

APPLICATION OF PROBABILISTIC METHODS FOR THE DETERMINATION OF AN ECONOMICALLY ROBUST HSCT CONFIGURATION

Dr. Dimitri N. Mavris[†]
Associate Director & Manager
Aerospace Systems Design Laboratory (ASDL)

Mr. Oliver Bandte[¥]
Graduate Research Assistant, ASDL

Dr. Daniel P. Schrage[†]
Director, ASDL

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150

Abstract

This paper outlines an approach for the determination of economically viable robust design solutions using the High Speed Civil Transport (HSCT) as a case study. Furthermore, the paper states the advantages of a probability based aircraft design over the traditional point design approach. It also proposes a new methodology called Robust Design Simulation (RDS) which treats customer satisfaction as the ultimate design objective. RDS is based on a probabilistic approach to aerospace systems design, which views the chosen objective as a distribution function introduced by so called noise or uncertainty variables. Since the designer has no control over these variables, a variability distribution is defined for each one of them. The cumulative effect of all these distributions causes the overall variability of the objective function. For cases where the selected objective function depends heavily on these noise variables, it may be desirable to obtain a design solution that minimizes this dependence. The paper outlines a step by step approach on how to achieve such a solution for the HSCT case study and introduces an evaluation criterion which guarantees the highest customer satisfaction. This customer satisfaction is expressed by the probability of achieving objective function values less than a desired target value.

[†] Member, AIAA

[¥] Graduate Student Member, AIAA

Paper presented at the AIAA/USAF/NASA/ISSMO
Multidisciplinary Analysis and Optimization Conference,
September 1996, Bellevue, Washington.

Introduction

The work presented in this paper describes elements of an overall aerospace system design methodology that proposes the use of probabilistic methods to meet some of the modern challenges in aircraft design. This methodology has been motivated by demands for future aircraft, like the High Speed Civil Transport (HSCT), to become economically competitive with current long range subsonic transports. Hence, the economic viability of modern aircraft is an essential although not the only concern of this methodology. Recognizing the presence of uncertainty in the assumptions made as to the number of paying passengers, fluctuations in fuel price, or travel distance, more emphasis has been put on replacing "point" by probabilistic estimates that account and quantify uncertainty of the prediction outcome. In order to implement this objective, a methodology called Robust Design Simulation (RDS)^{1,2,3,4}, that is based on a Concurrent Engineering (CE)/Integrated Product and Process Development (IPPD) approach has been introduced. The procedure for conducting this IPPD approach employs the use of a Design of Experiments (DOE) to facilitate the development of Response Surface Equations (RSEs)^{5,6,7} which approximate sophisticated, computationally intense disciplinary analyses tools with second (or higher) order polynomial equations. Furthermore, under this new way of thinking the design focus has shifted from optimizing to 'compromising'. Compromising describes a decision process that yields a robust solution^{8,9} i.e. a design that is insensitive to the variation of those parameters that are difficult or impossible to control. Such a design might be preferable to a true optimum which exhibits low confidence of achieving that optimum consistently.

Robust Design Simulation

Robust Design Simulation is a multidisciplinary approach to aircraft design within a so called CE/IPPD environment.³ This method concurrently considers product and process characteristics that are subject to planned or anticipated technology infusions. Product characteristics usually embrace such discipline characteristics as lift and drag for aerodynamics, moments of inertia and structural weight for structures, fuel flow and thrust for propulsion, and so forth. On the other hand, process characteristics capture the effects of producibility, supportability, reliability, and affordability.

An aircraft synthesis and sizing process, utilizing appropriate analytical tools, evaluates the system value to the customer for each aircraft configuration through selected objectives such as performance, cost, profit, or quality/reliability. Regardless of the defined objective, customer satisfaction can only be achieved if all system design and environmental constraints are met. This algorithm is displayed in Figure 1, depicting the dependence of the objective on economic and discipline uncertainties as well as technological and schedule risk.

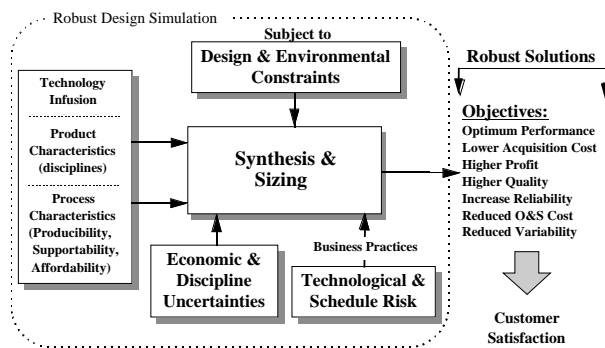


Figure 1: Robust Design Simulation

The uncertainties and risks associated with the system are usually accounted for in the form of variability distributions for system inherent random variables. These random variables introduce an undesired variability in the objective that can be modeled as a probability distribution. So far, most robust design methodologies^{3,9,10} strived to reduce the variability of the objective, assuming that one can reach a higher customer satisfaction with such reduction. In contrast, an alternate approach is proposed here where customer satisfaction is achieved by considering an evaluation criterion in terms of probability, P , of achieving objective function values, Y , that are smaller (or greater) than a desired target value, T . In addition, the method relates this probability qualitatively and

quantitatively back to design and control factors of the system. Hence, by maximizing the evaluation criterion, probability of achieving objective function values that are smaller (or greater) than a desired target value, $P(Y \leq T)$ (or $P(Y \geq T)$), a design can be found that guarantees the highest customer satisfaction while satisfying all imposed design and environmental constraints.

Traditionally, a designer's goal is to optimize a given objective, e.g. performance, cost, or reliability, that will satisfy a set of given customer requirements. Commonly applied optimization procedures focus on finding the optimum settings which define a single design point without providing any insight into product performance or cost at off-design conditions¹¹. Robust design, on the other hand, tries to overcome this deficiency by optimizing the objective while also accounting for off-design conditions. In other words, a robust design, as defined by Kapur⁸, is that design in which control factors minimize the effect of the noise factors on the objective of the customer. This general idea of a robust versus an optimal solution is displayed in Figure 2 in terms of an L/D distribution over a range of C_L for two different hypothetical designs.

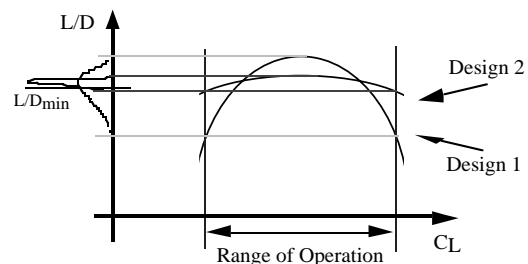


Figure 2: Change in Sensitivity of L/D for Two Hypothetical Designs

For the same optimal C_L value, Design 1 yields a higher maximum L/D . However, since the aircraft will inevitably be flown at off-design conditions, C_L will vary over the assumed operating range. Thus, each design has an associated variation of L/D . As depicted in Figure 2, this variation depends on the setting of design variables, causing Design 1 to yield a larger variation in L/D than Design 2. If the objective is to create a robust design, then Design 2 may become more desirable, since it yields a higher probability of achieving values for L/D greater than a minimum value during operation. In effect, the better design point performance is traded for the superior off-design performance. This trade-off is captured by the Robust Design Simulation proposed in this paper. The execution of this methodology is described in Figure 3.

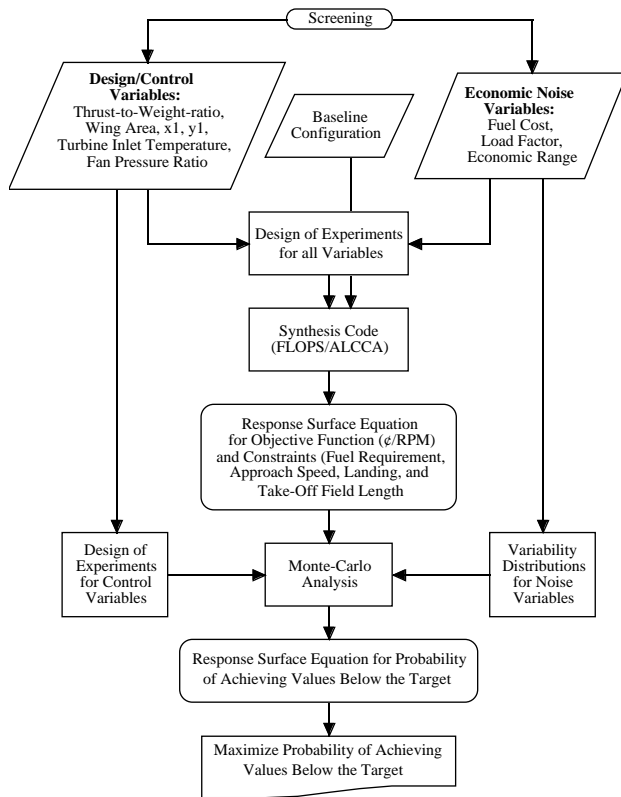


Figure 3: Methodology Execution

First, a screening process has to be performed to determine the most important set from the complete list of parameters that describe the system. The screening for the system investigated in this paper, the High Speed Civil Transport, has been described in various previous papers.^{1,3,4} The variables selected for this study, as displayed in Figure 3, are: Thrust-to-weight ratio, wing area, kink location (x_1 , y_1), see Figure 4, turbine inlet temperature, fan pressure ratio, fuel cost, load factor, and economic range. A description of these variables will be provided in the following section. The selected set of variables is usually composed of design/control and uncertainty/noise variables. However, for this study the assumption is made that all noise is generated by economic uncertainty variables.

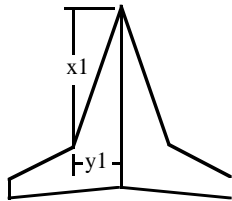


Figure 4: Illustration of the Kink Location

Next, a design of experiments table is created for all variables. The significance of this table is to allow for an estimation of main effects, interactions, and quadratic effects of the selected set of variables. As exercised in many previous studies, References 3,4,12,13,14,15,16, a response surface equation can be generated, utilizing this DOE table, that approximates the selected synthesis/economic analysis code, FLOPS¹⁷/ALCCA¹⁸. This equation is an integral part of the proposed methodology, since it enables the use of a Monte Carlo simulation to determine the probability function of the objective. Without this equation a Monte Carlo simulation approach would have been impractical, since the synthesis code would have to be executed 5,000 times.

So far, an RSE has been established that links the objective function, average yield per Revenue Passenger Mile, ϕ /RPM, to discipline level parameters, some of which are controllable by the designer while others are not. In order to identify a robust solution for the posed problem, one has to identify those control variables that minimize the influence of noise variables. In other words, one has to identify the interaction between control and noise variables, a task that is particularly easy for the second order equation modeled here. An example, provided in Figure 5, how fuel cost, an uncontrollable noise variable for the designer, has a strong influence on the objective, average yield per Revenue Passenger Mile, ϕ /RPM, for a given low setting of the longitudinal location, x_1 , of the kink of the wing. However, by increasing the value of x_1 , the influence of fuel cost on the objective can be decreased. On the other hand, this can very well mean that an increase in x_1 also increases the ϕ /RPM, a very undesirable change for the objective. Hence, in order to obtain a robust solution some trade of performance versus cost may have to occur.

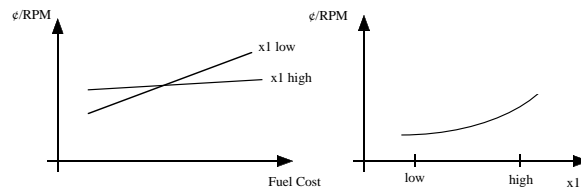


Figure 5: Trade-off Between a Fictitious Optimal and Robust Solution

The following steps describe the extension to previous studies made in this paper. Similar to the process described in Ref. 3, a new design of experiments table is employed for the design/control variables only. For each setting of the design variables a Monte Carlo

Simulation is being executed based on the assumptions made for the economic noise variables, which are in general nonnormal distributions. Note, that this process encompasses 58 simulations each of which executes 5,000 function calls. Without the facilitation of RSEs, this is an almost impossible task to accomplish. In addition, the Monte Carlo Simulation has been chosen over an analytical method for the distribution generation, since it does not need the simplifying assumption of normal distributions for the noise variables. Each of the simulations generates a distribution for the objective, ϕ /RPM, similar to the one in Figure 6.

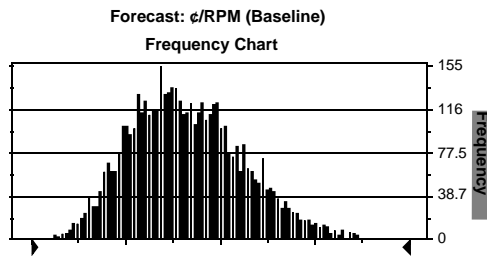


Figure 6: Example Distribution for the Objective

In order to ease the computational effort, each distribution is approximated by one of the standard probability distributions. This distribution fitting process, employed by Crystal Ball^{®19}, yielded for all cases of this study a gamma distribution, displayed in Equation 1, as the best approximation (Chi-Square Ranking Method). By keeping track of the parameters, location (L), scale (α), and shape (β), for each distribution of each run in the DOE table, a response surface equation in terms of the control variables in the table can be fitted for each parameter.

$$f(x) = \frac{\left(\frac{x-L}{\alpha}\right)^{\beta-1} \cdot e^{-\left(\frac{x-L}{\alpha}\right)}}{\alpha \cdot \Gamma(\beta)} \quad (1)$$

However, the results from this study showed that the fit of a quadratic equation for the distribution parameters is generally very poor (R-square between 40 and 80%). Hence, the proposed methodology employs the fit of an RSE for the probability of achieving objective function values below a desired target value $P(Y \leq T)$ that can generally be fit much better to the obtained data (R-square of 92 to 98%). In other words, the obtained equation links a customer's objective of achieving values smaller than a target to design or control variables that allow the designer to optimize the objective in order to find the design solution that guarantees the maximum customer satisfaction. After

having obtained this equation in terms of the design variables, an optimal solution can easily be found by maximizing $P(Y \leq T)$ while satisfying all imposed design and environmental constraints. This optimal solution corresponds to a shift of the objective distribution for the objective as displayed in Figure 7. The ability to perform a constrained optimization is one of the advantages of this method over, for example, the Taguchi method.^{10,20}

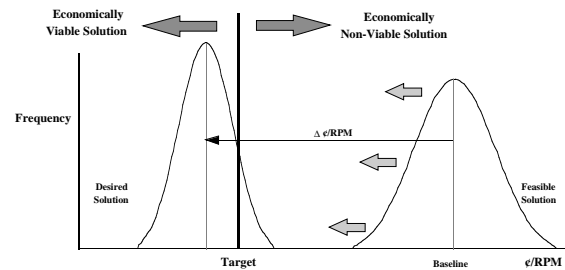


Figure 7: Distribution Shift to Maximize Probability of Achieving a Target

As shown in Figure 7, a feasible but economically non-viable solution becomes economically viable by shifting the distribution beyond the target value, i.e. yielding a probability of achieving values below the target of more than 50%. However, the optimized solution may not satisfy a required minimal probability (usually 75 to 90%) in all cases. If so, several options are left to achieve a higher customer satisfaction: relaxing the stringent target requirement by introducing a fare premium; change some of the economic assumptions by guaranteeing a higher market share or schedule provisions; design constraint relaxation; changing the baseline by technology infusion; or if everything else fails accepting the solution at a lower rate of probability of achieving objective function values below the target, hence taking a greater risk.

A Case Study: HSCT

Vehicle Description

As previously mentioned the applicability of the proposed approach to robust design was demonstrated during the design of a High Speed Civil Transport, which is displayed in Figure 8. The HSCT is envisioned to be an aircraft capable of flying supersonically (~Mach 2.4) and carrying 300 passengers to destinations up to 5,000 nautical miles. Furthermore, stringent requirements are being placed on this aircraft to make it economically viable and environmentally compatible. On that account, it must,

for example, abide restrictive FAR Stage III or IV noise regulations, be comparable in safety and comfort to the current long range subsonic fleet, and provide economic benefits for airline and manufacturer at an affordable ticket price, ϕ /RPM.



Figure 8: Georgia Tech HSCT Configuration

The mission profile used to size the HSCT configuration, is depicted in Figure 9. In order to model the economics of the aircraft correctly, a distinction must be made between the economic and the design range. The first represents the distance a plane will fly from one airport to another during its life, while the latter is the maximum distance a plane is able to fly by design. Table I summarizes the most important mission as well as geometric parameters of the baseline HSCT configuration.

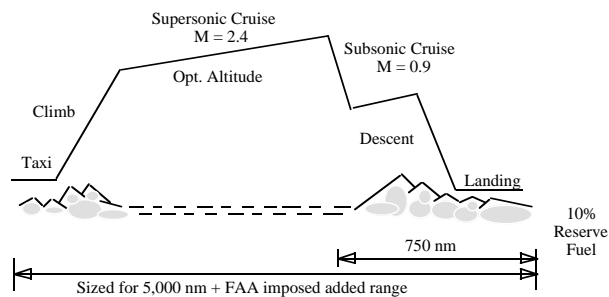


Figure 9: Baseline Mission Profile

Table I: Summary of the Baseline HSCT

Parameter	Baseline
M	2.4
Cruise Altitude	~63,000 ft.
Design Range	5000 nm
Payload	300 Passengers
Sustained Load	2.5 g
Fuselage length	280 ft.
Span	77.5 ft.
Sweep 1	74 deg.
Sweep 2	45 deg.
S_{ref}	9,000 ft ²

Finding an optimal configuration for a supersonic transport vehicle, however, is a

multidisciplinary and very difficult task. Choosing a wing planform shape, for example, is driven by the need for efficient performance at both sub- and supersonic cruise conditions, a conflicting design objective in itself already^{4,21}. Furthermore, the trades involved in planform selection are being complicated by different discipline considerations for aerodynamics, structures, propulsion, etc., and the presence of design and performance constraints at the system level which are directly related to the wing. The limit on approach speed, for example, is mostly a function of wing loading. Fuel volume requirements impact the wing size and shape. Both become sizing criteria and are treated as constraints that tend to increase the wing in size. On the other hand, increased wing area yields higher induced and skin friction drag, increasing fuel consumption. Based on the way FLOPS models an aircraft and its mission, a fuel requirement, R_f , can be constructed in the form of a ratio of available fuel to required fuel for completing the mission. Hence, R_f has to have a value greater than one to satisfy the fuel requirement. Additional design challenges are presented by takeoff and landing field length limitations (less than 11,000ft) that are also modeled as design constraints for this study. Table II summarizes the design objective, ϕ /RPM, and all constraints considered for this study that need to be satisfied during an optimization.

Table II: Summary of Objective and Constraints

Response	Requirement
ϕ /RPM	minimize
Acquisition Cost	minimize
Gross Weight	N/A
Fuel Requirement R_f	> 1
Approach Speed	< 154 kts
Takeoff Field Length	< 10,500 ft
Landing Field Length	< 11,000 ft

All responses presented in Table II are modeled by FLOPS/ALCCA as functions of design/control and noise variables. In order to facilitate the Monte Carlo simulation without executing the actual synthesis code for each simulation run, each of the responses is approximated by an RSE in terms of the most important design/control and noise variables, according to the Pareto Principle⁷.

For this study the six most important control factors, presented in Table III, were selected together with the three most influential economic noise.^{1,4} These noise variables as presented in Table III are responsible for the observed objective function, ϕ /RPM, distribution. Essential for a meaningful constrained

Figure 10: Prediction Profiles for Objectives and Constraints

The importance of the selected set of variables lies in the multidisciplinary nature of the problem. Four major disciplines contribute at least two variables to a multidisciplinary representation of the HSCT during the synthesis phase. In order to obtain this equation, a face-centered Central Composite Design of experiments is employed for all nine variables. This requires 531 function calls that execute FLOPS/ALCCA, and a second order polynomial is fitted to these obtained data. These equations are displayed in Figure 10 in the form of prediction profiles for each variable. Note that the results for ϕ /RPM, acquisition cost, and gross weight have been normalized with respect to the baseline.

Even though, the fit for all responses was very good (R-square value between 99.1% and 99.9%), a major concern is the performance of this equation at off-design points, i.e. at points the equation has not been fitted through. For this purpose an additional DOE table has been employed, executing the synthesis code 500 times at random over the design space. These

results were compared with the results which the previously obtained RSEs yield at those random design points. These results are presented in Figures 11 to 16, for each of the responses, in the form of correlation graphs as an off-design point performance measure for the RSE. These graphs compare the results from the synthesis code with the by the RSE predicted values. A perfect correspondence of both data sets would be indicated by a 45 degree line through all data points from the bottom left to the top right corner and by a correlation value of 100%. As the graphs indicate, the correlation between the two data sets differ from 99.65% for the objective function ϕ /RPM to 86.4% for the gross weight. These differences can be explained by the way each of the responses is simulated. The economics are usually comprised of regression equations that are well behaved (e.g. no discontinuities). Gross weight that embodies several nonmonotonic equations and table look up procedures that are not being fit as accurately as it is for ϕ /RPM. The graphs also display a prediction ellipse, as a confidence interval, that encloses 95% of the data points in each graph.

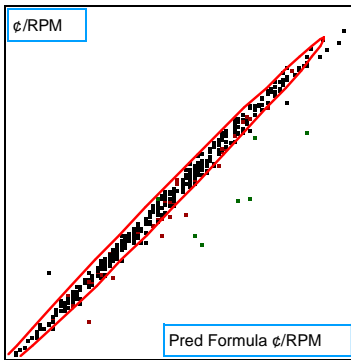


Figure 11: Correlation of Actual with Predicted Values for ϕ /RPM

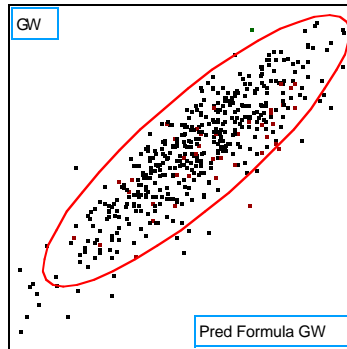


Figure 12: Correlation of Actual with Predicted Values for Gross Weight

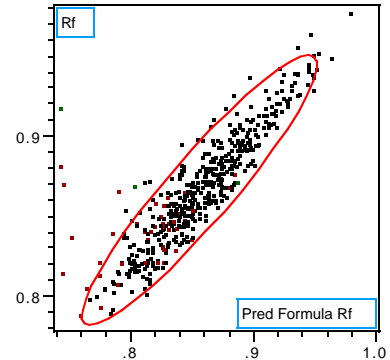


Figure 13: Correlation of Actual with Predicted Values for Rf

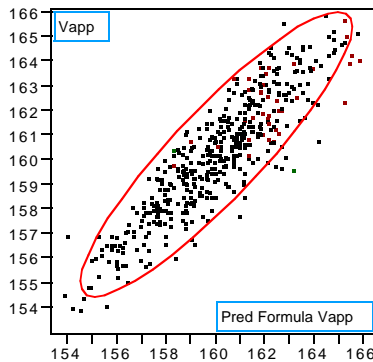


Figure 14: Correlation of Actual with Predicted Values for Approach Speed

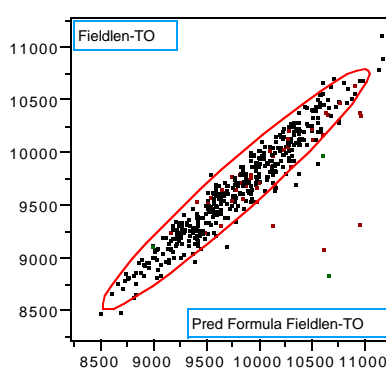


Figure 15: Correlation of Actual with Predicted Values for Takeoff Field Length

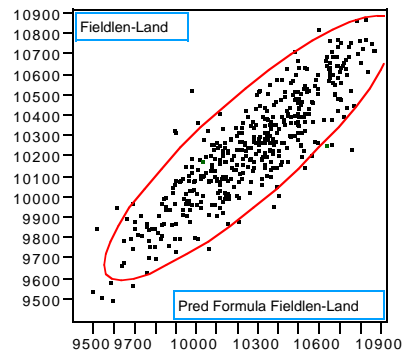


Figure 16: Correlation of Actual with Predicted Values for Landing Field Length

Robust Design Simulation Results

The obtained RSE for the objective function, ϕ /RPM, can now be employed in a Monte Carlo Simulation. The purpose of this simulation is to obtain a distribution for ϕ /RPM that is introduced by the intrinsic variability of the economic noise variables. Due to lack of more precise knowledge about the economic variables, triangular distributions are assumed. These three distributions are depicted in Figures 17 to 19, marking the range and mode for each variable. In order to identify the dependence of the objective distribution on the design variables, another face centered Central Composite Design is set up, this time for the six control variables only. A Monte Carlo Simulation is now executed for each of the cases listed in Table IV. Each case sets the control variables to a fixed value while the noise

variables are varied according to their distributions as depicted in Figures 17 to 19. Hence, each case yields a distribution for ϕ /RPM that can be fit by a standard gamma distribution that is uniquely defined by its three parameters: location, shape, and scale (Figure 20). Since a fit for these parameters is usually rather poor (see previous section) the probability of achieving values below four sample targets, A, B, C, and D, is collected for each distribution and regressed by a second order polynomial (R-Square value of 0.98 to 0.92). These equations allow the designer to relate $P(Y \leq T)$ back to design variables and enable the designer to maximize the probability based on the selected design parameters. Since all constraints are a function of the same design variables, the optimization process is now able to find a solution that also satisfies all imposed constraints.

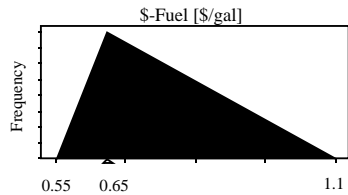


Figure 17: Distribution for Fuel Cost

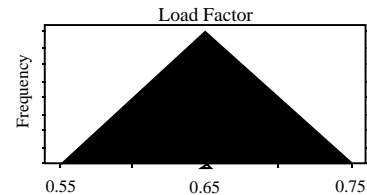


Figure 18: Distribution for Load Factor

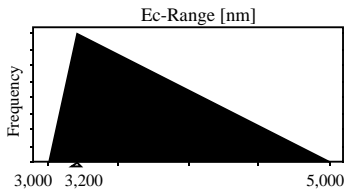


Figure 19: Distribution for Economic Range

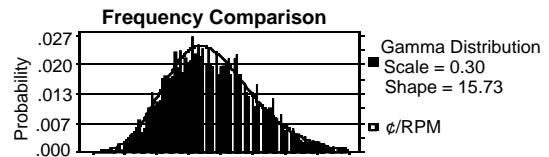


Figure 20: Objective Distribution Fit

Table IV: DOE Table and Noise Variable Distributions to Obtain Objective Distribution

Exp.#	Control Variables						Noise Variables			Response		
	T/W-Ratio	S-Wing	x1	y1	TIT	FPR	\$-Fuel	Load Factor	Ec-Range	ϕ /RPM	Shape	Scale
1	0.28	8.5	1.54	0.5	3	3.5					0.3	14.86
2	0.28	8.5	1.54	0.5	3.25	4.5					0.33	13.24
3	0.28	8.5	1.54	0.58	3	4.5					0.34	11.77
4	0.28	8.5	1.54	0.58	3.25	3.5					0.28	15.5
5	0.28	8.5	1.62	0.5	3	4.5					0.35	13.25
6	0.28	8.5	1.62	0.5	3.25	3.5					0.31	15.38
7	0.28	8.5	1.62	0.58	3	3.5					0.31	14.81
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
57	0.3	9	1.58	0.54	3.125	4					0.32	14.02
58	0.3	9	1.58	0.54	3.125	4					0.33	13.85

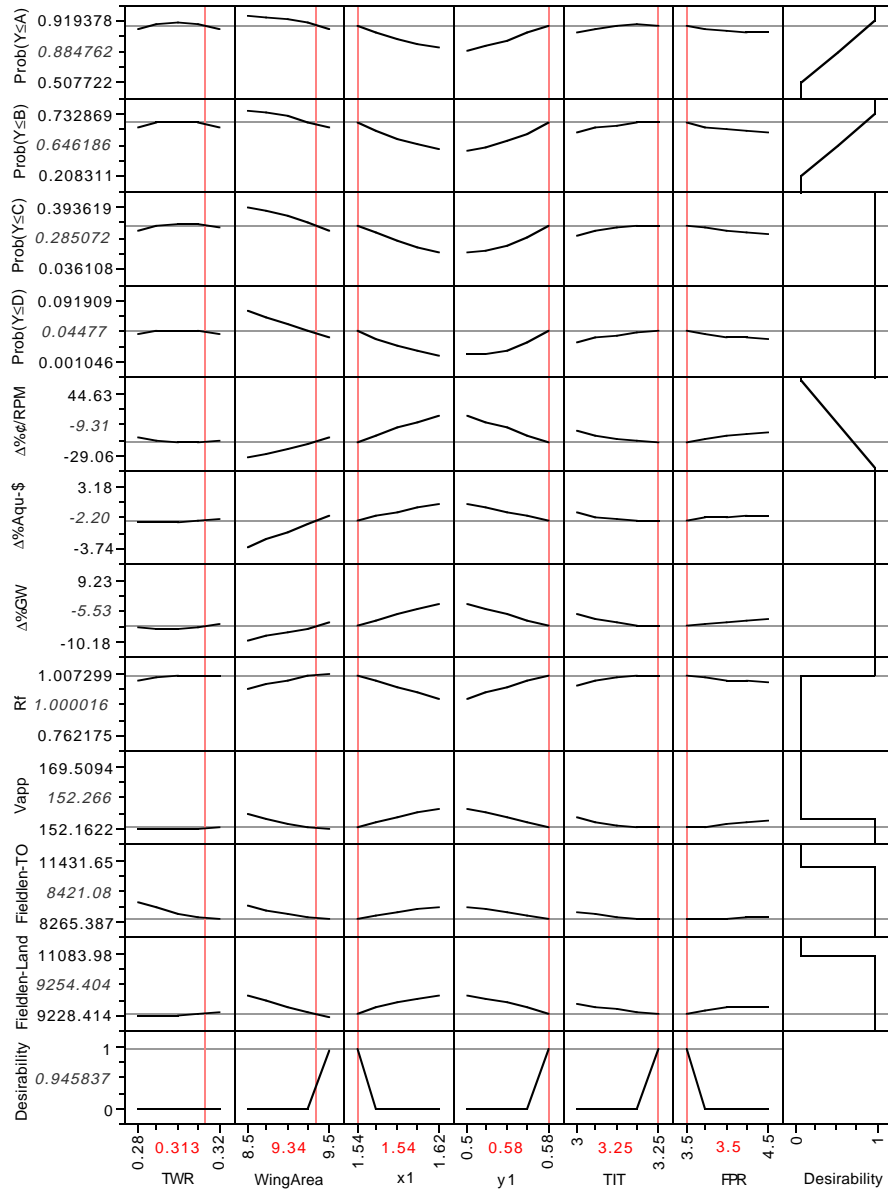


Figure 21: Prediction Profiles for Probabilities, Objectives, and Constraints

To complete the methodology described in Figure 3, the equations for probability of achieving objective values below the target values A, B, C, and D together with the objective function, acquisition cost, gross weight, and the constraints in the form of prediction profiles, as displayed in Figure 21. Also shown is the robust design solution based on its so called desirability, a feature of JMP^{®22}, the statistical package used to generate the DOEs, equations, and all graphs presented in this paper. JMP[®] assigns desirability values between zero and one (bottom row of Figure 21), one being the most desirable, to all design variable settings based on assigned desirability values for each response. For example, if a response is

supposed to be maximized, like $\text{Prob}(Y \leq A)$, high values of that response are assigned high desirability values. As displayed in the last column of Figure 21. If a response is supposed to be minimized, like the objective function ϕ/RPM , high desirability values are assigned to low values for that response. By perturbing the variable setting, each outcome of a response yields a desirability for that setting based on the assigned desirability. If more desirabilities are being assigned to different responses, all desirabilities are multiplied with each other. This allows a multiple objectives optimization that is translated into a single objective, called desirability.

Additionally, this feature is able to handle constraints by assigning a desirability of zero to all constraint response values that violate their requirement and one to those that satisfy it. Hence, all variable settings that violate a constraint will have a desirability of zero since the desirabilities of all responses are multiplied. On the other hand, if a variable setting satisfies the constraint, the solution will not be influenced since it is multiplied by one. Refer to Rf in Figure 21 as an example, where all values for Rf below one are assigned a desirability of zero while values greater than one are assigned a desirability of one. If a response, such as gross weight, should not influence the desirability value of the solution, all values are being assigned a desirability of one. This feature enables the designer to obtain a solution to an optimization problem quickly and very visibly on the screen without the need for a separate optimization execution. Table V summarizes the obtained robust design solution and compares it with the baseline.

Table V: Robust Design Solution Summary

Parameter	Robust Solution	Baseline
T/W Ratio	0.313	0.300
Wing Area	9340 ft ²	9000 ft ²
x1	1.54 x span	1.58 x span
y1	0.58 x span	0.54 x span
TIT	3250°F	3125°F
FPR	3.5	4.0

Based on the results presented in Table V a Monte Carlo simulation was employed one more time in order to compare the cumulative distribution for ϕ /RPM of this robust design solution to the original one of the baseline, as displayed in Figure 22. The robust design solution yields for all targets a higher probability of achieving values below that target. Naturally the probability increases with increasing values for the target. However, it can be seen that for “small” and “very large” target values the difference in probability between the robust design solution and the baseline is very small. The difference increases,

¹ Mavris, D.N., Bandte, O., and Schrage, D.P., *Economic Uncertainty Assessment of an HSCT Using a Combined Design of Experiments/ Monte Carlo Simulation Approach*, 17th Annual Conference of the International Society of Parametric Analysts, San Diego, CA, June, 1995.

however, for values around the means of the distributions. Hence, one can also conclude from this study that the improvement of one solution over another does depend on the target itself. Nevertheless, the distribution and therefore the probability of achieving values below a target value $P(Y \leq T)$ are independent of the value of target T.

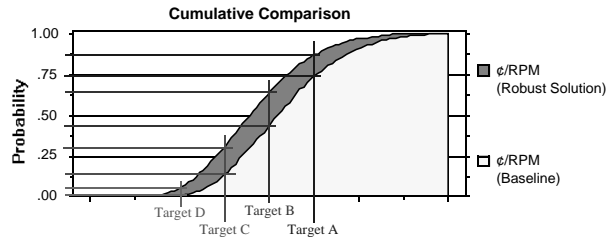


Figure 22: Distribution Comparison Between Baseline and Robust Design Solution

Conclusions

This paper presented a step by step approach to design for robustness. The advantages and differences of this new approach to other approaches were described and an application was presented. The example case, an HSCT design, demonstrated the ability of the approach to identify and quantify the effect of nine variables from four different disciplines on an objective function and imposed design constraints. The paper also identified and quantified how variations in parameters of a design process effect the objective. Further, it proposed a method on how to minimize the dependency of the objective on these varying uncertainty parameters. The paper also introduced probability of achieving objective values below a desired target as a new metric to evaluate the goodness of a design. The paper demonstrated how this evaluation criterion can be related back to the design variables to facilitate an optimization of the design problem. This optimum, or so called robust design solution, was identified and presented in this paper.

References

- Mavris, D.N., Bandte, O., and Brewer, J.T., *A Method for the Identification and Assessment of Critical Technologies Needed for an Economical Viable HSCT*, 1st AIAA Aircraft Engineering, Technology and Operations Congress, Los Angeles, CA, September, 1995, AIAA 95-3887.
- Mavris, D.N., Bandte, O., and Schrage, D.P., *Effect of Mission Requirements on the Economic*

- Robustness of an HSCT*, 18th Annual Conference of the International Society of Parametric Analysts, Cannes, France, June, 1996.
- ⁴ DeLaurentis, D.A., Mavris, D.N., and Schrage, D.P., *An IPPD Approach to the Preliminary Design Optimization of an HSCT using Design of Experiments*, accepted at the 20th ICAS Congress, Sorrento, Italy, September, 1996.
- ⁵ Box, G.E.P., Draper, N.R., Empirical Model-Building and Response Surfaces, John Wiley & Sons, Inc., New York, NY, 1987.
- ⁶ Box, G.E.P., Hunter, W.G., Hunter, J.S., Statistics for Experimenters, John Wiley & Sons, Inc., New York, NY, 1978.
- ⁷ Haaland, P.D., Experimental Design in Biotechnology, Marcel Dekker, Inc., New York, NY, 1989.
- ⁸ Kusiak, A., Concurrent Engineering, John Wiley & Sons, Inc., New York, NY, 1993.
- ⁹ Bras, B.A., and Mistree, F., *Robust Design Using Compromise Decision Support Problems*, Engineering Optimization, vol. 21, pp. 213-239, 1993.
- ¹⁰ Roy, R., A Primer on the Taguchi Method, Van Nostrand Reinhold, New York, NY, 1990.
- ¹¹ Dieter, G.E., Engineering Design, A Materials and Processing Approach, 2nd Edition, McGraw-Hill, Inc., New York, NY, 1991.
- ¹² DeLaurentis, D.A., Calise, A., Schrage, D.P., and Mavris, D.N., *Integrating Guidance Optimization into Vehicle Design Via Singular Perturbation Theory and Statistical Methods*, accepted at the 6th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, WA, September, 1996.
- ¹³ Kaufman, M., et al, *Variable-Complexity Response Surface Approximations for Wing Structural Weight in HSCT Design*, 34th Aerospace Sciences Meeting & Exhibit, Reno, NV, January, 1996.
- ¹⁴ Giunta, A.A., et al, *Wing Design for a High-Speed Civil Transport Using a Design of Experiments Methodology*, paper accepted at the 6th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, WA, September, 1996
- ¹⁵ Chen, W., et al., *Integration of Response Surface Method with the Compromise Decision Support Problem in Developing a General Robust Design Procedure*, Advances in Design Automation (Azarm, S., et al. Eds.), pp. 485-492. ASME DE-Vol. 82-2, ASME, New York, 1995
- ¹⁶ Unal, R., Stanley, D.O., Joyner, C.R., *Parameter Model Building and Design Optimization Using Response Surface Methods*, Journal of Parametrics, Washington, DC, May, 1994
- ¹⁷ McCullers, L.A., Flight Optimization System, Computer Program and Users Guide, Version 5.7, NASA Langley Research Center, Hampton, VA, December 1994.
- ¹⁸ Galloway, T.L., and Mavris, D.N., Aircraft Life Cycle Cost Analysis (ALCCA) Program, NASA Ames Research Center, September 1993.
- ¹⁹ Decisioneering, Inc., Crystal Ball Computer Program and User's Guide, Version 4.0, Aurora, CO, 1996
- ²⁰ Taguchi, G., *Introduction to Quality Engineering: Designing Quality into Products and Processes*, Asian Productivity Organization, available in the USA from American Supplier Institute, Dearborn, MI , 1986
- ²¹ Sakata, I.F. and Davis, G.W., *Evaluation of Structural Design Concepts for Arrow-Wing Supersonic Cruise Aircraft*, NASA CR-2667, May, 1977
- ²² SAS Institute Inc., JMP Computer Program and User's Manual, Cary, NC, 1994