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MULTI-LEVEL, PARTITIONED RESPONSE SURFACES FOR MODELING COMPLEX SYSTEMS

Patrick N. Koch[*]
Engineous Software, Inc.
1800 Perimeter Park West, Suite 275
Morrisville, North Carolina 27560, USA

Dimitri Mavris[†]
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150, USA

Farrokh Mistree[‡]
G. W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0405, U.S.A.

ABSTRACT

The most prevalent type of approximating functions employed for efficient engineering analysis and design integration are polynomial *response surfaces*. However, the construction of response surface approximations has been limited to problems with only a few variables, due to the number of analyses necessary to fit sufficiently accurate models. In this paper, an approach is presented for partitioning response surfaces and constructing multi-level approximations for problems with larger numbers of variables. Using this approach, the (computer) experimentation necessary for fitting response surface models is reduced tremendously. A modified composite experimental design is also presented for the construction of response models that are more consistently accurate across the range of the design variables. The multi-level, partitioned response surface modeling and modified composite design approaches are demonstrated for the preliminary design of a commercial turbofan engine, an example problem defined in collaboration with Allison Engine Company, Rolls-Royce Aerospace Group.

NOMENCLATURE

$\hat{\mu}$	mean response estimate
$\hat{\sigma}$	standard deviation estimate
$\hat{y}$	predicted response value
RSM	Response Surface Methodology
DOE	Design of Experiments
CCD	Central Composite Design
η	efficiency
SFC	Specific Fuel Consumption
OPR	Overall Pressure Ratio
SOT	Stator Outlet Temperature
HPC	High Pressure Compressor
HPT	High Pressure Turbine
LPT	Low Pressure Turbine

1 OVERVIEW OF PROBLEM

Statistical techniques are widely used in multidisciplinary design to construct *approximations* of expensive computer analyses that are much more efficient to run, easier to integrate, and yield insight into the functional relationship between design variables, **x,** and performance responses, **y**. A recent review of statistical experimentation (design of experiments-DOE) and approximation techniques for engineering design is given in Ref. 1. The most commonly used approximating functions are polynomial *response surface* equations. A recent review of several applications of response surface models in engineering design is also given in Ref. 1, and applications in structural design are presented in Ref. 2.

The creation of such response surface *metamodels*[3] to approximate detailed computer analysis codes is particularly appropriate in the preliminary design stages when comprehensive trade-offs of multiple performance and economic objectives is critical. Although computers continue to get faster, analysis codes always seem to keep pace so that their computational time remains non-trivial. Through metamodeling, approximations of these codes are built that are orders of magnitude cheaper to run. These metamodels can then be linked to optimization routines for fast analysis, or they can serve as a bridge for integrating analysis codes across multiple disciplines.

Many design methods have recently been developed incorporating response surface metamodeling techniques.[4-6] One significant limitation of response surface techniques is due to the *problem of size* for modeling large-scale, complex systems: the computational expense of experimentation associated with the combinatorial explosion in data points necessary for fitting models in a large number of variables. A detailed investigation into the problem of size is presented in Ref. 7 for the design of a high speed civil transport. For complex systems the number of variables affecting the system is often greater than desirable for response surface modeling with standard experiment designs (greater than ten variables—a

* Member AIAA. Formerly Graduate Research Fellow at Georgia Institute of Technology.
† Assistant Professor and Senior Member AIAA
‡ Professor and Senior Member AIAA. **Corresponding author.** Email: farrokh.mistree@me.gatech.edu.

standard central composite design for ten factors requires 1045 analyses). In addition, if small or minimum size experiments are designed to accommodate modeling for large numbers of variables, the resulting models are frequently not of sufficient predictive accuracy (sparse experiments do not capture the trends sufficiently). The question being addressed in this paper, then, is the following: *How can accurate response surface models be fit efficiently for large numbers of design factors?*

To address this question an approach for constructing *multi-level*, *partitioned* response surfaces is presented in this paper. Since this response modeling approach is implemented within the context of *robust design*, a review of approximation-based robust design and response surface metamodeling is first provided in Section 2. The multi-level, partitioned response surface metamodeling approach is then presented in Section 3 and demonstrated in Section 4 for the preliminary design of a commercial turbofan engine; closing remarks are presented in Section 5.

2 FRAME OF REFERENCE: APPROXIMATION-BASED ROBUST DESIGN

2.1 Robust Engineering Design

The basic approach to approximation-based robust design, illustrated in Figure 1, is a generalization of *response surface methodology* (RSM),[8-10] Taguchi's parameter design,[11] and related recent engineering design approaches[4,6] in the literature.[§] The ultimate goal with approximation-based robust design is to arrive at improved or *robust* solutions *efficiently*. In statistical terms, design variables are factors and design objectives and constraints are responses; the factors and responses for a particular design problem provide the input or starting point for the approach of Figure 1, and the solutions, improved or robust, become the outputs or results. To identify these solutions, this approach includes three sequential stages: *screening, model building,* and *model exercising*.

Screening is employed only if the problem includes a large number of factors (say > 10, which is usually the case for complex systems). When the number of factors is too large for comprehensive exploration and/or when experimentation is expensive, screening experiments are used to reduce the set of factors to those that are most important to the response(s) being investigated. DOE is used to define the appropriate design analysis cases to evaluate the desired effects of the factors. If the factor set can be reduced, this reduced set provides the input for the second stage.

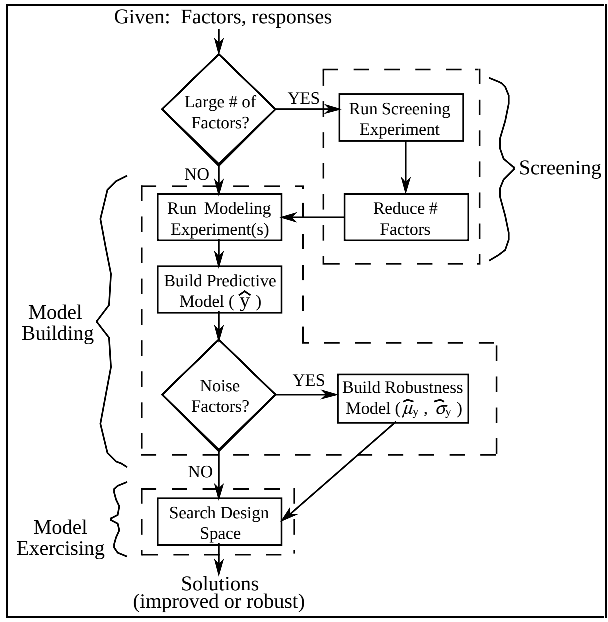


Figure 1 Basic Robust Design Approach

In the second stage (model building), predictive metamodels are created to replace computationally expensive codes. An experiment is run, and least squares regression is used to fit a surface model. If noise (uncontrollable) factors[12] are included in the experimentation, the mean and variance of each response must be estimated, and predictive models for both must be built. One approach for building these *robustness* metamodels is presented in Ref. 13, and three approaches are compared in Ref. 14.

In the third stage of the approach in Figure 1 the response models created in stage 2 are exercised to explore a design space *efficiently* (polynomial models are essentially free computationally) while including uncertainty (noise) to identify robust solutions.

The focus in this paper is on the model building stage of the approach of Figure 1, particularly on response surface approximating functions. A review of response surface modeling is thus provided next.

2.2 Response Surface Metamodeling

An experimental design formally represents a sequence of experiments to be performed, expressed in terms of *factors* (design variables) set at specified *levels*, or pre-defined values. Given a response of interest, y, and the vector of independent factors **x** included in the experimental design and thought to influence y, the relationship between y and **x** can be written as follows:

$$y(\mathbf{x}) = f(\mathbf{x}) + \varepsilon \qquad [1]$$

§ Although slight variations or a more detailed breakdown of this approach may exist, the basic approach of Figure 1 is used for discussion purposes.

where ε represents random error, which is assumed to be normally distributed with mean zero and standard deviation σ. Since the true response surface function f(**x**) is usually unknown, a response surface g(**x**) is created to approximate f(**x**). Predicted values are then obtained using $\hat{\mathbf{y}}$ = g(**x**).

The most widely used response surface approximating functions are simple low-order polynomials. If little curvature appears to exist, the first-order polynomial given in Eqn. 2 can be employed. If significant curvature exists, the second-order polynomial in Eqn. 3, including all two-factor interactions, can be used.

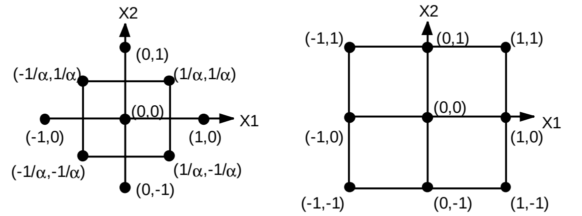
$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i \qquad [2]$$

$$\hat{y} = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i}\sum_{j>i} \beta_{ij} x_i x_j \qquad [3]$$

The parameters (β terms) of the polynomials in Equations 2 and 3 are calculated using a least squares regression analysis to fit these response surface approximations to existing data (empirical or generated from simulation/analysis routines). These approximations can then be used for prediction. A more complete discussion of response surfaces and least squares fitting can be found in Ref. 10, and of experimental design in Ref. 15.

Among the various types of experimental designs for fitting a second-order response surface model and studying second-order effects, the Central Composite Design (CCD) is probably the most widely used.[15] Central composite designs are first order fractional factorial designs augmented by an additional star and center points which allows for the creation of a second-order surface with fewer points than would be required for a three-level full factorial design.

The two CCD experiments commonly used for computer experiments are illustrated in Figure 2 for two factors; the factor ranges are generally normalized between -1 and 1. The CCI (central composite *inscribed*) experiment of Figure 2(a) is offered to contain the experiment within the defined factor ranges. For this composite experiment, the star points are defined by the factors ranges, and the factorial points are interior, defined by the combinations of $\pm 1/\alpha$. The value for α is normally chosen to ensure *rotatability*[10] of the design. The predictive capability of a response surface fit using the CCI design, however, is unknown at the extremes of the factor ranges since no data points are taken at these extremes ([1,1], [1,-1], [-1,1], and [-1,-1] in Figure (a)). In addition, the standard α value pushes the factorial points towards the center with increasing factors ($1/\alpha$ is 0.707 for two factors and 0.297 for eight factors). Thus the accuracy of a response surface fit using the CCI design is even more questionable at the extremes with increasing factors.



(a) CCI **(b) CCF**

Figure 2 Two Forms of Central Composite Experiment Designs

If the extreme points are known to be important, the central composite *face-centered* (CCF) design shown in Figure 2(b) is used. In this design, the factorial points are pushed to the corner points defined by the factor extremes, keeping the star points at the factor extremes along each factor axis. With this configuration, however, other than the center point no interior data points are taken; thus only three factor levels, rather than five, are used.

Unfortunately, with computer experiments, it is often not known in advance whether the interior points or extreme points are more important; a computer analysis code is often truly a "black-box" for which the input-output relationships are not clearly or completely understood. If the region where good solutions exist is known, the experiment should be designed around this region, and again it is unknown whether the CCI or CCF should be used. Of course it is undesirable and often not possible to run both the CCI and CCF experiments, fit models for both, and check the predictions from each.

3 MULTI-LEVEL, PARTITIONED RESPONSE SURFACES

In this section, an approach for constructing multi-level, partitioned response surfaces is presented (Section 3.1) to overcome the problem of size with large problems. A modified composite design is also presented (Section 3.2) to facilitate the construction of response models that are more consistently accurate across the range of the design variables.

3.1 Constructing Partitioned Response Surfaces

An approach for fitting second order polynomial response surface models in large numbers of design factors is shown in Figure 3. In this approach the factors and responses of a complex analysis code are grouped, and the response surface models themselves are partitioned to create multi-level models incorporating the effects of all factors.

Given:

n total number of factors
r total number of responses

Partition Factors and Responses:

k number of factors in set 1
n-k number of factors in set 2
s responses of set 1, $\hat{y}_1$
r-s responses of set 2, $\hat{y}_2$

Construct Partitioned Response Surface Equations:

For responses of set 1,

$$\hat{\mathbf{y}}_1 = \alpha_0 + \sum_{i=1}^{k} \alpha_i x_i + \sum_{i=1}^{k} \alpha_{ii} x_i^2 + \sum_{i<j}\sum \alpha_{ij} x_i x_j \qquad [4]$$

where,

$$\alpha_0 = \gamma_0 + \sum_{p=k+1}^{n} \gamma_p x_p + \sum_{p=k+1}^{n} \gamma_{pp} x_p^2 + \sum_{p<q}\sum \gamma_{pq} x_p x_q \qquad [5]$$

For responses of set 2,

$$\hat{\mathbf{y}}_2 = \beta_0 + \sum_{p=k+1}^{n} \beta_p x_p + \sum_{p=k+1}^{n} \beta_{pp} x_p^2 + \sum_{p<q}\sum \beta_{pq} x_p x_q \qquad [6]$$

where,

$$\beta_0 = \delta_0 + \sum_{i=1}^{k} \delta_i x_i + \sum_{i=1}^{k} \delta_{ii} x_i^2 + \sum_{i<j}\sum \delta_{ij} x_i x_j \qquad [7]$$

Figure 3 Approach for Constructing Partitioned Response Surfaces

Given a number of factors, *n*, and responses, *r*, associated with a particular complex analysis, the factors and responses are partitioned into two sets in Figure 3. This partitioning is ideally based on domain knowledge; the factors and responses should be grouped based on knowledge about the problem and particular analysis, based on which factors are believed to directly affect particular responses. The exact partitioning is not necessarily critical, however, since each response is made a function of all factors; this is accomplished by concurrently constructing two-part *partitioned* response surfaces. The first set of responses $\hat{y}_1$ are fit as a function of the *k* factors of set 1 (Eqn. 4), and the second set $\hat{y}_2$ is fit as a function of the *n-k* factors of set 2 (Eqn. 6). Two separate experiments are designed and run, ideally concurrently, to fit these two sets of response surfaces.

To capture the effects of the second set of factors (k+1 to n) on the responses $\hat{y}_1$ modeled using the first set (1 to k), the mean term of these responses (α_0 in Eqn. 4) is fit as a function of the second set of factors; this is shown in Eqn. 5. The same is done for the second set of responses, $\hat{y}_2$, using the first set of factors (Eqn. 7), and thus the two-level response surfaces are created. The effect of modeling the mean term of each set of responses as a response surface itself is essentially to allow the intercept of the primary response surface (Eqn. 4 or 6) to move along the response, or *y*, axis with the secondary effects of the factors used to model the mean term. It is important to note that in creating the second part of each set of response surfaces, the mean-term response models, no additional experimentation is necessary. Data gathered for fitting the response surfaces of Eqn. 6 is used in fitting the mean term response surfaces of Eqn. 5; data gathered for fitting Eqn. 4 is used in fitting Eqn. 7.

The advantage of this approach of Figure 3 is that the experimentation and model fitting expense is reduced tremendously. If, for example, 16 input factors are known to be important to the responses of a particular problem, fitting response models including all 16 factors using a standard CCI or CCF design would require 65,569 computer analysis cases. If the 16 factors are partitioned into two sets of eight, and the responses are also grouped according to the two sets of factors, two experiments of 273 runs are required for the standard CCD to fit the two part response surface models of Eqn. 4 - 7. These two experiments can be run concurrently, and the experimentation expense to construct the models is cut by more than two orders of magnitude. Even without concurrency, with this partitioning, doubling the number of factors increases the necessary experimentation by a factor of two rather than quadratically ($2^k + 2^k$ rather than 2^{2k}, or $(2^k)^2$).

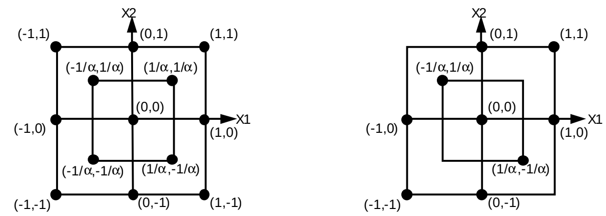
What is the limitation of this approach? The major assumption in this approach is that the interaction effects between the factors of each partitioned set are negligible or nonexistent. These interaction terms ($x_i x_p$ terms using the notation of Figure 3) are the only terms missing from the partitioned response surface models that would be present for response surfaces fit in all factors. The factors and responses must be partitioned such that these interaction terms are believed to be at least negligible. Many interactions are known to be nonexistent based on domain knowledge, or can be shown to be nonexistent through experimentation. The factors and responses can then be partitioned so that only the interactions that make sense and are known to exist are included.

In the approach of Figure 3, a set of factors and responses is partitioned into *two* groups, and the partitioned response surfaces then have *two* parts. This approach is not limited to two sets of factors and ranges, however; the factors and responses could be partitioned into more than two parts. Constructing the partitioned models would be increasingly more difficult to manage, but doable nonetheless. In this paper only two-level partitioned response surfaces are investigated. A modified composite design is used to construct these partitioned respones models.

3.2 Modified Composite Design

A modified composite design is illustrated in Figure 4. This experiment combines the factorial portion of both the CCI and CCF experiments (Figure 2), and is designed for the case in which it is unknown whether the interior or extreme points are most important. Two versions of this experiment are shown is Figure 4, the full combination of the CCI and CCF (Figure 4(a)) and a half fraction in the factorial points of the combined CCI/CCF (Figure 4(b)). With the full experiment of Figure 4(a), while for the two factor experiment only four points are added over the standard CCI or CCF designs of Figure 2, for an eight factor experiment 529 total points would be needed compared to the 273 for the standard CCI or CCF. As the number of factors increases, the factorial portion of the composite design (increasing exponentially, 2^k), dominates the experiment; thus with large factor numbers, the size of the combined experiment of Figure 4(a) is essentially twice the size of the standard CCI or CCF, which may be unmanageable. For the combined CCI/CCF of Figure 4(b), a half fraction of each factorial portion is taken. It is common to take a fraction of the factorial points with a composite design to reduce the number of experiments (many more points are defined in a standard composite design than are necessary for fitting the standard second order response surface of Eqn. 3). The experiment of Figure 4(b), then, is the same size as the standard CCI or CCF, but allows data to be gathered both in the extremes and the interior points in an attempt to obtain better model fits.

To evenly space the experiment points within the design space, the α value for the modified composite design is taken to be 2, placing the interior factorial points at the selected combinations of ±0.5 for all normalized factor ranges. Rotatability is based on statistical measures deemed inappropriate for deterministic computer experiments,[16] and thus rather than defining α to ensure this property, *space filling*[17] becomes appropriate and necessary to construct sufficiently accurate approximations. Evenly spaced design points are more appropriate to capture the effects of deterministic computer analyses; For more detailed discussions of deterministic computer experiments, see Refs. 1,17.



(a) Combined CCI and CCF **(b) Half Fraction, Combined CCI/CCF**

Figure 4 Modified Central Composite Experiments

The modified design is investigated in detail and compared to the standard CCI design in Ref. 18; more consistent accuracy of predictions across the ranges of design variables are observed with the modified design. The design of Figure 4(b) is employed with the approach for multi-level partitioned response surfaces of Figure 3 in the next section.

4 EXAMPLE: PRELIMINARY DESIGN OF A COMMERCIAL TURBOFAN ENGINE

The two-level partitioned response surface modeling approach is tested and demonstrated in this section for the preliminary design of a commercial turbofan engine. The example presented here is a 7000-lb. thrust class turbine engine designed for regional and business jet applications. This example problem was defined in collaboration with Allison Engine Company, Rolls-Royce Aerospace Group, and is based on the existing Allison AE3007 engine. The focus in this example is on the engine thermodynamic cycle design and the basic configuration design (weight, overall dimensions, and stage numbers). While the cycle design is independent of the configuration factors, the configuration responses are dependent on the cycle factors in addition to the configuration factors; thus two-level partitioned response surfaces are fit for the engine configuration design. The response models constructed are formulated within a compromise Decision Support Problem (DSP)[19] to evaluate multi-objective trade-offs; *robust* solutions are identified and response surface approximations are verified for this turbofan engine preliminary design example.¶

4.1 Commercial Turbofan Problem Definition: Requirements and Design factors

The primary concern for this commercial turbofan engine example is fuel consumption, followed by thrust,

¶ Robust design techniques are incorporated as noise factors (not controllable by a designer) are included in this preliminary design example. The objective in seeking robust solutions is to minimize performance variation in addition to attempting to achieve performance targets.

weight, and overall dimensions. The design point investigated in this study is a 40,000 ft. cruise at Mach 0.8 and 77°F (off-design is not investigated in this example). The Rolls Royce software package *Engine Maker* was employed for the cycle and configuration analysis. The commercial turbofan design requirements are given in Table 1.

The Engine Maker model of the AE3007 engine (AE3007 EM) are presented in Table 1 and are used as a basis for comparison. The thermodynamic cycle performance is captured through the engine specific fuel consumption (SFC) and the net thrust (at the 40,000 ft. cruise). Both of these performance characteristics are normalized in Table 1 (based on the actual AE3007 values) for proprietary reasons; they are thus nondimensionalized. The engine configuration is defined by the engine bare weight, and the overall dimensions (length and diameter, where the maximum fan diameter is used as representative of the maximum engine diameter for comparison purposes).

Table 1 Commercial Turbofan Requirements

	AE3007 EM	Constraint	Goal
Performance:			
SFC (normalized)	1.035	≤ 1.030	0.960
Net Thrust (normalized)	990	≥ 990	1052
(40,000 ft., 77°F day			
Configuration:			
Bare Engine Weight	1426 lbs	≤ 1425 lbs	1350 lbs
Max Length	6.89 ft	≤ 7 ft	6 ft
Max Fan Diameter	38.37 in	≤ 38.4 in	37 in
Other Constraint:			
Phot/Pcold	0.880	0.94-1.0	

The requirements of Table 1 are defined through both constraint (hard) values and goals (soft performance targets). Constraint values define feasibility while goals represent desired targets (define the ideal scenario). The constraint values in Table 1 are set to identify solutions that perform at least as well as the Engine Maker model of the AE3007 engine, while the goal values reflect the desire to achieve improved performance. The last requirement of Table 1 (Phot/Pcold – ratio of core exhaust and bypass exhaust pressures) is a constraint that ensures mixing for a mixed-flow turbofan engine. These requirements become the desired responses for which response surface equations are created.

After review of the Engine Maker inputs for cycle and configuration analysis, the relevant input parameters selected as design factors for this study are listed in Table 2 (cycle factors) and Table 3 (configuration factors). The ranges for experimentation for these factors (also shown in these tables) are defined based on experience and based on the AE3007 values for these factors (not shown), and are thus specific to this problem. The 12 cycle factors of Table 2 are classified as *8 control factors* and *4 noise factors* (the component efficiencies). One key cycle factor, bypass ratio (BPR), is not included in the list of independent control factors. This factor was made a function of fan outer pressure ratio (FOPR), overall pressure ratio (OPR), and stator outlet temperature (SOT) to ensure feasible mixed flow turbofan cycles during automated experimentation, and thus is not an independent factor here.

Table 2 Commercial Turbofan Cycle Design Factors and Ranges

CONTROL FACTORS		LOW	HIGH
Compression/Expansion Parameters			
1) Inlet Flow	(lb/s)	70	105
2) Fan Outer Pressure Ratio		1.5	1.85
3) Overall Pressure Ratio (OPR)		20	35
4) Stator Outlet Temp (SOT)	(F)	2000	2500
Pressure Losses			
5) Transition Duct Pressure Loss (DPP21)	(%)	0.5	5
6) Diffuser/Combustor Pressure Loss (DPP3)	(%)	2	6
7) LPT Exit Pressure Loss (DPP5)	(%)	0.5	3
8) Bypass Duct Pressure Loss (DPP13)	(%)	1	4
NOISE FACTORS		**LOW**	**HIGH**
9) Fan Tip Polytropic Efficiency (η_{fan})		0.84	0.88
10) HPC Polytropic Efficiency (η_{HPC})		0.86	0.90
11) HPT Isentropic Efficiency (η_{HPT})		0.85	0.92
12) LPT Isentropic Efficiency (η_{LPT})		0.88	0.92

The four component efficiencies of Table 2 are defined as noise factors for this study, not under the designer's control during *preliminary* design. These factors are not actually uncontrollable noise throughout the design process; they can be viewed as "technology level" factors (higher efficiencies represent higher component technology levels) used to evaluate the effects of changes in technology at the component level. When designing the cycle, efficiencies are needed to evaluate cycle performance and are required inputs within Engine Maker. However, insufficient component level detail is available at this stage to predict these efficiencies, and thus they are included as noise factors or *uncertainties* in the cycle design. The purpose of including these efficiencies as noise factors at the cycle level, then, is to reduce the effects on the cycle design of variation or uncertainty in component efficiency values.

It could be argued also that the five pressure loss factors of Table 2 are not control factors. The losses are related to the Mach number through the associated ducts and the size of these ducts. However, the way in which Engine Maker has been configured, these factors are needed as inputs for cycle and configuration design. Thus they are included as control factors, primarily to monitor their effects on the engine design.

Table 3 Commercial Turbofan Configuration Design Factors and Ranges

CONTROL FACTORS	LOW	HIGH
1) Fan Inlet Mn	0.5	0.7
2) Fan Entry Hub/Tip Ratio	0.2	0.4
3) Fan Tip Speed	1200	1850
4) Fan Hub Loading	1.0	2.0
5) HPC Tip Speed	900	1700
6) HPC Hub Loading	0.9	2.0
7) LPT Mean Loading	1.5	2.5

The 7 configuration factors of Table 3 are all classified as control factors. Thus 19 total factors are to be investigated with Engine Maker. These 19 factors are the results of previous screening experimentation[18]; each factor has been found to have a significant effect on at least one of the requirement responses of Table 1. Thus the problem size cannot be reduced further by removing additional factors. However, 19 factors are not manageable for the construction of second order response surfaces (524,327 analyses would be necessary for a standard central composite designed experiment). Thus the two-level partitioned response surface approach of Figure 3 is appropriate here. The partitioning necessary for the construction of two-level response surfaces is easily defined for this problem; the requirements and factors have already been classified for the thermodynamic cycle design and the mechanical configuration design. Thus the problem is partitioned into the 12 cycle factors (four of which are noise factors) of Table 2 and the 7 configuration factors of Table 3, even though Engine Maker is used for both cycle and configuration analyses. This partitioning makes the problem significantly more manageable, and the experimentation for the cycle and the configuration can be done concurrently (on different machines). Yet the approach of Figure 3 allows the appropriate responses to be modeled in all 19 factors. The construction of two-level partitioned response surfaces for this example is described in the next section.

4.2 Commercial Turbofan Two-Level Response Surface Construction

For the cycle design experimentation, a product array approach (crossed inner and outer arrays)[12] is implemented. The modified composite experiment of Figure 4(b) is designed in the eight cycle control factors of Table 2, with a total of 273 experiments ($1/2{\bullet}2^8+1/2{\bullet}2^8$ factorial points, interior and exterior, 2•8 star points, and one center point). This experiment, used as the inner control array in the product array, is crossed with the outer noise array given in Table 4. The four noise factors of Table 2 are varied over their ranges using a half fraction of a two-level factorial ($2^{4-1} = 8$) with an added center point for nine total cases. The total number of experiment cases for the cycle design sub-problem when crossing the two arrays, then, is 2457 cases (273*9). Note that if a full composite design were designed in all 12 cycle control and noise factors 4121 cases would be necessary.

Table 4 Outer, Noise Array for Cycle Experimentation

Run	Pattern	η_{Fan}	η_{HPC}	η_{HPT}	η_{LPT}
1	----	0.84	0.86	0.85	0.88
2	--++	0.84	0.86	0.92	0.92
3	-+-+	0.84	0.9	0.85	0.92
4	-++-	0.84	0.9	0.92	0.88
5	+--+	0.88	0.86	0.85	0.92
6	+-+-	0.88	0.86	0.92	0.88
7	++--	0.88	0.9	0.85	0.88
8	++++	0.88	0.9	0.92	0.92
9	0000	0.86	0.88	0.885	0.9

For each of the responses monitored during the cycle experimentation (the cycle responses as well as the configuration responses for fitting the second level of these partitioned response surfaces), the mean and standard deviation data are calculated for each run of the inner control array across the runs of the outer noise array. Response surface models for mean and standard deviation are then fit to this data; resulting model fits are summarized in Table 5. For each mean and standard deviation response, second and third order response surfaces are fit, and the best fit is chosen (the modified composite experiment is a five level experiment, and thus the third order terms can be added to the basic model of Equation 3 or Equations 4-7; three-way interaction terms are not added). The order of fit and R^2 values for each mean and standard deviation response are given in Table 5. Recall that the response models fit for the configuration mean responses in the cycle factors are actually the second portion of these models, the intercept term models (see Figure 3); the primary models for the configuration responses are fit in the configuration factors of Table 3.

Also, no model is fit for standard deviation of the fan diameter response as this response does not change with the noise factors; no deviation is observed.

Table 5 Response Surface Fits for Cycle Experimentation

	Response Name	Order of Fit	R^2 value
Performance:			
Mean SFC	SFC	3rd	0.998
Standard Deviation of SFC	STDSFC	3rd	0.986
Mean Thrust	THRUST	2nd	0.999
Standard Deviation of Thrust	STDTHR	2nd	0.991
Configuration (Intercept Term Response Models):			
Weight (intercept term)	B0WGT	2nd	0.998
Standard Deviation of Weight	STDWGT	3rd	0.838
Length (intercept term)	B0LNG	3rd	0.997
Standard Deviation of Length	STDLNG	3rd	0.963
Fan Diameter (intercept term)	B0DIA	2nd	1.000
Other Constraints:			
Mean Phot/Pcold	PHPC	3rd	0.998
Standard Deviation Phot/Pcold	STDPHC	2nd	0.995

In fitting models for prediction, R^2 values as close as possible to 1 are desirable. The fits are extremely good for all the mean response models, with R^2 values greater than 0.99 in every case. The standard deviation models are also very good for SFC, thrust, fan diameter, and Phot/Pcold. The R^2 values for the standard deviation of weight and length are lower however (0.838 and 0.963 respectively). Fitting models for these responses is more difficult since they change in discrete jumps with stage numbers, and this difficulty is magnified in calculating and modeling the standard deviation. However, the approach in robust design is to reduce the predicted variance, rather than actually achieve strict targets. Also the standard deviations for weight and length are smaller (relatively) than the deviations of the other responses. With these relatively small standard deviations these standard deviation approximations for the configuration responses are accepted.

In Figure 5 the response model fits for the SFC responses (mean and standard deviation) of Table 5 are shown graphically as actual response data (experiment points) versus predicted response values. In these plots, the angled line represents the ideal fit (actual and predicted values being equal) around which the predicted data is scattered; the horizontal dashed line represents the response mean value. Confidence bands for the predicted responses, indicating whether the model is significant at the 5% level, are also shown (curved dashed lines on either side of the ideal fit line). If these confidence curves completely contain the horizontal dashed line, the model is said to not be significant. Plots similar to those of Figure 5 are presented in Ref. 18 for the remaining responses of Table 5. The bands in these plots are very tight for most of the response fits. Since the confidence curves cross the horizontal line in every case, the models are significant at the 5% confidence level.

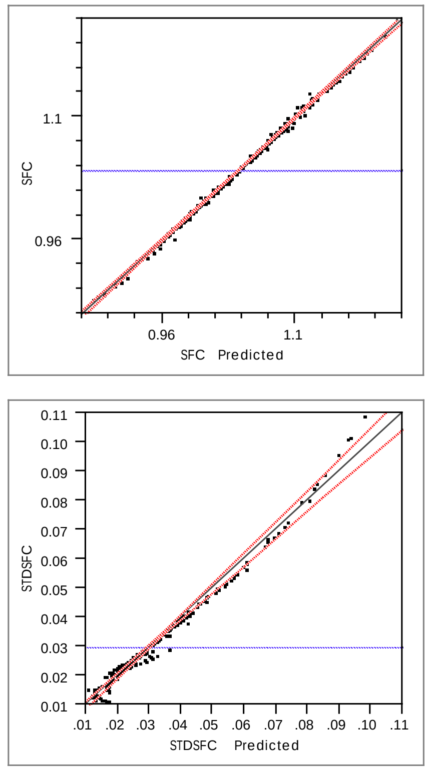


Figure 5 SFC Mean and Standard Deviation Response Fits (Actual vs. Predicted)

For the configuration experimentation, the modified composite experiment of Figure 4(b) is designed in the seven cycle control factors of Table 3. Since the configuration factors are all control factors, only the single modified composite experiment is needed, for a total of 143 cases. This experiment is run and again both the configuration and cycle responses are monitored. For this configuration experimentation, however, the cycle responses (SFC, thrust, as well as Phot/Pcold) do not vary with the configuration parameters (identical values are obtained for these responses for all cases). Models are thus only fit for the configuration responses, and two part partitioned response surfaces are not fit for the cycle responses.

The model fits obtained for the configuration responses are shown in Table 6. Third order fits are employed for weight and length. The R^2 values are very good for engine length and fan diameter (both over

0.99), but not quite as good for weight. The effects of the configuration factors on the number of stages for each component makes it more difficult to model the effects of these factors on the engine weight than for the cycle factors. The response model fit for engine weight is shown graphically in Figure 6; the larger scatter of data can be seen in the this plot when compared to the SFC plot in Figure 5. Since the weight is a rough approximation at the system level, this model is accepted.

Table 6 Response Surface Fits for Configuration Experimentation

	Response Name	Order of Fit	R^2 value
Engine Weight	WEIGHT	3rd	0.929
Engine Length	LENGTH	3rd	0.991
Fan Diameter	FANDIA	2nd	1.000

Fourteen response surface models have been constructed in this section: eleven through the cycle experimentation and three through the configuration experimentation. The configuration responses have been fit as two-level partitioned response surfaces, with the first surface fit in the configuration factors, and the second surface portion (mean term) for each configuration response fit in the cycle factors. Robustness response models (models for mean and standard deviation) have been constructed for all responses (with the exception of fan diameter). These response models are used to replace Engine Maker to allow efficient exploration of the preliminary design space to identify robust solutions. The compromise DSP and robust design solutions for this commercial turbofan example are presented in the next section.

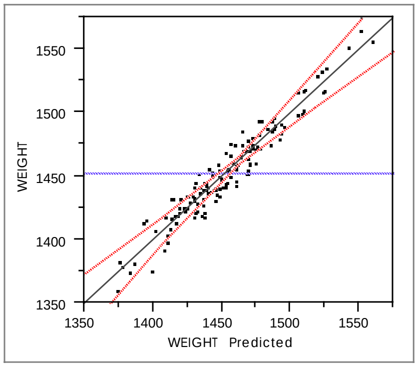


Figure 6 Engine Weight Response Fit (Actual vs. Predicted)

4.3 Commercial Turbofan Response Surface-Based Robust Design Exploration

The compromise DSP formulated for this example problem is presented in Table 7. This formulation incorporates all the commercial turbofan cycle and configuration information of the previous sections: the design requirements (constraint values and goal targets) given in Table 1, the eight cycle control factors and ranges (Table 2), the seven configuration control factors and ranges (Table 3), and the fourteen cycle and configuration response surface equations. A detailed description of the compromise DSP (goal formulations, deviation variables, deviation function, etc.) can be found in Ref. 19. The objective of this formulation is to *find* the values of the fourteen cycle and configuration control factors that *satisfy* the four cycle constraints (Eqns. 8-11), the three configuration constraints (Eqns. 16-18), and the cycle and configuration factor bounds, and *minimize* the deviation function (Eqn. 24) to achieve as closely as possible the four cycle goals (Eqns. 12-15) and the five configuration goals (Eqns. 19-23).

Included in the cycle and configuration goals are the standard deviations of SFC, thrust, weight, and length. With the component efficiencies varied over the outer, noise array, these standard deviation models are included to seek robust solutions (reduced variance, or standard deviation in this case). With the multi-objective formulation of the compromise DSP, however, realistic targets must be set for each goal, including the standard deviation robustness goals. The targets set for these goals, and given in Table 7, are determined based on the range of the response models created using the JMP® statistical package (the lower limits of the response models are used as targets to attempt to reduce the standard deviation as much as possible).

Table 7 Commercial Turbofan Cycle and Configuration Compromise DSP

Given:

- Cycle and configuration design requirements (Table 1)
- Cycle control factors and ranges (Table 2); configuration control factors and ranges (Table 3)
- 14 cycle and configuration response surface models

Find:

Cycle Design:

- The values of the 8 cycle control factors (Table 2)
- The values of the cycle deviation variables d_i^-, d_i^+ (i = 1-4; corresponding to goals, Eqns. 12-15)

Configuration Design:

- The values of the 7 configuration factors (Table 3)
- The values of the configuration deviation variables d_i^-, d_i^+ (i = 5-9; corresponding to goals, Eqns. 19 - 23)

Satisfy:

Cycle Design:

- The constraints:

SFC ≤ 1.030 lbm/h/lbf-normalized [8]

Thrust ≥ 990 lbf-normalized [9]

Mixing pressure ratio (Phot/Pcold) ≥ 0.94 [10]

Mixing pressure ratio (Phot/Pcold) ≤ 1.0 [11]

- The goals:

$SFC/0.96 + d_1^- - d_1^+ = 1$ [12]

$STDSFC/0.01 + d_2^- - d_2^+ = 1$ [13]

$THRUST/1052 + d_3^- - d_3^+ = 1$ [14]

$STDTHR/29 + d_4^- - d_4^+ = 1$ [15]

- Bounds on the control factors (Table 2)

Configuration Design:

- The constraints:

Weight ≤ 1425 lbs [16]

Length ≤ 7 ft [17]

Fan diameter ≤ 38.4 in [18]

- The goals:

$WEIGHT/1350 + d_5^- - d_5^+ = 1$ [19]

$STDWGT/0.005 + d_6^- - d_6^+ = 1$ [20]

$LENGTH/6 + d_7^- - d_7^+ = 1$ [21]

$STDLNG/0.02 + d_8^- - d_8^+ = 1$ [22]

$FANDIA/37 + d_9^- - d_9^+ = 1$ [23]

- Bounds on the control factors (Table 3)
- $d_i^- \cdot d_i^+ = 0$, with $d_i^-, d_i^+ \geq 0$

Minimize:

The deviation function:

$$Z = [f_1(d_1^+), f_2(d_2^+), f_3(d_3^-), f_4(d_4^+), f_5(d_5^+), f_6(d_6^+), f_7(d_7^+), f_8(d_8^+), f_9(d_9^+)] \quad [24]$$

Five design scenarios (different deviation function formulations) are evaluated for the commercial turbofan example, as shown in Table 8. In Scenarios 1 and 2 the Archimedean approach[19] is used with each deviation variable weighted equally. In Scenario 1 the mean response goals for SFC, thrust, weight, length, and fan diameter are included (no standard deviation goals). This scenario represents the case in which robustness is not addressed and is used as a baseline for comparison with robust solutions. Since thrust is to be maximized (up to its target of 1052), the under-achievement deviation variable, d_3^-, is included in the deviation function. The over-achievement deviation variables, d_i^+, are included for the other four responses, which are to be minimized to their respective targets (approach targets from above). In Scenario 2, the standard deviation goals are included in the formulation. Again the Archimedean approach is used with all nine deviation variables weighted equally. This scenario represents the case in which both achieving performance targets (mean value) and reducing variation associated with the uncertain efficiencies are equally important. In Scenarios 3, 4, and 5, a combination of the Archimedean and preemptive approaches[19] is employed. Two preemptive priority levels are used in each case, with the deviation variables placed at each level weighted equally. In Scenario 3, emphasis is placed on fuel consumption, with SFC and the standard deviation of SFC placed in the first priority level. The deviation variables for the remaining seven goals are placed in the second priority level. In Scenario 4, for comparison with Scenario 3, thrust and the standard deviation of thrust are placed in the first priority level. The SFC and thrust requirements are the primary concerns for this engine, and thus the trade-offs of focusing on each are evaluated in these two scenarios. For Scenario 5, robustness is given top priority with a focus on the robustness of these two primary requirements; the deviation variables associated with the standard deviations of SFC and thrust are thus placed in the top priority level. The focus with this scenario is achieving the engine design that is the most robust to the uncertain efficiencies, with respect to SFC and thrust.

The robust preliminary design exploration results obtained for the five design scenarios of Table 8 are given in Table 9. Included in this table, in addition to the predicted solutions for the five design scenarios (using the response models), are the cycle and configuration values for the Engine Maker model of the AE3007 engine (AE3007 EM). For all scenarios, feasible, converged solutions are obtained.

For all five scenarios the performance and configuration values achieved (response surface predicted values) are at least slightly better than those of AE3007 EM (SFC, weight, length, and fan diameter are less in every case, and thrust is greater for all five scenarios). For the five scenarios, OPR is pushed towards its lower bound of 20. SOT is pushed towards its upper bound of 2500°F when robustness (standard deviation goals) is included, Scenarios 2-5. The values of the other control factors vary between the scenarios in attempt to achieve the different trade-offs.

Table 8 Five Turbofan Cycle and Configuration Design Scenarios

Scenarios	Deviation Function: Priority Level 1	Deviation Function: Priority Level 2
1. Overall, no robustness (Equal Weights)	$(d_1^+ + d_3^- + d_5^+ + d_7^+ + d_9^+)/5$	N/A
2. Overall, with robustness (Equal Weights)	$(d_1^+ + d_2^+ + d_3^- + d_4^+ + d_5^+ + d_6^+ + d_7^+ + d_8^+ + d_9^+)/9$	N/A
3. SFC, standard deviation of SFC	$(d_1^+ + d_2^+)/2$	$(d_3^- + d_4^+ + d_5^+ + d_6^+ + d_7^+ + d_8^+ + d_9^+)/7$
4. Thrust, standard deviation of Thrust	$(d_3^- + d_4^+)/2$	$(d_1^+ + d_2^+ + d_5^+ + d_6^+ + d_7^+ + d_8^+ + d_9^+)/7$
5. Robustness: standard deviation of SFC, Thrust	$(d_2^+ + d_4^+)/2$	$(d_1^+ + d_3^- + d_5^+ + d_6^+ + d_7^+ + d_8^+ + d_9^+)/7$

The cycle and configuration solutions of the five design scenarios behave as expected; when possible, the goal deviations are reduced (values closer to target) for the goals placed in the first priority level. The results for each scenario are summarized as follows:

Scenario 1: Overall, no Robustness (equally weighted goals)

- represents equal tradeoff among the five mean response goals and a basis for comparison; robustness (standard deviation goals) is not included;
- all values are significantly better than AE3007 EM;
- length and fan diameter goals are achieved;
- standard deviation values are calculated for comparison (but are not included in deviation function).

Scenario 2: Overall with Robustness (equally weighted goals)

- represents equal trade-off among the five mean response goals and the four standard deviation goals (robustness introduced);
- results change as expected; all four standard deviation values are reduced from the values calculated for the solution to Scenario 1;
- thrust is sacrificed significantly; SFC and length are slightly sacrificed;
- length and fan diameter targets are achieved and weight target is nearly achieved.

Scenario 3: Fuel Consumption (SFC, standard deviation SFC)

- SFC is reduced significantly from Scenario 2, and the standard deviation of SFC is reduced slightly; neither target is achieved;
- all other mean goals are sacrificed (thrust, weight, length fan diameter); standard deviation goals for thrust and length are also sacrificed;
- no goals are achieved in second priority level (fan diameter nearly achieved).

Scenario 4: Thrust (thrust, standard deviation thrust)

- mean thrust is increased significantly; standard deviation of thrust is slightly larger than for Scenario 2, but less than for Scenario 1; both targets are very nearly achieved;
- mean SFC, weight, length, and fan diameter, and standard deviations of SFC, weight, and length, are all sacrificed (compared to Scenario 2);
- only the fan diameter target is achieved in the second priority level.

Scenario 5: Robustness

- represents solution that is most robust with respect to the primary requirements (SFC and thrust);
- lowest value for standard deviation of SFC is achieved (same as for Scenario 3); standard deviations of thrust are slightly higher than for Scenario 2, but lower than values for all other scenarios; neither target is achieved;
- solution is very similar to that of Scenario 2; mean weight and fan diameter, standard deviations of weight and length all slightly sacrificed;
- mean length and mean fan diameter are achieved in the second priority level.

The factor values for the five scenario solutions of Table 9 are entered back into Engine Maker to evaluate the response surface predictions. For this evaluation, 1000 random combinations of the four noise factors (component efficiencies) are run for each scenario; the mean and standard deviation are then calculated for these 1000 runs for each scenario. The errors of the predicted solutions of Table 9, when compared to the actual Engine Maker values, are given here in Table 10. The negative error values reflect a response result that is under-predicted (value from response surface is less than actual value).

Table 9 Commercial Turbofan Cycle and Configuration Robust Preliminary Design Exploration Results

		DESIGN SCENARIO				
		1	2	3	4	5
INPUTS		**Overall**	**Overall, Robust**	**SFC, STDSFC**	**THRUST STDTHR**	**Robust**
Inlet Flow	(lb/s)	90.98	88.09	90.05	91.89	88.93
Fan Outer Press. Ratio (FOPR)		1.67	1.67	1.65	1.67	1.67
Overall Pressure Ratio (OPR)		20.30	20.00	20.02	20.00	20.00
Stator Outlet Temp. (SOT)	(°F)	2336	2492	2497	2488	2497
Transition Duct Loss	(%)	0.79	3.45	4.00	1.40	3.74
Diffuser/Combustor Loss	(%)	3.37	2.00	2.00	3.24	2.02
LPT Exit Pressure Loss	(%)	1.08	2.05	0.66	1.68	1.49
Bypass Duct Loss	(%)	1.00	1.01	1.01	1.19	1.17
Fan Inlet Mn		0.70	0.69	0.64	0.70	0.70
Fan Entry Hub/Tip Ratio		0.20	0.26	0.25	0.22	0.28
Fan Tip Speed	(ft/s)	1850.0	1632.9	1498.7	1491.2	1741.0
Fan Hub Loading		2.000	1.235	1.312	1.220	1.035
HPC Tip Speed	(ft/s)	1700.0	1175.2	1087.6	1074.4	1154.1
HPC Hub Loading		0.900	1.163	1.216	1.314	1.215
LPT Mean Loading		2.161	1.981	2.069	1.897	1.889
OUTPUTS	**AE3007 EM**					
SFC (normalized)	1.035	1.002	1.005	0.988	1.008	1.005
STDSFC (lb/h/lb)		0.0152	0.0111	0.0110	0.0136	0.0110
THRUST (norm.)	990	1039.50	1014.97	994.08	1047.13	1018.95
STDTHR (lb)		32.28	29.28	29.61	29.79	29.36
WEIGHT (lb)	1426	1370.9	1357.7	1387.6	1425.0	1364.0
STDWGT (lb)		0.00812	0.00702	0.00691	0.00752	0.00707
LENGTH (ft)	6.89	5.90	6.00	6.41	6.41	5.72
STDLNG (ft)		0.0276	0.0250	0.0283	0.0266	0.0254
FANDIA (in)	38.37	36.12	36.10	37.02	36.50	36.38
PHPC	0.880	0.999	1.001	0.973	1.000	1.001

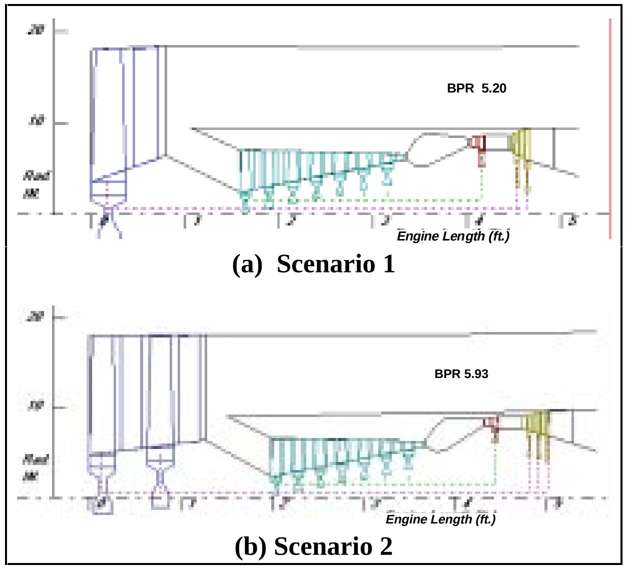
Table 10 Response Surface Errors for Results of Table 9

		% ERROR				
		1	2	3	4	5
ENGINE MAKER	**AE3007 EM**	**Overall**	**Overall, Robust**	**SFC, STDSFC**	**THRUST STDTHR**	**Robust**
SFC (normalized)	1.035	-0.398	-0.199	-0.504	-0.099	-0.298
STDSFC (lb/h/lb)		72.140	-40.684	36.986	76.166	40.306
THRUST (norm.)	990	-0.628	0.554	-0.087	-0.4485	0.088
STDTHR (lb)		63.941	77.025	78.373	76.064	76.761
WEIGHT (lb)	1426	-0.710	1.815	2.436	1.720	1.398
STDWGT (lb)		-24.112	-13.226	-28.542	-20.925	-12.824
LENGTH (ft)	6.89	13.680	8.108	10.708	8.460	2.656
STDLNG (ft)		226.63	123.21	141.88	131.30	124.78
FANDIA (in)	38.37	0.305	0.055	-0.189	0.413	0.110
PHPC	0.880	-0.299	0.100	-0.815	-1.283	-0.497

In Table 10 the errors for mean SFC, thrust, and fan diameter are less than one percent for all five scenarios; the errors for the Phot/Pcold mixing constraint are also less than one percent for all except Scenario 4 (slightly greater than one percent). As expected, the errors for mean weight and length are higher than those of the other mean responses (recall that these responses change discretely with component stage numbers). The errors for mean weight, however, are surprisingly very low, less than 2.5 percent for all scenarios. These errors are sufficiently small, and acceptable. The errors for mean engine length, however, are much larger, varying between two percent and fourteen percent across the five scenarios. Fitting a continuous surface to this data leads to less accuracy than for the continuous responses. However, the engine length is over-predicted in every case. The actual engine length for these five scenarios are all less than the goal target as desired. Since this requirement is not a primary concern, and does not lead to problems, the accuracy of the model is accepted for this demonstration.

The errors for the standard deviation predictions are significantly larger, ranging from twelve percent to over 200 percent. This result is consistent with the results of an investigation into noise modeling for approximation-based robust design, presented in Ref. 14--standard deviation is much more difficult to model than mean response; sufficient data points are needed to capture the standard deviation of a response. Here, the standard deviation data is gathered across the nine cases of the outer, noise array. If a full factorial were used for the noise array to gather more data, the size of the experiment would double; using a central composite design for the noise array would basically triple the total number of cases run. Rather than increasing the experiment size, however, it is important to view the trends of the standard deviation predictions. Even though the standard deviation predictions have high errors, the trends have been captured. The robust solutions of Scenarios 2-5 have actual standard deviation values less than the values for the AE3007 Engine Maker model (the standard deviations are reduced through this robust preliminary design exploration). Interestingly, even the standard deviations of Scenario 1 (for which robustness is not included in the deviation function) are lower than those of the AE3007 Engine Maker model (Scenario 1 is more robust than AE3007EM).

Since both mean performance and robustness are improved over the AE3007EM model (without using this engine as a starting point for robust preliminary design exploration), the solutions of Table 9 are accepted. The Engine Maker illustrations of the solutions for Scenario 1 (no robustness) and Scenario 2 (overall with robustness) are given in Figure 7(a) and 7(b) respectively. The Engine Maker model of the AE3007 engine is illustrated in Figure 8 for comparison.



(a) Scenario 1

(b) Scenario 2

Figure 7 Engine Maker Illustrations of Commercial Turbofan Configurations, Scenarios 1 and 2

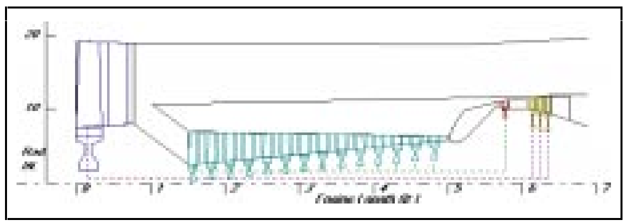


Figure 8 Engine Maker Model of AE3007 Engine Configuration (AE3007EM)

The engine configurations for the four scenarios that include robustness (Scenarios 2-5) are very similar, with two major differences from the configuration of Scenario 1 (no robustness): one fan stage and one LPT stage are added for the robust solutions of Scenarios 2-5. The additional fan stage reflects the reduced fan hub loading and reduced fan tip speed for these robust solutions. The additional LPT stage also results from reduced LPT mean loading.

Other than the second fan stage, the major difference between the robust preliminary design exploration solutions and the AE3007EM configuration illustrated in Figure 8 is the size of the compressor. The AE3007 engine has a fourteen stage compressor. For the solutions of Table 9 and the illustrations of Figure 7, the compressor is reduced to seven or eight stages.

Through further investigation, however, it was discovered that Engine Maker grossly under-predicts the compressor stage numbers and thus the compressor weight. In a follow-up study, a compressor subsystem design module was added to this problem for more detailed compressor performance and configuration design[18]. The resulting compressor design information is then used to improve the system level engine performance and configuration evaluation.

5 CLOSING REMARKS

The multi-level, partitioned response surface modeling approach of Section 3 has been successfully demonstrated in the previous section for the commercial turbofan preliminary design example problem. Using the two-level partitioned response surfaces, the configuration design responses are modeled in all 15 control factors (7 configuration factors and 8 cycle factors). With this approach, partitioning the factors and responses based on the configuration and cycle design reduces the necessary experimentation tremendously. If a single full composite experiment were design in all 15 control factors and crossed with the 4-factor noise array of Table 4, 295,191 analysis cases would have been required. With the partitioned response surface approach and concurrent cycle/ configuration experimentation, the experimentation is limited by the cycle experimentation (the crossed inner and outer arrays – 2457 cases). Experimentation expense is thus reduced by more than two orders of magnitude! By further increasing concurrency, running the cycle inner, control array concurrently (distributed computing) for each of the 9 cases of the noise array, the experimentation time can be reduced to the computational expense of the 273 cases of a single cycle control array.

The multi-level partitioned response surface metamodeling approach presented here is appropriate and useful for large-scale complex design problems in which:

1) the number of variables prohibits standard response surface experimentation and modeling,
2) the problem can be easily partitioned, and interactions between factors of partitioned sets can be neglected, and
3) second or third order polynomial response surfaces are adequate to model the factor/response relationships of a complex analysis code(s), over the desired factor ranges, with sufficient accuracy.

The limitation of response surface approximations that has not been discussed here is associated with the third condition. For highly nonlinear analyses, alternate approximation approaches must be investigated for efficient analysis and design space exploration. One approach in the current literature, *kriging*[16], is being tested for engineering applications[17,20] and compared to response surface approximations[21], and shows promise as a viable approximation option for nonlinear problems.

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