Use of Probability of Success as an Independent Variable for Decision Making

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Early phases of design are characterized by risk and uncertainty. Appropriate accounting for this uncertainty is an important requirement for any designer. This work suggests collapsing risk and uncertainty into a single metric called the probability of success, which accounts for the probability of a given design simultaneously meeting all of the design requirements. Optimal or lowest cost designs can then be found for various levels of probability of success. These designs can be compared to each other, creating a trade-off between the cost of a design and its risk. These risk versus cost figures can be generated before a decision-maker commits to the design. Thus, the decision-maker will have all the information regarding the cost and risk of potential designs before making any design decisions. The decision-maker can thus treat the probability of success, or risk, as an independent variable, choosing the level of risk that he or she finds acceptable based upon the cost of the system, with the corresponding "optimal" design already determined. This paper illustrates a process to generate these risk versus cost curves for conceptual design, and gives two examples of the implementation of the process.

I. Introduction

Early phases of decision-making in conceptual design are characterized by a high degree of uncertainty in the system. This uncertainty arises from multiple sources, including: not having detailed analysis tools early in the design process, incomplete knowledge of the operational environment or requirements, or uncertainty in the performance of potential technologies. Yet, in the midst of this uncertainty, decision-makers often are required to make commitments that permanently lock-in final system cost and performance. Figure 1 illustrates how, in today's design process, the cost is committed early in the design process while knowledge of the system is most limited. Today's system engineering world is driving to bring more knowledge forward in the design process so that better decisions can be made before a design's final cost is locked-in.

Using probabilistics to characterize and analyze uncertainty early in the design process is one method that helps bring this system knowledge forward in the design process. By estimating the uncertainty, the decision-maker gains better knowledge of the risks and rewards of various system configurations, thereby leading to better-informed decisions.

The fundamental goal of this research is to develop risk vs. reward curves for torpedo systems which will show the relations between performance, cost, and the probability of meeting design requirements, or probability of success, all in an environment that accounts for uncertainty. These risk vs. reward charts will be used to provide the decision-maker more insight into the tradeoffs between probability of success, performance, and cost. By generating these curves before any decisions are made, the decision-maker will be able to gain an understanding of the level of risk in a design versus the cost of the design. The decision-maker can then choose the best design based upon the level of risk that he or she is willing to accept, thereby treating risk as an additional independent variable in the system.

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II. Process

The first step in developing a design process is to define a measure of risk. In this case, risk, or, more accurately, it's opposite (the quantity "1.0 - risk"), is being defined as the probability of success. Probability of success (POS) is formally defined by Bandte as "the envelope objective function or overall evaluation criterion ... measuring the probability of satisfying all criteria[2]." It is the chance of a probabilistically designed vehicle simultaneously meeting all design requirements, whether they be given by physical constraints such as weight and length, performance constraints such as velocity and range, or systems effectiveness constraints such as probability of hit and probability of evade. The equation for probability of success, assuming N number of trials and M number of constraints, is given in Equation (1), where z_{jmin} and z_{jmax} are the minimum and maximum required values for each constraint. A POS of one indicates that the design is guaranteed to meet all the stated requirements (assuming that the uncertainty was adequately captured) and a POS of zero indicates that the system will never simultaneously meet all the design requirements. Probability of success is useful when dealing with uncertainty analysis, because it shows the likelihood of a given design to meet all the design constraints and collapses this information into a single quantitative value. A simplified equation for probability of success is shown in Equation (2). If a random sample of designs is being taken, probability of success is literally nothing more than the number of "successful" designs – those designs that meet all the requirements, over the total number of designs. The probability of success is used to show the likelihood that the design will remain inside of the area of design interest, as illustrated in Figure 2.

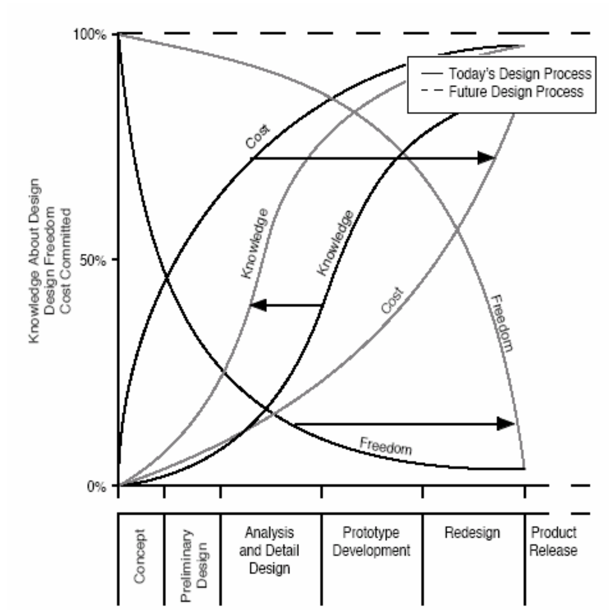

Figure 1: Design Knowledge vs. Cost Committed[1]

$$POS = \frac{1}{N}\sum_{i=1}^{N}\prod_{j=1}^{M}\begin{Bmatrix} 1 \text{ for } z_{j_{\min}} \le z_j \le z_{j_{\max}} \\ 0 \text{ otherwise} \end{Bmatrix} \quad \textbf{(Eq. 1)}$$

$$POS = \frac{\text{Number of Successful Designs}}{\text{Total Number of Designs}} \quad \textbf{(Eq. 2)}$$

Figure 3 shows the proposed probabilistic design process. The first step of the process is to define the system requirements, which consist of the system constraints that are used to calculate the probability of success parameter in Equation 1. Next, a concept is selected, along with corresponding design variables. Next, a modeling and simulation capability must be created so that the design space can be analyzed. A means to handle uncertainty must then be added to the model. Uncertainty analysis can be built directly into the original simulation model, or added as "k-factors" to the system. These "k-factors", which have been used in the past to model the effects of technology[3,4], "modify disciplinary technical metrics, such as specific fuel consumption, …, drag, and/or component weights that result from a sizing tool. The modification is essentially an incremental change in the technical metric, either enhancement or degradation[5]." "K-factors" mirror the effect of uncertainty, by randomly adding either benefit or degradation to system components.

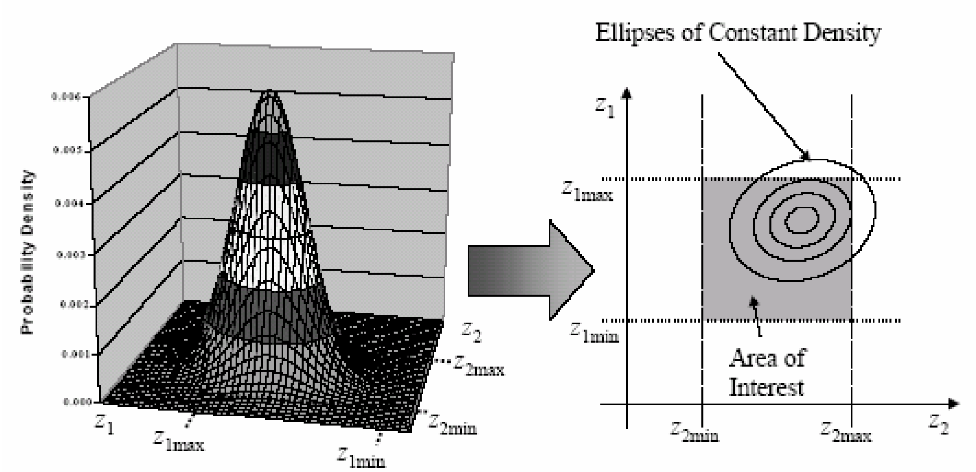

Figure 2: Area of Interest that Defines the Probability of Success[2]

In addition to using a full analysis tool, surrogate models, or metamodels, may also be used to replace the analysis tool. Metamodeling is a process by which a complex computer model is represented by a simpler analytical approximation. Metamodeling generally combines some type of Design of Experiments, which is an intelligently designed set of experiments[6], along with a polynomial approximation (commonly called a response surface equation) or other, more complex metamodel[3,7,8,9]. The actual probabilistic analysis can then be done around either the modeling and simulation tool or around the metamodel for the modeling and simulation tool. The simplest form of probabilistic analysis is called Monte Carlo. Monte Carlo methods consist of direct, random sampling involving large numbers of trials. If the number of trials in the random sampling are large enough, then the output distributions from the random trials will match the actual output distributions[10,11,12]. The greatest disadvantages of Monte Carlo techniques are that they are computationally intensive. For this reason, Monte Carlo techniques are often combined with metamodels to decrease the computational load[13,14,15,16]. In addition to using direct Monte Carlo simulation, other approximation techniques such as descriptive sampling[17,18] or advanced mean value methods[2,10,19,20] may also be used in the probabilistic analysis.

Once the modeling and simulation environment has been created, appropriate uncertainty distributions need to be defined. There has been significant research into the definition and characterization of uncertainty[21,22,23,24]. Once the uncertainty is defined, a single deterministic analysis is generally run to get a baseline solution for the problem. Next, a target value for probability of success is selected, such as 0.90. The "optimum" design is then found for this probability of success. The optimum design is defined by the design with the lowest value of objective function (usually a cost parameter) that meets the given probability of success. This design can be found using any constrained, non-linear optimization program, such as a gradient-based, sequential Quadratic Programming optimizer[25]. Once the optimum solution for a given probability of success is found, a new probability of success is chosen, and a new optimum found. This technique creates a range of optimum solutions for varying probability of success, or, a curve that matches risk versus rewards.

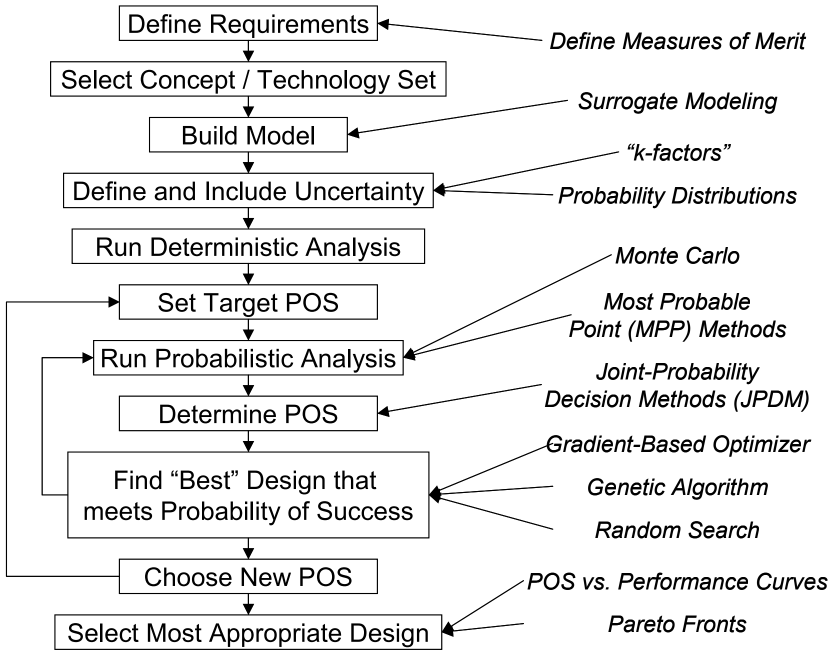

Figure 3: Process for Creation of Probability of Success versus Performance Curves

Figure 4 shows a hypothetical cost versus probability of success curve. Each design point on the design chart (left-hand side of Figure 4) corresponds to a single optimum solution for a given probability of success. By finding optimal solutions for multiple POS values, the Pareto Frontier for cost versus POS can be established. The probability of success chart (right-hand side of Figure 4) shows the optimum, or lowest cost solution, for each probability of success. By collecting this data, a POS-cost frontier can be created *before making a decision,* allowing the decision-maker to make a better informed tradeoffs as to what probability of success, or associated system risk, he or she is willing to accept, since the implications on cost are immediately known. This type of information will also be useful when comparing two potential design alternatives. As Figure 5 shows, the relative cost versus probability of success can be shown simultaneously for two alternatives. With this information, the decision-maker can quickly determine which alternative is the best to pursue, depending upon the amount of risk that he or she is willing to accept.

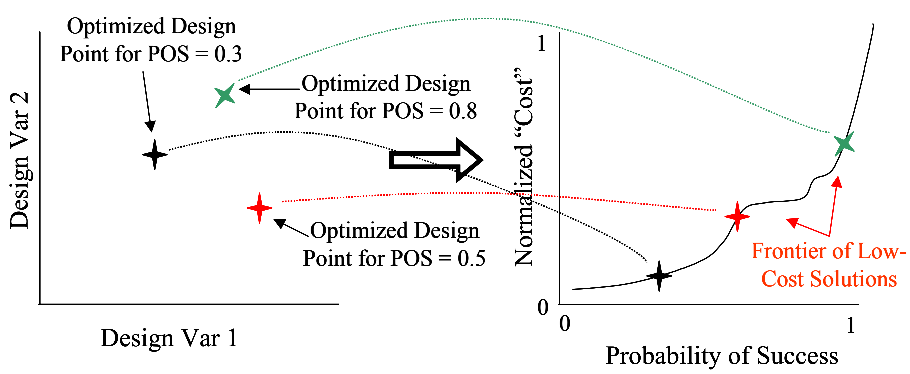

Figure 4: Translation of Design Space to Cost versus Probability of Success Curve

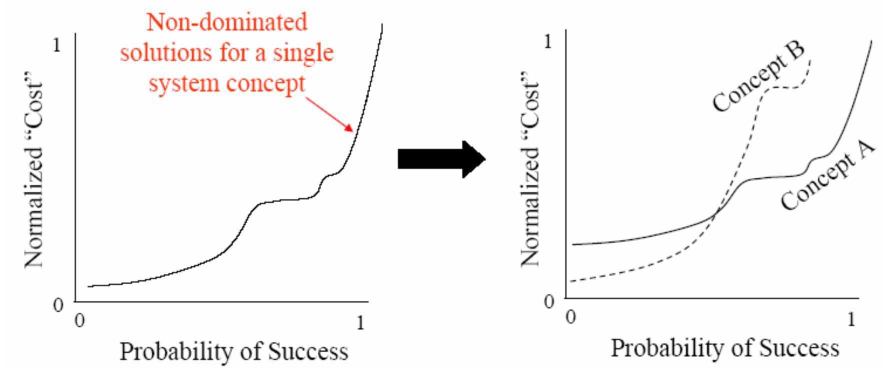

Figure 5: Hypothetical Cost versus POS for Multiple Alternatives

In addition to using a gradient-based optimizer, random searches or genetic algorithms are also well suited for this problem[26,27,28]. These techniques can be used to find the Pareto optimal front, or the set of non-dominated solutions for the problem. The set of Pareto solutions found by random searches or genetic algorithms should be equivalent to the set of optimal solutions at multiple probabilities of success found by a traditional gradient-based optimizer.

III. Traveller Example

A. Setup

A simple problem was used to help conceptualize this approach. The problem consists of a hypothetical traveller desiring to cross the region between two mountains. The topography of the scenario, with an optimal path illustrated by a dashed line, is shown in Figure 6. The goal of the traveller is to move from the lower-left corner (0,0) location to the upper-right corner (1,1) location, spending the least amount of energy possible. The traveller is penalized both for height increases along the route and for any additional horizontal distance. An objective function for the problem is shown in Equation (3). By changing the relative weightings of α and β in this function, the traveller can be driven towards preferring a longer, less hilly path (low α) or a shorter, hillier path (low β). In this case, the objective function is analogous to a travel time, with extra time being required to both travel vertical distances and to travel in a round-about manner, with the traveller desiring to minimize the time required to cross the topography. The weightings on the objective function would then relate to how quickly a traveller can travel vertically (β) versus how quickly a person could travel horizontally (α). The final addition to the example problem is the inclusion of a maximum-height constraint, prohibiting a path from entering a region greater than a specific height.

$$objective = \min(\alpha \cdot \Delta Dis\tan ce + \beta \cdot \Delta Height) \qquad \textbf{(Eq. 3)}$$

The design variables for this problem were, for ease of implementation, limited to two. In order to fully define a path using only two variables, it was decided that the path would consist of two straight-line segments, one beginning at the start location, one ending at the end location, and both meeting at a 'kink' location in the middle. The x- and y-location of this kink point is defined by the two design variables. Figure 6 shows the optimal path for traversing the mountains, with a 'kink' location located at x=0.61, y=0.28. Using this approach, two design variables, each ranging from zero to one, can capture a large field of potential paths through the mountains.

This design problem was chosen as a testbed for these techniques because of many promising features. For one, since the problem is two-dimensional, it is easily visualizable. Secondly, and just as importantly, the presence of two mountains in the topography makes the problem inherently multi-modal. Figure 7 shows contours of constant objective function. As can be seen, there is a local minimum in the 'ridge' formed between the two mountains, along with local minimums in the corners of the design space. The minimums in the corners of the space correspond to a situation in which the traveler was moving around the mountains. Finally, constraints can be added to the problem. For this example, a maximum height constraint was added, preventing the traveller from taking a path with a height greater than 0.4.

Once the deterministic problem was defined and subsequently solved, uncertainty was added to the problem. In this case, uncertainty was added to the heights of the mountains. This uncertainty would be characteristic of a scenario in which the traveller was not fully aware of the mountain heights before beginning travel. Each mountain height was varied randomly based upon a normal distribution, centered on the original height (0.6 for the wider mountain and 1.0 for the larger mountain). The normal distribution was given a standard deviation of 0.125. With the addition of uncertainty, Monte Carlo methods must now be used to evaluate each path. A 10,000 run Monte Carlo is sufficient to calculate a mean travel time (or the mean of the objective function), a standard deviation for travel time, and the probability that the path does not violate the maximum height constraint. Since there is only a single constraint for this problem, the probability of the path not exceeding the maximum height constraint is equivalent to the probability of success for the system.

In order to speed analysis of the design space, a metamodel was created of the design space. Since the design space consists of only two dimensions, a simple 36x36 point grid-search was used. Monte Carlo runs were made for each of these points to generate a grid of mean objective function, signal-to-noise-ratio, and probability of success for each sampled design point. The metamodel then consisted of a cubic spline interpolation of the 1296 points in the 36x36 grid.

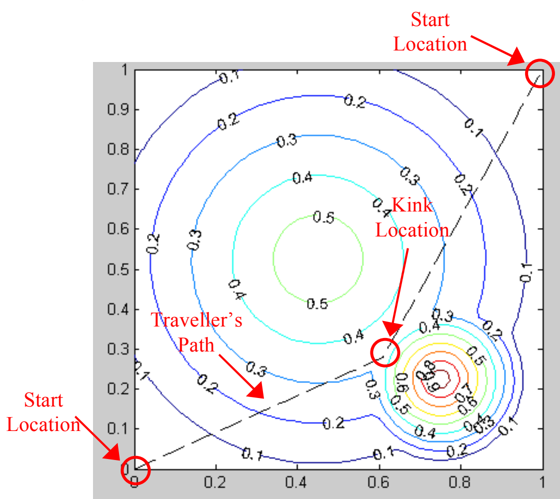

Figure 6: Mountain Topography Showing Optimal Path as Dashed Line

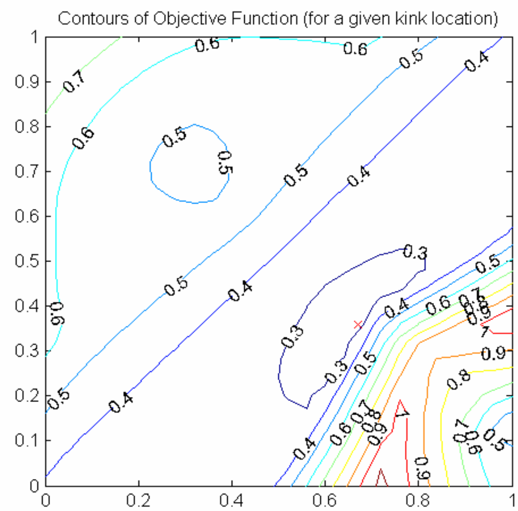

Figure 7: Contours of Constant Objective Function

B. Results

The built-in Matlab optimizer, called 'fmincon', which is a constrained, gradient-based, sequential quadratic programming tool, was used to perform optimizations on the metamodel of the probabilistic design space[29]. The goal of the optimizer was to find the location of the path with the smallest mean objective function while remaining within a specified probability of success. For instance, for a required probability of success 0.8, the analysis program would return a mean objective function of 0.43. Thus, if the designer wishes to be 80% confident of reaching his or her goal without violating the maximum height constraint, the best value for the mean of objective function that can be obtained is 0.43. The implementation of the example problem in the proposed process is shown in Figure 8.

The resulting probability of success versus performance curve (or POS vs. objective function) is shown in Figure 9. Note that as the probability of success decreases, or relaxes, the objective function improves. Remember that for these figures a lower mean travel time is better and a higher probability of success is less risky. Thus, the perfect solution would be a travel time of zero with a 100% probability of success, which corresponds to a point in the lower right corner of the graph.

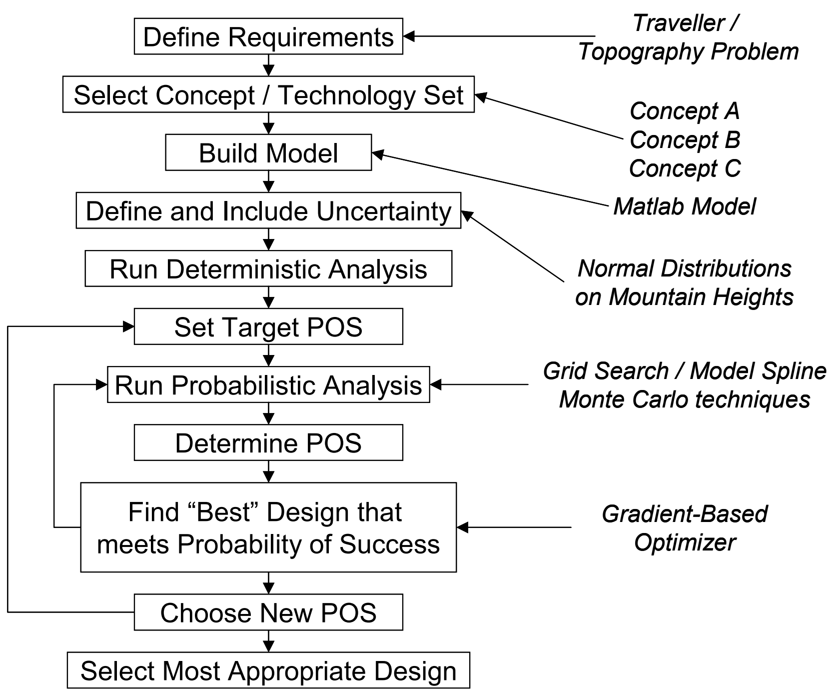

Figure 8: Implementation of the Design Process for the Traveller Example Problem

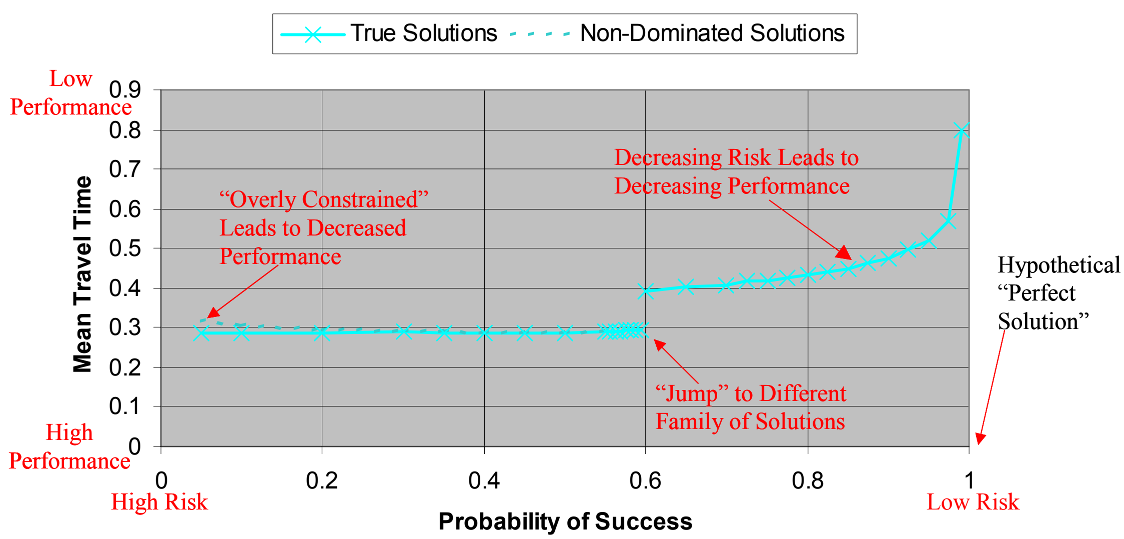

Figure 9: Traveller Problem Results

Several interesting features are apparent in Figure 9. Looking at the dashed line on the left hand side of the figure, the performance of the system actually increases slightly as the risk is increased, as indicated by the dashed line. This behavior most likely occurs due to the use of an equality constraint on meeting the specified probability of success, i.e., at a POS=0.2, the probability of success must be exactly 20%, even if a higher POS (or less risky) design gives a better solution. These equality constraints may ‘over-constrain’ the solution, driving the solution to a non-optimal answer. Two lines are used in Figure 9 and Figure 10. The dashed line represents the sometimes non-optimal equality constraint values, while solid lines represent the ‘best’ solution for a given minimum probability of success. These non-dominated solutions are found when inequality constraints are used; they represent the non-dominated set of solutions.

The second oddity in Figure 9 is the discontinuous 'jump' that occurs at a probability of success equal to 0.6. Each point on the figure represents an optimized solution. Therefore, it would be expected that as the probability of success increases slightly, the optimum would also change slightly, resulting in only a small improvement in the performance of the system. This smooth, continuous improvement occurs throughout most of the figure, which results in the smooth curvature of the line. However, since the example problem is multi-modal in nature, the jump in the figure corresponds to the solution discontinuously changing from one region of the design space to a completely different region. The reason for the discontinuous behavior is because the best solution meets a constraint boundary where it can no longer increase in probability of success. Thus, when a higher probability of success is called for, a feasible solution in this region of the design space is not available, so the solution must be found in a different region of the design space, with less optimal characteristics. In many respects, it can be thought of as a completely different 'family' of solutions in the design space.

Figure 10 is a path-inset figure that better illustrates this point. The figure shows the same set of optimal solutions as the previous figures, but also contains insets showing the actual path for the traveller at each optimized probability of success. Note that there is not much variation between the first two insets in the figure. The optimal solution stays in the same general area, which explains why the curve is smooth between these points. However, in the third inset, the optimal solution has jumped from the valley between the two mountains to a path moving around the mountains. This jump, resulting from the multi-modal nature of the design space, is what generates the discontinuous region in the objective function vs. POS curve. The next inset shows that the design point moves a little further, but still remains in the same neighborhood, again illustrating the fact that when the solution remains in the same neighborhood, there is a smooth curve for POS versus travel time. The last inset indicates yet another jump in the design space, which explains the strong kink at the end of the function. This kink at the right-hand side of the figure might better be represented by another discontinuous break, representing the jump in the optimal solution.

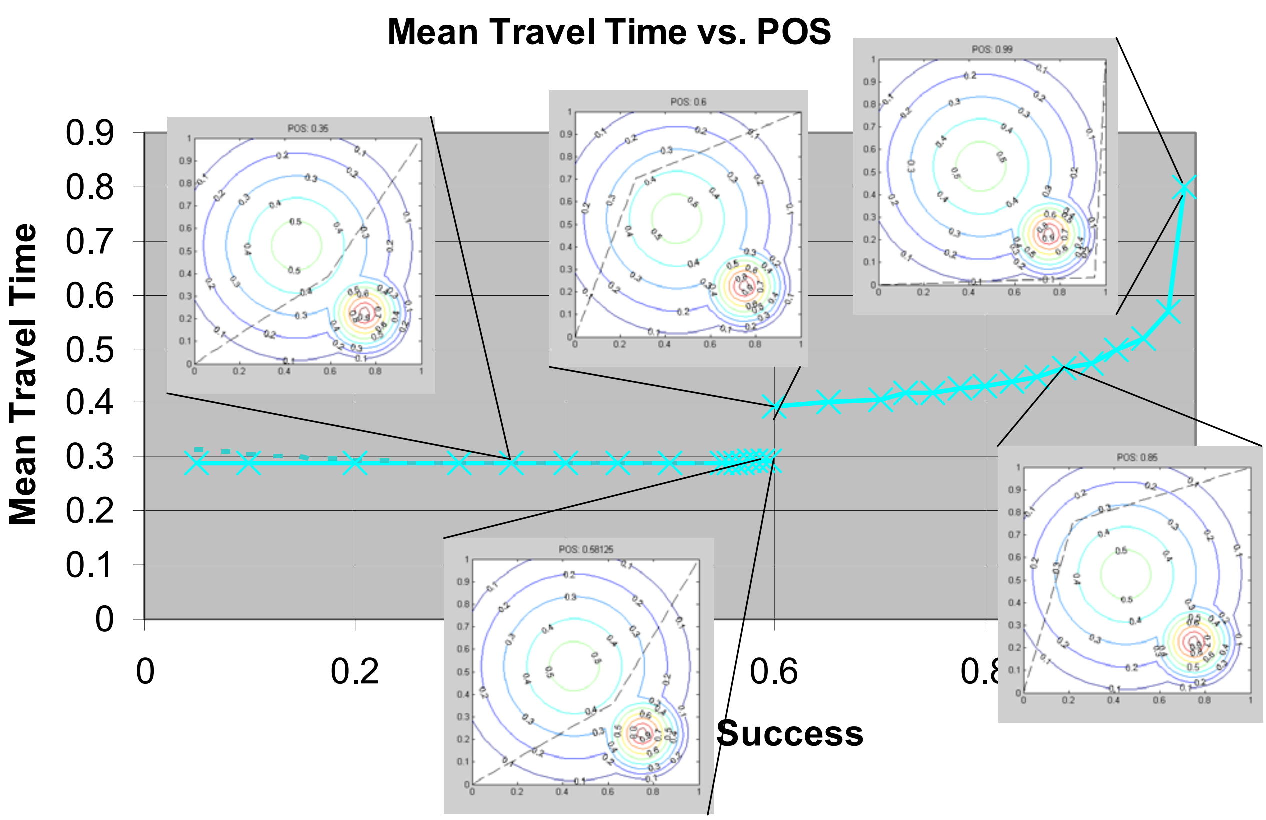

Figure 10: Traveller Problem Results with Path Insets

Essentially, Figure 10 illustrates to the decision-maker the following point: the fastest route across the terrain is to travel in the region between the two mountain peaks. But, since the traveller isn't certain about the true mountain heights, this is the riskiest path to take, as he or she may violate the maximum height constraint. A safer path (with a correspondingly higher probability of success) is to go around the larger mountain, but it is likely that this path will be slower than moving through the valley. Finally, if the traveller wishes to be certain that the trip is successful, going completely around both mountains will insure a successful trip, but will take the longest time. Thus, a decision-maker can look at the information and then make the decision as to what probability of success he or she is willing to accept, and then find the corresponding optimal design.

C. Multiple Alternatives

To further examine options for the decision-maker, the mountain topography was again optimized with a different set of parameters for the objective function. Having a different objective function is akin to using an alternative solution or concept for solving the problem. The new concepts, and their corresponding objective function parameters, are shown in Table I. The previous results were for an objective function where parameters (α and β) had α heavily penalizing the traveler for going long distances and only lightly penalizing the traveler for going uphill. The second concept has the majority of the penalty for going uphill, with small penalties for extra horizontal distance. The first concept can be thought of as a slow-moving design, but one that is designed for mountainous terrain, while the second concept could be though of as a fast-moving vehicle that has very poor performance climbing. As a further example, a third concept was added, with an evenly weighted objective function. This objective setting corresponds to having neither a preference for going over the mountains or around the mountains; it is a compromise between the two systems.

Table I: Objective Function Weightings for Alternatives

		α	β	
Alternative	Preference	Unwillingness to Travel Extra Distance	Unwillingness to Travel Vertically	Maximum Height Constraint
One	Climbing	1.2	0.6	0.4
Two	Flat Land	0.6	1.4	0.4
Three	Compromise	1.0	1.0	0.4

The probability of success versus optimal solution for the three different concepts is shown in Figure 11. As is evident in the figure, concept two is little more than a straight line. This straight line is due to the fact that the concept's best travel time occurs when it moves completely around the mountains, which also corresponds to the highest probability of success for the problem, since there is no danger of accidentally climbing too high and violating the maximum height constraint. The dashed line for concept two was generated by constraining the concept to travel a path with a probability of success exactly equal to the value specified, i.e., an equality constraint was used for POS instead of an inequality constraint. The dashed line shows a tremendous detriment in performance as the POS increases, showing again that the use of equality constraints often forces the traveller to use sub-optimal paths around the topography. The travel time of concept two over these sub-optimal paths increases greatly, as the traveller is forced to travel over the mountain, as opposed to the optimal route, around the mountain. The straight, solid line is therefore representative of the set of non-dominated solutions for concept 2, showing that the concept always prefers the high probability of success path around the mountain. If a path-inset figure, similar to Figure 10, were shown for this concept, then every route would be identical, with the path going around the mountain. The third concept, the compromise solution, always performs worse then either of the other two alternatives. However, this fact provides valuable information to the decision-maker, who now realizes that there is no benefit in looking at the compromise alternative, as both of the other choices are better performers over the entire range of possible risk.

In summary, Figure 11 is very useful to a decision-maker, outlining the relative advantages (or lack thereof) of three distinct concepts. Clearly, if the decision-maker is willing to accept some measure of risk, which translates to a probability of success lower than 1.0, the concept one is the most beneficial, since it has a lower travel time for points where the POS is less than 1.0. However, if the decision-maker is not willing to accept any risk, then concept two is by far the most beneficial concept, since it has the lowest travel time at the probability of success closest to 1.0.

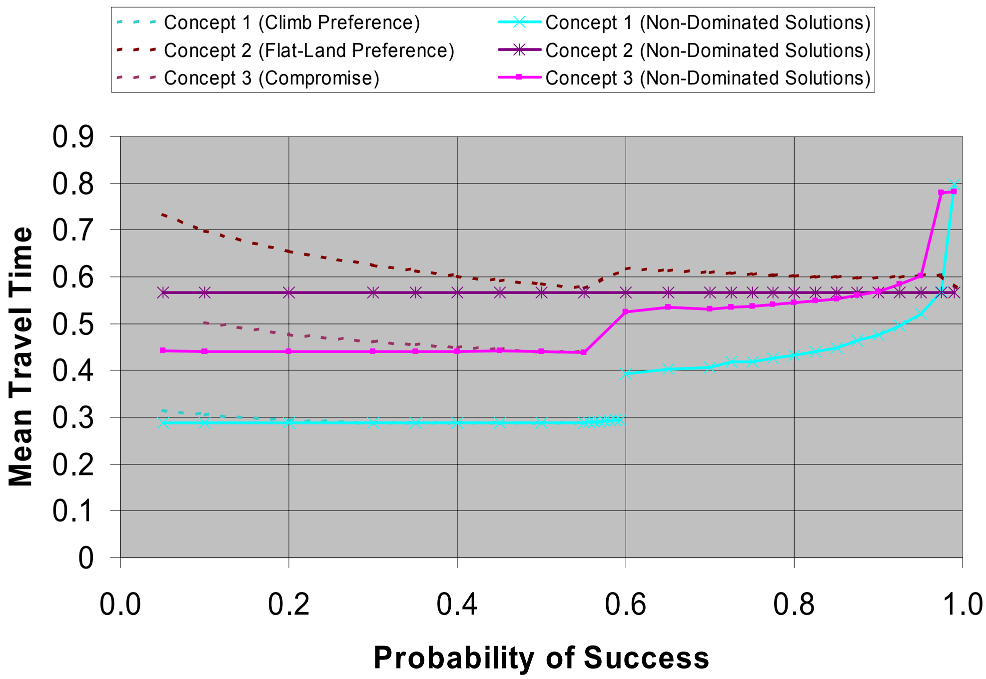

Figure 11: Traveller Problem Results for Multiple Alternatives

IV. Torpedo Example

A. Setup

A torpedo example was conducted for a heavyweight torpedo (torpedoes in the 21 inch diameter class). The analysis tool used for this study is the Torpedo Optimization, Analysis, and Design (TOAD) program, developed co-operatively between the Aerospace Systems Design Laboratory and the Naval Undersea Warfare Center, with additional collaboration from several other Navy entities. TOAD is an object-oriented, parametric sizing and synthesis program for both lightweight and heavyweight torpedo systems. It handles all-electric torpedoes, piston and turbine powered systems, and stored chemical-energy propulsion systems. It has been validated against existing torpedo systems[30] and used in research analysis comparing alternative torpedo concepts[31]. A list of the inputs and outputs for TOAD are shown in Figure 12; a typical torpedo layout with individual sections is shown in Figure 13.

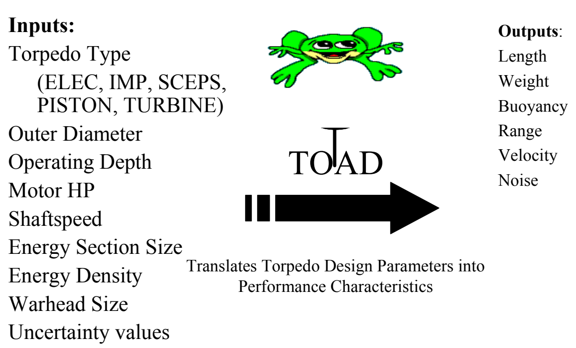

Figure 12: Inputs and Outputs for TOAD Analysis Tool[32]

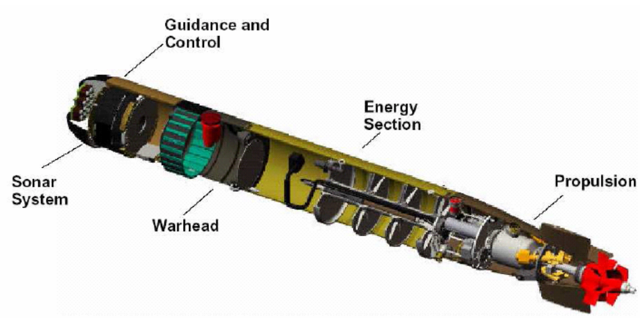

Figure 13: Typical Internal Layout of a Torpedo[33]

In order to handle uncertainty, “k-factors” were added to the TOAD analysis tool. “K-factors” are often used to model the use of future technologies or uncertainty[3]. The factors work as multipliers to interim values in the code, such as estimated drag and system efficiencies. The factors thus affect the predicted performance of the system by changing internal values. In this form, the “k-factors” represent the inability of the individual analysis tools to predict the final results, i.e., the inability of the motor module to accurately predict the motor efficiency. The “k-factors” thus can model the uncertainty between the proposed design and the actual torpedo system. A list of the “k-factors” used in TOAD is given as Table II.

The example design problem has six design variables, or control variables, as listed in Table III. The problem also has five noise variables, which correspond to the “k-factors” as shown in Table II. Normal distributions were added to each of these noise variables. The goal of the design problem was to design a large diameter torpedo system, carrying a 1,000-lb_m warhead that meets the minimum performance characteristics listed in Table IV. An individual torpedo design corresponds to a single deterministic setting of the design variables, with a probabilistic distribution associated for each response. The probability of success for each torpedo system is the probability of *simultaneously* meeting all of the performance characteristics listed in Table IV.

Table II: Uncertainty "k-factors" Applied in TOAD

Uncertainty “k-factors”
Estimated Drag
Battery Efficiency
Motor Efficiency
Propulsor Efficiency
Radiated Noise

Table III: Design Variables for the Heavyweight Torpedo Example

Design Variables
Diameter
Energy Length
Horsepower
Decoupling Layer Thickness
Damping Layer Thickness
Motor/Propulsor RPM

Table IV: Design Requirements for Torpedo Example

Performance Requirements		
Max. Vel	*at least*	45 kts
Max. Range	*at least*	15 nmi
Noise	*at most*	45 dB

In order to speed the analysis, a Design of Experiments, coupled with a response surface equation, was used around the design space. The ensuing metamodel, or response surface equation, takes as inputs both the design variables (Table III) and noise variables (Table IV), and quickly generates a response. With this technique, the TOAD analysis program could be replaced by a series of simple polynomial expression, and large Monte Carlo analyses could be run quickly.

Two separate algorithms were used to find the Pareto Frontier. The first algorithm is the same gradient-based ‘fmincon’ optimizer in Matlab that was used for the traveller example. The gradient-based optimizer was used in conjunction with the response surface equations and Monte Carlo techniques with a very large number of runs per case. The large numbers of Monte Carlo runs were required in order to get accurate gradient information. In addition to using a gradient-based optimizer, a random search was also employed, with a smaller number of Monte Carlo runs being used. Random search requires significantly more function calls in order to get a good solution, but, since highly precise Monte Carlo results are not required, does not need as many total runs. A summary of the optimization techniques is shown in Table V. The complete process by which the design methodology was implemented is shown in Figure 18.

Table V: Summary of Optimization Technique

	Optimizer	Model	Uncertainty Analysis Technique	# of Function Calls
Approach 1	Gradient-Based SQP Optimizer	Response Surface Equation	50,000 Monte Carlo Trials per Call	Dozens
Approach 2	Random Search	Response Surface Equation	1,000 Monte Carlo Trials per Call	Thousands

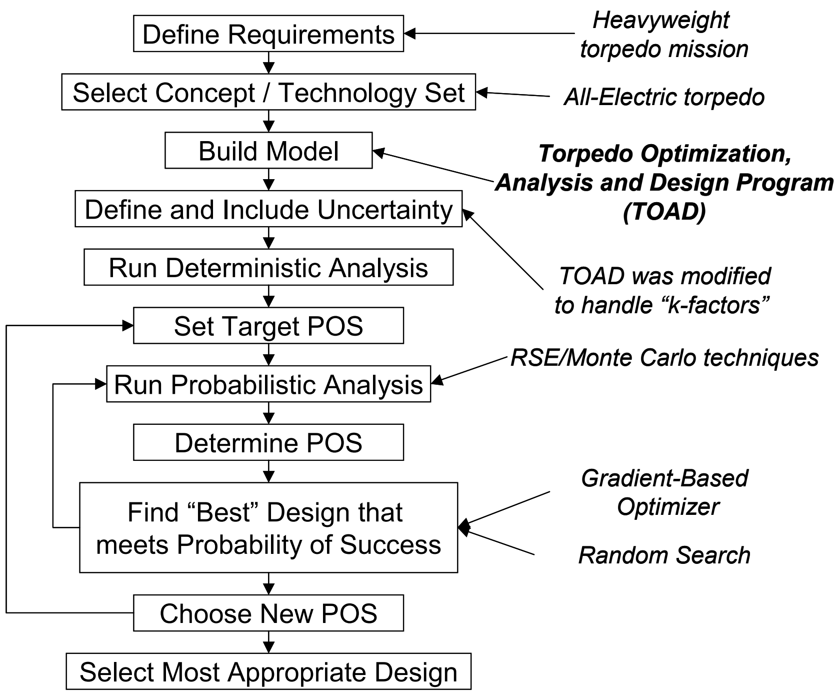

Figure 14: Design Process Implementation for a Heavyweight Torpedo Problem

B. Results

The results of the random search and the gradient-based optimizer are shown in Figure 15. The figure shows that, individually, the gradient-based optimizer and the random search only partially captured the front of Pareto points. The gradient-based approach found the best solutions at smaller probabilities of success values, while the random search found better solutions at higher probabilities of success values. Perhaps if more random searches were conducted, or more starting points for the gradient-based optimizer were used, both techniques would be able to capture the entirety of the Pareto Front.

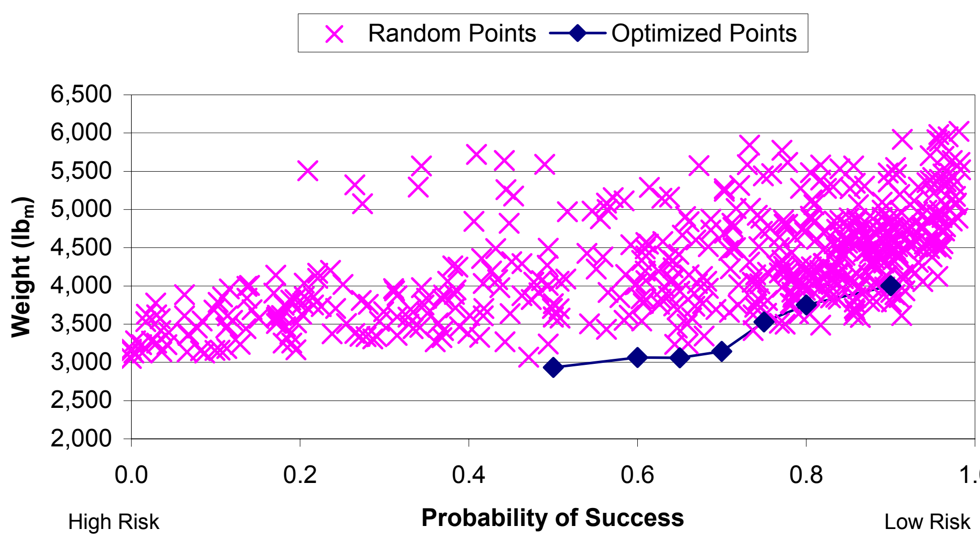

Figure 15: Comparison of Optimized Points vs. Random Points for Torpedo Example

The complete Pareto front of non-dominated points, formed from the aggregate of the two approaches, is shown in Figure 16. The results in this chart behaved similarly to the results in the traveller problem, with increasing probability of success (or decreasing risk) leading to heavier, more costly solutions. Note that in these figures there are no discontinuities in the design space like those seen in the traveller example (Figure 9). One reason for this lack of discontinuities is because of the use of response surface equations for the torpedo problem. By using second-order equations, the multi-modal nature of the problem is removed, so that large 'jumps' in the probability of success versus weight curve do not exist. If this work were repeated with direct function calls of the analysis program, then the more complex behavior of the previous problem might become visible.

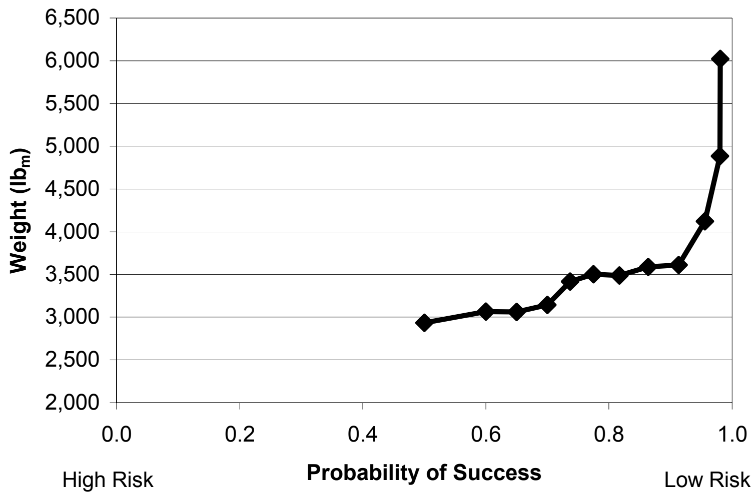

Figure 16: Probability of Success vs. Weight for an All-Electric, Heavyweight Torpedo

C. Additional Comparisons

To further illustrate the advantages of the technique, two alternatives were compared. A piston-driven system was compared to the previous results for an all-electric system. The design process in Figure 14 was repeated for the new piston concept and the results compared in Figure 17. The results indicate that the two concepts are fairly similar in performance. However, if a lower probability of success (higher risk) is acceptable, then the electric system is the preferred alternative. However, if a very high probability of success is required, then the piston system is definitely the best choice. Thus, these figures allow the decision-maker to assess the relative merits of alternatives based upon his or her choice of the level of acceptable risk.

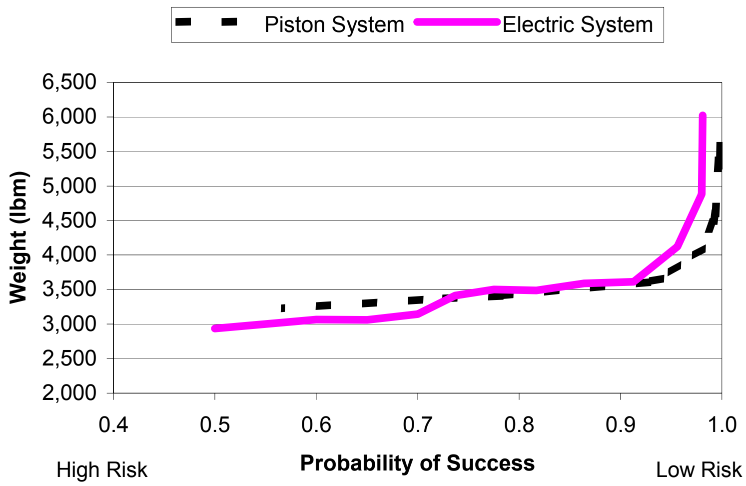

Figure 17: Probability of Success vs. Weight for Multiple Alternatives

Figure 18 shows one more useful application of this design technique. In Figure 18, the effect of changing requirements is shown for the electric torpedo system. The design process in Figure 14 was repeated three times, each with a separate set of design requirements. The results show that decreasing the performance requirements of the system increases the probability of success, or, stated another way: if the requirements are relaxed, there is less risk that a system of given weight will not meet the requirements. Figure 18 would be useful to a decision-maker who wanted to look at the effect of changing requirements on the risk and cost of the system.

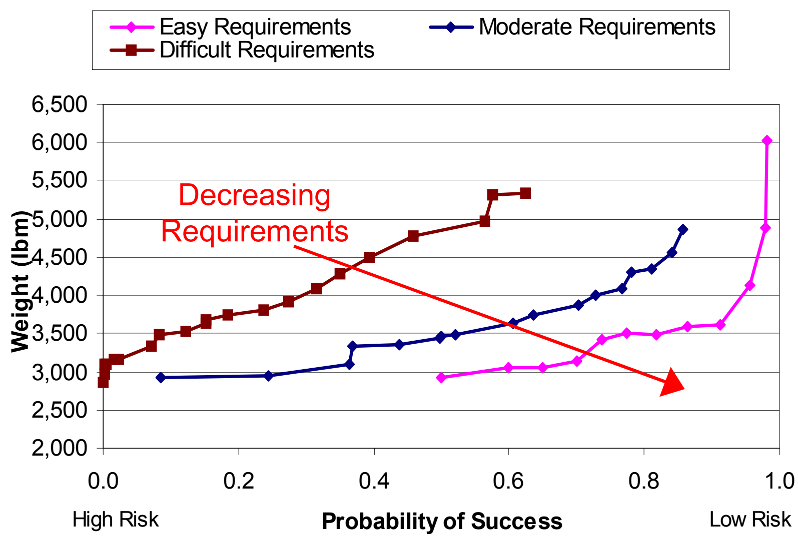

Figure 18: Probability of Success vs. Weight for Decreasing Requirements

V. Conclusion

Accounting for uncertainty in designs is crucial, however, bringing this uncertainty information into the design process in an intelligent manner poses new dilemmas. Using the design process outlined in this paper, the decision-maker is able to construct an inherent tradeoff between probability of success and any desired system metrics. Put another way, the decision-maker is able to compare the risk of accomplishing the mission requirements versus the cost of the system. By looking at this information, *a priori*, the decision-maker can then make an informed decision as to the amount of risk that is considered acceptable for the program. The decision-maker can thus treats the probability of success, or risk, as an independent variable – first examining the tradeoff between risk and cost, choosing a value for risk, and then selecting the optimal design that falls out from the decision.

The test examples given illustrate that highly informative probability of success versus cost curves can be created that will readily assist in decision-making. In addition to these curves, multiple alternatives can be compared to determine which system performs best at the desired level of risk. Tradeoffs between risk and requirements can also be quickly performed. The methods outlined are scaleable, and can be used for any number of complex engineering systems.

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