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Estimation of Launch Vehicle Propellant Tank Structural Weight Using Simplified Beam Approximation

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Many conceptual launch vehicles are designed through the integration of various disciplines, such as aerodynamics, propulsion, trajectory, weights, and aeroheating. In the determination of the total vehicle weight, a large percentage of the vehicle weight is composed of the structural weight of the vehicle subsystems, such as propellant tanks. Empirical mass estimating relations (MERs) and multi-dimensional finite element analysis (FEA) are two methods commonly used by the aerospace industry to estimate the load-bearing structural weight. MERs rapidly estimate the weight by evaluating empirical equations and the high-fidelity techniques of FEA accurately calculates the structural weight. The extreme inability for either method to provide both rapid and accurate weight estimations warrants an investigation into developing an improved, intermediate method.

A methodology based on fundamental beam structural analysis has been developed for the rapid estimation of the load-bearing structural weight of the launch vehicle fuselage and integral propellant tanks. By creating a simplified beam approximation model of the vehicle, the method utilizes the vehicle component weights, load conditions, and basic material properties to analytically estimate the structural shell and stability frame weight. Implementation of this methodology into a fast-acting software tool allowed for rapid estimation of the component structural weight. Using statistical techniques, an empirical relationship between the estimated and actual load-bearing structure weights was determined. The method was utilized to estimate the liquid hydrogen (LH_2) and liquid oxygen (LOX) propellant tanks for an existing Evolved Expendable Launch Vehicle (EELV) and the Space Shuttle External Tank (ET) for verification and correlation.

Nomenclature

A = cross-sectional area (in^2)
a = semi-major axis (in)
A_f = stability frame cross-sectional area (in^2)
axial_accel = axial acceleration (g's)
b = semi-minor axis (in)
c = farthest from the neutral axis along the y-axis (in)
C_f = Shanley constant (1/16,000)
I_y = Area Moment of Inertia with respect to y-axis
norm_accel = normal acceleration (g's)
P_{ell} = perimeter of ellipse (in)
p_{head} = head pressure (lb/in^2)
prop_ullage = ullage pressure (lb/in^2)
p_{ullage} = ullage pressure (lb/in^2)
t_f = smeared equivalent stability frame thickness (in)
t_s = equivalent shell thickness (in)

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x1, x2	=	simple support reaction location (in)
ρ_f	=	density of stability frame material (lb/in^3)
ρ_p	=	propellant density (lb/in^3)
ρ_s	=	density of shell material (lb/in^3)
σ_{axial}	=	axial stress (lb/in^2)
σ_{bend}	=	bending stress (lb/in^2)
σ_{hhead}	=	normal stress due to head pressure in hoop (circumferential) direction (lb/in^2)
$\sigma_{hullage}$	=	normal stress due to ullage pressure in hoop (circumferential) direction (lb/in^2)
σ_{lhead}	=	normal stress due to head pressure in axial (longitudinal) direction (lb/in^2)
$\sigma_{lullage}$	=	normal stress due to ullage pressure in axial (longitudinal) direction (lb/in^2)
σ_{UTS}	=	ultimate tensile strength (lb/in^2)
σ_{YS}	=	yield strength (lb/in^2)
%_fuel	=	percent fuel remaining

I. Introduction

Conceptual launch vehicle design involves the integration of various disciplines to generate a complete vehicle design. Disciplines included in the conceptual design synthesis are aerodynamics, propulsion, trajectory, weight and sizing, and aeroheating. Vehicle weight is an important parameter used in acquiring the required information from each discipline. Aerodynamic coefficients, required thrust, projected trajectory, and sized propellant masses are all direct and indirect functions of the vehicle weight. In the determination of the total vehicle weight, a large percentage of the vehicle weight is composed of the structural weight of the vehicle subsystems, such as propellant tanks, interstages, and fuselage structure. The weight of each subsystem is derived from the material composition and structural configuration required to withstand the load conditions it experiences during the vehicle operation.

There are two methods commonly used by the aerospace industry to estimate the load-bearing structural weight of launch vehicle components: empirical mass estimating relations (MERs) determined from regressing existing vehicle data and detailed finite element structural analysis. Preliminary subsystem weights of conceptual launch vehicles are conventionally obtained from MERs based on the empirical regressions of existing vehicle components. Though this method results in rapid weight estimation, it is not always preferred and reliable for studies of unconventional vehicle concepts. Since the weight estimations are based upon existing vehicles, their application to unconventional configurations and loading conditions are questionable. For instance, the use of aircraft MERs to determine the structural weight of a horizontal take-off and landing reusable launch vehicle may be suspect due to the fact that the configuration and loading conditions of the vehicle with an orbital trajectory will be vastly different than that of a conventional aircraft.

Finite element analysis (FEA) methods for determining structural weight are often inappropriate for conceptual design. The idealized, multi-dimensional structural model of the vehicle must be created off-line and is incapable of being subjected to dynamic changes due to modifications in other vehicle parameters. The analysis of a moderately complex finite-element model will provide a highly accurate structural weight, but it can require a large amount of computational effort. The analysis can also be very time consuming, which can lead to a bottleneck in the vehicle design synthesis. For these reasons, the finite-element method is more relevant for use in detailed vehicle design.

The inability for either common weight estimation methods to provide both accuracy and speed warrants an investigation into developing an intermediate, improved method that can accurately determine structural weight of the launch vehicle components at a minimized cost of time and computational effort. A methodology based on fundamental beam structural analysis has been developed for the rapid estimation of the load-bearing structural weight of the launch vehicle fuselage and its associated components. By creating a simplified beam approximation model of the vehicle, the method utilizes the vehicle component weights, load conditions, and basic material properties to analytically estimate the structural shell and stability frame weight. Implementation of this methodology into a fast-acting software tool for conceptual design resulted in the creation of a computer program, Georgia Tech Structural Tool for Rapid Estimation of Shell Sizes (GT-STRESS). The input format and basic operation of GT-STRESS is derived from RL, a computer program to calculate fuselage running loads, which was developed by Jeff Cerro, formerly of Lockheed Martin Engineering & Science Services. The method was applied to an existing Evolved Expendable Launch Vehicle (EELV) and the External Tank (ET) of the Space Shuttle. The liquid hydrogen (LH_2) and liquid oxygen (LOX) propellant tanks of the launch systems were estimated for verification and correlation of the methodology. Using statistical techniques, the relationship between the estimated load-bearing structure weight calculated by GT-STRESS and the actual structure weights were determined.

II. Overview of Methodology

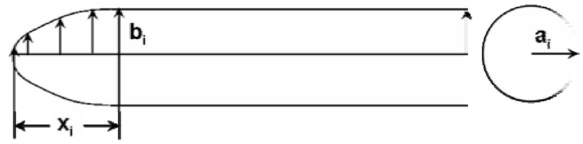

Figure 1. Vehicle Geometry Approximation

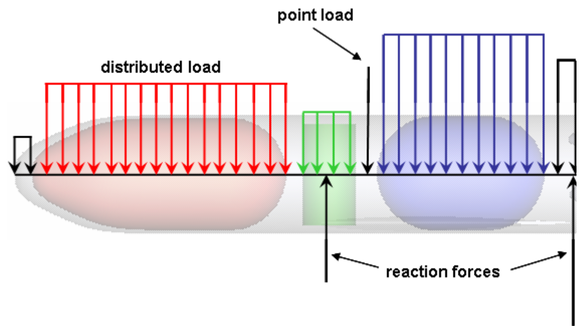

Figure 2. Vehicle Weight Distribution

Prior to the start of the actual analysis, the vehicle geometry and preliminary subsystem weights are defined along the fuselage. The vehicle geometry is modeled by a sequence of elliptical cross sections, which are defined by their location, semi-major axis, and semi-minor axis along the longitudinal axis of the vehicle as depicted in Fig. 1. Cross sections not defined are determined from linear interpolation of the semi-major and minor axes between the two defined boundary sections. Inert masses and propellant masses are modeled as point and distributed loads over their position along the longitudinal axis of the fuselage in both the normal and axial directions. The weights are defined by the starting and ending position of the loading, and the total weight to be distributed over the range of the load, as depicted in Fig. 2. This methodology simulates liquid propellant contained in an integral tank structure arrangement. Also, the method does not analytically model the stress and structure weight involved in the propellant tank end closures (i.e. hemispherical, elliptical). Instead an effective tank length is employed, which accounts for the distance of the tank end closure and models the tank as a cylinder. The effective tank length for cylindrical tanks with end closures in the form of hemispherical or elliptical shape is equal to the tank barrel length plus one-third the depth of the end closures.[1]

Structural analysis of the beam approximated fuselage begins by determining the external load distribution experienced by the vehicle at the selected load conditions. Each defined load condition provides the location of the two simple support reaction points along with the axial acceleration, normal acceleration, propellant ullage pressure, and percent of remaining fuel at the particular point in the trajectory. Reaction loads at each of the simple supports are determined from the calculation of vehicle center of gravity. Calculation of the external loads involves the combined load distribution of the vehicle subsystem weights and the reaction loads, and account for the experienced accelerations and amount of propellant available at each load condition. The external stress resultants are calculated at each cross-section, or station, along the length of the fuselage using a discretized form of equilibrium equations from Euler-Bernoulli beam theory for axial force, shear force, and bending moment.[2] Equations (1)-(3) are the equilibrium equations for axial force (P), shear force (V), and bending moment (M), respectively, where w_x is the axial distributed load and w_y is the normal distributed load. A factor of safety of 1.5 is applied to each external load.

$$-w_x = \frac{dP}{dx} \tag{1}$$

$$-w_y = \frac{dV}{dx} \tag{2}$$

$$V = \frac{dM}{dx} \tag{3}$$

After determining the external load distribution, the internal stress resultants, or running loads, used to size the thickness of the fuselage/component shell. Running loads are computed by the product of the shell thickness and the stresses derived from the external loads (bending moment, axial force, and shear force) and the internal tank pressure (ullage pressure and head pressure). The presence of shell thickness in the area and moment of inertia terms of the stresses removes the thickness from the computation. The running loads in the fuselage/component shell are a function of axial and circumferential position and are determined on a station-by-station basis. The top and bottom sections of the shell are loaded predominately in bending stress, the side sections are loaded mainly in shear stress, and the axial stress is distributed over the entire cross section. The longitudinal bending moment (N_{xbend}), longitudinal axial (N_{xaxial}), and transverse (shear) (N_{xy}) running loads are functions of their associated external loads and the cross section parameters, as given in Eqs. (4)-(6), respectively.

$$N_{x\,bend} = \sigma_{bend} t_s = \frac{Mc}{I_y} t_s = \frac{Mc}{\frac{\pi}{4} b^2 (b + 3a)} \tag{4}$$

$$N_{xaxial} = \sigma_{axial} t_s = \frac{P}{A} t_s = \frac{P}{P_{ell} t_s} t_s = \frac{P}{P_{ell}} \tag{5}$$

$$N_{xy} = \frac{2V}{A} t_s = \frac{2V}{P_{ell}} \tag{6}$$

The contributions to the running loads by the propellant ullage and head pressures are determined from the product of the shell thickness and the normal stresses in the hoop (circumferential) and axial (longitudinal) directions for a cylindrical tank. From membrane stresses in pressure vessel theory, the radius of curvature (R_t) for the elliptical cross section can replace the circular radius in the calculation of the hoop and axial pressure stresses as defined in ref. 3. Running load contributions derived from the internal tank pressures are given in Eqs. (7)-(10).

$$N_{xullage} = \sigma_{l\,ullage} t_s = p_{ullage} R_t / 2 \tag{7}$$

$$N_{xhead} = \sigma_{l\,head} t_s = p_{head} R_t / 2 \tag{8}$$

$$N_{yullage} = \sigma_{h\,ullage} t_s = p_{ullage} R_t \tag{9}$$

$$N_{yhead} = \sigma_{h\,head} t_s = p_{head} R_t \tag{10}$$

The individual contributions from the external loads and internal tank pressures are summed to obtain the total longitudinal (N_x), circumferential (N_y), and transverse running loads (Eq. 6).

$$N_x = N_{xbend} + N_{xaxial} + N_{xullage} + N_{xhead} \tag{11}$$

$$N_y = N_{yullage} + N_{yhead} \tag{12}$$

After determining the running loads at each fuselage station for each load condition, the maximum running loads from the entire set of defined load conditions are selected to be used to determine the amount of shell material required at each section based on a *worst-case scenario*.

The maximum running loads determined at each fuselage station are used to calculate the amount of shell material required to preclude failure. The most critical point of the shell thickness is assumed to be the outermost location of the circumference, which is the position of the maximum stress experienced. The failure modes considered are ultimate strength, yield strength, and buckling. A material minimum gage restriction is also imposed as a final failure criterion. The shell thickness is selected as the maximum thickness from the failure modes at each fuselage station.

The equivalent isotropic thicknesses of the shell material determined for failure limited by ultimate tensile strength ($t_{s,UTS}$), yield strength ($t_{s,YS}$), and minimum gage thickness ($t_{s,mg}$) are given by Eqs (13)-(15), respectively:

$$t_{s,UTS} = N_1 / \sigma_{UTS} \tag{13}$$

$$t_{s,YS} \geq N_{eq} / \sigma_{yield} \tag{14}$$

$$t_{s,mg} = K_{mg} t_{mg} \tag{15}$$

The maximum principal running load (N_1) and equivalent running load (N_{eq}) are defined by the following:

$$N_1 = \frac{N_x + N_y}{2} + \sqrt{\left(\frac{N_x - N_y}{2}\right)^2 + N_{xy}{}^2} \tag{16}$$

$$N_{eq} = \left[N_x{}^2 + N_y{}^2 + N_z{}^2 - N_y N_z - N_z N_x - N_x N_y + 3\left(N_{yz}{}^2 + N_{xz}{}^2 + N_{xy}{}^2\right)\right]^{1/2} \tag{17}$$

In Eq. (15), K_{mg} is the minimum gage parameter that relates the shell thickness ($t_{s,mg}$) to the minimum material thickness (t_{mg}). This parameter is derived from the fuselage skin and shell arrangement for various stiffened shell configurations typically used in aerospace vehicles.[4]

The maximum running loads determined at each fuselage station are used to size both the fuselage stiffened shell and general-stability frames required to preclude buckling failure and general instability, respectively. The calculations to size the fuselage shell assume a wide column behavior of the shell, and the required stability ring frames are sized using the Shanley criterion.[5] Expressions were derived to determine the equivalent isotropic thickness of the shell and ring frames. Assumptions for the analysis are that the structural shell behaves as an Euler beam, all structural materials behave elastically, and shell shapes are restricted to the case of cylindrical shells.

Minimum weight equations determined in Ref. 6 present the following expression for buckling of wide column stiffened shells:

$$\frac{N_x}{LE} = \varepsilon\left(\frac{t_s}{L}\right)^m \tag{18}$$

where ε is the shell buckling efficiency, m is the equation exponent, L is the frame spacing, and E is the modulus of elasticity for the shell material. The shell buckling efficiency and equation exponent are a function of certain proportions of the stiffened shell configurations under consideration. The shell buckling efficiency and equation exponent values are given for each shell configuration in Table 1.[4] All of the shell configurations used within this study has an equation exponent equal to 2, which then solving for the shell thickness leads to the following equation:

$$t_{s,B} = \sqrt{\frac{N_x L}{E\varepsilon}} \tag{19}$$

Table 1. Stiffened Shell Configuration Factors for Wide Column Shell.[4]

Shell Configuration	ε	m	K_{mg}
Simple unflanged integrally stiffened	0.656	2	2.463
Z-stiffened	0.911	2	2.475
Truss-core sandwich	0.605	2	4.310

In addition to the stiffened shell, ring frames are sized to prevent general instability failure of the fuselage using the Shanley criterion. The Shanley criterion is based on the principle that the frames act as elastic supports for the wide column shell[4], which results in the following equation for *smeared* equivalent thickness of the frames.

$$t_f = \frac{k_c D^2}{2}\sqrt{\frac{C_f \pi N_x}{L^3 k_f E_f}} \tag{20}$$

where k_f is frame stiffness coefficient, k_c is the shape correction factor for circumference of non-circular shell cross-sections, C_f is the Shanley constant, D is the depth of the cross section, and E_f is the modulus of elasticity for the frame. (See ref. 5 for a discussion of the application of this criterion and detailed derivation of the equations presented here.) Assuming that the shell is buckling critical, the total thickness is the sum of the buckling shell thickness and the *smeared* frame thickness.

$$t = t_{s,b} + t_f = \sqrt{\frac{N_x L}{E\varepsilon}} + \frac{k_c D^2}{2}\sqrt{\frac{C_f \pi N_x}{L^3 k_f E_f}} \tag{21}$$

Minimizing the total thickness with respect to the frame spacing and solving for the frame spacing yields an expression for the frame spacing that is a function of the coefficient parameters and cross-section depth.

$$L = \left(\frac{3}{2} k_c D^2 \left(\frac{\rho_f}{\rho_s} \right) \sqrt{\frac{C_f \pi E_s \varepsilon}{k_f E_f}} \right)^{1/2} \tag{22}$$

The fuselage shell must satisfy all failure criteria at each station. The shell thickness was determined by selecting the maximum thickness according to the ultimate strength, yield strength, buckling, and minimum gage failure.

$$t_s = \max\left(t_{s,UTS}, t_{s,YS}, t_{s,B}, t_{s,mg} \right) \tag{23}$$

If $t_s = t_{s,B}$, the shell structure is buckling critical, then the equivalent isotropic thickness of the frames (t_f) is computed using the given equation from Shanley. If $t_s > t_{s,B}$, the shell structure is not buckling critical at the optimum frame sizing. The frames are resized to make the selected shell thickness buckling critical ($t_s = t_{s,B}$). New frame spacing is computed using the shell buckling thickness equation as

$$L = \frac{t_s^{\,2} E_s \varepsilon}{N_x} \tag{24}$$

This new frame spacing is used with the frame thickness equation to resize the frame.

From utilizing the failure criterion and selecting the appropriate shell configuration and material, the equivalent isotropic shell and *smeared* frame thicknesses at each fuselage station are determined, and the total thickness is calculated by their summation. Station-by-station integration of the equivalent shell and frame thicknesses generate the structural weight of the vehicle fuselage and components

$$W_T = \sum P_{ell\,i} \left(\rho_s t_{s\,i} + \rho_f t_{f\,i} \right) \Delta x_i \tag{25}$$

where the parameters subscripted *i* depend on position along the length of the fuselage, *x*.

The approach of an analysis based exclusively on fundamental structural principals will result in an accurate estimation of the vehicle structural weight only. Non-optimum weights for fuselage and component primary structure, such as bulkheads, minor frames, coverings, fasteners, and joints, are not estimated within the structural analysis and must be predicted from correlation to existing vehicles. The analysis only accounts for the external loads and do not consider any vibration effects experienced by the vehicle during flight.

III. Implementation of Analytical Methodology into GT-STRESS Computer Program

The methodology developed from fundamental beam structural analysis was implemented into a computer program to allow for the rapid estimation of the load-bearing structural weight of the launch vehicle fuselage and its associated components. Rapid approximation of the vehicle structural weight permits this design tool is useful for conceptual vehicle design studies.

The Georgia Tech Structural Tool for the Rapid Estimation of Shell Sizes (GT-STRESS) is a C++ constructed computer program that utilizes the previously described fundamental beam structural analysis to calculate the required running loads for sizing the fuselage shell and frames based on selected material and shell structure properties. From the determined shell thickness and selected material properties, the structural weight is calculated. The program simulates a launch vehicle fuselage fueled by liquid propellant contained in an integral tank structure arrangement.

The information input and basic operation of GT-STRESS are derived from RL, a computer program to calculate fuselage running loads, which was developed by Jeff Cerro, formerly of Lockheed Martin Engineering & Science Services.[7] GT-STRESS accepts a specified input text file that describes the geometry, preliminary subsystem weights, propellant and material properties, and the load conditions experienced by the vehicle. Keywords located within the input file are utilized by the program to recognize the relevant information required to run the program. The propellant densities used to calculate the head pressure load and the material properties used to size the fuselage shell and frame structures are located in text files that are external to the GT-STRESS program. The advantage of external files is the ability for the addition and modification of material and propellant keywords and properties within the database without affecting the functionality of the program or changing the program source code. An example input file is presented in Table 2.

Table 2. Example GT-STRESS Input File

EELV (with LRBs)	**Title (max 100 characters)**
oal 2285	**Overall length**
geom	**Geometry (max 10 sections)**
0 1 1	**x, semi-major axis, semi-minor axis**
20 18 18	**…**
198 100 100	**…**
2285 100 100	**…**
end_geom	
weights	**Weights (max 35)**
lh2_tank 1388 2196 9709.37	**description, start, end, weight (lbs)**
lox_tank 943 1218 3977.71	**…**
intertank 1219 1361 7860	**…**
2nd_stage 463 942 68662	**…**
propellant 943 1218 377143 lox	***propellant*, start, end, weight, type**
propellant 1388 2196 62857 lh2	**…**
...	**…**
end_weights	
structure	**Structure Component Sizing (max 35)**
lox_tank	**description (same as weights description)**
lh2_tank	**…**
...	**…**
end_structure	
material	**Materials (max 35)**
default_shell sandwich	**default shell configuration**
default_mat aluminum	**default shell/frame material**
composite 1219 1361 z-stiffened	**material, start, end, shell configuration**
...	**…**
end_material	
loadcase 1	**Loadcases (max 15)**
title Max q-alpha	**loadcase title (max 80 characters)**
x1 0	**simple support reaction location**
x2 2285	**…**
axial_accel 2.193	**axial acceleration (g's)**
normal_accel 0.514	**normal acceleration (g's)**
prop_ullage 30 30 30 30	**ullage pressure for each propellant tank (psi)**
pct_fueled 53.3 53.3 36 36	**percent fuel remaining for each propellant tank**
end_loadcase	
...	**…**

At the execution, the GT-STRESS program prompts the user to enter the name of the input file, the root name of the output file, and the value of the convergence relaxation factor. After entering the relaxation factor value GT-STRESS starts operation by reading in the geometry, preliminary weight, material, and loadcase data from the input file. The program continues by initiating the analysis to determine the external loads and running loads required to size the shell and frame material and ascertain the fuselage structure weight.

After the first analysis of the vehicle by GT-STRESS, the initial weight values of the components defined in the *structure* section of the input file are replaced by the structural weight calculated using the analytical method within the program. Typically there are 3-5 structural components for each stage of an expendable liquid propellant launch vehicle and 5-10 components for reusable launch vehicles. Once the component weight values are replaced, GT-STRESS runs another iteration of analysis and calculates new values of the components weight and vehicle weight based on the new initial values taken from the last iteration. After each iteration GT-STRESS outputs the current calculated vehicle structural weight and iteration number to the screen. This fixed point iteration (FPI) process continues until the difference between the previous and present values of the total vehicle structural weight reaches absolute convergence, which is less than or equal to 1×10^{-4} pounds. A flow chart of the convergence process to obtain the structural component weight is displayed in Fig. 3.

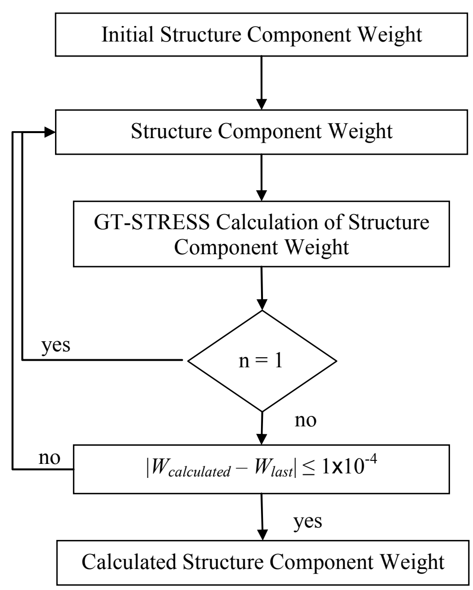

Figure 3. GT-STRESS Convergence Process

If the convergence process of the vehicle becomes unstable or the value of the component initial masses are vastly different from the converged computed values, reaching convergence for the vehicle structural weight can require a larger amount of iterations. Therefore relaxation was integrated into the FPI process to introduce damping into the convergence process and improve the stability. A relaxation factor (α) is introduced into the feedback variables of the FPI process by the following expression:

$$W_{next} = \alpha W_{calculated} + (1-\alpha) W_{last} \tag{26}$$

where W_{last} is the component weight from the previous iteration, $W_{calculated}$ is the component weight from the current iteration, and W_{next} is the component weight value fed back to the weight definition. Relaxation essentially takes a weighted average value of the component weight calculated from the previous and present iterations and feeds back this value to the weight definition. These averaged values of the feedback variables allows the vehicle weight to reach convergence quicker for a stable problem with extreme initial masses or reach a happy medium for an unstable problem. The relaxation factor value is between 0 and 1. A value of zero will only feedback the initial component masses, a value of one will continue feeding back the recent calculated value for each iteration as the basic FPI process, and ½ is an average value. A relaxation factor value of ½ was used for this study. An equivalent Multidisciplinary Analysis (MDA) model of the program operation is presented in Fig. 4.

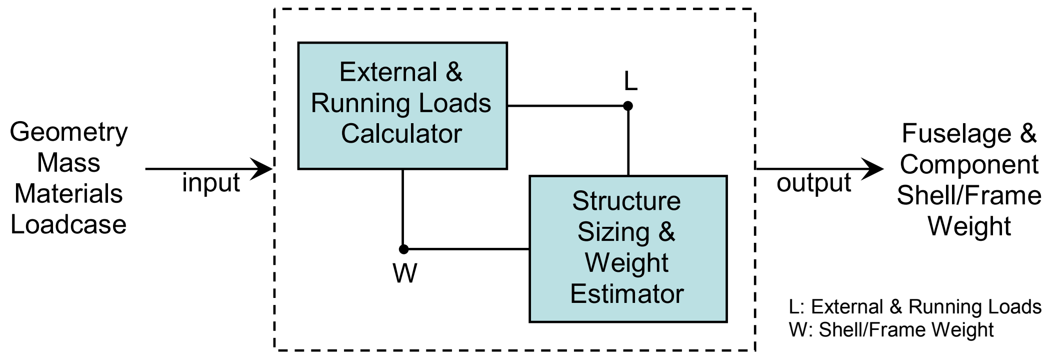

Figure 4. MDA Model of GT-STRESS Program Operation.

After operation the program computes the fuselage structure weight and other vehicle component weights (i.e. propellant tanks, interstages) as specified in the input file. Along with the resulting structural weight, the program will also generate output files that contain the summary of the information received from the input file, external loads over the vehicle length for each load condition, running loads for the overall vehicle, shell and frame thickness for the overall vehicle, and a structural weight breakdown based on fuselage and structural components. The average operation time for the process ranges from 60-90 seconds on a 2.26 GHz Pentium 4 computer, and the time increases/decreases with variation in vehicle definition complexity, amount of loadcases, and difficulty in determining a converged weight.

IV. Verification and Correlation with Existing Launch Vehicle Propellant Tanks

The previously described analytical methodology was applied to the determination of the liquid propellant tanks structural weight of an existing Evolved Expendable Launch Vehicle (EELV) and the Space Shuttle External Tank (ET) for verification and correlation. These two vehicles were selected for validation of the methodology because extensive non-proprietary weight breakdown statements for the vehicles were available and the required information for the load cases could be determined from their predicted trajectories. After calculating the load-bearing structural weight of the vehicle components, statistical techniques were used to estimate the relationship between the weight calculated by GT-STRESS and the actual vehicle load-bearing structural weights.

A. Evolved Expendable Launch Vehicle Analysis

The EELV used for verification of the methodology investigated in this study is based on the Boeing Delta-IV Heavy EELV. The launch vehicle geometry, inert masses, propellant masses, material type, and structural configuration are very similar to that of the Delta-IV Heavy. The trajectory for the EELV was modeled after the Geostationary Transfer Orbit (GTO) mission for the Delta-IV Heavy and simulated using POST, a trajectory optimization program. The majority of the information used to estimate the values for the vehicle parameters and trajectory was provided by Ref. 8. Dimensions, masses, and structure properties of the EELV are presented in Table 3.

Table 3. Dimensions, Masses, and Structure Properties of the EELV and ET.[8,9]

	EELV - Stage 1	STS External Tank
Dimensions		
Length	133.9 ft	154.2 ft
Diameter	16.7 ft	27.6 ft
Mass		
Propellant Mass	440 klb	1589 klb
Inert Mass	59 klb	59.5 klb
Gross Mass	499 klb	1648 klb
Structure		
Type	Tanks: isogrid Interstage: skin-stringer Intertank: skin-stringer	skin-stringer
Material	Tanks: aluminum Interstage: graphite-epoxy Intertank: graphite-epoxy	aluminum

Table 4. EELV Load Cases and Required Parameters.

Load case	1	2	3	4	5
title	liftoff	max q	max q-alpha	max thrust	max axial accel
x1 (in)	2284	0	0	0	0
x2 (in)	2285	2285	2285	2285	2285
axial_accel (g's)	1.1945	1.44	2.193	5.6162	6.0
normal_accel (g's)	6.92e-5	0.0001	0.514	0.0012	6.4e-4
prop_ullage (psi)*	30	30	30	10	10
	30	30	30	10	10
	30	30	30	10	
	30	30	30	10	
pct_ fuel (%)*	100	71	53.3	7.28	5.26
	100	71	53.3	7.28	5.26
	100	67	36	10	
	100	67	36	10	

*order of propellant tanks: CCB LOX, CCB LH_2, LRB LOX, LRB LH_2

After collecting all of the required information, a GT-STRESS input file was created for the vehicle. The load conditions examined for the vehicle were liftoff, maximum dynamic pressure (max q), maximum dynamic pressure and angle of attack (max q-alpha), maximum thrust, and maximum axial acceleration. The required parameters for each load case were obtained from the simulated trajectory determined by POST and are listed in Table 4. Since GT-STRESS's modeling capability is limited to a single fuselage with all of its associated components arranged in-line throughout the length of the vehicle, the Liquid Rocket Boosters (LRBs) were modeled as point loads at their attachment location to the core booster. The LRB structure weight remains a constant load at its point of connection to the CCB, and the propellant loads are modeled by their percentage with each load condition. In the final load condition the LRBs are not modeled with the vehicle since they have already separated, and this load condition has an independent input file from the others since the weight statement for the input file was different from the others.

The focus of the EELV analysis was the determination of the common core booster (CCB) liquid hydrogen and liquid oxygen tank structural weight. Analysis was not conducted to determine the upper stage and fairing weight due to the lack of detail geometry and weight information. The propellant tanks structure type was substituted with the truss-core sandwich configuration since GT-STRESS could not accommodate the isogrid structure type. The graphite-epoxy for the interstage and intertank were substituted with the composite material defined in the material database since the properties of that particular graphite-epoxy were unknown. Actual structural weights of the propellant tanks are listed in Table 6.

B. Space Shuttle External Tank

The entire inert mass, propellant mass, material, and geometry information for the space shuttle external tank was made available from Ref. 9. Dimensions, masses, and structure properties of the ET are presented in Table 3. After accumulating all of the required information, a GT-STRESS input file was created for the ET. The trajectory for the Space Shuttle ascension was simulated using the POST. Similar to the LRBs for the EELV, the Solid Rocket Boosters (SRBs) were modeled as point loads at their attachment location to the ET. Since the amount of propellant for the SRBs change at each load condition, each load condition was run individually in GT-STRESS because the weight statement for each input file was different. The orbiter is modeled as two point loads at the locations of the orbiter attachment bars on the ET. The load conditions examined for the ET and their required parameters are presented in Table 5. The focus of the analysis for the ET was determining the structural weight of the liquid hydrogen tank and the liquid oxygen tank. The actual weights for the propellant tanks are listed in Table 6.

Table 5. ET Load Cases and Required Parameters

Load case	1	2	3	4
title	liftoff	max q	max q-alpha	max thrust
x1 (in)	1847	666	666	690
x2 (in)	1848	1372	1372	1723
axial_accel (g's)	1.2356	1.3193	1.462	2.9976
normal_accel (g's)	0	0.3857	0.3477	6.2E-6
prop_ullage (psi)*	31	31	31	30
	36	36	36	30
pct_ fuel (%)*	100	83	87	3.0
	100	83	87	3.0

C. GT-STRESS Result Data for Validation Cases

The total operation time for each vehicle analysis was about 100 seconds. The propellant tank structural weights calculated by GT-STRESS for the EELV and ET are given in Table 6. Graphs of the Axial Load Magnitude, Shear Load, and Bending Moment along the vehicle length for each load condition of the EELV and ET are located in Figs. 5-10. Graphs of the shell and frame thicknesses along the vehicle length for the EELV and ET are presented in Figs. 11 and 12.

Table 6. Actual and Calculated Structural Weights for the EELV and ET.

	Actual Weight (lb)	GT-STRESS Weight (lb)	% Error
ET LOX tank	12520	8970	28.35
ET LH_2 tank	31739	22502	29.10
EELV LOX tank	4926	3977	19.27
EELV LH_2 tank	10937	9709	11.22

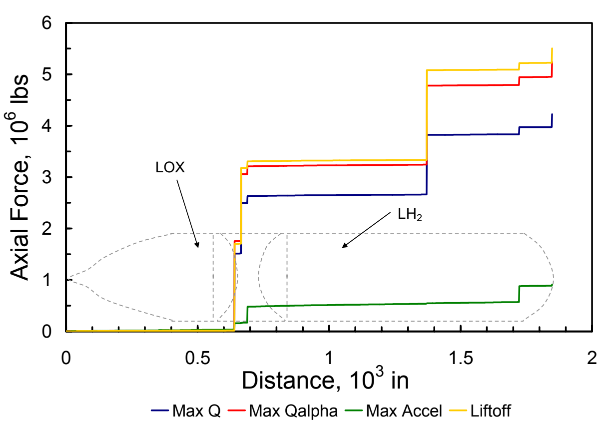

Figure 5. Axial Load Magnitude Variation along the ET for Each Load Condition.

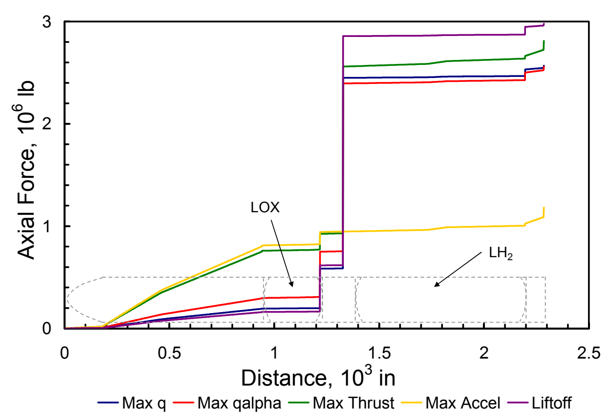

Figure 6. Axial Load Magnitude Variation along the EELV Fuselage for Each Load Condition.

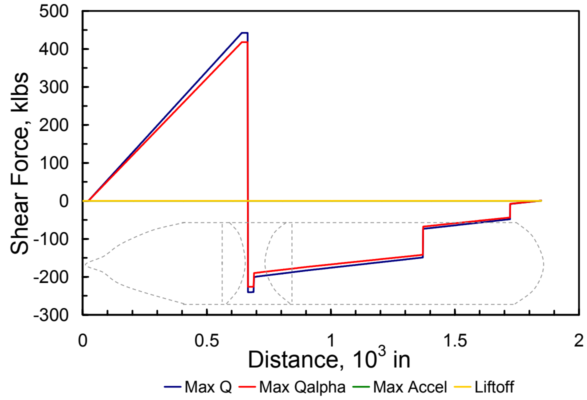

Figure 7. Shear Force Variation along the ET for Each Load Condition.

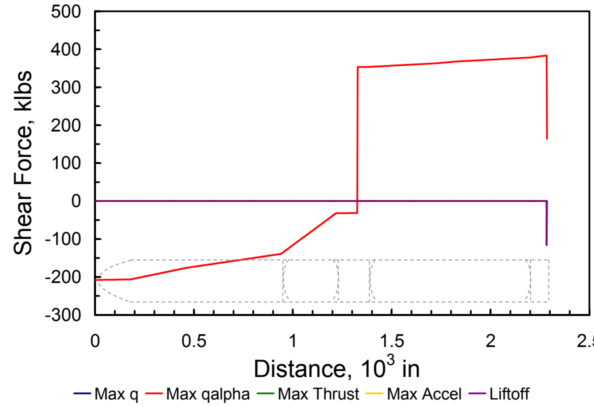

Figure 8. Shear Force Variation along the EELV Fuselage for Each Load Condition.

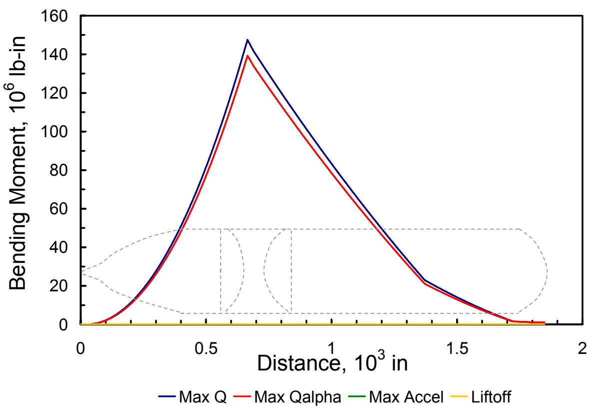

Figure 9. Bending Moment Variation along the ET for Each Load Condition.

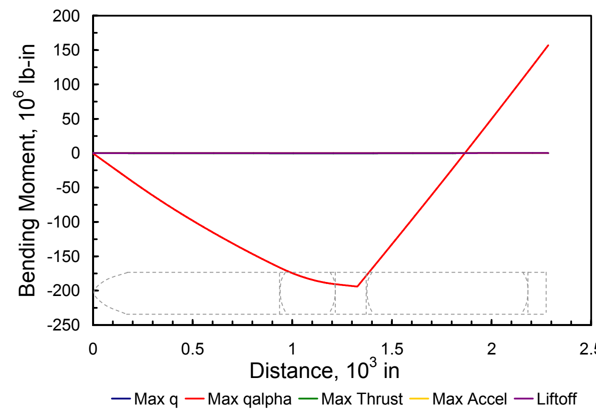

Figure 10. Bending Moment Variation along the EELV Fuselage for Each Load Condition.

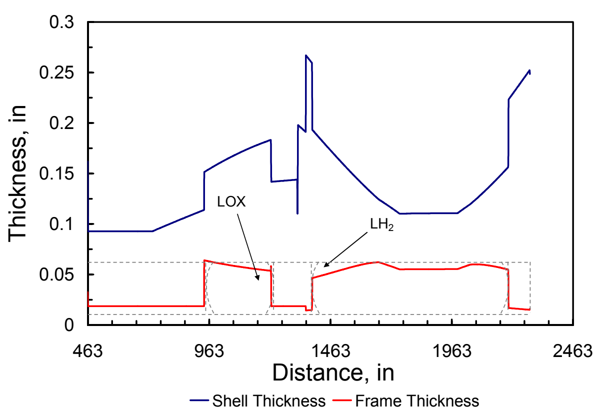

Figure 11. Shell and Frame Thickness Variation along the 1st stage fuselage of the EELV.

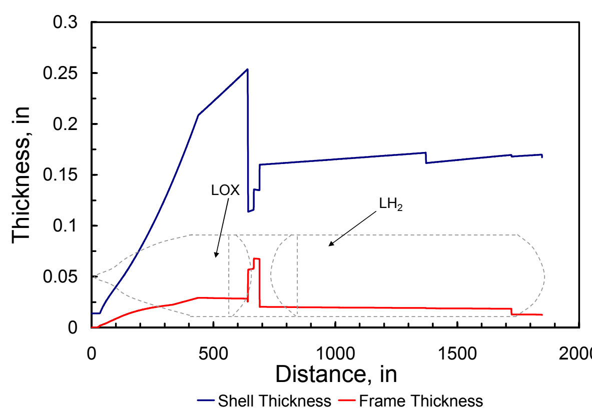

Figure 12. Shell and Frame Thickness Variation along the length of the ET.

The percent error between the actual tank structural weight and the weight calculated from GT-STRESS ranges from 11.22% to 29.10%. The large percent error overall indicated that the structural analysis based only upon the external loads experienced by the vehicle to estimate the fuselage stiffened shell and stability frame weight was unable to account for the total propellant tank structural weight. In order to resolve the large error, a linear regression of the actual structural weight by the calculated weight was conducted in order to determine a factor that accounts for the percentage of the structure weight not represented in the analytical method.

D. Regression Analysis

By obtaining the actual fuselage component weights from the weight statements of the two launch vehicles, a relation between the calculated load-bearing structure weights obtained from GT-STRESS and the actual load-bearing structure weights and primary structure weights are determined using linear regression. Applying linear regression develops the relation of the estimated component weights of the launch vehicle to the calculated weights from GT-STRESS using a straight line.

$$y = \beta_1 x + \beta_0 \tag{27}$$

where y is the value of the estimated weight, β_1 is the slope of the regression line, x is the weight value obtained from GT-STRESS, and β_0 is the y-intercept. The regression line is determined by using the *method of least squares*, where the sum of the squares of the residual errors between the actual data points and the estimated data points on the regression line is minimized. Therefore a straight line is drawn through the ordered pairs of weight data so that the collective deviation of the actual weight above or below the line is minimized. Using the regression technique allows for the formation of an expression for the estimated weight as a function of the calculated weight from GT-STRESS. For the regression line the y-intercept term is set to zero knowing that a calculated weight of zero will result in a true actual weight of zero. This simplified version of the linear equation allows the expression to be applied to a large spread of weights and compared with other regression data for analytical weight estimation.

The accuracy of the regression in the prediction of the estimated component structural weight from the GT-STRESS calculated weight is represented by the coefficient of variation, which is also denoted as the R^2 value. The R^2 value is interpreted as the reduction in residual error due to the regression technique.[4] An R^2 value of 1 represents a perfect fit of the regression line to the data while an R^2 value of zero represents denotes that regression analysis does not provide any improvement in fitting the data.

The analytical methodology implemented into the GT-STRESS program only accurately predicts the load-bearing structure of the shell and stability frames. Other load-carrying members included in the structural weight of the integral propellant tanks, such as bulkheads, fasteners, minor frames, covering, and covering stiffeners are not included. Also the analysis only accounts for the external loads in the weight determination and does not consider any vibration effects experienced by the vehicle during flight. Applying linear regression to the actual and calculated values of the propellant tanks of the launch vehicles used for verification yields an empirical relationship that estimates the total structural weight as a function of the program calculated value (W_{STRESS}).

$$W_{actual} = 1.3665 W_{STRESS} \tag{28}$$

The R^2 value for this linear curve-fit is 0.9948. Based on the linear regression, the calculated weight from GT-STRESS must be increased by about 36.7% to get the actual structure weight. The linear regression of the structural weight is displayed in Fig. 13. The actual and correlated structural weights for the EELV and ET propellant tanks are listed in Table 7.

Table 7. Actual and Correlated Structural Weights for the EELV and ET.

	Actual Weight (lb)	Correlated GT-STRESS Weight (lb)	% Error
ET LOX tank	12520	12257	2.10
ET LH_2 tank	31739	30749	3.12
EELV LOX tank	4926	5436	10.35
EELV LH_2 tank	10937	13268	21.31

Due to the limited quantity of data points for the regression analysis of the structural weight, the resulting equations relating the component weight calculated in GT-STRESS to the actual structural weight are questionable. Validation of the resulting correlation between the calculated and actual component structure weight required either a larger quantity of data points to generate a regression or that the current regression followed a trend of a larger data set that conducted a very similar analysis. Within Ref. 4 a computer program, PDCYL, which employed the same basic fundamental beam structure analysis used within GT-STRESS, was used by authors to determine the structural weight of eight conventional transport aircraft fuselage.[4] The classification of the structural weight within Ref. 4 is equivalent to the structural members that comprise the structures of the integral propellant tanks. Linear regression analysis of the data generated from PDCYL and the actual values yielded the following correlation for the fuselage structure weights:

$$W_{actual} = 1.3503 W_{PDCYL} \quad (56)$$

The correlation between the actual and calculated structure weights from the regression of the aircraft fuselage data is very similar to the regression equation for the launch vehicle fuselage data. The trends from the regression of the structure weight for both data sets are very comparable, as displayed in Fig. 14. Therefore the close resemblance of the trends and correlation of the estimated structural weight from GT-STRESS to PDCYL validates that the launch vehicle propellant tank structural weight are accurately represented.

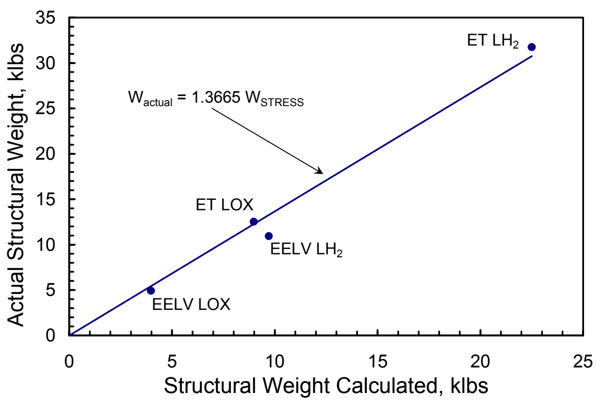

Figure 13. Linear Regression of Structural Weight.

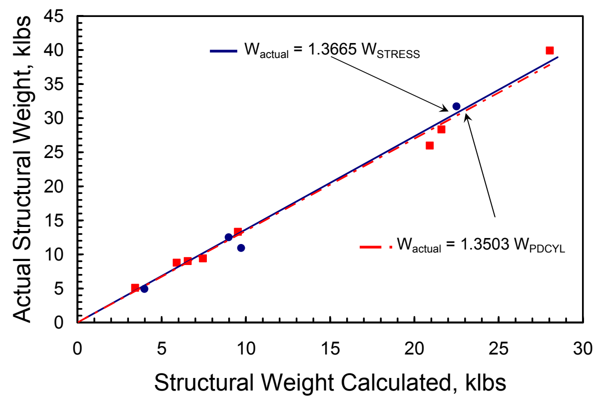

Figure 14. Regression Comparison of Structural Weight Results from GT-STRESS and PDCYL.

V. Conclusion

A method based on fundamental beam structure analysis to accurately determine structural weight of the launch vehicle fuselage and components at a minimized cost of time and computational effort was developed. The simplified beam approximation model of the vehicle was utilized by the fast-acting software tool GT-STRESS for the rapid estimation of the fuselage & component load-bearing structural weight. The method was applied for the estimation of the structural weight of the liquid propellant tanks of an EELV and the Space Shuttle ET. Due to GT-STRESS's inability to account for the vehicle vibration loads and additional load-bearing structural members involved in the total structural weight, a linear correlation between the actual and calculated tank weights was developed to resolve the larger percent weight difference. The correlation of the GT-STRESS data for the propellant tanks was verified by comparison to the resulting trend of a similar methodology employed for transport aircraft fuselage weight estimation.

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