An Evolutionary Framework for AS-Level Internet Topology Modeling

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Abstract— **Models for network topology form a crucial component in the analysis of protocols. This paper systematically investigates a variety of *evolutionary* models for autonomous-system (AS) level Internet topology. Evolution-based models produce a topology incrementally, attempting to reflect the growth patterns of the actual topology. While evolutionary models are appealing, they have generally agreed less closely with measurements of real data than non-evolutionary models. We attempt to understand what contributes to a "good" evolutionary model. Our systematic study consists of a relatively generic evolutionary model framework, which we populate with different choices for the components. This allows us to compare a variety of instances of models to measurements from real data sets. We study issues such as the initial topology, the type of preferential connectivity used in adding edges, and the role of "growth" edges added between existing nodes. We find that appropriate instantiation of the framework can provide topologies that agree closely with real data. We also use our work to highlight several crucial open problems in topology modeling.**

I. Introduction

Models for network topology form a crucial component in the analysis of protocols. Typically, a model for topology is combined with a model for traffic to define an operational environment for the protocol under study. In some cases, a fairly simple topology (e.g., a single bottleneck link) is used; such models may admit analytic solutions. Increasingly, however, the evaluation of protocols includes simulation-based studies using topologies that are larger, in an attempt to more accurately reflect the properties of networks like the Internet.

Attempts to understand and model the topology of the Internet have traditionally been limited by the difficulty of obtaining real data. Network providers are reluctant to reveal details of internal topology, for obvious reasons of security. While internal topology is a necessary component of router-level topology (where the topological entities are routers and their interconnections), it is hidden in autonomous system (AS) level topology (where the topological entities are autonomous systems and their BGP interconnections). Fortunately, data on the AS-level topology can be inferred using the tables built by the interdomain routing protocol BGP [1]. Daily snapshots of AS-level topology since November 1997 are available as part of the U. of Oregon Route Views project [2] and archived at NLANR [3].[1]

Faloutsos et al. analyzed the AS-level topology data, focusing on three instances over a period of one year [4]. They observed that several properties of the topology can be described using power laws, of the form $y = x^{\alpha}$. In particular, these power laws concern the outdegree of nodes, the neighborhood size around a node (defined by reachable nodes within h hops), and the eigenvalues of the adjacency matrix.

The Faloutsos observations have inspired the analysis of older models for topology, which are largely intended for router-level modeling (e.g., Tiers [5], GT-ITM [6], Waxman [7]), as well as development of newer models that clearly target AS-level representation (e.g., BRITE [8] and Inet [9]). Generally speaking, the newer models have been demonstrated to agree more closely with real topological data than the older models, on a variety of measures [8], [9].

One type of newer model, typified by the BRITE work, is evolution-based in the sense that it produces a topology incrementally, by adding one node at a time to an existing topology. Evolution-based models have appeal for several reasons. First, they explicitly model, to some extent, the growth of the actual Internet. In doing so, they have the potential to lead to insights about why particular charac-

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[1] It should be noted that these snapshots are more accurately called *approximations* of the AS-level topology. They are based on merging views from a number of BGP routers, but do not claim complete coverage. Other limitations in the real data are discussed in Section II.

teristics are present in any snapshot of a topology. Second, because they maintain a current topology, they can be used to produce a series of snapshots for the study of a protocol under a dynamic, time-varying topology. Third, they are naturally amenable to "what-if" studies that consider the effect a change in technology might have on an evolving topology.

To date, however, the evolutionary models have generally agreed less closely with real data than models such as Inet, that are driven by achieving a particular degree distribution. Thus, though evolutionary models are appealing for the reasons listed above, there is a gap in the quality of the topologies they produce. The goal of this paper is to systematically investigate a variety of evolutionary models. We do this via a relatively generic evolutionary model framework, which we populate with different options for the components. Our choices for specific components are in part driven by observations taken from the real data set over time. That is, we attempt to use the real data, not in single snapshot form, but viewed as a historical record of actual evolution.

Our work is close in spirit to the BRITE paper, which also uses a framework to study the effect of different choices in an evolutionary model. We differ from BRITE in the types of components we choose to focus upon. For example, we explore several different types of preferential connectivity of quite different flavor than the BRITE choices. We also consider a greater number of basic evolutionary events than BRITE, including the addition of edges between existing nodes. Finally, we use measurements from the real data to drive choices in our evolutionary model.

The remainder of the paper is organized as follows. Section II describes the real data set and presents evaluation of properties of many instances of the real data. This sets the stage for the target of modeling efforts. Section III describes the two most relevant related models, Inet and BRITE, and evaluates topologies generated by these models. We find room for improved models. Section IV presents our generic framework, and Section V lists the ways in which we can instantiate the framework. Section VI evaluates several key components of our framework. We conclude in Section VII and include a list of key open problems.

II. Real Topology Data

A. Description of data set

We use the NLANR data sets, which merge BGP data from a collection of BGP routers at different locations. The intent of the merged views is to produce a fairly complete version of the Internet AS-level topology. Though this data is widely used for topology research, it has several types of inaccuracy:

- **Incompleteness.** The data is currently collected from 52 BGP routers distributed geographically. There is no guarantee that their merged views include all ASes in the Internet. Some AS information may be hidden intentionally; other AS information may simply be out of the regions covered by the collection routers. It is difficult to quantify the extent of incompleteness, and we do not attempt to do so. As in much other work, we use the data as is. An open and interesting question is to quantify the incompleteness and consider improved methods for capturing the full Internet topology. Some progress in router level modeling can be found in Govindan et al. [10].
- **Collection failures.** The data sets contain occasional instances of unusually small data files. We attribute these to collection failures, either at BGP collection points or in the process of recording the merged data. We are careful not to use any instance that is unusually small.
- **Instability.** The data sets viewed over time contain instabilities, in the form of nodes that are intermittently present in the data. That is, an AS number may be present in the data set one day, absent for a short number of subsequent days, then reappear. This sort of instability could be caused by failure of the BGP-speaking router in the AS or flapping of routes that causes this AS to be bypassed intermittently. Though BGP4 [11] provides solutions to dampen route flapping, flapping routes are still very common [12]. Instability causes difficulties in certain measures. For example, it is difficult to measure the number of original edges that appear with a node (as opposed to growth edges that are added later), when the node has no single point of entry into the topology.

B. Evaluation of real data

Before evaluating models for generating topologies, we consider the values of a set of metrics on instances from the NLANR data. We have selected 10 instances from the data set, covering a doubling in growth from about 3000 nodes to 6000 nodes [2]

An open question in the area of topology modeling concerns the evaluation of proposed models. The approach currently taken by most researchers is to evaluate the generated topologies using a variety of graph-based metrics, some focusing on fine-grained properties of the graph (e.g., the degree sequence [4]) and others attempting to capture more coarse-grained structure (e.g., expansion [13], clustering [8], [14]).

[2] The data sets chosen represent snapshots that increment in size by approximately 500 nodes.

size	3503	4008	4502	5003	5500	6023	6505	6996	7505	*Legend of abbreviations*
edgenum	6302	7308	8371	9514	10570	11682	12600	13908	15007	edgenum: *number of edges*
avgdgr	3.60	3.65	3.72	3.80	3.84	3.88	3.87	3.98	4.00	avgdgr: *average degree*
clustcoeff	0.77	0.76	0.87	0.74	0.74	0.74	0.74	0.74	0.74	clustcoeff: *clustering coefficient*
diameter	10	11	10	9	9	9	9	9	10	
asp	3.76	3.78	3.74	3.72	3.72	3.70	3.71	3.67	3.65	asp: *average shortest path length*
eccentri	7.44	7.49	7.01	6.87	6.67	6.74	6.76	6.64	6.66	eccentri: *eccentricity*
dgrfreq(b)	-1.22	-1.23	-1.20	-1.18	-1.21	-1.17	-1.18	-1.15	-1.16	dgrfreq(b): *slope of linear regression result on degree-frequence sequence*
dgrfreq(r)	0.851	0.860	0.843	0.838	0.849	0.835	0.85	0.84	0.84	dgrfreq(r): *correlation coefficient of linear regression result above*
pw2(b)	-2.16	-2.19	-2.21	-2.22	-2.16	-2.34	-2.21	-2.27	-2.15	pw2(b): *slope of linear regression result of sequence above excluding 2% largest nodes*
pw2(r)	0.98	0.98	0.98	0.97	0.98	0.97	0.97	0.98	0.98	pw2: *correlation coefficient of linear regression result as above*

TABLE I
SNAPSHOT OF REAL TOPOLOGY

Our intuition is that measures of coarse-grained structure are more practically important than measures of fine-grained structure. However, we do not currently have research results to substantiate that intuition. In the absence of a solid understanding of which metrics matter (and under what circumstances), we take a broad brush approach and evaluate a fairly large collection of metrics, all previously proposed for evaluation of topologies. The metrics are defined in the Appendix. In fact, our work makes clear that this approach leads to great difficulty in reaching conclusions about models, and hence points to evaluating topologies as a critical and unsolved open problem.

Table I shows the sizes of each instance and the values of the metrics. As the topology gets larger, the average node degree increases, and hence the graph is more dense. This increase in density is also evident in the diameter (which tends to remain constant or decrease as size increases), the average shortest path length (which tends to decrease slightly, despite a much larger topology), and the eccentricity (which tends to decrease).

Other metrics show no clear trend as the topology increases. For example, the clustering coefficient increases and then decreases. Measures with this sort of variability are both interesting and problematic. The fact that the real data exhibits non-monotonic values for certain metrics seems to provide evidence that the underlying growth mechanisms are not always well-behaved. In turn, this presents a challenge for any modeling method that is attempting to capture this behavior. Variable measures also lead to difficulty in assessing a model, since the value observed in the real data is sensitive to the particular choice of instance.

Many of these measures are actually surprisingly constant, varying by about 10% even as the topology size doubles. This perhaps points to additional invariants (driven by performance considerations?) in the growth of the Internet topology, beyond those observed in the focus on power law behavior.

III. Evaluation of Inet and BRITE

In this section, we compare topologies generated by Inet and BRITE to the real topology data. We observe that while each model agrees with the real data on some measures, both exhibit disagreement on other measures, hence indicating room for improved models.

In BRITE, the initial topology is formed by random connection of a fixed number of backbone nodes. Nodes are placed in the plane either all at once or incrementally. In the case of placement all at once, nodes are then considered in random order to be connected to exactly m neighbors selected from all other nodes. In the case of incremental placement, new nodes are added one at a time and connect to exactly m neighbors from those nodes present in the topology. Three forms of connectivity are supported: Waxman-style, where the probability of an edge is proportional to the distance between the two nodes; degree-based preferential connectivity, where the probability of an edge is proportional to the current outdegree of the target node; and weighted preferential connectivity, where the probability of an edge is proportional to the outdegree weighted by the distance between the two nodes. The primary conclusion of the BRITE study is that preferential connectivity and incremental growth are key factors contributing to agreement between generated topologies and power-law measurements on real AS-level topologies.

The Inet approach represents another class of models, which are not evolutionary. Instead Inet generates a target degree for each node and then interconnects the nodes so as to achieve the target for each node. Preferential connectivity is used in deciding the connections when there are multiple nodes with unfilled outdegrees.

In our comparison, we use an instance of size 5003 nodes and 9514 edges, and use each generator to produce a topology with a similar number of nodes and edges. Table II summarizes the comparison.

	RealTopo	Inet	BRITE
size	5003	5000	5009
edgenum	9514	9852	9364
avgdgr	3.80	3.94	3.74
clustcoeff	0.74	0.99	0.82
diameter	9	12	9
asp	3.72	3.74	5.12
eccentri	6.87	8.29	7.34
dgrfreq(b)	-1.18	-1.30	-1.81
dgrfreq(r)	0.84	0.86	0.86
pw2(b)	-2.22	-2.25	-0.57
pw2(r)	0.97	0.99	0.98

TABLE II
EVALUATION OF INET AND BRITE

The Inet data agrees fairly closely with the real topology on the measures of average degree and average shortest path length. It also agrees with two power law measures: dgrfreq(b), which is the slope of the linear regression for the degree-frequency power law, and pw2(b), which is the slope of the linear regression for the degree-frequency data excluding the largest 2% of ndoes. Since Inet explicitly uses a degree sequence derived from the real data, the agreement with degree-based measures is not surprising.

Inet disagrees with the real data, however, on the measures of clustering coefficient, diameter and eccentricity. In all cases the Inet results are larger than the real data. Examining the Inet procedure closely, we see that it first assigns each node a degree according to an exponential function inferred from the real data, and then connects all those nodes whose degree is larger than one into a spanning tree. The order in which nodes are selected to connect to the spanning tree is based on the assigned degree: higher degree nodes tend to be added to the spanning tree early, while lower degree nodes tend to be connected into the spanning tree later. As a result, lower degree nodes tend to form the leaves of the tree. After forming the initial spanning tree, degree one nodes are added to form a spanning tree that includes all nodes. Although additional edges are added to fulfill the pre-assigned degrees, there remain low degree nodes at the leaves that cause the large diameter. This spreading of the topology also affects clustering coefficient and eccentricity.

Though BRITE also uses a spanning tree initially, it doesn't produce topologies with overly large diameter. There are two reasons for this. First, the spanning tree is typically a small fraction of the whole topology, used as an initial topology before the incremental growth is applied. Second, the spanning tree nodes are connected based on geometric distance, with nodes near by in Euclidean space connected together. This tends to result in tree paths without too many hops.

We might expect diameter and average shortest path length to be correlated. That is, topologies with low (high) diameter would be expected to have low (high) shortest path length. The BRITE and Inet topologies reverse this intuition. Inet's average shortest path length agrees closely with the real data, despite the larger diameter. This occurs because while a few nodes are far apart, most nodes are fairly tightly connected. In BRITE, the average shortest path length is a good deal higher than the real data, despite the similar small diameter. In BRITE, the incremental growth nodes are connected to the initial topology using degree preference, however the nodes in the initial topology tend to have similar degrees. Hence, the additional nodes tend to be distributed somewhat evenly over the existing nodes. The result is much less of a "center" to the graph and more long paths.

In addition to the average shortest path length, BRITE also disagrees with the real data on the power law measures. The slope of the linear regression is significantly different than the real data for both versions of degree-frequency data. This is caused by the well-known lack of large degree nodes in BRITE.

We conclude from this analysis that while both methods are able to match the real data on some measures, there is room for improved models and particularly for improved evolutionary models.

IV. A GENERIC FRAMEWORK

We desire the ability to study a variety of different evolutionary models. As in BRITE, our approach is to use a relatively generic framework that can be populated with different choices for the basic components to produce different evolutionary models. The basic evolutionary framework is straightforward: we begin with an initial topology, then apply discrete events to the topology until a stop requirement is met. (See Figure 1.)

A key issue in the design of the framework is the choice of events that can be applied. The BRITE model essentially includes only an AddNode event, which introduces a new node into the graph and connects the new node to existing nodes with some number of edges. Our observations from the real data set lead us to include three additional events (though any particular instantiation of the framework could choose to omit any of these). First, we include an AddEdge event, which adds an edge between two existing nodes. We thus distinguish between two types of edges, *original* edges that enter the topology with a new node, and *growth* edges that are added between existing nodes.

```
Main()
{
    Initialization();
    /* Read the argument file, initialize
     * the data structure and variables */
    GenInitTopo();
    /* Generate Initial Topology */
    while (stop condition is not met )
    {
        EventType = SelectEvent();
        switch (EventType)
        {
            case 1:
                AddNode();
                break;
            case 2:
                DelNode();
                break;
            case 3:
                AddEdge();
                break;
            case 4:
                DelNode();
                break;
        }
    }
}
```

Fig. 1. Basic procedure of the framework

The real data set also supports the inclusion of DelNode and DelEdge events, which, respectively, remove a node plus incident edges and a single edge. These events are probably most important if one desires to use the evolutionary model to produce a series of snapshots for studying the behavior of a protocol in a time-varying environment, where failure or removal of nodes and edges may be quite important. On the other hand, these two events pose difficulties in the generation process since either one has the potential to disconnect the graph. In our implementation of the framework we include a check for disconnection that is performed on deletion. If the deletion disconnects the graph, it is not performed.

The choice of specific event at each step is controlled by a SelectEvent function. For example, this function might use probabilities to determine the mix of the four different types of events.

Though not highlighted in the figure, we keep a notion of *time* in our evolution. In each time unit, a certain number of events occur, each associated with the current time stamp. This allows us to associate an *age* with a node, which we use in implementing some of the preferential connectivity methods. This could also be used in analyzing time-varying behavior of the topology, though we do not pursue this evaluation here.

V. Population of the Framework

The generic framework is populated by instantiating each of the relevant functions. This involves supplying (or choosing) a method to generate the initial topology, a function to control the frequency of different event types, a method for each event type and a stopping condition. This section describes the functions that we provide and use in our experiments.

A. Initial Topology

We have implemented four methods to generate the initial topology. Each method has its own set of control parameters.

1. **Incremental with degree preference.**
This method uses incremental creation to generate the initial topology. Starting from one node, additional nodes are added one at a time. The edges associated with the new node are connected to the existing nodes with probability that is linearly proportional to the degree of the existing node. The number of edges that come with a new node can be distributed in one of three ways: exponential, deterministic or uniform. This method is essentially equivalent to the generation method of BRITE [8].

2. **Degree-based topology.**
This method uses pre-assigned degrees for each of the nodes, with degree chosen from an exponential distribution. To generate a connected initial topology, nodes are added one at a time and connected (with a single edge) to the existing nodes, with equal probability. This produces a spanning tree over all the nodes. Finally, edges are added between nodes to fill the assigned original degrees. This is close to the method used by Inet to generate a complete topology [9]. However, there are two major differences between this method and Inet's. First, Inet uses degree preferential connectivity when building the spanning tree. In our case, we connect the nodes of the spanning tree purely randomly, i.e., every node has equal probability. Second, Inet assigns the degree sequence according to the real topology, i.e., it first assigns 98% nodes according to the second power law in Faloutsos' paper, then assigns top 2% large nodes. We assign the degree according to an exponential distribution.

3. **Random graph.**
The initial topology is a random graph with a specified average degree. Thus far, we have implemented only "pure" random graphs, where each pair of nodes is connected with fixed probability.

4. **Snapshot.**
The initial topology is taken from a specified graph file, which might be a snapshot from a real network or the

topology generated by another model.

B. Edge Connectivity Method

Both the AddNode and AddEdge events require a method to determine the endpoint or endpoints of the edge. We provide three methods to determine edge connectivity, all based on some form of preferential connectivity. Specifically, each existing node is assigned a weight according to an attribute (e.g., degree, activity or age). When a new node or edge is added into the topology, it connects to an existing node with probability proportional to the weights. Specifically, our methods are:

1. **Degree preference.**
This is the common preferential connectivity function in which the weight assigned to existing nodes is linearly proportional to the degree of the nodes at that time [9], [8]. This tends to cause high degree nodes to develop higher degree. We also implement a variation in which the weight is assigned to a power of the degree of nodes.

2. **Activity preference.**
In this method, the weight assigned is linearly proportional to the number of growth edges that have been added to the node. Nodes without growth edges are assigned a minimal weight to give new nodes a chance to grow. We include this method because we observed that the real data shows some preference based on activity: an edge is more likely to connect to a more "active" node than a less active node.

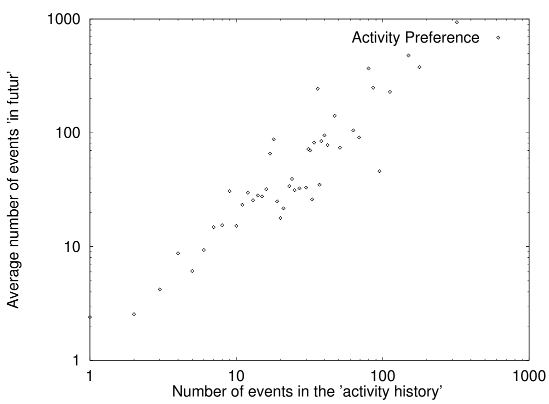

Fig. 2. Activity preference of events

Specifically, we measure preference for active nodes as follows. We measure the activity during the first 200 days, and call this the "activity history". Nodes with an equal number of events E_p during the activity history period are grouped into equivalent sets. The value of E_p associated with an equivalent set is an "activity index" in the past (x-axis). The higher the value of E_p the more active are the nodes. The number of events E_f that occur after the 200-day period denote the activity in the future of a node. An average of all values of E_f across nodes in an equivalent set correspond to the activity in future (y-axis). The clear rising trend in Figure 2 shows that node active in past will still be active in future. In other words, edges tend to be added to active nodes.

A variation of the activity preference includes a *sliding window*. In this variation only those edges added within the time window (e.g., 20 time units before the "current" time) are counted. This simulates variable periods of growth for nodes.

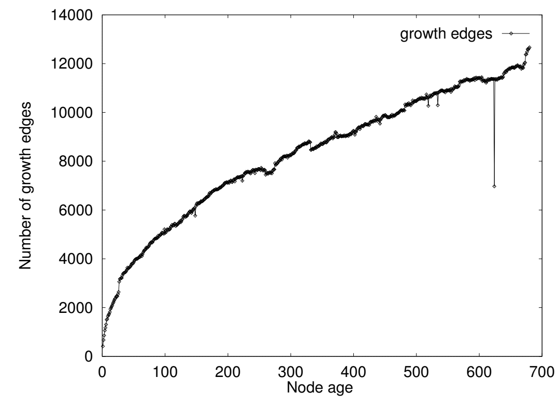

Fig. 3. Age of nodes selected by growth edges

3. **Age preference.**
In this method, the weight assigned is equal to the age of the node, which means the older nodes tend to get more edges. Figure 3 shows the age of the nodes selected by growth edges. This illustrates that growth edges prefer connectivity to older nodes.

We should note that there are, of course, correlations between degree, activity, and age. Older nodes and nodes subject to more activity will tend to have higher degree. Thus these three categories of preferential connectivity are not entirely independent.

C. Additional comments

There are other functions in the framework for which we currently provide limited options. For example, in the AddNode method, we set the node degree to have an exponential distribution. (See Figure 4.)

Many other topology generators associate nodes with a location in a 2-d plane to model geographic location. The location (and, more precisely, the distance between two nodes) is then used as a parameter in determining the probability of an edge between two nodes. We do not use geographic location in our model because the nodes in our model represent ASes, which are not typically associated with a single location in geographic space. More properly they might be represented as "spread" across a region of space, but even that does not reflect semantic reality espe-

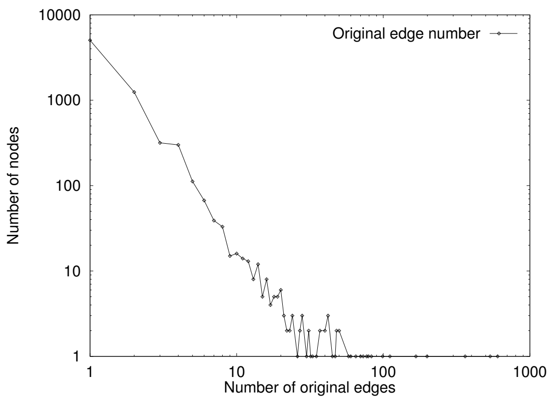

Fig. 4. The distribution of original degree of new nodes

cially well. A better (but significantly more complex) approach might be to select a number of peering locations for each AS and explicitly model each with a vertex, including auxiliary information that would group together the set of peering locations for the same AS. This approach represents a sort of hybrid between a router-level model and an AS-level model.

VI. Evaluation

We examine three areas where we may improve upon the BRITE method: initial topology alternatives, preferential connectivity functions, and the inclusion of growth edges.

A. *Effect of initial topology*

We compare three methods for generating the initial topology: random, degree-based and incremental with degree preference. We consider four sizes for the initial topology: 50, 100, 200 and 500 nodes. All three methods have parameters that control the number of edges. We use these parameters to produce three different densities: sparse, medium and well-connected. In all experiments, we run the evolution process until the topology has 5000 nodes and about 9500 edges. Note that this means we adjust some of the evolution parameters to achieve approximately the same number of edges in the final topology, regardless of the density of the initial topology.

Appendix II summarizes the results. Of particular note are the following:

- The random method generally has larger average shortest path length and smaller clustering coefficient than the other methods. The degrees of the nodes in the random initial topology will tend to be quite similar, and this is generally preserved in the final topology leading to relatively long paths and less clustering of nodes.
- The size of the initial topology has surprisingly little effect on the comparison between methods. We might have expected that larger initial topologies would result in greater differences in the methods, but that hypothesis is not supported by the data. Our largest initial topology is 10% of the size of the final topology; certainly we might see greater differences if we increased the initial topology size even further. We later explore using a relatively large snapshot from the real topology as an initial topology.
- The density of the initial topology does matter. Because we require the final topologies to have the same number of edges, the density of the initial topology affects the evolution process. If the initial topology is sparse, the evolution process adds more edges; if the initial topology is dense, the evolution process adds fewer edges. In the extreme case, during evolution all new nodes are connected with just a single edge to the topology, leading to a tree-like structure surrounding a much more dense core. A more dense initial topology leads to a better connected final topology (e.g., smaller average shortest path length). A more dense initial topology also makes the differences in the methods more clear.

Finding: Initial topology (even if small) affects the final topology. The method and density of the initial topology are more important than the size, at least for small to moderate sizes.

B. *Preferential connectivity*

In addition to degree-based preferential connectivity, recall that our framework includes preferential connectivity based on activity (where more active nodes are more likely to be the endpoint of an edge) and preferential connectivity based on age (where more recently added nodes are more likely to be the endpoint of an edge). We further include a generalized version of degree-based preference, where the weight associated with a node is raised to a power. If the power is greater than 1.0, this will further increase the preference associated with large degree nodes.

The purpose of this set of experiments is to evaluate the effect of these different forms of preferential connectivity. We choose one version of each method for experimentation, selected based on our experience with methods that tend to work fairly well. Specifically, we use a Poisson distributed number of events per day in the age-based method with mean of 10, a sliding window in the activity-based method with window size 20 days, and an exponent of 1.2 in the degree-based method. To eliminate the effect of other parameters, all the experiments have similar final topology size and the same initial topology (50 nodes and incremental degree preference).

Table III summarizes the comparison. We find that the degree-based method comes closest to the real data, and the age-based method is least like the real data. The activity-based method falls in between. Both the age and activity-based methods tend to have larger diameters, average shortest path lengths and eccentricities than the real data. This indicates topologies that are less closely connected. We note that the age-based method will treat all nodes of the same age equally, without regard to degree. Hence, there is limited opportunity for a single node to develop large degree. The activity-based method also contains a notion of time since activity is measured within a sliding age window. However this method is more similar to degree-based connectivity. Indeed, one can view the activity-based method as equivalent to degree-based preference with truncation, where the truncation causes only the most recently added edges to be counted in assessing degree.

Preference Method	Real Topology	Activity: SlidingWindow	Age: Poisson Distribution	Power of Degree
edgenum	9514	9493	9588	9375
avgdgr				
clustcoeff	0.74	0.82	0.82	0.88
diameter	9	12	16	9
avgshtpth	3.72	4.97	6.06	3.76
eccentri	6.87	8.68	11.32	6.67
dgrfreq(b,r)	-1.18,0.84	-2.01,0.97	-2.47,0.93	-1.29,0.87
pw2(b,r)	-2.22,0.97	-2.03,0.98	-2.22,0.90	-2.27,0.98

TABLE III

DIFFERENT PREFERENTIAL CONNECTIVITY

Finding: Degree-based preferential connectivity (especially with an exponent slightly greater than 1.0) produces better topologies than using age or activity as the basis for preference. These age and activity methods suffer because they only weakly prefer higher degree nodes and therefore produce more uniform and loosely connected topologies.

C. Event mix

Recall that our framework allows specification of the frequency of different types of events. The purpose of this set of experiments is to evaluate the effect of varying the mix of events. In particular, we focus only on the two addition events (AddNode and AddEdge) since deletion events are primarily interesting when considering a series of topologies. We vary the percentage of AddNode events from 60% to 100%; the remaining events are AddEdge events. We use degree-based preferential connectivity; a new node has an exponentially distributed number of original edges. We vary the mean of the exponential distribution to achieve a similar number of edges in the final topology. When there are more AddNode events, we use a larger mean.

Table IV summarizes the comparison. We see that as the event mix contains more AddNode events, the clustering coefficient decreases, the average shortest path length increases, and the resilience decreases. Consistent with intuition, AddEdge events tend to produce more closely connected graphs, while AddNode events tend to produce longer paths. New edges add short-cuts in the topology that make paths shortest, while new nodes cause a more tree-like structure since most new nodes have just one original edge.

ratio	60%	70%	80%	90%	100%
parameter	6:4, 0.5	7:3, 0.8	8:2, 1.0	9:1, 1.1	10:0,1.25
edgenum	9377	9386	9519	9445	9473
avgdgr	3.75	3.75	3.81	3.78	3.79
clustcoeff	0.85	0.83	0.82	0.81	0.80
diameter	10	9	9	10	10
asp	3.93	4.01	4.08	4.14	4.22
eccentri	7.36	6.70	6.91	7.16	7.53
dgrfreq(b,r)	-1.38,0.90	-1.41,0.88	-1.47,0.91	-1.49,0.90	-1.53,0.90
pw2(b,r)	-2.24,0.99	-2.32,0.98	2.34,0.97	-2.32,0.96	-2.33,0.96

TABLE IV

DIFFERENT ADDNODE:ADDEDGE RATIOS

More AddEdge events also tend to help generate large degree nodes, as can be seen from the linear regression of the degree frequency plot. This is also consistent with intuition: a new edge selects *two* endpoints, using degree-based preference for both endpoint. Hence a new edge can increase the degree of two large nodes. A new node will add edges to the topology, but one endpoint is fixed at the (relatively low degree) new node.

Finding: Increasing the percentage of AddEdge events yields more closely connected graphs and larger nodes with large-degree. Both trends are present in the real data, hence the inclusion of AddEdge events appears important in evolutionary models.

VII. CONCLUSIONS

We conclude by first presenting an example instantiation of our framework which does especially well in comparison to the real data set. Table V shows the results; the topology has 10500 nodes and 22500 edges. The initial topology is a snapshot of the real topology, with about 3500 nodes (35% of the final total). The ratio of AddNode and AddEdge events is 6:4, and edges are added using degree preferential connectivity raised to a power. For this instance, the agreement between model and reality is quite close, and better than the BRITE or Inet comparison shown earlier for a 5000 node topology.

This example illustrates the positive result that evolutionary models, if properly instantiated, can produce topologies that agree with real data on a number of mea-

	RealTopology	Generated Topology
node num	10494	10500
edge num	21890	22550
clustcoeff	0.732	0.717
diameter	10	9
asp	3.63	3.70
eccentri	7.18	6.75
dgrfreq(b,r)	-1.14,0.838	-1.32,0.842
pw2(b,r)	-2.17,0.982	-2.40,0.962

TABLE V

REAL TOPOLOGY AND GENERATED TOPOLOGY

sures.

However, our study also leads us to conclude that the following open problems are critical to the field of topology modeling:

• **Evaluation.** It is clear that we need a much better understanding of how to evaluate models. Using a collection of graph theoretic measures is problematic both because the level of agreement may be mixed across the metrics (e.g., model A does better on metric 1 than model B, while model B does better on metric 2 than model A), and because it is unclear which metrics matter most (e.g., if metric 1 matters, model A may be "better"). Better understanding of how metrics relate to issues such as protocol performance are crucial, but appear to be difficult to tackle in a general way. (See [13] and [15] for initial work in this direction.)

• **Modeling.** Our analysis of the real data indicates that the Internet graph has some properties that are surprisingly constant, yet others that behave non-monotonically over time. None of the existing models have sufficient time-varying dynamics to capture non-monotonic behavior. There is some work in modeling the WWW that may be relevant in this area [16], [17].

• **Theory.** The theory community has engaged in the problem of WWW modeling; there is also a role for theoretical foundations in the AS and router-level modeling arena. For example, one can consider the initial topology question in theoretical terms: how large can the initial topology be before it is "detectable" in the final topology (for some theoretical definition of detectable, perhaps cast as a game with an adversary). We are currently working with theoreticians on several problems in topology modeling [18].

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Appendix

I. Metrics

A. Terminology

Terms we used are clarified below.

- **Nodes** and **Edges** are the basic elements in the topology. If we consider the topology of the network as a graph, each AS is a node, and the BGP sessions between ASes are edges.
- The **Degree** of a node is the number of edges incident to the node. we use an undirected graph representation.
- The **Distance** between a pair of nodes is the length of the shortest path between the two nodes. In a graph without any weights on the edges, the length equals the number of hops of the path.
- **Neighborhood(h)** of node i, is the sub-graph formed by the nodes that can be reached from node i within h hops.

B. Metrics

Our metrics can be categorized into five groups: basic, degree oriented, path/connection oriented, sub-graph oriented and others.

1. Basic

This group includes the most basic metrics of the topology: the number of nodes n, the number of edges e, diameter D, and the average degree $d = e/n$. The diameter of a graph is the maximum distance between any two nodes in the graph. These metrics only provide a coarse picture of the whole topology.

2. Degree oriented

These metrics focus on the degree of each node: degree-frequency distribution and rank-degree distribution. *Degree-frequency* is the relationship between the degree of a node and the number of nodes that have this degree. A variation of degree-freqeuency is the sequence with the 2% largest node taken off. We refer to this as power law 2 (pw2). The comparison of the results of the two methods helped us understand the role of the large nodes. *Rank-degree* distribution describes the relationship between degree rank and degree value. The degree rank of a node is the rank by sorting nodes according to their degrees. These are local metrics, in the sense they are measured at a node.

3. Path oriented

The metrics in this group describe how the nodes connect to each other. The metrics include: *hop-plot*, *average shortest path length*, and *eccentricity*. All of these metrics are based on the distance between nodes. Hop-plot(h) is the number of node pairs that have a distance shorter than h. Eccentricity of node i is the longest distance from i to any other node. Basically, if there is a "center" of the graph, eccentricity describes how far the node is from the center. For the whole graph, we define the eccentricity as the average eccentricity across all the nodes.

4. Sub-graph oriented

The metrics in this group consider the neighborhood of the nodes. *Expansion*(h) [13]is the size of the neighborhood within h hops. For the graph, we take the average neighborhood size of all nodes. This metric is quite similar to the hop-plot metric mentioned above, but with increasing h it tries to capture the growth of the neighborhood from one node. *Clustering coefficient* [14] is defined as the number of edges in the 1-hop neighborhood, normalized by the number of edges in a full mesh graph with the same number of nodes (i.e., for node i: n, the number of nodes in the neighborhood; e, the number of edges; cc, the clustering coefficient of the node. $cc = e/(n(n+1))$) We take the average of cc of all nodes, to define the clustering degree of the whole topology.

II. Results from experiments with different initial topologies

Tables VI to Table IX show the evaluation results of different experiments with different initial topologies. In the tables, the Initial topologies have 3 densities: S:sparse, M:Medium, W:Well-Connected.

	Incremental Degree			Random			Degree-base		
InitDensity	S	M	W	S	M	W	S	M	W
edgenum	9338	9377	9565	9481	9550	9634	9371	9465	9461
avgdgr	3.74	3.75	3.83	3.79	3.82	3.85	3.75	3.79	3.78
clustcoeff	0.95	0.80	0.87	0.75	0.73	0.98	0.88	0.89	0.96
diameter	11	10	10	12	10	10	10	10	10
asp	4.33	4.14	3.85	4.36	4.24	3.79	4.31	4.19	3.89
eccentri	7.52	7.37	7.02	8.45	7.35	7.25	7.08	7.16	7.11
dgrfreq(b)	-1.62	-1.49	-1.30	-1.69	-1.61	-1.26	-1.60	-1.54	-1.34
dgrfreq(r)	0.91	0.89	0.90	0.94	0.92	0.89	0.93	0.92	0.91
pw2(b)	-2.09	-2.16	-2.15	-2.21	-2.16	-2.12	-2.14	-2.15	-2.17
pw2(r)	0.98	0.98	0.99	0.98	0.98	0.99	0.99	0.99	0.99

TABLE VI
INITIAL TOPOLOGY WITH 50 NODES

	Incremental Degree			Random			Degree-base		
InitDensity	S	M	W	S	M	W	S	M	W
edgenum	9364	9525	9514	9455	9696	9455	9303	9693	9396
avgdgr	3.74	3.81	3.81	3.87	3.88	3.78	3.72	3.88	3.75
clustcoeff	0.94	0.81	0.87	0.70	0.70	0.97	0.86	0.87	0.95
diameter	11	10	9	11	11	9	11	10	10
asp	4.31	4.12	3.75	4.42	4.25	3.86	4.38	4.14	3.8
eccentri	7.57	6.32	6.59	7.74	7.57	6.78	7.88	7.49	7.32
dgrfreq(b)	-1.62	-1.46	-1.23	-1.73	-1.63	-1.25	-1.64	-1.54	-1.22
dgrfreq(r)	0.92	0.90	0.88	0.94	0.91	0.87	0.93	0.91	0.85
pw2(b)	-2.18	-2.3	-2.27	-2.11	-2.31	-2.46	-2.16	-2.17	-2.47
pw2(r)	0.98	0.98	0.99	0.98	0.97	0.99	0.98	0.98	0.98

TABLE VII

INITIAL TOPOLOGY WITH 100 NODES

	Incremental Degree			Random			Degree-base		
InitDensity	S	M	W	S	M	W	S	M	W
edgenum	9396	9597	9511	9599	9612	9502	9429	9682	9437
avgdgr	3.75	3.84	3.80	3.84	3.84	3.80	3.77	3.87	3.77
clustcoeff	0.80	0.81	0.86	0.71	0.75	0.81	0.87	0.87	0.93
diameter	10	11	11	10	11	9	10	10	10
asp	4.22	4.10	3.84	4.39	4.34	4.07	4.28	4.17	3.93
eccentri	7.46	7.71	7.54	7.51	7.49	6.92	7.35	7.52	7.15
dgrfreq(b)	-1.55	-1.50	-1.16	-1.73	-1.70	-1.15	-1.59	-1.57	-1.18
dgrfreq(r)	0.92	0.91	0.84	0.95	0.94	0.75	0.93	0.91	0.85
pw2(b, r)	-2.26	-2.31	-2.48	-1.91	-2.08	-3.02	-2.23	-2.30	-2.52
pw2(r)	0.98	0.98	0.97	0.96	0.99	0.97	0.98	0.97	0.97

TABLE VIII

INITIAL TOPOLOGY WITH 200 NODES

	Incremental Degree			Random			Degree-base		
InitDensity	S	M	W	S	M	W	S	M	W
edgenum	9405	9540	9414	9584	9499	9335	9577	9466	9423
avgdgr	3.76	3.82	3.77	3.83	3.80	3.73	3.83	3.79	3.77
clustcoeff	0.91	0.84	0.91	0.69	0.76	0.87	0.85	0.89	0.93
diameter	11	10	12	10	11	12	11	11	11
asp	4.33	4.14	4.06	4.72	4.55	4.41	4.59	4.30	4.14
eccentri	7.9	7.44	8.28	7.86	7.96	8.23	8.09	7.64	7.40
dgrfreq(b)	-1.69	-1.48	-1.12	-1.98	-1.63	-1.11	-1.83	-1.59	-1.23
dgrfreq(r)	0.94	0.93	0.91	0.97	0.96	0.73	0.98	0.94	0.90
pw2(b, r)	-2.06	-1.90	-1.51	-1.83	-1.52	-0.90	-1.87	-1.84	-1.30
pw2(r)	0.99	0.96	0.91	0.99	0.94	0.66	0.98	0.96	0.89

TABLE IX

INITIAL TOPOLOGY WITH 500 NODES