$

The block describing the light fields ν_α and χ and the off-diagonal mixing block have respectively the following structure,

$$m_{4\times 4} = \begin{pmatrix} 0_{3\times 3} & m_{\beta\chi} \\ m_{\alpha\chi} & m_\chi \end{pmatrix} , \qquad \mu_{4\times 4} = \begin{pmatrix} 0_{1\times 3} & 0 \\ m_{Z\beta} & m_{Z\chi} \\ -\mu_\beta & m_{u\chi} \\ 0 & m_{d\chi} \end{pmatrix} . \tag{4.2.43}$$

As expected, the parameters appearing in the blocks μ and m are much smaller than those of the upper-left block M, allowing to perform a seesaw expansion. After this step, the 8×8 matrix M_N is block-diagonal, and the mass matrix m_ν of neutrinos, defined as the four light eigenstates ν_α and χ, is given by

$$m_\nu = m - \mu^T M^{-1} \mu . \tag{4.2.44}$$

The final expression in terms of the original parameters is rather complicated. Defining instead the following quantities,

$$C_{\gamma\chi} = \mu_0 M_{12} \left(\mu_0\, m_{Z\chi} + M_d m_{u\chi} - M_u m_{d\chi}\right) , \tag{4.2.45}$$

$$C_{Z\chi} = -\mu_0 M_{11} \left(\mu_0\, m_{Z\chi} + M_d m_{u\chi} - M_u m_{d\chi}\right) , \tag{4.2.46}$$

$$C_{u\chi} = -\mu_0 M_{11} M_d m_{Z\chi} - M_{11} M_d^2 m_{u\chi} + \left(\mu M_1 M_2 - M_{11} M_u M_d\right) m_{d\chi}, \tag{4.2.47}$$

$$C_{d\chi} = \mu_0 M_{11} M_d m_{Z\chi} - M_{11} M_u^2 m_{d\chi} + \left(\mu M_1 M_2 - M_{11} M_u M_d\right) m_{u\chi} , \tag{4.2.48}$$

$$D_{\chi\chi} = C_{Z\chi} m_{Z\chi} + C_{u\chi} m_{u\chi} + C_{d\chi} m_{d\chi} , \tag{4.2.49}$$

the neutrino mass matrix can be written as

$$m_\nu = \begin{pmatrix} \dfrac{M_{11}}{\det M}(M_d\, \mu_\alpha - \mu_0\, m_{Z\alpha})(M_d\, \mu_\beta - \mu_0\, m_{Z\beta}) & \dfrac{C_{u\chi}\mu_\alpha - C_{Z\chi} m_{Z\alpha}}{\det M} + m_{\alpha\chi} \\ \dfrac{C_{u\chi}\mu_\beta - C_{Z\chi} m_{Z\beta}}{\det M} + m_{\beta\chi} & -\dfrac{D_{\chi\chi}}{\det M} + m_\chi \end{pmatrix} . \tag{4.2.50}$$

This expression is still quite involved, but what really matters here is the structure of the matrix, that leads to interesting patterns when investigating the possible values of neutrino mixing parameters.

4.3 The neutrino mass matrix

In this section, we will focus on the 4×4 neutrino mass matrix. Here, the pseudo-Goldstone fermion χ plays the role of a sterile neutrino. In particular, its mass and mixing with the active neutrinos can be compared to present and future experimental bounds.

4.3.1 Parametrization

For numerical purpose, it is more convenient to write m_ν under a modified form involving effective parameters, that retains the essential features of the expression displayed in eq. (4.2.50)

$$m_\nu = \begin{pmatrix} A\epsilon_\alpha\epsilon_\beta & B\eta_\alpha \\ B\eta_\beta & F \end{pmatrix} , \tag{4.3.1}$$

where $\vec{\epsilon}$ and $\vec{\eta}$ are unit vectors. A and B can both be chosen real. Then, through rephasings of the four neutrinos, it is possible to eliminate for instance the phases of $\vec{\epsilon}$ and F, so that, in full generality, there are three independent physical phases only. The determinant of this matrix turns out to be zero, which means that the lightest neutrino, ν_1 in the normal hierarchy or ν_3 in the inverted hierarchy, is necessarily massless. This can be made explicit by a unitary transformation. For this, we define $\vec{\zeta}$ as

$$\vec{\zeta} = \frac{\vec{\epsilon} \wedge \vec{\eta}}{|\vec{\epsilon} \wedge \vec{\eta}|} . \tag{4.3.2}$$

Extending $\vec{\zeta}$ to a dimension 4 vector, we get the eigenvector associated to the eigenvalue 0, $V = (\vec{\zeta}, 0)$. In particular, we can see that $\vec{\zeta}$ coincides with a column of the 3×3 PMNS matrix: in the normal hierarchy, $m_{\nu 1} = 0$ and $\zeta_\alpha = U_{\alpha 1}$, whereas in the inverted hierarchy, $m_{\nu 3} = 0$ and $\zeta_\alpha = U_{\alpha 3}$.

Defining the following parameters,

$$c = (\vec{\epsilon}\,)^* . \vec{\eta} \,, \tag{4.3.3}$$

$$s = \sqrt{1 - |c|^2} \,, \tag{4.3.4}$$

$$\eta' = \frac{\vec{\eta} - c\vec{\epsilon}}{s} \,, \tag{4.3.5}$$

we can now build a unitary matrix that brings m_ν to a simpler form, namely

$$\hat{U}^T m_\nu \hat{U} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Bs \\ 0 & 0 & A & Bc \\ 0 & Bs & Bc & F \end{pmatrix} , \qquad \hat{U} = \begin{pmatrix} \zeta_e & \eta_e'^* & \epsilon_e^* & 0 \\ \zeta_\mu & \eta_\mu'^* & \epsilon_\mu^* & 0 \\ \zeta_\tau & \eta_\tau'^* & \epsilon_\tau^* & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{4.3.6}$$

It is now explicit that one neutrino is massless, and that the nonzero 3×3 block mixes the massive active neutrinos (ν_2 and ν_3 in the normal hierarchy, ν_1 and ν_2 in the inverted one) with the sterile neutrino. In what follows, we will specialize ourselves to the real case.

4.3.2 Numerical search for solutions

In order to find authorized values of the parameters, as well as the associated predictions for the sterile neutrino mass and active-sterile mixing angles, we performed a random scan of parameter space. For each set of parameters $\{A, B, F, \epsilon_\alpha, \eta_\beta\}$, we computed and diagonalized the neutrino mass matrix, after what we imposed the following constraints on the masses m_i and on the mixing angles $U_{\alpha i}$.

1. First of all, one should make sure that the model reproduces the known features of the 3×3 mass matrix for active neutrinos only. Thus, the mass squared differences Δm^2_{ji}, $i, j = 1, 2, 3$ and the PMNS matrix entries $|U_{\alpha i}|$ for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$ should lie within the 3σ ranges quoted in ref. [263] for normal and inverted ordering, respectively. In the following, all the points displayed satisfy these constraints.

2. Then, the mass squared difference $\Delta m^2_{41} \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$ and the active-sterile mixing parametrized by the $U_{\alpha 4}$'s ($\alpha = e, \mu, \tau$) should satisfy the constraints from negative searches for active-sterile neutrino oscillations.

   (i) The effective mixing angle $\sin^2 2\theta_{ee} \equiv \sin^2 2\theta_{14} = 4|U_{e4}|^2(1 - |U_{e4}|^2)$ is constrained by $\bar{\nu}_e$ disappearance experiments at nuclear reactors. We use the 90% C.L. exclusion contours of Bugey-3 [264] and Daya Bay [265], which provide the best constraints on $\sin^2 2\theta_{ee}$ in the range explored here (10^{-3} eV$^2 < \Delta m^2 < 2$ eV^2).

   (ii) Regarding $|U_{e4}|$, we use the strongest bound, which, according to ref. [104], comes from long baseline reactor experiments and is given by $|U_{e4}|^2 \leq 0.055$ at 95% C.L..

   (iii) ν_μ and $\bar{\nu}_\mu$ disappearance experiments are sensitive to the effective mixing angle $\sin^2 2\theta_{\mu\mu} \equiv 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$. Here, we use the combined 99% C.L. constraint on $|U_{\mu 4}|^2$ from ref. [104] in the range $0.05\,\text{eV}^2 \leq \Delta m^2 \leq 20\,\text{eV}^2$, while below $\Delta m^2 = 0.05\,\text{eV}^2$ we impose $|U_{\mu 4}|^2 \leq 0.04$.

   (iv) Finally, $|U_{\tau 4}|$ is constrained by MINOS and Super-Kamiokande. Ref. [104] gives a 99% C.L. combined upper limit $|U_{\tau 4}|^2 \leq (0.24 - 0.30)$ in the range $0.05\,\text{eV}^2 \leq \Delta m^2 \leq 20\,\text{eV}^2$, while Super-Kamiokande gives $|U_{\tau 4}|^2 \leq 0.23$ at 99% C.L. for $\Delta m^2 \geq 0.1\,\text{eV}^2$. In our numerical study, for simplicity, we impose the constraint $|U_{\tau 4}|^2 \leq 0.23$ for all values of Δm^2_{41}.

   In what follows, we will use the following conventions: points that do not satisfy the constraints from negative searches for sterile neutrino oscillations will be displayed in orange, those that do in green or blue. The points displayed in green are furthermore compatible with the allowed regions at 95% CL for 3+1 oscillations that could explain the Gallium and reactor anomalies: more precisely, we use the combined data from ν_e and $\nu_{\bar{e}}$ disappearance experiments quoted in ref. [104].

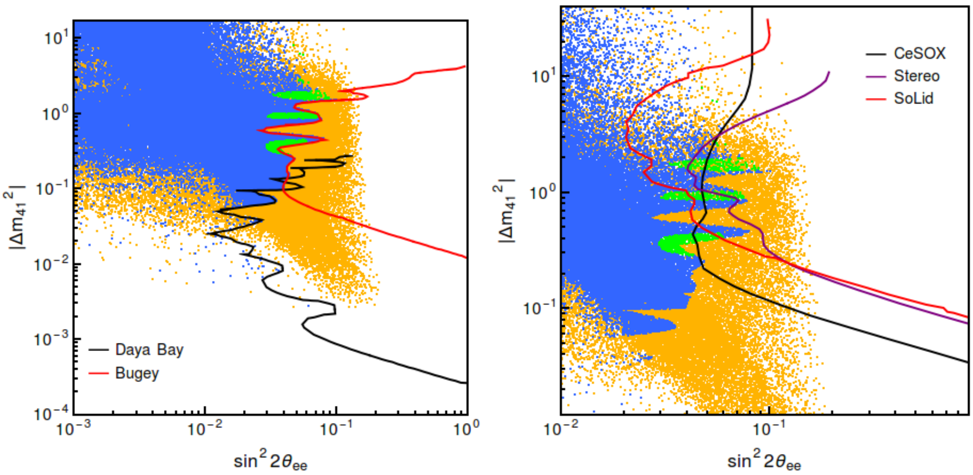


Figure 4.1: Predicted values of $\sin^2 2\theta_{ee}$ and Δm^2_{41} in the case of normal hierarchy. Left: Comparaison with the exclusions of Daya Bay and Bugey. Right: Comparaison with the expected sensitivity of Sox and Stereo.

Normal hierarchy

In fig. 4.1, we compare the predictions of the model to the sensitivity of present and future experiments in the plane $\sin^2 2\theta_{ee} - \Delta m^2_{41}$, for the case of normal hierarchy. After diagonalizing the neutrino mass matrix, the mixing angle $\theta_{ee} \equiv \theta_{14}$ is simply given by $\sin\theta_{ee} = U_{e4}$, while the squared mass difference Δm^2_{41} reduces to $m^2_{\nu 4}$ because, in our scenario, the lightest neutrino is massless.

In fig. 4.1 left, we compare the values obtained to the exclusion limits of Bugey [264] and Daya Bay [265]. This highlights how the constraints apply. For instance, below $\Delta m^2 \simeq 2 - 3\,\text{eV}^2$, it is clear that the upper bound on the mixing $\sin^2 2\theta_{ee}$ is set by the exclusion limits of Bugey and Daya Bay, while for larger values of Δm^2, $\sin^2 2\theta_{ee}$ cannot be very large as a result of the various conditions imposed. One can also see that, once all the constraints are taken into account, the new splitting cannot be much smaller than $\Delta m^2 \sim 0.05$ eV^2.

Fig. 4.1 right shows how the predicted values for the new squared mass splitting and the active-sterile mixing angle θ_{ee} compare to the expected sensitivity of the future experiments CeSOX [266], Stereo [267] and SoLid [267]. They would slightly improve the constraints on the high mass splitting region, while for Δm^2 smaller than 1 eV^2, Bugey and Daya Bay would still provide better constraints. In this case of normal hierarchy, the model provides a small number of points around 1 eV^2 in the region compatible with 3+1 oscillations from ν_e and $\bar{\nu}_e$ disapperance experiments, which are all close to the present and future experimental limits.

Inverted hierarchy

We turn now to inverted hierarchy. Fig. 4.2 shows again the parameters predicted by the model in the plane $\sin^2 2\theta_{ee} - \Delta m^2_{41}$ together with the experimental limits of Bugey and Daya Bay (left) and the expected sensitivity of CeSOX, Stereo and SoLid (right). In the case of inverted hierarchy, there is a stronger lower bound on the sterile

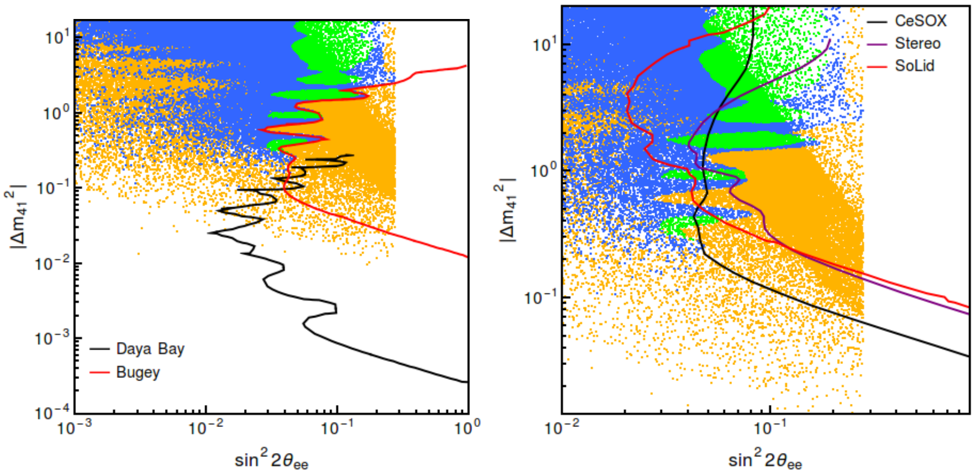


Figure 4.2: Predicted values of $\sin^2 2\theta_{ee}$ and Δm_{41}^2 in the case of inverted hierarchy. Left: Comparaison with the exclusions of Daya Bay and Bugey. Right: Comparaison with the expected sensitivity of Sox and Stereo.

neutrino mass than in the normal one. The constraints imposed by the 3×3 PMNS matrix entries and the active neutrino squared mass splittings already impose a lower bound of approximately 10^{-2} eV^2 on Δm^2, and after all the experimental exclusions from negative searches for sterile neutrinos are taken into account, we get

$$\Delta m_{41}^2 > 1 - 2 \times 10^{-1} \ \text{eV}^2 \ . \tag{4.3.7}$$

On the other hand, for large values of Δm_{41}^2, the mixing $\sin^2 2\theta_{ee}$ appears to be less constrained than in the normal hierarchy and can be as large as approximately 0.25.

Here, the new exclusion limits of CeSOX, SoLid and Stereo would provide interesting informations on this high mass region, where the mixing is little constrained. In particular, for the inverted hierarchy, this scenario gives a large number of points compatible with the allowed region for 3+1 oscillations from ν_e and $\nu_{\bar{e}}$ disappearance. Most of these points are beyond the reach of Bugey but could be tested by the next generation of experiments.

Conclusion

In this chapter, we studied a model which could explain the existence of a sterile neutrino and its mixing with standard neutrinos. The sterile neutrino is (mostly) the fermionic partner of a pseudo-Goldstone boson associated to a broken symmetry, which is described by an effective theory. Because of R-parity-violating interactions in the superpotential and in the non-canonical Kähler potential, active and sterile neutrinos mix with the higgsinos and the neutral gauginos $\tilde{\gamma}$ and $\tilde{Z}^0$. Consequently, there are four heavy mass eigenstates with masses above the electroweak scale, and four light eigenstates, which are the three standard neutrinos and the sterile neutrino. Requiring the model to reproduce the solar and atmospheric masses as well as the mixings of active neutrinos, we studied the possible values for the third squared mass splitting Δm^2_{41} and active-sterile mixing. We compared our results with current experimental constraints and anomalies, which had been considered previously mostly in phenomenological frameworks.

The number of parameters in the original theory makes it difficult to relate them closely to the expression of neutrino parameters. However, the structure of the mass matrix is enough to establish that the lightest neutrino has to be massless (or at least much lighter than the others once the loop corrections are included in the neutrino mass matrix). Furthermore, normal and inverted hierarchy lead to different predictions for the active-sterile mixing. For instance, inverted hierarchy offers more room to a sterile neutrino that would fit the gallium and reactor anomalies. The next generation of experiments looking for sterile neutrinos will shed new light on these anomalies and constrain further the parameter space of this model. Of course, an exhaustive study should include the possibility of nonzero CP-violating phases. We are currently working on this extension, and the first results seem to indicate that nonvanishing phases do not significantly extend the allowed region in the plane $\sin^2 2\theta_{ee} - \Delta m^2_{41}$, but only modify the local density of points, so that the main conclusions remain valid.

Chapter 5

Supersymmetric seesaw and gauge mediation

5.1 Introduction to Supersymmetry breaking

As explained in the previous chapter, supersymmetry has to be broken to be compatible with the non-observation of squarks, sleptons, higgsinos and gauginos. Soft terms provide a simple and convenient parametrization of supersymmetry breaking, but introduce a lot of arbitrariness. Thus, a self-consistent theory should include an explicit mechanism to break supersymmetry. A way to ensure that the theory where supersymmetry is broken is still free of quadratic divergences is to break supersymmetry spontaneously: the Lagrangian of the theory is manifestly supersymmetric, but the ground state is not. Remarkably, such models generate supersymmetry-breaking terms which are indeed soft according to the definition of 4.1.2. However, difficulties comes from the fact that it is not possible to achieve a satisfactory supersymmetry breaking within the MSSM itself.

5.1.1 The vacuum of supersymmetric theories

Taking the trace over spinor indices of eq. (4.1.1) and using $\mathrm{tr}(\sigma^\mu) = 2\eta^{\mu 0}$, one gets

$$4P_0 = 4\mathcal{H} = \left[Q_1\bar{Q}_1 + \bar{Q}_1 Q_1 + Q_2\bar{Q}_2 + \bar{Q}_2 Q_2\right] \;, \tag{5.1.1}$$

where $\mathcal{H}$ is the hamiltonian of the system. The vacuum energy can therefore be written as

$$E_{\mathrm{vac}} = \langle 0|\,\mathcal{H}\,|0\rangle = \frac{1}{4}\left[\|\bar{Q}_1\,|0\rangle\,\|^2 + \|Q_1\,|0\rangle\,\|^2 + \|\bar{Q}_2\,|0\rangle\,\|^2 + \|Q_2\,|0\rangle\,\|^2\right] \;, \tag{5.1.2}$$

which is obviously always positive. As long as supersymmetry remains unbroken, the vacuum energy should be zero. Indeed, in this case, the ground state $|0\rangle_{\mathrm{SUSY}}$ has to be invariant under supersymmetry transformations,

$$Q_\alpha\,|0\rangle_{\mathrm{SUSY}} = \bar{Q}_{\dot\alpha}\,|0\rangle_{\mathrm{SUSY}} = |0\rangle_{\mathrm{SUSY}} \Rightarrow E_{\mathrm{vac}} = 0 \;. \tag{5.1.3}$$

Conversely, a breaking of supersymmetry implies that the vacuum is not invariant, and therefore has a strictly positive energy $E_{\mathrm{vac}} > 0$.

Assuming that the other generators of the super-Poincaré algebra P_μ and $M_{\mu\nu}$ remain unbroken, only spin-0 fields can be non-vanishing in the vacuum. As a consequence, the vacuum energy is determined by the scalar potential. Since it can be written in terms of the auxiliary fields as follows,

$$\mathcal{V}(\phi_i) = F_i^\dagger F_i + \frac{1}{2} D_A D_A \ , \tag{5.1.4}$$

the breaking of supersymmetry requires at least one of the auxiliary fields to get a nonzero vacuum expectation value.

5.1.2 Explicit Models of supersymmetry breaking

Historically, two classes of models have been proposed to generate a non-vanishing expectation value for an auxiliary field.

In the Fayet-Iliopoulos model [268], a nonzero expectation value for a D-term is generated by the introduction of the following contribution to the Lagrangian,

$$\mathcal{L}_{\text{FI}} = -[\kappa V]_D = -\int D^4\theta\, \kappa V(x, \theta, \bar{\theta}) = -\kappa D \ . \tag{5.1.5}$$

Unlike in eq. (4.1.36), V is now a fundamental vector superfield, and κ has dimension 2. Gauge invariance implies that this vector superfield is associated to an abelian $U(1)$ gauge theory. The equation of motion for the D-term (4.1.50) is modified to become

$$\frac{\partial \mathcal{L}}{\partial D} = 0 = g q_i \phi_i^\dagger \phi_i - \kappa + D \Rightarrow D = \kappa - g q_i \phi_i^\dagger \phi_i \ , \tag{5.1.6}$$

where q_i is the charge of the chiral superfield Φ_i under the $U(1)$ symmetry. A simple version of this mechanism involves only a pair of chiral superfields (Φ^+, Φ^-) with a supersymmetric mass term $W_{\text{mass}} = M\Phi^+\Phi^-$, so that they necessarily have opposite charges. Through a rephasing of the fields, M can be chosen real and positive. The scalar potential is then

$$\mathcal{V}(\phi^+, \phi^-) = M^2 \left(|\phi^+|^2 + |\phi^-|^2\right) + \frac{1}{2}\left[\kappa + gq\left(|\phi^-|^2 - |\phi^+|^2\right)\right]^2 \ . \tag{5.1.7}$$

Since both terms are positive and cannot be simultaneously zero, the scalar potential is always strictly positive. As explained previously, this implies that supersymmetry is spontaneously broken in the vacuum.

In the O'Raiferteaigh model [269], supersymmetry is broken by the nonzero expectation value of an F-term. A minimal implementation of this mechanism involves three chiral superfields, Φ_1, Φ_2 and Φ_3, which are gauge singlets. The superpotential is

$$W = \lambda \Phi_1 (\Phi_3^2 - \mu^2) + M \Phi_2 \Phi_3 \ . \tag{5.1.8}$$

λ, μ and M can be chosen real and positive through a rephasing of the superfields. The scalar potential is

$$\begin{aligned} \mathcal{V}(\phi_1, \phi_2, \phi_3) &= F_1^\dagger F_1 + F_2^\dagger F_2 + F_3^\dagger F_3 \\ &= |2\lambda \phi_1 \phi_3 + M\phi_2|^2 + M^2 |\phi_3|^2 + |\lambda(\phi_3^2 - \mu^2)|^2 \ . \end{aligned} \tag{5.1.9}$$

Again, it is not possible to cancel simultaneously the second and third term, so that the scalar potential is strictly positive and supersymmetry is spontaneously broken.

Both models exhibit a sum rule, which generalizes to any scenario in which supersymmetry is broken at tree-level by renormalizable interactions. If one calls M_J the mass matrix of particles with spin J, the squared mass matrices of the scalars, fermions and gauge bosons satisfy [270]

$$\sum_J (-1)^{2J}(2J+1)\mathrm{tr}\left(M_J^2\right) = \mathrm{tr}\left(M_S^2\right) - 2\mathrm{tr}\left(M_F^\dagger M_F\right) + 3\mathrm{tr}\left(M_V^2\right) = 0 \ . \tag{5.1.10}$$

In other words, the supertrace of the total squared mass matrix of the theory vanishes. To prove this, let us start with the mass matrix of the scalars, which can be obtained from the scalar potential. By definition, the squared mass for $\phi_i^\dagger \phi_j$[1] is

$$\left(M_S^2\right)_{ij} = \left\langle \frac{\partial^2 \mathcal{V}}{\partial \phi_i^\dagger \phi_j} \right\rangle \ . \tag{5.1.11}$$

The scalar potential is given in eq. (4.1.57),

$$\mathcal{V} = F_k^\dagger F_k + \frac{1}{2} D_A D_A = W_k W_k^\dagger + \frac{1}{2} D_A D_A \ , \tag{5.1.12}$$

Using the fact that W is holomorphic so that $\partial W/\partial \phi_i^\dagger = \partial W^\dagger/\partial \phi_j = 0$, the squared mass matrix can be written as

$$\left(M_S^2\right)_{ij} = \left\langle W_{ik}^\dagger W_{jk} + D_A \frac{\partial^2 D_A}{\partial \phi_i^\dagger \partial \phi_j} + \frac{\partial D_A}{\partial \phi_i^\dagger} \frac{\partial D_A}{\partial \phi_j} \right\rangle \ , \tag{5.1.13}$$

The derivatives of the auxiliary field are given by the following formulae,

$$\frac{\partial D_A}{\partial \phi_i^\dagger} = -g(t_A \phi_i) \ , \qquad \frac{\partial D_A}{\partial \phi_j} = -g(\phi_j^\dagger t_A) \ , \qquad \frac{\partial^2 D_A}{\partial \phi_i^\dagger \partial \phi_j} = -t_A \ , \tag{5.1.14}$$

Using this, one can express the trace as

$$\mathrm{tr}\left(M_S^2\right) = 2\sum_i \langle W_{ik}^\dagger\rangle \langle W_{ik}\rangle + 2g^2 \sum_i C(\Phi_i) \langle \phi_i \rangle \langle \phi_i^\dagger \rangle \ . \tag{5.1.15}$$

$C(\Phi_i)$ is the Casimir of the representation containing ϕ_i, defined by $(t_A t_A)_{ab} = C(\Phi_i)\delta_{ab}$ Here, we used the fact that for a non-abelian gauge theory $\mathrm{tr}(t_A) = 0$ while for an abelian gauge theory, $\mathrm{tr}(t_A)$ is the sum of the charges of the chiral superfields, which has to vanish to avoid the presence of an anomaly. As can be seen from eq. (4.1.55), the mass matrix of the chiral fermions arises both from the supersymmetric mass terms and from the Yukawa-like couplings if scalars get a v.e.v., so in the most general case it is given by

$$(M_F)_{ij} = \langle W_{ij} \rangle \ . \tag{5.1.16}$$

[1] Two scalars $\phi_{\bar{i}}$ and ϕ_j belonging to conjugate representations can also have a mass term of the form $\left(M_S^2\right)_{\bar{i}j} \phi_{\bar{i}} \phi_j$. However, if one writes the total squared mass matrix in the basis $(\phi_i, \phi_j^\dagger)$, this type of contribution appears as an off-diagonal term, so that it does not contribute to the trace.

In full generality, there can also be mass terms mixing a chiral fermion ψ_{ia} with a gaugino λ_A. According to eq. (4.1.42), this take the form

$$(M_F)_{iA} = -i\sqrt{2}g\left(\langle\phi_i\rangle t_A\right) \tag{5.1.17}$$

As a consequence, in the basis (ψ_i, λ_A), the fermion mass matrix reads

$$M_F = \begin{pmatrix} \langle W_{ij}\rangle & -i\sqrt{2}g\left(\langle\phi_i\rangle t_B\right) \\ -i\sqrt{2}g\left(\langle\phi_j\rangle t_A\right) & 0 \end{pmatrix} , \tag{5.1.18}$$

so that

$$\mathrm{tr}\left(M_F^\dagger M_F\right) = \sum_i \langle W_{ik}^\dagger\rangle\langle W_{ik}\rangle + 4g^2\sum_i C(\Phi_i)\langle\phi_i\rangle\langle\phi_i^\dagger\rangle \ . \tag{5.1.19}$$

Finally, the squared masses of the vector bosons arise from the kinetic term of the scalars in eq. (4.1.42), and are given by

$$\left(M_V^2\right)_{AB} = g^2\left\langle \sum_i \phi_i^\dagger\{t_A, t_B\}\phi_i \right\rangle , \tag{5.1.20}$$

so that

$$\mathrm{tr}\left(M_V^2\right)_{AB} = 2g^2\sum_i C(\Phi_i)\langle\phi_i^\dagger\rangle\langle\phi_i\rangle \ . \tag{5.1.21}$$

From eq. (5.1.15), (5.1.19) and (5.1.21) we can check that indeed,

$$\mathrm{tr}\left(M_S^2\right) - 2\mathrm{tr}\left(M_F^\dagger M_F\right) + 3\mathrm{tr}\left(M_V^2\right) = 0 \ . \tag{5.1.22}$$

This sum rule is a problem when it comes to generating a realistic spectrum for Standard Model fields and their superpartners, because it implies that some of the superpartners of Standard Model fields should be light enough to be observable [234].

5.1.3 Supersymmetry broken in a separate sector

The models of the previous section provide explicit examples of supersymmetry breaking, but cannot work within the MSSM, because they are not able to reproduce a realistic mass spectrum. Moreover, they do not generate Majorana masses for gauginos. Thus, one has to introduce a hidden supersymmetry-breaking sector involving new superfields, then find a way to communicate this breaking to the MSSM. Even then, the sum rule of eq. (5.1.10) makes it difficult to obtain phenomenologically acceptable predictions as long as the supersymmetry-breaking effects are communicated to the MSSM by tree-level renormalizable interactions. A way to overcome this issue is to communicate the breaking to the MSSM radiatively or via nonrenormalizable interactions.

(i) In Planck-scale mediated or gravity mediated supersymmetry breaking [271–275], the mediation is carried by nonrenormalizable interactions arising from new physics at the Planck scale, including gravitational interactions and new heavy states. These interactions appear in the superpotential and in the Kähler potential and couple the supersymmetry-breaking sector directly to the MSSM fields. If supersymmetry breaking arises from the v.e.v. of an F-term, the scale of soft masses should be around $\langle F\rangle/M_P$ (and therefore, for soft masses around the TeV scale, $\sqrt{\langle F\rangle}$ should lie between 10^{10} and 10^{11} GeV). The main drawback of this class of models is that the exact form of the new interactions is not known, and therefore the results depend crucially on the assumptions made about the structure of the soft terms at the Planck scale.

(ii) In gauge mediation [225, 226, 276–281], supersymmetry-breaking effects are communicated to the visible sector radiatively. One of the main appeals of this kind of models is that they are very predictive, because every soft term can be computed from renormalizable gauge interactions, and there are few free parameters.

(iii) Another possibility is extra-dimensional mediated supersymmetry breaking [282], which relies on the universe having more than four spacetime dimensions (usually five). For instance, the chiral superfields of the MSSM and the supersymmetry-breaking sector can be confined in two parallel four-dimension branes. Then, there are two different possibilities. If gauge superfields live in the bulk, they can transmit supersymmetry-breaking effects to the MSSM [283–287]. Alternatively, the gauge superfields can also be confined with the chiral superfields of the MSSM in a four-dimensional brane and only supergravity mediates supersymmetry-breaking effects [288, 289]. The latter scenario is referred to as anomaly mediation.

5.1.4 Minimal gauge mediation

We focus now on gauge mediation, and more precisely on its minimal realization, where gauge interactions alone are responsible for mediating supersymmetry breaking. In the next section, we will consider extensions in which additional interactions are involved, leading to a richer phenomenology.

For simplicity, the hidden sector that breaks supersymmetry is parametrized by a spurion X. The scalar and F-term components of the chiral superfield X get a v.e.v.,

$$\langle X\rangle = M_X + \theta^2 F_X \ . \tag{5.1.23}$$

X does not have any direct coupling to the MSSM, but to chiral superfields called messengers. In minimal gauge mediation, direct couplings between messengers and ordinary MSSM fields are forbidden by a symmetry called messenger parity [290], so that the two sectors only communicate through gauge interactions. In order to give a mass to the gauginos of the three subgroups $U(1)$, $SU(2)$ and $SU(3)$, there should be messengers charged under every one of them. A usual choice, motivated by the will of preserving the unification of gauge couplings provided by supersymmetry, is to take a pair of doublets (Φ_u, Φ_d) with the same quantum numbers as the Higgs doublets (except for messenger parity) and a pair of color triplets $(\Phi_T, \Phi_{\bar{T}})$: $\Phi_M = (\Phi_u, \Phi_T)$ and

$\Phi_{\bar{M}} = (\Phi_d, \Phi_{\bar{T}})$ are a vectorlike pair of $SU(5)$ 5-plets, and therefore their introduction does not spoil the unification. There could be several pairs of messengers, but here we restrict ourselves to the minimal case. Keeping the $SU(5)$ notation, the coupling between the spurion and the messengers is described by the following superpotential

$$W_{\rm GM} = X\Phi_{\bar{M}}\Phi_M \ . \tag{5.1.24}$$

In components, the Lagrangian derived from this superpotential reads

$$\mathcal{L}_{\rm GM} = -M_X^2\left(|\phi_M|^2 + |\phi_{\bar{M}}|^2\right) - \left(M_X\psi_{\bar{M}}\psi_M + F_X\phi_{\bar{M}}\phi_M + \text{h.c.}\right) \ . \tag{5.1.25}$$

The physical states are a four-component Dirac fermion $(\psi_M, \psi^\dagger_{\bar{M}})$ with mass M_X, and two complex scalars,

$$\phi_{M\pm} = \frac{\phi_M \pm \phi^\dagger_{\bar{M}}}{\sqrt{2}} \ , \qquad M_\pm^2 = M_X^2 \pm F_X \ . \tag{5.1.26}$$

The fact that the scalars and fermions are no more degenerate is a clear indication that supersymmetry is indeed broken. One can also check that the sum rule of eq. (5.1.10) is satisfied.

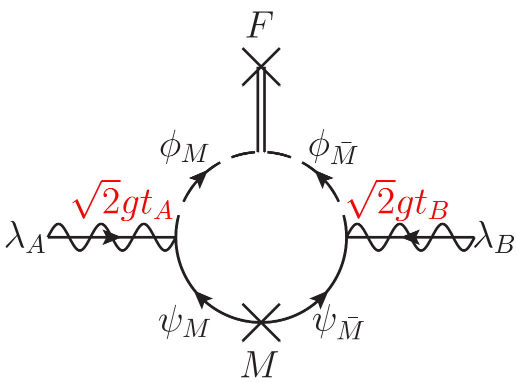


Figure 5.1: One-loop contribution to gaugino masses in the spurion insertion approximation.

Gaugino Majorana masses are then generated at one loop. The exact computation should be performed in the mass eigenstate basis for the scalars (ϕ_{M+}, ϕ_{M-}), but as long as $F_X \ll M_X^2$, which is a good approximation if the messenger scale M_X is large enough, one can conveniently work in the original basis $(\phi_M, \phi_{\bar{M}})$ and perform a spurion insertion, as shown in fig. 5.1. One can check that this is equivalent to expanding the propagator of ϕ_{M+} and ϕ_{M-} at first order in F_X/M_X^2. Explicitly, for a given gauge group, the computation gives

$$\begin{aligned} -iM_{AB} &= 2g^2\text{tr}(t_At_B)\int\frac{d^4k}{2\pi^4}\frac{M_X}{k^2 - M_X^2 + i\epsilon}\frac{F_X}{(k^2 - M_X^2 + i\epsilon)^2} \\ &= -i\frac{\alpha}{4\pi}S(\Phi_M)\delta_{AB}\Lambda \ , \end{aligned} \tag{5.1.27}$$

where $\alpha = g^2/(4\pi)$, $\Lambda = F_X/M_X$ and $S(\Phi_M)$ is the Dynkin index of the representation containing Φ_M. From this, it appears that gaugino masses are roughly of the order of $m_{\rm soft} \sim \alpha\Lambda/(4\pi)$, which should be around a TeV. Thus, the assumption $F_X \ll M_X^2$ should hold as long as M_X is much larger than $4\pi m_{\rm soft}/\alpha$. On the other hand, gauge bosons remain massless because their masses are forbidden by gauge symmetry. The formula above is valid at a scale $\mu \simeq M_X$ and is modified by the running of the renormalization group.

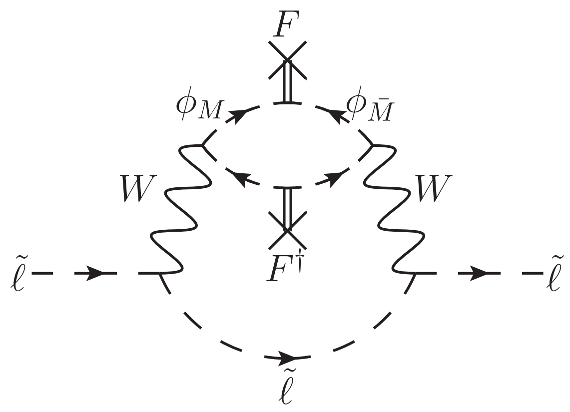


Figure 5.2: Two-loop contribution to the slepton squared mass.

Squared masses for the scalars are generated at two loops only because the chiral superfields have no direct coupling to the messengers. One example of diagram contributing to the slepton squared mass is displayed in fig. 5.2. Summing the contributions of the various diagrams involved leads to the following result for the squared masses of the scalars [291],

$$m_\phi^2 = 2|\Lambda|^2 \left[C_3(\phi)\left(\frac{\alpha_s}{4\pi}\right)^2 + C_2(\phi)\left(\frac{\alpha_{EW}}{4\pi}\right)^2 + \frac{3Y_\phi^2}{5}\left(\frac{\alpha_Y}{4\pi}\right)^2 \right] . \qquad (5.1.28)$$

where $C_3(\phi) = 4/3$ if ϕ is a color triplets ($\tilde{Q}$, $\tilde{u}^c$ or $\tilde{d}^c$) and $C_3(\phi) = 0$ otherwise, $C_2(\phi) = 3/4$ if ϕ is an electroweak doublet ($\tilde{Q}$, $\tilde{\ell}$, H_u or H_d) and $C_2(\phi) = 0$ otherwise, and Y_ϕ is the hypercharge. This is of the order of $m_{\rm soft}^2$, so that the scalars and gauginos masses are of the same order of magnitude. On the other hand, the A-terms, which have mass dimension 1, are also generated at the two-loop order, so they scale roughly like $A \sim \Lambda(\alpha/4\pi)^2$, which is much smaller than $m_{\rm soft}$. They can therefore be neglected in a first approximation. However, this is true only at the scale $\mu \sim M_X$ and A-terms are generated by the running. The chiral fermions do not receive any contribution to their mass: the quarks and leptons remain massless and get their mass after the electroweak symmetry breaking, like in the Standard Model.

An important feature of gauge mediation is that, because gauge interactions are flavour blind, the squared masses of the squarks and sleptons are proportional to the identity matrix in flavour space, for instance

$$(m_{\tilde\ell})^2_{\alpha\beta} = m_{\tilde\ell}^2 \delta_{\alpha\beta} \ , \qquad (5.1.29)$$

which is precisely what is needed to avoid flavour-changing neutral currents that could be in conflict with the experimental bounds quoted in table 2.2. Thus, contrary to Planck scale mediated supersymmetry breaking, gauge mediation leads to suppressed flavour-changing neutral currents without requiring any particular assumption on the structure of soft terms.

5.2 Extended gauge mediation

From now on, we will focus on gauge mediation and its extensions. As already said, minimal gauge mediation is predictive and naturally forbids flavour violation. However, the measured mass of the Higgs boson turns out to be problematic, because it is a bit too large to fit naturally in this framework. This stems from the fact that, if one wants to restrict fine-tuning, a large mass for the Higgs boson requires large A-terms, and this condition is not satisfied within minimal gauge mediation. Extended gauge mediation, in which direct couplings between messengers and matter superfields are allowed, is a way to overcome this issue.

5.2.1 The electroweak symmetry breaking in the MSSM

Compared to the Standard Model, the electroweak symmetry breaking in the MSSM is slightly complicated by the fact that there are two Higgs, and therefore two expectation values, already encountered in chapter 4,

$$\langle H_u^0 \rangle = v_u \ , \qquad \langle H_d^0 \rangle = v_d \ . \tag{5.2.1}$$

This generates the following masses for the Standard Model fermions,

$$m_{ij}^u = y_{ij}^u v_u \ , \qquad m_{ij}^d = y_{ij}^d v_d \ , \qquad m_{\alpha\beta}^e = y_{\alpha\beta}^e v_d \ . \tag{5.2.2}$$

The Z boson, on the other hand, couples to both Higgs doublets, and has therefore the following squared mass,

$$M_Z^2 = \frac{g^2 + g'^2}{2}(v_u^2 + v_d^2) \ , \tag{5.2.3}$$

which, together with eq. (1.2.23), gives the relation between v_u, v_d and the Standard Model v.e.v. $v = 174$ GeV,

$$v_u^2 + v_d^2 = v^2 \ . \tag{5.2.4}$$

The parameter β is defined by $\tan\beta = v_u/v_d$, so that

$$v_u = v\sin\beta \ , \qquad v_d = v\cos\beta \ . \tag{5.2.5}$$

In the MSSM, once supersymmetry is broken, the potential for neutral Higgs scalars, that triggers the electroweak symmetry breaking, is given by

$$\begin{aligned} V(H_u^0, H_d^0) = & \left(|\mu|^2 + m_{H_u}^2\right)|H_u^0|^2 + \left(|\mu|^2 + m_{H_d}^2\right)|H_d^0|^2 \\ & - \left(B_\mu H_u^0 H_d^0 + \text{h.c.}\right) + \frac{1}{8}\left(g^2 + g'^2\right)\left(|H_u^0|^2 - |H_d^0|^2\right) \ . \end{aligned} \tag{5.2.6}$$

Through a rephasing of H_u^0 or H_d^0, B_μ can be chosen real and positive. Then, at the minimum of the potential, the product $H_u^0 H_d^0$ should be positive. At this stage, one still has the freedom to perform another rephasing to have them both real.

Here, contrary to the Standard Model, the quartic term of the Higgs potential, which comes from integrating out the D terms, is entirely determined by the gauge interactions.

The μ and $\mu - B_\mu$ problems

The minimum of the potential is defined by the following equations,

$$\frac{\partial V}{\partial H_u^0} = 0 \Rightarrow 2\left(|\mu|^2 + m_{H_u}^2\right) v_u - 2B_\mu v_d + \frac{1}{2}\left(g^2 + g'^2\right) v_u \left(v_u^2 - v_d^2\right) = 0 \ , \qquad (5.2.7)$$

$$\frac{\partial V}{\partial H_d^0} = 0 \Rightarrow 2\left(|\mu|^2 + m_{H_d}^2\right) v_d - 2B_\mu v_u - \frac{1}{2}\left(g^2 + g'^2\right) v_d \left(v_u^2 - v_u^2\right) = 0 \ . \qquad (5.2.8)$$

Dividing the first equality by v_u and the second one by v_d (which are required to be nonzero in order to break the electroweak symmetry and give mass to all the fermions), one gets

$$2|\mu|^2 + 2m_{H_u}^2 - 2B_\mu \cot\beta - M_Z^2 \cos 2\beta = 0 \ , \qquad (5.2.9)$$

$$2|\mu|^2 + 2m_{H_d}^2 - 2B_\mu \tan\beta + M_Z^2 \cos 2\beta = 0 \ . \qquad (5.2.10)$$

From these equations, one can derive the following relations between the parameters [292],

$$\sin 2\beta = \frac{2B_\mu}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu^2|} \ , \qquad (5.2.11)$$

$$M_Z^2 = \frac{|m_{H_u}^2 - m_{H_d}^2|}{\sqrt{1 - \sin^2 2\beta}} - m_{H_u}^2 - m_{H_d}^2 - 2|\mu^2| \ . \qquad (5.2.12)$$

This reveals the so-called μ problem of supersymmetric theories. Eq. (5.2.12) implies that, unless there is a cancellation, all parameters should be roughly of the order of magnitude of M_Z. On one hand, the squared masses $m_{H_u}^2$, $m_{H_d}^2$ and B_μ come from supersymmetry breaking, so it means that the supersymmetry-breaking scale should not be too far from the electroweak scale. On the other hand, μ is a fundamental parameter of the supersymmetric theory, and there is a priori no reason why it should be of the order of magnitude of M_Z. A solution to this problem is provided by the NMSSM, in which the μ-term is generated dynamically as the v.e.v. of a gauge singlet [220, 293–296].

Another option is to forbid the μ-term in the superpotential by the mean of some Peccei-Quinn symmetry. Then, both μ and B_μ are generated by effective operators coupling the Higgs to the supersymmetry-breaking spurion [297]. However, in the context of gauge mediation, this does not really solve the problem because μ and B_μ are typically generated at the same loop order [298]. Therefore, they scale like

$$\mu \sim \left(\frac{\alpha}{4\pi}\right)^n \Lambda \ , \qquad B_\mu \sim \left(\frac{\alpha}{4\pi}\right)^n \Lambda^2 \ , \qquad (5.2.13)$$

where n is the loop order. This implies in particular $B_\mu \sim \mu\Lambda$. Since, as previously said, Λ should be of the order of $4\pi m_{\text{soft}}/\alpha \sim 10 - 100$ TeV, it seems that μ and B_μ cannot have simultaneously the right order of magnitude which is approximately $\mathcal{O}(1\ \text{TeV})$. This is the $\mu - B_\mu$ problem, which is peculiar to gauge mediation.

The Higgs boson mass

Together, the two Higgs doublets contain eight real scalar degrees of freedom, which can be parametrized as follows,

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_{\text{CP}^+} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_{\text{CP}^-} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} , \tag{5.2.14}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-\dagger} \end{pmatrix} = \frac{1}{\sqrt{2}} R_+ \begin{pmatrix} H^+ \\ G^+ \end{pmatrix} , \tag{5.2.15}$$

where R_{CP^+}, R_{CP^-} and R_+ are 2×2 orthogonal matrices. G^0, G^+ and G^- are the Goldstone particles absorbed by the gauge bosons W^+, W^- and Z^0, which makes them massive. The remaining five degrees of freedom are two charged scalars H^+ and H^-, two CP-even neutral scalars h^0 and H^0 and one CP-odd neutral scalar A^0. By convention, h^0 is chosen to be the lightest CP-even scalar. At tree-level, its mass is bounded from above [299, 300],

$$m_h < M_Z |\cos 2\beta| . \tag{5.2.16}$$

This is obviously in contradiction with the measured values of the Higgs and Z boson masses. Of course, this relation only holds at tree-level, and should be modified by loop corrections, that can in principle relax the upper bound on the Higgs mass up to 135 GeV [301–303].

Thus, large radiative corrections are needed in order to reproduce the measured value $m_h = 125.6 \pm 0.3$ GeV. For this, either stops should be very heavy (with $m_{\tilde{t}} \gtrsim 10$ TeV), or there should be a large mixing between the right-handed and left-handed stops [304–307]. Unfortunately, the first option leads to a larger fine-tuning, which can be seen by computing the Barbieri-Giudice measure [308],

$$\Delta = \max \Delta_a , \qquad \Delta_a = \frac{\partial \log M_Z^2}{\partial \log a} , \tag{5.2.17}$$

where a are fundamental parameters of the theory. This is precisely what supersymmetric theories aim to avoid. The second option requires a large A-term for the stop, which seems to rule out gauge mediation. However, it is still possible to achieve large A-terms by allowing direct couplings between messengers and matter which will contribute to the mediation of supersymmetry breaking together with gauge interactions.

Before turning to models of extended gauge mediation, we will present the general method used here to compute soft terms.

5.2.2 Soft terms from wavefunction renormalization

A convenient way to compute soft terms was introduced by Gian Francesco Giudice and Riccardo Rattazzi in the context of minimal gauge mediation [309]. The principle is the following: one computes the wavefunction renormalization of chiral and gauge superfields in the supersymmetric theory. The results obtained are functions of the messenger mass M_X, treated as a complex number. Then, one performs an analytic continuation in superspace by replacing M_X with $\langle X \rangle = M_X + \theta^2 F_X$.

Here, similarly, we extract soft terms from wavefunction renormalization. Let us summarize this method in a general framework where, apart from the messenger sector, the superpotential is

$$W = \frac{1}{2} M_{ij} \Phi_i \Phi_j + \frac{1}{6} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \ , \tag{5.2.18}$$

After computing the wavefunction renormalization, the kinetic terms for the gauge and chiral superfields are the following

$$\mathcal{L}_{\text{kin}} = \int d^2\theta S(M_X, \mu) \text{tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha] + \text{h.c.} + \int d^4\theta Z_{ij}(M_X, M_X^\dagger, \mu) \Phi_i^\dagger \Phi_j \ , \tag{5.2.19}$$

where μ is the renormalization scale. Replacing M_X with $\langle X \rangle$ in the wavefunction renormalization of the gauge superfield and performing the expansion gives

$$S(\langle X \rangle, \mu) = S(M_X, \mu) + \theta^2 \frac{\partial S}{\partial M_X} F_X \ . \tag{5.2.20}$$

The kinetic term for the gauge superfield becomes

$$\int d^2\theta S(M_X, \mu) \left[1 + \theta^2 \frac{\partial \log S}{\partial \log M_X} \Lambda \right] \text{tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha] + \text{h.c.} \ . \tag{5.2.21}$$

In order to recover a canonical form for the kinetic term, one performs the following rescaling on the field strength, $\mathcal{W}_\alpha \to S(M_X, \mu)^{1/2} \mathcal{W}_\alpha$. Finally, the integration of the term in θ^2 gives

$$\int d^2\theta \theta^2 \frac{\partial \log S}{\partial \log M_X} \Lambda \, \text{tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha] = -\frac{1}{4} \frac{\partial \log S}{\partial \log M_X} \Lambda \, \lambda_A^\alpha \lambda_{A\alpha} \ , \tag{5.2.22}$$

so that one can identify the gaugino mass,

$$M = \frac{1}{2} \frac{\partial \log S}{\partial \log M_X} \Lambda \ . \tag{5.2.23}$$

Similarly, after expanding the wavefunction renormalization of the chiral superfields, the second term of eq. (5.2.19) reads

$$\int d^4\theta \left[Z_{ij|_1} + \theta^2 Z_{ij|_{\theta^2}} + \bar{\theta}^2 Z^\dagger_{ij|_{\theta^2}} + \theta^4 Z_{ij|_{\theta^4}} \right] \Phi_i^\dagger \Phi_j \ , \tag{5.2.24}$$

where we used the following notations

$$Z_{ij|_1} = Z_{ij}(M_X, M_X, \mu) \ , \quad Z_{ij|_{\theta^2}} = \frac{\partial Z_{ij}}{\partial M_X} F_X \ , \quad Z_{ij|_{\theta^4}} = \frac{\partial^2 Z_{ij}}{\partial M_X \partial M_X^\dagger} |F_X|^2 \ . \tag{5.2.25}$$

The first term gives the ordinary kinetic terms, while the other ones will give rise to A-terms, B-terms and squared masses for the scalars. Indeed, one has

$$\int d^4\theta\, \theta^2 \Phi_i^\dagger \Phi_j = F_i^\dagger \phi_j \ , \tag{5.2.26}$$

$$\int d^4\theta\, \theta^4 \Phi_i^\dagger \Phi_j = \phi_i^\dagger \phi_j \ . \tag{5.2.27}$$

One should redefine the chiral superfields to recover a canonical form for the kinetic terms. Using $Z_{|_1} = V^\dagger V$, the suitable rescaling is

$$\Phi_i \to V_{ij}\Phi_j \ . \tag{5.2.28}$$

Once this is done, after integrating out the auxiliary F fields, one finds the following A- and B-terms (using the same conventions as in eq. (4.1.62)),

$$B_{ij} = V_{i'i}^{-1} V_{j'j}^{-1} \left[M_{ki'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{kj'} + M_{kj'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{ki'} \right] \ , \tag{5.2.29}$$

$$A_{ijk} = V_{i'i}^{-1} V_{j'j}^{-1} V_{k'k}^{-1} \left[\lambda_{li'j'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{lk'} + \lambda_{lk'i'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{lj'} + \lambda_{lj'k'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{li'} \right] \ , \tag{5.2.30}$$

as well as the following scalar squared masses,

$$(m^2)_{ij} = V_{i'i}^{-1*} V_{j'j}^{-1} \left[-Z_{i'j'|_{\theta^4}} + Z^\dagger_{i'k|_{\theta^2}} Z^{-1}_{kl|_1} Z_{lj'|_{\theta^2}} \right] \ . \tag{5.2.31}$$

Finally, if both the μ and B_μ-terms are to be generated by the supersymmetry breaking, one should also mention the holomorphic wavefunction renormalization for Higgs doublets, defined by the following term in the Kähler potential.

$$K_{\text{holomorphic}} = \mathcal{H}_{ud} H_u H_d + \text{h.c.} \tag{5.2.32}$$

In the previous chapter, we discarded this term because, in a supersymmetric theory, the θ^4 component of an holomorphic function of chiral superfields is a total derivative. However, in the context of supersymmetry breaking, such considerations do not apply anymore. In particular, in the present method, $\mathcal{H}_{ij}$ is promoted to a superfield, so that

$$[\mathcal{H}_{ud} H_u H_d]_D = \mathcal{H}_{ud|_{\bar\theta^2}} \left(H_u F_{H_d} + H_d F_{H_u} - \tilde{H}_u \tilde{H}_d \right) + \mathcal{H}_{ud|_{\theta^4}} H_u H_d \ . \tag{5.2.33}$$

Thus, after integrating out the F-terms, one can identify the mass parameters μ and B_μ,

$$\mu = V_{uu}^{-1} V_{dd}^{-1} \mathcal{H}_{ud|_{\bar\theta^2}} \ , \qquad B_\mu = -V_{uu}^{-1} V_{dd}^{-1} \mathcal{H}_{ud|_{\theta^4}} \ . \tag{5.2.34}$$

5.2.3 General matter-messenger mixing

In extended gauge mediation, the superpotential contains new couplings between messengers and matter fields [310–321]. This generates in particular A-terms at one loop, needed to raise the upper bound on the Higgs mass. In models usually considered, messengers are the only heavy fields, while all other fields are light MSSM fields that can be considered massless. Thus, there are two kinds of new couplings in the superpotential, that can be referred to as messenger-light-light and messenger-messenger-light [322]. In some particular models, such as the one considered in [323], the messengers are also fields involved in the seesaw mechanism. Note that, in order to give masses to gauginos at one loop, these fields should be involved in the type II or type III seesaw, because the right-handed neutrinos involved in type I are gauge singlets.

We consider here a general scenario involving not only light MSSM fields and messengers, but also heavy fields that are not messengers. In the following section, we will consider the specific cases of the type I and type II seesaw. In the general case, the superpotential reads

$$W = X\Phi_{\bar{M}}\Phi_M + \frac{1}{2}M_{ij}\Phi_i\Phi_j + \frac{1}{6}\lambda_{ijk}\Phi_i\Phi_j\Phi_k \ . \tag{5.2.35}$$

We want to compute soft terms at the order of two-loop, so that the wavefunction renormalization of chiral superfields can be expanded as

$$Z_{ij} = \delta_{ij} + Z^{(1)}_{ij|_1} + Z^{(2)}_{ij|_1} + \left[\theta^2\left(Z^{(1)}_{ij|_{\theta^2}} + Z^{(2)}_{ij|_{\theta^2}}\right) + \text{h.c.}\right] + \theta^4\left(Z^{(1)}_{ij|_{\theta^4}} + Z^{(2)}_{ij|_{\theta^4}}\right) \ . \tag{5.2.36}$$

At the one-loop order, previous formulae (5.2.29), (5.2.30) and (5.2.31) give the following expressions,

$$B^{(1)}_{ij} = M_{ki}Z^{(1)}_{kj|_{\theta^2}} + (i \leftrightarrow j) \ , \tag{5.2.37}$$

$$A^{(1)}_{ijk} = \lambda_{ljk}Z^{(1)}_{li|_{\theta^2}} + (i \leftrightarrow j) + (i \leftrightarrow k) \ , \tag{5.2.38}$$

$$(m^2)^{(1)}_{ij} = -Z^{(1)}_{ij|_{\theta^4}} \ , \tag{5.2.39}$$

while at two loops, the squared masses of scalars are

$$(m^2)^{(2)}_{ij} = -Z^{(2)}_{ij|_{\theta^4}} + Z^{(1)\dagger}_{ik|_{\theta^2}}Z^{(1)}_{kj|_{\theta^2}} + \frac{1}{2}Z^{(1)\dagger}_{ik|_1}Z^{(1)}_{kj|_{\theta^4}} + \frac{1}{2}Z^{(1)\dagger}_{ik|_{\theta^4}}Z^{(1)}_{kj|_1} \ . \tag{5.2.40}$$

At lowest order, the gaugino masses are given by the same formulae as in minimal gauge mediation, but they may differ at two loops.

As explained previously, we will compute the wavefunction renormalization in the fully supersymmetric theory, and only in the end extract the θ^2 and θ^4 components to recover soft terms. More precisely, we will match the theory above M_X, that contains messengers, with the low-energy theory where they have been integrated out. At one loop, this matching reads schematically

$$\sum D^{(1)} + i\delta Z^{(1)} = iZ^{(1)} + \sum_{\text{light}} D^{(1)} + i\delta Z^{(1)}_{\text{light}} \ , \tag{5.2.41}$$

The left-hand side contains a sum of all one-loop diagrams $D^{(1)}$ of the high-energy theory, plus a counterterm that cancel their divergences, while the right-hand side contains the one-loop wavefunction renormalization of the low-energy theory, the sum of one-loop diagrams involving light fields only (the heavy ones have been integrated out) and the counterterm $\delta Z^{(1)}_{\text{light}}$ that cancels their divergences. The contributions involving light fields only are the same in both the high-energy and the low-energy theory, so that they cancel out. This gives for the one-loop wavefunction

$$\sum_{\text{heavy}} D^{(1)} + i\delta Z^{(1)}_{\text{heavy}} = iZ^{(1)} \ , \tag{5.2.42}$$

where, on the left-hand side, we keep only diagrams involving at least one heavy field.

At two loops, similarly, we get

$$\sum_{\text{heavy}} D^{(2)} + \sum_{\text{heavy}} D^{(1)}_{\delta Z} + i\delta Z^{(2)}_{\text{heavy}} = iZ^{(2)} + \sum_{\text{light}} D^{(1)}_{Z} + i\delta \tilde{Z}^{(1)}_{\text{light}} \ , \tag{5.2.43}$$

where the $D^{(2)}$ are two-loop diagrams, the $D^{(1)}_{\delta Z}$ are one-loop diagrams with a one-loop counterterm, while the D_Z are one-loop diagrams with a one-loop wavefunction insertion, and $\delta\tilde{Z}^{(1)}$ is the conterterm that cancels their divergences. This is summarized diagramatically in fig. 5.3.

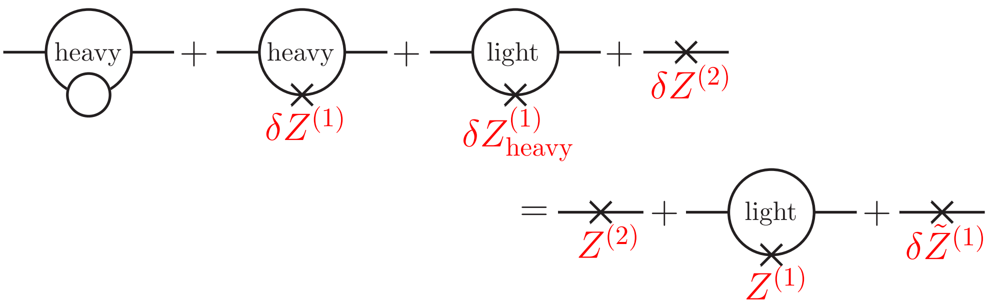


Figure 5.3: Diagrammatic representation of the two-loop matching procedure. The label "heavy" in the right-hand side means that at least one heavy field is involved in the loop, while the label "light" means that all the fields in the loop are light. The countertem $\delta Z^{(1)}_{\text{heavy}}$ was defined in eq. (5.2.42).

In what follows, the indices i, j, k, ... will run over every fields, while greek indices α, β, γ, ... will run over light fields only.

From now on, we work in a basis where the matrix M_{ij} is diagonal, so that the mass terms can be rewritten as

$$W_{\text{mass}} = M_i \Phi_{\bar{i}} \Phi_i \ , \tag{5.2.44}$$

where Φ_i and $\Phi_{\bar{i}}$ are chiral superfields belonging to conjugate representations of the gauge group.

One-loop order

Superdiagrams contributing to the one-loop wavefunction renormalization of chiral superfields are displayed in fig. 5.4. In ordinary gauge mediation, since there is no direct

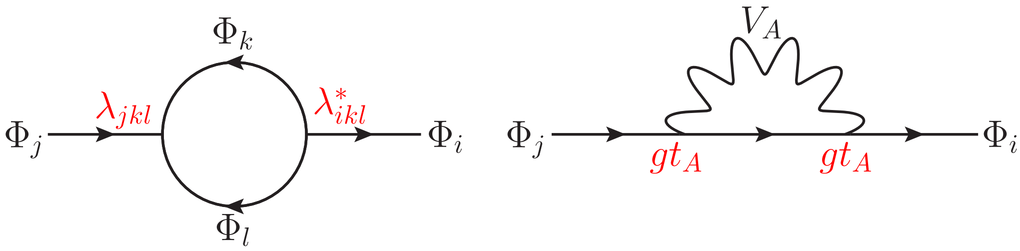


Figure 5.4: Superdiagrams contributing to the wavefunction renormalization at one loop.

couplings between the messengers and the other chiral superfield, none of these diagrams can generate soft terms by itself. However, in the present scenario, the diagram on the left can generate soft terms if at least one of the fields in the loop is a messenger. Consistently with the results presented in 5.1.4, the diagram on the right does not generate any soft term, but we should nevertheless compute it because the result will be needed when performing two-loop computations. Although we are interested in the soft terms for light fields only, we should compute the 1-loop wavefunction and counterterm for every kind of field because these results will be needed when performing two-loop calculations.

The left-hand diagram involves only interactions coming from the superpotential, so we label it for short λ^2. It gives the following contribution to the wavefunction renormalization,

$$iZ^{(1)}_{ij|_{\lambda^2}} = \frac{1}{2}\sum_{k,l} d_i^{kl}\lambda^*_{ikl}\lambda_{jkl}\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k-M_k^2)(k-M_l^2)} + i\delta Z^{(1)}_{ij|_{\lambda^2}}\ , \tag{5.2.45}$$

where d_i^{kl} is a gauge multiplicity factor. The computation of the integral in dimensional reduction[2] in $d = 4 - 2\epsilon$ gives

$$\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k-M_k^2)(k-M_l^2)} = \frac{i}{16\pi^2}\left[\frac{1}{\epsilon} + \frac{x_kf_1(x_k,s)-x_lf_1(x_l,s)}{\Delta_{kl}}\right.$$
$$\left. + \,\epsilon\,\frac{x_kf_{1\epsilon}(x_k,s)-x_lf_{1\epsilon}(x_l,s)}{\Delta_{kl}} + \mathcal{O}(\epsilon^2)\right]\ , \tag{5.2.46}$$

with $x_n = M_n^2/M_X^2$, $\Delta_{kl} = x_k - x_l$, $s = \mu^2/M_X^2$ (where μ is the renormalization scale), and the loop functions f_1 and $f_{1\epsilon}$ are defined by

$$f_1(x,s) = 1 - \log\frac{x}{s}\ , \tag{5.2.47}$$

$$f_{1\epsilon}(x,s) = \frac{1}{2}\left[1 + \frac{\pi^2}{6} + \left(1-\log\frac{x}{s}\right)^2\right]\ . \tag{5.2.48}$$

[2]Here, dimensional reduction with $\overline{\mathrm{DR}}$ is equivalent to dimensional regularization with $\overline{\mathrm{MS}}$.

We can now identify the counterterm which, in the $\overline{\mathrm{DR}}$ scheme, is

$$\delta Z^{(1)}_{ij|_{\lambda^2}} = -\frac{1}{32\pi^2\epsilon} d_i^{kl} \lambda^*_{ikl} \lambda_{jkl} \ , \tag{5.2.49}$$

and extract the θ^2 and θ^4 components at order zero in ϵ by performing the following replacement,

$$x_M \to \sqrt{x_M^\dagger x_M} = 1 + \Lambda\theta^2 + \Lambda^\dagger\bar{\theta}^2 + |\Lambda|^2\theta^4 \ , \tag{5.2.50}$$

(we recall here that Φ_M and $\Phi_{\bar{M}}$ are the messenger superfields). After this last step, the x_i can be treated as real variables in the expressions of $Z_{ij|_{\theta^2}}$ and $Z_{ij|_{\theta^4}}$.

$$Z^{(1)}_{ij|_{\theta^2}} = -\frac{\Lambda}{16\pi^2}\left[d_i^{kA} \lambda^*_{ikA} \lambda_{jkA} g_2(x_k) + d_i^{AB} \lambda^*_{iAB} \lambda_{jAB} \right] \ , \tag{5.2.51}$$

$$Z^{(1)}_{ij|_{\theta^4}} = \frac{|\Lambda|^2}{16\pi^2} d_i^{kA} \lambda^*_{ikA} \lambda_{jkA} g_4(x_k) \ , \tag{5.2.52}$$

where uppercase indices A and B run over messengers only and the functions g_2 and g_4 are given by

$$g_2(x) = \frac{1 - x + x\log x}{(1-x)^2} \ , \tag{5.2.53}$$

$$g_4(x) = \frac{x\,(2 - 2x + (1+x)\log x)}{(1-x)^3} \ . \tag{5.2.54}$$

The computation of the gauge superdiagram (right of fig. 5.4) gives

$$iZ^{(1)}_{ij|_{g^2}} = -2g^2 C(\Phi_i)\delta_{ij} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2(k^2 - M_i^2)} + \delta Z^{(1)}_{ij|_{g^2}} \ , \tag{5.2.55}$$

The loop integral can be expressed in terms of the same functions f_1 and $f_{1\epsilon}$ as before,

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2(k^2 - M_i^2)} = \frac{i}{16\pi^2}\left[\frac{1}{\epsilon} + f_1(x_i, s) + \epsilon x_i f_{1\epsilon}(x_i, s) + \mathcal{O}(\epsilon^2) \right] \ . \tag{5.2.56}$$

The counterterm of eq. (5.2.55) can be identified,

$$\delta Z^{(1)}_{ij|_{g^2}} = \frac{g^2}{8\pi^2} \delta_{ij} C(\Phi_i) \frac{1}{\epsilon} \ . \tag{5.2.57}$$

As expected, if Φ_i is not a messenger, this contribution does not have any θ^2 or θ^4 component, and hence does not participate in soft terms.

Two-loop order

At two loops, in addition to the usual contribution of gauge interactions to the scalar squared masses, already given in eq. (5.1.28), the wavefunction renormalization depends on two kinds of superdiagram: diagrams involving only superpotential interactions, and diagrams involving both superpotential and gauge interactions. The former

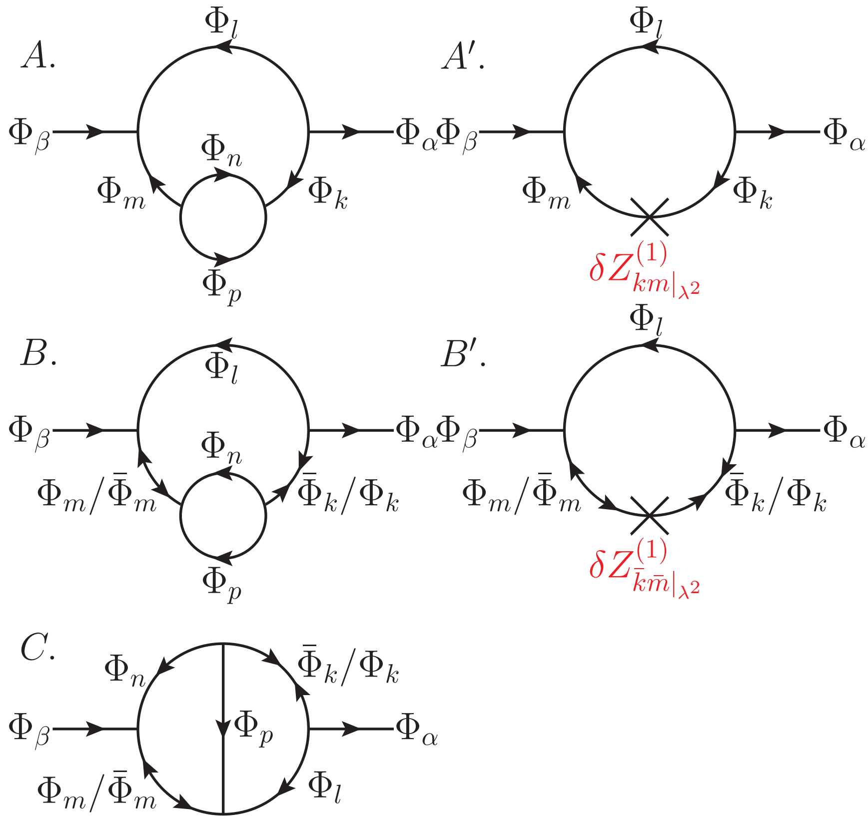


Figure 5.5: Superdiagrams involving superpotential interactions only contributing to the wavefunction renormalization at two loop. Double arrows indicate mass insertions. The diagrams labelled with a prime involve the one-loop counterterm from eq. (5.2.49).

will scale like λ^4 and the latter like $\lambda^2 g^2$. This time, we are only interested in the wavefunction renormalization of light fields Φ_α, Φ_β.

Let us focus on the first kind of contribution. As can be seen from fig. 5.5, there exists five possible topologies, leading to the following contribution to the left-hand side of eq. (5.2.43) (here, it is understood that at least one of the fields Φ_k, Φ_l, Φ_m,

Φ_n or Φ_p has to be heavy),

$$\begin{aligned}\Big[\sum D^{(2)}_{\alpha\beta} + i\delta Z^{(2)}_{\alpha\beta}\Big]_{|\lambda^4} &= \frac{i}{1024\pi^4} d^{kl}_\alpha d^{np}_k \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda^*_{mnp}\lambda_{knp}\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right] \\ &+ \frac{i}{512\pi^4} d^{kl}_\alpha d^{np}_k \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda^*_{mnp}\lambda_{knp} f_A(x_k, x_l, x_m, x_n, x_p) \\ &+ \frac{i}{512\pi^4} d^{kl}_\alpha d^{np}_k \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda_{\bar{m}np}\lambda^*_{\bar{k}np} f_B(x_k, x_l, x_m, x_n, x_p) \\ &+ \frac{i}{256\pi^4} d^{kl}_\alpha d^{mn}_\beta \lambda^*_{\alpha kl}\lambda_{\beta mn}\lambda^*_{\bar{k}np}\lambda_{l\bar{m}p} f_C(x_k, x_l, x_m, x_n, x_p) \\ &+ i\delta Z^{(2)}_{\alpha\beta|_{\lambda^4}} \ . \end{aligned} \tag{5.2.58}$$

The expression of the loop functions f_A, f_B and f_C is given in appendix B. This gives the following two-loop counterterm,

$$\delta Z^{(2)}_{\alpha\beta|_{\lambda^4}} = -\frac{1}{1024\pi^4} d^{kl}_\alpha d^{np}_k \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda^*_{mnp}\lambda_{knp}\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right] \ . \tag{5.2.59}$$

The corresponding contribution to the right-hand side of eq. (5.2.43) is given by

$$\begin{aligned}&\left[iZ^{(2)}_{\alpha\beta} + \sum_{\text{light}} \left(D^{(1)}_Z\right)_{\alpha\beta} + i\left(\delta\tilde{Z}^{(1)}_{\text{light}}\right)_{\alpha\beta}\right]_{|\lambda^4} \\ &= iZ^{(2)}_{\alpha\beta|_{\lambda^4}} + \frac{i}{512\pi^4} d^{\gamma\delta}_\alpha \lambda^*_{\alpha\gamma\sigma}\lambda_{\beta\gamma\delta}\lambda^*_{\delta np}\lambda_{\sigma np} f_{A'}(0, x_n, x_p) f_{A'}(0,0,0) \ ,\end{aligned} \tag{5.2.60}$$

where we recall that Φ_γ, Φ_δ and Φ_σ are light fields, and we defined

$$f_{A'}(x_i, x_j, x_k) = \frac{x_i^2 f_1(x_i,s)}{\Delta_{ij}\Delta_{ik}} - \frac{x_j^2 f_1(x_j,s)}{\Delta_{ij}\Delta_{jk}} + \frac{x_k^2 f_1(x_k,s)}{\Delta_{ik}\Delta_{jk}} \ . \tag{5.2.61}$$

$f_{A'}$ diverges when its three arguments go to zero, so in principle it should be regularized. As explained in appendix B, the loop functions f_B and and f_C have no infrared divergence. On the other hand, f_A diverges when $x_k = x_l = x_m = 0$, but this divergence is precisely cancelled by the one coming from $f_{A'}(0,0,0)$ in eq. (5.2.60). Thus, when writing the expression of $Z^{(2)}$, we split the terms in f_A and regroup the infrared-divergent contributions, leading to the following expression for the two-loop wavefunction renormalization,

$$\begin{aligned}Z^{(2)}_{\alpha\beta|_{\lambda^4}} &= \frac{1}{512\pi^4} d^{\gamma\sigma}_\alpha d^{np}_\gamma \lambda^*_{\alpha\gamma\sigma}\lambda_{\beta\gamma\delta}\lambda^*_{\delta np}\lambda_{\sigma np}\left[f_A(0,0,0,x_n,x_p) - f_{A'}(0,x_n,x_p) f_{A'}(0,0,0)\right] \\ &+ \frac{1}{512\pi^4} d^{kl}_\alpha d^{np}_k \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda^*_{mnp}\lambda_{knp} f_A(x_k, x_l, x_m, x_n, x_p) \\ &+ \frac{1}{512\pi^4} d^{kl}_\alpha d^{np}_k \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda_{\bar{m}np}\lambda^*_{\bar{k}np} f_B(x_k, x_l, x_m, x_n, x_p) \\ &+ \frac{1}{256\pi^4} d^{kl}_\alpha d^{mn}_\beta \lambda^*_{\alpha kl}\lambda_{\beta mn}\lambda^*_{\bar{k}np}\lambda_{l\bar{m}p} f_C(x_k, x_l, x_m, x_n, x_p) \ .\end{aligned} \tag{5.2.62}$$

where, in the second line, at least one of the fields Φ_k, Φ_l and Φ_m is massive so that this term is infrared-safe.

Schematically, we extract the θ-components in the following way. If one of the fields appearing in the loop function f is a messenger, for instance $\Phi_p \equiv \Phi_M$, then we obtain the θ^2 and θ^4 component as

$$f_{|\theta^2}(x_k, x_l, x_m, x_n) = \Lambda \frac{\partial}{\partial x_M} f(x_k, x_l, x_m, x_n, x_M)_{|x_M=1} \;, \tag{5.2.63}$$

$$f_{|\theta^4}(x_k, x_l, x_m, x_n) = \Lambda^2 \frac{\partial^2}{\partial x_M^2} f(x_k, x_l, x_m, x_n, x_M)_{|x_M=1} \;. \tag{5.2.64}$$

The second kind of contribution is summarized by the diagrams displayed in fig. 5.6. There are now eight different topologies, but, as explained in appendix B.2, the diagrams F, G, F', G' (obtained from the former by moving the gauge propagator to the right) and H can be gathered as a result of gauge invariance, which reduces the number of contributions. The remaining integrals can be expressed in terms of the same functions f_A and f_B as those appearing in eq. (5.2.58),

$$\begin{aligned}
\left[\sum D^{(2)}_{\alpha\beta} + i\delta Z^{(2)}_{\alpha\beta}\right]_{|\lambda^2 g^2} = &-\frac{i}{512\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 \left(2C(\Phi_l) - C(\Phi_a)\right)\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right] \\
&- \frac{i}{128\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 C(\Phi_l) f_A(x_k, x_l, x_l, x_l, 0) \\
&- \frac{i}{128\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 C(\Phi_l) f_B(x_k, x_l, x_l, x_l, 0) \\
&+ \frac{i}{256\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 C(\Phi_\alpha) f_A(0, 0, 0, x_k, x_l) \\
&+ i\delta Z^{(2)}_{\alpha\beta|_{\lambda^2 g^2}} \;.
\end{aligned} \tag{5.2.65}$$

We can now identify the counterterm,

$$\delta Z^{(2)}_{\alpha\beta|_{\lambda^2 g^2}} = \frac{i}{512\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 \left(2C(\Phi_l) - C(\Phi_a)\right)\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right] \;. \tag{5.2.66}$$

The corresponding contribution to the right-hand side of eq. (5.2.43) is given by

$$\begin{aligned}
&\left[iZ^{(2)}_{\alpha\beta} + \sum_{\text{light}} \left(D^{(1)}_Z\right)_{\alpha\beta} + i\left(\delta\tilde{Z}^{(1)}_{\text{light}}\right)_{\alpha\beta}\right]_{|\lambda^2 g^2} \\
&= iZ^{(2)}_{\alpha\beta|_{\lambda^2 g^2}} + \frac{i}{256\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 C(\Phi_a) f_{A'}(0, x_k, x_l) f_{A'}(0,0,0) \;.
\end{aligned} \tag{5.2.67}$$

Again, the infrared divergences from eqs. (5.2.65) and (5.2.67) cancel, so that the gauge-superpotential contribution to the two-loop wavefunction renormalization is

$$\begin{aligned}
Z^{(2)}_{\alpha\beta|_{\lambda^2 g^2}} = &-\frac{1}{256\pi^4} d^{kl}_\alpha \lambda^*_{\alpha kl}\lambda_{\beta kl} g^2 \\
&\times \Big\{ 2\, C(\Phi_l)\left[f_A(x_k, x_l, x_l, x_l, 0) + f_B(x_k, x_l, x_l, x_l, 0)\right] \\
&- C(\Phi_a)\left[f_A(0,0,0,x_k,x_l) - f_{A'}(0, x_k, x_l) f_{A'}(0,0,0)\right]\Big\} \;.
\end{aligned} \tag{5.2.68}$$

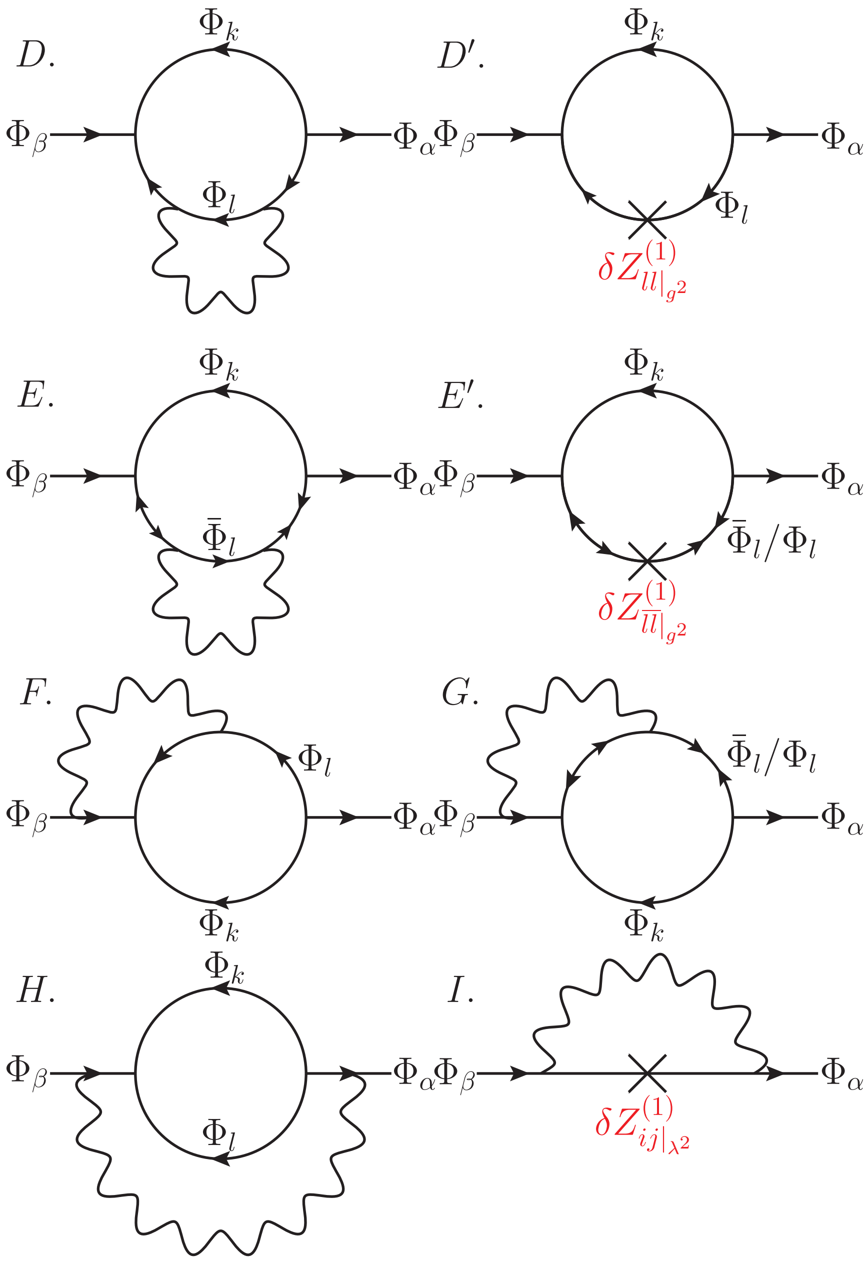


Figure 5.6: Superdiagrams involving superpotential and gauge interactions contributing to the wavefunction renormalization at two loop. Double arrows indicate mass insertions. There are also two diagrams similar to F and G but with the gauge propagator on the right. The diagrams labelled with a prime involve the one-loop counterterm from eq. (5.2.57), while the diagram I involves the counterterm from eq. (5.2.49).

Holomorphic wavefunction

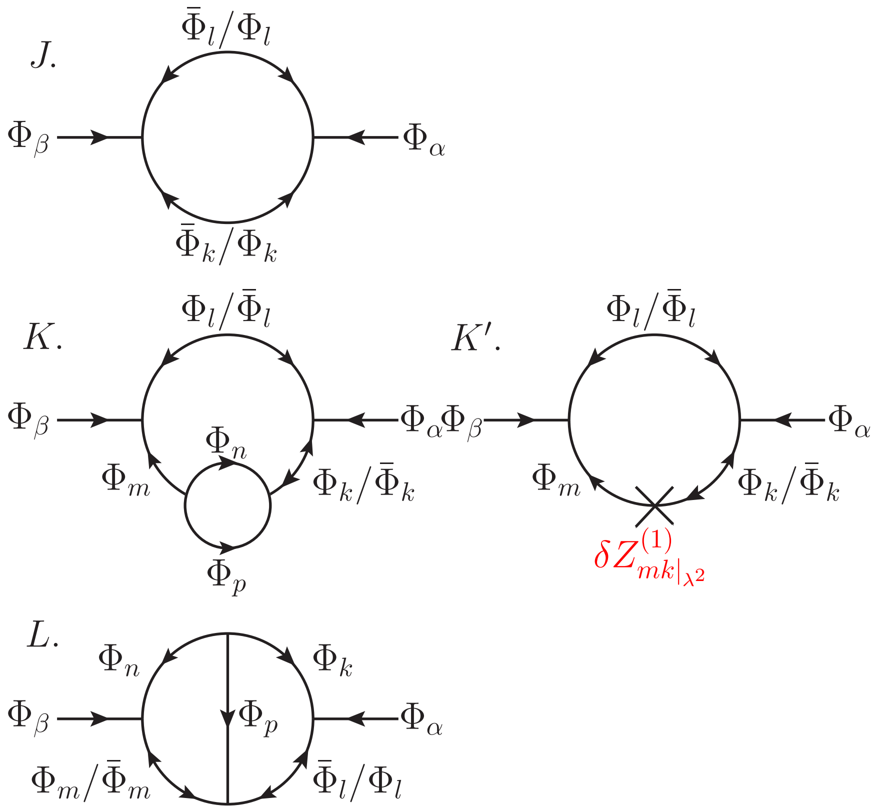


Figure 5.7: One and two-loop contributions to the holomorphic wavefunction renormalization. Diagrams obtained from K and K' by mirror symmetry are also involved.

The contributions to the holomorphic wavefunction renormalization are displayed in fig. 5.7. Since only the $\bar{\theta}^2$ and θ^4 components play a physical role, we do not compute other possible contributions that have no θ dependence. We focus here on the leading order, so we do not consider mixed gauge-superpotential contributions at two loops. Indeed, they contribute to soft terms only if the one-loop wavefunction already does so. At one loop, we find the following result,

$$i\mathcal{H}^{(1)}_{\alpha\beta} = d^{kl}_\alpha \lambda_{\alpha kl}\lambda_{\beta\bar{k}\bar{l}} M^\dagger_k M^\dagger_l \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2(k-M_k^2)(k-M_l^2)} \; . \tag{5.2.69}$$

This is both IR- and UV-convergent, so we can take $d=4$ and compute the integral straightforwardly. This gives the following result,

$$\mathcal{H}_{\alpha\beta} = -\frac{1}{16\pi^2} d^{kl}_\alpha \lambda_{\alpha kl}\lambda_{\beta\bar{k}\bar{l}} \frac{\sqrt{x^\dagger_k x^\dagger_l}}{\Delta_{kl}} \log\left(\frac{x_k}{x_l}\right) \; . \tag{5.2.70}$$

If at least one of the fields running in the loop is a messenger, the θ components are

$$\mathcal{H}_{\alpha\beta|_{\bar\theta^2}} = -\frac{\Lambda}{16\pi^2}\left[d_\alpha^{kA}\lambda_{\alpha kA}\lambda_{\beta\bar k\bar A}h_2(x_k) + d_\alpha^{AB}\lambda_{\alpha AB}\lambda_{\beta\bar A\bar B}\right] , \tag{5.2.71}$$

$$\mathcal{H}_{\alpha\beta|_{\theta^4}} = \frac{\Lambda}{16\pi^2}\left[d_\alpha^{kA}\lambda_{\alpha kA}\lambda_{\beta\bar k\bar B}h_4(x_k) + \frac{2}{3}d_\alpha^{AB}\lambda_{\alpha AB}\lambda_{\beta\bar A\bar B}\right] , \tag{5.2.72}$$

where again, uppercase indices run over messenger fields only.

$$h_2(x) = \frac{\sqrt{x}\,(-1+x-x\log x)}{(1-x)^2} , \tag{5.2.73}$$

$$h_4(x) = \frac{\sqrt{x}\,(1-x^2+2x\log x)}{(1-x)^3} . \tag{5.2.74}$$

Unfortunately, it turns out that the two functions h_2 and h_4 always have the same order of magnitude. This illustrate the $\mu - B_\mu$ problem: if both parameters μ and B_μ are to be generated this way, it is impossible to overcome the relation $B_\mu \sim \mu\Lambda$. Actually, this problem remains even if one introduces additional pairs of messengers with different couplings to the spurion.

If there is no direct coupling between the fields Φ_a and Φ_b and the spurion, the relevant contributions to the holomorphic wavefunction renormalization arises at two loops at least. They lead to the following expression,

$$\begin{aligned}\mathcal{H}^{(2)}_{\alpha\beta} &= \frac{1}{512\pi^4}d_\alpha^{kl}d_k^{np}\lambda_{\alpha\bar k\bar l}\lambda_{\beta lm}\lambda_{knp}\lambda^*_{mnp}\sqrt{\frac{x_l^\dagger}{x_m}}f_B(x_k,x_l,x_m,x_n,x_p)\\ &+ \frac{1}{512\pi^4}d_\alpha^{kl}d_k^{np}\lambda_{\alpha kl}\lambda_{\beta\bar l\bar m}\lambda^*_{knp}\lambda_{mnp}\sqrt{\frac{x_l^\dagger}{x_k}}f_B(x_k,x_l,x_m,x_n,x_p)\\ &+ \frac{1}{256\pi^4}d_\alpha^{kl}d_\beta^{mn}\lambda_{\alpha kl}\lambda_{\beta mn}\lambda^*_{knp}\lambda_{\bar l\bar m p}\sqrt{\frac{x_l^\dagger}{x_k}}f_C(x_k,x_l,x_m,x_n,x_p) .\end{aligned} \tag{5.2.75}$$

5.3 Application: the supersymmetric seesaw

5.3.1 Flavour violation

When considering specific models of extended gauge mediation, the new couplings between matter superfields and messengers should not generate flavour-changing neutral currents that exceed the experimental bounds, otherwise one of the main assets of gauge mediation would be lost. For instance, considering a messenger Φ_d with the same quantum numbers as the down-type Higgs, one should in principle be able to write the following term in the superpotential

$$W_{\text{new}} = \lambda^e_{\alpha\beta}\Phi_d\ell_\alpha e^c_\beta . \tag{5.3.1}$$

If the new couplings are fully generic, this will typically generate squared mass matrices for the slepton proportional to $\lambda^{e\dagger}\lambda^e$. This in turn will give rise to flavour-changing neutral currents in the charged lepton sector, in contradiction with the bounds quoted in table 2.2, unless the new couplings are small enough or have a very peculiar structure.

A solution to this problem was introduced in ref. [314]. Matter-messenger mixing is initially forbidden by messenger parity. We call $\tilde{\Phi}_u$ and $\tilde{H}_u$ the up-type messenger and Higgs superfields that appear in this superpotential. Then, messenger parity is broken by the expectation value of a field S such that the following interaction is allowed,

$$\frac{S}{\Lambda_S} X \Phi_d \tilde{H}_u \ . \tag{5.3.2}$$

Once S gets a v.e.v. $\langle S \rangle \neq 0$, the superpotential contains the following contributions,

$$W_{\text{GM}} = X\Phi_d \left(\tilde{\Phi}_u + s\tilde{H}_u\right) + X\Phi_T\bar{\Phi}_T \ , \tag{5.3.3}$$

$$W_{\text{MSSM}} = \tilde{y}^u_{ij}\tilde{H}_u Q_i u^c_j + y^d_{ij} H_d Q_i d^c_j + y^e_{\alpha\beta} H_d \ell_\alpha e^c_\beta \ . \tag{5.3.4}$$

Now, we redefine fields in such a way that only one vectorlike pair (Φ_u, Φ_d) couples to the supersymmetry-breaking spurion. We define Θ such that $t_\Theta = \tan\Theta = \langle S \rangle / \Lambda_S$, and

$$\Phi_u = c_\Theta \tilde{\Phi}_u + s_\Theta \tilde{H}_u \tag{5.3.5}$$

$$H_u = -s_\Theta \tilde{\Phi}_u + c_\Theta \tilde{H}_u \ . \tag{5.3.6}$$

In the new basis, the previous contributions to the superpotential read

$$W_{\text{GM}} = \kappa X \Phi_d \Phi_u + X\Phi_T \Phi_{\bar{T}} \ , \tag{5.3.7}$$

$$W_{\text{MSSM}} = y^u_{ij} H_u Q_i u^c_j + y^d_{ij} H_d Q_i d^c_j + y^e_{\alpha\beta} H_d \ell_\alpha e^c_\beta \ . \tag{5.3.8}$$

with $\kappa = 1/\cos\Theta = \sqrt{1+s^2}$ and $y^u_{ij} = c_\Theta \tilde{y}^u_{ij}$. The superpotential also contains the following new term,

$$W_{\text{new}} = \lambda^u_{ij} \Phi_u Q_i u^c_j \ , \tag{5.3.9}$$

with $\lambda^u_{ij} = s_\Theta \tilde{y}^u_{ij} = t_\Theta y^u_{ij}$. Because the coupling matrix λ^u is proportional to the Yukawa coupling matrix y^u, this model does not involve any new source of flavour violation. In the next paragraphs, when considering the type I and type II seesaw, we will find similarly that the new contributions to the slepton squared mass matrix can be expressed in terms of the neutrino mass matrix.

The operator of eq. (5.3.9) is present in most models of extended gauge mediation. Using eqs. (5.2.38), (5.2.53) and (5.2.54), one can see that it gives the following contribution to the one-loop A-terms of squarks,

$$A^u_{ij} = -\frac{\Lambda}{16\pi^2}\left(\lambda^u \lambda^{u\dagger} y^u + 2 y^u \lambda^{u\dagger}\lambda^u\right)_{ij} \simeq -\frac{3\Lambda}{16\pi^2} t^2_\Theta y^3_t \ , \tag{5.3.10}$$

$$A^d_{ij} = -\frac{\Lambda}{16\pi^2}\left(\lambda^u \lambda^{u\dagger} y^d\right)_{ij} \simeq -\frac{\Lambda}{16\pi^2} t^2_\Theta y^2_t y_b \ . \tag{5.3.11}$$

Extracting soft terms from wavefunction renormalization, one finds vanishing contributions to the squared masses of squarks at one loop. However, this method only gives access to the leading order in F_X/M^2_X. There are actually nonzero one-loop contributions to the squared masses of squarks, but they are suppressed by additional powers of F_X/M_X^2, so that, unless the messenger scale is very low, they will be negligible compared to two-loop contribution with no such suppression [314].

In addition to the flavour blind contributions from ordinary gauge mediation, the 2-loop squared masses of scalars include the following terms [314], where we keep only the dominant entries of y^u and y^d like in eqs. (5.3.10) and (5.3.11),

$$\Delta m_{\tilde{b}}^2 \simeq -\frac{\Lambda}{128\pi^4} t_\Theta^2 y_t^2 y_b^2 \ , \tag{5.3.12}$$

$$\Delta m_{\tilde{t}}^2 \simeq \frac{\Lambda}{128\pi^4} t_\Theta^2 y_t^2 \left[6y_t^2 \left(1 + t_\Theta^2\right) + y_b^2 - \left(\frac{13}{15} g_1^2 + 3g_2^2 + \frac{16g_3^2}{3}\right)\right] \ , \tag{5.3.13}$$

$$\Delta m_{\tilde{Q}_3}^2 \simeq \frac{\Lambda}{256\pi^4} t_\Theta^2 y_t^2 \left[6y_t^2 \left(1 + t_\Theta^2\right) - \left(\frac{13}{15} g_1^2 + 3g_2^2 + \frac{16g_3^2}{3}\right)\right] \ , \tag{5.3.14}$$

$$\Delta m_{H_u}^2 \simeq -\frac{9\Lambda^2}{256\pi^4} t_\Theta^2 y_t^4 \ , \tag{5.3.15}$$

$$\Delta m_{H_d}^2 \simeq -\frac{3\Lambda^2}{256\pi^4} t_\Theta^2 y_t^2 y_b^2 \ . \tag{5.3.16}$$

When considering specific models, additional terms appear, including in particular A-terms and one-loop squared masses for the sleptons.

5.3.2 Type I seesaw

We consider now the type I seesaw scenario. The mechanism does not differ much from the non-supersymmetric case. Apart from the messenger sector, the superpotential is given by

$$W_{\text{seesaw}} = \frac{1}{2} M_i N_i N_i + y_{i\alpha}^\nu H_u N_i \ell_\alpha \ , \tag{5.3.17}$$

and the neutrino mass matrix has the same expression as in the non-supersymmetric case (apart from the substitution $v \to v_u$),

$$m_{\nu\alpha\beta} = \sum_i \frac{y_{i\alpha}^\nu y_{i\beta}^\nu}{M_i} v_u^2 \ . \tag{5.3.18}$$

Ref. [324] studied how integrating out right-handed neutrinos affects soft terms. However, the mediation mechanism was not explicitly described, and instead an effective parametrization of soft terms at a high scale was chosen. In [325], right-handed neutrinos played the role of messengers in addition to the usual charged messengers of minimal gauge mediation: as was previously mentioned, right-handed neutrinos cannot by themselves transmit supersymmetry-breaking to all MSSM fields because they are gauge singlets. However, promoting them as messengers allows to generate an A-term for the stops, because of the neutrino Yukawa $y_{i\alpha}^\nu H_u N_i \ell_\alpha$ which is a direct matter-messenger coupling. Here, in contrast, we consider right-handed neutrinos that do not couple directly to the spurion, while other matter-messenger couplings are allowed. More precisely, the mechanism of section 5.3.1 gives rise to the following new terms,

$$W_{\text{new}} = \lambda_{i\alpha}^\nu \Phi_u N_i \ell_\alpha + \lambda_{ij}^u \Phi_u Q_i u_j^c \ , \tag{5.3.19}$$

with $\lambda^{\nu}_{i\alpha} = t_\Theta y^{\nu}_{i\alpha}$. Because of this direct coupling with messengers, the sleptons receive the following one-loop soft terms,

$$A^e_{\alpha\beta} = -\frac{\Lambda}{16\pi^2} t_\Theta^2 \sum_i y^{\nu}_{i\alpha} \left(y^{\nu\dagger} y^e\right)_{i\beta} g_2(x_i) \ , \tag{5.3.20}$$

$$(\Delta m^2_{\tilde{\ell}})_{\alpha\beta} = -\frac{\Lambda^2}{16\pi^2} t_\Theta^2 \sum_i y^{\nu *}_{i\alpha} y^{\nu}_{i\beta} g_4(x_i) \ , \tag{5.3.21}$$

with $x_i = M_i^2/M_X^2$ and the functions g_2 and g_4 are defined in eqs. (5.2.53) and (5.2.54) respectively.

At two loops, the squared masses of squarks, slepton and Higgs doublets receive new contributions, while the slepton singlets get their mass only at this order. Regarding the latter, in addition to the ordinary pure gauge contribution, the following term is generated,

$$\begin{aligned}(\Delta m^2_{\tilde{e}})_{\alpha\beta} = {} & \frac{\Lambda^2}{256\pi^4} t_\Theta^2 \sum_i y^{e*}_{\gamma\alpha} y^{e}_{\sigma\beta} y^{\nu}_{i\gamma} y^{\nu *}_{i\sigma} \\ & \times \left[\partial_5^2 f_A(0,0,0,x_i,1) - f_{A'}(0,0,0)\partial_3^2 f_{A'}(0,x_i,1)\right] \ , \end{aligned} \tag{5.3.22}$$

where $\partial_n f$ means that we take the derivative of f with respect to its n^{th} variable in order to extract the θ components as in eqs. (5.2.63) and (5.2.64).

These results can be compared to those of ref. [325], where right-handed neutrinos were messengers. The first difference is that, in this paper, soft terms depended only on F_X/M_X, while in our scenario, they also depend on the mass ratios $x_i = M_i^2/M_X^2$, because right-handed neutrino masses are not related in any way to the supersymmetry-breaking scale. The flavour structure of soft terms is also different, and in ref. [325], slepton masses appeared only at two loop, while here a contribution already arises at one loop. This is due to the fact that, according to eqs. (5.2.52) and (5.2.54), the θ^4 component of the one-loop wavefunction is nonzero only if the fields running in the loop are a messenger and a massive field, which are respectively Φ_u and N_i in the present model.

In general, the supersymmetric type I seesaw gives rise to flavour violation in the charged lepton sector, usually because neutrino Yukawa couplings contribute to the running of slepton masses. Here, in addition to that, right-handed neutrinos contribute directly to the generation of slepton masses and slepton A-terms from supersymmetry-breaking. This is in contrast with the non-supersymmetric type I seesaw, which does not give a significant contribution to such phenomena. The lepton flavour-violating slepton masses depend on the same Yukawa couplings as neutrino masses but unfortunately, as already mentioned in the context of leptogenesis, the coupling matrix y^ν cannot be expressed in a simple way in terms of the neutrino mass matrix, so that the relation between lepton flavour violation and neutrino masses is not straightforward. In order to make more precise predictions on flavour-changing observables, one should select a flavour model to parametrize the neutrino Yukawa couplings. The type II seesaw is more predictive with respect to this matter.

5.3.3 Type II seesaw

The supersymmetric type II seesaw differs from the non-supersymmetric case in the fact that there must be two triplet superfields Δ and $\bar{\Delta}$ for the same reason that there are two Higgs doublet. Indeed, the scalar triplet comes with a fermionic superpartner, that should be part of a vectorlike pair to avoid anomalies. Moreover, as can be seen from eq. (2.1.57), in the non-supersymmetric case, the generation of neutrino masses requires Δ to couple to lepton doublets and $\Delta^\dagger$ to couple to Higgs doublets, but in a supersymmetric theory, the superpotential is an holomorphic function of superfields and therefore cannot depend on $\Delta^\dagger$.

The most general type II seesaw scenario involves a pair of triplet superfields with opposite hypercharges, and the following superpotential,

$$W_{\text{seesaw}} = M_\Delta \bar{\Delta}\Delta + \frac{1}{2} f_{\alpha\beta} \Delta \ell_\alpha \ell_\beta + \frac{1}{2}\lambda^u_{HH} \bar{\Delta} H_u H_u + \frac{1}{2}\lambda^d_{HH} \bar{\Delta} H_d H_d \ . \tag{5.3.23}$$

The neutrino mass matrix of eq. (2.1.61) becomes

$$m_{\nu\alpha\beta} = \frac{1}{2}\lambda^u_{HH} f_{\alpha\beta} \frac{v_u^2}{M_\Delta} \ . \tag{5.3.24}$$

Here, we consider a model that differs from this generic case. We assume that the μ-term is forbidden by a Peccei-Quinn symmetry. The coupling, $\bar{\Delta} H_u H_u$ should be allowed in order to generate neutrino masses, but then the coupling $\Delta H_d H_d$ is forbidden by the same symmetry as the μ-term. We also consider the mechanism of 5.3.1 to avoid large flavour-changing neutral currents in the quark sector. Thus, apart from the messenger and MSSM parts, the superpotential contains the following contributions,

$$W_{\text{seesaw}} = \frac{1}{2} f_{\alpha\beta} \Delta \ell_\alpha \ell_\beta + \frac{1}{2}\lambda_{HH} \bar{\Delta} H_u H_u \ , \tag{5.3.25}$$

$$W_{\text{new}} = \lambda_{\Phi H} \bar{\Delta} H_u \Phi_u + \frac{1}{2}\lambda_{\Phi\Phi} \bar{\Delta} \Phi_u \Phi_u + \lambda^u_{ij} \Phi_u Q_i u^c_j \ , \tag{5.3.26}$$

with $\lambda_{\Phi H} = t_\Theta \lambda_{HH}$, $\lambda_{\Phi\Phi} = t_\Theta^2 \lambda_{HH}$.

At one loop, the new soft terms with respect to 5.3.1 are

$$\Delta A^u_{ij} = -\frac{3\Lambda}{32\pi^2} \lambda^2_{\phi H} y^u_{ij} g_2 \left(x_\Delta\right) \ , \tag{5.3.27}$$

$$\Delta m^2_{H_u} = -\frac{3\Lambda^2}{32\pi^2} \lambda^2_{\phi H} g_4 \left(x_\Delta\right) \ . \tag{5.3.28}$$

No soft terms for sleptons are generated at this order, because there is no direct coupling between lepton and messenger superfields. However, soft terms are generated at two loops. In particular, in addition to the usual flavour blind contribution from minimal gauge mediation (5.1.28), the slepton squared mass matrix includes the following term,

$$\begin{aligned}(\Delta m^2_{\tilde{\ell}})_{\alpha\beta} = \frac{\Lambda^2}{256\pi^4} \left(f^\dagger f\right)_{\alpha\beta} \Big[& 2\lambda^2_{\phi H} \partial_5^2 f_B \left(x_\Delta, 0, x_\Delta, 0, 1\right) \\ & + \lambda^2_{\phi\phi} (\partial_4 + \partial_5)^2 f_B \left(x_\Delta, 0, x_\Delta, 1, 1\right) \Big] \ . \end{aligned} \tag{5.3.29}$$

This is particularly interesting, because it connects flavour violation to the neutrino mass matrix. Indeed, as can be seen from eq. (2.2.22), the mass insertion parameter $\delta^{LL}_{\alpha\beta}$ ($\alpha \neq \beta$) involved in flavour-violating processes is proportional to the contribution $(\Delta m^2_{\tilde{\ell}})_{\alpha\beta}$ displayed here, which, using eq. (5.3.24), can be expressed as

$$(\Delta m^2_{\tilde{\ell}})_{\alpha\beta} = \frac{\Lambda^2}{64\pi^4}\frac{M^2_\Delta}{v_u^4}\left(m_\nu^\dagger m_\nu\right)_{\alpha\beta} f_{\tilde{\ell}}(t_\Theta, x_\Delta) \ , \tag{5.3.30}$$

with

$$f_{\tilde{\ell}}(x_\Delta, t_\Theta) = \left[2t_\Theta^2 \partial_5^2 f_B\left(x_\Delta, 0, x_\Delta, 0, 1\right) + t_\Theta^4 (\partial_4 + \partial_5)^2 f_B\left(x_\Delta, 0, x_\Delta, 1, 1\right)\right] \ . \tag{5.3.31}$$

This can be compared to the results obtained in ref. [323], where the scalar triplet plays the role of a messenger. When the scalar triplet is a messenger, it gives sev-

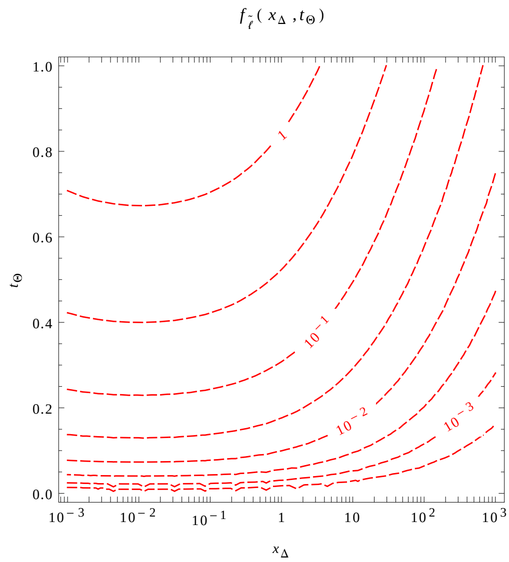


Figure 5.8: Loop function $f_{\tilde{\ell}}(t_\Theta, x_\Delta)$ appearing in the expression of slepton masses.

eral more contributions to slepton masses. However, keeping only the dominant term among these, the flavour structure is very similar to that of our scenario and can be conveniently related to that of the neutrino mass matrix. The main novelty here comes from the presence of two different mass scales. Indeed, one can play with the ratio $x_\Delta = M^2_\Delta / M^2_X$ and use the variations of the function $f_{\tilde{\ell}}(x_\Delta, t_\Theta)$, displayed in fig. 5.8, to control the size of flavour-changing rates in the lepton sector.

5.3.4 Additional comments

The $\mu - B_\mu$ problem

In both scenarios above, the μ- and B_μ-term cannot be generated because of the symmetry we imposed (or at least not with the minimal field content). We already showed that, if they are to be generated at one loop, it is not possible to have them both with the right order of magnitude. In the type I and type II seesaw, a way to generate μ and B_μ at two loop would be to introduce a new vectorlike pair of chiral superfields $(\chi, \bar{\chi})$, whose couplings break the Peccei-Quinn symmetry.

For instance, in the type I, one would add the following terms to the superpotential,

$$W_{\cancel{\text{sym}}} = y_{i\chi} N_i H_u \chi + \lambda_{i\chi} N_i \phi_u \chi + y_{i\bar{\chi}} N_i H_d \bar{\chi} + M_\chi \chi \bar{\chi} \ , \tag{5.3.32}$$

while, in the type II, one could introduce the following terms,

$$W_{\cancel{\text{sym}}} = \lambda_{H\chi} \bar{\Delta} H_u \chi + \lambda_{\phi\chi} \bar{\Delta} \phi_u \chi + \lambda_{H\bar{\chi}} \Delta \bar{\chi} H_d + M_\chi \chi \bar{\chi} \ . \tag{5.3.33}$$

In both case, using the results of the previous sections, the parameters μ and B_μ have the following form,

$$\mu = \frac{\Lambda}{256\pi^4} \sum C_i h_{i2}(x_a, x_\chi) \ , \qquad a = N_i, \Delta \ , \tag{5.3.34}$$

$$B_\mu = \frac{\Lambda^2}{256\pi^4} \sum C_i h_{i4}(x_a, x_\chi) \ , \tag{5.3.35}$$

where C_i is a product of couplings. One could hope that, playing with the ratios of the different mass scales, it would be possible to make B_μ small enough compared to μ^2, in contrast with models in which the messenger mass M_X is the only scale. Unfortunately, this turns out not to work numerically: the functions h_{i2} and h_{i4} typically satisfy the following relation, $h_{i2}^2(x_a, x_\chi) \lesssim h_{i4}(x_a, x_\chi)$, so that unless we fine tune the couplings, we get $\mu^2 \ll B_\mu$.

Thus, it does not seem possible to use the results of this section to solve the $\mu - B_\mu$ problem in a minimal way, which would rely only on the presence of different mass scales to adjust the Higgs mass parameters.

Leptogenesis

Leptogenesis can easily be implemented in the type I seesaw scenario, just as in its non-supersymmetric version. It is also possible to play with the ratio of the different right-handed neutrino masses to lower the scale of leptogenesis.

In the type II seesaw, supersymmetry breaking can mix the scalar components of Δ and $\bar{\Delta}$. This generates two nearly degenerate states, which could lead to soft leptogenesis [326, 327]. However, this does not work within the present framework. Then, the problem is the same as that already encountered in 3.2: leptogenesis requires an additional source of CP-violation, typically a heavier pair of triplets or right-handed neutrinos. One can fear that this would spoil the relation between the slepton squared mass matrix and the neutrino mass matrix.

An exception occurs in the following case: if we introduce two pairs of triplets $(\Delta_i, \bar{\Delta}_i)$, $i = 1, 2$, with couplings labelled $\lambda_{HH}^{(i)}$ (which can be chosen real) and $f^{(i)}$, and with nearly degenerate masses $M_{\Delta_i} = M_\Delta \pm \delta M_\Delta$, $\delta M_\Delta \ll M_\Delta$, then their total contribution to the slepton mass matrix, obtained through a strightforward generalization of eq. (5.3.29), reads

$$\Delta \tilde{m}_\ell^2 = \frac{\Lambda}{256\pi^4} \left[f^{(1)\dagger} f^{(1)} \lambda_{HH}^{(1)\,2} + f^{(2)\dagger} f^{(2)} \lambda_{HH}^{(2)\,2} + \left(f^{(2)\dagger} f^{(1)} + f^{(1)\dagger} f^{(2)} \right) \lambda_{HH}^{(1)} \lambda_{HH}^{(2)} \right] \\ \times \left[f_{\tilde{\ell}}(t_\Theta, x_\Delta) + \mathcal{O}(\delta x_\Delta) \right] \ . \tag{5.3.36}$$

This can be rewritten as

$$\Delta\tilde{m}_{\ell}^2 \simeq \frac{\Lambda^2}{256\pi^4} f'^{\dagger} f' \lambda'^2_{HH} f_{\tilde{\ell}}(t_\Theta, x_\Delta) \ , \tag{5.3.37}$$

where we defined the following effective couplings,

$$\lambda'_{HH} = \sqrt{\lambda_{HH}^{(1)\,2} + \lambda_{HH}^{(2)\,2}} \ , \qquad f'_{\alpha\beta} = \frac{\lambda_{HH}^{(1)} f_{\alpha\beta}^{(1)} + \lambda_{HH}^{(2)} f_{\alpha\beta}^{(2)}}{\sqrt{\lambda_{HH}^{(1)\,2} + \lambda_{HH}^{(2)\,2}}} \ . \tag{5.3.38}$$

The same effective parameters can be used to express the neutrino mass matrix,

$$\begin{aligned} (m_\nu)_{\alpha\beta} &= \left[\frac{1}{2}\lambda_{HH}^{(1)} f_{\alpha\beta}^{(1)} \frac{v_u^2}{M_\Delta} + \frac{1}{2}\lambda_{HH}^{(2)} f_{\alpha\beta}^{(2)} \frac{v_u^2}{M_\Delta}\right] \times \left[1 + \mathcal{O}\left(\frac{\delta M_\Delta}{M_\Delta}\right)\right] \\ &\simeq \frac{1}{2}\lambda'_{HH} f'_{\alpha\beta} \frac{v_u^2}{M_\Delta} \ . \end{aligned} \tag{5.3.39}$$

Thus, the relation between the slepton and neutrino mass matrices is preserved, and the presence of two quasi-degenerate triplets allows to achieve leptogenesis at a much lower scale than what was observed in chapter 3. However, the drawback of this scenario is that it seems difficult to justify the existence of two fields very close in mass but with different couplings (in particular, $f^{(1)}$ and $f^{(2)}$ should have different phases in order to have a non-vanishing CP violation).

5.3.5 Conclusion

In this chapter, we derived expressions for supersymmetry-breaking terms in a general framework of extended gauge mediation. To achieve this, we used a method based on that initially proposed in ref. [309]: first, we computed wavefunction renormalization of superfields in the exactly supersymmetric theory, and then we extracted soft terms by promoting the messenger mass parameter to a chiral superfield $\langle X \rangle = M_X + F_X \theta^2$. This method allows to derive first order terms in F_X/M_X^2, which are dominant if the supersymmetry-breaking scale is not too low. Finally, we applied these general results to specific scenarios, which include both extended gauge mediation and the seesaw mechanism.

The new ingredients here with respect to previous works on extended gauge mediation is the presence of heavy fields that are not messengers, and the existence of direct couplings between messengers and these heavy fields. In particular, this case is not covered by ref. [322] which gave very general methods to study extended gauge mediation with several different kinds of couplings between messengers and matter fields. Because of this, there are several different mass scales and soft terms therefore depend on non-trivial kinematic functions.

The supersymmetric seesaw mechanism fits naturally in this framework, and allows to describe in a common model gauge mediation, neutrino masses and leptogenesis. Direct couplings between messengers and other fields generate A-terms at one-loop and new contributions to scalar masses, which give rise to flavour violation that may be in conflict with experimental bounds. Ref. [314] introduced a solution to this problem, that makes flavour violation minimal. Applying it in the context of the seesaw mechanism, this generates soft terms for the leptons which can be expressed in terms of the same couplings as the neutrino mass matrix. In the type I seesaw, unfortunately, the combination of couplings appearing in the slepton and neutrino mass matrices are different, and predictions on flavour violation depend strongly on the flavour model used to describe neutrino Yukawa couplings. In the type II seesaw, on the other hand, the flavour-violating contribution to the squared mass matrix of sleptons is simply proportional to $m_\nu^\dagger m_\nu$.

However, in the type I and type II seesaw, the fact that the existence of several different mass scales allow to adjust the magnitude of soft terms is not enough to solve the μ/B_μ problem of gauge mediation in a minimal way. If μ and B_μ are generated through this mechanism, B_μ is always too large compared to μ^2. Thus, it seems that the μ/B_μ problem needs a separate solution.

Conclusion

Ce n'est qu'en essayant
continuellement que l'on finit par
réussir. Autrement dit : plus ça rate,
plus on a de chances que ça marche.

Proverbe shadok

The Standard Model contains every elementary particle that has ever been seen experimentally. With the discovery of the Higgs boson, the converse is now also true: every particle predicted by the Standard Model has been discovered. However, a complete theory of nature should, among other things, account for neutrino masses, dark matter and dark energy and provide an explanation for the origin of the matter-antimatter asymmetry of the Universe. Thus, the Standard Model has to be embedded into a bigger picture. In the lepton sector, low energy phenomena could provide an interesting probe of new physics. For instance, the observation of neutrinoless double beta decay would be an unmistakable sign that neutrinos are Majorana particles, while flavour violating processes in the sector of charged leptons are so much constrained in the Standard Model that their observation would be a very clear signal of new physics.

A good example of the links that can exist between high energy and low energy physics is leptogenesis, that provides a common origin for neutrino masses and the matter-antimatter asymmetry of the Universe. Leptogenesis is in general difficult to probe directly due to the large mass of particles involved, but it can be related to properties of neutrinos: for instance, any indication that neutrinos are Majorana fermions would strongly advocate for such scenarios. In chapter 3, we showed the importance of flavour effects in the context of leptogenesis with a scalar triplet, even in a temperature regime in which the Yukawa couplings of leptons do not allow to distinguish them. With respect to that matter, leptogenesis with a scalar triplet differs from scenarios involving hierarchical right-handed neutrinos. These flavour effects significantly enlarge the parameter space available for successful leptogenesis. We also studied a model in which the CP violation responsible for the lepton asymmetry can be expressed straightforwardly in terms of neutrino parameters, which makes it very predictive.

In chapter 4, we saw that supersymmetry with broken R-parity can account for neutrino masses without resorting to new heavy states. In such a case, we showed how the fermionic partner of a pseudo-Goldstone scalar plays the role of a sterile neutrino that can explain the Gallium and reactor anomalies. Future neutrino experiments will shed new light on the anomalies and explore the parameter space allowed by this model. This illustrates how supersymmetry, usually considered from the high energy perspective, could manifest itself in the neutrino sector.

If supersymmetry is realized in nature, it has to be broken. Supersymmetry breaking in itself is not easy to model, but we know that it should happen in a separate sector, and be transmitted to the visible sector by some mean. Gauge mediated supersymmetry breaking is a very predictive scenario that allows to compute supersymmetry-breaking term from a small number of new parameters, but it is in tension with the measured Higgs boson mass, slightly larger than what minimal scenarios would predict. In chapter 5, we studied models of extended gauge mediation, that provide a solution to this problem. We derived formulae for the soft terms in a general framework involving messengers with direct couplings to both light and heavy fields. The extension of gauge mediation has a cost, because it requires to introduce new couplings, which can induce uncontrolled flavour violation. However, it is possible to align the flavour structure of the new couplings with that of the Standard Model Yukawa. The general formulae can then be applied to seesaw scenarios, where they lead to prediction for flavour-changing observables in the lepton sector. In particular, in the type II seesaw, the squared mass matrix of sneutrino has the same flavour structure as the neutrino mass matrix.

There is strong evidence of physical phenomena beyond the scope of the Standard Model, but it is still unclear what the new physics is and where it will show up. In particular, supersymmetric theories have interesting features but are more and more constrained by the non-observation of the superpartners of Standard Model fields. The next run of LHC at 14 TeV may unblock the situation if new sates are discovered. New physics could also manifest itself at low energy and it is therefore of great importance to distinguish the physical manifestations of the various possible scenarios. Precision measurements could then provide valuable information, and especially if new states are beyond the reach of colliders.

Appendix

A Boltzmann equations

A.1 Classical derivation

The equilibrium phase-space distribution function for a particle a is given by

$$\rho_a(\vec{p}) \;=\; \frac{1}{e^{\beta(E-\mu_a)} \pm 1} \;, \tag{A.1}$$

in which the plus sign applies for bosons and the minus sign to fermions. The density of particles a is related to their phase-space distribution (which does not necessarily have its equilibrium form) through

$$n_a = g_a \int \frac{d^3p}{(2\pi)^3} \rho_a(\vec{p}) \;, \tag{A.2}$$

where g_a is the number of degrees of freedom. Then, the Boltzmann equation describing the evolution of the density n_a stems from [328]

$$\frac{dn_a}{dt} + 3Hn_a = \sum \left\{ \gamma(b_1 \dots b_n \to a\, a_1 \dots a_m) - \gamma(a\, a_1 \dots a_n \to b_1 \dots b_n) \right\} \;, \tag{A.3}$$

which describes homogeneously distributed particles in an expanding universe. The first term on the right-hand side refers to reactions that create a particle a, and the second one to their counterparts that destroy a particle a. Formally, the space-time density of reaction for a process $a_1 \dots a_m \to b_1 \dots b_n$ reads

$$\begin{aligned} \gamma(a_1 \dots a_m \to b_1 \dots b_n) \;=& \int \prod_{i=1}^{m} \frac{d^3p_i}{(2\pi)^3 2\omega_{\vec{p}_i}} \rho_{a_i}(\vec{p}_i) \prod_{j=1}^{n} \frac{d^3q_j}{(2\pi)^3 2\omega_{\vec{q}_j}} \left[1 \pm \rho_{b_j}(\vec{q}_j)\right] \\ & |\mathcal{M}|^2 \,(2\pi)^4 \delta^{(4)}(p_1 + \dots + p_m - q_1 - \dots - q_n) \;, \end{aligned} \tag{A.4}$$

where $|\mathcal{M}|^2$ is the squared matrix element for this process, summed over the internal degrees of freedom of the initial and final states. To obtain Boltzmann equations, some approximations are performed. First, we neglect Bose enhancement and Pauli blocking factors $1 \pm \rho_{b_j}(\vec{q}_j)$ for the final states particles. Then, the Bose-Einstein and Fermi-Dirac distributions (A.1) are approximated by the Maxwell-Boltzmann distribution,

$$\rho_a(\vec{p}) \;=\; e^{-\beta(E-\mu_a)} \;, \tag{A.5}$$

We define the thermal average of the space-time density of reaction, which, for the abovementioned process, reads

$$\gamma^{\rm eq}(a_1 \dots a_m \to b_1 \dots b_n) \;=\; \int \prod_{i=1}^{m} \frac{d^3 p_i}{(2\pi)^3 2\omega_{\vec{p}_i}} \rho^{\rm eq}_{a_i}(\vec{p}_i) \prod_{j=1}^{n} \frac{d^3 q_j}{(2\pi)^3 2\omega_{\vec{q}_j}}$$
$$|\mathcal{M}|^2 \, (2\pi)^4 \delta^{(4)}(p_1 + \dots + p_m - q_1 - \dots - q_n) \;, \qquad \text{(A.6)}$$

where this time the $\rho^{\rm eq}$'s are the equilibrium phase-space distribution function at zero chemical potential,

$$\rho^{\rm eq}(\vec{p}) \;=\; e^{-\beta E} \;. \qquad \text{(A.7)}$$

Finally, we approximate eq. (A.3) by

$$\frac{dn_a}{dt} + 3Hn_a = \sum \left\{ \frac{n_{b_1}}{n^{\rm eq}_{b_1}} \dots \frac{n_{b_n}}{n^{\rm eq}_{b_n}} \gamma^{\rm eq}(b_1 \dots b_n \to a \, a_1 \dots a_m) \right.$$
$$\left. - \frac{n_a}{n^{\rm eq}_a} \frac{n_{a_1}}{n^{\rm eq}_{a_1}} \dots \frac{n_{a_m}}{n^{\rm eq}_{a_m}} \gamma^{\rm eq}(a \, a_1 \dots a_n \to b_1 \dots b_n) \right\} \;. \qquad \text{(A.8)}$$

From now on, we drop the superscript eq for the space-time densities of reactions, so that $\gamma(a_1 \dots a_m \to b_1 \dots b_n)$ will refer to the thermally averaged quantity defined in eq. (A.6).

When considering small asymmetries, it is convenient to linearize the Boltzmann equation for $\Delta n_a = n_a - n_{a^c}$. This is done as follows. The particles such as leptons and Higgs doublets remain close to equilibrium, thanks to fast gauge interactions, which ensure in particular $\mu_a = -\mu_{a^c}$. Then, performing the expansion of the density n_a in μ_a, we get

$$n_a(T,\mu) = \frac{g_a}{\pi^2} T^3 \begin{cases} \zeta(3) + \dfrac{\mu_a}{T}\zeta(2) + \mathcal{O}(\mu_a^2) & \text{for bosons ,} \\ \dfrac{3}{4}\zeta(3) + \dfrac{\mu_a}{T}\dfrac{\zeta(2)}{2} + \mathcal{O}(\mu_a^2) & \text{for fermions ,} \end{cases} \qquad \text{(A.9)}$$

so that the sum of the densities of a and a^c is then given by

$$n_a + n_a^c = 2n_a^{\rm eq} + \mathcal{O}\left(\mu_a^2\right) \;, \qquad \text{(A.10)}$$

where $n_a^{\rm eq}$ is the equilibrium density for $\mu_a = 0$, whereas the asymmetry is

$$\Delta n_a = n_a - n_a^c = \frac{g_a \zeta(2)}{\pi^2} \mu_a T^2 \begin{cases} 1 + \mathcal{O}(\mu_a^2) & \text{for bosons ,} \\ \dfrac{1}{2} + \mathcal{O}(\mu_a^2) & \text{for fermions .} \end{cases} \qquad \text{(A.11)}$$

For a CP-conserving process, i.e. satisfying

$$\begin{aligned} \gamma(a_1 \dots a_n \to b_1 \dots b_n) &= \gamma(a_1^c \dots a_n^c \to b_1^c \dots b_n^c) \\ &= \gamma(b_1 \dots b_n \to a_1 \dots a_m) \\ &= \gamma^{a_1 \dots a_n}_{b_1 \dots b_n} \;, \end{aligned} \qquad \text{(A.12)}$$

using eqs. (A.10) and (A.11), we see that at first order,

$$
\begin{aligned}
&\frac{n_{a_1}}{n^{\rm eq}_{a_1}}\dots\frac{n_{a_n}}{n^{\rm eq}_{a_n}}\gamma(a_1\dots a_m\to b_1\dots b_n)-\frac{n_{a^c_1}}{n^{\rm eq}_{a_1}}\dots\frac{n_{a^c_n}}{n^{\rm eq}_{a_n}}\gamma(a^c_1\dots a^c_m\to b^c_1\dots b^c_n)\\
&\qquad=\left(\frac{\Delta n_{a_1}}{n^{\rm eq}_{a_1}}+\dots+\frac{\Delta n_{a_n}}{n^{\rm eq}_{a_n}}\right)\gamma^{a_1\dots a_m}_{b_1\dots b_n}\ .
\end{aligned}
\tag{A.13}
$$

In leptogenesis, it is convenient to define the parameter $z = M/T$, where M is the mass of the decaying particle, and to use densities and asymmetries normalized over entropy, $Y_a = n_a/s$ and $\Delta_a = Y_a - Y_{a^c}$, so that the linearized Boltzmann equation for Δ_a is the following,

$$
sHz\frac{d\Delta_a}{dz}=S_{\not{CP}}+\sum\left\{\left(\frac{\Delta_{b_1}}{Y^{\rm eq}_{b_1}}+\dots+\frac{\Delta_{b_n}}{Y^{\rm eq}_{b_n}}-\frac{\Delta_a}{Y^{\rm eq}_a}-\frac{\Delta_{a_1}}{Y^{\rm eq}_{a_1}}\dots-\frac{\Delta_{a_m}}{Y^{\rm eq}_{a_m}}\right)\gamma^{a\,a_1\dots a_n}_{b_1\dots b_n}\right\}\ ,
\tag{A.14}
$$

where $S_{\not{CP}}$ refers to CP-violating source terms, which will be dealt with in A.3.

A.2 Decays and scattering rates

Before going further, we display the typical form of the space-time densities of reactions appearing in Boltzmann equations.

General formulae

When considering a 2-body decay such as $a \to b_1 b_2$, eq. (A.6) reduces to

$$
\gamma^a_{b_1 b_1} = sY^{\rm eq}_a\,\frac{K_1(z)}{K_2(z)}\,\Gamma(a\to b_1 b_2)\ .
\tag{A.15}
$$

In the leptogenesis scenarios studied here, we modified slightly this definition when considering the total space-time density of decays for the triplets Δ and Δ^c, namely

$$
\gamma^\Delta_D = s\Sigma^{\rm eq}_\Delta\,\frac{K_1(z)}{K_2(z)}\,\Gamma_\Delta\ ,
\tag{A.16}
$$

where $\Sigma_\Delta \equiv (n_\Delta + n_{\Delta^c})/s$ is the comoving number density of triplets and antitriplets, $K_{1,2}(z)$ are modified Bessel functions of the second kind, $z \equiv M_\Delta/T$ and Γ_Δ is the triplet decay width.

For $2 \to 2$ scatterings, eq. (A.6) becomes

$$
\gamma(a_1+a_2\to b_1+b_2)\ =\ \frac{T}{64\pi^4}\int_{s_{\rm min}}^\infty ds\, s^{1/2}\hat\sigma(s)\,K_1\left(\frac{\sqrt{s}}{T}\right)\ ,
\tag{A.17}
$$

where the reduced cross section $\hat\sigma$ is defined in terms of the usual dimensionful cross-section as

$$
\hat\sigma(s) = 2s\,\lambda(1,\, m^2_{a_1}/s,\, m^2_{a_2}/s)\,\sigma(s)\ ,
\tag{A.18}
$$

$$
\lambda(a,\,b,\,c) = (a-b-c)^2 - 4bc\ ,
\tag{A.19}
$$

and is summed over the internal degrees of freedom of initial and final particles.

Substraction of on-shell intermediate states

A well-known problem is the possible redundancy due to the contribution of on-shell intermediate states to scatterings [329]. For instance, an s-channel scattering such as $\ell\ell \to H^c H^c$ involves a contribution with an on-shell intermediate triplet, which is nothing but an inverse decay $\ell\ell \to \Delta^c$ followed by a decay $\Delta^c \to H^c H^c$. Since these are already taken into account in Boltzmann equations by terms describing decays and inverse decays of the triplet, if we include scatterings without further care, they will be counted twice.

Therefore, when computing $2 \to 2$ scattering rates, one must take care to properly subtract the contribution of on-shell intermediate particles. When the resonance occurs in the s-channel, one can compute the subtracted rate by taking away the resonant part from the squared propagator in the narrow-width approximation [330]:

$$|D|^2 \;\to\; |D|^2 - \frac{\pi}{M^3\Gamma}\delta\left(\frac{s-M^2}{M^2}\right), \tag{A.20}$$

where M and Γ are the mass and width of the intermediate particle, and D is the propagator of the intermediate state in the Breit-Wigner form,

$$D \;=\; \frac{1}{s - M^2 + iM\Gamma}\;. \tag{A.21}$$

Similarly, when computing $\gamma(\ell\Delta^c \to \ell\Delta^c)$, which occurs in both scenarios presented in 3.2 and 3.3, one has to subtract the contribution of real intermediate leptons in the u-channel, corresponding to the two processes $\Delta^c \to \ell\ell$ and $\ell\ell \to \Delta^c$. Following [187], we perform the subtraction in the following way:

$$|D|^2 \;\to\; |D|^2 - \frac{\pi}{E^3\Gamma_{\rm th}}\delta\left(\frac{u}{E}\right), \tag{A.22}$$

where E is the energy of the intermediate lepton, $\Gamma_{\rm th}$ its thermal width due to the interactions with the dense plasma, and the propagator of the intermediate lepton is

$$D \;=\; \frac{1}{u + iE\Gamma_{\rm th}}\;. \tag{A.23}$$

As in [187], we use the following representation of $\delta(x)$ in numerical computations,

$$\delta(x) \;=\; \frac{2}{\pi}\frac{\epsilon^3}{(x^2+\epsilon^2)^2}\;, \tag{A.24}$$

where ϵ is a small number. In the limit of small width, which we assume to be valid, one can simply set $\epsilon = \Gamma/M$ for a resonance in the s-channel with an intermediate particle of mass M and width Γ, and $E\Gamma_{\rm th} = \epsilon M_\Delta^2$ for a resonance in the u-channel, with any small value for ϵ. This amounts to perform the replacements

$$\frac{1}{\left(s-M^2\right)^2 + M^2\Gamma^2} \longrightarrow \frac{\left(s-M^2\right)^2 - M^2\Gamma^2}{\left[\left(s-M^2\right)^2 + M^2\Gamma^2\right]^2}\;, \tag{A.25}$$

$$\frac{1}{u^2 + M^2\Gamma_{\rm th}^2} \longrightarrow \frac{u^2 - M^2\Gamma_{\rm th}^2}{\left[u^2 + M^2\Gamma_{\rm th}^2\right]^2}\;. \tag{A.26}$$

With this choice, we can compute the subtracted rates for the various $2 \to 2$ scatterings.

A.3 CP violating terms

In A.1, we showed the derivation of CP-conserving collision terms in classical Boltzmann equations, but let aside the CP-violating source terms.

Let us consider the case of the general leptogenesis scenario involving scalar triplets 3.2. In this scenario, the CP asymmetry in the decay of Δ is due to an interference between the tree-level diagram and the diagram with a Higgs loop, displayed in fig. 3.3. First, we derive explicitly a Boltzmann equation for a very simplified model including only decays and inverse decays of the triplet. We neglect flavour effects and spectator processes. Using CPT invariance, the relevant scattering rates satisfy

$$\gamma(\Delta^c \to \ell\ell) = \gamma(\ell^c\ell^c \to \Delta) = \frac{B_\ell}{2}\gamma_D + \frac{1}{4}\epsilon^\Delta\gamma_D \ , \tag{A.27}$$

$$\gamma(\Delta \to \ell^c\ell^c) = \gamma(\ell\ell \to \Delta^c) = \frac{B_\ell}{2}\gamma_D - \frac{1}{4}\epsilon^\Delta\gamma_D \ . \tag{A.28}$$

the Boltzmann equation for the lepton asymmetry derives from

$$\begin{aligned} sHz\frac{d\Delta_\ell}{dz} &= 2\left(\frac{Y_{\Delta^c}}{Y_\Delta^{\rm eq}} + \frac{Y_{\ell^c}^2}{Y_\ell^{\rm eq2}}\right)\left(\frac{B_\ell}{2}\gamma_D + \frac{1}{4}\epsilon^\Delta\gamma_D\right) \\ &\quad - 2\left(\frac{Y_\Delta}{Y_\Delta^{\rm eq}} + \frac{Y_\ell^2}{Y_\ell^{\rm eq2}}\right)\left(\frac{B_\ell}{2}\gamma_D - \frac{1}{4}\epsilon^\Delta\gamma_D\right) \ , \end{aligned} \tag{A.29}$$

which, after linearizing, gives for the source and washout terms

$$sHz\frac{d\Delta_\ell}{dz} = \epsilon^\Delta\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} + 1\right)\gamma_D - 2B_\ell\left(\frac{\Delta_\Delta}{\Sigma_\Delta^{\rm eq}} + \frac{\Delta_\ell}{Y_\ell^{\rm eq}}\right)\gamma_D \ . \tag{A.30}$$

This cannot possibly be correct, because it gives rise to an asymmetry even when the system is completely in equilibrium, with $\Sigma_\Delta = \Sigma_\Delta^{\rm eq}$ and $\Delta_\Delta = \Delta_\ell = 0$, which is in contradiction with Sakharov's third condition.

This happens because we neglected a source of CP violation occurring at the same order [331]. To understand the problem, we include now the scattering $\ell\ell \to H^cH^c$. In this scenario, it can be mediated by the exchange of a triplet in the s-channel, or by the effective operator with coupling κ. We focus on the interference between the two amplitudes in the effective cross-section, which reads

$$\left(\hat\sigma_{H^cH^c}^{\ell\ell}\right)^{\mathcal{I}} = \frac{3x}{16\pi}\left\{\frac{\Re\left[\mu^*\,{\rm tr}(f^\dagger\kappa)\right]}{\Lambda}\frac{1-x}{(1-x)^2+\epsilon^2} + \frac{\Im\left[\mu^*{\rm tr}(f^\dagger\kappa)\right]}{\Lambda}\frac{\epsilon}{(1-x)^2+\epsilon^2}\right\} \ , \tag{A.31}$$

where we defined the dimensionless parameter $x = s/M_\Delta^2$, whereas, for the CP-conjugate process $\ell^c\ell^c \to HH$, this becomes

$$\left(\hat\sigma_{HH}^{\ell^c\ell^c}\right)^{\mathcal{I}} = \frac{3x}{16\pi}\left\{\frac{\Re\left[\mu\,{\rm tr}(f\kappa^\dagger)\right]}{\Lambda}\frac{1-x}{(1-x)^2+\epsilon^2} - \frac{\Im\left[\mu^*{\rm tr}(f^\dagger\kappa)\right]}{\Lambda}\frac{\epsilon}{(1-x)^2+\epsilon^2}\right\} \ . \tag{A.32}$$

The sign of the second term is different in the two cases, which means that this scattering is actually another source of CP violation, which has to be included in the Boltzmann equations. In terms of space-time densities of reaction,

$$\begin{aligned}&\gamma'(\ell\ell\to H^cH^c)-\gamma'(\ell^c\ell^c\to HH)\\&\qquad=\frac{M_\Delta^4}{64\pi^4 z}\int dx\sqrt{x}K_1(z\sqrt{x})\left[\hat\sigma(\ell\ell\to H^cH^c)-\hat\sigma(\ell^c\ell^c\to HH)\right]\ ,\end{aligned}\tag{A.33}$$

where the prime means that we have subtracted the contribution with an on-shell intermediate state $\ell\ell\to\Delta\to H^cH^c$, which is already accounted for in decays and inverse decays, as mentioned in A.2. In the narrow-width approximation, we can perform the following replacement,

$$\frac{\epsilon}{(1-x)^2+\epsilon^2}\to\pi\,\delta(x-1)\ ,\tag{A.34}$$

so that the previous term becomes

$$\begin{aligned}&\gamma'(\ell\ell\to H^cH^c)-\gamma'(\ell^c\ell^c\to HH)\\&\quad=\frac{M_\Delta^4}{64\pi^4 z}\int dx\ x\sqrt{x}\ K_1(z\sqrt{x})\times 2\times\frac{3x}{16\pi M_\Delta^2}\frac{\Im\left[\mu^*\mathrm{tr}(f^\dagger\kappa)\right]}{\Lambda}\times\pi\,\delta(x-1)\\&\quad=\frac{3M_\Delta^4}{512\pi^4 z}K_1(z)\,\frac{\Im\left[\mu^*\mathrm{tr}(f^\dagger\kappa)\right]}{\Lambda}.\end{aligned}\tag{A.35}$$

We can compare this to the term accounting for the CP asymmetry in the decay of the triplet, which reads

$$\begin{aligned}&\gamma(\Delta^c\to\ell\ell)-\gamma(\Delta\to\ell^c\ell^c)\\&\quad=\frac{1}{2}\epsilon^\Delta\gamma_D=\frac{1}{2}\times\frac{1}{8\pi\Lambda}\frac{\Im\left[\mu^*\mathrm{tr}(f^\dagger\kappa)\right]}{\lambda_\ell^2+\lambda_H^2}\times\frac{1}{32\pi}(\lambda_\ell^2+\lambda_H^2)M_\Delta\times s\,\Sigma_\Delta^{\mathrm{eq}}\frac{K_1(z)}{K_2(z)}\ .\end{aligned}\tag{A.36}$$

Using the explicit expression of the thermal distribution,

$$s\,\Sigma_\Delta^{\mathrm{eq}}=n_\Delta^{\mathrm{eq}}+n_{\Delta^c}^{\mathrm{eq}}=\frac{3M_\Delta^3}{\pi^2 z}K_2(z)\ ,\tag{A.37}$$

this becomes

$$\gamma(\Delta^c\to\ell\ell)-\gamma(\Delta\to\ell^c\ell^c)=\frac{3M_\Delta^4}{512\pi^4 z}K_1(z)\frac{\Im\left[\mu^*\mathrm{tr}(f^\dagger\kappa)\right]}{\Lambda}\ .\tag{A.38}$$

Thus, $\gamma'(\ell\ell\to H^cH^c)-\gamma'(\ell^c\ell^c\to HH)=\epsilon^\Delta\gamma_D$, as can be seen by comparing eq. (A.35) and eq. (A.38). Then, using again the CPT invariance, we can express the space-time densities of scatterings as

$$\gamma'(\ell\ell\to H^cH^c)=\gamma'(HH\to\ell^c\ell^c)=\gamma'^{\ell\ell}_{H^cH^c}+\frac{1}{2}\epsilon^\Delta\gamma_D\ ,\tag{A.39}$$

$$\gamma'(\ell^c\ell^c\to HH)=\gamma'(H^cH^c\to\ell\ell)=\gamma'^{\ell\ell}_{H^cH^c}-\frac{1}{2}\epsilon^\Delta\gamma_D\ ,\tag{A.40}$$

where we separated for each process the CP-conserving part and the CP-violating part. Using this, we write down explicitly the derivation of the corresponding terms appearing in Boltzmann equations,

$$sHz\frac{d\Delta_\ell}{dz} = \epsilon^\Delta \left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\text{eq}}} + 1\right)\gamma_D - 2B_\ell \left(\frac{\Delta_\Delta}{\Sigma_\Delta^{\text{eq}}} + \frac{\Delta_\ell}{Y_\ell^{\text{eq}}}\right)\gamma_D$$
$$+ 2\left[\left(\frac{Y_{H^c}^2}{Y_H^{\text{eq}2}} + \frac{Y_{\ell^c}^2}{Y_\ell^{\text{eq}2}}\right)\left(\gamma'^{\ell\ell}_{H^cH^c} - \frac{1}{2}\epsilon^\Delta\gamma_D\right) - \left(\frac{Y_H^2}{Y_H^{\text{eq}2}} + \frac{Y_\ell^2}{Y_\ell^{\text{eq}2}}\right)\left(\gamma'^{\ell\ell}_{H^cH^c} + \frac{1}{2}\epsilon^\Delta\gamma_D\right)\right] . \tag{A.41}$$

Again, this can be simplified by linearizing and using the fact that because of gauge scatterings, the Higgs and lepton densities are maintained very close to equilibrium,

$$sHz\frac{d\Delta_\ell}{dz} = \epsilon^\Delta \left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\text{eq}}} - 1\right)\gamma_D - 2B_\ell \left(\frac{\Delta_\Delta}{\Sigma_\Delta^{\text{eq}}} + \frac{\Delta_\ell}{Y_\ell^{\text{eq}}}\right)\gamma_D - 4\left(\frac{\Delta_H}{Y_H^{\text{eq}}} + \frac{\Delta_\ell}{Y_\ell^{\text{eq}}}\right)\gamma'^{\ell\ell}_{H^cH^c} . \tag{A.42}$$

The previous problem is solved because the right-hand side contains precisely what is needed to ensure that the asymmetry remains zero when everything is in equilibrium. This means that even if the off-shell scattering rate $\gamma'^{\ell\ell}_{H^cH^c}$ is small enough to be neglected, its CP-violating part should always be included in Boltzmann equation, which is usually done implicitly.

This result is actually a consequence of CPT invariance and is more general than this. At any order in perturbation theory, the sum of all processes creating a given state must be CP-conserving, which implies for us

$$\sum_a |\mathcal{M}(a \to \ell\ell)|^2 = \sum_a |\mathcal{M}(a^c \to \ell^c\ell^c)|^2 \ , \tag{A.43}$$

where $a = \Delta^c$, H^cH^c,... can be any multi-particle state.

A.4 The closed time-path formalism

To derive the the flavour-covariant Boltzmann equation for the density matrix, we used the Closed Time-Path (CTP) formalism [332–335]. The CTP formalism is a powerful tool that allows to describe out-of-equilibrium quantum phenomena. However, since we worked in the Boltzmann approximation, we used it principally to read the flavour structure of the source and washout terms, which we reported then in the classical Boltzmann equation.

The framework

The CTP formalism relies on the introduction of an oriented closed time-path, $\mathcal{C}$, that goes from 0 to ∞ and back (fig. a). The upper branch, $\mathcal{C}_+$, is time-ordered, whereas the lower branch, $\mathcal{C}_-$, is anti-time-ordered. We define Green's functions $\tilde{G}(x, y)$ that are ordered following the contour. This means that, since both x^0 and y^0 can lie either

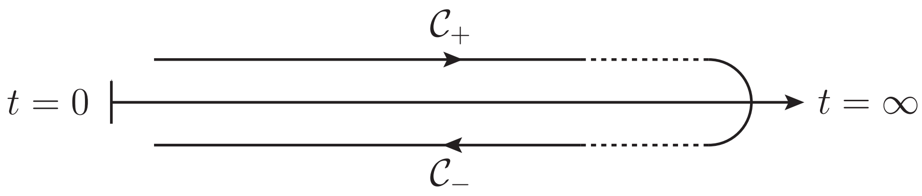


Figure a: Schematical representation of the time-path $\mathcal{C}$.

on the upper branch or on the lower branch, there are four possible orderings. $\tilde{G}(x, y)$ can therefore be defined as a 2×2 matrix

$$\tilde{G}(x,y) = \begin{pmatrix} G^t(x,y) & \pm G^<(x,y) \\ G^>(x,y) & -G^{\bar{t}}(x,y) \end{pmatrix} , \tag{A.44}$$

where the element $G^<$ comes with a plus sign for bosons and a minus sign for fermions. This is known as the doubling of the degrees of freedoms [336]. G^t, that refers to the case where both x^0 and y^0 lie on the upper branch, is the usual time-ordered Green's function, whereas $G^{\bar{t}}$, that refers to the case where x^0 and y^0 lie on the lower branch, is anti-time-ordered. The explicit expression of the various components of the left-handed lepton doublet Green's functions are the following,

$$G^>_{\alpha\beta}(x,y) = -i\langle \ell_\alpha(x)\bar{\ell}_\beta(y)\rangle , \tag{A.45}$$
$$G^<_{\alpha\beta}(x,y) = i\langle \bar{\ell}_\beta(y)\ell_\alpha(x)\rangle , \tag{A.46}$$
$$G^t_{\alpha\beta}(x,y) = \theta(x^0-y^0)G^>_{\alpha\beta}(x,y) + \theta(y^0-x^0)G^<_{\alpha\beta}(x,y) , \tag{A.47}$$
$$G^{\bar{t}}_{\alpha\beta}(x,y) = \theta(y^0-x^0)G^>_{\alpha\beta}(x,y) + \theta(x^0-y^0)G^<_{\alpha\beta}(x,y) , \tag{A.48}$$

where the brackets indicate that we take the average over all available states of the system and α, β are lepton flavour indices, whereas for a scalar field $\phi(x)$ (representing a Higgs doublet or a scalar triplet),

$$G^>_{\phi}(x,y) = -i\langle \phi(x)\phi^\dagger(y)\rangle , \tag{A.49}$$
$$G^<_{\phi}(x,y) = -i\langle \phi^\dagger(y)\phi(x)\rangle , \tag{A.50}$$
$$G^t_{\phi}(x,y) = \theta(x^0-y^0)G^>_{\phi}(x,y) + \theta(y^0-x^0)G^<_{\phi}(x,y) , \tag{A.51}$$
$$G^{\bar{t}}_{\phi}(x,y) = \theta(y^0-x^0)G^>_{\phi}(x,y) + \theta(x^0-y^0)G^<_{\phi}(x,y) . \tag{A.52}$$

Explicit expression of the Green's functions

In order to express the Green's functions more explicitly, we write the Dirac field as

$$\psi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3\sqrt{2\omega_{\vec{p}}}} \sum_s \left(u(p,s)b(\vec{p},s)e^{-ip\cdot x} + v(p,s)d^\dagger(\vec{p},s)e^{ip\cdot x} \right) , \tag{A.53}$$

where $\omega_{\vec{p}} = \sqrt{\vec{p}^{\,2} + m^2}$. The phase-space distribution functions of lepton and antilepton doublets $\rho_{\alpha\beta}(\vec{p})$ and $\bar{\rho}_{\alpha\beta}(\vec{p})$ are matrices in flavour space, defined by

$$\langle b^\dagger_\alpha(\vec{p})b_\beta(\vec{p'})\rangle = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{p'})\rho_{\alpha\beta}(\vec{p}) , \tag{A.54}$$
$$\langle d^\dagger_\beta(\vec{p})d_\alpha(\vec{p'})\rangle = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{p'})\bar{\rho}_{\alpha\beta}(\vec{p}) . \tag{A.55}$$

The reversed order of the flavour indices α and β in the definition of $\bar{\rho}_{\alpha\beta}(\vec{p})$ ensures that the distribution functions of lepton and antilepton doublets transform in the same way under a rotation in flavour space, $\ell \to V^*\ell$,

$$\rho \;\to\; V\rho V^\dagger \;, \tag{A.56}$$

$$\bar{\rho} \;\to\; V\bar{\rho} V^\dagger \;, \tag{A.57}$$

which is essential for flavour covariance, since the density matrix of lepton asymmetry $(\Delta_\ell)_{\alpha\beta}$ will be expressed in terms of $\rho_{\alpha\beta} - \bar{\rho}_{\alpha\beta}$. Similarly, the phase-space distribution functions of a charged scalar ϕ and of its antiparticle are defined by

$$\langle a_\phi^\dagger(\vec{p}) a_\phi(\vec{p'})\rangle \;=\; (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p'})\rho_\phi(\vec{p})\,, \tag{A.58}$$

$$\langle b_\phi^\dagger(\vec{p}) b_\phi(\vec{p'})\rangle \;=\; (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p'})\bar{\rho}_\phi(\vec{p})\,, \tag{A.59}$$

where $a_\phi(\vec{p})$ and $b_\phi^\dagger(\vec{p})$ are the annihilation and creation operators appearing in the definition of the free charged scalar field,

$$\phi(x) \;=\; \int \frac{d^3\vec{p}}{(2\pi)^3\sqrt{2\omega_{\vec{p}}}} \left(a_\phi(\vec{p}) e^{-ip\cdot x} + b_\phi^\dagger(\vec{p}) e^{ip\cdot x} \right). \tag{A.60}$$

With these definitions, the Green's functions $G^>$ and $G^<$ for left-handed lepton doublets can be written as (neglecting lepton masses)

$$iG^>_{\beta\alpha}(x,y) \;=\; P_L \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec{p}}} \; \not{p} \left\{ \left[\delta_{\alpha\beta} - \rho_{\alpha\beta}(\vec{p})\right] e^{-ip\cdot(x-y)} + \bar{\rho}_{\alpha\beta}(\vec{p})\, e^{ip\cdot(x-y)} \right\} P_R \;, \tag{A.61}$$

$$iG^<_{\beta\alpha}(x,y) \;=\; -P_L \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec{p}}} \; \not{p} \left\{ \rho_{\alpha\beta}(\vec{p})\, e^{-ip\cdot(x-y)} + \left[\delta_{\alpha\beta} - \bar{\rho}_{\alpha\beta}(\vec{p})\right] e^{ip\cdot(x-y)} \right\} P_R \;. \tag{A.62}$$

The time-ordered Green's function can be deduced from the previous expressions,

$$\begin{aligned} iG^t_{\beta\alpha}(x,y) \;=&\; \delta_{\alpha\beta}\, P_L S(x,y) P_R \\ &- P_L \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec{p}}} \; \not{p} \left\{ \rho_{\alpha\beta}(\vec{p})\, e^{-ip\cdot(x-y)} - \bar{\rho}_{\alpha\beta}(\vec{p})\, e^{ip\cdot(x-y)} \right\} P_R \;, \end{aligned} \tag{A.63}$$

as well as the anti-time-ordered one,

$$\begin{aligned} iG^{\bar{t}}_{\beta\alpha}(x,y) \;=&\; -\delta_{\alpha\beta}\, P_L S(y,x) P_R \\ + \; P_L \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec{p}}} \; \not{p} & \left\{ \left[\delta_{\alpha\beta} - \rho_{\alpha\beta}(\vec{p})\right] \; e^{-ip\cdot(x-y)} - \left[\delta_{\alpha\beta} - \bar{\rho}_{\alpha\beta}(\vec{p})\right] \; e^{ip\cdot(x-y)} \right\} P_R \;, \end{aligned} \tag{A.64}$$

where $S(x,y)$ is nothing but the usual propagator of the (massless) Dirac field,

$$S(x,y) = \int \frac{d^4p}{(2\pi)^4} \frac{i\not{p}}{p^2 + i\epsilon} e^{-ip\cdot(x-y)} \;. \tag{A.65}$$

For the scalars, the various Green's functions are given by the following expressions,

$$iG^{>}_{\phi}(x,y) \;=\; \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec p}} \left\{ [1+\rho_\phi(\vec p)]\, e^{-ip\cdot(x-y)} + \bar\rho_\phi(\vec p)\, e^{ip\cdot(x-y)} \right\} \;, \tag{A.66}$$

$$iG^{<}_{\phi}(x,y) \;=\; \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec p}} \left\{ \rho_\phi(\vec p)\, e^{-ip\cdot(x-y)} + [1+\bar\rho_\phi(\vec p)]\, e^{ip\cdot(x-y)} \right\} \;, \tag{A.67}$$

$$iG^{t}_{\phi}(x,y) \;=\; D(x,y) + \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec p}} \left\{ \rho_\phi(\vec p)\, e^{-ip\cdot(x-y)} + \bar\rho_\phi(\vec p)\, e^{ip\cdot(x-y)} \right\} \;, \tag{A.68}$$

$$iG^{\bar t}_{\phi}(x,y) \;=\; -D(x,y) + \int \frac{d^3p}{(2\pi)^3 2\omega_{\vec p}} \left\{ [1+\rho_\phi(\vec p)]\; e^{-ip\cdot(x-y)} + [1+\bar\rho_\phi(\vec p)]\; e^{ip\cdot(x-y)} \right\} \;, \tag{A.69}$$

where $D(x,y)$ is the Klein-Gordon propagator,

$$D(x,y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip\cdot(x-y)} \;. \tag{A.70}$$

Notice that, under CP conjugation, considering for instance $G^{>}$, the Green's function for lepton doublets transforms as

$$(\mathcal{CP})\, G^{>}_{\beta\alpha}\, (\mathcal{CP})^{-1}\,(x,y) = C G^{<T}_{\alpha\beta}(y,x) C^{-1} = -G^{<}_{\alpha\beta,\,RL}(y,x) \;, \tag{A.71}$$

where the subscript RL indicates that the order of the projectors P_L and P_R is reversed with respect to eq. (A.61). For scalars, this transformation takes the following simple form,

$$(\mathcal{CP})\, G^{>}_{\phi}\, (\mathcal{CP})^{-1}\,(x,y) = G^{<}_{\phi}(y,x) \;. \tag{A.72}$$

From the Schwinger-Dyson equation to the Boltzmann equation

The Green's function $\tilde G$ is a solution of the Schwinger-Dyson equation,

$$\tilde G(x,y) \;=\; \tilde G^0(x,y) + \int_{\mathcal C} d^4z_1 \int_{\mathcal C} d^4z_2\, \tilde G^0(x,z_1)\tilde\Sigma(z_1,z_2)\tilde G(z_2,y)\,. \tag{A.73}$$

$\tilde\Sigma$ is a 2×2 matrix containing the self-energy functions $\Sigma^{>}$, $\Sigma^{<}$, Σ^{t} and $\Sigma^{\bar t}$, defined in an analogous way to the Green's functions $G^{>}$, $G^{<}$, G^{t} and $G^{\bar t}$,

$$\tilde\Sigma \;=\; \begin{pmatrix} \Sigma^t & \pm\Sigma^{<} \\ \Sigma^{>} & -\Sigma^{\bar t} \end{pmatrix} \;, \tag{A.74}$$

where again, the plus sign in front of $\Sigma^{<}$ applies for bosons and the minus sign for fermions, and $\tilde G^0$ is the free 2-point correlation function. The Schwinger-Dyson equation can also be written as

$$\tilde G(x,y) \;=\; \tilde G^0(x,y) + \int_{\mathcal C} d^4z_1 \int_{\mathcal C} d^4z_2\, \tilde G(x,z_1)\tilde\Sigma(z_1,z_2)\tilde G^0(z_2,y) \;. \tag{A.75}$$

From now on, we focus on lepton doublets. Acting on eqs. (A.73) and (A.75) with the operators $i\overrightarrow{\partial\!\!\!/}_x$ and $i\overleftarrow{\partial\!\!\!/}_y$, respectively, gives the following equations of motion,

$$i\overrightarrow{\partial\!\!\!/}_x\tilde{G}_{\beta\alpha}(x,y) \;=\; \delta^{(4)}(x-y)\,\delta_{\alpha\beta}\,\tilde{I} \,+\, \sum_\gamma \int_{\mathcal{C}} d^4z\,\tilde{\Sigma}_{\beta\gamma}(x,z)\tilde{G}_{\gamma\alpha}(z,y)\,, \tag{A.76}$$

$$\tilde{G}_{\beta\alpha}(x,y)\,i\overleftarrow{\partial\!\!\!/}_y \;=\; -\,\delta^{(4)}(x-y)\,\delta_{\alpha\beta}\,\tilde{I} \,+\, \sum_\gamma \int_{\mathcal{C}} d^4z\,\tilde{G}_{\beta\gamma}(x,z)\tilde{\Sigma}_{\gamma\alpha}(z,y)\;, \tag{A.77}$$

where we used the fact that the free Green's function for massless fermions satisfies, by definition, $i\partial\!\!\!/_x\tilde{G}^0_{\alpha\beta}(x,y) = \delta^{(4)}(x-y)\,\delta_{\alpha\beta}\,\tilde{I}$, with $\tilde{I}$ the identity matrix in both spinor and CTP spaces. This is nothing but the Dirac equation for massless free fermions.

We will use the Schwinger-Dyson equation to derive a Boltzmann equation for the density matrix. First of all, applying eq. (A.2) to lepton doublets gives

$$(n_\ell)_{\alpha\beta} = s\times(Y_\ell)_{\alpha\beta} = 2\int\frac{d^3p}{(2\pi)^3}\rho_{\alpha\beta}(\vec{p})\;, \tag{A.78}$$

$$(n_{\ell^c})_{\alpha\beta} = s\times(Y_{\ell^c})_{\alpha\beta} = 2\int\frac{d^3p}{(2\pi)^3}\bar{\rho}_{\alpha\beta}(\vec{p})\;, \tag{A.79}$$

$$(\Delta n_\ell)_{\alpha\beta} = s\times(\Delta_\ell)_{\alpha\beta} = 2\int\frac{d^3p}{(2\pi)^3}\left[\rho_{\alpha\beta}(\vec{p}) - \bar{\rho}_{\alpha\beta}(\vec{p})\right]\;. \tag{A.80}$$

Defining the current $J^\mu_{\alpha\beta}$ as

$$J^\mu_{\alpha\beta} =\, :\bar{\ell}_\alpha\gamma^\mu\ell_\beta:\;, \tag{A.81}$$

where the colons refer to the normal orderings of the operators, the expression of eq. (A.80) can be seen as the expectation value of the zero component of this current,

$$(\Delta n_\ell)_{\alpha\beta} = \langle J^0_{\alpha\beta}\rangle = \langle :\ell^\dagger_\alpha\ell_\beta:\rangle\;. \tag{A.82}$$

The divergence of this current (or more precisely of its expectation value) can be conveniently expressed in terms of the Green's function $G^>_{\beta\alpha}$,

$$\langle\partial_\mu J^\mu_{\alpha\beta}\rangle = -\;\mathrm{tr}\left[(i\overrightarrow{\partial\!\!\!/}_x + i\overleftarrow{\partial\!\!\!/}_y)G^>_{\beta\alpha}(x,y)\right]_{y=x}\;, \tag{A.83}$$

where the trace is taken over spinorial and $SU(2)_L$ indices. This is precisely what we need, because we can use the Schwinger-Dyson, and more precisely the expressions derived in eqs. (A.76) and (A.77), to express the right-hand side of eq. (A.83) in terms of the self-energy functions $\Sigma^>$ and $\Sigma^<$,

$$\begin{aligned}\langle\partial_\mu J^\mu_{\alpha\beta}\rangle \;&=\; -\int_{\mathcal{C}} d^4z\;\mathrm{tr}\Big[\Sigma^>_{\beta\gamma}(x,z)G^t_{\gamma\alpha}(z,x) - \Sigma^{\bar{t}}_{\beta\gamma}(x,z)G^>_{\gamma\alpha}(z,x)\\ &\qquad -G^>_{\beta\gamma}(x,z)\Sigma^t_{\gamma\alpha}(z,x) + G^{\bar{t}}_{\beta\gamma}(x,z)\Sigma^>_{\gamma\alpha}(z,x)\Big]\\ &=\; -\int d^3z\int_0^t dt_z\;\mathrm{tr}\Big[\Sigma^>_{\beta\gamma}(x,z)G^<_{\gamma\alpha}(z,x) - \Sigma^<_{\beta\gamma}(x,z)G^>_{\gamma\alpha}(z,x)\\ &\qquad -G^>_{\beta\gamma}(x,z)\Sigma^<_{\gamma\alpha}(z,x) + G^<_{\beta\gamma}(x,z)\Sigma^>_{\gamma\alpha}(z,x)\Big]\,.\end{aligned} \tag{A.84}$$

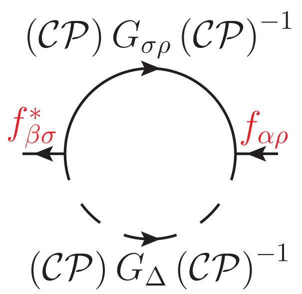


Figure b: One-loop contribution of the triplet to the self-energy $\Sigma_{\beta\alpha}$ of lepton doublets.

Since we consider a homogeneous and isotropic medium, the divergence of the current is equal to its zero component $\partial_0 J^0_{\alpha\beta}$. Finally, we incorporate the expansion of the universe by making the following replacement in the above equation,

$$\langle \partial_0 J^0_{\alpha\beta} \rangle \quad \to \quad \frac{d\Delta n_{\alpha\beta}}{dt} + 3H\Delta n_{\alpha\beta} \,=\, sHz\frac{d(\Delta_\ell)_{\alpha\beta}}{dz} \ , \tag{A.85}$$

which amounts to replacing the ordinary derivative by a covariant derivative in the Friedmann–Lemaître–Robertson–Walker metric. We thus obtain the quantum Boltzmann equation for the density matrix $(\Delta_\ell)_{\alpha\beta}$,

$$\begin{aligned} sHz\frac{d(\Delta_\ell)_{\alpha\beta}}{dz} \,=\, &- \int d^3z \int_0^t dt_z \ \mathrm{tr}\Big[\Sigma^>_{\beta\gamma}(x,z)G^<_{\gamma\alpha}(z,x) - \Sigma^<_{\beta\gamma}(x,z)G^>_{\gamma\alpha}(z,x) \\ &- G^>_{\beta\gamma}(x,z)\Sigma^<_{\gamma\alpha}(z,x) + G^<_{\beta\gamma}(x,z)\Sigma^>_{\gamma\alpha}(z,x)\Big] \ . \end{aligned} \tag{A.86}$$

This expression is rather abstract, so we focus on a simple explicit example to illustrate the link between this equation and the Boltzmann equations previously written. From now on, we focus on the general approach to leptogenesis with a scalar triplet presented in sec 3.2. In order to simplify the discussion, we forget spectator processes in this derivation. Fig. b shows the contribution of the triplet to the one-loop self-energy for the lepton. This is the equivalent in the CTP formalism of the washout term due to tree-level decays and inverse decays in ordinary Boltzmann equations. From this figure, we can read for instance the first term on the right-hand side of eq. (A.86),

$$\begin{aligned} \mathrm{tr}\Big[\Sigma^>_{\beta\gamma}(x,z)G^<_{\gamma\alpha}(z,x)\Big] \,=\, &(-if^*_{\beta\sigma})(-if_{\gamma\rho})\,\mathrm{tr}\Big[iG^<_\Delta(z,x)(-i)G^<_{\sigma\rho,\,RL}(z,x)iG^<_{\gamma\alpha}(z,x)\Big] \\ =\, &3\int\frac{d^3p}{(2\pi)^3 2\omega_{\vec{p}}}\int\frac{d^3k}{(2\pi)^3 2\omega_{\vec{k}}}\int\frac{d^3l}{(2\pi)^3 2\omega_{\vec{\ell}}}\ f^*_{\beta\sigma}f_{\gamma\rho}\,2(k.l) \\ &\times\Big\{\rho_\Delta(\vec{p})\,e^{-ip\cdot(z-x)} + \Big[1+\bar{\rho}_\Delta(\vec{p})\Big]\,e^{ip\cdot(z-x)}\Big\} \\ &\times\Big\{\rho_{\sigma\rho}(\vec{k})\,e^{-ik\cdot(z-x)} + \Big[\delta_{\sigma\rho}-\bar{\rho}_{\sigma\rho}(\vec{k})\Big]\ e^{ik\cdot(z-x)}\Big\} \\ &\times\Big\{\rho_{\alpha\gamma}(\vec{l})\,e^{-il\cdot(z-x)} + \Big[\delta_{\alpha\gamma}-\bar{\rho}_{\alpha\gamma}(\vec{l})\Big]\ e^{il\cdot(z-x)}\Big\} \ , \end{aligned} \tag{A.87}$$

where the factor 3 comes from the trace over $SU(2)$ indices (roughly speaking, it can be understood from the fact that this is an interaction between two $SU(2)$ triplets, $\ell\ell$

and Δ). This expression begins to look like the collision term of eq. (A.4), which is indeed what we are aiming for. However, the expression of eq. (A.87) (or rather its integral over z) includes non-markovian memory effects that are not present in classical Boltzmann equations.

Reporting this expression in the right-hand side of eq. (A.86), it is easy to get rid of the integral over the spatial coordinates of z, which gives simply a momentum-conserving delta function. The integral over t_z is more subtle. The classical limit is obtained by assuming that t is much smaller than the relaxation time of phase-space distribution functions, which can therefore be factorized out of the integral, but much larger than the typical duration of a collision, so that the time integral can be extended to infinity. This is actually an assumption that is usually implicitly made when writing Boltzmann equation. Indeed, Boltzmann equations rely on the fact that the successive collisions affecting a particle are independent from one another, so that there are no memory effects.

Concretely, this assumption amounts to replacing the integral of oscillating exponentials by energy-conserving delta functions, which are exactly what we need to recover classical Boltzmann equations, plus terms proportional to the principal value of $1/(\omega_{\vec{p}} \pm \omega_{\vec{k}} \pm \omega_{\vec{l}})$, where p, k and l are the momenta of the scalar triplet and of the two leptons, respectively. The latter terms, however, can be neglected because they arise at second order in the CP asymmetry. Indeed, all terms involving a principal value are proportional to $\eta_{\rho\sigma}(\vec{k})+\bar{\eta}_{\rho\sigma}(\vec{k})$, where $\eta_{\rho\sigma}(\vec{k})$ (resp. $\bar{\eta}_{\rho\sigma}(\vec{k})$) parametrizes the departure of the phase-space density $\rho_{\rho\sigma}(\vec{k})$ (resp. $\bar{\rho}_{\rho\sigma}(\vec{k})$) from its equilibrium value,

$$\rho_{\rho\sigma}(\vec{k}) \;=\; \rho_\ell^{\rm eq}(\vec{k}) \left[\delta_{\rho\sigma} + \eta_{\rho\sigma}(\vec{k})\right] \;, \quad \bar{\rho}_{\rho\sigma}(\vec{k}) \;=\; \rho_\ell^{\rm eq}(\vec{k}) \left[\delta_{\rho\sigma} + \bar{\eta}_{\rho\sigma}(\vec{k})\right] \;. \tag{A.88}$$

Since the imbalance between the lepton and antilepton densities is generated by the asymmetries in triplet decays, which are small numbers of order ϵ, while the lepton and antilepton populations are maintained close to equilibrium by fast electroweak interactions, one has[3]

$$\eta_{\rho\sigma}(\vec{k}) - \bar{\eta}_{\sigma\rho}(\vec{k}) \;=\; \mathcal{O}(\epsilon) \;, \tag{A.89}$$

$$\eta_{\rho\sigma}(\vec{k}) + \bar{\eta}_{\sigma\rho}(\vec{k}) \;=\; \mathcal{O}(\epsilon^2) \;. \tag{A.90}$$

After dropping these terms proportional to $\eta_{\rho\sigma} + \bar{\eta}_{\rho\sigma}$, we are left with energy-conserving delta functions only. These delta functions ensure that only terms which are kinematically allowed for on-shell particles remain. In the case of eq. (A.87), the remaining terms are precisely the ones describing the decay $\Delta \to \ell^c \ell^c$ and inverse decay $\ell\ell \to \Delta^c$,

$$\begin{aligned} &\int d^3z \int_0^t dt_z \, {\rm tr}\left[\Sigma^>_{\beta\gamma}(x,z) G^<_{\gamma\alpha}(z,x)\right] \\ &\qquad = -\int \frac{d^3p}{(2\pi)^3 2\omega_{\vec{p}}} \int \frac{d^3k}{(2\pi)^3 2\omega_{\vec{k}}} \int \frac{d^3l}{(2\pi)^3 2\omega_{\vec{\ell}}} \; 3 f^*_{\beta\rho} f_{\gamma\sigma}(k.l)\,(2\pi)^4 \delta^{(4)}(p-k-l) \\ &\qquad \times \left\{ \rho_{\rho\sigma}(\vec{k}) \rho_{\alpha\gamma}(\vec{\ell}) \left[1 + \bar{\rho}_\Delta(\vec{p})\right] + \rho_\Delta(\vec{p}) \left[\delta_{\sigma\rho} - \bar{\rho}_{\sigma\rho}(\vec{k})\right] \left[\delta_{\alpha\gamma} - \bar{\rho}_{\alpha\gamma}(\vec{k})\right] \right\} \;. \end{aligned} \tag{A.91}$$

[3] Eqs. (A.89) and (A.90) generalize the relations between the number densities derived in eqs. (A.10) and (A.11).

whereas the terms describing off-shell processes are eliminated. For instance, there is a term proportional to

$$\rho_\Delta(\vec{p}) \times \rho_{\sigma\rho}(\vec{k}) \times \left[\delta_{\alpha\gamma} - \bar{\rho}_{\alpha\gamma}(\vec{l})\right] \delta^{(4)}(p+k-l) \ , \tag{A.92}$$

describing an off-shell process of the type $\ell\Delta \to \ell^c$, which disappears because the delta function is always zero when k, l and p are on-shell.

Eq. (A.91) has exactly the same form as eq. (A.4), except that there is now a nontrivial flavour structure. In particular, it is not possible to identify the factor $f^*_{\beta\rho} f_{\gamma\sigma}(k.l)$ with the modulus squared of a matrix element. Computing the other contributions from fig. b to the right-hand side of eq. (A.86), and making the same approximations as in section A.1 when we derived Boltzmann equations[4], we obtain the washout term $W^{ID}_{\alpha\beta}$ associated with triplet decays and inverse decays:

$$W^{ID}_{\alpha\beta} = \frac{2B_\ell}{\lambda_\ell^2}\left[(ff^\dagger)_{\alpha\beta}\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} + \frac{1}{4Y^{\rm eq2}_\ell}\Big(Y_\ell f Y_\ell^T f^\dagger + f Y_\ell^T f^\dagger Y_\ell - (Y_\ell \leftrightarrow Y_{\ell^c})\Big)_{\alpha\beta}\right]\gamma^\Delta_D \tag{A.93}$$

One can linearize this expression, using again the fact that flavour-blind gauge interactions keep the lepton densities close to their equilibrium values, so that eqs. (A.10) and (A.11) give

$$\begin{aligned}(Y_\ell)_{\alpha\beta} - (Y_{\ell^c})_{\alpha\beta} &= (\Delta_\ell)_{\alpha\beta}\,,\\ (Y_\ell)_{\alpha\beta} + (Y_{\ell^c})_{\alpha\beta} &= 2Y^{\rm eq}_\ell\left[\delta_{\alpha\beta} + \mathcal{O}(\epsilon^2)\right] \ .\end{aligned} \tag{A.94}$$

Finally, the term which appears on the right-hand side of the Boltzmann equation for the density matrix $(\Delta_\ell)_{\alpha\beta}$ reads

$$W^{ID}_{\alpha\beta} = \frac{2B_\ell}{\lambda_\ell^2}\left[(ff^\dagger)_{\alpha\beta}\frac{\Delta_\Delta}{\Sigma^{\rm eq}_\Delta} + \frac{1}{4Y^{\rm eq}_\ell}\Big(2f\Delta_\ell^T f^\dagger + ff^\dagger\Delta_\ell + \Delta_\ell ff^\dagger\Big)_{\alpha\beta}\right]\gamma^\Delta_D \ . \tag{A.95}$$

A similar reasoning could be applied to derive other washout terms as well as the flavour-covariant source terms, although things become more complicated when dealing with scatterings and one-loop decays, because these expressions, which arise typically at the order f^4 or $f^2\lambda_H^2$, are described by two-loop self-energy diagrams in the CTP formalism. However, since we know that the CTP formalism leads to (flavoured) Boltzmann equations in the classical limit, it is possible to avoid such technicalities. Working on an example, we show how here the effective method that can be used to derive scattering terms.

We consider the triplet-mediated contribution to the lepton-Higgs scatterings, which, in the CTP formalism, involves the 2-loop diagrams displayed in fig. c. The flavour structure can be read directly on the diagram, using the following rules: a lepton propagator going from α to β yields a factor $(Y_\ell)_{\alpha\beta}/Y^{\rm eq}_\ell$ for an incoming lepton, and $\delta_{\alpha\beta}$ for outgoing leptons, whereas a Higgs propagator yields a factor $Y_H/Y^{\rm eq}_H$ for an incoming Higgs and a factor 1 for an outgoing Higgs. Rules are similar for antiparticles.

[4]More explicitly, we neglect Bose enhancement and Fermi blocking factors, for instance $\left[\delta_{\alpha\beta} - \rho_{\alpha\beta}(\vec{k})\right] \to \delta_{\alpha\beta}$, and we factorize the particle densities out of the integral.

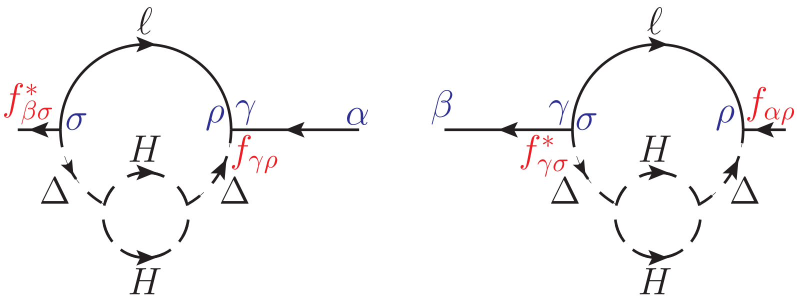


Figure c: Two-loop contributions to $\Sigma_{\beta\gamma}G_{\gamma\alpha}$ (left) and $G_{\beta\gamma}\Sigma_{\gamma\alpha}$ (right), that give the flavour structure of the scalar-mediated lepton-Higgs scatterings $\ell\ell \leftrightarrow H^c H^c$ and $\ell H \leftrightarrow \ell^c H^c$ (and their CP-conjugate) in the Boltzmann equation for $(\Delta_\ell)_{\alpha\beta}$.

Focusing on the s-channel scatterings $\ell\ell \leftrightarrow H^c H^c$ and $\ell^c\ell^c \leftrightarrow HH$, we see that the flavoured scattering term will have a contribution proportional to

$$\begin{aligned} & f_{\gamma\rho} f^*_{\beta\sigma} \left[\frac{(Y_\ell)_{\alpha\gamma}}{Y_\ell^{\rm eq}} \frac{(Y_\ell)_{\sigma\rho}}{Y_\ell^{\rm eq}} - \frac{Y^2_{H^c}}{Y^{\rm eq2}_{H^c}} \delta_{\alpha\gamma}\delta_{\rho\sigma} - \frac{(Y_{\ell^c})_{\alpha\gamma}}{Y_\ell^{\rm eq}} \frac{(Y_{\ell^c})_{\sigma\rho}}{Y_\ell^{\rm eq}} + \frac{Y^2_H}{Y^{\rm eq2}_H} \delta_{\alpha\gamma}\delta_{\rho\beta} \right] \\ & + f_{\alpha\rho} f^*_{\gamma\sigma} \left[\frac{(Y_\ell)_{\sigma\rho}}{Y_\ell^{\rm eq}} \frac{(Y_\ell)_{\gamma\beta}}{Y_\ell^{\rm eq}} - \frac{Y^2_{H^c}}{Y^{\rm eq2}_{H^c}} \delta_{\beta\gamma}\delta_{\rho\sigma} - \frac{(Y_{\ell^c})_{\sigma\rho}}{Y_\ell^{\rm eq}} \frac{(Y_{\ell^c})_{\gamma\beta}}{Y_\ell^{\rm eq}} + \frac{Y^2_H}{Y^{\rm eq2}_H} \delta_{\beta\gamma}\delta_{\rho\sigma} \right] \\ & = 4(ff^\dagger)_{\alpha\beta} \frac{\Delta_H}{Y_H^{\rm eq}} + \frac{1}{Y_\ell^{\rm eq}} \left(2f\Delta_\ell^T f^\dagger + ff^\dagger\Delta_\ell + \Delta_\ell ff^\dagger \right)_{\alpha\beta} . \end{aligned} \tag{A.96}$$

This is only the contribution of s-channel scalar-mediated scatterings. Taking now into account all the contributions to lepton-Higgs scatterings, and normalizing properly[5], the washout term accounting for lepton-Higgs scatterings in the Boltzmann equation for $(\Delta_\ell)_{\alpha\beta}$ reads

$$\begin{aligned} W^{\ell H}_{\alpha\beta} = & \frac{2}{\lambda_\ell^2} \left[(ff^\dagger)_{\alpha\beta} \frac{\Delta_H}{Y_H^{\rm eq}} + \frac{1}{4Y_\ell^{\rm eq}} \left(2f\Delta_\ell^T f^\dagger + ff^\dagger\Delta_\ell + \Delta_\ell ff^\dagger \right)_{\alpha\beta} \right] \gamma^{\Delta}_{\ell H} \\ & + \frac{2}{\Re\left[{\rm tr}(f\kappa^\dagger)\right]} \left[(f\kappa^\dagger)_{\alpha\beta} \frac{\Delta_H}{Y_H^{\rm eq}} + \frac{1}{4Y_\ell^{\rm eq}} \left(2f\Delta_\ell^T \kappa^\dagger + f\kappa^\dagger\Delta_\ell + \Delta_\ell f\kappa^\dagger \right)_{\alpha\beta} \right] \gamma^{\mathcal{I}}_{\ell H} \\ & + \frac{2}{\Re\left[{\rm tr}(f\kappa^\dagger)\right]} \left[(\kappa f^\dagger)_{\alpha\beta} \frac{\Delta_H}{Y_H^{\rm eq}} + \frac{1}{4Y_\ell^{\rm eq}} \left(2\kappa\Delta_\ell^T f^\dagger + \kappa f^\dagger\Delta_\ell + \Delta_\ell \kappa f^\dagger \right)_{\alpha\beta} \right] \gamma^{\mathcal{I}}_{\ell H} \\ & + \frac{2}{\lambda_\kappa^2} \left[(\kappa\kappa^\dagger)_{\alpha\beta} \frac{\Delta_H}{Y_H^{\rm eq}} + \frac{1}{4Y_\ell^{\rm eq}} \left(2\kappa\Delta_\ell^T \kappa^\dagger + \kappa\kappa^\dagger\Delta_\ell + \Delta_\ell \kappa\kappa^\dagger \right)_{\alpha\beta} \right] \gamma^{\kappa}_{\ell H} . \end{aligned} \tag{A.97}$$

[5] For this example, it can be done very simply by considering the single-flavour case, in which we know that we should recover $W^{\ell H} = 2\left(\frac{\Delta_\ell}{Y_\ell^{\rm eq}} + \frac{\Delta_H}{Y_H^{\rm eq}}\right)\left(2\gamma'^{\ell\ell}_{H^cH^c} + \gamma^{\ell H}_{\ell^c H^c}\right)$.

B Loop integrals in wavefunction renormalization

In this section, we give the expression of the loop integrals involved in the computation of the wavefunction renormalization in section 5.2.3.

B.1 Contribution of the superpotential

The λ^4 contribution to the two-loop wavefunction renormalization, summarized in fig. 5.5, can be written as

$$\begin{aligned}\left[\sum D^{(2)}_{\alpha\beta} + i\delta Z^{(2)}_{\alpha\beta}\right]_{|_{\lambda^4}} &= d^{kl}_{\alpha} d^{np}_{k} \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda^*_{mnp}\lambda_{knp}\left[\frac{i}{2} I^{klmnp}_{A} + \frac{1}{32\pi^2\epsilon} I^{klm}_{A'}\right] \\ &+ d^{kl}_{\alpha} d^{np}_{k} \lambda^*_{\alpha kl}\lambda_{\beta lm}\lambda_{\bar{m}np}\lambda^*_{\bar{k}np}\left[\frac{i}{2} I^{klmnp}_{B} + \frac{1}{32\pi^2\epsilon} I^{klm}_{B'}\right] \\ &+ i d^{kl}_{\alpha} d^{mn}_{\alpha} \lambda^*_{\alpha kl}\lambda_{\beta mn}\lambda^*_{\bar{k}np}\lambda_{l\bar{m}p} I^{klmnp}_{C} + i\delta Z^{(2)}_{\alpha\beta|_{\lambda^4}} \ .\end{aligned} \tag{B.1}$$

The loop integrals are the following ones,

$$I^{klmnp}_{A} = \int \frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{p^2}{D_k(p)D_l(p)D_m(p)D_n(k)D_p(p+k)} \ , \tag{B.2}$$

$$I^{klmnp}_{B} = M_k M^\dagger_m \int \frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{1}{D_k(p)D_l(p)D_m(p)D_n(k)D_p(p+k)} \ , \tag{B.3}$$

$$I^{klmnp}_{C} = M_k M^\dagger_m \int \frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{1}{D_k(p)D_l(p)D_m(k)D_n(k)D_p(p+k)} \ , \tag{B.4}$$

$$I^{klm}_{A'} = \int \frac{d^dp}{(2\pi)^d}\frac{p^2}{D_k(p)D_l(p)D_m(p)} \ , \tag{B.5}$$

$$I^{klm}_{B'} = M_k M^\dagger_m \int \frac{d^dp}{(2\pi)^d}\frac{1}{D_k(p)D_l(p)D_m(p)} \ . \tag{B.6}$$

with $D_x(q) = q^2 - M^2_x$. It should be noticed that I_B, $I_{B'}$ and I_C are free of infrared divergences, because if M_k or M_m goes to zero these integrals simply vanish. I_A, I_B and I_C can be expressed in terms of a single two-loop integral as follows,

$$I^{klmnp}_{A} = \frac{M^2_k}{\Delta M^2_{kl}\Delta M^2_{km}} I^{knp}_{\text{2-loop}} - \frac{M^2_l}{\Delta M^2_{kl}\Delta M^2_{lm}} I^{lnp}_{\text{2-loop}} + \frac{M^2_m}{\Delta M^2_{km}\Delta M^2_{lm}} I^{mnp}_{\text{2-loop}} \ , \tag{B.7}$$

$$I^{klmnp}_{B} = \frac{M_k M^\dagger_m}{\Delta M^2_{kl}\Delta M^2_{km}} I^{knp}_{\text{2-loop}} - \frac{M_k M^\dagger_m}{\Delta M^2_{kl}\Delta M^2_{lm}} I^{lnp}_{\text{2-loop}} + \frac{M_k M^\dagger_m}{\Delta M^2_{km}\Delta M^2_{lm}} I^{mnp}_{\text{2-loop}} \ , \tag{B.8}$$

$$I^{klmnp}_{C} = \frac{M_k M^\dagger_m}{\Delta M^2_{kl}\Delta M^2_{mn}}\left[I^{kmp}_{\text{2-loop}} - I^{lmp}_{\text{2-loop}} - I^{knp}_{\text{2-loop}} + I^{lnp}_{\text{2-loop}}\right] \ , \tag{B.9}$$

with $\Delta M^2_{xy} = M^2_x - M^2_y$ and

$$I^{ijk}_{\text{2-loop}} = \int \frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{1}{[p^2 - M^2_i][k^2 - M^2_j][(p+k)^2 - M^2_k]} \ . \tag{B.10}$$

This integral can be computed by using the methods presented in ref. [337]. The expansion in powers of ϵ gives the following result,

$$
\begin{aligned}
I^{ijk}_{\text{2-loop}} = &-\frac{M_X^2}{512\pi^4\epsilon^2}\sum_{n=i,j,k} x_n - \frac{M_X^2}{256\pi^4\epsilon}\sum_{n=i,j,k} x_n\left[\frac{1}{2}+f_1(x_n,s)\right] \\
&- \frac{M_X^2}{256\pi^4}\left\{\sum_{n=i,j,k} x_n\left[\frac{1}{2}+2f_1(x_n,s)+f_{1\epsilon}^{\epsilon}(x_n,s)\right]+g(x_i,x_j,x_k,s)\right\} \\
&+\mathcal{O}(\epsilon)\ .
\end{aligned}
\tag{B.11}
$$

f_1 and $f_{1\epsilon}$ are the functions defined in eqs. (5.2.47) and (5.2.48), while the expression of g depends on the region of parameter space. For $\lambda(x_i,x_j,x_k)>0$ and $\sqrt{x_k}>\sqrt{x_i}+\sqrt{x_j}$, g is given by

$$
\begin{aligned}
&g(x_i,x_j,x_k,s) \\
&= (x_i+x_j-x_k)\log\frac{x_i}{s}\log\frac{x_j}{s} + (x_i-x_j+x_k)\log\frac{x_i}{s}\log\frac{x_k}{s} \\
&+ (-x_i+x_j+x_k)\log\frac{x_j}{s}\log\frac{x_k}{s} \\
&- \sqrt{\lambda(x_i,x_j,x_k)}\left\{\log\frac{x_i}{x_k}\log\frac{x_j}{x_k}-\frac{\pi^2}{3}+2\mathrm{Li}_2\left(\frac{x_i-x_j+x_k-\sqrt{\lambda(x_i,x_j,x_k)}}{2x_k}\right)\right. \\
&+ 2\,\mathrm{Li}_2\left(\frac{-x_i+x_j+x_k-\sqrt{\lambda(x_i,x_j,x_k)}}{2x_k}\right) \\
&- 2\log\left(\frac{x_i-x_j+x_k-\sqrt{\lambda(x_i,x_j,x_k)}}{2x_k}\right) \\
&\left.\times\log\left(\frac{-x_i+x_j+x_k-\sqrt{\lambda(x_i,x_j,x_k)}}{2x_k}\right)\right\}\ .
\end{aligned}
\tag{B.12}
$$

The cases $\lambda(x_i,x_j,x_k)>0$ and $\sqrt{x_i}>\sqrt{x_j}+\sqrt{x_k}$ or $\sqrt{x_j}>\sqrt{x_i}+\sqrt{x_k}$ can be recovered from this by permuting indices, while, in the case $\lambda(x_i,x_j,x_k)<0$ and $\sqrt{x_i}+\sqrt{x_j}>\sqrt{x_k}$, the function g becomes

$$
\begin{aligned}
&g(x_i,x_j,x_k,s) \\
&= (x_i+x_j-x_k)\log\frac{x_i}{s}\log\frac{x_j}{s} + (x_i-x_j+x_k)\log\frac{x_i}{s}\log\frac{x_k}{s} \\
&+ (-x_i+x_j+x_k)\log\frac{x_j}{s}\log\frac{x_k}{s} \\
&- 2\sqrt{-\lambda(x_i,x_j,x_k)}\left\{\mathrm{Cl}_2\left[2\arccos\left(\frac{x_i+x_j-x_k}{2\sqrt{x_ix_j}}\right)\right]\right. \\
&\left.+\mathrm{Cl}_2\left[2\arccos\left(\frac{x_i-x_j+x_k}{2\sqrt{x_ix_k}}\right)\right]+\mathrm{Cl}_2\left[2\arccos\left(\frac{-x_i+x_j+x_k}{2\sqrt{x_jx_k}}\right)\right]\right\}\ ,
\end{aligned}
\tag{B.13}
$$

Where Cl_2 is the Clausen function of order 2. Again, the cases $\lambda(x_i,x_j,x_k)<0$ and $\sqrt{x_i}+\sqrt{x_k}>\sqrt{x_k}$ or $\sqrt{x_j}+\sqrt{x_k}>\sqrt{x_i}$ can be recovered from this by permutation.

The loop functions f_A, f_B and f_C that appear in eq. (5.2.58) are defined by

$$I_A^{klmnp} - \frac{i}{16\pi^2\epsilon} I_{A'}^{klm} = \frac{i}{512\pi^4}\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right] + \frac{1}{256\pi^4} f_A(x_k, x_l, x_m, x_n, x_p) \ , \tag{B.14}$$

$$I_B^{klmnp} - \frac{i}{16\pi^2\epsilon} I_{B'}^{klm} = \frac{1}{256\pi^4} f_B(x_k, x_l, x_m, x_n, x_p) \ , \tag{B.15}$$

$$I_C^{klmnp} = \frac{1}{256\pi^4} f_C(x_k, x_l, x_m, x_n, x_p) \ . \tag{B.16}$$

In full generality, they are given by

$$\begin{aligned} &f_A(x_k, x_l, x_m, x_n, x_p, s) \\ &\quad = \left[-\frac{5}{2} + 2\left(\frac{x_k^2 \log(x_k/s)}{\Delta_{kl}\Delta_{km}} - \frac{x_l^2 \log(x_l/s)}{\Delta_{kl}\Delta_{lm}} + \frac{x_m^2 \log(x_m/s)}{\Delta_{km}\Delta_{lm}}\right)\right. \\ &\quad \left. - 2\left(\frac{x_k\, g(x_k, x_n, x_p, s)}{\Delta_{kl}\Delta_{km}} - \frac{x_l\, g(x_l, x_n, x_p, s)}{\Delta_{kl}\Delta_{lm}} + \frac{x_m\, g(x_m, x_n, x_p, s)}{\Delta_{km}\Delta_{lm}}\right)\right] \ , \end{aligned} \tag{B.17}$$

$$\begin{aligned} &f_B(x_k, x_l, x_m, x_n, x_p, s) \\ &\quad = \sqrt{x_m x_k^\dagger} \times \left[2\left(\frac{x_k \log(x_k/s)}{\Delta_{kl}\Delta_{km}} - \frac{x_l \log(x_l/s)}{\Delta_{kl}\Delta_{lm}} + \frac{x_m \log(x_m/s)}{\Delta_{km}\Delta_{lm}}\right)\right. \\ &\quad \left. - 2\left(\frac{g(x_k, x_n, x_p, s)}{\Delta_{kl}\Delta_{km}} - \frac{g(x_l, x_n, x_p, s)}{\Delta_{kl}\Delta_{lm}} + \frac{g(x_m, x_n, x_p, s)}{\Delta_{km}\Delta_{lm}}\right)\right] \ , \end{aligned} \tag{B.18}$$

$$\begin{aligned} f_C(x_k, x_l, x_m, x_n, x_p, s) &= -\frac{\sqrt{x_k x_m^\dagger}}{\Delta_{kl}\Delta_{mn}} \left[g(x_k, x_m, x_p, s) - g(x_l, x_m, x_p, s)\right. \\ &\quad \left. - g(x_k, x_n, x_p, s) + g(x_l, x_n, x_p, s)\right] \ . \end{aligned} \tag{B.19}$$

We also define $f_{A'}$ through

$$I_{A'}^{klm} = \frac{i}{16\pi^2\epsilon} + \frac{i}{16\pi^2} f_{A'}(x_k, x_l, x_m) + \mathcal{O}(\epsilon) \ , \tag{B.20}$$

which gives, explicitly,

$$\begin{aligned} f_{A'}(x_i, x_j, x_k) &= \frac{x_i^2 f_1(x_i,s)}{\Delta_{ij}\Delta_{ik}} - \frac{x_j^2 f_1(x_j,s)}{\Delta_{ij}\Delta_{jk}} + \frac{x_k^2 f_1(x_k,s)}{\Delta_{ik}\Delta_{jk}} \\ &= -\frac{x_i^2 \log(x_i/s)}{\Delta_{ij}\Delta_{ik}} + \frac{x_j^2 \log(x_j/s)}{\Delta_{ij}\Delta_{jk}} - \frac{x_k^2 \log(x_k/s)}{\Delta_{ik}\Delta_{jk}} \ . \end{aligned} \tag{B.21}$$

In particular, the one-loop wavefunction renormalization $Z^{(1)}_{|_{\lambda^4}}$ can be expressed as

$$Z^{(1)}_{\alpha\beta|_{\lambda^4}} = \frac{1}{32\pi^2} d_\alpha^{kl} \lambda^*_{\alpha kl} \lambda_{\beta kl} f_{A'}(0, x_k, x_l) \ . \tag{B.22}$$

B.2 Mixed gauge-superpotential contribution

The mixed contribution from the superpotential and gauge interactions to the two-loop wavefunction renormalization, summarized in fig. 5.6, reads

$$\begin{aligned}\left[\sum D^{(2)}_{\alpha\beta}+i\delta Z^{(2)}_{\alpha\beta}\right]_{|\lambda^2 g^2} =& -2d^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_l)\left[iI^{kl}_D+\frac{1}{16\pi^2\epsilon}I^{kll}_{A'}\right]\\ &-2d^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_l)\left[iI^{kl}_E+\frac{1}{16\pi^2\epsilon}I^{kll}_{B'}\right]\\ &-id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2\left[t_A(\Phi_l)t_A(\Phi_b)+t_A(\Phi_l)t_A(\Phi_a)\right]I^{kl}_F\\ &+id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2\left[t_A(\Phi_l)t_A(\Phi_b)+t_A(\Phi_l)t_A(\Phi_a)\right]I^{kl}_G\\ &-id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 t_A(\Phi_a)t_A(\Phi_b)I^{kl}_H\\ &-2d^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_a)\frac{1}{32\pi^2\epsilon}I^{ab}_I+\delta Z^{(2)}_{\alpha\beta|_{\lambda^2g^2}}\ .\end{aligned} \tag{B.23}$$

The loop integrals appearing here are ($I_{A'}$ and $I_{B'}$ were defined previously)

$$I^{kl}_D=\int\frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{p^2}{D_k(p)D_l(p)^2D_l(k)D_0(p+k)}\ , \tag{B.24}$$

$$I^{kl}_E=M_lM_l^\dagger\int\frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{p^2}{D_k(p)D_l(p)^2D_l(k)D_0(p+k)}\ , \tag{B.25}$$

$$I^{kl}_F=\int\frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{k^2}{D_0(p)^2D_k(k)D_l(k)D_l(p+k)}\ , \tag{B.26}$$

$$I^{kl}_G=M_lM_l^\dagger\int\frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{1}{D_0(p)^2D_k(k)D_l(k)D_l(p+k)}\ , \tag{B.27}$$

$$I^{kl}_H=\int\frac{d^dp}{(2\pi)^d}\frac{d^dk}{(2\pi)^d}\frac{1}{D_0(p)^2D_k(k)D_l(p+k)}\ , \tag{B.28}$$

$$I^{ij}_I=\int\frac{d^dp}{(2\pi)^d}\frac{1}{D_i(p)D_j(p)}\ , \tag{B.29}$$

This gives a rather complicated expression. However, from gauge invariance, one can simplify the result by reorganizing the terms involving I_F, I_G and I_H,

$$\begin{aligned}&-id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2\left[t_A(\Phi_l)t_A(\Phi_\beta)+t_A(\Phi_l)t_A(\Phi_\alpha)\right]I^{kl}_F\\ &+id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2\left[t_A(\Phi_l)t_A(\Phi_\beta)+t_A(\Phi_l)t_A(\Phi_\alpha)\right]I^{kl}_G\\ &-id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 t_A(\Phi_\alpha)t_A(\Phi_\beta)I^{kl}_H\\ &\qquad=id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_\alpha)I^{kl}_H\ ,\end{aligned} \tag{B.30}$$

so that eq. (B.23) reduces to

$$\begin{aligned}\left[\sum D^{(2)}_{\alpha\beta}+i\delta Z^{(2)}_{\alpha\beta}\right]_{|\lambda^2 g^2} =& -d^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_l)\left[2iI^{kl}_D+\frac{1}{8\pi^2\epsilon}I^{kll}_{A'}\right]\\ &-d^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_l)\left[2iI^{kl}_E+\frac{1}{8\pi^2\epsilon}I^{kll}_{B'}\right]\\ &+id^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_a)I^{kl}_H+d^{kl}_{\alpha}\lambda^*_{\alpha kl}\lambda_{\beta kl}g^2 C(\Phi_\alpha)\frac{1}{16\pi^2\epsilon}I^{ab}_I\\ &+\delta Z^{(2)}_{\alpha\beta|_{\lambda^2g^2}}\ .\end{aligned} \tag{B.31}$$

Finally, the integrals appearing here can be computed by noticing that they are special cases of the previous integrals I_A, $I_{A'}$, I_B and $I_{B'}$.

B.3 Holomorphic wavefunction

The two-loop contributions to the holomorphic wavefunction renormalization, summarized in fig. 5.7, can be written as

$$\begin{aligned} i\mathcal{H}_{\alpha\beta} &= d_\alpha^{kl} d_k^{np} \lambda_{\alpha\bar{k}\bar{l}} \lambda_{\beta lm} \lambda_{knp} \lambda^*_{mnp} M_k^\dagger M_l^\dagger \left[\frac{i}{2} I_K^{klmnp} + \frac{1}{32\pi^2\epsilon} I_{K'}^{klm} \right] \\ &+ d_\alpha^{kl} d_k^{np} \lambda_{\alpha kl} \lambda_{\beta\bar{l}\bar{m}} \lambda^*_{knp} \lambda_{mnp} M_l^\dagger M_m^\dagger \left[\frac{i}{2} I_K^{klmnp} + \frac{1}{32\pi^2\epsilon} I_{K'}^{klm} \right] \\ &+ i d_\alpha^{kl} d_\beta^{mn} \lambda_{\alpha kl} \lambda_{\beta mn} \lambda^*_{knp} \lambda_{\bar{l}\bar{m}p} M_m^\dagger M_l^\dagger I_L^{klmnp} \end{aligned} \tag{B.32}$$

with the following integrals,

$$I_K^{klmnp} = \int \frac{d^d p}{(2\pi)^d} \frac{d^d k}{(2\pi)^d} \frac{1}{D_k(p) D_l(p) D_m(p) D_n(k) D_p(p+k)} \ , \tag{B.33}$$

$$I_L^{klmnp} = \int \frac{d^d k}{(2\pi)^d} \frac{d^d p}{(2\pi)^d} \frac{1}{D_k(p) D_l(p) D_m(k) D_n(k) D_p(p+k)} \ , \tag{B.34}$$

$$I_{K'}^{klm} = \int \frac{d^d p}{(2\pi)^d} \frac{1}{D_k(p) D_l(p) D_m(p)} \ . \tag{B.35}$$

Again, it should be noticed that these integrals can be expressed in terms of the same function as the integrals I_B, $I_{B'}$, and I_C.

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Synopsis

Le Modèle Standard de la physique des particules a permis de classifier l'ensemble des particules élémentaires découvertes jusqu'ici. Il s'agit d'un succès sans précédent dans l'histoire de la physique. Cependant, certains faits inexpliqués semblent indiquer qu'il existe une théorie plus vaste. Celle-ci devrait bien sûr englober le Modèle Standard, tout en apportant une réponse satisfaisante aux questions non résolues, qui concernent par exemple la matière noire, les masses des neutrinos ou encore le problème de la hiérarchie. À l'heure actuelle, il n'y a toutefois pas d'indice permettant de déterminer la nature de cette théorie

L'étude du secteur leptonique peut s'avérer intéressante dans ce contexte. Les neutrinos sont encore peu connus et peuvent fournir des informations décisives : par exemple, nous savons déjà que le Modèle Standard doit être étendu pour inclure leurs masses. Si de plus les neutrinos sont des particules de Majorana, cette extension peut ouvrir des perspectives intéressantes, en permettant par exemple d'expliquer l'asymétrie entre matière et antimatière via la leptogenèse. D'autre part, les phénomènes violant la saveur leptonique sont extrêmement rares dans le Modèle Standard. Leur observation, en plus d'être un signal clair de l'existence d'une nouvelle physique, pourrait donner des indications sur la nature de celle-ci.

Ce résumé suit globalement la structure de la thèse. Dans un premier temps, nous introduisons le Modèle Standard et le problème de la masse des neutrinos. Ensuite, nous revenons sur les scénarios de leptogenèse étudiés dans cette thèse. Puis, après une introduction à la supersymétrie, nous présentons un scénario où le partenaire fermionique d'un pseudo-boson de Goldstone joue le rôle d'un neutrino stérile. Enfin, nous abordons le sujet du seesaw supersymétrique dans le cadre des scénarios de médiation de jauge étendue.

I Le Modèle Standard

Le Modèle Standard est une théorie de jauge, basée sur le groupe de symétrie $SU(3) \times SU(2)_L \times U(1)_Y$, où $SU(3)$ est le groupe de l'interaction forte, $SU(2)_L$ celui de l'isospin faible et $U(1)_Y$ celui de l'hypercharge. À chaque générateur d'un groupe de symétrie est associé un boson de jauge : les bosons vecteurs de l'interaction forte sont les gluons, ceux de l'isospin faible sont notés W_A et celui de l'hypercharge B. Dans le Modèle Standard, il est possible de distinguer deux types de fermions. Les quarks sont des triplets de couleur et de ce fait sensibles à l'interaction forte, tandis que les leptons ne sont sensible qu'aux interactions électrofaibles. D'autre part, les fermions de chiralité droite et gauche, définis respectivement par

$$\psi_R = \left(1 + \frac{\gamma_5}{2}\right)\psi = P_R\,\psi\ , \qquad \psi_L = \left(1 - \frac{\gamma_5}{2}\right)\psi = P_L\,\psi\ , \tag{I.1}$$

se transforment différemment sous $SU(2)$: les fermions de chiralité gauche forment des doublets, tandis que les fermions de chiralité droite sont des singlets. Le contenu

	Champs	$SU(3) \times SU(2)_L \times U(1)_Y$	B	L
Quarks	$Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	$(3,\ 2,\ +1/6)$	$1/3$	0
	u_{Ri}	$(3,\ 1,\ +2/3)$	$1/3$	0
	d_{Ri}	$(3,\ 1,\ -1/3)$	$1/3$	0
Leptons	$\ell_\alpha = \begin{pmatrix} \nu_{L\alpha} \\ e_{L\alpha} \end{pmatrix}$	$(1,\ 2,\ -1/2)$	0	1
	$e_{R\alpha}$	$(1,\ 1,\ -1)$	0	1

Table i: Contenu fermionique du Modèle Standard. B dénote le nombre baryonique et L le nombre leptonique.

fermionique du Modèle Standard est résumé dans le tableau i.

À ce stade, le Lagrangien du modèle est

$$\begin{aligned}\mathcal{L} = &\sum_{i=1,2,3} i\left(\bar{Q}_i \not{D} Q_i + \bar{u}_{Ri}\not{D} u_i + \bar{d}_{Ri}\not{D} d_{Ri}\right) + \sum_{\alpha=e,\mu,\tau} i\left(\bar{\ell}_\alpha \not{D}\ell_\alpha + \bar{e}_{R\alpha}\not{D} e_{R\alpha}\right) \\ &- \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a\ .\end{aligned} \tag{I.2}$$

Comme les fermions de chiralité droite et gauche appartiennent à des représentations différentes, il n'est pas possible d'inclure des termes de masse pour les fermions dans le lagrangien, car ceux-ci prendraient la forme suivante,

$$\mathcal{L} = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)\ , \tag{I.3}$$

ce qui violerait l'invariance de jauge sous $SU(2)$. De plus, les bosons de jauge de l'interaction faibles ne peuvent pas non plus avoir de masse, car elle aussi violerait $SU(2)$. Ces deux points sont en contradiction flagrante avec les observations expérimentale. La solution à ce problème est fournie par le mécanisme de Higgs, qui repose sur l'introduction d'un champ scalaire complexe, doublet de $SU(2)$,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}\ . \tag{I.4}$$

Le potentiel du champ de Higgs est choisi de telle manière que, au minimum du potentiel (qui définit le vide), la composante neutre de celui-ci prend une valeur non nulle $\langle H^0\rangle = v \neq 0$. Le terme cinétique du champs de Higgs génère des masses pour les bosons vecteurs de l'interaction faible. En effet, celui-ci s'écrit sous la forme suivante,

$$D_\mu H^\dagger D^\mu H = \left(\partial^\mu + igW^\mu_A\tau_A + i\frac{g'}{2}B^\mu\right)H^\dagger\left(\partial^\mu - igW^\mu_A\tau_A - i\frac{g'}{2}B^\mu\right)H\ . \tag{I.5}$$

Après remplacement $H \to \langle H \rangle$, il apparaît que les deux bosons chargés W^+ et W^-, définis comme $W^\pm = (W_1 \pm i W_2)/\sqrt{2}$, obtiennent la même masse $M_W^2 = g^2 v^2/2$, tandis que les bosons neutres W^3 et B se mélangent pour former un état lourd Z^0, de masse $M_Z^2 = (g^2 + g'^2)v^2/2$, et un boson de masse nulle, que l'on identifie comme le photon, vecteur de l'interaction électromagnétique.

Les fermions deviennent également massifs grâce à leurs couplages de Yukawa avec le champs de Higgs,

$$\mathcal{L}_{\text{Yukawa}} = -\sum_{i,j} \left(y_{ij}^u \bar{Q}_i \tilde{H}\, u_{Rj} + y_{ij}^d \bar{Q}_i H\, d_{Rj} \right) - \sum_{\alpha,\beta} y_{\alpha\beta}^e \bar{\ell}_\alpha H\, e_{R\beta} + \text{h.c.}\ , \tag{I.6}$$

qui donne les matrices de masses suivantes, $m_{ij}^a = y_{ij}^a v$.

II Le Mécanisme de seesaw

Le modèle Standard ne contient pas de neutrinos droits : pendant longtemps, les neutrinos ont été considérés comme des particules de masse nulle, ce qui rendait des neutrinos droits non nécessaires, et ce d'autant plus que les neutrinos droits seraient des singlets de jauge et donc indétectables.

Le phénomène d'oscillation des neutrinos, imaginé dès 1957 par Bruno Pontecorvo [76] et découvert à la fin des années 1990 par l'expérience Super Kamiokande [79], nécessite toutefois que les neutrinos soient massifs. La solution la plus simple pour cela semble être d'introduire les neutrinos droits initialement non inclus dans le Modèle Standard (que nous noterons ici N en prévision de la suite), et d'écrire un couplage de Yukawa pour les neutrinos comme pour les autres fermions,

$$\mathcal{L}_{\text{Yukawa}} = -y_{i\alpha}^\nu \bar{N}_i H \ell_\alpha + \text{h.c.}\ . \tag{II.1}$$

En revanche, ceci n'explique pas pourquoi les neutrinos sont si légers en comparaison des autres fermions : bien que les valeurs exactes de leurs masses soient toujours inconnues, les bornes supérieures fournies par la cosmologie indiquent qu'ils sont au moins un million de fois plus légers que l'électron. Ici, cette hiérarchie nécessite un couplage de Yukawa extrêmement petit. De plus, les neutrinos droits étant des singlets de jauge contrairement aux autres fermions, rien n'interdit a priori de leur donner un terme de masse de Majorana puisque celui-ci respecte automatiquement l'invariance de jauge,

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2} M_i N_i^T C N_i + \text{h.c.}\ . \tag{II.2}$$

Ce terme de masse n'est en rien relié à la brisure de symétrie électrofaible. Cette masse peut donc être arbitrairement grande, et en particulier beaucoup plus grande que l'échelle définie par $v = 174$ GeV. Le mécanisme de seesaw de type I, proposé initialement par Peter Minkowski [114], exploite précisément cette idée. Il repose sur le Lagrangien suivant,

$$\mathcal{L}_{\text{seesaw}} = -y_{i\alpha}^\nu \bar{N}_i H \ell_\alpha - \frac{1}{2} M_i N_i^T C N_i + \text{h.c.}\ , \tag{II.3}$$

où les masses M_i sont très grande devant v. Ceci devient, après la brisure de symétrie électrofaible

$$\mathcal{L}_{\text{seesaw}} = -y^{\nu}_{i\alpha} v \bar{N}_i \nu_\alpha - \frac{1}{2} M_i N_i^T C N_i + \text{h.c.} \; . \tag{II.4}$$

Les neutrinos ont maintenant une matrice de masse de Majorana 6×6. Après diagonalisation, il y a trois états propres lourds qui correspondent en bonne approximation aux N_i, et trois états propres légers qui sont essentiellement issus du mélange des ν_α. La matrice de masse des neutrinos légers est donnée par

$$m_{\nu\alpha\beta} = \sum_i \frac{y^{\nu}_{i\alpha} y^{\nu}_{i\beta}}{M_i} v^2 \; . \tag{II.5}$$

La légèreté des neutrinos du Modèle Standard s'explique maintenant par les très grandes masses des neutrinos droits qui apparaissent ici au dénominateur, sans nécessiter des couplages de Yukawa particulièrement petits.

D'autres mécanismes similaires peuvent donner de petites masses de Majorana aux neutrinos. Dans le mécanisme de seesaw de type II [117, 119, 120], il n'y a pas de neutrinos droits mais un triplet électrofaible scalaire Δ d'hypercharge 1. La partie du Lagrangien responsable des masses des neutrinos est

$$\mathcal{L}_{\text{seesaw II}} = -\frac{1}{2} \left(f_{\alpha\beta} \ell_\alpha^T C i\sigma_2 \Delta \ell_\beta + \mu H^T i\sigma_2 \Delta^\dagger H + \text{h.c.} \right) - M_\Delta^2 \, \text{tr}(\Delta^\dagger \Delta) \; . \tag{II.6}$$

Les doublets de Higgs et de leptons peuvent maintenant interagir via l'échange d'un triplet. En conséquence, après la brisure de symétrie électrofaible, ceci induit la matrice de masse suivante pour les neutrinos,

$$m_{\nu\alpha\beta} = \frac{1}{2} \mu f_{\alpha\beta} \frac{v^2}{M_\Delta^2} \; . \tag{II.7}$$

Là encore, la petitesse des masses des neutrinos est reliée à la très grande masse du triplet.

Enfin le seesaw de type III [123] repose sur l'introduction de triplets fermioniques auto-conjugués. Ce mécanisme est très similaire au type I : le Lagrangien impliqué dans le seesaw de type III est

$$\mathcal{L}_{\text{seesaw III}} = -y^{\Sigma}_{i\alpha} \bar{\vec{\Sigma}}_i . (H \vec{\sigma} \ell_\alpha) - \frac{1}{2} M_i \, (\vec{\Sigma}_i^T C) . \vec{\Sigma}_i + \text{h.c.} \; , \tag{II.8}$$

ce qui donne pour la matrice de masse des neutrinos

$$m_{\nu\alpha\beta} = \sum_i \frac{y^{\Sigma}_{i\alpha} y^{\Sigma}_{i\beta}}{M_i} v^2 \; . \tag{II.9}$$

Une observation expérimentale permettrait d'affirmer avec certitude que les neutrinos sont des fermions de Majorana. Certains noyaux d'atomes sont instables et se désintègrent en émettant deux particules beta. Cela peut s'accompagner de l'émission de deux antineutrinos, que les neutrinos soient des fermions de Dirac ou de Majorana. Cela peut aussi se faire sans émission d'antineutrino (on parle alors de désintégration beta sans neutrino), mais seulement si les neutrinos sont des particules de Majorana. Le mécanisme est montré par la figure i. En revanche, si les neutrinos sont des particules de Dirac, il n'existe malheureusement pas de moyen de le prouver.

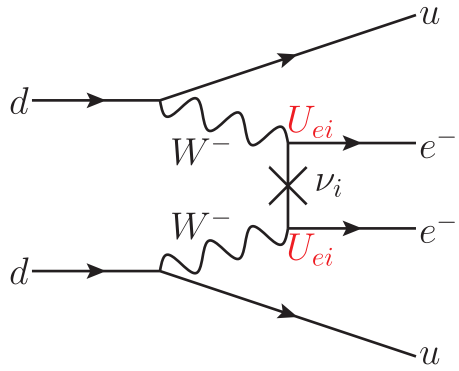

Figure i: Double désintégration beta sans neutrino. La croix indique une insertion de masse de Majorana.

III La leptogenèse avec un triplet scalaire

En dépit de l'apparente similitude entre particules et antiparticules, ces dernières sont presque absentes de notre univers. Cette différence entre les densités de matière et d'antimatière est mesurée par l'asymétrie baryonique, pour laquelle l'expérience Planck [113] donne la mesure la plus précise à ce jour,

$$\frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.10 \pm 0.06) \times 10^{-10} \qquad (68\% \text{ C.L.}) \ . \tag{III.1}$$

Ceci n'est pas sans poser problème : en effet, on pourrait imaginer que cette asymétrie n'est que le résultat des conditions initiales de notre univers. Cependant, il existe de nombreux indices en faveur de l'inflation, une phase lors de laquelle l'expansion de l'univers aurait tellement accéléré que le facteur d'échelle aurait été multiplié par un facteur gigantesque (typiquement e^{50}). Ce phénomène aurait tellement dilué toutes les particules que leur densité serait devenue négligeable et que n'importe quelle asymétrie pré-existante aurait été effacée. Dans ce cas, l'asymétrie baryonique ne pourrait avoir été créée qu'après l'inflation. Il s'agirait alors d'un processus dynamique, la baryogenèse. Andreï Sakharov a établi en 1967 les trois conditions nécessaires pour générer une asymétrie entre baryons et antibaryons [137].

1. Le nombre baryonique ne doit pas être conservé.

2. Les symétries discrètes C et CP, qui relient particules et antiparticules, doivent être brisées.

3. La création de l'asymétrie doit avoir lieu hors de l'équilibre thermodynamique.

Dans le modèle Standard, il est connu que C et CP sont violées. Le nombre baryonique, bien qu'apparemment conservé, est en fait également violé par des effets non perturbatifs, les sphalérons, qui sont liés aux anomalies électrofaibles [139]. Plus précisément, les sphalérons violent le nombre baryonique et le nombre leptonique mais conservent la combinaison $B - L$. Cependant, le Modèle Standard ne réunit pas les conditions nécessaires à la création d'une asymétrie baryonique : en effet, pour que les sphalérons électrofaibles agissent hors d'équilibre, la masse du boson de Higgs devrait être inférieure à environ 45 GeV [144, 145], ce qui est très en-deçà de sa valeur mesurée au LHC.

Ceci indique que la baryogenèse dépasse le cadre du Modèle Standard. Parmi les scénarios possibles, la leptogenèse [163] est un mécanisme élégant qui relie l'asymétrie baryonique aux masses des neutrinos : en effet, les divers mécanismes de seesaw évoqués auparavant font intervenir des champs lourds dont la désintégration viole le nombre leptonique. Si leur désintégration viole également CP et a lieu hors d'équilibre, elle peut générer une asymétrie entre leptons et antileptons, que les sphalérons vont ensuite partiellement convertir en une asymétrie entre baryons et antibaryons. Le scénario historiquement le plus étudié fait intervenir des neutrinos droits dans le cadre du seesaw de type I [163, 165].

Dans cette thèse, nous nous intéressons aux scénarios de leptogenèse faisant intervenir un triplet scalaire Δ, par ailleurs responsable des masses des neutrinos via le mécanisme de seesaw de type II [199–204]. Malheureusement, en l'absence d'autres nouveaux champs, la désintégration du triplet scalaire viole bien le nombre leptonique, mais pas CP (ou du moins pas suffisamment). Pour remédier à ceci, nous supposons dans un premier temps qu'il existe d'autres nouveaux états plus lourds que le triplet (un autre triplets, des neutrinos droits...), et dont la principale manifestation est l'opérateur effectif suivant,

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\frac{\kappa_{\alpha\beta}}{\Lambda}(\ell_\alpha^T i\sigma_2 H)C(H^T i\sigma_2 \ell_\beta)\ . \tag{III.2}$$

Ceci contribue à la matrice de masse des neutrinos, qui s'écrit maintenant

$$m_{\nu\alpha\beta} = \frac{1}{2}\mu f_{\alpha\beta}\frac{v^2}{M_\Delta^2} + \frac{1}{2}\kappa_{\alpha\beta}\frac{v^2}{\Lambda}\ . \tag{III.3}$$

Avec cet opérateur et en raison de l'interférence entre les deux diagrammes de la figure

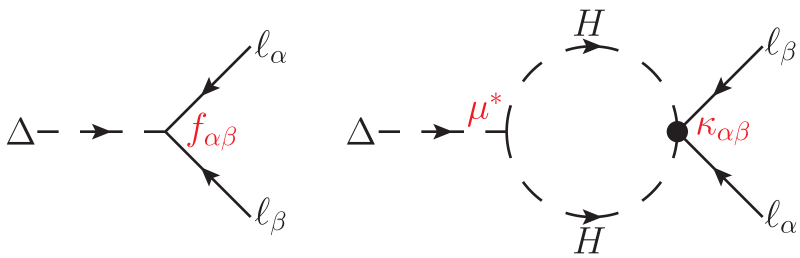

Figure ii: Diagrammes responsables de la violation de CP dans la désintégration de Δ.

ii, la désintégration du triplet viole CP. L'asymétrie CP est définie comme

$$\begin{aligned}\epsilon^\Delta &= 2\frac{\Gamma(\Delta^c \to \ell\ell) - \Gamma(\Delta \to \ell^c\ell^c)}{\Gamma_{\Delta^c} + \Gamma_\Delta} \\ &= \frac{1}{8\pi\Lambda}\frac{\Im\left[\mu^* \text{tr}(f^\dagger\kappa)\right]}{\lambda_\ell^2 + \lambda_H^2}\ ,\end{aligned} \tag{III.4}$$

où nous avons défini $\lambda_\ell = \sqrt{\mathrm{tr}(ff^\dagger)}$ et $\lambda_H = \mu/M_\Delta$. Le déroulement de la leptogenèse est décrit par un système d'équations de Boltzmann, décrivant l'évolution de la densité de triplets $\Sigma_\Delta = Y_\Delta + Y_{\Delta^c}$ ainsi que des asymétries $\Delta_a = Y_a - Y_{a^c}$ (où Y_a est la densité numérique normalisée par rapport à la densité d'entropie $Y_a = n_a/s$). L'équation décrivant la densité de triplets est la suivante [203],

$$sHz\frac{d\Sigma_\Delta}{dz} = -\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - 2\left[\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}}\right)^2 - 1\right]\gamma_A^\Delta \ . \tag{III.5}$$

Le premier terme représente les désintégrations, tandis que le second représente les annihilations dues aux interactions de jauge.

Une question importante ici concerne le traitement des différentes saveurs [178–182]. Un traitement naïf consisterait à écrire trois équations différentes pour les trois asymétries leptoniques stockées dans les saveurs e, μ et τ. Cependant, aux échelles de températures où la leptogenèse prend place, les interactions induites par les couplages de Yukawa ne sont pas nécessairement à l'équilibre thermodynamique. Or, dans le Modèle Standard, seules ces interactions distinguent les trois générations de leptons. Si elles sont trop lentes, les trois familles deviennent indiscernables. En conséquence, la désintégration de Δ crée des états qui sont des superpositions quantiques des trois saveurs leptoniques, qui restent cohérentes sur les échelles de temps typiques du problème. Dans ce cas, l'asymétrie leptonique devrait être décrite par une matrice densité plutôt que par trois quantités numériques indépendantes. Une façon d'obtenir cette matrice densité est de remarquer que l'asymétrie stockée dans une certaine saveur ℓ_α peut s'écrire de la manière suivante,

$$\Delta_{\ell_\alpha} = \frac{1}{s}\langle :\ell_\alpha^\dagger \ell_\alpha: \rangle \ . \tag{III.6}$$

On peut généraliser cette expression pour en faire une matrice dans l'espace des saveurs

$$(\Delta_\ell)_{\alpha\beta} = \frac{1}{s}\langle :\ell_\alpha^\dagger \ell_\beta: \rangle \ . \tag{III.7}$$

Ceci permet de voir que les éléments diagonaux de la matrice densité correspondent aux asymétries “classiques” stockées dans les différentes saveurs. En revanche, les éléments hors-diagonale représentent des corrélations purement quantiques entre les différentes saveurs. L'asymétrie leptonique totale est donnée par la trace de cette matrice, qui est invariante sous changement de base dans l'espace des saveurs : sous une transformation $\ell \to U\ell$ (U étant une matrice unitaire), la matrice densité se transforme selon $\Delta_\ell \to U^\dagger \Delta_\ell U$, donc sa trace est inchangée. Cette propriété est importante, car dans un régime où l'on ne peut distinguer les saveurs, il doit être possible de choisir n'importe quelle base pour mener le calcul, sans que le résultat dépende de ce choix. Cette condition constitue également un test de cohérence important pour l'équation de Boltzmann décrivant la matrice densité, qui doit être covariante de saveur, c'est-à -dire qu'elle doit se transformer comme Δ_ℓ sous la transformation $\ell \to U\ell$ évoquée précédemment.

Nous sommes a priori intéressés par les asymétries stockées dans les leptons, les doublets de Higgs et les triplets. Cependant, ces quantités ne sont pas indépendantes et l'équation décrivant l'asymétrie Δ_H peut être éliminée. Lorsque la température de l'univers est supérieure à 10^{12} GeV, les trois saveurs sont indiscernables et les équations décrivant les asymétries prennent la forme très générale suivante

$$sHz\frac{d(\Delta_\ell)_{\alpha\beta}}{dz} = \epsilon_{\alpha\beta}\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - W_{\alpha\beta} \ , \tag{III.8}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -W_\Delta \ . \tag{III.9}$$

$\epsilon_{\alpha\beta}$ est la généralisation de l'asymétrie CP, responsable du terme source pour l'asymétrie leptonique. Les W sont des termes de dilution, qui tendent à faire diminuer les asymétries. La covariance implique que, sous une transformation unitaire $\ell \to U\ell$, ϵ et W se transforment selon

$$\epsilon \to U^\dagger \epsilon U \ , \qquad W \to U^\dagger W U \ . \tag{III.10}$$

Lorsque la température devient inférieure à 10^{12} GeV, la saveur τ devient discernable, tandis que e et μ restent indiscernables. En conséquence, l'asymétrie Δ_{ℓ_τ} peut être décrite par une équation à part, tandis que la matrice densité se réduit au bloc 2×2 décrivant l'espace $e - \mu$,

$$sHz\frac{d(\Delta_\ell)_{\alpha\beta}}{dz} = \epsilon_{\alpha\beta}\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - W_{\alpha\beta} \ , \qquad \alpha, \beta = e, \mu \tag{III.11}$$

$$sHz\frac{d\Delta_{\ell_\tau}}{dz} = \epsilon_\tau\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - W_\tau \ , \tag{III.12}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -W_\Delta \ . \tag{III.13}$$

Enfin, en-dessous de 10^9 GeV, les trois saveurs leptoniques sont discernables et toutes les corrélations quantiques disparaissent. Dans ce régime, les équations de Boltzmann prennent un forme plus intuitive,

$$sHz\frac{d\Delta_{\ell_e}}{dz} = \epsilon_e\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - W_e \ , \tag{III.14}$$

$$sHz\frac{d\Delta_{\ell_\mu}}{dz} = \epsilon_\mu\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - W_\mu \ , \tag{III.15}$$

$$sHz\frac{d\Delta_{\ell_\tau}}{dz} = \epsilon_\tau\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\mathrm{eq}}} - 1\right)\gamma_D^\Delta - W_\tau \ , \tag{III.16}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -W_\Delta \ . \tag{III.17}$$

Dans ces équations, les termes de dilution font intervenir de nombreuses réactions, et en particulier les diffusions qui modifient le nombre leptonique, telles que $\ell\ell \leftrightarrow HH$. Jusqu'à maintenant, nous avons négligé d'autres processus, désignés comme "processus spectateurs" [188–190]. La raison de cette appellation est que ces réactions, provenant du Modèle Standard, ne modifient pas directement $B - L$. Cependant, elles influent tout de même sur la dynamique du problème. Les processus spectateurs sont par exemple les interactions de Yukawa, qui transfèrent une partie de l'asymétrie dans les leptons droits, ou les sphalérons électrofaibles qui entrent en jeu lorsque la température devient inférieure à 10^{12} GeV. Décrire exactement toutes ces réactions est une tâche difficile, mais une bonne approximation consiste à considérer que, selon le domaine de température, elle sont soit très lentes et donc négligeables, soit suffisamment rapides pour atteindre l'équilibre thermodynamique. Dans ce dernier cas, elles imposent des relations entre les différents potentiels chimiques qui permettent de les prendre en compte sans calculer de taux de réaction [207]. Il faut toutefois remarquer que, si l'on veut prendre en compte les processus spectateurs, il est préférable de considérer l'asymétrie contenue dans $B - L$, qui n'est pas affectée par ces processus, plutôt que l'asymétrie stockée dans les doublets de leptons. Dans le domaine de température $T > 10^{12}$ GeV par exemple, on considèrera la matrice densité suivante,

$$\Delta_{\alpha\beta} = \frac{\Delta_B}{3}\delta_{\alpha\beta} - (\Delta_\ell)_{\alpha\beta} \ , \tag{III.18}$$

qui satisfait l'équation de Boltzmann,

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = \epsilon_{\alpha\beta}\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\rm eq}} - 1\right)\gamma_D^\Delta - W_{\alpha\beta} \ . \tag{III.19}$$

L'étude numérique de ce modèle montre que la prise en compte des effets de saveurs peut avoir des conséquences importantes, en particulier si l'on compare les résultats avec ceux obtenus en faisant l'approximation à une seule saveur : dans certains cas, les résultats diffèrent d'un facteur 100 (voir par exemple la figure 3.8). Dans tous les cas, pour reproduire l'asymétrie baryonique observée, ce scénario nécessite un triplet scalaire très lourd, avec une masse au moins de l'ordre de 10^{10} GeV, comme le montre la figure 3.9 où l'asymétrie prédite est tracée en fonction de $\lambda_\ell = \sqrt{\mathrm{tr}(ff^\dagger)}$ et de M_Δ.

Nous étudions également dans cette thèse un scénario inspiré d'une théorie de grande unification basée sur le groupe de jauge $SO(10)$ [211]. Le contenu en particules de cette théorie est résumé dans le tableau ii. Les particules impliquées dans la leptogenèse sont, en plus des particules du Modèle Standard, des paires vectorielles de doublets de leptons lourds notées $(\mathcal{L}_i, \bar{\mathcal{L}}_i)$, un triplet scalaire qui participe au mécanisme de seesaw de type II, ainsi que deux scalaires supplémentaires, un triplet T et un singlet S. Contrairement au scénario générique exposé précédemment, dans ce modèle, la masse des neutrinos provient uniquement du seesaw de type II. Ceci est d'autant plus intéressant qu'en raison de la symétrie $SO(10)$, les couplages des différents scalaires aux doublets de leptons sont tous proportionnels à une même matrice $f_{\alpha\beta}$, qui est elle-même reliée à la matrice de masse des neutrinos. Ainsi il est possible d'exprimer l'asymétrie CP ϵ dans la désintégration des trois scalaires Δ, S et T en fonction des masses des neutrinos du modèle Standard, des angles de mélange et des phases de la matrice PMNS.

Ce modèle contient en revanche un plus grand nombre de particules, ce qui rend l'étude des équations de Boltzmann plus compliquée. Nous faisons ici l'hypothèse que les différents scalaires ont la même masse $M_\Delta = M_S = M_T$. Pour que l'asymétrie CP soit non nulle, il est nécessaire qu'au moins une paire de leptons lourds ait une masse

Représentation	Champ	$SU(3) \times SU(2)_L \times U(1)_Y$	B	L
16_i	Q_i	$(3, 2, +1/6)$	$+1/3$	0
	u_i^c	$(3, 1, -2/3)$	$-1/3$	0
	e_i^c	$(1, 1, +1)$	0	-1
	$\mathcal{L}_i$	$(1, 2, -1/2)$	0	$+1$
	$\bar{\mathcal{D}}_i$	$(3, 1, +1/3)$	$-1/3$	0
10_i	ℓ_i	$(1, 2, -1/2)$	0	$+1$
	d_i^c	$(3, 1, +1/3)$	$-1/3$	0
	$\bar{\mathcal{L}}_i$	$(1, 2, +1/2)$	0	-1
	$\mathcal{D}_i$	$(3, 1, -1/3)$	$+1/3$	0
54	Δ	$(1, 3, +1)$	0	0
	Δ^c	$(1, 3, -1)$	0	0
	T	$(1, 3, 0)$	0	0
	S	$(1, 1, 0)$	0	0

Table ii: Les différentes représentations et particules du modèle.

inférieure à M_Δ, et que les troix paires ne soient pas simultanément beaucoup plus légères que M_Δ. Nous supposons ici que la paire $(\mathcal{L}_1, \bar{\mathcal{L}}_1)$ a une masse nettement plus petite que M_Δ, tandis que les autres sont bien plus lourdes et découplées à l'échelle de la leptogenèse. Ceci permet à la fois de garantir que l'asymétrie CP est non nulle et de simplifier les équations de boltzmann. Sous ces conditions, l'asymétrie CP dans la désintégration de Δ est donnée par

$$\epsilon^\Delta = \frac{B_L}{4\pi\lambda_\ell^2} \Im\left[f_{11}\left(f^\dagger f f^\dagger\right)_{11}\right]\left[c_S^2 \tilde{G}\left(\frac{M_S}{M_\Delta}\right) + \frac{c_T^2}{2}\tilde{G}\left(\frac{M_T}{M_\Delta}\right)\right] , \qquad \text{(III.20)}$$

et l'on peut réécrire ceci en utilisant

$$\Im\Big[f_{11}(f^\dagger f f^\dagger)_{11}\Big] = \frac{\lambda_\ell^4}{\bar{m}_\nu^4}\Big(-m_{\nu 1}m_{\nu 2}\Delta m_{21}^2 c_{12}^2 c_{13}^4 s_{12}^2 \sin 2\rho$$
$$+m_{\nu 1}m_{\nu 3}\Delta m_{31}^2 c_{12}^2 c_{13}^2 s_{13}^2 \sin 2(\sigma-\rho) + m_{\nu 2}m_{\nu 3}\Delta m_{32}^2 c_{13}^2 s_{12}^2 s_{13}^2 \sin 2\sigma\Big) \ . \qquad \text{(III.21)}$$

Un grand nombre de processus de diffusions participent à la dilution de l'asymétrie, mais si les couplages ne sont pas trop grands (ce qui est les cas en général), ces processus peuvent être négligés, et les équations de Boltzmann prennent la forme suivante (on se concentre ici sur le régime de haute température $T > 10^{12}\,\text{GeV}$),

$$sHz\frac{d\Sigma_\Delta}{dz} = -\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\text{eq}}} - 1\right)\gamma_D^\Delta - 2\left[\left(\frac{\Sigma_\Delta}{\Sigma_\Delta^{\text{eq}}}\right)^2 - 1\right]\gamma_A^\Delta , \tag{III.22}$$

$$sHz\frac{dY_T}{dz} = -\left(\frac{Y_T}{Y_T^{\text{eq}}} - 1\right)\gamma_D^T - 2\left[\left(\frac{Y_T}{Y_T^{\text{eq}}}\right)^2 - 1\right]\gamma_A^T , \tag{III.23}$$

$$sHz\frac{dY_S}{dz} = -\left(\frac{Y_S}{Y_S^{\text{eq}}} - 1\right)\gamma_D^S , \tag{III.24}$$

$$sHz\frac{d\Delta_{\alpha\beta}}{dz} = -S_{\alpha\beta} + W_{\alpha\beta}^{ID} , \tag{III.25}$$

$$sHz\frac{d\Delta_{\bar{\mathcal{L}}_1}}{dz} = \text{tr}\,[S] - W_{\bar{\mathcal{L}}}^{ID} , \tag{III.26}$$

$$sHz\frac{d\Delta_\Delta}{dz} = -\frac{1}{2}\,\text{tr}\left[W^{ID}\right] + \frac{1}{2}W_{\bar{\mathcal{L}}}^{ID} - B_H\left(\frac{\Delta_\Delta}{\Sigma_\Delta^{\text{eq}}} - \frac{C_H^k\Delta_k}{Y_H^{\text{eq}}}\right)\gamma_D^\Delta . \tag{III.27}$$

L'exposant ID signifie que nous ne retenons ici que les processus de dilutions dûs aux désintégrations inverses.

Ici, la leptogenèse ne génère une asymétrie baryonique suffisament grande qu'à condition que le triplet soit plus lourd que 10^{12} GeV. Ce modèle mène par ailleurs à des prédictions intéressantes. Par exemple, il est possible de tracer l'asymétrie baryonique obtenue en fonction de la masse du neutrino le plus léger ou de la masse de Majorana effective impliquée dans la double désintégration beta sans neutrino. La condition que la leptogenèse permette de reproduire l'asymétrie baryonique observée permet de placer une borne supérieure sur ces paramètres autours de $10^{-1}\,\text{eV}$ dans le cas de la hiérarchie normale, comme on peut le voir sur la figure 3.13. La figure 3.14, qui montre l'asymétrie baryonique en fonction de la masse du plus léger neutrino m_{ν_1} et de $\sin^2\theta_{13}$, indique que la valeur mesurée récemment de $\sin^2\theta_{13} \simeq 0.023$ est favorable à ce scénario, alors qu'une valeur plus petite l'aurait exclu. Enfin, la hiérarchie inverse serait très défavorable à ce scénario puisque la région de l'espace des paramètres qui permet de reproduire l'asymétrie baryonique y est très restreinte (voir figure 3.15).

IV Introduction à la supersymétrie

Dans la suite, nous nous plaçons dans le cadre des théories supersymétriques. La supersymétrie relie bosons et fermions [212–217, 220–223]. Ses générateurs sont donc des spineurs, qui étendent l'algèbre de Poincaré en superalgèbre de Poincaré. Ces générateurs, que l'on note ici Q_α et $\bar{Q}^{\dot{\alpha}}$ ($\alpha, \dot{\alpha} = 1, 2$) satisfont les relations d'anticommutation suivantes [219],

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}}P_\mu , \tag{IV.1}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 , \tag{IV.2}$$

les $\sigma^\mu_{\alpha\dot{\alpha}}$ étant les matrices de Pauli. Il peut en principe y avoir plusieurs paires de tels générateurs, mais on se restreint ici au cas dit $N = 1$. Une représentation explicite de ces générateurs peut être obtenue en ajoutant des coordonnées grassmanniennes aux coordonnées d'espace-temps usuelles. Ces coordonnées, notées θ^α et $\bar{\theta}_{\dot{\alpha}}$, sont des quantités spinorielles qui anticommutent. À cause de ceci, n'importe quelle fonction définie sur le superespace peut s'écrire comme un polynôme en $\theta, \bar{\theta}$ d'ordre au plus $\theta^4 = \epsilon_{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\theta^\alpha\theta^\beta\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}}$.

Les générateurs de la supersymétrie peuvent s'écrire de la manière suivante,

$$Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu \ , \tag{IV.3}$$

$$\bar{Q}^{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + (\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu \ . \tag{IV.4}$$

On définit également des dérivées chirales,

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu \ , \tag{IV.5}$$

$$\bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} + i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu \ . \tag{IV.6}$$

Bosons et fermions sont regroupés au sein de superchamps, qui comptent le même nombre de degrés de liberté fermioniques et bosoniques. Un superchamp chiral Φ est défini par la condition $\bar{D}^{\dot{\alpha}}\Phi = 0$. Il prend une expression plus simple en fonction des coordonnées $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ et θ:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \ . \tag{IV.7}$$

ϕ est un champ scalaire, ψ un fermion de Weyl et F un champ auxiliaire qui ne se propage pas. Un superchamp réel ou vectoriel V est quant à lui défini par la condition $V = V^\dagger$. En plus de cette condition, il est possible de procéder à une transformation de jauge, en remarquant que la condition de réalité $V = V^\dagger$ n'est pas affectée par la transformation $V \to V + i\left[\Lambda^\dagger - \Lambda\right]$, où Λ est un superchamp chiral. Cette transformation permet d'éliminer certaines composantes. Dans la jauge de Wess-Zumino, le superchamp vectoriel prend l'expression suivante,

$$V_{WZ}(x,\theta,\bar{\theta}) = \bar{\theta}\bar{\sigma}^\mu\theta A_\mu(x) - i\bar{\theta}^2\theta\lambda(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) + \frac{1}{2}\theta^4 D(x) \ . \tag{IV.8}$$

A est un champ vectoriel (le boson de jauge), λ un fermion de Weyl (le jaugino) et D un champ auxiliaire semblable à F. Cette transformation de jauge généralise les transformations de jauge des champs vectoriels dans les théories non supersymétriques. En particulier, on peut se ramener à ce dernier cas en choisissant un superchamp chiral Λ dont seule la composante scalaire est non nulle.

Un lagrangien supersymétrique fait intervenir trois fonctions des superchamps.

(i) Le potentiel de Kähler $K(\Phi^\dagger, \Phi)$ est une fonction des superchamps chiraux, qui détermine leurs termes cinétiques. En particulier, dans une théorie exactement supersymétrique et renormalisable, la forme canonique du potentiel de Kähler est simplement

$$K = \Phi_i^\dagger\Phi_i \ . \tag{IV.9}$$

L'intégrale du potentiel de Kähler sur les coordonnées grassmaniennes donne les termes cinétiques du lagrangien,

$$\int d^4\theta \Phi_i^\dagger \Phi_i = F_i^\dagger F_i + \partial_\mu \phi_i^\dagger \partial^\mu \phi_i - i\bar{\psi_i}\bar{\sigma}^\mu \partial_\mu \psi_i \ . \tag{IV.10}$$

Dans une théorie de jauge supersymétrique, les termes décrivant les interactions de jauge ainsi que les interactions jaugino-scalaire-fermion se déduisent également du potentiel de Kähler. Il faut alors généraliser l'expression précédente de la manière suivante,

$$\begin{aligned}\int d^4\theta K(\Phi_i^\dagger, e^{2gt_A V_A}\Phi_i) = {} & F_i^\dagger F^i + D_\mu \phi_i^\dagger D^\mu \phi_i - i\psi_i^\dagger \bar{\sigma}^\mu D_\mu \psi_i \\ & + i\sqrt{2}g\phi_i^\dagger t_A \psi_i \lambda_A - i\sqrt{2}g\bar{\lambda}_A^\dagger \psi_i^\dagger t_A \phi_i + g\phi_i^\dagger t_A \phi_i D_A \ ,\end{aligned} \tag{IV.11}$$

où les t_A sont les générateurs de l'algèbre de Lie.

(ii) La fonction cinétique de jauge détermine les termes cinétique des superchamps vectoriels. Elle se construit à partir d'un superchamp chiral $\mathcal{W}_\alpha$, qui s'exprime en fonction du superchamp vectoriel et dont l'expression varie selon que la théorie est abélienne ou non. Dans une théorie abélienne, $\mathcal{W}_\alpha$ est défini de la manière suivante

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \ . \tag{IV.12}$$

Les termes cinétiques du champs de jauge et du jaugino s'obtiennent de la manière suivante,

$$\int d^2\theta \mathrm{tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha]_F + \mathrm{h.c.} = \frac{1}{2}D^2 - i\lambda\sigma^\mu D_\mu \bar{\lambda} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ . \tag{IV.13}$$

Dans une théorie non-abélienne, ces expressions sont légèrement modifiées pour tenir compte du fait que dans ce cas, les champs de jauge interagissent entre eux.

(iii) Le superpotentiel est une fonction holomorphe des superchamps chiraux. Il détermine les interactions de type Yukawa ainsi qu'une partie du potentiel scalaire de la théorie, l'autre partie provenant de l'intégration des D-termes des champs de jauge. Dans une théorie renormalisable, le superpotentiel est donné par

$$W = L_i \Phi_i + \frac{1}{2}M_{ij}\Phi_i\Phi_j + \frac{1}{6}\lambda_{ijk}\Phi_i\Phi_j\Phi_k \ . \tag{IV.14}$$

Le premier terme n'est possible que pour les singlets de jauge. Nous n'en tenons pas compte ici. Le Lagrangien associé est

$$\int d^2\theta W + \mathrm{h.c.} = W_i F_i - W_{ij}\psi_i\psi_j + \mathrm{h.c.} \ , \tag{IV.15}$$

où nous avons utilisé les notations suivantes,

$$W_i = \frac{\partial W}{\partial \Phi_i}_{|\Phi_i = \phi_i}, \qquad W_{ij} = \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}_{|\Phi_i = \phi_i, \Phi_j = \phi_j} \ . \tag{IV.16}$$

Nous pouvons déduire de ceci ainsi que des termes cinétiques l'équation qui permet d'éliminer les F-termes, ce qui permet d'écrire le potentiel scalaire de la théorie comme

$$\mathcal{V}(\phi_i) = F_i^\dagger F_i + \frac{1}{2} D_A D_A \ . \tag{IV.17}$$

Le superpotentiel a une propriété très importante : les paramètres qui y apparaissent n'ont pas besoin d'être renormalisés, ainsi que l'affirme un théorème de non-renormalisation [230, 231]. En conséquence, les divergences dans les diagrammes de boucle impliquant des superchamps chiraux ne peuvent provenir que de la renormalisation de fonction d'onde, et sont au plus logarithmiques.

La supersymétrie, si elle est effectivement réalisée, doit être brisée. En effet, dans le cas contraire, les particules d'un même supermultiplet ont nécessairement la même masse, ce qui implique que les superpartenaires scalaires des fermions du Modèle Standard (par exemple) auraient dû être découvert depuis longtemps. La brisure de supersymétrie est un sujet complexe, mais elle peut être paramétrisée de manière simple en introduisant dans le Lagrangien des termes de brisure douce : ces termes impliquent seulement une des composantes de chaque superchamp. De plus, ils doivent préserver la non-renormalisation des paramètres du superpotentiel (d'où l'appellation brisure douce). Les termes de brisure douce les plus génériques sont les suivants [232],

$$\mathcal{L}_{\text{soft}} = -(m^2)_{ij} \phi_i^\dagger \phi_j - \left(\frac{1}{2} B_{ij} \phi_i \phi_j + \frac{1}{6} A_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} M_A \lambda_A \lambda_A + \text{h.c.} \right) \ . \tag{IV.18}$$

Les deux premiers termes entre parenthèses sont appelés B-termes et A-termes. Nous mentionnons également ici une autre possiblité : s'il existe un superchamp chiral dans la représentation conjuguée d'un groupe de jauge, il peut y avoir terme de masse mélangeant jaugino et fermion (on parle alors de jaugino de Dirac) [233],

$$\mathcal{L} = M_A \bar{\lambda}_A \psi_A + \text{h.c.} \ . \tag{IV.19}$$

Dans le MSSM, les fermions du Modèle Standard font partie de superchamps chiraux, tandis que les bosons de jauge font partie de superchamps vectoriels. Il y a deux superchamps de Higgs H_u et H_d, qui sont des superchamps chiraux. Le premier a les même nombres quantiques que le doublet de Higgs du Modèle Standard, tandis que le second a les mêmes nombres quantiques qu'un doublet de lepton. La présence de deux doublets de Higgs est nécessaire entre autres raisons pour garantir que la théorie n'a pas d'anomalie. Le superpotentiel renormalisable le plus général que l'on peut en principe écrire avec ce contenu est le suivant,

$$\begin{aligned} W = {} & y^u_{ij} H_u Q_i u^c_j + y^d_{ij} H_d Q_i d^c_j + y^e_{\alpha\beta} H_d \ell_\alpha e^c_\beta + \mu H_u H_d \\ & + \frac{1}{2} \lambda_{\alpha\beta\gamma} \ell_\alpha \ell_\beta e^c_\gamma + \lambda'_{\alpha ij} \ell_\alpha Q_i d^c_j + \frac{1}{2} \lambda''_{ijk} u^c_i d^c_j d^c_k + \mu_\alpha H_u \ell_\alpha \ . \end{aligned} \tag{IV.20}$$

Les termes de la première ligne donnent lieu aux couplages de Yukawa qui génèrent les masses des fermions du Modèle Standard, ainsi que le terme de masse du potentiel de Higgs. Les termes de la seconde ligne sont problématiques, car ils induisent la désintégration du proton qui est un processus sévèrement contraint. Une des solutions à ce problème est d'introduire une symétrie discrète, la R-parité [222], qui les interdit. Alternativement, on peut aussi interdire un seul de ces types de couplage, comme nous allons le voir par la suite.

V Pseudo-fermions de Goldstone et neutrinos stériles

Dans cette partie, nous étudions un scénario où une symétrie $U(1)$ brisée à la fois explicitement et spontanément à très haute énergie génère un pseudo-superchamp de Goldstone,

$$A = \frac{s + ia}{\sqrt{2}} + \sqrt{2}\theta\chi + \theta^2 F \ . \tag{V.1}$$

Comme la symétrie est brisée explicitement, les composantes de ce superchamp sont massives. La brisure de supersymétrie lève ensuite la dégénérescence entre les masses du scalaire s du pseudo-boson de Goldstone a et du fermion χ. En particulier, on suppose que ce dernier a une masse de l'ordre d'un électronvolt, tandis que les scalaires sont bien plus lourds.

La R-parité est brisée mais le nombre baryonique est préservé. Par conséquent, seuls le couplage λ'' de l'équation (IV.20) est interdit. En l'absence de R-parité, rien ne distingue le superchamp H_d des superchamps leptonique, aussi il est pratique d'introduire la notation suivante, $\ell_a = (H_d, \ell_\alpha)$ $(\alpha = e, \mu, \tau)$. En partant de ces principes, le potentiel de Kähler non-renormalisable le plus général est le suivant à l'ordre $1/f$,

$$\begin{aligned} K = {} & A^\dagger A + H_u^\dagger H_u + \ell_a^\dagger \ell_a + C_u H_u^\dagger H_u \frac{A + A^\dagger}{f} \\ & + C_{ab} \ell_a^\dagger \ell_b \frac{A + A^\dagger}{f} + \left(C_{ua} H_u . \ell_a \frac{A + A^\dagger}{f} + \text{h.c.} \right) + \mathcal{O}\left(\frac{1}{f^2}\right) \ , \end{aligned} \tag{V.2}$$

tandis que le superpotentiel est donné par

$$W = y_{ij}^u H_u Q_i u_j^c + y_{aij}^d \ell_a Q_i d_j^c + y_{ab\gamma}^e \ell_a \ell_b e_\beta^c + \mu_a H_u \ell_a \ . \tag{V.3}$$

Il faut cependant noter que les couplages qui violent la R-parité doivent rester suffisamment petits pour des raisons phénoménologiques.

En raison de la brisure de R-parité, les leptons chargés se mélangent avec le jaugino $\tilde{W}^-$, ainsi qu'avec les Higgsinos $\tilde{H}_d^-$ et $(\tilde{H}_u^+)^\dagger$ (les états propres de masses issus de ce mélange sont appelés charginos), tandis que les neutrinos se mélangent avec les jauginos $\tilde{Z}^0$ et $\tilde{\gamma}$, ainsi qu'avec les higgsinos $\tilde{H}_d^0$ et $\tilde{H}_u^0$ (les états propres de masses issus de ce mélange sont appelés neutralinos). Le singlet fermionique χ participe également à ce mélange et joue le rôle d'un neutrino stérile. À l'ordre des arbres, les matrices de masses des charginos et neutralinos reçoivent trois types de contributions.

1. Certains termes peuvent s'obtenir directement à partir du Lagrangien. Le superpotentiel donne les contributions suivantes,

$$M_{ij}^W = \langle W_{ij} \rangle \ , \tag{V.4}$$

tandis que les jauginos et le singlet χ reçoivent leur masse des termes de brisure douce.

2. Le potentiel de Kähler contribue également aux masses des fermions, car il contient des termes non-canoniques.

3. En raison de la brisure de symétrie électrofaible, les jauginos de $SU(2) \times U(1)$ se mélangent avec les higgsinos et les leptons. De plus, à cause des termes non canoniques du potentiel de Kähler, le pseudo-fermion de Goldstone χ se mélange également avec les jauginos.

En fin de compte, il est possible de définir une base où la matrice de masse des leptons chargés est diagonale et les sneutrinos ont une valeur moyenne nulle dans le vide. Dans cette base, la matrice de masse 8×8 des neutralinos ($\tilde{Z}^0$, $\tilde{\gamma}$, $\tilde{H}_u^0$, $\tilde{H}_d^0$, ν_α, χ) a la structure suivante,

$$M_N = \begin{pmatrix} M_{4\times4} & \mu_{4\times4} \\ \mu^T_{4\times4} & m_{4\times4} \end{pmatrix} . \tag{V.5}$$

Cette matrice a une structure de seesaw car les entrées du bloc M, qui mélange les higgsinos et les jauginos, sont bien plus grandes que celles des autres blocs. Les 4 états propres de masse légers, que l'on peut assimiler aux neutrinos actifs accompagnés du pseudo-fermion de Goldstone χ (le neutrino stérile), ont donc la matrice de masse suivante,

$$m_\nu = m - \mu^T M^{-1} \mu \ , \tag{V.6}$$

que l'ont peut réécrire en termes de paramètres effectifs,

$$m_\nu = \begin{pmatrix} A\epsilon_\alpha\epsilon_\beta & B\eta_\alpha \\ B\eta_\beta & F \end{pmatrix} \ , \tag{V.7}$$

où $\vec{\epsilon}$ et $\vec{\eta}$ sont deux vecteurs unitaires. Cette structure permet de prédire que le neutrino le plus léger est de masse nulle. Nous cherchons ensuite les valeurs de A, B, F, $\vec{\epsilon}$ et $\vec{\eta}$ qui satisfont les contraintes suivantes : le résultat doit reproduire les Δm^2_{ij} et les éléments de la matrice PMNS $U_{\alpha i}$ pour les neutrinos actifs (i, $j = 1$, 2, 3 et $\alpha, = e$, μ, τ), et il doit satisfaire les contraintes provenant des recherches négatives de neutrinos stériles. Nous nous intéressons ici au cas de figure où la matrice de masse est réelle.

Sur les figures 4.1 et 4.2 , les points satisfaisant uniquement la première contrainte sont tracés en orange, les points satisfaisant les deux en bleu, tandis que les points verts sont en plus compatibles avec les anomalies du gallium et des réacteurs [106, 107, 110]. Ces figures montrent les résultats dans le plan $\sin^2 2\theta_{ee} - \Delta m^2_{41}$, avec $\sin^2 2\theta_{ee} = 4|U_{e4}|^2(1 - |U_{e4}|^2)$, pour les hiérarchies normale et inverse respectivement. On observe en particulier que le neutrino stérile peut avoir une masse et un mélange avec les neutrinos actifs dans la région compatible avec les anomalies, qui pourraient donc être expliquées par ce modèle. La hiérarchie inverse est à ce titre plus favorable que la hiérarchie normale, car elle donne accès à une portion bien plus grande de la région des anomalies. Par ailleurs, la hiérarchie inverse favorise de plus grandes valeurs pour Δm^2_{41} et $\sin^2 2\theta_{ee}$. De futures expériences devraient permettre de confirmer ou d'infirmer l'existence de ces anomalies, et de restreindre l'espace des paramètres autorisés pour ce scénario.

VI Médiation de jauge étendue

Comme il a été mentionné auparavant, la supersymétrie, si elle est effectivement réalisée, doit être brisée. Si les termes de brisures douce fournissent une paramétrisation simple de ce phénomène, une théorie exhaustive devrait inclure un mécanisme complet de brisure de supersymétrie. Ceci n'étant pas sans présenter de difficulté, il est fréquent de paramétriser la brisure de supersymétrie en introduisant un spurion dans un secteur caché. Par un mécanisme de médiation, la brisure est ensuite transmise au secteur visible (le MSSM) et y génère précisément les termes de brisure douce.

On peut déduire l'inégalité suivante des relations de commutation des générateurs de supersymétrie,

$$4P_0 = 4\mathcal{H} = \left[Q_1\bar{Q}_1 + \bar{Q}_1Q_1 + Q_2\bar{Q}_2 + \bar{Q}_2Q_2\right] \ , \tag{VI.1}$$

$\mathcal{H}$ étant le hamiltonien du système. L'énergie du vide est donc

$$E_{\text{vac}} = \langle 0|\,\mathcal{H}\,|0\rangle = \frac{1}{4}\left[\|\bar{Q}_1\,|0\rangle\,\|^2 + \|Q_1\,|0\rangle\,\|^2 + \|\bar{Q}_2\,|0\rangle\,\|^2 + \|Q_2\,|0\rangle\,\|^2\right] \geq 0 \ . \tag{VI.2}$$

Si la supersymétrie est préservée, le vide doit être invariant sous les transformations de supersymétrie, donc l'énergie du vide doit être nulle. Inversement, une brisure de supersymétrie implique que le vide n'est pas invariant, et donc l'expression précédente doit être strictement positive. Comme l'énergie du vide est déterminée par le potentiel scalaire, qui peut s'écrire de la manière suivante,

$$\mathcal{V}(\phi_i) = F_i^\dagger F_i + \frac{1}{2}D_AD_A \ , \tag{VI.3}$$

la brisure de supersymétrie implique qu'au moins un champ auxiliaire prend une valeur non nulle dans le vide.

Les deux principaux modèles de brisure de supersymétrie sont le modèle de Fayet-Iliopoulos, qui implique un superchamp vectoriel associé à une symétrie abélienne [268]. Dans ce modèle, la brisure de supersymétrie peut provenir aussi bien d'un D-terme que d'un F-terme. Le modèle d'O'Raiferteaigh en revanche n'implique que des superchamps chiraux, et la supersymétrie y est brisée par la valeur moyenne non nulle d'un F-terme [269].

Dans les deux cas, il existe une règle de somme, qui se généralise à tout modèle où la supersymétrie est brisée à l'ordre des arbres par des interactions renormalisables. Cette règle impose une contrainte sur la supertrace de la matrice de masse de la théorie : M_J étant la matrice de masse des particules de spin J, on a la relation suivante [270],

$$\sum_J (-1)^{2J}(2J+1)\text{tr}\left(M_J^2\right) = \text{tr}\left(M_S^2\right) - 2\text{tr}\left(M_F^\dagger M_F\right) + 3\text{tr}\left(M_V^2\right) = 0 \ . \tag{VI.4}$$

Cette règle pose problème, car elle impose généralement un spectre de masse non réaliste, où certains superpartenaires sont suffisamment légers pour être observables [234]. Par conséquent, la brisure de supersymétrie devrait avoir lieux dans un secteur caché, mais ce n'est pas suffisant. En effet, la brisure de supersymétrie devrait en outre être transmise au secteur visible radiativement ou via des interactions non-renormalisables afin de contourner la règle de somme précédente. Les principaux mécanismes pour réaliser cela sont les suivants.

(i) Dans la médiation à l'échelle de Planck [271–275], la brisure de supersymétrie est transmise par des interactions non-renormalisable, provenant de la nouvelle physique apparaissant à l'échelle de Planck (et qui inclut la gravité quantique). Comme la forme exacte de ces nouvelles interactions n'est pas connue, les résultats dépendent fortement des hypothèses qui sont faites sur la structure des termes de brisure douce à l'échelle de Planck.

(ii) En médiation de jauge [225, 226, 276–281], les interactions de jauge ordinaires communiquent la brisure de supersymétrie au secteur visible. Ces modèles sont bien plus prédictifs, car la forme des interactions impliquées est connue.

(iii) Enfin, une troisième possibilité repose sur l'hypothèse que l'univers a plus de quatre dimensions d'espace-temps [282–289]. Les superchamps chiraux du MSSM et le secteur de brisure de supersymétrie peuvent par exemple être confinés dans deux hyperplans parallèles de dimension 4. Si les champs de jauge évoluent dans les autres dimensions, ils peuvent communiquer la brisure de supersymétrie, tandis que si les champs de jauge sont également confinés dans un hyperplan, la transmition se fait via les interactions gravitationnelles uniquement.

Nous nous intéressons maintenant plus particulièrement à la médiation de jauge. La brisure de supersymétrie est représentée par un spurion X dont les composantes scalaire et auxiliaire prennent une valeur moyenne non nulle dans le vide, $\langle X\rangle = M_X + F_X\theta^2$. Le spurion étant un singlet de jauge, ce scénario doit également contenir des champs appelés messagers, qui sont directement couplés au spurion et communiquent avec le MSSM via les interactions de jauge. Φ_M et $\Phi_{\bar{M}}$ étant deux messagers appartenant à des représentations conjuguées, ils se couplent au spurion de la manière suivante,

$$W_{\rm GM} = X\Phi_{\bar{M}}\Phi_M \ . \tag{VI.5}$$

En médiation de jauge minimale, les couplages directs entre messagers et champs de matière sont interdits par une symétrie, la parité des messagers [290]. Après remplacement du spurion par sa valeur d'équilibre, les états propres de masse sont un fermion de Dirac $(\psi_M, \psi^\dagger_{\bar{M}})$ de masse M_X, et deux scalaires complexes,

$$\phi_{M\pm} = \frac{\phi_M \pm \phi^\dagger_{\bar{M}}}{\sqrt{2}} \ , \qquad M^2_\pm = M^2_X \pm F_X \ . \tag{VI.6}$$

La supersymétrie est donc bien brisée puisque les masses des scalaires et des fermions ne sont plus égales. Des diagrammes tels que ceux des figs. 5.1 et 5.2 génèrent ensuite les masses des jauginos et des scalaires radiativement [291],

$$M_\lambda = i\frac{\alpha}{4\pi}S(\Phi_M)\Lambda \ , \tag{VI.7}$$

$$m^2_\phi = 2|\Lambda|^2\left[C_3(\phi)\left(\frac{\alpha_s}{4\pi}\right)^2 + C_2(\phi)\left(\frac{\alpha_{EW}}{4\pi}\right)^2 + \frac{3Y^2_\phi}{5}\left(\frac{\alpha_Y}{4\pi}\right)^2\right] \ . \tag{VI.8}$$

$S(\Phi_M)$ désigne l'indexe de Dynkin de la représentation à laquelle appartiennent les messagers. Il est important de noter qu'à l'échelle de brisure de supersymétrie, les

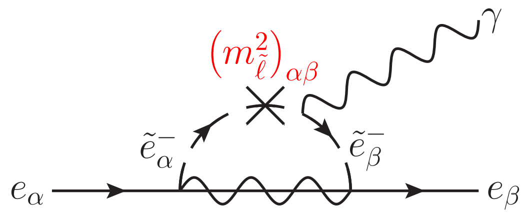

Figure iii: Contribution des sleptons gauches à $\mu \to e\gamma$, avec échange d'un photino $\tilde{\gamma}$ ou d'un zino $\tilde{Z}^0$.

A-termes générés par ce mécanisme sont négligeables, ce qui n'est pas sans poser problème, comme nous allons voir.

En revanche, un avantage important de la médiation de jauge est qu'elle permet d'éviter l'apparition de nouvelles sources de violation de saveur. En effet, en supersymétrie, les processus violant la saveurs sont reliés aux masses des sfermions : par exemple, la figure iii montre comment les sleptons interviennent dans le processus $\mu \to e\gamma$. On peut définir les paramètres suivants pour $\alpha \neq \beta$,

$$\delta^{LL}_{\alpha\beta} = \frac{(m^2_{\tilde{\ell}})_{\alpha\beta}}{\bar{m}^2_{\tilde{\ell}}} \ , \qquad \delta^{RR}_{\alpha\beta} = \frac{(m^2_{\tilde{e}})_{\alpha\beta}}{\bar{m}^2_{\tilde{e}}} \ , \qquad \delta^{LR}_{\alpha\beta} = \frac{A^e_{\alpha\beta} v_d}{\bar{m}_{\tilde{\ell}} \bar{m}_{\tilde{e}}} \ . \tag{VI.9}$$

Ici, $\bar{m}^2$ désigne la masse moyenne des sleptons. Le taux d'embranchement pour $\mu \to e\gamma$ est approximativement donné par [129–133]

$$\mathcal{B}r(\mu \to e\gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|\delta^{LL}_{e\mu}|^2}{m^4_{\tilde{\ell}}} \tan^2\beta \ . \tag{VI.10}$$

De tels processus sont sévèrement contraints expérimentalement (voir les bornes données dans le tableau 2.2), ce qui implique que les entrées hors diagonale de la matrice de masse des sleptons ne doivent pas excéder certaines limites. Or, les interactions de jauge étant indifférentes à la saveur, les masses des sleptons en médiation de jauge vérifient

$$(m_{\tilde{\ell}})^2_{\alpha\beta} = m^2_{\tilde{\ell}} \delta_{\alpha\beta} \ . \tag{VI.11}$$

Il en est de même pour les squarks. En conséquence, les éléments $\delta^{XY}_{\alpha\beta}$ sont nuls et il n'y a pas de nouvelle source de violation de saveur.

Le problème posé par de petits A-termes est lié à la brisure de symétrie électrofaible dans le MSSM. Comme il a été mentionné auparavant, la supersymétrie nécessite l'existence de deux doublets de Higgs, H_u et H_d. Les composantes neutres de ces deux doublets prennent des valeurs moyennes non nulles dans le vide, notées respectivement v_u et v_d, et qui sont reliées à la valeur moyenne du champ de Higgs dans le Modèle Standard par

$$v_u^2 + v_d^2 = v^2 \ . \tag{VI.12}$$

On définit également β via

$$\tan\beta = \frac{v_u}{v_d} \ . \tag{VI.13}$$

À l'ordre des arbres, la masse du boson de Higgs doit satisfaire la relation suivante [299, 300],

$$m_h < M_Z|\cos 2\beta| \ . \tag{VI.14}$$

Ceci est en contradiction flagrante avec les résultats des expériences Atlas [60] et CMS [61], qui donnent $m_h = 126$ GeV. La résolution de ce problème nécessite des corrections importantes aux ordres supérieurs. Pour cela, les stops devraient être très lourds, ou bien il devrait y avoir un mélange suffisant entre les stops associés aux quarks de chiralité gauche et droite [304–307]. Le problème de la première solution est qu'elle nécessite un "fine tuning" important. La seconde solution nécessite de grands A-termes, ce que la médiation de jauge dans sa version minimale ne fournit pas. Il est cependant possible de modifier ce scénario pour le rendre compatible avec la masse observée du boson de Higgs.

La résolution du problème repose sur la médiation de jauge étendue : en plus des interactions de jauge, des couplages directs entre messagers et champs du MSSM sont maintenant autorisés [310–321]. Ceci permet de générer des A-termes non négligeables. En revanche, cela a également pour effet de générer de nouvelles sources de violation de saveur, qui doivent être maitrisées.

Nous utilisons ici une méthode générale, basée sur celle présentée dans la référence [309], pour calculer les termes de brisure de supersymétrie. Supposons qu'en dehors du secteur de brisure de supersymétrie, le superpotentiel s'écrive de la manière suivante,

$$W = \frac{1}{2}M_{ij}\Phi_i\Phi_j + \frac{1}{6}\lambda_{ijk}\Phi_i\Phi_j\Phi_k \ , \tag{VI.15}$$

Après prise en compte de la renormalisation de fonction d'onde, les termes cinétique s'écrivent dans la limite supersymétrique (on note μ l'échelle de renormalisation)

$$\mathcal{L}_{\text{kin}} = \int d^2\theta S(M_X,\mu)\text{tr}[\mathcal{W}^\alpha\mathcal{W}_\alpha] + \text{h.c.} + \int d^4\theta Z_{ij}(M_X, M_X^\dagger,\mu)\Phi_i^\dagger\Phi_j \ , \tag{VI.16}$$

On remplace ensuite M_X par $\langle X\rangle$ et de procéder à l'expansion suivante,

$$S(\langle X\rangle,\mu) = S(M_X,\mu) + \theta^2\frac{\partial S}{\partial M_X}F_X \ . \tag{VI.17}$$

L'intégration sur les coordonnées fermionique fait apparaitre un terme de masse pour le jaugino, mais pas pour le boson de jauge. Dans la base où les termes cinétiques sont canonique, la masse du jaugino est

$$M = \frac{1}{2}\frac{\partial\log S}{\partial\log M_X}\Lambda \ . \tag{VI.18}$$

On procède de manière similaire avec les superchamps chiraux, ce qui permet d'obtenir les A-termes, les B-termes ainsi que les masses des scalaires grâce aux relations suivantes,

$$\int d^4\theta\, \theta^2 \Phi_i^\dagger \Phi_j = F_i^\dagger \phi_j \ , \tag{VI.19}$$

$$\int d^4\theta\, \theta^4 \Phi_i^\dagger \Phi_j = \phi_i^\dagger \phi_j \ . \tag{VI.20}$$

Finalement, dans la base où les termes cinétiques sont canoniques, on peut identifier les A-termes et B-termes,

$$B_{ij} = V_{i'i}^{-1} V_{j'j}^{-1} \left[M_{ki'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{kj'} + M_{kj'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{ki'} \right] \ , \tag{VI.21}$$

$$A_{ijk} = V_{i'i}^{-1} V_{j'j}^{-1} V_{k'k}^{-1} \left[\lambda_{li'j'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{lk'} + \lambda_{lk'i'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{lj'} + \lambda_{lj'k'} \left(Z_{|_1}^{-1} Z_{|_{\theta^2}} \right)_{li'} \right] \ , \tag{VI.22}$$

ainsi que les masses au carré des scalaires

$$(m^2)_{ij} = V_{i'i}^{-1*} V_{j'j}^{-1} \left[-Z_{i'j'|_{\theta^4}} + Z_{i'k|_{\theta^2}}^\dagger Z_{kl|_1}^{-1} Z_{lj'|_{\theta^2}} \right] \ . \tag{VI.23}$$

Enfin, il est à noter que les termes μ et B_μ peuvent être générés par le mécanisme de brisure de supersymétrie. Si tel est le cas, il faut aussi prendre en compte la renormalisation de fonctione d'onde holomorphe,

$$K_{\text{holomorphic}} = \mathcal{H}_{ud} H_u H_d + \text{h.c.}, \tag{VI.24}$$

qui est relié à ces termes de la manière suivante,

$$\mu = V_{uu}^{-1} V_{dd}^{-1} \mathcal{H}_{ud|_{\bar\theta^2}} \ , \qquad B_\mu = -V_{uu}^{-1} V_{dd}^{-1} \mathcal{H}_{ud|_{\theta^4}} \ . \tag{VI.25}$$

Nous utilisons cette méthode pour calculer les termes de brisure douce dans une théorie générale contenant des messagers, des champs lourds et des champs légers (comprenant les champs du MSSM). Nous effectuons les calculs à l'ordre de 2 boucles au maximum. Il faut procéder au raccordement de la théorie à haute énergie, qui contient les messagers, avec la théorie à basse énergie qui ne les contient plus. À une boucle, ce raccordement se fait de la manière suivante,

$$\sum_{\text{heavy}} D^{(1)} + i\delta Z_{\text{heavy}}^{(1)} = iZ^{(1)} \ . \tag{VI.26}$$

Le membre de droite contient une somme sur les diagrammes à une boucle qui impliquent au moins un champ lourd ainsi que le contreterme qui en annule les divergences.

À deux boucles, la situation est résumée par la figure iv. Nous employons ici génériquement les indices i, j, k, ... pour désigner l'ensemble des champs tandis que les indices grecs α, β, γ, ... désignent des champs légers seulement.

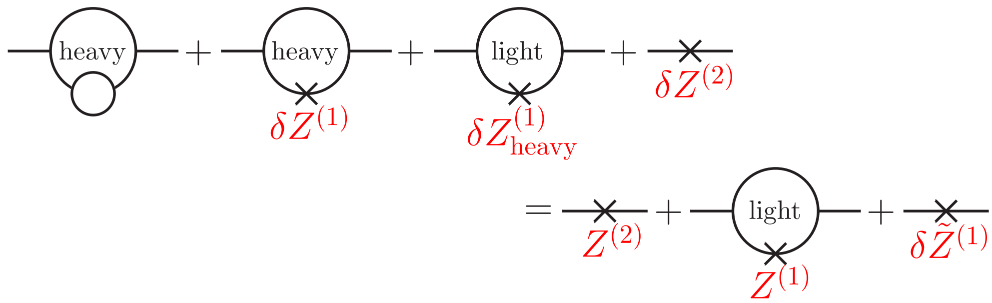

Figure iv: Représentation diagrammatique du raccordement à deux boucles. La mention “heavy” signifie qu'au moins un champ lourd est impliqué, tandis que la mention “light” signifie que tous les champs impliqués sont légers.

Les diagrammes impliqués dans la renormalisation de fonction d'onde à une boucle sont représentés dans la figure 5.4. Le premier type de diagramme implique uniquement des interactions du superpotentiel, tandis que le second implique les interactions de jauge : ce dernier ne contribue pas directement aux termes de brisure douce puisqu'il n'implique jamais de messagers, mais il est néanmoins nécessaire d'en déterminer l'expression puisque les contretermes à une boucle sont impliqués dans le calcul à deux boucles. À deux boucles, il existe trois types de contribution : les diagrammes n'impliquant que des interactions de jauge sont les mêmes que dans la médiation de jauge minimale et donnent des résultats connus. Nous ne les reproduisons pas ici. En médiation de jauge étendue, il faut ajouter les diagrammes impliquant des interactions du superpotentiel, représentés par la figure 5.5, et les diagrammes “mixtes” jauge-superpotentiel, représentés par la figure 5.6. Enfin, la fonction d'onde holomorphe reçoit les contributions indiquées par la figure 5.7.

Le calcul de ces différentes contributions fournit des résultats très généraux, qui peuvent être appliqués à des modèles spécifiques de médiation de jauge étendue. Le principal défi de ces modèles est de réconcilier la médiation de jauge avec la masse du boson de Higgs mesurée au LHC, tout en maintenant la violation de saveur dans des limites acceptables. En effet, s'il existe par exemple un messager ϕ_d avec les mêmes nombres quantiques que le Higgs de type “down” du MSSM (comme c'est généralement le cas en médiation de jauge), on peut en principe écrire le terme suivant dès lors que les couplages directs entre messagers et matière sont autorisés,

$$W_{\text{new}} = \lambda^e_{\alpha\beta} \Phi_d \ell_\alpha e^c_\beta \; . \tag{VI.27}$$

Ceci génère une matrice de masse au carré pour les sleptons proportionnelle à $\lambda^{e\dagger}\lambda^e$, qui donne lieu à des courants neutres violant la saveur. Or, comme il a été mentionné précédemment, de tels processus sont fortement contraints expérimentalement. La médiation de jauge étendue soulève donc de nouveaux problèmes. Une solution, introduite dans la référence [314], consiste à contraindre la forme des couplages directs autorisés entre messagers et champs du MSSM.

Par exemple, supposons que de tels couplages sont interdits par une symétrie dans une base initiale, où on note $\tilde{\Phi}_u$ et $\tilde{H}_u$ le messager et le doublets de Higgs de type "up" respectivement. Cette symétrie est brisée par la valeur moyenne d'un champ S dont les nombres quantiques sont tels que le couplage suivant est autorisé,

$$\frac{S}{\Lambda_S} X \Phi_d \tilde{H}_u \ . \tag{VI.28}$$

Grâce à la condition $\langle S \rangle \neq 0$, le superpotentiel contient les termes suivants,

$$W_{\mathrm{GM}} = X \Phi_d \left(\tilde{\Phi}_u + s \tilde{H}_u \right) + X \Phi_T \bar{\Phi}_T \ , \tag{VI.29}$$

$$W_{\mathrm{MSSM}} = \tilde{y}^u_{ij} \tilde{H}_u Q_i u^c_j + y^d_{ij} H_d Q_i d^c_j + y^e_{\alpha\beta} H_d \ell_\alpha e^c_\beta \ . \tag{VI.30}$$

On redéfinit ensuite les champs de telle sorte que seule une paire vectorielle (Φ_u, Φ_d) interagit directement avec le spurion X. Pour cela, on définit Θ tel que $t_\Theta = \tan \Theta = \langle S \rangle / \Lambda_S$, et

$$\Phi_u = c_\Theta \tilde{\Phi}_u + s_\Theta \tilde{H}_u \ , \tag{VI.31}$$

$$H_u = -s_\Theta \tilde{\Phi}_u + c_\Theta \tilde{H}_u \ . \tag{VI.32}$$

Ceci permet de réécrire le superpotentiel comme suit,

$$W_{\mathrm{GM}} = \kappa X \Phi_d \Phi_u + X \Phi_T \Phi_{\bar{T}} \ , \tag{VI.33}$$

$$W_{\mathrm{MSSM}} = y^u_{ij} H_u Q_i u^c_j + y^d_{ij} H_d Q_i d^c_j + y^e_{\alpha\beta} H_d \ell_\alpha e^c_\beta \ . \tag{VI.34}$$

avec $\kappa = 1/\cos\Theta = \sqrt{1+s^2}$ and $y^u_{ij} = c_\Theta \tilde{y}^u_{ij}$. Le superpotentiel contient maintenant le terme suivant,

$$W_{\mathrm{new}} = \lambda^u_{ij} \Phi_u Q_i u^c_j \ , \tag{VI.35}$$

avec $\lambda^u_{ij} = s_\Theta \tilde{y}^u_{ij} = t_\Theta y^u_{ij}$. Il existe donc dans la nouvelle base un couplage direct entre messagers et champs du MSSM. Ce couplage génère en particulier un A-terme pour les stops, qui contribue à corriger la masse du boson de Higgs. De plus, ce couplage est proportionnel à la matrice de Yukawa, et n'induit donc pas de nouvelle source de violation de saveur. La référence [314] montre qu'il est possible de donner une masse réaliste au boson de Higgs en utilisant ce mécanisme.

Nous utilisons ici ce mécanisme dans le cadre des scénarios de seesaw de type I et II. Dans sa version supersymétrique, le seesaw de type I est décrit par le superpotentiel suivant,

$$W_{\mathrm{seesaw}} = \frac{1}{2} M_i N_i N_i + y^\nu_{i\alpha} H_u N_i \ell_\alpha \ , \tag{VI.36}$$

Dans le contexte de la médiation de jauge étendue, le mécanisme décrit précédemment donne naissance à l'opérateur suivant,

$$W_{\mathrm{new}} = \lambda^\nu_{i\alpha} \Phi_u N_i \ell_\alpha \ , \tag{VI.37}$$

Ceci génère des A-termes et des contributions aux masses pour les sleptons (qui s'additionnent aux contributions générées par la médiation de jauge minimale) à partir d'une boucle,

$$A^e_{\alpha\beta} = -\frac{\Lambda}{16\pi^2} t_\Theta^2 \sum_i y^\nu_{i\alpha} \left(y^{\nu\dagger} y^e\right)_{i\beta} g_2\left(x_i\right) \ , \tag{VI.38}$$

$$(\Delta m^2_{\tilde{\ell}})_{\alpha\beta} = -\frac{\Lambda^2}{16\pi^2} t_\Theta^2 \sum_i y^{\nu *}_{i\alpha} y^{\nu\dagger}_{i\beta} g_4\left(x_i\right) \ , \tag{VI.39}$$

avec $x_i = M_i^2/M_X^2$ et les fonctions g_2 et g_4 sont définies explicitement dans les équations (5.2.53) et (5.2.54) respectivement. D'autres contributions aux termes de brisure douce sont générées à partir de deux boucles, telles que par exemple

$$\begin{aligned}(\Delta m^2_{\tilde{e}})_{\alpha\beta} = &\frac{\Lambda^2}{256\pi^4} t_\Theta^2 \sum_i y^{e*}_{\gamma\alpha} y^e_{\sigma\beta} y^\nu_{i\gamma} y^{\nu *}_{i\sigma} \\ &\times \left[\partial_5^2 f_A(0,0,0,x_i,1) - f_{A'}(0,0,0)\partial_3^2 f_{A'}(0,x_i,1)\right] \ ,\end{aligned} \tag{VI.40}$$

où f_A et $f_{A'}$ sont des fonctions issues du calcul des diagrammes à deux boucles des figures 5.5 et 5.6 et $\partial_n f$ signifie qu'on dérive f par rapport à sa n^{e} variable. Les contributions aux masses des sleptons générées par le seesaw de type I impliquent que la saveur leptonique est violée par des processus tels que $\mu \to e\gamma$, ce qui est une nouveauté par rapport au scénario non supersymétrique.

La version supersymétrique du seesaw de type II implique deux triplets scalaires, Δ et $\bar{\Delta}$, appartenant à des représentations conjuguées. Après avoir procédé au changement de base $(\tilde{\phi}_u, \tilde{H}_u) \to (\phi_u, H_u)$, le superpotentiel contient les termes suivants,

$$W_{\text{seesaw}} = \frac{1}{2} f_{\alpha\beta} \Delta \ell_\alpha \ell_\beta + \frac{1}{2}\lambda_{HH} \bar{\Delta} H_u H_u \ , \tag{VI.41}$$

$$W_{\text{new}} = \lambda_{\Phi H} \bar{\Delta} H_u \Phi_u + \frac{1}{2}\lambda_{\Phi\Phi} \bar{\Delta} \Phi_u \Phi_u \ , \tag{VI.42}$$

avec $\lambda_{\Phi H} = t_\Theta \lambda_{HH}$, $\lambda_{\Phi\Phi} = t_\Theta^2 \lambda_{HH}$. La masse des neutrinos est donnée par

$$(m_\nu)_{\alpha\beta} = \frac{1}{2}\lambda_{HH} f_{\alpha\beta} \frac{v_u^2}{M_\Delta^2} \tag{VI.43}$$

Les termes spécifiques au seesaw de type II qui apparaissent à une boucle sont les suivants,

$$\Delta A^u_{ij} = -\frac{3\Lambda}{32\pi^2} \lambda^2_{\phi H} y^u_{ij} g_2\left(x_\Delta\right) \ , \tag{VI.44}$$

$$\Delta m^2_{H_u} = -\frac{3\Lambda^2}{32\pi^2} \lambda^2_{\phi H} g_4\left(x_\Delta\right) \ . \tag{VI.45}$$

Les nouveaux termes de brisure douce impliquant les leptons n'apparaissent qu'à deux boucles, car il n'y a pas de couplage direct entre leptons et messagers,

$$\begin{aligned}(\Delta m^2_{\tilde{\ell}})_{\alpha\beta} = &\frac{\Lambda^2}{256\pi^4} \left(f^\dagger f\right)_{\alpha\beta} \Big[2\lambda^2_{\phi H} \partial_5^2 f_B\left(x_\Delta, 0, x_\Delta, 0, 1\right) \\ &+ \lambda^2_{\phi\phi} (\partial_4 + \partial_5)^2 f_B\left(x_\Delta, 0, x_\Delta, 1, 1\right)\Big] \ .\end{aligned} \tag{VI.46}$$

Cette forme est intéressante car elle permet de relier les masses des sleptons impliquées dans les processus violant la saveur, à la matrice de masse des neutrinos. En effet, toutes deux s'expriment en fonction des mêmes couplages $f_{\alpha\beta}$. Par conséquent, il est possible d'écrire une relation de la forme suivante,

$$(\Delta m_{\tilde{\ell}}^2)_{\alpha\beta} = \frac{\Lambda^2}{64\pi^4} \frac{M_\Delta^2}{v_u^4} \left(m_\nu^\dagger m_\nu \right)_{\alpha\beta} f_{\tilde{\ell}}(t_\Theta, x_\Delta) \ . \tag{VI.47}$$

Ceci peut être comparé aux résultats obtenus dans la référence [323], où le triplet scalaire est lui-même messager. La structure de saveur est similaire, mais la différence est ici la présence de différentes échelles de masse qui permet de contrôler la violation de saveur leptonique.

Pour résumer, les couplages directs entre messagers et matière permettent de rendre la médiation de jauge cohérente avec la masse relativement élevée du boson de Higgs. Dans ce contexte, le mécanisme de seesaw apporte de nouvelles contributions aux termes de brisure de supersymétrie et fournit une origine commune aux masses des neutrinos et à la violation de saveur leptonique. Les phénomènes violant la saveur étant extrêmement contraints dans le Modèle Standard, ils peuvent dans ce contexte constituer un test du mécanisme de seesaw supersymétrique.

Nous nous sommes intéressés dans cette thèse à quelques scénarios de nouvelle physique en lien avec le secteur leptonique. La leptogenèse, par exemple, fournit une explication élégante à l'asymétrie entre matière et antimatière, qui est de plus reliée aux masses des neutrinos par le mécanisme de seesaw. De tels scénarios sont difficiles à tester directement, mais seraient néanmoins fortement motivés s'il s'avérait que les neutrinos sont des particules de Majorana. Les expériences de détection de la double désintégration beta sans neutrino sont pour cela d'une grande importance. Les masses des neutrinos peuvent également provenir d'une brisure de R-parité en supersymétrie. Nous avons vu que dans ce cas, certaines anomalies expérimentales peuvent s'expliquer par la présence d'un neutrino stérile léger, qui est en fait le fermion associé à un pseudo-boson de Goldstone. Enfin nous avons étudié la médiation de jauge étendue et ses conséquences pour le mécanisme de seesaw, qui permet alors de mettre en relation les masses des neutrinos avec la violation de saveur leptonique.

Sujet : Nouvelle physique dans le secteur des leptons

Résumé : Cette thèse aborde quelques scénarios de nouvelle physique, ainsi que leurs manifestations dans le secteur des leptons.

Le fait que les neutrinos soient massifs est un des problèmes non élucidés par le Modèle Standard. Une des solutions possibles est le mécanisme de seesaw, qui fait intervenir de nouveaux états lourds dont la désintégration viole le nombre leptonique. À cause de ce dernier point, ces états peuvent jouer un rôle clé dans la leptogenèse, un des scénarios susceptibles d'expliquer l'origine de l'asymétrie observée entre matière et antimatière dans l'univers. Nous étudions ici la leptogenèse avec un triplet scalaire et nous intéressons tout particulièrement l'impact des effets de saveur.

Dans un second temps, nous considérons des théories supersymétriques. Nous étudions un modèle où le partenaire fermionique d'un pseudo-boson de Goldstone joue le rôle d'un neutrino stérile, qui pourrait expliquer certaines anomalies expérimentales. Enfin, pour être viable, la supersymétrie doit être brisée dans un secteur caché, et cette brisure doit ensuite être transmise aux champs de la théorie à basse énergie. Un des scénarios les plus élégants pour cela est la médiation de jauge. Malheureusement, celle-ci peine à reproduire la masse du boson de Higgs mesurée au LHC. Nous nous intéressons ici à des extensions susceptibles de réhabiliter ce scénario tout en le reliant mécanisme de seesaw.

Mots clés : Nouvelle physique, neutrinos, mécanisme de seesaw, leptogenèse, supersymétrie, médiation de jauge

Subject : New physics in the lepton sector

Résumé : This thesis addresses some scenarios of new physics as well as their consequences on the lepton sector.

The fact that neutrinos are massive is one of the problems left unsolved by the Standard Model. One of the possible solutions is the seesaw mechanism, that involves new heavy states whose decay violates lepton number. Because of this, these states can participate in leptogenesis, one of the scenarios to explain the origin of the asymmetry between matter and antimatter in our universe. Here, we study leptogenesis with a scalar triplet and consider especially the impact of flavour effects.

After that, we turn to supersymmetric theories. We study a scenario in which the fermionic partner of a pseudo-Goldstone boson, associated to a symmetry broken at high energy, plays the role of a sterile neutrino, that could explain some experimental anomalies. Finally, to be viable, supersymmetry should be broken in a hidden sector, and this breaking should be transmitted to the fields of the low energy theory. One of the most elegant scenarios for this is gauge mediation. Unfortunately, it cannot easily reproduce the mass of the Higgs boson measured at LHC. We consider here extensions that could rehabilitate this scenario and relate it to the seesaw mechanism.

Keywords : New physics, neutrinos, seesaw mechanism, leptogenesis, supersymmetry, gauge mediation